The Impact of Market Characteristics on Price Setting and Market Outcomes: Four Essays

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Dipl.-Kfm. (Int.) Alexander Rasch

aus

Nürnberg

Referent:	Prof. Achim Wambach, PhD
Korreferent:	Prof. Dr. Axel Ockenfels

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to my parents

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1 Introduction

This introductory chapter briefly reviews the motivations behind the models which are laid out in this thesis as well as the main results. The present thesis deals with the question how market characteristics and the change thereof affect market players' price-setting decisions and market outcomes. The market characteristics which are analyzed here are the possibility to multihome, market transparency, differentiation, and inequity aversion. In chapters two through four, models of horizontal product differentiation are used. The first model focuses on competitive price setting whereas the models in chapters three and four analyze collusive outcomes. The fifth chapter looks at bilateral bargaining in a double-auction framework where inequity-averse sellers and buyers set prices. Each chapter is written in such a way that it can be read on its own.

The model in the second chapter entitled *"Platform competition with partial multihoming under differentiation"* deals with competition in a two-sided market which is characterized by singlehoming on one side of the market and multihoming on the second one.[1] In this kind of market, platforms bring together two types of customers (sides) each of which is interested in the platforms' products or services only if the other side is "on board" at the same time. Typical examples include video-game consoles or software (where the two sides are game or application developers and users) or payment-card systems (merchants and cardholders). The model is an extension of the seminal contribution by Armstrong (2006) who considers sin-

1 The main results of the model were presented in Rasch (2007) which is a shorter version of the chapter. That version benefited from comments by an anonymous referee as well as Matthew Mitchell, the Associate Editor of the Economics Bulletin.

glehoming on both sides of the market. Singlehoming describes a situation where the customers on the respective side do not want to or cannot join more than one platform. On the other hand, multihoming, which is a typical feature of many two-sided markets, is present whenever customers may go to more than one platform. In the benchmark case without differentiation, customers on one side are homogeneous in a sense that they all make the same multihoming decision. In such a situation, platforms charge the multihoming side a high price for providing an exclusive access to the singlehoming side. As a result, the multihoming side is left with no surplus from trade. This price structure, however, is not necessary what is observed in real life where the multihoming side sometimes even does not have to pay anything to get access to the platforms' services or products. The model presented here shows that due to product differentiation, platforms are no longer local monopolists on the multihoming side which benefits the latter as indeed it may end up paying a lower price than the singlehoming side.

The model in the third chapter entitled *"Customer-side transparency, elastic demand, and tacit collusion under differentiation"* is joint work with Jesko Herre. It analyzes the impact of market transparency on the customer side on collusive stability in a market where customers have elastic demand.[2] The model is motivated by experimental as well as by empirical evidence that policies mainly designed to improve customers' position vis à vis firms by giving them more information often have failed. Instead of inducing a price reduction as a result of a tougher competitive environment due to better informed customers, these policies lead to the opposite outcome. This is usually explained by pointing out that giving customers access to more information also provides firms with better information about their competitors' activities at the same time which makes it easier for firms to monitor each other. As a consequence, collusive agreements are easier to sustain. Contrary to that, we are interested in whether the unexpected and undesirable implications of

2 I presented the model at the Brown-Bag Seminar of the Economics Department at the University of Cologne (2007), at the *35th Conference of the European Association for Research in Industrial Economics* (Toulouse, France, 2008), and at the 7th *Annual International Industrial Organization Conference* (Boston, MA, USA, 2009).

the above-mentioned policies may be explained by explicitly analyzing the customer side. Different from the standard explanation with respect to the firms' information level, our model shows that an increase in market transparency on the customer side may itself have a stabilizing effect for a collusive agreement: Depending on the degree of differentiation and on the level of transparency already achieved, a more transparent market may be bad news for customers as collusion may be facilitated.

The fourth chapter entitled *"Internal decision-making rules and collusion"* is joint work with Achim Wambach. It introduces a model to analyze the implications of different decision-making structures within a holding company for collusive stability.[3] While previous contributions focused on the analysis of the underlying principal-agent problem, we are interested in the results from an industrial-organization point of view. The motivation behind this work is that antitrust and competition authorities have to consider the implications for post-merger competition before allowing a proposed merger to go ahead. Clearly, an important aim in this context is to hinder firms from coordinating their decisions (e.g., setting joint prices or quantities) at the expense of their customers. In practice, authorities have ordered the acquiring firm to maintain the acquired firm as a separate entity in several cases. In our framework, there is a holding company (the headquarters) which owns two local outlets. We model different internal decision-making structures by assuming that the holding company must decide whether to give away its price-setting powers (decentralization) or not (centralization). The reasoning that underlies the authorities' orders seems to make sense at first sight: By keeping merging firms separate, the number

3 A shorter version of the model has been accepted for publication in the *Journal of Economic Behavior & Organization*. That version benefited from comments by two anonymous referees. I presented the model at the following conferences: *34th Conference of the European Association for Research in Industrial Economics* (Valencia, Spain, 2007), at the *Annual Meeting of the German Economic Association of Business Administration* (Tübingen, 2007), *NAKE Research Day* (Utrecht, The Netherlands, 2007), *Annual Meeting of the Spanish Economic Association* (Granada, 2007), 1st *Doctoral Meeting of Montpellier* (France, 2008), 4th *IUE International Student Conference: Cooperation, Coordination and Conflict* (Izmir, Turkey, 2008), *XIII. Spring Meeting of Young Economists* (Lille, France, 2008), 6th *Annual International Industrial Organization Conference* (Arlington, USA, 2008), *Annual Meeting of the Austrian Economic Association* (Vienna, 2008), *Annual Meeting of the German Economic Association* (Graz, Austria, 2008).

of decision makers in the market increases which destabilizes a collusive agreement. However, the centralization of decision making affects the other firms' incentives to collude at the same time. It is shown that although the merged entity has a greater incentive to deviate from a collusive agreement if the decision powers are passed on to the local units, the opposite may be true for the other firms in the market. The exact outcome depends on the level of differentiation in the market.

The fifth chapter entitled *"The double auction with inequity aversion"* is joint work with Achim Wambach and Kristina Kilian. The double auction is a simple model of bargaining with two-sided incomplete information where a seller and a buyer submit bids. If the seller's bid is smaller than the bid submitted by the buyer, trade will take place at a price that is equal to the average bid (split-the-difference rule). Typically, the market outcome without inequity aversion is characterized by inefficiencies as both sides have an incentive not to tell the truth in order to get a better price. The model is motivated by experimental evidence which suggests that a more efficient outcome than predicted by theory is possible. Our approach tries to explain these differences by allowing for inequity-averse sellers and buyers where inequity aversion includes envy and compassion. We show that if compassion is rather important, then there is a (separating) equilibrium which is efficient in the sense that all gains of trade are realized. We also show that there may exist further (though less efficient) separating equilibria with a fair allocation of the gains from trade even if compassion is not as important. However, if these equilibria do not exist and if envy is very strong, there is a maximum of inefficiency as trade breaks down. Moreover, if envy is not as strong, we present a numerical solution of the (non-linear) symmetric equilibrium. It turns out that the more important the envy (compassion) element, the less (more) efficient the outcome in the bargaining situation. For pooling (or price) equilibria, it is shown that inequity aversion always reduces the bargaining efficiency.

The last chapter provides some concluding remarks on the models developed in this thesis. It discusses their implications and limitations and outlines related aspects that should be analyzed in future work.

2 Platform competition with partial multihoming under differentiation

A model of a two-sided market with two horizontally differentiated platforms and multihoming on one side is developed. In contrast to recent contributions, it is shown that platforms do not necessarily generate all revenues on the multihoming side by charging it a higher price than the singlehoming side. Also, whether platforms' pricing structures favor exclusivity over multihoming is ambiguous.

2.1 Introduction

In two-sided markets, platforms try to bring together two groups of customers each of which is interested in the participation by the other side.[4] In most two-sided markets, multihoming by at least one side plays an important role. Multihoming (unlike singlehoming) describes a situation where customers join more than one platform.

Multihoming is a distinctive feature of most two-sided markets. It is present in markets like apartment brokerage, media, online shopping portals, operating systems, payment cards, video game consoles, etc.[5] These industries generate revenues of several hundred billions of dollars each year. Platforms often generate most of

4 For an introduction, see, e.g., Rochet and Tirole (2003, 2005), Roson (2005b), and Peitz (2006).
5 For these and other examples, see Evans (2003).

their revenues from the multihoming side. Apartment brokers, for instance, tend to charge (potentially) multihoming buyers/renters and not singlehoming owners. Contrary to that, the online auction house eBay only charges singlehoming sellers a certain percentage of the sales price when a transaction takes place whereas (potentially) multihoming buyers receive services for free.[6] Given the importance of the markets involved and the different pricing structure that can be observed, it is necessary to better understand the relevant factors that affect firms' pricing decisions.

The issue of (endogenous) multihoming has been dealt with by a number of authors (Caillaud and Jullien, 2001, 2003; Gabszewicz and Wauthy, 2004; Armstrong, 2006; Armstrong and Wright, 2007; Roson, 2005). These contributions differ from the present one with respect to the aspect of differentiation on the multihoming side. Without differentiation, *all* customers on one side make the same decision and platforms generate all revenues on the multihoming side by charging it a higher price than the singlehoming side and thus leaving it with no surplus from trade. Contrary to that, the present note shows that due to product differentiation (i.e. heterogeneous preferences among customers), partial multihoming arises. As a result, platforms neither always charge this side a higher price nor leave it without any surplus from trade. This is intuitive as partial multihoming implies that platforms are no longer local monopolists on the multihoming side which results in a price reduction. However, when it comes to the relative prices on both sides, there are ambiguous effects as to whether platforms prefer multihoming (which is equal to lowering the respective price even more in order to boost overall demand) or whether they do not (which is equal to making services more exclusive).

Belleflamme and Peitz (2006) simultaneously developed a model which partly follows the same logic and setup as the present one. However, they focus on the implications of different platform types (free/not-for-profit/public vs. for-profit) for the incentives to innovate. They find that for-profit intermediation may increase or

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6 See `www.ebay.com`.

decrease investment incentives depending on which side of the market singlehomes. Moreover, Poolsombat and Vernasca (2006) also simultaneously developed a similar version of the present model with two heterogeneous groups of agents who only differ in their valuation of the network benefit. They also find that platforms do not always generate all their revenues from the partially multihoming side but their results differ from the ones presented here in some important aspects which will become clear below.

In the next section, the model is presented. The third section discusses the results.

2.2 The model

The basic setup follows Armstrong (2006) who uses a Hotelling (1929) specification. Platform 1 is located at 0 and platform 2 is located at 1 on the linear city of unit length. Platforms incur marginal costs c^k per side-k customer served. Fixed costs are normalized to 0.

There are two groups of customers ($k \in \{a, b\}$) with mass 1 each. Customers are uniformly distributed on their side of the linear city. Different from Armstrong (2006), side-b customers are assumed to have the opportunity to multihome whereas side-a customers singlehome.[7] As Armstrong and Wright (2007) point out, multihoming on only one side makes sense especially in a situation where the multihoming side *as a whole* joins two platforms, as the other side then will not have an incentive to multihome as well. Instead, side-a customers would gain access to all side-b customers without joining two platforms, so that they would rather free-ride on the side-b customers' decision.[8]

7 This setup is justified for the examples of apartment brokerage and online auctioning mentioned before: Placing an object with two or more platforms is not possible (from a legal point of view) as it cannot be rented/sold to more than one renter/buyer.

8 In the present setup, it turns out that not all side-b customers will opt for multihoming which would leave some room for multihoming for side-a customers. However, the focus here is on the different treatment of singlehoming and multihoming customers which makes the above setup necessary (see *Assumption 2.2* below).

The contributions cited in the introduction all deal with the case where only one side is able to multihome. Contrary to that, Kind, Nilssen, and Sørgard (2005), Ambrus and Reisinger (2006), and Doganoglu and Wright (2006) additionally allow for multihoming on both sides.

Utility for some side-a/side-b customer who is located at a distance Δ^a/Δ^b from platform i ($i \in \{1, 2\}$) and who joins this platform (and possibly platform j) is defined as follows (where n_i^k denotes platform i's [overall] demand on side k [singlehoming and multihoming customers]):

$$U_i^a = u^a + \xi^a n_i^b - p_i^a - \tau^a \Delta^a \tag{2.1}$$

and

$$U_i^b = \begin{cases} u^b + \xi^b n_i^a - p_i^b - \tau^b \Delta^b & \text{when singlehoming} \\ u^b + \xi^b - p_i^b - p_j^b - \tau^b & \text{when multihoming.} \end{cases} \tag{2.2}$$

Note that joining platform j only leads to a travel distance of $1 - \Delta^k$ and that $n_j^a = 1 - n_i^a$ as well as $n_j^b = 1 - n_{i,s}^b$ where $n_{i,s}^b$ denotes the number of side-b customers who join platform i exclusively. Moreover, customers derive some basic utility u^k which is independent of whether they join one or two platform(s). Both sides benefit from the participation of the other side the extent of which is measured by the two-sided network externality ξ^k ($\xi^k > 0$). Customers incur linear transportation costs τ^k per unit of distance traveled ($\tau^k > 0$). The market is depicted in *Figure 2.1.*

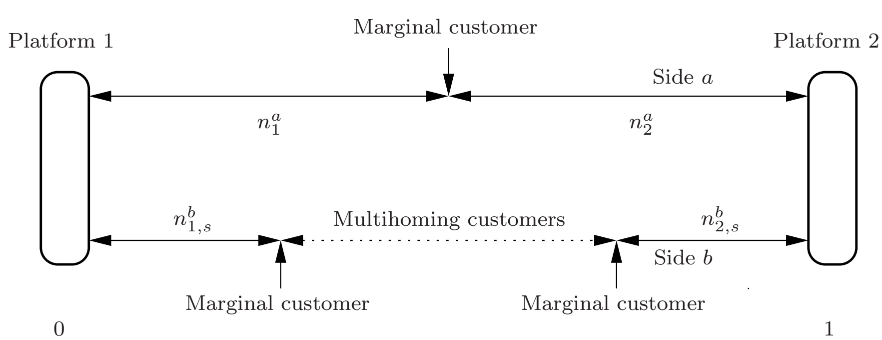

Figure 2.1: Differentiated two-sided market with partial multihoming

Before turning to the equilibrium analysis, the following assumptions are made:

Assumption 2.1 *u^k is sufficiently high such that the market is covered on both sides.*

Assumption 2.2 $\tau^a > \xi^a$.

This assumption is due to Armstrong and Wright (2007)[9] and ensures that side-a customers have indeed no incentive to multihome.

Assumption 2.3 $\tau^b + c^b < \frac{\xi^a + \xi^b}{2} < 2\tau^b + c^b$.

This assumption ensures that multihoming demand does not exceed 1 and that the market is covered on side b.

Assumption 2.4 $8\tau^a\tau^b > \xi^{a2} + \xi^{b2} + 6\xi^a\xi^b$.

This is the necessary and sufficient condition for a market-sharing equilibrium with multihoming to exist.

Turning to the equilibrium analysis, the indifferent side-a customer is determined in the standard way by equating the utility derived from joining platforms 1 and 2, respectively, and solving for the location variable. The same is done on side b for the two indifferent customers who are indifferent between joining only one of the two platforms on the one hand and joining both on the other hand.

From the resulting implicit expressions, the following explicit expressions can be derived:

$$n_i^a = \frac{1}{2} - \frac{\xi^a\left(p_i^b - p_j^b\right) + \tau^b\left(p_i^a - p_j^a\right)}{2(\tau^a\tau^b - \xi^a\xi^b)}, \tag{2.3}$$

$$n_{i,s}^b = 1 - \frac{\xi^b}{2\tau^b} - \frac{\xi^b\left(p_i^a - p_j^a\right)}{2\left(\tau^a\tau^b - \xi^a\xi^b\right)} - \frac{\xi^a\xi^b p_i^b - \left(2\tau^a\tau^b - \xi^a\xi^b\right)p_j^b}{2\tau^b\left(\tau^a\tau^b - \xi^a\xi^b\right)}, \tag{2.4}$$

and

$$n_i^b = \frac{\xi^b}{2\tau^b} - \frac{\xi^b\left(p_i^a - p_j^a\right)}{2\left(\tau^a\tau^b - \xi^a\xi^b\right)} - \frac{\left(2\tau^a\tau^b - \xi^a\xi^b\right)p_i^b - \xi^a\xi^b p_j^b}{2\tau^b\left(\tau^a\tau^b - \xi^a\xi^b\right)}. \tag{2.5}$$

The profit for platform i amounts to

9 See their assumption *A2*. They provide a proof for this assumption in their *Lemma 1*.

$$
\begin{aligned}
\pi_i = \left(p_i^a - c^a\right)\left(\frac{1}{2} - \frac{\xi^a\left(p_i^b - p_j^b\right) + \tau^b\left(p_i^a - p_j^a\right)}{\tau^a\tau^b - \xi^a\xi^b}\right) + \left(p_i^b - c^b\right) \\
\times\left(\frac{\xi^b}{2\tau^b} - \frac{\xi^b\left(p_i^a - p_j^a\right)}{2\left(\tau^a\tau^b - \xi^a\xi^b\right)} + \frac{\xi^a\xi^b\left(p_i^b + p_j^b\right) - \left(2\tau^a\tau^b - \xi^a\xi^b\right)p_i^b}{2\tau^b\left(\tau^a\tau^b - \xi^a\xi^b\right)}\right).
\end{aligned}
\tag{2.6}
$$

Differentiating the profits with respect to prices and setting the resulting first-order conditions equal to 0 leads to a system of equations with four unknowns.[10] Solving for (symmetric) prices yields (where N denotes the competitive Nash outcome):

Proposition 2.1 *With the possibility of multihoming for side-b customers, platforms will charge the following equilibrium prices:*

$$
p^{a,N} = \tau^a + c^a - \frac{\xi^b\left(3\xi^a + \xi^b - 2c^b\right)}{4\tau^b} \tag{2.7}
$$

and

$$
p^{b,N} = \frac{c^b}{2} + \frac{\xi^b - \xi^a}{4}. \tag{2.8}
$$

Hence, platforms charge side-a customers the Hotelling (1929) price $\tau^a + c^a$ which is reduced by a term reflecting the importance of the network externalities involved. On the second side, platforms charge a price consisting of—like in previous contributions without differentiation on the multihoming side[11]—a monopolistic term $\frac{c^b}{2} + \frac{\xi^b}{4}$ (assuming that each side-b customer reaches half the customers on the other side which implies a gross willingness to pay of $\frac{\xi^b}{4}$) which is, however, adjusted downward due to the network externality on the other side.[12]

Comparing equilibrium prices yields:

Corollary 2.1 *With the possibility of multihoming for side-b customers, $p^{a,N} > p^{b,N}$ may hold.*

10 Note that the second-order conditions are satisfied due to *Assumption 2.4*.
11 This is not the case in the Poolsombat and Vernasca (2006) model as they assume *equal* transportation costs on *both* sides.
12 See also Belleflamme and Peitz (2006).

Hence, partial multihoming may lead to a situation where platforms will not always generate all revenues from the multihoming side by setting a higher price on that side.[13] Let $P := p^{a,N} - p^{b,N}$. Then, $\frac{\partial P}{\partial \xi^a} = \frac{\tau^b - 3\xi^b}{4\tau^b} \lesseqgtr 0$, $\frac{\partial P}{\partial \xi^b} = -\frac{\tau^b + 3\xi^a + 2\xi^b - 2c^b}{4\tau^b} < 0$ (due to *Assumption 2.2*), $\frac{\partial P}{\partial \tau^a} = 1 > 0$, and $\frac{\partial P}{\partial \tau^b} = \frac{\xi^b(3\xi^a + \xi^b - 2c^b)}{4{\tau^b}^2} > 0$ (due to *Assumption 2.2*).

Equilibrium prices will lead to market shares of

$$n^{a,N} = \frac{1}{2}, \tag{2.9}$$

$$n_s^{b,N} = 1 + \frac{c^b}{2\tau^b} - \frac{\xi^a + \xi^b}{4\tau^b}, \tag{2.10}$$

and

$$n^{b,N} = \frac{\xi^a + \xi^b - 2c^b}{4\tau^b}. \tag{2.11}$$

Due to *Assumptions 2.2* and *2.4*, $\frac{1}{2} < n^{b,N} < 1$ holds, i.e. some side-b customers will multihome. Note that $\frac{\partial n^{b,N}}{\partial \xi^a} = \frac{\partial n^{b,N}}{\partial \xi^b} = \frac{1}{4\tau^b} > 0$ and $\frac{\partial n^{b,N}}{\partial \tau^b} = -\frac{\xi^a + \xi^b - 2c^b}{4{\tau^b}^2} < 0$.

The profit for platform i amounts to

$$\pi^N = \frac{\tau^a}{2} + \frac{{c^b}^2}{4} - \frac{{\xi^a}^2 + 6\xi^a\xi^b + {\xi^b}^2}{16\tau^b}.^{14} \tag{2.12}$$

2.3 Discussion

The pricing decision here is in contrast to a *competitive-bottleneck*[15] scenario *without* differentiation on the multihoming side which is the driving force behind the

13 Unlike in the Poolsombat and Vernasca (2006) model, this is not necessarily related to the scope of multihoming. They derive this result only for a low degree of multihoming by agents with a high network benefit. Here, a large τ^a is sufficient.

14 Note that the profit increases with the costs on the multihoming side. This is reminiscent of the result in Caillaud and Jullien (2003) where the profit increases with an increase in the cost for the singlehoming side.

15 See Armstrong (2002) and Wright (2002) for telecommunication services as well as Armstrong (2006) and Armstrong and Wright (2007) for two-sided markets. See also Jullien (2005).

results in the contributions mentioned in the introduction. In such a situation, the singlehoming side, which is critical for attracting the other side, is left with (some) surplus from trade. On the other hand, platforms do not compete for multihoming customers and generate all revenues from this side. There, once the singlehoming side is attracted by the platforms, the latter have some form of local monopoly power to connect the multihoming side to the singlehoming base. This means that the multihoming side is left with no surplus from trade. Due to the lack of differentiation, *all*—if any—agents on one side multihome.[16] On the other hand, singlehoming customers benefit from an increased competition among platforms to get them on board.[17]

The local monopoly element is still present here. However, due to differentiation on the multihoming side, there is only *partial* multihoming, i.e. some customers do not multihome because of the increased overall transportation costs.[18] This means that customers are no longer captive which reduces platforms' local monopoly power. As a result, instead of leaving customers with no surplus from trade, the price for side-b customers may be lower than the one for side-a customers.

Consider first the implications of an increase in τ^b which makes multihoming less attractive. There are two opposing strategic considerations: First, it may make sense for platforms to reduce their price on the multihoming side in order to increase their demand and boost their attraction on the other side. Second, less multihoming means that there are more singlehoming customers left on the multihoming side. Singlehoming customers, however, can only be accessed through the respective platform which makes the platforms' services more exclusive. Thus, it is

16 In the Caillaud and Jullien (2003) as well as in the Armstrong and Wright (2007) framework, this is the case since side-b customers consider platforms to be homogeneous, i.e. $\tau^b = 0$. Therefore, customers do not have to incur different levels of overall transportation costs which means that they all make the same decision.

17 One example put forward in the articles cited above is that of *call termination*. Whereas telecommunication services providers compete *for* subscribers, they often enjoy a monopoly position when it comes to providing communication services *to* their subscribers once they have decided to sign up with the provider.

18 See also Ambrus and Reisinger (2006) and Poolsombat and Vernasca (2006) as well as the setup by Doganoglu and Wright (2006) with two different types of customers.

possible to charge the singlehoming side a higher price. From the equilibrium prices, it becomes clear that the second effect is stronger which means that exclusivity is more important than demand expansion and that multihoming customers are less valuable. Not surprisingly, this is true for an increase in τ^a and a decrease in ξ^b too.

However, the opposite may be true when considering an increase in ξ^a which leads to a decrease in prices on both sides. The (exclusive) singlehoming side benefits from the fact that an increase in ξ^a makes the externality relatively more important compared to τ^a, i.e. competition is increased. With respect to relative prices, the following reasoning applies: If τ^b is very large ($> 3\xi^b$), i.e. if there are hardly any multihoming customers, the price decrease is stronger on the multihoming side. In such a situation, delivering a great number of side-b customers is more important to the singlehoming side. Hence, it makes sense for platforms to increase the overall demand on the potentially multihoming side through a greater price cut. This is in contrast to the model of media markets by Ambrus and Reisinger (2006) where multihoming viewers are always less valuable.

3 Customer-side transparency, elastic demand, and tacit collusion under differentiation

We analyze the effects of a change in customer-side price transparency on horizontally differentiated firms' possibility to sustain collusion when customers' demand is elastic. It is shown that there is a non-monotone relationship for low levels of differentiation: A higher level of transparency stabilizes collusion if the market is not very transparent. The opposite is true as the market becomes more transparent. If, however, the degree of differentiation is high, a more transparent market unambiguously makes collusion easier to sustain. We also show that competition authorities favor a fully transparent market only for low degrees of differentiation.

3.1 Introduction

The question of whether more information on the customer side is good for competition is of great importance both for competition authorities as well as for consumer protection agencies. Practitioners seem to consider an increased market transparency on the customer side as an appropriate means to promote competition. E.g., in Germany the Bundeskartellamt (German Competition Authority) emphasizes the unambiguously positive effects of a higher degree of customer information on competition.[19] In the same vein, it is often argued that the undesirable consequences with respect to coordinated behavior stemming from an increased transparency among firms may be alleviated if customers gain access to more information at the same time. As Capobianco and Fratta (2005) report, the Autorità Garante per la Concorrenza ed il Mercato (Italian Competition Authority) holds the opinion that a higher elasticity of demand in a situation where customers are better informed "may, in a dynamic context, undermine any potential collusive practice" (p. 6) resulting from the exchange of information between firms.

In this chapter, we show that this intuition does not always hold. To this end, we use a specification à la Hotelling (1929) with two firms and with customers that have an elastic demand. We study the effects of a higher degree of price transparency among customers on collusive stability. Generally speaking, there are two opposing effects due to an increase in transparency: On the one hand, undercutting the other firm's price increases the profit of the deviating firm as more customers will actually take notice of the price cut. On the other hand, this increased price awareness leads to a tougher price competition, i.e. the potential punishment is harder. Applying the concept of grim-trigger strategies, we show that for a relatively low degree of differentiation, the implications of an increase in market transparency are ambiguous and full transparency may be desirable in order to destabilize collusion. If, however, the degree of differentiation is sufficiently high, a greater market transparency stabilizes collusion. This can be explained by pointing out that with elastic demand, a higher degree of differentiation in the market leads to a situation where firms focus more on local demand which means that there is less competition for the marginal customer. This results in less tough competition and hence a weaker punishment for a deviating firm. As a consequence, collusion is destabilized. It can then be argued that an increase in market transparency has a similar effect like a decrease in differentiation as more customers learn about firms'

19 See, e.g., "Bundeskartellamt veröffentlicht bundesweiten Gaspreisvergleich für Haushaltskunden [German Competition Authority publishes countrywide gas-price comparison for households]," press release, January 3, 2007 (document available from `www.bundeskartellamt.de`).

prices which toughens competition.

Unlike the literature dealing with the effects of information exchange between firms on the likelihood of collusion, there are only a few contributions that look at the implications of different levels of market transparency on the customer side.[20] Our approach is closely related to the contribution by Schultz (2005). He sets up a Hotelling (1929) model with inelastic demand to analyze the implications of customer-side price transparency for the stability of tacit collusion. The author shows that—contrary to what we find—a higher degree of transparency unambiguously destabilizes collusion.

A very different approach to dealing with customer-side transparency is suggested by Nilsson (1999): He develops a model with unit demand and homogeneous products. In his model, the majority of the customers account for the expected benefits from searching and decide whether to search or not on that basis. Contrary to that, a fraction of the customers always search. A higher degree of transparency here translates into lower search costs. Most customers thus no longer search if firms set the same price which is true for the (high-price) collusive phase. As a consequence, deviation leads to a moderate increase in demand only which stabilizes the collusive agreement. In the punishment phase of the collusive equilibrium, firms set different (mixed-strategy) prices which means that the majority of customers do search. Clearly, if transparency increases, there will be more search activity and hence tougher competition. Since an increase in transparency only affects the punishment profits—unlike in the Schultz (2005) as well as the present setup—, it helps stabilize the collusive agreement.

Møllgaard and Overgaard (2001) define market transparency as customers' ability to compare the products' characteristics or quality. Products are actually homogeneous but are perceived as differentiated due to a lack of rationality on the customer side. The authors show that for the case of trigger strategies, the optimal degree of

20 For an overview of the implications of information exchange for coordinated behavior, see, e.g., Kühn and Vives (1995) as well as Kühn (2001). Concerning the issue of market transparency on the customer side, see Møllgaard and Overgaard (2006) for an overview.

transparency to make collusion as difficult to sustain as possible is interior in the duopoly case. The implication of their analysis to maintain some degree of intransparency in the market in order to make collusion harder to sustain contrasts with the results in the present model for the case of high differentiation.[21]

From an empirical point of view, Albæk, Møllgaard, and Overgaard (1997) as well as Wachenheim and DeVuyst (2001) provide two studies where a policy mainly directed at improving customers' level of information resulted in higher prices.[22] The argument often put forward to explain this outcome is that by giving customers more information, firms learn about competitors' prices at the same time. This, however, makes punishment easier and therefore facilitates collusion. The analysis in our setup where firms are fully informed about their competitor's price suggests a different—or complementary—explanation for the observation of increased prices: A higher degree of transparency on the customer side may have a direct stabilizing effect for collusion as well.

In their experimental study, Hong and Plott (1982) analyze the possible consequences of a proposed rate publication policy for the domestic barge industry on inland waterways in the United States. Back then, rates on tows were set through individual negotiations, and the terms of each contract were private knowledge of the contracting parties only. Therefore, there were calls for a requirement that a carrier had to announce a rate change with the Interstate Commerce Commission (ICC) at least fifteen days before the new rate was to become effective. The authors find that a publication policy resulted in higher prices, lower volume, and reduced efficiency in the laboratory. Moreover, the introduction of such a policy hurt the small participants.[23]

In the next section, the model along with the main result is presented. The last

21 Full transparency is shown to be optimal for five or more firms. Moreover, they find that full transparency is unambiguously optimal with two firms when applying optimal symmetric penal codes following Abreu (1986, 1988).

22 Albæk, Møllgaard, and Overgaard (1997) analyze the Danish market for concrete. Wachenheim and DeVuyst (2001) look at the U.S. livestock and meat industries.

23 Note that it is true that conversations on price collusion were strictly forbidden but clearly, there was room for tacit collusion.

section concludes.

3.2 The model

There are two firms which are located at the two extremes of the Hotelling (1929) line of unit length. Customers of mass 1 are uniformly distributed along the line. To include different transparency levels, only a share θ of the customers are assumed to be informed about the prices charged by the firms. As we are interested in the implications of a change in price transparency under elastic demand, let q denote the quantity a customer demands at a given price and location. Then, a customer located at x derives the following utility

$$U = \begin{cases} q - \frac{q^2}{2} - q\left(\tau x + p_1\right) & \text{when buying from firm 1} \\ q - \frac{q^2}{2} - q\left(\tau\left(1-x\right) + p_2\right) & \text{when buying from firm 2,} \end{cases} \tag{3.1}$$

where τ measures the degree of differentiation between the two firms (transportation costs) and where p_i denotes the price charged by firm i (with $i \in \{1,2\}$). Note that the way the utility is defined implies that a customer incurs the transportation costs for every unit purchased. As customers are assumed to be utility maximizers, the above utility specification is equivalent to a local demand function of

$$q_i(p_i, x) = 1 - \tau x - p_i.^{24} \tag{3.2}$$

As a consequence, the indifferent (informed) customer located at $\tilde{x}$ is given by

$$1 - \tau\tilde{x} - p_i = 1 - \tau(1-\tilde{x}) - p_j \Leftrightarrow \tilde{x} = \frac{1}{2} - \frac{p_i - p_j}{2\tau}, \tag{3.3}$$

24 Rothschild (1997) has a comparable specification. His results are the same like ours for the limit case with informed customers only ($\theta = 1$). Puu (2002) also uses a similar setup in the context of a price-then-location game. Gupta and Venkatu (2002) develop a model with horizontally differentiated firms, elastic demand, and fully informed customers to analyze the stability of collusion under quantity competition and delivered pricing (i.e. in a situation where firms bear the transportation costs).

where $i \neq j$ (with $j \in \{1,2\}$). On the other hand, those customers belonging to the share $1-\theta$ of the population that are uninformed always buy from the closest firm. Thus, the indifferent uninformed customer is given by $\tilde{x} = \frac{1}{2}$. Note that the uninformed customers have the same elastic demand.

Before continuing with the analysis, the following assumption is made:

Assumption 3.1 $\underline{\tau} := \frac{4(\sqrt{121\theta^2+128\theta}-4\theta)}{128+105\theta} \leq \tau \leq \frac{4}{3} =: \bar{\tau}$.

This assumption ensures that a deviating firm's market share is always smaller than or equal to 1 and that each firm will target both groups of customers (lower bound; see below). At the same time, applying an upper bound on this parameter ensures that the market is covered and that a customer's utility is non-negative, i.e. every customer will go to either of the two firms even if firms charge the monopoly price.[25]

To analyze the effects of transparency on the sustainability of collusion, we focus on the standard grim-trigger strategies defined by Friedman (1971). Thus, collusion is stable as long as the discount factor is higher than the critical one defined by the profits in the competitive, collusive, and deviating cases (superscripts N, C, and D, respectively):[26]

$$\delta \geq \bar{\delta} := \frac{\pi^D - \pi^C}{\pi^D - \pi^N}. \tag{3.4}$$

Next, we derive the profits in the three scenarios.

3.2.1 Punishment: competition in prices

In the competitive case, demand at firm i is given by

25 Note that only the lower bound depends on the market transparency parameter. This is due to the fact that the deviating price is a function of θ whereas the monopoly price is not (see below).

26 The critical discount factor is due to the requirement that profits from collusion must be higher than those from deviation and the ensuing punishment phase, i.e. $\frac{\pi^C}{1-\delta} \geq \pi^D + \frac{\delta\pi^N}{1-\delta}$.

$$Q_i^N = \theta \int_0^{\frac{1}{2}-\frac{p_i^N - p_j^N}{2\tau}} \left(1 - \tau x - p_i^N\right) dx + (1-\theta) \int_0^{\frac{1}{2}} \left(1 - \tau x - p_i^N\right) dx. \tag{3.5}$$

Firms' profits are thus $\pi_i^N = p_i^N Q_i^N$. Proceeding in the standard way to derive the optimal symmetric price, we get

$$p^N = \frac{4\tau - \theta\tau + 2\theta - \sqrt{A}}{4\theta} \tag{3.6}$$

where $A := 16\,\tau^2 - 4\,\theta\,\tau^2 + \theta^2\tau^2 - 4\theta^2\tau + 4\theta^2 > 0 \;\forall \theta \in [0,1], \tau \in [\underline{\tau}, \bar{\tau}]$.[27] The resulting profit for each firm then equals

$$\pi^N = \frac{(4\tau - \theta\tau + 2\theta + \sqrt{A})(-4\tau + 2\theta + \sqrt{A})}{32\theta^2}. \tag{3.7}$$

Note that the assumption also ensures that a firm is not better off when catering to the uninformed customers only. Denote by π^a firm i's profit for this case. Then, the maximization problem yields the following price:

$$\max_{p_i^a} \pi_i^a = p_i^a\,(1-\theta) \int_0^{\frac{1}{2}} (1 - \tau x - p_i^a)\, dx \Leftrightarrow p^a = \frac{1}{2} - \frac{\tau}{8} \tag{3.8}$$

The associated profit is given by

$$\pi^a = \frac{(1-\theta)(4-\tau)^2}{128}. \tag{3.9}$$

Having a closer look at the profits in both scenarios reveals that $\pi^N \lesseqgtr \pi^a \Leftrightarrow \tau \lesseqgtr \frac{4(1-\theta)(\theta^2 - 3\theta + 2\sqrt{\theta^3 - 16\theta^2 + 64\theta})}{256 - \theta^3 + 10\theta^2 - 73\theta} < \frac{4(\sqrt{121\theta^2 + 128\theta} - 4\theta)}{128 + 105\theta} = \underline{\tau}$. Hence, given our initial assumption, it is never optimal to target the uninformed customers exclusively.

Next, we turn to the case of collusion.

27 Note that $\frac{\partial p}{\partial \theta} < 0$ and that—applying de l'Hôpital's rule—$\lim_{\theta \to 0} p = \frac{1}{2} - \frac{\tau}{8}$ which is equal to the collusive price p^C (see below).

3.2.2 Collusive profits

In the case of tacit collusion, firms coordinate their price-setting decision and share the market equally. This leads to an individual total demand of

$$Q_i^C = \int_0^{\frac{1}{2}} \left(1 - \tau x - p_i^C\right) dx. \tag{3.10}$$

The optimal collusive price is set at

$$p^C = \frac{1}{2} - \frac{\tau}{8}. \tag{3.11}$$

The associated profit for each firm is then given by

$$\pi^C = \frac{(4-\tau)^2}{128}. \tag{3.12}$$

3.2.3 One-period deviation profits

Given that the other firm sticks to the collusive price, the optimal deviating price is given by

$$p^D = \frac{32\tau - 18\theta\tau + 40\theta - \sqrt{B}}{72\theta} \tag{3.13}$$

where $B := 1024\tau^2 + 189\,\theta^2\tau^2 - 576\theta\tau^2 + 256\theta\tau - 648\,\theta^2\tau + 592\theta^2 > 0\ \forall\theta \in [0,1], \tau \in [\underline{\tau}, \bar{\tau}]$.[28] The deviating profit thus amounts to

$$\begin{aligned} \pi^D &= (-32\,\tau - 40\,\theta + 18\,\theta\tau + \sqrt{B}) \\ &\times \frac{(512\tau^2 + 27\,\theta^2\tau^2 - 1024\,\theta\,\tau + 72\,\theta^2\tau - 208\,\theta^2 - \sqrt{B}(16\tau + 20\theta - 9\theta\tau))}{497664\theta^2\tau}. \end{aligned} \tag{3.14}$$

3.2.4 Critical discount factor

Making use of the results from the three different cases, we can calculate the critical discount factor. We find the following relationship:

28 Note that $\frac{\partial p^D}{\partial \theta} < 0$ and that $\lim_{\theta \to 0} p^D = p^C$.

Proposition 3.1 *There exist a* $\tilde{\theta}$ *and a* $\tilde{\tau}$ *with* $0 < \theta \leq 1$ *and* $\underline{\tau} < \tilde{\tau} < \bar{\tau}$ *such that*

$$\frac{\partial \bar{\delta}}{\partial \theta} \begin{cases} \geq 0 & \textit{if } \tau \leq \tilde{\tau} \textit{ and } \theta \geq \tilde{\theta}, \\ < 0 & \textit{otherwise.} \end{cases} \tag{3.15}$$

Proof See the appendix. ■

As the proposition suggests, the relationship between collusive stability and market transparency is non-monotonous for the case of low transportation costs.

Figure 3.1 depicts the degree of market transparency which yields the lowest discount factor for a given differentiation parameter (solid line). For low values of θ, it also shows the degree of differentiation resulting in the lowest discount factor (dotted line).

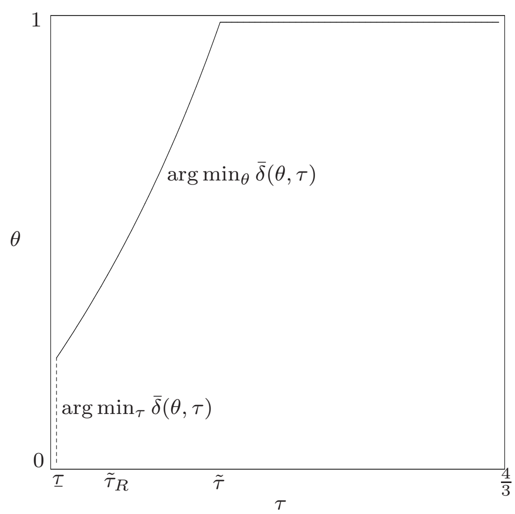

Figure 3.1: Characterization of $\arg\min_\tau \bar{\delta}(\theta, \tau)$ and $\arg\min_\theta \bar{\delta}(\theta, \tau)$

Interestingly, an increase in market transparency always leads to a lower critical discount factor if the degree of differentiation is relatively high. This result is in contrast to the findings in Schultz (2005) with inelastic demand. In order to understand the reason for this difference, we need to distinguish between those effects stemming from a change of the transportation-cost level and those that are due to different degrees of market transparency.

Note first that differentiation has two opposing effects for competing firms which can be seen from equation (3.5): On the one hand, a higher degree of differentiation (like in the inelastic-demand case) is good for firms as it weakens the competitive pressure in the sense that there is less competition for the marginal customer. On the other hand (and different from the inelastic-demand case), an increase in the level of differentiation means that customers incur a higher loss of utility per unit purchased which leads to a decrease in local demand.

We start with the case where the level of market transparency is fixed and where differentiation increases. Consider a situation with full transparency and fairly moderate differentiation: This case is similar to a situation with inelastic demand as local demand is high and there is tough competition for the marginal customer. Hence, if the level of differentiation increases, we have the same effect compared to the inelastic case, i.e. collusion is stabilized as deviation is not sufficiently attractive. Now as firms become more differentiated, this means that local demand both for competing as well as for deviating firms goes down and firms have to lower their prices. In this situation, competing firms are more concerned with attracting sufficient local demand which means that there is (even) less competition for the marginal customer. At the same time, a deviating firm must extend its market share through a lower price. It is true that it is harder for a deviating firm to do so but the lower price here also has a positive effect since local demand goes up. As a result, a deviating firm is hit less hard by an increase in differentiation. Put together, these arguments imply that collusion is destabilized for high degrees of differentiation.

Turning to the change in the degree of market transparency, we point out that transparency affects the market in a similar way like differentiation. To see this, consider first an increase in transparency for a (fixed) high level of differentiation: In this situation, giving more information to customers means that a larger number of them actually become aware of the prices set by the two firms which toughens competition. We can thus argue that increasing market transparency on the customer side has a similar effect like a decrease in differentiation. As the above reasoning showed, if highly differentiated firms become closer substitutes (when differentiation decreases), collusion is stabilized under elastic demand. In line with this argument, the proposition showed that rendering a market with very differentiated firms more transparent facilitates collusion.

Turning to an increase in market transparency for a (fixed) low degree of differentiation, we point out that if the market is quite transparent already, then a further increase in market transparency has the same effect like a decrease in differentiation in a situation where firms are close substitutes: Collusion will be harder to sustain. The opposite is true for the case where the market is very intransparent: Even though firms are close substitutes, the competition in the market is not very intense due to a lack of information on the customer side, i.e. customers behave as if firms were very differentiated: Since they have hardly any information about prices, their purchasing decision is based purely on product preference (or characteristics)—irrespective of its price. Now if customers have access to more information, this has the same effect explained above, i.e. collusion is easier to sustain.

Figures 3.2 and *3.3* illustrate these findings for different values of the transparency as well as the differentiation parameter.[29]

29 Note that $\hat{\underline{\tau}}$ in the figure is defined as the maximum value of the lower bound $\underline{\tau}$ ($\hat{\underline{\tau}} \approx 0.20223$).

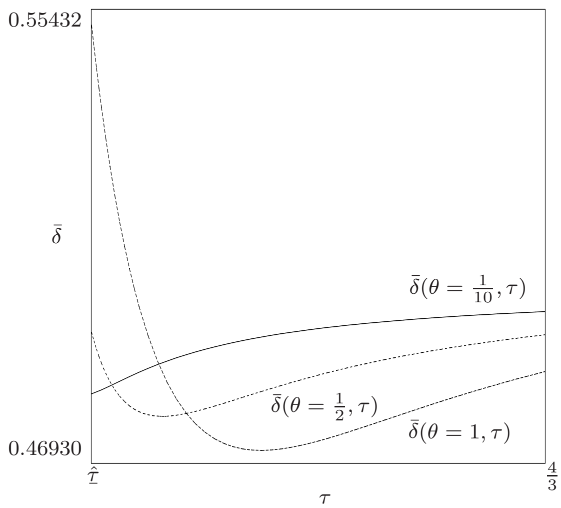

Figure 3.2: Impact of τ on $\bar{\delta}(\theta, \tau)$ (for different values of θ)

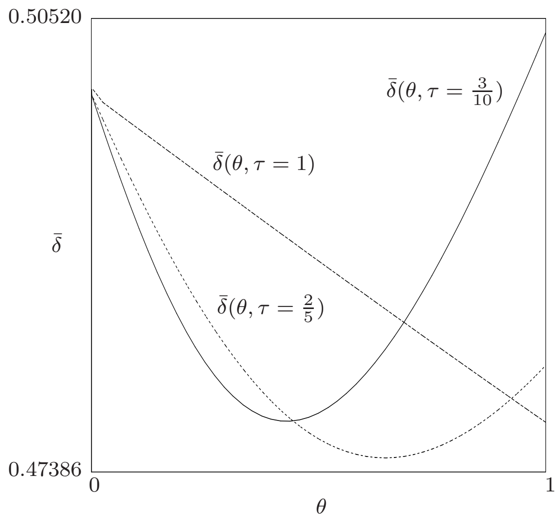

Figure 3.3: Impact of θ on $\bar{\delta}(\theta, \tau)$ (for different values of τ)

Given the examples in the introduction, it seems interesting to consider the role of a competition authority as well. Obviously, a competition authority is interested in a discount factor as high as possible. To this end, we define $\tilde{\tau}_R$ such that $\bar{\delta}(\theta = 0, \tau = \tilde{\tau}_R) = \bar{\delta}(\theta = 1, \tau = \tilde{\tau}_R)$. Then, making use of the result from the proposition, we can derive the following result:

Corollary 3.1 *From the perspective of the competition authority, full transparency is optimal only if* $\tau \leq \tilde{\tau}_R < \tilde{\tau}$.

Hence, a competition authority would want a fully transparent market only if the degree of differentiation is sufficiently low.[30] Note, however, that due to the ambiguous impact of θ on the critical discount factor for low levels of differentiation, increasing the market transparency only a little bit may actually have detrimental effects for customers as firms may be enabled to charge higher (collusive) prices. If the degree of differentiation is above this threshold, then it may make sense not to favor more transparency on the customer side as this may enable firms to collude. Nevertheless, rendering the market completely intransparent is not an option as this would imply monopoly prices for customers as well.

3.3 Conclusions

This chapter addresses the question whether increased market transparency on the customer side stabilizes collusion when horizontally differentiated firms face an elastic demand. It is shown that the answer depends on the degree of differentiation: For low levels of differentiation, there is an ambiguous effect with the lowest critical discount factor being no corner solution. If, on the other hand, firms are very differentiated, full transparency implies the highest degree of collusive stability.

Competition authorities therefore have to take into account several important market features. Given the results mentioned in the introduction, the type of decision variable or parameter affected by a change in market transparency on the customer side seems to be crucial. With respect to price transparency, demand characteristics seem to play an important role. At the same time, it is important to assess the degree of differentiation in the market. More generally, the level of transparency that has already been achieved is important as well since an increase may facilitate collusion.

30 More precisely, $\tilde{\tau}_R \approx 0.31319$.

3.4 Appendix

Proof of *Proposition 3.1*

Proof An obvious approach to investigating the impact of θ on $\bar{\delta}$ would be to consider $\frac{\partial\bar{\delta}}{\partial\theta}$. However, as it turns out, this expression is not tractable which is why we follow a different route: We first set $\frac{\partial\bar{\delta}}{\partial\tau} = 0$ and solve for θ in order to get the extremal values of the critical discount factor for *any combination* of θ and τ denoted by $\tilde{\theta}(\tau)$.[31] As *Figure 3.1* suggests, $\tilde{\theta}(\tau)$ is an increasing function of τ. As $0 \leq \tilde{\theta}(\tau) \leq 1$ must hold, we need to check whether this condition is satisfied. It holds that $\tilde{\theta}(\tau) > 0\, \forall \tau \in [\underline{\tau}, \bar{\tau}]$. Note that $\tilde{\theta}(\tau = \underline{\tau}) \approx 0.23769$. Solving $\tilde{\theta}(\tau) = 1$ for τ gives $\tilde{\tau} \approx 0.62060$ as a solution. As $\frac{\partial^2\bar{\delta}}{\partial\tau^2}\big|_{\theta=\tilde{\theta}(\tau)} > 0$ (see *Figure 3.4* below), the critical discount factor reaches a minimum for $\theta = \tilde{\theta}(\tau)$. Therefore, we can conclude that if $\theta \leq \tilde{\theta}(\tau)$, then $\frac{\partial\bar{\delta}}{\partial\theta} < 0$ and vice versa. Moreover, it must hold that $\frac{\partial\bar{\delta}}{\partial\theta} < 0\, \forall \tau \in (\tilde{\tau}, \bar{\tau}]$. ■

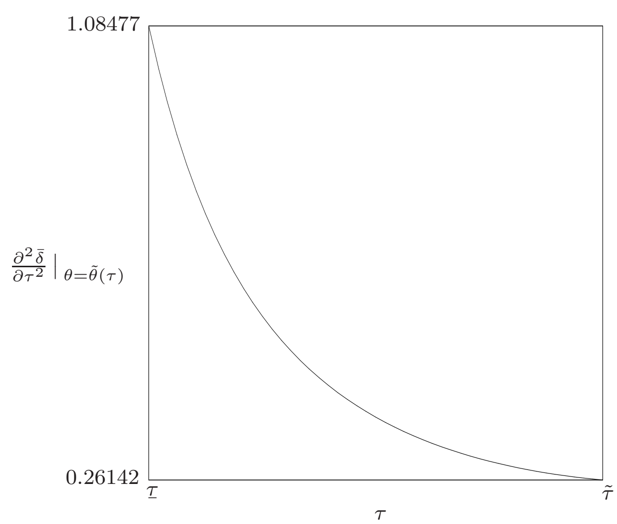

Figure 3.4: Characterization of $\frac{\partial^2\bar{\delta}}{\partial\tau^2}\big|_{\theta=\tilde{\theta}(\tau)}$

31 We used Maple to derive this solution. Note that $\tilde{\theta}(\tau)$ is the unique solution for the parameter space we are interested in.

4 Internal decision-making rules and collusion

We study the impact of internal decision-making structures on the stability of collusive agreements. To this end, we use a three-firm spatial competition model where two firms belong to the same holding company. The holding company can decide to set prices itself or to delegate this decision to its local units. It is shown that when transportation costs are high, collusion is more stable under delegation. Furthermore, collusion with maximum prices is more profitable if price setting is delegated to the local units. Profitability is reversed for low discount factors.

4.1 Introduction

Antitrust and competition authorities have to evaluate the implications of a merger for post-merger competition. To prevent coordinated effects in an industry, authorities have two types of remedies at their disposal: structural and non-structural (behavioral) remedies. The former are often associated with (partial) divestiture of assets owned by the acquiring firm. However, they may also involve a so-called spin-off where the shares of an acquired company are transferred to the stockholders of the acquiring firm and where the acquired firm is maintained as a separate entity.[32] Non-structural remedies may involve the delegation of decision-making powers. For instance, as Campbell and Halladay (2005) report, authorities in Canada accepted

32 This was the case in *In re Procter & Gamble Co* (see Shelanski and Sidak, 2001). As Timmins (1986) points out, a spin-off may be required if there is no viable purchaser in the market in the wake of a divestiture order by the court.

the "maintenance of the acquired firm as a separate and distinct corporate entity with a separate board of directors" (p. 10) in some cases[33] as a remedy to prevent anti-competitive behavior. In this chapter, we are interested in the implications for collusion when the (acquired) firms are kept as separate entities with certain decision-making powers.

At first sight, the above logic behind such remedies seems compelling: Giving decision-making powers away tends to have a similar effect as an increase in the number of firms in the market which makes it harder to sustain collusive behavior. However, this standard reasoning ignores the effect the centralization of decision powers has on the other firms in the market. We show that while it turns out to be true that from the point of view of a holding company, its incentive to deviate from a collusive agreement are increased if it delegates decision powers to the local units, the other firms in the market may be less likely to deviate if decisions are made locally.

We use a three-firm spatial model à la Salop (1979). In order to account for different decision-making structures, there is a holding company which owns two of the firms. The holding company decides whether to delegate the price setting to its local units or not. As shown in various contributions in the literature on mergers, the holding company indeed prefers to keep the pricing decision by itself in the competitive case. This is easily understood by noting that when delegating the price setting to the local units, there are three firms competing in the market instead of two (which is the case when the holding company keeps authority). This implies stronger competition and thus lower prices and lower profits. So if discount rates are low and collusion with maximum prices is not possible (although a weaker form of collusion is still possible, see the third section), the holding company will set prices itself. If, however, the discount factor is sufficiently high and collusion at maximum prices is possible, the picture changes: We assume that the colluding parties bargain over the collusive prices as in Harrington (1991) and Harrington et al. (2005). Then,

33 See *Canada Post/Purolator* and *Avis/Budget*.

although the disagreement payoff (which is the competitive outcome) is better for the holding company if it keeps decision powers, the overall outcome is better for the holding company if the local units negotiate (*Proposition 4.1*). Intuitively, this can be understood by noting that in the latter case, the 'collusive cake' has to be shared by three players, two of which belong to the holding company. In the former case, there are two players trying to get their share of the cake.[34]

These results have consequences for the stability of collusion at maximum prices. It is well known from the literature that collusion becomes less stable if there are more parties to it. So one would expect that also here collusion will break down more easily if the pricing is delegated to the local units as then three parties are involved in the collusion. In line with intuition, *Lemma 4.2* shows that the critical discount rate, i.e. the discount factor where a party is just indifferent between colluding and deviating, is smaller for the holding company for the case where it keeps decision powers compared to the case where it delegates the pricing decision. However, in order to determine whether collusion is sustainable in the market as a whole, the outside firm's incentive to deviate from collusion becomes relevant. Indeed, as *Lemma 4.3* shows, the critical discount factor of the outside firm is greater than the one for the holding company. *Proposition 4.2* shows that maximum prices in the market are more likely to be sustained if the holding company delegates decision-making powers and if transportation costs are high. For low transportation costs, the result is reversed. Finally, *Proposition 4.3* summarizes the results and discusses the case for intermediate and low discount rates where, as shown by Chang (1991), collusion does not break down completely but is still possible at lower collusive prices.

There has been a growing interest in understanding the effects that different internal firm structures have on market outcomes. However, the literature focuses

34 This result resembles the classical case of quantity competition with three firms where the two merging firms usually lose out as they have the highest incentive to reduce their quantity after the merger (so-called merger paradox). So a holding company prefers to merge but would rather leave decision powers with the local units. In our case, the holding company is weaker with respect to negotiating good conditions for itself as its marginal profits count less compared to a situation where the two local units bargain instead.

mainly on strategic implications of divisionalization for competition and the resulting profits as well as on relative profits of insiders and outsiders in the context of mergers. What has been neglected in the contributions so far is an analysis of the implications of different decision-making regimes on the profitability and stability of collusive agreements. Baye, Crocker, and Ju (1996) assume divisionalization by one firm which increases the number of quantity-setting firms producing the same output in the industry. This drives down industry profit but the multidivisional firm enjoys a greater share of these profits. The authors show that the second effect is stronger which means that divisionalization is profitable. Huck, Konrad, and Müller (2004) develop a model where a joint headquarter lets two divisions decide on output quantities. Additionally, it sets up an internal sequential game in which the divisions compete against one another and where there are internal information flows due to the sequential nature of the game.[35] This implies that the market resembles a Stackelberg rather than a Cournot market. As a result, mergers may be profitable and welfare-improving even if the linear cost function is the same for all firms. At the same time, competitors are worse off as their profits decrease. In a similar setup, Creane and Davidson (2004) come to the same conclusion.

The approach by Levy and Reitzes (1992) is related to the present one in that it analyzes the implications of mergers for the stability of collusion. Unlike the present article, they look at the conditions under which firms actually find it profitable to merge. Contrary to the analysis in this article, firms are assumed to maximize industry profits in a collusive phase which means that firms' asymmetry is not accounted for. As a consequence, the profitability of a merger once collusion can be achieved is not an issue. In addition, a comparison between the deviating incentives of the merged firm and of the other firms in the market is not made.

The articles by Lambertini and Trombetta (2002) and Spagnolo (2005) are related to the present one with respect to the delegation of decision powers and its implications for collusion. Contrary to these contributions, we do not compare the

35 The authors show that this sequential structure also arises endogenously when the divisions may make their own timing decisions.

effects of different (publicly observable) contract schemes for the stability of collusion but consider different decision-making scenarios.[36] When it comes to the internal decision-making structure actually preferred by the merging firms, Prechel, Boies, and Woods (1999) show in an empirical study that these firms tend to opt for decentralized decision making.

The rest of the chapter is organized as follows. In the next section, we set up the model and look at the competitive case. We then analyze collusion as well as the incentives to deviate before we derive the critical discount factors. This is followed by a comparison of the outcomes and the profitability for the two decision-making regimes. The last section concludes.

4.2 The model

We want to capture the trade-off a holding company faces when having to decide about delegating decision-making powers under market collusion. To this end, we use the well known Salop (1979) setup reflecting a market of spatial competition with horizontal product differentiation. There are three firms with two firms (firms 2 and 3) being governed by a holding company. All three firms are located equidistantly from each other along a circular city with a circumference of 1. In order to capture different internal decision-making structures, the holding company may either keep the pricing decision or give it away to both of its local units.[37]

We model collusive behavior following Friedman (1971): Firms tacitly agree to collude. So, in every period, the holding company and the third firm set collusive prices whenever no firm deviated from the agreement in the previous period. If there is deviation, this triggers a price war and there is competition forever. A problem with the modeling by Friedman (1971) and with most models on collusive outcomes

36 Relatedly, Olaizola (2007) studies the effects of delegation on endogenous cartel formation among symmetric firms.

37 Note that one may also think of a situation where the pricing decision is partially delegated only. This could be done by having local units take into account the other unit's (weighted) profits as well.

is that there are many prices which can be sustained in equilibrium. Usually the literature concentrates on those prices which maximize the overall collusive profit. This procedure might be considered reasonable as long as firms are symmetric. Here, however, the outside firm and the holding company differ. Therefore, we follow Harrington (1991) and Harrington et al. (2005) and assume that at an initial stage, the price-setting parties bargain over the collusive prices.

As is common for the analysis of collusion, we will later make use of the so-called critical discount factor in order to define the range of discount factors for which maximum collusive prices are sustainable. Remember that critical discount factor according to Friedman (1971) is characterized as follows:

$$\delta \geq \bar{\delta} := \frac{\pi^D - \pi^C}{\pi^D - \pi^N} \tag{4.1}$$

where superscripts C, D, and N again denote profits in the collusive, the deviating, and the punishment (competitive Nash price equilibrium) cases, respectively.[38]

In the following, we will derive the profits for all three situations. We start with the punishment case as the profits under price competition will be needed to derive the optimal collusive prices.[39]

4.2.1 Punishment: competition in prices

Customers of mass 1 are uniformly distributed along the unit circle and are interested in buying one product or none. They are assumed to derive a basic utility of 1 from consuming the product offered by firm i. Furthermore, a customer located at some distance Δ_i from firm i ($i \in \{1, 2, 3\}$) has to incur (quadratic) transportation costs

38 An alternative option would be the use of optimal punishments as introduced by Abreu (1986). As Häckner (1996) shows, applying optimal punishments in the context of a Hotelling (1929) setup with quadratic transportation costs and symmetric firms gives qualitatively the same results with respect to the impact of product differentiation on the collusive price as in Chang (1991) who uses grim-trigger strategies. Therefore, we conjecture that our results would hold qualitatively even for optimal punishments. In the present context with asymmetric firms, the derivation of such optimal punishments becomes non-tractable.

39 The following section is a special case of the analysis in Posada and Straume (2004) who analyze firms' incentives to merge under a partial merger and relocation.

of $\tau\Delta_i^2$ when traveling to that firm. p_i denotes the price charged by firm i. Hence, a customer derives the following net utility U_i when buying from this firm:

$$U_i = 1 - p_i - \tau\Delta_i^2. \tag{4.2}$$

We make the following assumption with respect to the transportation costs:

Assumption 4.1 $0 < \tau \leq \frac{7}{2} =: \bar{\bar{\tau}}$.

This assumption ensures that it is optimal for the market to be covered.[40]

Centralized decision making

We start by considering the case where the holding company makes the pricing decision. Denote by x_i the position of the marginal customer located in between firm i and the next firm in the clockwise direction (where subscript c stands for centralization). Then, the indifferent customer between firm 1 (price $p_{1,c}^N$) and 2 (price $p_{h,c}^N$ where subscript h stands for the holding company) can be obtained by solving

$$p_{1,c}^N + \tau {x_{1,c}^N}^2 = p_{h,c}^N + \tau\left(\frac{1}{3} - x_{1,c}^N\right)^2 \tag{4.3}$$

for $x_{1,c}^N$ which gives

$$x_{1,c}^N = \frac{1}{6} - \frac{3p_{1,c}^N - 3p_{h,c}^N}{2\tau}. \tag{4.4}$$

Both marginal and fixed costs are normalized to 0. Hence, from the profits for firm 1 and the holding company, i.e.

$$\pi_{1,c}^N = 2p_{1,c}^N\left(\frac{1}{6} - \frac{3p_{1,c}^N - 3p_{h,c}^N}{2\tau}\right) \tag{4.5}$$

and

40 This result is derived in the appendix as part of the derivation of the collusive prices.

$$\pi_{h,c}^N = 2p_{h,c}^N \left(\frac{1}{3} + \frac{3p_{1,c}^N - 3p_{h,c}^N}{2\tau} \right), \tag{4.6}$$

we can derive the optimal prices in the usual way. We then find the following equilibrium prices and profits:

$$p_{1,c}^N = \frac{4\tau}{27}, \tag{4.7}$$

$$p_{h,c}^N = \frac{5\tau}{27}, \tag{4.8}$$

$$\pi_{1,c}^N = \frac{16\tau}{243}, \tag{4.9}$$

and

$$\pi_{h,c}^N = \frac{25\tau}{243}. \tag{4.10}$$

We now turn to the case where the holding company gives away the pricing decision.

Decentralized decision making

Decentralized decision making is characterized by a situation where the two local units are in charge of price setting (subscript d). As a result, the punishment involves price competition between all three firms. We assume that the managers of the local units maximize the profits of their individual unit.

Proceeding in the same way as before gives

$$p_{i,d}^N = \frac{\tau}{9}, \tag{4.11}$$

$$\pi_{i,d}^N = \frac{\tau}{27}, \tag{4.12}$$

and

$$\pi_{h,d}^N = \frac{2\tau}{27}. \tag{4.13}$$

To summarize, we restate a familiar result that in the competitive case, the holding company will set higher prices. By setting prices itself, the holding company can avoid competing with itself for the customers located in between its two local units. As prices are strategic complements, it follows immediately that firm 1 will charge a higher price if the holding company sets the prices for its units. These higher prices then translate into higher profits for the holding company as well as for firm 1.

We now proceed with analyzing the different scenarios under collusion.

4.2.2 Collusive outcome

In this section, it is assumed that collusion is feasible. The incentives to defect from collusion are analyzed in the next section. Again, we have to distinguish between the two cases of internal decision making.

Centralized decision making

We start with the situation where all decisions are made by the holding company. Like Harrington (1991) and Harrington et al. (2005), we assume that under collusion prices will be set according to the Nash bargaining solution[41], i.e. the maximization problem can be written as follows:

$$\max_{p_{1,c}^C, p_{h,c}^C} \Pi_c^C := \left(\pi_{1,c}^C - \pi_{1,c}^N\right)\left(\pi_{h,c}^C - \pi_{h,c}^N\right) \tag{4.14}$$

where $\pi_{1,c}^C$ and $\pi_{h,c}^C$ are calculated by replacing $p_{1,c}^N$ and $p_{h,c}^N$ in equations (4.5) and

41 This solution was proposed by Nash (1950) (see also Osborne and Rubinstein, 1990, chapter 2).

(4.6) by $p^C_{1,c}$ and $p^C_{h,c}$, respectively. In the appendix, we provide a formal proof for the derivation of the optimal collusive prices. It is shown that the holding company will set its collusive price at the highest possible level such that the market is still covered. This means that the customer halfway between the holding company's local units will be left with zero utility, i.e. $p^C_{h,c} = 1 - \frac{\tau}{36}$. At the same time, the outside firm charges a lower collusive price. This is due to the fact that starting from $p^C_{1,c} = p^C_{h,c}$, $\pi^C_{1,c} - \pi^N_{1,c}$ is lower than $\pi^C_{h,c} - \pi^N_{h,c}$. Thus, a small increase in $\pi^C_{1,c}$ (through a decrease of $p^C_{1,c}$) with a corresponding small reduction in $\pi^C_{h,c}$ increases the product.

Decentralized decision making

Next, we are interested in the collusive prices if the holding company does not set the prices for its two local units. We assume that collusion is feasible only if all three firms participate, i.e. there is a grand coalition. The optimal outcome is therefore the solution to the following maximization problem:

$$\max_{p^C_{i,d}} \Pi^C_d := (\pi^C_{1,d} - \pi^N_{1,d})(\pi^C_{2,d} - \pi^N_{2,d})(\pi^C_{3,d} - \pi^N_{3,d}) = (\pi^C_{i,d} - \pi^N_{i,d})^3. \tag{4.15}$$

Due to the symmetric structure, the optimal collusive price is the same for all firms and is such that the indifferent customer in between any two firms is left with a utility of zero which implies

$$p^C_{i,d} = 1 - \frac{\tau}{36}. \tag{4.16}$$

Profits thus amount to

$$\pi^C_{i,d} = \frac{1}{3} - \frac{\tau}{108} \tag{4.17}$$

and

$$\pi^C_{h,d} = 2\pi^C_{i,d} = \frac{2}{3} - \frac{\tau}{54}. \tag{4.18}$$

We summarize our findings from this section in the following proposition:

Proposition 4.1 *Comparing both decision-making regimes under collusion yields:*

(a) $p^C_{1,c} < p^C_{h,c}$,

(b) $p^C_{i,d} = p^C_{h,c}$,

(c) $\pi^C_{1,c} > \pi^C_{1,d}$, *and*

(d) $\pi^C_{h,c} < \pi^C_{h,d}$.

Proof Ad (a) and (b): Follow from the derivation of the maximum collusive prices which is relegated to the appendix.

Ad (c) and (d): Follow from (a) and (b). ■

If pricing decisions are delegated to the local units, all three firms charge the monopoly price of $1 - \frac{\tau}{36}$ and share the market equally (result (b)). If, however, the holding company keeps authority over prices, then it will be worse off. While its units still charge $1 - \frac{\tau}{36}$ (result (b)), firm 1 charges a lower price (result (a)), leaving less profit to the holding company. This result is not straightforward from the point of view of the holding company: The analysis of the competitive case showed that in the case of breakdown of collusion, a holding company fares better by keeping decision powers. This effect of a more favorable disagreement payoff leads to an increase in bargaining power which tends to lead to a higher collusive profit. On the other hand, as the parties bargain over the collusive prices, the holding company fares worse in bilateral negotiations. This is intuitively clear as by direct negotiations, the 'collusive cake', i.e. the profits generated by collusion, has to be divided among two players. If, however, the two local units negotiate for themselves, this cake is divided among three firms two of which are owned by the holding company.[42] More

42 One may argue that such a distribution of profits in the present scenario is due to the fact that all parties have the same bargaining power. Indeed, giving the holding company (sufficiently) more bargaining power when negotiating with the outside firm would result in equal profits for all three (local) firms. However, we think that our specification is suitable insofar as factors that usually lead to the different degrees of bargaining power like impatience, risk of breakdown, (legally binding) commitment tactics, etc. (see Muthoo, 1999) are not relevant here.

formally, in equilibrium the profits of the holding company's local units enter the maximization problem additively in the former case (see equation (4.14)) while in the latter case, they enter quadratically (see equation (4.15)). A marginal increase in profits increases this expression more if profits enter quadratically.

Below, the following observation will be relevant: $\frac{\partial(\pi^C_{1,c}-\pi^C_{1,d})}{\partial\tau} < 0$, i.e. an increase in the transportation costs reduces the difference in the outside firm's collusive profits under both regimes. This can be explained by pointing out that the outside firm captures a market share of more than $\frac{1}{3}$ by charging a lower price than the holding company. However, as transportation costs increase, customers incur a higher disutility which means that this price would have to be lowered to a (much) larger extent—compared to the actual price—in order to reach the same market share. As a consequence, the outside firm's market share is reduced which leads to (relatively) lower profits since in the case of decentralization, there is only a price-reducing effect whenever transportation costs increase.

Having determined the collusive prices, we next turn to firms' deviation incentives.

4.2.3 One-period deviation incentives

We start with the simpler case where firm i faces two rivals which charge a collusive price of $1-\frac{\tau}{36}$. Then, a deviating firm may either seize only a share of the market by setting a moderate deviating price or it may cover the whole market through a very low price. Now consider the first situation: Determining firm i's deviation profit in the usual way, differentiating with respect to the deviating price, and solving for the optimal price for firm i gives

$$p^D_{1,c} = p^D_{i,d} = \frac{1}{2} + \frac{\tau}{24}. \tag{4.19}$$

In this case, the deviating firm's market share amounts to

$$x^D_{1,c} = x^D_{i,d} = \frac{12+\tau}{16\tau}. \tag{4.20}$$

Hence, the deviating firm covers the whole market whenever $x^D_{1,c} = x^D_{i,d} \geq \frac{1}{2} \Leftrightarrow \tau \leq \frac{12}{7} =: \hat{\tau}$, i.e. when transportation costs are low. As the deviating firm cannot extend its market share further than that, it will charge a deviating price of

$$\frac{1}{6} - \frac{3p^D_{1,c} - 3(1 - \frac{\tau}{36})}{2\tau} = \frac{1}{2} \Leftrightarrow p^D_{1,c} = p^D_{i,d} = 1 - \frac{\tau}{4}. \tag{4.21}$$

We thus have the following outcome:

$$p^D_{1,c} = p^D_{i,d} = \begin{cases} 1 - \frac{\tau}{4} & \text{if } \tau \leq \hat{\tau} \\ \frac{1}{2} + \frac{\tau}{24} & \text{if } \hat{\tau} < \tau \leq \bar{\bar{\tau}} \end{cases} \tag{4.22}$$

and

$$\pi^D_{1,c} = \pi^D_{i,d} = \begin{cases} 1 - \frac{\tau}{4} & \text{if } \tau \leq \hat{\tau} \\ \frac{(12+\tau)^2}{192\tau} & \text{if } \hat{\tau} < \tau \leq \bar{\bar{\tau}}. \end{cases} \tag{4.23}$$

Performing the same analysis for the holding company's deviating incentives under centralized decision making leads to the following lemma:[43]

Lemma 4.1 *One-period deviation incentives imply that* $p^D_{h,c} > p^D_{1,c} = p^D_{i,d}$.

Proof See the appendix. ■

If the holding company keeps pricing power and deviates, it will charge a higher price compared to the case where local units set prices themselves and deviate. At first glance, this may not seem obvious as the collusive price by firm 1 is lower in the case without the delegation of the pricing decision, so one would expect the best reply to that price to be a lower price too. Whenever a firm deviates, however, it has to find an optimal balance between a reduction in price and an increase in market share. Therefore, a holding company charges a higher deviating price as a price cut applies to a greater market share.[44]

43 Note that the holding company would want to serve the whole market for any $\tau \lesssim 2.01244 =: \check{\tau}$.
44 Note also that if the holding company seeks to cover the whole market, the customer that is furthest away from each of the local units is located at a distance of only $\frac{1}{3}$.

Given the results from the analysis so far, it is now possible to calculate the critical discount factors in order to compare the profitability of collusive agreements under different decision-making rules.

4.3 Sustainability of maximum prices under collusion

In order to analyze the overall incentives to deviate from the maximum collusive prices, discount factors as mentioned above are calculated. They define the range for which the maximum collusive prices can be sustained: The higher the critical discount factor, the smaller this range. First, we analyze how the internal decision-making process affects the deviation incentives for the holding company and its two local units. Let $\bar{\delta}_{h,c} := \frac{\pi^D_{h,c} - \pi^C_{h,c}}{\pi^D_{h,c} - \pi^N_{h,c}}$ and $\bar{\delta}_{i,d} := \frac{\pi^D_{i,d} - \pi^C_{i,d}}{\pi^D_{i,d} - \pi^N_{i,d}}$. We obtain:

Lemma 4.2 *On the holding-company level, $\bar{\delta}_{h,c} < \bar{\delta}_{i,d}$ holds.*

Proof See the appendix. ■

On the holding-company level, collusion is more stable under a centralized decision-making structure. Giving away the pricing decision to the local units will make deviation from the collusive agreement more likely when considering maximum collusive prices. The reason for this is similar to the classic argument of why collusion becomes less stable if there are more parties to it.[45] Deviating is much more attractive for a local unit as it also steals customers from the other local unit and not just from firm 1. In addition, firm 1 charges a higher price for the case where pricing power is with the local units which makes deviation even more profitable. So although future punishment is harder, i.e. the profits in future periods are lower if pricing rests with the local units, the first two effects outweigh the latter to give the result of *Lemma 4.2*.

45 See, e.g., Majerus (1988).

Next, we look at the outside firm and the holding company. Let $\bar{\delta}_{1,c} := \frac{\pi^D_{1,c} - \pi^C_{1,c}}{\pi^D_{1,c} - \pi^N_{1,c}}$. Then, we can state the following result:

Lemma 4.3 *For the case where the holding company sets prices, it holds that* $\bar{\delta}_{1,c} > \bar{\delta}_{h,c}$.

Proof See the appendix. ■

The outside firm is more likely to deviate whenever the holding company does not delegate the pricing decision. Note that the holding company charges the higher collusive price. Hence, deviating is quite attractive for the outside firm as it is able to capture a large share of the market (or all of it, for that matter).

From a market point of view, we have to find the weakest link for the collusion with maximum prices, i.e. the firm with the largest critical discount rate, to determine whether collusion is possible or not. *Lemma 4.3* and the fact that the deviation incentives are the same for all firms under decentralization shows that it is indeed the outside firm's behavior which is crucial for the stability of a collusive agreement from the perspective of the market as a whole. This is given in the next proposition:

Proposition 4.2 *There exists a* $\tilde{\tilde{\tau}} \leq \bar{\bar{\tau}}$ *such that* $\bar{\delta}_{1,c} \lesseqgtr \bar{\delta}_{1,d} \Leftrightarrow \tau \lesseqgtr \tilde{\tilde{\tau}}$, *i.e. for low (high) transportation costs, the critical discount factor for the outside firm is smaller (greater) under centralization than under decentralization.*

Proof See the appendix. ■

Proposition 4.2 shows that maximum collusive prices can be sustained for a broader range of discount factors if the holding company gives away (does not give away) the pricing decision and if the transportation costs are high (low). The intuition behind the above result is as follows: Both under centralized and decentralized decision making, the outside firm has the same deviation incentives, so it is sufficient to focus on its competitive and collusive profits. With respect to these profits, there are two effects which work in opposite directions: On the one hand, due to the bargaining process under centralization, the outside firm generates a higher collusive profit which stabilizes collusion. On the other hand, however, it faces a less severe punishment. For low values of the transportation costs, the latter effect is negligible as competitive profits for the outside firm are rather low both under centralization as well as decentralization. At the same time, the difference between the outside firm's collusive profit under both regimes is rather large (see remarks on *Proposition 4.1*).[46] As a result, the outside firm is more likely to deviate under delegation, thus destabilizing collusion to a larger extent. The strength of these effects is turned around if transportation costs are high as the difference with respect to the competitive profits increases with the transportation costs. As a consequence, the centralization of decision-making powers leads to a destabilization of the collusive agreement. This is not necessarily what one expects as usually a lower number of firms in the market means that collusion can be sustained more easily. Here, however, the less severe punishment under centralized decision making turns this result around.

Figure 4.1 compares all three critical discount factors for different values of the transportation-cost parameter.

Next, we turn to the case where the industry's discount factor is below the critical discount factor.

Collusion for discount factors below the critical one

As shown in Chang (1991), a lower discount factor than the critical one does not necessarily mean that collusion is not sustainable in a situation where firms are (horizontally) differentiated. Firms may not be able to charge their maximum collusive prices but prices higher than the competitive prices can still be supported as a collusive outcome as long as the discount factor is strictly positive. In order to understand the basic mechanism, consider the case of decentralized decision-making powers. Consider a situation where $\tau > \hat{\tau}$, i.e. a deviating firm does not want to capture the whole market. Clearly, if the collusive price has to be adjusted down-

46 E.g., for $\tau \to 0$ we get $\pi_{1,c}^N \to 0$ and $\pi_{i,d}^N \to 0$ whereas collusive profits are strictly positive and $\pi_{1,c}^C > \pi_{i,d}^C$.

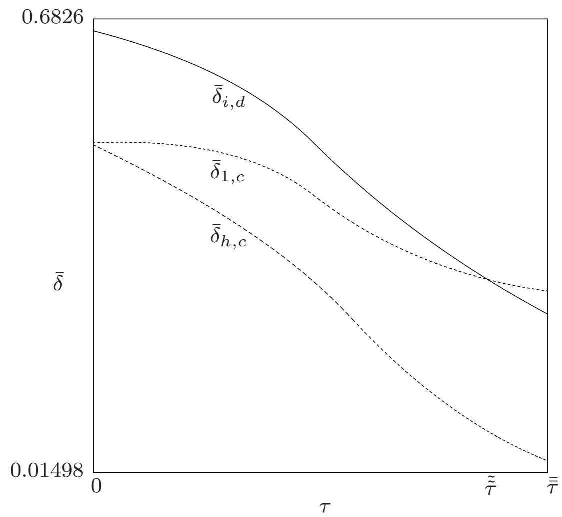

Figure 4.1: Comparison of the critical discount factors

wards, the incentive to seize all of the market is further reduced. Therefore, collusion will be stable as long as

$$\frac{p^C_{i,d}}{3(1-\delta)} \geq \frac{(9p^C_{i,d}+\tau)^2}{108\tau} + \frac{\delta\pi^N_{i,d}}{1-\delta} \Leftrightarrow p^C_{i,d}(\delta) \leq \frac{\tau(1+3\delta)}{9(1-\delta)} \tag{4.24}$$

holds.[47]

Hence, one arrives at a δ-adjusted collusive price which is strictly increasing in the discount factor. If the discount factor converges to 0, then the price converges to the competitive price. We will not derive the explicit collusive prices for the scenario with centralized price setting as computations are not tractable.

From the competitive case, we know that centralizing the price-setting decision is more profitable, i.e. $\pi^N_{h,c} > \pi^N_{h,d}$. *Proposition 4.1* states that when comparing maximum collusive profits, delegating price-setting power is more profitable for the holding company, i.e. $\pi^C_{h,c} < \pi^C_{h,d}$. Combining these results with the general insights from the approach by Chang (1991), we can state the following proposition:

Proposition 4.3 *For any* $\tau \in (0, \bar{\bar{\tau}}]$*, there exist* δ^* *and* δ^{**} *with* $0 < \delta^* \leq \delta^{**} \leq \bar{\delta}_{i,d}$

47 See the proof of *Proposition 4.3* in the appendix for further details as well as an analysis of the case where $\tau \leq \hat{\tau}$.

such that if $\delta \leq \delta^* \Rightarrow \pi^C_{h,c} \geq \pi^C_{h,d}$ *and if* $\delta \geq \delta^{**} \Rightarrow \pi^C_{h,c} \leq \pi^C_{h,d}$.

Proof See the appendix. ■

Hence, the profitability of collusion depends on the centralization of the price-setting decision. Whereas centralization makes sense for lower discount factors, intermediate and high discount factors suggest the decentralization of internal decision responsibilities. For two values of the transportation costs (τ small and τ large), we derived the respective profits numerically. The results are given in *Figures 4.2* and *4.3*.

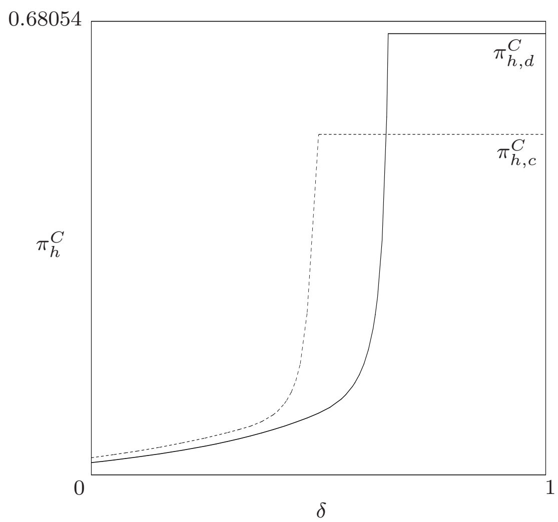

Figure 4.2: Comparison of collusive profits under both regimes (for $\tau = \frac{1}{4}$)

4.4 Conclusions

In this chapter, we analyze how different internal decision-making structures affect the stability of collusive agreements. Our findings can be summarized as follows. Making use of a three-firm spatial model à la Salop (1979), we find that on the holding-company level, (a) the maximum profit from collusion is higher if the holding company delegates the pricing decision and (b) if it does so, however, the incentives for its local units to deviate from the collusive agreement are increased. From the

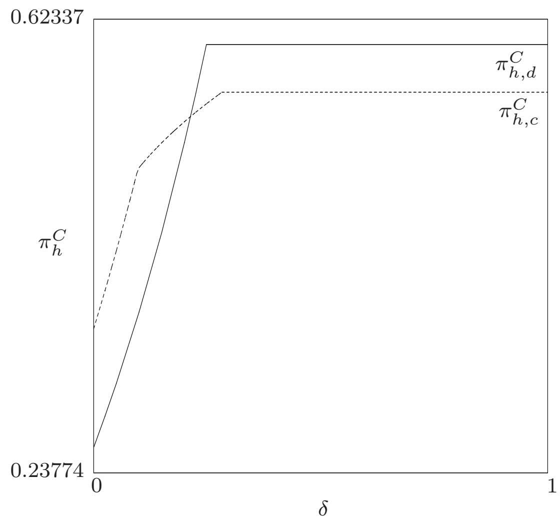

Figure 4.3: Comparison of collusive profits under both regimes (for $\tau = \frac{7}{2}$)

perspective of the market as a whole, however, we find that (c) the critical discount factor is determined by the outside firm: It is higher (lower) in the centralized decision-making regime if transportation costs are high (low).

Summing up, antitrust and competition authorities should be careful when imposing certain restrictions with respect to the decision-making structure on merging firms. While the "maintenance of the acquired firm as a separate and distinct corporate entity with a separate board of directors" (see the introduction) might restrain the merged firm in some of its detrimental economic behavior, it might, however, enable the market as a whole to collude.

4.5 Appendix

4.5.1 Derivation of maximum collusive prices under centralization

Proof We divide the proof into two parts. The first one deals with the derivation of the maximum collusive prices assuming full market coverage. The second part shows that partial market coverage cannot be optimal.

Maximum collusive prices under full market coverage

As explained above, the maximization problem concerning collusive prices amounts to $\max\limits_{p^C_{1,c},p^C_{h,c}} \Pi^C_c := (\pi^C_{1,c} - \pi^N_{1,c})(\pi^C_{h,c} - \pi^N_{h,c})$. Note first that if both collusive prices are below $1 - \frac{\tau}{36}$, then increasing these prices simultaneously by the same amount such that the market shares are not changed leads to an increase in the product. There are two cases: The price charged by the outside firm may either be (a) higher or (b) lower than the one charged by the holding company.

Ad (a): As argued above, this case implies that $p^C_{1,c} \geq 1 - \frac{\tau}{36}$ must hold. At the same time, $p^C_{h,c} < 1 - \frac{\tau}{36}$ must be true in order for the market to be covered. However, $p^C_{h,c}$ cannot be too small as the outside firm needs to have an incentive to participate. More precisely, it must hold that $\pi^C_{1,c} - \pi^N_{1,c} \geq 0 \Leftrightarrow p^C_{h,c} \geq p^C_{1,c} + \frac{16\tau^2}{729 p^C_{1,c}} - \frac{\tau}{9}$. Note that this expression attains a minimum when $p^C_{1,c}$ is equal to the competitive price $p^N_{1,c}$. This implies that $p^C_{h,c} \geq p^N_{h,c}$. Next, we get $\frac{\partial \Pi^C_c}{\partial p^C_{1,c}} = \frac{p^C_{1,c}(-19683 p^C_{1,c} p^C_{h,c} + 26244 {p^C_{h,c}}^2 - 1458 p^C_{h,c}\tau + 450\tau^2) - 6561 {p^C_{h,c}}^3 + 729 {p^C_{h,c}}^2 \tau - 207 p^C_{h,c}\tau^2 - 25\tau^3}{729\tau^2}$. Solving $\frac{\partial \Pi^C_c}{\partial p^C_{1,c}} = 0$ for $p^C_{1,c}$ gives $p^C_{1,c\,[1]} = \frac{2p^C_{h,c}}{3} - \frac{\tau}{27} + \frac{25\tau^2 - \sqrt{531441 {p^C_{h,c}}^4 - 59049 {p^C_{h,c}}^3 \tau + 29160 {p^C_{h,c}}^2 \tau^2}}{2187 p^C_{h,c}}$ and $p^C_{1,c\,[2]} = \frac{2p^C_{h,c}}{3} - \frac{\tau}{27} + \frac{25\tau^2 + \sqrt{531441 {p^C_{h,c}}^4 - 59049 {p^C_{h,c}}^3 \tau + 29160 {p^C_{h,c}}^2 \tau^2}}{2187 p^C_{h,c}}$ as the two solutions. From the second-order condition given by $\frac{\partial^2 \Pi^C_c}{\partial {p^C_{1,c}}^2} = -\frac{2(2187 p^C_{1,c} p^C_{h,c} - 1458 {p^C_{h,c}}^2 + 81 p^C_{h,c}\tau - 25\tau^2)}{81\tau^2}$, we find that $\frac{\partial^2 \Pi^C_c}{\partial {p^C_{1,c}}^2}\Big|_{p^C_{1,c} = p^C_{1,c\,[1]}} = \frac{2\sqrt{531441 {p^C_{h,c}}^4 - 59049 {p^C_{h,c}}^3 \tau + 29160 {p^C_{h,c}}^2 \tau^2 - 10125 p^C_{h,c}\tau^3 + 625\tau^4}}{81\tau^2} > 0$. In a similar way, one can show that the second-order condition is strictly smaller than 0 for $p^C_{1,c\,[2]}$. For $p^C_{1,c\,[2]}$ to be part of the equilibrium defined above, it must hold that $p^C_{1,c\,[2]} \geq 1 - \frac{\tau}{36}$. However, for any $p^C_{h,c} \in [p^N_{h,c}, 1 - \frac{\tau}{36}) \Rightarrow p^C_{1,c\,[2]} < 1 - \frac{\tau}{36}$. Thus, $p^C_{1,c} > p^C_{h,c}$ cannot be the solution.

Ad (b): From (a) and the argument put forward above, we know that $p^C_{1,c} < p^C_{h,c} = 1 - \frac{\tau}{36}$ must hold. Now let $E := 136048896 - 30233088 + 9354528\tau^2 - 3053376\tau^3 + 238165\tau^4$.[48] Then, plugging $p^C_{h,c} = 1 - \frac{\tau}{36}$ into $p^C_{1,c\,[2]}$ gives the optimal price for the outside firm: $p^C_{1,c} = \frac{23328-2592\tau+454\tau^2+\sqrt{E}}{972(36-\tau)}$. Thus, profits are given by $\pi^C_{1,c} = \frac{(23328-2592\tau+454\tau^2+\sqrt{E})(11664+4536\tau-535\tau^2-\sqrt{E})}{314928\tau(36-\tau)^2}$ and $\pi^C_{h,c} = \pi^C_{2,c} + \pi^C_{3,c} = \frac{-11664+7128\tau+211\tau^2+\sqrt{E}}{11664\tau}$.

Partial market coverage

When it comes to partial market coverage, there are three cases which are relevant: (a) no full coverage between any two firms, (b) no coverage between the outside firm and the local units, and (c) no coverage between the local units.

Ad (a): Note first that in this case, we must have $p^C_{h,c} > 1 - \frac{\tau}{36}$ so that the market between the two local units is not covered. If the market is not covered between any two of the three firms, the maximization problem boils down to $\max\limits_{p^C_{1,c}, p^C_{h,c}} \Pi^C_{c\,[\text{i}]} := (2p^C_{1,c}\sqrt{\frac{1-p^C_{1,c}}{\tau}} - \pi^N_{1,c})(4p^C_{h,c}\sqrt{\frac{1-p^C_{h,c}}{\tau}} - \pi^N_{h,c})$. Hence, one gets $\frac{\partial \Pi^C_{c\,[\text{i}]}}{\partial p^C_{h,c}} = (2p^C_{1,c}\sqrt{\frac{1-p^C_{1,c}}{\tau}} - \pi^N_{1,c})(-\frac{3p^C_{h,c}-2}{\sqrt{\tau(1-p^C_{h,c})}})$. Note that $\frac{\partial \Pi^C_{c\,[\text{i}]}}{\partial p^C_{h,c}} > 0$ only holds if $p^C_{h,c} < \frac{2}{3}$ which contradicts the condition that $p^C_{h,c} > 1 - \frac{\tau}{36}$ must hold.

Ad (b): Note that we must have $p^C_{h,c} \leq 1 - \frac{\tau}{36}$. This implies that $p^C_{1,c} > 1 - \frac{\tau}{36}$ must hold to make sure that the market between the outside firm and one of the two local units will indeed not be covered. The maximization problem can be written as $\max\limits_{p^C_{1,c}, p^C_{h,c}} \Pi^C_{c\,[\text{ii}]} := (\frac{2p^C_{1,c}(1-p^C_{1,c})}{\tau} - \pi^N_{1,c})(2p^C_{h,c}(\frac{1}{6} + \sqrt{\frac{1-p^C_{h,c}}{\tau}}) - \pi^N_{h,c})$. Thus, we get $\frac{\partial \Pi^C_{c\,[\text{ii}]}}{\partial p^C_{1,c}} = (-\frac{3p^C_{1,c}-2}{\sqrt{\tau(1-p^C_{1,c})}})(2p^C_{h,c}(\frac{1}{6} + \sqrt{\frac{1-p^C_{h,c}}{\tau}}) - \pi^N_{h,c})$. Again, $\frac{\partial \Pi^C_{c\,[\text{ii}]}}{\partial p^C_{1,c}} > 0$ is only true if $p^C_{1,c} < \frac{2}{3}$ which is not compatible with the condition that we must have $p^C_{1,c} > 1 - \frac{\tau}{36}$.

Ad (c): Note that we must have $p^C_{h,c} > 1 - \frac{\tau}{36}$ which implies that $p^C_{1,c} < 1 - \frac{\tau}{36}$ must hold. We then face the following maximization problem: $\max\limits_{p^C_{1,c}, p^C_{h,c}} \Pi^C_{c\,[\text{iii}]} := (2p^C_{1,c}(\frac{1}{6} -$

48 Note that $E \geq 0 \, \forall \tau \in (0, \bar{\bar{\tau}}]$.

$\frac{3p^C_{1,c}-3p^C_{h,c}}{2\tau}) - \pi^N_{1,c})(2p^C_{h,c}(\sqrt{\frac{1-p^C_{h,c}}{\tau}} + \frac{1}{6} + \frac{3p^C_{1,c}-3p^C_{h,c}}{2\tau}) - \pi^N_{h,c})$. We start by differentiating with respect to $p^C_{1,c}$ and find that $\frac{\partial \Pi^C_{c\,[\text{iii}]}}{\partial p^C_{1,c}} = 0$ and $\frac{\partial^2 \Pi^C_{c\,[\text{iii}]}}{\partial {p^C_{1,c}}^2} < 0$ hold for some solution denoted ${p^C_{1,c}}^*$. Next, we check whether an improvement is possible for any $p^C_{h,c} > 1 - \frac{\tau}{36}$. To this end, solve $\Pi^C_{c\,[\text{iii}]}\big|_{p^C_{1,c}={p^C_{1,c}}^*} > \Pi^C_c\big|_{p^C_{1,c}=\frac{23328-2592\tau+454\tau^2+\sqrt{E}}{972(36-\tau)},\, p^C_{h,c}=1-\frac{\tau}{36}}$ for $p^C_{h,c}$. We find that for any $\tau \leq \bar{\bar{\tau}}$, there is no solution to the problem which satisfies $p^C_{h,c} > 1 - \frac{\tau}{36}$. ■

4.5.2 Proof of *Lemma 4.1*

Proof Note first that for $\tau = \bar{\bar{\tau}}$, it is true that $p^D_{1,c} < p^D_{h,c}$. Next, consider $\tau < \bar{\bar{\tau}}$: We look for a solution to $p^D_{1,c} = p^D_{h,c}$ with respect to τ in the three relevant regions defined by the deviating prices, i.e. (a) $\tau \in (0, \hat{\tau}]$, (b) $\tau \in (\hat{\tau}, \check{\tau}]$, and (c) $\tau \in (\check{\tau}, \bar{\bar{\tau}}]$. In cases (a) and (c), solving $p^D_{1,c} = p^D_{h,c}$ for τ does not give a solution at all. Turning to case (b), we find that $p^D_{1,c} = p^D_{h,c}$ only holds for $\tau \in \{-0.92087, 2.49001, 9.90141\}$ which contradicts the condition that $\tau \in (\hat{\tau}, \check{\tau}]$. As the deviating prices are continuous, we can thus conclude that $p^D_{1,c} < p^D_{h,c}$ must hold for any $\tau \in (0, \bar{\bar{\tau}}]$. ■

4.5.3 Proof of *Lemma 4.2*

Proof For $\tau = \bar{\bar{\tau}}$, it holds that $\bar{\delta}_{h,c} < \bar{\delta}_{i,d}$. Now consider $\tau < \bar{\bar{\tau}}$: We check whether a solution to $\bar{\delta}_{h,c} = \bar{\delta}_{i,d}$ with respect to τ exists in the three relevant regions defined by the deviating prices (see proof of *Lemma 4.1*). In case (a), we find that $\bar{\delta}_{h,c} = \bar{\delta}_{i,d} \Leftrightarrow \tau \in \{2.76923, 4.43725\}$ which contradicts the condition that $\tau \in (0, \hat{\tau}]$. Proceeding with case (b), one gets $\bar{\delta}_{h,c} = \bar{\delta}_{i,d}$ only if $\tau \in \{2.16384, 9.66200\}$ which conflicts with the condition that $\tau \in (\hat{\tau}, \check{\tau}]$. Finally, in case (c), we find that $\bar{\delta}_{h,c} = \bar{\delta}_{i,d}$ is true only if $\tau \in \{-5.08691, -1.58949\}$ which contradicts the condition that $\tau \in (\check{\tau}, \bar{\bar{\tau}}]$. As the discount factors are continuous, we know that $\bar{\delta}_{h,c} < \bar{\delta}_{i,d}$ must hold for any $\tau \in (0, \bar{\bar{\tau}}]$. ■

4.5.4 Proof of *Lemma 4.3*

Proof We find that if $\tau = \bar{\bar{\tau}}$, then $\bar{\delta}_{1,c} > \bar{\delta}_{h,c}$. Turning to $\tau < \bar{\bar{\tau}}$, we need to analyze if a solution to $\bar{\delta}_{1,c} = \bar{\delta}_{h,c}$ with respect to τ exists in the three relevant regions (see proof of *Lemma 4.1*). For case (a), one gets $\bar{\delta}_{1,c} = \bar{\delta}_{h,c}$ for $\tau \in \{0, 2.54409, 4.69565, 8.67054\}$ which conflicts with the condition that $\tau \in (0, \hat{\tau}]$. In case (b), we find that $\bar{\delta}_{1,c} = \bar{\delta}_{h,c}$ is true only if $\tau \in \{-7.75837, 1.64988, 9.84511\}$ which contradicts the condition that $\tau \in (\hat{\tau}, \check{\tau}]$. Finally, in case (c), solving $\bar{\delta}_{1,c} = \bar{\delta}_{h,c}$ for τ does not give any solution. Since the discount factors are continuous, we can therefore conclude that $\bar{\delta}_{1,c} > \bar{\delta}_{h,c}$ must hold for any $\tau \in (0, \bar{\bar{\tau}}]$. ∎

4.5.5 Proof of *Proposition 4.2*

Proof For $\tau = \bar{\bar{\tau}}$, $\bar{\delta}_{1,c} > \bar{\delta}_{i,d}$ is true. Proceeding with the analysis of the situation where $\tau < \bar{\bar{\tau}}$, we look for a solution to $\bar{\delta}_{1,c} = \bar{\delta}_{i,d}$ with respect to τ in the two relevant regions defined by the deviating prices, i.e. (a) $\tau \in (0, \hat{\tau}]$ and (b) $\tau \in (\hat{\tau}, \bar{\bar{\tau}}]$. In case (a), we find that $\bar{\delta}_{1,c} = \bar{\delta}_{i,d}$ only holds if $\tau \in \{-24.77747, 2.51752\}$ which contradicts the condition that $\tau \in (0, \hat{\tau}]$. Turning to case (b), we find that $\bar{\delta}_{1,c} = \bar{\delta}_{i,d}$ holds for $\tau \approx 3.04168 =: \tilde{\bar{\tau}}$. As the discount factors are continuous, the proposition follows. ∎

4.5.6 Proof of *Proposition 4.3*

Proof The proof is divided into two parts: We (a) complete the analysis for the situation with decentralization and (b) analyze the centralization case.

Ad (a): Consider the decentralization case where $\tau \leq \hat{\tau}$. If the collusive price has to be adjusted downwards, a deviating firm may no longer try to seize all of the market. If it still does, it will set a deviating price of $\frac{1}{6} - \frac{3p_{i,d}^D - 3p_{i,d}^C}{2\tau} = \frac{1}{2} \Leftrightarrow p_{i,d}^D = p_{i,d}^C - \frac{2\tau}{9}$. Given this deviating price, the following condition must hold for collusion to be sustainable: $\frac{p_{i,d}^C}{3(1-\delta)} \geq p_{i,d}^C - \frac{2\tau}{9} + \frac{\delta\pi_{i,d}^N}{1-\delta} \Leftrightarrow p_{i,d}^C(\delta) \leq \frac{\tau(6-7\delta)}{9(2-3\delta)}$. Given these two prices and the resulting profits from deviation, we can summarize the adjusted collusive prices under decentralization as follows:[49]

$$p_{i,d}^{C}(\delta)=\begin{cases}\frac{\tau(6-7\delta)}{9(2-3\delta)} & \text{if } \tau\leq\hat{\tau}\wedge\frac{1}{2}\leq\delta\leq\bar{\delta}_{i,d}\\ \frac{\tau(1+3\delta)}{9(1-\delta)} & \text{else.}\end{cases}\tag{4.25}$$

Ad (b): We prove that a solution to the maximization problem exists.[50] The maximization problem under centralized decision making is given by $\max\limits_{p_{1,c}^{C},p_{h,c}^{C}}\Pi_{c}^{C}$ subject to $\frac{2p_{1,c}^{C}(\frac{1}{6}-\frac{p_{1,c}^{C}-p_{h,c}^{C}}{2\tau})}{1-\delta}\geq\frac{(9p_{h,c}^{C}+\tau)^2}{108\tau}+\frac{\delta\pi_{1,c}^{N}}{1-\delta}$ and $\frac{2p_{h,c}^{C}(\frac{1}{3}-\frac{p_{h,c}^{C}-p_{1,c}^{C}}{2\tau})}{1-\delta}\geq\frac{(9p_{1,c}^{C}+2\tau)^2}{108\tau}+\frac{\delta\pi_{h,c}^{N}}{1-\delta}$ if a deviating firm does not want to cover the whole market. If it does, however, the incentive constraints change to $\frac{2p_{1,c}^{C}(\frac{1}{6}-\frac{p_{1,c}^{C}-p_{h,c}^{C}}{2\tau})}{1-\delta}\geq p_{h,c}^{C}-\frac{2\tau}{9}+\frac{\delta\pi_{1,c}^{N}}{1-\delta}$ and $\frac{2p_{h,c}^{C}(\frac{1}{3}-\frac{p_{h,c}^{C}-p_{1,c}^{C}}{2\tau})}{1-\delta}\geq p_{1,c}^{C}-\frac{2\tau}{9}+\frac{\delta\pi_{h,c}^{N}}{1-\delta}$. According to the Weierstrass Theorem, a solution to the maximization problem exists if (i) Π_{c}^{C} is continuous on its domain and (ii) if the (non-empty) set of inequality constraints is compact (i.e. closed and bounded).[51] Clearly, (i) is satisfied. To check the second condition, we need to verify that the relevant inequalities can be satisfied at the same time. This, however, is always the case as any incentive constraint is satisfied if $p_{1,c}^{C}=p_{1,c}^{N}$ and $p_{h,c}^{C}=p_{h,c}^{N}$. Hence, the lowest collusive profit is equal to the competitive profit.

As $\pi_{i,d}^{C}(\delta)\leq\pi_{h,c}^{N}\Leftrightarrow\delta\leq\frac{4}{13}$, δ^{*} exists. From *Proposition 4.1* we know that $\pi_{h,c}^{C}<\pi_{h,d}^{C}$ if δ is large. Hence, δ^{**} exists. ■

49 Note that $\bar{\delta}_{i,d}\lesseqgtr\frac{1}{2}\Leftrightarrow\tau\gtreqless\hat{\tau}$.

50 Due to the complexities of the nonlinear optimization problem in the case of centralized price setting, we cannot derive the explicit expressions for the adjusted collusive prices.

51 See, e.g., Sundaram (1996), chapter 3.

5 The double auction with inequity aversion

We analyze the double auction with two-sided incomplete information under the assumption that the parties are inequity averse. We show that if compassion is sufficiently strong, an efficient equilibrium exists, i.e. all gains of trade are realized. For the case where compassion is not as strong, trade may break down completely in the limit of infinitely strong envy. The analysis shows that bids in a separating equilibrium are further away from (closer to) truth-telling, the greater the importance of envy (compassion). Moreover, pooling equilibria are shown to be always more inefficient compared to the case without inequity aversion.

5.1 Introduction

The double auction as a simple model of bargaining with two-sided incomplete information was first analyzed by Chatterjee and Samuelson (1983). In a double auction, a seller and a buyer independently submit a bid. If the price submitted by the seller is smaller than the buyer's bid, trade will take place. If the opposite is true, no trade will occur. Chatterjee and Samuelson (1983) show that a pricing strategy which is linear in costs (for the seller) or valuation (for the buyer) is an equilibrium among others. More generally, Myerson and Satterthwaite (1983) show for uniformly distributed costs and valuations that the linear equilibrium in the double auction is the most efficient mechanism which, however, is ex post inefficient. Inefficiency arises because both the seller as well as the buyer tend to overstate their costs and understate their valuation, respectively, in order to get a better deal. Contrary to that, experimental evidence (Rapoport and Fuller, 1995) suggests that subjects seem to come to a more efficient outcome than theory would predict. We take this as an indication that people do not behave purely selfish as assumed by theory but take into account aspects other than price as well. In this article, we investigate what happens in a double auction with two-sided asymmetric information if people are averse toward inequality.

The basis of our analysis is the approach by Fehr and Schmidt (1999).[52] In their model, the utility of a person decreases whenever the final split is unfair. Their model of inequity aversion is made up of two parts: an envy as well as a compassion factor. Applying this concept to the double-auction framework, we show that if compassion is sufficiently important, then there exists an equilibrium in the double auction which is efficient, i.e. all gains of trade are realized. However, if compassion plays a less crucial role and if envy is very strong, we show that in the limit, trade breaks down. If the envy factor is not as strong, then—different from the case without inequity aversion—a symmetric equilibrium in linear strategies does not exist if costs and valuation are uniformly distributed on the unit interval. For this case, we analyze the symmetric equilibrium numerically and find that an increase in the envy factor leads to optimal bidding strategies which are further away from truth-telling. The opposite is true for an increase in the compassion factor. As a result, the lower (higher) the degree of envy (compassion), the more efficient the bargaining outcome and vice versa.

The reasoning for the truth-telling equilibrium is as follows: If a seller expects the buyer to bid a price equal to his valuation, then by also revealing her costs truthfully, the resulting price lies halfway between costs and valuation if trade occurs. As a consequence, both parties are equally well off. If the seller were to overstate her costs, this might lead to a better price from her point of view but at the same time this renders the final allocation more unequal. With compassion being sufficiently

52 For an alternative approach to modeling fair-minded behavior, see Bolton and Ockenfels (2000).

strong, the seller then prefers to state her costs truthfully.

The result where trade breaks down completely can be explained as follows: If there is no fair allocation of the surplus from trade, then there are types that will be worse off with strictly positive probability if trade occurs. Hence, the more crucial envy, the less attractive it will be for the types who are worse off to participate in trading.

We also show that another type of equilibrium, the pooling (step-function or price) equilibrium, becomes less efficient if both sides are inequity averse. In a partial-pooling equilibrium, both parties either agree to trade at the equilibrium price or, if at least one party refuses to trade, no trade will take place. In this type of equilibrium, an inequity-averse seller whose costs are close to the price might reason as follows: By accepting the price, trade will take place with some probability and some gains from trade will occur. On the buyer side, however, those buyers with a very high valuation are most likely to trade at this price. Thus, in expectation the seller will be worse off than her counterpart in case of trade which reduces the seller's utility from trading if envy is important. As a consequence, it might be better for the seller not to accept the price. The effect here works similar to the well-known lemons' model by Akerlof (1970): Sellers with costs close to the price—just like buyers with a valuation close to this price—do not want to trade. Thus the average seller (buyer) that accepts the deal has even lower costs (an even higher valuation) which makes the final outcome potentially less attractive for the other side. Similar to the market breakdown in the lemons' model, we show that there exists a minimum and maximum price strictly larger than zero and strictly smaller than one (for the uniform distribution on the unit interval) such that no trade occurs if prices are below (above) the minimum (maximum) price.

There exists some theoretical work on bargaining where inequity aversion is considered. The articles differ from the present work with respect to both the bargaining setup and the informational assumptions. The existing literature either analyzes the ultimatum game (see, e.g., Fehr and Schmidt, 1999) or deals with two-stage Stahl

(1972)-type bargaining or bargaining in an infinite-horizon model (Lopomo and Ok, 2001; Ewerhart, 2006). To the best of our knowledge, there is no paper that analyzes the double auction. If informational asymmetries are considered, they usually relate to the degree of inequity aversion (see Fehr and Schmidt, 1999; Lopomo and Ok, 2001; Ewerhart, 2006). Informational asymmetries concerning costs and valuations have not been considered so far.

In many experimental studies, it is shown that players do not behave as rational as predicted by theory. Especially experiments capturing the ultimatum game, a straightforward bargaining game, provide some evidence that players bear fairness considerations in mind. A strong result in these experiments is that individuals reject inequitable divisions although the outcome is profitable to them.[53] There are only a few experiments which deal with the double auction. In their experiment, Rapoport and Fuller (1995) report that although the players bid strategically, their behavior does not fully match standard theoretical predictions, i.e. it does not completely reflect the linear equilibrium strategy. Rapoport and Fuller (1995) find a tendency to bid toward a truthful revelation. Radner and Schotter (1989) provide some evidence that while sellers tend to bid according to their predicted linear bidding strategy, the buyers deviate from the predicted linear bidding strategy toward truthful bidding.[54]

The organization of the chapter is as follows. In the next section, we develop the model which applies inequity aversion to the double-auction framework. In section 3, we investigate separating equilibria. In particular, we look at the truth-telling equilibrium and analyze whether there exist similar equilibria with a fair allocation of the gains from trade. We then look at more general equilibrium candidates for the case where envy becomes very important and present the results from a numerical simulation of a symmetric equilibrium if compassion is not very important. In section 4, we consider pooling equilibria. The fifth section discusses an extension where we allow for heterogeneous sellers and buyers. The sixth section concludes.

53 See, e.g., Roth (1995).
54 See also Daniel, Seale, and Rapoport (1998).

5.2 The model

We model inequity aversion in a two-person bargaining problem with two-sided incomplete information. The seller produces a good the buyer wants to buy. We denote by c the costs the seller has to incur when producing the good and by v the value of the object for the buyer. Both costs and valuations are assumed as independent random variables, distributed over a given interval $c \in [\underline{c}, \bar{c}]$ and $v \in [\underline{v}, \bar{v}]$ with $\underline{c} < \bar{v}$. $f(v)$ and $g(c)$ are the respective probability density functions which are assumed continuous and positive on their domains. The cumulative distribution functions corresponding to $f(c)$ and $g(v)$ are $F(c)$ and $G(v)$.[55] Each party has private information about their reservation price and considers the other side's reservation price as a random variable distributed as above.

Double auctions are a simple way to capture the bargaining situation between the two parties. In this framework, both sides simultaneously bid a price, denoted by b_S and b_B for the seller and the buyer, respectively. If $b_S \leq b_B$, then trade takes place at a price $p = \frac{b_S + b_B}{2}$, i.e. the difference between the two bids is assumed to be split equally.[56] If $b_S > b_B$, there is no trade.

Suppose the price is p and assume for the moment that costs and valuations are known to both sides. Following Fehr and Schmidt (1999), we model the utility function for the seller and the buyer in a double-auction environment allowing for inequity aversion as

$$U_S(c, v) = \begin{cases} p - c - \alpha \max\{0, -(p-c) + (v-p)\} & \\ \quad -\beta \max\{0, (p-c) - (v-p)\} & \text{if } b_S \leq b_B \text{ and} \\ 0 & \text{else} \end{cases} \tag{5.1}$$

55 For the analysis of a situation where the (privately known) valuations are affiliated, see Kadan (2007).

56 Hence, we consider a special case (i.e. $k = \frac{1}{2}$) of the general k-double auction where k represents the seller's bargaining position (with $k \in [0, 1]$).

and

$$U_B(c,v) = \begin{cases} v - p - \alpha \max\{0, -(v-p) + (p-c)\} & \\ \quad -\beta \max\{0, (v-p) - (p-c)\} & \text{if } b_S \leq b_B \text{ and} \\ 0 & \text{else.} \end{cases} \tag{5.2}$$

If trade takes place, the first two terms represent the gross gains of trade to the seller/buyer. The third and the fourth term reflect the disutility the parties receive from an unfair division of the surplus where the third term gives the disadvantage weighted with the envy factor α. The fourth term accounts for the advantage to the seller/buyer weighted with the compassion factor β. Furthermore, we assume that $\alpha \geq \beta$ and $0 \leq \beta \leq 1$.[57]

In what follows, we will analyze separating and pooling equilibria. We start with the separating equilibria.

5.3 Separating equilibria

In this section, we analyze a type of equilibrium where both parties bid according to a strictly increasing bidding strategy, i.e. where $b_S(c)', b_B(v)' > 0$.[58]

The first subsection looks at fair allocations. We then analyze non-linear equilibrium strategies numerically in the second subsection.

5.3.1 Fair allocations

This part of the analysis deals with fair allocations in the sense that neither envy nor compassion are relevant for both sides' equilibrium utility. More precisely, bidding strategies are characterized by $b_S(c) - c = v - b_B(v)$ and $b_S(c)' = b_B(v)' = 1$ for

57 For a discussion of these assumptions, see Fehr and Schmidt (1999).

58 More precisely, we consider bidding strategies that are strictly increasing for those types who trade with positive probability.

all types who actually trade with positive probability. Since we consider inequity-averse agents, these equilibrium candidates where the gains of trade are allocated fairly seem to be worth a closer look.

The most appealing equilibrium from an efficiency point of view clearly is the equilibrium where both sides reveal their types truthfully. Such a bidding behavior implies that trade occurs whenever the seller's costs are equal to or smaller than the buyer's valuation. As the following proposition shows, such an equilibrium exists for general distributions of costs and valuations if compassion plays a crucial role in a double-auction environment.

Proposition 5.1 *If $\beta \geq \frac{1}{2}$, there exists an efficient equilibrium in the double auction where the two parties bid according to $b_S(c) = c$ and $b_B(v) = v$.*

Proof We prove the proposition for the seller only. The case for the buyer can be analyzed in an analogous fashion and is therefore omitted. Assume that the buyer sticks to the truth-telling strategy, i.e. $b_B(v) = v$. Consider the case where the seller submits a price $s = c + \epsilon > c$. If trade occurs, this will lead to a price which is larger than $\frac{c+v}{2}$. Therefore, we only need to take into account the advantage term with the parameter β in the expected-utility function. Trade occurs whenever $v \geq c + \epsilon$. Hence, the expected utility of a seller with costs c who submits a price $s = c + \epsilon$, which we denote by $\mathrm{E}[U_S(c, c + \epsilon)]$, is given by:

$$\begin{aligned}
\mathrm{E}[U_S(c, c+\epsilon)] &= \int_{c+\epsilon}^{\bar{v}} \left(\frac{c+v+\epsilon}{2} - c \right) g(v)dv \\
&\quad - \beta \int_{c+\epsilon}^{\bar{v}} \left(\left(\frac{c+v+\epsilon}{2} - c \right) - \left(v - \frac{c+v+\epsilon}{2} \right) \right) g(v)dv \\
&= \int_{c+\epsilon}^{\bar{v}} \left(\frac{c+v+\epsilon}{2} - c - \beta\epsilon \right) g(v)dv \qquad (5.3) \\
&= \frac{1}{2} \int_{c+\epsilon}^{\bar{v}} v g(v)dv - \frac{c}{2}(1 - G(c+\epsilon)) \\
&\quad + \epsilon \left(\frac{1}{2} - \beta \right) (1 - G(c+\epsilon)).
\end{aligned}$$

In order to give the seller an incentive to bid c, it must hold that $\mathrm{E}[U_S(c,c)] \geq \mathrm{E}[U_S(c,c+\epsilon)]\ \forall\epsilon > 0$. Taking the derivative of $\mathrm{E}[U_S(c,c+\epsilon)]$ with respect to ϵ gives

$$\begin{aligned}\frac{\partial \mathrm{E}[U_S(c,c+\epsilon)]}{\partial\epsilon} &= -\frac{1}{2}(c+\epsilon)g(c+\epsilon) + \frac{c}{2}g(c+\epsilon) \\ &\quad + \left(\frac{1}{2}-\beta\right)(1-G(c+\epsilon)) - \epsilon\left(\frac{1}{2}-\beta\right)g(c+\epsilon) \\ &= \left(\frac{1}{2}-\beta\right)(1-G(c+\epsilon)) - \epsilon\,(1-\beta)\,g(c+\epsilon).\end{aligned} \tag{5.4}$$

For truth-telling to be optimal, this expression has to be smaller than zero for any value of ϵ. Since $1 - G(c+\epsilon) \geq 0$, the seller has no incentive to bid more than c whenever $\beta \geq \frac{1}{2}$.

Now consider the case where $s = c - \epsilon < c$. Under the split-the-difference rule, claiming to have lower costs not only leads to a reduction in the seller's payoff by passing more to the buyer but also yields additional disutility by favoring the buyer. Therefore, the seller would never claim to have lower costs. ■

The intuition behind the truth-telling equilibrium is as follows: Consider the seller side. If a seller expecting the buyer to submit a bid equal to his valuation reveals her true costs, the resulting price lies halfway between costs and valuation if trade occurs. Under these circumstances, both parties are equally well off for any value of both costs and valuation. If, on the other hand, the seller asks for a price that lies above her actual costs, it is true that this may lead to a better price from her point of view. At the same time, however, this renders the final bargaining outcome less equal. Now if compassion is of sufficiently great importance, the seller would rather tell the truth concerning her costs.[59]

We next concentrate on further equilibria where both sides bid according to a strategy with slope 1 with the same markup Δ, i.e. where $b_S(c) = c + \Delta$ and $b_B(v) = v - \Delta$. To this end, we make the following two assumptions:

59 In their setup, Chatterjee and Samuelson (1983) show that truth-telling is also an equilibrium if both sides are very (infinitely) risk averse.

Assumption 5.1 *The reverse hazard rate, defined as* $\phi(c) := \frac{f(c)}{F(c)}$*, has the following properties*

(a) $\dfrac{1}{\phi(\underline{c})} = 0$ *and*

(b) $\phi'(c) \leq 0$.

Assumption 5.2 *The hazard rate, defined as* $\gamma(v) := \frac{g(v)}{1-G(v)}$*, has the following properties*

(a) $\dfrac{1}{\gamma(\bar{v})} = 0$ *and*

(b) $\gamma'(v) \geq 0$.

Given these assumptions[60], we can establish the following result:

Proposition 5.2 *Consider the class of bidding strategies where* $b_S(c) = c + \Delta$ *and* $b_B(v) = v - \Delta$*. For this class, it holds that*

(a) *if* $\underline{v} \leq \underline{c}$ *and/or* $\bar{v} \leq \bar{c}$*, then*

  (i) *there exists no such equilibrium if* $\beta < \frac{1}{2}$ *and*

  (ii) $\Delta = 0$ *is the unique equilibrium if* $\beta \geq \frac{1}{2}$*;*

(b) *if* $\underline{c} < \underline{v}$ *and* $\bar{c} < \bar{v}$*, then*

  (i) *there may exist equilibria with* $\Delta > 0$ *if* $\beta < \frac{1}{2}$ *and*

  (ii) *there exist equilibria with* $\Delta < 0$ *and* $\Delta > 0$ *if* $\beta \geq \frac{1}{2}$.

Proof Ad (a): We consider cases (i) and (ii) together. Note that this part of the proposition is to hold for three different scenarios with respect to the intervals on which c and v are distributed: 1. $\underline{v} \leq \underline{c} < \bar{v} \leq \bar{c}$, 2. $\underline{v} \leq \underline{c} < \bar{c} \leq \bar{v}$, and 3. $\underline{c} \leq \underline{v} < \bar{v} \leq \bar{c}$.

60 The assumptions are slightly more restrictive than the ones in Myerson and Satterthwaite (1983) and Satterthwaite and Williams (1989) who assume the so-called *virtual reservation values* (defined as $c + \frac{1}{\phi(c)}$ and $v - \frac{1}{\gamma(v)}$, respectively) to be increasing in c and v. The properties of the derivatives assumed here are sufficient conditions for this to hold (see also Krishna, 2002, p. 69).

We start with the seller's incentive to deviate. To this end, assume that the buyer submits a bid equal to $v - \Delta$. Note that $\Delta < 0$ cannot be part of a (symmetric) equilibrium. To see this, take scenario 1. and consider the seller with the highest costs for which there is trade denoted by $\hat{c} \leq \bar{c}$. This seller only trades with the buyer of type $\bar{v}$ $(= \hat{c} + 2\Delta)$. As both sides submit the same bid, the resulting price equals these bids, i.e. $p = b_S(\hat{c}) = b_B(\bar{v}) = \hat{c} + \Delta < \hat{c}$. Since this price does not cover the seller's costs, i.e. $\mathrm{E}[U_S(\hat{c}, \hat{c} + \Delta)] < 0$ in the case of trade, the seller would prefer not to trade. The same argument holds for case 3. An analogous reasoning for the buyer applies in cases 1. and 2.

Turning to the case where $\Delta \geq 0$, consider first the situation where the seller submits a price $s = c + \Delta - \epsilon < c + \Delta$. If trade occurs, this will lead to a price which is smaller than $\frac{c+v}{2}$. Hence, only the disadvantage term with the parameter α is relevant in the expected-utility function. Trade occurs whenever $v \geq c + 2\Delta - \epsilon$. Then, the expected utility of a seller with costs c who submits a price $s = c + \Delta - \epsilon$, denoted by $\mathrm{E}[U_S(c, c + \Delta - \epsilon)]$, is given by

$$\begin{aligned} \mathrm{E}[U_S(c, c + \Delta - \epsilon)] &= \int_{c+2\Delta-\epsilon}^{\bar{v}} \left(\frac{c + v - \epsilon}{2} - c - \alpha\epsilon \right) g(v)dv \\ &= \frac{1}{2} \int_{c+2\Delta-\epsilon}^{\bar{v}} vg(v)dv - \frac{c}{2}(1 - G(c + 2\Delta - \epsilon)) \\ &\quad - \epsilon \left(\frac{1}{2} + \alpha \right) (1 - G(c + 2\Delta - \epsilon)). \end{aligned} \tag{5.5}$$

In order to give the seller an incentive to bid $c+\Delta$, it must hold that $\mathrm{E}[U_S(c, c+\Delta)] \geq \mathrm{E}[U_S(c, c + \Delta - \epsilon)]$ $\forall \epsilon > 0$. Taking the derivative of $\mathrm{E}[U_S(c, c + \Delta - \epsilon)]$ with respect to ϵ yields

$$\begin{aligned} \frac{\partial \mathrm{E}[U_S(c, c + \Delta - \epsilon)]}{\partial \epsilon} &= - \left(\frac{1}{2} + \alpha \right) (1 - G(c + 2\Delta - \epsilon)) \\ &\quad - \epsilon(1 + \alpha)g(c + 2\Delta - \epsilon) + \Delta g(c + 2\Delta - \epsilon). \end{aligned} \tag{5.6}$$

Performing a similar analysis on the buyer side gives

$$\begin{aligned}\frac{\partial \mathrm{E}[U_B(v, v-\Delta+\epsilon)]}{\partial \epsilon} = & -\left(\frac{1}{2}+\alpha\right) F(v-2\Delta+\epsilon) \\ & - \epsilon(1+\alpha) f(v-2\Delta+\epsilon) + \Delta f(v-2\Delta+\epsilon).\end{aligned} \tag{5.7}$$

If the proposed bidding behavior is to be optimal for each side, the above derivatives have to be smaller than or equal to 0. Given *Assumptions 5.1* and *5.2*, we can conclude that expressions (5.6) and (5.7) divided by $g(c+2\Delta-\epsilon)$ and $f(v-2\Delta+\epsilon)$, respectively, decrease in ϵ. We therefore only have to consider the case where $\epsilon \to 0$. The derivatives (5.6) and (5.7) must also hold in the most critical cases, i.e. for the highest value of c and for the lowest value of v for which trade occurs.[61] At this value, the seller (buyer) will only trade with a buyer (seller) with the highest valuation $\bar{v}$ (lowest cost $\underline{c}$) such that $G(\bar{v}) = 1$ ($F(\underline{c}) = 0$) becomes relevant. Given that $\epsilon \to 0$ and $G(\bar{v}) = 1$ ($F(\underline{c}) = 0$), the first two terms of derivative (5.6) (derivative 5.7) are equal to 0. We can thus conclude that only $\Delta = 0$ can be part of an equilibrium. Note that this is true independent of the value of α.

Ad (b): In this situation, the most critical cases do not imply $G(v) = 1$ ($F(c) = 0$).[62] We relegate the proof to the appendix where we derive the lower and upper bounds for Δ. We give an example below. ■

If compassion is strong enough, then for boundaries of the intervals as laid out in part (a) of the proposition, truth-telling is the unique equilibrium in the class of equilibrium candidates where the parties bid according to a strategy with slope 1.

Note that part (b) of the proposition implies that an equilibrium with $\Delta > 0$ may exist even in a situation where $\beta < \frac{1}{2}$. Clearly, these equilibria are less efficient than the truth-telling equilibrium as high costs and low valuations do not necessarily trade. Interestingly, if compassion is important, there are even equilibria where the seller (buyer) underbids her costs (overbids his valuation). By doing so, both sides

61 See also *Figures 5.18–5.20* in the appendix.
62 See *Figure 5.21* in the appendix.

extend the scope of trade. The derivation of the exact conditions for these equilibria to exist is relegated to the appendix. To illustrate these findings, we consider the following example instead:

Example for the case where $\underline{c} < \underline{v}$ and $\bar{c} < \bar{v}$. *Suppose c is uniformly distributed on the $[0, \frac{3}{4}]$ interval and let v be uniformly distributed on $[\frac{1}{4}, 1]$. Clearly,* Assumptions 5.1 *and* 5.2 *are satisfied.*

Since a seller (buyer) must not have an incentive to submit a lower (higher) bid (see also inequalities (5.26) and (5.27)), we have

$$-\left(\frac{1}{2}+\alpha\right)\frac{16(1-c-2\Delta)}{9}+\Delta \leq 0 \tag{5.8}$$

and

$$-\left(\frac{1}{2}+\alpha\right)\frac{16(v-2\Delta)}{9}+\Delta \leq 0. \tag{5.9}$$

The first (second) expression increases (decreases) in c (v), i.e. we must set $c = \bar{c} = \frac{3}{4}$ and $v = \underline{v} = \frac{1}{4}$, respectively, which in both cases gives an upper bound of

$$\Delta \leq \frac{2(1+2\alpha)}{25+32\alpha} =: \hat{\Delta}. \tag{5.10}$$

Note that this upper bound is always smaller than $\frac{\underline{v}-\underline{c}}{2} = \frac{\bar{v}-\bar{c}}{2} = \frac{1}{8}$ which is the condition that ensures that the seller with costs $\bar{c}$ (the buyer with valuation $\underline{v}$) indeed does not trade with the buyer with valuation $\bar{v}$ (the seller with costs $\underline{c}$). Note further that $\lim_{\alpha\to\infty}\hat{\Delta} = \frac{1}{8}$. Figure 5.1 *illustrates the upper bound $\hat{\Delta}$.*

Similarly, we need to make sure that the seller (buyer) does not find it profitable to submit a higher (lower) bid (see also inequalities (5.33) and (5.34)), i.e. it must hold that

$$\left(\frac{1}{2}-\beta\right)\frac{16(1-c-2\Delta)}{9}-\Delta \leq 0 \tag{5.11}$$

and

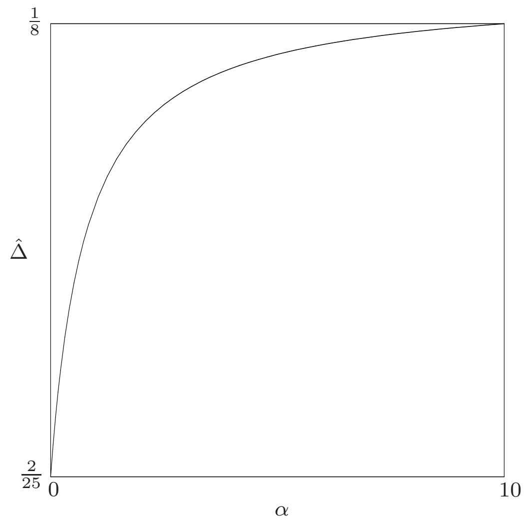

Figure 5.1: Impact of α on $\hat{\Delta}$

$$\left(\frac{1}{2} - \beta\right) \frac{16(v - 2\Delta)}{9} - \Delta \leq 0. \tag{5.12}$$

We now have to look at the following two cases: (a) $\beta < \frac{1}{2}$ *and (b)* $\beta \geq \frac{1}{2}$.

Ad (a): Since the left-hand side of inequality (5.11) (inequality (5.12)) decreases (increases) in c (v), we need to consider the lowest (highest) value of c (v) for which trade takes place. This cost (valuation) is equivalent to $\underline{v} - 2\Delta = \frac{1}{4} - 2\Delta$ $(\bar{c} + 2\Delta = \frac{3}{4} + 2\Delta)$*. As a result, the lower bound for the case where* $\beta < \frac{1}{2}$ *is then given by*

$$\Delta \geq \frac{2(1 - 2\beta)}{3} =: \check{\Delta}. \tag{5.13}$$

This lower bound, which is greater than 0, must not be greater than $\frac{\underline{v} - \underline{c}}{2} = \frac{1}{8}$ *either, i.e. we have* $\check{\Delta} \leq \frac{1}{8} \Leftrightarrow \beta \geq \frac{13}{32} = 0.40625$*. This is in accordance with the observation that we must have* $\check{\Delta} \leq \hat{\Delta} \Leftrightarrow \beta \geq \frac{11+13\alpha}{25+32\alpha} =: \tilde{\beta}$*. Note that* $\lim_{\alpha \to \infty} \tilde{\beta} = \frac{13}{32}$. Figure 5.2 *illustrates.*

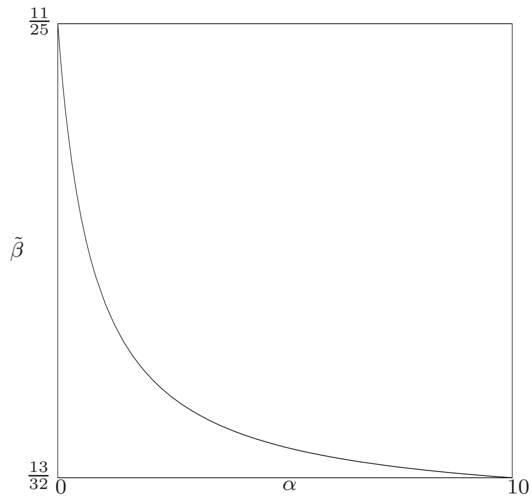

Figure 5.2: Impact of α on $\tilde{\beta}$

Result 5.1 If $\tilde{\beta} \leq \beta < \frac{1}{2}$ holds, there is a class of equilibria in the example where the seller and the buyer bid according to $b_S(c) = c + \Delta$ and $b_B(v) = v - \Delta$ with $\check{\Delta} \leq \Delta \leq \hat{\Delta}$.

Ad (b): Now the left-hand side of inequality (5.11) (inequality (5.12)) increases (decreases) in c (v). Therefore, we need to consider $c = \bar{c}$ and $v = \underline{v}$. The lower bound is then given by

$$\Delta \geq -\frac{2(2\beta - 1)}{25 - 32\beta} =: \check{\check{\Delta}}. \tag{5.14}$$

Obviously, $\check{\check{\Delta}} \leq 0$. Moreover, for the above analysis to be valid, it must hold that $\underline{v} - 2\Delta \leq \bar{c} \Leftrightarrow \Delta \geq -\frac{\bar{c}-\underline{v}}{2} = -\frac{1}{4}$. Last, we need to make sure that the seller with costs $\bar{c}$ has a positive expected utility:

$$E\left[U_S\left(\frac{3}{4}, \frac{3}{4} + \Delta\right)\right] = \frac{1}{2}\int_{\frac{3}{4}+2\Delta}^{1} \left(v - \frac{3}{4}\right) dv \geq 0 \Leftrightarrow \Delta \geq -\frac{1}{8}. \tag{5.15}$$

Hence, we get $\check{\check{\Delta}} \geq -\frac{1}{8} \Leftrightarrow \beta \leq \frac{41}{64} = 0.640625$.[63] Note further that for any $\frac{41}{64} < \beta \leq$

63 Note that $25\Delta - 32\beta\Delta \geq 0 \Leftrightarrow \beta \leq \frac{25}{32} = 0.78125$ which is relevant when deriving $\check{\check{\Delta}}$.

1, *the lower bound is given by* $-\frac{1}{8}$. Figure 5.3 *illustrates.*

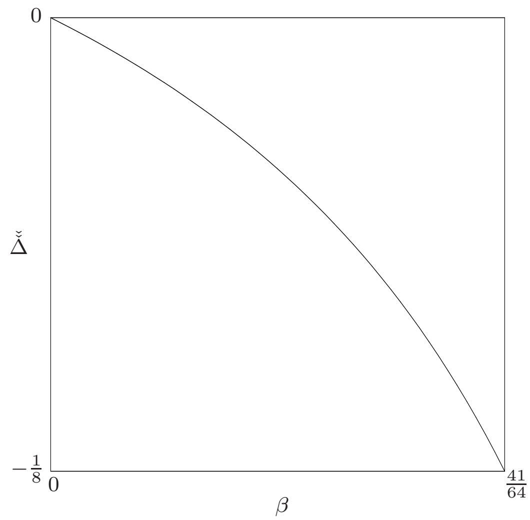

Figure 5.3: Impact of β on $\check{\check{\Delta}}$

Result 5.2 If $\frac{1}{2} \leq \beta \leq \frac{41}{64}$ holds, there is a class of equilibria in this example where the bidding strategies are given by $b_S(c) = c + \Delta$ and $b_B(v) = v - \Delta$ with $\check{\check{\Delta}} \leq \Delta \leq \hat{\Delta}$. In the case where $\frac{41}{64} < \beta \leq 1$, Δ must satisfy $-\frac{1}{8} \leq \Delta \leq \hat{\Delta}$.

Hence, we can conclude that in this example where $\underline{c} < \underline{v}$ *and* $\bar{c} < \bar{v}$, *there exist equilibria which are characterized by a below-cost (above-valuation) bidding strategy on the seller (buyer) side if compassion is strong enough.*

So far, we have considered the case where compassion is of great importance. In such a situation, fair allocations exist. The situation becomes very much different if compassion is weak and envy plays a crucial role.

Proposition 5.3 *Suppose that* $\underline{v} \leq \underline{c}$ *and/or* $\bar{v} \leq \bar{c}$ *and that* $\beta < \frac{1}{2}$. *Then, in the limit where* $\alpha \to \infty$, *trade occurs with probability 0 in equilibrium.*

Proof See the appendix. ■

The proposition therefore suggests a maximum of inefficiency: As envy becomes extremely important and if compassion is low, trade no longer takes place. This is rather intuitive: If a fair allocation is not possible, then one trading partner will be worse off than the other. Now as the importance of envy increases, the less attractive will be the trade outcome for the side that is worse off.

We next turn to non-linear bidding strategies.

5.3.2 Non-linear equilibrium strategies

Throughout this subsection, we assume a situation where c and v are uniformly distributed on the unit interval. As a benchmark, consider first the equilibrium without inequity aversion: Myerson and Satterthwaite (1983) show that in this case, the most efficient outcome in a double auction is the linear equilibrium strategy. To maximize the total gains from trade, the seller and the buyer bid according to a (piecewise) linear strategy. Between 0 and $\frac{3}{4}$, the seller overbids her true costs and from $\frac{3}{4}$ up to 1, the bidding strategy reflects the true costs. The buyer bids his true valuation from 0 up to $\frac{1}{4}$ and between $\frac{1}{4}$ and 1, the bid lies below the true valuation. More precisely, the bidding functions for the seller and the buyer are given by

$$b_S(c) = \begin{cases} \frac{1}{4} + \frac{2c}{3} & \text{if } c \leq \frac{3}{4} \\ c & \text{if } c > \frac{3}{4} \end{cases} \tag{5.16}$$

and

$$b_B(v) = \begin{cases} v & \text{if } v < \frac{1}{4} \\ \frac{1}{12} + \frac{2v}{3} & \text{if } v \geq \frac{1}{4}. \end{cases} \tag{5.17}$$

As a result, trade occurs if and only if $v \geq c + \frac{1}{4}$. The resulting trade region is shown in *Figure 5.4*. The dotted line represents the frontier above which trade is efficient since $v \geq c$.

Hence, the equilibrium in (symmetric) linear strategies for the benchmark case

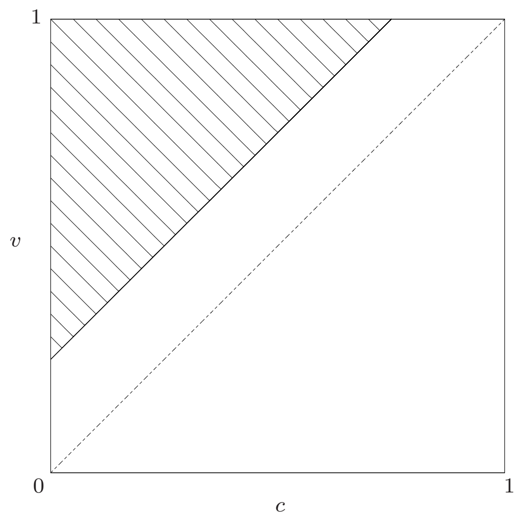

Figure 5.4: Trade region in a symmetric linear-strategy equilibrium without inequity aversion

without inequity aversion is very appealing from an efficiency point of view. We are therefore interested in whether there exists a symmetric equilibrium in linear strategies in the present setup as well. Symmetry implies that if a buyer bids according to $b_B(v) = \lambda + \mu v$, then in a symmetric equilibrium, a seller's best response is to submit a bid $b_S(c) = 1 - \lambda - \mu(1 - c)$ in the case of trade (see also the proof in the appendix).[64] For the case where the seller and the buyer are inequity averse, we establish the following result:

Lemma 5.1 *Suppose that c and v are uniformly distributed on $[0, 1]$ and that $\beta < \frac{1}{2}$. Then, a symmetric equilibrium where both parties bid according to linear strategies and where trade occurs does not exist in the double auction with inequity aversion.*

Proof See the appendix. ■

Note that if trade occurs in a symmetric equilibrium, there are types (high costs and low valuations, respectively) who are always worse off than their counterpart

64 More generally, in a symmetric equilibrium, the seller's best response to the buyer's strategy $b_B(v)$ is to bid $b_S(c) = 1 - b_B(1 - c)$. This observation is also used in the numerical simulation below.

(i.e. only envy is relevant for them) and there are types who may be better or worse off. One may argue that this should be accounted for in the linear strategy by allowing for a change in the bidding behavior for the different types. However, it can be shown that if both sides bid according to such a (symmetric) piecewise linear strategy, this cannot be an equilibrium either.

As an analytical solution is not tractable for the derivation of symmetric non-linear equilibrium strategies even in the relatively simple case of uniform distributions on the unit interval, we derive an equilibrium numerically. Given that we consider a special version of the double auction where the price is equal to the average bid and in line with the analysis above, we focus on a symmetric equilibrium. As before, we assume that both c and v are uniformly distributed on the unit interval. From the results derived so far, we know that in this case the equilibrium—if it exists—must consist of non-linear bidding strategies. The following figures depict both parties' symmetric equilibrium strategies for different degrees of inequity aversion as well as the resulting region where trade occurs: In the figures illustrating the equilibrium bidding strategies, the bold lines represent the equilibrium result under inequity aversion. As a comparison, the dotted lines give the linear strategies absent any inequity aversion as well as the truth-telling outcome. In the figures illustrating the scope of trade, the bold lines analogously depict the trade regions for the case in question. Also, the upper dotted lines represent the benchmark case without inequity aversion and the lower dotted lines give the efficient trading areas.

Compared with the situation where there is no inequity aversion, for low values of the cost (for a high valuation), the seller (buyer) bids less aggressively if envy is not too strong. This can be explained as follows. Consider the seller: If the costs for supplying the product are high, the seller bids more aggressively due to envy. Now if envy becomes more important, the bid will be even higher. On the other hand, if the costs are low, obtaining more trade through a lower bid is optimal. As the importance of compassion rises, the seller will bid even less aggressively in order to avoid being better off. This can be seen from *Figures 5.5–5.8* where we set $\alpha = \frac{1}{4}$

and $\beta = 0$ as well as $\alpha = \beta = \frac{1}{4}$.

In line with *Proposition 5.1*, *Figure 5.9* shows that if the compassion factor is close to $\frac{1}{2}$, then both sides' bidding strategies are indeed not far away from truth-telling. As a consequence, there is only a slight loss in efficiency (see *Figure 5.10*). In line with *Proposition 5.3*, *Figure 5.11* shows that as envy becomes very important, both sides bid very aggressively which results in lesser trade (see *Figure 5.12*).

From the results of the numerical simulation we can conclude that a higher envy factor and a lower compassion factor both result in reduced trade activity, i.e. envy renders the outcome less efficient while compassion leads to more (efficient) trade.

We next turn to pooling equilibria.

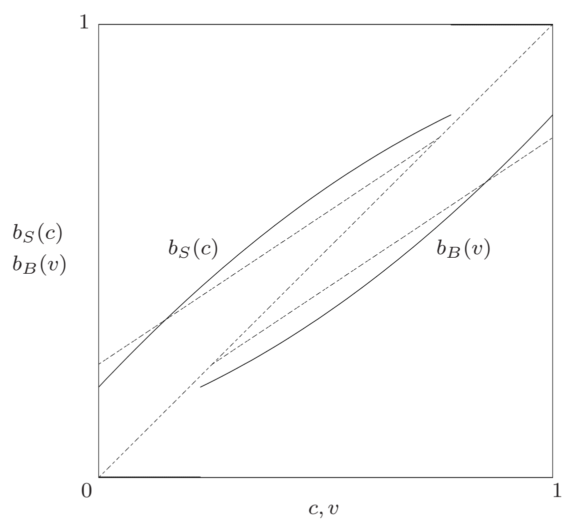

Figure 5.5: Optimal symmetric bidding strategies (for $\alpha = \frac{1}{4}$, $\beta = 0$)

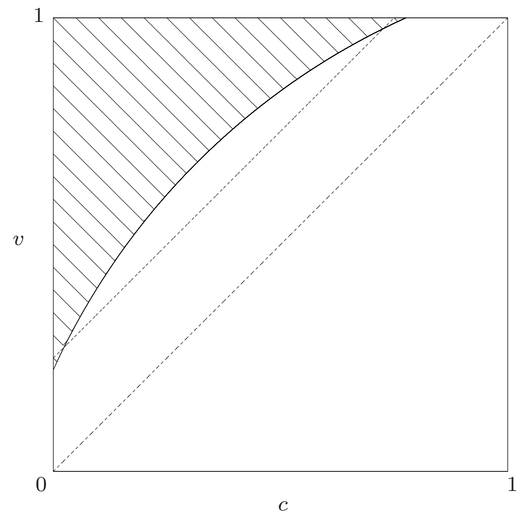

Figure 5.6: Trade region in a symmetric equilibrium (for $\alpha = \frac{1}{4}$, $\beta = 0$)

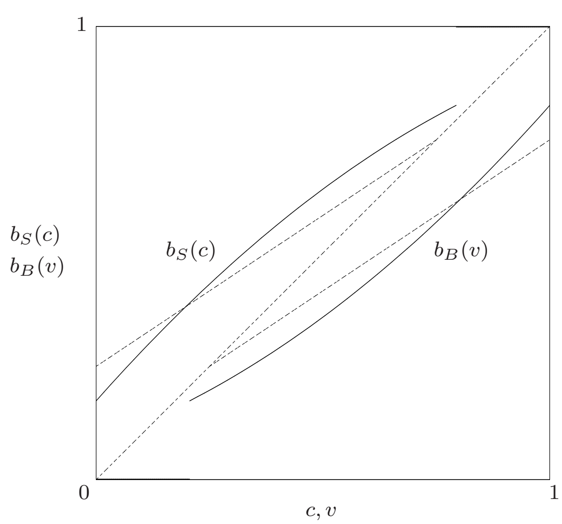

Figure 5.7: Optimal symmetric bidding strategies (for $\alpha = \beta = \frac{1}{4}$)

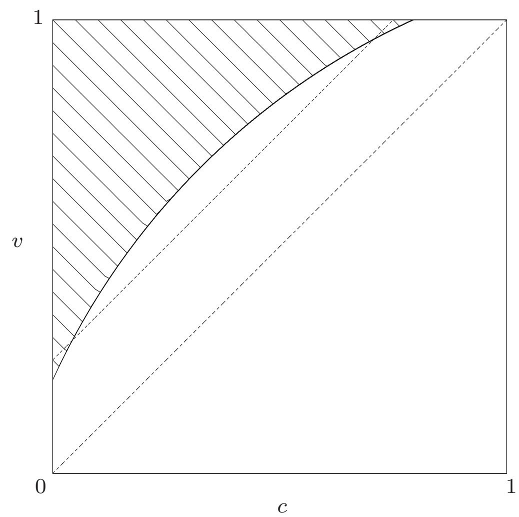

Figure 5.8: Trade region in a symmetric equilibrium (for $\alpha = \beta = \frac{1}{4}$)

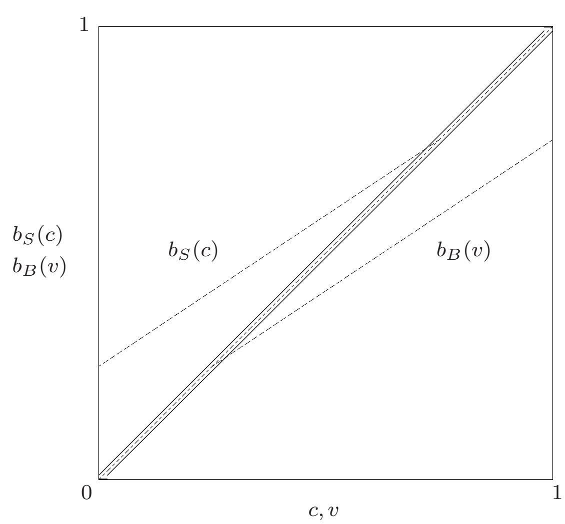

Figure 5.9: Optimal symmetric bidding strategies (for $\alpha = \frac{1}{2}$ and $\beta = 0.495$)

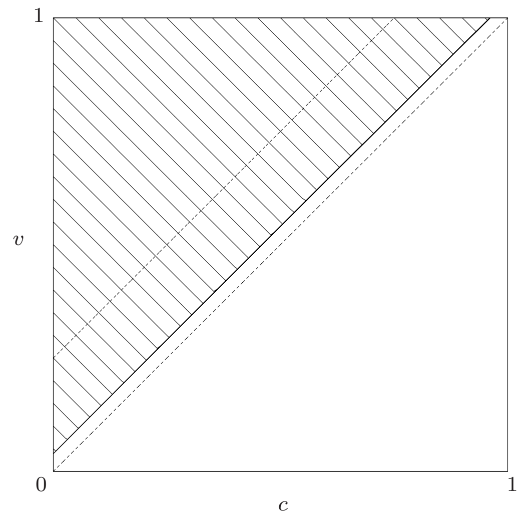

Figure 5.10: Trade region in a symmetric equilibrium (for $\alpha = \frac{1}{2}$ and $\beta = 0.495$)

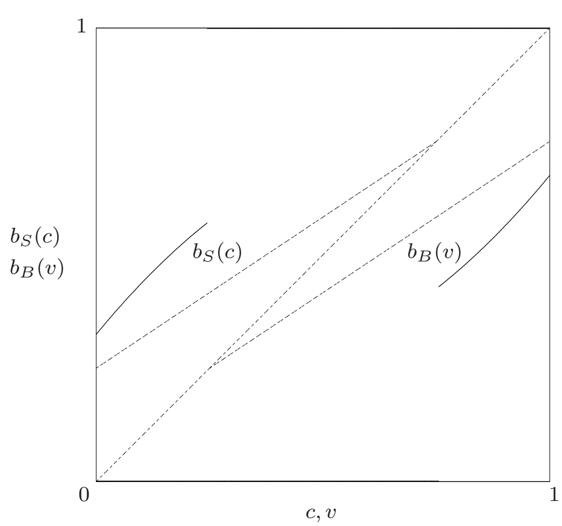

Figure 5.11: Optimal symmetric bidding strategies (for $\alpha = 10$, $\beta = 0$)

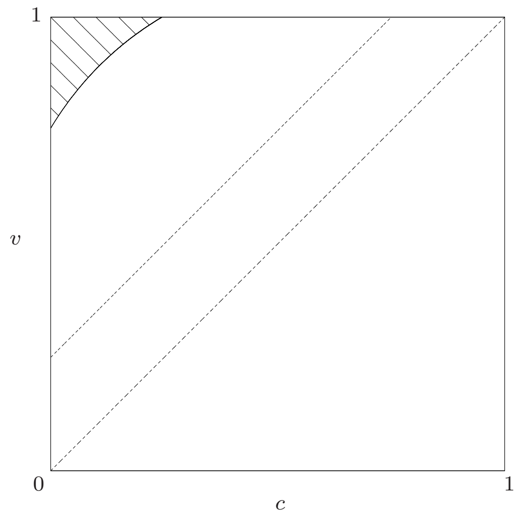

Figure 5.12: Trade region in a symmetric equilibrium (for $\alpha = 10$, $\beta = 0$)

5.4 Pooling equilibria

Apart from equilibria with strictly increasing bidding strategies, there also exist equilibria where types pool, i.e. where $b_S(c)' = b_B(v)' = 0$.[65] In one such equilibrium, both sides either submit the same bid or bid 1 and 0, respectively. Such a bidding behavior is equivalent to a situation where the good is sold at a fixed price p. Consider a situation where costs and valuations are uniformly distributed on the unit interval. Then, the pooling (price) equilibrium in the double auction without inequity aversion is characterized by the following equilibrium strategies:

$$b_S(c) = \begin{cases} p & \text{if } c \leq p \\ 1 & \text{if } c > p \end{cases} \tag{5.18}$$

and

65 More precisely, we consider equilibria where those types who trade with positive probability pool.

$$b_B(v) = \begin{cases} 0 & \text{if } v < p \\ p & \text{if } v \geq p. \end{cases} \tag{5.19}$$

For any price $p \in [0, 1]$ these strategies describe an equilibrium of the double auction without inequity aversion. Trade occurs if and only if $c \leq p$ and $v \geq p$ and does not take place if $c > p$ and/or $v < p$. As can be seen in *Figure 5.13*, this price equilibrium is not efficient either.

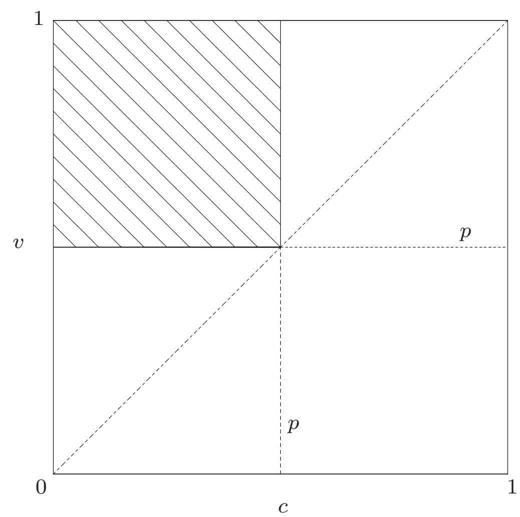

Figure 5.13: Trade region in a pooling equilibrium without inequity aversion

Now consider a pooling equilibrium with inequity aversion in the same setup. In order to derive explicit expressions and to make clear the underlying effects, we assume that c and v are uniformly distributed between zero and one. Interestingly, the outcome becomes worse with inequity aversion—even if compassion is of great importance. The results are given in the following proposition:

Proposition 5.4 *Suppose that c and v are uniformly distributed on $[0,1]$. The price equilibria can then be characterized as follows: There exist a $\underline{p}(\alpha,\beta) > 0$ and a $\bar{p}(\alpha,\beta) < 1$ such that*

(a) if $\underline{p}(\alpha,\beta) \leq p \leq \bar{p}(\alpha,\beta)$, then there exist a $c^*(\alpha,\beta) < p$ and a $v^*(\alpha,\beta) > p$ such that only sellers with $c \leq c^*(\alpha,\beta)$ and buyers with $v \geq v^*(\alpha,\beta)$ trade and

(b) if $p < \underline{p}(\alpha,\beta)$ or $p > \bar{p}(\alpha,\beta)$, there is no trade.

A change in the degree of inequity aversion leads to

(c) $\frac{\partial \underline{p}}{\partial \alpha} > 0$, $\frac{\partial \underline{p}}{\partial \beta} > 0$, $\frac{\partial \bar{p}}{\partial \alpha} < 0$, and $\frac{\partial \bar{p}}{\partial \beta} < 0$, and

(d) $\frac{\partial c^*}{\partial \alpha} < 0$, $\frac{\partial c^*}{\partial \beta} < 0$, $\frac{\partial v^*}{\partial \alpha} > 0$, and $\frac{\partial v^*}{\partial \beta} > 0$.

The general proof is relegated to the appendix. Here we discuss the steps necessary to determine the minimum and maximum prices $\underline{p}$ and $\bar{p}$. Consider the case where $p < \frac{1}{2}$. This price favors the buyer. So the seller is less willing to accept the deal. We show below that this implies that the marginal seller has costs c^* such that $p - c^* > v^* - p$, where v^* is the marginal buyer (see *Figure 5.14*). Hence, there must exist a $\tilde{v}$ such that the marginal seller is better off if the buyer's valuation lies in between v^* and $\tilde{v}$, otherwise the marginal seller is worse off.

We can write the expected utility functions for the marginal buyer and for the marginal seller as

$$\mathrm{E}[U_S(c^*)] = \int_{v^*}^{\tilde{v}} (p - c^* - \beta((p - c^*) - (v - p)))\, dv + \int_{\tilde{v}}^{1} (p - c^* - \alpha(-(p - c^*) + (v - p)))\, dv \quad (5.20)$$

and

$$\mathrm{E}[U_B(v^*)] = \int_{0}^{c^*} (v^* - p - \alpha(-(v^* - p) + (p - c)))\, dc. \quad (5.21)$$

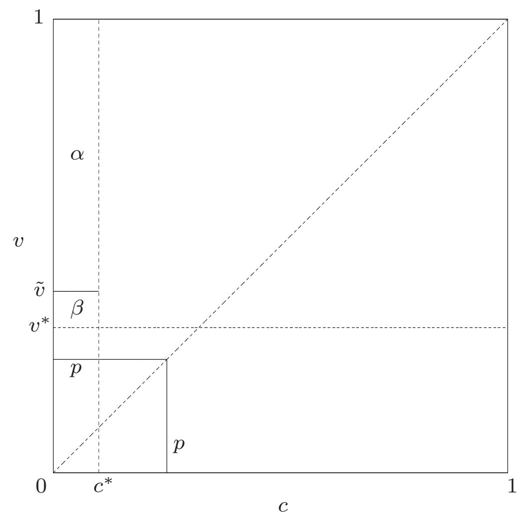

Figure 5.14: Pooling equilibrium with inequity aversion for $p < \frac{1}{2}$ (seller of type c^*: β relevant if $v \in [v^*, \tilde{v})$ and α relevant if $v \in (\tilde{v}, 1]$)

Next, it is shown that there exists a threshold below which no trade takes place. We call this threshold the minimum price. The minimum price is such that the marginal buyer has $c^* = 0$. Formally, this results in

$$\mathrm{E}[U_S(c^* = 0)] = \int_{v^*}^{\tilde{v}} \big(\underline{p} - \beta(2\underline{p} - v)\big)\, dv + \int_{\tilde{v}}^{1} \big(\underline{p} - \alpha(v - 2\underline{p})\big)\, dv. \tag{5.22}$$

Setting $\mathrm{E}[U_S(c^* = 0)] = 0$ and $\mathrm{E}[U_B(v^*)] = 0$ for the limit $c^* \to 0$, the minimum price can be derived as $\underline{p} = \frac{1}{2} - \rho$ where ρ is given by[66]

$$\rho = \frac{2(1+\alpha)(\alpha + \sqrt{1 + 3\alpha^2 + 4\alpha - \alpha\beta}) + \beta}{2(2 + 4\alpha^3 + 12\alpha^2 + 10\alpha + \beta)}. \tag{5.23}$$

The analogous analysis on the buyer side is omitted. The maximum price denoted by $\bar{p}$ is given by $\bar{p} = \frac{1}{2} + \rho$.

The existence of the minimum price can be explained as follows: Consider a seller with very low costs in a situation where envy is important. Now if the price is very

66 The formal derivation is given in the appendix.

low too, the seller will end up trading with a buyer whose expected valuation is greater than $\frac{1}{2}$. This, however, means that in expectation the seller is worse off than the buyer. As envy plays an important role, the seller would rather stay away from trade.

A result which can be seen from *Figure 5.15* and which is stated in the proposition is that the greater the envy factor α, the greater (lower) the minimum (maximum) price.

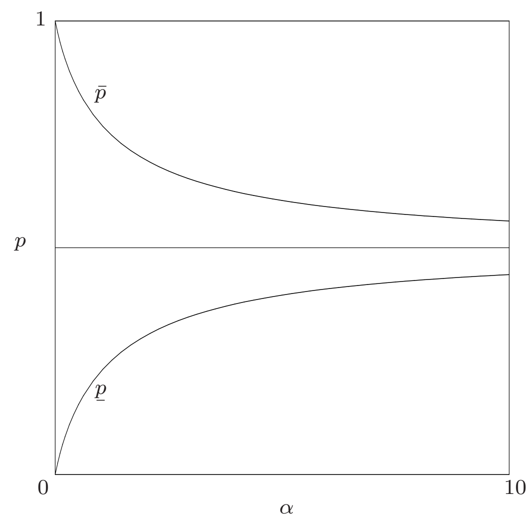

Figure 5.15: Impact of α on $\underline{p}$ and $\bar{p}$ (for $\beta = 0$)

Last, *Figure 5.16* illustrates the implications of a change in the envy factor for the critical costs (valuations) above (below) which the seller (buyer) is not willing to trade. Note that in the case depicted in the figure, i.e. $p = \frac{1}{2}$, the compassion factor is not relevant as only envy plays a role when considering the critical value (see also the proof of *Proposition 5.4*).

In this type of equilibrium, an inequity-averse seller whose costs are close to the price faces the following decision problem: Trade will occur with some probability and some gains from trade will be realized if the seller accepts the price. This, however, means that the seller will most likely trade with those buyers that have a very high valuation for the product. As a consequence, whenever trade takes place, the seller will be worse off than her counterpart in expectation. Now if envy is important, the seller's utility from trading is reduced.

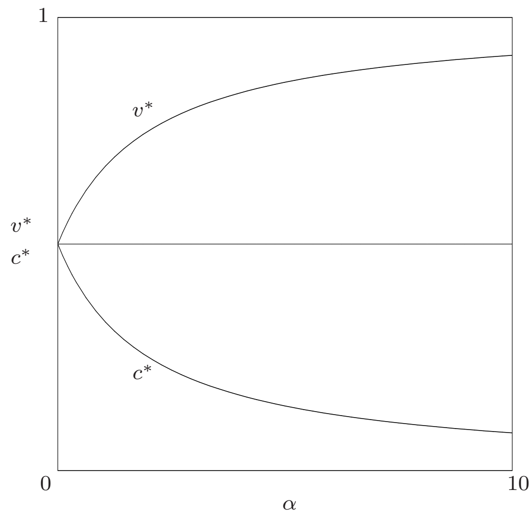

Figure 5.16: Impact of α on c^* and v^* (for $p = \frac{1}{2}$)

We conclude this section by pointing out the similarities and differences with respect to the inefficiencies in the separating and pooling equilibria. Comparing both types of equilibria, we observe that the greater the importance of envy, the smaller the trade region, i.e. the more inefficient the outcome. However, different from the separating equilibria analyzed in the previous section, an increase in the importance of compassion reduces the efficiency in the pooling equilibrium. This is due to the fact that by accepting a price, there is always a chance that the seller or the buyer will end up trading with someone on the other side who is worse off. This reduces the expected utility such that both sides are less willing to accept a given price.

5.5 Extension: heterogeneous sellers and buyers

So far, we have assumed that all sellers and buyers have the same preferences. We now consider different utility function across and within the two sides and discuss how the above results change. Suppose therefore that there are two groups of sellers and buyers: Agents of the first group are inequity averse whereas those in the second group are not.

Now if compassion is very relevant such that truth-telling is an equilibrium in the case of homogeneous (inequity-averse) sellers and buyers, the picture changes with heterogeneous groups. Clearly, the inequity-averse groups anticipate that those agents that are not inequity averse bid more aggressively. This, in turn, implies that inequity-averse agents are more likely to be worse off if they stick to a truth-telling strategy, i.e. they will move away from truth-telling as well. As a result, the truth-telling equilibrium will no longer exist and there is less trade than would be efficient.

The contrary is true for the case where compassion is not as important and where envy is of great relevance. With homogeneous groups, trade may break down completely, i.e. there is a maximum of inefficiency. However, if some agents are not inequity averse, they will not be affected by envy and will still be happy to trade, i.e. their bidding behavior will be less aggressive. As a consequence, at least some trade will occur and efficiency increases.

A similar argument holds for the pooling equilibrium: For a given price, those sellers and buyers that are not inequity averse rationally accept this price if their costs/valuations are below/above the price. This, in turn, induces more inequity-averse agents to accept the price as well. As a consequence, there is more efficient trade. More precisely, consider a situation where there is a share ζ of sellers/buyers who are not inequity averse. The remaining share $1-\zeta$ is inequity averse. *Figure 5.17* then illustrates the critical cost for the marginal inequity-averse seller c^* (the marginal inequity-averse buyer v^*) for given values of p and β: Increasing ζ leads to an increase in c^* (v^*).

The less inequity averse sellers and buyers there are, the more likely it is that an inequity-averse seller or buyer accepts the price.

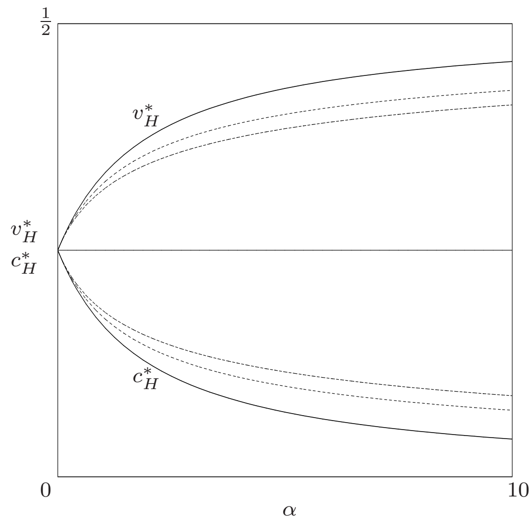

Figure 5.17: Impact of ζ on c_H^* and v_H^* for different values of ζ (for $\beta = 0$ and $p = \frac{1}{2}$): $\zeta = 0$ (bold line), $\zeta = \frac{1}{2}$ (dotted line), and $\zeta \to 1$ (dashed line)

5.6 Conclusions

In this chapter, we apply the concept of inequity aversion consisting of envy and compassion to the double-auction model by Chatterjee and Samuelson (1983). We find that truth-telling is optimal for both sides if compassion is sufficiently important. If it is not, in the limit of infinitely strong envy, there is no trade at all. Numerical simulations show that an increase in envy (compassion) reduces (increases) the efficiency of a separating equilibrium. We further show that in a pooling equilibrium with inequity aversion, in which sellers and buyers either accept a price or do not trade, the outcome is more inefficient than without inequity aversion. The players only accept the price if the seller's cost (buyer's valuation) is sufficiently smaller (greater) than this price, depending on their envy and compassion factors.

A tentative conclusion from this theoretical result might be that in the case of bargaining with two-sided incomplete information where the bargaining parties are inequity averse, a personalized bargaining structure should be preferred to a market-like bargaining structure where the price does not depend on the individual valuations.

5.7 Appendix

5.7.1 Proof of *Proposition 5.2* (continued)

Proof Ad (b): In order to simplify the characterization of the lower and upper bounds for Δ, we define the following:

Definition 5.1 *Consider the solutions to the following equations:*

1. $\bar{\Delta}'$ *is defined such that* $-(\frac{1}{2}+\alpha)\frac{1}{\gamma(\bar{c}+2\bar{\Delta}')}+\bar{\Delta}'=0$.

2. *Analogously,* $\bar{\Delta}''$ *is defined such that* $-(\frac{1}{2}+\alpha)\frac{1}{\phi(\underline{v}-2\bar{\Delta}'')}+\bar{\Delta}''=0$.

3. $\underline{\Delta}'$ *is defined such that* $(\frac{1}{2}-\beta)\frac{1}{\gamma(\bar{c}+2\underline{\Delta}')}-\underline{\Delta}'=0$.

4. *Analogously,* $\underline{\Delta}''$ *is defined such that* $(\frac{1}{2}-\beta)\frac{1}{\phi(\underline{v}-2\underline{\Delta}'')}-\underline{\Delta}''=0$.

5. $\underline{\Delta}''' \geq -\frac{\bar{c}-\underline{v}}{2}$ *is defined such that* $\frac{1}{2}\int_{\bar{c}+2\underline{\Delta}'''}^{\bar{v}} v g(v)dv - \frac{\bar{c}}{2}(1-G(\bar{c}+2\underline{\Delta}''')) = 0$.

Note that all solutions depend on the boundaries of the intervals and that all but the last solution depend on the inequity-aversion parameters (see also the example in the main text). Due to *Assumptions 5.1* and *5.2*, we can conclude that if a solution exists, it is unique in cases 1.–4. as the expressions on the left-hand sides are either strictly increasing (cases 1. and 2.) or strictly decreasing (cases 3. and 4.) in Δ. As will be shown, case 5. is relevant only for $\Delta < 0$. The expression on the left-hand side is strictly increasing in Δ (see also below). Thus, if there is a solution in this case, it is also unique.

Consider now the deviation incentives: From derivatives (5.6) and (5.7) we get

$$-\left(\frac{1}{2}+\alpha\right)\frac{1}{\gamma(c+2\Delta)}+\Delta \leq 0 \tag{5.24}$$

and

$$-\left(\frac{1}{2}+\alpha\right)\frac{1}{\phi(v-2\Delta)}+\Delta \leq 0. \tag{5.25}$$

Different from the cases considered so far, it now holds that $\frac{1}{\gamma(c+2\Delta)}, \frac{1}{\phi(v-2\Delta)} > 0$ (see also *Figure 5.21*). As a result, $\Delta = 0$ may no longer be the unique equilibrium. Note that both inequalities imply an upper bound on Δ as the left-hand sides increase in Δ (given *Assumptions 5.1* and *5.2*). Together with the observation that the left-hand side of condition (5.24) (condition (5.25)) increases in c (decreases in v) (see also 1. and 2. in the definition above), the upper bound is defined as the minimum of

$$-\left(\frac{1}{2}+\alpha\right)\frac{1}{\gamma(\bar{c}+2\Delta)}+\Delta \leq 0 \tag{5.26}$$

and

$$-\left(\frac{1}{2}+\alpha\right)\frac{1}{\phi(\underline{v}-2\Delta)}+\Delta \leq 0. \tag{5.27}$$

Note that the (non-negative) upper bound implicitly defined by the above conditions is always smaller than (or equal to) $\min\{\frac{\underline{v}-\underline{c}}{2}, \frac{\bar{v}-\bar{c}}{2}\}$. If this was not the case, $\Delta = 0$ would emerge as the equilibrium condition since $\frac{1}{\gamma(c+2\Delta)} = 0$ and/or $\frac{1}{\phi(v-2\Delta)} = 0$ would be true (see proof of *Proposition 5.2*).

To see whether $\Delta > 0$ can indeed be part of an equilibrium, we also need to look at the case where the seller deviates and sets a higher price. Thus, suppose that the seller submits a price $s = c + \Delta + \epsilon > c + \Delta$. If trade occurs, this will lead to a price which is larger than $\frac{c+v}{2}$. Therefore, only the advantage term with the parameter β is relevant in the expected-utility function. Trade takes place whenever $v \geq c + 2\Delta + \epsilon$. Then, the expected utility of a seller with costs c that submits a price $s = c + \Delta + \epsilon$, $\mathrm{E}[U_S(c, c + \Delta + \epsilon)]$, amounts to

$$
\begin{aligned}
\mathrm{E}[U_S(c, c+\Delta+\epsilon)] &= \int_{c+2\Delta+\epsilon}^{\bar{v}} \left(\frac{c+v+\epsilon}{2} - c - \beta\epsilon \right) g(v)dv \\
&= \frac{1}{2} \int_{c+2\Delta+\epsilon}^{\bar{v}} vg(v)dv - \frac{c}{2}(1 - G(c+2\Delta+\epsilon)) \\
&\quad + \epsilon \left(\frac{1}{2} - \beta \right) (1 - G(c+2\Delta+\epsilon)).
\end{aligned} \tag{5.28}
$$

In order to give the seller an incentive to bid $c+\Delta$, it must hold that $\mathrm{E}[U_S(c, c+\Delta)] \geq \mathrm{E}[U_S(c, c+\Delta+\epsilon)]\ \forall \epsilon > 0$. Taking the derivative of $\mathrm{E}[U_S(c, c+\Delta+\epsilon)]$ with respect to ϵ gives

$$
\begin{aligned}
\frac{\partial \mathrm{E}[U_S(c, c+\Delta+\epsilon)]}{\partial \epsilon} &= \left(\frac{1}{2} - \beta \right) (1 - G(c+2\Delta+\epsilon)) \\
&\quad - \epsilon (1-\beta) g(c+2\Delta+\epsilon) - \Delta g(c+2\Delta+\epsilon).
\end{aligned} \tag{5.29}
$$

Turning to the buyer side, an analogous analysis gives

$$
\begin{aligned}
\frac{\partial \mathrm{E}[U_B(v, v-\Delta-\epsilon)]}{\partial \epsilon} &= \left(\frac{1}{2} - \beta \right) F(v-2\Delta-\epsilon) \\
&\quad - \epsilon (1-\beta) f(v-2\Delta-\epsilon) - \Delta f(v-2\Delta-\epsilon).
\end{aligned} \tag{5.30}
$$

Consider now the two cases distinguished in the proposition.

Ad (i): Under the above assumptions, both derivatives decrease in ϵ, i.e. we must look at the case where $\epsilon \to 0$. Then, the following conditions have to be met if the proposed strategies are to be part of an equilibrium:

$$
\left(\frac{1}{2} - \beta \right) \frac{1}{\gamma(c+2\Delta)} - \Delta \leq 0 \tag{5.31}
$$

and

$$
\left(\frac{1}{2} - \beta \right) \frac{1}{\phi(v-2\Delta)} - \Delta \leq 0. \tag{5.32}
$$

As the left-hand side of (5.31) decreases in c, the critical c is the lowest c for which trade takes place, i.e. we must plug $c = \underline{v} - 2\Delta \geq \underline{c}$ into the above expression. A similar argument holds for v such that the relevant v is given by $\bar{c} + 2\Delta \leq \bar{v}$. As a consequence, conditions (5.31) and (5.32) can be rewritten as

$$\left(\frac{1}{2} - \beta\right) \frac{1}{g(\underline{v})} - \Delta \leq 0 \tag{5.33}$$

and

$$\left(\frac{1}{2} - \beta\right) \frac{1}{f(\bar{c})} - \Delta \leq 0. \tag{5.34}$$

The maximum of these inequalities defines a (non-negative) lower bound for Δ.

Ad (ii): Obviously, expressions (5.29) and (5.30) are both smaller than or equal to 0 irrespective of $\epsilon, \Delta > 0$, i.e. the relevant condition for any $\Delta > 0$ is the upper bound specified above.

It remains to be checked whether $\Delta < 0$ can be part of an equilibrium. As conditions (5.26) and (5.27) imply a non-negative upper bound, setting a lower price would never be optimal. Consider therefore the situation where the seller deviates and sets a higher price. Since the left-hand side of (5.31) now increases in c, we must consider $c = \bar{c}$ in the above expression. Analogously, we need to set $v = \underline{v}$. Then, conditions (5.31) and (5.32) can be rewritten as

$$\left(\frac{1}{2} - \beta\right) \frac{1}{\gamma(\bar{c} + 2\Delta)} - \Delta \leq 0 \tag{5.35}$$

and

$$\left(\frac{1}{2} - \beta\right) \frac{1}{\phi(\underline{v} - 2\Delta)} - \Delta \leq 0. \tag{5.36}$$

Note that conditions (5.35) and (5.36) imply a lower bound which is non-positive and that they are relevant as long as $\bar{c} + 2\underline{\Delta}' \geq \underline{v} \Leftrightarrow \underline{\Delta}' \geq -\frac{\bar{c} - \underline{v}}{2}$. The same argument holds for $\underline{\Delta}''$. If this is not the case, conditions (5.33) and (5.34) define the lower bound (which is then negative).

Given this bidding strategy where $\Delta < 0$, the seller with costs $\bar{c}$ will end up trading with buyers of types $v < \bar{c}$. As this leads to prices which are lower than the seller's costs $\bar{c}$, we have to check whether the expected utility from trading with all types of buyers is indeed greater than 0. We thus have

$$\begin{aligned} \mathrm{E}[U_S(\bar{c}, \bar{c}+\Delta)] &= \int_{\bar{c}+2\Delta}^{\bar{v}} \left(\frac{\bar{c}+v}{2} - \bar{c} \right) g(v) dv \\ &= \frac{1}{2} \int_{\bar{c}+2\Delta}^{\bar{v}} v g(v) dv - \frac{\bar{c}}{2} \left(1 - G(\bar{c}+2\Delta) \right). \end{aligned} \tag{5.37}$$

Now consider

$$\begin{aligned} \frac{\partial \mathrm{E}[U_S(\bar{c}, \bar{c}+\Delta)]}{\partial \Delta} &= -\frac{1}{2} \left(\bar{c}+2\Delta \right) g(\bar{c}+2\Delta) + \frac{\bar{c}}{2} g(\bar{c}+2\Delta) \\ &= -\Delta g(\bar{c}+2\Delta) > 0 \ \forall \Delta < 0. \end{aligned} \tag{5.38}$$

We therefore have to set $\Delta = -\frac{\bar{c}-\underline{v}}{2}$ in which case the seller has a utility of

$$\mathrm{E}[U_S(\bar{c}, \bar{c}+\Delta)] = \int_{\underline{v}}^{\bar{v}} \left(\frac{\bar{c}+v}{2} - \bar{c} \right) g(v) dv = \frac{1}{2} \int_{\underline{v}}^{\bar{v}} v g(v) dv - \frac{\bar{c}}{2}. \tag{5.39}$$

Clearly, if this expression is greater than or equal to 0, then any $\Delta < 0$ satisfying the conditions derived above can be part of an equilibrium and the seller with costs $\bar{c}$ will trade with any buyer type (i.e. even with $\underline{v}$). If the opposite is true, then—as $\frac{\partial \mathrm{E}[U_S(\bar{c}, \bar{c}+\Delta)]}{\partial \Delta} > 0$—there exists a $\underline{\Delta}'''$ such that the seller of type $\bar{c}$ only trades with buyers of types $v \geq \bar{c} + 2\Delta$, i.e. $\mathrm{E}[U_S(\bar{c}, \bar{c}+\underline{\Delta}''')] = 0$.

We can thus summarize the above results as follows: There exist a class of symmetric bidding strategies where the two parties bid according to $b_S(c) = c + \Delta$ and $b_B(v) = v - \Delta$ as long as Δ satisfies

1. $\max\{(\frac{1}{2} - \beta)\frac{1}{g(\underline{v})}, (\frac{1}{2} - \beta)\frac{1}{f(\bar{c})}\} \leq \Delta \leq \min\{\bar{\Delta}', \bar{\Delta}''\}$ if $\beta < \frac{1}{2}$ and
2. $\max\{\underline{\Delta}', \underline{\Delta}'', -(\beta - \frac{1}{2})\frac{1}{g(\underline{v})}, -(\beta - \frac{1}{2})\frac{1}{f(\bar{c})}\} \leq \Delta \leq \min\{\bar{\Delta}', \bar{\Delta}''\}$ if $\beta \geq \frac{1}{2}$. ■

The following *Figures 5.18–5.21* illustrate the different cases with respect to the boundaries of the intervals on which c and v are distributed (see proofs of *Propositions 5.1* and *5.2*):

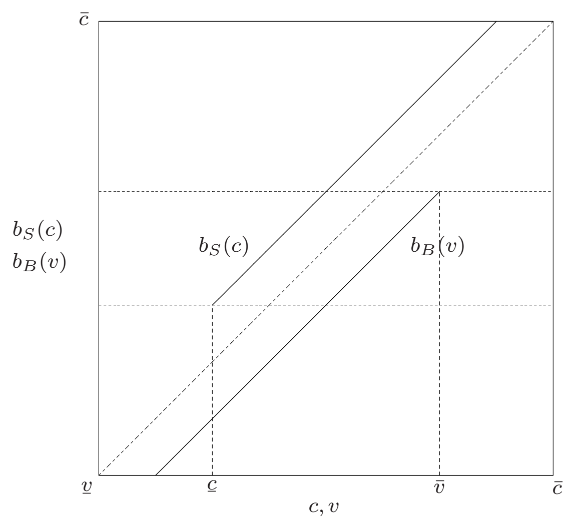

Figure 5.18: Equilibrium candidate with $b_S(c) = c + \Delta$, $b_B(v) = v - \Delta$, and $\Delta > 0$ ($\underline{v} < \underline{c} < \bar{v} < \bar{c}$)

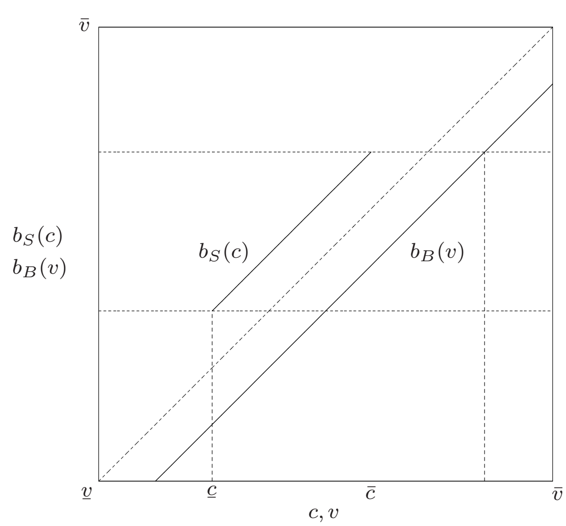

Figure 5.19: Equilibrium candidate with $b_S(c) = c + \Delta$, $b_B(v) = v - \Delta$, and $\Delta > 0$ ($\underline{v} < \underline{c} < \bar{c} < \bar{v}$)

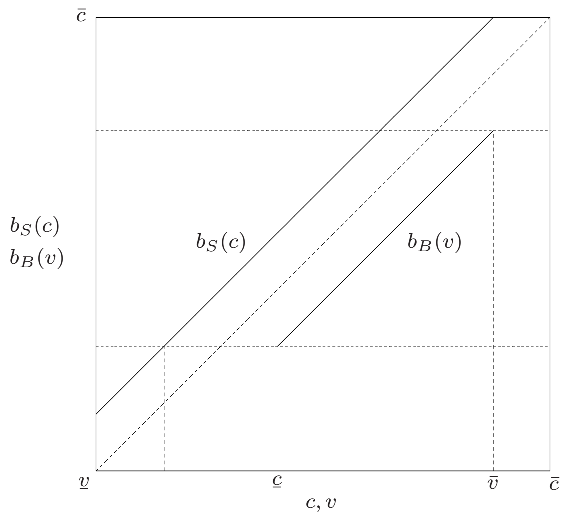

Figure 5.20: Equilibrium candidate with $b_S(c) = c + \Delta$, $b_B(v) = v - \Delta$, and $\Delta > 0$ $(\underline{c} < \underline{v} < \bar{v} < \bar{c})$

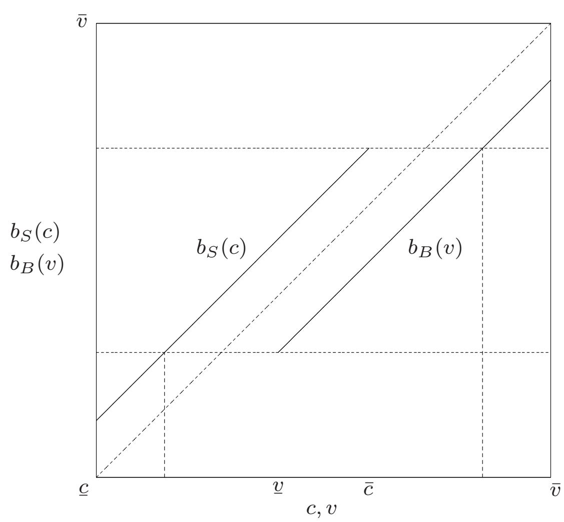

Figure 5.21: Equilibrium candidate with $b_S(c) = c + \Delta$, $b_B(v) = v - \Delta$, and $\Delta > 0$ $(\underline{c} < \underline{v}$ and $\bar{c} < \bar{v})$

5.7.2 Proof of *Proposition 5.3*

Proof We prove the proposition by contradiction: Assume that there exists an equilibrium where trade occurs with strictly positive probability for any value of α. Each side's bidding strategy may depend on α which implies that the bargaining price $p(c, v, \alpha)$ may depend on α as well.

As we assumed $b'_S(c), b'_B(v) > 0$, we can conclude that if there is trade, both $\underline{c}$ and $\bar{v}$ trade. Note further that if trade is to occur with strictly positive probability, there must exist intervals $[\underline{c}, c'(\alpha)]$ and $[v'(\alpha), \bar{v}]$ where $\underline{c} < c'(\alpha)$ and $v'(\alpha) < \bar{v}$ such that any $c \in [\underline{c}, c'(\alpha)]$ and $v \in [v'(\alpha), \bar{v}]$ trade with positive probability.

Define now $\bar{c}' := \lim_{\alpha\to\infty} \sup\{c'(\alpha)\}$. In what follows, we will show that if $\alpha \to \infty \Rightarrow \underline{c} = \bar{c}'$. An analogous reasoning holds for the buyer side where we get $\underline{v}' := \lim_{\alpha\to\infty} \inf\{v'(\alpha)\}$.

Assume the opposite holds, i.e. $\underline{c} < \bar{c}'\ \forall\alpha \geq 0$. Then, there exists a seller with costs $c \in [\underline{c}, \bar{c}']$ who trades with buyers of types $v \in [\underline{v}', \bar{v}]\ \forall\alpha \geq 0$. In order to determine this seller's expected utility from trade, we point out that she may be worse off, equally well off, or better off compared to her trading partner. We thus define the following sets which correspond to these cases: $\mathcal{A}_S(c, \alpha) := \{v \in [\underline{v}', \bar{v}] | \frac{c+v}{2} > p(c, v, \alpha)\}$, $\mathcal{B}_S(c, \alpha) := \{v \in [\underline{v}', \bar{v}] | \frac{c+v}{2} = p(c, v, \alpha)\}$, and $\mathcal{C}_S(c, \alpha) := \{v \in [\underline{v}', \bar{v}] | \frac{c+v}{2} < p(c, v, \alpha)\}$. Such sets can be defined for the buyer side analogously. The seller then gets an expected utility of

$$\begin{aligned} \mathrm{E}[U_S(c, v)] = {} & \mathrm{E}[U_S(c, v) | v \in \mathcal{A}_S(c, \alpha)] + \mathrm{E}[U_S(c, v) | v \in \mathcal{B}_S(c, \alpha)] \\ & + \mathrm{E}[U_S(c, v) | v \in \mathcal{C}_S(c, \alpha)]. \end{aligned} \tag{5.40}$$

Let $\Pr(\cdot)$ be the probability measure associated with $G(\cdot)$. Consider first the case where $\lim_{\alpha\to\infty} \Pr(\mathcal{A}_S(c, \alpha)) = 0\ \forall c \in [\underline{c}, \bar{c}']$ and $\lim_{\alpha\to\infty} \Pr(\mathcal{A}_B(v, \alpha)) = 0\ \forall v \in [\underline{v}', \bar{v}]$ which implies that types who trade must bid their costs or valuation ($\pm\Delta$), respectively. This, however, cannot be an equilibrium as shown above.

Assume therefore that $\lim_{\alpha\to\infty} \Pr(\mathcal{A}_S(c, \alpha)) \geq \underline{k}_S$ with $\underline{k}_S > 0$. The expected utility for a seller who trades amounts to

$$\begin{aligned}\mathrm{E}[U_S(c,v)] = &\int_{\mathcal{A}_S(c,\alpha)} (p(c,v,\alpha))g(v)dv - \alpha \int_{\mathcal{A}_S(c,\alpha)} (c+v-2p(c,v,\alpha))g(v)dv \\ &+ \mathrm{E}[U_S(c,v)|v \in \mathcal{B}_S(c,\alpha)] + \mathrm{E}[U_S(c,v)|v \in \mathcal{C}_S(c,\alpha)].\end{aligned} \tag{5.41}$$

Given that $\Pr(\mathcal{A}_S(c,\alpha)) \geq \underline{k}_S$, it then holds that $\alpha \to \infty \Rightarrow \mathrm{E}[U_S(c,v)] < 0$ if and only if $\int_{\mathcal{A}_S(c,\alpha)}(c+v-2p(c,v,\alpha))g(v)dv > 0$ for $\alpha \to \infty$. Assume that the opposite holds, i.e. that

$$\int_{\mathcal{A}_S(c,\alpha)} (c+v-2p(c,v,\alpha))g(v)dv = 0 \text{ for } \alpha \to \infty. \tag{5.42}$$

Let $\mathcal{A}'_S(c,\alpha) := \{v \in \mathcal{A}_S(c,\alpha)|c+v-2p(c,v,\alpha) > \epsilon\} \subset \mathcal{A}_S(c,\alpha)$. We can thus conclude that

$$\lim_{\alpha\to\infty} \int_{\mathcal{A}'_S(c,\alpha)} (c+v-2p(c,v,\alpha))g(v)dv = 0 \;\forall \epsilon > 0 \tag{5.43}$$

and hence

$$\lim_{\alpha\to\infty} \Pr(\mathcal{A}'_S(c,\alpha)) = 0 \;\forall \epsilon > 0. \tag{5.44}$$

Then, assuming that $||\cdot||_\infty$ is the supremum norm for set $\mathcal{A}_S(c,\alpha)$, it must hold that

$$\lim_{\alpha\to\infty} ||c+v-2p(c,v,\alpha)||_\infty = 0 \qquad \text{a.s.} \tag{5.45}$$

Then, it is true that $\forall\epsilon > 0\; \exists\tilde{\alpha}$ such that $\forall\alpha > \tilde{\alpha}$,

$$||c+v-2p(c,v,\alpha)||_\infty < \epsilon \qquad \text{a.s.} \tag{5.46}$$

which is equal to

$$\sup_{v \in \mathcal{A}_S(c,\alpha)} |c + v - 2p(c,v,\alpha)| < \epsilon \qquad \text{a.s.} \tag{5.47}$$

From this, it follows that $\forall \epsilon > 0$ $\exists \tilde{\alpha}$ such that $\forall \alpha > \tilde{\alpha}$ and for almost all $v \in \mathcal{A}_S(c,\alpha)$, it must hold that

$$|c + v - 2p(c,v,\alpha)| < \epsilon. \tag{5.48}$$

Then, however, we have $\lim_{\alpha\to\infty} p(c,v,\alpha) = \frac{c+v}{2}$ for almost all $v \in \mathcal{A}_S(c,\alpha)$. Therefore, we get $\lim_{\alpha\to\infty} \Pr(\mathcal{A}_S(c,\alpha)) = 0$ which contradicts our initial assumption that $\Pr(\mathcal{A}_S(c,\alpha)) \geq \underline{k}_S$ $\forall \alpha \geq 0$. An analogous argument holds on the buyer side. ■

5.7.3 Proof of *Lemma 5.1*

Proof From *Proposition 5.2* we know that a symmetric equilibrium where both parties stick to a bidding strategy with slope 1 does not exist. We now generalize the result for any linear bidding strategy. We prove the result by contradiction. To this end, consider the case where the buyer follows the following linear strategy:

$$b_B(v) = \lambda + \mu v. \tag{5.49}$$

Trade will then occur whenever the seller's bid s is not greater than this bid, i.e. whenever

$$s \leq b_B(v) \Leftrightarrow v \geq \frac{s-\lambda}{\mu} =: \check{v}. \tag{5.50}$$

Note that if both sides follow a symmetric (linear) equilibrium strategy, then there is exactly one occasion (a pair of types denoted by $\tilde{c}$ and $\tilde{v}$) where both sides submit the same bid ($b_S(\tilde{c}) = b_B(\tilde{v}) = \frac{1}{2}$) and where $b_S(\tilde{c}) - \tilde{c} = \tilde{v} - b_B(\tilde{v})$ holds. As a consequence, the seller faces two possible scenarios depending on c: In the first scenario, the seller has costs $c \leq \tilde{c}$ such that both envy and compassion may be relevant depending on the buyer type. In the second situation, the seller has costs $c > \tilde{c}$ which means that only envy plays a role in the case of trade.

Remember that $p = \frac{s+b_B(v)}{2}$ and hence

$$p - c \lesseqgtr v - p \Leftrightarrow v \gtreqless \frac{s + \lambda - c}{1 - \mu} =: \tilde{\tilde{v}}. \tag{5.51}$$

As a result, the corresponding expected utilities in the two cases are given by

$$\mathrm{E}[U_S(c,s)] = \int_{\check{v}}^{1} \left(\frac{s + b_B(v)}{2} - c\right) dv - \alpha \int_{\tilde{\tilde{v}}}^{1} (c + v - s - b_B(v)) dv \\ - \beta \int_{\check{v}}^{\tilde{\tilde{v}}} (s + b_B(v) - c - v)\, dv \tag{5.52}$$

if $c \leq \tilde{c}$ and

$$\mathrm{E}[U_S(c,s)] = \int_{\check{v}}^{1} \left(\frac{s + b_B(v)}{2} - c\right) dv - \alpha \int_{\check{v}}^{1} (c + v - s - b_B(v)) dv \tag{5.53}$$

if $c > \tilde{c}$. From the first-order conditions, we can derive the following optimal bidding behavior for the seller in both cases:

$$s = \frac{\mu(1+\alpha)(\lambda + \mu^2 - \mu) - \lambda\mu + 4\beta\lambda\mu - 2\beta\lambda}{3\mu^2 - 2\alpha\mu^2 + 8\beta\mu^2 - 3\mu + 8\beta\mu - 2\beta} \\ + \frac{3\mu c(-1 + \mu - \alpha\mu - 2\beta\mu + \beta)}{3\mu^2 - 2\alpha\mu^2 + 8\beta\mu^2 - 3\mu + 8\beta\mu - 2\beta} \tag{5.54}$$

and

$$s = \frac{\mu(\lambda + \mu)(1 + 2\alpha) - 2\alpha\lambda}{3\mu(1 + 2\alpha) - 2\alpha} + \frac{2\mu c(1 + \alpha)}{3\mu(1 + 2\alpha) - 2\alpha}. \tag{5.55}$$

Given that the buyer sticks to the linear bidding strategy specified above, a symmetric equilibrium requires that both expressions in equations (5.54) and (5.55) are the same. To check if this is indeed the case, we set them equal and solve for β which gives $-\alpha$ as a solution. As this contradicts the assumption that $0 \leq \alpha$, the above scenario cannot be an equilibrium. ■

5.7.4 Proof of *Proposition 5.4*

Proof Ad (a): We focus on the case where $p < \frac{1}{2}$ which favors the seller. The case where $p > \frac{1}{2}$ is omitted since it is symmetric to the first case. The starting points are the expected-utility functions for the marginal seller and the marginal buyer given in equations (5.20) and (5.21).

Taking into account the participation constraints, $\mathrm{E}[U_S(c^*)]$ changes to

$$p(1+2\alpha) - c^*(1+\alpha) - 2p\tilde{v}(\alpha+\beta) + c^*\tilde{v}(\alpha+\beta) + \frac{\tilde{v}^2(\alpha+\beta)}{2} - pv^*(1-2\beta) + c^*v^*(1-\beta) - \frac{\beta v^{*2}}{2} - \frac{\alpha}{2} = 0. \quad (5.56)$$

Solving $\mathrm{E}[U_B(v^*)] = 0$ for v^* gives

$$v^* = \frac{p(1+2\alpha) - 2\alpha c^*}{1+\alpha}. \quad (5.57)$$

Plugging v^* from equation (5.57) and $\tilde{v} = 2p - c^*$ into equation (5.56) and solving for c^* yields

$$\begin{aligned} c^* \quad = \quad & \frac{1}{4\alpha^3 + 12\alpha^2 + 8\alpha + \alpha^2\beta + 4\alpha\beta + 4\beta} \qquad (5.58) \\ & \times(p(22\alpha + 26\alpha^2 + 8\alpha^3 + 2\alpha\beta + 4\beta + 4) - 12\alpha - 12\alpha^2 - 4\alpha^3 - 4 \\ & +2(p^2(28\alpha + 69\alpha^2 + 78\alpha^3 + 41\alpha^4 + 8\alpha^5 - 8\alpha\beta - 28\alpha^2\beta - 36\alpha^3\beta \\ & -20\alpha^4\beta - 4\alpha^5\beta + 4) \\ & +p(-52\alpha - 120\alpha^2 - 128\alpha^3 - 64\alpha^4 - 12\alpha^5 + 12\alpha\beta + 38\alpha^2\beta \\ & +44\alpha^3\beta + 22\alpha^4\beta + 4\alpha^5\beta - 8) \\ & +24\alpha + 52\alpha^2 + 52\alpha^3 + 24\alpha^4 + 4\alpha^5 - 4\alpha\beta - 12\alpha^2\beta - 13\alpha^3\beta \\ & -6\alpha^4\beta - \alpha^5\beta + 4)^{\frac{1}{2}}). \end{aligned}$$

Next, we show that $c^* < p$ for all $p < \frac{1}{2}$. After some rearrangements $c^* < p$ can be rewritten as

$$\begin{aligned} 0 \; < \; & p^2(8\alpha^2 + 28\alpha^3 + 36\alpha^4 + 20\alpha^5 + 4\alpha^6 + 4\alpha\beta + 12\alpha^2\beta + 15\alpha^3\beta + 9\alpha^4\beta \\ & +2\alpha^5\beta + \alpha^2\beta^2 + \alpha^3\beta^2 + \frac{\alpha^4\beta^2}{4}) \\ & +(1-2p)(8\alpha^2 + 28\alpha^3 + 36\alpha^4 + 20\alpha^5 + 4\alpha^6 + 4\alpha\beta + 12\alpha^2\beta + 13\alpha^3\beta \\ & +6\alpha^4\beta + \alpha^5\beta). \end{aligned} \quad (5.59)$$

The first term on the right-hand side is always positive. The second term is always positive due to the assumption that $p < \frac{1}{2}$, i.e. $c^* < p$ indeed holds.

From equation (5.57) we observe that for $c^* < p \Rightarrow v^* > p$.

Ad (b): First we consider the lower threshold $\underline{p}$ below which no trade takes place ($p < \underline{p}$). The expected utility of the marginal seller with $c^* = 0$ is given in equation (5.22).

Setting $\mathrm{E}[U_S(c^* = 0)] = 0$ yields

$$\underline{p}(1+2\alpha) - 2\underline{p}\tilde{v}(\alpha+\beta) + \frac{\tilde{v}^2(\alpha+\beta)}{2} - v^*\underline{p}(1-2\beta) - \frac{\beta v^{*2}}{2} - \frac{\alpha}{2} = 0. \quad (5.60)$$

Solving $\mathrm{E}[U_B(v^*)]$ from equation (5.21) for v^* and letting $c^* \to 0$ yields

$$v^* = \frac{\underline{p}(1+2\alpha)}{1+\alpha}. \quad (5.61)$$

$\tilde{v}$ is such that the marginal seller is better (worse) off if the buyer has a v smaller (greater) than $\tilde{v}$. Formally, this yields

$$\tilde{v} - p = p - c^*. \quad (5.62)$$

Solving for $\tilde{v}$ and using $p = \underline{p}$ and $c^* = 0$ yields

$$\tilde{v} = 2\underline{p}. \tag{5.63}$$

Replacing v^* and $\tilde{v}$ in $\mathrm{E}[U_S(c^* = 0)]$ by the above expressions and solving for $\underline{p}$ yields

$$\underline{p} = \frac{1}{2} - \frac{2(1+\alpha)(\alpha + \sqrt{1 + 3\alpha^2 + 4\alpha - \alpha\beta}) + \beta}{2(2 + 4\alpha^3 + 12\alpha^2 + 10\alpha + \beta)}. \tag{5.64}$$

Ad (c) and (d): Follow from the results derived above. ■

6 Concluding remarks

This chapter provides a few concluding remarks on the topics presented in this thesis. The main implications of the theoretical models developed in the previous chapters as well as their limitations and possible extensions are discussed.

The model in the second chapter helps explain a common real-life phenomenon. In markets which are characterized by two-sided network externalities in the sense that each of two distinct sides is interested in gaining access to a large number of customers on the other side, it is not always the case that customers that have the possibility to go to different platforms face a higher price than the singlehoming side. The following extension seems to be worth a closer look: Obviously, network size plays a crucial role in two-sided markets (as in other network industries). However, it is also important to consider customers' usage behavior when they are time constrained. For example, an online dating community does not only have to make sure that it attracts a large number of women and men, but also must have an interest in their members using the platform. Often, such Internet platforms show the number of members currently logged on to the service to indicate frequency/intensity of usage. As a matter of fact, time-constrained users that sign up to more than just one service must decide how much time to spend on the different platforms which may further increase the competitive pressure among platforms. Interestingly, such issues have hardly been addressed in the literature despite the number of real-life applications.[67]

67 An exception is Gabszewicz, Laussel, and Sonnac (2004) who analyze the location choices of broadcasting stations on the linear line of horizontal product differentiation. They find that broadcasting stations will move closer toward the center of the line, the greater the viewers' disutility from being exposed to commercials.

The third chapter introduces a model which analyzes whether collusion is stabilized or destabilized when customers are better informed about the prices that firms set. It is shown that such a policy is not always in the interest of customers as collusion may indeed be facilitated. We point out that antitrust authorities as well as consumer protection agencies need to account for the different market characteristics (such as demand elasticity, product homogeneity, and the like) that affect the stability of collusion. Ignoring these important market features may have just the opposite effect of what is intended, i.e. prices may go up instead. As pointed out, this has been the case in some instances where antitrust and competition authorities tried to make the market more transparent. One aspect that is not incorporated in the model and which has not been touched upon by any of the previous contributions is the observation that a higher degree of transparency on the customer side may also mean that firms will be better informed about their competitors' prices. As mentioned in the introductory section of the second chapter, this aspect is usually known as *information exchange* among competitors and is typically conceived as bad news for customers as collusive agreements can be stabilized more easily. This is due to the fact that an exchange of pricing information helps detect deviating firms much faster which renders punishment more effective. It would be interesting to investigate the implications of a simultaneous increase in transparency both on the customer as well as on the firm side for the stability of collusive agreements.

The model in the fourth chapter points to another important issue in the context of coordinated behavior: the delegation of decision-making powers within firms. In our model, a holding company that has two outlets may decide about prices itself or may delegate the pricing decision to these outlets. It is shown that the well established remedy in merger cases to maintain the merging firms as separate market players may be detrimental if collusion is an issue in the market. Again, competition and antitrust authorities are well advised to account for the specific market characteristics in order not to bring about an undesirable outcome. Clearly, the setup models internal decision making in a very simple and stylized way. Typically, firms have to decide about different variables. There are many industries where the headquarters are responsible for advertising, administrative support, etc. whereas price (or quantity) competition is up to the local businesses. Considering a richer set of decision parameters may help figure out which of these have a greater impact on collusive stability and which ones are less critical.

The model in the fifth chapter deals with a bargaining situation where a seller and a buyer submit a bid each. Different from previous contributions, we analyze the two sides' bidding behavior under the assumption that they are inequity averse in the sense that they care about whether their counterpart is relatively better (envy) or worse (compassion) off whenever trade takes place. In contrast to what is true for the case without inequity aversion, we find that there is a separating equilibrium where both sides truthfully reveal their type (i.e. costs and valuation, respectively) if compassion is sufficiently important. This equilibrium is efficient as trade occurs whenever the buyer's valuation is greater than the seller's costs. As far as separating equilibria in a situation where compassion is not as important are concerned, we find that the efficiency of the bargaining outcome depends both on the strength of the envy and the compassion factor in most circumstances. It turns out that as the importance of envy (compassion) increases, the outcome becomes less (more) efficient. With respect to the envy factor, this result also holds for pooling equilibria. Moreover, trade may break down completely if envy is very strong. These findings have interesting implications for the market design in the context of such bargaining situations. Obviously, in the presence of strong inequity aversion, both sides should be given the opportunity to submit a price themselves as this will lead to a more efficient outcome than proposing a price which may be accepted or rejected by the two sides. Clearly, it would be interesting to look at this issue from an empirical or experimental point of view to see whether a more personalized bargaining setup indeed fares better. From a theoretical perspective, it would be worthwhile to consider a situation where the two sides are no longer symmetric with respect to their respective bargaining power. In the above model, we assumed that the price is the average of both bids (split-the-difference rule). Now if one party has a better bargaining position, there are two opposing effects concerning efficiency: On the one hand, the other side is less interested in trading as the inequality between the two sides is increased through the unequal splitting rule. On the other hand, it is not in the interest of the side with the better bargaining position either to end up with an unequal outcome. Therefore, it may be willing not to make full use of its bargaining powers.

Bibliography

Abreu, D. (1986): “Extremal equilibria of oligopolistic supergames,” *Journal of Economic Theory*, 39(1), 191–225.

——— (1988): “On the theory of infinitely repeated games with discounting,” *Econometrica*, 56(2), 383–396.

Akerlof, G. A. (1970): “The market for lemons: quality uncertainty in the market mechanism,” *The Quarterly Journal of Economics*, 84(3), 488–500.

Albæk, S., P. Møllgaard, and P. B. Overgaard (1997): “Government-assisted oligopoly coordination? A *concrete* case,” *The Journal of Industrial Economics*, 45(4), 429–443.

Ambrus, A., and M. Reisinger (2006): “Exclusive vs. overlapping viewers in media markets,” Munich Economics Discussion Paper 2006-25.

Armstrong, M. (2002): “The theory of access pricing and interconncetion,” in *Handbook of Telecommunications Economics*, ed. by M. E. Cave, S. K. Mujumdar, and I. Vogelsang, vol. 1, pp. 297–384. North-Holland, Amsterdam.

——— (2006): “Competition in two-sided markets,” *The RAND Journal of Economics*, 37(3), 668–691.

Armstrong, M., and J. Wright (2007): “Two-sided markets, competitive bottlenecks and exclusive contracts,” *Economic Theory*, 32(2), 353–380.

BAYE, M. R., K. J. CROCKER, AND J. JU (1996): "Divisionalization, franchising and divestiture incentives in oligopoly," *The American Economic Review*, 86(1), 223–236.

BELLEFLAMME, P., AND M. PEITZ (2006): "Intermediation and investment incentives," CORE Discussion Paper 2006-94 and Working Paper 47/2006, International University in Germany.

BOLTON, G. E., AND A. OCKENFELS (2000): "ERC: a theory of equity, reciprocity, and competition," *The American Economic Review*, 90(1), 166–193.

CAILLAUD, B., AND B. JULLIEN (2003): "Chicken & egg: competition among intermediation service providers," *The RAND Journal of Economics*, 34(2), 309–328.

CAMPBELL, A. N., AND C. W. HALLADAY (2005): "The use of behavioural remedies in Canadian merger law," mimeo.

CAPOBIANCO, A., AND S. FRATTA (2005): "Information exchanges between competitors: the Italian Competition Authority's recent practice," *Competition Law Insight*, 4(1), 3–6.

CHANG, M.-H. (1991): "The effects of product differentiation on collusive pricing," *International Journal of Industrial Organization*, 9(3), 453–469.

CHATTERJEE, K., AND W. SAMUELSON (1983): "Bargaining under incomplete information," *Operations Research*, 31(5), 835–851.

CREANE, A., AND C. DAVIDSON (2004): "Multidivisional firms, internal competition, and the merger paradox," *Canadian Journal of Economics*, 37(4), 951–977.

DANIEL, T. E., D. A. SEALE, AND A. RAPOPORT (1998): "Strategic play and adaptive learning in the sealed-bid bargaining mechanism," *Journal of Mathematical Psychology*, 42(2–3), 133–166.

Doganoglu, T., and J. Wright (2006): "Multihoming and compatibility," *International Journal of Industrial Organization*, 24(1), 45–67.

Evans, D. S. (2003): "The antitrust economics of multi-sided platform markets," *Yale Journal on Regulation*, 20(2), 325–381.

Ewerhart, C. (2006): "The effect of sunk costs on the outcome of alternating-offers bargaining between inequity-averse agents," *Schmalenbach Business Review*, 58(2), 184–203.

Fehr, E., and K. M. Schmidt (1999): "A theory of fairness, competition, and cooperation," *The Quarterly Journal of Economics*, 114(3), 817–868.

Friedman, J. (1971): "A non-cooperative equilibrium for supergames," *The Review of Economic Studies*, 38(1), 1–12.

Gabszewicz, J. J., D. Laussel, and N. Sonnac (2004): "Programming and advertising competition in the broadcasting industry," *Journal of Economics & Management Strategy*, 13(4), 657–669.

Gabszewicz, J. J., and X. Y. Wauthy (2004): "Two-sided markets and price competition with multi-homing," CORE Discussion Paper 2004/30.

Gupta, B., and G. Venkatu (2002): "Tacit collusion in a spatial model with delivered pricing," *Journal of Economics*, 76(1), 49–64.

Häckner, J. (1996): "Optimal symmetric punishments in a Bertrand differentiated products duopoly," *International Journal of Industrial Organization*, 14(5), 611–630.

Harrington, J. E. (1991): "The determination of price and output quotas in a heterogeneous cartel," *International Economic Review*, 32(4), 767–792.

Harrington, J. E., B. F. Hobbs, J. S. Peng, A. Liu, and G. Roth (2005): "Collusive game solutions via optimization," *Mathematical Programming (Series B)*, 104(2–3), 407–435.

Hong, J. T., and C. R. Plott (1982): "Rate filing policies for inland water transportation: an experimental approach," *The Bell Journal of Economics*, 13(1), 1–19.

Hotelling, H. (1929): "Stability in competition," *The Economic Journal*, 39(153), 41–57.

Huck, S., K. A. Konrad, and W. Müller (2004): "Profitable horizontal mergers without cost advantages: the role of internal organization, information and market structure," *Economica*, 71(284), 575–587.

Jullien, B. (2005): "Two-sided markets and electronic intermediaries," *CESifo Economic Studies*, 51(2–3), 233–260.

Kadan, O. (2007): "Equilibrium in the two-player, k-double auction with affiliated private values," *Journal of Economic Theory*, 135(1), 495–513.

Kind, J. K., T. Nilssen, and L. Sørgard (2005): "Financing of media firms: Does competition matter?," mimeo.

Krishna, V. (2002): *Auction Theory*. Academic Press, San Diego et al.

Kühn, K.-U. (2001): "Fighting collusion by regulating communication between firms," *Economic Policy*, 16(32), 167–204.

Kühn, K.-U., and X. Vives (1995): *Information Exchanges Among Firms and Their Impact on Competition*. Office for Official Publications of the European Communities, Luxemburg.

Lambertini, L., and M. Trombetta (2002): "Delegation and firms' ability to collude," *Journal of Economic Behavior & Organization*, 47(4), 359–373.

Levy, D. T., and J. D. Reitzes (1992): "Anticompetitive effects of mergers in markets with localized competition," *Journal of Law, Economics, & Organization*, 8(2), 427–440.

Lopomo, G., and E. A. Ok (2001): "Bargaining, interdependence, and the rationality of fair division," *The RAND Journal of Economics*, 32(2), 263–283.

Majerus, D. W. (1988): "Price vs. quantity competition in oligopoly supergames," *Economics Letters*, 27(3), 293–297.

Møllgaard, H. P., and P. B. Overgaard (2001): "Market transparency and competition policy," *Rivista di Politica Economica*, 91(4–5), 11–58.

——— (2006): "Transparency and competition policy," in *The Pros and Cons of Information Sharing*, ed. by M. Bergman, pp. 155–162. Swedish Competition Authority.

Muthoo, A. (1999): *Bargaining Theory with Applications*. Cambridge University Press, Cambridge et al.

Myerson, R. B., and M. A. Satterthwaite (1983): "Efficient mechanisms for bilateral trade," *Journal of Economic Theory*, 29(2), 265–281.

Nash, J. F. (1950): "The bargaining problem," *Econometrica*, 18(2), 155–162.

Nilsson, A. (1999): "Transparency and competition," SSE/EFI Working Paper Series in Economics and Finance No. 298.

Olaizola, N. (2007): "Cartel formation and managerial incentives," *Spanish Economic Review*, 9(3), 219–236.

Osborne, M. J., and A. Rubinstein (1990): *Bargaining and Markets*. Academic Press, San Diego et al.

Peitz, M. (2006): "Marktplätze und indirekte Netzwerkeffekte," *Perspektiven der Wirtschaftspolitik*, 7(3), 317–333.

Poolsombat, R., and G. Vernasca (2006): "Partial multihoming in two-sided markets," Discussion Paper in Economics No. 2006/10, The University of York.

Posada, P., and O. R. Straume (2004): "Merger, partial collusion and relocation," *Journal of Economics*, 83(3), 243–265.

Prechel, H., J. Boies, and T. Woods (1999): "Debt, mergers and acquisitions, institutional arrangements and change to the multilayered subsidiary form," *Social Sciences Quarterly*, 80(1), 115–135.

Puu, T. (2002): "Hotelling's 'Ice cream dealers' with elastic demand," *The Annals of Regional Science*, 36(1), 1–17.

Radner, R., and A. Schotter (1989): "The sealed-bid mechanism: an experimental study," *Journal of Economic Theory*, 48(1), 179–220.

Rapoport, A., and M. A. Fuller (1995): "Bidding strategies in bilateral monopoly with two-sided incomplete information," *Journal of Mathematical Psychology*, 39(2), 179–196.

Rasch, A. (2007): "Platform competition with partial multihoming under differentiation: a note," *Economics Bulletin*, 12(7), 1–8.

Roson, R. (2005a): "Platform competition with endogenous multihoming," FEEM Working Paper No. 20.05.

Roson, R. (2005b): "Two-sided markets: a tentative survey," *Review of Network Economics*, 4(2), 142–160.

Roth, A. E. (1995): "Bargaining experiments," in *Handbook of Experimental Economics*, ed. by J. Kagel, and A. E. Roth, vol. 1, pp. 253–348. Princeton University Press.

Rothschild, R. (1997): "Product differentiation and cartel stability: Chamberlin versus Hotelling," *The Annals of Regional Science*, 31(3), 259–271.

Salop, S. C. (1979): "Monopolistic competition with outside goods," *The Bell Journal of Economics*, 10(1), 141–156.

Satterthwaite, M. A., and S. R. Williams (1989): "Bilateral trade with the sealed bid k-double auction: existence and efficiency," *Journal of Economic Theory*, 48(1), 107–133.

Schultz, C. (2005): "Transparency on the consumer side and tacit collusion," *European Economic Review*, 49(2), 279–297.

Shelanski, H. A., and J. G. Sidak (2001): "Antitrust divestiture in network industries," *The University of Chicago Law Review*, 68(1), 1–99.

Spagnolo, G. (2005): "Managerial incentives and collusive behavior," *European Economic Review*, 49(6), 1501–1523.

Stahl, I. (1972): *Bargaining Theory*. Stockholm School of Economics, Stockholm.

Sundaram, R. K. (1996): *A First Course in Optimization Theory*. Cambridge University Press, Cambridge et al.

Timmins, P. V. (1986): "Divestiture as a remedy in private actions brought under Section 16 of the Clayton Act," *Michigan Law Review*, 84(7), 1579–1597.

Wachenheim, C. J., and E. A. DeVuyst (2001): "Strategic response to mandatory reporting legislation in the U.S. livestock and meat industries: Are collusive opportunities enhanced?," *Agribusiness*, 17(2), 177–195.

Wright, J. (2002): "Access pricing under competition: an application to cellular networks," *The Journal of Industrial Economics*, (3), 289–315.

Curriculum vitae

Personal details

Place of birth	Nuremberg, Germany
Date of birth	June 17, 1979
Nationality	German

Education

04/2006–03/2009	Ph.D. student University of Cologne, Germany Supervisor: Prof. A. Wambach, Ph.D. *Dissertation (Dr. rer. pol.)*
09/2005–03/2006	Visiting Ph.D. student Midi-Pyrénées School of Economics (M.P.S.E.) Université Toulouse 1 (Sciences Sociales), France
10/2004–03/2006	Ph.D. student University of Erlangen-Nuremberg, Germany Supervisor: Prof. A. Wambach, Ph.D.
09/2001-07/2002	Visiting student University of International Business and Economics (UIBE), Beijing, China
10/1999–03/2004	Studies in International Business Administration University of Erlangen-Nuremberg, Germany *Diplom*
09/1989–06/1998	Sigmund-Schuckert-Gymnasium, Nuremberg, Germany *Abitur*

Professional experience

Since 10/2004	Scientific employee University of Erlangen-Nuremberg, Germany, and University of Cologne, Germany
06–09/2004	Internship TWS Partners Munich, Germany
04–06/2004	Student employee Chair of Economic Theory University of Erlangen-Nuremberg, Germany
Summer 2001, Winter 2002/03, and Summer 2003	Tutor (microeconomics, undergraduate level) Chair of International Economic Relations University of Erlangen-Nuremberg, Germany
04–09/2001	Student employee Chair of Information Systems I University of Erlangen-Nuremberg, Germany
09–10/2000	Internship MAN Vehículos Industriales España (After Sales) Madrid, Spain
08–09/1999	Internship MAN UK (Boat and Industrial Engines) Swindon, UK
06–09/2004	Community service Malteser Hilfsdienst Nuremberg, Germany