On Equilibria in Insurance Markets with Asymmetric Information

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1 Introduction

Existence and efficiency of equilibria in markets with asymmetric information have been studied in great depth ever since the seminal Akerlof (1970) article. Whether asymmetric information per se has a strong impact on market equilibria thereby crucially depends on whether the asymmetry of information pertains to private or, as in Akerlof (1970), common values. When values are private, that is when the payoff that the uninformed party receives from a given contract does not depend on the other party's private information, competitive equilibria exist and are efficient under fairly mild assumptions.[1] However, in markets with common values, a competitive equilibrium might not exist at all. This is the result in the famous Rothschild and Stiglitz (1976) model on competitive insurance markets with adverse selection. Rothschild and Stiglitz (1976) show that when insurers offer contracts to consumers that have private information about their risk type which is payoff-relevant for a given insurance contract, an equilibrium in pure strategies might fail to exist: If the share of low risks is high, a profitable pooling contract would be preferred by both high and low risk types over the candidate separating, zero-profit-making Rothschild-Stiglitz (RS) contracts. However, a pooling contract cannot be tendered in equilibrium as insurers would try to cream skim low risks.

This equilibrium nonexistence result is not merely a theoretical oddity. In fact, equilibrium nonexistence is sometimes brought forward as an efficiency reason for regulation of insurance markets, notably social insurance. However, equilibrium inexistence itself is not a sensible reason to call for regulation: It cannot

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[1] See Pouyet, Salanié, and Salanié (2008). An earlier work with more restrictive assumptions is Fagart (1996).

be determined whether regulation, resulting in a particular market allocation, improves efficiency in a market where it is not even clear what the market allocation is without regulation. Rather, the equilibrium inexistence result points out that it is necessary to examine whether the RS model is the correct model to describe behavior in insurance markets and consequently to think of alternative models. Whether adverse selection provides an efficiency reason for regulation might then be determined based on equilibrium properties in these alternative models.

Not surprisingly, the Rothschild and Stiglitz (1976) result has spurred extensive research. In chapter 2, the nonexistence problem and ensuing debate is reviewed in more detail. In terms of the resulting market allocation, the contributions following RS can with a few exceptions be classified in two broad categories: Models that yield the RS allocation, even for the case in which there is no equilibrium in the original RS model, and models that yield the so-called Wilson-Miyazaki-Spence (WMS) allocation. The crucial difference between these allocations is that the RS allocation is only second-best efficient if an equilibrium exists in the original RS model, whereas the WMS allocation is generally second-best efficient. Thus, there is an efficiency reason for regulation due to adverse selection if insurance markets are considered correctly captured in models of the first, but not the second class. However, although several modifications to the RS model have been brought forward, there is still no generally agreed upon solution. This is because proposed solutions either lack sound game-theoretic foundation, introduce somewhat arbitrary changes to the RS model or impose exogenous constraints. Chapters 3 and 4 therefore propose solutions to the RS puzzle that tackle these problems.

The third chapter introduces a dynamic model that allows insurers to withdraw contracts in reaction to their competitors.[2] This is the logic suggested in

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[2] The model in this chapter is joint work with Achim Wambach. An earlier version was presented at the following conferences: *36th Conference of the European Association for Research in Industrial Economics* (Ljubljana, Slowenia, 2009), *XIV. Spring Meeting of Young Economists* (Istanbul, Turkey, 2009), *24th Annual Meeting of the European Economic Assocation* (Barcelona, Spain, 2009), *Annual Meeting of the European Group of Risk and Insurance Economists* (Bergen, Norway, 2009), *Annual Meeting of the German*

Wilson (1977)'s "anticipatory equilibrium concept" that, in spite of departing from Nash equilibrium, to date is one of the most appealed to solutions. In our model an equilibrium with the WMS allocation always exists, thus providing a game-theoretic foundation for the Wilson (1977) equilibrium. However, jointly profit-making contracts can as well be sustained in equilibrium. We then allow for entry and show that the WMS allocation is the unique equilibrium allocation under entry.

In the fourth chapter we endogenize insurer capital: Instead of being assumed to be exogenously endowed with large financial assets as in the RS model, insurers can choose their level of capital and consequently go insolvent.[3] With this endogenous insolvency risk, an equilibrium with the WMS allocation always exists. Interestingly, solvency regulation might have unintended consequences as strong capital requirements impede the existence of a second-best efficient equilibrium.

Whereas the consequences of asymmetric information on competition in insurance markets with adverse selection have thus been thoroughly analyzed, little is known about the impact of asymmetric information on oligopoly behavior.[4] In the fifth chapter we therefore depart from the assumption of competition and analyze the impact of asymmetric information on the ability of insurance firms to collude.[5] As insurance markets tend to be highly concentrated and there have been several antitrust cases, notably the recent German case in which 17 insurers for industrial insurance were fined € 140 million in 2010 on the grounds of collusive behavior, an analysis of factors that influence collusion in insurance markets seems warranted.[6] It is shown that asymmetric information destabi-

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Economic Association (Magdeburg, Germany, 2009).

[3] The model in this chapter is joint work with Achim Wambach. An earlier version of the model was presented at the following conferences: *10th Econometric Society World Congress* (Shanghai, China, 2010), *25th Annual Meeting of the European Economic Association* (Glasgow, UK, 2010), *World Risk and Insurance Economics Conference* (Singapore, 2010), *Annual Meeting of the German Association for insurance science* (Düsseldorf, Germany, 2010).

[4] An exception is Olivella and Vera-Hernández (2007) in which horizontally differentiated health plans are considered.

[5] The model in this chaper is joint work with Alexander Rasch.

[6] According to Buzzacchi and Valletti (2005), concentration indices for the top 5 insurance companies in non-life business in Europe range from 27 % in Germany to 89% in Finland.

lizes collusion, however, this is not a result of asymmetric information per se, but stems from the common value property of the market. Thus, on a general note, we identify a new factor that destabilizes collusion: payoff-relevant private information.

2 Competitive Insurance Markets with Adverse Selection

This chapter lays out the Rothschild-Stiglitz model and discusses the literature on existence of a pure strategy equilibrium. Besides the Rothschild-Stiglitz contracts, the Wilson-Miyazaki-Spence contracts are introduced.

2.1 The Rothschild-Stiglitz model

In their seminal work on competition in adverse selection insurance markets, Rothschild and Stiglitz (1976) (hereafter RS) analyze the market by defining a particular notion of equilibrium. In this section, we spell out the game behind the analysis in RS. While RS consider single contract offers by firms, we allow insurers to offer contract menus. Consider the following game:

There is a continuum of individuals with mass 1. Each individual faces two possible states of nature: In state 1, no loss occurs and the endowment is w_{01}, in state 2 a loss occurs and the endowment is w_{02} with $w_{01} > w_{02} > 0$. There are two types of individuals, an individual may be a high risk type (H) with loss probability p^H, or a low-risk type (L) with loss probability p^L, with $0 < p^L < p^H < 1$. Insurance is provided by firms in the set $F := \{1, ..., f, ...n\}$. Firms do not know, ex ante, any individual's type. If an individual buys insurance, then the endowment $\omega_0 = (w_{01}, w_{02})$ is traded for another state-contingent endowment

$\omega = (w_1, w_2)$, we say the individual buys insurance contract ω. The set of feasible contracts, Ω, is given by $\Omega := \{(w_1, w_2) \,|\, (w_1 \geq w_2 > 0\}$ where $w_1 < w_2$ is ruled out for moral hazard considerations.

The expected utility of a J-type individual, $J \in \{H, L\}$ from chosing a contract $\omega \in \Omega$ is abbreviated by $u^J(\omega) := (1 - p^J)v(w_1) + p^J v(w_2)$ where v is a strictly increasing, twice continuously differentiable and strictly concave von Neumann-Morgenstern utility function. Note that, since consumers only differ in risk, the single-crossing property holds naturally as $\frac{(1-p^L)v'(w_1)}{p^L v'(w_2)} > \frac{(1-p^H)v'(w_1)}{p^H v'(w_2)}$.[7]

The RS model is a screening game, i.e. the timing is the following:

Stage 0: The risk type of each individual is chosen by nature. Each individual has a chance of γ, $0 < \gamma < 1$ to be a H-type, and of $(1-\gamma)$ to be a L-type.

Stage 1: Each firm $f \in F$ offers a finite set of contracts $\Omega^f \subset \Omega$.

Stage 2: Individuals choose an insurance contract among offered contracts.[8]

In this game, the only candidate equilibrium contracts are separating and separately zero-profit making. In particular, candidate equilibrium contracts are such that the high risk type is fully insured at her fair premium, and the low risk type is partially insured at her respective fair premium such that contracts are incentive compatible. These contracts are the famous Rothschild-Stiglitz contracts, shown below in Figure 2.1. The straight lines correspond to the H-type and L-type fair insurance contracts with the dashed one indicating the fair insurance contracts for an average risk.

The intuition why an equilibrium in pure strategies, if it exists, yields the RS al-

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7   Wambach (2000), Smart (2000) and Villeneuve (2003) introduce an additional dimension of asymmetric information by assuming that consumers furthermore differ in wealth/risk aversion. With two dimensions of asymmetric information, the single crossing property may be violated. Wambach (2000), Smart (2000) and Villeneuve (2003) all consider single contract offers and show that due to violation of single crossing there might be contracts with positive profits offered in equilibrium. However, Snow (2009) argues that this is simply a consequence of restricting insurer strategies to single contract offers as contracts with positive profits cannot be tendered in equilibrium when contract menus can be offered.
8   Insurance purchase is exclusive, i.e. a consumer is assumed to buy at most one insurance contract at one firm.
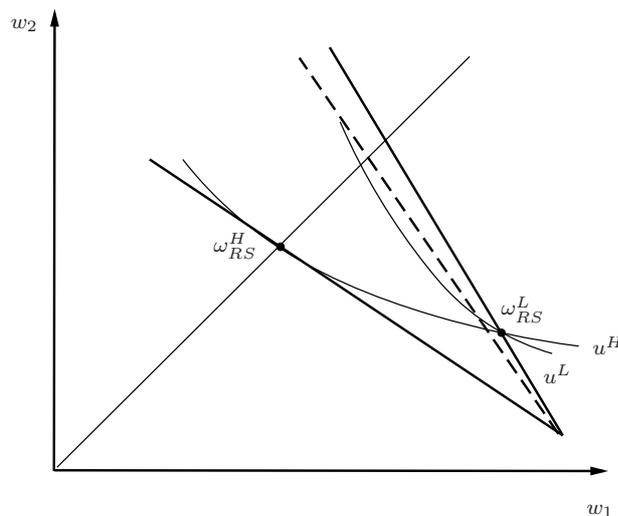
Figure 2.1: RS contracts

location, is the following: Pooling cannot be an equilibrium, as for any possible pooling contract that yields nonnegative profits on the whole population, a profitable deviation exists in either attracting all customers, if profits on each type in the pooling contract are positive, or cream-skimming low risks, if high risks are cross-subsidized. In a separating equilibrium, competition always drives the H-type contract to but not beyond the full insurance contract at the fair H-type premium, where cross-subsidization is again not possible due to cream-skimming deviations. Finally, competition drives the L-type contract to the contract that, yielding zero profits, is the best possible contract for L-types given the incentive compatibility constraint on H-types. Note that the RS contracts can be obtained as the solution to the following maximization problem:

$$\max_{\omega^L, \omega^H} \ u^L(\omega^L)$$
$$\text{s.t.}$$
$$u^H(\omega^H) \geq u^H(\omega^L)$$
$$(1 - p_H)(w_{01} - w_1^H) + p_H(w_{02} - w_2^H) \geq 0$$
$$(1 - p_L)(w_{01} - w_1^L) + p_L(w_{02} - w_2^L) \geq 0$$

i.e. L-type utility is maximized subject to incentive compatibility for the H-type

and constraints specifying that each contract is separately zero profit-making. From the above maximization problem, it is easy to see that RS contracts are independent of the shares of risk types in the population. Then, an equilibrium in pure strategies with the RS allocation might not exist for the following reason: If the share of H-types is low, there are profitable pooling contracts or (cross-subsidizing) contract menus that are preferred by both risk types over their respective RS contracts. However, pooling, or a cross-subsidizing contract menu cannot be tendered in equilibrium as cream-skimming would occur. Hence, an equilibrium in pure strategies might fail to exist if the share of high risk types is low.

In Figure 2.1, the RS contracts can be overturned by a simple pooling deviation. Note that even if there is no pooling contract that is profitable and preferred by both risk types, still a cross-subsidizing contract menu can be profitable and preferred to the RS contracts, i.e. the equilibrium inexistence problem is aggravated when firms offer contract menus as in our specification instead of single contracts as in the original RS model. Furthermore, from the above discussion it is also clear that RS contracts are not necessarily second-best efficient: If there exists a profitable deviation overturning the RS contracts, the resulting allocation would be more efficient than the RS allocation.[9] We can summarize the above results as follows:

Result 2.1 (Rothschild-Stiglitz) *Any equilibrium in pure strategies yields the RS allocation. However, an equilibrium in pure strategies fails to exist if the share of high risk types is low. Furthermore, the RS allocation is not generally second-best efficient.*

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9   A second-best efficient allocation is Pareto efficient among those that satisfy self-selection conditions and resource constraints, see Crocker and Snow (1985). The resource constraint here translates to nonnegative profits on the whole population.

2.2 The equilibrium nonexistence debate

From the review of the Rothschild-Stiglitz model, we have established that an equilibrium in pure strategies might not exist. In an application on their general existence theorem of mixed-strategy equilibria in discontinuous games, Dasgupta and Maskin (1986) prove existence of a mixed-strategy equilibrium in the Rothschild-Stiglitz model and partially characterize the equilibrium: H-types are always fully insured, and firms mix between jointly zero-profit making contract menus.[10] However, although mixed strategies are a meaningful concept in several economic environments, an equilibrium in mixed strategies in the insurance market does not seem particularly appealing: For any contract menu offered with some probability, once it is offered, competitors would want to change their own contracts. Then it is not clear why insurers should only act simultaneously and not be allowed to react to their competitors contract offers once these are observed; or, put differently, this raises the question of when and how often insurers are allowed to modify contracts and thus react to their competitors.

In consequence, rather than settling on a mixed-strategy equilibrium, it seems sensible to rethink models of competition in adverse selection insurance markets. Early works by Wilson (1977) and Riley (1979) map the idea of a reaction to competitors by modifying the equilibrium concept to include expectations about competitor behavior. Instead of modifying the equilibrium concept, a second vein of research explicitly models dynamic insurance market interactions (Hellwig 1987; Engers and Fernandez 1987; Jaynes 1978; Asheim and Nilssen 1996; Ania, Tröger, and Wambach 2002). A third strand takes a different approach and does not include dynamics, but changes the contract or insurer characteristics (Inderst and Wambach 2001; Faynzilberg 2006; Picard 2009). Below, we

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10 For the screening version of Spence (1973)'s signalling model of education that exhibits a similar structure as the RS model, Rosenthal and Weiss (1984) derive an equilibrium in mixed strategies. A somewhat unsatisfying characteristic of equilibrium is that although there is no profitable deviation for existing firms in the market, an entrant can earn positive expected profits.

will discuss these contributions in more detail.[11]

2.2.1 Modification of the equilibrium concept

Wilson (1977) proposes the "anticipatory equilibrium" concept, or E2 equilibrium. In this concept, an expectation rule is imposed such that "each firm assumes that any policy will be immediately withdrawn which becomes unprofitable after that firm makes its own policy offer". Wilson (1977) considers single contract offers and shows that the anticipatory equilibrium concept leads to a pooling allocation in which low risk utility is maximized subject to an overall zero-profit condition, the so-called Wilson pooling contract. This pooling contract is tendered in equilibrium as with the above expectation rule, if a cream-skimming deviation is attempted, the Wilson contract would be assumed to be withdrawn such that the deviator expects all types to choose the deviating contract, which in turn renders it unprofitable.

Miyazaki (1977) and Spence (1978) extend the analysis to contract menus and show that the anticipatory equilibrium concept results in an allocation with separating, cross-subsidizing, jointly zero-profit making contracts that are second-best efficient, the Wilson-Miyazaki-Spence (WMS) contracts. Formally, consider the following maximization problem:

$$\max_{\omega^L, \omega^H} \; u^L(\omega^L)$$
$$\text{s.t.}$$
$$u^H(\omega^H) \geq u^H(\omega^L)$$
$$u^H(\omega^H) \geq u^H(\omega^H_{RS})$$

$$\gamma_H[(1-p_H)(w_{01} - w_1^H) + p_H(w_{02} - w_2^H)]+$$
$$(1-\gamma_H)[(1-p_L)(w_{01} - w_1^L) + p_L(w_{02} - w_2^L)] \geq 0$$

where ω^H_{RS} denotes the H-type RS contract. The above maximization problem

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11 In this review, we discuss literature that in the spirit of RS uses noncooperative game-theoretic methods and do not consider cooperative concepts as e.g. Lacker and Weinberg (1999) or RS economies in general-equilibrium frameworks as e.g. in Dubey and Geanakoplos (2002) or Bisin and Gottardi (2006).

has a unique solution.[12]

Definition 2.1 *The unique solution to the above maximization problem are the Wilson-Miyazaki-Spence contracts, denoted by ω_{WMS}^H and ω_{WMS}^L.*

Hence, the WMS contracts are obtained by maximizing U-type utility subject to an overall zero profit condition allowing for cross-subsidization. Note that this implies that the WMS contracts are second-best efficient.

Remark 2.1 (Crocker and Snow 1985) *The WMS contracts are second-best efficient.*

When the second constraint on H-type utility is binding, the WMS contracts correspond to the RS contracts. Note that since in the RS model there is no equilibrium if RS contracts can be overturned by a profitable deviating menu, an equilibrium exists in the RS model exactly when the WMS contracts coincide with the RS contracts, i.e. an equilibrium with the RS allocation exists if and only if the RS contracts are second-best efficient. When the second constraint on H-type utility is not binding, WMS contracts are such that the fully-insured H-types are subsidized by the partially insured L-types. The WMS contracts in this case are shown below in Figure 2.2. The dotted curve gives all L-types contracts that combined with a full insurance contract that lies on the same H-type indifference curve as the L-type contract jointly yield zero-profits if taken out by the whole population. Among these L-type contracts, the WMS L-type contract then is the contract that maximizes L-type utility.

At the heart of Wilson (1977)'s anticipatory equilibrium is the notion that contracts, if unprofitable, will be withdrawn. In Riley (1979)'s "reactive equilibrium" concept, instead of contract withdrawal, insurers expect competitors to make *new* contract offers, i.e. react by adding contracts. To be precise, "a set of offers is a reactive equilibrium if, for any additional offer which generates an expected gain to the agent making the offer, there is another which yields a gain to a second agent and losses to the first...". It is easy to see that the reactive

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12 See e.g. Asheim and Nilssen (1996).
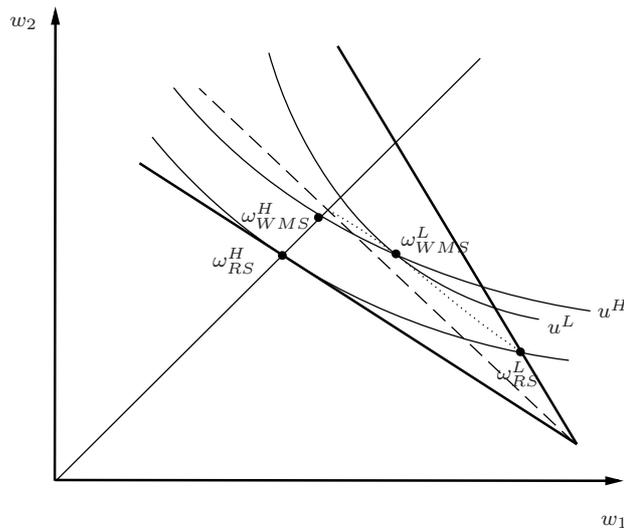
Figure 2.2: WMS contracts

equilibrium concept yields the RS allocation: For any profitable deviation on the RS contracts with a pooling contract or contract menu, the expectation rule implies that firms anticipate the addition of a cream-skimming contract by another competitor, rendering the deviation unprofitable.

2.2.2 Dynamic market interaction

The notion that insurance markets are more dynamic than in the RS model has been, instead of imposing expectation rules, translated to explicitly modelling dynamic insurance market interactions between firms. Engers and Fernandez (1987), besides generalizing Riley (1979)'s reactive equilibrium notion, provide a game-theoretic foundation for the reactive equilibrium with the following game: First, insurers set contracts. After observation of competitors' offers, firms may repeatedly add new contracts to their existing offers such that there is no exogenously determined last mover. If no more contracts are added, consumers choose contracts. This implements the Riley logic such that an equilibrium with the RS allocation exists as again cream-skimming on deviations renders any deviation unprofitable. However, with the same logic, different allocations than RS can be sustained as equilibrium allocations.

In Hellwig (1987), instead of adding contracts, insurers may decide to not fulfill a contract. In particular, firms first offer a single contract, consumers then choose their insurance contract in the second stage, and insurers can in a third stage decide not to fulfill the contract already chosen by a consumer. Note that, since insurers offer single contracts, this type of contract withdrawal corresponds to exit from the market. Hellwig (1987) shows that the Wilson pooling contract corresponds to a stable equilibrium of this three-stage game, as cream-skimming would lead insurers to exit the market. However, when allowing for contract menus and individual contract withdrawal, the WMS contracts do not constitute equilibrium contracts in Hellwig (1987)'s game as any firm would have an incentive to withdraw the high risk contract.

Whereas the above contributions add stages to the game in which insurers can revise their set of contract offers, Jaynes (1978) relaxes the assumption that contracts are exclusive and considers firm interaction in the form of sharing information. Jaynes (1978) argues that this leads to some firms offering the Wilson pooling contract which share information among them, and some firms offer contracts at the fair H-type premium rate who do not share information. Hellwig (1988) translates the idea into the following game: First, firms offer contracts and can attach an exclusivity condition on it. Then, consumers choose combinations of contracts. In the third stage, insurers decide what customer information to share with which insurers, and in the fourth stage firms receive information and decide on enforcing exclusivity conditions.[13] Hellwig (1988) shows that since insurers in the last stages can strategically condition information sharing and enforcement of exclusivity conditions on stage 1 offers, the equilibrium proposed in Jaynes (1978) corresponds to a sequentiel equilibrium of the game and cannot be upset by a cream-skimming offer: High risks would prefer to supplement their insurance purchase by this cream-skimming offer, rendering it unprofitable. As the firm offering this cream-skimming contract would not receive information from other firms, it could not enforce exclusivity

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13 Ales and Maziero (2009) and Attar, Mariotti, and Salanié (2010) consider non-exclusive contracting in RS environments without information sharing. In both models, a pure strategy equilibrium may fail to exist.

conditions to prevent high risks from taking the contract.

In Asheim and Nilssen (1996), insurers can renegotiate contracts with their own customers on a nondiscriminatory basis. Asheim and Nilssen (1996) show that WMS is the unique equilibrium allocation for the following reason: a cream-skimming contract that by construction of WMS contracts lies outside the efficiency region, would be renegotiated to an efficient contract that yields high risks higher utility than their respective WMS contract, even if all high risk types choose this contract. Then, high risks would choose the deviating contract, rendering it unprofitable. Note that the overall result depends on the assumption that insurers cannot renegotiate contracts individually, which seems unrealistic once consumers have chosen an insurance contract.

In a methodically different approach, Ania, Tröger, and Wambach (2002) model dynamics in insurance markets using evolutionary game theory. Insurers do not have perfect knowledge about the market and imitate successful behavior, i.e. they copy the most profitable contract on the market and in addition, they experiment with their own contracts. RS contracts are the long run outcome of the evolutionary game if insurers experiment only locally. This is because even if a pooling contract is preferred, the RS contracts cannot be destabilized by small changes in contracts, whereas pooling contracts can.

2.2.3 Changes to contract or insurer characteristics

Instead of introducing additional stages to the RS game to model dynamics, another strand of literature modifies the assumptions about insurer characteristics or initial contracts offered. In Inderst and Wambach (2001), insurers face capacity constraints such that the amount of offered contracts at an insurer is limited and consumers might be rationed. When consumers have search costs, the risk of being rationed and thus to incur search costs can lead to the 'wrong' type of consumers turning up at insurers. In particular, consider a pooling deviation on RS contracts: As the gain in utility at this deviating contract is higher

for high risk types due to the single-crossing property, for search cost sufficiently but not too high only high risks would choose the deviating contract, rendering it unprofitable. Thus, capacity constraints sustain an equilibrium with the RS allocation.[14]

Instead of capacity constraints, in an unpublished working paper Faynzilberg (2006) assumes that the financial resources of insurers are exogenously limited. Then, the expected utility that a customer receives from a contract might not correspond to the expected utility derived from the parameters of the contract as the insurer, due to lack of financial resources, might not be able to fulfill the contract. The analysis in Faynzilberg (2006) suggests that this leads to an equilibrium with the WMS allocation if the financial constraint is severe: Since high risk type utility from the WMS contract is lowered if the contract is not cross-subsidized by low risks, a cream-skimming deviation might be attractive to high risks as well.

In a recent working paper, in contrast to the two previous works that impose exogenous constraints on insurers, Picard (2009) endogenizes the contract structure by lifting the assumption that insurance contracts are nonparticipating. In particular, insurers may choose to offer contracts in which customers share the profits or losses of the insurer. Fully participating contracts are implemented in a mutual. Picard (2009) shows that this sustains the WMS allocation in equilibrium as a cream-skimming deviation imposes losses on the high risk type WMS contract such that high risks as well prefer the deviating contract.[15]

Note that all three models share the feature that the actual expected utility that customers receive from a contract at an insurer is modified and customers take this into account when choosing an insurance contract.

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14 Guerrieri, Shimer, and Wright (2010) combine adverse selection with competitive search theory. Principals post terms of trade, privately informed agents direct their search, and principals and agents are then matched bilaterally. An equilibrium exists in an RS-type application as with bilateral matching, a pooling offer to attract a cross-section of types is less attractive to good types for the following reason: the more agents search an offer, the less likely is a match. Now it is exactly the good types that are discouraged from searching as their outside option, the separating contract, is higher. Note that this works similarly to the above discussed capacity constraints.

15 Picard (2009) also analyzes the n-type case and thereby shows coexistence of mutuals and stock insurers.

2.2.4 Synthesis

Even more than 30 years after its first publication, the RS result that an equilibrium in pure strategies might not exist in a competitive insurance market with adverse selection is still puzzling. To solve the problem, besides early work on modifications of the equilibrium concept, research has either introduced dynamics into the RS model or modified assumptions about insurer or contract characteristics. In each of these different strands of research, there are models that yield the RS allocation, as well as models that yield the WMS allocation. The RS allocation has the feature intuitively associated with competition that each contract is separately zero-profit making; yet it is not generally second-best efficient. The WMS allocation is second-best efficient, however precisely because it is second-best efficient it requires cross-subsidization in equilibrium, which seems counterintuitive in a competitive market. This highlights again the crucial issue that is at the core of the original equilibrium nonexistence result: competition and efficiency do not necessarily go hand in hand in this market.

In extensions of the RS model these different allocations can be sustained in equilibrium as, put broadly, RS contracts cannot be overturned if a deviation on *all* risks is rendered unprofitable because a deviator ends up with high risks, and the WMS allocation can be sustained if a deviator can not offer contracts such that *only* low risks are attracted.

However, although the review shows that extensive research has been conducted to settle the equilibrium nonexistence problem, the debate is still open as there is no generally agreed upon solution. This stems from several reasons: Firstly, one of the most appealed to solutions, the Wilson concept, lacks sound game-theoretic foundation. Another issue is that some modifications to the RS model seem rather arbitrary: In e.g. Asheim and Nilssen (1996), it is not clear why when insurers can renegotiate, they cannot renegotiate on an individual basis. A related criticism is that with the exception of Picard (2009) who endogenizes contract characteristics, models impose exogenous constraints: In e.g. Inderst and Wambach (2001), the RS allocation is sustained as firms are assumed to be

capacity constrained, however if insurers were to choose their capacity, limited capacity would not be the outcome of the generalized game.

In light of the above discussion, the following two chapters aim to close the RS debate by providing solutions that address these problems: Chapter 3 provides a game-theoretic foundation for the Wilson concept. Chapter 4 considers an endogenous framework in which insurers can choose their capital level.

3 A Game-Theoretic Foundation for the Wilson Equilibrium

This chapter extends the RS model in the spirit of Wilson (1977)'s "anticipatory equilibrium" by introducing an additional stage in which initial contracts can be withdrawn repeatedly after observation of competitors' contract offers. We show that an equilibrium always exists where consumers obtain their respective Wilson-Miyazaki-Spence (WMS) contract. Jointly profit-making contracts can also be sustained as equilibrium contracts. However, the WMS allocation is the unique equilibrium allocation under entry.

3.1 Introduction

To date, one of the most appealed to solutions to the RS equilibrium nonexistence problem is still the Wilson anticipatory equilibrium concept. However surprisingly, despite its appeal the Wilson concept has never been formally modelled, i.e. although some research on the equilibrium inexistence problem yields the WMS allocation, the mechanisms in these models are quite different from the idea of withdrawing contracts in reaction to competitors.

The present chapter spells out the idea behind the Wilson "anticipatory equilibrium concept" by introducing an additional stage into the RS model in which firms can withdraw individual contracts (repeatedly) before consumers make their choice but after observing the contract offers of competitors. We show that an equilibrium always exists where every consumer obtains her respective WMS contract. Intuitively, the possibility of contract withdrawal prevents

cream-skimming deviations that upset the WMS contracts in the original RS set-up. However, to sustain the WMS equilibrium, not only the WMS, but also a continuum of low risk contracts as well as the RS contracts have to be on offer as latent contracts. The reason is that the possibility to withdraw contracts allows for sophisticated deviating strategies that are prevented by latent contracts that, off the equilibrium path, attract low risks away from such possible deviations.[16]

We show moreover that, besides the WMS contracts, profit-making contracts can also be enforced as equilibrium contracts as the possibility to retract contracts provides firms with adequate threat points. More generally, contract withdrawal leads to a multiplicity of equilibrium allocations. This multiplicity remains if, instead of only considering contract withdrawal, we allow for the addition of contracts in the second stage. We then extend the game to allow for entry as would be expected in a model of a competitive market. Then, positive profits cannot be sustained in equilibrium. More strongly, the WMS equilibrium is generically unique under entry.

There is a small literature where contract withdrawal is added to a market with adverse selection. This literature differs from the present work in that while we allow the withdrawal of individual contracts to model the logic behind the Wilson equilibrium, contract withdrawal in the literature so far implies exit from the market, i.e. only complete contracts withdrawal. Hellwig (1987), as discussed in chapter 2, only considers exit and the WMS equilibrium cannot be sustained when insurers offer contract menus and can withdraw individual contracts in his game in which the offered contract can be withdrawn after insurees choose. A study with a similar timing structure as ours is Netzer and Scheuer (2008). In a model of moral hazard without commitment, Netzer and Scheuer (2008) model competition after unobservable effort choice such that

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16 Latent contracts are not new in adverse selection environments. Attar, Mariotti, and Salanie (2009) model nonexclusive competition in an adverse selection market. In their model, infinitely many contracts need to be issued as latent contracts to sustain the equilibrium allocation as well.

firms offer contract menus and can, after observation of competitor's offers, decide to exit the market before consumers choose contracts. Again, contrary to our model, the restriction to exit does not allow to withdraw individual loss-making contracts. This is not a minor point but relates to a more general problem: why should a firm in a competitive market offer a loss-making contract. We show that, even when individual contracts can be withdrawn, firms may offer loss-making contracts in a competitive market.

3.2 The model

The set-up closely follows Rothschild and Stiglitz (1976) and Wilson (1977): There is a continuum of individuals with mass 1. Each individual faces two possible states of nature: In state 1, no loss occurs and the endowment is w_{01}, in state 2 a loss occurs and the endowment is w_{02} with $w_{01} > w_{02} > 0$. There are two types of individuals, an individual may be a high risk type (H) with loss probability p^H, or a low-risk type (L) with loss probability p^L, with $0 < p^L < p^H < 1$. Insurance is provided by firms in the set $F := \{1, ..., f, ...n\}$. Firms do not know, ex ante, any individual's type. If an individual buys insurance, then the initial endowment $\omega_0 = (w_{01}, w_{02})$ is traded for another state-contingent endowment $\omega = (w_1, w_2)$; we say the individual buys insurance contract ω. The set of feasible contracts, Ω, is given by $\Omega := \{(w_1, w_2) \,|\, w_1 \geq w_2 > 0\}$ where $w_1 < w_2$ is ruled out for moral hazard considerations.

The expected utility of a J-type individual, $J \in \{H, L\}$ from chosing a contract $\omega \in \Omega$ is abbreviated by $u^J(\omega) := (1 - p^J)v(w_1) + p^J v(w_2)$ where v is a strictly increasing, twice continuously differentiable and strictly concave von Neumann-Morgenstern utility function.

The timing of the game is as follows: First, firms set contracts simultaneously and observe their competitors' contract offers. Then, firms can withdraw contracts potentially repeatedly for several rounds whereby firms observe their competitors remaining contract offers after each round. Contract withdrawal is possible as long as at least one contract was withdrawn by any firm in the

previous round. After contract withdrawal ends, consumers make their contract choice. Formally, the game proceeds as follows:

Stage 0: The risk type of each individual is chosen by nature. Each individual has a chance of γ, $0 < \gamma < 1$ to be a H-type, and of $(1-\gamma)$ to be a L-type.

Stage 1: Each firm $f \in F$ offers a set of contracts $\Omega_1^f \subset \Omega$. The offered sets are observed by all firms before the beginning of the next stage.

Stage 2: Stage 2 consists of $t = 1, 2, ...$ rounds. In each round t, each firm $f \in F$ can withdraw a set from its remaining contracts. After each round, firms observe the remaining contract offers of all firms. Denote by $\Omega_{2,t}^f$ firm f's contract set on offer at the end of t. For notational convenience, we denote $\Omega_1^f := \Omega_{2,0}^f$. If, for any t, $\Omega_{2,t}^f = \Omega_{2,t-1}^f$ for all $f \in F$, this stage ends. Denote the final round in stage 2 by $\hat{t}$, i.e. $\Omega_{2,\hat{t}}^f = \Omega_{2,\hat{t}-1}^f$ for all $f \in F$.

Stage 3: Individuals choose among the remaining contracts $\bigcup_F \Omega_{2,\hat{t}}^f$ or remain uninsured.

Before proceeding, let us discuss the difference of our setup to Rothschild-Stiglitz and how this implements the Wilson concept: The Rothschild-Stiglitz game corresponds to stages $0, 1$ and 3. In this reduced game, a pooling contract or more generally cross-subsidizing contracts cannot be sustained as equilibrium contracts as insurers would always try to cream skim low risks. In Wilson's "anticipatory equilibrium" concept, such cream skimming deviations are not profitable because the expectation rule is that cross-subsidized contracts at non-deviating insurers would be withdrawn since they become unprofitable after introduction of the cream-skimming contract. We implement this concept by adding stage 2. However, when instead of imposing an expectation rule, firms are explicitly allowed to withdraw contracts after observation of competitor's contract offers, for the Wilson reasoning to hold in a game with contract menus contract

withdrawal has to end endogenously as in our model specification: with fixed withdrawal rounds, as e.g. only one round of contract withdrawals, a single firm would always be able to profitably deviate by withdrawing a cross-subsidized contract in the last round.

When stage 2 ends after round $\hat{t}$ and contract $\omega_j^f \in \Omega_{2,\hat{t}}^f$ is taken out by a mass of individuals λ_j^f among which the share of H-types is σ_j^f, then the expected profit of firm $f \in F$ is:

$$\pi^f = \int_{\Omega_{2,\hat{t}}^f} \lambda_j^f \left[(w_{01} - w_{j,1}^f) - (p^H \sigma_j^f + p^L(1 - \sigma_j^f))(w_{j,2}^f - w_{02} + w_{01} - w_{j,1}^f) \right] d\omega$$

As we did not restrict the sets of contract offers in stage 1 to be finite, stage 2 does not necessarily end. For $t \to +\infty$, we specify that firms make zero (expected) profits. Let us stress that it is solely out of simplicity that we do not restrict the set of feasible contracts Ω and hence do not assume contract offers to be finite such that stage 2 does not necessarily end. All our results hold if instead of Ω as defined above we would consider a discrete contract grid and thus a finite number of stage 1 contract offers.

3.3 Equilibrium with WMS allocation

We will show that the WMS allocation can be sustained as equilibrium allocation, however, in order to prevent such deviations, latent contracts have to be offered alongside the WMS contracts to support the WMS equilibrium.

Proposition 3.1 *There exists a symmetric equilibrium where every individual obtains her respective WMS contract in stage 3.*

Proof See Appendix. ∎

Consider the following firm strategy: In stage 1, firms offer the WMS contracts and additionally the RS contracts as well as a continuum of contracts that lie

on the L-type fair insurance line and give the L-type a lower expected utility
than her WMS contract but higher expected utility than her RS contract. We
name this continuum of contracts 'LR contracts'. These contracts are shown in
Figure 3.1.
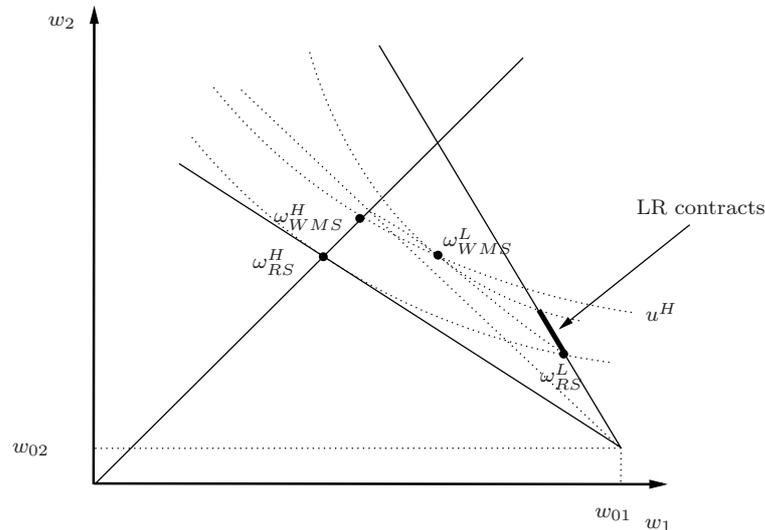
Figure 3.1: Contracts on offer in equilibrium

Then, in stage 2, after each round t, each firm computes the hypothetical profit
it would make if stage 2 ended after round t and, if it makes a loss, withdraws
the loss-making contract(s), but does not withdraw any contracts if it makes
zero expected profits.

This strategy supports the WMS allocation for the following reasoning: A simple
cream-skimming deviation is prevented by withdrawing WMS contracts as they
become unprofitable. Similarly, as stage 2 ends endogenously, a deviation that
involves the withdrawal of the H-type WMS contract in some round is prevented
as all other firms withdraw the loss-making H-type WMS contract subsequently.
Furthermore, a deviation aimed at forcing firms to withdraw WMS contracts and
making a positive profit on e.g. a pooling contract is prevented by withdrawing
those contracts from the LR contracts that would be taken up by H-types and
hence be loss-making, but leaving exactly those that would not be taken up
by H-types but only by L-types and hence cream-skim the L-types from any
deviating contract (menu). This type of deviation and the reaction according

to the equilibrium strategy is shown in Figure 3.2. Finally, the RS contracts always remain on offer since they are separately zero profit making.
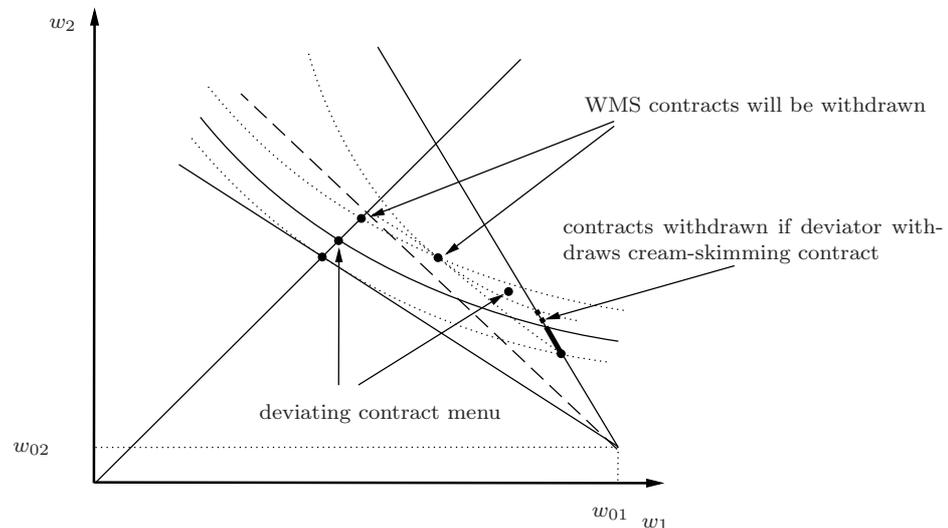
Figure 3.2: Reaction to a deviation

In this equilibrium, latent contracts are offered: the RS contracts and the LR contracts. A standard criticism of latent contracts is that they are loss-making off the equilibrium path. Note that this is not the case here: If, off the equilibrium path, latent contracts would be the best available contracts on offer for some type, they would either not be loss-making, or they would be withdrawn such that they cannot be chosen in stage 3. In particular, either the LR contracts are taken up only by low risks, or, potentially in more than one round, withdrawn. The other latent contracts, the RS contracts, will never be withdrawn, however, they are zero-profit making anyway.

The above Proposition provides an existence result for an equilibrium with an allocation that yields zero expected profits and is second-best efficient. However, this is not the unique equilibrium allocation. In the next section it will be shown that there also exist equilibria where firms share positive profits.

3.4 Equilibrium with positive profits

To show that equilibria exist in which firms share positive profits, we will first concentrate on a simple case: Consider the full insurance contract that extracts all consumer surplus from H-types. We denote this contract by ω_P. Now as ω_P just leaves H-types indifferent between purchasing insurance and remaining uninsured, ω_P will not be taken up by L-types, but it yields a per (H-type) customer profit equal to the H-type risk premium and hence, as $0 < \gamma$, strictly positive profits overall. Note that, if the share of H-types is sufficiently high, ω_P corresponds to the monopoly allocation.

Proposition 3.2 *The profit-making full insurance contract ω_P can be sustained as equilibrium contract in a symmetric equilibrium for any number of firms in the market.*

Proof See Appendix.   ■

The possibility to withdraw contracts allows firms to coordinate on a profit-making allocation: Consider offering ω_P and the set of contracts from the equilibrium strategy in Proof of Proposition 1, i.e. the WMS and RS contracts and LR contracts. If only those contracts are observed, all contracts different from ω_P are withdrawn sequentially in stage 2. In particular, firms first have to withdraw the H-type WMS and RS contracts first such that there is no pooling deviation on any of those contracts. After that, all remaining contracts different from ω_P are withdrawn since they are loss-making if taken out by both risk types. Initial contract offers and the equilibrium contract are shown in Figure 3.3.

Then, if any deviating, stand-alone profit-making contracts are observed, the WMS (and all other initial contracts) are not withdrawn. This intuition works as it is credible for firms not to withdraw the WMS contracts and make zero profits on WMS contracts when they observe deviation. It is credible because any profitable deviation from ω_P implies that if WMS, RS and LR contracts are withdrawn, insurers make zero expected profits, hence, it is sequentially ra-
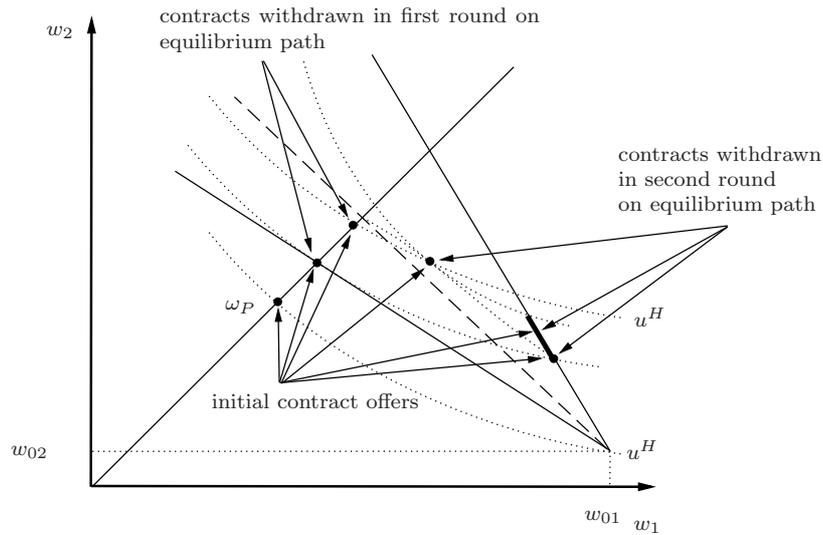
Figure 3.3: profit-making allocation

tional not to withdraw the WMS, RS and LR contracts and make zero expected profits on WMS contracts. Again, as was the logic in the Proof of Proposition 1, attempting a deviation by initially offering a cream-skimming deviation such that WMS contracts have to been withdrawn is prevented by the offer of RS and LR contracts.[17]

From the above intuition it is also clear why we consider the case in which the final allocation corresponds to a single contract only taken up by H-types: Any deviation on this contract leads to zero expected profits for the remaining firms. This is not necessarily the case for a contract menu, as each contract might be separately profit-making (or similarly a pooling contract without cross-subsidization). Then a deviator might only deviate on one contract/risk type, leaving firms with a positive profit on the other contract/risk type such that the threat of not withdrawing the WMS contracts is not credible.

However, although we have concentrated on a particular equilibrium allocation in Proposition 2, with the above logic it is simple to show that any pooling contract that involves cross-subsidization and lies below the H-type indifference curve through the H-type WMS contract as well as any profit-making separating

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17 Note that, as discussed in section 2, the possibility to sustain positive profits in equilibrium does not stem from the fact that stage 2 is potentially infinite.

contract menu that yields nonpositive profits on one risk type can be supported as equilibrium allocation using strategies analogous to those that support the equilibrium allocation from Proposition 2. Similarly, it is easy to show that the RS contracts can also be supported as equilibrium contracts.

Furthermore, note that even in the case of a contract menu with separately profit-making contracts, this contract menu might be supported as the equilibrium allocation in an asymmetric equilibrium of the following form: In stage 1, one firm offers intermediate contracts that yield positive and in particular more than $1/n$th of the profit of each contract from the menu separately, but less than $1/n$th of total profits from the menu. All other firms follow the strategy described previously. If there is no deviation, then all contracts except the profit-making contract menu will be withdrawn, and this is sequentially rational for all firms, even for the firm offering the intermediate contracts, as the intermediate contracts yield less than $1/n$th of total profits from the contract menu. If a deviation 'below' intermediate contracts is observed, the intermediate contracts will not be withdrawn, and this is again sequentially rational, as intermediate profits yield more than $1/n$th of the profit of each contract from the menu separately.[18]

3.5 Riley extension

So far, we have only considered contract withdrawal in stage 2 in the spirit of Wilson. However, if contracts can be withdrawn, it seems plausible to enlarge the action space and allow for also offering new contracts in stage 2 to add the dynamic proposed in Riley's reactive equilibrium. In a survey, Riley (2001) conjectures that in a game where firms are allowed to either add or drop offers in the second stage "both the Wilson and reactive equilibria are a Nash equilibrium of this new game". Consider the game with the following modification:

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18 Note that, with an analoguous contract set to the LR contracts, a deviation on the intermediate contracts can easily be prevented as well.

Stage 2': Stage 2 consists of $t = 1, 2, ...$ rounds. In each round t, each firm $f \in F$ can withdraw a set from its remaining contracts and add any set of contracts to the remaining contract. After each round, firms observe the contract offers of all firms. Again, denote by $\Omega_{2,t}^f$ firm f's contract set on offer at the end of t. If, for any t, $\Omega_{2,t}^f = \Omega_{2,t-1}^f \ \forall \ f \in F$, this stage ends.

We will now argue that this extension of the action space does not eliminate equilibrium allocations, in particular profit-making equilibria can still be sustained.

Proposition 3.3 *Any equilibrium allocation of the original game can be supported as an equilibrium allocation in the extended game that allows for additional contract offers in stage 2.*

Proof See Appendix. ∎

To see why, pick an equilibrium in the original game with the corresponding equilibrium strategy of firms. Then, consider that firms have the same strategy, with the addition that whenever they observe any new contract offer by any other firm in stage 2, round t, then in round $t + 1$ they add the complete set of contracts offered in stage 1 in the equilibrium strategy. That way, if, e.g. in a profit-making equilibrium, after WMS, RS and LR contracts have been withdrawn, a firm attempts to make a profit by offering a contract that profitably attracts the whole population, this strategy replicates, in round $t+1$, any possible configuration of contract offers at the end of stage 1 in the original game. However, then there is no profitable deviation since it was an equilibrium in the original game. This result allows us to formally confirm Riley (2001)'s conjecture:

Corollary 3.1 *In the game in which contracts can be withdrawn and added in stage 2, both the WMS and RS allocation can be sustained as equilibrium allocations.*

3.6 Entry

We will now allow for entry in any round in stage 2. In particular, entry takes the following form: There are $m \geq 2$ potential entrants. A potential entrant can decide in which round t to enter in stage 2 as long as in $t-1$ there was either entry or some contract withdrawn as otherwise stage 2 would end after $t-1$, and then to offer a set of contracts. Once an entrant has offered a nonempty set of contracts in some round t, he can, as incumbents, withdraw contracts from the offered contracts in subsequent rounds. We specify that if an entrant is indifferent between entering the market or not, the entrant enters. Formally, the game proceeds as follows:

Stage 2″: Stage 2″ consists of $t = 1, 2, \ldots$ rounds. There is a set of entrants $E := \{1, \ldots, f, \ldots m\}$ with $m \geq 2$. As long as firm $f \in E$ does not enter, we say that f offers $\Omega_{2,t}^f = \emptyset$ in round t. In any round t for which $\Omega_{2,t}^f \neq \Omega_{2,t-1}^f$ for some $f \in F \cup E$, any $f \in E$ with $\Omega_{2,j}^f = \emptyset$ for all $j = 1, \ldots, t-1$ can decide on entering the market and offer a set of contracts $\Omega_{2,t}^f \in \mathcal{P}(\Omega) \setminus \emptyset$ in t. We denote the round in which $f \in E$ enters by $\bar{t}^f$. In each round t, each firm $f \in F \cup E$ can withdraw a set from its remaining contracts. After each round, firms observe the contract offers of all firms. Denote by $\Omega_{2,t}^f$ firm f's contract set on offer at the end of t. If, for any t, $\Omega_{2,t}^f = \Omega_{2,t-1}^f$ for all $f \in F \cup E$, this stage ends. Define $\hat{t}$ by $\Omega_{2,\hat{t}}^f = \Omega_{2,\hat{t}-1}^f$ for all $f \in F \cup E$.

Stage 3″: Individuals choose among the contracts $\bigcup_{F \cup E} \Omega_{2,\hat{t}}^f$.

Now under entry, an equilibrium with the WMS allocation still always exists. More strongly, it is generically unique:

Proposition 3.4 *In the game with entry, an equilibrium with the WMS allocation exists and is generically unique.*

Proof See Appendix.   ∎

The proof proceeds in two steps. First, it is shown that an equilibrium with the WMS allocation always exists. In the second step, it is shown that any equilibrium yields the WMS allocation.

The reasoning why an equilibrium with the WMS allocation always exists is very similar to the one for existence of WMS equilibrium without entry: Assume that firms initially on the market follow the strategy specified in proof of Proposition 1, i.e. they offer the WMS, RS and LR contracts in stage 1 and, in case they would make a loss if stage 2 were to end after round $t-1$, they withdraw the loss-making contracts in round t and do not withdraw any contract if they make zero expected profits. The strategy of any entrant is the following: If, after any round $t-1$, incumbent firms (firms initially on the market and previous entrants) would either make zero or positive profits if stage 2 ended after round $t-1$, then the entrant enters the market in t. If incumbent firms make zero expected profits on WMS contracts, then the entrant offers WMS, RS and LR contracts in t, otherwise, the entrant offers the largest contract set that maximizes her expected profit given the contract offers of incumbents at the end of $t-1$. The entrant's strategy in all subsequent rounds is the same as that of initial firms. This constitutes an equilibrium with the WMS allocation since any entrant cannot profitably deviate from the WMS contracts: Firstly, some entrant will have to enter in $t=1$ as otherwise stage 2 ends. Secondly, as incumbent firms offer WMS, RS and LR contracts, there is no profitable deviation as shown in proof of Proposition 1.

For the second step, assume on the contrary that an equilibrium exists that yields an allocation that differs from WMS. Since it is an equilibrium, it yields nonnegative profits to all firms. Then, independent of whether it is (an) initial firm(s) or (an) entrant(s) that serve customers, since the allocation is not WMS, at least one entrant can profitably deviate by waiting to enter until the last round and, in the last round, offering a slightly better contract menu attracting all customers. Note that, as the deviating contract menu attracts all types, i.e. yields a utility for both types at least as high as that on the contracts that would have been the best on offer without the deviation, then there are

no latent contracts by incumbents (firms active in stage 1 or previous entrants) that can prevent this deviation.

Note that, although entry implies additional contract offers in stage 2, there is a subtle difference to allowing additional contract offers by incumbent firms: The situation under entry is asymmetric in the sense that, if there are positive profits to be made, a firm can enter without the possibility of incumbent firms to punish additional contract offers by own new contract offers.

3.7 Conclusion

We modify the seminal Rothschild and Stiglitz (1976) insurance model in the spirit of Wilson (1977)'s "anticipatory equilibrium" concept by introducing an additional stage in which firms can withdraw contracts (repeatedly) after observation of competitor's contract offers. It is shown that an equilibrium always exists where consumers obtain their respective Wilson- Miyazaki-Spence (WMS) contract, i.e. second-best efficiency can be achieved for any share of high-risk types in the population. However, contrary to intuition the game-theoretic analysis of the Wilson concept is not that straightforward: Latent contracts have to be offered alongside the WMS contracts for the Wilson logic to work as contract withdrawal allows for complex deviating strategies. Furthermore, equilibria exist in which firms share positive profits. When, besides contract withdrawal, additional contracts can be offered in stage 2, all equilibrium allocations in the game without adding contracts can be supported as equilibrium allocations in the extended game. In particular, as suggested by Riley (2001), both the WMS and RS allocations are equilibrium allocations in the extended game. However, if there is entry in the second stage, the WMS equilibrium is generically unique. The WMS equilibrium, both in the game with and without entry, is sustained by the offer of latent contracts. There are mainly two arguments against the possibility to offer latent contracts: The first one is that they might make losses off the equilibrium path, and the second one that they lead to a multiplicity of

equilibrium allocations.[19] Note that the criticism does not apply here: latent contracts either yield nonnegative profits, or will be withdrawn off the equilibrium path. Furthermore, in the game with entry latent contracts are also required to sustain the equilibrium allocation, however the equilibrium allocation is generically unique.

In this market, (non-linear) Bertrand competition is not sufficient to establish outcomes that would be associated with a competitive market, namely that firms make zero expected profits. To obtain zero profits, potential entry is required. However, interestingly, cross-subsidization prevails in equilibrium although there is entry.

3.8 Appendix

Proof of Proposition 1.

Let $\Omega_{WMS} := \left\{ \omega_{WMS}^H, \omega_{WMS}^L \right\}$ denote the set of WMS contracts, $\Omega_{RS} := \left\{ \omega_{RS}^H, \omega_{RS}^L \right\}$ denote the set of RS contracts and

$$\Omega_{LR} := \left\{ \omega \in \Omega \left| u^L(\omega) < u^L(\omega_{WMS}^L), u^L(\omega) > u^L(\omega_{RS}^L) \right. \right.$$
$$\left. \text{and } (1 - p_L)(w_{01} - w_1) + p_L(w_{02} - w_2) = 0 \right\}$$

the continuum of contracts that lie on the L-type fair insurance line and yield an L-type a higher expected utility than her RS contract but lower expected utility than her WMS contract.

Let $\Omega_1 := (\Omega_1^1, ... \Omega_1^n)$, $\Omega_{2,t} := (\Omega_{2,t}^1, ..., \Omega_{2,t}^n)$ and let $h_t = (\Omega_1, \Omega_{2,1}, ..., \Omega_{2,t-1})$ denote the history in the beginning of round t. Furthermore, $\Delta_{2,t} := \bigcup_F \Omega_{2,t}^f$. We denote by $\bar{\omega}_{2,t}^J$ the contract such that

$$\bar{\omega}_{2,t}^J \in \arg \max_{\omega \in \Delta_{2,t}} u^J(\omega)$$

and

$$\bar{w}_2^J \geq \tilde{w}_2^J \ \forall \ \tilde{\omega}^J \in \arg \max_{\omega \in \Delta_{2,t}} u^J(\omega)$$

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19 Criticism of latent contracts is e.g. reviewed in Attar, Mariotti, and Salanie (2009).

Let $\bar{k}^J$ denote the number of firms offering $\bar{\omega}_{2,t}^J$ at the end of t, i.e. $\bar{K}^J :=$ $\left\{ f \in F \mid \bar{\omega}_{2,t}^J \in \Omega_{2,t}^f \right\}$ and $\bar{k}^J := |\bar{K}^J|$.

The strategy of a consumer of type J is to choose $\bar{\omega}_{2,\hat{t}}^J$ at firm $f \in \bar{K}^J$ with probability $1/\bar{k}^J$.

A strategy of a firm f specifies a set of contracts in stage 1, and in stage 2, round t, a map from the history to a set of remaining contracts of firm f at the end of t in stage 2, i.e. $\alpha_t^f : h_t \longmapsto \Omega_{2,t}^f$.

We denote the hypothetical profit of firm f if stage 2 would end after $t-1$ by

$$\pi^f(\Omega_{2,t-1}) := \gamma_H[(1-p_H)(w_{01}-\bar{w}_1^H) + p_H(w_{02}-\bar{w}_2^H)](1/\bar{k}^H)\mathbb{1}_{\Omega_{2,t-1}^f}(\bar{\omega}_{2,t-1}^H) +$$
$$(1-\gamma_H)[(1-p_L)(w_{01}-\bar{w}_1^L) + p_L(w_{02}-\bar{w}_2^L)](1/\bar{k}^L)\mathbb{1}_{\Omega_{2,t-1}^f}(\bar{\omega}_{2,t-1}^L)$$

where $\mathbb{1}$ is an indicator function. Similarly, we denote by

$$\pi^{f,J}(\Omega_{2,t-1}) = \gamma_H[(1-p_H)(w_{01}-\bar{w}_1^J) + p_H(w_{02}-\bar{w}_2^J)](1/\bar{k}^J)\mathbb{1}_{\Omega_{2,t-1}^f}(\bar{\omega}_{2,t-1}^J)$$

the hypothetical profits on J-types respectively. Finally, let

$$A := \left\{ f \in F \,\middle|\, \Omega_1^f \subseteq \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} \right\}.$$

We propose that a possible equilibrium strategy of firms is the following: In stage 1, firm $f \in F$ offers $\Omega_1^f = \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR}$. In stage 2, round t the strategy of firm f specifies

$$\alpha_t^f(h_t) = \begin{cases} \Omega_{2,t-1}^f & \text{if } \pi^f(\Omega_{2,t-1}) = 0; \\ \Omega_{2,t-1}^f \setminus \left\{\hat{\Omega}_{2,t-1}^H\right\} & \text{if } \pi^f(\Omega_{2,t-1}) < 0 \text{ and } \pi^{f,L}(\Omega_{2,t-1}) \geq 0; \\ \Omega_{2,t-1}^f \setminus \left\{\bar{\omega}_{2,t-1}^L\right\} & \text{if } \pi^f(\Omega_{2,t-1}) < 0 \text{ and } \pi^{f,H}(\Omega_{2,t-1}) \geq 0; \\ \Omega_{2,t-1}^f \setminus \left\{\bar{\omega}_{2,t-1}^H, \bar{\omega}_{2,t-1}^L\right\} & \text{if } \pi^f(\Omega_{2,t-1}) < 0 \text{ and} \\ & \qquad\qquad \pi^{f,L}(\Omega_{2,t-1}), \pi^{f,H}(\Omega_{2,t-1}) < 0; \\ \bar{\Omega}_{2,t}^f & \text{if } \pi^f(\Omega_{2,t-1}) > 0. \end{cases}$$

where

$$\hat{\Omega}^H_{2,t-1} := \left\{ \omega \in \Omega^f_{2,t-1} \setminus \{\Omega_{RS}\} \text{ such that if } \Omega^j_{2,t-1} = \{\omega\} \; \forall j \in A, \right.$$

$$\left. \text{then } \omega = \bar{\omega}^H_{2,t-1} \text{ and } \pi^{f,H}(\Omega_{2,t-1}) < 0 \right\}$$

i.e. if firm f makes losses on H-types, it withdraws any contract that, if all firms that in stage 1 offered the contracts according to equilibrium strategy (or less contracts) only offered this one contract, would attract the H-types and be loss-making. Furthermore, $\bar{\Omega}^f_{2,t}$ denotes the *largest* set of contracts such that for $\Omega^j_{2,t} = \Omega^j_{2,t-1} \; \forall j \in F \setminus \{f\}$, then $\pi^f(\Omega_{2,t})$ is maximal.

If all firms follow the above strategy, no firm withdraws any contract in $t = 1$ and stage 2 ends after $t = 1$, firms make zero expected profit and a customer of type J receives her J-type WMS contract.

It remains to show that there is no profitable deviation. We will proceed in two steps: First, we show that a deviator serves some H-types. In a second step, we show that if the deviator serves some H-types, she cannot be making a strictly positive profit.

Consider firm $\bar{f}$ that offers $\Omega^{\bar{f}}_1$ in stage 1 and has a strategy $\hat{\alpha}^{\bar{f}} : h_t \longmapsto \Omega^{\bar{f}}_{2,t}$ in stage 2. Let $\Omega^f_{2,\hat{t}}$ be the final set of contract offers of firm f, i.e. $\Omega^f_{2,\hat{t}} = \Omega^f_{2,\hat{t}-1} \; \forall$ $f \in F$. Then it must be that $\pi^f(\Omega_{2,\hat{t}-1}) \geq 0 \; \forall \; f \in F \setminus \{\bar{f}\}$ as otherwise a firm $f \in F \setminus \{\bar{f}\}$ would withdraw a nonempty set of contracts in $\hat{t}$ and $\hat{t}$ would not be the last round in stage 2.

Now assume $\pi^{\bar{f}}(\Omega_{2,\hat{t}}) > 0$. As $\pi^f(\Omega_{2,\hat{t}-1}) \geq 0 \; \forall \; f \in F \setminus \{\bar{f}\}$, we will show that $\bar{\omega}^H_{2,\hat{t}} \in \Omega^{\bar{f}}_{2,\hat{t}}$: Since $\pi^{\bar{f}}(\Omega_{2,\hat{t}}) > 0$, $\bar{f}$ serves some customers. To show that it cannot be possible that $\bar{f}$ serves only L-types, assume on the contrary that $\bar{f}$ only serves L-types. If L-types prefer an insurance contract to remaining uninsured, than H-types prefer to be insured as well. As $\bar{f}$ only serves L-types, then at least one firm $f \in F \setminus \{\bar{f}\}$ serves H-types and the share of L-types among customers at f is less than $1 - \gamma$. There are three possible cases:

Case 1: $\bar{\omega}^H_{2,\hat{t}} = \omega^H_{WMS}$. Now any firm $f \in F \setminus \{\bar{f}\}$ that serves H-types with ω^H_{WMS} and has a share of L-types among customers that is less than $1 - \gamma$ does

not make a nonnegative profit.[20] This contradicts $\pi^f(\Omega_{2,\hat{t}-1}) \geq 0 \ \forall \ f \in F \backslash \{\bar{f}\}$.

Case 2: $\bar{\omega}_{2,\hat{t}}^H \in \Omega_{LR}$. Any contract $\omega \in \Omega_{LR}$ if taken up by some H-types is loss-making, independent of whether it is also taken up by some L-types. This contradicts $\pi_f(\Omega^{2,\hat{t}}) \geq 0 \ \forall \ f \in F \backslash \{\bar{f}\}$.

Case 3: $\bar{\omega}_{2,\hat{t}}^H = \omega_{RS}^H$. From the strategy of all $f \in F \backslash \{\bar{f}\}$, both RS contracts will never be withdrawn, i.e. the L-type contract is still on offer when $\bar{\omega}_{2,\hat{t}}^H = \omega_{RS}^H$. Then, there is no contract that $\bar{f}$ can offer attracting L-types and making a positive profit, which is a contradiction.

Hence, $\bar{\omega}_{2,\hat{t}}^H \in \Omega_{2,\hat{t}}^{\bar{f}}$. We will now show that if $\bar{\omega}_{2,\hat{t}}^H \in \Omega_{2,\hat{t}}^{\bar{f}}$, $\bar{f}$ cannot be making a positive profit. First, note that, the RS contracts are always, i.e. in any t, offered by each firm $f \in F \backslash \{\bar{f}\}$. Then, it follows that $u^H(\bar{\omega}_{2,\hat{t}}^H) \geq u^H(\omega_{RS}^H)$. There are again three possible cases:

Case 1: $u^H(\bar{\omega}_{2,\hat{t}}^H) \geq u^H(\omega_{WMS}^H)$. Then, ω_{WMS}^L will not have been withdrawn by any firm $f \in F \backslash \{\bar{f}\}$. As ω_{WMS}^L is on offer from firms $f \in F \backslash \{\bar{f}\}$, by construction of the WMS contracts, $\pi^{\bar{f}}(\Omega_{2,\hat{t}}) \leq 0$ for the cases that $\bar{f}$ only serves H-types or both types.

Case 2: $u^H(\bar{\omega}_{2,\hat{t}}^H) < u^H(\omega_{WMS}^H)$ and $\bar{\omega}_{2,\hat{t}}^L \in W_{CS}$. Hence, both WMS contracts are not on offer at any firm $f \in F \backslash \{\bar{f}\}$ and any firm $f \in F \backslash \{\bar{f}\}$ does not serve L-types since it does not offer any contract $w \in W_{CS}$. However, by construction of the WMS contracts, there is no incentive compatible menu of contracts with $\bar{w}_{2,\hat{t}}^L \in W_{CS}$ that is profit-making, hence $\pi_{\bar{f}}(\Omega^{2,\hat{t}}) < 0$.

Case 3: $u^H(\bar{\omega}_{2,\hat{t}}^H) < u^H(\omega_{WMS}^H)$ and $\bar{\omega}_{2,\hat{t}}^L \notin \Omega_{CS}$. If $\bar{\omega}_{2,\hat{t}}^H \in \Omega_{LR}$ or , then $\pi^{\bar{f}}(\Omega_{2,\hat{t}}) < 0$. If $\bar{\omega}_{2,\hat{t}}^H \notin \Omega_{LR}$, then from the strategy of any firm $f \in F \backslash \{\bar{f}\}$, $\bar{\omega}_{2,\hat{t}}^L \in \Omega_{2,\hat{t}}^f \ \forall \ f \in F \backslash \{\bar{f}\}$. Then, $\pi^{\bar{f}}(\Omega_{2,\hat{t}}) \leq 0$.

Hence, $\pi^{\bar{f}}(\Omega_{2,\hat{t}}) \leq 0$ which is a contradiction.

Proof of Proposition 2.

Let $\Omega_P := \{\omega_P\}$. Again, the strategy of a consumer of type J is to choose $\bar{\omega}_{2,\hat{t}}^J$ at firm $f \in \bar{K}^J$ with probability $1/\bar{k}^J$. Let $\Omega_{WH} := \{\omega \in \Omega \ | \ u^H(\omega) < u^H(\omega_P)\}$

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[20] This is because firm $f \in F \backslash \{\bar{f}\}$ at best serves some L-types with ω_{WMS}^L, however, since the share of L-types is less than $1 - \gamma$, this is loss-making.

and let

$$B := \left\{ f \in F \left| \Omega_1^f \subseteq \Omega_P \cup \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} \cup \Omega_{WH} \right. \right\}.$$

We claim that a possible equilibrium strategy of firms is the following: In stage 1, firm $f \in F$ sets $\Omega_1^f = \Omega_P \cup \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR}$. In stage 2, round t the strategy of firm f specifies

$$\beta_t^f(h_t) = \begin{cases} \Omega_{2,t-1}^f \setminus \left\{ \omega_{RS}^H, \omega_{WMS}^H \right\} & \text{if } \pi^f(\Omega_{2,t-1}) = 0 \text{ and } B = F; \\ \Omega_{2,t-1}^f & \text{if } \pi^f(\Omega_{2,t-1}) = 0 \text{ and } B \neq F; \\ \Omega_{2,t-1}^f \setminus \left\{ \hat{\Omega}_{2,t-1}^H \right\} & \text{if } \pi^f(\Omega_{2,t}) < 0 \text{ and } \pi^{f,L}(\Omega_{2,t-1}) \geq 0; \\ \Omega_{2,t-1}^f \setminus \left\{ \bar{\omega}_{2,t-1}^L \right\} & \text{if } \pi^f(\Omega_{2,t-1}) < 0 \text{ and } \pi^{f,H}(\Omega_{2,t-1}) \geq 0; \\ \Omega_{2,t-1}^f \setminus \left\{ \bar{\omega}_{2,t-1}^H, \bar{\omega}_{2,t-1}^L \right\} & \text{if } \pi^f(\Omega_{2,t-1}) < 0 \text{ and} \\ & \qquad \qquad \pi^{f,L}(\Omega_{2,t-1}), \pi^{f,H}(\Omega_{2,t-1}) < 0; \\ \bar{\Omega}_{2,t}^f & \text{if } \pi^f(\Omega_{2,t-1}) > 0. \end{cases}$$

$$\hat{\Omega}_{2,t-1}^H := \left\{ \omega \in \Omega_{2,t-1}^f \setminus \{\Omega_{RS}\} \text{ such that if } \Omega_{2,t-1}^j = \{\omega\} \ \forall j \in B, \right.$$
$$\left. \text{then } \omega = \bar{\omega}_{2,t-1}^H \text{ and } \pi^{f,H}(\Omega_{2,t-1}) < 0 \right\}$$

i.e. if firm f makes losses on H-types, it withdraws any contract that, if all firms that in stage 1 offered the contracts according to equilibrium strategy (or less contracts) only offered this one contract, would attract the H-types and be loss-making. Furthermore, $\bar{\Omega}_{2,t}^f$ denotes the *largest* set of contracts such that for $\Omega_{2,t}^j = \Omega_{2,t-1}^j \ \forall j \in F \setminus \{f\}$, then $\pi^f(\Omega_{2,t})$ is maximal.

The strategy thus specifies that if, after stage 1, there is no contract ω with $\omega \notin \{\Omega_P \cup \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} \cup \Omega_{WH}\}$ on offer, then the WMS, RS and LR contracts will be sequentially withdrawn, however, if a contract ω with $\omega \notin \{\Omega_P \cup \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} \cup \Omega_{WH}\}$ is observed after stage 1, the WMS contracts will not be withdrawn (if the firm's hypothetical expected profit is zero) and the strategy is the same as the stategy in proof of Proposition 1.

If all firms follow the above strategy, then all firms withdraw the H-type WMS

contract and the H-type RS contracts in $t = 1$ and the L-type WMS and RS contracts as well as LR contracts in $t = 2$. Stage 2 ends after $t = 3$, firms share the profit on ω_P, H-type customers buy ω_P and L-types remain uninsured.

It remains to show that there is no profitable deviation. First, note that, since firms share the profit from the single contract ω_P, any deviation yielding a higher profit than $1/n$th of the profit from ω_P necessarily yields zero expected profits to nondeviating firms as they do not serve any customer. Then, it is sequentially rational not to withdraw the WMS contracts as prescribed by the equilibrium strategy.

The rest of the proof proceeds along the same lines as proof of Proposition 1 and is therefore omitted.[21]

Proof of Proposition 3.

Fix an equilibrium in the original game. In this equilibrium, the equilibrium strategy of firm $f \in F$ specifies a contract offer Ω_1^f in stage 1 and in stage 2, round t a withdrawal strategy $\chi_t^f(h_t)$ specifying remaining contract offers in the end of t. Note that, we neither assume that equilibrium strategies are symmetric nor put any restrictions on stage 2 strategies.

In the extended game, a strategy specifies a contract offer in stage 1, and map from the history h_t to a contract offer in stage 2, round t.

Then consider the following strategy in the extended game: Firm $f \in F$ offers Ω_1^f in stage 1. In stage 2, round t the strategy specifies

$$
\bar{\chi}_t^f(h_t) = \begin{cases}
\chi_t^f(h_t) & \text{if } \Omega_{2,t-1}^j \subseteq \Omega_{2,t-2}^j \ \forall j \in F; \\
\widetilde{\Omega}_t^f & \text{if } \Omega_{2,t-1}^j \subseteq \Omega_{2,t-2}^j \ \forall j \in F \setminus f \text{ and there exists a contract} \\
& \quad \omega \in \Omega_{2,t-1}^f \text{ with } \omega \notin \Omega_{2,t-2}^f; \\
\Omega_1^f & \text{otherwise.}
\end{cases}
$$

where for notational simplicity, we denote stage 1 as round $t = 0$ and let $\Omega_{2,-1}^j = \emptyset \ \forall j \in F$ and $\widetilde{\Omega}^f$ is a set of contracts such that for $\Omega_{2,t}^j = \Omega_{2,t-1}^j \ \forall j \in F \setminus \{f\}$,

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21 Note that, in particular a deviation aiming at offering the H-type WMS or RS contract as a pooling contract is covered by proof of Proposition 1.

then $\pi^f(\Omega_{2,t})$ is maximal. Note that, in this case $\widetilde{\Omega}$ does not need to be the largest set such that the profit is maximal as if there are some contracts withdrawn or added in t, f can add contracts in $t + 1$.

This strategy implies that firms have the same strategy as in the original game, however, whenever a firm f observes another firm j adding contracts in the previous round, then f replicates contract offers after stage 1 as it throws all stage 1 contracts on the market.

Now firstly, this strategy yields the same equilibrium allocation as in the original game as on the equilibrium path, each firm $f \in F$ takes the same action in stage 1 and in all rounds of stage 2 as on the corresponding equilibrium path in the original game.

It remains to show that there is no profitable deviation. A profitable deviation here means a deviation such that profits are higher than in equilibrium in the original game. Assume a firm $\bar{f}$ offers $\widehat{\Omega}_1^{\bar{f}}$ in stage 1 and has a strategy that specifies some $\widehat{\chi}_t^{\bar{f}} : h_t \longmapsto \Omega_{2,t}^{\bar{f}}$ in stage 2 and makes a profit that is strictly higher than in the equilibrium in the original game. Firstly, note that this implies that stage 2 ends in some t. We need to distinguish 4 cases:

Case 1: $\widehat{\Omega}_1^{\bar{f}} \neq \Omega_1^{\bar{f}}$ and $\widehat{\chi}_t^{\bar{f}}(h_t) = \bar{\chi}_t^{\bar{f}}(h_t)$. This implies that no contract will be added by any firm $f \in F$ in any round in stage 2. However, then either it involves the same allocation and same profits for all firms as in the equilibrium in the original game, or $\chi^f(h_t)$ cannot have been part of an equilibrium strategy in the original game for some $f \in F$.

Case 2: $\widehat{\Omega}_1^{\bar{f}} = \Omega_1^{\bar{f}}$, $\widehat{\chi}_t^{\bar{f}}(h_t) \neq \bar{\chi}_t^{\bar{f}}(h_t)$ and $\bar{f}$ does not add any contract in any t. Again, this implies that no contract will be added by any firm $f \in F$ in any round in stage 2. As in Case 1, then either it involves the same allocation and same profits for all firms as in the equilibrium in the original game, or $\chi^f(h_t)$ cannot have been part of an equilibrium strategy in the original game for some $f \in F$.

Case 3: $\widehat{\Omega}_1^{\bar{f}} = \Omega_1^{\bar{f}}$, $\widehat{\chi}_t^{\bar{f}}(h_t) \neq \bar{\chi}_t^{\bar{f}}(h_t)$ and $\bar{f}$ adds at least one contract in some t. Assume first that a contract will only be added by $\bar{f}$ in at most one round t and let $\tilde{t}$ denote this round. Then, the strategy of firms $f \in F \setminus \bar{f}$ specifies

that $\Omega_{\tilde{t}+1}^f = \Omega_1^f \ \forall f \in F \setminus \bar{f}$. However, then, for $t \geq \tilde{t} + 1$ this replicates either Case 1 or 2 above. Now assume that $\bar{f}$ adds contracts in more than one round t. Let $\check{t}$ denote the last round in which a contract will be added by $\bar{f}$. Again, the strategy of firms $f \in F \setminus \bar{f}$ specifies that $\Omega_{\check{t}+1}^f = \Omega_1^f \ \forall f \in F \setminus \bar{f}$. However, then, this again replicates either Case 1 or 2 above.

Case 4: $\widehat{\Omega}_1^{\bar{f}} \neq \Omega_1^{\bar{f}}$ and $\widehat{\chi}_t^{\bar{f}}(h_t) \neq \bar{\chi}_t^{\bar{f}}(h_t)$. We can transform this case in the following way: Instead of $\widehat{\Omega}_1^{\bar{f}} \neq \Omega_1^{\bar{f}}$, let $\widehat{\Omega}_1^{\bar{f}} = \Omega_1^{\bar{f}}$ and $\bar{f}$ either adds or withdraws some contract in stage 2, round 1 and plays $\widehat{\chi}_t^{\bar{f}}(h_t)$ thereafter. However, then, this falls under one of the above cases.

Proof of Proposition 4.

We will proceed in two steps: First, we show that an equilibrium with the WMS allocation always exists. In the second step, we show that any equilibrium yields the WMS allocation.

For the first part, again, the strategy of a consumer of type J is to choose $\bar{\omega}_{2,\hat{t}}^J$ at firm $f \in \bar{K}^J$ with probability $1/\bar{k}^J$.

For any t, let $\Omega_{2,t}^{FE} := (\Omega_{2,t}^1, ..., \Omega_{2,t}^n ..., \Omega_{2,t}^{n+m})$ denote the observed contract offers of all firms, that is initial firms and (potential) entrants. We denote *initial* contract offers of all firms that are on the market in t by $\Omega_{1,t}^{FE} := (\Omega_1^1, ..., \Omega_1^n, \Omega_{2,\bar{t}^{n+1}}^{n+1}, ..., \Omega_{2,\bar{t}^{n+k}}^{n+k})$ where $\bar{t}^{n+i} < t \ \forall i = 1, ..., k$ and denote by M_t the set of firms on the market in t. We can then denote the history in t by $h_t^{FE} = (\Omega_{1,t}^{FE}, \Omega_{2,1}^{FE}, ..., \Omega_{2,t-1}^{FE})$.

A strategy of a firm $f \in F$ specifies a set of contracts in stage 1, and in stage 2, round t, a map from the history to a set of remaining contracts of firm f at the end of t in stage 2, i.e. $\alpha_t^f : h_t^{FE} \longmapsto \Omega_{2,t}^f$.

We propose that the equilibrium strategy of any firm $f \in F$ specifies the following: In stage 1, firm $f \in F$ offers $\Omega_f^1 = \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR}$. In stage 2, round t the strategy of firm f is

$$\alpha_t^f(h_t^{FE}) = \begin{cases} \Omega_{2,t-1}^f & \text{if } \pi^f(\Omega_{2,t-1}^{FE}) = 0; \\ \Omega_{2,t-1}^f \setminus \left\{ \hat{\Omega}_{2,t-1}^H \right\} & \text{if } \pi^f(\Omega_{2,t-1}^{FE}) < 0 \text{ and } \pi^{f,L}(\Omega_{2,t-1}) \geq 0; \\ \Omega_{2,t-1}^f \setminus \left\{ \bar{\omega}_{2,t-1}^L \right\} & \text{if } \pi^f(\Omega_{2,t-1}^{FE}) < 0 \text{ and } \pi^{f,H}(\Omega_{2,t-1}) \geq 0; \\ \Omega_{2,t-1}^f \setminus \left\{ \bar{\omega}_{2,t-1}^H, \bar{\omega}_{2,t-1}^L \right\} & \text{if } \pi^f(\Omega_{2,t-1}^{FE}) < 0 \text{ and} \\ & \qquad \pi^{f,L}(\Omega_{2,t-1}^{FE}), \pi^{f,H}(\Omega_{2,t-1}^{FE}) < 0; \\ \bar{\Omega}_{2,t}^f & \text{if } \pi^f(\Omega_{2,t-1}^{FE}) > 0. \end{cases}$$

$$\hat{\Omega}_{2,t-1}^H := \Big\{ \omega \in \Omega_{2,t-1}^f \setminus \{\Omega_{RS}\} \text{ such that if } \Omega_{2,t-1}^j = \{\omega\} \ \forall j \in C,$$
$$\text{then } \omega = \bar{\omega}_{2,t-1}^H \text{ and } \pi^{f,H}(\Omega_{2,t-1}) < 0 \Big\}$$

with
$$C := \left\{ f \in M_t \ \Big| \ \Omega_1^f, \Omega_{\bar{t}f}^f \subseteq \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} \right\}.$$

and $\bar{\Omega}_{2,t}^f$ denotes the *largest* set of contracts such that for $\Omega_{2,t}^j = \Omega_{2,t-1}^j$ for all $j \in F \cup E \setminus \{f\}$, then $\pi^f(\Omega_{2,t})$ is maximal.

The strategy of an entrant $f \in E$ in stage 2, round t specifies the following: As long as f has not entered, the strategy consists of a map from the history in t to a decision to enter the market in round t, i.e. $\theta_t^f : h_t^{FE} \longmapsto \eta \in \{entry, noentry\}$. When f enters in t, i.e. $\theta_t^f(h_t^{FE}) = entry$, the strategy specifies a map from the history to a set of contract offers Ω_t^f, $\gamma_t^f : h_t^{FE} \longmapsto \Omega_{2,t}^f$. For all subsequent rounds t, the strategy specifies a map from the history to a set of remaining contracts of firm f at the end of t i.e. $\phi_t^f : h_t^{FE} \longmapsto \Omega_{2,t}^f$.

We propose that a possible equilibrium strategy of an entrant $f \in E$ specifies the following:

$$\theta_t^f(h_t^{FE}) = \begin{cases} entry & \text{if } \pi^j(\Omega_{2,t-1}^{FE}) \geq 0 \ \forall j \in F \cup E \setminus f; \\ noentry & \text{otherwise.} \end{cases}$$

$$\gamma_t^f(h_t^{FE}) = \begin{cases} \Omega_{WMS} \cup \Omega_{RS} \cup \Omega_{LR} & \text{if } \pi^j(\Omega_{2,\bar{t}f-1}^{FE}) = 0 \text{ for all } j \in F \cup E \setminus f \text{ and} \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad C = M_t; \\ \bar{\Omega}^f & \text{otherwise.} \end{cases}$$

where $\bar{\Omega}^f$ denotes the *largest* set of contracts such that for $\Omega^j_{2,\bar{t}^f} = \Omega^j_{2,\bar{t}^f-1} \ \forall j \in F \cup E \setminus f$, then $\pi^f(\Omega_{2,t})$ is maximal and lastly,

$$\phi_t^f(h_t^{FE}) = \alpha_t^f(h_t^{FE}).$$

If all initial firms and entrants follow the above respective strategies, no initial firm withdraws any contract in $t = 1$ and all entrants enter in $t = 1$, no firm withdraws any contract in $t = 2$ and stage 2 ends after $t = 2$, firms make zero expected profit and a customer of type J receives her J-type WMS contract.

It remains to show that there is no profitable deviation. Firstly, note that, for the same reasoning as in proof of Proposition 1, no initial firm can deviate profitably. Then, assume that an entrant $\bar{f}$ in $\bar{t}^{\bar{f}}$ offers $\Omega^{\bar{f}}_{\bar{t}^{\bar{f}}}$ and has a withdrawal strategy $\hat{\phi}_t^{\bar{f}}$ and assume that $\bar{f}$ makes a strictly positive profit. By the strategy of all firms $f \in F \cup E \setminus \bar{f}$, it follows that $\bar{t}^{\bar{f}} \leq 2$. Then, however, independent of whether $\bar{t}^{\bar{f}} = 1$ or $\bar{t}^{\bar{f}} = 2$, from the strategy of all firms $f \in F \cup E \setminus \bar{f}$ it follows that round $\bar{t}^{\bar{f}}$ is equivalent to stage 1 in the game without entry. The rest of the proof corresponds to the proof of Proposition 1.

For the second part of the proof, assume that an equilibrium exists that yields an allocation different from WMS, i.e. $(\bar{w}^H_{2,\hat{t}}, \bar{w}^L_{2,\hat{t}}) \neq (\omega^H_{WMS}, \omega^L_{WMS},)$. Since it is an equilibrium, each firms $f \in F \cup E$ makes a nonnegative expected profit. Since we specified that if an entrant is indifferent between entering the market or not, the entrant enters, nonnegative expected profits imply that all firms $f \in E$ enter in some round $t < \hat{t}$.

Now since $(\bar{w}^H_{2,\hat{t}}, \bar{w}^L_{2,\hat{t}}) \neq (\omega^H_{WMS}, \omega^L_{WMS},)$ there exist contracts $\hat{\omega}^H, \hat{\omega}^L$ such that $u^H(\hat{\omega}^H) \geq u^H(\bar{w}^H_{2,\hat{t}})$, $u^H(\hat{\omega}^H) \leq u^H(\hat{\omega}^L)$, $u^L(\hat{\omega}^L) > u^L(\bar{w}^H_{2,\hat{t}})$ and for firm $\hat{f}$ offering $\Omega^{\hat{f}} = \{\hat{\omega}^H, \hat{\omega}^L\}$ and attracting the whole population, $\pi^{\hat{f}} > 0$ and $\pi^{\hat{f}} \geq \sum_{F \cup E} \pi^j$. As $\pi^{\hat{f}} \geq \sum_{F \cup E} \pi^j$, an entrant can profitably deviate by waiting to enter the market until $\hat{t}$ and offer $\hat{\omega}^H, \hat{\omega}^L$ in $\hat{t}$. Note that this entry cannot be prevented by firms as $u^L(\hat{\omega}^L) > u^L(\bar{w}^H_{2,\hat{t}})$, i.e. there exists no contract that cream skims low risks from $u^L(\hat{\omega}^L)$.

4 Endogenous Capital in the Rothschild-Stiglitz Model

We endogenize upfront capital of insurers in the RS model. Under limited liability, low upfront capital gives rise to an endogenous insolvency risk. This introduces an externality among customers of an insurer such that an equilibrium with the WMS allocation always exists. In this market, solvency regulation might have unintended consequences: If the required solvency capital is too high, an equilibrium with a second-best efficient allocation fails to exist.

4.1 Introduction

We propose a basic extension to the RS model to solve the equilibrium existence problem: Instead of being exogenously endowed with sufficiently high assets as in RS, insurers choose the level of upfront capital before entering the market stage. Now under limited liability, low upfront capital gives rise to an endogenous insolvency risk as, depending on contract offers and the distribution of risk types over the contracts of an insurer, there might not be sufficient assets to fulfill claims, as in Faynzilberg (2006).[22] Generally speaking, this introduces

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[22] Insolvency has been analyzed in insurance markets without adverse selection. Doherty and Schlesinger (1990) analyze insurance demand under an exogenous insolvency risk and show that less than full insurance will be purchased at the actuarially fair premium if default is total, however, if default is partial, overinsurance might occur and there is generally no monotonic relationship between default payout rate and insurance coverage. Agarwal and Ligon (1998) introduce an exogenous default risk into the RS model when consumers have CARA preferences and apply the Wilson anticipatory equilibrium concept. Comparing the situation with and without default risk, Agarwal and Ligon (1998) find that high risks

an externality among customers of a firm as an individual's expected utility now does not only depend on contract parameters but also on the distribution of risk types over the contracts of an insurer. This externality guarantees equilibrium existence - we show that with capital choice under limited liability, an equilibrium in pure strategies that yields the second-best efficient Wilson-Miyazaki-Spence (WMS) allocation always exists. The equilibrium is sustained as cream-skimming offers aimed at attracting low risk types lead to a deterioration of high risks' WMS contract due to insolvency when upfront capital is low. High risks then prefer to choose the deviating contract as well, rendering the deviation unprofitable. Now in terms of the strategic capital choice of firms, putting in more capital is not profitable for an insurer as this only increases incentives of competitors to cream skim low risks away from this insurer. Hence, an equilibrium with low upfront capital and the WMS allocation always exists. One implication of our analysis is that solvency regulation might have unintended consequences: If imposed capital requirements are too strict, a second-best efficient equilibrium fails to exist.

4.2 The model

There is a continuum of individuals of mass 1 in the market, representing a large population of consumers. Each individual faces two possible states of nature: In state 1 no loss occurs and the endowment is w_0, in state 2 a loss occurs and the endowment is $w_0 - L$ with $w_0 > w_0 - L > 0$. There are 2 types of individuals, an individual may be a high risk type (H) with loss probability p^H, or a low-risk type (L) with loss probability p^L, with $0 < p^L < p^H < 1$. Individuals have a twice continuously differentiable strictly concave von Neumann-Morgenstern utility function $v(w)$. Insurance is provided by firms in the set $F := \{1, ..., f, ...m\}$. Firms are risk neutral and do not know, ex ante, any individual's type.

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might be better of with default risk if the Wilson equilibrium changes from separating (no default) to pooling (default).

The timing of the market interaction follows RS, i.e. insurers offer contracts first, then insurees choose. The difference to RS is that while RS assume insurers to be exogenously endowed with sufficiently high capital, insurers in this model choose the level of upfront capital before entering the market stage. Firms will be subject to limited liability, i.e. if a loss occurs and the insurer does not have enough assets, full indemnity payment might not be possible. Formally, the timing of the game is as follows:

Stage 0: The risk type of each individual is chosen by nature. Each individual has a chance of γ, $0 < \gamma < 1$ to be a H-type, and of $(1 - \gamma)$ to be a L-type.

Stage 1: Each firm $f \in F$ decides on the level of its upfront capital K^f. There are nonnegative opportunity costs to holding capital.[23]

Stage 2: Each firm $f \in F$ offers a finite set of contracts $\Omega^f = \left\{\omega_1^f, \omega_2^f, ..., \omega_k^f\right\}$ where a contract $\omega_l^f = (P_l^f, I_l^f) \in \Omega^f$ specifies a premium P_l^f and an indemnity I_l^f.[24]

Stage 3: Individuals choose their insurance contract.[25]

Stage 4: Losses are realized.

Stage 5: Insurance firms pay out indemnities. If total claims C^f at firm f are less than total assets A^f, the insurance firm fully settles claims. If total claims C^f exceed total assets A^f, then the insurance firm pays out the assets and defaults on the remaining claims. The ex ante known insolvency rules specify proportional payout, i.e. a customer at firm f who has bought contract ω_g^f and has realized a loss receives a fraction $\beta^f = A^f/C^f$ of her indemnity claim I_g^f.[26]

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23 Equilibrium existence does not depend on whether opportunity costs of capital are zero or positive.

24 Each firm cannot offer more than k contracts. We restrict the set of possible contract offers to $\Omega := \{(P, I)|I \leq L\}$ where $I > L$ is ruled out for moral hazard considerations. In this chapter, we change notation for insurance contracts as the analysis of insolvency is more straightforward when contract paramaters are explicitly expressed as premia and indemnity payments.

25 An individual can only sign one contract with one firm, i.e. we consider exclusive contracting.

26 We will discuss the effects of alternative insolvency rules in Section 3.

For expositional convenience, we will henceforth call the subgame starting with firms' contract offers in stage 2 the 'market game'.

Firm assets are comprised of two components: upfront capital and premium income. When upfront capital is low, depending on the offered contracts and distribution of agents across the contracts, insolvency might occur. However, insolvency also determines an agent's expected utility from chosing a contract at a specific insurer. Formally, let λ_i^f denote the mass of individuals taking out contract $\omega_i^f \in \Omega^f$ and let σ_i^f denote the share of high risks in λ_i^f. Then the expected utility of a J-type individual $J \in \{H, L\}$ from choosing the contract $\omega_i^f \in \Omega_f$, $\lambda_i^f > 0$ is

$$u^J(\omega_i^f, \beta^f) := (1 - p^J)v(w_0 - P_i^f) + p^J v(w_0 - P_i^f - L + min\left\{1, \beta^f\right\} I_i^f)$$

where
$$\beta^f = \frac{K^f + \sum_{i=1}^{k} \lambda_i^f P_i^f}{\sum_{i=1}^{k} \lambda_i^f (p^H \sigma_i^f + p^L(1 - \sigma_i^f))I_i^f}.^{27}$$

Firms expected profits are given by

$$\pi^f = max\left\{0, \sum_{i=1}^{k} \lambda_i^f(P_i^f - (p^H \sigma_i^f + p^L(1 - \sigma_i^f))I_i^f)\right\}.$$

In the RS model, firms are assumed to be exogenously endowed with sufficient capital such that insolvency does not occur. The expected utility derived from a contract then depends solely on the contract parameters. With a slight abuse of notation, we denote expected utility of a J-type individual from contract w_l in the RS setting with large capital holdings by:

$$u^J(\omega_l) := (1 - p^J)v(w_0 - P_l) + p^J v(w_0 - P_l - L + I_l).$$

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27 i.e. we evoke the law of large numbers to identify the average indemnity at a contract with the expected indemnity of a customer randomly drawn at the contract. This representation of expected utility is valid as long as it is not the case that $\lambda_i^f = 0 \ \forall i$. Note that, if $\lambda_i^f = 0$ $\forall i$, the expected utility of a customer choosing any contract at firm f is less than her expected utility from remaining uninsured.

4.3 Equilibrium with WMS allocation

4.3.1 Market game with no upfront capital

Endogenous insolvency risk will play a key role in the analysis. To analyze the effect of endogenous insolvency on the expected utility of consumers, we will for now assume that no firm holds upfront capital, i.e. $K^f = 0 \ \forall f \in F$. This is similar to the situation discussed by Faynzilberg (2006). As our approach and modeling differ, we will derive the results in detail.

Under this assumption, suppose that firm f offers only the H-type WMS contract ω_{WMS}^H and attracts all H-types. Then this firm would go insolvent with final assets of $\gamma P_{\omega_{WMS}^H}$ and final claims of $\gamma p^H I_{\omega_{WMS}^H} = \gamma p^H L$. Thus every insured with a loss obtains $P_{\omega_{WMS}^H}/p^H$, so the expected utility of a customer of firm f is:

$$u^H(\omega_{WMS}^H, \beta^f) = (1 - p^H)v(w_0 - P_{\omega_{WMS}^H}) + p^H v(w_0 - L + \frac{(1 - p^H)}{p^H} P_{\omega_{WMS}^H})$$

Endogenous insolvency in this case lowers the indemnity such that the (H-type) customer's indifference curve shifts vertically downward to the point where it crosses the H-type fair insurance line. To see this, note that, due to insolvency, the insurer makes exactly zero profits as he pays out the assets and defaults on the remaining claims; however, the customer always has to pay the premium. The resulting expected utility is lower than that of the H-type RS contract as there is no full insurance. This deterioration of the contract is illustrated in Figure 4.1. We can now state our first result.

Proposition 4.1 *Let $K^f = 0 \ \forall f \in F$. Then there exists a symmetric equilibrium in the market game where every individual of type J obtains contract ω_{WMS}^J in stage 3 and no insurer goes insolvent.*

Proof See Appendix. ∎

The intuition behind Proposition 1 works as follows: As high risks are cross-subsidized and the WMS contracts maximize low risk utility subject to overall
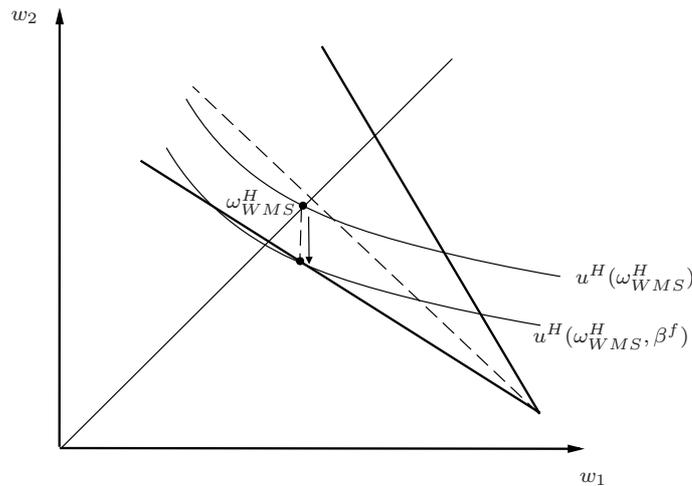
Figure 4.1: Contract deterioration with endogenous insolvency risk if a firm
has no upfront capital and only attracts high risk types which buy
ω_{WMS}^H

non-negative profits, a deviation has to aim at cream-skimming low risks. That
is an insurer offers a contract in the set Ω_{CS} as shown in Figure 4.2. In the RS
model, this deviation is profitable as high risk's expected utility is not affected by
cream-skimming. Here however, due to endogenous insolvency risk, a deviation
aimed at cream skimming low risks leads to a contract deterioration for high
risks.[28] When they correctly anticipate this contract deterioration, high risks
prefer the deviating contract as well, at least as long as the deviator makes a
profit and does not go insolvent. However, this implies that the deviator will go
insolvent, which renders a deviation unprofitable.

Remark: The result of Proposition 1 does not depend on the particular in-
solvency rule. Instead of the proportional insolvency rule where each customer
receives the same share of her indemnity claim, consider e.g. ex post efficient
rationing: if a loss occurs, each customer experiences the same loss independent

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28 In Picard (2009), sharing profits or losses in e.g. a mutual creates an analogous externality.
    In Picard (2009) the realized contract for the high risks moves along the diagonal rather
    than vertically. This is due as in his case those high risks who do not have a loss have to
    pay for those who experience a loss. A similar general logic also applies in Kosfeld and
    von Siemens (2009). In a labor market context, Kosfeld and von Siemens (2009) assume
    that high productivity workers prefer to be pooled with their own type.
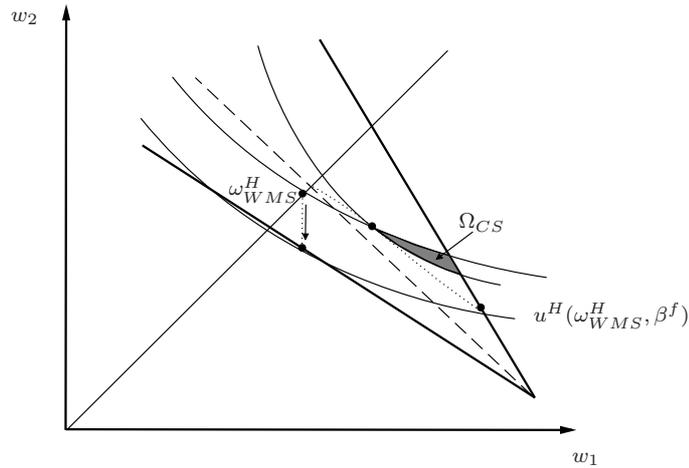
Figure 4.2: Cream-skimming and WMS insurer insolvency

of risk type, and thus it is ex post efficient to distribute assets equally among claimants. Now the expected utility of an H-type from his respective WMS contract when there is insolvency is never higher with ex post efficient rationing than with proportional rationing: Either there are only high risk claimants, then there is no difference, or there are also low risk claimants, but then high risks are worse of as under proportional rationing they have a higher indemnity claim and thus payout. As contract deterioration for high risks is the crucial part that guarantees equilibrium existence, ex post efficient rationing would thus not affect our results.

Next it is shown that the equilibrium allocation in the market game is unique if $K^f = 0 \ \forall f \in F$.

Proposition 4.2 *Let $K^f = 0 \ \forall f \in F$. Any equilibrium of the market game yields the WMS allocation.*

Proof See Appendix. ∎

The intuition for this result is the following: Suppose an equilibrium exists with an allocation that is different from WMS. This allocation either involves nonnegative profits or some insurers go insolvent. In the first case, there is always a profitable deviating contract menu. In the second case, the allocation

can be translated to an allocation without insolvency yielding any consumer the same expected utility and yielding firms zero profits. However, then there always exists a profitable deviating contract menu as well.

4.3.2 Endogenous capital

So far in the analyis we assumed that capital endowment is exogenous and in particular that firms do not own any assets. Now we are ready to analyze the complete game with upfront capital choice.

Proposition 4.3 *In the complete game with endogenous capital, an equilibrium always exists in which $K^f = 0 \; \forall \; f \in F$, every individual of type J obtains contract ω_{WMS}^J in stage 3 and no insurer goes insolvent.*

Proof See Appendix. ■

From Propositions 1 and 2 we know that if firms do not hold any capital, an equilibrium with the WMS allocation exists and is generically unique. Now consider the following strategy: firm f sets zero upfront capital in stage 1. If firm f observes that all firms set zero capital, then f offers the WMS contracts in stage 2. If a firm $j \neq f$ chooses a nonzero amount of capital, f still offers WMS contracts. If every firm follows this strategy, a deviator who chooses a different level of capital cannot make a positive profit for exactly the reasoning laid out in Proposition 1: Since high risks' WMS contracts would deteriorate if the deviator tries to cream skim low risks, high risks would choose the deviating contract offer, rendering it unprofitable. This reasoning holds as every firm except possibly the deviator does not hold any upfront capital.

Remark: We consider upfront capital choice but do not model the possibility to recapitalize ex post if claims exceed assets. However, note that, for the same reasoning that makes it unattractive for an insurer to put in upfront capital, an insurer does not have any incentive to commit to a recapitalization policy.

4.4 Solvency regulation

Insurance markets are subject to regulation. While regulation traditionally targeted the product level directly, e.g. via regulation of premia, liberalisation in the 1970's triggered a shift towards solvency requirements as the main regulatory tool. Solvency regulation itself saw a change from volume-based to risk-based solvency requirements in most major regulatory jurisdictions since the 1990's, more recently in Europe with Solvency II underway.[29] Although the aim of Solvency II is to ensure risk management that better fits an individual insurer's risk, practitioners argue that it will increase solvency capital requirements for most lines of business.[30]

The economic effects of solvency regulation have so far only been analyzed in insurance markets without adverse selection. Rees, Gravelle, and Wambach (1999) show that there is no efficiency rationale for solvency regulation when consumers are fully informed about the insurer's insolvency risk such that the role of regulation should consist in providing information rather than imposing capital requirements. However, when there is asymmetric information, we will show that if solvency capital requirements are too strong, regulation may actually impede the existence of a second-best efficient equilibrium.

For the analysis, assume that solvency regulation requires each firm to hold a minimum capital K^* at the beginning of stage 1. Let $\hat{K}$ be implicitly defined by

$$(1 - p^H)v(w_0 - P_{\omega_{WMS}^H}) + p^H v(w_0 - L + \frac{\hat{K}}{\gamma p^H} + \frac{(1 - p^H)}{p^H}P_{\omega_{WMS}^H}) = u^H(\omega_{LCS})$$

where $\omega_{LCS} = (P_{LCS}, I_{LCS})$ is the unique contract that satisfies

$$P_{LCS} = p^L I_{LCS}$$

$$u^L(\omega_{LCS}) = u^L(\omega_{WMS}^L),$$

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29 See e.g. Eling and Holzmuller (2008).
30 In a recent Economist article, an analyst at JPmorgan claimed that "the rules could increase the amount of capital that insurers need to hold by as much as 75 %".

i.e. in the set of cream skimming contracts that contract which is worst for a high risk type when there is no insolvency. This is illustrated in Figure 4.3.
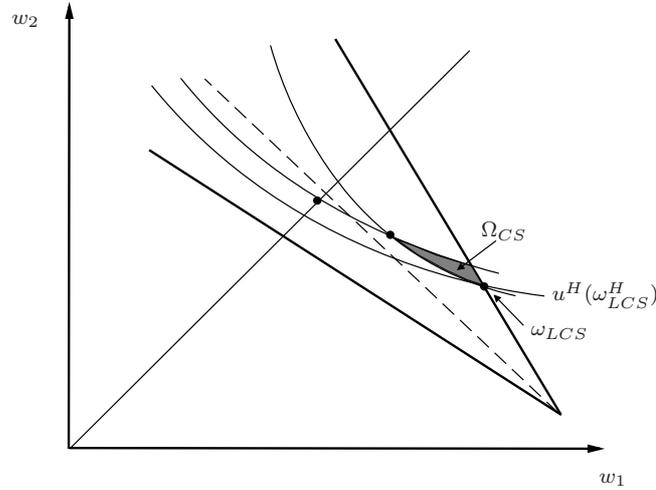
Figure 4.3: Contract deterioration if a firm has capital $\hat{K}$ and only attracts high risk types which buy ω_{WMS}^{H}

Corollary 4.1 *When solvency regulation requires firms to hold capital $K^* > \hat{K}$, an equilibrium with the WMS allocation fails to exist.*

Proof See Appendix.   ∎

When firms hold sufficiently large upfront capital, the high risk type WMS contract does not deteriorate strongly when there is a cream-skimming deviation. Hence, high risks might not opt for the deviating contract such that there are profitable deviations attracting only low risks. This impedes the existence of an equilibrium in pure strategies with the WMS allocation. Although we have concentrated on the WMS allocation, from standard reasoning it is easy to show that there is no equilibrium yielding a second-best efficient allocation. Thus, if there is adverse selection, ensuring solvency via regulation is not a good consumer protection policy as the externality from contracting disappears if solvency capital is too high.

4.5 Conclusion

To address the equilibrium non-existence result in competitive insurance markets with asymmetric information, we modify the Rothschild and Stiglitz (1976) model by allowing insurers to decide on the level of upfront capital and possibly go insolvent. This introduces an externality among the customers of a firm that guarantees equilibrium existence: An equilibrium with the second-best efficient Wilson-Miyazaki-Spence allocation always exists. In such a market, cream-skimming becomes unattractive as an insurer trying to attract low risks has to fear attracting high risks as well since the high risks' contract at another insurer deteriorates if low risks do not buy from that particular insurer. When insurers choose the level of their upfront capital, this externality is present because any insurer will opt for a low amount of capital simply because putting in more capital only increases the incentive of competitors to engage in cream skimming. Interestingly, solvency regulation aiming at minimizing insolvency risk leads to unintended consequences: If imposed solvency capital requirements are too strong, the externality from contracting disappears and there is no equilibrium with a second-best efficient allocation.

4.6 Appendix

Proof of Proposition 1.

We claim that a possible equilibrium strategy of firms is to offer both WMS contracts each. For the case that all firms $f \in F$ offer $\left\{\omega_{WMS}^H, \omega_{WMS}^L\right\}$, the strategy of a J-type individual specifies to choose ω_{WMS}^J at firm $f \in F$ with probability $1/k$. Then no individual has an incentive to deviate, as no insolvency occurs and all are served with the best possible contract on offer. It remains to show that there is no profitable firm deviation. Consider the case that all firms follow the above strategy apart from firm $\bar{f}$ which offers $\Omega^{\bar{f}} = \left\{\omega_1^{\bar{f}}, \omega_2^{\bar{f}}, ..., \omega_k^{\bar{f}}\right\}$. We will proceed in two steps. We will first show that there is no profitable

deviation if $\bar{f}$ does not offer a cream-skimming contract. We will then show that $\bar{f}$ offering a cream-skimming contract cannot be a profitable deviation either.

Assume that $\bar{f}$ is making a strictly positive profit.

For the first part, assume that $\Omega^{\bar{f}} \cap \Omega_{CS} = \emptyset$ where Ω_{CS} is defined as follows

$$\Omega_{CS} := \left\{\omega \in \Omega | u^L(\omega) \geq u^L(\omega^L_{WMS}) \text{ and } u^H(\omega) \leq u^H(\omega^L_{WMS})\right\}$$

i.e. Ω_{CS} is the cream-skimming region with respect to the WMS contracts as displayed in Figure 3.[31] There are two possible cases:

Case 1: There exists a contract $\omega \in \Omega^{\bar{f}}$ such that $u^L(\omega) > u^L(\omega^L_{WMS})$. Since $\Omega^{\bar{f}} \cap \Omega_{CS} = \emptyset$, this implies that $u^H(\omega) > u^H(\omega^H_{WMS})$ as well. Then, both L- and H-types would choose the deviating contract offer.[32] However, by construction of the WMS contracts, there is no contract (set) preferred by both types to the WMS contracts that is profitable.

Case 2: There does not exist a contract $\omega \in \Omega^{\bar{f}}$ such that $u^L(\omega) > u^L(\omega^L_{WMS})$. Then, no L-type chooses a contract offer of the deviator. As no L-type deviates, no WMS insurer goes insolvent. However, then either $u^H(\omega) < u^H(\omega^H_{WMS})$, i.e. there is no customer at $\bar{f}$ or $u^H(\omega) \geq u^H(\omega^H_{WMS})$ and some H-types choose the deviating contract. However, in this latter case, as no L-type deviates, $\bar{f}$ would not be making a strictly positive profit.

Now, the more interesting part, assume that $\Omega^{\bar{f}} \cap \Omega_{CS} \neq \emptyset$. We will consider three cases.

Case 1: WMS insurers do not sell any contract.

As $\Omega^{\bar{f}} \cap \Omega_{CS} \neq \emptyset$, each type prefers taking a contract at $\bar{f}$ to remaining uninsured. However, since $\Omega^{\bar{f}} \cap \Omega_{CS} \neq \emptyset$ and $\Omega^{\bar{f}} \neq \left\{\omega^H_{WMS}, \omega^L_{WMS}\right\}$, by construction of the WMS contracts $\bar{f}$ cannot be making a strictly positive profit when attracting the whole population.

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31 Note that Ω_{CS} includes ω^L_{WMS} and contracts that, under solvency, give the L-types the same expected utility as ω^L_{WMS}.

32 Note that, as $u^H(\omega^H_{WMS}) \geq u^H(\omega^H_{WMS}, \beta)$ and $u^L(\omega^L_{WMS}) \geq u^L(\omega^L_{WMS}, \beta)$, this is true irrespective of WMS insurer insolvency, i.e. for any possible β of WMS insurers.

Case 2: WMS insurers sell contracts and there is no insolvency.

As $\Omega^{\bar{f}} \cap \Omega_{CS} \neq \emptyset$, there exists a contract $\omega_i^{\bar{f}}$ such that $u^L(\omega_i^{\bar{f}}) \geq u^L(\omega_{WMS}^L)$ and $u^H(\omega_i^{\bar{f}}) \leq u^H(\omega_{WMS}^H)$. Then some L-types deviate from ω_{WMS}^L. It follows that either WMS insurers make losses, which is ruled out by assumption, or some H-types deviate as well. For some H-types to deviate as well, a contract $\omega_j^{\bar{f}}$ such that $u^H(\omega_j^{\bar{f}}) \geq u^H(\omega_{WMS}^H)$ has to exist. However, by construction of the WMS contracts, there is no profitable contract set preferred by both types to the WMS contracts, hence $\bar{f}$ does not make strictly positive profit.

Case 3: Some WMS insurers go insolvent with positive proability.

As $\Omega^{\bar{f}} \cap \Omega_{CS} \neq \emptyset$, there exists a contract $\omega_i^{\bar{f}}$ such that $u^L(\omega_i^{\bar{f}}) \geq u^L(\omega_{WMS}^L)$ and $u^H(\omega_i^{\bar{f}}) \leq u^H(\omega_{WMS}^H)$. Now as $u^L(\omega_i^{\bar{f}}) \geq u^L(\omega_{WMS}^L) \geq u^L(\omega_{WMS}^L, \beta)$ for any $\beta \leq 1$, it is optimal for an L-type to choose the deviating offer if $u^L(\omega_i^{\bar{f}}) > u^L(\omega_{WMS}^L, \beta)$ and it is optimal to either choose the deviating offer or the L-type WMS contract if $u^L(\omega_i^{\bar{f}}) = u^L(\omega_{WMS}^L, 1)$. We define the consumer strategy such that the deviating contract offer is chosen. Hence, all L-types deviate. As all L-types deviate, the H-type WMS contract at WMS insurers deteriorates such that $\forall f \in \hat{F}$

$$u^H(\omega_{WMS}^H, \beta^f) = (1-p^H)v(w-P_{\omega_{WMS}^H}) + p^H v(w-L+\frac{(1-p^H)}{p^H}P_{\omega_{WMS}^H}) < u^H(\omega_{RS}^H).$$

As $u^H(\omega_{WMS}^H, \beta^f) < u^H(\omega_{RS}^H) \; \forall f \in \hat{F}$, it follows that $u^H(\omega_i^{\bar{f}}) \geq u^H(\omega_{WMS}^H, \beta^f)$, $\forall f \in \hat{F}$ by construction of the WMS contracts: The WMS contracts maximize L-type utility. Hence when they do not coincide with the RS contracts, $u^L(\omega_{WMS}^L) > u^L(\omega_{RS}^L)$. As $u^H(\omega_{RS}^H) = u^H(\omega_{RS}^L)$ and using that H-type indifference curves are less steep than L-type indifference curves in the two-states wealth space, this implies that $u^H(\omega_i^{\bar{f}}) \geq u^H(\omega_{RS}^H)$ and thus $u^H(\omega_i^{\bar{f}}) \geq u^H(\omega_{WMS}^H, \beta^f)$ $\forall f \in \hat{F}$. Then all H-types would choose the deviating offer.[33] However, since all types deviate, this contradicts the assumption that some WMS insurers go insolvent.

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33 Note that expected utility for H-types at a WMS insurer would even be lower if some or all take out the L-type WMS contract.

Proof of Proposition 2.

Suppose that an equilibrium exists that does not yield the WMS allocation. From the set of equilibrium contracts, select $A = \{\bar{\omega}^H, \bar{\omega}^L\}$ with[34]

$$\bar{\omega}^H \in \arg \max_{\omega_i^j \in \bigcup \Omega^f} u^H(\omega_i^j, \bar{\beta}^j)$$

$$\bar{\omega}^L \in \arg \max_{\omega_i^j \in \bigcup \Omega^f} u^L(\omega_i^j, \bar{\beta}^j)$$

and

$$I_{\bar{\omega}^H} \geq I_{\tilde{\omega}^H} \; \forall \; \tilde{\omega}^H \in \arg \max_{\omega_i^j \in \bigcup \Omega^f} u^H(\omega_i^j, \bar{\beta}^j)$$

$$I_{\bar{\omega}^L} \geq I_{\tilde{\omega}^L} \; \forall \; \tilde{\omega}^L \in \arg \max_{\omega_i^j \in \bigcup \Omega^f} u^L(\omega_i^j, \bar{\beta}^j)$$

$j, k \in F$ and $\bar{\omega}^H \neq \omega_{WMS}^H$ and $\bar{\omega}^L \neq \omega_{WMS}^L$. From the set of equilibrium contracts, we select the H-type and L-type contracts that have the highest indemnity. These are precisely the contracts that yield insurers the highest per contract profit. We distinguish 2 cases:

Case 1: All insurers make nonnegative profits if the allocation is A.

Now as insurers make nonnegative profits, $\bar{\beta}^j = 1 \forall j \in F$ and $u^J(\bar{\omega}^J, \bar{\beta}) = u^J(\bar{\omega}^J)$. As $\bar{\omega}^H \neq \omega_{WMS}^H$ and $\bar{\omega}^L \neq \omega_{WMS}^L$, there exist contracts $\hat{\omega}^H$, $\hat{\omega}^L$ such that $u^H(\hat{\omega}^H) \geq u^H(\bar{\omega}^H)$, $u^H(\hat{\omega}^H) \leq u^H(\hat{\omega}^L)$, $u^L(\hat{\omega}^L) > u^L(\bar{\omega}^L)$ and for firm $\hat{f}$ offering $\Omega^{\hat{f}} = \{\hat{\omega}^H, \hat{\omega}^L\}$ and attracting the whole population, $\pi^{\hat{f}} > 0$ and $\pi^{\hat{f}} \geq \sum_F \pi^j$. As $\pi^{\hat{f}} > 0$, a deviating insurer offering $\hat{\omega}^H$, $\hat{\omega}^L$ would not go insolvent and all types would choose their respective deviating offer. Hence, there is a profitable deviation and A cannot be an equilibrium allocation.

Case 2: Some insurers go insolvent if the allocation is A.

As some insurers make negative profits, there is insolvency. Now since A are equilibrium contracts and customers take insolvency into account, i.e. they maximize expected utility with insolvency, A can be converted to a contract set $B = \{\check{\omega}^H, \check{\omega}^L\}$ without insolvency providing individuals the same expected utility and yielding zero expected profit for insurers with $I_{\check{\omega}^H} = \bar{\beta}^j I_{\bar{\omega}^H}^j$ and $I_{\check{\omega}^L} = \bar{\beta}^k I_{\bar{\omega}^L}^k$. Now we can apply the reasoning from *Case 1* above to show that

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34 It might well be the case that $\bar{\omega}^H$ and $\bar{\omega}^L$ coincide.

there is a profitable deviation.

Proof of Proposition 3.

We claim that a possible equilibrium strategy is the following: Firm f sets $K = 0$ in stage 0. In stage 1, if $K^j = 0 \ \forall j \in F$, firm f sets both WMS contracts. If $K^f > 0$ and $K^j = 0 \ \forall j \neq f$, then firm f sets the RS contracts. If $K^l > 0$, $l \neq f$ and $K^j = 0 \ \forall j \neq l$, firm f sets both WMS contracts.[35] For the case that all firms $f \in F$ offer $\left\{ \omega^H_{WMS}, \omega^L_{WMS} \right\}$ in stage 2, the strategy of a J-type individual specifies to choose ω^J_{WMS} at firm $f \in F$ with probability $1/k$. Then no individual has an incentive to deviate, as no insolvency occurs and all are served with the best possible contract on offer. Firms make zero expected profits. It remains to show that there is no profitable firm deviation. Consider the case that all firms follow the above strategy apart from firm $\bar{f}$ which sets $K^{\bar{f}} \geq 0$ and offers $\Omega^{\bar{f}} = \left\{ \omega^{\bar{f}}_1, \omega^{\bar{f}}_2, ..., \omega^{\bar{f}}_k \right\}$.

Now, as all firms $f \in F \setminus \bar{f}$ set both WMS contracts each, a profitable deviation has to involve cream-skimming, as shown in proof of Proposition 1. However, as also $K^f = 0 \ \forall f \in F \setminus \bar{f}$, cream-skimming is not profitable for $\bar{f}$ for any $K^{\bar{f}} \geq 0$ following the reasoning laid out in proof of Proposition 1. Hence, there is no profitable deviation.

Proof of Corollary 1.

Let required upfront capital be $K^* > \hat{K}$ for any firm operating in the market and assume an equilibrium exists that yields the WMS allocation. Then, there is at least one insurer who offers both WMS contracts, hereafter called a WMS insurer, sells contracts to both risk types and does not go insolvent.

Now consider contract $\bar{\omega} \in \Omega_{CS}$ with $u^L(\bar{\omega}) > u^L(\omega^L_{WMS})$ and

$$u^H(\bar{\omega}) < (1 - p^H)v(w_0 - P_{\omega^H_{WMS}}) + p^H v(w_0 - L + \frac{K^*}{\gamma p^H} + \frac{(1 - p^H)}{p^H} P_{\omega^H_{WMS}})$$

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35 A complete specification of the strategy includes the contract sets by firm f if two or more firms choose $K^j > 0$ at stage 1. As this is not required for the existence proof, we do not specify the strategy further.

i.e. contract $\bar{\omega}$ is preferred by L-types over their respective WMS contract even without WMS insurer insolvency but not preferred by H-types over their respective WMS contract even if there is WMS insurer insolvency. As $K^* > \hat{K}$, such a contract exists. Now consider a firm offering $\bar{\omega}$. As $u^L(\bar{\omega}) > u^L(\omega_{WMS}^L)$, all L-types prefer $\bar{\omega}$ to their respective WMS contract at the WMS insurer. As $u^H(\bar{\omega}) < (1-p^H)v(w_0 - P_{\omega_{WMS}^H}) + p^H v(w_0 - L + \frac{K^*}{\gamma p^H} + \frac{(1-p^H)}{p^H} P_{\omega_{WMS}^H})$, all H-types at the WMS insurer prefer to be insured with the WMS insurer and would not choose $\bar{\omega}$. A firm offering $\bar{\omega}$ would thus attract L-types and make a positive expected profit. However, then the WMS insurer does not sell contracts to both risk types, which is a contradiction.

5 Asymmetric Information and Collusive Stability

We compare the stability of collusive agreements in adverse selection insurance markets under symmetric and asymmetric information. We show that asymmetric information weakly destabilizes collusion. This is not a consequence of asymmetric information per se, but of the common value characteristic of this market. We furthermore analyze the effect of consumer information about their risk type on collusive stability. Generally, there is a non-monotonous relationship between consumer information and collusive stability under asymmetric information.

5.1 Introduction

Although a large theoretical literature is devoted to the study of competition in insurance markets under averse selection, the literature on oligopoly models is scarce.[36] Therefore, in this chapter, we depart from the assumption of competition and analyze the ability of insurance firms to engage in collusive behavior. In particular, we analyze whether asymmetric information impacts collusive stability, i.e. whether collusion is more or less stable in adverse selection insurance markets under symmetric or asymmetric information. There are several interpretations for the comparative statics that we consider: the first one is across insurance markets, i.e. asymmetric information might be less important in some

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36 An exception is Olivella and Vera-Hernández (2007) in which horizontally differentiated health plans are considered.

markets, as e.g. risk types are strongly correlated with (collectible) observables, but that it is more prevalent in other insurance markets. Another interpretation might be the introduction of a data collection or information technique in a particular market such that asymmetric information is lessened. An example are genetic tests, which can be requested by insurers depending on the regulatory environment.

To analyze the impact of asymmetric information on collusive stability, an infinitely repeated version of the Rothschild and Stiglitz (1976) model is considered. Under symmetric information, the analysis mirrors analysis of a standard Bertrand market, with the only exception that insurers offer nonlinear contract menus. However, under asymmetric information, deviation incentives and thus the stability of collusion depend on the shares of risk types in the population: If there are only a few high risk type, perfect collusion implies that high risks are cross-subsidized as firms want to extract a large surplus from the large share of low risks. Then, a deviator does not deviate on the complete set of collusive contracts, but only cream-skims low risks, thereby earning higher than total collusive profits which destabilizes collusion.

This effect can be particularly pronounced if there are consumers that do not have precise information abour their risk type. We show that collusion is destabilized under asymmetric information if there are only a few consumers informed about their risk type, even if the share of high risks is relatively high. Generally, the impact of information about risk type on collusive stability is non-monotonous.

On a general note, we thus contribute to the theory of collusion in terms of providing a new factor that destabilizes collusion: common values in asymmetric information markets.[37] Note that it is not asymmetric information per se that destabilizes collusion: In a standard private value asymmetric information case à la Maskin and Riley (1984), asymmetric information does not destabilize

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37 Furthermore, note that in contrast to previous work on asymmetry of information in the context of collusion in which the asymmetric information is between colluding firms, either in the form of moral hazard (Green and Porter, 1984) or adverse selection (Athey, Bagwell, and Sanchirico, 2004), we consider firm-symmetric asymmetric information.

collusion, as although collusive profits are always lower due to incentive compatibility, *relative* deviation incentives are not affected by private value asymmetric information. That is because under private values each type at worst yields zero profits, but there is never cross-subsidization.

Interestingly, in the insurance literature, the term collusion is generally not used to describe collusive practices in the sense of sustaining above Nash profits through cooperation in an infinite horizon setting, but to describe fraudulent behavior of a coalition of insurees and service providers vis-à-vis the insurer.[38] The lack of collusion models is striking as insurance markets has seen quite a few cases of cartel behavior over the past years. For example, the German market for industrial insurance was cartelized by a group of 17 companies.[39] The German Antitrust Authority looked into the market in 2002 and 2003 to find evidence that the insurance companies had been colluding since 1999. The firms had come to the agreement to, among others, stop competing with respect to prices as well as terms and conditions.[40] The firms as well as 23 representatives involved were found guilty and fined an amount of around € 140 million, the companies as well as their representatives ultimately accepted to pay this fine in 2010.[41] Another example is the well known so-called liability crisis that hit the insurance market in the mid-1980s and is often associated with collusive activity.[42] This crisis was characterized by a sharp increase in insurance premiums, cancellations of policies and massive withdrawal of insurers from some lines.[43] Attornies general in 19 states in the US filed charges against several insurance companies which faced claims to have been involved in "a 'global conspiracy' to limit or exclude certain types of liability coverage in an effort to cut competition,

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38 See e.g. Alger and Ma (2003) and Bourgeon, Picard, and Pouyet (2008). To our knowledge, the only study besides ours on collusion between insurers is Kesternich and Schumacher (2009), in which the focus is on stability of pooling under symmetric information.

39 The following is based on Bundeskartellamt (2010) (Bundeskartellamt, 2010, Fallbericht: Ende des OLG-Verfahrens wegen der Bußgelder gegen Industrieversicherer).

40 Another aspect had been the improvement of the communication among cartel members.

41 Note that insurance markets are however exempt from competition law in several ways: there is e.g. a block exemption in European competition law allowing insurers to share information.

42 See, e.g. Angoff (1988).

43 See Winter (1991).

increase prices, and make those who purchase policies pay more for less".[44]
Furthermore, Chiappori, Jullien, Salanié, and Salanié (2006) analyze French
automobile insurance and point out that a deeper data analysis suggests that
profits are higher for contracts with higher coverage, which is contrary to pre-
dictions of competitive models. We now turn to the model.

5.2 The model

Consider a discrete time setting, $t = 1, 2, ..., \infty$. In each period t, a continuum
of individuals of mass 1, representing a large population of consumers, enters
the market. Each individual faces two possible states of nature: In state 1 no
loss occurs and the endowment is w_0, in state 2 a loss occurs and the endowment
is $w_0 - l$ with $w_0 > w_0 - L > 0$. There are 2 types of individuals, an individual
may be a high risk type (H) with loss probability p^H, or a low-risk type (L)
with loss probability p^L, with $0 < p^L < p^H < 1$. Individuals have a twice
continuously differentiable strictly concave von Neumann Morgenstern utility
function $v(w)$. Individuals purchase insurance for one period and then exit the
market. Insurance is provided by risk neutral firms in the set $F := \{1, 2\}$, the
firms' common discount factor is $\delta \in (0, 1)$.

The stage game in each period t is the static game underlying the analysis in
Rothschild and Stiglitz (1976) with contract menus: First, the risk type of each
individual is chosen by nature. Each individual has a chance of γ, $0 < \gamma < 1$ to
be a H-type, and of $(1 - \gamma)$ to be a L-type. Then, each firm $f \in F$ can offer a set
of contracts Ω_f from the set of possible contract offers $\Omega := \{(P, I) | I + P \leq L\}$
where a contract (P, I) specifies a premium P and a net indemnity I.[45] Finally,
consumers choose an insurance contract. We denote

$$w_1(P) = w_0 - P$$

$$w_2(I) = w_0 - l + I$$

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44 See Reske (1993).

45 $I + P > L$ is ruled out for moral hazard considerations. In this chapter, we define I as the
net indemnity as this facilitates the analysis of optimal contracts lateron.

Expected utility of customer $J \in \{L, H\}$ from contract (P, I) is

$$u^J(P, I) = (1 - p^J)v(w_1(P)) + p^J v(w_2(I))$$

We will consider two informational settings: Symmetric information and asymmetric information, i.e. firms do not know, ex ante, any individual's type in any period t.

To illustrate the results we use an example throughout the paper. In the example, $v(w) = ln(w)$, $p^H = 1/4$, $p^L = 1/8$, $l = 1$ and $w_0 = 4$.

5.3 Analysis

We will analyze the necessary and sufficient conditions for collusion at maximal profits. We consider grim trigger strategies as defined by Friedman (1971), i.e. the strategy specifies that after a deviation firms revert to the Nash equilibrium of the static game for all subsequent periods.[46] Let π^D denote deviation profits, π^C the per period profit of a firm from collusion and π^N the profit from Nash play in the static game. Then, collusion is stable if

$$\frac{\pi^C}{1 - \delta} \geq \pi^D + \frac{\delta \pi^N}{1 - \delta}$$

i.e. if the discounted profits from collusion are higher than the profits from a one shot deviation followed by discounted profits from the punishment phase. Solving for δ gives the critical discount factor $\bar{\delta}$ such that for $\delta \geq \bar{\delta}$ collusion is stable:

$$\bar{\delta} := \frac{\pi^D - \pi^C}{\pi^D - \pi^N}$$

In the following, we will derive the respective profits for all three cases.

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46 We will show below that this corresponds to an optimal penal code.

Punishment profits

As discussed in chapter 2, an equilibrium in pure strategies might not exist. If a (Nash-)equilibrium in pure strategies exists, firms offer the separately zero-profit making RS contracts. However, even if an equilibrium in pure strategies fails to exist, Dasgupta and Maskin (1986) have shown that an equilibrium in mixed strategies always exists. In particular, firms mix between jointly zero-profit contract menus such that the equilibrium in mixed strategies yields zero expected profits to firms. Since we analyze collusion, we are not interested in the exact characterization of potential punishment contracts, we only note that a Nash equilibrium in the one stage game exists that yields zero expected profits to firms.

Remark 5.1 *For any share of high risk types in the population, a Nash equilibrium exists in the static game. Firms earn zero expected profits in equilibrium.*

In chapter 2 we discussed that mixed strategies might not be the best description of insurance markets. In this chapter we refer to Dasgupta and Maskin (1986) merely out of simplicity: The relevant aspect is that an equilibrium exists that yields zero profits. Since the extensions of the RS model as well yield equilibria with zero profits, we use the simplest possible version to describe the market. Note furthermore that, since the Nash equilibrium in the static game yields zero expected profits, grim-trigger strategies correspond to an optimal penal code.

Collusive profits

Since we want to analyse perfect collusion, i.e. collusion at maximal profits, let us review the monopoly solution. Under symmetric information, a monopolist fully insures each type such that they are indifferent to not purchasing insurance, i.e. the monopolist's profit per customer is just that customer's risk premium. Formally, total monopoly profits are

$$\pi_S^M = \gamma r^H + (1 - \gamma) r^L$$

where r^J is implicitly defined by

$$v(w_0 - p^J l - r^J) = u^J(0,0)$$

Note that, contrary to standard monopoly models, first-best profit is not necessarily increasing in type as the risk premium first increases but then decreases in risk type. Monopoly contracts are shown below in Figure 5.1.
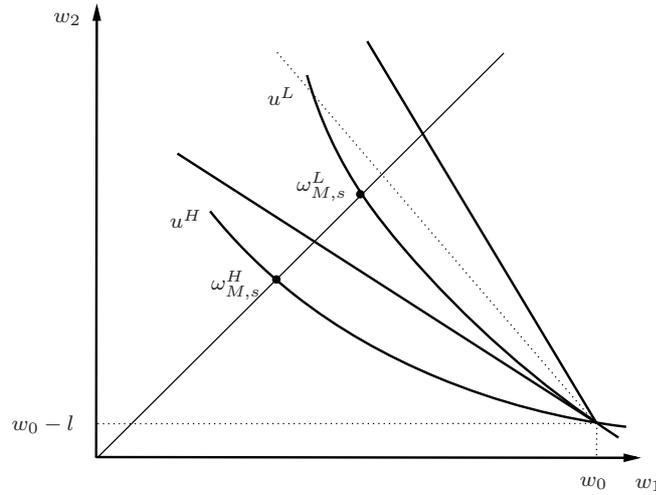
Figure 5.1: Monopoly contracts under symmetric information

Stiglitz (1977) gives the first characterization of the monopoly solution under asymmetric information. For analytical simplicity, we use the approach in Szalay (2008) and let insurers offer utility contracts. Therefore, let $v_1^L \equiv v(w_1(P^L))$ and $v_2^L \equiv v(w_2(I^L))$ and so forth where (P^L, I^L) is the contract intended for the L-type. We denote by $v_1^0 \equiv v(w)$ and $v_2^0 \equiv v(w - l)$ state-contingent utility in case of no insurance. Furthermore $z \equiv v^{-1}$ is the inverse of v.[47] A monopolistic insurer solves the following maximization problem:

$$\max_{v_1^L, v_2^L, v_1^H, v_2^H} \gamma(w - p^H l - (1 - p^H)z(v_1)^H - p^H z(v_2^H)) +$$

$$(1 - \gamma)(w - p^L l - (1 - p^L)z(v_1^L - p^L z(v_2^L))$$

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47 The inverse exists as v(w) is strictly increasing, $z' > 0$ and $z'' > 0$ from concavity of v.

$$\text{s.t.}$$

$$(1 - p^H)v_1^H + p^H v_2^H \geq (1 - p^H)v_1^L + p^H v_2^L$$

$$(1 - p^L)v_1^L + p^L v_2^L \geq (1 - p^L)v_1^H + p^L v_2^H$$

$$(1 - p^H)v_1^H + p^H v_2^H \geq (1 - p^H)v_1^0 + p^H v_2^0$$

$$(1 - p^L)v_1^L + p^L v_2^L \geq (1 - p^H)v_1^0 + p^H v_2^0$$

Monopoly contracts are separating and high-risk types are always fully insured, however low-risk types might not receive insurance at all. With the indirect utility approach, Szalay (2008) shows that optimal contracts, whenever both types receive insurance, are characterized by

$$v_1^{*H} = v_2^{*H}$$

$$\frac{1}{v'(z(v_2^{*H}))} = \frac{1-\gamma}{\gamma} \frac{p^L(1-p^L)}{p^H - p^L} \left[\frac{1}{v'(z(v_1^{*L}))} - \frac{1}{v'(z(v_2^{*L}))} \right]$$

and that

$$\frac{dv_1^{*H}}{d\gamma} = \frac{dv_2^{*H}}{d\gamma} < 0$$

What is more interesting for the analysis of collusion is that a monopolist might make losses on the H-type insurance contract. This is the case if the optimal menu specifies that the H-type utility is larger than H-type utility from the respective zero-profit making Rothschild-Stiglitz contract. For this to occur, types have to be sufficiently distinct such that the optimal L-type contract for $\gamma = 0$ yields an H-type a higher utility than the H-type Rothschild-Stiglitz contract:

Assumption 5.1 $p^H - p^L > r^L/l.$

Under Assumption 1, we can use the first order conditions and binding constraints to determine the share of H-types such that the optimal H-type contract corresponds to the H-type full insurance zero profit-making Rothschild-Stiglitz contract when both types buy insurance. This is the case for $\gamma = \hat{\gamma}$ with

$$\frac{\hat{\gamma}}{1-\hat{\gamma}} = \frac{p^L(1-p^L)}{p^H-p^L}\left[\frac{z'(c(v(w-p_Hl))) - z'(a(c(v(w-p^Hl))))}{z'(v(w-p_Hl))}\right]$$

where

$$a(x) \equiv v_2^0 + \frac{1-p^L}{p^L}(v_1^0 - x)$$

$$c(x) \equiv \frac{p^H}{p^H-p^L}u^L(v^0) - \frac{p^L}{p^H-p^L}x$$

Then, since $\frac{dv_2^{*H}}{d\gamma} < 0$, the H-type contract will yield losses to the monopolist for all $\gamma < \hat{\gamma}$. From concavity of v, $\hat{\gamma} < 1$ always holds and $0 < \hat{\gamma}$ follows from Assumption 1.

That even a monopolist might have to incur losses on a type is a result of the common value characteristic of this market: For a given contract, the profit from that contract depends on the type who takes up the contract, which is not the case in standard private value models of asymmetric information. Now if the share of low risks is high such that the monopolist tries to extract a large profit from these low risks, the corresponding incentive compatible high risk contract is loss-making. Monopoly contracts under asymmetric information for $\gamma < \hat{\gamma}$ are shown below in Figure 5.2.
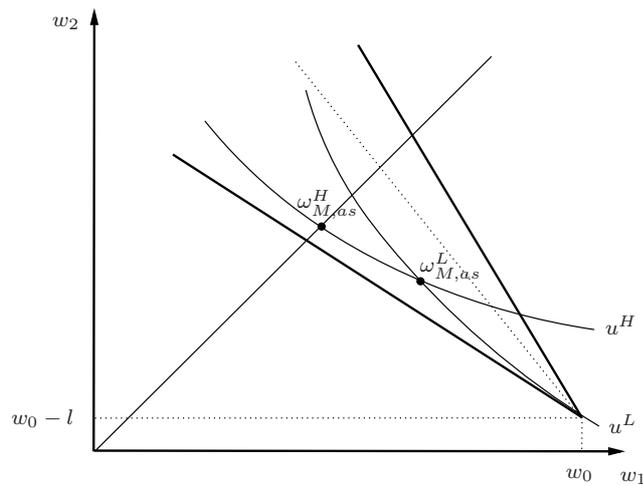
Figure 5.2: Monopoly contracts under asymmetric information for $\gamma < \hat{\gamma}$

Furthermore, monopoly profits per risk type under symmetric and asymmetric information are shown in Figure 5.3 the example in which $\hat{\gamma} = 0.15920$. The thick (thin) solid line corresponds to the profit on H-types under asymmetric (symmetric) information, and the thick dashed (thin dashed) line corresponds to profit on L-types uner asymmetric (symmetric) information.
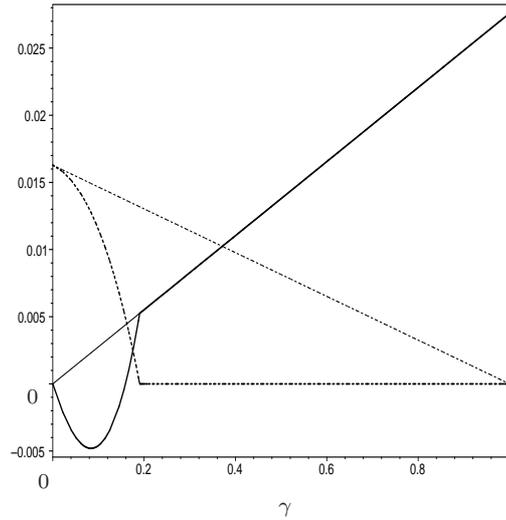
Figure 5.3: Monopoly profits per risk type under symmetric and asymmetric
information

Deviation profits

Under symmetric information, since collusive contracts are such that each contract is separately profit-making for any $\gamma \in [0, 1]$, a deviator would slightly undercut each monopoly contract separately and earn monopoly profits by deviating. Hence, as in the standard Bertrand case, we have $\pi_s^D(\gamma) = 2\pi_s^C(\gamma)$ for all $\gamma \in [0, 1]$.

Now under asymmetric information, as long as a monopolist makes profits on all offered contracts separately, i.e. if $\gamma \geq \hat{\gamma}$, a deviator would again slightly undercut each contract separately and earn monopoly profits by deviating. However, if $\gamma < \hat{\gamma}$, since a monopolist has to incur losses on high-risk types, a deviator would not deviate on *all* collusive contracts, but instead only cream skim low

risks, earning higher than total collusive profits in the deviation period. We thus have $\pi_{as}^D(\gamma) = 2\pi_{as}^C(\gamma)$ for all $\gamma \in [\hat{\gamma}, 1]$ and $\pi_{as}^D(\gamma) = \pi_{M,as}^L > 2\pi_{as}^C(\gamma)$ for all $\gamma \in [0, \hat{\gamma})$ where $\pi_{M,as}^L$ denotes monopoly profits on low risk types.

Stability of collusion

Since under symmetric information $\pi_s^D(\gamma) = 2\pi_s^C(\gamma)$ for all $\gamma \in [0, 1]$ and punishment profits are zero, it follows that collusion at maximal profits can be sustained as a subgame perfect equilibrium for all $\gamma \in [0, 1]$ if and only if $\delta \in [1/2, 1]$. Note that this is true irrespective of whether it is a high or low risk that yields higher profit to a monopolist, i.e. has a higher risk premium.

Under asymmetric information, since $\pi_{as}^D(\gamma) = 2\pi_{as}^C(\gamma)$ for all $\gamma \in [\hat{\gamma}, 1]$ and $\pi_{as}^D(\gamma) = \pi_{M,as}^L > 2\pi_{as}^C(\gamma)$ for all $\gamma \in [0, \hat{\gamma})$ and punishment profits are as well zero, it follows that collusion at maximal profits can be sustained as a subgame perfect equilibrium for all $\gamma \in [\hat{\gamma}, 1]$ if and only if $\delta \in (1/2, 1)$ and for all $\gamma \in [0, \hat{\gamma})$ if and only if $\delta \in [\hat{\delta}, 1]$ with $\hat{\delta} > 1/2$. This establishes our first result:

Proposition 5.1 *Collusion under asymmetric information is weakly less stable than collusion under symmetric information.*

Proof Follows immediately from comparison of the critical discount factors. ∎

Let us stress again that it is not asymmetric information and thus incentive compatibility constraints per se that destabilizes collusion, but common values combined with asymmetric information: Both in private and common value contexts, asymmetric information lowers monopoly and thus profits from perfect collusion. However, only under common values asymmetric information changes deviation incentives: Cross-subsidization occurs in perfect collusion, and thus the ratio between deviation and collusive profits increases under common values, whereas under private values profit from every type is nonnegative such that relative deviation incentives are not affected.[48]

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48 Although we consider the particular example of an insurance market, the above logic can easily be extended to other common value markets.

So far, we only consider a two-type distribution. Chade and Schlee (2008) characterize the monopoly solution in adverse selection insurance markets for an arbitrary type set $\Theta \subset (0, 1)$ and a general cumulative distribution function on it. Chade and Schlee (2008) show that, as for the two-type case, monotonicity holds, i.e. indemnity and premium increase in risk type, the highest risk type receives full coverage, there is no pooling at the top, and if there is exclusion, then it is low risks that are excluded. From analogous reasoning as in the two-type case, as incentive compatibility has to hold for the optimal menu, if the distribution function on the type set specifies that the mass of low enough risk types is sufficiently high, some of the highest risk types will be cross-subsidized in the optimal menu. Then, our reasoning holds that collusion under asymmetric information is (weakly) less stable.

Hence, access to information means good news for the insurance companies not just because this yields overall higher profits under collusion, but also because it stabilizes the collusive agreement.

5.4 Consumer information and collusive stability

So far, we have assumed that consumers are perfectly informed about their risk type. However, not all consumers might have that information. In this section, we will analyze the impact of consumer information about their risk type on collusive stability. A simple example is the health insurance and related markets with the recent availability of genetic tests. With genetic tests, customers have a higher precision of information about their risk type. We will analyze whether collusion is more or less stable when the share of informed customers, i.e. in the example customers that have genetic information available, increases. To this end, consider the following change in the stage game:

There is a continuum of individuals of mass 1, representing a large population of consumers. There are three types of customers, as before H,L but also U.

Type U customers do not know whether they are high or low risk. The share of uninformed U-types in the population is denoted by β, as before γ is the share of H-types among the $(1-\beta)$ informed customers. Furthermore, γ is also the probability of a U-type to be a high risk. The loss probability of U is therefore given by $p^U = \gamma p^H + (1-\gamma)p^L$. As in the baseline case in which all customers are informed, there is no possibility to signal either risk type or informational status. We also abstract from endogenous information acquisition, i.e. we will not analyze whether U-types have an incentive to acquire information. Rather, we assume that the share of informed customers is exogenously given and does not depend on available insurance contracts.[49]

Punishment profits

As for the two-type case, a Nash equilibrium in pure strategies might not exist. If it exists, in analogy to the Rothschild-Stiglitz equilibrium, H-types are fully insured at their fair premium, and U- and L-types are partially insured at their respective fair premiums such that contracts are incentive compatible such that firms make zero expected profits. An equilibrium in pure strategies might not exist as, depending on the share of informed customers and the share of high risks in the population, there might be e.g. profitable pooling deviations attracting U- and L-types, H- and U-types or even all 3 types. However, as in the two-type case, we can appeal to Dasgupta and Maskin (1986) to argue that an equilibrium in mixed strategies exists and that mixing is necessarily between jointly zero-profit making contract menus. Thus a Nash equilibrium always exists and yields zero-expected profits to firms.

Collusive profits

Under symmetric information, analogous to the two-type case, the monopolist offers each type a contract that extracts their risk premium. Total collusive profits are then

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49 For an analysis of incentives to acquire information and welfare consequences, see e.g. Doherty and Thistle (1996).

$$\pi_S^M = \beta(\gamma r^H + (1-\gamma)r^L) + (1-\beta)r^U$$

where r^U is implicitly defined by

$$v(w_0 - (\gamma p^H + (1-\gamma)p^L)L - r^U) = u^U(\omega^0)$$

Note that, $\qquad\qquad \dfrac{\partial \pi_S^M}{\partial \beta} = (\gamma r^H + (1-\gamma)r^L) - r^U$

i.e. since r is concave in risk type, a monopolist does not necessarily prefer informed customers.

Let us turn to asymmetric information. We will consider the case of full asymmetric information, i.e. insurers do not know risk types, and insurers as well do not know whether a customer is informed or not, i.e. they do not know informational status. A monopolistic insurer solves the following maximization problem:

$$\max_{v_1^L, v_2^L, v_1^H, v_2^H, v_1^U, v_2^U} (1-\beta)\left[w - p^U l - (1-p^U)z(v_1^U) - p^U z(v_2^U)\right] +$$

$$\beta\left[\gamma(w - p^H l - (1-p^H)z(v_1^H) - p^H z(v_2^H) + \right.$$

$$\left. (1-\gamma)(w - p^L l - (1-p^L)z(v_1^L) - p^L z(v_2^L))\right.$$

s.t.

$$(1-p^j)v_1^j + p^j v_2^j \geq (1-p^j)v_1^{j'} + p^j v_2^{j'} \ \forall j, j' \in \{L, U, H\}$$

$$(1-p^j)v_1^L + p^j v_2^j \geq (1-p^j)v_1^0 + p^j v_2^0 \ \forall j \in \{L, U, H\}$$

As this maximization problem can be analyzed as a standard problem, we will assume that the participation constraint of the L-type, and incentive constraints for the U-type with respect to the L-type contract and the H-type with respect to the U-type contract bind and that all other constraints are slack. We can rewrite the L-type participation constraint as

$$v_2^L = s(v_1^L) \equiv v_2^0 + \frac{1-p^L}{p^L}(v_1^0 - v_1^L) \tag{5.1}$$

Similarly, the U-type incentive constraint can be transformed to

$$v_2^U = t(v_1^L, v_1^U) \equiv v_2^0 + \frac{1-p^L}{p^L}(v_1^0 - v_1^L) + \frac{1-p^U}{p^U}(v_1^L - v_1^U) \qquad (5.2)$$

Finally, the H-type incentive constraint can be written as

$$v_2^H = k(v_1^L, v_1^U, v_1^H) \equiv t(v_1^L, v_1^U) + \frac{1-p^H}{p^H}(v_1^U - v_1^H) \qquad (5.3)$$

We can now rewrite the monopolist's maximization problem as the following reduced unconstrained problem:

$$\max_{v_1^L, v_1^H, v_1^U} (1-\beta)\left[w - p^U l - (1-p^U)z(v_1^U) - p^U z(t(v_1^L, v_1^U))\right] +$$

$$\beta\left[\gamma(w - p^H l - (1-p^H)z(v_1^H) - p^H z(k(v_1^L, v_1^U, v_1^H))) +\right.$$

$$\left. (1-\gamma)(w - p^L l - (1-p^L)z(v_1^L) - p^L z(s(v_1^L)))\right]$$
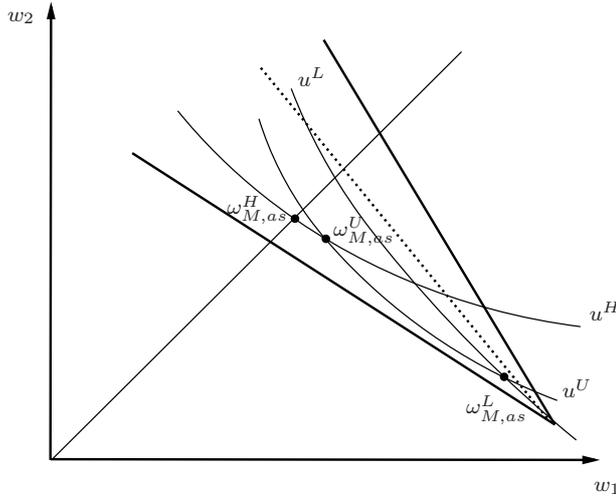
Then, whenever all three types buy insurance, optimal contracts are characterized by:

$$v_1^{*H} = v_2^{*H} \qquad (5.4)$$

$$\gamma\frac{1}{v'(z(v_2^{*H}))} = \frac{(1-\beta)}{\beta}\frac{(1-p^U)p^U}{p^H - p^U}\left[\frac{1}{v'(z(v_1^{*U}))} - \frac{1}{v'(z(v_2^{*U}))}\right] \qquad (5.5)$$

$$\gamma\frac{p^H}{p^U}\frac{1}{v'(z(v_2^{*H}))} + \frac{(1-\beta)}{\beta}\frac{1}{v'(z(v_2^{*U}))} = (1-\gamma)\frac{(1-p^L)p^L}{p^U - p^L}\left[\frac{1}{v'(z(v_1^{*L}))} - \frac{1}{v'(z(v_2^{*L}))}\right]$$

$$(5.6)$$

and (5.1)-(5.3). An example for the optimal contracts is shown below in Figure 5.4. Note that in Figure 5.4 the optimal H-type contract is such that H-types are cross-subsidized. As cross-subsidization drives our results on collusive stability, we want to analyze when cross-subsidization occurs if there are uninformed consumers and how cross-subsidization changes with the share in uninformed consumers. Let us first consider the extreme cases of either perfect or no consumer information. First assume $\beta = 1$, i.e. there are only informed customers. This corresponds to the two-type case analyzed in the previous section and there

Figure 5.4: Monopoly contracts when all three types buy insurance

is cross-subsidization of H-types if $\gamma < \hat{\gamma}$. We let $\hat{\gamma} \equiv \hat{\gamma}(1)$. Now consider $\beta = 0$, i.e. there are only uninformed customers. The optimal contract is pinned down by

$$v_1^{*U} = v_2^{*U}$$

$$v_1^{*U} = (\gamma p^H + (1 - \gamma)p^L)v_2^0 + (1 - (\gamma p^H + (1 - \gamma)p^L))v_1^0$$

Then, as the share of H-types in the whole population impacts the slope of U-type indifference curves, we can determine the share of H-types such that the optimal U-type contract for $\beta = 0$ corresponds to the H-type Rothschild-Stiglitz contract. This share is denoted as $\hat{\gamma}(0)$ and given by

$$\hat{\gamma}(0) \equiv \frac{u^L(0,0) - v(w - p^H l)}{(p^H - p^L)(v_1^0 - v_2^0)}$$

Note that $\hat{\gamma}(0) < 1$ due to concavity of v. Note also that, under Assumption 1, $u^L(0,0) - v(w - p^H l) > 0$ and hence $\hat{\gamma}(0) > 0$. Now since

$$\frac{dv_1^{*U}}{d\gamma}\bigg|_{\beta=0} = \frac{dv_1^{*U}}{d\gamma}\bigg|_{\beta=0} = (p^L - p^H)(v_1^0 - v_2^0) < 0$$

increasing β at $\beta = 0$ implies that H-types will be cross-subsidized for β greater but sufficiently close to 0 if $\gamma < \hat{\gamma}(0)$. This is because if there are only a few

high risk types, the optimal U-type contract is still close to the U-type contract for $\beta = 0$ such that the corresponding incentive compatible H-type contract is loss-making. In our example, $\hat{\gamma}(0) = 0.795$ such that even for a relatively large share of H-types in the overall population there is cross-subsidization if there are only a few informed customers.

In a similar manner, when all three types buy insurance, we can determine a critical $\gamma(\beta)$ such that the optimal H-type contract corresponds to the H-type Rothschild-Stiglitz contract, denoted by $\hat{\gamma}(\beta)$. From (5.3) and (5.4), define

$$v_1^{*U}\Big|_{v_1^{*H}=v(w-p^H l)} = \frac{p^U}{p^H - p^U}(p^H(v_2^0 + \frac{1-p^L}{p^L}v_1^0) - v(w-p^H l) - \frac{p^H(p^U - p^L)}{p^U p^L}v_1^{*L})$$

$$\equiv f^{\hat{\gamma}}(v_1^{*L}) \quad (5.7)$$

and
$$t^{\hat{\gamma}}(v_1^{*L}) \equiv t(v_1^{*L}, f^{\hat{\gamma}}(v_1^{*L})) \quad (5.8)$$

Then, rewriting (5.5) and (5.6), $\hat{\gamma}(\beta)$ (and v_1^{*L}) is implicitly given by the solution to the following system of equations:

$$p^U = \hat{\gamma}(\beta)p^H + (1 - \hat{\gamma}(\beta))p^L$$

$$\hat{\gamma}(\beta)z'(v(w - p^H l)) = \frac{(1-\beta)}{\beta}\frac{(1-p^U)p^U}{p^H - p^U}\left[z'(f^{\hat{\gamma}}(v_1^{*L})) - z'(t^{\hat{\gamma}}(v_1^{*L}))\right]$$

$$\frac{\hat{\gamma}(\beta)}{1 - \hat{\gamma}(\beta)}\frac{p^H}{p^U}z'(v(w - p^H l)) + \frac{(1-\beta)}{\beta(1-\hat{\gamma})}z'(t^{\hat{\gamma}}(v_1^{*L})) = \frac{(1-p^L)p^L}{p^U - p^L}\left[z'(v_1^{*L}) - z'(s(v_1^{*L}))\right]$$

and (5.7)-(5.8). The above equations give the critical share of high risks in the population for a given share of informed customers when parameters are such that all three types buy insurance. Note that, if β is small and γ large, optimal contracts will be such that L-types do not receive insurance, however, under Assumption 1 H-types will be cross-subsidized for small values of β even if L-types do not receive insurance. Thus, even if not all three types buy insurance, there is a critical share of high risks such that there is cross-subsidization. With a slight abuse of notation, we will denote this critical share for every $0 < \beta < 1$ by $\hat{\gamma}(\beta)$. We illustrate $\hat{\gamma}(\beta)$ for our example derived from numerical analysis in Figure 5.5.
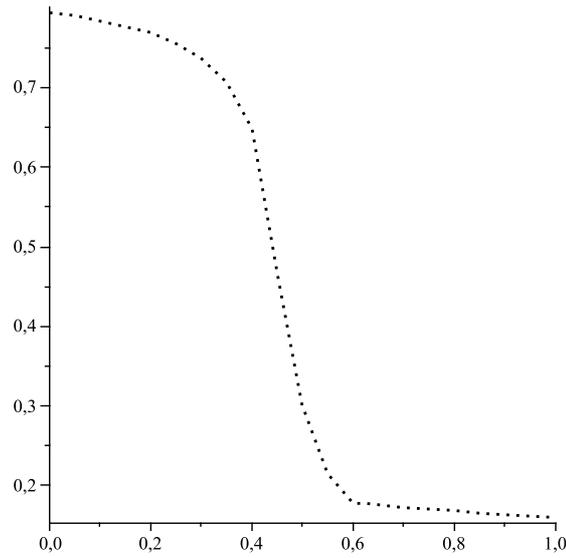
Figure 5.5: $\hat{\gamma}(\beta)$.

From the previous section $\hat{\gamma}(1) = 0.15920$. Note that $\hat{\gamma}(\beta)$ decreases in β. This is because for low values of β, U-types are important for profit-maximization and hence if optimal contracts are such that there is (hypothetical) cross-subsidization for $\beta = 0$, then if there are only a few informed types there will be cross-subsidization even if the share of high risks is fairly high. With an increase in informed consumers, the shares of high and low risks are increasingly important such that now for cross-subsidization to occur, the share of high risks needs to be lowered in order for the monopolist to be willing to cross-subsidize them to extract profit from low risks.

Deviation profits

Under symmetric information, as each contract from perfect collusion is separately profit-making for any $(\gamma, \beta) \in [0, 1]^2$, as in the two-type case a deviator would slightly undercut each monopoly contract separately and earn monopoly profits by deviating. Hence, independent of the share of informed customers, we have $\pi_s^D(\gamma, \beta) = 2\pi_s^C(\gamma, \beta)$ for all $(\gamma, \beta) \in [0, 1]^2$.

Under asymmetric information, a deviator only offers those contracts that yield positive profits and does not offer cross-subsidized contracts. Since from the

above analysis at least H-types might be cross subsidized for some $(\gamma, \beta) \in [0,1]^2$, there exist $(\gamma, \beta) \in [0,1]^2$ such that $\pi_{as}^D(\gamma, \beta) > 2\pi_{as}^C(\gamma, \beta)$. We also have $\pi_{as}^D(\gamma, 0) = 2\pi_{as}^C(\gamma, 0)$ for all $\gamma \in [0,1]$ and, for $\hat{\beta}$ sufficiently close to 0, $\pi_{as}^D(\gamma, \hat{\beta}) > 2\pi_{as}^C(\gamma, \hat{\beta})$ for $\gamma < \hat{\gamma}(0)$. Furthermore, from the previous section, $\pi_{as}^D(\gamma, 1) = 2\pi_{as}^C(\gamma, 1)$ for all $\gamma \in [\hat{\gamma}, 1]$ and $\pi_{as}^D(\gamma, 1) > 2\pi_{as}^C(\gamma, 1)$ for all $\gamma \in [0, \hat{\gamma})$.

Stability of collusion

Since under symmetric information $\pi_s^D(\gamma, \beta) = 2\pi_s^C(\gamma, \beta)$ for all $(\gamma, \beta) \in [0,1]^2$ and punishment profits are zero, it follows that collusion at maximal profits can be sustained as a subgame perfect equilibrium for all $(\gamma, \beta) \in [0,1]^2$ if and only if $\delta \in [1/2, 1]$.

Under asymmetric information, if $\beta = 0$, collusion at maximal profits can be sustained as a subgame perfect equilibrium for all $\gamma \in [0,1]$ if and only if $\delta \in [1/2, 1]$. Now for $\hat{\beta}$ sufficiently close to 0, collusion at maximal profits can be sustained as a subgame perfect equilibrium for all $\gamma \in [\hat{\gamma}(0), 1]$ if and only if $\delta \in [1/2, 1]$ and for all $\gamma \in [0, \hat{\gamma}(0))$ if and only if $\delta \in (\tilde{\delta}, 1)$ with $\tilde{\delta} > 1/2$. Combining the last two results, we have that a marginal increase in β from no informed consumers to some informed consumers always destabilizes collusion as long as $\gamma < \hat{\gamma}(0)$. For other values of β, the impact of an increase of β on collusive stability is nonmonotonous. We will discuss $\bar{\delta}(\beta)$ for different γ for our example.

First consider $\gamma = 0.1$. Figure 5.6 shows $\bar{\delta}(\beta)$ obtained from numerical analysis for $\gamma = 0.1$. For low values of β, $\bar{\delta}(\beta)$ increases as the loss on H-types increases since the monopolist wants to extract a large surplus from U-types. Furthermore, for low values of β, the L-type does not receive insurance. For higher values of β, H-types are still cross-subsidized, but the overall loss on them decreases such that $\bar{\delta}(\beta)$ decreases slightly. For even higher values of β, since γ is low, now for profit maximization the profit from L-types become increasingly important such that now H-types are increasingly cross-subsidized to extract profits from L-types instead of U-types and $\bar{\delta}(\beta)$ increases again.

Now consider $\gamma = 0.15$. Figure 5.7 shows $\bar{\delta}(\beta)$ obtained from numerical analysis for this case. The analysis for low and intermediate values of β is the same
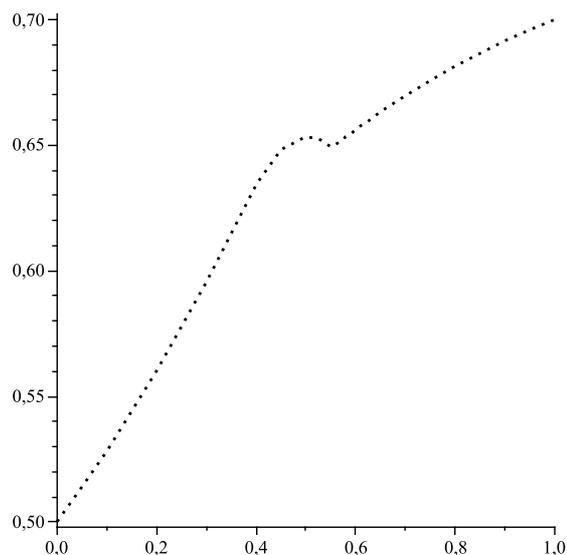
**Figure 5.6:** $\bar{\delta}(\beta)$ for $\gamma = 0.1$

as for $\gamma = 0.1$. However, for higher values of β, the critical discount factor now decreases. This is because as the share of high risks is sufficiently high, profit-maximization requires that although there is still cross-subsidization of high risks, it decreases in the share of informed consumers.
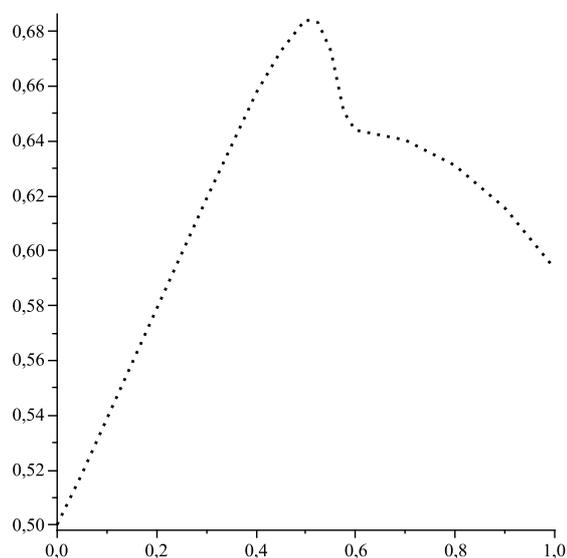
Figure 5.7: $\bar{\delta}(\beta)$ for $\gamma = 0.15$

The last examples is an even higher share of high risks with $\gamma = 0.2$. Figure

5.8 shows $\bar{\delta}(\beta)$ obtained from numerical analysis for this case. Now as the share of high risks is substantial, high risks will not be cross-subsidized any more if uninformed consumers become unimportant as now profit-maximization requires a large surplus to be extracted from high risks.
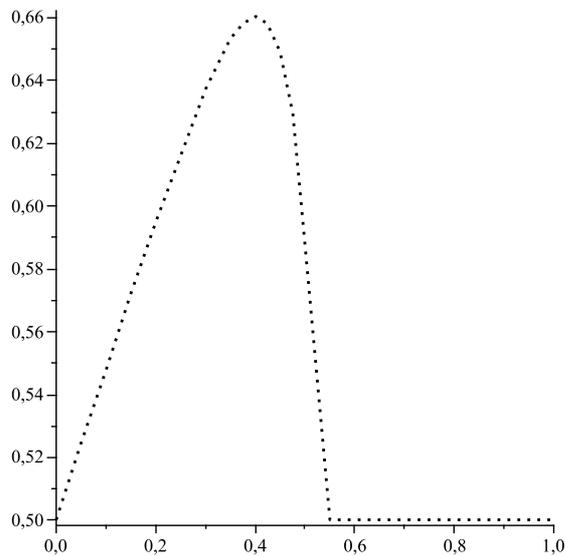
Figure 5.8: $\bar{\delta}(\beta)$ for $\gamma = 0.2$

We can summarize our results in the following proposition:

Proposition 5.2 *Consumer information about risk type does not affect collusive stability under symmetric information. Under asymmetric information, the impact of information on collusive stability is nonmonotonous. If there are only uninformed consumers and the share of high risks is not too high, some information about risk type always destabilizes collusion.*

Proof Follows immediately from comparison of the critical discount factors. ∎

We have analyzed the impact of consumer information on collusive stability under the assumption that insurers neither know risk type of informed consumers, nor know whether a consumer is informed, i.e. insurers cannot observe informational status. Note that, if e.g. in the case of genetic tests, insurers know whether a test has been taken or not, the analysis is similar to the two-type from the previous section as uninformed consumers would always obtain

the profit-making contract that extracts their risk premium and the adverse
selection problem remains only for informed types.

5.5 Demand shocks: Volatile risk composition

In the last section, we considered the impact of different demand conditions
in terms of the share of consumers that have precise information about their
risk type on collusive stability. We will now consider an insurance market in
which consumers are again perfectly informed about their risk type, however,
there might be intertemporal shocks to demand in the form of a volatile risk
composition of customers.

Consider the following change in the model: There are again only two types of
customers, H-types and L-types. In each period, with probability σ the share
of high risks in period t is $\bar{\gamma}$, and with probability $1 - \sigma$, the share is $\underline{\gamma}$ with
$0 \leq \underline{\gamma} < \bar{\gamma} \leq 1$.[50] We assume that σ is common knowledge. Furthermore,
in the beginning of each period before setting contracts, all firms observe the
realization of γ.[51] There are no deviation incentives for an individual firm, if

$$\pi^C(\gamma) + \frac{\delta}{1-\delta}(\sigma\pi^C(\bar{\gamma}) + (1-\sigma)\pi^C(\underline{\gamma})) \geq \pi^D(\gamma) \Leftrightarrow \delta \geq \frac{1}{1 + \frac{\sigma\pi^C(\bar{\gamma})+(1-\sigma)\pi^C(\underline{\gamma})}{\pi^D(\gamma)-\pi^C(\gamma)}} =: \bar{\delta}(\gamma)$$

We start again with symmetric information. Under symmetric information,
since for all γ each collusive contract is separately profit-making, deviation profit
always equals total collusive profits. Then deviation incentives only depend on
the ratio between the period collusive profit and the average collusive profit.
For $\underline{\gamma} = \bar{\gamma}$, $\bar{\delta}_s(\underline{\gamma}) = \bar{\delta}_s(\bar{\gamma}) = \frac{1}{2}$. Now fix $\underline{\gamma}$ and consider an increase in $\bar{\gamma}$:

$$\text{sgn}\left(\frac{\partial\bar{\delta}_s(\bar{\gamma})}{\partial\bar{\gamma}}\right) = \text{sgn}\left(\frac{\partial\pi_s^C(\bar{\gamma})}{\partial\bar{\gamma}}\right) \; \forall\bar{\gamma} \in \; [\underline{\gamma}, 1]$$

and

---

50 Hence, the demand shocks are iid and we do not consider correlation of demand shocks.
51 We thus model demand shocks in the spirit of Rotemberg and Saloner (1986) and do not
   introduce imperfect monitoring as in Green and Porter (1984).

$$\text{sgn}\left(\frac{\partial \bar{\delta}_s(\gamma)}{\partial \bar{\gamma}}\right) = -\text{sgn}\left(\frac{\partial \pi_s^C(\bar{\gamma})}{\partial \bar{\gamma}}\right) \forall \bar{\gamma} \in [\underline{\gamma}, 1]$$

It follows that for $\pi^C(0) < \pi^C(1)$ which is equivalent to $r^L < r^H$,

$$\bar{\delta}_s(\underline{\gamma}) < \frac{1}{2} < \bar{\delta}_s(\bar{\gamma}) \text{ for } 0 \le \underline{\gamma} < \bar{\gamma} \le 1.$$

Note that this corresponds to the destabilizing demand boom effect in Rotemberg and Saloner (1986) and the subsequent literature: If $\pi^C(0) < \pi^C(1)$, then a high share of high risk types implies high demand as they have the larger risk premium. From analogical reasoning, it follows that for $\pi^C(0) > \pi^C(1)$, $\bar{\delta}_s(\underline{\gamma}) > \frac{1}{2} > \bar{\delta}_s(\bar{\gamma})$ for $0 \le \underline{\gamma} < \bar{\gamma} \le 1$.

We now turn to the asymmetric information setting. Under asymmetric information, deviation profits might be higher than total collusive profits leading to a higher critical discount factor, as shown in section 3. We will refer to this as the cream-skimming effect. What is however more relevant for the analysis of volatile risk type shares is that collusive profits first decrease and then increase in the share of high risks. Let π_{as}^M denote monopoly profits under asymmetric information. Note that π_{as}^M is continuously differentiable w.r.t γ.

Lemma 5.1 $\partial \pi_{as}^M / \partial \gamma < 0 \ \forall \gamma < \check{\gamma}$ and $\partial \pi_{as}^M / \partial \gamma > 0 \ \forall \gamma > \check{\gamma}$ with $\hat{\gamma} < \check{\gamma} < \tilde{\gamma}$ where $\tilde{\gamma}$ is the share of high risks such that for $\gamma \ge \tilde{\gamma}$, $\omega_{M,as}^L = (0,0)$.

Proof See Appendix. ∎

Figure 5.9 shows monopoly profits under symmetric and asymmetric information. In our example, $\hat{\gamma} \approx 0.15920$, $\check{\gamma} \approx 0.16496$ and $\tilde{\gamma} \approx 0.19032$. As collusive profits first decrease and then increase in the share of high risks and because the cream-skimming effect might be relevant, the analysis for the asymmetric information setting depends on which parameter ranges $\underline{\gamma}$ and $\bar{\gamma}$ are drawn from. We will discuss basic cases and assume that $\pi^C(0) < \pi^C(1)$.
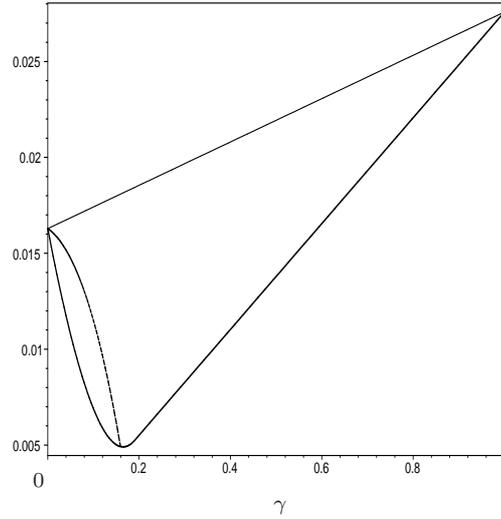
Figure 5.9: Monopoly and deviation profits under symmetric and asymmetric information.

Case 1: $\tilde{\gamma} < \underline{\gamma} < \bar{\gamma}$

This is the case in which, under asymmetric information, in both demand states low risks will not receive insurance. Collusive and deviation contracts coincide and $\partial\pi_{as}^M/\partial\gamma = r^H > r^H - r^L = \partial\pi_s^M/\partial\gamma$. Then, from analogous reasoning as above under symmetric information, it follows that $\bar{\delta}_{as}(\underline{\gamma}) < \bar{\delta}_{as}(\bar{\gamma})$ and

$$\bar{\delta}_{as}(\underline{\gamma}) < \bar{\delta}_s(\underline{\gamma}) < \bar{\delta}_s(\bar{\gamma}) < \bar{\delta}_{as}(\bar{\gamma}) \ \ \text{for } \tilde{\gamma} < \underline{\gamma} < \bar{\gamma} \le 1.$$

Here, the standard demand boom effect is at work both for symmetric and asymmetric information, however, although absolute profits are lower under asymmetric information, the relative demand boom effect is stronger under asymmetric information as low risks do not receive insurance anyways.

Case 2: $\underline{\gamma} < \bar{\gamma} < \check{\gamma}$

Contrary to Case 1, now for both possible realizations of γ the share of low risks is high and the cream-skimming effect can occur. What is interesting is that under asymmetric information there exist $\underline{\gamma}$ and $\bar{\gamma}$ such that $\bar{\delta}_{as}(\bar{\gamma}) < \bar{\delta}_{as}(\underline{\gamma})$. This is particularly easy to see when $\underline{\gamma} < \hat{\gamma}$ and $\hat{\gamma} < \bar{\gamma}$ as firstly collusive profits

decrease in γ and secondly there is no cream-skimmming effect on $\bar{\delta}_{as}(\bar{\gamma})$, but on $\bar{\delta}_{as}(\underline{\gamma})$. This is interesting as although $\bar{\gamma}$ implies a higher share of consumers with a higher willingness to pay, colluding firms and consequently a deviator cannot exploit the overall higher willingness to pay due to asymmetric information and actually realize a low profit such that the critical discount factor is lower.

Case 3: $\underline{\gamma} < \check{\gamma} < \bar{\gamma}$

In this case, collusive profits monotonically decrease in the region where $\underline{\gamma}$ is drawn from and monotonically increase in the region where $\bar{\gamma}$ is drawn from. Assume first that $\hat{\gamma} < \underline{\gamma}$ such that the cream-skimming effect is not present. Since collusive profits decrease for $\gamma < \check{\gamma}$ and increase for $\gamma > \check{\gamma}$, for a $\underline{\gamma}$ there exist a $\bar{\gamma}$ such that $\pi_{as}^{C}(\underline{\gamma}) = \pi_{as}^{C}(\bar{\gamma}) = \sigma\pi_{as}^{C}(\bar{\gamma}) + (1-\sigma)\pi_{as}^{C}(\underline{\gamma})$ and thus $\bar{\delta}_{as}(\underline{\gamma}) = \bar{\delta}_{as}(\bar{\gamma}) = 1/2$. Then, since under symmetric information, $\bar{\delta}_{s}(\underline{\gamma}) < \frac{1}{2} < \bar{\delta}_{s}(\bar{\gamma})$ for $0 \leq \underline{\gamma} < \bar{\gamma} \leq 1$, there exist $\underline{\gamma}$ and $\bar{\gamma}$ such that

$$\bar{\delta}_{s}(\underline{\gamma}) < \bar{\delta}_{as}(\underline{\gamma}) \lesseqgtr \bar{\delta}_{as}(\bar{\gamma}) < \bar{\delta}_{s}(\bar{\gamma})$$

i.e. the highest critical discount factor pertains to symmetric information. Now consider $\underline{\gamma} < \hat{\gamma}$ such that the cream-skimming effect destabilizing collusion under asymmetric information is present for $\underline{\gamma} \neq 0$. However, assume for the moment that $\underline{\gamma} = 0$. Then, from an analogical reasoning as above and since $\pi^{C}(0) < \pi^{C}(1)$, there exists a $\bar{\gamma}_0 < 1$ such that $\pi_{as}^{C}(0) = \pi_{as}^{C}(\bar{\gamma}_0)$ and thus $\bar{\delta}_{as}(0) = \bar{\delta}_{as}(\bar{\gamma}_0) = 1/2$. Then although

$$\operatorname{sgn}\left(\frac{\partial\bar{\delta}_{as}(\bar{\gamma}_0)}{\partial\underline{\gamma}}\right) = -\operatorname{sgn}\left(\frac{\partial\pi_{as}^{C}(\underline{\gamma})}{\partial\underline{\gamma}}\right) > 0 \,\forall\underline{\gamma} \in [0, \check{\gamma})$$

and

$$\operatorname{sgn}\left(\frac{\partial\bar{\delta}_{s}(\underline{\gamma})}{\partial\underline{\gamma}}\right) = \operatorname{sgn}\left[A\frac{\partial\pi_{as}^{D}(\underline{\gamma})}{\partial\underline{\gamma}} - \frac{\partial\pi_{as}^{C}(\underline{\gamma})}{\partial\underline{\gamma}}[A + (1-\sigma)(\pi_{as}^{D}(\underline{\gamma}) - \pi_{as}^{C}(\underline{\gamma}))]\right]$$
$$> 0 \,\forall\underline{\gamma} \in [0, \hat{\gamma})$$

i.e. although under asymmetric information both critical discount factors increase in $\underline{\gamma}$, as long as $\underline{\gamma}$ is sufficiently close to 0, it holds that $\bar{\delta}_s(\underline{\gamma}) < \bar{\delta}_{as}(\underline{\gamma}) \lesseqgtr$ $\bar{\bar{\delta}}_{as}(\bar{\gamma}_0) < \bar{\delta}_s(\bar{\gamma})$. Hence, although the cream-skimming effect destabilizing collusion under asymmetric information, for certain parameter ranges the demand boom effect under symmetric information dominates such that the highest critical discount factor pertains to symmetric information.
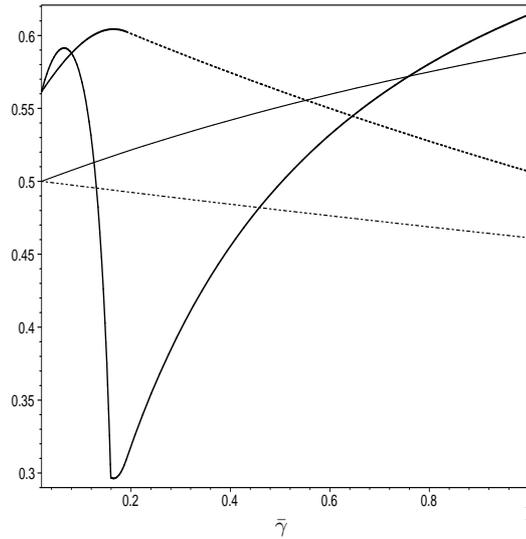
Figure 5.10: $\underline{\gamma} = 0.02, \sigma = 1/4$.

Figure 5.10 shows for $\underline{\gamma} = 0.02$ where the cream-skimming effect is present the critical discount factors for $\bar{\gamma} \geq 0.02$. The solid black (grey) curve depicts $\bar{\delta}(\bar{\gamma})$ under asymmetric (symmetric) information, and the dotted black (grey) curve depicts $\bar{\bar{\delta}}(\underline{\gamma})$ under asymmetric (symmetric) information. As was shown, the critical discount factors under symmetric information monotonically increase respectively decrease. Furthermore, it can be seen that for a range of $\bar{\gamma} > \check{\gamma}$, the highest critical discount factor is $\bar{\delta}_s(\bar{\gamma})$. This requires in particular for $\bar{\gamma}$ to be sufficiently but not too high as otherwise the demand boom effect under asymmetric information would dominate. Note also that Case 2 is illustrated, as $\bar{\bar{\delta}}_{as}(\bar{\gamma}) < \bar{\bar{\delta}}_{as}(\underline{\gamma})$ for $\bar{\gamma}$ sufficiently close to $\check{\gamma}$.

We can now summarize our findings. First, when the risk composition is volatile over time, collusion under asymmetric information is not always less stable than collusion under symmetric information:

Proposition 5.3 *If the risk composition is volatile, collusion under asymmetric information might be more stable than collusion under symmetric information.*

Proof Follows immediately from comparison of the critical discount factors. ∎

Note that this result is a consequence of asymmetric information per se and does not, contrary to the result with a constant risk composition, pertain to common values: Due to asymmetric information, collusive profits are nonmonotonous in the share of high risk types. Then under asymmetric information a change in the share of risk types might not change collusive profits strongly such that collusion is more stable.

We can furthermore relate our results to the literature on whether collusion breaks down in boom or bust phases. We will thereby define a boom phase by a period with a high share of costumers with high willingness to pay, i.e. if $r^H > r^L$, a boom phase is a period with high γ.

Proposition 5.4 *Under asymmetric information, collusion might break down (require prices to be lowered) in bust phases.*

Proof Follows immediately from comparison of the critical discount factors. ∎

The second result is interesting as we analyze a Rotemberg and Saloner (1986) set-up which does not involve any imperfect monitoring of competitors' actions. The result follows again from asymmetric information as although a higher share of customers with a higher willingness to pay would increase profits under symmetric information, incentive compatibility constraints prevent firms from exploiting the demand boom under asymmetric information such that collusive and consequently deviating profits are higher when the share of customers with a higher willingness to pay is low.

5.6 Conclusion

We study the ability of firms to sustain collusive agreements in insurance markets with adverse selection. In particular, we analyze whether collusion is more stable under symmetric or asymmetric information. It is shown that asymmetric information destabilizes collusive agreements, however this is not a consequence of asymmetric information per se but due to the fact that the customers' private information about risk types impacts an insurers profit from a particular contract: Since even a monopolist might want to cross-subsidize high risks a deviator can earn higher than total collusive profits which destabilizes collusive agreements. In terms of the industrial organization literature, we can thus contribute a new factor that destabilizes collusion: pay-off relevant private information.

Furthermore, we show that the impact of consumer information about risk type has a nonmonotonous effect on collusive stability. This is an interesting result as more consumer information about market parameters is typically assumed to deter collusion.

In this chapter, we only analyze the market with two respectively three customer types and consider the extreme cases of either complete symmetric or asymmetric information. One extension would be to conduct the analysis for more general type distributions. Furthermore, an interesting extension would be to consider a model where instead of either symmetric or asymmetric information firms receive a signal about a customer's risk type with varying precision such that more detailed comparative statics results can be derived.

5.7 Appendix

Proof of Lemma 1.

From the envelope theorem, we have:

$$\partial\pi_{AS}^M(\gamma)/\partial\gamma = (w - p^H l - (1 - p^H)z(v_1^{*H}(\gamma)) - p^H z(v_2^{*H}(\gamma)))$$
$$- (w - p^L l - (1 - p^L)z(v_1^{*L}(\gamma)) - p^L z(v_2^{*L}(\gamma))) \quad (5.9)$$

Rewriting (15) by substituting:

$$v_2^{*L} = v_2^0 + \frac{1 - p^L}{p^L}(v_1^0 - v_1^{*L}) \equiv a(v_1^{*L}) \quad (5.10)$$

$$v_1^{*H} = v_2^{*H} = p^H(v_2^0 + \frac{1 - p^L}{p^L}(v_1^0) + \frac{p^L - p^H}{p^L p^H}v_1^{*L}) \equiv k(v_1^{*L}) \quad (5.11)$$

from (2) and the binding constraints, we get

$$\partial\pi_{AS}^M(\gamma)/\partial\gamma = (w - p^H l - z(k(v_1^{*L}(\gamma)))) -$$
$$(w - p^L l - (1 - p^L)z(v_1^{*L}(\gamma)) - p^L z(a(v_1^{*L}(\gamma))) \quad (5.12)$$

Substituting (19) and (20) into (3), we have:

$$z'(k(v_1^{*L})) - \frac{1 - \gamma}{\gamma}\frac{p^L(1 - p^L)}{p^H - p^L}(z'(v_1^{*L}) - z'(a(v_1^{*L})) = 0 \equiv F(\gamma, v_1^{*L}) \quad (5.13)$$

Then, from the implicit function theorem:

$$\partial v_1^{*L}/\partial\gamma = -\frac{\partial F(\gamma, v_1^{*L})/\partial\gamma}{\partial F(\gamma, v_1^{*L})/\partial v_1^{*L}} \quad (5.14)$$

with

$$\partial F(\gamma, v_1^{*L})/\partial v_1^{*L} = z''(k(v_1^{*L}))\frac{p^H(p^L - p^H)}{p^L p^H} -$$
$$\frac{1 - \gamma}{\gamma}\frac{p^L(1 - p^L)}{p^H - p^L}(z''(v_1^{*L}) + z''(a(v_1^{*L}))(\frac{1 - p^L}{p^L})) < 0 \quad (5.15)$$

and

$$\partial F(\gamma, v_1^{*L})/\partial \gamma = \frac{\frac{p^L(1-p^L)}{p^H-p^L}(z'(v_1^{*L}) - z'(a(v_1^{*L})))}{\gamma^2} > 0 \qquad (5.16)$$

It follows, as long as both types buy insurance, that

$$\partial v_1^{*L}/\partial \gamma > 0$$

Since
$$\partial k(v_1^{*L})/\partial \gamma = \frac{p^L - p^H}{p^L} < 0$$

we have
$$\partial(w - p^H l - z(k(v_1^{*L}(\gamma))))/\partial \gamma > 0$$

as long as both types buy insurance. Furthermore, since

$$\partial a(v_1^{*L})/\partial \gamma = -\frac{1 - p^L}{p^L} < 0$$

we have

$$\partial(w - p^L l - (1 - p^L)z(v_1^{*L}(\gamma)) - p^L z(a(v_1^{*L}(\gamma)))/\partial \gamma =$$
$$(\partial v_1^{*L}/\partial \gamma)(1 - p^L)(z'(a(v_1^{*L})) - z'(v_1^{*L})) < 0 \quad (5.17)$$

as long as both types buy insurance. Now, as already shown in Stiglitz (1977), there exists a $\tilde{\gamma}$ such that for $\gamma < \tilde{\gamma}$ L-types do not buy insurance. With our approach, $\tilde{\gamma}$ is given by

$$\frac{\tilde{\gamma}}{1 - \tilde{\gamma}} = \frac{p^L(1-p^L)}{p^H - p^L}\left[\frac{z'(c(v(w - p_H l - r^H))) - z'(a(c(v(w - p^H l - r^H))))}{z'(v(w - p_H l - r^H))}\right]$$
$$(5.18)$$

Note that $\tilde{\gamma} > \hat{\gamma}$. Furthermore, we have $\partial \pi_{AS}^M(\gamma)/\partial \gamma = r^H \; \forall \; \gamma \in [\tilde{\gamma}, 1]$. Then, since $(w - p^H l - z(k(v_1^{*L}(\gamma)))) < 0$ for all $\gamma < \hat{\gamma}$ and $(w - p^L l - (1 - p^L)z(v_1^{*L}(\gamma)) - p^L z(a(v_1^{*L}(\gamma))) > 0 \forall \; \gamma \in [0, \check{\gamma})$, combining the above results gives

$$\partial \pi_{AS}^M/\partial \gamma < 0 \; \forall \gamma < \check{\gamma}$$
$$\partial \pi_{AS}^M/\partial \gamma > 0 \; \forall \gamma > \check{\gamma}$$

with $\hat{\gamma} < \check{\gamma} < \tilde{\gamma}$.

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