The Real Effects of Money Growth in the Long Run

An Analysis of a Schumpeterian Growth Model with Nominal Price Rigidity

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NNS	New Neoclassical Synthesis
RBC	Real Business Cycles

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Chapter 1

The real effects of money – an overview

1.1 Introduction

The following quotation from David Hume's essay "Of money" from the year 1752 nicely summarises a fundamental puzzle that has engaged the thoughts of many generations of economists: How is it that changes in money – an intrinsically valueless good that serves as a unit of account and medium of exchange in transactions – have an effect on the real level of activity in an economy? In the words of Hume:

> "It is indeed evident, that money is nothing but the representation of labour and commodities, and serves only as a method of rating or estimating them. Where coin is in greater plenty; as a greater quantity of it is required to represent the same quantity of goods; it can have no effect, either good or bad, taking a nation within itself; any more than it would make an alteration on a merchant's books, if, instead of the Arabian method of notation, which requires few characters, he should make use of the Roman, which requires a great many. (...) But notwithstanding this conclusion, (...) we find, that, in every kingdom, into which money begins to flow in greater abundance than formerly, every

> thing takes a new face: labour and industry gain life; the merchant becomes more enterprising, the manufacturer more diligent and skilful, and even the farmer follows his plough with greater alacrity and attention."[1]

Since the days of Hume, an enormous number of theories have been put forward to explain this puzzle.[2] The goal of the present thesis is to contribute to this literature by enhancing the understanding of money's effects on the real side of the economy in the long run. In particular, the following chapters will analyse the influence of the growth rate of money supply on an economy's growth rate of output and on its levels of output and employment in a framework of a Schumpeterian model of innovation-driven growth augmented with a New Keynesian specification of nominal price rigidity.

In the context of this project, the aim of the present chapter is threefold: The first aim is to present some empirical results about the relationship between monetary variables, in particular inflation, the level and growth rate of money supply, and real variables, in particular the levels of employment and output and the latter's growth rate. The second goal is to introduce the reader to the main explanations offered by the modern theoretical literature for the correlations found in the data between the mentioned nominal and real variables. Although the analysis of this thesis is restricted to the long-run influence of money, some stylised facts on the short-run influence of money and a presentation of the modern theoretical approach to analysing money's role in the business cycle are included. The reason for doing so is related to the third aim of the present chapter, which is to demonstrate that the subsequent analysis is actually a synthesis of elements from the New Keynesian literature dealing with the short-term effects of money and from long-run oriented endogenous growth theory. The reader will along the way be introduced to most of the central building blocks of the subsequent chapters models' while also learning about the methodological differences between the approach of this thesis and the existing literature.

[1] Hume [1955] p. 37, emphasis omitted.

[2] See Blanchard [1990] and Lucas [1996] for recent surveys.

The remainder of this chapter is organised as follows: Section 1.2 presents some helpful definitions. In section 1.3, the reader is introduced to the analysis of the relationship between the money supply and its growth rate on the one hand and the *levels* of output and employment on the other. In particular, in section 1.3.1 some stylised facts about money's role in the business cycle are presented and the New Keynesian business cycle model is introduced which as stated above contains several of the building blocks of the models of this thesis. Money's level effects in the long run are analysed in section 1.3.2 with an emphasis on the New Keynesian literature. Some empirical results complement the theoretical analysis. Section 1.4 is concerned with the effects of the growth rate of money supply on the *growth rate* of output. Theoretical explanations based on neoclassical and endogenous growth theory frameworks are discussed in section 1.4.1. Section 1.4.2 presents recent empirical findings. Finally, section 1.5 introduces the reader to the approach taken in this thesis. Section 1.5.1 discusses the motivation for the intended synthesis and presents some evidence on its empirical relevance. Section 1.5.2 gives an overview of the remaining chapters of this thesis.

1.2 The real effects of money – some definitions

The title of this chapter refers very generally to the real effects of "money". We now introduce some central definitions which help to clarify which specific changes in nominal variables are conjectured to have an effect on real variables.

If changes in both the level and the growth rate of the nominal money supply affect only nominal variables such as inflation and the nominal interest rate, money is said to be both neutral and superneutral.[3] Conversely, any changes in real variables caused by variations in the *level* of money supply are said to be evidence of money's *non-neutrality*, whereas any effects on

[3] See Orphanides and Solow [1990]. The nominal money supply is in the context of the literature understood to be a narrow monetary aggregate under direct control of the monetary authority.

the real side of the economy brought about by changes in the *growth rate* of money supply are manifestations of money's *non-superneutrality.*

Given that inflation is considered to be a monetary phenomenon entirely determined by money growth in the long run,[4] a shortcut is sometimes taken and the effects of inflation on the real economy are directly discussed. In particular, the term "*Phillips curve*" is used to refer to a conjectured effect of inflation on unemployment.[5]

Some theory and evidence concerning all these neutrality propositions will be discussed in this chapter.

In the models to be presented in the subsequent chapters of this thesis, money is neutral in the long run.[6] Consequently, our analysis focuses on the non-superneutrality of money with respect to the economy's output growth rate and on the effects of the money growth rate and inflation on the economy's levels of employment and output, where the latter relations will be discussed under the heading of a long-run Phillips curve.

1.3 Money and the levels of output and employment

1.3.1 Money in the business cycle

Various measures of money supply are positively correlated with contemporaneous output and employment in the short run. Also, output is positively correlated with lagged values of money supply, indicating that money leads output in the business cycle.[7]

[4] There is a nearly complete long-run correlation between various measures of the money supply growth rate and the inflation rate. E.g., McCandless and Weber [1995] report correlations between 0.92 and 0.96. Besides, money growth Granger-causes inflation, see Crowder [1998].

[5] Somewhat confusingly, inflation rate and growth rate of money supply are sometimes used synonymously in the literature studying whether money is superneutral.

[6] In the context of an empirical study, Fisher and Seater [1993] show that neutrality is a necessary but not sufficient condition for the superneutrality of money.

[7] See, e.g. Walsh [2003], pp. 12-13.

These are two of the well-documented empirical regularities that characterise business cycle fluctuations in real economies.[8]

By themselves, these correlations say nothing about the underlying causal relationship between money and output, i.e. about the neutrality of money. Starting with the seminal study of Friedman and Schwartz [1963], researchers have used a variety of methodological approaches to show that causality runs mainly from money to output.[9] While there are still some voices arguing for "reverse causation", i.e. changes in the money supply being caused by output,[10] there is now a broad consensus that money is non-neutral in the sense that increases in money supply cause a hump-shaped reaction in output over the following quarters.[11,12] In particular, the vector autoregression (VAR) framework has been used in recent years to investigate the influences of monetary policy shocks on output.[13] Christiano et al. [1999] (p. 69) summarise the consensus in this literature about the short-run non-neutrality of money:[14]

> "there is considerable agreement about the qualitative effects of a monetary policy shock (...): after a contractionary monetary policy shock, short term interest rates rise, aggregate output, employment, profits and various monetary aggregates fall, the aggregate price level responds very slowly, and various measures of wages fall, albeit by very modest amounts."

[8] See Stock and Watson [2000] for a detailed presentation of the stylised facts of the US business cycle. Cooley/Hansen [1995] focus on those regularities concerning nominal variables.

[9] See the survey and discussion in chapter 1 of Walsh [2003].

[10] See e.g. King and Plosser [1984].

[11] See e.g. Sims [1992] for the US and a number of European countries.

[12] In contrast, most empirical studies find support for the neutrality of money in the *long-run*, see Weber [1994], King and Watson [1997], Serletis and Koustas [1998] and the references in Bullard [1999].

[13] By identifying exogenous shocks to monetary policy, one excludes systematic variations in monetary policy that may be endogenous reactions to current or expected future economic conditions. For an overview of the VAR approach to the analysis of monetary policy and the business cycle see Christiano et al. [1999].

[14] While Christiano et al. [1999] summarise studies for the US, Peersman and Smets [2003] find very similar characteristics for the Euro area.

Turning to the theoretical side, the goal of modern business cycle theory is to explain the observed pattern of co-movements of real and nominal variables in the business cycle.[15]

In doing so, Real Business Cycle (RBC) theory, which was the dominant approach to the study of economic fluctuations in the 1980s assigned a very limited role to money and other nominal influences since the theory sought to explain business cycles as resulting from the optimal reactions of firms and households to *real* shocks in a Walrasian setting with flexible prices. In fact, the bulk of the literature abstracted from money entirely.[16]

Despite this, RBC theory is important for the modern explanation of the short-term effects of money in that it is one of the central building blocks of the New Keynesian or New Neoclassical Synthesis (NNS) literature,[17] the modern paradigm of business cycle analysis which in turn does attribute an important role to nominal variables in the explanation of cyclical movements. The New Neoclassical Synthesis literature owes its name to the fact that it integrates elements of Keynesian flavour into an optimisation-based Walrasian RBC model with rational expectations and infinitely-lived households.[18] These key New Keynesian elements are first, monopolistic competition in the goods or labour market. Second and most important, money matters in NNS models due to the introduction of nominal price rigidity as a source of non-neutrality.

Since the models to be presented in this thesis share the central building blocks of the New Neoclassical synthesis, these are discussed in some detail in

[15] Our presentation is focussed on the modern New Keynesian paradigm since it is beyond the scope of this introduction to give a complete account of the historical development of modern business cycle theory and money's role in it. For an exposition on this history see e.g. Mankiw [1990].

[16] If money was at all present in these models, in the absence of other nominal frictions the only source of non-neutrality was the "inflation tax on labour supply" which will be discussed in section 1.4.1.2. Simulations showed that money was not an important influence on the business cycle under these assumptions, see Cooley and Hansen [1989, 1995].

[17] The two terms are used synonymously in the literature.

[18] In the words of Goodfriend and King [1997] who first used the term, "Building on new classical macroeconomics and RBC analysis, it incorporates intertemporal optimization and rational expectations [...]. Building on New Keynesian economics, it incorporates imperfect competition and costly price adjustment [...]" (p. 255).

section 1.3.1.1. Since the Phillips curve relationship has long been considered one of the central trade-offs facing monetary policy makers, the idea is briefly discussed and the variant of the Phillips curve implied by the supply side of the NNS is introduced in section 1.3.1.2. Further, the main monetary transmission channel responsible for money's non-neutrality in the general equilibrium of NNS models is discussed in section 1.3.1.3.

1.3.1.1 The basic New Keynesian Business Cycle model

This section presents central elements of the standard New Keynesian or NNS business cycle: The assumption leading to a well-defined demand for money in equilibrium, the structure of the intermediate goods market with monopolistic competition and price rigidity, and the specification of monetary policy.

For the sake of better comparability with the models of the following chapters, the ensuing presentation of the New Keynesian model is set in continuous time while NNS models are usually presented in discrete time.

Money in the utility function Starting with the motive why money is held in equilibrium, a standard way to introduce money into the New Keynesian model is to assume that households derive utility from holding cash.[19] As shown by Feenstra [1986], for the case of non-separable utility for consumption and real balances assumed in later chapters of this thesis, this can be viewed as a shortcut to modelling the transaction cost reducing services of money. In NNS models, a simpler separable form of preferences is usually assumed:

$$u\left(c,m,l\right)=\frac{c^{1-\sigma}}{1-\sigma}+a\frac{m^{1-b}}{1-b}-d\frac{\left(1-l\right)^{1+\eta}}{1+\eta} \tag{1.1}$$

[19] This representation is used e.g. by Chari, Kehoe and McGrattan [2000], Kim [2000], Erceg, Henderson and Levin [2000]. A frequently used alternative is the assumption that a subset of goods can only be paid for with cash ("Cash in advance", CIA) which will be discussed in section 1.4.1.2. It is used in NNS models by e.g. Yun [1996].

where c denotes consumption, $m = M/P$ is real money balances held by the household, l is leisure l and σ, a, b, d, η are positive constants. The representative infinitely lived household chooses intertemporal paths for c, m and l to maximise the expected present value of utility subject to his budget constraint.[20] This results in the following conditions for the household's optimum at time τ:

$$d\frac{[1-l(\tau)]^{\eta}}{c(\tau)^{-\sigma}} = \frac{w(\tau)}{P(\tau)} \tag{1.2}$$

$$\frac{E_{\tau}\left\{\dot{c}(\tau)\right\}}{c(\tau)} = \frac{r(\tau)-\rho}{\sigma} \tag{1.3}$$

$$a\frac{m(\tau)^{-b}}{c(\tau)^{-\sigma}} = i(\tau) \tag{1.4}$$

where E is the expectations operator. These equations constitute the demand side of the New Keynesian model. Equations (1.2) and (1.3) are two standard efficiency conditions implying that the marginal utility ratio between leisure and consumption is equalised to the real wage w/P and that the expected intertemporal allocation of consumption depends on the difference between the real interest rate r and the household's discount factor ρ. Most importantly, equation (1.4) states that the marginal utility ratio between real balances and consumption in equilibrium equals the nominal interest rate i which is the opportunity cost of holding money: By holding money instead of real assets, the household foregoes interest income r. Further, the real value of nominally denominated cash depreciates at the rate of deflation. Thus the difference in return between holding real assets and money is r minus the deflation rate, i.e. $r + \pi = i$ where π is the inflation rate. This implies that an increase in inflation ceteris paribus has an effect on the household's optimal ratio of consumption to real balances.

[20] The only source of uncertainty are monetary policy shocks that will be discussed below. They determine the size of the seigniorage-financed lump-sum transfer the household receives.

Monopolistic Competition Households consume the economy's final good Y which is produced using a continuum of horizontally differentiated intermediate goods x_j according to the production function

$$Y(\tau) = \left[\int_{j=0}^{1} x_j(\tau)^{\frac{\alpha-1}{\alpha}} dj \right]^{\frac{\alpha}{\alpha-1}}$$

following Dixit and Stiglitz [1977]. The elasticity of substitution between any two types of intermediate good is $\alpha > 1$ implying that the types are imperfect substitutes in production, so that their producers act in an environment of monopolistic competition. This implies that the producer of any intermediate good type j can optimally choose a price subject to the demand function for his good

$$x_j(\tau) = \left[\frac{p_j(\tau)}{P(\tau)} \right]^{-\alpha} Y(\tau)$$

derived from the final good producers' cost minimisation, which is downward sloping in the good's relative price $\frac{p_j(\tau)}{P(\tau)}$.[21] For the sake of simplicity, production of intermediate goods is assumed to be linear in labour.[22] Optimal pricing then involves setting a positive mark-up over marginal cost, i.e. over the wage. Given positive marginal profits, firms are willing to increase supply at given prices in the face of an increase in nominal demand, creating the possibility for nominal shocks to affect output.[23]

Nominal Price Rigidity The key Keynesian assumption allowing money to have real effects is the introduction of nominal price rigidity in the goods market: It is assumed that intermediate good producers can only change their prices infrequently. The standard specification of price rigidity used is due to Calvo [1983] and implies that firms can adjust their prices whenever they receive a reset signal. Since this reset signal follows a Poisson distribution, a firm's price is fixed for an interval of random length.

The main consequence of price rigidity is that instead of setting the mark-

[21] For details on the derivation of the demand function see, e.g. Walsh [2003].

[22] The models abstract from capital.

[23] At the same time, monopolistic competition implies that equilibrium is not Pareto optimal and that fluctuations are asymmetric in their welfare consequences.

up that maximises current profits, $\alpha/(\alpha-1)$, firms, whenever they have the chance to readjust their price, choose a forward-looking mark-up that takes account of the expected future development of marginal cost and revenue while their price will be temporarily fixed. In particular, firms solve the problem

$$\max_{p_j} E_\tau \int_{s=\tau}^{\infty} e^{-\int_{t=\tau}^{s}[\chi(t)+\beta]dt} \left[p_j - w(s)\right] \left[\frac{p_j}{P(s)}\right]^{-\alpha} Y(s)\, ds$$

where the wage $w(s)$ is the firm's marginal cost and where future profits are discounted with the factor $\chi(s)$. The weight given to profits of the future period s further decreases in the flow probability of receiving a price adjustment signal β since only those profits are relevant for the maximisation that accrue to the firm *before* its next price adjustment opportunity. The optimal price $p^*(\tau)$ is then a weighted average of the optimal prices that maximise expected profits in the future periods s where the price is fixed:

$$0 = E_\tau \int_{s=\tau}^{\infty} e^{-\int_{t=\tau}^{s}[\chi(t)+\beta]dt} \left[p^*(\tau) - \frac{\alpha}{\alpha-1} w(s)\right] \left[\frac{p^*(\tau)}{P(s)}\right]^{-\alpha} Y(s)\, ds$$

which can be solved for the forward-looking optimal mark-up under price rigidity

$$p^*(\tau) = \frac{\alpha}{\alpha-1} \frac{E_\tau \int_{s=\tau}^{\infty} e^{-\int_{t=\tau}^{s}[\chi(t)+\beta]dt} \frac{w(s)}{w(\tau)} P(s)^\alpha Y(s)\, ds}{E_\tau \int_{s=\tau}^{\infty} e^{-\int_{t=\tau}^{s}[\chi(t)+\beta]dt} P(s)^\alpha Y(s)\, ds} w(\tau) \tag{1.5}$$

Given perfect competition in the final goods market, firms there make zero profits so that the output price $P(\tau) = \left[\int_{j=0}^{1} p_j(\tau)^{1-\alpha} dj\right]^{\frac{1}{1-\alpha}}$ is a weighted average of the intermediate good firms' prices. Calvo's specification of price rigidity is widely used because it facilitates aggregation of individual prices: Given that an intermediate good's effective price is the optimal price set when the firm last received a price adjustment signal and that the Poisson flow probability β of receiving a signal is independent of how long the price has been fixed, the output price can equivalently be expressed as a weighted average of past optimal prices, where the weights are determined by the

Poisson distribution:

$$P(\tau) = \left[\int_{s=-\infty}^{\tau} \beta e^{-\beta(\tau - s)} p^*(s)^{1-\alpha} ds \right]^{\frac{1}{1-\alpha}} \tag{1.6}$$

Together, equations (1.5) and (1.6) form the supply side of the New Keynesian model.

Monetary policy The standard New Keynesian model can be reduced to a system of two equations: A demand side relationship derived from the household's Euler equation and a supply side relationship known as the New Keynesian Phillips curve. The model is then closed by the specification of a monetary policy rule. Besides its positive goal – understanding the business cycle and money's role in it – the NNS literature has also made important contributions to the normative debate about the optimal conduct and goals of monetary policy in the presence of nominal price rigidity.[24] Since most real world monetary authorities' policy is aimed at control of a short-term nominal rate rather than a monetary aggregate, most NNS models describe monetary policy as following an interest rate rule.[25] In NNS models not concerned with optimal policy design, a stochastic autoregressive process for money growth is instead chosen.[26] For our discussion of the monetary transmission mechanism in section 1.3.1.3, we make the latter assumption: $\frac{dM_t}{dt}\frac{1}{M_t} = \psi + \vartheta_t$ where the disturbance ϑ_t is independently and identically distributed.

In the context of the deterministic models of the subsequent chapters of this thesis, which all contain the simple deterministic relationship between money demand and the nominal interest rate described by equation (1.4), monetary policy can equivalently be described by an interest rate rule or a money growth rule.[27]

[24] See Woodford [2003], Walsh [2003], Clarida et al. [1999], Galí [2003], Erceg et al. [2000] and Goodfriend and King [1997].

[25] See, e.g. Woolford [2003], Walsh [2003] and Clarida et al. [1999].

[26] See e.g. Chari, Kehoe and McGrattan [2000].

[27] See Ireland [2006], Clarida et al. [1999].

The models to be presented in this thesis are optimisation-based models with sticky prices set by intermediate goods firms acting in an environment of monopolistic competition and where households' utility depends positively on real money balances- i.e. the models share the central building blocks of the New Neoclassical synthesis we have just presented. However, some features of the methodological approach of the NNS literature sharply distinguish it from the approach of this thesis despite the mentioned similarities: The NNS literature is concerned with the explanation and replication of the fluctuations experienced by real economies in the short run. Concentration on the short-run implies firstly, that analysis focuses on models' behaviour off the steady state. In particular, the equations describing the economy's dynamic behaviour are linearly approximated around a zero-inflation steady state. Secondly, the analysis abstracts from secular growth, output is constant in the model's steady state. Thirdly, to be able to replicate real economies' fluctuations, both nominal and real shocks are introduced into the model which hit the economy in the short run.[28] The analysis usually does not involve explicitly solving the model but proceeds through the use of simulations. The reasons for this procedure are twofold: First, Lucas [1980] argued that the comparison of second moments generated by simulations of the model to those known from real world time series data is the preferable method for the empirical validation of a business cycle model.[29] Second, even if one preferred to proceed differently, New Keynesian Dynamic Stochastic General Equilibrium models are often too complex to lend themselves to non-numerical methods. The drawback to this methodological approach is that many "results" of the NNS literature are only shown with the help of simulations, meaning they only apply to a range of parameter calibrations considered plausible.

In contrast to the NNS literature, the focus of this thesis is on the effects of money on economies' long-run development in the absence of shocks at

[28] At the same time, the model's equations are approximated around the model's deterministic steady state.

[29] Although widely used, the approach is by no means universally accepted, see Hansen and Heckman [1996] for a critical discussion of calibrated simulations as a method of empirical validation.

the aggregate level. Results about the comparative static properties of our models' steady state equilibria are derived analytically; calibrated examples only serve to illustrate them. The issue of linearised dynamics in the neighbourhood of the steady state is only addressed in order to analyse the latter's stability properties.

Having described the New Keynesian model's building blocks, we now give a very brief description of the idea underlying the Phillips curve concept and show that the NNS business cycle model implies a forward-looking Phillips-curve relationship.

1.3.1.2 The Phillips curve and its New Keynesian variant

In an influential paper, Phillips [1958] described a strong negative relation between unemployment and the growth rate of nominal wages (wage inflation) for the UK.[30] By tying price movements to wage movements through firms' mark-up calculations, Samuelson and Solow [1960] reformulated the relationship in terms of inflation and output and emphasised the trade-off it involved for policy makers. The existence of a *long*-run trade-off was disputed by Friedman [1968] and Phelps [1967] who introduced the expectations-augmented Phillips Curve: In deciding about their nominal wage claims, workers take into account the expected price level. Generating surprise inflation can therefore in the short-run lower real wages, increasing employment and output. Yet in the *long* run, Friedman and Phelps argued, there is no money illusion: workers' expectations about future prices adapt to the monetary authority's behaviour, making the long-run Phillips Curve vertical – any mean rate of inflation is then compatible with the long-run "Natural Rate" of unemployment and output.[31] The long-run vertical Phillips curve or Natural Rate hypothesis has been a widely accepted tenet of Macroeconomics ever

[30] It is beyond the scope of this introduction to give an extensive overview of the development of the Phillips curve discussion. Such an overview of the development (prior to the NNS literature) is given by McCallum [1990], King and Watson [1994] and Romer [2001]. King and Watson [1994] also include an overview of some empirical studies.

[31] Holding employment and output above their natural rates is only possible by allowing for ever accelerating inflation.

since. Yet the reader will in section 1.3.2 be introduced to a small strand of literature originating in the NNS framework that has recently disputed the vertical slope of the long-run Phillips Curve in a world with nominal price rigidity. Chapter 5 contributes to this small literature in showing that the relationships between money growth and both employment and output are hump-shaped in the Schumpeterian growth model with price rigidity.

In contrast, the existence of a short-run trade-off between (unexpected) inflation and output has never been seriously questioned. Recently, interest in the Phillips Curve has augmented with the surge of the NNS literature which for all widely used specifications of price rigidity[32] implies a modern variant of the Phillips Curve reminiscent of the expectations-augmented formulation of Friedman and Phelps. We will now derive a continuous time version of this New Keynesian Phillips curve and briefly describe its properties.

Linearising the output price equation (1.6) around steady state equilibrium with zero inflation yields[33]

$$[\pi(\tau) - \pi] = \beta\left[\widetilde{p^*}(\tau) - \widetilde{p^*}\right] \tag{1.7}$$

where $\pi(\tau) = \dot{P}(\tau)/P(\tau)$, $\widetilde{p^*}(\tau) = p^*(\tau)/P(\tau)$ and $x(\tau) - x$ is the current deviation of the variable $x(\tau)$ from its steady state value.

Linearising the optimal price (1.5) yields

$$(\beta+\chi)\left[\widetilde{p^*}(\tau) - \widetilde{p^*}\right] = E_\tau\left\{\left[\widetilde{p^*}(\dot{\tau}) - \widetilde{p^*}\right]\right\} + \frac{\alpha(\beta+\chi)}{\alpha-1}\left[\widetilde{w}(\tau) - \widetilde{w}\right] + [\pi(\tau) - \pi] \tag{1.8}$$

where $\widetilde{w}(\tau) = w(\tau)/P(\tau)$ and where $\dot{x} = dx/dt$ is the time derivative of x. Finally using (1.7) to eliminate $\widetilde{p^*}(\tau)$ from equation (1.8), we have a continuous time version of the New Keynesian Phillips Curve (NKPC):

$$\chi[\pi(\tau) - \pi] = E_\tau[\pi(\dot{\tau}) - \pi] + \frac{\alpha\beta(\beta+\chi)}{\alpha-1}\left[\widetilde{w}(\tau) - \widetilde{w}\right] \tag{1.9}$$

[32] Roberts [1995] shows that NNS models based on the staggered contracts models of Taylor [1980] or Calvo [1983] or the quadratic price adjustment model of Rotemberg [1982] give rise to a common formulation of the Phillips Curve.

[33] Details on the linearisation of slightly more complicated versions of (1.5) and (1.6) can be found in chapter 3 which deals with the local stability of the steady state equlibrium.

Note first that in contrast to the traditional Phillips curve, the NKPC is forward-looking: Inflation today increases because future inflation is expected to increase, due to the forward-looking nature of optimal price setting under rigidity. Second, the driving force for the evolution of inflation are changes in marginal cost $\widetilde{w}(\tau)$ since monopolistically competitive firms as seen above optimally set their price as a mark-up over marginal cost. In the presence of a positive correlation between marginal cost and output $[\widetilde{w}(\tau) - \widetilde{w}] = g[Y(\tau) - Y]$ with $g'[Y(\tau) - Y] > 0$,[34] the NKPC can be recast in terms of an inflation-output relationship:

$$\chi[\pi(\tau) - \pi] = E_\tau[\dot{\pi}(\tau) - \pi] + \frac{\alpha(\beta + \chi)}{\alpha - 1}\frac{\beta}{g}[Y(\tau) - Y] \qquad (1.10)$$

Like traditional Phillips-curve specifications, equation (1.10) involves a short-run trade-off between inflation and output or equivalently, employment, since higher output is associated with higher employment in the model:[35] Inflation rates higher than their steady state level are for a given expected change in inflation associated with an output (employment) level above the steady state level since the coefficients on both variables are positive.

1.3.1.3 Transmission of monetary impulses: The interest rate channel

Putting together the Phillips curve and the demand side of the New Keynesian model, how are changes in the money supply transmitted to the real side of the economy in the NNS framework? The main transmission channel for monetary policy shocks in NNS models is the interest rate channel: An unexpected change in the nominal money supply or the nominal short-term

[34] A positive correlation can be established due to the fact that households are only willing to supply the labour needed for an increase in output under a higher wage. In the present context, see equation (1.2).

[35] However, the focus has been on the New Keynesian Phillips Curve as a description of inflation dynamics. Empirical evaluations have focussed on this aspect, see e.g. Galí and Gertler [1999], Galí, Gertler, and López-Salido [2001], Sbordone [2002] and Eichenbaum and Fisher [2003] Fuhrer [1997] and Bils and Klenow [2004] present a more critical view. For an overview of the development of the theoretical and empirical discussion about the Phillips Curve see King and Watson [1994].

interest rate under price rigidity causes a change in the real short-term interest rates which in turn affects agents' optimal decisions.[36] The effects of a monetary policy shock are usually derived in NNS models with the help of simulations. Thus even fundamental insights about transmission channels may only be valid for certain parameter constellations considered plausible. Insofar the intuition that will be given here applies to the transmission of a serially uncorrelated negative money shock in a standard NNS model with "typical" calibration as presented e.g. by Walsh [2003] pp. 247ff.[37]

The intuition can be illustrated with the help of equations (1.2) - (1.4) and (1.9). Given that the price level is predetermined in that only part of intermediate goods producers can change their prices at any given time, a decrease in the nominal money supply translates into a decrease of the real money supply m. By equation (1.4), for the household to be willing to reduce his real money demand, the nominal interest rate must rise.[38] Assuming that the rise in the nominal interest rate entails an increase in inflation that is less than proportionate, the real interest will rise.[39] Via an intertemporal substitution effect, this will lead the household to postpone consumption, i.e. current consumption falls relative to future consumption. For standard calibrations, the negative substitution effect on current consumption dominates the positive income effect of the interest rate increase so that current consumption decreases. By equation (1.2), this lowers demand for leisure (raises labour supply) at a given real wage. Since the fall in consumption entails a fall in equilibrium employment, the equilibrium real wage must fall. By equation (1.9), this decrease in marginal cost causes a decrease in the optimal price and in the inflation rate at given future inflation. Given that uncorrelated monetary policy shocks are examined, the money supply in the

[36] Regarding this transmission channel's empirical importance, Angeloni et al. [2003] find in a recent empirical survey of studies analysing monetary transmission channels in the Euro area that "while not dominant on the whole, the IRC [interest rate channel] is still a prominent channel in the transmission". (p. 25)

[37] See also Galí [2003] and Christiano, Eichenbaum and Evans [2005].

[38] Galí [2003] discusses the transmission mechanism when money supply shocks are highly correlated: Here the nominal interest rate may fall (absence of "liquidity effect") but the real interest rate will still rise.

[39] The effect on inflation is described below.

future will return to its original value, causing an increase in future inflation ($E_\tau \left[\dot{\pi} (\tau) - \pi \right] > 0$) that dampens the fall in current inflation.

Contrasting this intuition with the stylised facts about the effects of a monetary policy shocks cited at the beginning of this section shows that the baseline NNS model does quite well at capturing the non-neutrality of money at business cycle frequencies.

1.3.2 Inflation, employment and output in the long run: A non-vertical Phillips curve

As stated in section 1.3.1.2, it is the mainstream view in economics that inflation raises employment in the short-run but the Phillips curve is vertical in the long run. In the words of Taylor [1999b]:

> "Theoretical and empirical research on the relationship between inflation and unemployment has left little doubt that there is no long-run trade-off between the rate of inflation and the rate of unemployment. In other words, the unemployment rate will average about the same amount, whether the inflation rate is zero percent or, say, ten percent." (p. 31)

Recently, however, a variety of models has been put forward which contain mechanisms allowing money to affect output and employment in the long run. Put differently, the models imply the existence of a non-vertical Phillips Curve. As stated in section 1.3.1.2, Friedman [1968] and Phelps [1967] argued that in the long run, the Phillips curve has to be vertical due to the absence of long-run money illusion. So to reintroduce a long-run influence of inflation on real variables, one has to either argue that money illusion does exist in the long run or put forward arguments why money might have an influence despite perfectly rational expectations. We begin by presenting one paper taking the first approach in section 1.3.2.1 and then in section 1.3.2.2 concentrate on the second idea which is closer to the focus of this thesis since it has been explored in the sticky-price framework of the NNS business cycle models presented in section 1.3.1. Further, some empirical evidence on the long-run inflation-unemployment relationship will be presented in section 1.3.2.3.

1.3.2.1 Non-vertical long-run Phillips curve: Money illusion

Turning to the first strand of the literature, in Akerlof, Dickens and Perry [2000], the flexibility of both prices and wages is unrestricted. The existence of a non-linear long-run Phillips curve in their model is due to agents being subject to a kind of money illusion even in the long run.[40] The authors argue that this departure from rationality will vanish not over time as one moves from the short run to the long run but moving from low inflation rates – where the costs associated with the deviation from perfect rationality are low – to high inflation rates where the costs increase. In particular, deviation from perfect rationality is modelled in the form of agents either ignoring inflation or giving inflation less than the adequate weight in the price / wage setting process. As a consequence, at low inflation rates where inflation is not a "salient" (p. 39) factor in decision making, price and wage setting will respond less than proportionately to expected inflation: Near-rational firms may not adjust nominal wages enough to keep real wages constant and near rational workers may not realise their real wage has fallen. Prices are a mark-up over unit labour cost, so that near rational firms' relative prices fall in inflation, raising demand and employment. As inflation rises, so does the cost of near-rational behaviour for agents. Threshold levels above which an agent switches to fully rational behaviour are normally distributed, leading to a non-linear form of the long-run Phillips curve with employment being a hump-shaped function of inflation. At high inflation, all agents behave rationally and employment returns to the Natural Rate.

1.3.2.2 Non-vertical long-run Phillips curve: Sticky prices

A handful of papers analyse the long-run relationship between money growth and the levels of employment and output by comparing the steady state

[40] The authors themselves do not use the term money illusion but it is suggested by Blinder [2000] in his discussion of the paper.

equilibria of their NNS models associated with different constant money growth rates.[41] The pivotal assumption that generates the money non-superneutrality in these models is the existence of temporary stickiness in nominal wages or prices. As is apparent from equation (1.9), the mere presence of a positive discount rate χ used by economic agents is sufficient for the existence of a long-run Phillips-type trade-off in the standard NNS model.[42]

The existence of the long-run inflation-unemployment trade-off is shown and its properties are analysed by Graham and Snower [2004] (henceforth: GS) in a model with flexible prices and rigid wages. In their model, households that are monopolistically competitive suppliers of differentiated labour types are faced with nominal wage rigidity. In particular, following Taylor [1980], each period the *nth* part of all households get to set a nominal wage for the next n periods. During the interval when a household's nominal wage is fixed, inflation steadily erodes its real wage. Taking this into account, a household that readjusts its wage chooses a nominal wage that will *on average* lead to an optimal real wage while the nominal wage is fixed. Positive discounting implies that the average of real wages formed by households is weighted, with weights that decrease over time. This implies that erosion of the real wage has not proceeded much in the periods that receive the highest weight. Consequently, the optimal nominal wage set by the household is not high enough to offset the effect of inflation on its mean wage during the n periods. As all households set wages this way, the aggregate real wage is lowered by inflation. The reduction in the real wage raises demand for labour, equilibrium employment and output. Thus by reducing the monopolistic distortion that households' optimal wage setting implies, the time discounting effect allows for a substantial employment-increasing effect of inflation – a

[41] See Ascari [1998, 2004], Devereux and Yetman [2002], Graham and Snower [2002, 2004], Karanassou, Sala and Snower [2003,2005] and King and Wolman [1998].

[42] Defining the long-run by the fact that expected changes in inflation are zero, the short-run and long-run slopes of the Phillips curve in equations (1.9) or (1.10) would be equal. Note, however, that this is an artifact of the continuous time specification. The equivalent of equation (1.10) in discrete time is $\widehat{\pi}(\tau) = \widetilde{\chi} E_\tau \widehat{\pi}(\tau+1) + \widetilde{\alpha}\widehat{Y}(\tau)$ where a hat over a variable denotes deviation from its steady state value. In the long-run characterised by $\widehat{\pi}(\tau) = E_\tau \widehat{\pi}(\tau+1) = \widehat{\pi}$, we have $\widehat{\pi} = \frac{\widetilde{\alpha}}{1-\widetilde{\chi}}\widehat{Y}(\tau)$, implying that the Phillips curve is steeper in the long-run than in the short-run.

non-vertical long-run Phillips-Curve.[43] The simulations in Karanassou, Sala and Snower [2005] show that the trade-off may be substantial even at small discount rates when there is what Ball and Romer [1990] call "real rigidity" of wages, i.e. when the responsiveness of optimal wages to changes in real variables is small.[44]

In contrast to the monotone inflation-employment relationship,[45] the long-run relationship between inflation and *output* is non-monotone in the model of GS: Given staggered wage setting under price rigidity, there is dispersion in real wages which distorts demand towards labour with a relatively low real wage, i.e. whose wage was set in the past and has since been eroded by inflation. Given that the labour types are imperfect substitutes in output production, this reduces their average productivity and hence, output. An increase in inflation thus ceteris paribus increases output via the increase in employment, but labour is used more and more inefficiently as inflation and real wage dispersion rise, ceteris paribus reducing output. The interaction of both effects makes output a hump-shaped function of inflation.[46]

Interestingly, the authors' simulations also show that at all but extreme inflation rates, the qualitative relationship between inflation and real variables is unchanged when the contract length is endogenously chosen by

[43] GS solve for the steady state numerically and explicitly derive results only for the steady state of the linearised economy in the simplest case $n = 2$.

[44] Karanassou, Sala and Snower [2003] argue that the sluggish adjustment of prices and wages may be complementary in generating the long run inflation-unemployment trade-off.

[45] GS also find the inflation-*employment* relationship to be hump-shaped when introducing a second transmission channel for inflation, the empirical relevance of which GS themselves consider "doubtful" (p. 16): Given real wage dispersion, the demand for a household's labour is distributed unevenly over the contract period, increasing as the wage is eroded by inflation. Assuming that households have a preference for smoothing out leisure over time, they demand a higher wage for a given average amount of labour to be supplied, the more unevenly this labour is distributed over time. Inflation that raises real wage dispersion thus raises the wage and reduces employment. Together with the time discounting effect, a hump-shaped relationship emerges.

[46] In a similar model with two period wage staggering à la Taylor [1980], Ascari [1998] engages in a number of numerical exercises on the output and welfare effects of a reduction in steady state money growth. His sensitivity analysis shows that the output and welfare gains increase strongly in the degree of non-linearity of the model, in particular the degree of increasing marginal disutility of labour (households' preference for smooth labour supply paths) and the non-linearities in production which make real wage dispersion inefficient (imperfect substitutability of intermediate goods, decreasing returns to labour-types).

agents.[47]

The paper of King and Wolman [1998] is a complement to Graham and Snower [2004] and the approach of this thesis in that it analyses the effects of positive steady state inflation in a NNS model with flexible wages where monopolistic producers using labour to produce intermediate goods are faced with *price* rigidity à la Taylor [1980] where each period half of all firms get to set prices for two periods. The authors do not explicitly derive the function relating output to inflation but argue that output is maximised at positive rather than zero inflation. This is due to the net effect of inflation via its influence on relative prices and on monopolistic mark-ups. For the model of chapter 5, we are able to analytically show that employment and output are hump-shaped functions of output in a broader framework where a more general specification of price rigidity is used.[48]

Summing up, money under rigid prices or wages is non-superneutral due to inflation's effects on monopolistic distortions and on production efficiency. While the model presented in chapter 5 of this thesis shares this central idea with the papers presented here, there are several important differences that make the approach taken in this thesis stand out: First, the papers abstract from secular output growth. Hence, in contrast to this thesis they present no analysis of the interaction of the effects of inflation on employment and growth. Second, the literature mainly relies on simulation exercises instead of the analytical derivation of results whereas the approach of this thesis is analytical. Third, the bulk of the literature uses a particularly simple specification of price rigidity, whereas the more general specification used in this thesis allows for richer effects of money.[49]

[47] Graham and Snower endogenise price rigidity by allowing households to choose the length of their wage contract period assuming there is a fixed cost to changing wages. Their finding of a qualitatively unchanged relationship is confirmed by simulations of Devereux and Yetman [2002] of a model with Calvo [1983]-type price rigidity on the goods market.

[48] Ascari [2004] builds a model with capital that like ours features Calvo [1983] price rigidity in the goods market. Relying exclusively on simulations, he shows that the steady state level of output depends negatively on the inflation rate.

[49] In particular, the average mark-up is a non-linear function of the money growth rate under our specification of price rigidity. Among other things, this makes employment a hump-shaped function in our model, in contrast to the result of Graham and Snower [2004].

1.3.2.3 Non-vertical long-run Phillips curve: Empirical evidence

A number of studies investigate the slope of the long-run Phillips-curve for the US or Europe.[50] The evidence is very mixed: In many cases a vertical long-run Phillips curve cannot be rejected,[51] while some evidence points to a long-run trade-off between inflation and unemployment. For example, in the study of Watson and King [1994], different identifying assumptions about the short-run behaviour of the variables lead to either a vertical long-run Phillips curve or a substantial trade-off for the post-war US.[52] Koustas and Serletis [2003] find the same dependence of the long-run slope estimate on Keynesian versus Monetarist identifying assumptions using European data. Karanassou, Sala and Snower [2003] find support for a negative slope of the inflation-unemployment relationships using European data and give the following quantification: A "10 percent increase in long-run money growth (equal to long-run inflation) is associated with a 3.18 percentage point fall in the EU unemployment rate". At the same time, a limited number of recent studies report a positive slope of the long-run inflation-unemployment relationship: Beyer and Farmer [2002] and Russell and Banerjee [2006] find evidence of a significantly positive long-run relationship between inflation and unemployment in the US. In the latter paper, the effect is quantified as implying that "an increase in inflation of around 5 percentage points [...] would be associated with an increase in unemployment in the long-run of about 1 1/2 percentage points" (p. 14).

Summing up, the empirical relationship between inflation and unemployment in the long run is ambiguous, making it hard to reject any of the theories presented in section 1.3.2 or the approach of chapter 5.

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[50] Relevant recent studies are mostly based on two influential papers by Watson and King [1994, 1997].

[51] See e.g. Watson and King [1994, 1997] for the US and Weber [1994] on evidence for the G7 countries including Germany.

[52] Setterfield and Leblond [2003] find support for a long-run trade-off in postwar US data.

1.4 Money growth, inflation and the growth rate of output

In this section, we first present some of the most prominent theoretical explanations of the non-superneutrality of money in the neoclassical and endogenous growth frameworks. The mechanisms allowing for monetary non-superneutrality in these literatures differ fundamentally from the assumption of nominal price rigidity made in this thesis and in the NNS literature. We nevertheless review the explanations in some detail in section 1.4.1 because they share the central question about the long-run *growth* effect of money which is one of the principal concerns of this thesis. Subsequently, we review some recent results of the empirical literature investigating the superneutrality of money with respect to output growth in section 1.4.2.

1.4.1 Survey of the theoretical literature

Two different literatures have contributed to the theoretical literature on inflation and growth. The first and older strand of the literature analyses the effects of money growth in the neoclassical growth model of Solow [1956] and Swan [1956] or Ramsey [1928], Cass [1965] and Koopmans [1965]. Technically, this literature might as well be included in section 1.3.2, since- given that steady state output growth is due to *exogenous* technical progress in the neoclassical framework- the influence of inflation on capital accumulation is at steady state restricted to effects on the *level* the capital-labour ratio. Yet due to the fact that this literature's focus is on the effects on capital accumulation which is also the focus of the modern literature on inflation and growth, the neoclassical monetary growth models are generally subsumed to the inflation-growth literature. Since this literature also paved the way for modern analyses of the inflation-growth nexus in models with an endogenously determined output growth rate which are the main focus of this section, section 1.4.1.1 starts with the presentation of the most influential contributions from this literature. Section 1.4.1.2 then turns to endogenous growth theory.

1.4.1.1 Long-run effects of money growth in neoclassical monetary growth models

While the steady state output growth rate in the neoclassical growth model is exogenously given, money does influence the *levels* of real variables in steady state.[53] In particular, the issue of money's superneutrality can be evaluated by comparing the steady-state levels of the capital-labour ratio and of output across steady states associated with different money growth rates. We sketch the main arguments of the three most influential articles from this literature.[54]

Tobin [1965] in the neoclassical model with constant savings rate of Solow [1956] and Swan [1956] argued that the increase in inflation implied in the long run by an increase in money growth raises the opportunity cost for households of holding nominally denominated currency. As explained in section 1.3.1.1, the opportunity cost of holding money is the difference between the rates of return on capital and on money which in turn is given by the sum of the real interest rate and the inflation rate, i.e. the nominal interest rate. Assuming money and real capital are substitutes in the household's asset portfolio, an increase in the nominal interest rate induced by an increase in inflation then makes the household shift its portfolio towards higher capital holdings. Via this portfolio composition effect, an increase in money growth causes an increase in real capital investment that raises the steady-state levels of the capital-labour ratio and output. Although the *Tobin-effect* of inflation has received widespread attention, the model has been criticised for not making clear why households would choose to hold money in the first place, since money as a store of value is dominated by capital for any non-negative inflation rate.

To make explicit the preferences underlying the household's portfolio choice, Sidrauski [1967] integrated money into the Ramsey [1928]-Cass [1965]-Koopmans [1965] framework by assuming that households derive utility from

[53] *Outside* steady state, money growth for standard classes of utility functions does influence the growth rate of consumption and output, see e.g. Fischer [1979] for an analysis of the Sidrauski [1967] model.

[54] See Orphanides and Solow [1990] for a thorough survey of the literature.

consumption and from holding real money balances. He showed that when households optimally choose their intertemporal consumption path, the steady state real capital stock in efficiency units and therefore, the level of output in steady state are determined independently of the money growth rate. Intuitively, money is superneutral since the marginal productivity of capital at steady state is unaffected by inflation.[55] Since an increase in inflation raises the optimal level of consumption relative to money holdings, the positive portfolio composition effect of inflation on the capital stock is exactly offset by a negative effect on total savings.

Finally, Stockman [1981] instead of introducing money into households' utility function modelled the role of money as a means of exchange by following Clower [1967] in assuming that households are subject to a Cash-in-advance constraint.[56] In his specification, this means that households must pay for their consumption good purchases and capital investments with cash instead of using credit services. In this framework, an increase in inflation that raises the opportunity cost of money holdings also raises the cost of investment in real capital. While the productivity of capital is unaffected, the return on investment is reduced. The "inflation tax" on investment therefore lowers the steady state capital stock in efficiency units and the output level in stark contrast to the result of Tobin [1965].

Summing up, different assumptions about the role of money in the economy led to vastly different results concerning the level effect of the money growth rate on the steady state capital intensity.

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[55] Sidrauski's specification of the utility function excluded leisure. Superneutrality continues to hold at steady state when leisure enters the utility function in such a way that the marginal rate of substitution between consumption and leisure is independent of money holdings. But as noted by Blanchard [1990] among others, even if little weight is given to money in the utility function, the superneutrality result is only an approximation since the change in the opportunity cost of money does influence real money holdings and therefore, utility.

[56] The CIA-framework will be described in more detail in section 1.4.1.2.

1.4.1.2 Long-run growth effects of money growth in endogenous growth models

In the new generations of growth models,[57] the output growth rate of the economy at steady state is endogenously determined by economic agents' decisions and is thus subject to the influence of policy measures that change agents' incentives. The models to be reviewed below therefore compare the output growth rate across steady state equilibria associated with different inflation rates caused by policy makers' exogenous choice of a constant money growth rate. In the light of the recent empirical findings on the inflation-growth nexus to be reviewed in section 1.4.2, the focus is on finding plausible channels through which inflation harms long-run growth. The literature's models share a similar structure made up by the following elements: A **money demand** function is generated by assuming either that households derive utility from holding money or that some purchases can only be made with cash (Cash in advance constraint).[58] If necessary, a further assumption is made to allow for an influence of inflation on real variables. In contrast to the NNS models discussed in section 1.3, the **money supply** process is modelled in the simplest possible way: The monetary authority expands the nominal money supply at a constant rate and distributes the proceeds lump-sum to households. Finally, the accumulation of physical and/or human capital which is not subject to diminishing returns is the **mechanism driving growth** in nearly all examined models.[59] The arguments inducing non-superneutrality of money in this framework can be divided into two categories: First, mechanisms that allow inflation to affect the marginal productivity of the capital form whose accumulation drives growth. Second, mechanisms that while leaving the marginal productivity of capital unaltered

[57] For an introduction to endogenous growth theory, see Barro and Sala-i-Martin [2003] or Aghion and Howitt [1998].

[58] Two less common alternatives are to introduce money as an input in production or to explicitly model a transaction cost reducing role for money.

[59] The exception is the model by Marquis and Reffett [1994] who introduce an additional financial services sector (see section 1.4.1.2) into a Romer [1990]-type growth model with an increasing variety of intermediate goods to study the interaction of inflation, the economy's system of payments and growth.

influence the net real return of capital investment and thus, the incentive to accumulate capital.

Inflation affects the marginal productivity of capital A variety of arguments has been put forward why variations in the long-run inflation rate might influence the marginal productivity of either human or physical capital. In models where the accumulation of capital is the engine of growth, the resulting change in the incentive to invest automatically implies a growth effect of inflation. The two most important arguments in this context are the inflation tax on labour supply and the absorption of productive resources in the process of avoiding the cost of inflation.[60] Both arguments imply that inflation has an effect on the level of employment in the sector central to growth, and that this level effect in turn causes inflation to also affect the economy's growth rate. We briefly discuss each idea in turn.[61]

Inflation tax on labour supply Given standard preferences over consumption and labour, the introduction of a cash-in-advance (CIA) constraint on households' consumption purchases induces households to shift their consumption profile away from activities requiring cash (i.e., consumption) towards activities not subject to the cash-in-advance constraint (i.e., leisure). To see this, make the standard CIA assumption that in each period, consumption c takes place before households receive labour and rental income, so that money M for shopping has to be carried over from the previous period:

$$P_t c_t \leq M_{t-1}$$

[60] Alternatively, real money balances can be introduced as an input in the production process. Inflation then increases the cost of the input, so firms economise on their money holdings. In a two sector growth model with constant returns to physical and human capital in output production, this allows inflation to influence growth if money is essential in the production of the limiting factor human capital but only has a level effect if human capital production is unaffected by money, see Wang and Yip [1992], Pecorino [1995] and Chang [2002].

[61] Gillman and Kejak [2005a] present a detailed comparison of a number of models integrating these two effects in a variety of different endogenous growth models.

As argued above, since money in contrast to capital does not bear any real interest r and further, the real value of money depreciates at the inflation rate π, the opportunity cost of transferring one unit of goods across time in the form of money instead of capital is given by the nominal interest rate $R = r + \pi$. Then the first order condition governing the static optimisation concerning leisure l and consumption c from the maximisation of utility $U(c, l)$ subject to a standard budget constraint is

$$\frac{U_l}{U_c} = \frac{w}{1+R}$$

where U_x is marginal utility from consumption of x and w is the real wage.[62] The term $1+R$ is the effective cost of consumption under the CIA-constraint given by the goods price plus the opportunity cost of holding the money necessary for the transaction. As the effective cost of consumption rises in the inflation rate π, an increase in π decreases the optimal ratio U_l/U_c, which for standard preferences implies that leisure is increased relative to consumption.[63]

The "inflation tax on labour supply" then results in a negative effect of inflation on growth in any model where the following three conditions hold:[64,65] Capital accumulation results in sustained growth, the marginal

[62] For details see, e.g. Walsh [2003].

[63] Although we here follow the literature in introducing money via a CIA-constraint, the substitution from consumption to leisure induced by the rising opportunity cost of holding money with a utility can also be modelled in the money in the utility framework of section 1.3.1: A more general specification of utility than (1.1) that would allow the ratio U_l/U_c in the optimality condition (1.2) to depend negatively on money holdings at given c, l, would imply that demand for leisure increases at a given real wage when inflation rises. Using the CIA constraint makes it possible to avoid introducing such a more complex utility function.

[64] See De Gregorio [1992,1993], Jones and Manuelli [1995], Dotsey and Ireland [1996], Gillman, Harris and Mátyás [2004] and Gillman and Kejak [2005b]. Gomme [1993] also features the inflation tax on labour supply but focuses on the short run inflation-growth relationship off steady state.

[65] Note that in the model of Stockman [1981], there was a negative level effect of inflation through intertemporal substitution of consumption because the CIA constraint extended to purchases of consumption *and* investment goods. In the models of the present section, CIA on consumption alone is sufficient to generate a negative growth effect of inflation via the substitution of consumption and leisure given that labour supply is endogenous. The "inflation tax on investment"-setting of Stockman is extended to an endogenous growth

productivity of capital increases in equilibrium employment (i.e., labour has a scale effect on growth) and a decrease in labour supply results in a decrease in equilibrium employment.[66]

This is most straightforward in the setup of Jones and Manuelli [1995], where long-run growth of per capita production y is possible since returns to physical capital k and effective labour nh are constant. The output production function is $y = k^{\alpha}(hn)^{1-\alpha}$ where $0 < \alpha < 1$ and n is employment. Output can be costlessly transformed into either physical capital or human capital h for accumulation purposes. At any given steady state capital intensity k/h,[67] accumulated human capital is used less intensely when employment decreases, lowering the marginal products of both types of capital[68] and thus lowering the incentive to invest and the steady state growth rate. Therefore, economic growth is maximised when monetary policy authorities follow the *Friedman rule*:[69] Contracting the money supply at the rate that results in a zero nominal interest rate as prescribed by the Friedman rule eliminates the opportunity cost of holding money and thus does away with households' substitution towards leisure.

Existence of a resource-absorbing credit sector In a number of papers, households can endogenously determine the fraction of goods they want to pay for with cash while the rest is paid for with credit the production of which requires input of productive resources.[70] In Dotsey and Ireland [1996], for example, credit is produced using labour. An increase in inflation

model e.g. by Marquis and Reffett [1995].

[66] The latter condition does not generally hold in the models examined by Fukuda [1996] and Itaya and Mino [2003] where under strongly increasing returns to scale in production due to labour externalities, a decrease in labour supply may under certain transaction technologies and parameter constellations cause an increase in equilibrium employment and hence, a positive growth effect of inflation.

[67] To simplify matters, assume that the depreciation rates of h and k are identical. Then the steady state value k/h is determined by the technology parameter α independently of inflation.

[68] $\frac{\partial y}{\partial k} = \alpha n^{1-\alpha}\left(\frac{k}{h}\right)^{\alpha-1}$, $\frac{\partial y}{\partial h} = (1-\alpha)\, n^{1-\alpha}\left(\frac{k}{h}\right)^{\alpha}$.

[69] See Friedman [1969].

[70] See Marquis and Reffett [1994], Dotsey and Ireland [1996], Gillman, Harris and Mátyás [2004] and Gillman and Kejak [2005b]. See Temple [2000] for references on the interaction of inflation, the financial sector and growth.

raises the cost of using money relative to credit and thus raises demand for credit. The resulting reallocation of labour to the financial sector reduces the resources available for real production activities. Therefore as in the previous section, in any model of growth through capital accumulation where the marginal productivity of capital depends on employment (or more generally, on the amount used in production of the resource that also produces credit), an increase in inflation entails a lower output growth rate due to the reallocation of resources to the credit sector. Again, the steady state output growth rate is maximised when monetary authorities follow the Friedman rule.

Influence of inflation on the return to investment in spite of unchanged marginal productivity of capital The second group of arguments explains why inflation might affect the return on investment, and hence, capital accumulation and the growth rate, even when the marginal productivity of capital is independent of inflation. E.g., the growth effect of inflation via nominal rigidity in the tax system is analysed by Chari, Jones and Manuelli [1996] and Jones and Manuelli [1995]. They investigate the consequences of a non-indexed tax system with nominal depreciation allowances. By raising the nominal interest rate, inflation reduces the present value of depreciation allowances, raising the effective tax rate and reducing the after tax return on investment. Similarly, they show the spread between borrowing and lending rates of banks caused by cash reserve requirements on bank deposits increases in inflation. Assuming that part of capital investment has to be financed with bank loans, rising inflation thus again reduces the return on investment.[71]

The approach to monetary analysis taken in this thesis shares with this literature the way money demand generated by money in the utility function and money supply are modelled, and the methodology of comparative statics regarding deterministic steady state equilibria. What distinguishes our approach are both the modelling of endogenous growth as being due to

[71] In both the non-indexed tax system approach and the model with reserve requirements for banks, setting the growth rate of money supply according to the Friedman rule again maximises the rate of economic growth.

stochastic research and development activities instead of capital accumulation and the introduction of nominal price rigidity as a friction that allows money to have real effects. As our literature review has shown, both elements have thus far been neglected in the study of inflation and endogenous growth.

1.4.2 Empirical evidence on money growth and output growth

The effect of inflation or money growth on the real output growth rate has been investigated using a variety of methods and approaches.[72] While some early studies using mostly cross-country data found no significant correlation between inflation and growth,[73] some cross-sectional studies report a significantly negative effect of inflation on growth. E.g., Motley [1998] reports that "the coefficients [...] imply that in long-run steady state, a ten percent inflation rate will reduce annual per capita growth in an average country by about 1/4 percentage point" compared to a situation with zero inflation (p. 22).[74] The results of cross-country studies have however been criticised for their lack of robustness with regard to changes in the country samples, time period and regression specification.[75] Research has therefore turned to the use of panel data sets hoping to get more robust results by exploiting the information contained in the data's time series dimension.[76] Using 5-year averages, Barro [1996] finds a linear negative effect of inflation similar to that reported by Motley [1998]. In reaction to the critique that this finding may

[72] Temple [2000] contains a survey of recent empirical contributions as well as a discussion of the methodological difficulties involved. Summaries or overviews of empirical investigations are also given in Gillman and Kejak [2005a], Gylfason and Herbertsson [2001], Ghosh and Phillips [1998], Bruno and Easterly [1998] and Ragan [1998].

[73] The cross-country study of McCandless and Weber [1995] is a good example and contains further references. More recently, Judson and Orphanides [1999] find no significant relation in cross-country data but a negative relation when panel data are used. In a time–series setup, Geweke [1986] finds support for the superneutrality hypothesis using a century of annual U.S. data.

[74] The results of Fischer [1993] are of a similar magnitude.

[75] See Levine and Renelt [1992], Levine and Zervos [1993] and Clark [1997].

[76] Cf. e.g. to Barro [1996], Judson and Orphanides [1999], Gylfason and Herbertsson [2001] and Gillman, Harris and Mátyás [2004].

be mainly due to the effect of high-inflation countries, most recent studies try to both allow for non-linearity of the inflation-growth relationship and differentiate between industrialised and developing countries. While a negative relationship has been confirmed for most inflation rates, non-linearity seems indeed to be present in the data along two dimensions: Firstly, the negative inflation-growth relationship seems to be convex: there is evidence of decreasing marginal cost of inflation at high inflation levels.[77] Secondly, using spline techniques, several studies have allowed the coefficient on inflation to differ for observations above or below certain thresholds. There is indeed some evidence that below a certain threshold value, the influence of inflation on growth may be insignificant or even positive. E.g., Khan and Senhadji [2001] report that the inflation-growth relationship is weakly but significantly positive for industrialised countries below an inflation threshold of 1%. The threshold value found for industrialised countries in other studies are somewhat bigger.[78,79]

While the investigation of the non-linearity of the inflation-growth relationship seems promising, a general caveat concerning all mentioned studies is that the long-run growth rate of output is of course not observable in the data: Evidence from panel studies using 15-, 10-, 5-year averages or even annual data is used to make claims about the nature of the long-run inflation-growth relationship. While the main message- at least medium to large values of inflation are detrimental to growth- is mostly preserved when moving from high to lower frequency data, the levels of significance are sometimes lower.[80] So additional efforts need to be made to further disentangle the medium-run and long-run efforts of inflation on growth.

Despite these difficulties encountered in the empirical investigation of the inflation-growth relationship, the results can be summed up as follows:

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[77] See, e.g. Gylfason and Herbertsson [2001] and Gillman, Harris and Mátyás [2004] or Ghosh and Phillips [1998].

[78] Ghosh and Phillips [1998] report a threshold value of 2.5%, Sarel [1996] finds a threshold at 8% and Gylfason and Herbertsson [2001] reported values of 10% and a rather high 20% for two different data sets.

[79] The threshold for developing countries seems to be somewhat higher than for industrialised countries, see e.g. Khan and Senhadji [2001].

[80] See e.g. Ghosh and Phillips [1998].

There is considerable evidence that money is non-superneutral with respect to the growth rate of real GDP. The relationship seems to be non-linear: The influence of inflation on output growth is negative at large inflation rates, while being possibly insignificant or positive at low levels of inflation.

1.5 The long-run influence of money on innovation-driven growth and the level of employment under nominal price rigidity

1.5.1 Attempting a long-run synthesis

As the reader might have gathered from the preceding sections, the framework of analysis to be used in this thesis will be a synthesis of the approaches presented in this introduction: With the New Keynesian literature we share the conviction that price rigidity is a realistic friction that is central to the transmission of nominal impulses to the real side of the economy.

With the endogenous growth literature, we share the conviction that the output growth rate can be directly influenced by policy and that in particular, the impact of monetary policy is not restricted to the short run. Therefore close in spirit to the New Neoclassical Synthesis literature that introduced Keynesian frictions into the Standard Business Cycle model we attempt to build a long-run synthesis by introducing nominal price rigidity into one of the workhorse models of endogenous growth theory, the Schumpeterian quality ladder model of Grossman and Helpman [1991] and Aghion and Howitt [1992].

1.5.1.1 Price rigidity in a long-run model?

At first sight, the reader might be surprised by our conjecture that nominal price rigidity – which is usually assumed to be relevant at business cycle frequencies – should matter for an economy's long-run growth performance – given the elements' different time dimension, their joint analysis might seem disproportionate. To see that this is not the case and why we should on the contrary expect short-term price rigidity to indeed have an impact on long-term growth, consider the following points: First, although prices are fixed for only short periods of time, price rigidity is a permanent feature of the economy. In its presence, in the short run relative prices are constantly distorted at any non-zero inflation rate. Since relative prices steer the short-run allocation of real sources in the economy, the latter is consistently influenced by price rigidity. Thus, permanent price rigidity does influence the level of real economic variables in the short and long run. Accepting this level effect inevitably leads one to expect that price rigidity allows for an influence of inflation on the growth rate of output, too: Long-term growth is but repeated short-run growth originating from the investment decisions of economic agents. These decisions are in turn based on current and expected prices and levels of real variables. Thus there is no reason to believe that a feature of the economy that influences short-run levels would not influence long-run economic growth as well. Therefore, the inflation-growth literature's negligence of price rigidity as a transmission channel for monetary impulses on output growth is an unjustified gap that the present thesis aims to fill.

1.5.1.2 Why innovation-driven growth?

In the framework used in this thesis, economic growth is fuelled by stochastic research and development activities that lead to innovations. We now give a short description of this framework and then argue why it is particularly suitable for our analysis.[81]

There are three productive sectors in the economy: The economy's final good is produced in a perfectly competitive sector using a large number of differentiated intermediate goods that are imperfect substitutes. Each intermediate good is of a certain quality level (position on the "quality ladder") which determines its productivity. Since intermediate goods are produced one for one with labour, increases in the quantity of intermediates are limited. However, long-run output growth is possible due to increases in the quality of intermediate goods brought about by innovations of the Research

[81] For a more detailed presentation see Aghion and Howitt [1998] or Barro and Sala-i-Martin [2003] which is closest to our exposition.

and Development (R&D) sector. Every intermediate good type is produced by one firm which holds the exclusive right to produce this type thanks to the acquisition of the relevant patent from the R&D sector. Firms in the intermediate goods sector earn positive quasi-profits since they act in an environment of monopolistic competition due to the fact that intermediate good types are imperfect substitutes in production. The prospect of positive profits leads to buyers' competition in the market for patents. The price of a patent is then determined by the present value of profits to be made from sales of the corresponding innovative intermediate good, which replaces its predecessor of inferior quality in the market for intermediate goods. The patent price in turn determines the incentive to engage in research activities. There is free entry to the R&D sector, where an increase in the input of real resources raises the flow probability of making an innovation. As the incentive to innovate increases, so does the amount of resources devoted to research and hence, the economy's output growth rate.

This framework is suitable for our purposes for various reasons: First, as explained above growth is driven by research activities leading to the design of innovative goods – the market entry of new intermediate goods with corresponding prices is a permanent feature of the economy. Thus, pricing decisions – and therefore, price rigidity – are intimately and naturally connected to the incentive to innovate and the very heart of the economy's growth process. Second, price setting under rigidity is easily integrated into the framework and its results are well comparable to the NNS models of sections 1.3.1 and 1.3.2 since the structure of monopolistic firms selling differentiated intermediate goods to a competitive final good sector is identical. Third, it is not the aim of this thesis to analyse the inflationary problems in developing countries – where governments' financing constraints and underdevelopment of the financial sector might be more central to the transmission of monetary impulses. But in industrialised countries which are thus the focus of our attention, the generation of new ideas and goods is probably more important for growth than the accumulation of capital that the inflation-growth literature has concentrated on so far. For all of these reasons, the quality ladder model is an adequate framework for the analysis of the long-run consequences of money growth.

1.5.1.3 Empirical evidence on price rigidity

The results of a model based on nominal price rigidity can only be considered substantial if this transmission channel for monetary policy is empirically relevant, i.e., if there is evidence that quantities do change faster than prices. Summing up the relevant empirical literature, this is indeed the case: It is the consistent finding of all major studies investigating price setting behaviour that producer and consumer prices are fixed for periods that are not negligible both in the Euro area and the US.[82] Disagreement only arises with respect to first, the average duration of the period for which prices are fixed and second the determinants of price rigidity, i.e. the motives firms have for not changing their prices immediately in response to shocks.[83] Regarding the duration of price rigidity, Taylor [1999a] in a survey of the literature reports an average length of one year for the US. According to the influential study of Blinder et al. [1998], prices in the US are changed on average once in 9 months, while Bils and Klenow [2004] concentrating on consumption goods found a median length of less than 6 months.[84] According to a recent survey, price rigidity seems to be somewhat more severe in the Euro area where the average price duration is reported to be "close to one year" by Álvarez et al. [2006].[85] In order not to overstate the effects of price rigidity, the parameter choices for the calibration of examples in the subsequent chapters are in line with the smallest mentioned estimates – and indeed we find that adding a small

[82] See e. g. the survey of Taylor [1999a].

[83] It is beyond the scope of this exposition to discuss the underlying causes of price rigidity. Surveys of the theoretical explanations for price rigidity and their empirical relevance are presented by Wolman [2000], and by Álvarez et al. [2006] for the Euro area. Blinder et al. [1998] conducted an influential survey in the US.

[84] The median length was 4.3 months or 5.5 months excluding temporary price cuts, i.e. sales. Bils and Klenow note that the difference in results between their study and the one of Blinder et al. [1998] might be due to the fact that price rigidity might be more important for intermediate goods rather than consumer goods producers since "firms in the Blinder et al. [1998] survey sell mostly intermediate goods and services (...) rather than consumer items" (pp. 953-954).

[85] Summarising recent studies, they report an average (median) duration of 13.0 (10.3) months from micro data on consumer prices and 10.8 months from firm surveys.

amount of price rigidity is sufficient to generate a quantitatively significant degree of non-superneutrality of money in our Schumpeterian model.

A related question is how price rigidity should be modelled – is the price setting process of firms best described by a time dependent or state dependent schedule, i.e. do firms change prices when a certain amount of time has passed or based exclusively on the state of demand and other economic variables. The results reported by Álvarez et al. [2006] for the Euro area indicate that a time dependent schedule is a reasonable approximation: 34% percent of firms use pure time-dependent price reviewing rules, while among the rest, "firms that mainly follow time dependent rules, but change prices in the case of specific events dominate" (p. 581).[86] We therefore use the time-dependent pricing rule of Calvo [1983] that as discussed in section 1.3 is standard in the modern New Keynesian business cycle analysis.

1.5.2 Short outline of the remaining chapters

The remainder of this thesis is dedicated to the analysis of the long-run effects of money growth in the described Schumpeterian growth model with price rigidity. To concentrate fully on the effects of price rigidity, money is introduced into the model in a way consistent with the superneutrality of money under flexible prices. In particular, we follow the approach of Sidrauski [1967] traced out in section 1.4.1 in assuming that households derive utility from the consumption of goods and from holding real balances.

The survey of the inflation-growth literature in section 1.4.1 has shown that effects of employment are often important in the analysis of the inflation-growth nexus. In **chapter 2**, we abstract from this channel entirely and show that increases in money growth and inflation are detrimental to growth in the presence of price rigidity even at constant labour supply and employment. A brief introduction is given in section 2.1. The model is then presented in section 2.2, whereas section 2.3 shows existence and uniqueness of the steady state rational expectations equilibrium. The analysis of the steady

[86] Without referring to a specific study, the authors also report that "the share of firms following time-dependent rules is 40%" for the US (Álvarez et al. [2006], p. 581).

state's comparative static properties in section 2.4 shows that money growth at the steady state equilibrium has three effects on innovation driven growth under price rigidity: First, the initial relative price of an intermediate good under price rigidity increases in the money growth rate. This is due to money growth's effect on the real wage and on the optimal mark-up chosen by the good's producer: As seen in section 1.3.1.1, since firms anticipate their inability to offset the erosion of their relative price through inflation under price rigidity, they optimally set an initial mark-up that increases in the growth rate of marginal cost. Since the latter is in equilibrium identical to the money growth rate, the optimal mark-up and the initial relative price increase in the money growth rate. Demand for an intermediate good and hence, the firm's profits, decrease in the firm's relative price. Since profits from sales of an intermediate good determine the market value of a patent, inflation ceteris paribus reduces the patent price and hence, the incentive to innovate and economic growth. In contrast, erosion of the firm's relative price through inflation raises demand and profits while the nominal price is fixed. Via this second effect, an increase in money growth ceteris paribus raises growth. Third, as seen in section 1.3.2, inflation also has an effect on the efficiency with which resources whose relative prices are distorted are used. In particular, demand is distorted towards intermediate goods whose nominal prices have been fixed for a long time, implying their relative prices are low. Given the constant elasticity of substitution between intermediate goods in final good production, this reduces their average productivity and hence, profits. Via this channel, any deviation of money growth from zero reduces growth because it raises the dispersion of intermediate goods prices. Taking together the three influences, the economic growth rate decreases monotonically in the money growth rate. The model thus shares the policy recommendation given by the growth models reviewed in section 1.4.1.2 that from a growth perspective, the monetary policy authority should follow the Friedman rule, i.e. contract money supply at a rate that at the limit makes the nominal interest rate zero.[87] The comparative static results concerning

[87]This is the lower bound on admissible money growth rates in those models and ours: Equation (1.2) shows that equilibrium is only well-defined for positive interest rates.

the effects of money growth and of price rigidity are illustrated with a calibrated example where the quantitative effect of money growth is compatible with empirical estimates reported in section 1.4.2 of the present chapter.

To check whether the properties of the steady state that our analysis focuses on are indeed relevant as a description of the economy's long-run behaviour, **chapter 3** investigates whether the economy converges to steady state in the long run. To this end, the linearised economy's behaviour in the neighbourhood of the steady state is examined. As explained in the introductory section to the chapter (section 3.1), analysing the steady state's local stability turns out to be a challenging task due to the dynamic system's high dimensionality. We therefore restrict the analysis to the steady state's local stability properties in a large number of calibrated examples. The key equation determining the economy's behaviour off steady state are described in section 3.2. The following section 3.3 presents the reduced dynamical system and the results of the local stability analysis: Without any exception, the steady state is found to be locally stable, which justifies the concentration of our long-run analysis on steady state outcomes. Interestingly, the steady state equilibrium under price rigidity at the same time turns out to be indeterminate, i.e. the convergent solution is not unique.

In the previous steps of the analysis, it was assumed that parameters are such that remaining in the market is unprofitable for the incumbent intermediate good firm if a firm entering the market with an improved version of the same type sets the monopoly price. Relaxing this assumption, chapter 4 analyses the equilibrium under limit pricing: Innovative firms set a limit price to force the incumbent out of the market. After a brief introduction in section 4.1, section 4.2 presents the model. It is shown that there is a unique steady state equilibrium where money growth has an additional negative effect on intermediate firms' profits and the incentive to innovate: Innovators cannot choose a forward-looking initial price to optimally offset the erosion of their intermediate goods' relative price through inflation because they set the limit price that makes staying in the market immediately unprofitable for the incumbent firm. This further raises the negative impact of an increase in money growth on economic growth and turns out to be of importance quantitatively in a calibrated example. Thus, money growth is more detrimental to output growth when innovations are non-drastic.

In **chapter 5**, we relax the assumption that employment is independent of money growth and inflation. Instead of introducing a consumption-leisure choice in the household's utility maximisation problem, we directly introduce an aggregate labour supply function which increases in the level of the real wage in efficiency units. Introducing labour supply in this simple way helps to keep the model tractable and allows us to speak about involuntary unemployment. A short introduction is given in section 5.1. In section 5.2, the model is presented with a slight modification in the sectoral structure: Labour is used together with intermediate goods in the production of the economy's final good, whereas the input used in the intermediate goods sector and in the R&D sector is output. Steady state equilibrium is discussed in section 5.3 and its existence and uniqueness are shown in section 5.4. Section 5.5 presents comparative static results on the interaction of money growth, employment and output growth in steady state equilibrium that are illustrated with the help of a calibrated example. While it turns out that the growth rate has no significant effect on employment, an increase in employment does in turn raise the rate of economic growth.

Employment is influenced by money growth and inflation via two channels because the productivity of labour increases in the total amount of intermediate goods used in production and in the efficiency of their use. The total amount of intermediates used is influenced by the money growth rate since the latter affects the average mark-up charged by intermediate good producers, i.e. the degree of monopolistic distortion in the economy. Efficiency of the usage of intermediate goods decreases in the degree of relative price distortion which in turn increases in the absolute value of the inflation rate. We show that the interaction of the two effects gives rise to a non-linear long-run Phillips curve: Given the research intensity, we prove that the effects make employment a hump-shaped function of money growth peaking at a money growth rate associated with a positive inflation rate. Given that employment is approximately invariant to the growth rate, the result carries over to general equilibrium. The same applies to the hump-shaped form of the output level as a function of the money growth rate.

Given employment, output growth is a hump-shaped function of the money growth rate peaking at zero inflation.[88] In general equilibrium, employment's positive effect on the research intensity and the fact that employment increases in money growth at small inflation rates bring about an increase in the growth maximising money growth rate: Output growth is maximised at a positive inflation rate.

A monetary authority interested in stimulating employment or economic growth should therefore fight high inflation which reduces both variables, whereas tolerating a very moderate amount of inflation promotes both its goals.

Finally, **chapter 6** presents some concluding remarks.

[88] The difference concerning the growth effects of deflation between the results of chapter 2 and 5 is due to the different influence of the degree of monopolistic distortions. Section 5.5.5 elaborates on this.

Chapter 2

Money growth and Innovation-driven growth

2.1 Introduction[1]

In this chapter, the Schumpeterian growth model with nominal price rigidity is presented and the unique steady state equilibrium's comparative static properties are discussed. In particular, the issue of money's (non-) superneutrality with respect to the output growth rate and the latter's dependence on the degree of price rigidity are examined.

The real side of the model is the quality ladder model following Aghion and Howitt [1992] and Grossman and Helpman [1991] which was briefly introduced in the last section of chapter 1. In this framework, sustained output growth is possible due to the growing quality (i.e., productivity) of a large number of differentiated intermediate goods that are imperfect substitutes in the production of the economy's final good. Quality improvements are the result of the efforts of researchers in many small firms in the research and development sector where research success occurs in a random manner governed by a Poisson process. In case of a success, the research firm sells the patent for its invention to a firm that then holds the exclusive right to produce the innovative good. The incumbent firm producing the old inferior

[1]Parts of this chapter are based on Funk and Kromen [2005].

version of the intermediate good type is forced out of the market by the innovator's entry. Intermediate goods are produced one for one with labour. Due to the fact that intermediate goods are imperfect substitutes, producers act in a monopolistically competitive environment and optimally set their price as a markup over marginal cost, i.e. the wage. Under perfectly flexible prices, this profit maximising mark-up is constant across time and depends only on the degree of monopoly power.

The survey of the literature on endogenous growth and inflation in section 1.4.1.2 of chapter 1 showed that variations in employment may be an important channel for the transmission of inflation's effects on growth. In this chapter, we abstract from this channel to focus on the direct effect of price rigidity and relative price changes on the return to R&D.[2]

The nominal side of the model corresponds to that of a standard New Keynesian model as presented in section 1.3 of chapter 1, with a simplified specification of the money supply rule. Money is introduced into the model by assuming that households derive utility from holding real money balances, following Sidrauski [1967]. Money matters in the model because we assume the existence of nominal price rigidity in the intermediate goods market: When entering the intermediate goods market with a new good, the innovator chooses a price. Once in the market, firms can only change their prices infrequently, where price rigidity follows the standard structure of Calvo [1983] and Kimball [1995][3]. Following the literature on inflation and endogenous growth presented in section 1.4.1.2 of chapter 1, it is assumed that the money supply grows at an exogenously fixed constant rate so that there are no shocks at the aggregate level. We analyse Rational Expectations Equilibria in this model, where we restrict our attention to steady state equilibria with constant output growth.

To get an intuition for the non-superneutrality of money in this frame-

[2] Labour supply will be endogenised and the interaction of money growth, employment and growth will be examined in chapter 5 of this thesis.

[3] In Kimball's variation of Calvo's model, firms are assumed to set prices so as to maximise the present value of profits, instead of following a rule of thumb as modelled by Calvo.

work, note first that the growth rate of intermediate good firms' nominal cost, i.e., the wage inflation rate, depends positively on (in fact, is equal to) the growth rate of money supply in this model. Therefore, an intermediate goods producer would like to constantly change his price – but can only do so infrequently under price rigidity – at any non-zero money growth rate. The changes in the firm's effective mark-up and relative price brought about by non-zero money growth influence the profits which can be made selling an innovative intermediate good. These level effects of money growth lead to an influence on the output growth rate of the economy given that the present value of profits from sales of an intermediate good determines the price of a patent for an innovative intermediate good. Since the patent price determines the incentive to innovate, investment in R&D and economic growth are influenced by the money growth rate.

In particular, we identify three distinct effects of non-zero money growth and inflation[4] on the steady state research intensity.

First, non-zero money growth under price rigidity implies dispersion in the effective prices of intermediate goods. The optimal price grows at the constant money growth rate while each firm's effective price is the price that was optimal at the stochastic point in time the firm was last allowed to readjust its price. Both absolute and relative prices of intermediate goods differ, distorting demand towards goods with low relative prices. Given the constant elasticity of substitution between all intermediate good types in final good production, the distortion of demand reduces the average productivity of intermediate goods. This reduces aggregate demand for intermediate goods. Given positive marginal profits, the reduction in demand for the innovative intermediate good entails lower profits for the innovator. Via this *price dispersion effect*, an increase in the absolute value of the money growth rate that raises price dispersion lowers the return to R&D.

Second, money growth influences the incentive to innovate via the *relative price erosion effect*: Given that the intermediate firm's price is tem-

[4]We discuss the effects of an increase in the exogenous money growth rate which in equilibrium entails an increase in the endogenous inflation rate.

porarily fixed under price rigidity whereas its marginal cost grows at the money growth rate, the firm's mark-up is eroded by inflation. Mark-up erosion lowers the firm's relative price, thereby raising demand for its good and therefore raising profits. The extent of mark-up erosion increases in the money growth rate ψ, so that an increase in ψ ceteris paribus raises profits, the value of a patent and hence, the research intensity.

Third, the incentive to innovate is influenced by money growth via the *initial relative price effect*: The intermediate goods producer's initial relative price depends on the real wage level and on the firm's choice of an initial mark-up given the real wage. Firms anticipate that their nominal price will be temporarily fixed and that consequently, their mark-ups and relative prices will be eroded by inflation. To offset this, every firm optimally chooses a forward-looking mark-up that increases in the money growth rate whenever it gets the chance to readjust its price. At the same time, the real wage is also influenced by money growth. Taking account of both effects, under inflation and price rigidity the high initial mark-up translates into a high relative price that reduces demand for the intermediate producer's good and hence, his profits. The resulting decrease in the return to R&D caused by an increase in the money growth rate ceteris paribus lowers the incentive to innovate and growth.

It is shown that the negative effects of an increase in money growth on the incentive to innovate always dominate so that the output growth rate decreases in the money growth rate ψ at all admissible values of ψ. A realistically calibrated example illustrates the results and shows that adding an empirically plausible degree of nominal price rigidity to our Schumpeterian growth model is sufficient for the generation of a significant negative effect of inflation on growth which is quantitatively in line with estimates from the empirical literature. So indeed the effects of money growth and price rigidity are not restricted to the short run. Instead, the negative level effects on production efficiency and relative prices that inflation has under price rigidity translate into long-run growth effects via their influence on the incentive to innovate.

The remainder of the chapter is organised as follows: Section 2.2 describes the model. Section 2.3 presents the steady state equilibrium. The comparative static properties of the steady state equilibrium are discussed in section 2.4. Section 2.5 presents a calibrated example. Section 2.6 concludes.

2.2 The model

Intermediate goods are the only input in the production of the final good, while labour is used in the production of intermediate goods and in research. We first derive the final good sector's optimal demand for intermediate goods and the intermediate goods sector's optimal prices and resulting labour demand in a monopolistically competitive environment under price rigidity. This labour demand function depends on intermediate firms' current prices, which in the presence of price rigidity depend on the past distribution of price resetting signals. Analogously, the following calculation of the market value of a new intermediate goods producer at the time of his market entry takes account of the influence of the stochastic timing of future price resetting opportunities on his relative price and profits. The resulting market value is the price for a patent from the research sector. Free entry implies expected profit from research must be zero. Using the information about the patent price, the zero profit condition gives us the optimal research intensity μ as a function of the size of the final good sector. Using optimal labour demands, we then determine the size of the final good sector compatible with labour market equilibrium. With this information, production side equilibrium gives us the optimal research intensity as a function of L and other variables firms take as given. Analysis of the public sector that controls the money supply, of the asset market and of the optimal behaviour of households, which hold utility-yielding real balances, yields the remaining conditions necessary to discuss the general equilibrium.

2.2.1 Final good sector

A perfectly competitive final good sector assembles the economy's final good $Y(\tau)$ from a large number N of differentiated intermediate goods according to a constant-elasticity-of-substitution aggregator following Dixit and Stiglitz [1977]

$$Y(\tau) = \left[\sum_{j=1}^{N} \left(q^{k_j(\tau)} x_{k_j}(\tau)\right)^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} \tag{2.1}$$

where we assume that the elasticity of substitution between intermediate goods α is larger than unity and where x_{k_j} is the amount of sector j type k_j intermediate good used, and q^{k_j} is this type's productivity. We assume that only the highest quality k_j currently available in sector j is used in production.[5] Profits in the sector are given by

$$\Pi^Y(\tau) = P(\tau)Y(\tau) - \sum_{j=1}^{N} p_{k_j}(\tau) x_{k_j}(\tau) \tag{2.2}$$

where $P(\tau)$ is the final good price and $P_{k_j}(\tau)$ is the price charged for one unit of type k_j sector j intermediate good. Cost minimization leads to firms' optimal demand for intermediate good x_{k_j}, which depends negatively on the type's relative price and positively on its productivity $q^{k_j(\alpha-1)}$ and on aggregate demand $Y(\tau)$.

$$x_{k_j}(\tau) = \left(\frac{p_{k_j}(\tau)}{P(\tau)}\right)^{-\alpha} q^{k_j(\alpha-1)} Y(\tau) \tag{2.3}$$

Constant returns to scale and perfect competition prevent firms from making positive profits. Optimal demand for intermediate goods and the zero profit condition determine the final good price as a quality-weighted

[5] This can be achieved by assuming that entry of the innovator terminates the incumbent's production. Alternatively and less drastically, the incumbent will voluntarily exit the market if production is not profitable for him once the innovator enters the market. See footnote 8 for a discussion of the conditions which ensure that this holds when the innovator sets the monopoly price.

average of intermediate goods' prices:

$$P(\tau) = \left[\sum_{j=1}^{N} \left(\frac{p_{k_j}(\tau)}{q^{k_j(\tau)}} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \tag{2.4}$$

For later use, we define the economy's aggregate technology index, $Q(\tau)$, as the weighted sum of the productivities $q^{k_j(\tau)}$ associated with each sector's intermediate good

$$Q(\tau) = \left[\sum_{j=1}^{N} q^{(\alpha-1)k_j(\tau)} \right]^{\frac{1}{\alpha-1}} \tag{2.5}$$

2.2.2 Intermediate goods sector

Each of N intermediate goods is produced by one firm that bought the blueprint from the corresponding research firm. Good k_j is produced using a linear technology with labour as the only input:

$$x_{k_j}(\tau) = L_{k_j}(\tau) \tag{2.6}$$

2.2.2.1 An intermediate good producer's pricing problem

The instantaneous profit of a firm producing type k_j consists of the difference between its price P_{k_j} and marginal costs, given by the wage rate $w(\tau)$ determined in the perfectly competitive labour market, times the number of units sold.

$$\Pi_{k_j}(\tau) = \left[p_{k_j}(\tau) - w(\tau) \right] x_{k_j}(\tau)$$

As intermediate goods are imperfect substitutes in the final good production function, intermediate goods producers act in an environment of monopolistic competition and can choose an optimal price subject to the final good sector's demand function.

We assume the existence of nominal rigidities in the intermediate goods markets: Producers can only adjust their prices infrequently. Following the standard assumption in the literature due to Calvo [1983] and Kimball [1995], firms in the market at each point in time with an exogenous flow probability

β receive a signal allowing them to readjust their prices, where the signal is generated by a stochastic Poisson process with parameter β. In addition, in our model a firm naturally chooses a price for its new product when entering the intermediate goods market with an innovative good. As the arrival of inventions in the research sector is governed by a Poisson process with endogenously determined parameter μ, this does not change the basic structure of price resetting signals received by firms as modelled in the literature while partly endogenising the parameter in a natural way.

Knowing they will not be able to readjust their prices for some time, firms thus solve an intertemporal problem to find the optimal price: At the time of its market entry $\tau = t_{k_j}$ as well as at each later date τ where it receives a price resetting signal, firm j sets it price such as to maximise the expected present value $E\left[V\left(p_j,\tau\right)\right]$ of future profits. Only those future profits are relevant for the maximisation that are generated before the firm's next opportunity to readjust its price arises *and* before the next innovative good enters the market in sector j, which we assume drives the incumbent out of the market.

Therefore, profits of all future periods s are weighted with the probability that as of time s, the firm has not received a signal allowing to readjust prices and has not been replaced by a successor. This leads to a discount factor given by the sum of the nominal interest rate, i, the flow probability of being replaced by another firm, $\mu_{k_j}(\tau)$ and the flow probability of receiving a reset signal for the price, β.[6] In steady state equilibrium, the replacement probability will be constant and the same for all firms, so we set $\mu_{k_j}(\tau) = \mu(\tau) = \mu$. The resulting value is

$$E\left[V\left(p_j,\tau\right)\right] = \int_{\tau}^{\infty} \widetilde{B} e^{-(i+\mu+\beta)(s-\tau)} \left[p_j - w(s)\right] \left(\frac{p_j}{P(s)}\right)^{-\alpha} q^{k_j(\alpha-1)} Y(s) ds \tag{2.7}$$

$\widetilde{B}$ is a constant from the integration of the probability distribution of the price reset signal which will drop out during maximisation.[7] Maximising the

[6] While the parameter β is constant by definition, the interest rate i is constant at steady state.

[7] The discount factor at time τ for profits from future period s contains the product of the cumulative probability densities $B(\tau,s) = \widetilde{B}e^{-\beta(s-\tau)}$ and $D(\tau,s) = \widetilde{D}e^{-\mu(s-\tau)}$ for

value given in (2.7) with respect to the price p_j subject to the final good producing firm's demand function (2.3) and using that at steady state, the final good $Y(\tau)$, price level $P(\tau)$ and wage $w(\tau)$ grow at rates γ, π and ω, respectively, leads to the following expression for the optimal price at time τ:[8]

$$p_j^*(\tau) = \frac{\alpha}{\alpha-1}\frac{i+\mu+\beta-\alpha\pi-\gamma}{i+\mu+\beta-\alpha\pi-\gamma-\omega}w(\tau) \tag{2.8}$$

Note that $p_j^*(\tau)$ is the optimal price for firm j *whenever* it can set a new price, whether this is at market entry or at a later date when it receives a price resetting signal.[9] Note also that the optimal price is independent of a firm's position on the quality ladder, so we will rename it $p^*(\tau)$. The optimal price is a mark-up over marginal cost $w(\tau)$. In a world without rigidities ($\beta \to \infty$), a monopolistically competitive firm would optimally charge a price $p^*_{flex}(\tau) = \frac{\alpha}{\alpha-1}w(\tau)$, where the mark-up $\frac{\alpha}{\alpha-1}$ reflects the degree of monopoly

the probabilities that between τ and s, no price resetting is received and the firm is not replaced by the next innovation, respectively. The firm that sets a price at market entry at time τ knows that $D(\tau,\tau) = 1$ so $\widetilde{D} = 1$ given that the probability of two innovations occurring in an infinitesimal time interval is negligible. However, since the timing of the last price resetting signal is unknown, so is $\widetilde{B}$. The situation is reversed when the firm later resets its price due to a price resetting signal.

[8] Knowledge of the optimal monopolistic mark-up allows us to discuss the conditions under which monopoly pricing will prevail if the incumbent is not forced to terminate production upon the entry of an innovator: Taking the innovation as a price resetting signal for the incumbent firm, too, only the latest quality is available in each sector if the innovator's mark-up is such that production is not profitable for the incumbent. Given that his good is one quality rung below the innovator's, the incumbent can at most charge $\frac{1}{q}p_j^*(\tau)$. Given the steady state optimal monopolistic mark-up (2.8), $\frac{1}{q}p_j^*(\tau) - w(\tau) \leq 0$ holds when the sufficient conditions $1 \geq \frac{1}{q}\frac{\alpha}{\alpha-1}\frac{\rho+\beta+(1-\alpha)\psi}{\rho+\beta-\alpha\psi}$ and $1 \geq \frac{1}{q}\frac{\alpha}{\alpha-1}$ are jointly satisfied. While the second condition, which is identical to the condition from the underlying real model, ensures that the incumbent makes non-positive profits under negative money growth rates, the first condition must hold for $\psi = \omega > 0$ where firms set an additional forward-looking mark-up $\frac{i+\mu+\beta-\alpha\pi-\gamma}{i+\mu+\beta-\alpha\pi-\gamma-\omega} = \frac{\rho+\beta+(1-\alpha)\psi+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu}{\rho+\beta-\alpha\psi+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu} > 1$. Since this term decreases in μ for $\psi > 0$, $1 \geq \frac{1}{q}\frac{\alpha}{\alpha-1}\frac{\rho+\beta+(1-\alpha)\psi}{\rho+\beta-\alpha\psi}$ is a sufficient condition. The conditions are satisfied at all examined money growth rates in our calibrated examples in section 2.5. If production continued to be profitable for the incumbent under monopoly pricing, the innovator would set the limit price to drive the incumbent out of the market. The equilibrium under limit pricing is examined in chapter 4.

[9] The maximisation problem has a well-defined solution for $i+\mu+\beta-\alpha\pi-\gamma > 0$, $i+\mu+\beta-\alpha\pi-\gamma-\omega > 0$. Assumption (2.38) and $\eta \geq 1$ from the household's problem guarantee that these inequalities hold in equilibrium.

power and decreases in the elasticity of substitution between intermediate goods, α.

Under price rigidity, positive growth of marginal costs ($\omega > 0$) will erode the firm's relative price in future periods while its price is fixed. Anticipating this, the firm chooses an initial mark-up that is higher than the flexible optimum $p^*_{flex}(\tau)$ under these circumstances.[10] Given that at steady state the wage inflation rate ω equals the growth rate of money supply, ψ,[11] we therefore have that the initial mark-up increases with money growth, allowing it to influence real activity.

The size of the additional mark-up further depends on the importance of future relative to current profits in the firm's optimisation at time τ. Increases in factors that reduce the weight of future profits reduce the mark-up, drawing it closer to the static optimum $\alpha/(\alpha-1)$.[12] In contrast, an increase in variables that accelerates future profit growth raises the initial mark-up, reducing the deviation from optimal price in future periods.[13]

Equilibrium in the market for intermediate goods Supply in the market for intermediate goods equals demand as production of each good j is determined by the final good sector's demand function for good j given in equation (2.3).

2.2.2.2 An intermediate good producer's market value at market entry

The value of a patent developed in the R&D-sector will be equal to the market value $E\left(V_{k_j}(\tau)\,\middle|\, t_{k_j}=\tau\right)$ at the time of market entry t_{k_j} of the firm

[10] In New Keynesian business cycle models without real growth, the optimal price at steady state would be $p^*_j(\tau) = \frac{\alpha}{\alpha-1}\frac{i+\beta-\alpha\psi}{i+\beta-(\alpha+1)\psi}w(\tau)$.

[11] This will be shown in section 2.3.1.1.

[12] These variables are the nominal interest rate i, the rate of obsolescence μ and the probability of receiving a price reset signal β. Remember that only profits obtained while this price has not been replaced by a newer one count in the choice of today's optimal price. This is why the price resetting probability β reduces the weight of future profits.

[13] Future demand and profits increase in the future growth rate of aggregate demand, γ, and inflation π. Remember from equation (2.3) that demand for the intermediate good is ceteris paribus a function $[p_j(s)/P(s)]^{-\alpha}$ of its relative price, which under price rigidity erodes at rate $\alpha\pi$.

using the patent. This market value is the expected present value at time τ of all future profits of the type k_j, given that $t_{k_j} = \tau$. Taking into account stationary growth of Y, P and w, the probability of obsolescence before time s, $e^{-\mu(s-\tau)}$, and the development of the firm's price, this value is:

$$E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right) = A(\tau)\, E \int_{\tau}^{\infty} \left[e^{-\chi(s-\tau)} p_j(s)^{1-\alpha} - w(\tau)\, e^{-(\chi-\omega)(s-\tau)} p_j(s)^{-\alpha}\right] ds \tag{2.9}$$

where $A(\tau) = q^{k_j(\alpha-1)} Y(\tau)\, P(\tau)^{\alpha}$ and $\chi = i + \mu - \alpha\pi - \gamma$. Note that unlike in the optimal pricing problem leading to equation 2.8, here all future profits enter the maximisation. Thus the calculation involves expectations not only about the life-span of the firm, but also about its price at any future date. This makes the analysis considerably more complicated than in the model without money. In particular, in any future period $s > \tau$ where it is still active, the firm's price is still $p^*(\tau)$ if no price reset signal has been received between periods τ and s, which has probability $\left(1 - \int_{\tau}^{s} \beta e^{-\beta(s-\theta)} d\theta\right)$. Otherwise, $p_j(s)$ is equal to $p^*(\theta)$ where θ is the last period where a reset signal was received. θ can take any value between τ and s, weighted with the corresponding probability $\beta e^{-\beta(s-\theta)}$. Using this and evaluating the integrals, the firm's market value is[14]

$$E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right) = \frac{q^{k_j(\alpha-1)} Y(\tau) \left(\frac{p^*(\tau)}{P(\tau)}\right)^{-\alpha} \frac{w(\tau)}{\alpha-1}}{\left[i + \mu - \alpha\pi - \gamma - (1-\alpha)\,\omega\right] \frac{i+\mu+\beta-\alpha\pi-\gamma-\omega}{i+\mu+\beta-\alpha\pi-\gamma-(1-\alpha)\omega}} \tag{2.10}$$

Like the expected market value of a firm with totally flexible prices, $q^{k_j(\alpha-1)} Y(\tau) \left(\frac{p^*(\tau)}{P(\tau)}\right)^{-\alpha} \frac{w(\tau)}{\alpha-1} / [i + \mu - \alpha\pi - \gamma - (1-\alpha)\omega]$, the firm's value under rigidity can be interpreted as the present value of an infinite stream of profits growing at a constant rate.[15] The flex-price firm's instantaneous profit is $q^{k_j(\alpha-1)} Y(\tau) \left(\frac{p^*(\tau)}{P(\tau)}\right)^{-\alpha} \frac{w(\tau)}{\alpha-1}$, while its discount rate is the difference

[14] Derivation of the market value is described in more detail in Appendix 2.A.1.

[15] Remember that the present value at time τ of a an infinite stream of steadily growing profits starting at τ is $\Pi(\tau)/(R-x)$, where $\Pi(\tau)$ is instantaneous profit at τ, x is the profit growth rate and R is the interest rate.

of the obsolescence-adjusted interest rate $i + \mu$ and its profit growth rate $(\alpha\pi + \gamma + (1 - \alpha)\,\omega)$. The flex-price firm's profit growth rate is determined by the growth rates of aggregate demand γ, of its price and wages ω, and of demand for the good at rate $\alpha\,(\pi - \omega)$ caused by the change of its relative price at rate $(\omega - \pi)$.[16]

The market value of a firm under price rigidity differs from this value in two respects: First, as explained in the discussion of equation 2.8, in anticipation of the effects of price rigidity, the firm's initial price $p^*(\tau)^{-\alpha}$ contains an additional mark-up that increases in ω (ψ), reducing demand and instantaneous profits. Second, due to the infrequent price adjustment, the growth rate of profits and hence the discount rate is different from the flex-price case. The flex-price discount rate is therefore corrected with the factor $(i + \mu + \beta - \alpha\pi - \gamma - \omega) \,/\, (i + \mu + \beta - \alpha\pi - \gamma - (1 - \alpha)\,\omega)$ which accounts for the difference in profit growth rates between periods where prices are fixed versus flexible.[17] This influence of rigidity on profit growth is one of the mechanisms that allow money growth and inflation to influence the incentive to innovate in general equilibrium.[18]

Note that in contrast, the effect of inflation and relative price erosion on the firm's profit *per unit* is completely offset by the firm's optimal choice of its initial mark-up: From Appendix 2.A.1, the average size of profits per unit sold is $\frac{p^*(\tau)}{i+\mu+\beta-\alpha\pi-\gamma} - \frac{w(\tau)}{i+\mu+\beta-\alpha\pi-\gamma-\psi}$, which is of course also influenced by inflation and money growth. The denominators reflect the different growth rates of revenues $(\alpha\pi + \gamma)$ and costs $(\alpha\pi + \gamma + \psi)$. Yet this difference in growth rates is taken account of in the endogenous choice of optimal price by the firm: The initial mark-up $\frac{\alpha}{\alpha-1}\frac{i+\mu+\beta-\alpha\pi-\gamma}{i+\mu+\beta-\alpha\pi-\gamma-\psi}$ over wages is chosen such that the present value of revenues is identical to what it would have been if revenues had grown at the same constant rate as costs.

[16] See equation (2.3) for the determinants of demand for the firm's good.

[17] Note that both discount rates featured in the correction factor increase in the frequency of the price adjustment signal β, such that as $\beta \to \infty$, the discount rate reduces to the flex-price rate.

[18] A detailed intuition is given in section 2.4 where we discuss the equilibrium's comparative static properties.

2.2.2.3 The Intermediate Goods sector's labour demand

We next derive the sector's labour demand which will later be used to determine the size of the final good sector compatible with labour market equilibrium. Due to the linear production function (2.6), firm j's labour demand is equal to the final goods sector's demand for the firm's good. Inserting the optimal price into the demand function (2.3) and aggregating leads to intermediate good producers' labour demand

$$L^X(\tau) = \frac{Y(\tau)}{Q(\tau)} \sum_{j=1}^{N} \left(\frac{p_j(\tau)}{P(\tau)Q(\tau)} \right)^{-\alpha} \left(\frac{q^{k_j}}{Q(\tau)} \right)^{\alpha-1} \tag{2.11}$$

Obviously, aggregate demand for intermediate goods depends negatively on the average relative price of intermediate goods (in efficiency units). This average price effective at time τ can be expressed as a weighted average of past optimal prices set by firms at the last (stochastic) point in time s where they could readjust their prices. The weights $f(s,\tau)$ thus refer to the probability that a price valid at time τ has not been changed since time s:

$$\sum_{j=1}^{N} p_j(\tau)^{-\alpha} = \int_{-\infty}^{\tau} f(s,\tau)\,[p^*(s)]^{-\alpha}\,ds$$

More specifically, the weights reflect the probability that a price reset signal was received *or* an innovation made at time s, $(\mu+\beta)$ *and* that no such event took place between times s and τ, $e^{(\mu+\beta)(s-\tau)}$. Thus, we have $f(s,\tau) = (\mu+\beta)\,e^{(\mu+\beta)(s-\tau)}$. Using this and evaluating the integral, we have[19]

$$L^X(\tau) = \frac{Y(\tau)}{Q(\tau)} \left[\frac{p^*(\tau)}{P(\tau)Q(\tau)} \left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega} \right)^{-\frac{1}{\alpha}} \right]^{-\alpha}$$

[19] A more detailed derivation of equation (2.12) can be found in Appendix 2.A.2. Note that convergence of the integral is ensured by condition (2.38), see footnote 40.

Inserting the general equilibrium value of the current the optimal price from equation (2.15) gives

$$L^X(\tau) = \frac{Y(\tau)}{Q(\tau)} \left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\,\omega} \right)^{\frac{1}{\alpha-1}} \left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega} \right)^{-\frac{1}{\alpha}} \right]^{-\alpha} \tag{2.12}$$

Given the constant elasticity of substitution between intermediate goods in final good production, the quality-weighted amounts of all intermediate goods used should be equal. Only in this case is production efficient and the amount of intermediate goods used corresponds exactly to the amount of output in efficiency units to be produced, $L^X(\tau) = Y(\tau)/Q(\tau)$. The term $\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega} \right)^{\frac{1}{\alpha-1}} \left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega} \right)^{-\frac{1}{\alpha}} \right]^{-\alpha} \geq 1$ is therefore a measure of static production inefficiency caused by *price dispersion*: The fact that intermediate goods have different effective relative prices under price rigidity causes them to be demanded and used in different amounts. Since this causes inefficient production, the higher price dispersion, the more intermediate goods $L^X(\tau)$ are needed to produce a given level of output in efficiency units $Y(\tau)/Q(\tau)$. This static production inefficiency is completely unrelated to the inefficiency caused by monopolistic producers' positive mark-ups since it is the dispersion in mark-ups and relative prices, not the absolute level of the mark-up, that causes the inefficiency.[20]

Going into more detail, note that price dispersion is caused by the concurrence of two elements: Price rigidity and non-zero growth of marginal cost. In the absence of price rigidity, all firms charge the same price, so there is no price dispersion. Under rigidity, each intermediate producer effectively

[20] Regarding the components of the price dispersion measure, remember that as seen above the term $\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega} \right)^{\frac{1}{\alpha-1}} \left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega} \right)^{-\frac{1}{\alpha}} \right]^{-\alpha}$ is the average relative price charged by intermediate goods producers: While $\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega} \right)^{\frac{1}{\alpha-1}}$ is the current optimal relative price, $\left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega} \right)^{-\frac{1}{\alpha}}$ is a measure of the deviation of the average relative price from the optimal value. If prices could be reset each period, all firms would charge the same optimal price and both terms would equal unity. The average relative price would then equal the common optimal relative price and therefore be unity. Under price rigidity, firms' actual prices differ and hence the average relative price in equation (2.12) differs from one.

charges the price that was optimal at the time when he could last readjust prices. Given that optimal prices are a mark-up over marginal cost, past optimal prices only differ from the current one when marginal cost has a non-zero growth rate. Since the growth rate of marginal cost ω equals the money growth rate ψ as will be shown in equation (2.34) in section 2.3.1.1, money growth influences price dispersion. In particular, price dispersion and production inefficiency increase in the distance of the money growth rate ψ from zero because the larger the absolute value of the money growth rate, the faster is the change in optimal prices and the bigger the difference in prices between goods with old prices and to goods with new prices.[21] It is also intuitive price that dispersion decreases in μ and β because the higher these variables, the more often prices are adjusted.[22]

2.2.3 Price level and real wage

Given that firms in the final good sector make zero profit in equilibrium, the output price (2.4) is a quantity-weighted average of the prices of the intermediate goods that are the only input in production. Analogously to the procedure in section 2.2.2.3, this average intermediate good price can be rewritten as a weighted average of past optimal prices, since every effective intermediate good price is a past optimal price dating from the time the firm could last readjust its price. The weights are again given by $f(s,\tau) = (\mu+\beta)\, e^{(\mu+\beta)(s-\tau)}$ from section 2.2.2.3. Using these weights and the growth rate ω of optimal prices the weighted average of past optimal prices can be

[21] $\frac{\partial D}{\partial \omega} = \frac{\alpha\omega D}{(\mu+\beta-\alpha\omega)[\mu+\beta-(\alpha-1)\omega]} \gtreqless 0$ for $\omega = \psi \gtreqless 0$ with $D = \left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}$.

[22] $\frac{\partial D}{\partial \mu} = \frac{\partial D}{\partial \beta} = \frac{-\alpha D\omega^2}{(\mu+\beta)[\mu+\beta-(\alpha-1)\omega](\mu+\beta-\alpha\omega)} < 0$ where $D = \left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}$. Remember that price rigidity decreases when the price adjustment parameter β increases or the extent of market entry with new prices increases due to a rise in the research intensity μ.

rewritten as[23]

$$P(\tau) = \left(\frac{p^*(\tau)}{Q(\tau)}\right)\left[\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\,\omega}\right]^{\frac{1}{1-\alpha}} \tag{2.13}$$

Equivalently,

$$1 = \left(\frac{\frac{\alpha}{\alpha-1}\frac{i+\mu+\beta-\alpha\pi-\gamma}{i+\mu+\beta-\alpha\pi-\gamma-\omega}w(\tau)}{P\,(\tau)\,Q(\tau)}\right)\left[\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\,\omega}\right]^{\frac{1}{1-\alpha}} \tag{2.14}$$

The last equation shows that the average real intermediate good price must equal one. It determines the real wage in efficiency units as one over the average intermediate goods mark-up $\frac{\alpha}{\alpha-1}\frac{i+\mu+\beta-\alpha\pi-\gamma}{i+\mu+\beta-\alpha\pi-\gamma-\omega}\left[\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right]^{\frac{1}{1-\alpha}}$. The average mark-up is the higher and the real wage is the lower, first, the lower the initial mark-up chosen by intermediate good firms, $\frac{\alpha}{\alpha-1}\frac{i+\mu+\beta-\alpha\pi-\gamma}{i+\mu+\beta-\alpha\pi-\gamma-\omega}$, and second, the higher the term $\left[\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right]^{\frac{1}{1-\alpha}}$. This term captures the fact that the average mark-up is smaller (higher) than the current optimal mark-up under price rigidity and positive (negative) growth of the optimal price: At $\omega > 0$ ($\omega < 0$), past optimal mark-ups are lower (higher) than the current value and since prices are temporarily fixed due to price rigidity, a lot of weight is put on past values. Since $\omega = \psi$ at steady state equilibrium, an increase in money growth influences the average mark-up and the wage in countervailing ways via its effects on the initial mark-up and the deviation term. The wage increases in the money growth rate at $\psi \leq 0$ and thus takes its maximum value at some $\psi > 0$.[24]

Finally, note that rewriting equation 2.13 as

$$\frac{p^*(\tau)}{P(\tau)Q(\tau)} = \frac{\alpha}{\alpha-1}\frac{i+\mu+\beta-\alpha\pi-\gamma}{i+\mu+\beta-\alpha\pi-\gamma-\omega}\frac{w(\tau)}{P\,(\tau)\,Q\,(\tau)} = \left[\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\,\omega}\right]^{1/(\alpha-1)} \tag{2.15}$$

shows that regardless of the offsetting effects, the firm's initial relative price under price rigidity always increases in $\omega = \psi$.[25]

[23] Details can be found in Appendix 2.A.3.

24 $\frac{\partial \frac{w}{PQ}}{\partial \psi} = \frac{\frac{w}{PQ}}{\mu+\beta-(\alpha-1)\psi}\left[-\frac{\rho+\beta+\widetilde{\eta}\mu}{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi}\frac{\mu+\beta-(\alpha-1)\psi}{\mu+\beta-(\alpha-1)\psi+\rho+(\widetilde{\eta}-1)\mu}+1\right] > 0$ for $\psi \leq 0$.

[25] Note that price dispersion is necessary for this effect: If there were no price dispersion

2.2.4 Patents and the R&D sector

There is free entry to R&D. Research is undertaken by small profit maximising research firms trying to improve the quality of existing intermediate goods. The flow probability of an invention being made is governed by a Poisson process with parameter $\mu_{k_j}(\tau)$ for the firm trying to improve intermediate good k_j. For a given quality rung k_j (i.e., current position of sector j), the probability of success depends linearly on the amount of research labour $L_j^R(\tau)$:

$$\mu_{k_j}(\tau) = \phi(k_j(\tau))L_j^R(\tau) \tag{2.16}$$

For any given level of research, the probability of success decreases in the number of innovations that have already been made in that particular sector. This idea is captured by assuming $\phi'(k_j(\tau)) < 0$. In case of success for research firm j, the design of the new, improved good is sold to a new intermediate goods firm replacing the incumbent in sector j. A potential producer's maximum willingness to pay is the present value of all future profits at market entry $E\left[V_{k_j+1}(\tau)\,\middle|t_{k_j} = \tau\right]$, as given in equation (2.10). Due to free entry, the patent price will be equal to this expected value.

Thus, sector j research firm's expected profit at time τ is

$$E\left[\Pi_j^R(\tau)\right] = \mu_{k_j}(\tau)E\left[V_{k_j+1}(\tau)\,\middle|t_{k_j} = \tau\right] - w(\tau)L_j^R(\tau)$$

Because of free entry into the research sector, firms' expected profit is zero at every instant which using (2.16) implies that either no research is undertaken ($L_j^R(\tau) = 0$) or

$$\phi(k_j(\tau))E\left[V_{k_j+1}(\tau)\,\middle|t_{k_j} = \tau\right] - w(\tau) = 0 \tag{2.17}$$

holds. Thus expected profit from research per researcher equals the wage.

For any given research effort, the probability of making an innovation

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(i.e., the deviation term $\left[\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right]^{\frac{1}{1-\alpha}}$ were one), all firms would charge the same price, $p^*(\tau)/Q(\tau)$. The relative price of any firm is then $p^*(\tau)/\left[P(\tau)Q(\tau)\right] = 1$ *regardless* of the level of the mark-up charged uniformly by all firms. The relative price would thus be independent of money growth's influence on the chosen mark-up.

decreases in the sector's position on the quality ladder. We choose a specification for $\phi(k_j(\tau))$ that implies the existence of spillovers in research: The lower the sector's quality level in comparison to aggregate quality, i.e. the further away the sector is from the research frontier, the easier is making an innovation:

$$\phi(k_j(\tau)) = (\alpha - 1)\,\frac{1}{\lambda}\left(\frac{q^{k_j+1}}{Q(\tau)}\right)^{1-\alpha} \tag{2.18}$$

where $1/\lambda$ is the productivity of labour in research. This standard razor's edge specification is chosen to make sure that the optimal research intensity μ can be constant and independent of a sector's position.

2.2.4.1 The R&D sector's labour demand

Research firm k_j's labour demand is found by rearranging (2.16) and inserting $\phi(k_j(\tau))$ as defined in equation (2.18). Aggregating over all research firms, total demand for research labour thus is $L^R = \sum_{j=1}^{N} L^R_{k_j}(\tau) = \sum_{j=1}^{N} \frac{\mu}{\frac{1}{\lambda}\frac{\alpha-1}{q^{\alpha-1}}\left(\frac{q^{k_j(\tau)}}{Q(\tau)}\right)^{1-\alpha}}$ or

$$L^R = \mu\lambda\frac{q^{\alpha-1}}{\alpha-1} \tag{2.19}$$

2.2.4.2 Behaviour of the aggregate quality index $Q(\tau)$ and the growth rate

The innovations made in the R&D sector determine the evolution of the quality index defined in equation (2.5). In case of an innovation occurring in sector j, the sector's quality increases from $q^{k_j(\tau)}$ to $q^{k_j(\tau)+1}$, which increases the sector's contribution to the aggregate quality index from $q^{(\alpha-1)k_j(\tau)}$ to $q^{(\alpha-1)[k_j(\tau)+1]}$. The expected change in sector j's contribution can be found by weighting the quality improvement in sector j with the flow probability that an innovation will occur there at time τ. Assuming again that the flow probability of an innovation occurring, μ, is constant and equal across sectors, the expected growth rate of the quality index (2.5), γ_Q, can be calculated from the sum of these expected contributions. Further, the law of large numbers implies that the actual steady state growth rate of $Q(\tau)$, γ_Q, equals its expected value for $N \to \infty$.[26] Thus we have:

$$\gamma_Q = \mu \frac{q^{\alpha-1} - 1}{\alpha - 1} \tag{2.20}$$

It will be shown in section (2.2.5) that at steady state, the growth rate of output γ equals the growth rate of the aggregate quality index, so we have

$$\gamma = \frac{q^{\alpha-1} - 1}{\alpha - 1} \mu \tag{2.21}$$

2.2.5 Labour market equilibrium

The variables determining the model's production side equilibrium, most notably expected profit in the research sector, depend on the size of the final good sector. Using equilibrium in the labour market we now pin down this variable as a function of endogenous variables and employment L and then proceed to the production side equilibrium.

Equilibrium in the labour market requires that the sum of the labour demands of the intermediate goods sector, L^X, and of the research sector, L^R, equal labour supply L:

$$L = L^X + L^R \tag{2.22}$$

Inserting the labour demands from the intermediate goods sector, (2.12), and the research sector, (2.19), determines the size of the final good sector[27]

$$\frac{Y(\tau)}{Q(\tau)} = \frac{L - \mu\lambda\frac{q^{\alpha-1}}{\alpha-1}}{\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}} \tag{2.23}$$

Note that for a given amount of labour employed in the intermediate goods sector, price dispersion reduces the amount of output in efficiency unit pro-

[26] For details see Appendix 2.A.4.

[27] Note that from Appendix 2.A.3, $\frac{p^*(\tau)}{P(\tau)Q(\tau)}$ is constant. Hence, we have that for constant μ, $\frac{Y(\tau)}{Q(\tau)}$ is constant and $\gamma = \gamma_Q$ at steady state, as asserted above.

duced $Y(\tau)/Q(\tau)$.

2.2.6 Production side equilibrium

Inserting $\phi(k_j(\tau))$ from equation (2.18) and a new firm's expected market value
$E\left(V_{k_j+1}(\tau)\left|t_{k_j}=\tau\right.\right)$ (2.10) into the zero profit condition (2.17), we have the zero profit condition in the production side equilibrium:

$$\frac{\frac{1}{\lambda}\frac{Y(\tau)}{Q(\tau)}\left[\frac{p^*(\tau)}{P(\tau)Q(\tau)}\right]^{-\alpha}w(\tau)}{\left[\chi-(1-\alpha)\,\omega\right]\frac{\chi+\beta-\omega}{\chi+\beta-(1-\alpha)\omega}}=w(\tau) \tag{2.24}$$

with $\chi = i+\mu-\alpha\pi-\gamma$ and where final good production $Y(\tau)/Q(\tau)$ is the value we have determined in the equation describing labour market equilibrium, (2.23). This equation determines the research intensity μ which makes current research firms indifferent with regard to the amount of research labour used.[28] As can be seen, the resulting research intensity is the same for all firms making an innovation at time τ, regardless of the sector's current position on the quality ladder, consistent with our assumption in section 2.2.2.

2.2.7 Public Sector

As in earlier monetary growth models, the public sector is modelled choosing the most parsimonious specification: The state expands the money supply at a constant rate ψ and distributes seigniorage to the households in form of a lump-sum transfer.[29] In particular, the independent central bank perfectly controls the money supply, $M^s(\tau)$, by setting the constant exogenous rate ψ:

$$\frac{\dot{M}^s(\tau)}{M(\tau)}=\psi \tag{2.25}$$

[28] Note that the firm's value is $E\left(V_{k_j+1}(\tau)\left|t_{k_j}=\tau\right.\right)$ because it will produce the *next* quality, k_j+1 for the sector, which is about to be developed.

[29] Cf., e.g., Gillman and Kejak [2005b], Chang [2002], Marquis and Reffett [1995], Orphanides and Solow [1990].

This simple money supply rule is sufficient for our purposes because we are not concerned with either the replication of actual data of central bank behaviour or the design of optimal monetary policy.

All revenue from money creation is allocated to households in form of a lump-sum cash transfer, $T(\tau)$

$$\dot{M^s}(\tau) = T(\tau) \tag{2.26}$$

The state does not levy taxes and there is no government spending apart from the transfer of seigniorage to households.

2.2.8 Assets and Households

There is a positive number of investment funds in the economy which finance research activities. Each fund is of sufficient size to diversify the risk associated with its investments, such that they only care about the expected profit of each investment.[30] Funds flowing to the investment funds at time τ are used to finance current research activities. At a given real interest rate $r(\tau)$, research investment must be equal to the desired change in non-monetary savings of households, whom we now proceed to describe.

There is a continuum of households with mass one distributed uniformly on the interval $[0, 1]$. The infinitely lived representative household is assumed to maximise the present value of utility from consumption of the final good $c(\tau)$ and real balances $m(\tau) = \frac{M(\tau)}{P(\tau)}$ over his lifetime, where future flows of instantaneous utility are discounted with the factor $\rho > 0$.[31] Assuming the rate of population growth to be zero, a standard specification for households' utility is

$$U = \int_{s=0}^{\infty} e^{-\rho s} \frac{(c(s)^{1-\theta} m(s)^{\theta})^{1-\eta} - 1}{1-\eta} ds \tag{2.27}$$

[30] At the same time, funds are not big enough to internalise existing distortions.

[31] The household needs money for transaction purposes. Instead of modelling the transactions services of money explicitly or introducing a cash-in-advance-constraint we adopt the widespread shortcut-assumption that households derive utility from holding real balances $m = \frac{M}{P}$. Feenstra [1986] shows that our case of non-separable utility for consumption and real balances is equivalent to the explicit modelling of cash holdings' transaction cost reducing function, which is the standard justification for choosing this shortcut.

where we assume $\theta \in [0,1)$ and $\eta \geq 1$, where the latter condition is sufficient for convergence of the interval.[32,33] The representative household maximises (2.27) subject to his budget constraint. Combining the budget constraint with a no-arbitrage-condition for the asset market, we get:

$$\dot{v(\tau)} = \frac{w(\tau)}{P(\tau)}L + \frac{T(\tau)}{P(\tau)} + r(\tau)v(\tau) - c(\tau) - [\pi(\tau) + r(\tau)]\, m(\tau) \tag{2.28}$$

where v is the real value of the household's monetary and non-monetary wealth, $\frac{w}{P}L$ is the real wage income from inelastically supplying L units of labour, $\frac{T(\tau)}{P(\tau)}$ is the real value of the transfer received from the government and r is the real interest rate.[34]

The first-order conditions resulting from the maximization of the household's utility (2.27) subject to (2.28) are:

$$\frac{\theta}{1-\theta}\frac{c(\tau)}{m(\tau)} = r(\tau) + \pi(\tau) \tag{2.29}$$

and

$$[\eta + \theta(1-\eta)]\frac{\dot{c(\tau)}}{c(\tau)} - \theta(1-\eta)\frac{\dot{m(\tau)}}{m(\tau)} = r(\tau) - \rho \tag{2.30}$$

Since real interest rate r and inflation π are constant at steady state, equation (2.29) implies that $c(\tau)/m(\tau)$ is constant and thus, the growth rates of consumption and real balances must be equal. Using this in equation (2.30), we have the familiar Euler equation:

$$\frac{\dot{c(\tau)}}{c(\tau)} = \frac{r-\rho}{\eta} \tag{2.31}$$

[32] This assumption is not restrictive. Kimball [1995], for example argues that the value one for the intertemporal elasticity of substitution $1/\eta$ that is assumed in most of the RBC literature and many New Keynesian models is implausibly high given empirical estimates.

[33] At the same time, the assumption ensures that the transversality condition $\lim_{t\to\infty}\xi_t v_t e^{-\rho t} = \lim_{t\to\infty} e^{-rt}v_t = 0$ holds where ξ is the shadow value of wealth. The condition holds since the household's real wealth v grows at rate γ at steady state and $r - \gamma = \rho + (\eta - 1)\gamma > 0$ for $\eta \geq 1$.

[34] The household receives real interest payments of $r(\tau)$ on his non-monetary assets, $v(\tau) - m(\tau)$ while the value of real money holdings depreciates at rate $\pi(\tau)$, where $\pi(\tau)$ is the rate of inflation.

Equation (2.29) states that in equilibrium the ratio of consumed goods' marginal utilities equal their cost ratio with the opportunity cost of money given by the sum of real interest rate $r(\tau)$ and inflation rate $\pi(\tau)$, i.e. the nominal interest rate.[35]

Equilibrium in the final goods market requires that the household's optimal choice of consumption equal production:

$$c(\tau) = Y(\tau) \tag{2.32}$$

2.3 General equilibrium

We will first use households' optimal behaviour and information from the public sector to determine equilibrium in the money market. We then introduce the compiled information into the production side equilibrium to analyse the model's general equilibrium.

2.3.1 Closing the model

2.3.1.1 Money market equilibrium

Money demand must equal supply, $M^s(\tau) = M^d(\tau)$ or, given the initial money stock owned by households $M(0)$, the growth rate of real money supply, $\frac{\dot{m}^s(\tau)}{m^s(\tau)} = \psi - \pi$,[36] must equal the growth rate of demand for real balances $\frac{\dot{m}^d(\tau)}{m^d(\tau)}$. Using final goods market equilibrium and households' optimal behaviour, we have $\gamma = \frac{\dot{c}(\tau)}{c(\tau)} = \frac{\dot{m}^d(\tau)}{m^d(\tau)}$.[37]

Equalizing the growth rates of real money demand and supply shows that money market equilibrium implies that the inflation rate at steady state is the difference between the money growth rate and the economy's output

[35] As there is no aggregate uncertainty in the model, there is no need to distinguish between expected and actual inflation, such that $i = r + \pi$.

[36] Remember that the nominal supply is expanded at the constant rate $\frac{\dot{M}^s(\tau)}{M^s(\tau)} = \psi$.

[37] See the household's static optimality condition (2.29).

growth rate

$$\pi = \psi - \gamma \tag{2.33}$$

In equation (2.15) we see that the wage $w(\tau)$ grows at rate $\gamma + \pi$.[38] Using (2.33) we thus have that the growth rate of marginal cost equals the growth rate of money supply:

$$\psi = \gamma + \pi = \omega \tag{2.34}$$

2.3.1.2 Research intensity in General Equilibrium

Inserting the value for $Y(\tau)/Q(\tau)$ from the labour market equation (2.22), using equilibrium in the money market (2.33), the equality of wage growth and money growth (2.34) and $i = r + \pi$, and rearranging, the R&D zero profit condition (2.24) can be rewritten as:

$$\frac{\left(\frac{L}{\lambda} - \mu\frac{q^{\alpha-1}}{\alpha-1}\right)\left(\frac{p^*(\tau)}{P(\tau)Q(\tau)}\right)^{-\alpha}}{\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\psi}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\psi}\right)^{\frac{-1}{\alpha}}\right]^{-\alpha}} = \left[r + \mu - (2-\alpha)\gamma\right]\frac{r + \mu + \beta - (2-\alpha)\gamma - \alpha\psi}{r + \mu + \beta - (2-\alpha)\gamma} \tag{2.35}$$

The LHS of equation (2.35) shows the instantaneous profits for a firm entering the market with a new patent. The RHS represents the compound discount rate for this firm's future profit streams.

Further using the value of the current optimal relative price from equation (2.15) in Appendix (2.A.3), the Euler equation (2.31) and the equation relating economic growth to research intensity (2.21), we get an equation in μ and the model's parameters:

$$\frac{\left(\frac{L}{\lambda} - \mu\frac{q^{\alpha-1}}{\alpha-1}\right)\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\psi}\right)^{-\alpha/(\alpha-1)}}{\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\psi}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\psi}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}} = (\rho + \widetilde{\eta}\mu)\frac{\rho + \beta + \widetilde{\eta}\mu - \alpha\psi}{\rho + \beta + \widetilde{\eta}\mu} \tag{2.36}$$

[38] As the RHS of (2.15) is constant at steady state, so must be the left hand side of the equation. Thus, with $p^*(\tau) = \frac{\alpha}{\alpha-1}\frac{i+\mu+\beta-\alpha\pi-\gamma}{i+\mu+\beta-\alpha\pi-\gamma-\omega}w(\tau)$, we have $\frac{\dot{w(\tau)}}{w(\tau)} = \frac{\dot{P(\tau)}}{P(\tau)} + \frac{\dot{Q(\tau)}}{Q(\tau)}$ which using that $\gamma_Q = \gamma$ at the steady state implies $\omega = \pi + \gamma$.

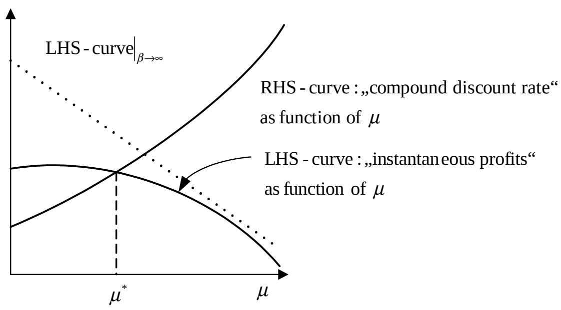

Figure 2.1: Steady state equilibrium research intensity μ

with $\widetilde{\eta} = 1 + (\eta + \alpha - 2)\,\frac{q^{\alpha-1}-1}{\alpha-1}$. Both sides of equation (2.36) reflect the dependence of the optimal research intensity μ on the new firm's value. The equation is represented graphically in the solid lines in figure 2.1.

2.3.2 Existence and uniqueness of the steady state equilibrium

Assumptions We prove the existence and uniqueness of a steady state under the following two standard conditions:

$$\frac{L}{\lambda} > \rho \qquad (2.37)$$

$$\psi \leq \alpha^{-1}\beta\frac{L/\lambda - \rho}{L/\lambda - \rho\frac{\beta}{\rho+\beta}} \qquad (2.38)$$

Condition (2.37) is the usual no-growth-trap-condition familiar from the underlying real growth model.[39] Condition (2.38) implies that price rigidity cannot be too strong or that, for any given β, there exists an upper bound on the growth rate of money supply ψ compatible with steady state equilibrium.[40] The existence of such an upper bound is necessary in all standard

[39] See for example Barro and Sala-i-Martin [2003].

[40] Note that since $L/\lambda > \rho > 0$ (condition 2.37), condition 2.38 implies $\mu + \beta > \alpha\psi$ and hence is sufficient for aggregate intermediate good demand L^X to be well-defined (see footnote 19). Together with the assumption $\eta \geq 1$ made in section 2.2.8, it also ensures that $\mu + \beta - \alpha\psi + (\widetilde{\eta} - 1)\,\mu + \rho > 0$ which means that the conditions from footnote 9 hold

New-Keynesian business cycle models with price rigidity.[41]

Proposition 1 *Under conditions (2.37)-(2.38), the economy has a unique steady state equilibrium with* $\mu > 0$.

The proofs to this and the following propositions can be found in Appendix 2.A.5.

Intuition Consider equation (2.36). The RHS of the equation shows the effects of the future research intensity on the compound discount rate of the new firm's entering the market for intermediate goods.[42] Given the assumption $\eta \geq 1$ from section 2.2.8, the discount rate rises in the research intensity μ as in the model without money. This is intuitive since an increase in μ means that the probability of being replaced increases, lowering the weight attached to potential future profits.[43]

Now turn to the LHS of equation (2.36) which shows how the instantaneous profit associated with the production of the new intermediate good depends on the current and past values of μ. In the case without effective price rigidity,[44] the LHS simplifies to $L/\lambda - \mu\frac{q^{\alpha-1}}{\alpha-1}$ and linearly decreases in μ (see LHS$|_{\beta\to\infty}$-curve in figure 2.1). This is because an increase in current research intensity μ spurs demand for research labour $\frac{q^{\alpha-1}}{\alpha-1}\mu$ ("investment"),

such that a firm's optimal price p^* is well-defined.

[41] The existence of an upper bound is commented on by, e.g. Ascari [2004] and King and Wolman [1996]. In the baseline example we introduce in section 2.5, the maximum money growth rate is $\psi = 0.180$, see footnote 62.

[42] To help intuition, we stress the fact that the compound discount rate depends on the *future* value of μ while instantaneous profits depend on the variable's *current* and *past* values. Equation (2.36) shows that these values are of course identical at steady state equilibrium.

[43] In addition to this direct effect of an increase in μ there are also several indirect effects of an increase in μ on the discount rate because μ is proportional to the growth rate γ: The real interest rate rises in γ while because of the decrease in inflation caused by a rising γ the nominal interest rate $i = \rho + \psi + (\eta - 1)\gamma$ may rise or fall in γ. Demand for the good and hence, the growth rate of profits falls in γ because although aggregate demand increases proportionately in γ, the decrease in inflation slows down the erosion of the good's relative price. For $\eta + \alpha - 2 > 0$, the positive effects of γ on the discount rate outweigh the negative effects.

[44] That is, either in the absence of price rigidity ($\beta \to \infty$) or in case existing price rigidity is irrelevant because money supply is constant, $\psi = 0$.

which for a given size of the labour force reduces production of the final good ("consumption") and hence, demand for the new firm's good and its profits. Under price rigidity, this linear curve is shifted by the two already known terms depending on money growth ψ and price rigidity β that reflect the initial mark-up effect, $\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\psi}\right)^{-\alpha/(\alpha-1)}$, and the price dispersion effect $\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\psi}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\psi}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}$, respectively, which will be important for our discussion of comparative statics.

Both these effects depend on the total amount of price rigidity or flexibility, which depends not only on β but also on the innovation rate μ. We show in Appendix 2.A.5 that as depicted in figure 2.1, these influences of μ on price rigidity make the LHS a concave function for $\beta < \infty$.[45] Yet note that the frequency of innovation μ is small relative to the frequency with which incumbents can change prices β, so that the effect of μ on $\mu + \beta$ is small. Therefore, these effects are quantitatively small both in reality and in our calibrated examples.

Existence and uniqueness follow from the shapes of the two curves.

Note that given assumptions (2.37)-(2.38) and concavity of the LHS-curve, the slope at the steady state equilibrium of the LHS-curve is smaller than that of the RHS-curve. This implies that in spite of the discussed positive effect of μ on instantaneous profits, the incentive to innovate at equilibrium decreases in μ, as in the model without money.

2.4 Comparative statics

The comparative static properties of the steady state equilibrium will be analysed using the zero profit condition (2.36) and figures 2.1 and 2.2.[46]

[45] Note that the slope of the LHS-curve is positive for small μ as in figure 2.1 if $0 < \lim_{\mu\to 0}\frac{\partial LHS}{\partial\mu} = -\frac{q^{\alpha-1}}{\alpha-1}\frac{\beta-\alpha\psi}{\beta} + \frac{L}{\lambda}\frac{\alpha\psi}{\beta^2}$. Whether this is the case is irrelevant for the existence and uniqueness of the equilibrium (concavity of the LHS-function is sufficient) and for the later discussion of comparative statics, where it is sufficient that at the equilibrium, $\partial LHS/\partial\mu < \partial RHS/\partial\mu$, which is assured by the additional assumption (2.38) and $\eta \geq 1$

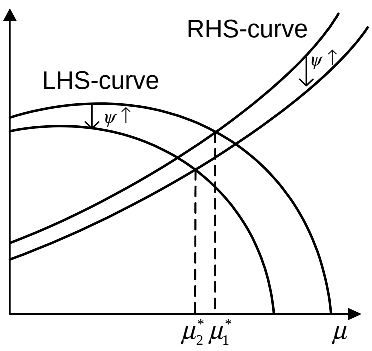

Figure 2.2: Comparative statics for $\beta < \infty$: money growth rate

2.4.1 Economy without price rigidity: Superneutrality of money

Proposition 2 *In the limiting case without rigidities ($\beta \to \infty$), money is superneutral. In partucular,* $\lim_{\beta\to\infty} \frac{\partial\mu}{\partial\psi} = \lim_{\beta\to\infty} \frac{\partial\gamma}{\partial\psi} = 0$.

The proof can be found in Appendix 2.A.5.

Intuition Without price rigidity, relative prices are undistorted such that the size of the final good sector, the firm's relative price and the only discount rate relevant for the firm are independent of money growth. The zero profit condition reduces to the one from the model without money and all other comparative static properties are those of the model without money.

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from the household's problem.

[46]We concentrate on comparative statics with respect to ψ and β since the effects on growth of the other parameters are standard: An increase in the household's rate of time preference ρ raises the interest rate r and the discount rate for a new firm's profits, which reduces the R&D-incentive and growth. In figure 2.1, the increase in ρ causes an upward movement of the RHS curve, lowering μ. The size of the research-productivity-adjusted labour force L/λ has a scale effect of on the firm's profits and growth. In figure 2.1, an increase in L/λ causes an upward movement of the LHS curve, which leads to a higher research intensity μ.

2.4.2 Economy with price rigidity: Negative growth effect of money growth

For $\beta < \infty$, the money growth rate ψ affects the zero profit equation (2.36) via three channels: The *price dispersion effect* and the *initial price effect* affect the innovating firm's instantaneous profits on the LHS of equation (2.36), while the *price erosion effect* changes the firm's compound discount rate on the RHS of the equation. All these effects on economic growth operate through money growth's influence on relative prices.

Negative price dispersion effect of non-zero money growth The price dispersion effect of money growth is captured in the term $\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\psi}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\psi}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}$ on the LHS of equation (2.36). The firm's instantaneous profits decrease in price dispersion which in turn increases in the absolute value of the money growth rate ψ.[47] This is because an increase in the absolute value of ψ raises the absolute value of the growth rate of the optimal price which under price rigidity increases price dispersion regarding intermediate goods. This increases the disparity in the amounts of intermediate goods used in the final good sector, which in turn reduces production efficiency and thus lowers the intermediate good producer's instantaneous profits. Figure 2.2 depicts the case $\psi > 0$, where an the increase in ψ ceteris paribus causes a downward shift of the LHS-curve, reducing the research intensity μ.[48]

To get a more detailed intuition, refer back to equation (2.12). Remember that the price dispersion in the intermediate goods sector caused by a non-zero growth rate ψ of marginal costs and optimal prices distorts demand towards intermediate goods with old (new) low prices when money growth is positive (negative).[49] Due to the concavity in intermediate goods of the

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[47] $\frac{\partial\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}}{\partial\psi} \gtreqless 0$ for $\psi \gtreqless 0$, see footnote 21.

[48] At $\psi < 0$, the LHS-curve would be shifted upwards by an increase in ψ.

[49] For $\psi = 0$, marginal cost and optimal prices are constant over time. Intermediate goods producers have no desire to readjust their initial prices so that there is no price dispersion despite the presence of nominal rigidity.

final good production function, this leads to inefficient production: Given current demand for research labour, and given inelastic labour supply L, the resulting labour employed in the intermediate goods sector indirectly produces the smaller an amount of final goods, the higher the deviation of ψ from zero.[50] As demand for the new intermediate good is proportional to the amount of output in efficiency units produced, the new producer's instantaneous profits and the incentive to innovate decrease (increase) in ψ for $\psi > 0$ ($\psi < 0$).

Negative initial relative price effect of an increase in money growth The term $\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\psi}\right)^{-\alpha/(\alpha-1)}$ on the LHS of equation (2.36) shows that the new intermediate good firm's instantaneous profits depend negatively on the firm's initial relative price because demand for the good falls in the good's relative price. An increase in ψ raises the initial relative price, reducing demand for the good and therefore profits. The increase in ψ thus ceteris paribus reduces the incentive to innovate, reflected in the downward shift of the LHS-curve in figure 2.2.

As explained in section (2.2.2.1), for $\psi \neq 0$ intermediate goods firms set a forward-looking initial mark-up differing from the static optimum one to offset the effect of the money growth rate, which is also the growth rate of marginal cost, on their relative price while their price is fixed. An increase in the money growth rate raises the extent of relative price erosion and hence, the optimal forward-looking initial mark-up. Also, the level of the real wage is influenced by an increase in ψ as discussed in section 2.2.3. Taking both effects into account, in general equilibrium with price rigidity an increase in ψ raises the relative price of the innovating firm at market entry which reduces demand for the firm's good and therefore, profits.[51]

Taking together the two effects, the LHS of equation (2.36) always decreases in ψ: At $\psi > 0$, both effects reduce the firm's instantaneous profits, at

[50] Remember that since intermediate goods are produced one for one with labour, labour indirectly produces the final good.

[51] Remember from the discussion of equation (2.10) that the firm's optimal price was chosen so as to offset the effect of money growth on the firm's profit *per unit* so that the effect of ψ on the relative price only affect profits via the quantity demanded.

$\psi < 0$, the negative effect via the relative price dominates the price dispersion effect.

Positive relative price erosion effect of an increase in money growth The RHS of equation (2.36) is the new firm's compound discount rate that was first discussed in section 2.2.2.2. An increase in ψ lowers the compound discount rate because faster relative price erosion increases demand and the profit growth rate, ceteris paribus raising the incentive to innovate. In figure 2.2, the RHS-curve is shifted down by an increase in ψ, ceteris paribus increasing μ.

Going into more detail, remember that the compound discount rate consists of the discount rate of a firm under price flexibility corrected by a factor that contains the appropriate discount rates for a firm under price rigidity for periods where prices can be changed or are fixed, respectively. The discount rates for periods where prices can be changed (both under flexibility and under rigidity) are unaffected by money growth. Yet money growth increases profit growth in periods where prices are fixed, reducing the associated discount rate:[52] While prices are fixed, the new good's relative price erodes at rate $-\pi$, leading to a growth rate $\alpha\pi$ of demand for the good. The rising demand translates into a higher growth rate of the new intermediate firm's profits.[53] An increase in ψ ceteris paribus raises inflation π and therefore reduces the discount rate for periods where prices are fixed and the compound discount rate, ceteris paribus raising the incentive to innovate and the research intensity μ.

Negative net effect of money growth on economic growth We have discussed that for $\psi > 0$, the negative effects of an increase in money growth ψ cause a downward shift of the LHS-curve while the positive relative price erosion effect entails a downward shift of the RHS-curve. Given the shapes of the curves discussed in section 2.3.2, it remains only to clarify which effect

[52] Remember that the discount rate is the obsolescence-adjusted interest rate minus the profit growth rate.

[53] See equation (2.3) for the determinants of demand faced by the firm.

on the research intensity μ is stronger. The following proposition is proven in Appendix 2.A.5:

Proposition 3 *An increase in the steady state money growth rate ψ decreases the research intensity μ and the real growth rate γ: $\frac{d\mu}{d\psi} < 0$, $\frac{d\gamma}{d\psi} < 0$.*

Money growth and inflation thus reduce economic growth in our model, in line with the stylised fact presented in the introduction. To get an intuition why growth depends negatively on the money growth rate over the entire range of admissible values, note first that *holding constant* the level of the real wage, an increase in ψ lowers (raises) the incentive to innovate at $\psi > 0$ ($\psi < 0$). This is due to the fact that the price dispersion effect increases in the absolute value of ψ and due to money growth's effect on the optimal mark-up (initial price effect given $w/(PQ)$) and effective mark-ups (relative price erosion effect): Holding constant the real wage, the firm cannot be made better of by imposing a restriction on the choice and adjustment of its mark-up, so profits are highest when the restriction does not bind, which is the case at $\psi = 0$ where due to constant marginal cost there is no incentive to change prices. Profits decrease as the restriction on individual price setting becomes more binding, i.e. as the absolute value of the growth rate of marginal cost ψ rises.[54]

Since the level of the real wage also influences the initial relative price effect,[55] the fact that growth decreases in ψ at $\psi < 0$, too, must be due to this effect. From the discussion in section 2.2.3, we know that the wage increases in ψ at $\psi \leq 0$. While we have not shown this analytically, in calibrated examples it decreases in the money growth rate at large admissible values of ψ. Since an increase in the real wage raises the firm's initial relative price, lowering demand and profits, it remains only to clarify why the effect of an increase in ψ on the real wage level dominates at $\psi < 0$ but not at

[54] The sign of $d\mu/d\psi$ for given $w/(PQ)$ is again determined by the sign of $\frac{\partial LHS}{\partial \psi} - \frac{\partial LHS}{\partial \psi}$. Straightforward calculations yield $\left(\frac{\partial LHS}{\partial \psi} - \frac{\partial LHS}{\partial \psi}\right)\Big|_{\overline{\frac{w}{PQ}}} = -\psi \left[\frac{\alpha(\alpha-1)LHS}{(\rho+\beta+\widetilde{\eta}\mu-\alpha\psi)[\rho+\beta-(\alpha-1)\psi+\widetilde{\eta}\mu]} + \frac{\alpha LHS}{[\mu+\beta-(\alpha-1)\psi](\mu+\beta-\alpha\psi)}\right] \lesseqgtr 0$ for $\psi \gtreqless 0$.

[55] As opposed to the mark-up erosion effect, where only the *growth rate* of the relative price and hence of wages matters.

positive values of ψ. To see this, note that since the value of the elasticity of substitution between intermediate goods α exceeds unity, the effect of any change in the initial relative price on demand is stronger when the relative price is smaller than one than when it exceeds unity- and given price rigidity, the initial price is smaller (bigger) than one when marginal cost shrinks (grows) over time, i.e. $\psi < 0$ ($\psi > 0$).[56]

2.4.3 Economy with price rigidity: Negative growth effect of the exogenous level of rigidity under inflation

We now use equation (2.36) to discuss the comparative statics of the steady state for $\beta < \infty$ with regard to the level of exogenous rigidity, which is inversely related to β.

Proposition 4 *An increase in the level of rigidity (i.e., decrease in β) decreases (increases) the research intensity μ and the real growth rate γ when money growth is positive (negative): $\frac{d\gamma}{d\beta} \gtreqless 0$ for $\psi \gtreqless 0$.*

Intuition The effect of a higher β on γ is the result of three already familiar effects: Firstly, for $\psi \neq 0$ a higher β (less rigidity) reduces the *price dispersion effect*: A higher value of β means that prices can be changed more often and thus are less dispersed, so as reflected in the term $\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\psi}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\psi}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}$, demand is less distorted towards relatively cheap goods, raising production efficiency. Via the mechanism explained above, an increase in β therefore raises the incentive to innovate. Secondly, by shifting more weight to present as opposed to future profits and reducing price dispersion, an increase in β reduces the deviation of the firm's initial mark-up and relative price from their values under flexibility. An increase in β thus reduces the *initial relative price effect.* The decrease (increase) in the initial relative price raises (lowers) demand, profit and thus, ceteris paribus the incentive to innovate at $\psi > 0$ ($\psi < 0$). The effect on price dispersion

[56] See equation (2.15).

(the initial relative price) dominates at $\psi > 0$ ($\psi < 0$) so that an increase in β shifts the LHS-curve in figure 2.1 upward (downward), ceteris paribus increasing (decreasing) μ.

At the same time, the RHS of equation (2.35) increases in β (decreases in the level of rigidity), so that an increase in β causes an upward shift of the RHS-curve in figure 2.1, ceteris paribus reducing μ. This is because a higher degree of price flexibility β reduces the extent of *relative price erosion* which only takes place while prices are fixed. Thus the increase in β shifts more weight to the discount rate for periods when prices are flexible. As this rate is higher than the fix-price discount rate due to the relative price erosion effect of money growth, an increase in β raises the compound discount rate, ceteris paribus reducing μ.[57]

The intuition for the net effect of β on μ is closely connected to the effects of money growth in that a decrease in β magnifies the effect of a deviation of ψ from zero: At $\beta \to \infty$, all discussed effects of money on growth disappear. Distortion of relative prices is the more pronounced, the higher rigidity. So when at $\psi > 0$ this deviation of money growth from zero is detrimental to growth (growth-enhancing at $\psi < 0$), an increase in β that mitigates the consequences of money growth raises (lowers) growth.

2.5 Calibrating the model: Comparative Statics

We calibrate the model to see if the magnitude of the growth-reducing net effect of money growth is consistent with the findings of empirical studies. We use standard parameter values from the literature and use steady state considerations to calibrate our model. Our calibration of the household's intertemporal rate of substitution η^{-1} and discount rate ρ and money's re-

[57] The compound discount rate is given by the discount rate for a firm under total price flexibility times a factor that corrects for price rigidity's effect on the discount rate. As seen above, through the mark-up erosion effect rigidity leads to a lower discount rate in fix-price periods than when prices can be changed. An increase in β (less rigidity) means that this rate is relevant less often, so that the downward correction of the flex-price rate is reduced and the compound discount rate increases.

latively small weight θ in the household's utility function follow standards from the calibration of New Keynesian business cycle models. For the same reason and in line with empirical estimates, α is chosen such that the mark-up in a steady state with constant marginal cost $\alpha/(\alpha-1)$ is between 10% and 20%.[58] The values for q imply that innovations bring about quality improvements of 1% to 50% relative to existing products. Our choice of β is based on empirical findings that prices are fixed for two to five quarters in the literature,[59] and entails an upper bound on (partly endogenous) price rigidity of one year when growth is at its minimum value zero.[60] Using data on average US M1 growth,[61] we further use the baseline money growth rate $\psi = 0.055$ in all examples. Table 2.1 lists parameters' values in our baseline case and the ranges wherein parameters were varied:[62,63]

In the baseline case, firms' optimal price implies a mark-up of 15.2% over marginal cost, while the average time during which prices are fixed is 0.54 years, i.e. 6.5 months. Both results are compatible with the discussed

[58] Cf. e.g. to Chari, Kehoe and McGrattan [2000]. Markups of this dimension are reported for the US by Basu and Fernald [1995, 1997] and for Germany by Linnemann [1999].

[59] See section 1.5.1.3 of the introductory chapter.

[60] Remember that due to innovating firms' market entry with new prices, the average interval of fixed prices is partly endogenous in our model and given by $(\beta+\mu)^{-1} = \left(\beta + \frac{\alpha-1}{q^{\alpha-1}-1}\gamma\right)^{-1}$.

[61] We calculated the average growth rate of the monetary aggregate M1 in the US between 1979 and 2004 based on data from www.federalreserve.gov/releases/h6/hist/. The narrow aggregate was chosen because money is modelled as yielding utility to the household. This can best be motivated with the transaction services of money as opposed to its services as a store of value. By definition of the monetary aggregates, the transaction function is best reflected by M1.

[62] In all checked examples, we examined the growth rate effects of money growth rates ψ with $\psi^0 < \psi < \psi^{\max}$. Here, $\psi^{\max}$ is the maximum money growth rate compatible with a given level of β in constraint (2.38) calculated in footnote 40: $\psi^{\max} = \frac{\beta\left(\frac{L}{\lambda}-\rho\right)}{\alpha\left(\frac{L}{\lambda}-\rho\frac{\beta}{\rho+\beta}\right)}$. In the baseline example, $\psi^{\max} = 0.180$. As our analysis applies to developed countries, this is not a severe restriction. The lower bound on ψ is implied by the household's static optimality condition (2.29) where the nominal interest rate $i = \rho + (\eta-1)\gamma + \psi$ must be positive for the existence of equilibrium. As $\gamma = \gamma(\psi)$, the implied value of ψ is endogenous. We therefore choose $\psi^0 = -\rho$ which is sufficient for $i > 0$ for any $\eta \geq 1$.

[63] The range of values for L/λ is determined by the fact that that we use the degree of freedom in choosing this variable so as to always make sure that $\gamma = 0.02$ at $\psi = 0.055$ given our other parameter choices.

Parameter	Baseline	Range	Parameter	Baseline	Range
η	2	$[1, 2]$	β	1.8	$[1, 3]$
α	10	$[6, 11]$	q	1.2	$[1.01, 1.5]$
ρ	0.015	$[0.01, 0.03]$	ψ	0.055	$\left(\psi^0, \psi^{\max}\right)$
θ	0.01	$[0.01, 0.5]$	L/λ	0.29455	

Table 2.1: Parameter values used for calibration

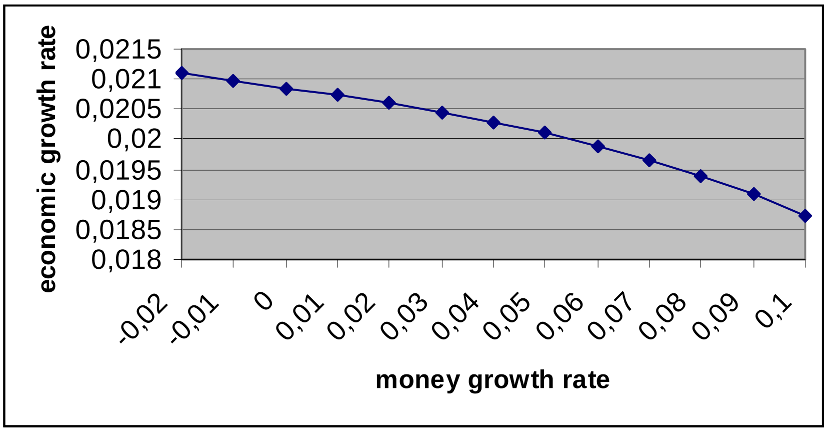

Figure 2.3: Function $\gamma(\psi)$ in baseline calibration

empirical estimates. The output growth rate is 2% per annum. At the same time, the average frequency of innovation $(1/\mu)$ is 23.1 years. So indeed we are investigating the effects of recurrent *short*-term frictions on *long*-term innovation.

Varying the policy parameter ψ shows that the size of the effect of an increase in the money growth rate corresponds closely to the findings of the empirical literature: Increasing the annual money growth rate from 1% to 10% reduces economic growth by some 0.2 percentage points from 2.07% to 1.87% per annum.[64] Figure 2.3 shows the rate of economic growth as a function of the money growth rate ψ in our baseline example.

The negative effect of an increase in money growth from $\psi = 0.01$ to

[64] As reported in the introduction, Motley [1998], Barro [1996] Fischer [1993] all report results of a similar magnitude. In particular, Barro [1996] finds that a 10 percentage point increase in M1 growth (inflation) reduces annual GDP growth by about 0.23 (0.24) percentage points.

$\psi = 0.1$ on economic growth is the net effect of quantitatively larger individual effects: The countervailing relative price erosion effect and initial relative price effect are quite sizeable individually, each changing the growth rate by over one percentage point. In our baseline example where price rigidity is not very severe, the effect through price dispersion is quantitatively small (the growth rate changes by -0.007 percentage points for the described money growth variation) in comparison to the other effects. Yet the relative importance of the three individual effects varies considerably under different parameter constellations: The importance of the production inefficiency channel rises considerably as rigidity becomes more severe.

The net effect of price rigidity on growth at positive money growth rates is negative but small: Decreasing β from $\beta = 1.8$ to $\beta = 1.5$ at $\psi = 0.055$, the growth rate drops from 2.00% to 1.96%.[65] The effect of rigidity on growth is stronger for higher rates of money growth and vice versa: The drop in the growth rate resulting from an increase of ψ from 1% to 10% growth is .33 percentage points at $\beta = 1.5$, compared to .20 percentage points at $\beta = 1.8$.[66]

2.6 Conclusion

The analysis of the Schumpeterian growth model with small nominal frictions has shown that indeed, the influence of price rigidity is not restricted to the short-run. At the unique steady state equilibrium, it allows money growth to have a level effect on production efficiency and on the relative prices charged by intermediate goods producers. Since an intermediate good producer's profits determine the incentive to engage in research activities for the development of improved intermediate goods, this implies a level effect on the total amount of resources devoted to R&D. Finally, since R&D activities leading to growth in the quality of intermediates are the engine of growth in the economy, the level effects of inflation induced by nominal price rigidity

[65] The implied change in the average period where prices are fixed is from 6.5 to 7.8 months.

[66] For $\psi = 0.1$, the growth rate drops from 1.87% to 1.74% when β is decreased from $\beta = 1.8$ to $\beta = 1.5$.

also influence the economy's long-run output growth rate.

The interaction of money growth's influences on intermediate good firms' relative prices (via optimal and effective mark-ups and the real wage level) and production efficiency (via the degree of dispersion in intermediate good prices) makes the economic growth rate a decreasing function of the money growth rate over the entire range of admissible money growth rates. Although the examined transmission mechanisms differ, this result is shared by most of the papers on endogenous growth and inflation presented in section 1.4.1.2 of the introductory chapter. While the quantitative results of our model concerning the growth effects of discrete large increases in inflation are in line with the results of the empirical literature, neither our model nor those discussed in section 1.4.1.2 are compatible with the pieces of evidence discussed in section 1.4.2 which suggest that the effect of small positive inflation rates on growth may be insignificant or even positive. In chapter 5, the influence of money growth on endogenous labour supply is at the origin of a non-monotone money-growth relationship, where economic growth increases in the money growth rate ψ at small values of ψ. Before we turn to this variation of our model, an issue more closely connected to this chapter's analysis deserves further investigation: Our analysis so far has focused exclusively on the steady state equilibrium of the Schumpeterian model with price rigidity. This is a valid simplification for the analysis of the model economy's long-run behaviour only in case the steady state is stable, such that the economy for any given initial conditions converges to the steady state in the long-run. Therefore, chapter 3 is dedicated to the analysis of the steady state's stability.

2.A Appendix to chapter 2

2.A.1 Derivation of a new firm's market value

The firm's market value is the discounted sum of profits from future periods s where the profits are weighted due to two independent sources of uncertainty: The first weight is given by the probability $e^{-\mu(s-\tau)}$ of not having been replaced by time s. The second source of uncertainty is given by the firm's price in period s: The price charged can be any past optimal price $p^*(\theta)$ with $\theta \in (\tau, s)$ depending on when the last reset signal for the price was received between τ and s. Thus, the price charged at time s can be represented as a weighted sum of the past optimal prices, where the weights are as follows: The flow probability that a signal to reset prices was received in period θ is β. With probability $e^{-\beta(s-\theta)}$, no signal was received between θ and s given that a signal was received in θ.[67] As these two events are independent, the probability of having last reset one's price due to a price reset signal at $\theta \in (\tau, s)$ is $\beta e^{-\beta(s-\theta)}$. Additionally, if no reset signal has been received up to period s, the firm's price will continue to be $p^*(\tau)$, which has probability $(1 - \int_\tau^s \beta e^{-\beta(s-\theta)} d\theta)$. Since the processes for innovations and reset signals are independent, the joint probability of the described events takes on a multiplicative form:

$$\begin{aligned} E\left(V_{k_j}\left|\tau\right.\right) =& A\left(\tau\right) \int_\tau^\infty e^{-\chi(s-\tau)} \left[\int_\tau^s \beta e^{-\beta(s-\theta)} p^*\left(\theta\right)^{1-\alpha} d\theta + \left(1 - \int_\tau^s \beta e^{-\beta(s-\theta)} d\theta\right) p^*\left(\tau\right)^{1-\alpha} \right] ds \\ & - A\left(\tau\right) w\left(\tau\right) \int_\tau^\infty e^{-(\chi-\omega)(s-\tau)} \left[\int_\tau^s \beta e^{-\beta(s-\theta)} p^*\left(\theta\right)^{-\alpha} d\theta + \left(1 - \int_\tau^s \beta e^{-\beta(s-\theta)} d\theta\right) p^*\left(\tau\right)^{-\alpha} \right] ds \end{aligned}$$

where we define $\chi = i + \mu - \alpha\pi - \gamma$ and $A\left(\tau\right) = q^{k_j(\alpha-1)} Y\left(\tau\right) P\left(\tau\right)^\alpha$ and where $E\left(V_{k_j}\left|\tau\right.\right)$ is a shorthand form for $E\left(V_{k_j}\left(\tau\right)\left|t_{k_j} = \tau\right.\right)$.

Making use of the fact that the optimal price $p^*(\theta)$ grows with the growth rate of marginal cost, ω and solving the integrals associated with the prob-

[67] Here, we have been able to definitize the constant $\widetilde{B} = 1$ since we know that the probability of receiving two or more signals at time θ is negligible (see footnote 7) for details.

ability of receiving a price resetting signal yields:

$$
\begin{aligned}
E(V_{k_j}|\tau) = {} & A(\tau)p^*(\tau)^{1-\alpha}\frac{\beta\int_\tau^\infty e^{-[\chi-(1-\alpha)\omega](s-\tau)}\left[1-e^{-[\beta-(1-\alpha)\omega](s-\tau)}\right]ds}{\beta-(\alpha-1)\omega} \\
& +A(\tau)p^*(\tau)^{1-\alpha}\int_\tau^\infty e^{-(\chi+\beta)(s-\tau)}ds \\
& -A(\tau)p^*(\tau)^{-\alpha}w(\tau)\frac{\beta}{\beta-\alpha\omega}\int_\tau^\infty e^{-[\chi-(1-\alpha)\omega](s-\tau)}\left(1-e^{-(\beta-\alpha\omega)(s-\tau)}\right)ds \\
& -A(\tau)p^*(\tau)^{-\alpha}w(\tau)\int_\tau^\infty e^{-(\chi+\beta-\omega)(s-\tau)}ds
\end{aligned}
$$

Calculating the value of the integrals yields

$$
\begin{aligned}
E(V_{k_j}|\tau) = {} & A(\tau)\,p^*(\tau)^{1-\alpha}\left\{\frac{\beta}{\beta-(\alpha-1)\,\omega}\left(\frac{1}{\chi-(1-\alpha)\,\omega}-\frac{1}{\chi+\beta}\right)+\frac{1}{\chi+\beta}\right\} \\
& -A(\tau)\,w(\tau)\,p^*(\tau)^{-\alpha}\left\{\frac{\beta}{\beta-\alpha\omega}\left(\frac{1}{\chi-(1-\alpha)\,\omega}-\frac{1}{\chi+\beta-\omega}\right)+\frac{1}{\chi+\beta-\omega}\right\}
\end{aligned}
$$

Multiplying out the terms in curly brackets, we have

$$
E\left(V_{k_j}(\tau)\,|t_{k_j}=\tau\right)=A(\tau)\,p^*(\tau)^{-\alpha}\,\frac{\chi+\beta+(\alpha-1)\,\omega}{\chi-(1-\alpha)\,\omega}\left[\frac{p^*(\tau)}{\chi+\beta}-\frac{w(\tau)}{\chi+\beta-\omega}\right]
$$

Finally using the equation for the optimal price (2.8) and reinserting $\chi = i+\mu-\alpha\pi-\gamma$ and $A(\tau)=q^{k_j(\alpha-1)}Y(\tau)\,P(\tau)^{\alpha}$ we have equation (2.10) in the main text.

2.A.2 Derivation of the intermediate goods sector's labour demand

Equation (2.11) can be rewritten as

$$
L^X(\tau)=Y(\tau)P(\tau)^{\alpha}Q(\tau)^{\alpha-1}\sum_{k=1}^{k^{\max}}d_k(\tau)\left(\frac{q^k}{Q(\tau)}\right)^{\alpha-1}\sum_{\{j|k_j=k\}}\left(p_{k_j}(\tau)\right)^{-\alpha}
$$

where p_{k_j} is the price of sector j that is at quality rung k and $d_k(\tau)$ is the number of sectors at quality rung k at time τ.

Following Benhabib, Schmitt-Grohé and Uribe [2001a] and [2001b], Leith and Wren-Lewis [2000] and Wolman [1999], the average price effective at time τ can be expressed as a weighted average of past optimal prices, where the weights $f(s,\tau)$ refer to the probability that a price valid at time τ has not been changed since time s. As the timing of innovations is independent of a sector's position on the quality ladder q^{k_j}, the structure of prices for a given q^k is the same as the structure for all sectors. Thus we have

$$\begin{aligned}\sum_{k=1}^{k^{\max}} d_k(\tau)\left(\frac{q^k}{Q(\tau)}\right)^{\alpha-1}\sum_{\{j|k_j=k\}} p_{k_j}(\tau)^{-\alpha} &= \sum_{k=1}^{k^{\max}} d_k(\tau)\left(\frac{q^k}{Q(\tau)}\right)^{\alpha-1}\sum_{j=1}^{N} p_j(\tau)^{-\alpha}\\ &= \int_{-\infty}^{\tau} f(s,\tau)\left[p^*(s)\right]^{-\alpha} ds\end{aligned}$$

with $\sum_{k=1}^{k^{\max}} d_k(\tau)\left(\frac{q^k}{Q(\tau)}\right)^{\alpha-1} = \frac{\sum_{k=1}^{k^{\max}} d_k(\tau)q^{k(\alpha-1)}}{Q(\tau)^{\alpha-1}} = 1$ by equation (2.5) and therefore

$$L^X(\tau) = P(\tau)^\alpha Q(\tau)^\alpha \frac{Y(\tau)}{Q(\tau)}\int_{-\infty}^{\tau} f(s,\tau)\,p^*(s)^{-\alpha}ds \tag{2.39}$$

A price in effect at time τ dates from time s if there was either an innovation *or* a price reset signal at time s *and* if there was no innovation between times s and τ *and* if no price reset signal was received in the same period. As explained in Appendix 2.A.1, the probability of no innovation *and* no price reset signal between times s and τ is $e^{-(\mu+\beta)(\tau-s)}$. The flow probability of an innovation *or* a price reset signal occurring at time s is $\mu+\beta$. Thus, we have $f(s,\tau) = (\mu+\beta)\,e^{-(\mu+\beta)(\tau-s)}$.[68] Using this and steady growth of p^* at rate ω in equation (2.39), we have

$$L^X(\tau) = \left[\frac{p^*(\tau)}{P(\tau)Q(\tau)}\right]^{-\alpha}\frac{Y(\tau)}{Q(\tau)}\int_{-\infty}^{\tau}(\mu+\beta)\,e^{-(\mu+\beta)(\tau-s)}e^{(\alpha\omega)(\tau-s)}ds$$

[68] Note that the two Poisson processes governing innovations and the occurrence of price adjustment sigals are stochastically independent so that that the joint probability is the product of the individual probabilities.

Solving the integral which converges for $\mu + \beta > \alpha\omega$ leads to (2.12) in the main text.

2.A.3 Derivation of equation (2.13)

Analogously to the procedure in Appendix 2.A.2, the final good price (2.4) at steady state can be rewritten as

$$P(\tau)^{1-\alpha} = Q(\tau)^{(\alpha-1)} (\mu+\beta) \int_{-\infty}^{\tau} e^{-(\mu+\beta)(\tau-s)} p^{*}(s)^{1-\alpha} ds$$

From (2.8), we have that the optimal price at steady state grows at rate ω. Convergence of the integral is then ensured by assumption (2.38). Using the growth rate of the optimal price ω, we can calculate the integral's value and rearrange terms yielding equation (2.13).

2.A.4 Derivation of the growth rate of the Quality index

The expected growth rate of the aggregate quality index is

$$E\left[\widehat{Q(\tau)}\right] = \frac{1}{\alpha-1} E\left[\widehat{\sum_{j=1}^{N} q^{(\alpha-1)k_j(\tau)}}\right] \tag{2.40}$$

where a hat denotes the proportional growth rate of variable x, $\widehat{x} = \frac{dx/dt}{x}$. The growth rate of the sum in equation (2.40) is equal to the sum of the individual growth rates weighted with the sector's share in the aggregate quality index[69]:

$$E\left[\widehat{Q(\tau)}\right] = \frac{1}{\alpha-1} \sum_{j=1}^{N} E\left[\widehat{q^{(\alpha-1)k_j(\tau)}} \frac{q^{(\alpha-1)k_j(\tau)}}{\sum_{j=1}^{N} q^{(\alpha-1)k_j(\tau)}}\right]$$

Using that the expected value of the product of two random variables is given by $E[XY] = E[X]E[Y] + Cov[XY]$, where $Cov[XY]$ denotes the

[69] For example, $\widehat{X+Y} = \widehat{X}\frac{X}{X+Y} + \widehat{Y}\frac{Y}{X+Y}$.

covariance between X and Y, gives

$$E\left[\widehat{Q(\tau)}\right] = \frac{1}{\alpha-1}\sum_{j=1}^{N} E\left[\widehat{q^{(\alpha-1)k_j(\tau)}}\right] E\left[\frac{q^{(\alpha-1)k_j(\tau)}}{\sum_{j=1}^{N} q^{(\alpha-1)k_j(\tau)}}\right]$$
$$+\frac{1}{\alpha-1}\sum_{j=1}^{N} Cov\left[\widehat{q^{(\alpha-1)k_j(\tau)}}\frac{q^{(\alpha-1)k_j(\tau)}}{\sum_{j=1}^{N} q^{(\alpha-1)k_j(\tau)}}\right]$$

The expected proportional change at time τ of sector j's contribution to the index is

$$E(\Delta\text{qual}_\tau^j) = \mu\left(\frac{q^{(\alpha-1)(k_j(\tau)+1)} - q^{(\alpha-1)k_j(\tau)}}{q^{(\alpha-1)k_j(\tau)}}\right) = \mu\left(q^{\alpha-1}-1\right)$$

Using this, we have

$$E\left[\widehat{Q(\tau)}\right] = \sum_{j=1}^{N}\mu\frac{q^{\alpha-1}-1}{\alpha-1}E\left[\frac{q^{(\alpha-1)k_j(\tau)}}{\sum_{j=1}^{N} q^{(\alpha-1)k_j(\tau)}}\right]$$
$$+\frac{1}{\alpha-1}\sum_{j=1}^{N} Cov\left[\widehat{q^{(\alpha-1)k_j(\tau)}}\frac{q^{(\alpha-1)k_j(\tau)}}{\sum_{j=1}^{N} q^{(\alpha-1)k_j(\tau)}}\right]$$

As was shown in section 2.2.4, the probability of an innovation being made and thus $\widehat{q^{(\alpha-1)k_j(\tau)}}$ is independent of sector j's position relative to the other sectors, $\frac{q^{(\alpha-1)k_j(\tau)}}{\sum_{j=1}^{N} q^{(\alpha-1)k_j(\tau)}}$. Thus, $Cov\left[\widehat{q^{(\alpha-1)k_j(\tau)}}\frac{q^{(\alpha-1)k_j(\tau)}}{\sum_{j=1}^{N} q^{(\alpha-1)k_j(\tau)}}\right] = 0$. Consequently,

$$E\left[\widehat{Q(\tau)}\right] = \mu\frac{q^{\alpha-1}-1}{\alpha-1}$$

The Law of large numbers implies that for a large number N of sectors, the actual growth rate of the quality index, $\gamma_Q(\tau)$, converges to the expected growth rate, $E\left[\widehat{Q(\tau)}\right]$. More precisely, a standard version of the law of large numbers for independent and identically distributed variables $\{Y_t\}$ with $E[Y_t] = \mu$ and variance $E\left[(Y_t-\mu)^2\right] = \sigma^2$ states that the sample mean $\overline{Y}_T = (1/T)\sum_{t=1}^{T} Y_t$ converges in probability to the population mean μ. $\overline{Y}_T$

has expectation μ and variance $E\left[\left(\overline{Y}_T-\mu\right)^2\right]=\sigma^2/T$.[70] This variance goes to zero as $T\to\infty$ implying $\overline{Y}_T\xrightarrow{p}\mu$. In the present context, the weighting factors $(q^{(\alpha-1)k_j(\tau)}/Q(\tau)^{(\alpha-1)})$ are not constant as in the standard case $(1/T)$. Yet their being small and summing up to unity suffices for the result to carry over to our case. With this, we have equation (2.20) in the main text.

2.A.5 Proofs of propositions 1-4

Proof of proposition 1. Given that we assumed $\eta\geq 1$, the RHS of equation (2.36) increases in μ as depicted in fig. 2.1. Further, the LHS of equation (2.36) is concave in μ for $\psi>0$, $\frac{\partial^2 LHS}{\partial\mu^2}=-\frac{2\alpha\psi}{(\mu+\beta)^2}\left[\frac{q^{\alpha-1}}{\alpha-1}+\left(\frac{L}{\lambda}-\mu\frac{q^{\alpha-1}}{\alpha-1}\right)\frac{1}{\mu+\beta}\right]<0$, while it decreases in μ at non-positive money growth rates, $\frac{\partial LHS}{\partial\mu}=\left[-\frac{q^{\alpha-1}}{\alpha-1}(\mu+\beta)+\frac{\alpha\psi}{\mu+\beta-\alpha\psi}\left(\frac{L}{\lambda}-\mu\frac{q^{\alpha-1}}{\alpha-1}\right)\right]\frac{\mu+\beta-\alpha\psi}{(\mu+\beta)^2}<0$ for $\psi\leq 0$. With conditions (2.37) and (2.38) we make sure that the value for $\mu\to 0$ of the LHS of equation (2.36) is larger than that of the RHS. Further, note that for $\mu\to\infty$, the value of the RHS exceeds that of the LHS: $\lim\limits_{\mu\to\infty}\left(\frac{L}{\lambda}-\mu\frac{q^{\alpha-1}}{\alpha-1}\right)\left(1-\frac{\alpha\psi}{\mu+\beta}\right)=-\infty<\infty=\lim\limits_{\mu\to\infty}(\rho+\widetilde{\eta}\mu)\left(1-\frac{\alpha\psi}{\rho+\beta+\widetilde{\eta}\mu}\right)$ where $\widetilde{\eta}=\left(1+(\eta+\alpha-2)\frac{q^{\alpha-1}-1}{\alpha-1}\right)$. Thus the two functions have a unique intersection with $\mu>0$. ■

For the sake of completeness, we give a short description of the two additional effects on instantaneous profits of an increase in μ under price rigidity ($\beta<\infty$, $\psi\neq 0$): Firstly, an increase in the frequency of innovation reduces price dispersion because any innovative firm replacing the incumbent to produce a new variety of intermediate good j can set a new price. This reduces production inefficiency, raising the average productivity of intermediates in final good production and therefore, demand for intermediate goods and a new intermediate producer's profits. Secondly, at $\psi>0$ ($\psi<0$) the increase in μ reduces (increases) the firm's initial relative price since a higher probability of being replaced shifts more weight to present as opposed to future profits in the firm's mark-up. This increases (reduces) demand for the good and profits.

Proof of proposition 2. For $\beta\to\infty$, the zero profit condition (2.36)

[70] See e.g. Hamilton [1994].

reduces to $\frac{L}{\lambda} - \mu\frac{q^{\alpha-1}}{\alpha-1} = \rho + \left(1 + (\eta + \alpha - 2)\frac{q^{\alpha-1}-1}{\alpha-1}\right)\mu$ so that the growth rate of money ψ has no influence on the equilibrium research intensity μ. Since by equation (2.21) $\gamma = \frac{q^{\alpha-1}-1}{\alpha-1}\mu$, the economy's real growth rate γ is independent of ψ, too. ■

Proof of proposition 3. Consider equation (2.36). To see that $\frac{d\mu}{d\psi} = -\frac{\frac{\partial LHS}{\partial\psi} - \frac{\partial RHS}{\partial\psi}}{\frac{\partial LHS}{\partial\mu} - \frac{\partial RHS}{\partial\mu}} < 0$, first refer to figure 2.1 to see that given assumptions (2.37)-(2.38) and concavity of the LHS-curve, the latter's slope is always smaller than that of the RHS-curve at the equilibrium ($\frac{\partial LHS}{\partial\mu} - \frac{\partial RHS}{\partial\mu} < 0$). Further, $\frac{\partial LHS}{\partial\psi} - \frac{\partial RHS}{\partial\psi} = -\frac{\alpha(\frac{L}{\lambda} - \mu\frac{q^{\alpha-1}}{\alpha-1})[\rho + (\widetilde{\eta}-1)\mu]}{(\mu+\beta)(\rho+\beta-\alpha\psi+\widetilde{\eta}\mu)} < 0$ since $1 < \widetilde{\eta} = \left(1 + (\eta + \alpha - 2)\frac{q^{\alpha-1}-1}{\alpha-1}\right)$ given $\eta \geq 1$, $\beta > \alpha\psi$ from assumption (2.38) and $L/\lambda - \mu q^{\alpha-1}/\left(\alpha - 1\right) > 0$ since final good production is positive. Thus the negative effects of an increase in money growth dominate for the very reasons (assumption (2.38)) that ensure the uniqueness of the steady state equilibrium. Further, since $\gamma = \frac{q^{\alpha-1}-1}{\alpha-1}\mu$, $\frac{d\gamma}{d\psi} = \frac{q^{\alpha-1}-1}{\alpha-1}\frac{d\mu}{d\psi} < 0$. ■

Proof of proposition 4. $\frac{d\mu}{d\beta} = -\frac{\frac{\partial LHS}{\partial\beta} - \frac{\partial RHS}{\partial\beta}}{\frac{\partial LHS}{\partial\mu} - \frac{\partial RHS}{\partial\mu}} \gtreqless 0$ for $\psi \gtreqless 0$ since $\frac{\partial LHS}{\partial\beta} - \frac{\partial RHS}{\partial\beta} = \alpha\psi\frac{\rho+\widetilde{\eta}\mu}{\rho+\widetilde{\eta}\mu+\beta}\frac{[(\rho+\beta+\widetilde{\eta}\mu)+(\mu+\beta-\alpha\psi)][\rho+(\widetilde{\eta}-1)\mu]}{(\mu+\beta-\alpha\psi)(\mu+\beta)(\rho+\widetilde{\eta}\mu+\beta)} \gtreqless 0$ for $\psi \gtreqless 0$ is positive as $\beta > \alpha\psi$ and $\widetilde{\eta} > 1$ given assumption (2.38). ■

Chapter 3

Local Stability

3.1 Introduction

In the preceding chapters, the Schumpeterian model with nominal price rigidity was analysed with regard to the comparative static properties of its unique steady state equilibrium. This analysis delivers relevant predictions for the model economy's long run behaviour only if the economy converges to the steady state in the long run, i.e. if the steady state is stable. To underscore the relevance of our approach, we therefore examine the stability of the steady state in this chapter. Given that the dynamical system describing the economy's behaviour under price rigidity and endogenously determined growth off steady state contains a large number of variables, this is a very challenging task. Neither can its global behaviour be analysed nor can we derive analytical conditions for its stability in the neighbourhood of the steady state. We therefore proceed by reporting the local stability properties of a large number of calibrated examples. In order to derive the local stability properties of the model several steps are necessary: In section 3.2, the model's key equations are adjusted to take account of the off-steady state behaviour of the variables. In the process, variables which have positive growth rates in the model's original steady state are transformed by normalising them such that all relevant variables are constant at steady state. As we limit our analysis to the local dynamics, the equations are linearised around the steady state using a first order Taylor approximation. As discussed in chapter 1, this is the standard approach used in the analysis of New Keynesian Business Cycle or NNS models. Note, however, that in contrast to the NNS approach, our interest here lies not in generating impulse responses to simulated shocks in order to characterise the model economy's behaviour off steady state quantitatively. Our exclusive aim is the qualitative characterisation of the steady state's local stability through the analysis of the dynamical system's eigenvalues. For this purpose, the eight equations describing the economy's behaviour in the neighbourhood of the steady state are reduced to a dynamical system in six variables in the Appendix. Section 3.3 presents the reduced dynamical system and the results of our stability analysis: The eigenvalues associated with the reduced system determine the steady state's local stability. We find that for all parameter constellations examined, the steady state is locally stable, so that our analysis of the steady state's comparative static properties is indeed relevant for the description of the model economy's long-run behaviour. Interestingly, for all parameter constellations the equilibrium is also indeterminate, i.e. for given initial conditions the convergent path to the steady state is not unique.

For the linearisation, we use the following notation: The steady state value of the variable $x(\tau)$ is denoted by dropping the time index, so that $[x(\tau)-x]$ denotes the deviation of the variable from steady state and $\overset{\cdot}{[x(\tau)-x]}$ is the time derivative of this expression.

3.2 Key equations for the economy's off-steady-state behaviour

3.2.1 Firms

3.2.1.1 Intermediate goods sector: Optimal intermediate goods price

Relaxing the assumptions that the interest rate $i(\tau)$, the intensity of innovation $\mu(\tau)$ and the rates of growth of output, wages and prices, $\gamma(\tau)$, $\omega(\tau)$

and $\pi(\tau)$ are all constant at steady state, the optimal price chosen by an intermediate goods firm allowed to reset its price is given by[1]

$$P^*(\tau) = \frac{\alpha}{\alpha-1} \frac{\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}[i(\theta)+\mu(\theta)+\beta]d\theta} P(s)^{\alpha} w(s) Y(s) ds}{\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}[i(\theta)+\mu(\theta)+\beta]d\theta} P(s)^{\alpha} Y(s) ds} \tag{3.1}$$

We aim to linearise the model around a steady state where all relevant variables are constant. We therefore define the normalised variables $\widetilde{w}(\tau) = \frac{w(\tau)}{P(\tau)Q(\tau)}$ and $Y(\tau) = \frac{Y(\tau)}{Q(\tau)}$ which are constant at steady state. Using these and dividing both sides of equation (3.1) by $P(\tau)Q(\tau)$ yields

$$\widetilde{P^*}(\tau) = \frac{\alpha}{\alpha-1} \frac{\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}[i(\theta)+\mu(\theta)+\beta]d\theta} \left(\frac{P(s)}{P(\tau)}\right)^{\alpha} \widetilde{w}(s) \frac{P(s)Q(s)}{P(\tau)Q(\tau)} \frac{Y(s)}{Q(s)} \frac{Q(s)}{Q(\tau)} ds}{\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}[i(\theta)+\mu(\theta)+\beta]d\theta} \left(\frac{P(s)}{P(\tau)}\right)^{\alpha} \frac{Y(s)}{Q(s)} \frac{Q(s)}{Q(\tau)} ds}$$

Using the fact that $\frac{X(s)}{X(\tau)} = \frac{X(0)e^{\int_{\theta=0}^{s}\gamma_X(\theta)d\theta}}{X(0)e^{\int_{\theta=0}^{\tau}\gamma_X(\theta)d\theta}} = e^{\int_{\theta=\tau}^{s}\gamma_X(\theta)d\theta}$, where $\gamma_X(\theta)$ is the growth rate of X at time θ, this can be rewritten as

$$\widetilde{P^*}(\tau) = \frac{\alpha}{\alpha-1} \frac{\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}\left[i(\theta)+\mu(\theta)-(\alpha+1)\pi(\theta)+\beta-2\gamma_Q(\theta)\right]d\theta} \widetilde{w}(s) \frac{Y(s)}{Q(s)} ds}{\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}\left[i(\theta)+\mu(\theta)+\beta-\alpha\pi(\theta)-\gamma_Q(\theta)\right]d\theta} \frac{Y(s)}{Q(s)} ds} \tag{3.2}$$

[1] Note that the relevant probability that the firm has not been replaced and has not received a pricing signal by time τ given that the last innovation was at time t_{k_j}, $\widetilde{B}e^{-\int_{\theta=t_{k_j}}^{\tau}(\mu(\theta)+\beta)d\theta}$, is unchanged, except for the fact that $\mu(\tau)$ need not be constant outside steady state. Note that the unchanged razor's edge condition from the research sector continues to hold, so that $\mu(\tau)$ is the same for all sectors j in equilibrium, which we use here.

The equation is linearised in Appendix 3.A.1,[2] yielding

$$
\begin{aligned}
\overset{\cdot}{\left[\widetilde{P^*}(\tau)-\widetilde{P^*}\right]} = & +\left(\frac{\widetilde{P^*}^{1-\alpha}}{L_2}\frac{Y}{Q}-\alpha\psi\right)\left[\widetilde{P^*}(\tau)-\widetilde{P^*}\right] \\
& -\frac{\widetilde{P^*}}{L_2}\psi\left[L_2(\tau)-L_2\right] \\
& +\widetilde{P^*}\frac{1}{\frac{Y}{Q}}\psi\left[\frac{Y(\tau)}{Q(\tau)}-\frac{Y}{Q}\right] \\
& -\frac{\widetilde{P^*}^{1-\alpha}}{L_2}\frac{Y}{Q}\frac{\alpha}{\alpha-1}\left[\widetilde{w}(\tau)-\widetilde{w}\right] \\
& -\widetilde{P^*}\left[\pi(\tau)-\pi\right] \\
& -\widetilde{P^*}\frac{q^{\alpha-1}-1}{\alpha-1}\left[\mu(\tau)-\mu\right]
\end{aligned}
\tag{3.3}
$$

where $L_2(\tau) = \widetilde{P^*}(\tau)^{1-\alpha}\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}\left[i(\theta)+\mu(\theta)+\beta-\alpha\pi(\theta)-\gamma_Q(\tau)\right]d\theta}\frac{Y(s)}{Q(s)}ds$ and $\frac{L_2(\tau)}{\widetilde{P^*}(\tau)}$ reflects the development of the firm's revenues.[3] For later reference, the linearised version of $L_2(\tau)$ is[4]

$$
\begin{aligned}
\overset{\cdot}{\left[L_2(\tau)-L_2\right]} = & L_2(1-\alpha)\widetilde{P^*}^{-1}\overset{\cdot}{\left[\widetilde{P^*}(\tau)-\widetilde{P^*}\right]} \\
& +L_2\left[r(\tau)-r\right] \\
& +L_2\left(1-\frac{q^{\alpha-1}-1}{\alpha-1}\right)\left[\mu(\tau)-\mu\right] \\
& -L_2(\alpha-1)\left[\pi(\tau)-\pi\right] \\
& +\left[\beta+h-(\alpha-1)\psi\right]\left[L_2(\tau)-L_2\right] \\
& -\widetilde{P^*}^{1-\alpha}\left[\frac{Y(\tau)}{Q(\tau)}-\frac{Y}{Q}\right] \\
& -(1-\alpha)\widetilde{P^*}^{-\alpha}\frac{Y}{Q}\left[\widetilde{P^*}(\tau)-\widetilde{P^*}\right]
\end{aligned}
$$

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[2] Note that for this and the following linearisations, the fact that $\gamma_Q(\tau)$ is proportional to $\mu(\tau)$ by equation (3.13) is used to replace the former variable.

[3] See the denominators in the fractions in equations (3.1) and (3.2).

[4] See the Appendix for details on the linearisation.

3.2.1.2 An intermediate good producer's market value at market entrance

Normalising variables as before and using the shorthand notation $E(V_\tau) = E\left(V_{k_j}(\tau)\,\middle|\,t_{k_j} = \tau\right)$, the market value given by the present value at market entrance in t_{k_j} of all future profits of the firm making an innovation at time τ is

$$\begin{aligned}\frac{E(V_\tau)}{P(\tau)Q(\tau)} &= \left(\frac{q^{k_j+1}}{Q(\tau)}\right)^{\alpha-1}\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{Y(s)}{Q(s)}\left(\frac{P_{k_j}(s)}{P(s)Q(s)}\right)^{1-\alpha} ds \\ &- \left(\frac{q^{k_j+1}}{Q(\tau)}\right)^{\alpha-1}\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{Y(s)}{Q(s)}\widetilde{w}(s)\left(\frac{P_{k_j}(s)}{P(s)Q(s)}\right)^{-\alpha} ds\end{aligned} \tag{3.4}$$

where $h(\theta) = i(\theta) + \mu(\theta) - \pi(\theta) + (\alpha - 2)\gamma_Q(\theta)$.

Taking account of the future development of the firm's normalised price $P_{k_j}(s)/[P(s)Q(s)]$ by using the probability density for pricing signals, we can rewrite equation (3.4) as a function of the development of the normalised optimal price $\widetilde{P^*}(s)$:[5]

$$\begin{aligned}\frac{E(V_\tau)}{A_1(\tau)} &= \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{Y(s)}{Q(s)}\left[\int\limits_{\zeta=\tau}^{s} \beta e^{-\int_{t=\zeta}^{s}[\beta-(\alpha-1)\widetilde{\pi}(t)]dt}\widetilde{P^*}(\zeta)^{1-\alpha} d\zeta\right] ds \\ &+ \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{Y(s)}{Q(s)}\left[e^{-\int_{t=\tau}^{s}[\beta-(\alpha-1)\widetilde{\pi}(t)]dt}\widetilde{P^*}(\tau)^{1-\alpha}\right] ds \\ &- \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{Y(s)}{Q(s)}\widetilde{w}(s)\left[\int\limits_{\zeta=\tau}^{s} \beta e^{-\int_{t=\zeta}^{s}[\beta-\alpha\widetilde{\pi}(t)]dt}\widetilde{P^*}(\zeta)^{-\alpha} d\zeta\right] ds \\ &- \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{Y(s)}{Q(s)}\widetilde{w}(s)\left[e^{-\int_{t=\tau}^{s}[\beta-\alpha\widetilde{\pi}(t)]dt}\widetilde{P^*}(\tau)^{-\alpha}\right] ds\end{aligned} \tag{3.5}$$

[5] As this involves a number of tedious steps we present these calculations in Appendix 3.A.2.

where $A_1(\tau) = \left(\frac{q^{k_j+1}}{Q(\tau)}\right)^{\alpha-1} P(\tau) Q(\tau)$, $\widetilde{\pi}(\tau) = \pi(\tau) + \gamma_Q(\tau)$ and h is the constant steady state value of $h(\tau)$.

3.2.1.3 Final good sector: Price level

Taking into account that $\mu(\tau)$ is not constant outside steady state, the price level (2.4) can analogously to the procedure in Appendix 2.A.3 be rewritten as

$$P(\tau)^{1-\alpha} = Q(\tau)^{(\alpha-1)} \int_{-\infty}^{\tau} [\mu(s) + \beta] e^{-\int_{\theta=s}^{\tau} [\mu(\theta)+\beta] d\theta} P^*(s)^{1-\alpha} ds$$

In this, we follow the literature[6] in making the assumption that the price level can in and near steady state be rewritten as a weighted average of past optimal prices, which here implies that the structure of past prices for a given quality level q^k is approximately the same as the structure for all quality levels.[7]

We rewrite the equation in terms of the normalised optimal price $\widetilde{P^*}(s) = P^*(s) / [P(s) Q(s)]$ which is constant at steady state:

$$1 = \int_{-\infty}^{\tau} [\mu(s) + \beta] e^{-\int_{\theta=s}^{\tau} [\mu(\theta)+\beta] d\theta} \left[\frac{P^*(s)}{P(s)Q(s)}\right]^{1-\alpha} \left[\frac{P(s)Q(s)}{P(\tau)Q(\tau)}\right]^{1-\alpha} ds$$

which can be rewritten as

$$1 = \int_{-\infty}^{\tau} [\mu(s) + \beta] e^{-\int_{\theta=s}^{\tau} \left[\mu(\theta)+\beta-(\alpha-1)\left(\pi(\theta)+\gamma_Q(\theta)\right)\right] d\theta} \widetilde{P^*}(s)^{1-\alpha} ds \tag{3.6}$$

where $\gamma_Q(\tau)$ is the off-steady-state growth rate of the quality index Q at time τ.[8]

[6] See Benhabib, Schmitt-Grohé and Uribe [2001a,2001b], Leith and Wren-Lewis [2000] and Wolman [1999]).

[7] While this is indeed a simplification for a general local stability analysis, it is a natural assumption for the analysis of local stability in the following situation: Think of an economy that has been in steady state – where the incentive to innovate and the past distribution of prices are indeed independent of a sector's quality rung – for a long time and is then moved into the neighbourhood of the steady state by a slight perturbation.

[8] Note that off the steady state, $\gamma(\tau) = \gamma_Q(\tau)$ need not hold.

Differentiating equation (3.6) with respect to time results in

$$0 = \left[\mu(\tau) + \beta\right] \widetilde{P^*}(\tau)^{1-\alpha} - \left[\mu(\tau) + \beta - (\alpha - 1)\left(\pi(\tau) + \gamma_Q(\tau)\right)\right]$$

Linearising this equation around the steady state yields

$$\begin{aligned}\left[\widetilde{P^*}(\tau) - \widetilde{P^*}\right] = {} & \frac{\widetilde{P^*}}{\mu + \beta - (\alpha - 1)\psi}\left[\pi(\tau) - \pi\right] \\ & + \frac{\widetilde{P^*}}{\mu + \beta - (\alpha - 1)\psi}\left(\frac{q^{\alpha-1} - 1}{\alpha - 1} - \frac{\psi}{\mu + \beta}\right)\left[\mu(\tau) - \mu\right] \end{aligned} \quad (3.7)$$

3.2.1.4 The R&D zero profit condition

Inserting $\phi(k_j(\tau))$ from equation (2.18) and a new intermediate firm's expected market value (3.5) into the zero profit equation in the research sector (2.17) and linearising, we have[9]

$$\begin{aligned}\left[\dot{\widetilde{w}}(\tau) - \widetilde{w}\right] = {} & \left[h + (\alpha - 1)\frac{1}{\lambda}\frac{Y}{Q}\widetilde{P^*}^{-\alpha}\right]\left[\widetilde{w}(\tau) - \widetilde{w}\right] \\ & + (\alpha - 1)\frac{1}{\lambda}\frac{Y}{Q}\widetilde{P^*}^{-\alpha}\left[\alpha\widetilde{P^*}^{-1}\left(\widetilde{P^*} - \widetilde{w}\right) - 1\right]\left[\widetilde{P^*}(\tau) - \widetilde{P^*}\right] \\ & + \widetilde{w}\left[1 + (\alpha - 2)\frac{q^{\alpha-1} - 1}{\alpha - 1}\right]\left[\mu(\tau) - \mu\right] \\ & - (\alpha - 1)\frac{1}{\lambda}\widetilde{P^*}^{-\alpha}\left(\widetilde{P^*} - \widetilde{w}\right)\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \\ & + \widetilde{w}\left[r(\tau) - r\right] \end{aligned} \quad (3.8)$$

where $h = r + \mu + (\alpha - 2)\gamma$ is the firm's steady state flex-price discount rate.

3.2.2 Labour market equilibrium

Equilibrium in the labour market requires that the sum of the labour demands of the intermediate goods sector, $L^X(\tau)$, and of the research sector,

[9] For details on the linearisation see Appendix 3.A.3.

$L^R(\tau)$, equal labour supply L. Demand for research labour is given by

$$L^R(\tau) = \lambda \frac{q^{\alpha-1}}{\alpha - 1} \mu(\tau) \tag{3.9}$$

The intermediate sector's labour demand is

$$L^X(\tau) = \frac{Y(\tau)}{Q(\tau)} \int_{-\infty}^{\tau} [\mu(s) + \beta] \, e^{-\int_{\theta=s}^{\tau}[\mu(\theta)+\beta]d\theta} \widetilde{P*}(s)^{-\alpha} \left[\frac{P(s)Q(s)}{P(\tau)Q(\tau)}\right]^{-\alpha} ds \tag{3.10}$$

Using the labour demands (3.9) and (3.10), the labour market equilibrium equation is

$$L - \mu(\tau)\lambda \frac{q^{\alpha-1}}{\alpha-1} = \frac{Y(\tau)}{Q(\tau)} \int_{-\infty}^{\tau} [\mu(s)+\beta] \, e^{-\int_{\theta=s}^{\tau}\left[\mu(\theta)-\alpha\pi(\theta)-\alpha\gamma_Q(\theta)+\beta\right]d\theta} \widetilde{P*}(s)^{-\alpha} ds \tag{3.11}$$

It is shown in Appendix 3.A.3.1 that linearising this equation gives

$$\begin{aligned}
\dot{[\mu(\tau) - \mu]} &= -(\lambda\widetilde{q})^{-1} \frac{\widehat{\beta}}{\widetilde{\mu}} \widetilde{P*}^{-\alpha} \dot{\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right]} \\
&\quad -\alpha(\lambda\widetilde{q})^{-1} \frac{\widehat{\beta}}{\widetilde{\mu}} \frac{Y}{Q} \widetilde{P*}^{-\alpha} [\pi(\tau) - \pi] \\
&\quad - \left\{ \widetilde{\mu} + (\lambda\widetilde{q})^{-1} \alpha \frac{\widehat{\beta}}{\widetilde{\mu}} \frac{Y}{Q} \widetilde{P*}^{-\alpha} \left(\widehat{q} - \widehat{\psi}\right) \right\} [\mu(\tau) - \mu] \\
&\quad -(\lambda\widetilde{q})^{-1} \widehat{\beta} \widetilde{P*}^{-\alpha} \left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \\
&\quad +\alpha(\lambda\widetilde{q})^{-1} \frac{Y}{Q} \widehat{\beta} \widetilde{P*}^{-\alpha-1} \left[\widetilde{P*}(\tau) - \widetilde{P*}\right]
\end{aligned} \tag{3.12}$$

where we introduce the shorthand notation $\widehat{q} = \frac{q^{\alpha-1}-1}{\alpha-1}$, $\widetilde{q} = \frac{q^{\alpha-1}}{\alpha-1}$, $\widetilde{\mu} = (\beta + \mu - \alpha\psi)$, $\widehat{\psi} = \frac{\psi}{\mu+\beta}$, $\widehat{\beta} = \mu + \beta$.

3.2.2.1 The aggregate quality index $Q(\tau)$

Taking into account that $\mu(\tau)$ is not constant outside steady state, the equation relating the growth rate of Q to the research intensity μ is:

$$\gamma_Q(\tau) = \frac{q^{\alpha-1} - 1}{\alpha - 1}\mu(\tau) \tag{3.13}$$

In deriving this, we make the simplifying assumption that the Law of Large Numbers[10] holds approximately in the neighbourhood of the steady state, implying that for a large number N of sectors, the actual growth rate of the quality index, γ_Q, converges to the expected growth rate. We use equation (3.13) to replace γ_Q in the other equilibrium relationships.

3.2.3 Households

Both consumption $c(\tau)$ and real money holdings $m(\tau)$ grow at rate γ at steady state. We normalise by dividing through the quality index $Q(\tau)$, such that we have the new variables $\frac{c(\tau)}{Q(\tau)}$ and $\frac{m(\tau)}{Q(\tau)}$ which are constant at steady state. The household's off-steady-state static optimality condition then reads:

$$\frac{\theta}{1-\theta}\frac{\frac{c(\tau)}{Q(\tau)}}{\frac{m(\tau)}{Q(\tau)}} = r(\tau) + \pi(\tau) \tag{3.14}$$

Linearly approximating both sides yields

$$\begin{aligned} \frac{\theta}{1-\theta}\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] &= (r+\pi)\left[\frac{m(\tau)}{Q(\tau)} - \frac{m}{Q}\right] \\ &\quad + \frac{m}{Q}\left[r(\tau) - r\right] \\ &\quad + \frac{m}{Q}\left[\pi(\tau) - \pi\right] \end{aligned} \tag{3.15}$$

where equilibrium in the output market was used to replace c/Q with equilibrium in the output market Y/Q. Noting that $\frac{\dot{c}(\tau)}{c(\tau)} - \gamma_Q(\tau) = \left(\frac{\dot{c(\tau)}}{Q(\tau)}\right)\left(\frac{c(\tau)}{Q(\tau)}\right)^{-1}$

[10] See Appendix 2.A.4 in chapter 2.

and $\frac{\dot{m(\tau)}}{m(\tau)} - \gamma_Q(\tau) = \left(\frac{m(\tau)}{Q(\tau)}\right)^{\cdot} \left(\frac{m(\tau)}{Q(\tau)}\right)^{-1}$, the household's dynamic efficiency condition (2.30) can be rewritten in terms of the normalised variables c/Q and m/Q. Linearising both sides, we have

$$\begin{aligned}
\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right]^{\cdot} \;&=\; \frac{\theta(1-\eta)}{\eta+\theta(1-\eta)} \frac{\frac{Y}{Q}}{\frac{m}{Q}} \left[\frac{m(\tau)}{Q(\tau)} - \frac{m}{Q}\right]^{\cdot} \\
&\quad + \frac{\frac{Y}{Q}}{\eta+\theta(1-\eta)} \left[r(\tau) - r\right] \\
&\quad - \frac{\eta}{\eta+\theta(1-\eta)} \frac{Y}{Q} \widehat{q} \left[\mu(\tau) - \mu\right] \qquad (3.16)
\end{aligned}$$

where Y/Q was again substituted for c/Q.

3.2.4 Money market equilibrium

Normalising money demand as $\frac{m(\tau)}{Q(\tau)}$, equilibrium in the money market is characterised by

$$\frac{\left(\frac{m(\tau)}{Q(\tau)}\right)^{\cdot}}{\frac{m(\tau)}{Q(\tau)}} = \psi - \pi(\tau) - \gamma_Q(\tau) \qquad (3.17)$$

Linearising this equation yields

$$\left[\frac{m(\tau)}{Q(\tau)} - \frac{m}{Q}\right]^{\cdot} = -\frac{m}{Q}\left[\pi(\tau) - \pi\right] - \frac{m}{Q}\widehat{q}\left[\mu(\tau) - \mu\right] \qquad (3.18)$$

3.3 The linearised dynamical system and local stability of the steady state equilibrium

The eight equations describing the linearised off-steady state behaviour of the optimal price (3.3), and the revenue variable L_2 (3.22), the price index (3.7), the research intensity (3.8), the labour market (3.12), the household's behaviour (3.16) and (3.15) and the money market (3.18), respectively, form a system in the eight variables $\frac{Y(\tau)}{Q(\tau)}$, $\frac{m(\tau)}{Q(\tau)}$, $L_2(\tau)$, $\mu(\tau)$, $\widetilde{P}^*(\tau)$, $\widetilde{w}(\tau)$, $r(\tau)$ and

$\pi(\tau)$.[11] Given that the two equations (3.7) and (3.15) are static, however, we can use them to eliminate $\frac{m(\tau)}{Q(\tau)}$ and $\widetilde{P}^*(\tau)$ from the system, resulting in a dynamical system in six variables. To achieve this and in order to write the system in the form $\dot{[\mathbf{x}-\overline{\mathbf{x}}]} = f[\mathbf{x}-\overline{\mathbf{x}}]$, where $\mathbf{x}$ is the vector of variables contained in the system and $\overline{\mathbf{x}}$ is the steady state value of $\mathbf{x}$, some cumbersome manipulations are necessary which can be found in Appendix 3.A.6. Resulting from these is the following system describing the dynamical behaviour of the model economy:

$$\begin{bmatrix} \dot{\overbrace{\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}}} \\ \dot{\overbrace{r(\tau) - r}} \\ \dot{\overbrace{\mu(\tau) - \mu}} \\ \dot{\overbrace{\pi(\tau) - \pi}} \\ \dot{\overbrace{\widetilde{w}(\tau) - \widetilde{w}}} \\ \dot{\overbrace{L_2(\tau) - L_2}} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q} \\ r(\tau) - r \\ \mu(\tau) - \mu \\ \pi(\tau) - \pi \\ \widetilde{w}(\tau) - \widetilde{w} \\ L_2(\tau) - L_2 \end{bmatrix}$$

where the elements of the **6x6** matrix $\mathbf{M}$ can be found in Appendix 3.A.7. The Hartman-Grobman theorem states that the local properties of the linearised dynamical system also hold for the non-linear system if the matrix $\mathbf{M}$ has no root with zero real part.[12] In this case, the steady state $\left(\frac{Y}{Q}, r, \mu, \pi, \widetilde{w}, L_2\right)$ is called a non-degenerate fixed point. Due to the system's complexity, it is not possible to describe the linearised dynamical system's stability properties analytically. We therefore investigate local stability in a number of calibrated examples. In all examined examples, the steady state was non-degenerate so that the linearised system's local properties carry over to the non-linearised system.

Parameters for the examples were chosen from the ranges indicated in

[11] Note that since the household's current wealth does not influence the equilibrium allocation, it is not necessary to keep track here of the evolution of the household's wealth (2.28) which can be determined separately. The transversality condition $\lim_{\tau\to\infty} v(\tau)\, e^{-\int_{\theta=0}^{\tau} r(\theta)d\theta} = 0$ derived from (2.28) holds for any stable steady state equilibrium since at steady state, $v(\tau)$ grows at rate γ and $r - \gamma = \rho + (\eta - 1)\gamma > 0$ given our assumption $\eta \geq 1$.

[12] See Gandolfo [1996], p. 362.

table 3.1:

Parameter	Range	Parameter	Range
α	$6-11$	η	$1-2$
β	$1-2.5$	ρ	$0.01-0.05$
q	$1.01-1.5$	θ	$0.01-0.5$
ψ	$-0.015-0.15$		

Table 3.1: Parameter values used in local stability analysis

The eigenvalues of the matrix **M** were computed for a total of 113 parameter constellations.[13] Of the model's variables, only one is predetermined: Real money supply $m(\tau)/Q(\tau)$. Therefore for the steady state to be locally stable, at least one of the eigenvalues needs to have a negative real part. In order for the system to display saddle-path-stable behaviour, i.e. for the convergent solution to be unique, there needs to be exactly one eigenvalue with a negative real part.[14] In all examples where the eigenvalues were computed, there were two eigenvalues with negative real parts, implying that the steady state is locally stable but the equilibrium is indeterminate, i.e. the convergent solution is not unique.

The existence of many equilibrium paths for the model's variables is an interesting feature since multiple solutions can imply the existence of self-fulfilling expectations which allow for endogenous fluctuations.[15]

Unlike in the two-dimensional case it is not possible to classify the dynamic behaviour of the system according to the sign and real or complex

[13]Note that it can be shown that in the case without price rigidity ($\beta \to \infty$), the dynamical system can be reduced to two dimensions. Since prices are flexible, the variables $m(\tau)/Q(\tau)$ and $Y(\tau)/Q(\tau)$ are both non-predetermined such that the steady state under flexibility is locally stable for any choice of parameters.

While we have not examined other parameter constellations, for logarithmic utility ($\eta = 1$) it can be shown analytically that the steady state is determinate, i.e. saddle-path stable (for systems that contain only jump variables, the constellation with all eigenvalues being positive is the equivalent of saddle path stability, although the stable arm is of dimension zero. See Benhabib, Schmitt-Grohé and Uribe [2001]).

[14]Stability conditions for dynamic systems are presented in Gandolfo [1996]. See also Buiter [1984].

[15]For a careful survey on the causes and consequences of indeterminacy in monetary and more general macroeconomic models, see Benhabib and Farmer [1999].

character of the eigenvalues.[16] Even so, it is interesting to note that the number of pairs of complex eigenvalues increases with the money growth rate.[17] This might imply that the complexity of the model's off-steady state behaviour increases with the size of the distortions caused by price rigidity.[18]

By way of conclusion, irrespective of these considerations, the main result that the steady state is locally stable assures us of the relevance of our analysis' focus on the steady state properties of the Schumpeterian growth model with price rigidity.

3.A Appendix to chapter 3

3.A.1 Linearising the optimal price

We define new variables $H(\tau) = \widetilde{P^*}(\tau) \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}\left[i(\theta)+\mu(\theta)+\beta-\alpha\pi(\theta)-\gamma_Q(\tau)\right]d\theta} \frac{Y(s)}{Q(s)} ds$ and

$K(\tau) = \frac{\alpha}{\alpha-1} \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}\left[i(\theta)+\mu(\theta)-(\alpha+1)\pi(\theta)+\beta-2\gamma_Q(\tau)\right]d\theta} \widetilde{w}(s) \frac{Y(s)}{Q(s)} ds$. Equation (3.2) that can now be rewritten as

$$H(\tau) = K(\tau)$$

In order to derive the linearised version of this equation that depends only on the relevant variables time τ values, we first take the derivative with respect to time τ of both sides of the equation separately and then linearise the

[16] See Gandolfo [1996], pp. 346-358 for the two-dimensional case.

[17] In 21 out of 22 parameter constellations with $\psi = 0$ all the system's eigenvalues were real. The exception ocurred in a case when the intertemporal elasticity of substitution $1/\eta$ took on its maximum value one. Here, there was one pair of complex eigenvalues. In contrast, at the maximum money growth rate investigated for each of the parameter constellations, the negative real parts of eigenvalues were indeed in all cases associated with complex roots.

[18] This observation is limited to the effect of large *positive* money growth rates since the zero bound on nominal interest rate prevents us from analysing large negative money growth rates.

resulting equation in a second step.[19]

Taking the time derivative of $H(\tau)$ yields

$$\begin{aligned}
\dot{H}(\tau) &= \frac{1}{\widetilde{P^*}(\tau)}\dot{\widetilde{P^*}}(\tau)\widetilde{P^*}(\tau)\int_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}\left[i(\theta)+\mu(\theta)+\beta-\alpha\pi(\theta)-\gamma_Q(\theta)\right]d\theta}\frac{Y(s)}{Q(s)}ds \\
&\quad -\widetilde{P^*}(\tau)\frac{Y(\tau)}{Q(\tau)} \\
&\quad +\iota(\tau)\widetilde{P^*}(\tau)\int_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}\left[i(\theta)+\mu(\theta)+\beta-\alpha\pi(\theta)-\gamma_Q(\tau)\right]d\theta}\frac{Y(s)}{Q(s)}ds
\end{aligned}$$

where $\iota(\tau) = \left[i(\tau)+\mu(\tau)+\beta-\alpha\pi(\tau)-\gamma_Q(\tau)\right]$. Taking account of the definition of $H(\tau)$, this reduces to

$$\begin{aligned}
\dot{H}(\tau) &= \left[\frac{1}{\widetilde{P^*}(\tau)}\dot{\widetilde{P^*}}(\tau)+\left[i(\tau)+\mu(\tau)+\beta-\alpha\pi(\tau)-\gamma_Q(\tau)\right]\right]H(\tau) \\
&\quad -\widetilde{P^*}(\tau)\frac{Y(\tau)}{Q(\tau)}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\dot{K}(\tau) &= -\frac{\alpha}{\alpha-1}\widetilde{w}(\tau)\frac{Y(\tau)}{Q(\tau)} \\
&\quad +\left[i(\tau)+\mu(\tau)-(\alpha+1)\pi(\tau)+\beta-2\gamma_Q(\tau)\right]K(\tau)
\end{aligned}$$

With $\dot{H}(\tau)=\dot{K}(\tau)$ we have

$$\begin{aligned}
&\left[\frac{1}{\widetilde{P^*}(\tau)}\dot{\widetilde{P^*}}(\tau)+\left[i(\tau)+\mu(\tau)+\beta-\alpha\pi(\tau)-\gamma_Q(\tau)\right]\right]H(\tau)-\widetilde{P^*}(\tau)\frac{Y(\tau)}{Q(\tau)} \\
&= -\frac{\alpha}{\alpha-1}\widetilde{w}(\tau)\frac{Y(\tau)}{Q(\tau)}+\left[i(\tau)+\mu(\tau)-(\alpha+1)\pi(\tau)+\beta-2\gamma_Q(\tau)\right]K(\tau)
\end{aligned}$$

[19]Note that the result is unchanged by the sequence of steps taken, in particular we could first linearise the equation and then take its time derivative.

Using $H(\tau) = K(\tau)$ and replacing $\gamma_Q(\tau)$ by $\frac{q^{\alpha-1}-1}{\alpha-1}\mu(\tau)$, this reduces to

$$\frac{\dot{\widetilde{P^*}}(\tau)}{\widetilde{P^*}(\tau)} = \frac{1}{H(\tau)}\left[\widetilde{P^*}(\tau) - \frac{\alpha}{\alpha-1}\widetilde{w}(\tau)\right]\frac{Y(\tau)}{Q(\tau)} - \left[\pi(\tau) + \frac{q^{\alpha-1}-1}{\alpha-1}\mu(\tau)\right]$$

We introduce an additional variable $L_2(\tau)$ with $L_2(\tau)\widetilde{P^*}(\tau)^{\alpha} = H(\tau)$ which allows us to eliminate $H(\tau)$:[20]

$$\frac{\dot{\widetilde{P^*}}(\tau)}{\widetilde{P^*}(\tau)} = \frac{\widetilde{P^*}(\tau)^{-\alpha}}{L_2(\tau)}\left[\widetilde{P^*}(\tau) - \frac{\alpha}{\alpha-1}\widetilde{w}(\tau)\right]\frac{Y(\tau)}{Q(\tau)} - \left[\pi(\tau) + \frac{q^{\alpha-1}-1}{\alpha-1}\mu(\tau)\right] \tag{3.19}$$

Linearising, we have

$$\begin{aligned}
\left[\dot{\widetilde{P^*}(\tau) - \widetilde{P^*}}\right] \quad = \quad & \left\{\frac{\widetilde{P^*}^{1-\alpha}}{L_2}\frac{Y}{Q} - \alpha\frac{\widetilde{P^*}^{-\alpha}}{L_2}\frac{Y}{Q}\left(\widetilde{P^*} - \frac{\alpha}{\alpha-1}\widetilde{w}\right)\right\}\left[\widetilde{P^*}(\tau) - \widetilde{P^*}\right] \\
& -\frac{\widetilde{P^*}^{1-\alpha}}{(L_2)^2}\frac{Y}{Q}\left(\widetilde{P^*} - \frac{\alpha}{\alpha-1}\widetilde{w}\right)\left[L_2(\tau) - L_2\right] \\
& +\frac{\widetilde{P^*}^{1-\alpha}}{L_2}\left(\widetilde{P^*} - \frac{\alpha}{\alpha-1}\widetilde{w}\right)\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \\
& -\frac{\widetilde{P^*}^{1-\alpha}}{L_2}\frac{Y}{Q}\frac{\alpha}{\alpha-1}\left[\widetilde{w}(\tau) - \widetilde{w}\right] \\
& -\widetilde{P^*}\left[\pi(\tau) - \pi\right] \\
& -\widetilde{P^*}\frac{q^{\alpha-1}-1}{\alpha-1}\left[\mu(\tau) - \mu\right]
\end{aligned}$$

Using the fact that at steady state, $0 = \frac{\dot{\widetilde{P^*}}}{\widetilde{P^*}} = \frac{\widetilde{P^*}^{-\alpha}}{L_2}\frac{Y}{Q}\left(\widetilde{P^*} - \frac{\alpha}{\alpha-1}\widetilde{w}\right) - \psi$ to simplify the coefficients in the first three lines, we have equation (3.3) in the text.

[20] For details on $L_2(\tau)$ refer to section 3.2.1.1.

3.A.2 A new intermediate good firm's market value

Defining $h(\theta) = r(\theta) + \mu(\theta) + (\alpha - 2)\gamma_Q(\theta)$ and $A_1(\tau) = \left(\frac{q^{k_j}}{Q(\tau)}\right)^{\alpha-1} P(\tau)\,Q(\tau)$, equation (3.4) from the text can be rewritten as

$$E(V_\tau) = A_1(\tau)\left\{\begin{array}{c} \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{Y(s)}{Q(s)}\left(P(s)Q(s)\right)^{\alpha-1} P_{k_j}(s)^{1-\alpha}\,ds \\ -\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{Y(s)}{Q(s)}\widetilde{w}(s)\left(P(s)Q(s)\right)^{\alpha} P_{k_j}(s)^{-\alpha}\,ds \end{array}\right\}$$

Like at steady state, the probability of having last reset one's price due to a pricing signal at time $\zeta \in (\tau, s)$ is $\beta e^{-\beta(s-\zeta)}$. Thus, we have

$$E(V_\tau) = A_1(\tau)\left\{\begin{array}{c} \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{\frac{Y(s)}{Q(s)}}{(P(s)Q(s))^{1-\alpha}}\left[\int\limits_{\zeta=\tau}^{s}\beta e^{-\beta(s-\zeta)}P^*(\zeta)^{1-\alpha}\,d\zeta\right]ds \\ +\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{\frac{Y(s)}{Q(s)}}{(P(s)Q(s))^{1-\alpha}}(1-\int\limits_{\zeta=\tau}^{s}\beta e^{-\beta(s-\zeta)}d\zeta)P^*(\tau)^{1-\alpha}\,ds \\ -\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{\frac{Y(s)}{Q(s)}\widetilde{w}(s)}{(P(s)Q(s))^{-\alpha}}\left[\int\limits_{\zeta=\tau}^{s}\beta e^{-\beta(s-\zeta)}P^*(\zeta)^{-\alpha}\,d\zeta\right]ds \\ -\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{\frac{Y(s)}{Q(s)}\widetilde{w}(s)}{(P(s)Q(s))^{-\alpha}}(1-\int\limits_{\zeta=\tau}^{s}\beta e^{-\beta(s-\zeta)}d\zeta)P^*(\tau)^{-\alpha}\,ds \end{array}\right\}$$

Rewriting the equation in terms of the normalised optimal price

$\widetilde{P^*}(\tau) = P^*(\tau)/[P(\tau)Q(\tau)]$ we have

$$\frac{E(V_\tau)}{A_1(\tau)} = \left\{ \begin{array}{c} \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta} \frac{Y(s)}{Q(s)} \left[\int\limits_{\zeta=\tau}^{s} \beta e^{-\beta(s-\zeta)} \widetilde{P^*}(\zeta)^{1-\alpha} \left(\frac{P(s)Q(s)}{P(\zeta)Q(\zeta)} \right)^{\alpha-1} d\zeta \right] ds \\ + \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta} \frac{Y(s)}{Q(s)} e^{-\beta(s-\tau)} \widetilde{P^*}(\tau)^{1-\alpha} \left(\frac{P(s)Q(s)}{P(\tau)Q(\tau)} \right)^{\alpha-1} ds \\ - \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta} \frac{Y(s)}{Q(s)} \widetilde{w}(s) \left[\int\limits_{\zeta=\tau}^{s} \beta e^{-\beta(s-\zeta)} \widetilde{P^*}(\zeta)^{-\alpha} \left(\frac{P(s)Q(s)}{P(\zeta)Q(\zeta)} \right)^{\alpha} d\zeta \right] ds \\ - \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta} \frac{Y(s)}{Q(s)} \widetilde{w}(s) \left[e^{-\beta(s-\tau)} \widetilde{P^*}(\tau)^{-\alpha} \left(\frac{P(s)Q(s)}{P(\tau)Q(\tau)} \right)^{\alpha} \right] ds \end{array} \right\}$$

which can be rearranged to yield equation (3.5) in the text. For further use in this appendix, we define new variables $L_1(\tau)$, $L_3(\tau)$ and $L_4(\tau)$ which refer to lines one, three and four of the term in curly brackets in $E(V_\tau)$ in equation (3.5):

$$L_1(\tau) = \int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta} \frac{Y(s)}{Q(s)} \left[\int\limits_{\zeta=\tau}^{s} \beta e^{-\int_{t=\zeta}^{s}[\beta-(\alpha-1)\widetilde{\pi}(t)]dt} \widetilde{P^*}(\zeta)^{1-\alpha} d\zeta \right] ds,$$

$$L_3(\tau) = -\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta} \frac{Y(s)}{Q(s)} \widetilde{w}(s) \left[\int\limits_{\zeta=\tau}^{s} \beta e^{-\int_{t=\zeta}^{s}[\beta-\alpha\widetilde{\pi}(t)]dt} \widetilde{P^*}(\zeta)^{-\alpha} d\zeta \right] ds \text{ and}$$

$$L_4(\tau) = -\int\limits_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta} \frac{Y(s)}{Q(s)} \widetilde{w}(s) \left[e^{-\int_{t=\tau}^{s}[\beta-\alpha\widetilde{\pi}(t)]dt} \widetilde{P^*}(\tau)^{-\alpha} \right] ds.$$

3.A.3 Deriving the linearised zero-profit-condition

Inserting the market value (3.5) using the variables L_1-L_4 defined in Appendix 3.A.2 and section 3.2.1.1 and $\phi(k_j(\tau))$ from equation (2.18) into the zero profit equation in the research sector (2.17) and rearranging yields

$$\left[(\alpha-1)\frac{1}{\lambda} \right] [L_1(\tau) + L_2(\tau) + L_3(\tau) + L_4(\tau)] = \widetilde{w}(\tau) \tag{3.20}$$

The derivative of $L_1(\tau)$ with respect to time τ is

$$\begin{aligned}
\dot{L_1}(\tau) &= h(\tau)\int_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s} h(\theta)d\theta}\frac{Y(s)}{Q(s)}\left[\int_{\zeta=\tau}^{s}\beta e^{-\int_{t=\zeta}^{s}[\beta-(\alpha-1)\widetilde{\pi}(t)]dt}\widetilde{P^*}(\zeta)^{1-\alpha}d\zeta\right]ds \\
&\quad -\widetilde{P^*}(\tau)^{1-\alpha}\int_{s=\tau}^{\infty}\frac{Y(s)}{Q(s)}\beta e^{-\int_{\theta=\tau}^{s}[\beta+h(\theta)-(\alpha-1)\widetilde{\pi}(\theta)]d\theta}ds
\end{aligned}$$

or, using the definitions of $L_1(\tau)$ and $L_2(\tau)$,

$$\dot{L_1}(\tau) = h(\tau)L_1(\tau) - \beta L_2(\tau)$$

Analogously, the derivative of
$L_2(\tau) = \widetilde{P^*}(\tau)^{1-\alpha}\int_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}\left[i(\theta)+\mu(\theta)+\beta-\alpha\pi(\theta)-\gamma_Q(\tau)\right]d\theta}\frac{Y(s)}{Q(s)}ds$ with respect to time is

$$\begin{aligned}
\dot{L_2}(\tau) &= (1-\alpha)\frac{\dot{\widetilde{P^*}}(\tau)}{\widetilde{P^*}(\tau)}\int_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}[\beta+h(\theta)-(\alpha-1)\widetilde{\pi}(\theta)]d\theta}\frac{Y(s)}{Q(s)}\widetilde{P^*}(\tau)^{1-\alpha}ds \\
&\quad +[\beta+h(\tau)-(\alpha-1)\widetilde{\pi}(\tau)]\int_{s=\tau}^{\infty} e^{-\int_{\theta=\tau}^{s}[\beta+h(\theta)-(\alpha-1)\widetilde{\pi}(\theta)]d\theta}\widetilde{P^*}(\tau)^{1-\alpha}\frac{Y(s)}{Q(s)}ds \\
&\quad -\widetilde{P^*}(\tau)^{1-\alpha}\frac{Y(\tau)}{Q(\tau)}
\end{aligned}$$

which can be simplified to

$$\dot{L_2}(\tau) = \left\{(1-\alpha)\frac{\dot{\widetilde{P^*}}(\tau)}{\widetilde{P^*}(\tau)} + [\beta+h(\tau)-(\alpha-1)\widetilde{\pi}(\tau)]\right\}L_2(\tau) - \widetilde{P^*}(\tau)^{1-\alpha}\frac{Y(\tau)}{Q(\tau)} \tag{3.21}$$

Linearising this equation yields

$$\begin{aligned}
\left[\dot{L_2}(\tau) - L_2\right] &= L_2 (1-\alpha)\,\widetilde{P^*}^{-1}\left[\dot{\widetilde{P^*}}(\tau) - \widetilde{P^*}\right] \\
&\quad + L_2\left[r(\tau) - r\right] \\
&\quad + L_2\left(1 - \frac{q^{\alpha-1}-1}{\alpha-1}\right)\left[\mu(\tau) - \mu\right] \\
&\quad - L_2(\alpha-1)\left[\pi(\tau) - \pi\right] \\
&\quad + \left[\beta + h - (\alpha-1)\psi\right]\left[L_2(\tau) - L_2\right] \\
&\quad - \widetilde{P^*}^{1-\alpha}\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \\
&\quad - (1-\alpha)\,\widetilde{P^*}^{-\alpha}\frac{Y}{Q}\left[\widetilde{P^*}(\tau) - \widetilde{P^*}\right] \qquad (3.22)
\end{aligned}$$

where $h = r + \mu + (\alpha - 2)\gamma$.

Using the definitions of $L_3(\tau)$ and $L_4(\tau)$, the time derivative of $L_3(\tau)$ is

$$\dot{L_3}(\tau) = h(\tau)\,L_3(\tau) - \beta L_4(\tau)$$

Lastly,

$$\begin{aligned}
\dot{L_4}(\tau) &= \frac{Y(\tau)}{Q(\tau)}\widetilde{w}(\tau)\widetilde{P^*}(\tau)^{-\alpha} \\
&\quad + \left[(-\alpha)\frac{\dot{\widetilde{P^*}}(\tau)}{\widetilde{P^*}(\tau)} + h(\tau) + \beta - \alpha\widetilde{\pi}(\tau)\right] L_4(\tau)
\end{aligned}$$

can be derived analogously.

Taking into account that $\dot{L_1}(\tau) + \dot{L_2}(\tau) + \dot{L_3}(\tau) + \dot{L_4}(\tau) = \left[(\alpha-1)\frac{1}{\lambda}\right]^{-1}\dot{\widetilde{w}}(\tau)$

we have

$$\begin{aligned}\left[(\alpha-1)\frac{1}{\lambda}\right]^{-1}\dot{\widetilde{w}}(\tau) = & \\ & h(\tau)\left[L_1(\tau)+L_2(\tau)+L_3(\tau)+L_4(\tau)\right] \\ & -(\alpha-1)\frac{\dot{\widetilde{P^*}}(\tau)}{\widetilde{P^*}(\tau)}\left[L_2(\tau)+\frac{\alpha}{\alpha-1}L_4(\tau)\right] \\ & -(\alpha-1)\widetilde{\pi}(\tau)\left[L_2(\tau)+\frac{\alpha}{\alpha-1}L_4(\tau)\right] \\ & -\frac{Y(\tau)}{Q(\tau)}\widetilde{P^*}(\tau)^{-\alpha}\left[\widetilde{P^*}(\tau)-\widetilde{w}(\tau)\right]\end{aligned}$$

Inserting the optimal price (3.2) into the definition of $L_2(\tau)$ shows that $L_2(\tau)+\frac{\alpha}{\alpha-1}L_4(\tau)=0$. Using this and the fact that $L_1(\tau)+L_2(\tau)+L_3(\tau)+L_4(\tau)=\left[(\alpha-1)\frac{1}{\lambda}\right]^{-1}\widetilde{w}(\tau)$, we have

$$\dot{\widetilde{w}}(\tau)=h(\tau)\widetilde{w}(\tau)-\left[(\alpha-1)\frac{1}{\lambda}\right]\frac{Y(\tau)}{Q(\tau)}\widetilde{P^*}(\tau)^{-\alpha}\left[\widetilde{P^*}(\tau)-\widetilde{w}(\tau)\right]$$

Linearising this equation yields

$$\begin{aligned}\left[\dot{\widetilde{w}(\tau)}-\widetilde{w}\right] &= \left[h+(\alpha-1)\frac{1}{\lambda}\frac{Y}{Q}\widetilde{P^*}^{-\alpha}\right]\left[\widetilde{w}(\tau)-\widetilde{w}\right] \\ &\quad +\widetilde{w}\left[h(\tau)-h\right] \\ &\quad -(\alpha-1)\frac{1}{\lambda}\widetilde{P^*}^{-\alpha}\left(\widetilde{P^*}-\widetilde{w}\right)\left[\frac{Y(\tau)}{Q(\tau)}-\frac{Y}{Q}\right] \\ &\quad +(\alpha-1)\frac{1}{\lambda}\frac{Y}{Q}\widetilde{P^*}^{-\alpha}\left[\alpha\widetilde{P^*}^{-1}\left(\widetilde{P^*}-\widetilde{w}\right)-1\right]\left[\widetilde{P^*}(\tau)-\widetilde{P^*}\right]\end{aligned}$$

We get equation (3.8) in the text by using that
$h(\tau)=i(\tau)+\mu(\tau)-\pi(\tau)+(\alpha-2)\gamma_Q(\tau)=r(\tau)+\left[1+(\alpha-2)\frac{q^{\alpha-1}-1}{\alpha-1}\right]\mu(\tau)$.

3.A.3.1 Linearising Labour Market equilibrium

Taking the derivative with respect to time τ of both sides of equation (3.11) leads to

$$
\begin{aligned}
-\dot{\mu}(\tau)\lambda\widetilde{q} = & \dot{\overline{\frac{Y(\tau)}{Q(\tau)}}} \int_{-\infty}^{\tau} \left[\mu(s)+\beta\right] e^{-\int_{\theta=s}^{\tau}[\mu(\theta)-\alpha\widetilde{\pi}(\theta)+\beta]d\theta}\widetilde{P^*}(s)^{-\alpha} ds \\
& + \frac{Y(\tau)}{Q(\tau)}\left[\mu(\tau)+\beta\right]\widetilde{P^*}(\tau)^{-\alpha} \\
& - \left[\mu(\tau) - \alpha\widetilde{\pi}(\tau) + \beta\right]\frac{Y(\tau)}{Q(\tau)} \int_{-\infty}^{\tau} \left[\mu(s)+\beta\right] e^{-\int_{\theta=s}^{\tau}[\mu(\theta)-\alpha\widetilde{\pi}(\theta)+\beta]d\theta}\widetilde{P^*}(s)^{-\alpha} ds
\end{aligned}
$$

Noting that $\widetilde{\pi}(\theta) = \pi(\theta) + \gamma_Q(\theta)$ and using again equation (3.11), this can be rewritten as

$$
\begin{aligned}
-\dot{\mu}(\tau)\lambda\widetilde{q} = & \left\{ \frac{1}{\frac{Y(\tau)}{Q(\tau)}} \dot{\overline{\frac{Y(\tau)}{Q(\tau)}}} - \left[\mu(\tau) - \alpha\pi(\tau) - \alpha\gamma_Q(\tau) + \beta\right] \right\} \left[L - \mu(\tau)\lambda\widetilde{q}\right] \\
& + \frac{Y(\tau)}{Q(\tau)}\left[\mu(\tau)+\beta\right]\widetilde{P^*}(\tau)^{-\alpha}
\end{aligned}
$$

Linearising the equation, we have

$$
\begin{aligned}
-\lambda\widetilde{q}[\dot{\mu}(\tau) - \mu] = & \left\{ \frac{\left[\dot{\overline{\frac{Y(\tau)}{Q(\tau)}}} - \frac{Y}{Q}\right]}{\frac{Y}{Q}} - \left[(1-\alpha\widehat{q})\left[\mu(\tau)-\mu\right] - \alpha\left[\pi(\tau)-\pi\right]\right] \right\} (L - \mu\lambda\widetilde{q}) \\
& + \left[(1-\alpha\widehat{q})\mu - \alpha\pi + \beta\right]\lambda\widetilde{q}\left[\mu(\tau)-\mu\right] \\
& + \frac{Y}{Q}\widetilde{P^*}^{-\alpha}\left[\mu(\tau)-\mu\right] \\
& + \widehat{\beta}\widetilde{P^*}^{-\alpha}\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \\
& - \alpha\frac{Y}{Q}\widehat{\beta}\widetilde{P^*}^{-\alpha-1}\left[\widetilde{P^*}(\tau) - \widetilde{P^*}\right]
\end{aligned}
$$

where we have again used that $\gamma_Q(\tau) = \widehat{q}\mu(\tau)$. Collecting terms, using that at steady state, the equation for labour market equilibrium (3.11) reduces to $L - \mu\lambda\widetilde{q} = \frac{\widehat{\beta}}{\mu - +\beta\alpha\psi}\frac{Y}{Q}\widetilde{P^*}^{-\alpha}$ and rearranging, we have equation (3.12) in the text.

3.A.4 Eliminating $\widetilde{P}^*(\tau)$ from the system

Using equation (3.7), we can eliminate $\widetilde{P}^*(\tau)$ from the system of equations.

Turning first to the linearised equation for the optimal price (3.3), eliminating $\left[\widetilde{P^*}\dot{(\tau)} - \widetilde{P^*}\right]$ and $\left[\widetilde{P^*}(\tau) - \widetilde{P^*}\right]$ yields

$$
\begin{aligned}
\left(\widehat{q} - \widehat{\psi}\right)\left[\mu\dot{(\tau)} - \mu\right] + \left[\pi\dot{(\tau)} - \pi\right] = & \\
& + \left[\left(\frac{\widetilde{P^*}^{1-\alpha}}{L_2}\frac{Y}{Q} - \alpha\psi\right) - \widehat{\mu}\right]\left[\pi(\tau) - \pi\right] \\
& + \left[\left(\frac{\widetilde{P^*}^{1-\alpha}}{L_2}\frac{Y}{Q} - \alpha\psi\right)\left(\widehat{q} - \widehat{\psi}\right) - \widehat{q\mu}\right]\left[\mu(\tau) - \mu\right] \\
& - \widehat{\mu}\frac{\widetilde{P^*}^{-\alpha}}{L_2}\frac{Y}{Q}\frac{\alpha}{\alpha - 1}\left[\widetilde{w}(\tau) - \widetilde{w}\right] \\
& + \widehat{\mu}\frac{\psi}{\frac{Y}{Q}}\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \\
& - \widehat{\mu}\frac{\psi}{L_2}\left[L_2(\tau) - L_2\right] \qquad (3.23)
\end{aligned}
$$

Similarly, the linearised zero profit equation (3.8) now reads

$$
\begin{aligned}
\left[\widetilde{w}\dot{(\tau)} - \widetilde{w}\right] = & \left[h + (\alpha - 1)\frac{1}{\lambda}\frac{Y}{Q}\widetilde{P^*}^{-\alpha}\right]\left[\widetilde{w}(\tau) - \widetilde{w}\right] \\
& + \widetilde{w}\left[r(\tau) - r\right] \\
& + a_{\widetilde{w}\mu}\left[\mu(\tau) - \mu\right] \\
& - (\alpha - 1)\frac{1}{\lambda}\frac{Y}{Q}\left[1 + \alpha\widetilde{P^*}^{-1}\left(\widetilde{w} - \widetilde{P^*}\right)\right]\frac{\widetilde{P^*}^{1-\alpha}}{\widehat{\mu}}\left[\pi(\tau) - \pi\right] \\
& + (\alpha - 1)\frac{1}{\lambda}\widetilde{P^*}^{-\alpha}\left(\widetilde{w} - \widetilde{P^*}\right)\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \qquad (3.24)
\end{aligned}
$$

where $a_{\widetilde{w}\mu} = \left\{\widetilde{w}\left[1 + (\alpha - 2)\widehat{q}\right] - \left[1 + \frac{\alpha}{\widetilde{P^*}}\left(\widetilde{w} - \widetilde{P^*}\right)\right]\frac{(\alpha-1)\frac{1}{\lambda}\frac{Y}{Q}\widetilde{P^*}^{1-\alpha}}{\widehat{\mu}}\left(\widehat{q} - \widehat{\psi}\right)\right\}$.

Eliminating $\left[\widetilde{P^*}(\tau) - \widetilde{P^*}\right]$ from the labour market equilibrium condition (3.12) gives:

$$
\begin{aligned}
\left[\mu(\dot{\tau}) - \mu\right] = &- (\lambda\widetilde{q})^{-1}\frac{\widehat{\beta}}{\widetilde{\mu}}\widetilde{P^*}^{-\alpha}\left[\frac{Y(\tau)}{Q(\tau)}^{\cdot} - \frac{Y}{Q}\right] \\
&+ \alpha(\lambda\widetilde{q})^{-1}\frac{Y}{Q}\widetilde{P^*}^{-\alpha}\left(\frac{\widehat{\beta}}{\widehat{\mu}} - \frac{\widehat{\beta}}{\widetilde{\mu}}\right)\left[\pi(\tau) - \pi\right] \\
&+ \left\{\alpha(\lambda\widetilde{q})^{-1}\frac{Y}{Q}\left(\widehat{q} - \frac{\psi}{\widetilde{\beta}}\right)\widetilde{P^*}^{-\alpha}\left(\frac{\widehat{\beta}}{\widehat{\mu}} - \frac{\widehat{\beta}}{\widetilde{\mu}}\right) - \widetilde{\mu}\right\}\left[\mu(\tau) - \mu\right] \\
&- (\lambda\widetilde{q})^{-1}\widehat{\beta}\widetilde{P^*}^{-\alpha}\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right]
\end{aligned} \tag{3.25}
$$

In the linearised equation describing revenues, (3.22), eliminating $\left[\widetilde{P^*}(\tau) - \widetilde{P^*}\right]$ and $\left[\widetilde{P^*}(\dot{\tau}) - \widetilde{P^*}\right]$ leads to

$$
\begin{aligned}
\left[L_2(\dot{\tau}) - L_2\right] = &-\widetilde{P^*}^{1-\alpha}\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \\
&+ \left[L_2(1-\widehat{q}) - \frac{Y}{Q}\frac{(1-\alpha)\widetilde{P^*}^{1-\alpha}}{\widehat{\mu}}\left(\widehat{q} - \widehat{\psi}\right)\right]\left[\mu(\tau) - \mu\right] \\
&+ \frac{(1-\alpha)L_2}{\widehat{\mu}}\left\{\left(\widehat{q} - \widehat{\psi}\right)\left(\mu(\dot{\tau}) - \mu\right) + \left(\pi(\dot{\tau}) - \pi\right)\right\} \\
&+ L_2\left[r(\tau) - r\right] \\
&+ (\alpha - 1)\left[\frac{Y}{Q}\frac{\widetilde{P^*}^{1-\alpha}}{\widehat{\mu}} - L_2\right]\left[\pi(\tau) - \pi\right] \\
&+ \left[h - (\alpha - 1)\psi + \beta\right]\left[L_2(\tau) - L_2\right]
\end{aligned} \tag{3.26}
$$

3.A.5 Eliminating $\frac{m(\tau)}{Q(\tau)}$ from the system

We first eliminate $\frac{m(\dot{\tau})}{Q(\tau)}$ from the household's dynamic efficiency equation (3.16) by plugging in the value of $\frac{m(\dot{\tau})}{Q(\tau)}$ given in the money market equation

(3.18). Rearranging, this yields

$$
\begin{aligned}
\left(\overset{\cdot}{\frac{Y(\tau)}{Q(\tau)}} - \frac{Y}{Q}\right) \quad = \quad & -\frac{\theta(1-\eta)}{\widehat{\eta}}\frac{Y}{Q}\left(\pi(\tau)-\pi\right) \\
& -\widehat{q}\frac{Y}{Q}\left(\mu(\tau)-\mu\right) \\
& +\frac{\frac{Y}{Q}}{\eta+\theta(1-\eta)}\left(r(\tau)-r\right)
\end{aligned}
\tag{3.27}
$$

where $\widehat{\eta} = \eta + \theta(1-\eta)$. We then use the household's static efficiency condition (3.15) to eliminate $\frac{\dot{m}(\tau)}{Q(\tau)}$ from equation (3.18), which results in:

$$
\begin{aligned}
\frac{\theta}{1-\theta}\frac{1}{r+\pi}\left(\overset{\cdot}{\frac{Y(\tau)}{Q(\tau)}} - \frac{Y}{Q}\right) = & \frac{\frac{m}{Q}}{r+\pi}\left(\dot{r}(\tau)-r\right) \\
& + \frac{\frac{m}{Q}}{r+\pi}\left(\dot{\pi}(\tau)-\pi\right) \\
& - \frac{m}{Q}\left(\pi(\tau)-\pi\right) \\
& - \frac{m}{Q}\widehat{q}\left(\mu(\tau)-\mu\right)
\end{aligned}
\tag{3.28}
$$

3.A.6 Rewriting the system

The system of differential equations is still in the form $\dot{y}(\tau) = f\left[\dot{z}(\tau), y(\tau)\right]$, where $y(\tau)$ is the vector containing the deviations of all variables in the system from their steady state values and $z(\tau)$ is a subset of $y(\tau)$. We therefore need to rearrange the system to write it in the form $\dot{y}(\tau) = f\left[y(\tau)\right]$.

We first solve equation (3.23) for the expression $\left(\widehat{q}-\widehat{\psi}\right)\left(\dot{\mu}(\tau)-\mu\right)+\left(\dot{\pi}(\tau)-\pi\right)$ and insert this into equation (3.26) which gives us $\left[\dot{L}_2(\tau)-L_2\right]$ as a function of current level deviations of variables only:

$$
\begin{aligned}
\dot{[L_2(\tau) - L_2]} &= -\widetilde{P^*}^{1-\alpha}\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \\
&+ \left[L_2(1-\widehat{q}) - (1-\alpha)\frac{Y}{Q}\frac{\widetilde{P^*}^{1-\alpha}}{\widehat{\mu}}\left(\widehat{q} - \widehat{\psi}\right)\right][\mu(\tau) - \mu] \\
&+ \frac{(1-\alpha)L_2}{\widehat{\mu}}\left\{\left(\frac{\widetilde{P^*}^{1-\alpha}}{L_2}\frac{Y}{Q} - \alpha\psi\right) - \widehat{\mu}\right\}[\pi(\tau) - \pi] \\
&+ \frac{(1-\alpha)L_2}{\widehat{\mu}}\left\{\left(\frac{\widetilde{P^*}^{1-\alpha}}{L_2}\frac{Y}{Q} - \alpha\psi\right)\left(\widehat{q} - \widehat{\psi}\right) - \widehat{q\mu}\right\}[\mu(\tau) - \mu] \\
&- \frac{(1-\alpha)L_2}{\widehat{\mu}}\left\{\widehat{\mu}\frac{\widetilde{P^*}^{-\alpha}}{L_2}\frac{Y}{Q}\frac{\alpha}{\alpha - 1}[\widetilde{w}(\tau) - \widetilde{w}]\right\} \\
&+ \frac{(1-\alpha)L_2}{\widehat{\mu}}\widehat{\mu}\frac{\psi}{\frac{Y}{Q}}\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \\
&- \frac{(1-\alpha)L_2}{\widehat{\mu}}\widehat{\mu}\frac{\psi}{L_2}[L_2(\tau) - L_2] \\
&+ L_2[r(\tau) - r] \\
&+ (\alpha - 1)\left[\frac{Y}{Q}\frac{\widetilde{P^*}^{1-\alpha}}{\widehat{\mu}} - L_2\right][\pi(\tau) - \pi] \\
&+ [h - (\alpha - 1)\psi + \beta][L_2(\tau) - L_2]
\end{aligned}
$$

Collecting terms, we have

$$
\begin{aligned}
\dot{[L_2(\tau) - L_2]} = &+ (\alpha - 1)\alpha\psi\frac{L_2}{\widehat{\mu}}[\pi(\tau) - \pi] \\
&+ L_2\left\{\left(q^{\alpha-1} - \widehat{q}\right) + \left(\widehat{q} - \widehat{\psi}\right)\frac{\alpha\psi(\alpha - 1)}{\widehat{\mu}}\right\}[\mu(\tau) - \mu] \\
&+ \alpha\widetilde{P^*}^{-\alpha}\frac{Y}{Q}[\widetilde{w}(\tau) - \widetilde{w}] \\
&+ \left[(1-\alpha)L_2\frac{\psi}{\frac{Y}{Q}} - \widetilde{P^*}^{1-\alpha}\right]\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \\
&+ L_2[r(\tau) - r] \\
&+ (h + \beta)[L_2(\tau) - L_2] \qquad (3.29)
\end{aligned}
$$

Similarly, in a second step we use equation (3.27) to substitute for $\overset{\cdot}{\left(\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right)}$ in equation (3.25), which gives

$$
\begin{aligned}
&\overset{\cdot}{\left(\mu(\tau) - \mu\right)} = \\
&\quad \left\{ -\widetilde{\mu} + \left(\widetilde{q}\lambda\right)^{-1} \widetilde{P^*}^{-\alpha} \frac{Y}{Q} \left[\frac{\widehat{\beta}}{\widetilde{\mu}} \widehat{q} + \alpha \left(\widehat{q} - \widehat{\psi}\right) \left(\frac{\widehat{\beta}}{\widehat{\mu}} - \frac{\widehat{\beta}}{\widetilde{\mu}} \right) \right] \right\} \left(\mu\left(\tau\right) - \mu\right) \\
&\quad - \left(\widetilde{q}\lambda\right)^{-1} \widehat{\beta} \widetilde{P^*}^{-\alpha} \left(\frac{Y(\tau)}{Q\left(\tau\right)} - \frac{Y}{Q} \right) \\
&\quad + \left(\widetilde{q}\lambda\right)^{-1} \frac{Y}{Q} \widetilde{P^*}^{-\alpha} \left[\frac{\widehat{\beta}}{\widetilde{\mu}} \frac{\theta(1-\eta)}{\widehat{\eta}} + \alpha \left(\frac{\widehat{\beta}}{\widehat{\mu}} - \frac{\widehat{\beta}}{\widetilde{\mu}} \right) \right] \left(\pi\left(\tau\right) - \pi\right) \\
&\quad - \left(\widetilde{q}\lambda\right)^{-1} \widetilde{P^*}^{-\alpha} \frac{\widehat{\beta}}{\widetilde{\mu}} \frac{\frac{Y}{Q}}{\widehat{\eta}} \left(r\left(\tau\right) - r\right)
\end{aligned}
\tag{3.30}
$$

We then use equation (3.30) to eliminate $\overset{\cdot}{[\mu(\tau) - \mu]}$ from equation (3.23), which yields

$$
\begin{aligned}
\overset{\cdot}{\left[\pi\left(\tau\right) - \pi\right]} = {} & \widetilde{a}_{\pi\pi} \left[\pi\left(\tau\right) - \pi\right] \\
& + \widetilde{a}_{\pi\mu} \left[\mu\left(\tau\right) - \mu\right] \\
& + \left\{ \widehat{\mu} \frac{\psi}{\frac{Y}{Q}} + \left(\widehat{q} - \widehat{\psi}\right) \left(\widetilde{q}\lambda\right)^{-1} \widehat{\beta} \widetilde{P^*}^{-\alpha} \right\} \left[\frac{Y(\tau)}{Q\left(\tau\right)} - \frac{Y}{Q} \right] \\
& + \left(\widehat{q} - \widehat{\psi}\right) \left(\widetilde{q}\lambda\right)^{-1} \widetilde{P^*}^{-\alpha} \frac{\widehat{\beta}}{\widetilde{\mu}} \frac{\frac{Y}{Q}}{\widehat{\eta}} \left[r\left(\tau\right) - r\right] \\
& - \widehat{\mu} \frac{\widetilde{P^*}^{-\alpha}}{L_2} \frac{Y}{Q} \frac{\alpha}{\alpha - 1} \left[\widetilde{w}(\tau) - \widetilde{w}\right] \\
& - \widehat{\mu} \frac{\psi}{L_2} \left[L_2\left(\tau\right) - L_2\right]
\end{aligned}
\tag{3.31}
$$

where $\widetilde{a}_{\pi\mu} = \left(\widehat{q} - \widehat{\psi}\right) \left(\frac{\widetilde{P^*}^{1-\alpha}}{L_2} \frac{Y}{Q} - \alpha\psi \right) - \widehat{q}\widehat{\mu} +$ $\left(\widehat{q} - \widehat{\psi}\right) \left\{ \widetilde{\mu} - \left(\widetilde{q}\lambda\right)^{-1} \frac{Y}{Q} \widetilde{P^*}^{-\alpha} \left[\alpha \left(\widehat{q} - \widehat{\psi}\right) \left(\frac{\widehat{\beta}}{\widehat{\mu}} - \frac{\widehat{\beta}}{\widetilde{\mu}} \right) + \frac{\widehat{\beta}}{\widetilde{\mu}} \widehat{q} \right] \right\}$ and $\widetilde{a}_{\pi\pi} = \left(\frac{\widetilde{P^*}^{1-\alpha}}{L_2} \frac{Y}{Q} - \alpha\psi \right) - \widehat{\mu} - \left(\widehat{q} - \widehat{\psi}\right) \left(\widetilde{q}\lambda\right)^{-1} \frac{Y}{Q} \widetilde{P^*}^{-\alpha} \left[\frac{\widehat{\beta}}{\widetilde{\mu}} \frac{\theta(1-\eta)}{\widehat{\eta}} + \alpha \left(\frac{\widehat{\beta}}{\widehat{\mu}} - \frac{\widehat{\beta}}{\widetilde{\mu}} \right) \right]$.

In a last step, we use this equation (3.31) and the household's dynamic efficiency condition (3.27) to substitute for $[\dot{\pi(\tau)} - \pi]$ and $\left[\dot{\frac{Y(\tau)}{Q(\tau)}} - \frac{Y}{Q}\right]$ in equation (3.28), respectively. To this end, we first rewrite equation (3.28) as

$$
\begin{aligned}
(\dot{r(\tau)} - r) = &-(\dot{\pi(\tau)} - \pi) \\
&+ \frac{\theta}{1-\theta} \frac{1}{\frac{m}{Q}} \left(\dot{\frac{Y(\tau)}{Q(\tau)}} - \frac{Y}{Q} \right) \\
&+ (r+\pi)(\pi(\tau) - \pi) \\
&+ (r+\pi)\,\widehat{q}\,(\mu(\tau) - \mu)
\end{aligned}
$$

Plugging in (3.27) for $\left[\dot{\frac{Y(\tau)}{Q(\tau)}} - \frac{Y}{Q}\right]$ yields

$$
\begin{aligned}
(\dot{r(\tau)} - r) = &-(\dot{\pi(\tau)} - \pi) \\
&+ \left[(r+\pi) - \frac{\theta}{1-\theta} \frac{\frac{Y}{Q}}{\frac{m}{Q}} \frac{\theta(1-\eta)}{\widehat{\eta}} \right] (\pi(\tau) - \pi) \\
&+ \widehat{q} \left[r + \pi - \frac{\theta}{1-\theta} \frac{\frac{Y}{Q}}{\frac{m}{Q}} \right] (\mu(\tau) - \mu) \\
&+ \frac{\theta}{1-\theta} \frac{\frac{Y}{Q}}{\frac{m}{Q}} \frac{1}{\widehat{\eta}} (r(\tau) - r)
\end{aligned}
$$

where the coefficient on the deviation $(\mu(\tau) - \mu)$ is zero given equation (3.14).

Further using equation (3.31) to eliminate $(\dot{\pi(\tau)} - \pi)$ from this equation and collecting terms yields

$$\begin{aligned}
\overset{\cdot}{(r(\tau) - r)} &= a_{r\mu}\left[\mu(\tau) - \mu\right] \\
&\quad - \left\{\widehat{\mu}\frac{\psi}{\frac{Y}{Q}} + \left(\widehat{q} - \widehat{\psi}\right)(\widetilde{q}\lambda)^{-1}\widehat{\beta}\widetilde{P^{*}}^{-\alpha}\right\}\left[\frac{Y(\tau)}{Q(\tau)} - \frac{Y}{Q}\right] \\
&\quad + a_{r\pi}\left[\pi(\tau) - \pi\right] \\
&\quad + \frac{1}{\widehat{\eta}}\left[r + \pi - \left(\widehat{q} - \widehat{\psi}\right)(\widetilde{q}\lambda)^{-1}\widetilde{P^{*}}^{-\alpha}\frac{\widehat{\beta}}{\widetilde{\mu}}\frac{Y}{Q}\right]\left[r(\tau) - r\right] \\
&\quad + \widehat{\mu}\frac{\widetilde{P^{*}}^{-\alpha}}{L_2}\frac{Y}{Q}\frac{\alpha}{\alpha - 1}\left[\widetilde{w}(\tau) - \widetilde{w}\right] \\
&\quad + \widehat{\mu}\frac{\psi}{L_2}\left[L_2(\tau) - L_2\right]
\end{aligned} \tag{3.32}$$

with

$a_{r\mu} = -\left(\widehat{q} - \widehat{\psi}\right)\left(\frac{\widetilde{P^{*}}^{1-\alpha}}{L_2}\frac{Y}{Q} + \widetilde{\mu} - \alpha\psi\right) + \widehat{q}\widehat{\mu} + \left(\widehat{q} - \widehat{\psi}\right)\frac{\frac{Y}{Q}}{\widetilde{q}\lambda}\widetilde{P^{*}}^{-\alpha}\left[\alpha\left(\widehat{q} - \widehat{\psi}\right)\left(\frac{\widehat{\beta}}{\widehat{\mu}} - \frac{\widehat{\beta}}{\widetilde{\mu}}\right) + \frac{\widehat{\beta}}{\widetilde{\mu}}\widehat{q}\right]$

and

$a_{r\pi} = (r + \pi)\frac{\eta}{\widehat{\eta}} - \left(\frac{\widetilde{P^{*}}^{1-\alpha}}{L_2}\frac{Y}{Q} - \alpha\psi\right) + \widehat{\mu} + \left(\widehat{q} - \widehat{\psi}\right)(\widetilde{q}\lambda)^{-1}\frac{Y}{Q}\widetilde{P^{*}}^{-\alpha}\left[\frac{\widehat{\beta}}{\widetilde{\mu}}\frac{\theta(1-\eta)}{\widehat{\eta}} + \alpha\left(\frac{\widehat{\beta}}{\widehat{\mu}} - \frac{\widehat{\beta}}{\widetilde{\mu}}\right)\right]$

3.A.7 Elements of the linearised system's matrix

m_{ij} refers to the element in the ith row and jth column of the matrix $\mathbf{M}$.

$m_{11} = 0$, $m_{12} = \frac{Y}{Q}\frac{1}{\widehat{\eta}}$, $m_{13} = -\frac{Y}{Q}\widehat{q}$, $m_{14} = -\frac{Y}{Q}\frac{\theta(1-\eta)}{\widehat{\eta}}$, $m_{15} = 0$, $m_{16} = 0$,
$m_{21} = -\widetilde{P^{*}}^{-\alpha}\left\{\widehat{\beta}\left(\widehat{q} - \widehat{\psi}\right)(\widetilde{q}\lambda)^{-1} + \frac{\widehat{\mu}}{L_2}\left(\widetilde{P^{*}} - \frac{\alpha}{\alpha-1}\widetilde{w}\right)\right\}$, $m_{22} = \frac{r+\pi}{\widehat{\eta}} - \frac{\widehat{q}-\widehat{\psi}}{\widetilde{q}\lambda\widetilde{P^{*}}^{\alpha}}\frac{\widehat{\beta}}{\widetilde{\mu}}\frac{Y}{Q}\frac{1}{\widehat{\eta}}$,
$m_{23} = -\frac{\widehat{\mu}\left(\frac{1}{L_2}\widetilde{P^{*}}^{1-\alpha}\frac{Y}{Q} - \alpha\psi\right)\left(\widehat{q} - \widehat{\psi}\right) - \widehat{q}\widehat{\mu}^2}{\widehat{\mu}} + \left(\widehat{q} - \widehat{\psi}\right)\left\{\frac{\frac{Y}{Q}}{\widetilde{q}\lambda\widetilde{P^{*}}^{\alpha}}\left[\alpha\left(\frac{\widehat{\beta}}{\widetilde{\mu}} - \frac{\widehat{\beta}}{\widehat{\mu}}\right)\left(\widehat{\psi} - \widehat{q}\right) + \frac{\widehat{\beta}}{\widetilde{\mu}}\widehat{q}\right] - \widetilde{\mu}\right\}$,
$m_{24} = (r + \pi)\frac{\eta}{\widehat{\eta}} - \frac{\widehat{\mu}\left(\frac{1}{L_2}\widetilde{P^{*}}^{1-\alpha}\frac{Y}{Q} - \alpha\psi\right) - \widehat{\mu}^2}{\widehat{\mu}} + \widetilde{P^{*}}^{-\alpha}\left(\widehat{q} - \widehat{\psi}\right)(\widetilde{q}\lambda)^{-1}\frac{Y}{Q}\left[\frac{\widehat{\beta}}{\widetilde{\mu}}\frac{\theta(1-\eta)}{\widehat{\eta}} + \alpha\left(\frac{\widehat{\beta}}{\widehat{\mu}} - \frac{\widehat{\beta}}{\widetilde{\mu}}\right)\right]$,
$m_{25} = \frac{\widehat{\mu}}{L_2}\frac{\alpha}{\alpha-1}\frac{Y}{Q}\widetilde{P^{*}}^{-\alpha}$, $m_{26} = \widehat{\mu}\psi\frac{1}{L_2}$,
$m_{31} = -(\widetilde{q}\lambda)^{-1}\widehat{\beta}\widetilde{P^{*}}^{-\alpha}$, $m_{32} = -\frac{\widetilde{P^{*}}^{-\alpha}}{\widetilde{q}\lambda}\frac{\widehat{\beta}}{\widetilde{\mu}}\frac{Y}{Q}\frac{1}{\widehat{\eta}}$,
$m_{33} = (\widetilde{q}\lambda)^{-1}\widetilde{P^{*}}^{-\alpha}\frac{Y}{Q}\left[\alpha\left(\frac{\widehat{\beta}}{\widetilde{\mu}} - \frac{\widehat{\beta}}{\widehat{\mu}}\right)\left[\widehat{\psi} - \widehat{q}\right] + \frac{\widehat{\beta}}{\widetilde{\mu}}\widehat{q}\right] - \widetilde{\mu}$,
$m_{34} = (\widetilde{q}\lambda)^{-1}\widetilde{P^{*}}^{-\alpha}\frac{Y}{Q}\left\{\frac{\widehat{\beta}}{\widetilde{\mu}}\frac{\theta(1-\eta)}{\widehat{\eta}} + \alpha\left[\frac{\widehat{\beta}}{\widehat{\mu}} - \frac{\widehat{\beta}}{\widetilde{\mu}}\right]\right\}$, $m_{35} = 0$, $m_{36} = 0$,
$m_{41} = \widetilde{P^{*}}^{-\alpha}\left\{\widehat{\beta}\frac{\widehat{q}-\widehat{\psi}}{\widetilde{q}\lambda} + \frac{\widehat{\mu}}{L_2}\left(\widetilde{P^{*}} - \frac{\alpha}{\alpha-1}\widetilde{w}\right)\right\}$, $m_{42} = \widetilde{P^{*}}^{-\alpha}\frac{\widehat{q}-\widehat{\psi}}{\widetilde{q}\lambda}\frac{\widehat{\beta}}{\widetilde{\mu}}\frac{Y}{Q}\frac{1}{\widehat{\eta}}$,

$$m_{43} = \frac{\widehat{\mu}\left(\frac{1}{L_2}\widetilde{P^*}^{1-\alpha}\frac{Y}{Q}-\alpha\psi\right)\left(\widehat{q}-\widehat{\psi}\right)-\widehat{\mu}^2\widehat{q}}{\widehat{\mu}} - \left(\widehat{q}-\widehat{\psi}\right)\left\{\frac{\frac{Y}{Q}}{\widetilde{q}\lambda\widetilde{P^*}^{\alpha}}\left[\alpha\left(\frac{\widehat{\beta}}{\widetilde{\mu}}-\frac{\widehat{\beta}}{\widehat{\mu}}\right)\left(\widehat{\psi}-\widehat{q}\right)+\frac{\widehat{\beta}}{\widetilde{\mu}}\widehat{q}\right]-\widetilde{\mu}\right\},$$

$$m_{44} = \frac{\widehat{\mu}\left(\frac{1}{L_2}\widetilde{P^*}^{1-\alpha}\frac{Y}{Q}-\alpha\psi\right)-\widehat{\mu}^2}{\widehat{\mu}} - \widetilde{P^*}^{-\alpha}\left(\widehat{q}-\widehat{\psi}\right)\left(\widetilde{q}\lambda\right)^{-1}\frac{Y}{Q}\left\{\frac{\widehat{\beta}}{\widetilde{\mu}}\frac{\theta(1-\eta)}{\widehat{\eta}}+\alpha\left[\frac{\widehat{\beta}}{\widehat{\mu}}-\frac{\widehat{\beta}}{\widetilde{\mu}}\right]\right\},$$

$$m_{45} = -\frac{\widehat{\mu}}{L_2}\frac{\alpha}{\alpha-1}\frac{Y}{Q}\widetilde{P^*}^{-\alpha},\ m_{46} = -\widehat{\mu}\psi\frac{1}{L_2},$$

$$m_{51} = (\alpha-1)\frac{1}{\lambda}\widetilde{P^*}^{-\alpha}\left(\widetilde{w}-\widetilde{P^*}\right),\ m_{52} = \widetilde{w},$$

$$m_{53} = \widetilde{w}\left[1+(\alpha-2)\widehat{q}\right] - \frac{\alpha-1}{\lambda}\frac{Y}{Q}\left[\frac{\alpha}{\widetilde{P^*}}\left(\widetilde{w}-\widetilde{P^*}\right)+1\right]\frac{\widetilde{P^*}^{1-\alpha}}{\widehat{\mu}}\left(\widehat{q}-\widehat{\psi}\right),$$

$$m_{54} = -(\alpha-1)\frac{1}{\lambda}\frac{Y}{Q}\left[\alpha\left(\widetilde{w}-\widetilde{P^*}\right)\frac{1}{\widetilde{P^*}}+1\right]\frac{\widetilde{P^*}^{1-\alpha}}{\widehat{\mu}},$$

$$m_{55} = \left[\widehat{r}-\beta\right]+(\alpha-1)\frac{1}{\lambda}\frac{Y}{Q}\widetilde{P^*}^{-\alpha},\ m_{56} = 0,$$

$$m_{61} = \widetilde{P^*}^{-\alpha}\left[(1-\alpha)\left(\widetilde{P^*}-\frac{\alpha}{\alpha-1}\widetilde{w}\right)-\widetilde{P^*}\right],\ m_{62} = L_2,$$

$$m_{63} = L_2\left[\frac{\alpha-1}{\widehat{\mu}}\alpha\psi\left(\widehat{q}-\widehat{\psi}\right)+\left(q^{\alpha-1}-\widehat{q}\right)\right],$$

$$m_{64} = \alpha\frac{\alpha-1}{\widehat{\mu}}\psi L_2,\ m_{65} = \alpha\frac{Y}{Q}\widetilde{P^*}^{-\alpha},\ m_{66} = \widehat{r}$$

Chapter 4

Limit Pricing

4.1 Introduction

In the first chapters of this thesis, it was assumed that the incumbent intermediate firm's production is terminated when an innovator enters the market with an improved version of the good produced by the incumbent.[1] We now relax this assumption: Existing firms *may* leave the market whenever an innovator enters but are *not* required to do so. If the incumbent makes non-positive profits in case the innovator sets the monopoly price, the latter will do so and we are back in the model of chapter 2. If in contrast exogenous parameters are such that the incumbent would make positive profits under monopoly pricing, the innovator will set a limit price to drive the incumbent out of the market. In this chapter, we examine the model's steady state equilibrium under limit pricing.

First, it is important to clarify under which conditions limit pricing will prevail. Remember that we interpret an innovation in sector j as a signal that allows both the innovator and the incumbent(s) in sector j to choose a new price. Equation (2.1) shows that in the production of the final good, used goods from sector j are weighted with their respective quality grades. Being one quality rung below the innovator, the incumbent can at most

[1] Footnotes 5 and 8 explained that alternatively, it could be assumed that parameters are such that the incumbent makes non-positive profit when the innovator sets the monopoly price for his good and that the relevant conditions hold in all examined numerical examples.

charge a price that is a fraction q of the innovator's price. If charging $1/q$ times the monopoly price allows the incumbent to make a positive profit and thus to stay in the market, the innovator will not charge the monopoly price. Instead, he will choose the limit price $p^{\lim}(\tau)$ which makes the incumbent indifferent about exiting the market. In the model without price rigidity, the incumbent can make a positive profit under monopoly pricing whenever $(1/q) * \alpha/(\alpha-1) > 1$. Thus under this condition limit pricing prevails, while the innovator sets the optimal monopolistic price whenever $(1/q) * \alpha/(\alpha-1) \leq 1$. Now in the model with rigidity, the optimal monopolistic mark-up depends on endogenous variables such as the rate of inflation. Therefore, knowing the model's exogenous parameters is not in all cases sufficient to establish whether limit pricing or monopoly pricing will prevail. We will here present conditions that are sufficient for the applicability of monopoly and limit pricing, respectively. As stated in section 2.2.2.1, monopoly pricing will prevail when the sufficient conditions $\frac{1}{q}\frac{\alpha}{\alpha-1} \leq 1$ and $\frac{1}{q}\frac{\alpha}{\alpha-1}\frac{\rho+\beta+(1-\alpha)\psi}{\rho+\beta-\alpha\psi} \leq 1$ hold.[2] Conversely, when the two following conditions both hold, firms will set limit prices:

$$\frac{1}{q}\frac{\alpha}{\alpha-1} > 1 \tag{4.1}$$

$$\frac{1}{q}\frac{\alpha}{\alpha-1}\frac{\rho+\beta+(1-\alpha)\psi}{\rho+\beta-\alpha\psi} > 1 \tag{4.2}$$

If $\psi \geq 0$, the endogenous part of the firm's mark-up $\frac{\rho+\beta+\widetilde{\eta}\mu+(1-\alpha)\psi}{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi}$ is equal to or bigger than one, such that the condition $\frac{1}{q}\frac{\alpha}{\alpha-1} > 1$ from the underlying real model is sufficient to ensure that limit prices will be charged. When $\psi < 0$, the endogenous part of the mark-up is smaller than one. Since it increases in μ, $\frac{1}{q}\frac{\alpha}{\alpha-1}\frac{\rho+\beta+(1-\alpha)\psi}{\rho+\beta-\alpha\psi} > 1$ is a sufficient condition for limit pricing in this case. We therefore assume in this chapter that both (4.1) and (4.2) hold.[3]

[2] For $\psi \leq 0$, the condition $\frac{1}{q}\frac{\alpha}{\alpha-1} \leq 1$ is sufficient for monopoly pricing since in this case we have for the endogenous part of the mark-up $\frac{\rho+\beta+\widetilde{\eta}\mu+(1-\alpha)\psi}{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi} \leq 1$. For $\psi > 0$, though, the endogenous part of the mark-up is bigger than one and decreases in μ. Therefore, $\frac{1}{q}\frac{\alpha}{\alpha-1}\frac{\rho+\beta+(1-\alpha)\psi}{\rho+\beta-\alpha\psi} \leq 1$ is a sufficient condition for monopoly pricing for $\psi > 0$.

[3] Note that this chapter makes no statement as to which pricing regime will hold in

The remainder of this chapter is organised as follows: In section 4.2, the general equilibrium of the model under limit pricing is derived, where only the elements that differ from the monopoly case will be discussed in detail. It is shown that, under some additional assumptions, the model with limit pricing has a unique steady state equilibrium. Section 4.4 then discusses changes in the comparative statics of the equilibrium and presents a calibrated example. Section 4.5 concludes.

4.2 The model with limit pricing

4.2.1 The limit price

To prevent the incumbent from staying in the market, the innovator chooses the limit price $p^{\lim}(\tau)$ which given marginal cost $w(\tau)$ makes the incumbent

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the case where none of the sets of conditions sufficient for either limit pricing or monopoly pricing holds. The following intuition implies that in these cases non-zero money growth might give rise to multiple equilibria: Take a situation where $\frac{1}{q}\frac{\alpha}{\alpha-1} < 1$ and $\psi > 0$. Since the optimal monopolistic mark-up $\frac{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu+(1-\alpha)\psi}{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu-\alpha\psi}$ decreases in μ, whether the incumbent can make a positive profit at the monopoly price (i.e., whether $\frac{1}{q}\frac{\alpha}{\alpha-1}\frac{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu+(1-\alpha)\psi}{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu-\alpha\psi} > 1$) depends on μ. Two scenarios are possible: First, all firms expect that μ will be high so that $\frac{1}{q}\frac{\alpha}{\alpha-1}\frac{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu+(1-\alpha)\psi}{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu-\alpha\psi} \leq 1$ and therefore set the optimal monopoly price. Second, firms expect that μ will be low so that $\frac{1}{q}\frac{\alpha}{\alpha-1}\frac{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu+(1-\alpha)\psi}{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu-\alpha\psi} > 1$ and therefore set the limit price. This chapter's analysis of steady state equilibrium under limit pricing will show that by taking away firms' opportunity to offset the mark-up erosion effect by choosing a correspondingly higher initial mark-up, limit pricing creates an additional channel for positive money growth to reduce profits and the incentive to innovate. Therefore, profits and the equilibrium value of μ will be lower (say, μ^{lim}) in the second case where all firms set limit prices relative to the first case (say, μ^{mon}) where firms set the optimal monopoly price. This implies that $\frac{1}{q}\frac{\alpha}{\alpha-1}\frac{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu^{mon}+(1-\alpha)\psi}{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu^{mon}-\alpha\psi} < 1 < \frac{1}{q}\frac{\alpha}{\alpha-1}\frac{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu^{lim}+(1-\alpha)\psi}{\rho+\beta+\left[(\alpha-2+\eta)\frac{q^{\alpha-1}-1}{\alpha-1}+1\right]\mu^{lim}-\alpha\psi}$ could hold, confirming firms' expectations in both cases and making both limit pricing and monopoly pricing an equilibrium for the given money growth rate $\psi > 0$.

indifferent about exiting the market:[4]

$$p^{\lim}(\tau) = qw(\tau) \tag{4.3}$$

Analogously to the optimal monopoly price, the optimal limit price grows at the same rate as marginal cost. Yet unlike in the case with monopoly pricing, when quality increments q are small and limit pricing prevails, the innovator has no margin to choose a forward-looking mark-up.

4.2.2 A new firm's market value at entry

As in the monopoly case, the market value of a new firm at entry is

$$E\left(V_{k_j}(\tau)\left|t_{k_j} = \tau\right.\right) = A(\tau)\,P^*(\tau)^{-\alpha}\,\frac{\chi+\beta+(\alpha-1)\,\omega}{\chi-(1-\alpha)\,\omega}\left[\frac{p_j(\tau)}{\chi+\beta} - \frac{w(\tau)}{\chi+\beta-\omega}\right]$$

where $A(\tau) = q^{k_j(\alpha-1)}Y(\tau)\,P(\tau)^{\alpha}$, $\chi = i+\mu-\alpha\pi-\gamma$ and $p_j(\tau)$ is the price chosen by the innovator at the time of his market entry $\tau = t_{k_j}$. Inserting the limit price (4.3) gives the innovator's market value under limit pricing:

$$E\left(V_{k_j}(\tau)\left|t_{k_j} = \tau\right.\right) = \frac{A(\tau)\,w(\tau)}{P^*(\tau)^{\alpha}}\,\frac{\chi+\beta+(\alpha-1)\,\omega}{\chi-(1-\alpha)\,\omega}\left[\frac{q}{\chi+\beta} - \frac{1}{\chi+\beta-\omega}\right] \tag{4.4}$$

The term in square brackets can be interpreted as the present value of the firm's average profit per unit. Since the growth rate of marginal cost is still given by the money growth rate ψ, this term is influenced by ψ. In the monopoly case, firms choose a forward-looking mark-up that neutralises inflation's negative effect on the present value of average profits per unit (see equation (2.8) and the discussion of equation (2.10) in section 2.2.2.2). Under limit pricing, this is not possible. The average profit per piece will therefore decrease in ψ because while the price is fixed under price rigidity, unit cost grows at the rate ω which at steady state equals the money growth rate. In comparison to the monopoly case, this negative unit-profit effect creates an

[4]Firms producing qualities k_{j-n} with $n > 1$ could charge a price $P_{k_j-n} = \frac{P_{k_j}}{q^n} < w(\tau)$ such that they would exit the market.

additional channel for money to have real effects.

In equilibrium, the firm's market value must nevertheless be positive, i.e. $\frac{q}{\chi+\beta} > \frac{1}{\chi+\beta-\omega}$ is a necessary condition for equilibrium. Assumption (4.10) is a sufficient condition for this inequality to hold.

4.2.3 Labour demands and labour market equilibrium

The structure of the intermediate sector's labour demand remains unchanged: Since the limit price chosen by firms readjusting their prices grows at rate ω as does the optimal monopoly price, the average price at time τ charged by intermediate producers is still given by $\left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega}\right)^{-\frac{1}{\alpha}}$ times the price chosen at time τ. Thus, the only change is that the time τ limit price $p^{\lim}(\tau)$ replaces the optimal monopoly price $p^*(\tau)$ in formula (2.11).

The demand for research labour is still given by (2.19). Inserting the intermediate sector's labour, this gives us labour market equilibrium:

$$\frac{Y(\tau)}{Q(\tau)} = \frac{L - \mu\lambda\frac{q^{\alpha-1}}{\alpha-1}}{\left[\frac{p^{\lim}(\tau)}{P(\tau)Q(\tau)}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}}$$

As already explained in sections 2.4 and 2.2.3, the *level* of the mark-up charged by intermediate goods firms is irrelevant for price dispersion. Therefore, using that analogously to Appendix 2.A.3 we can rewrite the current relative limit price in efficiency units as $\frac{p^{\lim}(\tau)}{P(\tau)Q(\tau)} = \left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right)^{\frac{1}{\alpha-1}}$, the price dispersion term is unchanged relative to the model under monopoly pricing:

$$\frac{Y(\tau)}{Q(\tau)} = \frac{L - \mu\lambda\frac{q^{\alpha-1}}{\alpha-1}}{\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}} \tag{4.5}$$

4.2.4 The zero-profit-condition in general equilibrium

Inserting the unchanged razor's edge condition (2.18), the equations for the firms's market value (4.4) and the size of the final good sector (4.5) into the unchanged zero profit condition (2.17) from the monopoly pricing case, we have

$$\frac{(\alpha-1)\left(\frac{L}{\lambda}-\mu\frac{q^{\alpha-1}}{\alpha-1}\right)\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right)^{\frac{-\alpha}{\alpha-1}}}{\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}}\frac{\chi+\beta+(\alpha-1)\,\omega}{\chi-(1-\alpha)\,\omega}\left(\frac{q}{\chi+\beta}-\frac{1}{\chi+\beta-\omega}\right)=1 \tag{4.6}$$

4.3 Existence and uniqueness of steady state equilibrium under limit pricing

First, using $\chi=i+\mu-\alpha\pi-\gamma$, $i=r+\pi$, $\pi=\psi-\gamma$ and the unchanged Euler equation from the household's problem (2.31) and rearranging, we rewrite the zero profit condition (4.6) as

$$\begin{aligned}&\frac{(\alpha-1)\left(\frac{L}{\lambda}-\mu\frac{q^{\alpha-1}}{\alpha-1}\right)\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right)^{\frac{-\alpha}{\alpha-1}}}{\left[\left(\frac{\mu+\beta}{\mu+\beta-(\alpha-1)\omega}\right)^{\frac{1}{\alpha-1}}\left(\frac{\mu+\beta}{\mu+\beta-\alpha\omega}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha}}\left(\frac{q\left(\rho+\beta+\widetilde{\eta}\mu-\alpha\psi\right)}{\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\,\psi}-1\right)\\ =\ &(\rho+\widetilde{\eta}\mu)\,\frac{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi}{\rho+\beta+\widetilde{\eta}\mu}\end{aligned} \tag{4.7}$$

where $\widetilde{\eta}=\left[1+(\eta-2+\alpha)\,\frac{q^{\alpha-1}-1}{\alpha-1}\right]$. We show that equation (4.7) has a unique positive solution for μ under the following set of assumptions:

$$\frac{L}{\lambda} > \frac{\rho}{(\alpha-1)(q-1)} \tag{4.8}$$

$$\psi < \frac{\rho+\beta}{\alpha+(q-1)^{-1}} \tag{4.9}$$

$$\psi < \frac{\beta}{\alpha}\frac{\frac{q^{\alpha-1}}{\alpha-1}}{\frac{q^{\alpha-1}}{\alpha-1}+\frac{L}{\lambda}\frac{1}{\beta}} \tag{4.10}$$

$$\psi < \frac{1}{2}a-\left\{\frac{1}{4}a^2-\frac{(\rho+\beta)\,\beta\left[(\alpha-1)\,\frac{L}{\lambda}\,(q-1)-\rho\right]}{(\alpha-1)\,\alpha\left[\frac{L}{\lambda}\left[(q-1)\,(\alpha-1)+q\right]-\frac{\rho\beta}{\rho+\beta}\right]}\right\}^{1/2} \tag{4.11}$$

where a in Condition (4.11) is given by $a = \frac{\frac{\alpha}{\alpha-1}\beta\left[\frac{L}{\lambda}(\alpha-1)(q-1)-\rho\right]+(\rho+\beta)\left[\frac{L}{\lambda}(\alpha-1)(q-1)-\frac{\rho\beta}{\rho+\beta}\right]+\frac{L}{\lambda}[(q-1)(\rho+\beta)+\beta]}{\alpha\left[\frac{L}{\lambda}[(\alpha-1)(q-1)+q]-\frac{\rho\beta}{\rho+\beta}\right]}$.[5] Condition (4.8) replaces the no-growth-trap condition (2.37) that is required to hold under monopoly pricing. Similarly, the upper bound on money growth under monopoly pricing (2.38) is replaced by conditions (4.9)-(4.11). Condition (4.9) is sufficient for the market value of the firm, and thus, the return to R&D, to be positive in spite of money's influence on the present value of the average profit per unit. Condition (4.10) is sufficient for the LHS of equation (4.7) to be concave in μ. Conditions (4.8) and (4.11) are jointly sufficient for the value of the LHS of equation (4.7) to exceed that of the RHS for $\mu \to 0$.[6] In numerical examples, the restrictions on ψ turn out to be somewhat more restrictive than their equivalent in the monopoly pricing case.[7]

Since the form of the LHS-curve and the RHS-curve from equation (4.7) is qualitatively equivalent to the monopoly case, we use figure (2.2) from the monopoly pricing case in the ensuing discussion.

Proposition 5 *Under conditions (4.8)-(4.11), the economy under limit pricing has a unique steady state equilibrium with $\mu > 0$.*

The proof can be found in the Appendix to this chapter.

4.4 Comparative statics and a calibrated example

For the discussion of comparative statics, refer back to equation (4.7). We first discuss comparative statics with respect to the money growth rate and then turn to the effect of an increase in the degree of price rigidity before presenting a calibrated example.

[5] Condition (4.11) defines a positive upper bound on ψ whenever the term in curly brackets is non-negative. When it is negative, the restriction is non-binding.

[6] Besides the solution given in (4.11), this inequality also holds for $\psi > \psi_1$, where ψ_1 is given by the term of the RHS of inequality (4.11), with a plus in front of the term in brackets instead of a minus. Yet these solutions are negligible since they violate condition (4.10).

[7] See section 4.4.3.

4.4.1 Unit profit effect reinforces negative net effect of money growth on real growth

On the RHS of equation (4.7), we have the firm's compound discount rate which is unchanged relative to the case with monopoly pricing. An increase in money growth reduces the compound discount rate via the already discussed *mark-up erosion effect*, raising the incentive to innovate and causing an upward shift of the LHS-curve in figure 2.2. The LHS of the equation reflects the firm's instantaneous profits. The *price dispersion effect* and the *initial relative price effect* of money growth which were discussed for the monopoly case in section (2.4) persist in unchanged form. Additionally, the new term $\left(q\frac{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi}{\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\psi}-1\right)$ can be interpreted as the present value of the firm's average profit per unit divided by the present value of its average unit cost. An increase in money growth has an additional negative effect on the firm's instantaneous profit via this measure of *profit per unit*:[8] Since limit pricing prevents firms from choosing a mark-up that would offset the effect of money growth ψ, the profit per unit term is reduced by an increase in ψ because while costs per unit grow at rate ω where $\omega = \psi$ at steady state, revenue per unit is fixed under price rigidity.[9] Thus via this reduction of instantaneous profits, an increase in ψ ceteris paribus lowers the incentive to innovate, which ceteris paribus translates into a downward shift of the LHS-curve. This reinforces the downward shift of the LHS-curve that an increase in ψ causes via the price dispersion effect and the initial relative price effect.[10] Since the net effect of an increase in money growth ψ on economic growth γ was already negative under monopoly pricing and limit pricing causes an

[8] Note that we are slightly simplifying in stating the term represents the present value of average profits per unit since the current real wage, to which unit profit is proportional, has already cancelled out against the cost of research in the zero profit equation. Yet our interpretation is correct insofar as the negative effect is indeed caused by the difference in growth rates between unit revenues and unit costs. Note also that the unit profit term is a *weighted* average of unit profits where the discount factor and the growth rate of demand for the good determine the weighting factor.

[9] $\frac{\partial\left(q\frac{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi}{\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\psi}-1\right)}{\partial\psi} = -q\frac{\rho+\beta+\widetilde{\eta}\mu}{[\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\psi]^2} < 0$. Note that at $\psi < 0$, profit per unit increases over time since wages grow at rate ψ while the price is fixed. An increase in ψ here *decreases* this effect thus lowering the present value of average unit profit.

[10] See section 2.4.2 of chapter 2.

additional negative effect, it is intuitive that the total effect of an increase in money growth ψ on the research intensity μ and economic growth γ is negative under limit pricing:

Proposition 6 *For $\beta < \infty$, an increase in the money growth rate ψ lowers the research intensity μ and real growth rate γ.*

The proof can be found in the Appendix.

Note also that for $\beta \to \infty$, all effects of money growth vanish, such that money is superneutral in the model with flexible prices regardless of whether monopoly pricing or limit pricing prevails.

4.4.2 Effect of price rigidity on growth depends on money growth rate

Unsurprisingly, the sign of the effect of price rigidity on growth is unchanged:

Proposition 7 *For $\psi > 0$ ($\psi < 0$), a decrease in β, i.e. an increase in the level of rigidity, decreases (increases) research intensity μ and the real growth rate γ.*

The proof can be found in the Appendix. A reduction in the level of rigidity that acts like a reduction in the absolute value of the money growth rate ψ is conducive (detrimental) to growth when an increase in the absolute value of ψ decreases (increases) growth at $\psi > 0$ ($\psi < 0$).

4.4.3 Calibrated example

We use the baseline example from the calibration given in table (2.1) of the monopoly pricing case in section (2.5) but lower the parameter q indicating the size of improvements brought about by innovations to $q = 1.1$. With this change, we have that in the baseline case with $\psi = 0.055$, the mark-up is 14.9%. At all examined money growth rates, conditions (4.1) and (4.2) hold such that limit pricing prevails. We find that the restrictions on the money growth rate imposed by conditions (4.9)-(4.11) are rather more restrictive than in the case with monopoly pricing: We can only investigate money growth rates lower than 7.4 percent in our baseline example.[11]

This reflects that the negative effect of money growth ψ on economic growth γ is strongly reinforced by the effect on the firm's profit per unit: While under monopoly pricing, increasing ψ from 0% to 10% reduced growth from 2.09% to 1.87% (0.22 percentage point reduction), increasing ψ from 0% to 5% already entails a reduction of the real growth rate from 3.22% to 2.15%, i.e. growth is reduced by 1.07 percentage points. At the maximum money growth rate of 7.4% implied by condition (4.10), the growth rate is 1.24%. Clearly, the effect of money growth is unrealistically large given empirical estimates.

Accordingly, the effect of an increase in price rigidity is more sizeable than under monopoly pricing: Decreasing β from 1.8 to 1.6 (i.e., increasing the average period where prices are fixed in a sector from 6.2 to 7.0 months) at $\psi = 0.055$ reduces growth by 0.2 percentage point from 2% to 1.80%, while the corresponding decline was 0.03 percentage point under monopoly pricing.

4.5 Conclusion

It has been shown that under additional restrictions on ψ, a unique steady state equilibrium under limit pricing exists. The additional restriction of the money growth rate is necessary because under limit pricing, the innovating firm cannot choose a forward-looking mark-up to correct the erosion of its profit per unit through money growth. This additional negative effect of money growth on the new firm's profits and the incentive to innovate leads to a stronger reduction of economic growth through a given increase in ψ than in the case of monopoly pricing. Thus high inflation is especially damaging to growth when the innovation size is small given the intensity of competition as measured in the elasticity of substitution between intermediate goods.

[11] The maximum money growth rates implied by conditions (4.9) and (4.11) are nearly identical at 9.1% and 8.9%, respectively. Condition (4.10) is the most restrictive and implies a maximum money growth rate of 7.4% in the baseline example.

4.A Appendix: Proofs of the propositions

Proof of Proposition 5. Consider equation (4.7). Condition (4.10) ensures that the LHS of the equation is concave in μ for $\psi > 0$ ($\frac{\partial^2 LHS}{\partial \mu^2} = -\frac{2\alpha\psi}{(\mu+\beta)^2}\left[q^{\alpha-1} + \frac{(\alpha-1)\frac{L}{\lambda}-\mu q^{\alpha-1}}{\mu+\beta}\right] < 0$ for $\psi > 0$), while the RHS of the equation increases in μ at an increasing rate for $\psi > 0$ ($\frac{\partial RHS}{\partial \mu} = \widetilde{\eta}\frac{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi}{\rho+\beta+\widetilde{\eta}\mu} + (\rho+\widetilde{\eta}\mu)\,\widetilde{\eta}\frac{\alpha\psi}{(\rho+\beta+\widetilde{\eta}\mu)^2} = \frac{\widetilde{\eta}\left[(\rho+\beta+\widetilde{\eta}\mu)^2-\alpha\psi\beta\right]}{(\rho+\beta+\widetilde{\eta}\mu)^2} > 0$ for all admissible ψ, $\frac{\partial^2 RHS}{\partial \mu^2} = \psi\frac{2\widetilde{\eta}^2\alpha\beta}{(\rho+\beta+\widetilde{\eta}\mu)^3} > 0$ for $\psi > 0$).[12] At $\psi \leq 0$, the LHS of equation (4.7) decreases in μ

($\frac{\partial LHS}{\partial \mu} = LHS\left\{\frac{-\frac{q^{\alpha-1}}{\alpha-1}}{\left(\frac{L}{\lambda}-\mu\frac{q^{\alpha-1}}{\alpha-1}\right)} + \frac{\alpha\psi}{(\mu+\beta)(\mu+\beta-\alpha\psi)} + \frac{\widetilde{\eta}q\frac{\psi}{[\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\psi]^2}}{\left(q\frac{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi}{\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\psi}-1\right)} < 0 \text{ for } \psi \leq 0\right\}$)
while the RHS increases in μ. Furthermore, it is trivial to show that $\lim_{\mu\to\infty} LHS = -\infty < \infty = \lim_{\mu\to\infty} RHS$ and conditions (4.8) and (4.11) ensure that the value for $\lim_{\mu\to 0} LHS > \lim_{\mu\to 0} RHS$ where LHS and RHS still refer to equation (4.7). Thus the two functions always have a unique intersection with $\mu > 0$ under conditions (4.8)-(4.11). ■

Proof of Proposition 6. Consider equation (4.7). To see that $\frac{d\mu}{d\psi} = -\frac{\frac{\partial LHS}{\partial\psi}-\frac{\partial RHS}{\partial\psi}}{\frac{\partial LHS}{\partial\mu}-\frac{\partial RHS}{\partial\mu}} < 0$, first refer to figure 2.2 to see that given assumptions (4.8) and (4.11) and concavity of the LHS-curve, the latter's slope is always smaller than that of the RHS-curve at the equilibrium ($\frac{\partial LHS}{\partial\mu} - \frac{\partial RHS}{\partial\mu} < 0$). Further,
$\frac{\partial LHS}{\partial\psi} - \frac{\partial RHS}{\partial\psi} = -\frac{(\rho+\widetilde{\eta}\mu)(\rho+\beta+\widetilde{\eta}\mu-\alpha\psi)}{\rho+\beta+\widetilde{\eta}\mu}\left\{\frac{\frac{q(\rho+\beta+\widetilde{\eta}\mu)}{[\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\psi]^2}}{q\frac{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi}{\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\psi}-1} + \frac{\alpha[\rho+(\widetilde{\eta}-1)\mu]}{(\rho+\beta+\widetilde{\eta}\mu-\alpha\psi)(\mu+\beta-\alpha\psi)}\right\} < 0$:
The fraction in front of the square brackets is the positive compound discount rate, further by assumption (4.9) $q\frac{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi}{\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\psi} - 1 > 0$, while $1 < \widetilde{\eta} = \left(1+(\eta+\alpha-2)\frac{q^{\alpha-1}-1}{\alpha-1}\right)$ given $\alpha > 1$, $\eta \geq 1$ and $\beta > \alpha\psi$ given assumption (4.10) ensure that the remaining expressions are positive. Further, since $\gamma = \frac{q^{\alpha-1}-1}{\alpha-1}\mu$, $\frac{d\gamma}{d\psi} = \frac{q^{\alpha-1}-1}{\alpha-1}\frac{d\mu}{d\psi} < 0$. ■

Proof of Proposition 7. Consider again equation (4.7). We have $\frac{d\mu}{d\beta} =$

[12] $\frac{\partial^2 LHS}{\partial\mu^2} = -\frac{2\alpha\psi}{(\mu+\beta)^2}\left[q^{\alpha-1} + \frac{(\alpha-1)\frac{L}{\lambda}-\mu q^{\alpha-1}}{\mu+\beta}\right] < 0$ for $\psi > 0$. For $\psi \leq 0$, $\frac{\partial LHS}{\partial\mu} < 0$ and $\frac{\partial^2 LHS}{\partial\mu^2} \geq 0$ which together with the condition (4.11) is sufficient for the existence of a unique steady state in the case $\psi \leq 0$, too.

$-\frac{\frac{\partial LHS}{\partial \beta}-\frac{\partial RHS}{\partial \beta}}{\frac{\partial LHS}{\partial \mu}-\frac{\partial RHS}{\partial \mu}} \gtreqless 0$ for $\psi \gtreqless 0$ since

$$\frac{\partial LHS}{\partial \beta}-\frac{\partial RHS}{\partial \beta}=\psi\frac{(\alpha-1)\left(\frac{L}{\lambda}-\mu\frac{q^{\alpha-1}}{\alpha-1}\right)}{\frac{\mu+\beta}{\mu+\beta-\alpha\psi}}\frac{\frac{q}{[\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\psi]^2}}{q\frac{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi}{\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\psi}-1}$$
$$+\psi\frac{(\alpha-1)\left(\frac{L}{\lambda}-\mu\frac{q^{\alpha-1}}{\alpha-1}\right)}{\frac{\mu+\beta}{\mu+\beta-\alpha\psi}}\frac{\alpha\left[\left(\rho+\left(\widetilde{\eta}-1\right)\mu\right)\left(\rho+\beta+\widetilde{\eta}\mu+\beta+\mu-\alpha\psi\right)\right]}{\left(\mu+\beta\right)\left(\mu+\beta-\alpha\psi\right)\left(\rho+\beta+\widetilde{\eta}\mu\right)\left(\rho+\beta+\widetilde{\eta}\mu-\alpha\psi\right)} \tag{4.12}$$

depends on the sign of ψ as positive final good production ($\frac{L}{\lambda}-\mu\frac{q^{\alpha-1}}{\alpha-1}>0$), $\beta>\alpha\psi$ by condition (4.10), $q\frac{\rho+\beta+\widetilde{\eta}\mu-\alpha\psi}{\rho+\beta+\widetilde{\eta}\mu-(\alpha-1)\psi}>1$ by condition (4.9) and $\widetilde{\eta}>1$ ensure that all terms except ψ on the right hand side of equation 4.12 are positive. Further, it was discussed above that $\frac{\partial LHS}{\partial \mu}-\frac{\partial RHS}{\partial \mu}<0$. ■

Chapter 5

Money growth, employment and output growth

5.1 Introduction[1]

In the preceding chapters, we have abstracted from the effects of money growth on employment. Yet these effects are of interest for two reasons: First, the level of employment is itself one of the most important performance figures of the economy and sustaining a high level of employment is a major goal of economic policy. Secondly, if the level of employment has a scale effect on an economy's output growth rate, the inflation-employment relationship is of additional interest for our analysis of the long-run effects of money growth on output growth.

As reported in the introductory chapter of this thesis, the mainstream view is that although inflation does raise employment in the short-run, the Phillips curve is vertical in the long run. However, the survey of the literature on endogenous growth and inflation in chapter 1 showed that changes in employment originating from households' substitution between goods and activities are regarded as an important transmission channel for the influence of inflation on long-run growth. Further, the literature survey showed that a small literature has recently emerged in the context of the New Keynesian

[1] Parts of this chapter are based on Funk and Kromen [2006].

business cycle model which analyses how nominal price rigidity might lead to a non-vertical Phillips curve in the long run.

In the same spirit, we now endogenise labour supply in our Schumpeterian growth model with price rigidity in order to study the effect of money growth on the level of employment and the latter's interaction with growth. In order not to complicate the analysis unduly, labour supply is endogenised in a very simple and straightforward way: We assume the existence of a labour supply function that increases in the wage. The positive correlation between wages and employment implied by this shortcut can be justified e.g. as the outcome of the optimisation of a central labour union which sets the wage and whose wage claims are less ambitious (or less effective) when unemployment is high than when it is low. In addition to keeping the model tractable, endogenising labour supply in this way – rather than introducing leisure in the household's utility function as is usually done in the New Keynesian business cycle model – has the additional benefit of allowing us to discuss unemployment that is involuntary for the individual worker rather than being his optimal choice.

To facilitate the analysis of the model with endogenous labour supply, we slightly modify the structure of the model: In the preceding chapters, labour was used as an input both in the production of intermediate goods and in the R&D sector. We now assume that labour is only used in the final goods sector where it is combined with intermediate goods in the production of output, implying that there is a single aggregate wage. Labour being used in the representative perfectly competitive final good firm is more readily compatible with our underlying story of one central union than labour being employed in the intermediate goods or R&D sectors where – due to price rigidity and the different positions of goods and the corresponding firms on the quality ladder – the economic conditions faced by individual firms differ. Therefore, the final good is now used as an input in the intermediate goods sector and the R&D sector.

In our analysis of the interaction of money growth, the employment level and the output growth rate at steady state, we find that the endogeneity of labour matters for the inflation-growth relationship whereas the inflation-employment relationship is approximately invariant to changes in the output growth rate. Higher employment raises growth because the production technology is such that the marginal productivity of intermediate goods in final good production increases in the amount of labour used in production. The higher productivity of intermediate goods raises profits to be made with their sales and thus raises the incentive to invent improved ones. Since the growing quality of intermediate goods resulting from R&D activities is the engine of growth, employment thus raises growth in the model.

Conversely, we find that employment is approximately unchanged by variations in the economy's growth rate or, equivalently, research intensity: The level of employment is influenced by the aggregate degree of rigidity or flexibility in prices and the degree of price flexibility is in turn influenced by the research intensity since every firm entering the market with an innovative good gets to set a new price. However, innovations are rare events compared to the frequency of price adjustment implied by the Poisson parameter β, so that the total degree of price flexibility is approximately unchanged by variations in the research intensity - and hence, so is the level of employment.

Turning to the effect of money growth on employment, employment and the level of output in efficiency units are hump-shaped functions of the money growth rate which peak at a money growth rate associated with a positive inflation rate. Labour's marginal productivity in final good production increases in the total amount of intermediates used in production and on the efficiency with which they are used. The nonlinearity of the employment-money growth relationship is therefore due to the interaction of the money growth rate's effects on the efficiency and on the total amount of intermediate goods usage in production. We now give a short intuition for these effects.

Any increase in the absolute value of the money growth rate that raises the absolute value of inflation causes a reduction in production efficiency. This is because an increase in the absolute value of inflation, which is the growth rate of marginal cost and hence, of the optimal price, will under rigidity lead to an increase in *relative price dispersion* concerning intermediate goods. Price dispersion in turn distorts demand for intermediate goods towards those with low prices. As first shown in chapter 2, given that intermediate goods are imperfect substitutes, an increase in the distortion of quantities causes a decrease in production efficiency. This in turn reduces labour's productivity and hence, the wage and employment. Via this effect, an increase in the money growth rate raises (lowers) employment if it raises (lowers) the absolute value of the inflation rate.

At the same time, an increase in the money growth rate also has an impact on employment via its effect on the average mark-up charged by intermediate goods producers, which in turn determines the total amount of intermediates used in final good production and hence, the marginal productivity of labour. Money growth influences the *average mark-up*, i.e. the average monopolistic distortion present in the economy, via two channels: First, an increase in money growth and inflation raises marginal cost while prices are fixed under rigidity, eroding effective mark-ups. Second, in anticipation of this effect, the initial mark-ups set by firms increase in inflation, which tends to increase the average mark-up.

As a result of the interaction of the relative price dispersion effect and the average mark-up effect of an increase in money growth, employment increases in money growth at negative and small positive inflation rates and decreases in the money growth rate at high inflation – that is, there is a non-linear long-run Phillips curve. In calibrated examples, the range of positive inflation rates where the average mark-up effect is positive and dominates, i.e. where employment increases in the money growth rate, is very small. At most values, an increase in inflation raises unemployment. Therefore, the model's main messages for monetary policy are that high inflation rates are bad for employment and that very moderate inflation is better than price stability from an employment perspective.

The relationship between the growth rates of money and output *given employment* is determined by money growth's two countervailing effects on the mark-up charged by an intermediate good firm: Given infrequent price adjustment, a firm's optimal *initial mark-up* increases in money growth and inflation. This is in anticipation of the fact that inflation (deflation) later leads to *mark-up erosion* (appreciation) while the firm's price is fixed. Given that the firm's mark-up thus generally differs from its optimum under flexible prices for any non-zero growth rate of marginal cost (i.e., inflation rate), demand for the good, the profits accruing to an innovator and hence, the incentive to innovate are lowered by non-zero inflation: The incentive to innovate given employment is a hump-shaped function of money growth that reaches its maximum at zero inflation.

Note that this implies that the main message from the previous chapters – that is, given employment, positive inflation is detrimental to growth – is unchanged by our change in the model's structure. The models do, however, differ with respect to the growth consequences of deflation in that in the present context, growth is (given employment) maximised at zero inflation whereas in the model of the preceding chapters growth decreases monotonically in the money growth rate over the entire range of admissible values. A short section is dedicated to the explanation of this difference.

The result that zero inflation maximises growth is no longer valid once it is taken into account that inflation's level effect on employment indirectly has an effect on the growth rate, too: Since the incentive to innovate increases in the level of employment and the latter is maximised at a positive inflation rate, the incentive to innovate and the output growth rate reach their maximum under positive inflation, too. The model is thus compatible with empirical evidence indicating a negative growth effect of medium and high inflation as well as the fact that this effect may only be present once inflation has reached a small positive threshold value. In a realistically calibrated numerical example, the effect of money growth on economic growth is quantitatively in line with the results of the empirical literature.

Summing up, short-term price rigidity allows inflation to affect the long-run levels of employment and output in a way that is consistent with a non-linear long-run Phillips curve. Taking account of the relationship between money growth and output growth which is influenced by inflation's level effect on employment, any monetary authority interested in fostering economic growth or employment should avoid high inflation rates by choosing a money growth rate that leads to very moderate but positive inflation.

The remainder of this chapter is organised as follows: Section 5.2 presents the model, while section 5.3 discusses the general equilibrium. Comparative statics and a calibrated example are presented in section 5.5 and section 5.6

concludes.

5.2 The model

5.2.1 Final good sector

With our modification of the sectoral structure of the mode, in the perfectly competitive final goods sector, the economy's final good Y is now produced using labour L and N varieties of differentiated intermediate goods. Intermediate goods continue to be combined according to the constant-elasticity-of-substitution aggregator of Dixit and Stiglitz [1977]:

$$Y(\tau) = AL(\tau)^{1/\alpha} \sum_{j=1}^{N} \left(q^{k_j(\tau)} x_j(\tau)\right)^{(\alpha-1)/\alpha} \tag{5.1}$$

where L is labour, x_j is the amount of sector j intermediate good used, q^{k_j} is this type's productivity and we assume $\alpha > 1$. We again assume that only the highest quality k_j available of intermediate good j is produced in equilibrium.[2]

The representative firm's profits are given by

$$\Pi^Y(\tau) = P(\tau)Y(\tau) - \sum_{j=1}^{N} p_j(\tau)x_j(\tau) - w(\tau)L(\tau) \tag{5.2}$$

where $P(\tau)$ is the final good price, $P_j(\tau)$ is the price charged for one unit of sector j intermediate good and $w(\tau)$ is the nominal wage. The firm's optimal

[2] We make sure that only the latest quality is available in each sector by assuming that parameters are such that the innovator's monopolistic mark-up makes production unprofitable for the incumbent. Given the steady state mark-up from (5.22), $q > \frac{\alpha}{\alpha-1}\frac{\rho+\beta-(\alpha-1)\psi}{\rho+\beta-\alpha\psi}$ is a sufficient condition. This condition is satisfied at the examined money growth rates in our calibrated examples. An analysis of the equilibrium under limit pricing would again worsen the impact of inflation on profits by removing firms' opportunity to set a forward-looking price but promises no further insights.

demand for labour and for intermediate good j, respectively, are given by

$$\frac{1}{\alpha}\frac{Y(\tau)}{L^{d}(\tau)}=\frac{w(\tau)}{P(\tau)} \tag{5.3}$$

and

$$x_j(\tau)=\left(\frac{p_j(\tau)}{P(\tau)}\right)^{-\alpha}\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}L(\tau)\,q^{(\alpha-1)k_j} \tag{5.4}$$

Optimal demand for the type j intermediate good depends negatively on the type's relative price and positively on its productivity $q^{k_j(\alpha-1)}$ and on employment $L(\tau)$.

5.2.2 Intermediate goods sector

It is assumed that the firm that bought the patent for intermediate good j from the research firm which developed the innovation produces the intermediate good one for one with output:

$$x_j(\tau)=h_j(\tau) \tag{5.5}$$

where h_j is the quantity of output used for production. Given the linear production function, the development of marginal cost is given by the development of the economy's output price level $P(\tau)$.

An intermediate good producer's pricing problem The firms producing the N intermediate goods act in an environment of monopolistic competition. That is, they maximise the present value of profits by choosing a price while taking as given the final good sector's demand function (5.4) and Calvo [1983]-type price rigidity. Whenever they have the opportunity to readjust prices, firms choose a price to maximise the expected present value of nominal profits obtained while their price is fixed, which is given by

$$E\left[V(p_j,\tau)\right]=\int_{\tau}^{\infty}e^{-\int_{\tau}^{s}\left[i(\theta)+\mu_{k_j}(\theta)+\beta\right]d\theta}\left[p_j-P(s)\right]x_j(s)\,ds \tag{5.6}$$

where $[p_j - P(s)]\, x_j\,(s)$ is the firm's profit at time s and i is the nominal interest rate.[3] Remember that the term $e^{-\int_\tau^s \left[i(\theta)+\mu_{k_j}(\theta)+\beta\right] d\theta}$ is the discount factor which is adjusted for the probability of obsolescence facing the firm in two different ways: Firstly, $e^{-\int_\tau^s \beta d\theta}$ represents the probability of not receiving a price setting signal before time s in the future. Secondly, the research intensity μ_{k_j} in research firm j determines the intermediate firm's probability $e^{-\int_\tau^s \mu_{k_j}(\theta) d\theta}$of not having being replaced by a successful innovator by time s. Since profits accruing after either of these two events occurs are irrelevant for the firm's pricing decision at time τ, discounting of future profits is the stronger, the higher β and μ_{k_j}.

5.2.3 Patents and the R&D sector

We continue to assume free entry to the research sector. Due to our modification of the sectoral structure, for each small research firm j, the value $\mu_{k_j}\,(\tau)$ of the Poisson parameter governing the probability of making an innovation now depends linearly on the amount of *final good* used, $z_j(\tau)$, for a given quality rung k_j (i.e., current position of sector j):

$$\mu_{k_j}(\tau) = \phi(k_j(\tau))z_j(\tau) \tag{5.7}$$

Sector j research firm's expected profit at time τ is given by the expected revenue
$\mu_{k_j}(\tau)E\left[V_{k_j+1}\,(\tau)\,\middle|t_{k_j} = \tau\right]$, where $E\left[V_{k_j+1}\,(\tau)\,\middle|t_{k_j} = \tau\right]$ is the expected present value at market entry of all future profits accruing to a potential producer of the new intermediate good, as given in equation (5.24), minus the input cost $P(\tau)z_j(\tau)$.

Due to the free entry assumption, firm j's expected profit is zero at every instant which using (5.7) implies that

$$\phi(k_j(\tau))E\left[V_{k_j+1}\,(\tau)\,\middle|t_{k_j} = \tau\right] - P(\tau) = 0 \tag{5.8}$$

[3] A constant from the integration of the probability distribution of the price reset signal has already been eliminated.

holds for all active research firms.

The standard knife-edge specification for $\phi(k_j(\tau))$ makes sure that the optimal research intensity μ can be constant and independent of a sector's position and which implies the existence of spillovers in research. Specifically, the lower the sector's quality level, the easier is making an innovation:

$$\phi(k_j(\tau)) = \frac{1}{\lambda} q^{-(\alpha-1)(k_j+1)} \tag{5.9}$$

where $1/\lambda$ is the productivity of labour in research.

5.2.4 Public Sector

The specification of the public sector is unchanged: The state does not levy taxes or issue bonds. Its only policy instrument is the money supply, $M^s(\tau)$ which is perfectly controlled by an independent central bank by setting the constant exogenous money growth rate ψ:

$$\frac{\dot{M^s}(\tau)}{M(\tau)} = \psi \tag{5.10}$$

All revenue from money creation is allocated to households in form of a lump-sum cash transfer, $T(\tau)$

$$\dot{M^s}(\tau) = T(\tau) \tag{5.11}$$

There is no government spending apart from the transfer of seigniorage to households.

5.2.5 Consumption and money demand

Since we do not introduce a consumption-leisure choice at the household level, households' consumption and money demand decisions are unchanged. Remember that the representative infinitely lived household maximises the discounted present value of his lifetime utility flows, where $\rho > 0$ is the discount factor. Households derive utility both from consumption $c\,(\tau)$ of the economy's final good and from holding real balances $m(\tau) = \frac{M(\tau)}{P(\tau)}$. Specifically, the household maximises

$$U = \int_{s=0}^{\infty} e^{-\rho s} \frac{(c(s)^{1-\theta} m(s)^{\theta})^{1-\eta} - 1}{1-\eta} ds \tag{5.12}$$

with $\eta > 1$[4], $\theta \in [0,1)$ subject to his budget constraint given labour:

$$\dot{v(\tau)} = \frac{w(\tau)}{P(\tau)} L(\tau) + \frac{T(\tau)}{P(\tau)} + r(\tau)v(\tau) - c(\tau) - [\pi(\tau) + r(\tau)]\, m(\tau) \tag{5.13}$$

where v is the real value of the household's monetary and non-monetary wealth, $\frac{w}{P}L$ is the household's real wage income from being employed $L \leq \overline{L}$ hours, $\frac{T}{P}$ is the real value of the transfer received from the government, and r is the real interest rate which is paid on the firms real holdings of shares in investment funds that finance R&D firms' activities.

The unchanged first-order conditions of the household's maximisation problem are:

$$\frac{\theta}{1-\theta} \frac{c(\tau)}{m(\tau)} = r(\tau) + \pi(\tau) \tag{5.14}$$

$$[\eta + \theta(1-\eta)] \frac{\dot{c(\tau)}}{c(\tau)} - \theta(1-\eta) \frac{\dot{m(\tau)}}{m(\tau)} = r(\tau) - \rho \tag{5.15}$$

Equation (5.14) is a static efficiency condition requiring that the ratio of marginal utilities from money holdings and consumption equal their cost ratio, where the opportunity cost of holding cash is the nominal interest rate $i(\tau) = r(\tau) + \pi(\tau)$. Equation (5.15) governs the utility-maximising allocation of the household's resources over time and will in steady state equilibrium reduce to the familiar Ramsey rule.

5.2.6 Labour supply

Labour supply is introduced in the simplest possible way as an *exogenously* given function $L^s(\widetilde{w})$ of real wages per efficiency unit $\widetilde{w}(\tau) = \frac{w(\tau)}{P(\tau)Q(\tau)}$,

[4] This is again sufficient for the transversality condition $\lim_{t\to\infty} \xi_t v_t e^{-\rho t} = \lim_{t\to\infty} e^{-rt} v_t = 0$ to hold.

where $L^S(\widetilde{w})$ is strictly increasing in $\widetilde{w}$ from $L^s(0) = L^{min} > 0$ to $\overline{L} = \lim_{\widetilde{w}\to\infty} L^s(\widetilde{w})$.[5]

For the sake of concreteness we assume that

$$L^s(\widetilde{w}) = \overline{L}\left(1 - \frac{e^{-\sigma\widetilde{w}}}{2}\right) \tag{5.16}$$

where $\overline{L} > 0$ is the maximal employment (full employment), $L^{min} = \overline{L}/2$ and $\sigma > 0$ is a parameter reflecting the reactiveness of employment with respect to the wage per efficiency unit $\widetilde{w}$ where at time τ we have $\widetilde{w}(\tau) = w(\tau)/(P(\tau)Q(\tau))$. L^s will be constant in steady state equilibrium where $\widetilde{w}(\tau)$ is constant. The strength of labour supply's reaction with respect to the wage depends on the parameter σ.

5.3 Steady state equilibrium

We now analyse the model's general equilibrium restricting our attention to Rational Expectations steady state equilibria with constant output growth. In doing so, we pay special attention to the partial equilibrium analysis of the labour market equilibrium given the research intensity μ and of the research

[5] One way to think about $L^s(\widetilde{w})$ is to assume that it results from the utility maximization of households with extremely separable preferences: The household's "worker" maximises a function $v(\widetilde{w}_\tau l_\tau, l_\tau)$ facing a trade-off between the disutility of too much work and bringing home high labour income $\widetilde{w}_\tau l_\tau$. The household's "shopper" receives $(\widetilde{w}_\tau l_\tau)_{\tau\geq 0}$ maximises (5.12) given $\{l_\tau\}_{\tau\geq 0}$ since he does not interfere with the "worker's" decision.

Assuming $v_1 > 0$, $v_{11} < 0$, $v_2 \gtrless 0$ for $L \lesseqgtr L^{min}$ and $v_2 \to -\infty$ for $L \to \overline{L}$, the worker's choice of $L^s(\widetilde{w})$ has the desired form.

Another way to think about the inverse of $L^s(\widetilde{w})$ is to assume that wages $w(\tau)$ are set by a centralised labour union. The union's real wage claims per efficiency unit are moderated by a high level of unemployment – leading to a positive relation between wages and employment. This may either reflect the union's genuine interest in low unemployment together with its belief that a moderation of wage claims reduces unemployment or it may directly reflect the waning of the union's power to implement high wages when unemployment rises. Note that in the present setting control over nominal wages $w(\tau)$ in fact allows to control real wages per efficiency unit $\widetilde{w}$ and also that the union's belief of a negative short-run relation between $\widetilde{w}(\tau)$ (and $w(\tau)$) and employment is warranted.

Note that only the second interpretation allows us to discuss unemployment that is involuntary for the individual worker.

market equilibrium given employment L. Understanding the corresponding partial equilibrium effects facilitates our ensuing discussion of the joint determination of L and μ under price rigidity and non-zero inflation in general equilibrium.

5.3.1 Households

From the household's static optimality condition (5.14), we have that the growth rates of consumption and real money holdings are equal at steady state equilibrium. Using this in the household's dynamic optimality condition and rearranging yields the familiar Ramsey rule:

$$\frac{\dot{c}(\tau)}{c(\tau)} = \frac{r - \rho}{\eta} \tag{5.17}$$

5.3.2 Money market equilibrium

Using $\psi - \pi = \frac{\dot{m}^s(\tau)}{m^s(\tau)} = \frac{\dot{m}^d(\tau)}{m^d(\tau)} = \frac{\dot{c}(\tau)}{c(\tau)} = \gamma$ at steady state equilibrium,[6] money market equilibrium is characterised by equality of the inflation rate and the output-growth adjusted money growth rate

$$\pi = \psi - \gamma \tag{5.18}$$

5.3.3 Behaviour of the aggregate quality index $Q(\tau)$ and the growth rate

The change in the final good sector's production technology leads to a slightly modified definition of the economy's aggregate technology index, $Q(\tau)$, as the weighted sum of the productivities $q^{k_j(\tau)}$ associated with each sector's intermediate good

$$Q(\tau) = \sum_{j=1}^{N} q^{(\alpha-1)k_j(\tau)} \tag{5.19}$$

[6] See the household's static optimality condition (5.14) and equation (5.26) in section 5.3.5.

The expected growth rate of the quality index Q at time τ, $E\left[\gamma_Q(\tau)\right]$, can be found by aggregating over j the changes in sector $j's$ quality brought about by an innovation, weighted with the flow probability that an innovation will occur in sector j in the infinitesimal time interval beginning at τ. In steady state equilibrium, this probability will be constant and the same for all sectors, so we set $\mu_{k_j}(\tau) = \mu(\tau) = \mu$. Using the law of large numbers, the expected and actual growth rates of the quality index coincide. Following these steps gives us

$$\gamma_Q = \left(q^{\alpha-1} - 1\right)\mu \tag{5.20}$$

Since at steady state, the growth rate of output γ again equals the growth rate of the aggregate quality index,[7] we have

$$\gamma = \left(q^{\alpha-1} - 1\right)\mu \tag{5.21}$$

5.3.4 Equilibrium in the market for intermediate goods

We now derive the optimal mark-up chosen by profit-maximising firms in the equilibrium under price rigidity. Together with the final sector's demand function (5.4), this allows us to derive the market value of an intermediate goods firm at market entry, which will determine the equilibrium patent price charged by successful R&D firms. We further use the optimal initial mark-up and (5.4) to find the quantity of intermediates produced in steady state equilibrium.

5.3.4.1 Optimal price at steady state equilibrium

We find the optimal price for an intermediate goods firm that first sets or readjusts its price by maximising the expected value of profits given in (5.6) with respect to the price p_j subject to the final good producing firms' demand function (5.4). Using that at steady state, the price level $P(\tau)$ grows at rate π, and the research intensity μ is equal for all sectors and constant leads to

[7] See equation (5.27) in section 5.3.5.

the following expression for the optimal price at time τ:[8]

$$p^*(\tau) = \frac{\alpha}{\alpha - 1} \frac{r + \mu + \beta - (\alpha - 1)\pi}{r + \mu + \beta - \alpha\pi} P(\tau) \tag{5.22}$$

where r is the real interest rate.

The optimal price is again a mark-up over marginal cost, with marginal cost now given by the output price $P(\tau)$. When prices can be constantly readjusted ($\beta \to \infty$), the optimal mark-up reduces to it flex-price value $\alpha/(\alpha - 1)$ from static profit maximisation. Under price rigidity, the mark-up is higher (lower) than the optimal flex-price mark-up when the growth rate of marginal cost, the inflation rate π, is positive (negative). This higher (lower) mark-up is chosen by the firm in anticipation of the fact that while its price is fixed, the firm's revenue per unit will be constant while unit cost grows at rate π- i.e., inflation (deflation) will lead to erosion (appreciation) of the firm's mark-up. The mark-up is chosen as to offset this effect of inflation on the expected present value of profit per unit. Further, under inflation (deflation) the optimal mark-up ceteris paribus decreases (increases) in the real interest rate r, the research intensity associated with the probability of being replaced by a successful innovator μ and the flow probability of receiving a price resetting signal β. This is because an increase in any of these variables reduces the weight given to future profits relative to current ones, drawing the mark-up closer to the static optimum.

Given that at steady state the inflation rate π ceteris paribus increases in the growth rate of money supply, ψ,[9] we therefore have that the initial mark-up increases ceteris paribus with money growth, allowing it to influence real activity.

[8] The maximisation problem has a well-defined solution for $r + \mu + \beta - (\alpha - 1)\pi > 0$. Assumption (5.37) guarantees that this inequality holds in equilibrium.

[9] See section 5.3.2.

5.3.4.2 An intermediate good producer's market value at market entry

The market value $E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)$ at the time of market entry t_{k_j} of a new intermediate goods firm j determines the value of the patent for the good developed in the R&D-sector. This market value is the expected present value at time τ of of all future profits of the firm, given that $t_{k_j}=\tau$:

$$E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)=\widetilde{A}L\int_{\tau}^{\infty}e^{-(i+\mu)(s-\tau)}\left[p_j(s)-P(s)\right]\left(\frac{p_j(s)}{P(s)}\right)^{-\alpha}ds \quad (5.23)$$

with $\widetilde{A}=\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}q^{(\alpha-1)k_j}$.[10]

In the absence of price rigidity when firms can constantly readjust their prices (i.e., $\beta\to\infty$), $p_j(s)=\alpha/(\alpha-1)P(s)$ so that the innovating firm's market value at market entry is given by

$$E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)_{\beta\to\infty}=\frac{\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}q^{(\alpha-1)k_j}\frac{1}{\alpha-1}P(\tau)\left(\frac{p^*_{flex}(\tau)}{P(\tau)}\right)^{-\alpha}L(\tau)}{r+\mu}$$

The real market value $E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)_{\beta\to\infty}/P(\tau)$ can again be interpreted as the properly discounted present value of an infinite stream of profits growing at a constant rate: The numerator of this term corresponds to the firm's instantaneous profit, while the obsolescence-adjusted discount rate is given in the denominator. Since $p^*_{flex}(\tau)/P(\tau)$ and employment L are constant at steady state, the firm's profit growth rate is zero, implying that the discount factor is $r+\mu-0$.[11] Note that instead of increasing in total final good production Y as in chapter 2, the firm's value is proportional to the amount of *labour* L employed in final good production since intermediate goods' productivity increases in L. The firm's value is also proportional to $P(\tau)$ since the price level determines both the firm's revenues and costs.

In the presence of Calvo-type price rigidity, deriving the firm's expected

[10] Note that the wage adjusts freely to clear the labour market such that in equilibrium, employment in the final good sector equals the constant labour supply L at all times.

[11] Remember that the appropriate discount rate for an infinite stream of profits that grows at constant rate x is $r-x$.

market value at market entry again requires going through a number of steps, resulting in the following equation:[12]

$$E\left(V_{k_j}(\tau)\left|t_{k_j}=\tau\right.\right)=\frac{\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}q^{(\alpha-1)k_j}\frac{1}{\alpha-1}P(\tau)\left(\frac{p^*(\tau)}{P(\tau)}\right)^{-\alpha}}{(r+\mu)\frac{r+\mu+\beta-\alpha\pi}{r+\mu+\beta}}L(\tau) \tag{5.24}$$

Equation (5.24) differs from the flex-price market value in two respects: First, as seen in equation (5.22) in the previous section, with positive inflation the initial mark-up $p^*(\tau)/P(\tau)$ chosen by the firm under price rigidity is higher than the optimal mark-up under flexibility, $p^*_{flex}(\tau)/P(\tau) = \alpha/(\alpha-1)$. This reduces demand for the good (see equation (5.4)) and therefore, the firm's instantaneous profits. Secondly, the discount rate under flexibility $r+\mu$ is replaced by a compound discount rate where the *flex-price* discount rate is corrected with the factor $(r+\mu+\beta-\alpha\pi)/(r+\mu+\beta)$ that consists of the appropriate discount rates for a firm under price *rigidity* for periods where prices can be changed or are fixed, respectively. The discount rate for periods where prices are fixed decreases in inflation. This is because while prices are fixed, the new good's mark-up and relative price erode at rate $-\pi$, which by equation (5.4) leads to a growth rate $\alpha\pi$ of demand for the good. Given positive profits per unit, the rising demand translates into a higher growth rate of the new intermediate firm's profits. Since the discount rate is the obsolescence-adjusted interest minus the profit growth rate, an increase in inflation thus reduces the discount rate for periods where prices are fixed and the compound discount rate.

An increase in the frequency of price adjustment, $\beta+\mu$ reduces the weight given to periods where prices cannot be changed and therefore reduces the necessary correction.[13]

[12] Derivation of the market value is described in more detail in Appendix 5.A.1.

[13] Note that at $\pi < 0$, the flex-price discount rate has to be corrected *upwards* for the negative growth rate of profits in periods where the mark-up appreciates. An increase in β here means that the correction term *rises* to reduce the extent of correction.

5.3.4.3 Intermediate goods production in steady state equilibrium

The final good sector's demand for intermediate goods can now be found by using the final good sector's demand function (5.4) for good j and aggregating over all intermediate goods:[14]

$$X(\tau) = \left(\frac{\alpha-1}{\alpha}A\right)^{\alpha} LQ(\tau) \sum_{j=1}^{N} \left(\frac{p_j(\tau)}{P(\tau)}\right)^{-\alpha} \frac{q^{(\alpha-1)k_j}}{Q(\tau)}$$

Aggregate demand for intermediate goods grows with the technology aggregate $Q(\tau)$ and depends negatively on the average relative price of intermediate goods. Using that the average price effective at time τ can again be expressed as a weighted average of past optimal prices set by firms at the last (stochastic) point in time s where they could change their prices and going through a number of steps, we can rewrite this equation as[15]

$$\left(\frac{X(\tau)}{Q(\tau)}\right)^{*} = \left(\frac{\alpha-1}{\alpha}A\right)^{\alpha} L \left[\frac{p^{*}(\tau)}{P(\tau)} \left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{-\frac{1}{\alpha}}\right]^{-\alpha} \tag{5.25}$$

where the term $\frac{p^{*}(\tau)}{P(\tau)} \left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{-1/\alpha}$ is the average relative price, or equivalently, the *average mark-up*, which under flexible prices reduces to, $\alpha/(\alpha-1)$. Since both components of the term depend on the inflation rate π, the average mark-up and hence, total demand for intermediate goods can be influenced by monetary policy.[16] The average mark-up increases in the optimal initial mark-up $p^{*}(\tau)/P(\tau)$ whose determinants were discussed in section 5.3.4.1. At the same time, the influence of past mark-ups on the average mark-up, which as explained above is a weighted average of the current and past values of the optimal mark-up, is captured in the term $[(\beta+\mu)/(\beta+\mu-\alpha\pi)]^{-1/\alpha}$: It implies that the average mark-up is lower (higher) than the current value

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[14] Note that due to the linear production function (5.5), the total production of intermediate goods equals both the final goods sector's demand for intermediate goods and.the intermediate goods sector's demand for the final good as an input.

[15] For details on the derivation of equation (5.25), see Appendix 5.A.2.

[16] Details will be discussed in section 5.3.6.1.

under inflation (deflation) because past optimal mark-ups are lower (higher). The weight of past mark-ups decreases in the frequency of price adjustments $\beta + \mu$: The higher the price setting signal or the frequency of market entry with new prices, the closer the average mark-up to its current value.

Since $p^* (\tau) / P(\tau)$ is constant at steady state equilibrium, $X(\tau)$ grows at the same rate as $Q(\tau)$. Bearing in mind that intermediate goods are produced one to one with output, $X(\tau)$ is also the intermediate sector's total demand for output.

5.3.5 Equilibrium in the final good market

Market equilibrium For the final good market to be in equilibrium, households' consumption must equal the difference between total final good production $Y(\tau)$ and the sum of the demands for final good by the intermediate goods and research sectors, which are $X(\tau)$ and $Z(\tau)$, respectively. In efficiency units, this is

$$\left(\frac{c(\tau)}{Q(\tau)}\right)^* = \left(\frac{Y(\tau)}{Q(\tau)}\right)^* - \left(\frac{X(\tau)}{Q(\tau)}\right)^* - \left(\frac{Z(\tau)}{Q(\tau)}\right)^* \tag{5.26}$$

Having already determined $(X(\tau)/Q(\tau))^*$ in the last section, we now turn to the steady state value of final good production in efficiency units, $(Y(\tau)/Q(\tau))^*$.[17]

Final good production in steady state equilibrium Now that we know both the final good sector's demand function for intermediate goods (5.4) and the optimal price chosen by intermediate goods producers (5.22), we can insert those equations into the final good production function to find that total production is

$$Y(\tau) = AL(\tau)^{\frac{1}{\alpha}} \sum_{j=1}^{N} \left(q^{\alpha k_j(\tau)} \left(\frac{p_j(\tau)}{P(\tau)}\right)^{-\alpha} \left(\frac{\alpha-1}{\alpha}A\right)^{\alpha} L(\tau) \right)^{\frac{\alpha-1}{\alpha}}$$

[17] The value $(Z(\tau)/Q(\tau))^*$ will be determined in equation (5.31) of section 5.3.7.

Going through similar steps as in the derivation of total intermediate good production (5.25) and some additional steps, this can be rewritten as[18]

$$\left(\frac{Y(\tau)}{Q(\tau)}\right)^{*} = AL(\tau)^{\frac{1}{\alpha}} \left(\frac{X(\tau)}{Q(\tau)}\right)^{\frac{\alpha-1}{\alpha}} \frac{\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}}{\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}} \tag{5.27}$$

where the total amount of intermediate goods produced $X(\tau)/Q(\tau)$ is given in equation (5.25). Note that since $X(\tau)/Q(\tau)$ is constant at steady state equilibrium, $Y(\tau)$ grows at the same rate as $Q(\tau)$. Output production in equation (5.27) is the product of two terms: The term $AL(\tau)^{\frac{1}{\alpha}}[X(\tau)/Q(\tau)]^{\frac{\alpha-1}{\alpha}}$ shows production when a total of $X(\tau)/Q(\tau)$ quality-weighted intermediate goods is employed efficiently. In contrast, the term $\left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}\right)/\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}} \leq 1$ represents the production inefficiency due to *price dispersion* under price rigidity: When inflation, i.e. the growth rate of marginal cost, is zero, all intermediate goods prices are equal in spite of price rigidity because the optimal price does not change over time. Given equation (5.4), the goods are then demanded in (quality-weighted) equal amounts, which given the constant elasticity of substitution between individual quality-weighted intermediates in the Dixit-Stiglitz final good production function means production is efficient.[19] Any non-zero inflation rate in contrast implies that the optimal price changes over time so that there is dispersion in prices and demanded quantities of intermediates. The production inefficiency term $\left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}\right)/\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}$ consists of the ratio of output actually produced with a given total amount of intermediate goods given current relative prices and output that could be produced with this input spread efficiently over the intermediate goods types.[20] Price dispersion and production inefficiency are the more pronounced, the higher the absolute value of the growth rate of optimal prices π, and the higher price rigidity, i.e. the lower $\beta+\mu$.

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[18] See Appendix 5.A.3.

[19] The term $\left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}\right)/\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}$ reaches its maximum value, unity, for $\pi=0$.

[20] For details see Appendix 5.A.3.

5.3.6 Labour market equilibrium given the innovation rate μ

By introducing the equilibrium amount of final goods produced (5.27) into the wage equation (5.3), we get the equilibrium real wage in efficiency units $\widetilde{w}(\tau) = w(\tau)/[P(\tau)Q(\tau)]$:

$$\widetilde{w}(\tau) = \frac{1}{\alpha} AL(\tau)^{\frac{1-\alpha}{\alpha}} \left(\frac{X(\tau)}{Q(\tau)}\right)^{\frac{\alpha-1}{\alpha}} \frac{\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}}{\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}}$$

which inserting the equilibrium amount of intermediate goods produced X/Q from equation (5.25), inserting the optimal initial mark-up $p^*(\tau)/P(\tau) = \frac{\alpha}{\alpha-1}\frac{r+\mu+\beta-(\alpha-1)\pi}{r+\mu+\beta-\alpha\pi}$ from equation (5.22), using the Euler equation (5.17), and using equation (5.18) from money market equilibrium and $\gamma = \overline{q}\mu$ can be rewritten as

$$\widetilde{w} = \overline{A}\left[\frac{\rho+\beta+\mu(1+\alpha\overline{q})-(\alpha-1)\psi}{\rho+\beta+\mu[1+(1+\alpha)\overline{q}]-\alpha\psi}\left(\frac{\beta+\mu}{\beta+\mu-\alpha(\psi-\overline{q}\mu)}\right)^{-\frac{1}{\alpha}}\right]^{-(\alpha-1)} \frac{\frac{\beta+\mu}{\beta+\mu-(\alpha-1)(\psi-\overline{q}\mu)}}{\left(\frac{\beta+\mu}{\beta+\mu-\alpha(\psi-\overline{q}\mu)}\right)^{\frac{\alpha-1}{\alpha}}} \tag{5.28}$$

with $\overline{A} = \frac{\left(\frac{\alpha}{\alpha-1}\right)^{-2(\alpha-1)}}{\alpha}A^{\alpha}$.

5.3.6.1 Properties of the real wage function

From equation (5.28), the steady state real wage in efficiency units is a function of the research intensity μ and of exogenous parameters, in particular of the money growth rate ψ and of the price rigidity parameter β:

$$\widetilde{w}(\mu, \psi, \beta) \tag{5.29}$$

We now discuss the properties of this function in some detail because equilibrium employment has qualitatively the same properties.[21] By equation (5.3) the real wage is determined by output per unit of labour $Y(\tau)/[Q(\tau)L(\tau)]$. Thus any influence of parameters on the total input of intermediate goods

[21] See section 5.3.6.2.

and on the efficiency with which this amount is used affects the wage and employment. First note that since $Y(\tau)/[Q(\tau)L(\tau)]$ is independent of total employment,[22] so is $\widetilde{w}$. This facilitates our analysis considerably.

Wage is a hump-shaped function of money growth ψ The two influences of money growth on the wage via the *average mark-up* and on *price dispersion* make $\widetilde{w}(\mu, \psi, \beta)$ a hump-shaped function of money growth. We discuss both influences in turn.

Price dispersion effect As explained in section 5.3.5, any increase in the money growth rate that increases inflation (decreases deflation) raises (lowers) the absolute value of the growth rate of optimal prices and therefore raises (lowers) price dispersion and production inefficiency $\frac{\beta+\mu}{\beta+\mu-(\alpha-1)(\psi-\bar{q}\mu)}\left(\frac{\beta+\mu}{\beta+\mu-\alpha(\psi-\bar{q}\mu)}\right)^{-\frac{\alpha-1}{\alpha}}$. Since the productivity of labour increases in the average productivity of intermediate goods, this lowers (raises) the wage.[23]

Average mark-up effect As explained in the last part of section 5.3.4.3, total demand for intermediate goods according to equation (5.25) depends negatively on the average mark-up charged by intermediate goods firms, $\frac{p^*(\tau)}{P(\tau)}\left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}\right)^{-\frac{1}{\alpha-1}}$ which is altered by an increase in money growth in two ways: Firstly, as discussed in section (5.3.4), an intermediate good firm's optimal *initial mark-up* $p^*(\tau)/P(\tau)$ increases in ψ since the growth rate of marginal cost π ceteris paribus rises in ψ, accelerating (slowing down) the future mark-up erosion (appreciation) under inflation (deflation). At the same time, the weight of past mark-ups in the average mark-up, $\{(\beta+\mu)/[\beta+\mu-(\alpha-1)\pi]\}^{-\frac{1}{\alpha-1}}$, decreases in ψ since the higher inflation (the smaller deflation), the lower are past mark-ups relative to the current one and thus the smaller the average mark-up relative to the current one. A

[22] This can be seen by inserting the value for X/Q from equation (5.25) into equation (5.27).

[23] $\frac{\partial\left[\left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}\right)/\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}\right]}{\partial\psi} = \frac{-(\alpha-1)\pi\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\pi}}{\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{\frac{\alpha-1}{\alpha}}[\beta+\mu-(\alpha-1)\pi](\beta+\mu-\alpha\pi)} \lesseqgtr 0$ as $\pi \gtreqless 0$.

sufficient condition for the average mark-up to decrease in ψ is $\pi \leq 0$.[24] In calibrated examples, the average mark-up is minimised at a small positive inflation rate.

This explains why the wage is maximised at a money growth rate associated with a positive inflation rate despite the negative price dispersion effect of positive inflation:

Net effect of money growth on employment:

Lemma 8 *The wage $w(\mu, \psi, \beta)$ is a hump-shaped function of the money growth rate ψ with a maximum at $\psi_1 > 0$ where ψ_1 is given in Appendix 5.A.4. At this unique maximum, the inflation rate $\pi(\psi_1)$ is strictly positive.*

The proof to the lemma can be found in Appendix 5.A.4.

Thus holding constant the research intensity, the wage is a hump-shaped function of the money growth rate with its peak at a money growth rate associated with a positive inflation rate.

An increase in price rigidity can increase the wage An increase in β, i.e. a decrease in price rigidity, affects employment via the same channels as does money growth. Through all these channels, the effect of an increase in flexibility is very similar to the effect of a decrease in the *absolute value* of the money growth rate: The fact that prices can be changed more often reduces *price dispersion* and thus increases the productivity of labour and hence, employment. At the same time, the average mark-up depends on β in two different ways: First, the initial mark-up decreases (increases) in β under inflation (deflation) since an increased probability that prices may be readjusted soon draws the mark-up closer to the static optimum. Second, the deviation of the average mark-up from the current optimal mark-up is smaller when β increases since effective prices were on average set more recently. Therefore, an increase in β raises (lowers) the average mark-up under inflation

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24 $\frac{\partial MU^{avg}}{\partial \psi} = \frac{\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{-\frac{1}{\alpha}}[\rho+\beta+\mu(1+\overline{q})]}{\{\rho+\beta+\mu[1+(1+\alpha)\overline{q}]-\alpha\psi\}^2}\left\{-\frac{\rho+\beta+\mu(1+\overline{q})-(\alpha-1)\pi}{\rho+\beta+\mu(1+\overline{q})}\frac{\rho+\beta+\mu(1+\overline{q})-\alpha\pi}{\beta+\mu-\alpha\pi}+1\right\} < 0$ for $\pi \leq 0$ where $MU^{avg} = \frac{\rho+\beta+\mu(1+\alpha\overline{q})-(\alpha-1)\psi}{\rho+\beta+\mu[1+(1+\alpha)\overline{q}]-\alpha\psi}\left(\frac{\beta+\mu}{\beta+\mu-\alpha\pi}\right)^{-\frac{1}{\alpha}}$.

(deflation). Thus, the fact that prices can be readjusted more frequently reduces the strength of both price dispersion and the average mark-up effect.

Concerning the net effect of an increase in β, we have the following lemma:

Lemma 9 *The wage and employment increase in the price flexibility parameter β under deflation. Under small positive inflation rates, wage and employment decrease in β.*

Intuitively, an increase in price rigidity promotes employment under moderate inflation since under these circumstances, an increase in (the absolute value of) money growth and inflation raises employment. Since an increase in price flexibility mitigates the effects of money growth, it reduces employment.[25]

Wage is approximately unchanged by the research intensity μ The effects of an increase in μ on the wage are qualitatively nearly identical to the effects of an increase in β since an increase in the innovation frequency reduces price rigidity, as does an increase in β.[26] Yet since the frequency of innovation μ is small compared to the frequency of Calvo-price adjustments β, its contribution to the degree of price flexibility $\beta + \mu$ is small. Therefore, the elasticities of price dispersion, the initial mark-up and the deviation of the average mark-up from the initial mark-up with respect to μ are very small. In fact, all the aforementioned elasticities with respect to μ go to zero for $\mu/\left(\mu+\beta\right) \to 0$ which holds approximately for all reasonable calibrations so that the effects of an increase in μ on the wage are quantitatively negligible.[27]

[25] It is intuitive that for the same reasons under high inflation rates where an increase in the money growth rate reduces employment, an increase in the price flexibility parameter β raises employment. While we cannot prove this analytically, it is confirmed by all our numerical examples.

[26] The effects of increases in μ and β are perfectly identical regarding price dispersion and the deviation of the average mark-up from the initial mark-up. In contrast, the initial mark-up decreases in μ not only due to the latter's influence on the degree of price flexibility $\beta+\mu$ but also via its indirect influence via the growth rate γ that increases the real interest rate r and lowers the inflation rate π. See section 5.3.4.1 for a description of the effects of r and π on the initial mark-up. Yet these indirect influences are not important numerically since the elasticity of the initial mark-up with respect to μ vanishes for $\mu/\left(\mu+\beta\right) \to 0$.

[27] E. g., in the baseline case of our leading example, $\mu/\left(\mu+\beta\right) = 0.007$.

5.3.6.2 Equilibrium employment $L(\mu, \beta, \psi)$

Given that labour supply from equation (5.16) increases monotonically in $\widetilde{w}$, the employment function (5.30) preserves the above-discussed properties of the wage function (5.29).

$$L(\mu, \beta, \psi) = L^s[\widetilde{w}(\mu, \psi, \beta)] \tag{5.30}$$

In particular, employment given the innovation rate μ is a hump-shaped function of money growth peaking at a value of ψ associated with a positive inflation rate, may be increased by an increase in rigidity under small positive inflation rates and is approximately invariant to the innovation rate μ.

5.3.7 Research market equilibrium given employment L

5.3.7.1 Equilibrium in the market for patents

The prospect of positive profits in intermediate goods production leads to buyers' competition in the market for patents in the course of which the price is bidden up to the market value of the new firm using the patent, (5.24). Given that research firms charge exactly this price, all new patents will be bought and the market for patents clears.

5.3.7.2 The R&D sector's demand for the final good at steady state equilibrium

The research sector's demand for the final good is found by rearranging (5.7), inserting $\phi(k_j(\tau))$ as defined in equation (5.9) and aggregating over all research firms. In efficiency units, this yields

$$\left(\frac{Z(\tau)}{Q(\tau)}\right)^* = \lambda\mu q^{\alpha-1} \tag{5.31}$$

The constant steady state equilibrium demand $\left(\frac{Z(\tau)}{Q(\tau)}\right)^*$ depends on the value of the equilibrium research intensity μ that we determine next.

5.3.7.3 Equilibrium research intensity

Using a new firm's expected market value $E\left(V_{k_j+1}(\tau)\,\middle|\,t_{k_j}=\tau\right)$ (5.24) and $\phi(k_j(\tau))$ from equation (5.9) in the zero profit condition (5.8) gives us an equation determining the equilibrium research intensity μ which makes current research firms indifferent with regard to the amount of research input used.

$$\frac{L}{\lambda}\frac{\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}\frac{1}{\alpha-1}P(\tau)\left(\frac{p^*(\tau)}{P(\tau)}\right)^{-\alpha}}{(r+\mu)\,\frac{r+\mu+\beta-\alpha\pi}{r+\mu+\beta}}=P(\tau) \tag{5.32}$$

Note that consistent with the assumption first made in section 5.3.3, the resulting steady state research intensity μ is the same for all research firms regardless of their sector's current position on the quality ladder.

Further using the optimal initial mark-up $\frac{\alpha}{\alpha-1}\frac{r+\mu+\beta-(\alpha-1)\pi}{r+\mu+\beta-\alpha\pi}$ from equation (5.22), the Euler equation (5.17), the equation relating economic growth to research intensity (5.21), using that equilibrium in the money market implies $\pi=\psi-\gamma$ and rearranging, we get an equation in μ, employment L and the model's parameters:

$$\begin{aligned}&\frac{L}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-\alpha}A^{\alpha}\frac{1}{\alpha-1}\left(\frac{\alpha}{\alpha-1}\frac{\rho+\beta-(\alpha-1)\,\psi+(\overline{\eta}-\overline{q})\,\mu}{\rho+\beta-\alpha\psi+\overline{\eta}\mu}\right)^{-\alpha}\\ =\;&\left[\rho+(\eta\overline{q}+1)\,\mu\right]\frac{\rho+\beta-\alpha\psi+\overline{\eta}\mu}{\rho+\beta+(\eta\overline{q}+1)\,\mu}\end{aligned} \tag{5.33}$$

where $\overline{q}=(q^{\alpha-1}-1)>0$, $\overline{\eta}=[(\eta+\alpha)\,\overline{q}+1]$ and $\overline{\eta}>\overline{q}$.

Both sides of the equation show the dependence of the optimal research intensity μ on the new firm's value: The LHS of equation (5.33) shows the instantaneous profits for a firm entering the market with a new patent as a function of the research intensity while the RHS represents the compound discount rate for this firm's future profit streams as a function of μ. Figure 5.1 depicts the LHS and the RHS of this equation.

The solution to equation (5.33) is a function of employment, money growth and rigidity:

$$\mu\,(L,\psi,\beta) \tag{5.34}$$

Lemma 10 *Under conditions (5.35)-(5.37), there is a unique steady state*

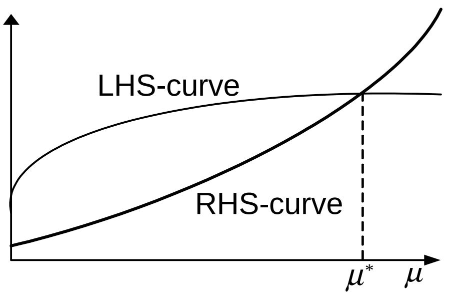

Figure 5.1: Partial equilibrium research intensity μ given employment $\overline{L}$

equilibrium research intensity $\mu(L, \psi, \beta)$ *for any* $L \in \left(\frac{1}{2}\overline{L}, \overline{L}\right)$.[28]

$$\frac{1}{2}\frac{\overline{L}}{\lambda} > \frac{\rho^{\frac{\rho+\beta-\alpha\psi}{\rho+\beta}}}{\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha} A^{\alpha} \frac{1}{\alpha-1}} \left(\frac{\rho+\beta-(\alpha-1)\psi}{\rho+\beta-\alpha\psi}\right)^{\alpha} \quad (5.35)$$

$$-\frac{\overline{q}}{\eta\overline{q}+1}(\rho+\beta) < \psi < \frac{1}{\alpha}\frac{(\rho+\beta)\left[2(\eta\overline{q}+1)+(\alpha-1)\overline{q}\right]}{\left(2+\frac{\alpha-1}{\alpha}\right)(\eta\overline{q}+1)+2(\alpha-1)\overline{q}} \quad (5.36)$$

$$\psi < \frac{1}{\alpha}\beta \quad (5.37)$$

where $\overline{q} = q^{\alpha-1}-1$. Condition (5.35) ensures that $\lim_{\mu\to 0} LHS > \lim_{\mu\to 0} RHS$ in equation (5.33). It implies that the efficiency weighted labour force cannot be too small. For $\beta < \infty$, conditions (5.36) and (5.37) are jointly sufficient for the LHS of equation (5.33) to be concave in μ, while condition (5.37) and the first inequality in condition (5.36) are sufficient to ensure that the RHS of the equation is convex in μ as depicted in fig. 5.1. Condition (5.36) can always be satisfied since the term to the very left is negative while the expression on the right hand side is positive. Conditions (5.36) and (5.37) imply that for any given β, there exist a lower and an upper bound on the growth rate of money supply ψ compatible with steady state equilibrium.

All conditions are easily satisfied in all our numerical examples.[29]

[28] All proofs can be found in Appendix 5.A.4.

[29] In the leading example we introduce in section 5.5, condition 5.35 implies $\frac{\overline{L}}{\lambda} > 2.28$ while we choose $\frac{\overline{L}}{\lambda} = 4.725$. Condition 5.37 is less restrictive than condition 5.36 which

Intuition For intuition concerning the form of the LHS-curve, first note that in the case without price rigidity ($\beta \to \infty$), the LHS of equation (5.33), which represents the instantaneous real profit associated with the production of the new good, simplifies to the constant $\frac{L}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha} A^{\alpha} \frac{1}{\alpha-1}$. For $\beta < \infty$, the curve has a positive slope in μ since as discussed in section 5.3.4.1 the forward-looking initial mark-up chosen by firms under price rigidity decreases in μ.[30] Since demand for the new firm's good is inversely related to its mark-up and relative price, an increase in μ increases the instantaneous profits associated with its invention, hence the positive slope of the LHS-curve.

The RHS of equation (5.33) represents the compound discount rate applicable to the new firm's profits. For $\beta \to \infty$, the discount rate reduces to $r + \mu$ which increases linearly in μ since the probability of being replaced increases. Under price rigidity ($\beta < \infty$), this effect of of an increase in μ is reinforced through an increase in the correction factor $(\rho + \beta - \alpha\psi + \overline{\eta}\mu) / [\rho + \beta + (\eta\overline{q} + 1)\mu]$.[31]

Note that given assumptions (5.35)-(5.37) and concavity of the LHS-curve, the slope at the steady state equilibrium of the LHS-curve is smaller than that of the RHS-curve. Intuitively, the increase in the discount rate caused by an increase in μ is bigger than the associated increase in instantaneous profits implying that expected profit from an innovation decreases in μ, as in the model without money.

implies $-1.12 < \psi < 0.14$. The upper bound, which corresponds to an inflation rate of $\pi = 12.5\%$, does not restrict our analysis of innovation-driven growth unduly.

[30] Note that in addition to the direct effect on the mark-up of an increase in the probability of being replaced by an innovator, an increase in μ has several indirect effects on the mark-up through its proportionality to the output growth rate γ and through the latter's effect on the interest rate $r = \rho + \eta\gamma$ and on the inflation rate $\pi = \psi - \gamma$ (see section 5.3.4.1 for an analysis of the influence of r and π on the mark-up). For $\pi > 0$, the net indirect effect is negative and thus reinforces the direct effect of an increase in μ.

At $\pi < 0$, the rigidity-caused part of the initial mark-up is *smaller* than unity because the mark-up will appreciate under deflation. An increase in μ further decreases the initial mark-up due to the indirect effect that $\pi = \psi - \gamma$ becomes even more negative, such that the future growth rates of revenues and costs diverge even further.

[31] This implies that the extent of correction decreases (increases) at $\pi > 0$ ($\pi < 0$) where the correction factor is smaller (bigger) than unity: The main reason is that through its proportionality to γ, an increase in μ lowers inflation (increases deflation) $\pi = \psi - \gamma$, thereby lowering (increasing) the positive (negative) profit growth rate in periods where the erosion (appreciation) of the firm's mark-up through inflation (deflation) leads to an increase (decrease) in demand for the good. Thus the deviation of the profit growth rate from its flex-price value that that requires correction decreases (increases) in μ.

We now discuss the properties of the research intensity function (5.34) with the help of figure 5.1.

5.3.7.4 Standard scale effect of employment L on the innovation rate μ

Lemma 11 *The innovation rate* $\mu(L, \psi, \beta)$ *increases monotonically in* L.

An increase in L raises instantaneous profits, shifting up the LHS-curve in figure 5.1. Given that the RHS-curve is unaffected by the change in L and given the curves' shapes, the increase in $\overline{L}$ results in a higher partial equilibrium innovation rate. The positive scale effect on growth of an increase in employment is a is a well-known feature of the underlying real growth model. In general equilibrium, this will allow for additional influences of exogenous parameters on the growth rate via their influence on employment.

5.3.7.5 Innovation rate μ depends negatively on absolute value of inflation $\pi = \psi - \gamma$ under price rigidity

Using equation (5.33), we note first that it is the presence of price rigidity that allows money to have an impact on $\mu(L, \psi, \beta)$:

Lemma 12 *In the limiting case without rigidities, money is superneutral:* $\lim_{\beta \to \infty} \frac{\partial \mu(L, \psi, \beta)}{\partial \psi} = 0$.

Intuitively, when prices are perfectly flexible, relative prices and mark-ups are independent of inflation, so that demand and hence, a research firm's profits are unaffected by money growth.

In contrast for $\beta < \infty$, the money growth rate ψ has two clear-cut countervailing effects on the innovation rate $\mu(L, \psi, \beta)$ which operate through money growth's influence on the firm's mark-up and relative price:

Negative initial mark-up effect of money growth under price rigidity As explained in section 5.3.4 an increase in ψ that raises the growth rate of marginal cost π raises the initial mark-up and relative price chosen by an

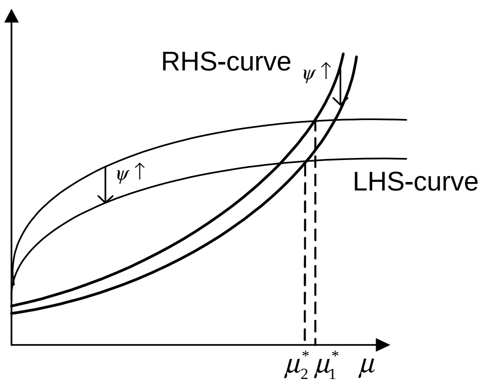

Figure 5.2: Effects on the partial equilibrium research intensity given employment $\mu\left(\psi, \overline{L}, \beta\right)$ of an increase in ψ when inflation is positive.

intermediate good firm under price rigidity. The increase in the relative price lowers demand for the firm's good and hence, its instantaneous profit which in turn determines the incentive to innovate.[32,33] In figure 5.2, the increase in ψ causes a downward shift of the LHS-curve which ceteris paribus reduces the innovation rate $\mu(L, \psi, \beta)$.

Positive mark-up erosion effect of money growth under price rigidity The RHS of equation (5.33) is the new firm's compound discount rate. As first discussed in section 5.3.4.2, the compound discount rate decreases in the inflation rate which determines the rate of demand- and profit-raising mark-up erosion.[34] Since an increase in the money growth rate ψ ceteris paribus raises inflation, it therefore ceteris paribus raises the incentive to innovate and the innovation rate $\mu(L, \psi, \beta)$ via a decrease in the compound discount rate.

Graphically, the increase in ψ causes a downward shift in the RHS-curve

[32] $\frac{\partial LHS}{\partial \psi} = -\frac{\alpha LHS[(\eta\overline{q}+1)\mu+(\rho+\beta)]}{(\rho+\beta-\alpha\psi+\overline{\eta}\mu)[\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu]} < 0$ given condition (5.37) and $\overline{\eta} - \overline{q} = [\eta + (\alpha-1)]\left(q^{\alpha-1} - 1\right) + 1 > 0$.

[33] Note that inflation only has an effect on profits through its influence on demand since the initial mark-up under price rigidity is optimally chosen by the firm to offset the direct effect of the changing mark-up on the firm's profit *per unit*. See Appendix 5.A.1.

[34] Remember that the discount rate is the obsolescence-adjusted interest minus the profit growth rate and that the present value of unit profit is unaffected by inflation since the initial mark-up is chosen optimally to offset this effect.

in figure 5.2, which ceteris paribus raises μ.

Net effect of money growth on economic growth depends on whether inflation is positive or negative. The negative price dispersion effect of an increase in money growth ψ shifts the LHS-curve of equation (5.33) downward while the positive mark-up erosion effect shifts the RHS-curve downward. Which effect is stronger, i.e. the sign of the net effect of money growth on $\mu(L, \psi, \beta)$ depends on whether inflation is positive or negative:

Lemma 13 *An increase in the money growth rate ψ decreases (increases) the innovation rate $\mu(L, \psi, \beta)$ when inflation is positive (negative).*

This is intuitive since both the discussed effects describe the impact on a firm's effective mark-up of a restriction on its price setting. This restriction which leads to suboptimal mark-ups cannot make the firm better off. Now while at $\pi = 0$, rigidity is ineffective since marginal cost is constant over time so that firms have no desire to readjust prices, for any departure from price stability, price rigidity becomes binding. At $\pi < 0$, an increase in ψ moves inflation closer to $\pi = 0$, reducing the distortion of the firm's mark-up and therefore increasing profits and the incentive to innovate which determines the growth rate. In contrast, at $\pi > 0$, an increase in ψ raises inflation and thus exacerbates the effects of rigidity, reducing profits and economic growth.

5.3.7.6 Innovation rate μ depends negatively on price rigidity $1/\beta$

Lemma 14 *An increase in the level of rigidity (i.e., decrease in β) decreases the innovation rate $\mu(L, \psi, \beta)$ for $\beta < \infty$.*

Analogously to the discussion of β's effect on the wage in section 5.3.6.1, an increase in the frequency of price adjustments β has qualitatively the same effects as a reduction in the inflation rate π: It reduces the need to have a forward-looking initial mark-up, reducing the *initial mark-up* effect of money growth by drawing the initial mark-up chosen closer to the static optimum. Graphically, an increase in β shifts the LHS-curve upward (downward) in figure 5.1 when inflation is positive (negative), which ceteris paribus decreases (increases) the partial equilibrium innovation rate.

At the same time, an increase in the frequency of price adjustment via β reduces the *mark-up erosion* effect of money growth since it shifts more weight to the discount rate for periods when prices are flexible, reducing the weight of the correction factor. Graphically, an increase in β shifts the RHS-curve upward (downward) in figure 5.1 when inflation is positive (negative), ceteris paribus increasing (decreasing) μ.

The intuition for the negative net effect of rigidity regardless of whether inflation is positive or negative is closely connected to the intuition concerning the effect of ψ: An intermediate good producer's profit is affected by rigidity only through the latter's effect on the firm's optimal and effective price. If changing prices infrequently were a profit-maximising strategy, the firm would have chosen this pricing strategy under flexibility, so there is no scope for price rigidity to increase the return to R&D.

5.4 Existence and uniqueness of the steady state equilibrium

Any solution to the equation:

$$\mu\left[L\left(\mu,\psi,\beta\right),\psi,\beta\right]=\mu \tag{5.38}$$

is a steady state equilibrium innovation rate and $L^*\left(\psi,\beta\right):=L\left[\mu\left(L,\psi,\beta\right),\psi,\beta\right]$ is the corresponding equilibrium employment level.

Remark 15 *In the leading example to be presented in the next section, the steady state equilibrium is unique.*

More generally, we can say the following:

Proposition 16 *Given conditions (5.35)-(5.37), a steady state equilibrium exists. If the maximum feasible* μ *in the economy,* $\mu^{max}:=\mu\left(\overline{L},\psi,\beta\right)$ *is sufficiently small, then there is a unique steady state equilibrium.*

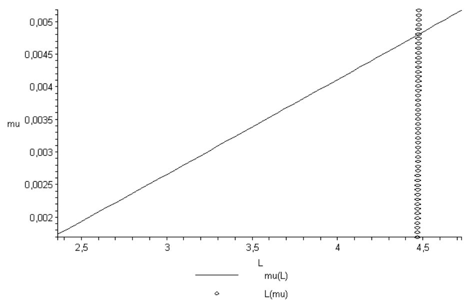

Figure 5.3: Existence and uniqueness of general steady state equilibrium research intensity mu in our leading example

Remember that we explained in section 5.3.6 that the wage and employment are approximately invariant to changes in μ when μ is small in relation to $\beta + \mu$. So given continuity of $L(\mu, \psi, \beta)$ in $\mu / (\beta + \mu)$, sufficiently small in this context means that the maximum feasible innovation rate μ^{max} must be small in relation to the frequency of price adjustment β, which is the case for all plausible economies. The $L(\mu)$-curve is then approximately linear and crosses the $\mu(L)$-curve once. Figure 5.3 illustrates this for our leading example.

5.5 Comparative statics: Employment level and economic growth rate in general equilibrium

In this section, we discuss the comparative static properties of the output growth rate, which is proportional to the innovation rate, and of the levels of employment and output in steady state equilibrium. These properties are determined by four effects we have already discussed in the partial equilibrium analysis of sections 5.3.6 and 5.3.7: The *Initial mark-up effect*, the *Mark-up erosion effect*, the *Average mark-up effect* and the *Price dispersion effect*. Using a calibrated example, we will in particular discuss which monetary policies would be chosen by monetary authorities interested in promoting employment and economic growth, respectively, and analyse the effect of price rigidity on growth and employment. Table 5.1 lists the parameter values chosen for our leading example.

parameter	value	parameter	value
q	1.2	ρ	0.015
α	10	σ	150
β	2.5	$\overline{L}$	4.7250
η	2		

Table 5.1: Parameter values used for calibration of the model with endogenous labour supply

The calibration is chosen to yield realistic and empirically plausible values for the economy's endogenous variables at a baseline money growth rate $\psi = 0.055$ per annum: The rate of economic growth is 2% while the unemployment rate is 5.3%. The mark-up chosen by firms amounts to 12.9%, while the average period during which prices are fixed is 0.40 years or 4.8 months.[35]

5.5.1 Inflation and employment: Monetary policy for promoting employment

First, note that superneutrality continues to hold:

Proposition 17 $\lim_{\beta\to\infty} \frac{dL}{d\psi} = \lim_{\beta\to\infty} \frac{d\mu}{d\psi} = \lim_{\beta\to\infty} \frac{d\gamma}{d\psi} = 0.$

The intuition remains unchanged: With perfectly flexible prices, inflation has no influence on relative prices or average mark-ups and therefore does not influence real variables.

Regarding the effect of money growth on employment in the presence of price rigidity ($\beta < \infty$), we first present the following proposition:

Proposition 18 *Starting from an equilibrium with price stability, an increase in the money growth rate ψ increases employment.*

[35] Both values are well in line with empirical estimates, see Basu and Fernald [1995, 1997] and Bils and Klenow [2004], respectively.

When we take into consideration the indirect effect of money growth on employment via the research intensity as well as the direct effects, then starting from an equilibrium with price stability, an increase in the money growth rate ψ lowers the average mark-up, so that output per labour unit, the wage and employment increase in ψ. Thus a monetary policy entailing moderate inflation is preferable to price stability for a monetary authority that wants to increase employment.

Before turning to the intuition for this result, consider more generally the shape of the function $L\left[\mu\left(\psi, L, \beta\right), \psi, \beta\right]$. Note that the total derivative of employment with respect to the money growth rate can be written as $\frac{dL}{d\psi} = \frac{L}{\psi}\left[\varepsilon_{L,\psi} + \varepsilon_{L,\mu} * \varepsilon_{\mu,\psi}\right]$ where $\varepsilon_{x,y}$ is the partial elasticity of x with respect to y. As argued in section 5.3.6.1, the elasticity of employment with respect to the innovation rate μ vanishes for $\mu/\left(\mu+\beta\right) \rightarrow 0$ and is indeed very small for all sensible calibrations since the contribution of the innovation rate to the degree of price flexibility $\beta + \mu$ is small.[36] Thus, $\varepsilon_{L,\mu} * \varepsilon_{\mu,\psi}$ is very small and the indirect effect of money growth on employment is negligible. We then have that Lemma (8) holds in general equilibrium:

Corollary 19 *For sufficiently small* $\varepsilon_{L,\mu} * \varepsilon_{\mu,\psi}$*, employment is a hump-shaped function of money growth with a maximum at a money growth rate* $\psi_2 > 0$ *associated with a positive inflation rate* $\pi\left(\psi_2\right) > 0$.

Figure 5.4 reflects this result for our leading example: The solid line depicts the function $L\left(\psi\right)$ in general equilibrium. The pointed line, which shows only the partial equilibrium effect of ψ on employment L given μ, is virtually indistinguishable from the solid line for negative and small ψ. For bigger values of ψ, the figure shows that the indirect effect through the research intensity reinforces the direct effect of money growth on employment, yet to a quantitatively negligible degree.

Thus through an increase in money growth starting from small rates of inflation, the monetary authority is successful in lowering the average

[36] At the same time, the point elasticity of the research intensity μ with respect to the money growth rate ψ also has to be small in all realistic examples- remember that a large discrete increase in the money growth rate from 1 percentage point to 10 percentage points should lower growth by not significantly more than a quarter percentage point.

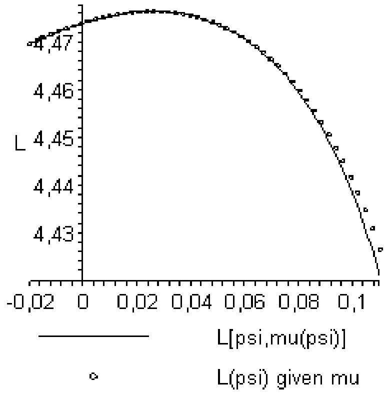

Figure 5.4: Partial and total effect of money growth on employment

mark-up in spite of the fact that intermediate good firms raise their initial mark-up in anticipation of price rigidity. This means monetary policy in this range successfully raises aggregate demand for intermediate goods which is inefficiently low due to monopolistic competition. The positive effect on aggregate demand via a lower average mark-up dominates money growth's negative effect on production efficiency at low levels of inflation, leading to a higher real wage and higher equilibrium employment than under price stability.

There is thus a range of money growth rates that entails a *Phillips-Curve*-trade-off for the monetary authority: Higher employment is only to be had at the price of higher inflation. In our leading example, this is true for money growth rates up to 2.60% or equivalently, positive inflation rates of up to 0.57%. In the range of money growth rates between 2.60% and 3.15% (inflation rate of 1.12%), employment again declines in money growth but is still higher than under price stability. Yet the effect is quantitatively small: At its maximum, employment is less than 0.01% higher than in the case of flexible prices. At the same time, the effect of an increase in money growth from $\psi = 0.01$ to $\psi = 0.1$ is quite sizeable: It increases the unemployment rate by over 0.8 percentage points from 5.28% to 6.08%.

Therefore monetary policy aimed at fostering employment should feature a moderate inflation rate, while high inflation should be avoided since it significantly reduces employment.

5.5.2 Inflation and the output level

By equation (5.3), we have

$$\frac{Y(\tau)}{Q(\tau)} = \alpha \widetilde{w}(\tau) L(\tau)$$

This implies that $\frac{d\frac{Y(\tau)}{Q(\tau)}}{d\psi} = \alpha \left(L(\tau) \frac{d\widetilde{w}(\tau)}{d\psi} + \widetilde{w}(\tau) \frac{\partial L(\tau)}{\partial \widetilde{w}} \frac{d\widetilde{w}(\tau)}{d\psi} \right) = \alpha \left(L + \widetilde{w} \frac{\partial L}{\partial \widetilde{w}} \right) \frac{d\widetilde{w}}{d\psi}$ or $\varepsilon_{\frac{Y}{Q},\psi} = (1 + \varepsilon_{L,\widetilde{w}}) \varepsilon_{\widetilde{w},\psi}$ where $\varepsilon_{x,y}$ is again the elasticity of x with respect to y. Given $\partial L / \partial \widetilde{w} > 0$, the steady state level of output in efficiency units $[Y(\tau)/Q(\tau)]^*$ as a function of money growth ψ has the same shape as the wage and employment functions. In particular, it reaches its maximum at the same money growth rate associated with positive inflation where employment peaks, and it is strongly reduced by high inflation. Starting from the maximum value at $\psi = 2.6\%$ and increasing ψ to $\psi = 0.1$ in our baseline numerical example, the reaction of output to changes in money supply is much more sizeable than the reaction of employment: Output drops by 7.22%, whereas employment decreases by only 0.86%. Figure 5.5 shows $Y(\tau)/Q(\tau)$ as a function of the money growth rate in the leading example.

5.5.3 Inflation and economic growth: Monetary policy for promoting growth

Remember that in section 5.3.7.5 holding constant employment, we found a hump-shaped relationship between the innovation rate and money growth or inflation, respectively. The innovation rate peaked at zero inflation. Our subsequent analysis of how the additional influence of money growth on the wage and employment changes this result shows that while the hump-shaped relationship persists, the best policy for a monetary authority interested in promoting economic growth features a positive rate of inflation.

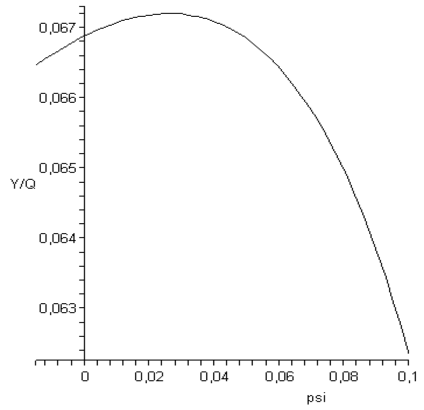

Figure 5.5: Output as a function of the money growth rate

Regarding the hump-shaped relationship between money growth and economic growth, which by equation (5.21) is a linear function of the innovation rate, we have the following proposition:

Proposition 20 *For small values of the money growth rate ψ, the economic growth rate γ increases in ψ, for large values of ψ, the growth rate γ decreases in ψ.*

Thus the qualitative relation is similar in partial equilibrium and general equilibrium. Yet there is one qualitative difference:

Proposition 21 *The economic growth rate reaches its maximum at a positive rate of inflation.*

As shown in section 5.3.7.5, *holding constant employment*, the incentive to innovate is reduced by nonzero inflation because an intermediate good producer's profits decreases due to the restriction on his price setting imposed by price rigidity: In anticipation of price rigidity, the initial mark-up chosen is higher than the static optimum. During the firm's life time, inflation erodes the mark-up. Consequently, the mark-up generically does not correspond to the optimal one, lowering profits and thus, the patent price.

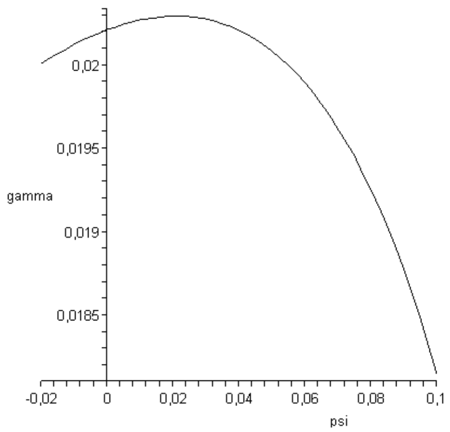

Figure 5.6: Economic growth rate as a function of the money growth rate

In general equilibrium, these effects are still present. Yet at the same time, lemma 11 shows that the innovation rate increases monotonically in employment. As seen in section 5.3.6.1, at zero inflation and small positive inflation rates the wage and employment increase in the money growth rate ψ. At small positive rates of inflation, the positive indirect effect of an increase in ψ on growth via employment is stronger than the negative direct effect so that the incentive to innovate and the growth rate increase in the money growth rate.

Yet as the money growth rate ψ continues to rise, distortions increase and the wage begins to fall in ψ, which adds to the mark-up distorting effects of positive money growth in causing a fall in the economic growth rate.

Figure (5.6) shows the economic growth rate as a function of money growth in our leading example.

The economic growth rate is maximised at $\psi = 0.021$, which corresponds to the positive inflation rate $\pi = 0.07\%$. At this inflation rate, the economic growth rate is 2.03% compared to 2.0% in the baseline case. While this effect is rather small, the effect of inflation on growth is more drastic at inflation rates that are further away from the optimum: When the money growth rate increases from $\psi = 0.01$ to $\psi = 0.1$, the growth rate decreases by 0.21 percentage point, which corresponds closely to empirical estimates mentioned in the introductory chapter.

Thus while a monetary authority that wants to promote growth should allow for moderate inflation rather than aim at price stability, it should also be aware of the growth depressing effects of high inflation.

5.5.4 Limited trade-off for monetary policy between employment and growth

In the preceding two sections, we examined which monetary policy would be optimal for a monetary authority interested in promoting either employment or economic growth. We found that some inflation raises the wage and employment. Due to its effect on employment, a small positive inflation rate also fosters economic growth. At the same time, too much inflation proved to reduce both employment and inflation.[37]

There is no strong conflict between promoting growth and raising employment for the central bank: Given perfect information about the central union's policy and given any preference structure involving the goals of employment and economic growth, the monetary authority will always choose a money growth rate from the range $\psi \in \left(\psi_\gamma, \psi_L\right)$ where ψ_γ (ψ_L) maximises economic growth (employment). Within this range, an increase in ψ always increases employment and lowers economic growth.[38] The trade-off is limited in our calibrated examples where ψ_γ and ψ_L are very close. Figure 5.7 illustrates the trade-off for our leading example, where the range of money growth rates involved is $\psi \in (0.021, 0.026)$ which corresponds to inflation rates between 0.07% and 0.57%.

[37] This implies that a long-run version of *Okun's law*, according to which an increase in economic growth is always accompanied by a decrease in the unemployment rate, holds in our model for most money growth rates.

[38] This follows from the fact that the inequality $\psi_\gamma < \psi_L$ always holds. To get intuition for this fact, remember that the total effect of an increase in money growth on economic growth comprises the sum of non-employment related effects and the employment related effect, where we know that the former are negative at positive rates of inflation. Thus, the total effect of ψ on γ $(d\gamma/d\psi)$ is always smaller than the employment related effect. Therefore, at the money growth rate that maximises employment $(dL/d\psi = 0)$, $d\gamma/d\psi < 0$ so $\gamma(\psi)$ reaches its maximum at a smaller ψ.

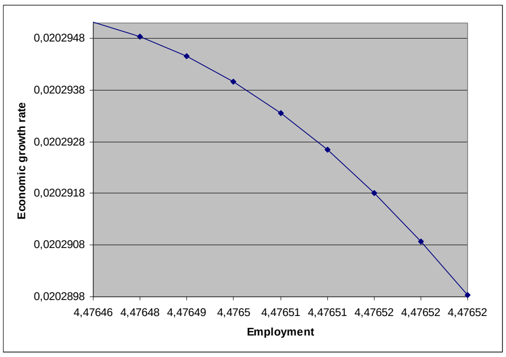

Figure 5.7: Trade-off between employment and growth

5.5.5 A digression on the difference in results concerning the effect of an increase in ψ between the present model and the model of chapter 2

The comparative static results of the models of chapters 2 and 5 regarding the effects of a change in the money growth rate differ in two respects due to the modification of the sector structure: First, when intermediate goods are produced using output (labour) as in the present chapter (chapter 2), the relevant growth rate of marginal cost is the inflation rate π (the money growth rate ψ) so that the outcomes of the model under price rigidity are identical to those of the model with flexible prices only at $\pi = 0$ ($\psi = 0$). Second, in the present chapter an increase in ψ raises the incentive to innovate when the growth rate of marginal cost is negative ($\pi < 0$), whereas in chapter 2 an increase in ψ lowers growth *regardless* of the initial value of the growth rate of marginal costs.

This second more substantial difference in results vanishes when one holds constant the real wage level in efficiency units $w/(PQ)$ in both models. Given the real wage, the incentive to innovate in both models increases (falls) in the money growth rate when the growth rate of marginal cost is negative (positive),[39] because any departure from constant marginal cost ($\psi = 0$ and

[39] See the intuition for proposition 3 in section 2.4.2 of chapter 2 and lemma 13 in section 5.3.7.5 of the present chapter.

$\pi = 0$, respectively) raises the distortion of the mark-up of the intermediate goods firm.[40]

In both model specifications, the real wage level depends negatively on the average monopolistic mark-up.[41] The average mark-up in turn depends on the money growth rate ψ via the initial mark-up and via the deviation between initial and average mark-up caused by the fact that existing mark-ups are eroded by money growth.[42] At negative growth rates of marginal cost, an increase in ψ causes a decrease in the average mark-up which in turn raises the real wage given Q.[43] In calibrated examples of the models of chapters 2 and 5, the average mark-up (the wage) is minimised (maximised) at a small positive growth rate of marginal cost.

In the present chapter, growth depends positively on employment which in turn increases in the real wage in efficiency units. Given that the latter is maximised at a (small) positive inflation rate, so is the output growth rate.

In contrast, in the model specification of chapter 2, an intermediate goods producer upon market entry sets his price as a mark-up over marginal cost, the nominal wage, implying that the initial relative price increases in the real wage. Since demand for the new good and hence, profits fall in the initial relative price, the incentive to innovate decreases in the real wage level. Given that the real wage decreases in the money growth rate at $\psi < 0$ and at the same time, the effect of a change in the initial relative price is stronger at $\psi < 0$ than at $\psi > 0$, the economic growth rate is maximised at the lowest

[40] In the context of chapter 2, the production inefficiency resulting from non-zero growth of marginal cost and optimal prices aggravates the fall in profits and hence, in the incentive to innovate.

[41] In the model specification of chapter 2, intermediate goods firms set prices as a mark-up over the nominal wage, the final good sector is perfectly competitive with zero profits and only intermediate goods are used in final good production. Hence, the real wage is one over the average mark-up.

In the specification of the present chapter, the real wage is the productivity of labour in final good production which depends positively on the amount of intermediate goods used. This amount (and therefore, the real wage level) depends negatively on the average mark-up charged by intermediate goods producers. The fact that the wage also depends on the degree of price dispersion in the specification of the present chapter does not change the results of our analysis here.

[42] See the discussion in section 5.3.6.

[43] See footnote 24 in chapter 2 and footnote 24 in section 5.3.6.1 of the present chapter.

admissible value of ψ.

Which specification is more plausible? Producing intermediate goods with constant returns to a single input is a strong assumption. Compared to this simplification, whether the unique input is labour or output (capital with full depreciation) seems to be of secondary importance. Investigating whether lower monopolistic distortions are empirically associated with higher (chapter 5) or lower growth (chapter 2) might help to distinguish between the specifications.

Note, however, that a positive influence of the real wage on labour supply in the model specification of chapter 2 would reduce the difference in results: The growth maximising value of the money growth rate would rise in this model, too, since firstly, growth depends positively on employment,[44] and secondly, the real wage is maximised at a positive value of ψ just as in the model of the present chapter.

Note finally that while there are differences concerning the implications of deflation for economic growth, the much more central policy implication that high inflation is detrimental to growth emerges consistently from both model specifications.

5.5.6 Comparative statics regarding the level of price rigidity

Unlike in partial equilibrium with constant employment where the innovation rate growth increased in β whenever $\pi \neq 0$, with endogenous labour supply price rigidity is not universally bad for innovation and economic growth. In spite of the distortions it entails, the very presence of price rigidity allows the monetary authority to implement a policy which through the lowering of the average mark-up raises employment and with it, the economic growth rate beyond its level in a world without rigidities. This is summarised in the following proposition which we prove in Appendix 5.A.4:

Proposition 22 *At sufficiently low levels of positive inflation, employment*

[44] See equation 2.36 and footnote 46 in chapter 2.

output and economic growth are higher under price rigidity than in a world without price rigidity.

5.6 Conclusion

Endogenising employment has shown that there is a non-linear long-run Phillips-curve in the Schumpeterian model with price rigidity: Employment and output are hump-shaped functions of money growth. This result both complements and generalises the results concerning the long run found by the New Keynesian literature and reported in section 1.3.2 of the introductory chapter since this literature relied mainly on simulation exercises and on a less general specification of price rigidity. At the same time, we find that the endogeneity of growth does not significantly change the qualitative or quantitative relationship between inflation and employment or output.

In contrast, the endogeneity of employment does matter for the inflation-growth relationship: Higher employment raises growth because the marginal productivity of intermediate goods, whose growing quality is the engine of growth, increases in employment. Thus as in the endogenous growth literature investigating the consequences of introducing money via a cash in advance constraint discussed in section 1.4.1.2 of chapter 1, inflation impacts on growth via employment because the latter influences the marginal productivity of investment. Yet fundamentally different policy recommendations emerge from their analysis of the leisure-substitution channel than from the present analysis of the sticky price transmission channel: In the models of section 1.4.1.2, growth is maximised when the monetary authority follows the Friedman rule, i.e. contracts the money supply at a rate that makes the nominal money supply zero. In contrast, in the present context where employment is influenced by inflation via the latter's influence on the average monopolistic mark-up and on price dispersion, employment and therefore growth are maximised at small positive growth rates.

Are these effects empirically relevant? A look at the data suggests they are: There are several studies investigating the relationship between inflation and mark-ups,[45] which find evidence of a negative influence of inflation on mark-ups. At the same time, higher inflation also seems to be associated with more price dispersion empirically.[46]

5.A Appendix to chapter 5

5.A.1 A new firm's market value

Following analogous reasoning about the development of the firm's price given the process for price reset signals β as in Appendix 2.A.1, the firm's market value is

$$\begin{aligned}
&E\left(V_{k_j}(\tau)\,\middle|\,t_{k_j}=\tau\right)\\
=&\widetilde{A}\int_{\tau}^{\infty} e^{-\widetilde{\chi}(s-\tau)}\left[\int_{\tau}^{s}\beta e^{-\beta(s-\theta)}p^*(\theta)^{1-\alpha}\,d\theta+(1-\int_{\tau}^{s}\beta e^{-\beta(s-\theta)}d\theta)p^*(\tau)^{1-\alpha}\right]ds\\
&-\widetilde{A}w(\tau)\int_{\tau}^{\infty} e^{-(\widetilde{\chi}-\omega)(s-\tau)}\left[\int_{\tau}^{s}\beta e^{-\beta(s-\theta)}p^*(\theta)^{-\alpha}\,d\theta+(1-\int_{\tau}^{s}\beta e^{-\beta(s-\theta)}d\theta)p^*(\tau)^{-\alpha}\right]ds
\end{aligned}$$

where $\widetilde{A}=\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}q^{(\alpha-1)k_j}L(\tau)$ and $\widetilde{\chi}=i+\mu-\alpha\pi$. Going through a number of steps reported in more detail in Appendix 2.A.1 yields

$$E\left(V_{k_j}|\tau\right)=\widetilde{A}\left(\frac{p^*(\tau)}{P(\tau)}\right)^{-\alpha}\frac{\widetilde{\chi}+\beta+(\alpha-1)\pi}{\widetilde{\chi}+(\alpha-1)\pi}\left[\frac{p^*(\tau)}{\widetilde{\chi}+\beta}-\frac{P(\tau)}{\widetilde{\chi}+\beta-\pi}\right]$$

Note that again, the initial mark-up $\frac{\alpha}{\alpha-1}\frac{\widetilde{\chi}+\beta}{\widetilde{\chi}+\beta-\pi}$ is again chosen such that the present value of revenues is identical to what it would have been if revenues had grown at the same constant rate as costs, implying that changes in relative prices only affect profits through their impact on demand, not via their effect on unit profit. Using the equation for the optimal price $p^*(\tau)$ (5.22) and reinserting $\widetilde{\chi}=i+\mu-\alpha\pi$ and $\widetilde{A}=\left(\frac{\alpha-1}{\alpha}A\right)^{\alpha}q^{(\alpha-1)k_j}L$ we have

[45] E. g., Benabou [1992], Banerjee and Russell [2005] and Banerjee, Mizen and Russell [2006]. More references can be found in the last mentioned paper.

[46] Parks [1978] is a seminal paper in this literature. For recent contributions see Banerjee, Mizen and Russell [2006] and the references therein.

equation (5.24) in the main text.

5.A.2 Total intermediate good production

The total production of intermediate goods at time τ can be rewritten as

$$X(\tau) = \left(\frac{\alpha-1}{\alpha}A\right)^{\alpha} L(\tau)\, Q(\tau)\, P(\tau)^{\alpha} \sum_{k=1}^{k^{\max}} d_k(\tau)\, \frac{q^{(\alpha-1)k}}{Q(\tau)} \sum_{\{j|k_j=k\}} \left(p_{k_j}(\tau)\right)^{-\alpha}$$

where p_{k_j} is the price of sector j that is at quality rung k and $d_k(\tau)$ is the number of sectors at quality rung k at time τ. Analogously to the procedure of Appendix 2.A.2, the average price effective at time τ can be expressed as a weighted average of past optimal prices, using the same weights $f(s,\tau) = (\mu+\beta)\, e^{-(\mu+\beta)(\tau-s)}$ reflecting the probability that a price valid at time τ has not been changed since time s. Using the steady state growth rate π of p^*, we have

$$X(\tau) = \left(\frac{\alpha-1}{\alpha}A\right)^{\alpha} L(\tau) \left[\frac{p^*(\tau)}{P(\tau)}\right]^{-\alpha} Q(\tau) \int_{-\infty}^{\tau} (\beta+\mu)\, e^{-(\beta+\mu-\alpha\pi)(\tau-s)} ds$$

Solving the integral which converges for $\beta > \alpha\psi$ leads to (5.25) in the main text. Convergence of the integral is ensured by condition (5.37).

5.A.3 Total final good production

Total final good production can be rewritten as

$$Y(\tau) = A^{\alpha} \left(\frac{\alpha-1}{\alpha}\right)^{\alpha-1} L(\tau)\, P(\tau)^{\alpha-1} Q(\tau) \sum_{j=1}^{N} \frac{q^{(\alpha-1)k_j(\tau)}}{Q(\tau)} p_j(\tau)^{-(\alpha-1)}$$

As in Appendix 5.A.2, the average intermediate good price effective at τ can be expressed as a weighted average of past optimal prices with the weights $\widetilde{f}(s,\tau)$ defined in Appendix 5.A.2.

$$Y(\tau) = A^{\alpha} \left(\frac{\alpha-1}{\alpha}\right)^{\alpha-1} L(\tau)\, P(\tau)^{\alpha-1} Q(\tau) \int_{-\infty}^{\tau} p^*(s)^{-(\alpha-1)}\, \widetilde{f}(s,\tau)\, ds$$

Inserting $\widetilde{f}(s,\tau) = (\mu+\beta)\,e^{-(\mu+\beta)(\tau-s)}$ and using that the optimal price grows at rate π in equilibrium, we can calculate the integral's value which gives

$$\frac{Y(\tau)}{Q(\tau)} = \left(\frac{\alpha-1}{\alpha}\right)^{\alpha-1} A^{\alpha} L(\tau) \left[\frac{p^*(\tau)}{P(\tau)} \left(\frac{\beta+\mu}{\beta+\mu-(\alpha-1)\,\pi}\right)^{-\frac{1}{\alpha-1}}\right]^{-(\alpha-1)}$$

Note that convergence of the integral is ensured by assumption (5.37).

Next, we want to rewrite Y/Q as a product of efficient production with a given X/Q and a term that describes production inefficiency due to price dispersion. To find the maximum amount of final goods Y^{eff} that can be produced with a given amount X of intermediate goods when X is distributed efficiently among the intermediate good types x_j, we solve the following problem:

$$\max_{x_j} Y + \varpi \left[X - \sum_j x_j\right]$$

subject to the final good production function (5.1). The first order condition for x_j can be rewritten as

$$\left(\frac{\frac{\alpha-1}{\alpha} A L^{1/\alpha} q^{k_j \frac{\alpha-1}{\alpha}}}{\varpi}\right)^{\alpha} = x_j$$

Aggregating over the intermediate good types and solving for ϖ gives

$$\frac{\alpha-1}{\alpha} A L^{1/\alpha} \left(\frac{X}{Q}\right)^{-1/\alpha} = \varpi$$

Reinserting this into the first order condition gives

$$q^{k_j(\alpha-1)} \left(\frac{X}{Q}\right) = x_j$$

which can in turn be reinserted in the final good production function (5.1), yielding

$$Y^{eff}(\tau) = AL(\tau)^{1/\alpha} \left(\frac{X(\tau)}{Q(\tau)}\right)^{\frac{\alpha-1}{\alpha}} Q(\tau)$$

Now multiplying and dividing actual total final good production (5.27) by Y^{eff}, replacing $X(\tau)/Q(\tau)$ in the denominator with the amount of intermediate goods actually used (5.25) and simplifying, we have equation (5.27) in the text.

5.A.4 Proofs of propositions, lemmata and corollaries (8)-(22)

Proof of lemma 8. The derivative with respect to the money growth rate ψ of the function $\widetilde{w}(\psi)$ from equation (5.28) for a given innovation rate μ is $\partial\widetilde{w}/\partial\psi = \frac{(\alpha-1)\widetilde{w}}{\beta+\mu-(\alpha-1)\pi}\left\{-\frac{r+\beta+\mu}{r+\beta+\mu-\alpha\pi}\frac{\beta+\mu-(\alpha-1)\pi}{r+\beta+\mu-(\alpha-1)\pi}+1\right\}$. Examining the nulls of the derivative shows that the the function has extrema at

$$\psi_{1/2} = \frac{1}{2}\frac{1}{\alpha-1}\left\{[r+\beta+\mu+2(\alpha-1)\gamma] \begin{smallmatrix}-\\+\end{smallmatrix} (\beta+\mu+r)^{1/2}\left(\beta+\mu+\frac{4-3\alpha}{\alpha}r\right)^{1/2}\right\}$$

associated with

$$\pi_{1/2} = \frac{1}{2}\frac{1}{\alpha-1}(\beta+\mu+r)^{1/2}\left\{(\beta+\mu+r)^{1/2} \begin{smallmatrix}-\\+\end{smallmatrix} \left(\beta+\mu+\frac{4-3\alpha}{\alpha}r\right)^{1/2}\right\}$$

with $\pi(\psi_2) > \pi(\psi_1) > 0$.

Examining the second derivative at ψ_1 and ψ_2 shows that $\widetilde{w}(\psi)$ has a maximum (minimum) at ψ_1 (ψ_2) because

$$\left.\frac{\partial^2 \frac{w(\tau)}{P(\tau)Q(\tau)}}{\partial\psi^2}\right|_{\psi=\psi_1} = \frac{-4\alpha\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}(\beta+\mu+r)^{-1/2}(\alpha-1)\widetilde{w}}{\left\{(\beta+\mu+r)^{1/2}+\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}\right\}^2\left\{\frac{1}{2}\frac{\alpha-2}{\alpha-1}(\beta+\mu+r)^{1/2}+\frac{1}{2}\frac{\alpha}{\alpha-1}\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}\right\}^2} < 0$$

and

$$\left.\frac{\partial^2 \frac{w(\tau)}{P(\tau)Q(\tau)}}{\partial\psi^2}\right|_{\psi=\psi_2} = \frac{+4\alpha(\beta+\mu+r)^{-1/2}\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}(\alpha-1)\widetilde{w}}{\left\{\frac{1}{2}\frac{\alpha-2}{\alpha-1}(\beta+\mu+r)^{1/2}-\frac{1}{2}\frac{\alpha}{\alpha-1}\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}\right\}^2\left\{(\beta+\mu+r)^{1/2}-\left[\beta+\mu+\frac{4-3\alpha}{\alpha}r\right]^{1/2}\right\}^2} > 0.$$

Further, we find that $\psi_2 > \frac{1}{\alpha-1}\beta$, which is the maximum admissible money growth rate from condition (5.37), since

$$\frac{1}{2}\frac{1}{\alpha-1}\left\{[r+\beta+\mu+2(\alpha-1)\gamma]+(\beta+\mu+r)^{1/2}\left(\beta+\mu+\frac{4-3\alpha}{\alpha}r\right)^{1/2}\right\} > \frac{1}{\alpha-1}\beta.$$

Therefore, $\widetilde{w}(\psi)$ increases (decreases) in ψ for all admissible money growth rates ψ with $\psi < \psi_1$ ($\psi > \psi_1$). The inflation rate $\pi(\psi_1) = \frac{1}{2}\frac{1}{\alpha-1}(\beta+\mu+r)^{1/2}\left\{(\beta+\mu+r)^{1/2}-\left(\beta+\mu+\frac{4-3\alpha}{\alpha}r\right)^{1/2}\right\}$ is positive since $\alpha > 1$ ensures that $1 > (4-3\alpha)/\alpha$. ■

Proof of lemma 9. Straightforward calculus shows that $\frac{\partial\widetilde{w}}{\partial\beta} = \frac{(\alpha-1)\widetilde{w}\pi}{(\rho+\beta-\alpha\psi+\overline{\eta}\mu)[\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu]} * \left[1-\frac{r+\beta+\mu-\alpha\pi}{\beta+\mu}\frac{r+\beta+\mu-(\alpha-1)\pi}{\beta+\mu-(\alpha-1)\pi}\right]$. The term in square brackets is always negative under deflation, while the fraction in front of the square brackets is negative (positive) for $\pi < 0$ ($\pi > 0$), so that $\frac{\partial \widetilde{w}}{\partial \beta} > 0$ for $\psi < \psi_0$ with $\pi(\psi_0) = 0$. A very strict sufficient condition for the term in square brackets to be negative is $r > \alpha\pi$, for which $\rho/\alpha > \psi$ is again a sufficient condition so that $\frac{\partial \widetilde{w}}{\partial \beta} > 0$ holds for $\rho/\alpha > \psi > \psi_0$. ■

Proof of lemma 10. Conditions (5.36) and (5.37) are jointly sufficient for the LHS of equation (5.33) to be concave in μ:
$\frac{\partial^2 LHS}{\partial \mu^2} = \frac{C_1[(\overline{\eta}-\alpha\overline{q})\psi+\overline{q}(\rho+\beta)]\{(1+\alpha)[(\eta\overline{q}+1)\psi+\overline{q}(\rho+\beta)]-2\overline{\eta}[\rho+\beta-(\alpha-1)\psi+[(\eta+\alpha-1)\overline{q}+1]\mu]\}}{(\rho+\beta-\alpha\psi+\overline{\eta}\mu)^3[\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu]\left(\frac{\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu}{\rho+\beta-\alpha\psi+\overline{\eta}\mu}\right)^{1+\alpha}}$ where $C_1 = \alpha\frac{L}{\lambda}A^{\alpha}\frac{1}{\alpha-1}\left(\frac{\alpha-1}{\alpha}\right)^{2\alpha}$, $\overline{q} = q^{\alpha-1} - 1 > 0$ and $\overline{\eta} = [(\eta+\alpha)\overline{q} + 1]$. Given that $\overline{\eta} - \alpha\overline{q} = \eta\overline{q} + 1 > 0$ and $\beta > \alpha\psi$, $\frac{\partial^2 LHS}{\partial \mu^2} < 0$ when condition (5.36) holds. Also, $\frac{\partial^2 RHS}{\partial \mu^2} = \frac{2\alpha\beta(\eta\overline{q}+1)[(\overline{\eta}-\alpha\overline{q})\psi+\overline{q}(\rho+\beta)]}{[\rho+\beta+(\eta\overline{q}+1)\mu]^3} > 0$ when condition (5.37) and the first inequality in condition (5.36) hold, so these conditions are sufficient to ensure that the RHS of the equation is convex in μ. With condition (5.35) we make sure that the value for $\mu \to 0$ of the LHS of equation (5.33) is larger than that of the RHS. Further, note that the RHS of equation (5.33) goes to infinity as $\mu \to \infty$ ($\lim_{\mu\to\infty} \frac{\rho+\left[\eta\left(q^{\alpha-1}-1\right)+1\right]\mu}{\rho+\beta+[\eta(q^{\alpha-1}-1)+1]\mu}(\rho+\beta-\alpha\psi+\overline{\eta}\mu) = \infty$ since $\overline{\eta} > 0$) while the limit of the LHS is bounded ($\lim_{\mu\to\infty} LHS = \frac{L}{\lambda}A^{\alpha}\frac{1}{\alpha-1}\left(\frac{\alpha-1}{\alpha}\right)^{2\alpha}\left(\frac{\overline{\eta}-\left(q^{\alpha-1}-1\right)}{\overline{\eta}}\right)^{-\alpha} < \infty$). Thus the two functions have a unique intersection with $\mu > 0$. ■

Proof of lemma 12. For $\beta \to \infty$, the zero profit condition (5.33) reduces to $\frac{L}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha} A^{\alpha}\frac{1}{\alpha-1} = [\rho + (\eta\overline{q}+1)\mu]$ so that the growth rate of money ψ has no influence on the equilibrium research intensity μ. Since by equation (5.21) $\gamma = (q^{\alpha-1} - 1)\mu$, the economy's real growth rate γ is independent of ψ, too. ■

Proof of lemma 13. Consider equation (5.33). First refer to figure 5.1 to see that given assumptions (5.35)-(5.37) and concavity of the LHS-curve, the latter's slope is always smaller than that of the RHS-curve at the equilibrium ($\frac{\partial LHS}{\partial \mu} - \frac{\partial RHS}{\partial \mu} < 0$). Further, $\frac{\partial LHS}{\partial \psi} - \frac{\partial RHS}{\partial \psi} = -\frac{\alpha(\alpha-1)LHS(\psi-\overline{q}\mu)}{(\rho+\beta-\alpha\psi+\overline{\eta}\mu)[\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu]}$. From assumption (5.37) we have that $\beta > \alpha\psi$. Further, $\overline{\eta} = [(\eta+\alpha)\overline{q}+1] > \overline{q}$ and from equations (5.21) and (5.18) we have $\psi - \overline{q}\mu = \pi$, so that $\frac{\partial LHS}{\partial \psi} - \frac{\partial RHS}{\partial \psi} \lesseqgtr 0$ for $\pi \gtreqless 0$. Thus we have that $\frac{d\mu}{d\psi} = -\frac{\frac{\partial LHS}{\partial \psi} - \frac{\partial RHS}{\partial \psi}}{\frac{\partial LHS}{\partial \mu} - \frac{\partial RHS}{\partial \mu}} \lesseqgtr 0$ for $\pi \gtreqless 0$.

Further, since $\gamma = \left(q^{\alpha-1} - 1\right)\mu$, $\frac{d\gamma}{d\psi} = \left(q^{\alpha-1} - 1\right)\frac{d\mu}{d\psi} \lesseqgtr 0$ for $\pi \gtreqless 0$. ■

Proof of lemma 14.

From equation (5.33), $\frac{d\mu}{d\beta} = -\frac{\frac{\partial LHS}{\partial\beta} - \frac{\partial RHS}{\partial\beta}}{\frac{\partial LHS}{\partial\mu} - \frac{\partial RHS}{\partial\mu}} > 0$ since $\frac{\partial LHS}{\partial\beta} - \frac{\partial RHS}{\partial\beta} = LHS\frac{(\alpha-1)\alpha(\psi-\overline{q}\mu)^2\{\rho+\beta-(\alpha-1)\psi+(\overline{\eta}-\overline{q})\mu\}^{-1}}{(\rho+\beta-\alpha\psi+\overline{\eta}\mu)\{\rho+\beta+(\eta\overline{q}+1)\mu\}} > 0$ is positive for $\psi - \overline{q}\mu = \pi \neq 0$ as $\beta > \alpha\psi$ given assumption (5.37) and $\overline{\eta} = [(\eta+\alpha)\overline{q}+1] > \overline{q}$ and $\frac{\partial LHS}{\partial\mu} - \frac{\partial RHS}{\partial\mu} < 0$ from the proof of lemma 13. ■

Proof of proposition 16. Existence of a solution to equation (5.38) follows from the fact that for any $L \in \left(\frac{1}{2}\overline{L}, \overline{L}\right)$, the innovation rate $\mu(L,\psi,\beta)$ is uniquely defined (Lemma 10), and increases monotonically in L (Lemma 11) and that $L(\mu,\psi,\beta) \in \left(\frac{1}{2}\overline{L}, \overline{L}\right)$ is defined for each μ and is continuous in μ, with $L(\mu^{max},\psi,\beta) < \overline{L}$ since $\widetilde{w}(\mu^{max},\psi,\beta) < \infty$. Given $\lim\limits_{[\mu/(\mu+\beta)]\to 0}\frac{\partial\widetilde{w}(\mu,\psi,\beta)}{\partial\mu} = \lim\limits_{[\mu/(\mu+\beta)]\to 0}\frac{\partial L(\mu,\psi,\beta)}{\partial\mu} = 0$ (section 5.3.6) and continuity of L in $\mu/(\mu+\beta)$, there always exists a μ small enough to ensure uniqueness. ■

Proof of proposition 17. From equation (5.28), for $\beta \to \infty$ the wage $w^{flex} = \frac{1}{\alpha}A^{\alpha}\left(\frac{\alpha}{\alpha-1}\right)^{-2(\alpha-1)}$ is constant so employment is independent of the money growth rate ψ. The equation determining the research intensity (5.38) reduces to $\frac{L\left(w^{flex}\right))}{\lambda}\left[\overline{\alpha}\left(\frac{\alpha}{\alpha-1}\right)^{-\alpha}\right] = (\rho + \overline{\eta}_2\mu)$ which is independent of ψ, too. ■

Proof of proposition 18. We analyse the influence at $\pi = 0$ of ψ on $L^* = L\{\widetilde{w}[\mu^*(\psi)],\psi\}$ (equation (5.30)). Here μ^* is the solution to equation (5.33) where L has been replaced by the endogenous $L(\mu,\psi,\beta)$ from equation (5.30). We then have that $\frac{dL^*}{d\psi} = \frac{\partial L^*}{\partial\psi} + \frac{\partial L^*}{\partial\mu}\frac{\partial\mu^*(\psi)}{\partial\psi} = \frac{\partial L^*}{\partial\widetilde{w}}\frac{\partial\widetilde{w}}{\partial\psi} + \frac{\partial L^*}{\partial\widetilde{w}}\frac{\partial\widetilde{w}}{\partial\mu^*}\frac{\partial\mu^*(\psi)}{\partial\psi}$. From equation (5.16), $\frac{\partial L}{\partial\widetilde{w}} > 0$ for all values of $\widetilde{w}$. Further, taking the derivatives of the wage in equation (5.28), we find that $\left.\frac{d\widetilde{w}}{d\psi}\right|_{\pi=0} = \frac{(\alpha-1)C_1 r}{\left(\frac{\alpha}{\alpha-1}\right)^{\alpha-1}(\beta+\mu)(r+\beta+\mu)} > 0$ with $\left(C_1 = \frac{1}{\alpha}\left(\frac{\alpha-1}{\alpha}\right)^{\alpha-1}A^{\alpha}\right)$ and $\left.\frac{d\widetilde{w}}{d\mu}\right|_{\pi=0} = \frac{-rC_1(\alpha-1)\overline{q}}{\left(\frac{\alpha}{\alpha-1}\right)^{\alpha-1}(r+\beta+\mu)(\beta+\mu)} = -\overline{q}\left.\frac{d\widetilde{w}}{d\psi}\right|_{\pi=0} < 0$. Finally, $\left.\frac{\partial\mu^*(\psi)}{\partial\psi}\right|_{\pi=0} = \frac{\left.\frac{\partial L(\mu,\psi,\beta)}{\partial\psi}\right|_{\pi=0}\frac{1}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^{\alpha}\frac{1}{\alpha-1}}{-\left\{-(\eta\overline{q}+1)+\frac{1}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^{\alpha}\frac{1}{\alpha-1}\left.\frac{\partial L(\mu,\psi,\beta)}{\partial\mu}\right|_{\pi=0}\right\}}$ from equation (5.33) with endogenous $L(\mu,\psi,\beta)$. So

$$\left.\frac{dL^*}{d\psi}\right|_{\pi=0} = \left.\frac{\partial L}{\partial\widetilde{w}}\right|_{\pi=0}\left\{\left.\frac{\partial\widetilde{w}}{\partial\psi}\right|_{\pi=0} + \left.\frac{\partial\widetilde{w}}{\partial\mu}\right|_{\pi=0}\frac{\left.\frac{\partial L(\mu,\psi,\beta)}{\partial\psi}\right|_{\pi=0}\frac{1}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^{\alpha}\frac{1}{\alpha-1}}{-\left\{-(\eta\overline{q}+1)+\frac{1}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha}A^{\alpha}\frac{1}{\alpha-1}\left.\frac{\partial L(\mu,\psi,\beta)}{\partial\mu}\right|_{\pi=0}\right\}}\right\}$$

which using that $\left.\frac{\partial\widetilde{w}}{\partial\mu}\right|_{\pi=0} = -\overline{q}\left.\frac{d\widetilde{w}}{d\psi}\right|_{\pi=0}$ can be rewritten as

$$\left.\frac{dL^*}{d\psi}\right|_{\pi=0} = \left.\frac{\partial L}{\partial \widetilde{w}}\right|_{\pi=0} * \left.\frac{\partial \widetilde{w}}{\partial \psi}\right|_{\pi=0} \left\{ 1 - \overline{q} \frac{-\frac{1}{\overline{q}} \left.\frac{\partial L(\mu,\psi,\beta)}{\partial \mu}\right|_{\pi=0} \frac{1}{\lambda} \left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha} A^{\alpha} \frac{1}{\alpha-1}}{(\eta\overline{q}+1) - \frac{1}{\lambda}\left(\frac{\alpha}{\alpha-1}\right)^{-2\alpha} A^{\alpha} \frac{1}{\alpha-1} \left.\frac{\partial L(\mu,\psi,\beta)}{\partial \mu}\right|_{\pi=0}} \right\} > 0$$

since the term in curly brackets is positive given $\left.\frac{\partial L(\mu,\psi,\beta)}{\partial \mu}\right|_{\pi=0} < 0.$ ■

Proof of proposition 20. We know from the proof of lemma 8 that $\partial\widetilde{w}/\partial\psi > 0$ for all admissible $\psi < \psi_1$ with $\pi(\psi_1) > 0$ and $\partial\widetilde{w}/\partial\psi < 0$ for all $\psi > \psi_1$ that are compatible with the uniqueness condition (5.37) Since employment L increases monotonically in $\widetilde{w}$ by equation (5.16) , the same applies to employment as a function of money growth ψ. From the proof of lemma 13, we further have that *given* employment $\frac{\partial \mu}{\partial \psi} \gtreqless 0$ for $\psi \lesseqgtr \psi_0$ with $\pi(\psi_0) = 0$. Hence we have that $d\mu[L(\psi), \psi]/d\psi > 0$ for all $\psi \leq \psi_0$, since here, both the direct and effect of money growth and its indirect effect via employment on economic growth are positive, and $d\mu[L(\psi), \psi]/d\psi < 0$ for all $\psi \geq \psi_1$, since here both effects are negative. This completes the proof. ■

Proof of proposition 21. From the proof of lemma 13, we know that at $\pi = 0$ $(\pi < 0)$, the non-employment-related effects of money growth ψ on economic growth γ are zero (positive). At the same time, from the proof of lemma 8 $\frac{\partial L}{\partial \psi} > 0$ for $\psi < \psi_1$ with $\pi(\psi_1) > 0$. Therefore, the $d\gamma[L^*(\psi,\beta), \psi, \beta]/d\psi > 0$ for $\pi \leq 0$ and the maximum growth rate is reached at a $\pi > 0$. ■

Proof of proposition 22. The proposition follows from the facts that first, at $\beta < \infty$ and $\pi = 0$ we have $d\gamma/d\psi > 0$ and $dL/d\psi > 0$ and second, the real outcomes of the models with price rigidity and with flexibility are identical at $\pi = 0$, which can be seen by letting $\beta \to \infty$ or setting $\pi = 0$, respectively, in equations (5.28) and (5.38). ■

Chapter 6

Concluding remarks

The present thesis has proposed a Schumpeterian growth model with nominal price rigidity for the analysis of the effects of money growth on the real development of an economy in the long run.

Chapter 1 has given an overview over theoretical and empirical results about the influence of money on the real side of the economy in the short and long run. In particular, the aim of the presentation was to motivate our choice of framework for the analysis of the long-run effects of money growth, which has been shown to be a synthesis of the approach of the New Keynesian business cycle literature and the literature concerned with the relationship between inflation and endogenous growth.

In chapter 2, the baseline model has been spelled out in detail. Money's non-superneutrality results from the effects of the money growth rate on relative prices, which affects firms' optimal and effective mark-ups, the level of the real wage and production efficiency. Since these level effects change the incentive to innovate for R&D firms, they affect the economy's growth rate, too.

Chapter 3 has shown that concentrating our long-run analysis on the steady state is justified since the economy converges to the steady state in all examined parameter constellations, i.e. the steady state has been found to be locally stable.

Chapter 4 has relaxed the harsh assumption made in chapter 2 that incumbents' production is terminated when an innovator enters the market. It has been shown that for small quality improvements, which under non-negative money growth are sufficient to imply innovators cannot drive incumbents out of the market by setting a monopoly price, money growth is more detrimental to growth than under the parameter constellations where monopoly pricing leads to incumbents' making non-positive profits, i.e. the scenario of chapter 2.

Finally, endogenising labour supply in chapter 5 has shown that employment and output are hump-shaped functions of money growth due to the interaction of the latter's effects on production efficiency and the average monopolistic mark-up. The functions peak at positive inflation rates. This has consequences for economic growth: Since output growth depends positively on employment in the model, the economic growth rate is maximised at a positive inflation rate, too.

The central positive goal of the research underlying this thesis was to understand the long-run effects of money growth on the real side of the economy in the presence of nominal price rigidity. The examined model has indeed shown that even moderate price rigidity can lead to a sizeable influence of money growth on employment, output and growth. The identified effects allowed money growth to influence firstly, individual mark-ups, secondly, average mark-ups, i.e. the average level of monopolistic distortions or the real wage level, and thirdly, production efficiency via its effect on price dispersion. All these effects of inflation on relative prices seem plausible and there is some evidence supporting their empirical relevance.[1] Further, the effects of money growth on output growth have turned out to be quantitatively in line with empirical estimates. We therefore believe that our analysis does indeed contribute to the understanding of money's non-superneutrality concerning economic long-run performance.

The main policy implication consistently emerging from our analysis is the clear message that excessive inflation is undesirable both from a growth and from an employment/output perspective. Our calibration exercises showed

[1] See section 5.6.

that even inflation rates of around ten percent, which are not abnormal for industrialised countries, significantly reduce output growth, employment and the output level in efficiency units. Therefore, keeping inflation at low levels should be the primary goal of monetary policy concerned with the stimulation of employment and growth. While employment is maximised at a small positive inflation rate, the exact level of inflation to be targeted to maximise growth depends on the influence of the average monopolistic distortion on the incentive to innovate. If this distortion can be offset with other economic policy instruments, then there is no trade-off between the primary monetary policy goal of price stability and the fostering of high economic growth.

While we have throughout the thesis put forward policy recommendations for a monetary policy authority interested in promoting employment and growth, we have not shown that maximising growth is a welfare maximising strategy. In fact it is a priori unclear whether the output growth rate in the decentralised equilibrium is lower than the socially optimal rate that would be chosen by a benevolent central planner.

Empirically, there is some evidence that the social return to R&D is much higher than the private return and that consequently, investment in R&D and innovation-driven growth are lower than socially optimal in real economies.[2]

Despite this reassuring evidence, it would be a worthwhile extension to augment the model with an explicit welfare analysis that would allow to straightforwardly evaluate the consequences of different money growth rates with regard to the changes they induce in an explicit measure of welfare.

A second suggestion for further research is that it would be desirable to modify the model in a way that allows for the integration of labour supply based on first principles while preserving the model's tractability. While the current approach taken to the endogenisation of labour supply does yield some valuable insights about inflation's influence on the wage and employment, the ad-hoc and short-cut nature of the approach is somewhat unsatisfactory. Spelling out inflation's influence on the household's or labour union's

[2]See Jones and Williams [1998] and the references therein.

optimal behaviour would render these insights even more persuasive.

A third interesting extension would take account of the fact that the costs imposed by price rigidity under inflation might induce economic actors to find a way to change the economic environment, i.e. reduce the degree of price rigidity that we take as given in the analysis. A first possibility is the introduction of indexation of contracts, which would reduce the effects of inflation in models with price rigidity.[3] However, it has been shown that perfect indexation is not an optimal strategy.[4] In our context, the most straightforward option would therefore be to allow agents to determine the contract length endogenously against a fixed cost. As briefly discussed in the introductory chapter, this possibility has been explored by Devereux and Yetman [2002] and Graham and Snower [2004] in their analyses of the long-run behaviour of the New Keynesian model with positive steady state inflation. In their simulations, the endogenously chosen contract length implies the persistence of price rigidity – and hence, of the distorting effects of inflation – under all but extreme inflation rates. While we presume that this result carries over to our framework where distortions also affect the growth rate, it would be worthwhile to study the issue in detail.

[3] In fact, the effects would be eliminated in any deterministic model.
[4] See Gray [1978].

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