Essays in Applied Microeconomics With Application to Electricity Markets

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Contents

1 Introduction

In my thesis, I present three essays dealing with issues of applied microeconomics. All three essays were originally motivated by research questions emerging from liberalized electricity markets, but the models I discuss can be more or less applied to other industries as well.

Traditionally, the electricity industry has been regarded as a natural monopoly. The electricity grid constitutes an essential facility, meaning that duplication is impossible or inefficient from an economic perspective. Since 1997, the European Union has adopted a number of directives in order to establish an efficient internal electricity market and to increase competition in national markets. The subsequent deregulation of electricity markets in Europe relies on the idea that only the electricity grid itself needs to be regulated, whereas the generation of electricity and related services can be organized such that competition may arise.

Therefore, appropriate market designs have to be implemented. Naturally, the question arises as to whether these market designs provide the correct incentives for firms to achieve an efficient market outcome.

In the thesis at hand, I analyze the behavior of firms in two different areas of electricity trading: *cross-border electricity trading* and *reserve capacity markets*. In both cases, mechanisms have been implemented by the authorities that admonish the grid operator to ensure non-discriminating access to downstream markets. These mechanisms are aimed to induce competition and to increase efficiency.

In Chapter 2, I discuss an issue related to *cross-border electricity trading*. Naturally, the prices of two neighboring domestic markets may vary for a given hour. If these markets are physically connected, transmitting electricity between these markets decreases the price difference. Because the transmission capacity between two markets is bounded, the right to transmit electricity has a value. Therefore, the management of transmission capacity needs to be organized by an appropriate mechanism in order to allocate scarce transmission capacity. Preferably, the implemented mechanism should lead to an efficient outcome. At some borders in Europe, transmission rights are *explicitly auctioned* by the grid operator to electricity traders *ex-ante*. Traders then schedule their transmission flows by means of profit maximization. Empirical data indicates that cross-border capacity is not utilized efficiently if explicit auctions are in place. Together with my co-author JOHANNES VIEHMANN, we analyze the strategic behavior of traders in scheduling transmission flows, e.g. exercising their transmission rights. We adopt a *Bayesian-Cournot* model by perceiving the price difference between markets as an inverse demand function. Traders face *incomplete information* with respect to the transmission capacity with which the other traders are endowed. We analyze equilibrium strategies and market outcomes when the number of electricity traders is small. We find that besides Cournot behavior and capacity constraints, a lack of information with respect to the allocation of transmission rights also reduces social welfare. Thus, we provide new insights as to why explicit cross-border auction regimes are inefficient.

Chapter 3 presents a Bayesian-Cournot model in which firms face incomplete information with respect to the other firms' production capacities. I adopt the general model framework presented in Chapter 2, but analyze a different specification of the *common prior belief*: I assume that the firms' capacities are stochastically independent, whereas in Chapter 2 the firms' capacities are strongly interdependent because total capacity is fixed. Thus, the model discussed in Chapter 3 is *not* a special case of the model analyzed in Chapter 2. In contrast to Chapter 2, in Chapter 3 I provide analytical results on existence, uniqueness and shape of equilibrium strategies and discuss the impact of *sharing information ex-ante* on producer surplus, consumer surplus and social welfare. Thus, I provide results that are complementary to results previously established by other authors dealing with Bayesian-Cournot oligopolies, who solely focused on incomplete information with respect to inverse demand or production costs.

In Chapter 4, I address the issue of *reserve capacity*, which is an ancillary service needed to ensure grid stability. For technical reasons, the frequency in Europe's electricity grids needs to be equal to 50 Hz. If the actual frequency deviates from this reference value, the grid could collapse. Such a deviation arises when demand and supply are not balanced. In order to ensure that demand equals supply in the very short term, generation capacity is procured that can increase or decrease its generation on short notice. Typically, this reserve capacity is procured by the grid operator via an auction. In Germany, market prices for reserve capacity resulting from these procurement auctions fluctuated heavily over the past years. It is initially unclear what drives these fluctuations. In order to better understand market prices for incremental reserve, I discuss a *general equilibrium model* containing a spot market for electricity and a market for *incremental* reserve capacity. By characterizing equilibrium strategies, I provide a benchmark for the competitive market outcome and thus describe how competitive prices are formed. The model shows how strongly market prices are driven by a firm's opportunity costs arising from spot market participation.

The next three sections provide extended abstracts of the three papers presented in the thesis, which are non-technical but more detailed compared to the sketches provided above.

Chapter 2: The Value of Information in Explicit Cross-Border Capacity Auction Regimes in Electricity Markets (based on Richter and Viehmann (2013))

The paper discussed in Chapter 2 is joined work with my co-author Johannes Viehmann, who contributed to the paper in equal parts.

We address an issue arising from cross-border electricity trading by considering two spot electricity markets connected by a fixed amount of *cross-border capacity* that is *common knowledge*. We model the price difference between both markets via a decreasing function of total cross-border transmission. We assume that traders, or firms, are perfectly informed about this functional relationship. Moreover, firms can not influence each market's price by means other than transmitting electricity between markets. In particular, firms do not produce and sell electricity on any of the two spot markets.

Total cross-border capacity is split among a finite number of firms via an auction. Every firm receives some information on the auction outcome. When it comes to spot market clearing, firms utilize their share of capacity to some extent in order to generate profits arising from a price difference between the two markets. Since the number of firms is finite, we adopt a *game theoretic framework*, with the share of capacity utilization as the strategic variable. We do not consider the first step of the game in which the capacity shares itself are auctioned.

These assumptions allow us to perceive the problem as a *Bayesian-Cournot oligopoly*. The price difference between markets, depending on total cross-border transmission, is perceived as an inverse demand function. The *common prior belief* is a probability measure on a finite set of capacity configurations. The total capacity with which the industry is endowed is equal to the total cross-border capacity, which is fixed *ex-ante* and common knowledge. Therefore, the firms' capacities are not stochastically independent.

We analyze three different levels of information with which the firms may be endowed. The case of *complete information* is just the case without any uncertainty. In the case of *incomplete information*, firms only learn their own capacity. Finally, in the case of *partial information*, firms additionally learn the number of successful firms, i.e. the number of firms that are endowed with a share of capacity exceeding zero.

Due to the dependency structure of the firms' capacities, equilibrium strategies cannot be derived analytically. Thus, the model is solved by means of simulation. We show that in the case of three firms, the best response function is a contraction under standard assumptions (Theorem 1). Thus, the iterated best response function converges to the unique equilibrium.

In the case in which firms only learn their own capacity, the unique and thus symmetric equilibrium strategy is an increasing function in the amount of capacity with which the firm is endowed. More precisely, a firm fully utilizes its capacity up to a threshold. When the firm's capacity exceeds this threshold, the output of the firm is increasing in a convex manner up to the Cournot monopoly output. This is because if a firm is endowed with the total cross-border capacity, the firm knows that there are no competitors, meaning that the firm acts as a Cournot monopolist without uncertainty. The case in which firms also learn the number of competitors is similar, although the equilibrium strategy is a function of two arguments in this information setting.

A welfare analysis shows that social welfare is increasing with the level of information. This increase in welfare is driven by an increase in producer surplus. However, there are states of nature in which firms do not profit from more information. Nevertheless the result shows that the gain from having more information is dominant. The key issue is that the more information a firm receives, the better firms can coordinate total industry output. This reduces the variance of total industry output, which is beneficial for the firms, but harmful for consumers.

However, the effect on consumer surplus is ambiguous. Reducing the variance of total industry output reduces expected consumer surplus. Expected total industry output only changes slightly between the three information regimes. Depending on which information regimes are compared, expected output may increase or decrease, and, thus, may increase or decrease consumer surplus. However, the effect on consumer surplus is small and somewhat less interesting.

To sum up, we find three forces reducing social welfare when cross-border capacity is explicitly auctioned to the firms. First, the fact that firms play a Cournot game apparently reduces social welfare compared to the competitive market outcome. Second, capacity constraints reduce welfare, even in the presence of complete information. This is derived from the slope of the best response function. Third, a lack of information further diminishes social welfare. Thus, we provide new arguments as to why explicit auctioning of cross-border capacity between electricity leads to inefficient market outcomes.

Chapter 3: Incomplete Information in Cournot Oligopoly: The Case of Unknown Production Capacities (based on Richter (2013))

In this essay, I discuss the general *Bayesian-Cournot model* in which the firms face *incomplete information* about *production capacities* (as sketched in the previous section). However, the specification of the *common prior belief* is different: I assume here that the firms' capacities are *stochastically independent*. Thus, analytical results can be obtained.

Under standard assumptions ensuring that the expected payoff function of each firm is convex in its own output, equilibrium strategies are nondecreasing. More precisely, firms fully utilize their capacity up to some threshold. If the capacity with which a firm is endowed exceeds this threshold, the output remains constant (Theorem 1). In particular, any equilibrium strategy is completely defined by a firm's action when endowed with the maximum level of capacity available. Therefore, a firm's strategy space is essentially one-dimensional. This result holds for a general common prior belief as long as the firms' capacities are stochastically independent.

If, in addition, the firms' capacities are *identically distributed*, and if the inverse demand function is *concave*, only one symmetric equilibrium exists. This result is intuitively clear in the case of linear demand: A firm's best response function then only depends on the expected aggregate output of the other firms. Since a firm's strategy space is essentially one-dimensional, the firm's strategy can be scaled up to the point at which is a fixed point of the best response function. The existence of the fixed point follows from the continuity of the underlying functions.

A similar argument shows that every equilibrium is symmetric if *demand is linear*, implying that only one equilibrium exists (Theorem 3). Again, the best response function of each firm only depends on the expected aggregate output of the other firms. Due to Theorem 1, any two strategies coincide if and only if their expected values coincide, which is implied by linearity of demand.

In the second part of the paper, I address the issue of *information sharing*, meaning that firms have the option to commit *ex-ante* to an industry-wide agreement on information pooling. Because equilibrium strategies cannot be calculated explicitly, but are rather implicitly characterized, the general case is not tractable. Therefore, a duopoly with a simple common prior belief is analyzed.

I find that the effects on producer and consumer surplus depend on the horizontal demand intercept as long as the available capacity levels are fixed. If the demand intercept is sufficiently large, then firms have an incentive to exchange information, as the expected profits under complete information exceed expected profits under incomplete information (Theorem 4). Contrarily, if the demand intercept is sufficiently small, then consumers benefit from information sharing. Moreover, within a certain range, both producers and consumers benefit from information sharing. Social welfare increases in a large class of examples; however, I also give a simple example where social welfare decreases.

Standard results on information sharing in Cournot oligopoly state that the incentives for firms to share information are stable (the case of unknown costs) or that an increase of consumer surplus is stable (the case of unknown demand intercept). These results, however, are driven by the assumption that the common prior belief is normally distributed. This leads to affine equilibrium strategies. In particular, outputs are unbounded and may be negative. I establish that similar stable results on information sharing cannot be derived in the case of non-negative outputs and incomplete information with respect to production capacities, i.e. standard results can be reversed.

Chapter 4: On the Interaction Between Product Markets And Markets For Production Capacity: The Case of the Electricity Industry (based on Richter (2011))

In this essay, I consider two markets and analyze simultaneous equilibria. Both markets are supplied by the very same continuum of firms able to produce a homogeneous good up to production capacity, which is normalized to unity.

On the first market, the *capacity market*, firms may sell their production capacity. The buyer of this capacity may request the capacity he procured for spot market production. On the second market, the *spot market*, firms may sell the good itself. Offering production capacity on the capacity market decreases production opportunities on the spot market. The capacity market clears first, before the spot market is able to follow.

Both markets are characterized by an inelastic demand curve. The demand of the spot market is anticipated by the firms via an appropriate probability distribution, whereas the demand of the capacity market is perfectly known to the firms beforehand. Both markets are cleared by determining the intersection of the inelastic demand and the accumulated supply curve.

The variable costs of production are different for each firm. Therefore, when firms are sorted according to their marginal costs, the resulting marginal cost curve is increasing. Moreover, it is assumed that the marginal cost curve is convex.

Since firms are price takers, they bid according to their marginal costs on the spot market. Costs are different for every firm, so firms can generate revenues exceeding their marginal costs. Therefore, the expected spot market profits of each firm per unit of production exceed zero.

On the capacity market, the accumulated supply curve of the firms is driven by opportunity costs. These opportunity costs consist of two components: First, firms face foregone spot market profits, since the selling of capacity reduces potential spot market output and thus the expected spot market profits of a firm exceed zero. The higher the marginal costs of a firm are, the lower are the expected foregone spot market profits. Therefore, these opportunity costs are decreasing with marginal costs.

Second, when selling capacity on the capacity market, firms are faced with costs of keeping their capacity ready for production. It is assumed that if firms provide capacity, they are subject to a minimum production condition, meaning that a fixed share of their capacity needs to be utilized. I call this condition the *must-run condition*. The production arising from the must-run condition is sold on the spot market at any price. Thus, the market outcome of the capacity market transforms the accumulated supply curve of the spot market and in turn expected spot market profits of each firm. Since a firm's variable costs of production may, with positive probability, exceed the market clearing price, the expected losses a firm faces exceed zero. Thus, the higher the marginal costs of a firm are, the higher are the expected losses arising from the must-run condition. Therefore, these opportunity costs are increasing with marginal costs.

The leading example of this setting is the electricity industry. The capacity market corresponds to the market for incremental reserve capacity: On this market, production capacity is procured, which may be called upon short notice in order to compensate for short-term deviations in demand and supply. When providing incremental reserve, a power plant has to generate electricity at a minimum load level in order to be able to quickly increase its output. Speaking in terms of the model as sketched above, the spot market corresponds to a liquid day-ahead electricity market. Finally, the increasing marginal cost curve corresponds to the *merit order* of conventional power plants.

I find that it is sufficient to consider the sum of expected foregone spot market profits and expected must-run costs in order to analyze the bidding behavior of the firms (Proposition 1). Since the first cost component is decreasing, whereas the second cost component is increasing with marginal costs, the accumulated supply curve of the capacity market is u-shaped (Theorem 1). As previously mentioned, the market outcome of the capacity market transforms the accumulated supply curve of the spot market. Moreover, the accumulated supply curve on the capacity market is determined by spot market expectations. Therefore, the supply curves of both markets are interdependent. Thus, an equilibrium is a fixed point ensuring that firms bid according to the accumulated supply curve on the capacity market that is consistent with spot market expectations. Since the capacity market supply curve is u-shaped, the set of firms selling production capacity is an interval at equilibrium (Corollary 1).

Regardless of the parameters of the model, a unique equilibrium exists (Theorem 2). Lastly, the equilibrium is efficient, meaning that total expected costs of meeting spot market demand are minimized. Although this result may be derived from the *first welfare theorem*, I give an instructive proof that provides further insights on how strongly opportunity costs on the capacity market correspond to a consumption of resources (Theorem 3).

To sum up, the market coordinates at an efficient equilibrium in this special setting. In particular, the design of the market for balancing power in Germany induces an efficient outcome also, provided that suppliers are competitive.

2 The Value of Information in Explicit Cross-Border Capacity Auction Regimes in Electricity Markets

The content of this chapter is joined work with my co-author Johannes Viehmann, who contributed in equal parts.

We study two electricity markets connected by a fixed amount of cross-border capacity. The total amount of capacity is known to all electricity traders and allocated via an auction. The capacity allocated to each bidder in the auction remains private information. We assume that traders are faced with a demand function reflecting the relationship between electricity transmitted between the markets and the spot price difference. Therefore, traders act like Bayesian-Cournot oligopolists in exercising their transmission rights when presented with incomplete information about the competitors' capacities. Our analysis breaks down the welfare effect into three different components: Cournot behavior, capacity constraints, and incomplete information. We find that social welfare increases with the level of information with which traders are endowed.

2.1 Introduction

Efforts to liberalize European electricity markets led to unparalleled structural changes within the last 10 to 15 years. Directives and regulations issued by the European Commission aimed to open markets, ensure non-discriminatory third-party access to power grids[1] and enforce cross-border trading activities[2] in order to harmonize prices and to mitigate market power.

[1]European Union, Directive 54/EC (2003).
[2]European Union, Regulation EC No 1228 (2003).

Resulting from Article 6 of Regulation 1228/2003 –*"Network congestion problems shall be addressed with non-discriminatory market based solutions which give efficient economic signals [...]"*–, non-market-based congestion methods such as *first-come-first-serve* or *pro-rata* were replaced by market-based regimes like *implicit* and *explicit* auctions. In *explicit* auction regimes, the right to use cross-border capacity is sold first stage to market participants by a uniform-pricing auction. In a second stage, market participants then have to decide which share of their transmission rights to exercise in order to schedule a power flow from one market area to another.

Explicit auctions have been criticized mainly for two reasons. First, they might allow for exertion of market power. A firm might acquire capacity to block it or strategically misuse it to protect a dominant position in one regional market. Second, firms face incomplete information with respect to the demand for power transmission. Traders might just not know ex ante in which region excess demand (and therefore prices) are larger and might nominate capacity in the wrong direction. However, explicit auctions are still in place at many interconnectors.[3]

We add to the analysis of explicit auctions an additional source of inefficiency, namely the inefficiency arising from strategic usage of capacity under incomplete information with respect to the allocation of capacity among competing traders. To do so, we consider explicit auction regimes as two stage games: while transmission rights are sold to firms via an auction in the first step and auction results are made public, the actual utilization of transmission capacity is determined by firms in the second step, in which firms essentially play a Bayesian-Cournot game. The strategic variable is a firm's utilization of transmission rights. We solely focus on the second stage of the game and argue why this is sufficient to demonstrate the inefficiency of the auction regime.

Since the total cross-border capacity is fixed, there is a strong stochastic dependency structure between the firms' transmission rights. Consequently,

[3]Examples are, among others, the interconnectors between France and the UK, France and Italy, Germany and Switzerland and Czech Republic and Poland.

equilibrium strategies can not be derived analytically. Therefore, we solve the model numerically for the case of three firms, which is the simplest relevant model specification – in the case of two firms, the game is subject to complete information because total capacity is common knowledge.

It turns out that a unique equilibrium exists, provided that firms are symmetric. In particular, the equilibrium itself must be symmetric. This is achieved by showing that the best response function converges to a unique fixed point – as opposed to the standard form Cournot oligopoly, in which the best response function only converges as long as $n < 3$. This result enables us to implement a stable algorithm that converges to the unique symmetric Bayesian-Cournot equilibrium.

The simulation results show that in the unique Bayesian-Cournot equilibrium, firms fully exercise their transmission rights up to a certain threshold. When the transmission rights with which a firm is endowed exceed this threshold, a bend occurs, leaving afterwards the strategy increasing in a convex manner up to the firm's monopoly output.

Moreover, social welfare increases with the level of information. The increase in social welfare is driven by an increase in producer surplus – i.e., when firms have more information, they can coordinate better on total electricity transmission. In particular, firms have an incentive to commit on an industry-wide information sharing agreement *ex-ante*. Stabilizing total transmission reduces its variance, which in turn lowers consumer surplus. However, the effect on consumer surplus is small and can be ambiguous, depending on the model parameters.

The remainder of this paper is structured as follows. In Section 2.2, we provide a literature review. In Section 2.3, we explain cross-border economics, auction offices and further motivate the model. The model and analytical results are presented in Section 2.4. The results of the numerical solution are presented and discussed in Section 2.5. Finally, Section 2.6 concludes.

2.2 Related Literature

The inefficiency of explicit auction regimes is unchallenged and has been documented in recent studies. Meeus (2011) describes the transition from explicit to implicit market coupling of the so-called *Kontek*-cable connecting Germany and the Danish island Zealand. He shows that implicit price coupling clearly outperforms explicit auctions. Gebhardt and Höffler (2013) find that cross-border capacity prices (first stage of the two-stage game) at the German-Danish and German-Dutch borders predict on average spot price differentials correctly, but with a lot of noise. Similar arguments are provided by Dieckmann (2008) and Zachmann (2008) who show that uncertainty about spot prices and timing of explicit auction regimes lead to a poor performance. For the German power market, Viehmann (2011) empirically shows the high volatility of spot prices also in comparison to their expected values.

While some of the literature mentioned above identifies market abuse as one possible reason for the inefficiencies observed, Bunn and Zachmann (2010) analytically derive cases in which dominant players, such as national incumbents, can maximize their profits by deliberately misusing cross-border capacities. The authors then analyze empirical data from the *IFA*-interconnector between France and UK and disclose flows against price differentials as well as unused capacity in the profitable direction in a significant number of hours. Additionally, Bunn and Zachmann (2010) provide a list of various design deficiencies contributing to the poor performance of explicit auction regimes. Finally, Turvey (2006) provides a broad overview about non-market and market-based congestion management methods and detailed information about South Eastern European markets.

The issue of incomplete information with respect to production capacities in Cournot oligopolies has recently been discussed by Richter (2013), who provides a characterization of equilibrium strategies when a firm's capacities are stochastically independent. Moreover, sufficient conditions for the existence and uniqueness of a Bayesian-Cournot equilibrium are given.

Bounded capacity is modeled by curtailing the firm's strategy space. We adopt this approach, since it ensures that the strategy spaces are compact and the expected payoff function is concave given a linear demand function, ensuring the existence of an equilibrium by Nash's theorem.

Regarding the issue of information sharing in oligopolies, literature focuses on Bayesian Cournot models in which there are no non-negativity constraints and no capacity constraints with respect to outputs. Provided the common prior belief is normally distributed, equilibrium strategies are linear (or affine) and closed-form solutions can be derived. An overview of these models is provided by Raith (1996). In all such models, firms face uncertainty with respect to marginal costs, or inverse demand, or both.

Most similar to the setting discussed in the paper at hand is the case of unknown costs, since costs as well as capacities are private values in which equilibrium strategies should be monotonous. Shapiro (1986) finds that in this case, firms have an incentive to share information, meaning that sharing information increases expected producer surplus. Moreover, he finds that consumer surplus decreases, whereas social welfare increases as a result of a positive net effect.

As outlined in the previous section, we obtain similar results as Shapiro, although the impact on consumer surplus is not that clear in the model developed. This is due to non-negativity and capacity constraints on outputs, leading to equilibrium strategies that are not affine. Thus, well-known results regarding information sharing can be reversed by introducing constraints – an issue that was addressed earlier by Maleug and Tsutsui (1998) and recently by Richter (2013).

2.3 Power Interconnectors

To further justify the use of the Cournot approach, we provide insights into interconnector economics and briefly introduce European auction offices and their information policies.

2.3.1 Interconnector Economics

While pools like the *PJM Market* in the US deal with regional supply and demand imbalances via nodal pricing, the predominant system in Europe can be described as a connection of market areas. In most cases, market areas that are connected by power interconnectors are equivalent to national borders.[4]

Today, the two prevailing mechanisms to allocate scarce cross-border capacities in Europe are *implicit* and *explicit* capacity auctions. With *implicit* auctions, also referred to as *market coupling* or *market splitting*, the auctioning of transmission capacity is implicitly integrated into the day-ahead exchange auctions of the connected market areas. Power exchanges can ensure welfare-maximizing cross-border flows between the market areas as they possess full information about all hourly supply and demand curves in the connected market areas and the available cross-border capacity.

When explicit capacity auctions are in place, the right to use cross-border capacity is sold in a first stage to market participants by a uniform-pricing auction, usually on a yearly, monthly and daily basis. In daily auctions, firms can bid for each hour of transmission capacity separately. In a second stage, market participants have to decide which share of their transmission rights to exercise in order to schedule a power flow from one market area to another.[5]

The basic interconnector economics are pictured in Figure 2.1, in which the relation between the *used transmission capacity* Q and the price spread P between two market areas is shown. When no transmission capacity is utilized ($Q = 0$), the price spread is at its maximum. The more capacity is booked to flow power from the low price area to the high price area, the smaller the price spread becomes. When the total available cross-border capacity $\hat{t}$ is not sufficient to equalize prices (pictured left), total welfare is maximized at a price spread P^* and leads to $Q^* = \hat{t}$. However, provided the

[4]Exceptions are Italy, the United Kingdom and the Scandinavian countries.

[5]A comprehensive overview of explicit and implicit cross border auctions is given by Kristiansen (2007) and Jullien et al. (2012).

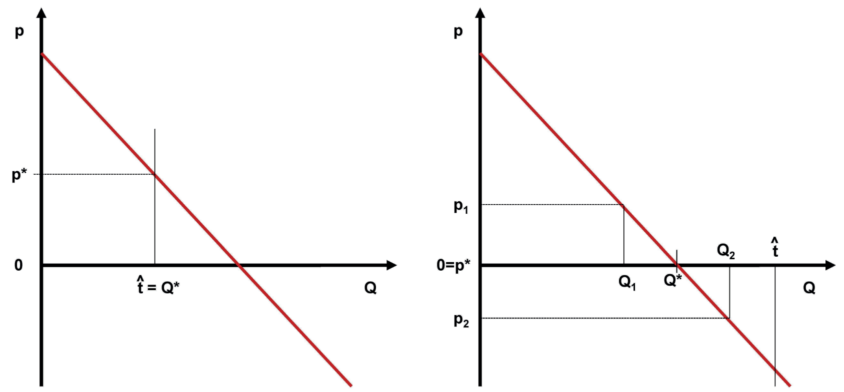

Figure 2.1: Basic economics of interconnectors

available cross-border capacity $\hat{t}$ is more than sufficient to equalize prices (pictured right), the price spread P equals zero and $Q^* < \hat{t}$.

If *implicit* market coupling or market splitting is in place and no further restrictions exist, the chosen quantity Q of cross border transmission flows is equal to Q^* for any given hour. The auction office knows the hourly aggregated supply and demand curves in both market areas and maximizes total welfare accordingly.

In the case of explicit auctioning, market participants who have acquired transmission rights determine the quantity Q. Empirical data shows that market participants do not choose the optimal quantity Q^*, especially when $Q^* < \hat{t}$ (Figure 2.1, right). As previously mentioned, there is a lot of noise in the empirical data due to the incomplete information about the demand for power transmission. However, when the assumption that firms play a Cournot game is valid, then firms must be undershooting on average, meaning that the outcome is *ex-ante* inefficient.

2.3.2 Auction Offices and Information Levels

Auction offices were recently subject to constant changes. Today, there are two main organizations in Europe, the *Capacity Allocating Service Com-*

pany (CASC) and the *Central Allocation Office (CAO)*.[6] Additionally, there are other platforms like *DAMAS*, *KAPAR* and the French TSO *RTE* that conduct daily cross-border auctions.[7]

In order to understand the inefficiencies in the second stage of explicit auction regimes, we first have a closer look at the auction offices and the information about the first-stage results passed to the traders. While some offices give detailed information about the number of successful bidders in the first stage (coincides with the number of firms in the second stage), others do not. The same holds true on how capacities are split among the firms. We analyze three explicit auction regime settings:

Complete information: The number of firms and their endowments with capacity are known to all firms,

Incomplete information: Each firm solely knows its own endowment, the number of competing firms is unknown,

Partial information: Each firm knows its own endowments and the number of other firms, but does not no know their rival's endowment.

There is at least one auction office providing *complete information* for day-ahead capacity auction results. Using the DAMAS System, the Romanian TSO *Transelectrica*, for example, currently publishes the number of successful auction participants, their names and their allocated capacities.[8] The

[6]CASC is currently operating daily cross-border capacity auctions at the Austrian-Swiss, Austrian-Italian, German-Swiss, French-Swiss, French-Italian, Greek-Italian and Swiss-Italian borders. Website: www.casc.eu. CAO is currently operating daily cross-border capacity auctions at the Austrian-Czech, Austrian-Hungarian, Austrian-Slovenian, Czech-German, Czech-Polish, German-Polish and the Polish-Slovakian borders. Website: www.central-ao.com. Last Update: 20th of September 2012.

[7]Daily cross-border auctions based on the DAMAS system are currently conducted at the French-English, Bulgarian-Romanian, and Hungarian-Romanian borders, among others. Daily cross-border auctions based on the KAPAR system operated by the Hungarian TSO MAVIR are currently conducted at the Hungarian-Croatian and the Hungarian Serbian borders. Last update: 20th of September 2012.

[8]Transelectrica is currently conducting explicit day-ahead auctions at the Bulgarian-Romanian and Hungarian-Romanian borders. https://www.markets.transelectrica.ro/public. Last update: 20th of September 2012.

incomplete information design, in which very little information about the number of successful bidders is published, is currently used by *RTE* at the French-Spanish Border and has been in operation at several other borders in the past. One prominent example was the German-French interconnector used before market-coupling started in November 2010. RTE merely publishes the number of successful bidders per day for daily auctions, meaning that firms know the maximum number of competitors for each hour but do not know how many competitors are endowed with a positive amount of capacity in a given hour. CASC and CAO currently publish *partial information*. They provide the number of successful bidders, but not precisely how capacities are split amongst them.

In the next section we present the general model framework, which is able to capture the information regimes as described above.

2.4 The Model

We consider a set of firms $N = \{1, 2, \ldots, n\}$. Firms may face uncertainty with respect to the other firm's endowment of transmission capacity. In a Bayesian approach, a strategy of firm i is a decision rule that specifies a firm's amount of transmitted electricity for every possible information set with which the firm may be endowed. The amount of transmitted electricity corresponds to a firm's output in the Cournot model setting, and we use the terms *transmission* and *output* interchangeably.

We denote $T \subset [0, \infty)$ as the finite set of possible capacity levels and $\Omega = \prod_{n \in N} T$ as the set of possible states of nature. We assume that $0 \in T$. The common prior belief μ is a probability measure on Ω. An element of Ω, which is a capacity allocation among all n firms, is denoted by $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$. We assume that every firm is endowed with a production capacity exceeding zero with positive probability. The information with which a firm is endowed when making its output decision is described by a random variable T_i on Ω.[9] A strategy is a function $q_i(T_i(\cdot))$ satisfying $q_i(T_i(\omega)) \leq \omega_i$. Lastly, we denote S_i as the strategy space of firm i and $S = \prod_{i=1}^{n} S_i$ as the space containing all strategy profiles.

As previously defined, $q_i(T_i(\omega))$ is the output of firm i. We let $Q(\omega) := \sum_{i=1}^{n} q_i(T_i(\omega))$ denote the overall output. The inverse demand function $P(Q)$ corresponds to the price difference between two electricity markets. We assume that P is linear and decreasing with total industry electricity transmission Q. We do not consider costs, since exercising transmission rights is costless.

The *state-dependent payoff function* u_i of firm i is given by

$$u_i(\omega, q_i, q_{-i}) = q_i(T_i(\omega))P(Q(\omega)). \tag{2.1}$$

A strategy profile $q \in S$ is a *Bayesian Cournot equilibrium* if for every i and $\tilde{q}_i \in S_i$ the *expected payoff function* is maximized,

$$E\left[u_i\left(\cdot, q_i, q_{-i}\right)\right] \geq E\left[u_i\left(\cdot, \tilde{q}_i, q_{-i}\right)\right], \tag{2.2}$$

meaning that in an equilibrium no firm has an incentive to unilaterally deviate from its strategy. Maximizing (2.2) is equivalent to maximizing the *conditional payoff expectation*, so that

$$E\left[u_i\left(\cdot, q_i, q_{-i}\right) \mid T_i(\omega)\right] \geq E\left[u_i\left(\cdot, \tilde{q}_i, q_{-i}\right) \mid T_i(\omega)\right] \tag{2.3}$$

for all $i \in N$ and all $\omega \in \Omega$.[10]

Remark 1. *Linearity of inverse demand ensures that the state-dependent payoff function (2.1) is concave in the output of firm i. Moreover, concavity is inherited by the expected payoff function (2.2) (Einy et al., 2010). Since a firm's strategy space is compact and convex, Nash's theorem implies the existence of an equilibrium.*

[9] The information sets of firm i are then the elements of the $\sigma-$algebra $\sigma(T_i)$ generated by T_i.

[10] See Harsanyi (1967) and Einy et al. (2002).

As previously mentioned, we analyze three schemes of information. In terms of the model formulation, the case of complete information corresponds to $T_i(\omega) = \omega$ for all $i \in N$ and all $\omega \in \Omega$. Thus, every firm is perfectly informed. When firms only know their own transmission capacity, then $T_i(\omega) = \omega_i$ holds. Finally, when information is partial, meaning that the number of active firms is known, then $T_i(\omega) = (\omega_i, F(\omega))$, where

$$F(\omega) = |\{i \in N : \omega_i > 0\}|.$$

In the next section we construct equilibrium strategies for the case of complete information. Moreover, for the case of three firms we provide a technique to numerically derive equilibrium strategies when information is incomplete.

2.4.1 Complete Information

This question of existence and uniqueness of equilibrium strategies in this setting is treated extensively in the literature.[11] However, we provide a constructive proof on existence and uniqueness, which coincidently is helpful for the simulations. Speaking in terms of the model formulation, we discuss the case of $T_i(\omega) = \omega$ for all i and all ω.

We arbitrarily choose a capacity configuration $\omega = (\omega_1, \omega_2, \dots, \omega_n)$. Without loss of generality, we assume that $\omega_i \leq \omega_j$ if $i < j$. We let q_i denote the output of firm i and write $q = (q_1, q_2, \dots, q_n)$. The firm's equilibrium strategy of the corresponding unrestricted Cournot oligopoly is denoted by q^C. We define

$$q_1(\omega_1, \omega_2, \dots, \omega_n) = \min\left\{\omega_1, q^C\right\}. \tag{2.4}$$

Firm 1 produces the $n-$firm Cournot quantity, whenever possible, and otherwise all of its capacity ω_1. If $q(\omega_1, \omega_2, \dots, \omega_n) = q^C$, we define

$$q_j(\omega_j, \omega_1, \omega_2, \dots, \omega_{j-1}, \omega_{j+1}, \dots, \omega_n) = q^C$$

[11] See for example Bischi et al. (2010).

for all $j \geq 1$. If not so, we consider the $n-1$-firm oligopoly in which firms $i = 2, 3, \ldots, n$ face residual demand resulting when firm 1 produces ω_1. We let q^C_{n-1} denote the Cournot output of the corresponding unrestricted oligopoly and define

$$q_2(\omega_2, \omega_1, \omega_3, \ldots, \omega_n) := \min\left\{\omega_2, q^C_{n-1}\right\}.$$

By iteration, we obtain a strategy for every firm with the following property: There exists a threshold $k \in N$ so that $q_i(\omega_i, \omega_{-i}) = \omega_i$ for all $i < k$ and $q_i(\omega_i, \omega_{-i}) = q_k(\omega_k, \omega_{-k}) < \omega_k$ for all $i \geq k$, following from the construction procedure.

If in equilibrium there is a firm with a binding capacity restriction, the total output of the industry is lower compared to the output of the standard form Cournot oligopoly. This property is derived from the slope of the best response function r, which exceeds -1. If one firm decreases its output due to its capacity restriction, then the corresponding increase of the other firms is smaller. The following proposition sums up the well-known results we reconsidered in this section.

Proposition 1. *The strategy constructed above is the unique and symmetric complete information equilibrium of the Cournot oligopoly. If there exists an $i \in N$ such that $\mu\left(T_i < q^C\right) > 0$, then the expected total output in the complete information equilibrium is smaller compared to the total output of the unrestricted Cournot oligopoly.*

All proofs are provided in the Appendix of the chapter.

2.4.2 Incomplete and Partial Information

The results provided in this section cover both the case of incomplete information and the case of partial information defined on page 18. Since we seek to solve the model numerically, we provide an algorithm converging to a unique equilibrium solution, which then must be symmetric.

While equilibrium strategies can be explicitly constructed in the case of complete information, as demonstrated in the last section, this task is challenging when information is incomplete. In the very general model setting presented on Page 19, equilibrium strategies can be of any shape since the common prior belief is left unspecified.[12]

However, in the context of exercising cross-border capacity, we can impose two restrictions on the common prior belief. First, firms are *ex-ante* symmetric by assumption. This leads to the following requirement:

$$\mu\left(T_i = t\right) = \mu\left(T_j = t\right) \text{ for all } t \in T \text{ and } i \neq j. \tag{2.5}$$

Second, we explicitly allow for firms to be endowed with zero capacity with positive probability. In particular, given that firm 1 is endowed with some capacity level t, then, with positive probability, firm 2 is endowed with zero capacity as long as there are at least three firms participating. This leads to

$$\text{If } n > 2, \text{ then } \mu\left(T_2 = 0 | T_1 = t\right) > 0 \text{ for all } t \in T. \tag{2.6}$$

Conditions (2.5) and (2.6) do not sufficiently specify the common prior belief to allow for an analysis of the shape of equilibrium strategies. To provide intuition for that, we consider the following construction procedure for the common prior belief. Let $\tilde{\mu}$ be an arbitrarily chosen probability measure on the product space $\prod_{i=1}^{n} T$ such that $\tilde{\mu}$ meets conditions (2.5) and (2.6). If T_i denotes the capacity with which firm i is endowed and if $\hat{t}$ denotes the overall cross-border capacity, we can define

$$\mu(\,\cdot\,) := \tilde{\mu}(\,\cdot\,\Big|\sum_{i=1}^{n} T_i = \hat{t}).$$

Thus, we can choose almost any distribution for $\tilde{\mu}$ and obtain the corresponding common prior belief μ. Even for a simple $\tilde{\mu}$, the conditional distribution μ is difficult to handle.

However, conditions (2.5) and (2.6) enable us to prove the existence of a unique Bayesian-Cournot equilibrium for the case of three firms. We show

[12] See Richter (2013) for an example.

that under conditions (2.5) and (2.6), the industry's best response function $\tilde{r}$ is a contraction mapping, meaning that if we iterate the best response function, then the sequence we obtain converges to the unique equilibrium solution.[13]

Therefore, we derive the best response function of the model. For a given strategy profile $q = (q_1, q_2, \ldots, q_n)$, we write $q_{-i} = \sum_{j \neq i} q_i$ and define for $t \in T$ and $i \in N$

$$\tilde{r}_i(t, q_{-i}) = \min \left\{ t, r \left(E \left[(q_{-i} | T_i = t \right] \right) \right\}.$$

Thus, $\tilde{r}_i(t, q)$ is the best response function of firm i when it is endowed with capacity t, given that the other firms apply q_{-i}. This stems from linear demand, since then the best reply function r of the unrestricted Cournot oligopoly only depends on the expected output of the other firms $j \neq i$. We define

$$\tilde{r}(q) := (\tilde{r}_i(t, q_{-i}))_{i \in N, t \in T}$$

to be the vector of best responses in each state and for each firm. Then a fixed point of $\tilde{r}$ is an equilibrium. Theorem 1 states that the iterated best response function converges to the unique fixed point. While we cannot derive equilibrium strategies analytically, Theorem 1 implies that we can numerically implement the iterated best response algorithm for any common prior belief and obtain the unique equilibrium solution.

Theorem 1. *Under conditions (2.5) and (2.6) and when $n \leq 3$, for any q_0 the sequence*

$$q(m) := \tilde{r}(q(m-1))$$

converges to the unique fixed point q that does not depend on the choice of q_0. In particular, a unique equilibrium exists, which then must be symmetric.

[13]More precisely, there exists $\theta < 1$ and a metric d on the space S of strategy profiles so that

$$d(\tilde{r}(q), \tilde{r}(q')) \leq \theta d(q, q')$$

for all strategy profiles q, q'. Moreover, S needs to be complete with respect to d. Then, the sequence $x_n := \tilde{r}(x_{n-1})$ converges to some element x that does not depend on x_0. Completeness with respect to d ensures that x is an element of S.

2.5 Numerical Solution to the Model

We solve the model numerically and compare the corresponding market outcomes by means of social welfare, producer surplus and consumer surplus for the three information regimes *incomplete information (II), partial information (PI)* and *complete information (CI)*. For the simulation, we assume that inverse demand is given by $p(q) = 6 - q$. We allow for 21 capacity levels, starting at 0 and ending at 5. The distance between any two capacity levels is constant and equal to 0.25. Lastly, we assume that μ is uniformly distributed on the set of feasible capacity levels.

2.5.1 Equilibrium Strategies

In Figure 2.2 A, the equilibrium strategy for the incomplete information setting is pictured. On the horizontal axis, the capacity with which a firm is endowed is plotted and on the vertical axis, we can see the corresponding output. The symmetric equilibrium strategy is strictly increasing with a firm's capacity. As in the i.i.d.-case analyzed by Richter (2013), firms fully utilize their capacity up to a threshold. Then, a bend occurs and the strategy is increasing up to the monopoly output in a convex manner. Indeed, a firm must produce its monopoly output when it is endowed with maximum capacity, since then the firm is facing a monopoly with complete information.

Next, we consider Picture B, in which the PI-equilibrium strategy q^{PI} is plotted (to some extent). Because q^{PI} is a function of two arguments (capacity of a firm and number of active players), we cannot directly plot it in Figure 2.2, and a three-dimensional chart is unfortunately not instructive. Therefore, we define $q^{PI}_{\min}$ to be

$$q^{PI}_{\min}(\omega_i) = \min\{q^{PI}(T_i(\tilde{\omega}))|\tilde{\omega}_i = \omega_i\}.$$

Thus, for a given capacity level ω_i, we pick the smallest equilibrium output among all possible numbers of active players given ω_i. The number $q^{PI}_{\max}$ is defined accordingly and, as seen in the example, $q^{PI}_{\max}$ equals q^{PI} if and only

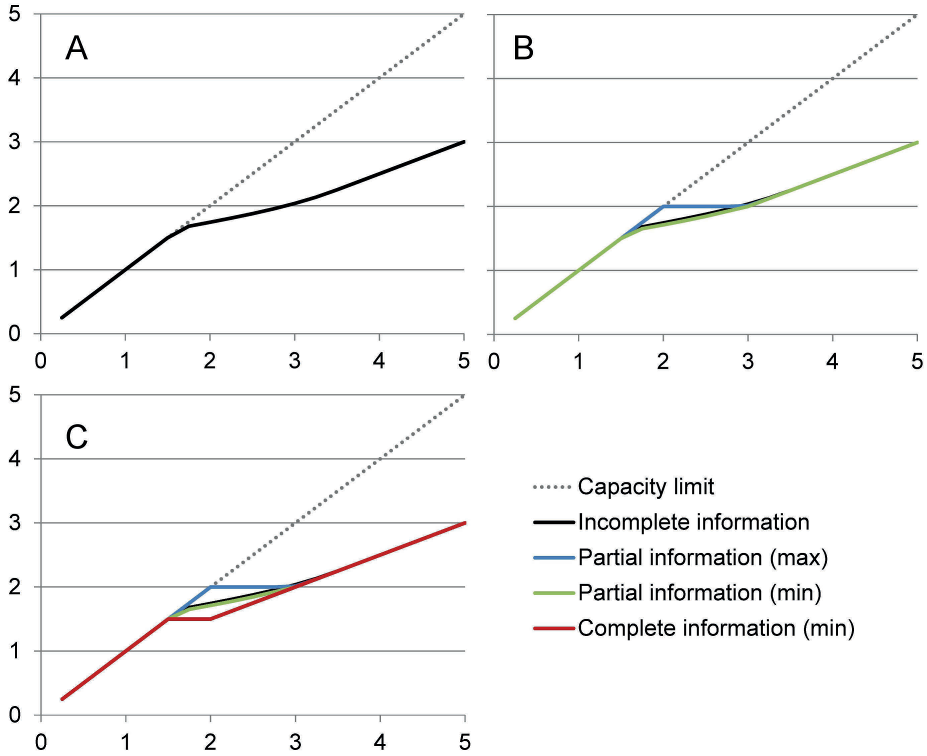

Figure 2.2: Numerically derived equilibrium strategies

if there are two or less active firms. In the example, a firm has complete information when knowing that there is only one competitor.

We can see that the PI-strategy exceeds the II-strategy on a certain range (if the number of active firms is low) and the other way around (if the number of active firms is small). The range $[2,3]$ corresponds to the event $(2,3,0)$ (or a permutation) in which two firms produce their two-player Cournot quantity. Moreover, for large capacity values, both strategies converge: If firm 1 is endowed with a sufficiently large amount of capacity, the other firms fully utilize their capacity in both information settings.

Lastly, we depict a similar modified strategy for the case of complete information in Picture C. The corresponding maximal strategy $q^{CI}_{\max}$ coincides with $q^{PI}_{\max}$ because in both cases, firms face complete information. The corresponding minimum strategy $q^{CI}_{\min}$ is smaller than the other strategies, since under complete information, a firm can protect itself against the case in which all three firms have roughly the same amount of capacity. In fact, in the range $[1.5,2]$, the strategy $q^{CI}_{\min}$ corresponds to the case in which every firm produces its Cournot quantity, which corresponds to the event $(2,1.5,1.5)$ (or a permutation).

2.5.2 Social Welfare

In this section, we analyze expected social welfare for the different information regimes and different demand intercepts. We express the expected welfare achieved under a given scheme of information and for a given demand intercept as a share of the maximal achievable welfare. When the demand intercept exceeds total capacity, welfare is maximized if and only if every firm utilizes all of its capacity. When the demand intercept is smaller than total capacity, welfare is maximized at the demand intercept.

As previously defined, the random variable $Q(\omega)$ denotes the industry's realized output. Consumer surplus is equal to $CS(\omega) := Q(\omega)^2/2$ and producer surplus is given by the aggregate industry profit $PS(\omega) := Q(\omega)P(Q(\omega))$. We define realized social welfare to be $CS(\omega) + PS(\omega)$.

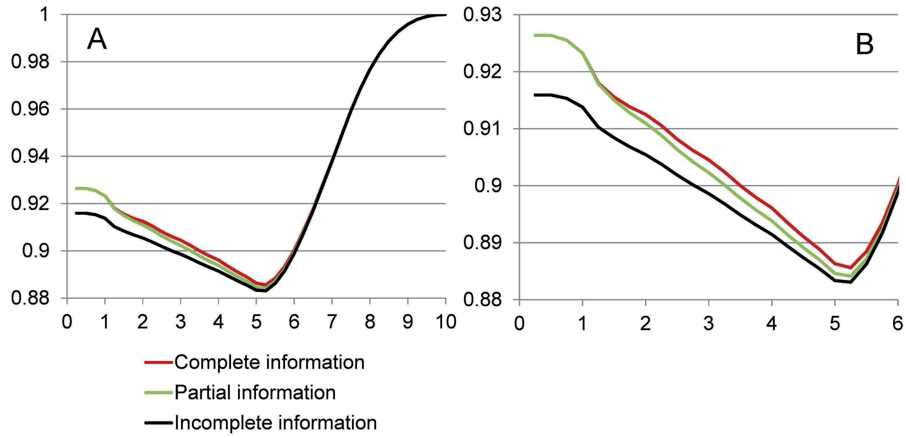

Figure 2.3: Effects of information sharing on social welfare

Figure 2.3 shows the expected welfare for the different schemes of information. On the horizontal axis, the demand intercept is plotted. On the vertical axis, we can see the expected share of maximum achievable welfare (Figure 2.3 B is an enlargement of Figure 2.3 A).

The expected welfare in the complete information regime and the partial information scheme coincide when the demand intercept is sufficiently small. In this setting, firms do not fully utilize their capacities (as long as capacity is exceeding zero). Therefore, firms have complete information when they are informed about the number of active firms.

Furthermore, relative expected welfare approaches unity as the demand intercept approaches 10 in all information regimes. Apparently, this is because then every firm fully utilizes its capacity in every information regime and in every state of nature. In this case, we have defined the maximum achievable welfare to be full utilization of total capacity. Via similar reasoning, the curve is increasing on the right-hand side of its local minimum. Therefore, relative expected social welfare is high when either capacity limits are rarely active (when the demand intercept is small, case 1) or when they are rarely redundant (when the demand intercept is high, case 2).

Equivalently speaking, expected social welfare is low if, with high probability, a firm with a large capacity can act as a monopolist on residual demand, since the other firms have little capacity and thus fully utilize it. In this case, the dominant firm leaves a large share of capacity unused. This follows from the slope of the best response function, which is equal to $-1/2$.

The impact of the slope of the best response function on total electricity transmission becomes smaller in case 1 and vanishes in case 2 as defined above. In case 1, in which the demand intercept is relatively small compared to total cross-border transmission capacity, firms do not fully utilize their capacity, since their capacity limits exceed the Cournot quantity of the unrestricted game. Therefore, if the demand intercept is sufficiently small, partial information is equivalent to complete information, whereas firms face uncertainty with respect to the number of active firms in the case of incomplete information.

In case 2, in which the demand intercept is relatively large compared to total cross-border transmission capacity, every firm fully utilizes its capacity, regardless of the observed capacity allocation. In this case, the equilibria of all three information regimes coincide.

Lastly, Figure 2.3 shows that social welfare increases with the level of information. This is the is the main result of the paper. Figure 2.4 compares expected welfare for different settings for the case in which the demand intercept equals 3. In the competitive market outcome, total output equals the demand intercept. Consumer surplus and social welfare coincide, since marginal costs are zero, and are equal to $3^2/2 = 4.5$. The outcome of the unrestricted Cournot oligopoly leads to an output of $9/4$. This leads to a dead weight loss of $(3 - 9/4)^2/2 = 9/32$, thus implying that social welfare is equal to $4.5 - 9/32 \approx 4.22$. To sum up, we can identify three driving forces reducing welfare.

First, Cournot behavior of firms reduces welfare, a well-known fact that holds in any Cournot oligopoly setting.

Second, capacity constraints reduce welfare, even when total capacity exceeds the demand intercept and firms have complete information. This result is already indicated by Proposition 1, which states that in the presence of capacity constraints, the expected total transmission of electricity declines.

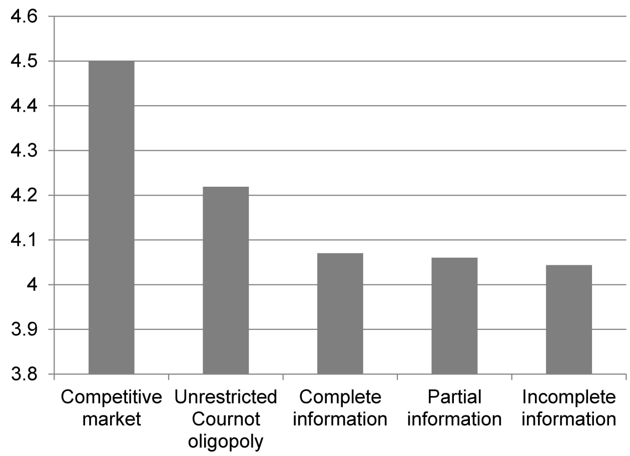

Figure 2.4: Expected welfare in different information regimes

The effect on welfare is initially unclear; however, Figure 2.4 shows that due to capacity constraints, welfare decreases.

Third, a reduction in information reduces welfare. The information effect is systematic but small; however, if we chose a common prior belief with a higher variance, the effect would probably become stronger.[14] The next two sections seek to explain the information effect on social welfare. The main driving force is the variance of total electricity transmission.

2.5.3 Consumer and Producer Surplus

We demonstrate that the increase of social welfare induced by information sharing is driven by an increase in producer surplus, whereas the effects on consumer surplus are small and partly ambiguous. When firms are better informed, they coordinate better on total industry output. This lowers the variance of total output, which decreases consumer surplus. This effect on consumer surplus is clearly observable when comparing the incomplete information equilibrium with the complete information equilibrium. However, the effect is less clear when we compare the partial information equilibrium with the complete information equilibrium.

[14] Richter (2013) discusses the impact of the variance of the common prior believe on results of information sharing in a similar context.

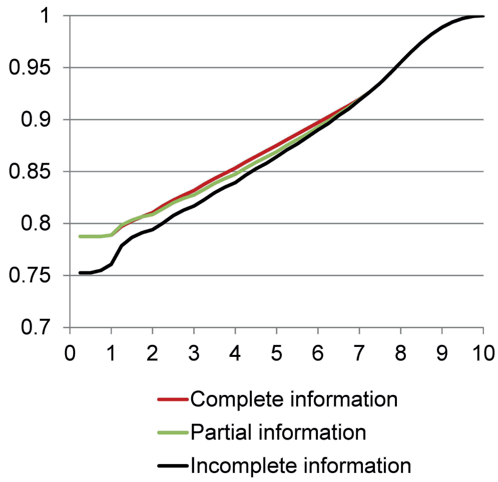

Figure 2.5: Effects of information sharing on producer surplus

Producer Surplus

As before, we calculate a relative number: We define the maximum achievable producer surplus to be the minimum of the maximal capacity and the aggregate industry output of the standard form Cournot oligopoly. Then, we consider the ratio of expected producer surplus and maximum achievable producer surplus.

Figure 2.5 shows that the effect on producer surplus is similar to the effect on social welfare – producer surplus increases with the level information. However, there are states of nature in which producer surplus can decrease due to information sharing: When there are two firms A and B that do not fully utilize their capacity in the incomplete information equilibrium, and when some firm C is endowed with zero capacity, then revealing this information induces firms A and B to increase their output. This is because the incomplete information output of firms A and B takes into account the possibility that there are three active firms rather than two. To give an example based on the simulation results, we consider the case in which the demand intercept is equal to 1. Firm A has a capacity that is equal to 2 and firm B has a capacity that is equal to 5. Under incomplete information, firm A produces 0.253, whereas firm B produces 0.296. That is to say, A and B take into account that the remaining capacity is (evenly) split up between two firms, which is why A and B produce less than the Cournot quantity, which is equal to 0.333. These equilibrium outputs lead *ex-post* to payoffs that are equal to 0.114 and 0.133, respectively. The complete information output of A and B equals 0.333, leading to a payoff that is equal to 0.112.

Similarly, there are states of nature in which producer surplus increases when information is shared. The simulation results show that this is always true as long as there are one dominant firm and two firms with little or zero capacity. Then, the small firms overestimate total industry output under incomplete information, and, as a consequence, their outputs are *ex-post* too low. Therefore, when information is shared, small firms increase their output. Because total industry output is relatively low due to the presence of a large firm, the marginal revenue of an increase of output is positive. Thus, the small firms gain from sharing.

Notice that in every information regime the outputs of the firms are negatively correlated. This is because if a firm is endowed with a large share of cross-border capacity, the other firms are endowed with little capacity. As a consequence, the variance of total output decreases.

Apparently, the absolute value of the correlation of outputs increases with the level of information, regardless of the choice of the common prior belief. This is because firms transmit some “average” amount of electricity when they have little information. Figure 2.6 shows that the variance of total output is decreasing with the level of information. Since consumer surplus is increasing with the variance of total industry output (see Richter (2013) or Shapiro (1986)), Figure 2.6 indicates that consumer surplus decreases with the information with which firms are endowed.

Consumer Surplus

Figure 2.7 A shows that consumer surplus varies with the demand intercept in a similar fashion as social welfare. Starting at 0.53, a local minimum of 0.45 is attained when the demand intercept equals 5. Apparently, the effect of different information regimes on consumer surplus is small.

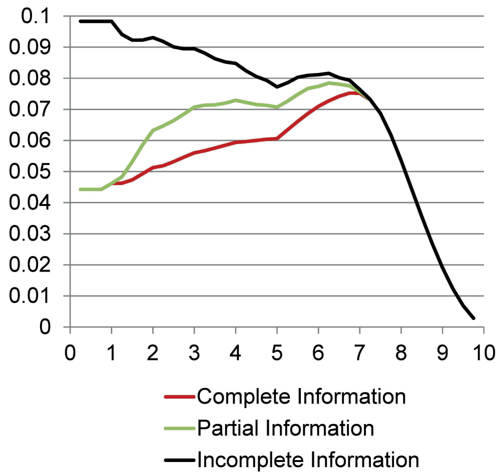

Figure 2.6: Effects of information sharing on the standard deviation of total industry output

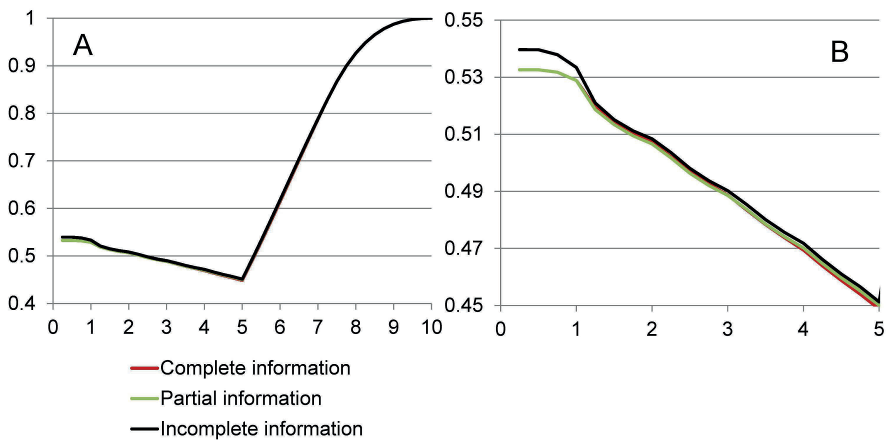

Figure 2.7: Effects of information sharing on consumer surplus

Figure 2.7 B enlarges the range $[0, 5]$. The expected consumer surplus in the incomplete information setting weakly exceeds both the complete information and partial information consumer surplus. However, in the case of partial information, consumer surplus can be above and below consumer surplus resulting from complete information. Thus, a clear statement regarding the impact of information sharing on consumer surplus can not be obtained.[15] However, Figure 2.6 shows that we can identify one stable result with respect to consumer surplus: The standard deviation of total output is decreasing with the level of information, which in turn decreases consumer surplus. To sum up, the impact on consumer surplus is small, and increasing information tends to reduce consumer surplus. The same holds true for expected electricity transmission. This follows from the fact that the variance of total output is decreasing and from the fact that consumer surplus is increasing with both variance of total output and expected total output.

2.6 Results and Discussion

We analyzed the strategic behavior of firms endowed with transmission rights that arises when transmission capacity between electricity markets is explicitly auctioned. In doing so, we perceived the strategic behavior of firms as a Cournot oligopoly in which firms face incomplete information with respect to the other firms' transmission rights.

Thereby, total cross-border capacity is common knowledge, which enables a firm to calculate the conditional distribution of the other firms' transmission rights given its own amount of transmission rights (the case of *incomplete information*). Moreover, we allow for an information regime in which the number of firms endowed with a positive amount of transmission rights is also revealed to the firms (the case of *partial information*).

For the case of three or less firms, we have shown that the best response function is a contraction, a result that is specific to the special setting under consideration. The best response function converges to the unique Bayesian

[15] This is a common issue, seen for example in Raith (1996).

Nash equilibrium, which, in particular, must then be symmetric. Because the best response function converges, we were able to calculate equilibrium solutions by means of simulation and to perform a sensitivity analysis with respect to the demand intercept. Moreover, we calculated the equilibrium for the case of complete information.

By comparing the equilibria for the three information regimes, we find that revealing information to firms increases social welfare. The increase of social welfare is driven by an increase in producer surplus. The states of nature that potentially diminish producer surplus are overcompensated by states of nature in which producer surplus increases. Since information sharing increases the negative correlation of the firms' outputs, the variance of total industry output decreases.

Although a decrease of the variance of total industry output in general decreases consumer surplus, the effect on consumer surplus is smaller than on producer surplus. We find that expected consumer surplus decreases when moving from the incomplete information equilibrium to the partial information or to the complete information equilibrium. However, when moving from the partial information equilibrium to the complete information equilibrium, the effect on consumer surplus is ambiguous. As a consequence, the same holds for total electricity transmission.

Thus, we identified three forces regarding capacity auctions that diminish social welfare: First, firms play a Cournot game, which prevents an efficient market outcome. Second, the presence of capacity constraints further reduces social welfare. This is derived from the slope of a firm's best response function, which exceeds -1: When a firm with little capacity fully exercises its transmission rights, its lack of transmission is not fully compensated by those firms endowed with a large amount of transmission rights. Third, incomplete information reduces welfare as well, as in the presence of incomplete information, firms exercise their transmission rights less aggressively.

As mentioned in the introduction, explicit capacity auctions are in fact a two-stage game. In the first stage, the transmission rights are auctioned. Then, firms are informed about their own amount of transmission rights (and, depending on the auction office, the number of active firms). In the second stage, firms exercise their transmission rights. The model analyzed in the paper at hand could be expanded to a two-stage game such as the following example.

Before the first step of the auction process is conducted, firms observe signals about a common value, for example the demand intercept of the inverse demand function. The action space of the first stage can be modeled via linear bidding functions that are decreasing, mapping transmission capacity to a price. The horizontal intercept of each firm's bidding function could be modeled as an increasing function of the firm's signal. The market operator then selects the highest bids and assigns transmission rights to the firms. When firms make their output decisions in the second step, the transmission rights of the other firms are stochastic – the corresponding distribution is induced by the distribution of the signals observed by the firms before the first step of the auction was conducted. Thus, the second stage game is equivalent to the game analyzed in the paper at hand. The results on the three driving forces diminishing social welfare should be stable even when the problem is modeled as a two-stage game.

As previously mentioned, implicit auction regimes clearly outperform explicit auction regimes. Nevertheless, as long as explicit auction regimes are still in place, we recommend that auction offices provide as much information as possible about the first stage results in order to maximize social welfare.

Appendix

Proof of Proposition 1

To show that q is an equilibrium, we choose the smallest number $k \in N$ so that $q_k(\omega_k, \omega_{-k}) < \omega_k$. Then $q_k(\omega_k, \omega_{-k})$ is firm k's best response by definition. Since a firm $i > k$ minimizes the same payoff function as firm k does, $q_i(\omega_i, \omega_{-i}) = q_k(\omega_k, \omega_{-k})$ is the best response of firm i as well. Any firm $i < k$ can not increase its output and does not have an incentive to decrease its output because $q_i(\omega_i, \omega_{-i}) < q_k(\omega_k, \omega_{-k})$. Furthermore, firm k does not have an incentive to decrease its output.

To show that the equilibrium is unique, we consider $\tilde{q} \neq q$ to be another equilibrium and denote i as the smallest number such that

$$\tilde{q}_i(\omega_i, \omega_{-i}) \neq q_i(\omega_i, \omega_{-i}).$$

Without loss of generality, we assume that $i = 1$. First, we consider the case in which

$$\tilde{q}_1(\omega_1, \omega_{-1}) < q_1(\omega_1, \omega_{-1}).$$

This implies

$$\tilde{q}_1(\omega_1, \omega_{-1}) < \omega_1,$$

which in turn leads to

$$\tilde{q}_j(\omega_j, \omega_{-j}) = \tilde{q}_j(\omega_i, \omega_{-i})$$

for all $j > i$. But then

$$q^C = \tilde{q}_1(\omega_1, \omega_{-1}) < q_1(\omega_1, \omega_{-1}),$$

contradicting (2.4).

Second, when

$$\tilde{q}_1(\omega_1, \omega_{-1}) > q_1(\omega_1, \omega_{-1}),$$

we conclude

$$q_1(\omega_1, \omega_{-1}) < \omega_1$$

and thus

$$q_j(\omega_j, \omega_{-j}) = q_i(\omega_i, \omega_{-i})$$

for all $j > i$, meaning that q is the standard form of the Cournot oligopoly equilibrium, which is unique, thus implying that $\tilde{q}$ can not be an equilibrium.

To show that the statement holds in the case of duopoly, we let r denote the best response function of the unrestricted Cournot duopoly. We choose $\omega \in \Omega$ arbitrarily and assume that firm 1 produces ω_1 and firm 2 produces $r(\omega_1) < \omega_2$ in the unique equilibrium. Then, since $r(q^C) = q^C$,

$$\omega_1 + r(\omega_1) \leq 2r(q^C)$$

if and only if

$$r(\omega_1) \leq 2q^C - \omega_1,$$

which is equivalent to

$$r(\omega_1) - r(q^C) \leq q^C - \omega_1. \tag{2.7}$$

The decrease of production by firm 1 must overcompensate the increase of production by firm 2, which is true: Equation (2.7) holds because $r' > -1$. Without loss of generality, we assume that $\mu\left(T_1 < q^C\right) > 0$, which yields the given statement.

The result easily translates to the case of an oligopoly. We arbitrarily choose a capacity configuration $(\omega_1, \omega_2, \ldots, \omega_n)$. Again, we assume that $\omega_i \leq \omega_j$ if $i \leq j$. Choose k so that $q(\omega_{k-1}, \omega_{-k-1}) = \omega_{k-1}$ and $q(\omega_k, \omega_{-k}) < \omega_k$. Define the capacity configuration $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)$ by $\tilde{\omega}_i := \omega_i$ if $i < k-1$ and for $i \geq k-1$ choose $\tilde{\omega}_i$ large enough so that in the corresponding equilibrium $q(\omega_{k-1}, \omega_{-k-1}) = \omega_{k-1} < \tilde{\omega}_{k-1}$, meaning that when moving from $(\omega_1, \omega_2, \ldots, \omega_n)$ to $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)$ the former active capacity restriction of firm $k-1$ becomes inactive, whereas all other active capacity restrictions remain as they are. Having established this, it is sufficient to show that the total output of the industry with respect to the former capacity configuration is smaller than the output of the industry with respect to the new capacity configuration $(\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_n)$. But this follows from the case of duopoly: We can either focus on the residual game in which we neglect firms $1, 2, \dots, k-2$ or we assume without loss of generality that $k = 2$.

Proof of Theorem 1

Any feasible strategy profile q is an element of

$$S = \prod_{j \in N} \{q : T \to \mathbb{R}_+ | q(t) \leq t \text{ for all } t \in T\}.$$

If we define

$$d(q, q') = \max_{j \in N, t \in T} |q_j(t) - q'_j(t)|,$$

then (S, d) is a complete metric space. Thus, it is sufficient to show that $\tilde{r}$ is a contraction with respect to d, since then Banach's fixed-point theorem establishes that $\tilde{r}$ has a unique fixed point. Therefore, it remains to be shown that there exists $0 \leq \theta < 1$ so that

$$d(\tilde{r}(q), \tilde{r}(q')) \leq \theta d(q, q')$$

for every $q' \in S$ such that $q' \neq q$.

We define

$$p := \min_{t \in T} \{\mu\left(T_2 = 0 | T_1 = t\right)\}$$

and

$$\theta := \frac{(1-p)(j-1)}{2}.$$

Clearly, if $j = 2$, then $\theta < 1$. If $j = 3$, then $p > 0$ due to (2.6) and thus $\theta < 1$

as well. We choose $s \in T$ and $i \in N$ such that

$$\begin{aligned} d(\tilde{r}(q), \tilde{r}(q')) =& |\tilde{r}_i(s, q_{-i}) - \tilde{r}_i(s, q'_{-i})| \\ =& \left| \min\left\{s, r\left(E\left[q_{-i}|T_i = s\right]\right)\right\} - \min\left\{s, r\left(E\left[q'_{-i}|T_i = s\right]\right)\right\}\right|. \end{aligned} \tag{2.8}$$

If (2.8) $= 0$, then $q = q'$, which contradicts the assumption that $q \neq q'$. Thus, we must have (2.8)>0. In particular, either

$$r\left(E\left[q_{-i}|T_i = s\right]\right) < s$$

or

$$r\left(E\left[q'_{-i}|T_i = s\right]\right) < s$$

or both. For the last case when both capacity limits are not active, we obtain

$$(2.8) = \frac{1}{2}\left|E\left[q_{-i} - q'_{-i}|T_i = s\right]\right| \leq \frac{(1-p)(j-1)}{2} d(q, q') = \theta d(q, q'),$$

since q_{-i} and q'_{-i} differ at most with probability $1-p$, and the difference can never exceed $(j-1)d(q, q')$ by definition. If only one capacity constraint is active, say $r\left(E\left[q_{-i}|T_i = s\right]\right) = s$ without loss of generality, we get

$$\begin{aligned} (2.8) =& s - r\left(E\left[q'_{-i}|T_i = s\right]\right) \\ \leq& r\left(E\left[q_{-i}|T_i = s\right]\right) - r\left(E\left[q'_{-i}|T_i = s\right]\right) \end{aligned}$$

and the proposed statement follows from the case case where both capacity limits are not active.

3 Incomplete Information in Cournot Oligopoly: The Case of Unknown Production Capacities

I study a Cournot oligopoly in which firms face incomplete information with respect to production capacities. For the case where the firms' capacities are stochastically independent, the functional form of equilibrium strategies is derived. If inverse demand is concave, a unique symmetric equilibrium exists, and if demand is linear, then every equilibrium is symmetric. In the case of duopoly, I analyze the impact on social welfare when firms commit *ex-ante* on exchanging information. Sharing information increases expected output and social welfare in a large class of models. If the demand intercept is sufficiently large, sharing information increases producer surplus and decreases consumer surplus (and vice versa).

3.1 Introduction

Previously conducted research on Bayesian-Cournot oligopolies deals with incomplete information with respect to inverse demand or production costs or both. In the paper at hand, a model in which the firms' production capacities are private information to the firms is analyzed. Models of this kind are not yet included in research concerning Bayesian Cournot oligopolies. In the well-known case where costs are unknown, the cost function is typically assumed to be convex; therefore, the model frameworks developed to deal with this source of uncertainty can not be applied to the case of unknown capacities (via production costs approaching infinity as output approaches the capacity limit). Alternatively, capacity constraints can be modeled via a penalty payment embedded in the firms' payoff function, such that firms receive a negative payoff if capacities are exceeded. However, this might destroy the quasi-concavity of the expected payoff function and thus may lead to the non-existence of equilibria.

Instead, we model a firm's capacity restriction by curtailing a firm's strategy space, ensuring that the existence of an equilibrium is implied by Nash's theorem under standard assumptions on inverse demand and costs if the common prior belief is probability measure on a finite space. For the case where capacities are stochastically independent and the state space may be infinite, we characterize the functional form of equilibrium strategies: In every equilibrium firms fully utilize their capacities up to some threshold. If the capacity with which firms are endowed exceeds this threshold, then firms produce a constant quantity that equals the inner maximum of the expected payoff function. This implies that a firm's strategy space is essentially one-dimensional.

Under the additional assumption that demand is strictly concave and that the firms' capacities are identically distributed, we show that a unique symmetric equilibrium exists. The expected output of the industry is smaller compared to the output of the standard form Cournot oligopoly. In case of linear demand, every equilibrium is symmetric. This is because each firm's best response function only depends on the expected aggregate output of the other firms and from the slope of the best response function (which exceeds -1).

For the special case of two firms and a simple common prior belief, we analyze the impact of information sharing on producer surplus, consumer surplus and social welfare. This is done by comparing the unique symmetric equilibrium when information is incomplete, the *private information equilibrium*, with the equilibrium of the corresponding complete information game, the *shared information equilibrium*.

In order to calculate expected profits and outputs, an explicit characterization of equilibrium strategies is required. However, due to non-negativity and capacity constraints standard techniques do not apply to derive closed-form solutions of equilibrium strategies. Therefore, we access the impacts on surplus and welfare by using inequality arguments.

While consumers benefit from an increase of expected output, they suffer from a decrease of the variance of outputs. Since under complete information equilibrium outputs are negatively correlated, the variance of total industry output is potentially reduced. We find that the net effect, which determines whether sharing information is beneficial for consumers, is ambiguous and depends on the horizontal demand intercept: While consumers benefit from sharing information when the horizontal demand intercept a is small, they increasingly suffer from information sharing when a increases. Thus, the change in consumer surplus is positive for small values of a and negative for sufficiently large values. This effect is driven by a constellation in which both firms are endowed with a large amount of capacity, thus leading to an "overproduction" under incomplete information as the total industry output exceeds the Cournot output. Due to information sharing, firms reduce their output accordingly. In contrast, the change of producer surplus may be negative for small values of a and is positive if a is sufficiently large.

3.2 Related Literature

Regarding Cournot oligopolies with complete information, a number of authors analyze equilibrium existence and uniqueness when production capacities are bounded and asymmetric. For example, Bischi et al. (2010) and Okuguchi and Szidarovszky (1999) discuss a wide range of oligopoly models and provide results on existence and uniqueness of equilibria. As in the paper at hand, production capacity is modeled by curtailing the strategy spaces.

For the case of incomplete information, Einy et al. (2010) provide a general framework of Bayesian-Cournot games and provide results of existence and uniqueness of Bayes-Nash equilibria. They allow for incomplete information with respect to the demand function as well as with respect to the cost function. However, the case of unknown capacities is not covered, and the model framework can not be applied.

The work on information sharing in oligopoly was pioneered by Novshek and Sonnenschein (1982), Clarke (1983) and Vives (1984). Novshek and Sonnenschein (1982) and Vives (1984) discuss a duopoly with uncertain linear demand, whereas Clarke (1983) analyzes an oligopoly where both demand and costs may be unknown.

Raith (1996) provides a general model that allows for Bertrand or Cournot competition and incomplete information with respect to costs or demand. If parameters are specified in an appropriate way, virtually all models on information sharing in oligopoly follow as special cases. As Clarke (1983), Raith (1996) applies a general result provided by Radner (1962): If the joint distribution of private values is normal, then equilibrium strategies are affine. Raith (1996) shows that if firms' signals are independent private values and if each firm perfectly learns its private value, meaning that no noise is added, then industry-wide information sharing is always profitable for firms.

However, equilibrium strategies are not affine in our model. In the case of uncertain demand, Maleug and Tsutsui (1998) find that standard results on information sharing can be reversed if equilibrium strategies are not affine. In their model, in which demand is random, non-linearity stems from non-negativity constraints or capacity limits. In particular, consumer surplus can decrease although firms have an incentive to share information – in contrast, Raith (1996) and the literature cited therein find that firms do not have an incentive to share information when demand is uncertain, but consumers would profit from sharing information. Moreover, Maleug and Tsutsui (1998) demonstrate that information sharing is profitable as long as the the variation of demand is sufficiently large. In the case of uncertain costs, Shapiro (1986) finds that firms have an incentive to share information, and that information sharing increases social welfare but decreases consumer surplus.

In the work at hand, I provide a result complementary to Maleug and Tsutsui (1998). I find that firms might not have an incentive to share their information, but consumers would benefit from sharing. Moreover, if the variance of the firms' capacities is sufficiently large, then firms have an incentive to share their information.

The remainder of the paper is structured as follows: Section 3.3 contains the model framework. Section 3.4 presents the private information equilibrium and provides a detailed characterization of the symmetric equilibrium strategy. Results on the impact of information sharing on producer and consumer surplus surplus as well as social welfare are derived in Section 3.5. Lastly, Section 3.6 concludes.

3.3 The Model

We consider a set $N = \{1, 2, \ldots, n\}$ of firms that may face uncertainty regarding the other firm's endowment with production capacity. Firms only differ with respect to their production capacities. In a Bayesian approach, a strategy of firm i is a decision rule that specifies a firm's output for every possible information set with which the firm might be endowed.

More formally, we denote $T = [0, \hat{t}] \subseteq [0, \infty]$ as the set of possible capacity levels and $\Omega = \prod_{n \in N} T$ as the set of possible states of nature. The common prior belief μ is a probability measure on Ω (with respect to some appropriate σ-field). An element of Ω is denoted as $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$. We assume that every firm is endowed with a production capacity exceeding zero with positive probability. The information with which a firm is endowed when making its output decision is described by a random variable $T_i : \Omega \to \Omega_i$, where Ω_i is chosen appropriately. Moreover, we assume that $E\left[|T_i|\right] < \infty$ for all i. The information sets of firm i are then the elements of the $\sigma-$algebra $\sigma(T_i)$ generated by T_i.[1] A strategy is an integrable function $q_i : \Omega_i \mapsto \mathbb{R}_+$ satisfying $q_i(T_i(\omega)) \leq \omega_i$.[2] Lastly, the strategy space of firm i is denoted by S_i and the space containing all strategy profiles is

[1]Following Einy et al. (2002), this is equivalent to the model by Harsanyi (1967) because each firm's σ-algebra is generated by a partition of Ω that is given by $\Pi_i = \{T_i^{-1}(\tilde{\omega}) | \tilde{\omega} \in \Omega_i\}$.

[2]Integrable means that q_i is Borel-measurable and satisfies $\int_\Omega |q_i(T_i(\omega))| d\mu < \infty$.

given by $S = \prod_{i=1}^{n} S_i$.

As defined above, $q_i(T_i(\omega))$ denotes the output of firm i. We let $Q(\omega) := \sum_{i=1}^{n} q_i(T_i(\omega))$ denote the overall production. The inverse demand function and the cost function are denoted by P and C, respectively. The *state-dependent payoff function* u_i of firm i is given by

$$u_i(\omega, q_i, q_{-i}) = q_i(T_i(\omega))P(Q(\omega)) - C(q_i(T_i(\omega))). \tag{3.1}$$

The strategy profile $q \in S$ is a *Bayesian Cournot equilibrium* if for every i and $\tilde{q}_i \in S_i$ the *expected payoff function* is maximized,

$$E\left[u_i\left(\cdot, q_i, q_{-i}\right)\right] \geq E\left[u_i\left(\cdot, \tilde{q}_i, q_{-i}\right)\right], \tag{3.2}$$

meaning that at equilibrium, no firm has an incentive to unilaterally deviate from its strategy. Maximizing (3.2) is equivalent to maximizing the *conditional payoff expectation*, so that

$$E\left[u_i\left(\cdot, q_i, q_{-i}\right)\middle|\sigma\left(T_i\right)\right](\omega) \geq E\left[u_i\left(\cdot, \tilde{q}_i, q_{-i}\right)\middle|\sigma\left(T_i\right)\right](\omega) \tag{3.3}$$

for all $i \in N$ and almost all $\omega \in \Omega$.[3]

Throughout the paper, we assume:

(A) The cost function C is convex, twice continuously differentiable and there are no fixed costs, meaning that $C(0) = 0$;

(B) Inverse demand P is nonincreasing and twice continuously differentiable;

(C) There exists $Z < \infty$ such that $qP(q) - C(q) \leq 0$ for all $q \geq Z$;

(D) The marginal revenue of firm i is strictly decreasing with the aggregate output of the other firms. This is equivalent to $P'(Q) + q_i P''(Q) < 0$ (the so-called *Novshek condition*). Notice that (B) and (D) imply that P is strictly decreasing.

Remark 1. *If $\mu\left(T_i \geq Z\right) = 1$, the model reduces to a standard form Cournot oligopoly with complete information in which firms face the capacity constraint*

[3]See Harsanyi (1967) and Einy et al. (2002).

Z, which is never exceeded due to assumption (C). In this case, assumptions (A) and (D) ensure the existence of a unique equilibrium (see Vives (1999), p.97). Throughout the paper, we denote the corresponding standard form Cournot oligopoly equilibrium quantity by q^C and the corresponding best response function by r.[4] *Under assumptions (A), (B) and (D), the best response r is twice continuously differentiable and $r' > -1$ (see Vives (1999), p.97).*

Remark 2. *Assumptions (A), (B) and (D) ensure that the state-dependent payoff function (3.1) is concave in the output of firm i. Moreover, concavity is inherited by the expected payoff function (3.2) (Einy et al., 2010). If Ω is finite, then a firm's strategy space is compact and convex, and Nash's theorem implies the existence of an equilibrium.*

Notice that we allow for negative prices in the model, which is arguable from an economic point of view but which is helpful when it comes to proving existence of equilibria. If demand is truncated where it intercepts the horizontal axis in order to avoid negative prices, a firm's payoff function is no longer concave but only quasi-concave. This is not a problem in the complete information case, however the argument for equilibrium existence may collapse if we allow for incomplete information. In this case, the quasi-concavity of the state-dependent payoff function does not necessarily translate into quasi-concavity of the expected payoff function (Einy et al., 2010). In contrast, allowing for negative prices ensures that the expected payoff function is concave.

3.4 Characterization of Equilibrium Strategies

First, we reconsider the case in which firms have asymmetric capacity constraints and share their information, meaning they are subject to complete information. The question of existence and uniqueness in this setting is treated extensively in the literature, as previously mentioned. In terms of the model formulation, we discuss the case where $T_i(\omega) = \omega$ for all i and

[4] Thus, we implicitly assume $Z \leq \hat{t}$, which is not a limitation.

all ω. If demand and costs are linear, then existence and uniqueness of an equilibrium are easily obtained.

In the remainder of the paper, we denote the shared information equilibrium strategy by q^S. For the duopoly case, the shared information equilibrium strategy q^S can be presented in a compact manner. As previously mentioned, r denotes the best response function of the unrestricted Cournot duopoly and q^c denotes the equilibrium strategy of the unrestricted Cournot duopoly.

$$q^S(\omega_1, \omega_2) = \begin{cases} \min\left\{\omega_1, q^C\right\}, \text{ if } \omega_1 \leq \omega_2, \\ \min\left\{\omega_1, r\left(q^S(\omega_2, \omega_1)\right)\right\} \text{ otherwise.} \end{cases} \tag{3.4}$$

It is easily demonstrated that q^S is the unique equilibrium strategy. We use this representation of q^S in Section 3.5.

Notice that if in an equilibrium there is a firm with a binding capacity restriction, the total output of the industry is lower compared to the output of the standard form Cournot oligopoly. This property derives from the slope of the best response function r which exceeds -1. If one firm decreases its output due to its capacity restriction, then the corresponding increase of the other firms is smaller (see also Remark 2).

In the private information setting, every firm perfectly learns its own capacity but receives no information about the other firms' capacities. Speaking in terms of the model, we analyze the case $T_i(\omega) = \omega_i$. A strategy of firm i is now a function on T. In the following, we write $q_i(t)$ instead of $q_i(T_i(\omega))$.

Recall that $\hat{t}$ denotes the maximal element in T. Theorem 1 states that an equilibrium strategy q_i is completely determined by $q_i(\hat{t})$ if the firms' capacities are independent. That is, the relevant strategy space is one-dimensional.

Theorem 1. *If the firms' capacities are stochastically independent, then in every equilibrium $q = (q_1, q_2, \ldots, q_n)$ and for every firm i the strategy q_i is nondecreasing. More precisely, for every i there exists a threshold $s_i \in T$ such that $q_i(t) = t$ for all $t \leq s_i$ and $q_i(t) < t$ for all $t > s_i$.*

Proof. We assume that q is an equilibrium and choose $i \in N$ arbitrarily. If $q_i(t) = t$ for all $t \in T$, then the proposed statement follows. Therefore, we denote s_i as the infimum of the set $\{t \in T | q_i(t) < t\}$. If $t, u \in T$ so that $u > t > s_i$, we must have $q_i(u) = q_i(t) < t$ since $q_i(t)$ maximizes the conditional payoff expectation (3.3), which is concave, and because $q_i(t)$ lies in the inner of $[0, t]$, implying that $q_i(t)$ is the global maximum. Notice that either $q_i(t) = q_i(s) < s$ or $s = q_i(s) < q_i(t)$. □

The result of Theorem 1 is driven by the independence of $T_1, T_2, \ldots, T_n$ and does not generally hold, as shown in the following example. We consider a duopoly in which the inverse demand function is given by $P(q) = 2 - q$. The set of possible capacity levels equals $T = \{0, 1, 2\}$. We assume that μ is symmetric, meaning that for all ω_1, ω_2

$$\mu\left(T_1 = \omega_1, T_2 = \omega_2\right) = \mu\left(T_1 = \omega_2, T_2 = \omega_1\right).$$

Moreover, we assume that $\mu\left(T_1 = 0 | T_2 = 1\right) = 1$ and $\mu\left(T_1 = 2 | T_2 = 2\right) = 1$.[5] Then, the unique symmetric equilibrium is given by

$$\begin{aligned} q(0) &= 0, \\ q(1) &= 1, \\ q(2) &= 2/3. \end{aligned}$$

The equilibrium strategy is neither increasing nor decreasing. In fact, when allowing for an arbitrary common prior belief, then we can say nothing about the shape of the equilibria.

Theorem 1 states that a firm's equilibrium strategy q_i is completely determined by $q_i(\hat{t})$, since $q_i(t) = \min\{t, q_i(\hat{t})\}$. If we restrict the analysis to symmetric equilibria and assume that the firms' capacities are identically distributed, then the space of feasible strategy profiles becomes one-

[5]This specification of the conditional probabilities implies that firms have complete information. However, this is just for convenience. We obtain similar results if we allow for the conditional probabilities to be close to 1.

dimensional. Next, we show that there exists a unique symmetric equilibrium if the inverse demand function is concave. We use two arguments in the proof: A fixed point argument applied to the one-dimensional space of feasible strategy profiles described above and the existence of a unique Cournot equilibrium in the unrestricted, standard form Cournot oligopoly, characterized by a smooth best reply function (see Remark 1). In order to ease notation, we write $r(q)$ instead of $r((n-1)q)$ if q is a symmetric equilibrium strategy or quantity in the remainder of the paper.

Theorem 1 shows that a firm produces some constant output $q(\hat{t})$ if the capacity level with which the firm is endowed exceeds a certain threshold. In Theorem 2 we show that in the case of independent and identically distributed capacities and concave inverse demand function, exactly one symmetric equilibrium exists. In this equilibrium, the output $q(\hat{t})$ exceeds the Cournot quantity q^C, but is smaller than the monopoly quantity q^M.[6] We characterize $q(\hat{t})$ via

$$q^C < q(\hat{t}) = r(\lambda q^C) < q^M$$

for an appropriate $0 < \lambda < 1$.

Clearly, $q(\hat{t})$ does not exceed the monopoly quantity, implying $\lambda > 0$. To encourage intuition why $q(\hat{t})$ exceeds the Cournot quantity, implying $\lambda < 1$, we consider a duopoly in which inverse demand is given by $P(q) = 1 - q$ and in which marginal costs are equal zero. Every firm's capacity may take values in $T = \{0, 1\}$, and each capacity level occurs with probability $p = 1/2$. If firm 1 is endowed with capacity 1, it maximizes

$$E\left[q_1(1 - q_2 - q_1)\right] = q_1(1 - E\left[q_2\right] - q_1)$$

subject to $q_1 \leq 1$. We let $\tilde{r}$ denote the best response function of firm 1. At

[6]Note that this result relies on the assumption of identically distributed capacities: Otherwise, some firms might have systematically small capacities, while other firms receive systematically large capacities. Firms with expected large capacities might then produce an expected quantity exceeding the Cournot quantity.

equilibrium,

$$q_1(1) = \tilde{r}(q_2) = \frac{1 - E\left[q_2\right]}{2} = \frac{1 - \frac{1}{2}q_2(1)}{2},$$

because $q_2(0) = 0$. Since $q_1(1) = q_2(1)$ in a symmetric equilibrium, we obtain $q_1(1) = 2/5 > 1/3 = q^C$. The equilibrium strategy is then

$$q_1(t) = \min\{t, 2/5\},$$

which may be written as

$$q_1(t) = \min\left\{t, r\left(\frac{3}{5}q^C\right)\right\}.$$

Theorem 2. *If capacities are i.i.d. and the inverse demand function is concave, there exists exactly one symmetric equilibrium and the equilibrium strategy* q^P *satisfies*

$$E\left[q^P\right] \leq q^C.$$

The inequality strictly holds if $\mu\left(T_i < q^C\right) > 0$.

Proof. We construct a symmetric equilibrium. Recall that firm i maximizes

$$E\left[u_i\left(\cdot, q_i, q_{-i}\right)\right] = E\left[q_i(T_i(\cdot))P(Q(\cdot)) - C(q_i(T_i(\cdot)))\right] \tag{3.5}$$

by choosing q_i. For every $t \in T$ and $\lambda \in \mathbb{R}$ we define the strategy q^λ by

$$q^\lambda(t) = \min\left\{t, r\left(\lambda q^C\right)\right\}. \tag{3.6}$$

Then, $q^\lambda(t)$ is continuous in λ since r is smooth (see Remark 1). We assume that the other firms $j \neq i$ apply q^λ for some $\lambda \in [0,1]$ and define

$$Q^\lambda_{-i}(\omega) := \sum_{j \neq i} q^\lambda(T_j(\omega)).$$

to be the corresponding, realized aggregate output, which is nonincreasing and continuous in λ. Consider the mapping $\phi : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$\begin{aligned}\phi(\lambda, x) :=& E\left[u_i\left(\cdot, x, q^\lambda\right)\right] \\ =& xE\left[P\left(Q^\lambda_{-i}(\cdot) + x\right)\right] - C(x).\end{aligned}$$

Then ϕ is continuous in λ as well. Due to the assumptions placed on P and C, the integrand

$$xP\left(Q^\lambda_{-i}(\omega) + x\right) - C(x)$$

is strictly concave in x (see Remark 2) and this implies that ϕ is concave in x as well. We let $\gamma(\lambda)$ denote the global maximizer of $\phi(\lambda, \cdot)$. Then γ is strictly decreasing with λ because Q^λ_{-i} is strictly decreasing with λ,[7] and this implies that the maximizer γ must increase since output decisions are strategic substitutes.

We prove indirectly that γ is continuous: Assume that γ has a discontinuity in μ. Then, since γ is nondecreasing, there exists an $\epsilon > 0$ and a sequence $\mu_n > \mu$ converging to μ such that

$$\gamma(\mu_n) - \gamma(\mu) > \epsilon \tag{3.7}$$

for all $n \in N$. Because $\gamma(\mu)$ maximizes $\phi(\mu, \cdot)$, which is strictly concave, we conclude

$$\phi(\mu, \gamma(\mu)) > \phi(\mu, \gamma(\mu) + \epsilon).$$

Because the sequence μ_n converges to μ and ϕ is continuous in its first argument, we can choose n^* large enough so that

$$\phi(\mu_{n^*}, \gamma(\mu)) > \phi(\mu_{n^*}, \gamma(\mu) + \epsilon).$$

This implies that $\gamma(\mu_{n^*}) < \gamma(\mu) + \epsilon$, since ϕ is strictly concave in its second

[7]More precisely, there exists a set $A \subset \Omega$ such that $Q^\lambda_{-i}(\omega)$ is strictly decreasing for almost all $\omega \in A$ and constant almost everywhere on A^c. In a non trivial setting, $\mu(A) > 0$, which is sufficient because γ does only depend on the expected value of Q_{-i}.

argument (and thus continuous as well). But this yields

$$\gamma(\mu_{n^*}) - \gamma(\mu) < \epsilon,$$

contradicting (3.7).

Next, we demonstrate that there exists $\lambda > 0$ such that $\gamma(\lambda) = r(\lambda q^C)$ by applying the intermediate value theorem. If $\lambda = 0$, then $r(\lambda q^C) = r(0) = q^M$, where q^M is the monopoly output. Clearly, we must have $\gamma(0) < q^M$: $\gamma(0)$ is the maximizer of

$$xE\left[P\left(Q^0_{-i}(\cdot) + x\right)\right] - C(x).$$

Since the inverse demand function is concave by assumption, we may apply Jensen's inequality and obtain

$$E\left[P\left(Q^0_{-i}(\cdot) + x\right)\right] \leq P\left(E\left[Q^0_{-i}\right] + x\right) < P(x),$$

meaning that the expected price is smaller than the monopoly price for any x, implying that $\gamma(0) < q^M$.

Similarly, if $\lambda = 1$, then $r(\lambda q^C) = r(q^C) = q^C$, and $\gamma(1)$ exceeds q^C: The expected price satisfies

$$E\left[P\left(Q^0_{-i}(\cdot) + x\right)\right] \geq E\left[P\left((n-1)q^C + x\right)\right] = P\left((n-1)q^C + x\right),$$

implying that the expected price exceeds the price of the unrestricted Cournot oligopoly for any x and further that $\gamma(1)$ must exceed q^C. Since both r and γ are continuous, we conclude that there exists a λ as claimed. Notice that the inequality above strictly holds if $\mu\left(T_i < q^C\right) > 0$.

Lastly, we denote $\tilde{r}(t, \cdot)$ as the best response of the restricted oligopoly when $T_i = t$, meaning that $\tilde{r}$ maximizes $E\left[u_i\left(\cdot, x, q_{-i}\right)\right]$ subject to $x \leq t$. When $q_j = q^\lambda$ for $j \neq i$, we obtain

$$\tilde{r}(t, q^\lambda) = \min\left\{t, \gamma(\lambda)\right\} = \min\left\{t, r(\lambda q^C)\right\} = q^\lambda(t).$$

This shows that q^λ is a fixed point of the best response function.

□

Ultimately, the result established in Theorem 2 stems from the slope of the best response function, which exceeds -1 (see Remark 1). If a firm's output is bounded with positive probability, then the remaining firms (state-wise) do not fully compensate this lack of production. It is easily verified that a similar result holds in the case complete information. Thus, Theorem 2 is a natural analog to the complete information case.

Remark 3. *Notice that if demand is linear, it follows $\gamma(\lambda) = r\left(E\left[q^\lambda\right]\right)$. This is because the expected payoff of firm i only depends on the expected aggregate output of the other firms. Since $\gamma(\lambda) = r(\lambda q^C)$ in the equilibrium, we conclude $E\left[q^\lambda\right] = \lambda q^C$.*

Since a firm's strategy is of the form $q_i(t) = \min\{t, q_i(\hat{t})\}$, the strategy is completely determined by its expected value, which is strictly increasing with $\hat{t}$. That is to say, a firm's decision variables are one-dimensional and the best response is of the form $\tilde{r}(t, Q_-) = \min\{t, r(Q_-)\}$ and thus depends only on the aggregate output. Under these conditions, only symmetric equilibria can exist if the slope of $\tilde{r}$ strictly exceeds -1.[8] In our case, $r' > -1$ (see Remark 1) and in fact, $r' > -1/2$ when demand is linear. Conversely, Theorem 3 may not hold if demand is not linear.

Theorem 3. *If capacities are i.i.d. and demand is linear, then every equilibrium is symmetric.*

Proof. First, we give a proof for the duopoly case. Second, we argue why the statement also holds true in an oligopoly. For an arbitrarily chosen equilibrium $q = (q_1, q_2)$ it is sufficient to show that $E[q_1] = E[q_2]$ due to Theorem 1. We define $x := E[q_1]$, $y := E[q_2]$ and

$$\phi_2(z) = E\left[u_2\left(\cdot, z, q_1\right)\right] = zP\left(E\left[q_1\right] + z\right) - C(z).$$

[8] See Vives (1999), p. 42–43, who discusses the complete information case.

Clearly, ϕ_2 is maximized by $r\left(E\left[q_1\right]\right) = r(x)$ because the expected payoff function of firm 2 does only depend on the expected quantity of firm 1 as a result of the linearity of P.

We let f denote a marginal probability density with respect to T_i, meaning that f is such that for all $c \in T$

$$\mu\left(T_i \leq c\right) = \int_0^c f(t)dt.$$

We write

$$\begin{aligned} y =& E\left[\min\left\{T_2, r(x)\right\}\right] && (3.8)\\ =& \int_0^{r(x)} tf(t)dt + \int_{r(x)}^{\infty} r(x)f(t)dt && (3.9)\\ =:& g(x). && (3.10)\end{aligned}$$

Similarly, we conclude $x = g(y)$.

Next, we demonstrate that $g(x) = y$ and $g(y) = x$ implies $x = y$, which yields the given statement. The strategy is to show that $g' > -1$, implying that g can not intersect a linear function with derivative -1 twice; however, this is a necessary condition for the existence of $x \neq y$ satisfying $g(x) = y$ and $g(y) = x$. We calculate[9]

$$\begin{aligned} g'(x) =& r'(x)r(x)f(r(x)) + r'(x)\int_{r(x)}^{\hat{t}} f(t)dt - r(x)r'(x)f(r(x)) \\ =& r'(x)\mu\left(T_i \geq r(x)\right) > -1. && (3.11)\end{aligned}$$

We suppose that there exist $0 \leq x < y$ such that $g(x) = y$ and $g(y) = x$. We define the linear function h by $h(z) = x + y - z$. Then $h(x) = y$ and $h(y) = h(h(x)) = x$. On one hand, this implies that h intersects g at x and y, so that

$$g(x) - h(x) = g(y) - h(y) = 0. \qquad (3.12)$$

On the other hand, $g' - h' > 0$, implying that $g - h$ is strictly increasing – a

[9]If the common prior belief is discrete, then g is piecewise linear and thus differentiable almost everywhere.

contradiction to (3.12).

It is important to notice that the proof does not rely on the demand function parameter a and b. This implies that for the oligopoly case we can define $\tilde{a} = a - b\sum_{j>2} E[q_j(T_j)]$ and apply the duopoly result to the residual demand function defined by $\tilde{a}$ (which is the same for both firm 1 and firm 2). Equivalently speaking, we proved that the firms' strategies are pair wise identical for any a, which is sufficient to prove the statement for the oligopoly case. $\square$

The result of Theorem 3 is driven by by the linearity of the demand function: If demand is linear, then the best response function of a firm does only depend on the expected output of the other firms. Thus, the maximizer of a firm's payoff function inherits the slope of the Cournot best response r to some extent (see equation (3.11)).

Lastly, Theorem 3 implies the existence of a unique symmetric equilibrium in the linear case: g has exactly one fixed point, and the fixed points of g correspond to symmetric equilibria. Figure 3.1 shows the symmetric equilibrium of the oligopoly for the case in which demand is linear and the common prior belief is discrete and uniformly distributed.

3.5 Information Sharing

We discuss the effects of information sharing on producer surplus, consumer surplus and social welfare. We consider the two extreme cases in which information is not shared at all, the *private information equilibrium*, and where firms commit *ex-ante* to an industry-wide information sharing agreement, e.g. via some trade association, the *shared information equilibrium*. We find that even in simple examples, the impact on both consumer and producer surplus is ambiguous. This ambiguity is driven by the concavity of the firms' payoff function and by the covariance of firms' equilibrium outputs. For a large class of examples, social welfare increases. However, we provide an example where social welfare decreases.

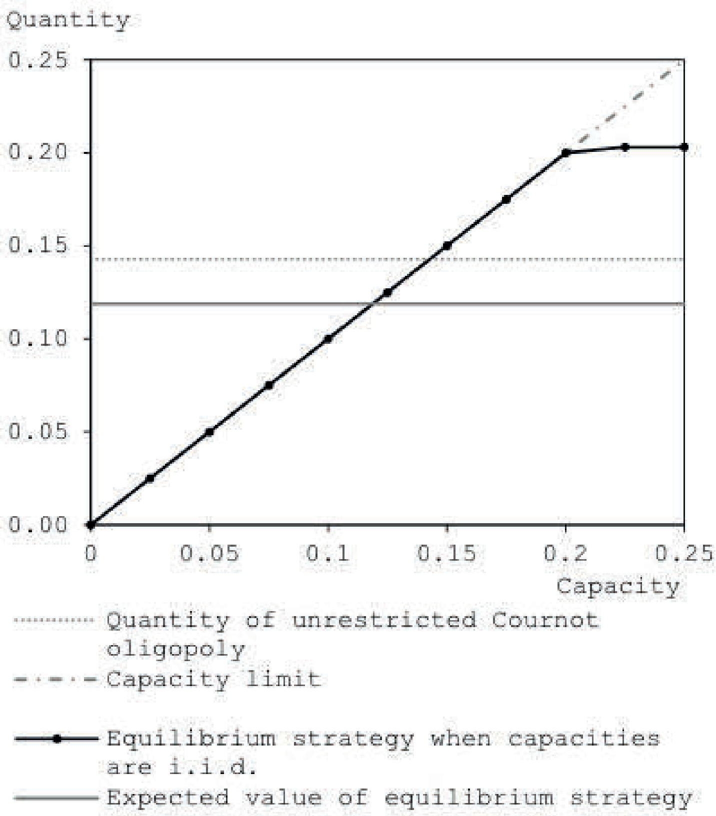

Figure 3.1: The unique symmetric equilibrium when capacities are stochastically independent and uniformly distributed ($a = 1$, $b = 1$, $c = 0$, $n = 6$, $\hat{t} = 0.25$, $|T| = 11$, $\lambda = 0.83$).

In the following, we discuss a simple duopoly. The common prior belief is discrete and there are two possible capacity levels $t_L < t_H$ that may each occur with probability $p = 1/2$. The symmetry of the common prior belief is just for convenience – the results are not driven by this assumption. Without loss of generality, we assume that costs equal zero and that inverse demand equals $p(q) = a - q$.[10] To avoid trivialities, we assume throughout the analysis that $t_L \leq q^C$, which is equivalent to $a \geq 3t_L$. In the limiting case $a = 3t_L$ the model reduces to a standard form Cournot duopoly in which both firms produce their Cournot quantity in both the private and shared information equilibrium.

Moreover, we assume that t_H is sufficiently large, meaning that t_H exceeds the monopoly output. This assumption simplifies the analysis, but we are still able to demonstrate the ambiguous effects on producer and consumer surplus. In contrast, under this assumption, social welfare increases when information is shared.

In this model specification we obtain (see Remark 3)

$$t_L < E\left[q^P\right] < r\left(E\left[q^P\right]\right).$$

This leads to

$$E\left[q^P\right] = pt_L + pr\left(E\left[q^P\right]\right).$$

Substituting $p = 1/2$ and solving for $E\left[q^P\right]$ yields

$$E\left[q^P\right] = \frac{2}{5}t_L + \frac{3}{5}q^C = \frac{2}{5}t_L + \frac{1}{5}a, \tag{3.13}$$

$$r\left(E\left[q^P\right]\right) = \frac{6}{5}q^C - \frac{1}{5}t_L = \frac{2}{5}a - \frac{1}{5}t_L. \tag{3.14}$$

We can see that the model becomes trivial if $a = 3t$: In this case, we obtain $q^C = t = E\left[q^P\right]$.

[10] If $c > 0$, we define $\tilde{a} = a - c$; if $b \neq 1$, we define $\overline{a} = \tilde{a}/b$. The payoff function of the case $b \neq 1$ is then a scaled version of the payoff function arising when demand equals $\overline{a} - q$. The same holds for consumer surplus.

3.5.1 Producer Surplus

Since both the shared and the private information equilibria are symmetric, firms have an incentive to share their information if and only if sharing information increases producer surplus (PS). Producer surplus equals the expected profit of the industry. Thus, firms have an incentive to share their information if the difference

$$E\left[\Delta PS\right] := 2E\left[u_i\left(\cdot, q^S, q^S\right)\right] - 2E\left[u_i\left(\cdot, q^P, q^P\right)\right]$$

exceeds zero.

Information sharing may *ex-post* lead to losses for firm i if and only if its capacity restriction is binding and the capacity restriction of firm 2 is not binding in the private information equilibrium, allowing firm 2 to increase its output when learning that firm 1 produces little, and the other way around. If both firms' capacity restriction are not binding, then sharing information induces both firms to decrease outputs and thus increases profits. The net effect depends on the demand intercept a.

We derive the effects of information sharing on producer surplus by analyzing the possible states of nature separately. *Ex-post*, information sharing leads to losses for firm 1 if and only if $\omega_1 = t_L$ and $\omega_2 = t_H$. In this case, firm 2 produces $r\left(E\left[q^P\right]\right)$ in the private information equilibrium and $r(t_L) > r\left(E\left[q^P\right]\right)$ in the shared information equilibrium, whereas the output of firm 1 remains constant. If we combine the events (t_L, t_H) and (t_H, t_L) and multiply the expected difference of a firm's payoff by 2, the decrease in producer surplus arising from asymmetric capacities $PS^-(a)$ equals

$$PS^-(a) = 2t_L\left(a - r\left(E\left[q^P\right]\right) - t_L\right) - 2t_L\left(a - r(t_L) - t_L\right) = t_L(E\left[q^P\right] - t_L).$$

Via (3.13) and (3.14) we calculate

$$PS^-(a) = \frac{1}{5}at_L - \frac{}{5}t_L^2$$

and

$$\frac{\partial}{\partial a} PS^{-}(a) = \frac{1}{10} t_L. \tag{3.15}$$

Thus, PS^{-} is linear and increasing. Clearly, if $a = 3t_L$, then $PS^{-}(a) = 0$.

Next, we examine two constellations that *ex-post* lead to an increase in producer surplus. The first is the counterpart of PS^{-}: If firm 1 is endowed with t_H and firm 2 is endowed with t_L, then firm 1 produces $r\left(E\left[q^P\right]\right)$ in the private information equilibrium and $r(t_L)$ in the shared information equilibrium. The output of firm 2 equals t_L in both equilibria. Again, we combine the events (t_L, t_H) and (t_H, t_L) and we denote the increase in producer surplus (when capacities are asymmetric) by $PS_1^{+}(a)$:

$$PS_1^{+}(a) = 2r(t_L)\left(a - t_L - r(t_L)\right) - 2r\left(E\left[q^P\right]\right)\left(a - t_L - r\left(E\left[q^P\right]\right)\right).$$

The expression PS_1^{+} has a zero at $a = 3t_L$. A straightforward calculation shows that

$$\frac{\partial}{\partial a} PS_1^{+}(a) = \frac{4}{25} a - \frac{12}{25} t_L. \tag{3.16}$$

This implies that PS_1^{+} is a parabola that has a local minimum at $a = 3t_L$.

Finally, both firms benefit *ex-post* from sharing information if $\omega_1 = \omega_2 = t_H$. In this case, both firms reduce their output to the Cournot quantity q^C when information is shared. The corresponding increase in producer surplus is denoted by PS_2^{+} and given by

$$PS_2^{+}(a) = 2q^C(a - 2q^C) - 2r\left(E\left[q^P\right]\right)(a - 2r\left(E\left[q^P\right]\right)).$$

Again, $PS_2^{+}(3t_L) = 0$. Moreover, a calculation shows that

$$\frac{\partial}{\partial a} PS_2^{+}(a) = \frac{140}{1125} a - \frac{6}{25} t_L. \tag{3.17}$$

Since

$$\frac{\partial}{\partial a} PS_2^{+}(3t_L) = \frac{2}{15} t_L > 0,$$

PS_2^{+} is increasing as long as $a \geq 3t_L$.

All three events $(t_H, t_L), (t_L, t_H)$ and (t_H, t_H) occur with probability p^2, leading to

$$\begin{aligned}
&E\left[\Delta PS\right](a) \\
=&p^2\left(\frac{1}{2}PS_1^+(a) - \frac{1}{2}PS^-(a)\right) + p^2\left(\frac{1}{2}PS_1^+(a) - \frac{1}{2}PS^-(a)\right) + p^2 PS_2^+(a) \\
=&p^2\left(PS_1^+(a) + PS_2^+(a) - PS^-(a)\right).
\end{aligned}$$

Using (3.15), (3.16) and (3.17), we obtain

$$\begin{aligned}
\frac{\partial}{\partial a}E\left[\Delta PS\right](3t_L) =& p^2\frac{\partial}{\partial a}\left(PS_1^+(3t_L) + PS_2^+(3t_L) - PS^-(3t_L)\right) \\
=& p^2\left(0 + \frac{2}{15}t_L - \frac{2}{10}t_L\right) < 0.
\end{aligned}$$

On one hand, since $E\left[\Delta PS\right](3t) = 0$ and $\partial/\partial a E\left[\Delta PS\right](3t) < 0$, we conclude that $E\left[\Delta PS\right](a) < 0$ if a is sufficiently small, meaning that firms do not have an incentive to share their information.

On the other hand, calculating the second derivative shows that

$$\partial/\partial^2 a E\left[\Delta PS\right] > 0.$$

This stems from the fact that PS^- is linear and implies $E\left[\Delta PS\right](a) > 0$ when a is sufficiently large. Thus, we have established:

Theorem 4. *If the demand intercept is sufficiently large, then firms have an incentive to exchange information.*

The result is driven by the concavity of the firms' payoff function. Consider that capacities are asymmetric and that $T_1 = t_L, T_2 = t_H$. Then firm 1 *ex-post* suffers from information sharing due to the price effect when firm 2 increases output, and theses losses are linear with respect to a. Conversely, firm 2 is subject to a price effect and a quantity effect. If a is large, then the quantity effect gains weight in a convex fashion, i.e. the marginal revenue of firm 2 is high (and vice versa).

3.5.2 Consumer Surplus

We let $Q(\omega, q^P)$ and $Q(\omega, q^S)$ denote the realized total output of the industry in the private information and shared information equilibria, respectively. Consumer surplus is given by $Q^2(\omega, q^P)/2$ and $Q^2(\omega, q^S)/2$. Sharing information leads to an increase in consumer surplus if and only if the expected difference

$$E[\Delta CS] = \frac{1}{2}E\left[Q^2(\cdot, q^S)\right] - \frac{1}{2}E\left[Q^2(\cdot, q^P)\right] \tag{3.18}$$

is positive.

Before we analyze the impact on consumer surplus, it is instructive to analyze the net effect on total industry output arising from information sharing. As performed in the last section, we can identify the states of nature that lead to a decrease or an increase of total output. A decrease can only occur if both firm 1 and firm 2 are endowed with t_H. We denote this quantity effect by Q^-. In this case, both firms produce $r\left(E\left[q^P\right]\right)$ in the private information equilibrium and $q^C < r\left(E\left[q^P\right]\right)$ in the shared information equilibrium. An increase, denoted by Q^+, occurs if both firms are endowed with different capacity levels: If firm 1 is endowed with t_L, then its outputs in both equilibria coincide. Firm 2 increases its output by $r(t_L) - r\left(E\left[q^P\right]\right)$. Notice that the same increase of output occurs if $\omega = (t_H, t_L)$.

The decrease in output *ex-post* amounts to

$$Q^- = 2r\left(E\left[q^P\right]\right) - 2q^C = q^C - E\left[q^P\right] = \frac{2}{15}a - \frac{2}{5}t_L.$$

The increase of output *ex-post*, multiplied by 2, is given by

$$2Q^+ = 2\left(r(t_L) - r\left(E\left[q^P\right]\right)\right) = E\left[q^P\right] - t_L = \frac{1}{5}a - \frac{3}{5}t_L.$$

Both the increase and the decrease of output equal zero if $a = 3t_L$, as expected. Apparently, the expected difference of total output exceeds zero as long as $a > 3t_L$:

$$E[\Delta Q] = p^2Q^+ + p^2Q^+ - p^2Q^- = \frac{p^2}{15}a - \frac{p^2}{5}t_L.$$

That is to say, sharing information always leads to an increase in expected output. This stems from the fact that equilibrium strategies are concave. Shapiro (1986) finds that in the presence of uncertain costs and linear equilibrium strategies, a firm's output does not change when information is shared.

Moreover, the variance of a firm's output increases. By applying (3.13), we see that the increase in the output of firm 1 when firm 2 is endowed with t_L exceeds the decrease in output of firm 1 when firm 2 is endowed with t_H:

$$\begin{aligned} &\left(q^S(t_H,t_L) - q^P(t_H)\right) - \left(q^P(t_H) - q^S(t_H,t_H)\right) \\ =&r(t_L) - r\left(E\left[q^P\right]\right) - r\left(E\left[q^P\right]\right) + r\left(q^C\right) \\ =&\frac{1}{15}a - \frac{1}{5}t_L \geq 0 \end{aligned} \tag{3.19}$$

if and only if $a \geq 3t_L$. Since the output of firm 1 remains constant when endowed with t_L and since $q^S(t_H,t_L)$ exceeds $q^P(t_H)$, the variance of outputs of firm 1 increases due to information sharing. Because equation (3.19) increases with a, the increase of variance, in turn, increases with a.

In order to examine consumer surplus, we calculate the realized consumer surplus of the shared information equilibrium when both firm 1 and firm 2 are endowed with t_H:

$$CS^S(a,t_H,t_H) = \frac{\left(2q^C\right)^2}{2} = \frac{2}{9}a^2.$$

For the private information equilibrium, we find

$$CS^P(a,t_H,t_H) = \frac{\left(2r\left(E\left[q^P\right]\right)\right)^2}{2} = \frac{2}{25}\left(4a^2 - 4at_L + t_L^2\right).$$

The *ex-post* decrease in consumer surplus when both firms are endowed with

t_H is then

$$CS^{-}(a) = CS^{P}(a, t_H, t_H) - CS^{S}(a, t_H, t_H) = \frac{2}{25}\left(\frac{11}{9}a^2 - 4at_L + t_L^2\right).$$

Similarly, if both firms have different capacity levels, we calculate the corresponding consumer surplus for both the shared and the private information equilibrium:

$$CS^{S}(a, t_L, t_H) = \frac{\left(t_L + r(t_L)\right)^2}{2} = \frac{1}{8}\left(a^2 + 2t_L a + t_L^2\right)$$

and

$$CS^{P}(a, t_L, t_H) = \frac{\left(t_L + r\left(E\left[q^P\right]\right)\right)^2}{2} = \frac{2}{25}\left(a^2 + 4at_L + 4t_L^2\right).$$

The *ex-post* increase in consumer surplus when firms have asymmetric capacities is then

$$CS^{+}(a) = CS^{S}(a, t_L, t_H) - CS^{P}(a, t_L, t_H) = \frac{1}{25}\left(\frac{9}{8}a^2 - \frac{7}{4}at_L - \frac{39}{8}t_L^2\right).$$

Since an increase in consumer surplus occurs in two states of nature, we may write

$$E\left[\Delta CS(a)\right] = 2p^2 CS^{+}(a) - p^2 CS^{-}(a).$$

Thus, it is sufficient to analyze the difference $2CS^{+} - CS^{-}$ in order to determine the sign of $E\left[\Delta CS\right](a)$. Note first that both CS^{+} and CS^{-} have a zero at $a = 3t$.

Differentiating with respect to a yields

$$2\frac{\partial}{\partial a}CS^{+}(a) = \frac{2}{25}\left(\frac{9}{4}a - \frac{7}{4}t_L\right)$$

and

$$\frac{\partial}{\partial a}CS^{-}(a) = \frac{2}{25}\left(\frac{22}{9}a - 4t_L\right).$$

Evaluating at $a = 3t$ shows

$$2\frac{\partial}{\partial a}CS^{+}(3t_L) = \frac{30}{75}t_L > \frac{20}{75}t_L = \frac{\partial}{\partial a}CS^{-}(a).$$

On one hand, this implies that $E\left[\Delta CS(a)\right]$ is positive when a is sufficiently small. On the other hand, calculating the second derivative yields

$$2\frac{\partial}{\partial^2 a}CS^{+}(a) = \frac{9}{100} < \frac{22}{225} = \frac{\partial}{\partial^2 a}CS^{-}(a).$$

This implies that $E\left[\Delta CS\right](a)$ is negative when a is sufficiently large. We have established:

Theorem 5. *If the demand intercept is sufficiently small, then information sharing increases consumer surplus.*

Ultimately, the result is due to the negative correlation of equilibrium outputs in the complete information case. This correlation effect decreases the variance of total industry output, which in turn lowers consumer surplus. When increasing a, the negative correlation of equilibrium outputs increases. If $q(T_1) + q(T_2)$ denotes the total industry output, we observe

$$\begin{aligned}
&E\left[\left(q(T_1)+q(T_2)\right)^2\right] \\
=&VAR\left[q(T_1)+q(T_2)\right] + E\left[q(T_1)+q(T_2)\right]^2 && (3.20)\\
=&2VAR\left[q(T_1)\right] + 2COV\left[q(T_1), q(T_2)\right] + E\left[q(T_1)+q(T_2)\right]^2. && (3.21)
\end{aligned}$$

As discussed on page 63, both expected output and variance of output of a *single* firm increase when information is shared. Theorem 5 and equation (3.20) imply that the variance of *total industry output* must decrease if a is sufficiently large. Lastly, equation (3.21) shows that the decrease of the variance of total industry output driven by a negative correlation of equilibrium outputs.

Apparently, we can easily construct an example in which firms do not have an incentive to share information but consumers nevertheless profit from an

Table 3.1: Equilibrium outputs for private (P) and shared (S) information equilibrium and effects on surplus and welfare ($a = 5, T = \{1, 5\}, \mu$ is uniformly distributed on T^2, implying $r\left(E\left[q^P\right]\right) = 9/5 = 1.8$)

ω	Output		P. surplus		C. surplus		Welfare	
	P	S	P	S	P	S	P	S
(1,1)	1.00	1.00	6.00	6.00	2.00	2.00	8.00	8.00
(1,5)	1.00	1.00	4.40	4.00	3.92	4.50	8.32	8.50
(5,1)	1.80	2.00	7.92	8.00	3.92	4.50	11.84	12.50
(5,5)	1.80	1.67	5.04	5.56	6.48	5.56	11.52	11.11
Expected Values	1.40	1.42	5.84	5.89	4.08	4.14	9.92	10.03
Variances	0.21	0.25	2.36	2.72	3.38	2.28	4.16	4.58

information sharing agreement, shown by choosing a sufficiently small a. Similarly, an example in which firms do have an incentive to share information, but the sharing of information in turn decreases consumer surplus, is easily obtained by choosing a sufficiently large a.

The example presented in Table 3.1 shows that we can choose a such that both producer and consumer surpluses increase, a result that is not implied by the analysis conducted above. We choose $a = t_H = 5$ and $t_L = 1$, implying $r\left(E\left[q^P\right]\right) = 9/5$.

3.5.3 Social Welfare

Finally, we look at the expected change of social welfare, given by

$$\begin{aligned} E\left[\Delta W(a)\right] =& E\left[\Delta PS(a)\right] + E\left[\Delta CS(a)\right] \\ =& E\left[PS_1^+(a) + PS_2^+(a) - PS^- + 2CS^+(a) - CS^-(a)\right]. \end{aligned}$$

Using the results previously established and differentiating with respect to a show that $E\left[\Delta W(a)\right]$ is a quadratic function that has a zero at $a = 3t_L$ and is increasing as long as $a \geq 3t_L$. This implies that information sharing increases social welfare.

Lastly, we demonstrate that social welfare may decrease if t_H is sufficiently

Table 3.2: Equilibrium outputs for private (P) and shared (S) information equilibrium when $t_H = r\left(E\left[q^P\right]\right) = 1.8$

ω	Output		P. surplus		C. surplus		Welfare	
	P	S	P	S	P	S	P	S
(1,1)	1.00	1.00	6.00	6.00	2.00	2.00	8.00	8.00
(1,1.8)	1.00	1.00	4.40	4.40	3.92	3.92	8.32	8.32
(1.8,1)	1.80	1.80	7.92	7.92	3.92	3.92	11.84	11.84
(1.8,1.8)	1.80	1.67	5.04	5.56	6.48	5.56	11.52	11.11
Expected Values	1.40	1.37	5.84	5.97	4.08	3.85	9.92	9.82
Variances	0.21	0.18	2.36	2.15	3.38	2.11	4.16	3.77

small. We discuss an example in which $t_H = r\left(E\left[q^P\right]\right)$. This implies that firms can never increase their outputs when moving from the private to the shared information equilibrium. The *ex-post* decrease in output that occurs when $\omega = (t_H, t_H)$ is not affected as long as $t_H \geq r\left(E\left[q^P\right]\right)$. Thus, both consumer surplus and social welfare decrease with t_H. Table 3.2 shows the equilibrium output and the corresponding surplus and welfare effects when we modify the example presented in Table 3.1 by defining $t_H = r\left(E\left[q^P\right]\right) = 9/5$.

Remark 4. *All results established in this section hold when we conduct the analysis in terms of t_L and keep the demand intercept constant. By lowering t_L, we increase the variance of μ. Theorem 4 implies that we can choose t_L small enough that firms have an incentive to share their information. This result is complementary to the results established by Maleug and Tsutsui (1998), who show that firms have an incentive to share their information if the variance of the common prior belief is sufficiently large.*

3.6 Results and Discussion

In the presence of uncertainty with respect to production capacities, equilibrium strategies are concave if capacities are stochastically independent. If firms are symmetric, a unique equilibrium exists. When inverse demand is linear, the best reply of a firm only depends on the expected output of the other firms, ensuring that every equilibrium is symmetric.

Consistent with the literature, we find that capacity constraints can reverse standard results on information sharing. These results are established by discussing a Cournot duopoly in which the common prior belief is discrete and there exist two capacity levels $t_L < t_H$ such that t_H is sufficiently large. Due to the concavity of equilibrium strategies, information sharing leads to an increase in the expected aggregate output of the industry. Moreover, the variance of each firm's output increases with the horizontal demand intercept a when information is shared. However, the variance of total industry might decrease when information is shared, which is due to the negative correlation of the firms' equilibrium outputs. The net effect can lead to an increase as well as to a decrease in producer surplus. The same is true for consumer surplus, which can decrease when information is shared although total output increases. However, social welfare increases when information is shared due to the sufficiently large value of t_H. This effect can be reversed by choosing t_H small enough.

The question as to whether antitrust authorities should either encourage firms to share information or if they should prohibit information exchange can not be answered clearly for two reasons. First, we needed to specify the weights an authority assigns to producer surplus and consumer surplus. In case an authority relies on social welfare as the appropriate measure, sharing information is beneficial for a large class of markets. In case an authority emphasizes consumer surplus, the question as to whether information should be shared depends on the market parameters.

One can think of a number of possible applications of the model. Consider, for example, two markets A and B, where market prices P_A and P_B are common knowledge. If the markets are physically separated, firms who possess transport capacity may take advantage of arbitrage profits. If we assume that the price difference $P := P_A - P_B$ is positive and decreasing in the quantity q bought on market B and sold on market A, we can perceive the problem as a Cournot oligopoly with capacity constraints. These capacity constraints may be unknown: Consider that A and B are two market places for natural gas that are connected via Liquefied Natural Gas (LNG) carriers. Since firms do not know their rival's operation strategies, they do not know the amount of carriers that are available to serve the route between A and B.

The model is limited to the case of stochastically independent capacities. However, the assumption on independence might not be reasonable in markets where the uncertainty is driven by a common source of risk. For example, local markets for agricultural products do not satisfy the assumption of independent signals, since the firms' harvest is determined by local weather conditions. However, independent capacities might be a suitable approximation.

4 On the Interaction Between Product Markets and Markets For Production Capacity: The Case of the Electricity Industry

We study the interdependency between two markets. In the first market, production capacity is offered; in the second, the produced commodity itself is sold. Selling capacity initially leads to foregone product market profits due to a lower output. These opportunity costs decrease with a firm's marginal costs. The key issue of the model is that there arises an additional cost component of selling capacity: Keeping capacity ready for delivery on demand induces ready-to-operate costs that increase with a firm's marginal costs.

It is shown that a competitive equilibrium not only exists, but is unique and efficient. In this equilibrium, the cumulative supply function of the capacity market is u-shaped, meaning that it is convex with respect to marginal costs. The leading example is the electricity industry, in which there is a capacity market that clears before the spot market is able to follow.

4.1 Introduction

Electricity markets are characterized by some properties that tend to complicate a matching of demand and supply. First, electricity is virtually nonstorable in large quantities from an economic perspective. Second, demand for and supply of electricity are not perfectly predictable. Third, supply has to equal demand at any time, since otherwise the electricity grid would collapse. Moreover, due to technical restrictions end-consumers cannot respond to real-time electricity prices, so demand for electricity is essentially inelastic in the short term (see, for example, Patrick and Wolak (2001)).

To ensure system stability, a network operator procures capacity to compensate for short-term prediction errors and to fill the gap between demand and supply.[1] If demand exceeds supply, capacity is called. The procurement of capacity is usually organized on a separate market platform.[2] Demand for capacity is defined by the transmission system operator to ensure a well-defined safety level regarding grid stability. In most European countries, a procurement auction is implemented in which the pricing mechanism can be uniform or pay-as-bid. The market for capacity clears before the spot market follows.

In the paper at hand, the analysis is conducted in terms of a day-ahead spot electricity market and a market for capacity.[3] We consider a continuum of firms that have different marginal costs of electricity generation.[4] Each firm has a fixed production capacity that can be split up allowing a firm to sell quantities on both markets. A technical restriction is imposed to ensure that if a firm wants to offer capacity electricity must be generated at a level greater than some minimum production level (the so-called "must-run" condition). A plant providing capacity must be running to guarantee a short response time when capacity is called. In the event of an unforeseen imbalance between demand and supply the plant's electricity generation can be increased quickly.

Selling capacity on the capacity market leads to foregone spot market profits due to a lower output of electricity. These opportunity costs are decreasing with a firm's marginal costs. The key issue of the model is the second cost component of selling capacity: Keeping capacity ready for de-

[1] In the electricity industry, capacity procured is called "incremental reserve", "incremental reserve capacity" or "positive balancing power".

[2] As is the case in Germany, for example.

[3] Here, it is important that the spot market is not a real-time market, since the time lag between gate closure and delivery necessitates a capacity market.

[4] Although it is generally not clear whether electricity markets are sufficiently competitive, the analysis tries to derive the competitive benchmark for the interaction between the two markets.

livery on demand induces ready-to-operate costs that are increasing with a firm's marginal costs since, with positive probability, the firm's marginal costs exceed the spot market clearing price.

The must-run condition leads to a strong interdependency between both markets, as quantities contracted on the capacity market induce quantities on the spot market. We see that in the setting sketched above, a unique equilibrium exists. The capacity market bidding function is u-shaped in this equilibrium, which stems from the must-run condition. This shows that in equilibrium, the set of firms supplying capacity constitutes an interval. Moreover, a welfare analysis shows that the equilibrium is efficient.

There are several other markets where at least one of these cost components as sketched above occur. Costs of foregone foregone profits from production always arise when assets are rented; costs of capacity provision arise when keeping capacity ready to operate is costly. The electricity industry, while an important example for the problem sketched in this paper, is not the only example where both effects occur simultaneously. Another example is presented in the discussion at the end of the paper.

There is little, but growing, literature available on capacity procurement in the electricity sector. One important line of research is instigated by the fact that capacity auctions are interpreted as a multi-unit auction with interdependent private values.[5] For example, the theory is applied to electricity markets in Hortacsu and Puller (2008). Swider (2007) introduces a model in which the spot market is competitive and the capacity market is not. The prices on the capacity market are modeled as random variables that bidders anticipate. Creti and Fabra (2006) model a short-term capacity market. Optimal bidding strategies for market participants are derived under consideration of opportunity costs that arise from previous sales on domestic and foreign electricity markets. It is assumed that all firms have identical marginal costs. The authors derive equilibrium strategies for both a monopolistic and a competitive market structure.

[5] The firms' signals are interrelated since the opportunity cost consideration of every firm depends on the stochastic spot market demand.

Closely related to the present paper is the work of Just and Weber (2008) and Just (2011). These papers, in turn, rely partly on Chao and Wilson (2002), who investigate optimal scoring rules on multi-dimensional procurement auctions for power reserves. Just and Weber (2008) model the interdependencies between markets for secondary reserve capacity and spot electricity to derive the price of capacity under equilibrium conditions in a uniform pricing setting. Just (2011) applies the same model setup and addresses questions on appropriate contract durations in the German market for reserve capacity. Both articles investigate the model numerically. The present work provides analytical results for a specific type of the stylized model developed by Just and Weber (2008), proving that a unique efficient competitive equilibrium exists.

The remainder of this paper is structured as follows. In Section 4.2, the model is explained and the equilibrium concept is introduced. Some properties of the model are derived, which allows for the model's strategy space to be narrowed down. In Section 4.3, the existence and uniqueness of an equilibrium is proved. Section 4.4 provides a welfare analysis that shows that the previously derived equilibrium is efficient. Finally, Section 4.5 summarizes the results.

4.2 The Model

The supply side is given by a continuum $X = [0, 1]$ of firms that have constant and different marginal costs. Firms are sorted by their marginal costs, so that the market's marginal cost curve $c : X \to \mathbb{R}_+$ is strictly increasing. For the sake of analytical convenience, c is assumed to be differentiable. Moreover, c is common knowledge.

The production capacity of every firm $x \in X$ equals one. Each firm bids quantities on both the spot and capacity market. First, capacity market bidding takes place. The result of the capacity market auction is revealed before the spot market bidding takes place. In the second step, the spot market clears. Some of the overall generation capacity is then no longer available, since it has been contracted on the capacity market.

We assume every firm x bids some price $b(x)$ on the capacity market and marginal costs $c(x)$ on the spot market. The share offered by every firm on the capacity market is fixed and given by $\alpha \in (0,1)$.[6] Thus, strategies based on the spot market only are excluded. This is not a limitation since every firm x may choose an arbitrarily high $b(x)$. An independent system operator (ISO) ensures that demand is met cost-efficiently, meaning the ISO selects the lowest bids on both markets.

Lastly, we reconsider the timing of the model as sketched above:

1. Each firm x bids some value $b(x)$ based on opportunity costs from spot market participation on the capacity market;

2. An ISO selects the lowest bids. Every firm is informed about the outcome of the capacity market;

3. Firms offer their remaining production capacity on the day-ahead spot market according to their marginal costs $c(x)$;

4. The day ahead spot market clears. (Whether procured capacity has to be called due to a gap in demand and supply during the short term is determined the following day and is not part of the model.)

4.2.1 Strategy Space and Payoff Function

The ISO ensures that demand for capacity is met cost-efficiently by selecting the lowest bids. This can be formalized by defining an *allocation*, which is an integrable function

$$s : [0,1] \longrightarrow \{0,1\}$$

[6]This share is determined by a power plant's minimum and maximum production level as well as the power plant's gradient (see Müsgens et al. (2011) for details). For simplicity we let α be the same for every firm. Typically, $\alpha \approx 0.1$ (see for example Stoft (2002), p. 307).

satisfying

$$\int_0^1 s(y)dy = D_c, \tag{4.1}$$

meaning that demand for capacity is met.

We choose a strategy profile $b : [0,1] \mapsto \mathbb{R}$ arbitrarily. We seek to consider an allocation s_b that is consistent with b, meaning that it ensures cost efficiency. That is to say, s_b is implicitly defined by the following condition:

$$\text{If } \; b(x) < b(y) \; \text{ and } \; s_b(y) = 1, \text{ then } s_b(x) = 1. \tag{4.2}$$

Therefore, we define the strategy space to be

$$\left\{ b : [0,1] \to \mathbb{R}_+ \mid \text{there exists a unique measurable } s_b \right\}.$$

Thus, the ISO provides the well-defined mapping $b \mapsto s_b$. As shown in the analysis, this definition of the strategy space does not exclude relevant strategy profiles.

The strategy profile b and the corresponding allocation s_b are interdependent. In order to be able to solve the model some results for an arbitrarily chosen s satisfying (4.1) but not necessarily (4.2) are provided. These results hold for all b and all s and thus, in particular, for a consistent pair (b, s_b). From now on, s is arbitrarily chosen, but fixed.

Firms selected to provide capacity are rewarded by the marginal bid, implying that the capacity market auction is uniform pricing.[7] For a given b and s, we denote by

$$\hat{b} := \inf_{x \in X} \{ b(x) | \, s(x) = 0 \}$$

the marginal bid. We do not indicate the dependency of $\hat{b}$ on s to ease notation.

[7] Since the marginal cost curve c is common knowledge, a pay-as-bid auction mechanism leads to the same market outcome (see Müsgens et al. (2011)). All results on existence, uniqueness and efficiency of equilibria translate to the pay-as-bid case.

An allocation s transforms the cumulative supply curve on the spot market for two reasons: First, we assume that demand for electricity on the spot market is high. Then, the firms providing capacity may happen to be inframarginal but can only generate electricity at the level $1-\alpha$. This leads to a higher price on the spot market compared to the case where no market for capacity is considered.

Second, we impose the technical restriction that a power plant needs to operate at a level of β in order to be able to provide capacity. Otherwise, a power plant cannot respond fast enough when capacity is actually called. If a power plant is providing capacity, then the cost of generating β are sunk, which implies that the firm bids the share β at a price of zero on the spot market. This leads to lower prices on the spot market when demand is low. As previously mentioned, we call this technical restriction the "must-run" condition.

In order to cover demand D_c for capacity, the accumulated must-run production amounts to $q_1 = \beta D_c/\alpha$, meaning that q_1 is a technical lower bound for the overall electricity generation in the model. Furthermore, the maximum electricity production in the market is given by $q_2 = 1-D_c$. We assume that the model parameters are such that $q_2 \geq q_1$. We define $Q = [q_1, q_2]$ and denote by D_e the random spot market demand, where the support of D_e equals Q.[8] We assume that D_e is distributed with respect to some probability measure P, and denote by E the expectation operator with respect to P. For a given s we seek to define the corresponding spot market price function $p_s : Q \mapsto [0,\infty)$ that maps quantities to prices and that is consistent with the following assumptions:

- A firm x that does not provide capacity bids all of its capacity at marginal costs into the spot market,
- A firm providing capacity is committed to bid its must-run share β at a price of zero into the spot market,

[8] We may also allow for the support of D_e to be an interval that is a subset of Q. All results remain, but the proofs become cumbersome.

- A firm bids the remaining share $1-\alpha-\beta$, which is assumed to strictly exceed zero, at marginal costs in the spot market.

For a given allocation s, we let $m_s(x)$ denote the aggregate amount of electricity firms bid into the market at a price not exceeding $c(x)$. Then m_s is the spot market's inverse cumulative supply curve resulting from s and following the restrictions described above. The function m_s can be written as

$$\begin{aligned} m_s : [0,1] &\longrightarrow Q, \\ m_s(x) &= q_1 + \int_0^x 1 - s(y)(\alpha+\beta)dy. \end{aligned} \tag{4.3}$$

Thus, the integrand equals 1 if and only if $s(y)=0$, meaning that firm y bids all of its capacity into the spot market. If $s(y)=1$, firm y bids only the share $1-\alpha-\beta$ at marginal costs, whereas the share β is bid at a price of zero and incorporated into q_1. Since q_1 is bid into the market at a price of zero $m_s(0)=q_1$ holds.

Note that the inverse supply curve m_s is continuous and invertible as long as $\alpha+\beta<1$. The spot market clearing price is given by

$$p_s(D_e) = c \circ m_s^{-1}(D_e). \tag{4.4}$$

Notice that if $D_c = 0$, then m_s is the identity and $p_s(D_e) = c(D_e)$. Figure 4.1 shows how the supply curve is transformed via s: Since the must-run capacity q_1 is bid into the spot market at a price of zero, electricity prices decrease when demand is low compared to the original supply curve c. Prices increase accordingly when demand is high, since firms with low marginal costs provide capacity and thus have to reduce electricity generation.

After having defined the spot market price function, we can express the expected payoffs of a firm. First, the relevant costs and profits are discussed. The must-run costs of a firm x are given by the expected difference of marginal costs and spot market prices, multiplied by the minimum load factor β:

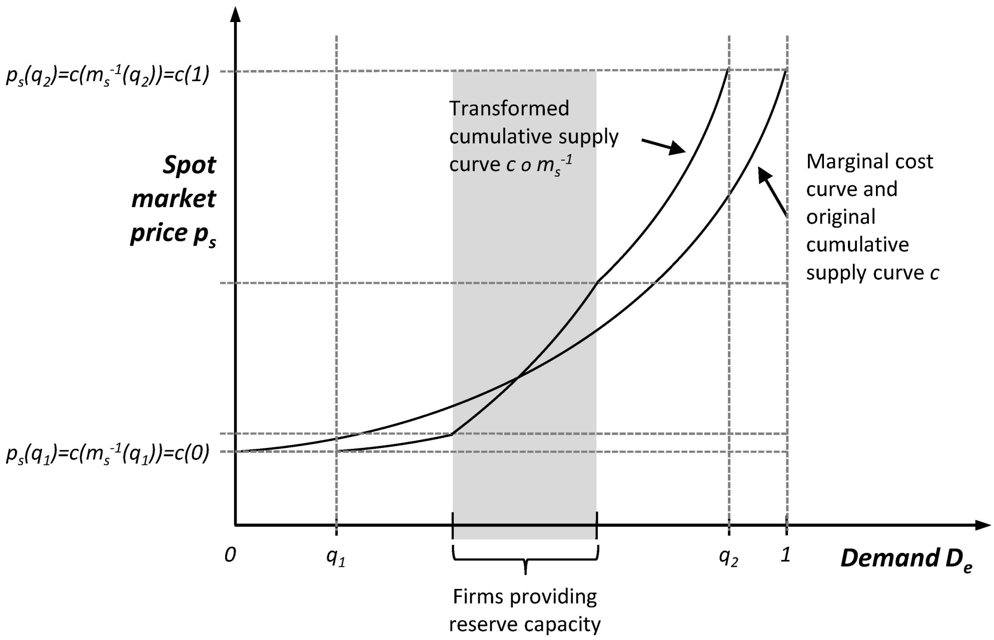

Figure 4.1: The transformation of the cumulative supply curve via s.

$$\beta E\left[\left(c(x)-p_s(D_e)\right)\mathbb{1}_{\{c(x)\geq p_s(D_e)\}}\right]=\beta E\left[\left(c(x)-p_s(D_e)\right)^+\right] \tag{4.5}$$

Thus, when providing capacity, firm x produces at least β due to the must-run property and sells this share on the spot market. This may lead to losses if the spot market price p_s exceeds the marginal costs of firm x. Expression (4.5) formalizes the expected value of these losses.

The spot market profits are given by the expected difference between spot market price and marginal costs. If this difference is positive, it is multiplied by the remaining share $1-\alpha$ that is not contracted on the capacity market and thus can be offered on the spot market. Expression (4.6) formalizes the expected profits from spot market participation:

$$(1-\alpha)E\left[\left(p_s(D_e)-c(x)\right)\mathbb{1}_{\{p_s(D_e)\geq c(x)\}}\right]=(1-\alpha)E\left[\left(p_s(D_e)-c(x)\right)^+\right]. \tag{4.6}$$

The function u expresses the profits of a risk neutral firm for fixed b and s and equals the sum of expected profits on both markets minus the expected costs of keeping the plant running. If a firm does not provide capacity, meaning that $b(x) > \hat{b}$, then u reduces to the expected spot market profits (the second case below):

$$u(x,b,s) :=$$
$$\begin{cases} \alpha\hat{b}+(1-\alpha)E\left[\left(p_s(D_e)-c(x)\right)^+\right]-\beta E\left[\left(c(x)-p_s(D_e)\right)^+\right], \text{ if } b(x)\leq\hat{b}, \\ E\left[\left(p_s(D_e)-c(x)\right)^+\right] \text{ otherwise.} \end{cases} \tag{4.7}$$

Here, $\alpha\hat{b}$ equals the profit from capacity market participation.

Since the allocation s is arbitrarily chosen and does not ensure that demand for capacity D_c is met cost-efficiently, u is not the payoff function but rather a helping function. The payoff function $\tilde{u}$ is then given by

$$\tilde{u}(x,b) := u(x,b,s_b). \tag{4.8}$$

An equilibrium is a strategy profile b if for any x and any $\tilde{b}$ satisfying $\tilde{b}(y) = b(y)$ as long as $x \neq y$ it holds true that $\tilde{u}(x,b) \geq \tilde{u}(x,\tilde{b})$.

4.2.2 Firms Bid Opportunity Costs

In this section, it is shown that we can restrict the analysis to an opportunity cost curve arising from expected gains and losses from spot market bidding. The basic argument is that given complete information, every firm bids its costs.

We see that the opportunity cost curve is u-shaped, which implies that those firms providing capacity constitute an interval in X in every equilibrium. This allows us to solve the interdependency of b and s_b. We now define b in a way ensuring that the marginal firm is exactly compensated for the expected foregone spot market profits. From now on, we explicitly indicate the dependency of b on s:

$$b(x,s) := E\left[\left(p_s(D_e) - c(x)\right)^+\right] + \frac{\beta}{\alpha} E\left[\left(c(x) - p_s(D_e)\right)^+\right]. \tag{4.9}$$

Note that $b(x,s)$ does not depend on the other firm's bids. Finding an equilibrium now reduces to finding the consistent allocation s, meaning that s must be the cost-efficient procurement of D_c if firms bid according to $b(\cdot,s)$.[9]

Recall that $\hat{b}$ denotes the marginal bid. If x places the highest accepted bid, if follows that $b(x,s) = \hat{b}$ and thus

$$u(x,b,s) = E\left[\left(p_s(D_e) - c(x)\right)^+\right], \tag{4.10}$$

which equals the expected profits generated by a spot market only strategy, implying that the marginal firm is indifferent between both markets. If firm x places a bid that is not accepted, the firm again generates profits at the amount of

$$u(x,b,s) = E\left[\left(p_s(D_e) - c(x)\right)^+\right]. \tag{4.11}$$

Any other firm places a bid that is lower than $\hat{b}$ and thus generates higher profits. The next proposition shows that every equilibrium b can be represented by a function of the form (4.9).

Proposition 1. *If a is an equilibrium strategy profile, then $b(\cdot,s_a)$ is also an equilibrium, and $s_a = s_b$ as well as $\hat{a} = \hat{b}$.*

Proof. We arbitrarily choose an equilibrium a and show that $b(x,s_a) \leq \hat{a}$ if and only if $a(x) \leq \hat{a}$: Choose $x \in X$ so that $s_a(x) = 1$ and $a(x) \neq b(x,s_a)$. Since a is an equilibrium, we must have $b(x,s_a) \leq \hat{a}$, since $b(x,s_a) > \hat{a}$

[9]Equivalently speaking, finding an equilibrium reduces to finding a fixed point of the mapping $s \mapsto s_{b(\cdot,s)}$.

implies $u(x,a,s) < b(x,s_a)$, which is impossible since a is an equilibrium. This implies $s_a = s_b$ and the statement follows. □

Thus, the equilibrium $b(\cdot,s_b)$ is equivalent to b, meaning that the market result does not change when moving from b to $b(\cdot,s_b)$. The basic intuition behind this result is that firms bid their costs in a uniform pricing auction if the industry's cost structure is common knowledge. Note that the proof does not rely on the continuity of c.

As explained above, the first summand of b describes the foregone spot market profits a firm faces when selling capacity. This cost component is decreasing with a firm's marginal costs. The second summand describes the costs of standby while offering capacity; these costs are increasing with a firm's marginal costs. Unsurprisingly, the sum of both cost components is a convex function, as Theorem 1 implies:

Theorem 1. *The opportunity cost function $b(\cdot,s)$ is continuous and u-shaped, meaning that $b(\cdot,s)$ there exists $\check{x}$ such that $b(\cdot,s)$ is strictly increasing for values smaller than $\check{x}$ and increasing for values exceeding $\check{x}$. The minimizer $\check{x}$ is defined by*

$$P\left(D_e \leq m_s(\check{x})\right) = \frac{\alpha}{\alpha+\beta}. \tag{4.12}$$

Proof. Since $b(\cdot,s)$ is an integral of a bounded function and since the marginal cost curve c is differentiable, $b(\cdot,s)$ is continuous everywhere and differentiable almost everywhere. We let $\tilde{X}$ denote the set of points where $b(\cdot,s)$ is not differentiable. We arbitrarily choose $x \in X\backslash\tilde{X}$ and calculate:

$$\frac{d}{dx}b(x,s) = c'(x)\left[(1+\beta/\alpha)P\left(D_e \leq m_s(x)\right) - 1\right].$$

Recall that $c' > 0$ by assumption. The term on the right-hand side is increasing with x and equals zero if and only if $P(D_e \leq m(x)) = (1+\beta/\alpha)^{-1}$. We denote by F the distribution function of D_e. Then F is invertible on $[0,1]$ since $f(x) > 0$ for all $x \in Q$. We define

$$\check{x} := m_s^{-1}\left(F^{-1}\left(\frac{1}{1+\beta/\alpha}\right)\right).$$

Since $b(\cdot,s)$ is not differentiable everywhere, it remains to be shown that $b(\cdot,s)$ is strictly decreasing on $[0,\check{x}]$ and strictly increasing on $[\check{x},1]$. We define

$$b'(x,s) := 0 \quad \forall x \in \tilde{X}.$$

As it is an antiderivative of a function that is integrable with respect to the Lebesgue measure, $b(\cdot,s)$ is absolutely continuous. Thus, we may express $b(\cdot,s)$ as

$$b(x,s) = b(0,s) + \int_0^x b'(t,s)dt.$$

If $x, y \in [0,\check{x}]$ and $x < y$, we conclude

$$b(y,s) - b(x,s) = \int_x^y b'(t,s)dt < 0.$$

A similar argument shows that $b(\cdot,s)$ is strictly increasing on $[\check{x},1]$. □

Theorem 1 states that firms at the boundary of X have high opportunity costs when bidding on the capacity market. For $x = 0$, expected losses from not bidding on the spot market are high, since the marginal costs are low. Moreover, the must-run costs equal zero. Conversely, $x = 1$ has high must-run costs due to high marginal costs, but the expected gains from spot market bidding are zero. If a firm's marginal costs are close to the expected spot price, the firm places a relatively low bid.

4.3 Existence and Uniqueness of an Equilibrium

The following Corollary 1 states that in every equilibrium the set of firms providing capacity is an interval in X, due to the shape of b. This result is the key to the solution procedure: It allows us to establish a one-to-one correspondence between X and the set of all allocations s that can eventually arise in an equilibrium.

Corollary 1. *In every equilibrium the set of firms providing capacity is an interval.*

Proof. The statement follows from the shape of $b(\cdot, s)$. □

We define $h := D_c/\alpha$ to ease notation. Thus, h is the length of the interval of firms providing capacity. By a slight abuse of notation we define s_x by

$$s_x(y) = 1 \text{ if and only if } y \in [x, x+h] .$$

Thus, the allocation s_x selects all firms located in $[x, x+h]$. One should keep in mind that from now on, x always equals the left boundary of this interval. According to the previous notation, the corresponding inverse supply curve should be denoted by m_{s_x}, but by a slight abuse of notation we identify m_{s_x} with m_x.

The strategy to prove the existence of equilibria relies on the observation that by restricting the shape of s to the functional form defined above we have established a mapping

$$x \mapsto b(\cdot, s_x)$$

that maps $[0, 1-h]$ bijective to the set of strategies in which every equilibrium must be located, as given by Corollary 1. We analyze the function g defined by

$$\begin{aligned} g : &[0, 1-h] \to \mathbb{R}, \\ &x \mapsto b(x+h, s_x) - b(x, s_x). \end{aligned}$$

Figure 4.2 provides the connection between g and the equilibrium solutions. The horizontal axis shows the continuum of firms. The interval $[x, x+h]$ contains those firms selected via s_x to provide capacity.

We consider the case where g has a zero. This corresponds to Fig. 4.2 B,

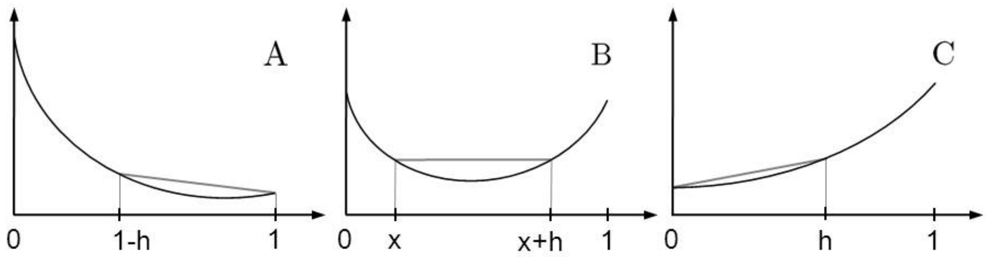

Figure 4.2: The three different possible types of equilibria.

where

$$b(x,s_x) = b(x+h,s_x)$$

holds true, and where every firm providing capacity is located in the inner of X. An obvious condition for g to possess a zero is if for the allocation s_0, the interval $[0,h]$ of firms providing capacity is located on the left-hand side of the minimum of $b(\cdot,s_0)$ and for the allocation s_{1-h}, the interval is located on the right-hand side of the minimum of $b(\cdot,s_{1-h})$. Since b is u-shaped, it follows that $g(0) < 0$ and $g(1-h) > 0$; and since g is continuous, a zero exists. The next proposition proves the existence of a zero under these two conditions mentioned above.

In the cases (A) and (C), the underlying model parameters are specified in a way that there does not exist an equilibrium in which $b(x,s_x) = b(x+h,s_x)$ holds, meaning g does not have a zero. In this case, it must be $g > 0$ or $g < 0$ everywhere, since g is continuous. If $g < 0$, we define $x = 1-h$, which corresponds to Fig. 4.2 A. If $g > 0$, we define $x = 0$, which corresponds to Fig. 4.2 C. Although it may not be apparent, we find that (A) and (C) constitute equilibria. In the remaining analysis, the equilibrium pictured in Fig. 4.2 B is sometimes referred to as an *inner equilibrium* or *inner solution*, since x lies in the inner of $[0,1-h]$. The next proposition establishes some properties of g and provides sufficient conditions for g to have a zero. This is intuitively the case when the ratio D_c/Q is sufficiently small and β is sufficiently large. Since this is typically the case in markets for capacity ($D_c/Q \leq 0.03, \beta \geq 0.3$), the inner equilibrium, as pictured in Fig. 4.2 B, can be seen as the typical equilibrium.

Proposition 2. *The function g has at most one zero. If g has a zero x_0, then g is strictly increasing in a neighborhood of x_0. A sufficient condition for g to have a zero is given by*

$$P\left(D_e \leq D_c\left(\frac{1}{\alpha}-1\right)\right) \leq \frac{\alpha}{\alpha+\beta}$$

and

$$P\left(D_e \leq 1-D_c\left(\frac{1-\beta}{\alpha}\right)\right) \geq \frac{\alpha}{\alpha+\beta}.$$

Proof. To see that g has a zero under the assumptions of Proposition 2, we show that $g(0) < 0$ and that $g(1-h) > 0$ holds true. Then the statement follows since g is continuous.

In order to prove $g(0) < 0$ we show that

$$h \leq \arg\min_{x\in X} b(x,s_0) =: \check{x},$$

implying that for the allocation s_0 the minimum $\check{x}$ of the corresponding bidding function $b(\cdot,s_0)$ is located on the right-hand side of the interval $[0,h]$, which is sufficient, since according to Theorem 1 $b(\cdot,s_0)$ is strictly decreasing on $[0,\check{x}]$.

A calculation shows that we have $m_0(h) = h - D_c$. Theorem 1 shows that

$$P\left(D_e \leq m_0(\check{x})\right) = (1+\beta/\alpha)^{-1}.$$

Since the mapping $x \mapsto P\left(D_e \leq m_0(x)\right)$ is strictly increasing with x, it is sufficient to show that

$$P\left(D_e \leq m_0(h)\right) \leq \frac{1}{1+\beta/\alpha},$$

which follows from the assumptions:

$$P\left(D_e \leq m_0(h)\right) = P\left(D_e \leq \frac{D_c}{\alpha} - D_c\right) \leq \frac{1}{1+\beta/\alpha}$$

The proof that $g(1-h) > 0$ holds is a similar calculation. We have to show that

$$1 - h \geq \arg\min_{x \in X} b(x, s_{1-h}) := \check{x},$$

meaning that for the allocation s_{1-h} the minimum of the corresponding bidding function is located on the left-hand side of the interval $[1-h, 1]$. It is sufficient to show that

$$P\left(D_e \leq m_{1-h}(1-h)\right) \geq \frac{1}{1+\beta/\alpha},$$

which follows again from the assumptions:

$$\begin{aligned} P\left(D_e \leq m_{1-h}(1-h)\right) &= P\left(D_e \leq q_1 + 1 - h\right) \\ &= P\left(D_e \leq 1 - D_c\left(\frac{1-\beta}{\alpha}\right)\right) \\ &\geq \frac{1}{1+\beta/\alpha}. \end{aligned}$$

Next, we see that we can find values $x_1, x_2 \in [0, 1-h]$ so that $g(x) < 0$ if $x \leq x_1$, $g(x) > 0$ if $x \geq x_2$ and so that g is strictly increasing on $[x_1, x_2]$. This is sufficient to prove the proposition. Note first that Theorem 1 implies that if the range $[x, x+h]$ of firms providing capacity moves to the right, then the minimum of $b(\cdot, s_x)$ moves to the left because the mapping $x \mapsto m_x(\cdot)$ is increasing with x. Therefore, under the assumptions of the proposition there exists a value x_1 so that the right edge of the interval $[x_1, x_1+h]$ and the minimum of the corresponding bidding function $b(\cdot, s_{x_1})$ coincide, meaning that $x_1 + h$ minimizes $b(\cdot, s_{x_1})$. Similarly, there exists x_2 so that x_2 minimizes $b(\cdot, s_{x_2})$.

The u-shape of the bidding function and the fact that $g(0) < 0$ imply that $g(x) < 0$ if $x \leq x_1$. Similarly, $g(1-h) > 0$ implies that $g(x) > 0$ if $x \geq x_2$. It remains to be shown that g is strictly increasing on $[x_1, x_2]$. We choose x and y satisfying $x_1 < x < y < x_2$ and show that $g(y) - g(x) > 0$. This expression can be written as

$$\begin{aligned} g(y) - g(x) &= b(y+h, s_y) - b(y, s_y) - (b(x+h, s_x) - b(x, s_x)) \\ &= \underbrace{b(y+h, s_y) - b(x+h, s_x)}_{(I)} + \underbrace{b(x, s_x) - b(y, s_y)}_{(II)}. \end{aligned}$$

First, we look at expression (II):

$$b(x, s_x) - b(y, s_y) = \underbrace{b(x, s_x) - b(x, s_y)}_{(A)} + \underbrace{b(x, s_y) - b(y, s_y)}_{(B)}$$

Expression (B) strictly exceeds zero because the function $b(\cdot, s_y)$ is u-shaped and the function's minimum strictly exceeds y by construction of $[x_1, x_2]$. It remains to be shown that expression (A) is non-negative. To prove this, we choose $z < x < y$ and consider the difference $b(z, s_x) - b(z, s_y)$. We show that this difference is non-negative and since the difference is continuous in z, the limit $z \to x$ is non-negative as well. The key to this result is the observation that the must-run costs of firm z are equal for both the allocation s_x and s_y because must-run costs only occur for z when a firm $\tilde{z} < z$ happens to be the marginal firm on the spot market. Since $\tilde{z} < z < x < y$, the must-run costs of z are not affected when moving from s_x to s_y. Furthermore, the foregone spot market profits for z decrease when the allocation moves from s_x to s_y because the spot market price (weakly) decreases. More formally (note that $m_x \leq m_y$), it can be outlined:

$$\begin{aligned} & b(z, s_x) - b(z, s_y) \\ =& E\left[(p_x(D_e) - c(z))^+\right] - E\left[(p_y(D_e) - c(z))^+\right] \\ =& \int_{m_x(z)}^{q_2} (c(m_x^{-1}(t)) - c(z)) f(t) dt - \int_{m_y(z)}^{q_2} (c(m_y^{-1}(t)) - c(z)) f(t) dt \\ \geq& \int_{m_x(z)}^{q_2} (c(m_x^{-1}(t)) - c(z)) f(t) dt - \int_{m_x(z)}^{q_2} (c(m_y^{-1}(t)) - c(z)) f(t) dt \\ \geq& \int_{m_x(z)}^{q_2} c(m_x^{-1}(t)) f(t) dt - \int_{m_x(z)}^{q_2} c(m_x^{-1}(t)) f(t) dt = 0. \end{aligned}$$

It remains to be shown that (I) is non-negative. The proof is similar to the proof that (II) exceeds zero: By construction, $x+h$ and $y+h$ are located on the right-hand side of the minimum of $b(\cdot, s_x)$ so that we can take advantage of the u-shape of $b(\cdot, s_x)$. Moreover, the foregone spot market profits of a firm $z > y+h$ are not affected when the allocation moves from s_x to s_y in analogy to the situation above. The details are omitted. □

Inequality (2) in Proposition 2 ensures that $g(0) < 0$ and inequality (2) leads to $g(1-h) > 0$. Since g is continuous, if follows that g has a zero.

Note that if $\beta = 0$, no costs of keeping the plant running arise, so that the first equality always holds true. Contrarily, since $\alpha < 1$, the second inequality does not hold true for any configuration of the model parameters as long as $\beta = 0$. This is consistent with the fact that b is strictly decreasing if $\beta = 0$, which is easily demonstrated. The next theorem is an immediate consequence of the proposition above.

Theorem 2. *A unique equilibrium exists.*

Proof. We split the existence proof into three parts.

First, we assume g has a zero x_0. It is apparent that $b(\cdot, s_{x_0})$ is an equilibrium in this case.

Second, we consider the case in which $g > 0$ everywhere. The proof of Proposition 2 shows that

$$P\left(D_e \leq m_0(h)\right) = P\left(D_e \leq D_c\left(\frac{1}{\alpha} - 1\right)\right) > \frac{\alpha}{\alpha + \beta}. \tag{4.13}$$

The first equation is a calculation. We observe that $b(\cdot, s_0)$ is an equilibrium, as pictured in Fig. 4.2 C: Combining expressions (4.12) and (4.13), we conclude that the minimum of $b(\cdot, s_0)$ is located in $[0, h]$, since the mapping $x \mapsto P\left(D_e \leq m_0(x)\right)$ is strictly increasing with x. Since $b(\cdot, s_0)$ is strictly increasing on $[h, 1]$, condition (4.2) is satisfied.

Third, we assume $g < 0$ everywhere. Then $b(\cdot, s_{1-h})$ is an equilibrium as pictured in Fig. 4.2 A. We argue by similar considerations as in the second case that $\min_X b(\cdot, s_{1-h}) \in [1-h, 1]$, which yields the given statement.

As in the existence proof, we examine three cases in order to prove uniqueness. First we assume $g > 0$ and that there exists $x > 0$ so that $b(\cdot, s_x)$ is an equilibrium. We denote by $\check{x}$ the minimizer of $b(\cdot, s_x)$. It follows that $\check{x} \in [x, x+h]$. Since $g > 0$, it holds true that $b(x, s_x) < b(x+h, s_x)$. Since $b(\cdot, s_x)$ is continuous and strictly decreasing on $[0, x]$, we may choose $y \in [0, x]$ so that $b(y, s_x) < b(x+h, s_x)$. But it also holds true that $s_x(x+h) = 1$, $s_x(y) = 0$, which is a contradiction to the cost-efficiency of s_x.

The second case in which $g < 0$ is similar to the first and is omitted.

Third, if there exists x_0 so that $g(x_0) = 0$, then x_0 is unique, which follows from Proposition 2.[10] We conclude that in this case, there exists exactly one equilibrium of the form pictured in Fig. 4.2 B. Moreover, $b(\cdot, s_0)$ and $b(\cdot, s_{1-h})$ also do not constitute equilibria since, according to Proposition 2, it holds true that $\min_X b(\cdot, s_0) \notin [0, h]$ and $\min_X b(\cdot, s_{1-h}) \notin [1-h, 1]$. Lastly, for any $x \in X$ satisfying $x \neq 0, x \neq 1-h$ and $g(x) \neq 0$, $b(\cdot, s_x)$ does not constitute an equilibrium by the arguments of the first case. $\square$

4.4 Welfare Analysis

Although the efficiency of the equilibrium may be derived from the first welfare theorem, I give an explicit proof. The aim is to show in an instructive manner that the cost minimizing problem of a central planner and the efficient equilibrium achieved by the market are equivalent.[11]

If the market equilibrium is attained, the cost-efficient firms that provide capacity are selected. As demonstrated in the analysis, a firm's costs are opportunity costs arising from foregone spot market profits and ready-to-operate costs. If we consider a central planner determining the allocation on both the capacity and the spot market, the goal is to minimize the expected

[10] If we allow for the support of D_e to be an interval that is a subset of Q, then g is no longer increasing but rather nondecreasing. However, if $[x, x+h] \cap supp(D_e)$ is empty, then $g(x) \neq 0$ because then $b(x, s_x) - b(x+h, s_x)$ has a very simple form and equals either $c(x+h) - c(x)$ or $c(x) - c(x+h)$. If $[x, x+h] \cap supp(D_e)$ is non-empty, then $g'(x) > 0$. Thus, the null of g remains unique.

[11] See also Müsgens et al. (2011) for a similar discussion.

costs of electricity generation by choosing an optimal interval of firms providing capacity.

We have to show that minimizing the firms' cumulative opportunity costs of capacity provision leads to the same allocation as minimizing the expected costs of electricity generation by the central planner. This implies that opportunity costs of capacity market bidding must be accompanied by real costs, meaning by an expected consumption of resources on the spot market.

To provide intuition, we discuss the issue in a heuristic manner. Since demand for capacity is inelastic, an efficient supply allocation is sufficient for efficiency. A supply allocation, in turn, is efficient if it minimizes overall costs. For the moment, we let demand D_e for electricity on the spot market be constant and be equal to some value d.

Any efficient allocation a central planner establishes is given by an interval $[x, x+h]$. If we denote by $\tilde{d} := m_x^{-1}(d)$ the firm that is price setting on the spot market, we must have $\tilde{d} \in [x, x+h]$. This is easily demonstrated: If the interval is located on the left-hand side of $\tilde{d}$, then there exists a firm $y \geq x+h$ that is inframarginal, meaning that $c(y) < c(\tilde{d})$. Clearly, we can reduce costs by shifting the interval to the right, so that y then provides capacity and a firm that has lower marginal costs compared to y can solely produce electricity. Conversely, $x \leq \tilde{d}$ is necessary condition for cost efficiency: If $\tilde{d} < x$, there exists y such that $c(\tilde{d}) < c(y) < c(x)$, which immediately implies that shifting the interval to the left does not change the spot market price but reduces must-run costs. Thus, shifting to the left decreases expected costs of electricity generation in this case.

Therefore, we assume that $\tilde{d} \in [x, x+h]$. In order to minimize total costs, the central planner has to minimize the marginal increase of costs when shifting the interval $[x, x+h]$ to the right. Next, we derive this marginal effect.

Note first that shifting to the right implies that x does not provide capacity anymore and can increase production by the share α. At the margin, this leads to negative additional costs of electricity generation that equal

$$\alpha[c(x) - c(\tilde{d})].$$

In turn, additional must-run costs emerge that equal

$$\beta c(x+h),$$

which reduce costs by

$$\beta c(\tilde{d}).$$

We denote by γ the total costs of electricity generation the central planner seeks to minimize. Shifting to the right changes costs according to

$$\begin{aligned}\gamma'(x) :=& \beta c(x+h) - \beta c(\tilde{d}) + \alpha[c(x) - c(\tilde{d})] \\ =& \beta[c(x+h) - p_x(d)] - \alpha[p_x(d) - c(x)] \\ =& \alpha g(x).\end{aligned} \tag{4.14}$$

Thus, the marginal effect of shifting the interval corresponds to the function g we previously analyzed – g is defined as the difference between the capacity market bids placed by $x+h$ and x. In the case of deterministic demand discussed here, g has a simple form: Firm x only faces costs of foregone spot market profits, whereas firm $x+h$ only faces ready-to-operate costs.

Recall that if s_x is the allocation of an inner equilibrium, then $g(x) = 0$. This implies $\gamma'(x) = 0$, and we conclude that x minimizes γ. This is because g and thus γ' are strictly increasing in a neighborhood of x as previously derived.

For the general case, we consider the expected costs of electricity production for a given s_x, which equal

$$\gamma(x) := E\left[\int_{q_1}^{D_e} p_x(q)dq\right] + \beta\int_x^{x+h} c(q)dq. \tag{4.15}$$

The first summand describes the the expected costs of generating electricity with respect to stochastic demand D_e and with respect to the allocation s_x. The transformation of the aggregated supply curve via s_x is incorporated in p_x. The second summand describes the costs that arise from generating q_1 units of electricity due to the must-run condition, which equal the integrated costs multiplied by β over the set $[x, x+h]$. The next proposition shows that equation (4.14) holds in the general case.

Proposition 3. *The overall cost function γ satisfies $\gamma' = \alpha g$.*

Proof. By applying Fubini's theorem to the first summand of (4.15) and then the transformation formula with transformation m_x we calculate (remember $m_x(0) = q_1$, $m_x(1) = q_2$ and the definition of m_x):

$$\begin{aligned}\gamma(x) &= \int_{q_1}^{q_2} p_x(q)P\left(q \leq D_e\right)dq + \beta\int_x^{x+h} c(y)dy \\ &= \int_0^1 c(y)\left(1 - s_x(y)(\alpha+\beta)\right)P\left(m_x(y) \leq D_e\right)dy + \beta\int_x^{x+h} c(y)dy.\end{aligned} \tag{4.16}$$

In order to be able to calculate the derivative of γ, we write (note that on the intervals $[0,x], [x,x+h], [x+h,1]$ the function s_x is constant and equals 0 or 1):

$$\gamma(x) = \int_0^x c(y) P\left(m_x(y) \le D_e\right) dy \tag{4.17}$$

$$+ \int_x^{x+h} c(y)(1-\alpha-\beta) P\left(m_x(y) \le D_e\right) dy \tag{4.18}$$

$$+ \int_{x+h}^{1} c(y)(1-\alpha-\beta) P\left(m_x(y) \le D_e\right) dy \tag{4.19}$$

$$+ \beta \int_x^{x+h} c(y) dy. \tag{4.20}$$

Notice that on $[0, x]$ and $[x+h, 1]$, the function $m_x(\cdot)$ does not depend on x, which makes it easy to differentiate expressions (4.17) and (4.19) with respect to x. Expression (4.18) is differentiated by applying the multi-dimensional chain rule to the function $\tilde{g}(\phi(x))$, where

$$\tilde{g}(x, z) := \int_x^{x+h} c(y)(1-\alpha-\beta) P\left(m_z(y) \le D_e\right) dy$$
$$\phi(x) := (x, x).$$

We calculate:

$$\begin{aligned}
\gamma'(x) =& c(x) P\left(m_x(x) \le D_e\right) - c(x+h) P\left(m_x(x+h) \le D_e\right) \\
& + (1-\alpha-\beta)\left[c(x+h) P\left(m_x(x+h) \le D_e\right) - c(x) P\left(m_x(x) \le +D_e\right)\right] \\
& + \beta\left(c(x+h) - c(x)\right) - (\alpha+\beta) \int_{m_x(x)}^{m_x(x+h)} f(y) p_x(y) dy \\
=& c(x)\left[(\alpha+\beta) P\left(m_x(x) \le D_e\right) - \beta\right] \\
& - c(x+h)\left[(\alpha+\beta) P\left(m_x(x+h) \le D_e\right) - \beta\right] \qquad (4.21) \\
& - (\alpha+\beta) \int_{m_x(x)}^{m_x(x+h)} f(y) p_x(y) dy. \qquad (4.22)
\end{aligned}$$

Second, we derive $\alpha g(x)$:

$$
\begin{aligned}
\alpha g(x) =& \alpha \int_{m_x(x+h)}^{q_2} f(y)\left(p_x(y) - c(x+h)\right) dy \\
& + \beta \int_{q_1}^{m_x(x+h)} f(y)\left(c(x+h) - p_x(y)\right) dy \\
& - \alpha \int_{m_x(x)}^{q_2} f(y)\left(p_x(y) - c(x)\right) dy \\
& - \beta \int_{q_1}^{m_x(x)} f(y)\left(c(x) - p_x(y)\right) dy \\
=& c(x)\left[\alpha P\left(m_x(x) \leq D_e\right) - \beta P\left(D_e \leq m_x(x)\right)\right] \\
& - c(x+h)\left[\alpha P\left(m_x(x+h) \leq D_e\right) - \beta P\left(D_e \leq m_x(x+h)\right)\right] \\
& - (\alpha+\beta) \int_{m_x(x)}^{m_x(x+h)} f(y) p_x(y) dy \\
=& c(x)\left[(\alpha+\beta) P\left(m_x(x) \leq D_e\right) - \beta\right] \\
& - c(x+h)\left[(\alpha+\beta) P\left(m_x(x+h) \leq D_e\right) - \beta\right] \\
& - (\alpha+\beta) \int_{m_x(x)}^{m_x(x+h)} f(y) p_x(y) dy \\
=& \gamma'(x).
\end{aligned}
$$

□

The factor α arises because the opportunity costs $b(\cdot, s_x)$ are per-unit costs, whereas γ describes the overall costs of production. Theorem 3 is an immediate consequence of Proposition 3, but in addition covers the case where an inner equilibrium does not exist.

Theorem 3. *The equilibrium is efficient.*

Proof. This follows from the proposition above: If there exists an inner equilibrium and if s_x denotes the equilibrium allocation, then $\gamma'(x) = 0$. Since g is strictly increasing in a neighborhood of x according to Proposition 2,

x is a local minimum of γ. If g does not have a zero, then s_0 or s_{1-h} is the equilibrium allocation. Since $\gamma'(x) \neq 0$ for all x, γ is minimized by 0 or $1-h$. Since the range of firms providing capacity must contain the bidding function's minimum in an efficient solution, γ is minimized by 0 if and only if s_0 is the equilibrium allocation. □

4.5 Results and Discussion

We analyzed a stylized model that accounts for the main interdependencies between a spot electricity market and a capacity market. We have seen that the strategy space of the firms may be restricted to an opportunity cost function that is u-shaped. Opportunity costs arise from the alternative of spot market participation instead of providing capacity. These opportunity costs are decreasing with marginal costs. Additional costs of capacity provision arise from the technical requirement that power plants must be running while providing capacity, and these ready-to-operate costs are increasing with a firm's marginal costs.

An immediate consequence of this result is Corollary 1, which states that in every equilibrium the set of firms providing capacity is an interval. This ensures that a unique equilibrium exists. Moreover, the equilibrium is efficient, since the opportunity costs a firm faces when placing a bid on the capacity market become true costs in the case of electricity generation on the spot market.

In the model, firms differ only by their marginal costs. In reality, there is a large number of different power plants that exhibit very different technical and economical properties. For example, the share of capacity a power plant can offer on the capacity market depends on the specific technology; and some technologies can not even meet the technical requirements for providing capacity at all. Moreover, the minimum load condition varies extensively and may even be zero (in the case of a pumped storage power station).

As previously mentioned, the results developed may translate to other markets where there is demand for products as well as for production capacity. There are three essential characteristics the market must possess: (i) The firms differ with respect to their marginal profits per unit, (ii) the overall profit that a firm generates is increasing with product market demand and (iii) a firm faces ready-to-operate that are decreasing with a firm's marginal profit per unit.

We consider, for example, two different restaurants A and B. Restaurant A has a reputation, whereas restaurant B does not. Every other restaurant in town is located between A and B with regard to its reputation. All restaurants have an identical cost structure and provide service of equal quality. Due to restaurant A's reputation, prices in restaurant A are higher than in restaurant B. The same translates for the profit per (customer). This yields the case described in property (i). If we assume that the potential customers are equally distributed across the restaurants that are open we yield property (ii).[12]

We next consider a small group of businessmen who want to rent a dining room in one of these two restaurants for a meeting. We analyze the costs of renting the dining room to the businessmen that both restaurants may face on a day with average demand.

Since restaurant A generates the highest profit per customer, it is the last restaurant in town to be closed when demand decreases. In particular, it is open when demand is on average. Since the group of businessmen is sufficiently small, restaurant A effectively looses customers when renting the dining room to the businessmen. Thus, there arise opportunity costs from sending customers away.

Conversely, due to the relatively low number of guests, restaurant B is not able to recover labor costs that evening and thus it is closed. If the group of businessmen is sufficiently small, the costs for restaurant B to rent a dining room are driven by the labor costs that have to be recovered, that is to say,

[12] We assume that this price structure and distribution of customers constitute a short-term equilibrium.

by the costs of keeping capacity ready for delivery on demand. If restaurant C has two dining rooms, it operates the one that is not rented and thus generates a contribution margin to cover labor costs. Thus, a restaurant with low marginal profits per unit has high ready-to-operate costs, which yields property (iii).

If prices decrease from A to B, then a restaurant with intermediate prices offers the dining room at the lowest price. We let C denote this restaurant. Notice that the allocation of the regular customers to the restaurants is transformed when C rents a dining room to the businessmen. If C was open anyway, then renting the dining room decreases supply on the product market. However, if C was originally meant to be closed, then renting the dining room leads to an increase of supply on the product market, as long as C has at least two dining rooms.

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