ESSAYS ON COMPETITION POLICY

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To my family.

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CHAPTER 1

INTRODUCTION

Competition policy can be defined as "the set of policies and laws which ensure that competition in the marketplace is not restricted in such a way as to reduce economic welfare" (Motta, 2004, p. 30). In the last decades, there has been a paradigm shift in the approach to competition policy in most developed economies. Under the heading of a "more economic approach", the basis of decision-making in competition cases has changed from a form-based approach where decisions are made based on relatively simple per-se rules to an effects-based approach building on case-specific economic analyses.

Under a form-based approach, certain *forms* of firm conduct are considered detrimental to welfare per-se and are therefore prohibited. An effects-based approach is based on the insight that the impact of firm behavior on welfare depends on the particular circumstances of each case, that is, characteristics of the market in question such as the type of products and competition as well as the structure of demand have to be taken into account when assessing whether certain firm behavior is detrimental to welfare. The *effect* of firm conduct on welfare can therefore hardly be generalized in a set of simple rules applying to all industries and cases but has to be analyzed on a case-by-case basis (Röller and Stehmann, 2006).

The increased influence of economic arguments in competition policy can to a large extent be attributed to the application of game theoretic methods to the analysis of (imperfectly) competitive markets. One of the pioneers in this area, Jean Tirole, received the 2014 Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel for his work on market power and regulation. Because markets and firms are constantly changing and evolving, the policy shift also necessitates further economic analyses in order to strengthen and deepen the understanding of imperfectly competitive markets and in order to be able to better identify and understand the effects of firm conduct on economic welfare.

This thesis consists of self-contained analyses of three current topics in the field of competition policy in order to contribute to the understanding of firm behavior and in order to derive policy implications based on rigorous economic arguments. In Chapter 2, the impact of (a reduction of) complexity on welfare in competition cases is analyzed in a setting where asymmetric parties can acquire information to help their case and submit it to the decision-maker. In Chapter 3, the price effects of horizontal mergers as well as the incentives of firms to merge are analyzed in a bidding market. Finally, in Chapter 4, the impact of different pricing schemes on firms' incentives to collude is analyzed in a differentiated-products market. In the remainder of this introduction, the motivation of the three main chapters of the present thesis is reviewed and the main results are presented.

The model in the second chapter entitled "*Searching for evidence: less can be more*" is joint work with David Kusterer and Achim Wambach. One consequence of the shift toward an effects-based approach in competition policy is the increased complexity of all cases. For example, concerning merger cases, the European Commission (EC) states: "[t]he recent trend that transactions become more complex has continued in 2013. Second phase investigations in particular generally require sophisticated quantitative and qualitative analyses involving large amounts of data" (European Commission, 2014, p. 25). Complexity itself may not be an issue, however it may become an issue if the involved firms and competition authorities cannot adjust to it in a similar fashion. This causes an imbalance or asymmetry between the parties (Neven, 2006): while firms can more easily increase their budget for legal and/or economic advice if necessary, government agencies face binding budget constraints and may be unable to increase their workforce or keep enough competent staff on their payroll. This asymmetry may in turn lead to decisions based on biased information and welfare losses.

In a setting where a decision-maker has to decide on an issue but is uninformed and has to rely on two biased parties that may search for multiple pieces of information and submit it to her, we find that reducing complexity may increase search activity and welfare. The two parties derive positive utility only if the decision is made in their favor and we assume the parties to be asymmetric in the utility they derive. Applied to a competition policy case, one party may be a group of firms filing for a merger and the other party may be the competition authority aiming to prevent the merger if it is detrimental to welfare. In this case, we assume that the (monetary) benefit of a cleared merger to the involved firms by far outweighs the (potentially non-monetary) benefit of the (bureaucrats of the) competition authority in case of a blocked merger.[1] Both parties can simultaneously search for information on multiple dimensions where the number of dimensions is interpreted as the complexity of a case. We assume that the privileged party always searches for information on all dimensions. If the utility of the disadvantaged party is too low to engage in any search for information initially, we show that a reduction of complexity, that is, a reduction of the number of dimensions evidence is accepted from, can increase search incentives of this party. The reduction of complexity attenuates the imbalance between the parties and makes search more attractive to the disadvantaged party.

The decision-maker aims to maximize welfare but is not informed about the state and also cannot observe search activity by the two parties. If the decision-maker is not fully informed because one of the parties does not search on all dimensions, she may make the wrong decision by deciding in favor of one party although more information exists but is not discovered in favor of the other party. We show that a reduction of complexity can lead to an increased search activity by the disadvantaged party which translates into more and more balanced information available to the decision-maker. For an initially large enough number of dimensions, we show that a reduction of complexity can lead to an increase of welfare.

The model in the third chapter entitled "*Mergers in bidding markets*" is joint work with Achim Wambach. In this chapter, we analyze the price effects of horizontal mergers and the incentives to merge in bidding markets. An ideal bidding market can be defined as a market in which (i) goods are traded by means of an auction, (ii) each contract is significant in size, (iii) each contract is awarded to one (winning) party only, and (iv) the fact that one player has won a

[1] Our model does not only apply to competition cases but also to regulation, white-collar crime, and informational lobbying with competing interest groups. Examples of lobbying cases where the benefit of a favorable decision may differ significantly between interest groups include tobacco companies competing with consumer protection groups or oil companies lobbying for drilling rights or the legalization of fracking against environmental protection interest groups.

previous auction does not improve (or worsen) his future position (Klemperer, 2007). In a number of recent cases, competition authorities have cleared mergers in bidding markets despite leading to high market shares of the merged firm with the argument that in bidding markets, it is not market shares that indicate market power but the (lack) of competitors.[2] This special treatment has been criticized by economists on the grounds that specific price-formation processes do not affect the implications of mergers for competition (see Klemperer, 2007; Rasch and Wambach, 2013).

We analyze the price effects of horizontal mergers as well as the incentives to merge in a dynamic bidding market with sequentially arriving consumers where firms have exogenous capacities. We find that a horizontal merger without efficiency gains increases current and future equilibrium prices only if the largest firm is involved in the merger or if a new largest firm is created through the merger. Current and future prices are not affected by a merger if the largest firm is not involved and even decrease temporarily in case of a 'catch-up' merger, that is, if a merger creates a firm just as large as the previously largest firm. We find that a large number of mergers does not have adverse effects on consumer welfare, in contrast to standard merger models. Our findings are also in contrast to the current practice of the EC of assessing market power in bidding markets. Our analysis suggests that the involvement of the firm with the largest capacity or the change of the identity of the largest firm is the key determinant of adverse price effects, independent of the number of active competitors.

Concerning the incentives to merge, we find that a merger between the largest firm and any number of rivals with a smaller capacity is always profitable for all firms in the market because the largest firm is able to sell each additional unit of capacity with a larger probability and at higher prices post-merger while the outsiders free ride on increased prices of the insiders. When smaller firms merge to become the new largest firm in the market, the profitability of the merger depends on a trade-off. The increase in equilibrium prices allows the merged firm to sell its units at higher prices than pre-merger, resulting in an increase in profit.

[2] Cases include Raytheon/Thales (European Commission, 2001), Metronet/Infraco (European Commission, 2002), Boeing/Hughes (European Commission, 2004), and Oracle/Peoplesoft (European Commission, 2005) on the EU level as well as Hochtief/Philipp Holzmann (Bundeskartellamt, 1998) and Webasto/Edscha (Bundeskartellamt, 2009) in Germany.

However, the probability of selling a unit for the merged firm is reduced by the merger, which is to the detriment of the merged firm. Depending on the strengths of the effects, the merger can either be profitable or unprofitable. All outsider firms profit due to the increased prices. In case of a 'catch-up' merger by smaller firms, the merger is unprofitable for the merging firms and the previously largest firm, but profitable for the outsiders. In traditional analyses, mergers without efficiency gains are unprofitable for the insiders unless large parts of the industry are involved in case of strategic substitutes (Salant et al., 1983), and to the benefit of all firms in case of strategic complements (Deneckere and Davidson, 1985). Different from these contributions, in our model with strategic complements, mergers may or may not be profitable depending on the capacity of the involved firms. However, as in the traditional models, we find that outsiders always profit from mergers in the industry.

The model in the fourth chapter entitled "*The scope for collusion under different pricing schemes*" is joint work with Alexander Rasch. In this model, we analyze the impact of different pricing schemes on firms' ability to collude. The question of how different pricing schemes or tariffs affect customer welfare has come to the attention of competition authorities and regulators in recent years for two main reasons. First, collusive agreements on a global scale involving almost all major airlines have been revealed in the air cargo industry where prices consist of multiple fixed and variable components.[3] In 2008, major airlines agreed to plead guilty and pay fines exceeding $500 million for fixing one or more components of total air cargo rates in the US alone (U.S. Department of Justice, 2008).[4] Second, it is an ongoing discussion of whether the simplification of tariff structures benefits customers. For example, the British Office of Gas and Electricity Markets (OFGEM) recently prohibited complex tariffs in the

[3] In this market, shipping or freight rates can be considered flat fees within a certain weight segment and/or type of commodity. Different from that, (per-kilo and/or per-km) surcharges typically depend on the exact chargeable weight and/or distance and can be considered linear.

[4] Involved airlines include Air Canada, Air France-KLM, British Airways, Cathay Pacific, Cargolux, Emirates, Japan Airlines, LAN Chile, Lufthansa, Martinair, SAS Cargo Group, Singapore Airlines, and Qantas Airways. Other jurisdictions where cartel members fined include the European Union (European Commission, 2010), Canada (Canadian Competition Bureau, 2013), Switzerland (Swiss Competition Commission, 2014), and New Zealand (Commerce Commission New Zealand, 2013).

markets for electricity and gas in the UK in order to enhance transparency and the comparability of prices to the benefit of consumers (Office of Gas and Electricity Markets, 2014).

We analyze the incentives to collude under different pricing schemes in a differentiated-products setting à la Hotelling (1929) with elastic demand. Total demand is elastic as local demand—the demand of a single customer—decreases in price and in the distance to the respective firm (i.e., transport cost). The three different pricing schemes we compare are (i) a linear per-unit price, (ii) a fixed fee independent of the quantity purchased, and (iii) a (nonlinear) two-part tariff consisting of a fixed (entry) fee *and* a linear per-unit price. We find that allowing firms to set two-part tariffs as opposed to linear prices facilitates collusion at maximum prices independent of the degree of product differentiation. However, compared to a situation where firms can only set fixed fees, collusion at maximum prices is less sustainable with two-part tariffs.

Collusion is more attractive under nonlinear pricing compared to linear pricing because of a relatively large profit from deviation under linear pricing. When a firm deviates in the linear-pricing scenario to increase market share, lowering its price has an additional positive effect on profits because it increases local demand. This effect is absent under nonlinear pricing which leads to a strong decrease of deviation profits as differentiation increases. Collusion is easiest to sustain in the fixed-fee scenario. This is also caused by relatively lower deviation profits under fixed fees relative to two-part tariffs. Under two-part tariffs, the deviating firm is able to fine-tune local demand using the linear part of the tariff especially when differentiation is large and optimal deviation does not entail covering the whole market, giving rise to larger profits from deviation and a lower incentive to collude.

When interpreting two-part tariffs as complex and fixed fees and linear prices as simplified tariffs, our model predicts an increase in customer surplus in a static, competitive environment when complex tariffs are prohibited. When considering the incentives to collude, however, the effects of simplification on customer surplus are unclear, suggesting the necessity of a careful approach to simplifying tariff structures. Allowing firms to charge simpler flat fees harms customers as it fosters collusion while the incentives to collude are reduced when firms can only set linear prices instead of two-part tariffs.

CHAPTER 2

SEARCHING FOR EVIDENCE: LESS CAN BE MORE

Abstract

We analyze a situation where an uninformed decision-maker has to decide on an issue. There are two parties with state-independent opposing interests who can acquire information in support of their cause through costly search. Information can be obtained across multiple dimensions. A decision is more complex the more dimensions are available for investigation. Each party has to decide on the number of searches it performs. If there is an asymmetry between the parties with regard to the utility they derive from decisions in their favor, we show that a reduction of complexity can lead to an overall increased and more balanced search which may improve welfare.

2.1 INTRODUCTION

Regulation and antitrust has become more complex. For example, in financial regulation in the United States, the Dodd-Frank Act which was signed into law in 2010 is "23 times longer than Glass-Steagall" (The Economist, 2012), the legislation passed in the 1930s as a response to the 1929 crash of Wall Street. In the European Union, the European Commission (EC) observes increased complexity in merger cases. The EC states: "The recent trend that transactions become more complex has continued in 2013. Second phase investigations in particular generally require sophisticated quantitative and qualitative analyses involving large amounts of data." (European Commission, 2014, p. 25).

Complexity itself may not be problematic, but it becomes an issue if the firms and government agencies involved in regulation and antitrust cases cannot adjust to it in a similar fashion. While firms can presumably more easily increase their budget for legal and/or economic advice if deemed necessary, government agencies face binding budget constraints and may be unable to increase their workforce or keep enough competent staff on their payroll. This asymmetry may lead to biased decisions and welfare losses, or, as Rogoff (2012) puts it for the case of financial regulation: "The problem, at least, is simple: As finance has become more complicated, regulators have tried to keep up by adopting ever more complicated rules. It is an arms race that underfunded government agencies have no chance to win."

In a setting where a decision-maker has to decide on an issue but is uninformed and has to rely on two biased parties that may search for multiple pieces of information and submit it to her, we find that reducing complexity may increase search activity and welfare if the two parties are asymmetric in the utility they derive from a favorable decision.

We assume that the two parties derive positive utility only if the decision is made in their favor. Applied to our initial example, one party may be interpreted as a firm filing for a merger with one of its competitors while the other party is the antitrust authority attempting to prevent a potential reduction of consumer surplus due to the merger. In this situation, we argue that it is natural to assume that both parties benefit from a favorable decision only and that the (monetary) benefit of a cleared merger to the involved firm(s) by far outweighs the (potentially non-monetary) benefit of the (bureaucrats of the) antitrust authority in case of a blocked merger.

Both parties can simultaneously search for information on multiple dimensions. We interpret the number of dimensions as the complexity of a case. If the utility of the disadvantaged party is too low to engage in any search for information initially, we show in a first step that a reduction of complexity, that is, a reduction of the number of dimensions available for investigation, may increase search incentives of this party holding constant full search by the other party. The reduction of complexity reduces the advantage of the privileged party which makes search more attractive to the disadvantaged party.

The decision-maker aims to maximize welfare but is neither informed about the state nor is she able to observe the search activity by the two parties. In a first-best world, the decision-maker is fully informed and does not generate welfare losses by wrong decisions. This could be reached in equilibrium if both parties search on all dimensions. In an equilibrium where one of the parties does not search on all dimensions, however, the decision-maker is not fully informed and cannot avoid decision errors. A reduction of complexity has in principle two effects: it makes it impossible to reach the first-best but it can at the same time lead to increased search activity by the disadvantaged party which translates into more and more balanced information available to the decision-maker. For an initially large enough number of dimensions, we show that this can lead to an increase of welfare.

Our results suggest that it may be beneficial for welfare to simplify procedures in competition and regulation cases if the involved agents are asymmetric. This finding is consistent with the Regulatory Fitness and Performance program (REFIT) initiated by the EU which, regarding merger review, aims "to make the EU merger review procedures simpler and lighter for stakeholders and to save costs." (European Commission, 2014, p. 24)

The decision-maker in our model could correspond to a judge deciding on an antitrust or regulation case in the US or to a judge presiding over a white-collar-crime case. In the EU, the EC has the hybrid role of a biased party *and* the decision-maker. On the one hand, its goal is to protect consumer interests, on the other hand, it decides on whether to allow or block a merger. Because 'wrong' decisions can be reviewed and overturned by the European Court of Justice, we argue that our setting also applies to the European case.[1]

Another prominent application of our model is informational lobbying with competing interest groups. Policy-makers who have to decide on whether to vote in favor of or against new legislation are potentially uninformed about the implications of the new legislation but can rely on lobby groups to feed them with (possibly biased) information. Lobby groups benefit from a policy change in their favor and can invest resources to search for arguments and information supporting their preferred outcome. If such information is discovered, the group has an incentive to inform the policy-maker about it. Examples where the benefit of a favorable decision may differ significantly between interest groups include tobacco companies competing with consumer protection groups in order to avoid sales and/or marketing restrictions or oil companies lobbying for drilling rights or the legalization of fracking against environmental protection interest groups.

[1]For a confirmation of this view from a law perspective, see Vesterdorf (2005).

Our model is related to the literature on lobbying[2]. There are two different channels through which interest groups can influence the political decision process: campaign contributions and informational lobbying.[3] Interest groups can either supply politicians with information pertinent to the policy decision (Milgrom and Roberts, 1986; Austen-Smith and Wright, 1992; Potters and van Winden, 1992) or donate money to swing policy in their favor or help the preferred candidate to get elected (Prat, 2002a,b; Coate, 2004a,b), or both (Bennedsen and Feldmann, 2006; Dahm and Porteiro, 2008; Cotton, 2012). It is argued that informational lobbying is more prevalent, especially in the EU (Chalmers, 2013; New York Times, 2013), and more important compared to contributions (Potters and van Winden, 1992; Bennedsen and Feldmann, 2002). Generally, the literature on informational lobbying shows that decision-makers can learn something about the state of the world even from biased experts and improve policy by taking their information into account.[4] We show in our paper that with asymmetric lobby groups and multiple searches, the decision-maker may only receive information from the stronger group and welfare-reducing decision errors can occur. Simplifying the decision process by restricting the number of dimensions where information is taken into account for the decision results in more balanced information provision and increased welfare. A similar result has been found in the literature on contribution limits. Exertion of political influence by means of contributions is seen critical by the general public which fears that wealthy groups can simply buy political favors (Prat, 2002b). In response, many countries use some form of contribution limits or try to reform campaign finance. Cotton (2012) analyzes a situation where a rich and a poor lobby group can pay contributions in order to get access to a decision-maker which is assumed to be essential for the transmission of information. In his model, limits can be beneficial and yield

[2]There is also a large political science literature on this topic, an overview is given by Woll (2006).

[3]For an overview, see Grossman and Helpman (2001).

[4]Our model is also related to the more general literature on strategic information transmission started by Crawford and Sobel (1982). In these models, an uninformed decision maker (receiver) makes a decision based on information presented by one or more informed expert(s) (sender). The messages in these games typically are cheap talk while in our model, messages are verifiable, and senders can only send hard information they have gathered at a cost beforehand. A more recent overview of this literature is provided by Sobel (2013).

more information transmission and better policy when interest groups can decide whether to form a lobby or not. Our model is complementary to that literature in that it shows that welfare can be improved by simplifying the decision process when two asymmetric interest groups compete.

In our model, the interest groups are only interested in finding evidence in favor of their cause and hence are advocates in the sense of Dewatripont and Tirole (1999) who have shown that when agents receive decision-based rewards, competition between opposed agents can increase information gathering or render it cheaper for the principal (see also Austen-Smith and Wright, 1992). Qualitatively, our information structure can be interpreted as an extension of the information structure of Dewatripont and Tirole (1999) to multiple dimensions. Similarly, Krishna and Morgan (2001) show that a decision-maker benefits from consulting two experts, but only when the experts' preferences are opposed. Bennedsen and Feldmann (2006) look at the interplay of informational lobbying and contributions and find that if contributions are available, less information is transmitted in equilibrium and competition between the groups cannot fully alleviate this result because search creates an information externality if it is unsuccessful which benefits the weaker group and thus decreases the incentives to search by the stronger group.

The positive effect of reducing the action space of the agents has also been shown in the literature on optimal delegation (e.g. Szalay, 2005; Alonso and Matouschek, 2008; Armstrong and Vickers, 2010). In these models, a principal delegates decision making authority to a self-interested agent. The principal has to decide how much liberty he wants to give to the agent. For example, in a model of interval delegation, Szalay (2005) shows that removing intermediate decisions from the agent's action set can improve his incentives for information gathering. In our model, the quality of decisions can be improved by restricting the information space through deliberate exclusion of one of the dimensions.[5]

[5] In the context of merger control, Dertwinkel-Kalt and Wey (2014) show that introducing remedies as an intermediate option can reduce the search incentives of the competition authority in a setting based on Szalay (2005).

Finally, our model is related to the contest literature.[6] Che and Gale (1998) show in an all-pay auction with asymmetric bidders that restricting the bid space by introducing a cap can increase competition and overall bids. Because the cap can also lower the winning probability of the high value bidder, welfare may decrease. One difference to our setting is the definition of welfare. In Che and Gale (1998), welfare is maximized if the bidder with the largest valuation wins the auction while in our model, welfare is maximized if the decision is made in favor of the party with more positive information, which is unrelated to valuation. Thus, for welfare maximization, both parties are ex-ante equally likely to win.

The remainder of this paper is organized as follows. In Section 2.2 we present the model. The analysis of the game in Section 2.3 starts in Subsection 2.3.1 with the case where search is unrestricted and proceeds with the case where search on one dimension is prohibited in Subsection 2.3.2. We then compare the search activity and the effects on welfare of the reduction in the number of dimensions in Subsection 2.3.3. A discussion follows in Section 2.4, and we conclude and provide an outlook in Section 2.5.

2.2 THE MODEL

A judge (she) has to make a decision on a case based on information available on multiple dimensions. The information is collected by two interested parties, the firm and the regulator. Information on all dimensions is weighted equally for the decision. More specifically, the judge can either accept or reject a proposal brought forward by the firm, denoted by d_f and d_r, respectively. The firm prefers decision d_f, while the regulator prefers d_r. The information on each dimension $i \in \{1,2,\ldots,n\}$ consists of the realization of two i.i.d. random variables $\theta_{i,j}$, $j \in \{f,r\}$. Each $\theta_{i,j}$ takes value 1 with probability p and value 0 with probability $1-p$ where $0<p<1$. $\theta_{i,f}=1$ can be interpreted as information in favor of the proposal while $\theta_{i,r}=1$ can be interpreted as information against the proposal

[6] In this literature, players exert effort which translates into a probability of winning the contest through the contest success function. It is a standard observation in this literature that a player's contest success function is at least weakly increasing in effort. In contrast, in our model the probability of winning may decrease in search activity due to the belief of the decision maker. For a recent overview of the contest literature, see Konrad (2009).

in dimension i. $\theta_{i,j} = 0$, $j \in \{f, r\}$ means that there is no information available either in favor or against the proposal in dimension i. The state of the world is defined as $\Theta = \{\sum_i \theta_{i,f}, \sum_i \theta_{i,r}\}$.

The two parties receive benefit $w_j \geq 0$ in case $d = d_j$ and zero otherwise.[7] Both parties maximize expected profits $u(w_j, d, c) = \Pr(d = d_j | E_f, E_r) w_j - E_j c$ where $c > 0$ are the marginal search costs and E_j, $j \in \{f, r\}$ is the number of searches by each party. To account for asymmetry between the two parties, we assume $w_f > w_r$. Furthermore we assume that the benefit accruing to the firm if the proposal is accepted is large enough such that full information collection on all dimensions always overcompensates the cost of doing so.[8]

The judge's aim is to maximize expected welfare based on the information available to her.[9] The welfare of a decision is given by $\sum_i \theta_{i,f} - \sum_i \theta_{i,r}$ if the proposal is accepted and $\sum_i \theta_{i,r} - \sum_i \theta_{i,f}$ if the proposal is rejected.[10]

Thus, in case of full information, if there is (weakly) more information in favor of the proposal, i.e., $\sum_i \theta_{i,f} \geq \sum_i \theta_{i,r}$, it is optimal to accept the proposal and reject it otherwise. Observe that in case of a tie, the proposal is accepted.[11]

In the situation we analyze there is incomplete information such that the state of the world is ex ante unknown. Without any information, the expected value of pro and contra information is the same in all dimensions and the judge accepts the proposal. In this situation, that decision reduces welfare whenever $\sum_i \theta_{i,r} > \sum_i \theta_{i,f}$. Hence, the judge is interested in gathering information. She cannot search for information herself but has to rely on information made available to her by the

[7] The subscript j is dropped later in the analysis whenever it is clear to which party w refers.

[8] It is easy to verify that there are values of w_f and c such that the firm wants to search n times as long as $\Pr(d = d_f | E_f, E_r) w_f$ is increasing in E_f, which holds as long as the judge believes that the firm conducts a full search on all dimensions.

[9] We assume that the decision-maker has no leeway and has to take the ex post optimal decision given the evidence presented to her. We believe that a judge or the legislature politically cannot implement a decision rule that is not welfare-optimal (for a similar argument, see Bennedsen and Feldmann, 2006).

[10] Our definition of welfare follows from the assumption that the benefits w_j as well as the search costs c are insignificant relative to the positive (negative) welfare effect of a decision in favor of the party for which more (less) supportive information exists and hence omit them from our welfare definition. A similar welfare function is used in Cotton (2012) when abstaining from the possibility of monetary contributions to the decision maker he analyzes.

[11] We argue that in case of a tie, there is no conclusive evidence against the proposal and thus, there is no obvious reason to decide against it. Our results do not change qualitatively if we use a tie breaking rule where the judge rejects the proposal or where she flips a fair coin.

firm and the regulator. At the beginning of the game, the judge chooses the number of dimensions which are relevant for the decision. The firm is interested in searching information in favor of the proposal only, i.e., $\theta_{i,f}$. The regulator only searches for information against the proposal $\theta_{i,r}$. Both firm and regulator simultaneously search in each dimension at cost c. If a party searches in a given dimension i and there exists evidence in this dimension, it learns $\theta_{i,j}$ with certainty. If a party does not search, it learns nothing, which we denote by 0. Hence, finding no information and not searching for information yield the same result.

After searching, both parties send a message $m_j \in \{\sum_k \theta_{k,j}, 0\}$ to the judge, where k denotes the number of dimensions where the respective party searched for information. We assume that parties cannot withhold information. This is natural in our case as parties only search for information that is beneficial to them and hence have no interest in holding it back.[12] The message either consists of the number of pieces of evidence that were found or 0. The information available to the judge therefore is $M = \{m_f, m_r\}$, which we also call outcome.

The judge holds two types of beliefs. First, the judge has an expectation $\mu_j \in \{0, 1, 2, \ldots, n\}$ about the number of searches of each party.[13,14] Second, she has a belief about the state of the world. Updating this belief not only depends on the messages received by the two parties but also on the expectation about the number of searches. Two cases are of particular interest to us. The first interesting case is a situation in which the judge expects the regulator not to search ($\mu_r = 0$) and receives $m_r = 0$, she learns nothing about the evidence against the proposal. The second case of interest is when the judge expects a full

[12]The only exception is the case where the judge believes that the regulator does not search. If the regulator does search and finds, say, one piece of information, he may prefer to withhold it. This hinges on the out-of-equilibrium belief of the judge. We argue that while the ability to withhold information does increase the incentives to search for the regulator in this case, it does not qualitatively change our results.

[13]Technically, the belief of the judge is a probability distribution over the number of searches of each party, that is, a vector including $n+1$ probabilities $\Pr(E_j = X)$ that party j's number of searches equals $X \in \{0, 1, 2, \ldots, n\}$. In pure-strategy equilibria, the judge expects the parties to search a specific number of times such that one probability equals 1 and all other probabilities equal zero.

[14]In what follows, we assume the judge's expectation about the number of searches by the firm μ_f to be equal to the number of admissible dimensions.

search by the regulator on all dimensions, i.e. $\mu_r = n$. The sequence of events is summarized in Figure 2.1.

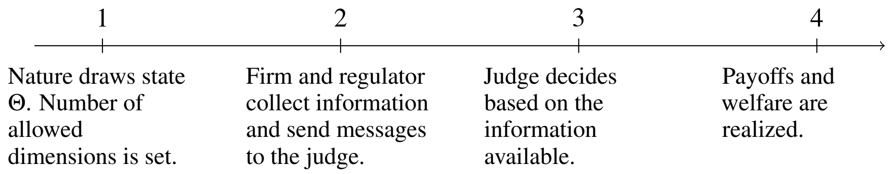

Figure 2.1: Sequence of events

2.3 ANALYSIS

We derive perfect Bayesian equilibria. An equilibrium consists of the number of searches by firm and regulator, the decision rule by the judge, and her beliefs μ_f and μ_r about the number of searches performed by the firm and regulator, respectively. In what follows, we fix the number of dimensions $n = 3$. We first analyze Situation 1, where searching in all three dimensions is allowed (the case of full complexity), and Situation 2, where the judge only accepts evidence from two of the three dimensions (reduced complexity).[15] All proofs are relegated to the Appendix.

2.3.1 SITUATION 1: INFORMATION IS ACCEPTED FROM ALL DIMENSIONS

Decision rules

We proceed backwards and first discuss the judge's decision rule for a given set of messages, M. The judge makes her decision based on the information submitted to her by the parties and her beliefs on the number of searches of firm

[15] Our analysis is only meaningful if the judge has access to information submitted to her by the parties only, ruling out the possibility that the judge might receive information from the prohibited dimension through other means. Given the confidential nature of most arguments in competition and regulation cases, publication or distribution via media outlets does not appear to be in the interest of the involved parties. This assumption is also common in the lobbying literature on access to legislators where information can only be submitted conditional on being granted access (see, for example, Austen-Smith (1998); Cotton (2009, 2012) and the references therein).

and regulator in order to maximize expected welfare. Given our assumption that the firm always searches on all available dimensions, two situations are of particular interest for us: the case where the judge believes that the regulator also searches on all available dimensions as a benchmark and the case where the judge believes that the regulator does not search at all. If the belief concerning the number of searches of party j is $\mu_j = 3$, the information submitted by j is believed to be the state with probability 1. As a consequence, if the submitted information is, say, $m_j = 1$ then $\sum_i \theta_{i,j} = 1$, i.e. the judge believes that (only) one piece of information in favor of j exists.

If the judge believes that both firm and regulator search three times ($\mu_f = \mu_r = 3$), the information available in equilibrium is equal to the state and she decides as in the case with full information according to the first-best decision rule: The decision is made in favor of the firm if it has found (weakly) more information than regulator and in favor of the regulator if it has found (strictly) more information. The optimal decision for all combinations of information for beliefs $\mu_f = \mu_r = 3$ is shown in Table 2.1.

Table 2.1: Decision rule for $\mu_f = 3$, $\mu_r = 3$

		Firm			
		$\{0,\cdot\}$	$\{1,\cdot\}$	$\{2,\cdot\}$	$\{3,\cdot\}$
Regulator	$\{\cdot,0\}$	F	F	F	F
	$\{\cdot,1\}$	R	F	F	F
	$\{\cdot,2\}$	R	R	F	F
	$\{\cdot,3\}$	R	R	R	F

If the judge believes the regulator did not search ($\mu_r = 0$), there are only four possible equilibrium outcomes M after the firm has searched for information: $\{0,0\}$, $\{1,0\}$, $\{2,0\}$, and $\{3,0\}$. If the regulator does not search, the judge cannot learn anything about the state and has to base her decision on the expected value of information existing in favor of the regulator. The expected value of information against the proposal is given by $3\times p^3 + 2\times 3(p^2(1-p)) + 1\times 3p(1-p)^2 = 3p$. The optimal decision is determined by comparing the information submitted by the firm (which is equal to the state in equilibrium) with the expected value of information of the regulator.

Clearly, the proposal is rejected (accepted) if the firm finds no evidence (evidence on all dimensions). The optimal decision rule for the two intermediate cases depends on p. If the firm finds one piece of evidence and the proposal is accepted, expected welfare is given by $1-3p$ which is positive only for $p<1/3$ and hence the judge will reject the proposal for values of p larger than $1/3$. Similarly, if the firm finds two pieces of evidence, expected welfare is given by $2-3p$.

We assume that if the judge receives a message other than 0 from the regulator when expecting him not to search, then she updates her (out-of-equilibrium) belief concerning the number of searches of the regulator to $\mu_r = 3$.[16] She updates her belief regarding the state such that the probability that it is equal to the message is equal to 1. It follows that the decision rule under full information applies out of equilibrium. The complete decision rule is given in Table 2.2.

Table 2.2: Decision rule for $\mu_f = 3$, $\mu_r = 0$

			Firm			
			$\{0,\cdot\}$	$\{1,\cdot\}$	$\{2,\cdot\}$	$\{3,\cdot\}$
Regulator	$\{\cdot\,,0\}$	$p\in(0,1/3]$	R	F	F	F
		$p\in(1/3,2/3]$	R	R	F	F
		$p\in(2/3,1)$	R	R	R	F
	$\{\cdot\,,1\}$		R	F	F	F
	$\{\cdot\,,2\}$		R	R	F	F
	$\{\cdot\,,3\}$		R	R	R	F

Incentives to search

We first analyze the equilibrium candidate where the judge has the belief $\mu_r = 3$ (and, as we assume throughout, $\mu_f = 3$). The regulator anticipates the decision by the judge dependent on the information submitted to her. This gives rise to the following probabilities $\Pr(d_r|E_r)$ of a decision against the proposal when he

[16]Because Bayes' rule does not apply in situations that occur with probability zero, there is no constraint on the out-of-equilibrium beliefs we specify. We choose the out-of-equilibrium belief that is least favorable for the regulator.

performs $E_r \in \{0,1,2,3\}$ searches:[17]

$$\Pr(d_r|0) = 0$$
$$\Pr(d_r|1) = (1-p)^3 p$$
$$\Pr(d_r|2) = (1-p)^3(2p(1-p)+p^2) + 3p(1-p)^2 p^2$$
$$\Pr(d_r|3) = (1-p)^3(1-(1-p)^3) + 3p(1-p)^2(3p^2(1-p)+p^3) + 3p^2(1-p)p^3.$$

For example, if the regulator searches one time, the decision is made in his favor if the firm does not find any information, which occurs with probability $(1-p)^3$, and if he finds one piece of information, which occurs with probability p. It is optimal for the regulator to search three times if the expected profit of searching three times is larger than the expected profit of searching twice (2.3.1), of searching once (2.3.2), and of not searching (2.3.3).

$$\Pr(d_r|3)w - 3c \geq \Pr(d_r|2)w - 2c \tag{2.3.1}$$
$$\Pr(d_r|3)w - 3c \geq \Pr(d_r|1)w - c \tag{2.3.2}$$
$$\Pr(d_r|3)w - 3c \geq \Pr(d_r|0)w \tag{2.3.3}$$

The following lemma gives a condition for the benefit of the regulator, w, under which these constraints hold such that the regulator matches the search effort by the firm.

Lemma 1. *There exists a critical value $\tilde{p} \in [0,1]$ such that if and only if $w \geq \overline{w}$, where*

$$\overline{w} = \begin{cases} c/\left(p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6\right) & \text{for } 0 < p \leq \tilde{p} \\ c/\left(p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6\right) & \text{for } \tilde{p} < p \leq 1, \end{cases}$$

there exists an equilibrium in which the regulator and the firm search three times and the judge has beliefs $\mu_f = \mu_r = 3$.

The lemma defines a lower bound on the benefit w as a function of p above which a full search by both parties constitutes an equilibrium. The binding incentive constraint is determined by the smallest increase in the probability of

[17] As $E_f = 3$ we omit the reference to the number of the searches by the firm.

a decision in favor of the regulator per unit of search cost when searching three times instead of two times, once or not all.

When p tends to zero, the probability of winning tends to zero regardless of the number of searches. Because searching is costly, it can never be implemented. For low values of p, the marginal cost-adjusted increase in the probability of winning is smallest when moving from two to three searches and IC (2.3.1) is binding. The probability of a favorable decision is maximal for intermediate values of p, resulting in the lowest benefit necessary to implement full search. As p grows large, the probability of winning tends to zero because it becomes more likely that the firm finds three pieces of information. The binding IC becomes (2.3.3) because in this range of p, the marginal probability of winning is convex for the regulator and hence the average increase when moving from no search to a full search is smaller than the increase from one or two searches to three searches. A graphical illustration of the lower bound $\overline{w}$ is shown in Figure 2.2.

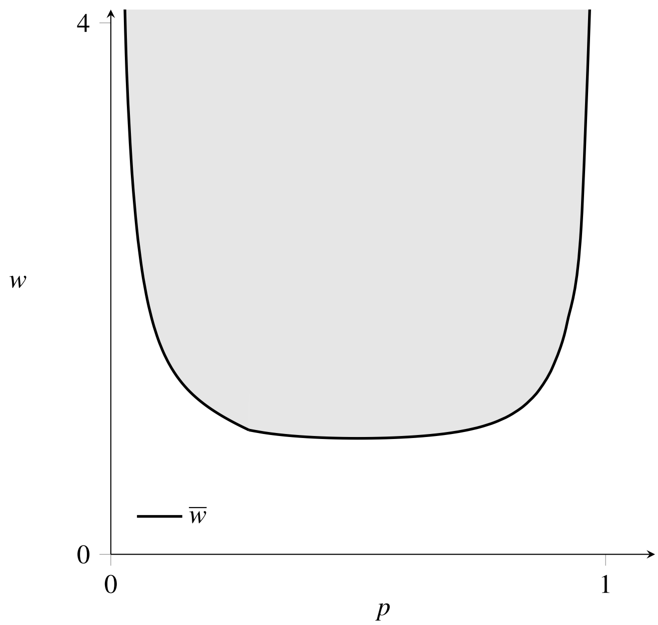

Figure 2.2: Lower bound for full-search equilibrium with $c = 1/10$. The grey area indicates the region where the full-search equilibrium exists.

As a next step, we show that an equilibrium where the regulator does not search also exists. If the judge has the belief $\mu_r = 0$, it is optimal for the regulator not to search if

$$\Pr(d_r|0)w \geq \Pr(d_r|3)w - 3c \tag{2.3.4}$$
$$\Pr(d_r|0)w \geq \Pr(d_r|2)w - 2c \tag{2.3.5}$$
$$\Pr(d_r|0)w \geq \Pr(d_r|1)w - c. \tag{2.3.6}$$

Clearly, these constraints can be satisfied by setting $w = 0$. The following lemma defines a threshold on w below which the regulator will not search for information.

Lemma 2. *There exists a critical value $\hat{p} \in [0,1]$ such that if and only if $w < \hat{w}$, where*

$$\hat{w} = \begin{cases} c/\left(3p^3 - 8p^4 + 8p^5 - 3p^6\right) & \text{for } 0 < p \leq \frac{1}{3} \\ \infty & \text{for } \frac{1}{3} < p \leq \hat{p} \\ -3c/\left(9p^2 - 36p^3 + 54p^4 - 39p^5 + 12p^6\right) & \text{for } \hat{p} < p \leq \frac{2}{3} \\ \infty & \text{for } \frac{2}{3} < p \leq 1, \end{cases}$$

there exists an equilibrium where the regulator does not search, the firm searches in all three dimensions, and the judge has beliefs $\mu_f = 3$ and $\mu_r = 0$.

Because the judge takes into account the information in favor of the regulator that may exist but is never discovered in equilibrium, searching only one or two times becomes increasingly unattractive for the regulator as p increases. Technically, the probability of winning decreases when searching once or twice instead of not searching. Additionally, searching is costly. Positive wages inducing a full search by the regulator exist only in regions of p just to the left of a change in the decision rule. The area in which not searching is an equilibrium is colored in grey in Figure 2.3.

As we are interested in a situation where three searches by the regulator cannot be implemented, we next show that an equilibrium in which the regulator does not search exists for benefits below $\overline{w}$, i.e. the lowest possible benefit inducing three searches by the regulator, by comparing the two critical values $\overline{w}$ and $\hat{w}$.

Lemma 3. *It holds that $\hat{w} \geq \overline{w}$.*

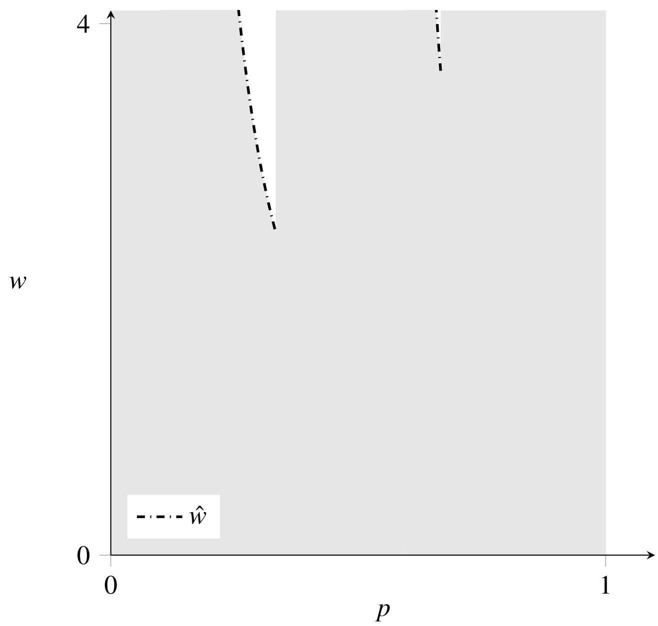

Figure 2.3: Upper bound for no-search equilibrium with $c = 1/10$. The grey area indicates the region where the no-search equilibrium exists.

The lemma establishes that the no-search equilibrium exists for benefits below the minimum benefit necessary to implement full search. When the judge holds the belief that the regulator does not search, searching is relatively unattractive for the regulator because the judge takes into account the information that may exist but remains undiscovered. When the judge believes the regulator performs a full search, she will not decide in favor of the regulator if he does not deliver information, increasing the intrinsic motivation to search. A graphical comparison of the two critical values $\overline{w}$ and $\hat{w}$ is given in Figure 2.4.

To complete the analysis of pure-strategy equilibria under full complexity, we note that equilibria where the regulator searches one or two times also exist for benefits below $\overline{w}$. A full characterization of these equilibria can be found in Appendix 2.6.2. In Lemma 4 we define constraints on w (and p) such that no search by the regulator is the unique equilibrium in pure strategies in Situation 1.

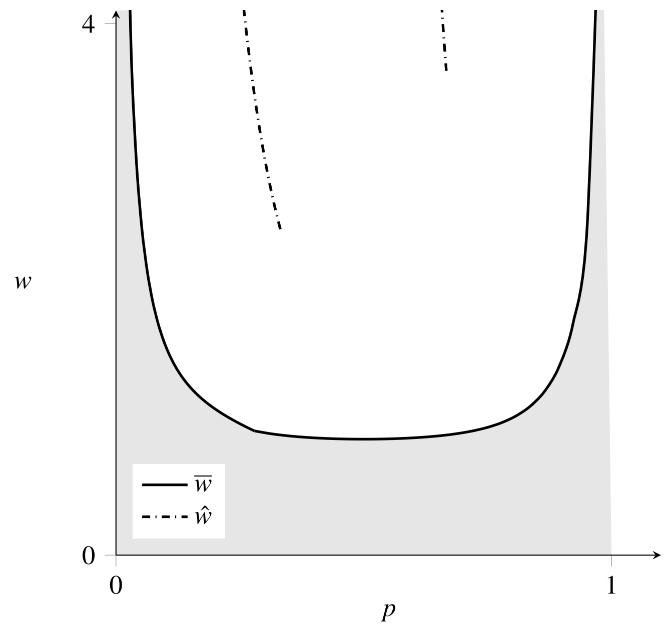

Figure 2.4: Comparison of thresholds for no-search and full-search equilibrium with $c = 1/10$. The grey area indicates the region where the full-search equilibrium does not exist while the no-search equilibrium exists.

Lemma 4. *There exist critical values $\overline{w}_2$ and $\ddot{p}$ such that if and only if either (a) $w < \tilde{w}$ or (b) $\overline{w}_2 < w < \overline{w}$ and $1/2 < p < \ddot{p}$, no search by the regulator is the unique equilibrium given three searches by the firm.*

If the judge accepts evidence from all three available dimensions and if the benefit w for the regulator is restricted, search activity is one-sided: the firm gathers evidence on all dimensions whereas the regulator does not search. The judge then only learns the arguments in favor of the firm but is not informed about arguments in favor of the regulator.

2.3.2 SITUATION 2: RESTRICTED SCOPE OF INFORMATION

In this section, we analyze the situation where the scope of information is restricted in the sense that the judge accepts information from two dimensions only. We characterize an equilibrium in which both parties search in all allowed dimensions. The probability that information exists in the third (unavailable) dimension is equal for both parties (and cases) and thus not relevant for the decision.

As in the case of full complexity, if the judge believes that both firm and regulator conduct a full search ($\mu_f = \mu_r = 2$) she takes the reported information to be equal to the true values and accepts the proposal if the firm has found (weakly) more information and rejects it otherwise. The decision rule by the judge is displayed in Table 2.3.

		Firm		
		$\{0,\cdot\}$	$\{1,\cdot\}$	$\{2,\cdot\}$
	$\{\cdot,0\}$	F	F	F
Regulator	$\{\cdot,1\}$	R	F	F
	$\{\cdot,2\}$	R	R	F

Table 2.3: Decision rule for $\mu_f = 2$, $\mu_r = 2$

The regulator has an incentive to search twice if

$$\Pr(d_r|2)w - 2c \geq \Pr(d_r|1)w - c \tag{2.3.7}$$

$$\Pr(d_r|2)w - 2c \geq \Pr(d_r|0)w. \tag{2.3.8}$$

The following lemma defines a range for the benefit w of the regulator in which an equilibrium where both regulator and firm search two times exists.

Lemma 5. *Suppose that search on one dimension is prohibited. If and only if $w \geq \underline{w}$, where*

$$\underline{w} = \begin{cases} c/\left(p - 3p^2 + 5p^3 - 3p^4\right) & \text{for } 0 < p < 1/3 \\ c/\left(p - \frac{5}{2}p^2 + 3p^3 - \frac{3}{2}p^4\right) & \text{for } 1/3 \leq p < 1, \end{cases}$$

there exists an equilibrium in which the regulator and the firm search on all two admissible dimensions and the judge has beliefs $\mu_f = \mu_r = 2$.

The shape of the lower bound on w defined in the lemma is qualitatively similar to $\overline{w}$ given in Lemma 1. As p tends to zero or one, the probability of winning tends to zero and a full search cannot be implemented. For small values of p the marginal increase in the probability of winning per unit of search cost is smallest for the second search while for large values of p, incentivizing full search as opposed to not searching at all leads to the lowest cost-weighted increase in the probability of a favorable decision. The lower bound $\underline{w}$ is displayed in Figure 2.5.

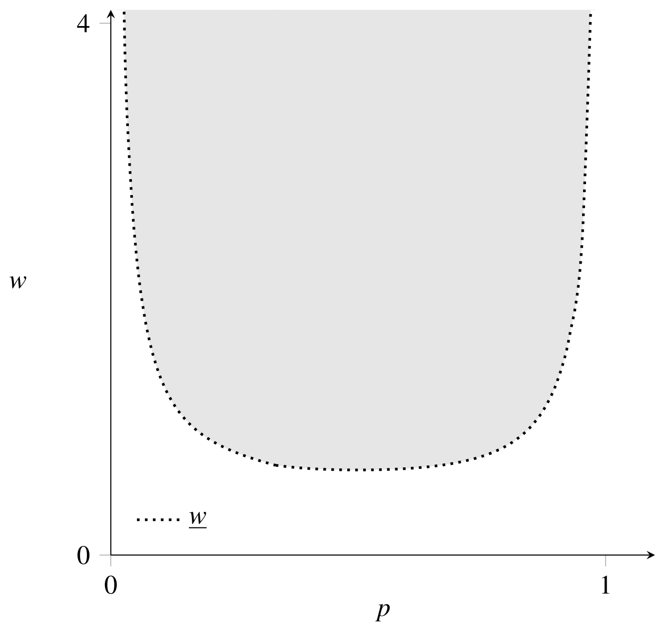

Figure 2.5: Lower bound for full-search equilibrium under reduced complexity with $c = 1/10$. The grey area indicates the region where the full-search equilibrium exists.

2.3.3 COMPARISON OF SITUATIONS 1 AND 2

Search activity

After solving the game separately in Situation 1, where search is unrestricted, and in Situation 2, where evidence on one dimension is not accepted by the judge, we now combine and summarize our previous results regarding the regulator's search activity in the following proposition. To set the stage for our first main result, we start by comparing the minimum wages necessary to implement the full-search equilibrium in both situations.

Proposition 1. *The minimum benefit $\overline{w}$ necessary to render three efforts optimal for the regulator when three dimensions are allowed is always larger than* *the minimum benefit $\underline{w}$ necessary to make two efforts optimal when only two dimensions are allowed.*

The proposition says that a range of benefits w exists where a full search by the regulator cannot be implemented when search is unrestricted, while a full search is an equilibrium when the scope of search is restricted to two dimensions. In particular, in connection with Lemma 3, which states that not searching is an equilibrium for benefits below $\overline{w}$, if $\underline{w} < w < \overline{w}$, i.e. if full search cannot be implemented when all dimensions are available, reducing complexity can increase the regulator's search activity. The range of benefits where full search is an equilibrium in Situation 2 but not in Situation 1 is depicted by the grey area in Figure 2.6.

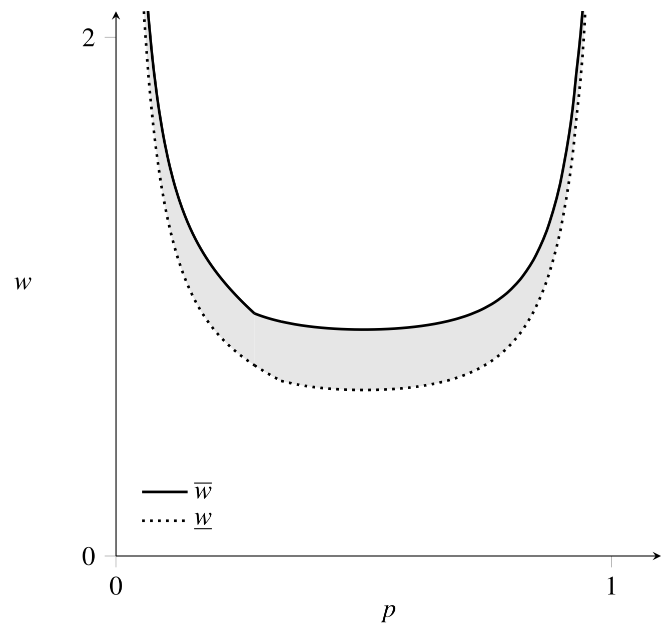

Figure 2.6: Comparison of lower bounds for full-search equilibria under full and reduced complexity with $c = 1/10$. The grey area indicates the region where the full-search equilibrium exists under reduced complexity but does not exist under full complexity.

Reducing the number of available dimensions increases the change in the probability of winning per unit of search costs in the binding incentive constraints, therefore decreasing the lowest benefit necessary to implement full search. If it is unlikely that information exists (small p), in both situations the binding constraint is given by comparing the expected profit from full search with the expected profit of one less search. With reduced complexity, the cost-weighted increase in the winning probability is larger because the second search is more likely to be decisive. For large probabilities that information exists, the relevant comparison is between full search and no search. The increase in the probability of a decision in favor of the regulator when moving from no search to full search is larger under full complexity than under reduced complexity. When taking into account the cost of searching, however, reducing complexity leads to an increase in the probability of winning per unit of search costs. For both constraints, the necessary benefit to induce full search by the regulator is therefore smaller compared to the case of full complexity.

Taken together, if the benefit of the regulator is bounded from above, particularly if the benefits are below $\overline{w}$, he will not search if the scope of search is unrestricted. Prohibiting to search for information on one of the three dimensions lowers the benefit that is necessary to make the regulator willing to search on all available dimensions. This levels the playing field such that the regulator is able to search on as many dimensions as the firm.

If $w < \overline{w}$ there are two other equilibria in Situation 1 where the regulator searches once and twice, respectively. The following proposition shows that further restrictions on w and p lead to an unambiguously one-sided search activity: Under the conditions specified in the following proposition, if search is unrestricted, the regulator will never search while he will conduct a full search if complexity is reduced.

Proposition 2. *There exist critical values $\overline{w}_2$ and $\ddot{p}$ such that if and only if either (a) $\underline{w} \leq w < \tilde{w}$ and $0 < p < 1/3$ or (b) $\max\{\overline{w}_2, \underline{w}\} < w < \overline{w}$ and $1/2 < p < \ddot{p}$, the regulator searches in two dimensions if the scope of search is limited to two dimensions while he does not search if searching on all three dimension is allowed.*

Next we show that besides potentially increasing the regulator's search activity, a reduction of the number of admissible dimensions can also be welfare-enhancing.

Welfare

We now determine the expected welfare losses L_k, where $k \in \{1,2\}$ refers to Situation 1 or 2, due to decision errors under incomplete information. These errors occur when the decision made by the judge given M does not match the optimal decision given Θ. For example, if the judge believes that the firm conducts a full search while the regulator does not search in situation 1 ($\mu_f = 3$, $\mu_r = 0$) and receives the message $M = \{1,0\}$, he decides in favor of the firm if $p \leq 1/3$. In this scenario, a loss of $3-1=2$ occurs if the state is $\Theta = \{1,3\}$.

In Situation 1, where we focus on the equilibrium where the regulator does not search, welfare losses can occur in the two intermediate outcomes, that is, if the firm has found evidence in one or two dimensions ($M = \{1,0\}$ or $M = \{2,0\}$). The cases in which welfare is reduced depend on the level of p as the judge's decision rule is different for different values of p. Each loss can be deconstructed into the probability of an outcome where a loss can occur (which we call error-prone messages) and the probability that a loss actually occurs.

For $0 < p < 1/3$, a wrong decision is made by the judge when the firm has found one piece of evidence but two or three pieces of evidence exist for the regulator, or when the firm has found two pieces but there are three pieces of evidence favoring the regulator's case. The expected losses in this range of p are given by

$$L_1\Big|_{p\in(0,\frac{1}{3}]} = 3p(1-p)^2 \times \left(3p^2(1-p)(2-1) + p^3(3-1)\right) + 3p^2(1-p) \times p^3(3-2).$$

If $1/3 < p < 2/3$, the judge decides in favor of the regulator if the the firm has found one piece of evidence. This decision rule results in a welfare loss if there exist no pieces of evidence for the regulator. A loss also occurs when the firm has found two pieces and there exist three pieces of evidence on the side of the regulator. In this case, the expected losses are

$$L_1\Big|_{p\in(\frac{1}{3},\frac{2}{3}]} = 3p(1-p)^2 \times (1-p)^3(1-0) + 3p^2(1-p) \times p^3(3-2).$$

For $2/3 < p < 1$, the decision is made in favor of the regulator when the firm has found two pieces of evidence. In this case, wrong decisions are made again in case of $\{1,0\}$, and when the firm has found two pieces but there exist zero or only one piece for the regulator. The expected losses are

$$\begin{aligned} L_1\Big|_{p\in(\frac{2}{3},1)} &= 3p(1-p)^2 \times (1-p)^3(1-0) \\ &\quad + 3p^2(1-p) \times \left((1-p)^3(2-0) + 3p(1-p)^2(2-1)\right). \end{aligned}$$

Simplifying leads to a total expected welfare loss in Situation 1 of

$$L_1 = \begin{cases} 9p^3 - 21p^4 + 18p^5 - 6p^6 & \text{for } 0 < p < 1/3 \\ 3p - 15p^2 + 30p^3 - 30p^4 + 18p^5 - 6p^6 & \text{for } 1/3 < p < 2/3 \\ 3p - 9p^2 + 15p^3 - 21p^4 + 18p^5 - 6p^6 & \text{for } 2/3 < p < 1. \end{cases}$$

In Situation 2, welfare losses can occur in all outcomes in which both parties have found the same amount of evidence, that is, $\{0,0\}$, $\{1,1\}$, and $\{2,2\}$. In these cases, it is possible that additional evidence in favor of the regulator but not in favor of the firm exists in the additional dimension but is not discovered. The expected losses in this situation are

$$\begin{aligned} L_2 &= (1-p)^4 \times (1-p)p(1-0) \\ &\quad + 4p^2(1-p)^2 \times (1-p)p(2-1) \\ &\quad + p^4 \times (1-p)p(3-2) \end{aligned}$$

or

$$L_2 = p - 5p^2 + 14p^3 - 22p^4 + 18p^5 - 6p^6.$$

A comparison of L_1 and L_2 gives the following result.

Proposition 3. *Under the conditions of Proposition 2 and for* $2-\sqrt{3} < p < \sqrt{3}-1$*, the reduction of the number of dimensions from three to two is welfare-enhancing.*

The proposition gives a condition under which it can be beneficial from a welfare perspective to reduce the complexity of the case and deliberately ignore evidence from one dimension when (i) one of the two parties who can search for information is disadvantaged in the sense that its benefit from the decision is smaller than the other party's, and (ii) the probability that evidence exists in a given dimension and in a given direction is intermediate. A graphical comparison of the expected losses in Situations 1 and 2 is given in Figure 2.7.

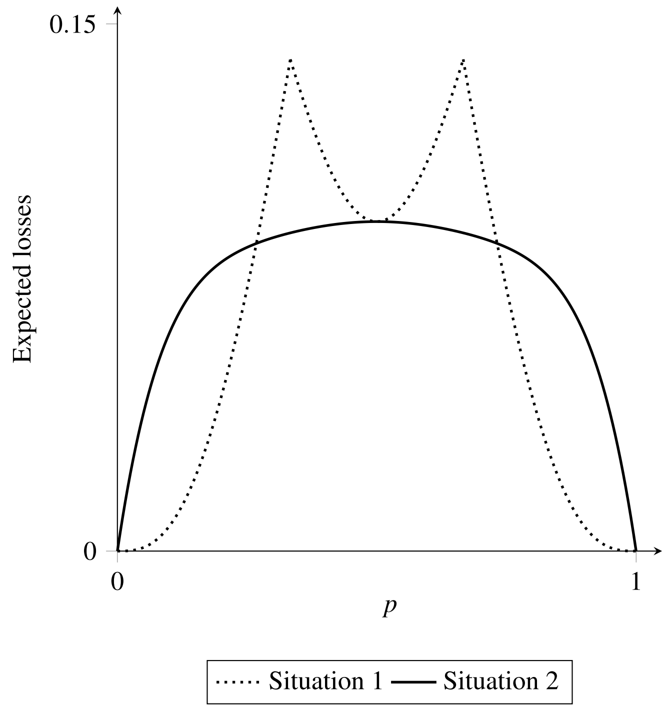

Figure 2.7: Expected losses in Situation 1 (where search activity is asymmetric) and Situation 2 (where search is symmetric but information from one dimension is omitted).

For both small and large probabilities that information exists, the expected losses are smaller in a situation of asymmetric search activity. For small values of p, in Situation 1, error-prone messages are very unlikely to occur because they require at least one piece of information. Conversely, in Situation 2, errors can occur if neither party finds any information ($M = \{0,0\}$), which is very likely for small p. Additionally, wrong decisions in Situation 1 occur if multiple pieces of information exist in favor of the regulator, which is also improbable. Conversely, in Situation 2 for errors to occur, only one additional piece of information is necessary, i.e., when there is information against the proposal but not in favor of it in the omitted third dimension, which is relatively likely. A similar argument can be made if the probability that information exists is large.

For intermediate probabilities that information exists, a situation of reduced complexity and symmetric search leads to lower expected losses. Two opposing effects are at work. On the one hand, the probability that an informational setting occurs in which wrong decisions can be made is large in Situation 1 and small in Situation 2, making losses more likely in the former situation; on the other hand the probability that a wrong decision is actually made is small in Situation 1 and large in Situation 2. We find that the first effect is stronger, resulting in larger expected losses when search is unrestricted but asymmetric. It is in this range of p where a reduction of complexity has an unambiguously welfare-enhancing effect.

2.4 DISCUSSION

In the following section we examine the robustness of our results. In particular, we discuss varying the number of initial dimensions as well as allowing the judge to ex ante commit to a decision rule.

Number of dimensions

Our analysis is based on initially three dimensions. We argue that this is the smallest number of dimensions where restricting search can increase welfare. The main difference between the two situations is that if the regulator does not search, the judge learns all arguments in favor of the proposal but none against it, while if search is restricted, she learns all pro and contra arguments on all but one dimensions. When there are two or less initial dimensions, knowing all evidence in one direction is better from a welfare perspective than not knowing any evidence from the excluded dimension. This is obvious if there is only one dimension. If there are more dimensions, the judge adjusts the decision rule in case of asymmetric search according to the expected value of information in the undiscovered direction which dampens welfare losses. If search is symmetric but restricted the expected value concerning the unavailable dimension is irrelevant for the decision and thus losses cannot be avoided. For two initial dimensions, the first effect dominates and welfare cannot be improved by restricting search.[18] As the number of dimensions increases, however, making the correct decision without information from the regulator becomes more and more problematic as the number of the intermediate outcomes where errors can occur and the size of the errors increase. In contrast, the losses when being fully informed about all but one dimensions decrease because the single omitted dimension's impact on the decision becomes increasingly smaller. Thus it appears intuitive that the effect exists and may be even stronger for a larger number of dimensions.

So far we have assumed that the number of dimensions is exogenous. However, in actual cases, the firm has some leeway in determining the number of dimensions, i.e., in choosing the degree of complexity. By hiring more lawyers and/or consultants, the firm might be able to file a larger report to the authorities. In this situation, the number of dimensions is determined endogenously. It is then to be expected that the firm will choose this number taking into account the incentives by the regulator to put effort into search. That is, the firm might strategically choose the size of complexity in order to minimize the search effort of the regulator. Then, again, reducing the number of dimensions from which information is accepted may have a positive effect on the search activity of the regulator. We leave the formal proof of these claims as open questions for future research.

Ex ante commitment to a decision rule

We assume throughout that the judge makes the ex-post-optimal decision given the information available to her. Alternatively, one could assume that the judge can commit herself to a decision rule before firm and regulator search for information. Two cases can be distinguished.

If the rational judge chooses to commit to the decision rule that maximizes her objective function ex post, ex ante commitment can be interpreted as an equilibrium selection device. The more interesting case arises if we assume that the judge commits ex ante to a decision rule which possibly violates ex post optimality. There is a considerable tension between inducing (optimal) search

[18] Reducing complexity also does not increase overall search activity and thus does not improve welfare.

activity and welfare losses. The judge can easily construct decision rules that support a full search by the regulator as an equilibrium for a larger range of w than discussed in the main text. As an example, consider a decision rule specifying a decision in favor of the regulator if he finds one or more pieces of information independent of the information submitted by the firm. It is straightforward to show that the minimum benefit necessary to induce a full search in this case is lower than $\overline{w}$ as specified in Lemma 1. However, there is a trade-off between enhancing the incentives to search by an ex ante commitment and possibly reducing welfare. In the current example, losses may be large because the judge commits to decide in favor of the regulator even if he only finds one piece of information while the firm finds three pieces. Deriving the optimal trade-off is beyond the scope of this article and may be an interesting direction for future research.

2.5 CONCLUSION

In merger cases, information about the potential effects on consumer surplus is essential for the decision-making of competition authorities. Similarly, in lobbying cases, policy-makers need access to information in order to draft sensible legislation. We analyze a model where a decision-maker has to decide on a proposal based on information imparted to her by two interested parties, the firm and the regulator. The firm prefers the proposal to be accepted while the regulator benefits from a rejection. The firm (regulator) only searches for information in favor (against) the proposal. Information is multidimensional in the sense that there is information in favor and against the proposal in several dimensions. The basis of our analysis is the assumption that the regulator receives a smaller benefit from winning than the firm. This assumption captures that the regulator typically consists of bureaucrats with fixed wages, while the firm employs consultants and lawyers with incentive contracts to defend their case. We show that this asymmetry between the two parties can lead to biased search activity where the firm searches for more information than the regulator. In this case, the decision-maker has to decide based on biased arguments a majority of which were obtained from only one party. We suggest to reduce the complexity of the decision-making process by reducing the number of dimensions that the decision maker takes into account. We show that this allows the disadvantaged regulator to catch up with the firm's search efforts. In turn, the decision-maker is provided with more and more balanced information which results in a welfare increase if the probability that information exists is neither too small nor too large.

At a first glance it is sensible to include as many relevant aspects as possible in merger cases or when new legislation is drafted. However, this aim might not be achievable when the parties who provide the decision-maker with information are very asymmetric, for example when small citizens' initiatives compete with large energy companies lobbying for fracking rights, or when a competition authority examines a proposed merger by companies with an unlimited budget for legal advice. In such cases it can be beneficial to reduce the complexity of the procedure in order to level the playing field. Our findings are in line with recent efforts by the EU as part of the REFIT programme which, regarding merger review, aims "to make the EU merger review procedures simpler and lighter for stakeholders and to save costs." (European Commission, 2014, p. 24)

More research on the topic of asymmetric parties and multidimensional information is needed. A natural next step would be generalize the present model to a setting including a variable number of initial dimensions. This makes it possible to study the optimal reduction of complexity depending on the number of initial dimensions and the degree of asymmetry. In a similar vein, allowing the firm to strategically choose the number of relevant dimensions could lead to interesting new results explaining the observation of asymmetric search effort.

2.6 APPENDIX

2.6.1 APPENDIX A: PROOFS

Proof of Lemma 1. For the moment, ignore (2.3.2). Rearranging (2.3.1) and (2.3.3) gives

$$\frac{c}{w} \leq p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6 \tag{2.6.1}$$

and

$$\frac{c}{w} \leq p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6, \tag{2.6.2}$$

respectively. The binding constraint is the stricter one, i.e., the one with the smaller RHS. Constraint (2.6.1) is binding if the RHS of (2.6.2) minus the RHS of (2.6.1) is positive, or $3-20p+45p^2-48p^3+20p^4 \geq 0$. Define the LHS as $g(p)$. The third derivative $g'''(p) = -288+480p$ is increasing and crosses the abscissa once from below. Hence, $g''(p) = 90-288p+240p^2$ is convex and has a local minimum at $p = 288/480$. As its value is positive at this local minimum, it is positive throughout the range of p. This implies that $g'(p) = -20+90p-144p^2+80p^3$ has a positive slope, and it crosses the abscissa once from below as $g'(0) < 0$ and $g'(1) > 0$. Therefore, $g(p)$ is convex and has a local minimum, and because $g(0) > 0$ but $g(1) = 0$ and $g'(1) > 0$, its local minimum must be negative. Hence, $g(p)$ has a root, and it can be shown that it lies at

$$\tilde{p} = -\frac{1}{30}(586+45\sqrt{271})^{1/3} + \frac{59}{30(586+45\sqrt{271})^{1/3}} + \frac{7}{15} \approx 0.2794.$$

Hence, for $0 < p < \tilde{p}$, the condition above is satisfied and (2.3.1) is binding. For $\tilde{p} < p < 1$, (2.3.3) is the relevant constraint.

It remains to be shown that (2.3.2) is slack. Rearranging gives

$$\frac{c}{w} \leq p - \frac{9}{2}p^2 + \frac{25}{2}p^3 - 19p^4 + 15p^5 - 5p^6 \tag{2.6.3}$$

It suffices to show that the RHS of (2.6.3) is larger than the RHS (2.6.1), which is equivalent to $g_a(p) := 1-7p+18p^2-22p^3+10p^4 \geq 0$ for $0 < p \leq \tilde{p}$ and larger than the RHS of (2.6.2) for $\tilde{p} < p < 1$, which is equivalent to $g_b(p) := -3+19p-36p^2+30p^3-10p^4 \geq 0$.

The third derivative $g_a'''(p) = -132+240p$ is increasing and negative in the relevant range as $g_a'''(\tilde{p}) < 0$. It follows that $g_a''(p) = 36-132p+120p^2$ has a negative slope and is positive as $g_a''(\tilde{p}) > 0$. Therefore, $g_a'(p) = -7+36p-66p^2+40p^3$ is increasing and negative because $g_a'(\tilde{p}) < 0$. The original function $g_a(p)$ is decreasing but positive, as $g_a(\tilde{p}) > 0$. Hence, the condition above holds and (2.3.2) is slack for $0 < p < \tilde{p}$.

For $\tilde{p} < p < 1$, the third derivative $g_b'''(p) = 180-240p$ is decreasing and crosses the abscissa once from above because $g_b'''(\tilde{p}) > 0$ and $g_b'''(1) < 0$. The second derivative $g_b''(p) = -72+180p-120p^2$ hence is concave and has a local maximum at $p = 3/4$. As it is negative at its local maximum it is negative throughout the relevant range of p and therefore $g_b'(p) = 19 - 72p + 90p^2 - 40p^3$ has a negative slope. From $g_b'(\tilde{p}) > 0$ and $g_b'(1) < 0$ follows that it crosses the abscissa once from above and that $g_b(p)$ is concave and has a local maximum in the range of interest. As $g_b(\tilde{p}) > 0$ and $g_b(1) = 0$ (and $g_b'(1) < 0$) we can infer that it is positive in the range of interest, the condition above holds and (2.3.2) is also slack for $\tilde{p} < p < 1$. □

Proof of Lemma 2. The proof is divided in three parts according to the three ranges of p which differ in the decision rule—and hence in the probabilities of winning $\Pr(d_r|E_r)$—as outlined in Table 2.2.

For $p \leq 1/3$,

$$\begin{aligned}
&\Pr(d_r|0) = (1-p)^3 \\
&\Pr(d_r|1) = (1-p)^3 \\
&\Pr(d_r|2) = (1-p)^3 + 3p(1-p)^2p^2 \\
&\Pr(d_r|3) = (1-p)^3 + 3p(1-p)^2(3p^2(1-p)+p^3) + 3p^2(1-p)p^3.
\end{aligned}$$

Observe that (2.3.6) holds as long as $c \geq 0$, which is given by definition. Plugging in the relevant probabilities $\Pr(d_r|E_r)$ in (2.3.4) and (2.3.5) and rearranging gives

$$\frac{c}{w} \geq 3p^3 - 8p^4 + 8p^5 - 3p^6 \tag{2.6.4}$$

and

$$\frac{c}{w} \geq \frac{3}{2}p^3 - 3p^4 + \frac{3}{2}p^5, \tag{2.6.5}$$

respectively. (2.3.4) is the binding constraint if the RHS of (2.6.4) is larger than the RHS of (2.6.5), which is equivalent to $\frac{3}{2} - 5p + \frac{13}{2}p^2 - 3p^3 > 0$. Define the LHS of this inequality as $g(p)$ where $g'(p) = -5 + 13p - 9p^2$. $g'(p)$ is strictly concave and takes a global maximum of $-11/36$ at $p = 13/18$. Hence $g(p)$ is decreasing, and with $g(0) > 0$ and $g(1) = 0$ we have shown that the sign of $g(p)$ is nonnegative in the relevant range of p. The wage in that range of p hence is given by (2.3.4), the incentive constraint preventing the regulator to conduct three instead of zero searches.

For $1/3 < p \leq 2/3$,

$$\Pr(d_r|0) = (1-p)^3 + 3p(1-p)^2$$
$$\Pr(d_r|1) = (1-p)^3 + 3p(1-p)^2(1-p)$$
$$\Pr(d_r|2) = (1-p)^3 + 3p(1-p)^2((1-p)^2 + p^2)$$
$$\Pr(d_r|3) = (1-p)^3 + 3p(1-p)^2((1-p)^3 + 3p^2(1-p) + p^3) + 3p^2(1-p)p^3.$$

We show that both incentive constraints (2.3.5) and (2.3.6) are always slack as the probability difference on the LHS is positive for all values of p. Observe that compared to zero searches, searching twice strictly reduces the winning probability and the regulator has to incur effort costs of $2c$. Searching twice hence can never be optimal. The same holds for searching once, which also never is optimal. It remains to be shown that (2.3.4) is slack for values of p below $\hat{p}$ and binding otherwise. Using the relevant winning probabilities in (2.3.4) and rearranging yields

$$\frac{c}{w} \geq -3p^2 + 12p^3 - 18p^4 + 13p^5 - 4p^6. \tag{2.6.6}$$

Define as $g(p) = -3 + 12p - 18p^2 + 13p^3 - 4p^4$, which is the RHS of (2.6.6) with p^2 factored out. The value of $g(0)$ is negative and the value of $g(1)$ is zero, so there can be at most three real roots in the range of p. We determine the actual number of roots between 0 and 1 by analyzing the derivatives of $g(p)$. $g'''(p) = 78 - 96p$ is a linear decreasing function with a positive value at $p = 0$ and a negative value at $p = 1$ and one root in between. Hence, $g''(p) = -36 + 78p - 48p^2$ is a concave function with one maximum in the relevant range. As $g''(p)$ is negative at the root of $g'''(p)$, its maximum, the second derivative of $g(p)$ is strictly negative in the domain from 0 to 1. Therefore, the first derivative $g'(p) = 12 - 36p + 39p^2 - 16p^3$ is decreasing in this range and has one root as $g'(0)$ is positive and $g'(1)$ is negative. Finally, this implies that $g(p)$ is concave and has a maximum in the domain from 0 to 1. Accordingly, there is one root at $p = \hat{p}$ where

$$\hat{p} = \frac{1}{4}3^{\frac{1}{3}} - \frac{1}{4}3^{\frac{2}{3}} + \frac{3}{4} \approx 0.5905$$

in that interval as the value of $g(1)$ is zero. Taken together, the RHS of (2.6.6) has roots at 0, $\hat{p}$, and 1, is negative for $0 < p < \hat{p}$ and positive for $\hat{p} < p < 1$.

Since both c and w are positive, (2.6.6) is slack for $0 < p < \hat{p}$ and no positive wage induces the regulator to search three times. For $\hat{p} < p < 1$, this constraint is binding and yields the upper bound for the wage in the lemma.

For $p > 2/3$,

$$\begin{aligned}
\Pr(d_r|0) &= (1-p)^3 + 3p(1-p)^2 + 3p^2(1-p) \\
\Pr(d_r|1) &= (1-p)^3 + 3p(1-p)^2(1-p) + 3p^2(1-p)(1-p) \\
\Pr(d_r|2) &= (1-p)^3 + 3p(1-p)^2((1-p)^2 + p^2) + 3p^2(1-p)(1-p)^2 \\
\Pr(d_r|3) &= (1-p)^3 + 3p(1-p)^2((1-p)^3 + 3p^2(1-p) + p^3) \\
&\quad + 3p^2(1-p)((1-p)^3 + p^3).
\end{aligned}$$

The probability of winning when not exerting effort $\Pr(d_r|0)$ consists of the probabilities that the firm finds zero, one, or two pieces of information. Searching once reduces the chances of winning because given one or two pieces of information found by the firm, the decision is now made against the proposal only if the regulator does not find information. A similar argument establishes that $\Pr(d_r|2)$ and $\Pr(d_r|3)$ are also smaller than $\Pr(d_r|0)$. Hence, all constraints (2.3.4), (2.3.5), and (2.3.6) are slack and no positive wage induces the regulator to search for evidence. □

Proof of Lemma 3. We need to show that $\hat{w} \geq \overline{w}$. Several cases have to be considered. For $0 < p \leq \tilde{p}$, $\hat{w} \geq \overline{w}$ is equivalent to

$$\frac{c}{3p^3 - 8p^4 + 8p^5 - 3p^6} \geq \frac{c}{p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6}$$

or

$$1 - 5p + 13p^2 - 20p^3 + 18p^4 - 7p^5 \geq 0. \tag{2.6.7}$$

Define the LHS of (2.6.7) as $g(p)$. The fourth derivative $g''''(p) = 432 - 840p$ is decreasing and positive in the relevant range as $g''''(\tilde{p}) > 0$. Hence, $g'''(p) = -120 + 432p - 420p^2$ is increasing and negative as $g'''(\tilde{p}) < 0$. It follows that $g''(p) = 26 - 120p + 216p^2 - 140p^3$ has a negative slope and is positive throughout the range of interest as $g''(\tilde{p}) > 0$. The first derivative $g'(p) = -5 + 26p - 60p^2 + 72p^3 - 35p^4$ therefore increases but is negative as $g'(\tilde{p}) < 0$. From this we know that $g(p)$ is decreasing and positive as $g(\tilde{p}) > 0$ and the LHS of (2.6.7) is positive in the relevant range.

For $p > \tilde{p}$, the relevant comparison is

$$\frac{c}{3p^3 - 8p^4 + 8p^5 - 3p^6} \geq \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6}$$

or

$$1 - 4p + (19/3)p^2 - 5p^3 + 2p^4 - (1/3)p^5 \geq 0. \tag{2.6.8}$$

Define the LHS of (2.6.8) as $g(p)$. The fourth derivative $g''''(p) = 48 - 40p$ is decreasing and positive. Hence, $g'''(p) = -30 + 48p - 20p^2$ is increasing and negative as $g'''(1) < 0$. Therefore, $g''(p) = 38/3 - 30p + 24p^2 - (20/3)p^3$ is decreasing and positive as $g''(1) = 0$. The first derivative $g'(p) = -4 + (38/3)p - 15p^2 + 8p^3 - (5/3)p^4$ hence is increasing and negative as $g'(1) = 0$. It follows that $g(p)$ is decreasing and positive as $g(1) = 0$, which implies that the LHS of (2.6.8) is positive in the relevant range.

For $1/3 < p \leq \hat{p}$, the regulator will not search for information for any positive value of w and hence, $\hat{w} \geq \overline{w}$ is always satisfied.

For $\hat{p} < p \leq 2/3$, the relevant comparison is

$$\frac{-3c}{9p^2 - 36p^3 + 54p^4 - 39p^5 + 12p^6} \geq \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6}$$

or

$$1 - p - (8/3)p^2 + 5p^3 - 3p^4 + (2/3)p^5 \geq 0. \tag{2.6.9}$$

Define as $g(p)$ the LHS of (2.6.9). The fourth derivative $g''''(p) = -72 + 80p$ is increasing and negative in the relevant range as $g''''(2/3) < 0$. Hence, $g'''(p) = 30 - 72p + 40p^2$ has a negative slope and crosses the abscissa once as $g'''(\hat{p}) > 0$ and $g'''(2/3) < 0$. It follows that $g''(p) = -16/3 + 30p - 36p^2 + (40/3)p^3$ is concave with a local maximum. As both $g''(\hat{p})$ and $g''(2/3)$ are positive, the second derivative is positive throughout the range of interest. The first derivative $g'(p) = -1 - (16/3)p + 15p^2 - 12p^3 + (10/3)p^4$ therefore is increasing and negative because $g'(2/3) < 0$. From this we know that $g(p)$ is decreasing and positive as $g(2/3) > 0$. This implies that inequality (2.6.9) strictly holds.

In the remaining interval $2/3 < p < 1$, the regulator will not search for any positive value of w and hence $\hat{w} \geq \overline{w}$ is satisfied. $\square$

Proof of Lemma 4. We show that $\overline{w}$ is either smaller than the lower bound or larger than the upper bound of the wage necessary for the other two equilibria where the regulator searches once or twice.

We start by showing that for $0 < p < 1/2$, where there are equilibria in which the regulator searches once or twice, $\underline{w}_1$ is smaller than $\underline{w}_2$ and hence is the relevant wage to be compared with $\overline{w}$ in order to determine $\min\{\overline{w}, \underline{w}_1, \underline{w}_2\}$. The comparison

$$\underline{w}_1 = \frac{c}{3p^2 - 6p^3 + 3p^4} \leq \frac{c}{3p^2 - 9p^3 + 12p^4 - 6p^5} = \underline{w}_2$$

can be simplified to $1 - 3p + 2p^2 \geq 0$. The LHS is decreasing in the relevant range as the derivative $-3 + 4p$ is negative for $0 < p < 1/2$. At $1/2$, the LHS is zero, and hence, the condition above holds.

The relevant upper bound is given by $\overline{w}_2$ if

$$\overline{w}_2 = \frac{c}{3p^2 - 12p^3 + 24p^4 - 24p^5 + 9p^6} \geq \frac{c}{3p^2 - 9p^3 + 9p^4 - 3p^5} = \overline{w}_1$$

or $3p^3(1 - 5p + 7p^2 - 3p^3) \geq 0$. Define the term in parentheses as $g(p)$. The second derivative $g''(p) = 14 - 18p$ is positive and decreasing in the relevant range as $g''(0.5)$ is positive. Hence, $g'(p) = -5 + 14p - 9p^2$ is increasing in the negative domain because $g'(0.5)$ is negative. This implies that $g(p)$ is decreasing, and we know further that it must have one root as $g(0)$ is positive but $g(0.5)$ is negative. It can be shown that $g(p)$ crosses the abscissa at $p = 1/3$. Hence, for $0 < p < 1/3$, the relevant upper bound is given by $\overline{w}_2$ and by $\overline{w}_1$ for $1/3 < p < 1/2$.

For $0 < p \leq \tilde{p}$, we start by determining the relevant lower bound below which no search by the regulator is the unique equilibrium. The relevant comparison is

$$\overline{w} = \frac{c}{p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6} \leq \frac{c}{3p^2 - 6p^3 + 3p^4} = \underline{w}_1$$

or $p(1 - 8p + 22p^2 - 31p^3 + 26p^4 - 10p^5) \geq 0$. Define the term in parentheses as $g(p)$. The fourth derivative $g''''(p) = 624 - 1200p$ is positive in the relevant range $[0, \tilde{p}]$. Therefore, $g'''(p) = -186 + 624p - 600p^2$ is increasing in this range and negative for both $p = 0$ and $p = \tilde{p}$ and hence has no root. The second derivative $g''(p) = 44 - 186p + 312p^2 - 200p^3$ is strictly decreasing in that interval and takes on a positive value both at $p = 0$ and at $p = \tilde{p}$, and thus has no root in the relevant interval. The first derivative $g'(p) = -8+44p-93p^2+104p^3-50p^4$ is negative for both $p = 0$ and $p = \tilde{p}$ and thus has no root as it is strictly increasing in that range of p. Hence, $g(p)$ is strictly decreasing in that interval and has one root as $g(0)$ is positive and $g(\tilde{p})$ is negative. It can be shown that the root lies at

$$\dot{p} = \frac{2}{5} + \frac{1}{10\sqrt{\frac{6}{-54+5\left(486-27\sqrt{323}\right)^{1/3}+15\left(18+\sqrt{323}\right)^{1/3}}}}$$
$$-\frac{1}{2}\left[-\frac{18}{25}-\frac{1}{30}\left(486-27\sqrt{323}\right)^{1/3}-\frac{1}{10}\left(18+\sqrt{323}\right)^{1/3}\right.$$
$$\left.+\frac{3}{25}\sqrt{\frac{6}{-54+5\left(486-27\sqrt{323}\right)^{1/3}+15\left(18+\sqrt{323}\right)^{1/3}}}\right]^{1/2} \approx 0.23802.$$

It follows that $\overline{w}$ is smaller than $\underline{w}_1$ up to that root. This implies that the relevant lower bound is given by $\overline{w}$ from $p = 0$ up to the root and then by $\underline{w}_1$ until $p = \tilde{p}$.

Next, we show that $\overline{w}$ lies below the relevant upper bound in this range given by $\overline{w}_2$. The comparison

$$\overline{w} = \frac{c}{p-5p^2+16p^3-28p^4+26p^5-10p^6} \leq \frac{c}{3p^2-12p^3+24p^4-24p^5+9p^6} = \overline{w}_2$$

can be simplified to $1-8p+28p^2-52p^3+50p^4-19p^5 \geq 0$. Define the LHS of the last inequality as $g(p)$. The fourth derivative $g''''(p) = 1200-1140p$ is positive throughout the relevant range and hence, $g'''(p) = -312+1200p-1140p^2$ is increasing. Observe that $g'''(\tilde{p})$ is negative which implies that the third derivative is negative throughout the range. Therefore, $g''(p) = 56-312p+600p^2-380p^3$ is decreasing and positive, as $g''(\tilde{p})$ is positive. This indicates that $g'(p) = -8+56p-156p^2+200p^3-95p^4$ increases in that range of p, and as $g'(\tilde{p})$ is negative, the first derivative is negative throughout. Taken together, we now know that $g(p)$ is decreasing, and as $g(\tilde{p})$ is positive, that it has no root in that range. The condition above holds.

For $\tilde{p} < p < 1/2$ we show first that

$$\underline{w}_1 = \frac{c}{3p^2-6p^3+3p^4} \leq \frac{c}{p-4p^2+\frac{28}{3}p^3-13p^4+10p^5-\frac{10}{3}p^6} = \overline{w}$$

or $-3+21p-46p^2+48p^3-30p^4+10p^5 \geq 0$. Define the LHS as $g(p)$. The fourth derivative $g''''(p) = -720+1200p$ is increasing and negative in the relevant range as $g''''(1/2) < 0$, implying a negative slope of $g'''(p) = 288-720p+600p^2$. The positive value of $g'''(1/2)$ shows that the third derivative is positive throughout. Hence, $g''(p) = -92+288p-360p^2+200p^3$ increases and is negative as $g''(1/2) < 0$, also implying that $g'(p) = 21-92p+144p^2-120p^3+50p^4$ is decreasing. As $g'(\tilde{p}) > 0$ but $g'(1/2) < 0$ the first derivative has one root in the range of interest. The function $g(p)$ hence first is increasing and then decreasing, and because both $g(\tilde{p})$ and $g(1/2)$ are positive, it is positive throughout the range of interest and hence the condition above holds. Therefore, $\underline{w}_1$ is the relevant lower bound for this range.

Second, for the same range of p we show that

$$\overline{w} = \frac{c}{p-4p^2+\frac{28}{3}p^3-13p^4+10p^5-\frac{10}{3}p^6} \leq \frac{c}{3p^2-9p^3+9p^4-3p^5} = \overline{w}_1$$

or $3-21p+55p^2-66p^3+39p^4-10p^5 \geq 0$. Define the LHS as $g(p)$. The fourth derivative $g''''(p) = 936-1200p$ is decreasing in the positive domain, and hence $g'''(p) = -396+936p+-600p^2$ is increasing. Because $g'''(1/2) < 0$ the third derivative is negative throughout, implying a negative slope of $g''(p) = 110-396p+468p^2-200p^3$. The second derivative is positive throughout as $g''(1/2) > 0$, and thus $g'(p) = -21+110p-198p^2+156p^3-50p^4$ is increasing. Since $g'(\tilde{p}) < 0$ and $g'(1/2) > 0$, the first derivative has one root, and hence $g(p)$ is decreasing first and then increasing. It can be easily verified that $g(p)$ is positive at this root of $g'(p)$ and hence is positive throughout, satisfying the condition above. Therefore, there is no range of p and w below $\overline{w}$ where not searching is the unique equilibrium.

Taken together, we can now define

$$\tilde{w} = \begin{cases} \overline{w} & \text{for} \quad 0 < p < \dot{p} \\ \underline{w}_1 & \text{for} \quad \dot{p} < p < 1/2 \\ \underline{w}_2 & \text{for} \quad 1/2 < p < 1 \end{cases}$$

as used in the proposition.

For $1/2 < p < 1$, where the equilibrium in which the regulator searches twice also exists, we now show that the lower bound is given by $\underline{w}_2$ and that there also exists an area below $\overline{w}$ and above $\underline{w}_2$ where not searching is the unique equilibrium. The first condition is

$$\underline{w}_2 = \frac{c}{3p^2 - (15/2)p^3 + (15/2)p^4 - 3p^5} \leq \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6} = \overline{w}$$

or $-12 + 84p - 202p^2 + 246p^3 - 156p^4 + 40p^5 \geq 0$. Define the LHS as $g(p)$. The fourth derivative $g''''(p) = -3744 + 4800p$ is increasing and crosses the abscissa once from below. This implies that $g'''(p) = 1476 - 3744p + 2400p^2$ first has a decreasing and then an increasing slope, and as it is positive at the root of $g''''(p)$, it is positive throughout the relevant range. From this fact we know that $g''(p) = -404 + 1476p - 1872p^2 + 800p^3$ is increasing, and it is negative and has no root in the relevant range as $g''(1) = 0$. Hence, $g'(p) = 84 - 404p + 738p^2 - 624p^3 + 200p^4$ is decreasing, and has one root as $g'(1/2) > 0$ but $g'(1) < 0$. The original function $g(p)$ thus has a local maximum at this root and is positive throughout as both $g(1/2)$ and $g(1)$ are positive, and the condition above is satisfied.

Lastly, we show that

$$\overline{w}_2 = \frac{c}{3p^2 - 12p^3 + 24p^4 - 24p^5 + 9p^6} \leq \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6} = \overline{w}$$

or $-3 + 21p - 64p^2 + 111p^3 - 102p^4 + 37p^5 \geq 0$ for some values of p. Define the LHS as $g(p)$. The fourth derivative $g''''(p) = -2448 + 4440p$ crosses the abscissa once from below, implying a local minimum of $g'''(p) = 666 - 2448p + 2220p^2$. As $g'''(1/2) < 0$ and $g'''(1) > 0$, the third derivative has one root in the range of interest. Hence, $g''(p) = -128 + 666p - 1224p^2 + 740p^3$ also has a local minimum in the relevant range. Similarly, $g''(1/2) < 0$ and $g''(1) > 0$, such that $g''(p)$ also crosses the abscissa once from below, implying one local minimum of $g'(p) = 21 - 128p + 333p^2 - 408p^3 + 185p^4$. As both $g'(1/2)$ and $g'(1)$ are positive but there are negative values of $g'(p)$ in between, the first derivative first crosses the abscissa from above and then again from below. Hence, $g(p)$ first has a local maximum and then a local minimum. As $g(1/2)$ is positive, the local maximum must be in the positive domain, and because $g(1) = 0$ and $g'(1) > 0$, the graph crosses the abscissa from below at $p = 1$ and hence the local minimum is in the negative domain. This implies that there must be a root in between. It can be shown that this root lies at

$$\ddot{p} = \frac{65}{148} + \frac{1}{148\sqrt{\frac{3}{-941-9176\left(\frac{2}{8561+9\sqrt{916593}}\right)^{1/3}+74\times 2^{2/3}\left(8561+9\sqrt{916593}\right)^{1/3}}}}$$
$$+\frac{1}{2}\left[-\frac{941}{8214}+\frac{62}{111}\left(\frac{2}{8561+9\sqrt{916593}}\right)^{1/3}-\frac{1}{111}\left(\frac{1}{2}\left(8561+9\sqrt{916593}\right)\right)^{1/3}\right.$$
$$\left.+\frac{29241\sqrt{\frac{3}{-941-9176\left(2/(8561+9\sqrt{916593})\right)^{1/3}+74\times 2^{2/3}\left(8561+9\sqrt{916593}\right)^{1/3}}}}{2738}\right] \approx 0.81216.$$

Therefore, $\overline{w}_2$ is smaller than $\overline{w}$ for $1/2 < p < \ddot{p}$ and there are levels of w between $\overline{w}_2$ and $\overline{w}$ for which not searching is the unique equilibrium. □

Proof of Lemma 5. The regulator's probability of winning contingent on the number of searches is given by

$$\begin{aligned}
&\Pr(d_r|0) = 0\\
&\Pr(d_r|1) = (1-p)^2 p\\
&\Pr(d_r|2) = 2p(1-p)p^2 + (1-p)^2\left(2p(1-p)+p^2\right).
\end{aligned}$$

Using the respective probabilities and rearranging (2.3.7) and (2.3.8) gives

$$\frac{c}{w} \leq p - 3p^2 + 5p^3 - 3p^4 \tag{2.6.10}$$

and

$$\frac{c}{w} \leq p - \frac{5}{2}p^2 + 3p^3 - \frac{3}{2}p^4, \tag{2.6.11}$$

respectively. (2.3.7) is the relevant constraint if the RHS of (2.6.10) is smaller than the RHS of (2.6.11) or $p^2 g(p) \geq 0$ where $g(p) = 1/2 - 2p + (3/2)p^2$. The first derivative $g'(p) = -2 + 3p$ crosses the abscissa once from below and hence, $g(p)$ is convex. It is easy to verify that $g(p)$ has roots at $p^* = 1/3$ and $p = 1$ and hence is positive for values of p below p^* and negative for values above. Thus, (2.3.7) is binding for small p while (2.3.8) is relevant for large p and the lemma follows. □

Proof of Proposition 1. We have to show that $\overline{w} > \underline{w}$ for all p. For $0 < p < \tilde{p}$, the relevant comparison is

$$\overline{w} = \frac{c}{p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6} \geq \frac{c}{p - 3p^2 + 5p^3 - 3p^4} = \underline{w}$$

or $2 - 11p + 25p^2 - 26p^3 + 10p^4 \geq 0$. Define the LHS as $g(p)$. The third derivative $g'''(p) = -156 + 240p$ is increasing and negative in the relevant range as $g'''(\tilde{p}) < 0$. This implies that $g''(p) = 50 - 156p + 120p^2$ is decreasing, and it is positive as its value at $\tilde{p}$ is positive. Hence, $g'(p) = -11 + 50p - 78p^2 + 40p^3$ is increasing, and from $g'(\tilde{p}) < 0$ we know that it is negative. Due to this fact, $g(p)$ is decreasing, and as $g(\tilde{p}) > 0$, it is positive and the condition above holds.

For $\tilde{p} < p < 1/3$, the relevant comparison is

$$\overline{w} = \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6} \geq \frac{c}{p - 3p^2 + 5p^3 - 3p^4} = \underline{w}$$

or $3 - 13p + 30p^2 - 30p^3 + 10p^4 \geq 0$. Define the LHS as $g(p)$. The third derivative $g'''(p) = -180 + 240p$ is increasing and negative in the range of interest as $g'''(1/3) < 0$. Hence, $g''(p) = 60 - 180p + 120p^2$ is decreasing and positive as $g''(1/3) > 0$. The slope of $g'(p) = -13 + 60p - 90p^2 + 40p^3$ thus is positive, and the first derivative is negative because $g'(1/3) < 0$. From this we know that $g(p)$ is decreasing. As $g(1/3) > 0$, the LHS is positive throughout the range of interest and the above condition is satisfied.

For $1/3 < p < 1$, the relevant comparison is

$$\overline{w} = \frac{c}{p - 4p^2 + \frac{28}{3}p^3 - 13p^4 + 10p^5 - \frac{10}{3}p^6} \geq \frac{c}{p - \frac{5}{2}p^2 + 3p^3 - \frac{3}{2}p^4} = \underline{w}$$

or $9 - 38p + 69p^2 - 60p^3 + 20p^4 \geq 0$. Define the LHS as $g(p)$. The third derivative $g'''(p) = -360 + 480p$ is increasing and crosses the abscissa once from below as $g'''(1/3) < 0$ and $g'''(1) > 0$. Hence $g''(p) = 138 - 360p + 240p^2$ is convex and has a local minimum at $p = 3/4$. As its value is positive at the local minimum it is positive throughout the range of interest. Therefore, $g'(p) = -38 + 138p - 180p^2 + 80p^3$ is increasing and negative as $g'(1/3) < 0$ and $g'(1) = 0$. That being the case, the LHS decreases in p and is positive as $g(1/3) > 0$ and $g(1) = 0$. The above condition holds. □

Proof of Proposition 2. First, we show that $\underline{w}$ is smaller than $\tilde{w}$ for $p < 1/3$. For $0 < p < \dot{p}$ where the relevant upper bound is $\overline{w}$, we check whether

$$\underline{w} = \frac{c}{p - 3p^2 + 5p^3 - 3p^4} \leq \frac{c}{p - 5p^2 + 16p^3 - 28p^4 + 26p^5 - 10p^6} = \overline{w}$$

or $2 - 11p + 25p^2 - 26p^3 + 10p^4 \geq 0$. Define the LHS as $g(p)$. Observe that $g'''(p) = -156 + 240p$ is increasing and negative throughout the relevant range, which means that $g''(p) = 50 - 156p + 120p^2$ is decreasing. As $g''(\dot{p}) > 0$, the second derivative is positive, which implies a positive slope for $g'(p) = -11 + 50p - 78p^2 + 40p^3$. As $g'(\dot{p}) < 0$ we know that the first derivative is negative and by that we know that $g(p)$ is decreasing. The fact that $g(\dot{p}) > 0$ implies that the LHS is positive throughout the relevant range and that the conditions above holds.

For $\dot{p} < p < 1/3$, the relevant upper bound for the wage is $\underline{w}_1$ and hence, the relevant comparison is

$$\underline{w} = \frac{c}{p - 3p^2 + 5p^3 - 3p^4} \leq \frac{c}{3p^2 - 6p^3 + 3p^4} = \underline{w}_1$$

or $1 - 6p + 11p^2 - 6p^3 \geq 0$. Define the LHS as $g(p)$. The second derivative $g''(p) = 22 - 36p$ is decreasing and positive for $p = 1/3$ and thus positive in the relevant range so that $g'(p) = -6 + 22p - 18p^2$ is increasing. As $g'(1/3) < 0$, the first derivative is negative which implies a negative slope of $g(p)$. Together with the facts that $g(\dot{p}) > 0$ and $g(1/3) = 0$ we know that $g(p)$ is positive in the relevant range and the condition above holds.

Next we show that for values of p above $1/3$, $\underline{w}$ is never below $\tilde{w}$. First, for $1/3 < p < 1/2$, the relevant comparison is

$$\underline{w} = \frac{c}{p - \frac{5}{2}p^2 + 3p^3 - \frac{3}{2}p^4} \geq \frac{c}{3p^2 - 6p^3 + 3p^4} = \underline{w}_1$$

or $-2 + 11p - 18p^2 + 9p^3 \geq 0$. Define the LHS as $g(p)$. The second derivative $g''(p) = -36 + 54p$ is increasing and negative for $p = 1/2$ and thus negative in the relevant range, implying a negative slope of $g'(p) = 11 - 36p + 27p^2$. As $g'(1/3) > 0$ and $g'(1/2) < 0$, there must be one root of the first derivative in the relevant range. This implies that $g(p)$ is concave and has a local maximum in that range, and because $g(1/3) = 0$ and $g(1/2) > 0$, it is positive throughout the range and the condition above holds. Second, for $1/2 < p < 1$, the relevant comparison is

$$\underline{w} = \frac{c}{p - \frac{5}{2}p^2 + 3p^3 - \frac{3}{2}p^4} \geq \frac{c}{3p^2 - \frac{15}{2}p^3 + \frac{15}{2}p^4 - 3p^5} = \underline{w}_2$$

or $-2 + 11p - 21p^2 + 18p^3 - 6p^4 \geq 0$. Define the LHS as $g(p)$. The third derivative $g'''(p) = 108 - 144p$ is decreasing and crosses the abscissa once because $g'''(1/2) > 0$ and $g'''(1) < 0$. This implies that $g''(p) = -42 + 108p - 72p^2$ is concave and has a local maximum at $p = 108/144$ in that range. As $g''(108/144) < 0$, the second derivative is negative throughout. The first derivative $g'(p) = 11 - 42p + 54p^2 - 24p^3$ hence is decreasing in the relevant range. As $g'(1/2) > 0$ and $g'(1) < 0$ the first derivative has one root and $g(p)$ is concave and has a local maximum. From $g(1/2) > 0$ and $g(1) = 0$ we can infer that $g(p)$ is positive in the relevant range and the condition above holds.

The second part of the proof follows from the earlier analysis. For $1/2 < p < \ddot{p}$, $\overline{w}_2 < \overline{w}$ from Lemma 4 and $\underline{w} < \overline{w}$ from Proposition 1. Hence, whenever $w > \max\{\overline{w}_2, \underline{w}\}$ it is below $\overline{w}$ and the unique equilibrium exists for this range of p. □

Proof of Proposition 3. There are three cases. For $0 < p < 1/3$, the losses in Situation 1 L_1 are larger than the losses in Situation 2 L_2 if

$$L_1 = 9p^3 - 21p^4 + 18p^5 - 6p^6 \geq p - 5p^2 + 14p^3 - 22p^4 + 18p^5 - 6p^6 = L_2$$

or $-1 + 5p - 5p^2 + p^3 \geq 0$. Define the LHS as $g(p)$. The second derivative $g''(p) = -10 + 6p$ is increasing and negative throughout the relevant range as $g''(1/3) < 0$. Hence $g'(p) = 5 - 10p + 3p^2$ is decreasing, and it is positive because $g'(1/3) > 0$. It follows that $g(p)$ has a positive slope. As $g(0) < 0$ but $g(1/3) > 0$ it crosses the abscissa once from below in the range of interest. It can be shown that this root lies at $p = 2 - \sqrt{3}$, and hence the condition above holds for values of p larger than that.

For $1/3 < p < 2/3$, the relevant comparison is

$$L_1 = 3p - 15p^2 + 30p^3 - 30p^4 + 18p^5 - 6p^6 \geq p - 5p^2 + 14p^3 - 22p^4 + 18p^5 - 6p^6 = L_2$$

or $1 - 5p + 8p^2 - 4p^3 \geq 0$. Define the LHS as $g(p)$. The second derivative $g''(p) = 16 - 24p$ is decreasing and positive as $g''(2/3) = 0$. Thus, $g'(p) =$

$-5+16p-12p^2$ is increasing and crosses the abscissa once from below as $g'(1/3)<0$ and $g'(2/3)>0$. It can be shown that $g'(1/2)=0$. From this we can infer that $g(p)$ is convex and has a local minimum. Its value at the local minimum is zero, so the condition above holds in the relevant range of p.

Lastly, for $2/3<p<1$, the relevant condition is

$$L_1 = 3p-9p^2+15p^3-21p^4+18p^5-6p^6 \geq p-5p^2+14p^3-22p^4+18p^5-6p^6 = L_2$$

or $2-4p+p^2+p^3 \geq 0$. Define the LHS as $g(p)$. The second derivative $g''(p)=2+6p$ is increasing and positive throughout the whole range of interest, implying a positive slope for $g'(p)=-4+2p+3p^2$. As $g'(2/3)<0$ and $g'(1)>0$, the first derivative crosses the abscissa once from below. From this we can infer that $g(p)$ is convex and has a local minimum. We know that this local minimum is negative as $g(2/3)>0$, $g(1)=0$, and $g'(1)>0$, and therefore, $g(p)$ crosses the abscissa once from above. It can be shown that this root lies at $p=\sqrt{3}-1$, and the condition above holds for values of p smaller than that. □

2.6.2 APPENDIX B: OTHER PURE-STRATEGY EQUILIBRIA

We show that equilibria in pure strategies exist in which the regulator searches once or twice given three searches by the firm.

	{0,0}	{1,0}	{2,0}	{3,0}	{0,1}	{1,1}	{2,1}	{3,1}
$0<p\leq 1/2$	R	F	F	F	R	R	F	F
$1/2<p\leq 1$	R	R	F	F	R	R	R	F

Table 2.4: Decision rule for $\mu_f=3$ and $\mu_r=1$.

We begin with the equilibrium where the regulator searches once. In this case, the judge decides according to the decision rule given in Table 2.4. If the expected value of information against the proposal on the two remaining dimensions where the regulator did not search, $2\times p^2+1\times p(1-p)=2p$, is larger than the value of evidence in favor of the proposal, then the decision is made against the proposal. In the two cases where the firm has one more piece of information than the firm ({1,0} and {2,1}), the expected value of contra information is large enough to tip the decision towards the regulator for $p > 1/2$. We again assume that when the regulator presents more than one piece of information out of equilibrium, the judge believes that the regulator did search in all other dimensions as well and did not discover any evidence. Hence, the decision rule under full information applies out of equilibrium.

The following conditions must hold such that searching once is optimal for the regulator, given three searches by the firm and the corresponding belief by the judge, $\mu_f = 3$, $\mu_r = 1$:

$$\Pr(d_r|1)w - c \geq \Pr(d_r|3)w - 3c \tag{2.6.12}$$

$$\Pr(d_r|1)w - c \geq \Pr(d_r|2)w - 2c \tag{2.6.13}$$

$$\Pr(d_r|1)w - c \geq \Pr(d_r|0)w. \tag{2.6.14}$$

We have the following result.

Lemma 6. *If and only if* $\underline{w}_1 \leq w \leq \overline{w}_1$ *and* $0 < p < 1/2$*, where* $\underline{w}_1 = c/(3p^2 - 6p^3 + 3p^4)$ *and* $\overline{w}_1 = c/(3p^2 - 9p^3 + 9p^4 - 3p^5)$*, there exists an equilibrium where the regulator searches once, the firm searches three times, and the judge has beliefs* $\mu_f = 3$ *and* $\mu_r = 1$*. For* $1/2 \leq p < 1$*, only one search by the regulator never is optimal.*

Proof. First, for $0 < p < 1/2$, the winning probabilities for the regulator are

$$\begin{aligned}
\Pr(d_r|0) &= (1-p)^3 \\
\Pr(d_r|1) &= (1-p)^3 + 3p(1-p)^2 p \\
\Pr(d_r|2) &= (1-p)^3 + 3p(1-p)^2 \left(2p(1-p) + p^2\right) \\
\Pr(d_r|3) &= (1-p)^3 + 3p(1-p)^2 \left(1 - (1-p)^3\right) + 3p^2(1-p)p^3.
\end{aligned}$$

The incentive constraint (2.6.14), which ensures that one search is better than no search, gives the lower bound for the wage $\underline{w}_1 = c/(3p^2 - 6p^3 + 3p^4)$. For the moment, ignore (2.6.12). Condition (2.6.13), which ensures that one search is more profitable than two searches, gives the upper bound for the wage, $\overline{w}_1 = c/(3p^2 - 9p^3 + 9p^4 - 3p^5)$. The upper bound $\overline{w}_1$ is above the lower bound $\underline{w}_1$ if

$$\frac{c}{3p^2 - 6p^3 + 3p^4} < \frac{c}{3p^2 - 9p^3 + 9p^4 - 3p^5}$$

or $p^3(3-(6p-3p^2)) > 0$. The term $6p-3p^2$ has its global maximum at $p=1$ with value 3 and is strictly concave, hence the condition is satisfied for the relevant range of p, $0 < p < 1/2$. It remains to be checked that (2.6.12) is slack. Plugging $\overline{w}_1$ into (2.6.12) yields

$$\frac{-6p^2+21p^3-27p^4+12p^5}{3p^2-9p^3+9p^4-3p^5}c \geq -2c$$

or $p^3\left(1-(3p-2p^2)\right) \geq 0$. The term $3p-2p^2$ is strictly concave and has its maximum at $p=3/4$. It is strictly increasing in the relevant range $0 < p < 1/2$ and takes on value 1 at $p=1/2$. Hence, the condition is satisfied for that range and (2.6.12) is slack.

For $1/2 \leq p < 1$, the winning probabilities for the regulator are

$$\begin{aligned}
\Pr(d_r|0) &= (1-p)^3+3p(1-p)^2\\
\Pr(d_r|1) &= (1-p)^3+3p(1-p)^2\\
\Pr(d_r|2) &= (1-p)^3+3p(1-p)^2+3p^2(1-p)2p(1-p)\\
\Pr(d_r|3) &= (1-p)^3+3p(1-p)^2+3p^2(1-p)(3p(1-p)^2+p^3).
\end{aligned}$$

It is obvious that for any positive value of search cost c and non-negative benefit w, the constraint (2.6.14) can never hold. Hence, an equilibrium with one search effort by the regulator does not exist for $1/2 \leq p < 1$. □

As the next step, we characterize an equilibrium where the regulator searches twice. The judge decides in favor of the party that delivers more pieces of information. In case of a tie ({0,0}, {1,1}, {2,2}) the decision is made in favor of the regulator as the expected value of contra information on the third dimension where no search has taken place is positive.[19] We again assume that when the regulator presents more than two pieces of information out of equilibrium, the judge believes that the regulator did search in the third dimension as well and did not discover any evidence. Hence, the decision rule under full information applies out of equilibrium.

[19]Note that when the judge receives the information {3,3} out of equilibrium, the decision is made for the firm.

The following conditions must hold such that two searches are optimal for the regulator, given three searches by the firm and the corresponding belief by the judge, $\mu_f = 3$, $\mu_r = 2$.

$$\Pr(d_r|2)w - 2c \geq \Pr(d_r|3)w - 3c \tag{2.6.15}$$
$$\Pr(d_r|2)w - 2c \geq \Pr(d_r|1)w - c \tag{2.6.16}$$
$$\Pr(d_r|2)w - 2c \geq \Pr(d_r|0)w \tag{2.6.17}$$

The three conditions guarantee that the regulator prefers two searches to three, one, and zero searches.

Lemma 7. *If and only if* $\underline{w}_2 \leq w \leq \overline{w}_2$, *where*

$$\underline{w}_2 = \begin{cases} c/\left(3p^2 - 9p^3 + 12p^4 - 6p^5\right) & \text{for } 0 < p < 1/2 \\ c/\left(3p^2 - (15/2)p^3 + (15/2)p^4 - 3p^5\right) & \text{for } 1/2 \leq p < 1, \end{cases}$$

and $\overline{w}_2 = c/\left(3p^2 - 12p^3 + 24p^4 - 24p^5 + 9p^6\right)$, *there exists an equilibrium where the regulator searches twice, the firm searches in all three dimensions and the judge has beliefs* $\mu_f = 3$ *and* $\mu_r = 2$.

Proof. The winning probabilities for the regulator are

$$\Pr(d_r|0) = (1-p)^3$$
$$\Pr(d_r|1) = (1-p)^3 + 3p(1-p)^2 p$$
$$\Pr(d_r|2) = (1-p)^3 + 3p(1-p)^2(2p(1-p) + p^2) + 3p^2(1-p)p^2$$
$$\Pr(d_r|3) = (1-p)^3 + 3p(1-p)^2(1-(1-p)^3) + 3p^2(1-p)(3p^2(1-p) + p^3).$$

The upper bound for the wage $\overline{w}_2 = c/\left(3p^2 - 12p^3 + 24p^4 - 24p^5 + 9p^6\right)$ is given by condition (2.6.15). Conditions (2.6.16) and (2.6.17) can be written as

$$w \geq \frac{c}{3p^2 - 9p^3 + 12p^4 - 6p^5}$$

and

$$w \geq \frac{c}{3p^2 - (15/2)p^3 + (15/2)p^4 - 3p^5},$$

respectively. (2.6.16) is binding if

$$\frac{c}{3p^2 - 9p^3 + 12p^4 - 6p^5} < \frac{c}{3p^2 - (15/2)p^3 + (15/2)p^4 - 3p^5} \tag{2.6.18}$$

or $p^3\left(-\frac{3}{2}+\frac{9}{2}p-3p^2\right)>0$. The polynomial in parentheses is negative at $p=1$ and equal to zero at $p=1$. It has another root at $p=1/2$ and hence is negative for values of p below $1/2$ and positive for values of p above. Taken together with the root at $p=0$ from the term p^3, condition (2.6.18) does not hold for $0<p\leq 1/2$ and holds for $1/2<p<1$. Hence, constraint (2.6.17) is binding for smaller p and (2.6.16) for larger p.

It remains to be shown that the upper bound $\overline{w}_2$ lies above the lower bound $\underline{w}_2$. For $0<p\leq 1/2$, we need to check whether

$$\frac{c}{3p^2-9p^3+12p^4-6p^5}<\frac{c}{3p^2-12p^3+24p^4-24p^5+9p^6},$$

which is equivalent to $p^3(3-12p+18p^2-9p^3)>0$. Define the term in parentheses as $g(p)=3-12p+18p^2-9p^3$. The second derivative $g''(p)=36-54p$ is positive for $p=0$ and negative for $p=1$ and has one root at $p=2/3$. Hence, $g'(p)=-12+36p-27p^2$ is strictly concave and takes on value 0 at its global maximum. The original function $g(p)$ is positive for $p=0$ and zero for $p=1$. It is non-increasing throughout $[0,1]$, convex up to the root of $g'(p)$ and concave thereafter. Hence it cannot have another root in the relevant range. Taken together, the condition is above satisfied and $\overline{w}_2$ lies above $\underline{w}_2$ for $0<p\leq 1/2$. For $1/2<p<1$, the comparison is

$$\frac{c}{3p^2-(15/2)p^3+(15/2)p^4-3p^5}<\frac{c}{3p^2-12p^3+24p^4-24p^5+9p^6}$$

or $p^3(3-11p+14p^2-6p^3)>0$. Let $g(p)=3-11p+14p^2-6p^3$. The second derivative $g''(p)=36-54p$ crosses the abscissa once from above. Hence, $g'(p)=-12+36p-27p^2$ is concave and has its global maximum at $p=7/9$ with a value of $-1/3$. Thus, the original function $g(p)$ is falling in the interval $[0,1]$, is positive at $p=0$ and equal to 0 at $p=1$, and therefore cannot have another root in that interval. Taken together, the condition above is also satisfied for values of p between $1/2$ and 1. $\square$

CHAPTER 3

MERGERS IN BIDDING MARKETS

Abstract

We analyze the effects of horizontal mergers in a dynamic market environment consistent with the features of a bidding market where firms have finite capacities and future demand is uncertain. We show that horizontal mergers can affect market prices but only if either the largest firm is involved in the merger or if it creates a new largest firm. A merger that increases the capacity of the largest firm is always profitable while a merger that creates a new largest firm may or may not be profitable.

3.1 INTRODUCTION

In recent years, there has been an active debate among economists as well as competition policy practitioners on whether markets using auctions or auction processes to allocate goods or services should be treated differently from “ordinary” markets in antitrust cases. Firms active in such “bidding markets” have repeatedly claimed that there is less need by competition authorities to evaluate them because market characteristics alone ensure efficient competition even if the relevant market is highly concentrated. A case in point is the Oracle/Peoplesoft merger in the EU (European Commission, 2005). After the initial concern of the European Commission (EC) that the merger would reduce the number of firms in the enterprise application software industry from three to two (the only major competitor being SAP), Oracle was able to successfully argue that the market definition determined by the EC was too narrow: because contracts are awarded by a bidding procedure, other competitors are able to enter specific tenders and pose a competitive constraint on the merging firms. Therefore, they are part of the market and have to be included when defining the market. A similar argument has also been used by the EC. For example, in the Raytheon/Thales case, the EC states that in a bidding market, "the conditions of competition are determined by the presence of credible suppliers, able to offer competitive alternatives to the parties' products". Therefore "even relatively high market shares may not necessarily translate into market power" (European Commission, 2001, rec. 40).[1]

On the other hand, Klemperer (2007) argues that most antitrust cases involving firms active in bidding markets should be treated no differently than other cases. Specifically, he states that "the key anti-trust challenge is simply to recognize that the particular method of price-formation in auctions and bidding processes does not affect the fundamental principles of antitrust" (p. 39).

In order to shed some light on the effects of mergers in bidding markets, we analyze merger incentives and unilateral effects of horizontal mergers in a dynamic market environment consistent with the features of a bidding market. Firms have fixed capacities and face an uncertain number of sequentially arriving consumers. In this framework which focuses on the interplay of capacity and demand uncertainty, we find that without efficiency gains, any horizontal merger increases prices from the date of the merger onwards if the largest firm is involved in it or if a new largest firm is created through the merger. Current and future prices are unaffected if the largest firm is not involved and decrease if a merger creates a firm which is as large as the previously largest firm. In contrast to traditional merger analysis, where all mergers without efficiency gains tend to increase prices, in our model a large number of mergers does not have adverse effects on consumer welfare. Our findings are also in contrast to the current practice of the EC of assessing market power in bidding markets based on the number of competitors. Our analysis suggests that the involvement of the firm with the largest capacity in a merger or the creation of a new largest firm due to merger is the key determinant of adverse price effects, independent of the number of active competitors.

Three features have been identified to define an ideal bidding market (see, e.g., Klemperer, 2007; Rasch and Wambach, 2013). First, competition is "winner

[1] Other European cases with the same line of reasoning include Metronet/Infraco (European Commission, 2002) and Boeing/Hughes (European Commission, 2004). The German competition authority ("Bundeskartellamt") also argues that high market shares are no indicator of market power in bidding markets in the Hochtief/Philipp Holzmann and Webasto/Edscha merger cases (see respectively Bundeskartellamt, 1998 and Bundeskartellamt, 2009).

take all" such that the winner of the auction receives the object for sale (or the supply contract in a procurement setting) while all other participants come away empty-handed. Second, each object or contract up for auction is significant in size relative to total sales in a given period. Third, in the terms of Klemperer (2007), "competition begins afresh for each contract, and each customer" (p. 5) if there is repeated interaction. This implies that in subsequent auctions the fact that one player has won a previous auction does not improve (or worsen) his position in the current contest. We stress that this third characteristic can only be meaningful if the market is a dynamic environment where firms interact repeatedly.

In our model, buyers arrive at the market sequentially and can buy one contract from one of n sellers via a first-price sealed-bid auction. Contracts cannot be split between sellers, resembling the "winner take all" feature of a bidding market. Sellers' capacity is constrained so that a seller's capacity is reduced each time it wins an auction. Future demand is uncertain. Demand uncertainty ensures the significance of each contract up for auction in the short-term, modeling the second characteristic of bidding markets. In line with the third feature of bidding markets, competition starts afresh for each contract in the sense that buyers' preferences and firms' costs are independent of past auctions. Any future auction is undertaken by a new buyer, and thus the preferences of each new buyer are independent of the result of previous auctions. Furthermore, firms have constant costs. Therefore, although the capacity of the winning firm is reduced permanently after winning a contract, this has no influence on the firm's cost function.

In this framework, we analyze the incentives to merge and find that a merger between the largest firm and any number of smaller rivals is always profitable for the merged firm as well as for all outsider firms. The largest firm is able to sell each additional unit of capacity with a larger probability and at higher prices than before the merger while the outsiders free-ride on increased prices of the insiders.

When a number of smaller firms merges and becomes the new largest firm in the market, whether the merger is profitable or not depends on a trade-off: on the one hand the increase in equilibrium prices allows the new largest firm to sell its units at higher prices than before the merger, resulting in an increase in profit. On the other hand, as opposed to before the merger, the probability of selling a unit for the merged firm is reduced by the merger, which is to the detriment of the merged firm. Depending on which effect dominates, the merger is either profitable or unprofitable. The outsider firms including the previously largest firm always profit due to the increased prices. If smaller firms merge such that their capacity is equal to the previously largest firm, the merger is unprofitable for the merging firms and the previously largest firm, but profitable for the outsiders.

The merger paradox, a situation where mergers without efficiency gains are unprofitable for the merged firm unless large parts of the industry are involved, occurs in markets with strategic substitutes, while with strategic complements, mergers are to the benefit of all firms.[2] Different from traditional analysis, in our framework with strategic complements, we find that mergers may or may not be profitable depending on the capacities of the involved firms. Instead, in line with static merger models, outsiders always profit from mergers in the industry in our model, yielding further support for this already very robust finding.

We model firms with a common marginal cost. While restrictive, there are numerous markets where a large fraction of total cost arises prior to the actual sale of the product. For example, in the testimony of the Oracle/PeopleSoft merger case in the US, a senior executive of Oracle explained that most of the cost accrues during the development of the software while the the cost of an additional sale of a license itself is negligible (Bengtsson, 2005).

In a dynamic market environment, it is plausible to assume demand to be uncertain. For example, firms offering management consulting services are often able to anticipate future tenders from specific firms or a small number of firms, however, overall demand for a period of time, e.g., the next year, is uncertain to some degree. Similarly, firms like Oracle or SAP in the market for enterprise application software—where the number of employees working to implement the

[2]The 'merger paradox' originates in the contribution of Salant et al. (1983). When merging without efficiency gains in a Cournot market, the merged firm internalizes the adverse effect of competition on their joint profit by reducing output. Due to strategic substitutability, this leads to the expansion of output by the outsiders to the detriment of the merged firm. Salant et al. (1983) show that mergers are unprofitable for the merged firm unless a large fraction of the firms in the industry are involved in it while the outsiders always profit. In contrast, Deneckere and Davidson (1985) show in a differentiated Bertrand setting that the merged firm as well as the outsiders profit from a merger because due to strategic complementarity, the reaction of the outsiders benefits the merged firm.

software is limited—may well know some of the upcoming tenders, however, it is impossible to perfectly predict how many firms will decide to invest into a new software system in the next years. As a result of demand uncertainty, capacity constraints may arise. In times of economic downturn, demand may have been overestimated such that capacity is abundant, while conversely, in situations of unexpectedly high demand, capacity in the market may be used to the full. For example, the strategy consulting industry in the United States was reported to have an overcapacity of 30% in 2002 as a result of the collapse of the dotcom bubble (The Economist, 2002).

Related literature

There is a large body of literature on the analysis of horizontal mergers starting with the seminal contributions of Salant et al. (1983) and Deneckere and Davidson (1985). Numerous other settings have since been analyzed, examples include efficiency gains (Farrell and Shapiro, 1990), entry (Pesendorfer, 2005; Marino and Zábojník, 2006), merger waves (Qiu and Zhou, 2007), remedies (Vasconcelos, 2010; Vergé, 2010), and strategic antitrust authorities (Nocke and Whinston, 2010).

Our paper is related to the literature on mergers in (static) auction markets. In these papers, one common assumption is that merging leads to a probabilistically lower cost post-merger, inducing an asymmetry among bidders. In this vein, Waehrer (1999) compares asymmetric open and first-price sealed-bid auctions where a merged firm's cost is the maximum of a number of draws from the same distribution where the number of draws depends on the size of the merged entity. Waehrer and Perry (2003) analyze mergers in a static asymmetric open procurement auction model where a merged firm's cost is the minimum of two draws from the identical cost distribution. Thomas (2004) examines mergers in a one-shot procurement setting where the merged firm can either become more or less efficient post-merger.[3] In contrast to this literature, we focus on a dynamic

[3]Other contributions concerning mergers in auction markets focus on the econometric estimation of price effects of mergers in auction markets (see, e.g., Dalkir et al., 2000 or Tschantz et al., 2000).

market environment with repeated auctions while assuming that the cost functions of a merged firm remains unchanged.

A number of papers analyze repeated procurement with capacity-constrained suppliers.[4] In this mostly empirical literature, it is typically assumed that a supplier's cost distribution in the current period depends on a backlog, i.e., the number of previous auctions it has won. It is particularly this assumption, which is in conflict with the third feature of a bidding market defined above. Jofre-Bonet and Pesendorfer (2000, 2003) propose a method to estimate a model of repeated first-price sealed-bid procurement auctions. Using data from California construction procurement auctions, they find that capacity constraints affect suppliers' bidding behavior. Suppliers with less available capacity (a larger backlog) may have higher costs in future auctions. On the other hand, previously won auctions may lead to a learning effect which reduces costs in later auctions. However, their focus is on the development of the estimation strategy and, in contrast to our paper, they do not solve their models analytically. In a similar vein, Saini (2012) numerically solves a duopoly model of repeated procurement auctions where a supplier's larger backlog leads to stochastically larger costs in the next period.[5]

Our paper is also related to the literature on revenue management in operations management. In this literature, pricing strategies are derived in an environment characterized by one firm or a small number of firms with a fixed capacity (inventory) facing customers arriving sequentially. Examples include the sale of airplane seats or hotel rooms. While a large number of papers focuses on pricing of a monopolist (see, e.g., Elmaghraby and Keskinocak, 2003 for a survey of the literature), there are a few more recent contributions who analyze situations with competition. For example, Lin and Sibdari (2009) prove the existence of a Nash equilibrium in a setting where n firms sell to sequentially arriving consumers using a multinomial logic choice model. Gallego and Hu (2014) analyze a stochastic game in continuous time where firms sell differentiated products.

[4]Contributions on sequential procurement without capacity constraints include Perry and Sákovics (2003) and Thomas (2010).

[5]Other empirical papers analyzing data from highway construction procurement auctions include De Silva et al. (2002, 2003) as well as De Silva et al. (2005).

The price equilibrium of our model, which is the basis for our analysis of horizontal mergers, is most closely related to two papers: Dudey (1992) and Martínez-de-Albéniz and Talluri (2011). The properties and intuition of the price equilibrium in these two contributions and in our model are qualitatively identical and carry over initially from Dudey (1992). Dudey (1992) characterizes the unique subgame perfect Nash equilibrium of a dynamic pricing game between two firms with fixed capacities facing sequentially arriving customers. In a situation where only one firm has enough capacity to serve the whole market (i.e. the capacity configuration where a pure strategy equilibrium fails to exist in Kreps and Scheinkman, 1983), he shows that the smaller firm sells its units first at the monopoly price, followed by the larger firm, which also sells its units as a monopolist. Martínez-de-Albéniz and Talluri (2011) extend the model of Dudey (1992) to an oligopoly with an arbitrary number of firms and stochastic demand and find similarly that in equilibrium, small firms have an incentive to slightly undercut the largest firm such that they sell their units first while the largest firm postpones sales until it is the monopolist. The reason is that the firm with most capacity has the largest probability of becoming the monopolist in the market and therefore has the largest opportunity cost of selling a unit today.

Martínez-de-Albéniz and Talluri (2011) specify demand via a stochastic process such that in each period, a consumer may or may not arrive. In order to maintain a finitely repeated game, they specify a maximum number of periods. This temporal restriction has an influence on firms' pricing behavior: as long as no customer arrives, equilibrium prices set once the next consumer arrives decrease as the endpoint of the game moves closer. We define demand via the probability that another costumer arrives without specifying a maximum number of periods.[6] Consumers arrive sequentially but it is not important when exactly they arrive, i.e., the time gaps between two consumers may vary without influencing prices. Besides substantially simplifying the analysis, this formulation allows us to avoid the effect of the endpoint of the game on pricing such that pricing behavior in our model is driven by the scarcity (or lack thereof) of capacity.

[6]In Martínez-de-Albéniz and Talluri (2011), the distribution of the expected number of consumers can be derived by combining the probability that a consumer arrives in a period with the maximum number of periods. In contrast, we directly define a distribution of the number of consumers via the probability that another consumer arrives.

The rest of this paper is organized as follows. In section 3.2, we present the setup of the model as well as a simple example of pricing behavior. We present the price equilibrium in section 3.3. In section 3.4, price effects of mergers as well as the incentives to merge are analyzed. Section 3.5 concludes.

3.2 THE MODEL

Consider the market for a homogenous good where n firms are active for $t = 1,2,\dots$ periods. In any period t, firm i, $i \in 1,\dots,n$ has a capacity $k_{i,t}$. The firms' capacities are ordered in descending sequence so that $k_{1,t} \geq k_{2,t} \geq \dots \geq k_{n,t}$.[7] Total capacity held by firms in the industry in period t is denoted by $K_t = \sum_{i=1}^{n} k_{i,t}$. Initially, overall capacity in the industry is (exogenously) given by $K_1 = \sum_{i=1}^{n} k_{i,1}$. The largest capacity by a firm in period t in the industry is denoted by κ_t. Each supplier faces the same constant marginal cost c, which we normalize to be zero.

A number of consumers wants to procure a unit of the good offered by the firms. Consumers have a common and constant willingness to pay v for one unit of the good. Consumers arrive in the market sequentially. While we assume without loss of generality that the first consumer arrives with certainty, whether additional consumers will arrive in the future is uncertain. For any period t denote by $\delta \in (0,1)$ the probability that another consumer arrives in the next period so that with probability $1-\delta$, there is no further demand and the game ends. This formulation of demand gives rise to a probability distribution of the number of consumers. In that regard, it is equivalent to the demand technology of Martínez-de-Albéniz and Talluri (2011) who indirectly model a stochastic process of the number of consumers by defining a probability that a consumer arrives in any period and a maximum number of periods. Besides simplifying the analysis with our formulation we avoid last round effects present in Martínez-de-Albéniz and Talluri (2011) in order to solely focus on the effect of capacity on pricing. If a consumer has arrived at the market in any period, it procures one unit via a first-price sealed-bid auction with (exogenous) reserve price v. Consumers are myopic and only come to the market once. They buy one unit from the seller with

[7] Technically, we reorder firms according to their current capacities in every period.

the lowest price if price does not exceed v. Otherwise, they do not buy a unit of the good, leave the market and do not return.[8]

If a consumer is in the market in period t, each firm i with $k_{i,t} > 0$ submits a price $p_{i,t}$ and the firm with the lowest price wins and sells one unit to the consumer. If multiple firms post the same lowest price, we assume that any firm but the firm with the largest capacity in the market wins with equal probability. In footnote 9 and in our discussion of Proposition 4 below we explain that this tie-breaking rule does not alter any property of the equilibrium and is merely made for technical reasons. When a firm wins a contract, it is paid its bid and its capacity is reduced by one unit. Capacity that has been occupied is assumed to remain so until the end of the game.

Because the game ends either if no additional consumer comes to the market or if firms run out of capacity, it is a finite dynamic game with complete information. We thus proceed by backward-induction to solve for a subgame perfect Nash equilibrium.

An example

We start the analysis by presenting an example where two firms with capacities of $k_{1,1} = 2$ and $k_{2,1} = 1$ are active in the market in period 1 such that $\kappa_1 = 2$ and $K_1 = 3$.

Assume for now that two consumers arrived previously and bought a unit each. If a third consumer arrives (in period 3), only one supplier with one unit of remaining capacity is active. Without competition, this supplier will set a price equal to v, resulting in a per-unit profit of v. Working backwards, in the second auction (period 2), two capacity constellations can arise. If $k_{1,2} = 2$ and $k_{2,2} = 0$, supplier 1 is the only active firm and will again bid v, resulting in a per-unit profit of v. The sum of future expected profits of this supplier when taking into account the per-unit profit today and the expected profit from a future third auction is $v + \delta v$.

[8] We argue that in equilibrium, forward-looking consumers who could potentially come to the market multiple times will always purchase on their first arrival because equilibrium prices increase as time progresses.

More interestingly, if $k_{1,2} = k_{2,2} = 1$, both firms are active. Firms face the following trade-off: Winning today results in a certain payoff in the current auction while losing it will lead to the monopoly profit in a future auction but it is uncertain if another customer will arrive. For each firm i, we derive a reservation price at which it is indifferent between winning and losing the current auction. For any lower price, firm i strictly prefers to lose today. Firm i is indifferent between winning and losing if the price it is paid in the current auction is equal to its expected future profits, that is, if $\bar{p}_{i,2} = \delta v$.

In this symmetric situation of price competition, each firm anticipates that it will be undercut by its rival as long as it does not bid the lowest (weakly) profitable price possible such that in equilibrium both firms end up bidding their reservation price: $p^*_{i,2} = \bar{p}_{i,2} = \delta v$. The winner of the auction is determined by a coin toss and is paid its bid δv. The loser of the auction will set a price v in case of another customer, resulting in an expected profit of δv as well.

In the first period, both firms possess their initial capacities, $k_{1,1} = 2$ and $k_{2,1} = 1$. A similar trade-off arises but now the firms have different amounts of capacity. Firm 2 knows that it will receive δv if it does not win today and if another customer arrives at the market. Because another customer arrives with probability δ, the reservation price for the current contract of firm 2 is $\bar{p}_{2,1} = \delta^2 v$.

If firm 1 is awarded the current contract at price p, it will still have capacity left to be active in future auctions. In this case and if there is an additional customer, both firms compete in two periods and firm 1 has a sum of future expected payoffs of $p + \delta^2 v$. On the other hand, both firms compete only once if firm 2 wins the current auction. Then, firm 1 is the monopolistic supplier in any future auction and a sum of future expected payoffs of $\delta v + \delta^2 v$. Putting the two situations together leads to a reservation price of firm 1 at which it is indifferent between winning and losing today of $p + \delta^2 v = \delta v + \delta^2 v \Leftrightarrow \bar{p}_{1,1} = \delta v$.

A comparison of both reservation prices shows that $\bar{p}_{1,1} > \bar{p}_{2,1}$. Knowing that firm 1 will not bid below its reservation price, the best response of firm 2 is to match it because of the tie-breaking rule.[9] As no firm has an incentive to deviate, the equilibrium strategy of both firms is to set a price $p^*_{i,1} = \delta v$. Firm 2

[9] If the smaller firm was not selected in case of a common price, firm 2 clearly has an incentive to marginally undercut the reservation price of firm 1 in this situation. Without a discrete price

wins the current auction and has a sum of future expected payoffs δv while firm 1 has a sum of future expected payoffs of $\delta v + \delta^2 v$.

By not winning the first auction, firm 1 becomes the monopolist in all remaining auctions and has to compete in one auction only. In comparison, winning today would result in facing a competing seller again in the next auction. While prices are determined by the reservation price of firm 1 only, such that it is indifferent between winning today or at a later date, firm 2 strictly prefers to win earlier. The reason is that if firm 1 wins today, the equilibrium price for the next customer will be the same as today, making winning today more attractive for firm 2 due to demand uncertainty.

3.3 BIDDING STRATEGIES

In the previous example, the equilibrium price was shown to be determined by the reservation price of the largest active firm. Additionally, in equilibrium the largest firm was shown to sell its capacity without competition as a monopolist. In this section, we show that the intuition of the example generalizes to a situation with n firms and describe the equilibrium bidding behavior of suppliers. For any exogenous distribution of the capacities in the industry, we have the following result.

Proposition 4. *(i) If the largest firm is strictly larger than all other firms, the equilibrium strategy of all firms is to set a price*

$$p^*_{i,t}(\kappa_t, K_t) = \delta^{K_t - \kappa_t} v \text{ for all } \kappa_t, K_t \text{ in any period } t. \tag{3.3.1}$$

(ii) If there are at least two firms j with the largest capacity κ_t in period t, the equilibrium strategy of each of these firms is to set a price

$$\bar{p}^*_{j,t}(\kappa_t, K_t) = \left(\frac{2 - \delta - \delta^{\kappa_t}}{1 - \delta} - \kappa_t \right) \delta^{K_t - \kappa_t} v \text{ for all } \kappa_t, K_t \tag{3.3.2}$$

while all other firms set price $p^*_{i,t}(\kappa_t, K_t)$ for all κ_t, K_t.

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grid, this incentive to undercut would lead to non-existence of equilibrium. Thus, we merely tweak the tie-breaking rule to 'replicate' rational behavior with continuous prices.

Proof. For part (i), we assume that all firms abide by the strategy specified in the proposition, i.e. they set the candidate equilibrium price $p^*_{i,t}(\kappa_t, K_t)$ in all future periods starting in some period t. We then calculate the sum of future expected payoffs of firms when using this strategy and show that no firm can increase payoffs by deviating from it.

Observe that this strategy in conjunction with the tie-breaking rule ensures that the smaller firms sell their capacity first, while the largest firm sells its units last. The candidate equilibrium pricing strategy has three properties that help simplify the analysis:

1. Starting in period t, the capacity of the largest firm does not change as long as multiple firms are active because the largest firm sells its capacity last: $\kappa_t = \kappa_{t+1} = \cdots = \kappa_{t+(K_t-\kappa_t-1)}$.
2. Industry capacity is reduced by one unit each period as long as the game lasts because a sale is made in each period: $K_{t+1} = K_t - 1; K_{t+2} = K_t - 2, \ldots$.
3. Using properties 1 and 2, it can be shown that the per-period profit (which equals the winning price) of the firm winning the auction in period t is equal to the expected per-period profit of winning an auction in any future period, say, $t+X$, if at least two firms are active:

$$\begin{aligned}\pi_{i,t+X}(\kappa_{t+X}, K_{t+X}) &= \delta^X p^*_{i,t+X}(\kappa_{t+X}, K_{t+X}) = \delta^X \delta^{K_{t+X}-\kappa_{t+X}} v \\ &= \delta^{X+K_t-X-\kappa_t} v = \delta^{K_t-\kappa_t} v = \pi_{i,t}(\kappa_t, K_t)\end{aligned}$$

In the next step, the sum of future expected payoffs of all firms is calculated. Any firm i with capacity $k_{i,t}$, $i > 1$, sells all its capacity under competition and before the largest firms sells its units as a monopolist. Thus, due to property 3, in period t each firm except the largest firm has a sum of future expected payoffs of

$$V_i^C(k_{i,t}, \kappa_t, K_t) = k_{i,t} \delta^{K_t-\kappa_t} v \text{ for all } i > 1.$$

No firm $i > 1$ has an incentive to deviate in one period: Undercutting $p^*(\kappa_t, K_t)$ cannot be profitable because it results in winning the current auction at a price $p^d < p^*(\kappa_t, K_t)$ which leads to a per-period profit lower than $\pi_{i,t}(\kappa_t, K_t)$ and hence to $V^d < V_i^C(k_{i,t}, \kappa_t, K_t)$. Increasing price does not increase profits for any supplier $i > 1$, either. This results in losing the current auction, but because of property 3, the deviating firm will still receive $V_i^C(k_{i,t}, \kappa_t, K_t)$ from future sales.

Finally the sum of future expected payoffs of the firm with the largest capacity in the industry, i.e., firm 1, is derived. Following the candidate equilibrium strategy, firm 1 sells its κ_t units when it is the sole supplier, bidding v in each of the κ_t remaining periods. From the perspective of period t, this results in a sum of future expected payoffs of

$$V_1^C(\kappa_t, K_t) = \delta^{K_t-\kappa_t}v + \delta\delta^{K_t-1-(\kappa_t-1)}v + \delta^2\delta^{K_t-2-(\kappa_t-2)}v \cdots + \delta^{\kappa_t-1}\delta^{K_t-\kappa_t-(\kappa_t-\kappa_t)}v,$$

which simplifies to

$$V_1^C(\kappa_t, K_t) = \sum_{j=K_t-\kappa_t}^{K_t-1} \delta^j v = \frac{\delta^{K_t-\kappa_t} - \delta^{K_t}}{1-\delta} v.$$

Deviating by increasing its price above $p^*(\kappa_t, K_t)$ is not profitable for firm 1 as this either has no effect (in case other firms are active) or leads to no sales (in case it is the sole supplier). Firm 1 also has no incentive to undercut $p^*(\kappa_t, K_t)$ as for the case that other suppliers are active, firm 1 is just indifferent between winning and losing the current auction at price $p^*(\kappa_t, K_t)$. Firm 1 is indifferent if

$$\begin{aligned}
&\hat{p}_{1,t} + \delta V_1^C(\kappa_t - 1, K_t - 1) = \delta V_1^C(\kappa_t, K_t - 1) \\
\Leftrightarrow &\hat{p}_{1,t} = \delta \left[V_1^C(\kappa, K_t - 1) - V_1^C(\kappa_t - 1, K_t - 1) \right] \\
\Leftrightarrow &\hat{p}_{1,t} = \delta \left[\sum_{j=K_t-\kappa_t-1}^{K_t-2} \delta^j v - \sum_{j=K_t-\kappa_t}^{K_t-2} \delta^j v \right] \\
\Leftrightarrow &\hat{p}_{1,t} = \delta^{K_t-\kappa_t} v = p^*(\kappa_t, K_t).
\end{aligned}$$

To prove part (ii), it is first established that $p^*(\kappa_t, K_t) > \bar{p}^*(\kappa_t, K_t)$ in any period t. This is the case if

$$\begin{aligned}
&\left(\frac{2-\delta-\delta^{\kappa_t}}{1-\delta} - \kappa_t \right) < 1 \\
\Leftrightarrow &\frac{1-\delta^{\kappa_t}}{1-\delta} < \kappa_t. \qquad (3.3.3)
\end{aligned}$$

The left-hand side (LHS) of inequality (3.3.3) is increasing in δ and takes its largest value when δ tends to 1. Then, by L'Hospital's rule, it tends to κ_t. Because $\delta < 1$ by definition, the maximum of the LHS of inequality (3.3.3) is never reached. It follows that strict inequality holds.

$p^*(\kappa_t, K_t) > \bar{p}^*(\kappa_t, K_t)$ implies that one of the firms with the largest capacity κ_t will win the auction in period t. If there are $1 < m \leq n$ firms with capacity κ_t in period t, bidding $\bar{p}^*(\kappa_t, K_t)$ will occur $m-1$ times, i.e., in the following $m-1$ periods. Any firm that bids $\bar{p}^*(\kappa_t, K_t)$ will win only one auction at that price because the firm selected to win by the coin toss will be strictly smaller than at least one competitor in all future periods and it will thus bid $p^*(\kappa_{t+1}, K_{t+1})$ after it has won the current auction. A firm with capacity κ_t bidding $\bar{p}^*(\kappa_t, K_t)$ thus has a sum of future expected payoffs of

$$V_m^C(\kappa_t, K_t) = \bar{p}^*(\kappa_t, K_t) + \delta(\kappa_t - 1)p^*(\kappa_t, K_t - 1),$$

which simplifies to

$$V_m^C(\kappa_t, K_t) = \sum_{j=K_t-\kappa_t}^{K_t-1} \delta^j v = \frac{\delta^{K_t-\kappa_t} - \delta^{K_t}}{1-\delta} v.$$

We now show that $\bar{p}^*(\kappa_t, K_t)$ is the price at which a firm with capacity κ_t is indifferent between winning and losing in period t and hence, does not have an incentive to deviate. If the firm wins the current auction, it is paid its bid, p^I and has a sum of future expected payoffs of a 'small' firm as defined in part (i) as $\delta V_i^C(k_{i,t} - 1, \kappa_t, K_t - 1) = \delta(k_{i,t} - 1)\delta^{K_t-1-\kappa_t} v$. If it loses the current auction, it is 'large' in future periods and accordingly receives future expected payoffs of $\delta V_1^C(\kappa_t, K_t - 1)$. The firm is indifferent between winning and losing if

$$\begin{aligned} p^I + \delta V_i^C(k_{i,t} - 1, \kappa_t, K_t - 1) &= \delta V_1^C(\kappa_t, K_t - 1) \\ \Leftrightarrow p^I &= \sum_{j=K_t-\kappa_t}^{K_t-1} \delta^j v - (k_{i,t} - 1)\delta^{K_t-\kappa_t} v = \bar{p}^*(\kappa_t, K_t). \end{aligned}$$

All other firms with capacity $k_{i,t} < \kappa_t$ set $p_{i,t}^*(\kappa_t, K_t)$ each period. We have already established that there is no deviation incentive when setting this price. □

According to Proposition 4 (i), all firms set a common price p_t^* in any period t if there is a single firm with the largest capacity κ_t.[10] Due to our tie-breaking rule, all firms setting a common price results in a pattern of sales where small firms sell out their capacities first and the largest firm selling its units last. While the tie-breaking rule drives this pattern, we argue that it is merely implemented to avoid non-existence of equilibrium caused by our assumption of continuous prices. Without the tie-breaking rule and with a discrete price grid, the same pattern of sales arises endogenously. This can be explained as follows. For each firm, we can determine an equilibrium reservation value below which it is unwilling to sell a unit in the current auction. Intuitively, if a firm waits until all competitors have sold out their capacity, it can sell its own units at the monopoly price v. Due to demand uncertainty, the probability of becoming the sole supplier and thus the equilibrium reservation value is lower the smaller the firm. For the current auction, this has the implication that any smaller firm has a incentive to marginally undercut any proposed price of the largest firm because the largest firm has the largest reservation value. With continuous prices, the result of marginal undercutting is the non-existence of a pure strategy equilibrium. With a discrete grid, which we do not use because it would substantially exacerbate our analysis, the smaller firms undercut the largest competitor by one increment in any period t. This results in the smaller firms selling their units first and the largest firm selling its units last. The sales pattern also implies that the number of periods with competition is minimized in equilibrium.

In part (i) of Proposition 4, the equilibrium price in period t is determined solely by the equilibrium reservation value of the largest firm. This firm has the largest opportunity cost of winning the current auction because winning reduces the probability that it becomes the sole supplier, and in addition, winning an auction under competition reduces the number of periods where it can enjoy this privilege. As the firm with the largest amount of capacity, it has the largest expected future payoffs to give up and hence prices least aggressive. Smaller firms therefore match the price of the largest firm (or undercut it with a discrete

[10]Firms not winning the current auction in equilibrium could set any price greater than the winning price. Without loss of generality and in order to reduce the multiplicity of equilibria we focus on equilibria where firms set reservation prices.

price grid) and sell their capacity earlier. In equilibrium, the firm with most capacity sells out as a monopolist.

An upward deviation by any small firm and optimal behavior from the next period onwards does not change the profit of the deviating firm as it will then simply sell its units under competition at a later date, which occurs with a lower probability but at a larger price, which makes all small firms indifferent towards selling in the current period. A downward deviation will result in winning the current auction but it will reduce overall expected profit of the deviator. If the largest firm is the sole supplier in the market, it sets the reserve price each period and clearly does not have an incentive to deviate in either direction. If smaller rivals are active, we show that the largest firm is just indifferent between winning and losing the current auction at the equilibrium price and thus does not have a deviation incentive.

If multiple firms share the largest capacity, one of these firms will win the auction in period t and become one of the small firms in the next auction (i.e. period $t+1$), as $\bar{p}^*_{j,t}(\kappa_t, K_t) < p^*_{i,t}(\kappa_t, K_t)$. The firms sharing the largest capacity compete head-on so that no smaller firm has an incentive to undercut them. Only when there is one firm left with the largest capacity, there is a switch in the equilibrium price structure back to part (i) of the Proposition so that small firms sell under competition and the remaining largest firm sells its units as a monopolist.

When there is a single largest firm, the equilibrium price is positive, leading to a positive per-period profit for any firm winning a contract. Additionally, the equilibrium price increases each period because the equilibrium marginal value of the largest firm increases as becoming a monopolist becomes more and more likely as time progresses. An example of the equilibrium price path with an industry capacity of $K_t = 16$ and a largest firm with capacity $\kappa_t = 4$ is displayed in Figure 3.1. The equilibrium price path is characterized first by a period of low prices due to a relative abundance of capacity and due to competition and a period of maximum prices when there is a single supplier who sells its capacity at the monopoly price. In the transition period in between, prices increase as concentration increases and as industry capacity decreases.

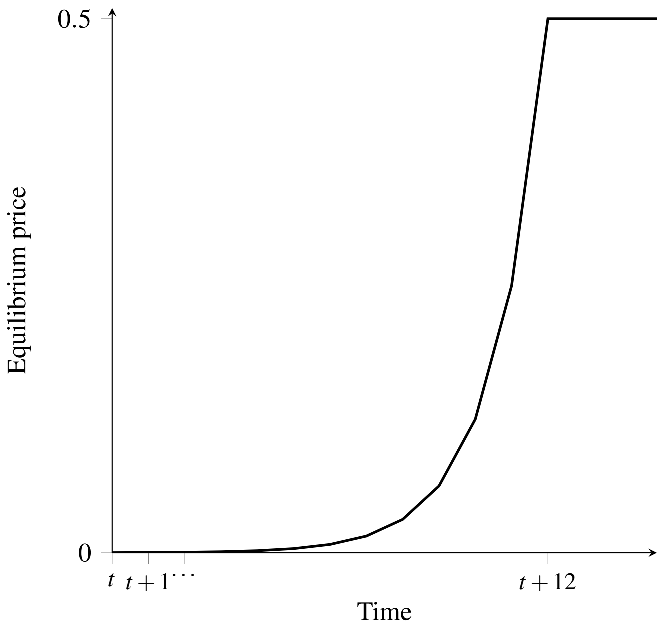


Figure 3.1: Equilibrium price path with one largest firm with $K_t = 16$, $\kappa_t = 4$, $v = 0.5$, $\delta = 0.5$.

If more than one firm share the largest capacity the equilibrium price in period t may even become zero or negative. Competition is so tough that the large firms may be willing to make a loss to win the current contract in order to improve upon their future position. In this situation, the large firms may prefer to become a smaller firm in future auctions: They trade off a higher price for the first unit they can sell as the sole supplier with a lower probability (if they lose the current auction and remain the largest firm) against a higher probability of making the first sale at competitive prices (if they win the current auction and become a smaller firm). Depending on parameter values, these firms are willing to 'buy' an increase of their probability of sale. An example of the negative part of the equilibrium price path with $m = 3$ firms with the largest capacity is given in Figure 3.2.

In order to analyze the profitability of horizontal mergers, we record the equilibrium sums of future expected payoffs derived in the proof of Proposition 4 in the following corollary.

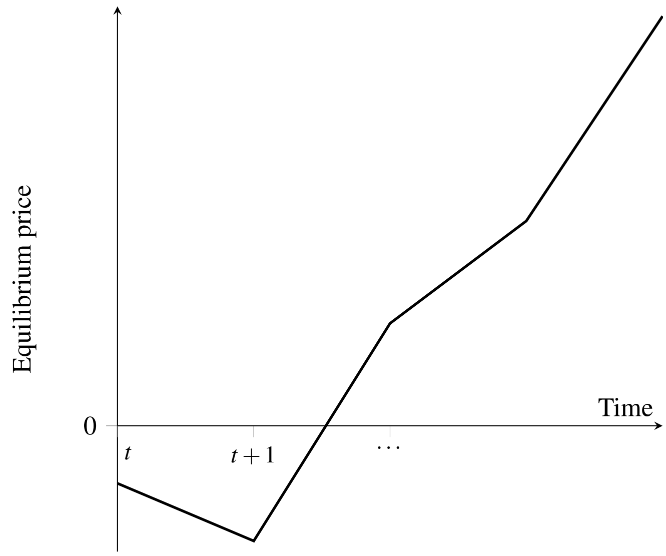


Figure 3.2: Equilibrium price path with $m = 3$ firms with the largest capacity $\kappa_t = 4$, $K_t = 16$, $v = 0.5$, $\delta = 0.5$.

Corollary 1. *In equilibrium, a firm with capacity $k_{i,t}$ in period t receives a sum of future expected payoffs of*

$$V_i^*(k_{i,t}, \kappa_t, K_t) = \begin{cases} \sum_{j=K_t-\kappa_t}^{K_t-1} \delta^j v & \text{if } k_{i,t} = \kappa_t \\ k_{i,t}\delta^{K_t-\kappa_t} v & \text{if } k_{i,t} < \kappa_t \end{cases} \tag{3.3.4}$$

for all $k_{i,t}, \kappa_t$.[11]

While it is relatively straight-forward that a firm with a larger capacity earns a higher sum of future expected payoffs than a smaller rival as long as both firms are smaller than the largest firm in the market, whether the largest firm earns a higher sum of future expected payoffs than the firm next in size is less obvious because the largest firm sells at higher prices but with a lower probability. As long as the largest firm does not sell, the expected per-period equilibrium profit in any future period $t+X$ remains constant because in each future period, the decrease in profit due to the uncertainty of demand is exactly cancelled out by the price-increasing effect of the reduction of overall capacity due to a sale by

[11] Observe that the profit of firm i if it is the largest firm can also be written in terms of a sum as $V_i^*(k_{i,t}, \kappa_t, K_t) = \frac{\delta^{K_t-\kappa_t} - \delta^{K_t}}{1-\delta} v$.

a smaller firm in the current period, i.e., $p^*_{i,t}(\kappa_t, K_t) = \delta^{t+X} p^*_{i,t+X}(\kappa_{t+X}, K_t - X)$ (see proof of Proposition 4). Thus, from an ex-ante view, the per-period profit of a small firm is equal across periods. This is different for the largest firm. In equilibrium, the largest firm sets a price equal to the customer's reservation value after all rivals have exhausted their capacity. As there is no incentive to raise prices beyond the reservation value further from that point onwards, the ex-ante expected per-period profit of largest firm decreases from period to period because of demand uncertainty. Despite its greater capacity, the largest firm may therefore earn a lower sum of future expected payoffs than a smaller rival. This property of the equilibrium sum of future expected payoffs of the largest firm also explains why competition is harshest if more than one firm share the largest capacity.

We next turn to the analysis of horizontal mergers based on firms' equilibrium pricing behavior.

3.4 HORIZONTAL MERGERS

We model horizontal mergers as the pooling of the capacity of the firms involved in it. In any period t, firms can decide to merge before pricing decisions are made. A merger between $i \leq n-1$ firms with a sum of capacities of $\sum_j k_{j,t}$ then results in a merged firm with capacity $k_{m,t} = \sum_j k_{j,t}$ in period t. Denote the capacities and sums of future expected payoffs before and after a merger by subscript b and a, respectively.

3.4.1 PRICE EFFECTS AND CONSUMER WELFARE

Competition authorities are interested in the expected changes of market prices induced by a merger. Our first result on merger price effects concerns mergers without the involvement or change of identity of the largest firm.[12]

Proposition 5. *A horizontal merger between $i \leq n-1$ firms in period t does not change the equilibrium price path from period t onwards as long as the capacity and the identity of the largest firm(s) are not changed by it.*

[12]Note that we focus on consumer welfare as total welfare remains constant in equilibrium in our framework as changes in equilibrium price do not affect demand but merely lead to a redistribution of surplus between buyers and suppliers.

Proof. The proof follows from the equilibrium prices derived in Proposition 4. Equilibrium prices in any period t are a function of κ_t, K_t, and the number of largest firms in the period but they are independent of any particular $k_{i,t}$. K_t remains constant because the merger only changes the distribution of capacity in the industry but not the total amount of capacity available in period t. As long as κ_t and the number of largest firms do not change, equilibrium prices and hence the equilibrium price path are not modified. □

This result holds independent of whether there are one or more firms with the largest capacity. In contrast to traditional merger analyses in Cournot or (differentiated) Bertrand markets, a horizontal merger that does not change the marginal cost of the involved firms does not influence equilibrium prices as long as the opportunity cost of winning by the largest firm(s) is not altered by it. As prices depend on the capacity of the largest firm(s), mergers have an effect on prices only if (one of) the largest firm(s) in the industry participates in it or if a merger changes the identity of the largest firm(s). We first analyze the case where only one firm with the largest capacity exists after a merger. We record this result in the following proposition.

Proposition 6. *If a supplier with the largest capacity κ_t is involved in a merger between $i \leq n-1$ firms in any period t or if there is a new single largest firm after a merger, prices increase.*

Proof. If there is a single largest firm, a merger in period t involving this firm results in an increase of κ_t. If a number of firms are sharing the largest capacity, a merger involving one of these firms will automatically lead to a single largest firm, resulting in an increase of κ_t. A merger between a number of small firms such that a new single largest firm is created through merging results in an increase of κ_t. If κ_t increases, it follows immediately from Proposition 4 that prices increase in all periods from period t onwards. □

Additional capacity in control of the largest firm increases the probability of the largest firm to become the sole supplier and sell out its capacity at the maximum (monopoly) price. This reduces the influence of demand uncertainty on the equilibrium reservation value of the largest firm and leads to less aggressive pricing behavior. As the equilibrium price in any period t is determined by the reservation value of the largest firm, this type of merger leads to an upward shift of the equilibrium price path. In Figure 3.3, we display an example of this shift of the equilibrium price path for an industry where the largest firm increases its capacity by merging with a smaller rival.

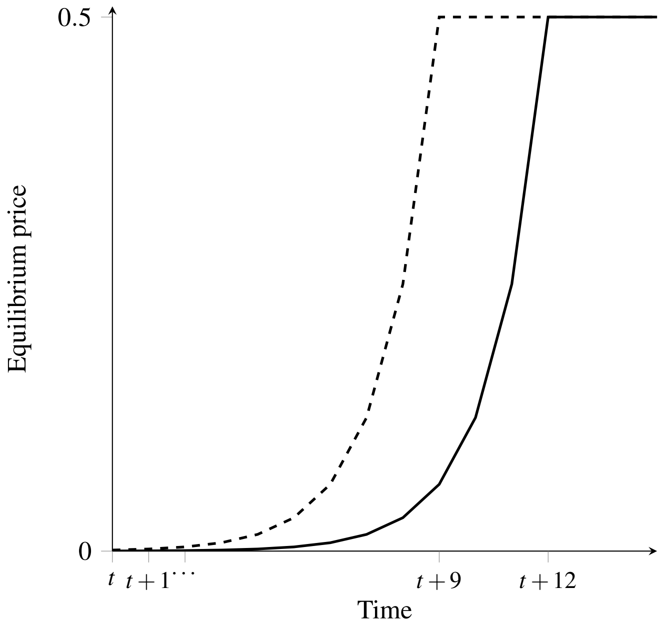


Figure 3.3: Equilibrium price paths pre-merger (solid) and post-merger (dashed) with $K_t = 16$, $\kappa_{b,t} = 4$, $\kappa_{a,t} = 7$, $v = 0.5$, $\delta = 0.5$.

Finally, an interesting case arises if a merger between a number of small firms leads to a merged firm with the same capacity as the previously largest firm or largest firms.

Proposition 7. *A merger resulting in a firm with the same capacity as the $m \geq 1$ previously largest firms in period t leads to a price drop in period $t+m-1$. In all other periods, prices are the same as in a situation without the merger.*

Proof. If a merger results in $m+1$ firms with the same largest capacity in period t, it follows from Proposition 4 that the equilibrium price decreases from $p^*_{i,t}(\kappa_t, K_t)$ (part (i) of Proposition 4) to $\bar{p}^*_{j,t}(\kappa_t, K_t)$ (part (ii) of Proposition 4) if $m = 1$. If $m > 1$, the equilibrium price in period t remains at $\bar{p}^*_{j,t}(\kappa_t, K_t)$. We have shown in the proof of Proposition 4 (ii) that $\bar{p}^*_{j,t}(\kappa_t, K_t) < p^*_{i,t}(\kappa_t, K_t)$ for all κ_t, K_t. It follows that one of the firms with capacity κ_t wins the auction in period t.

If there are $m+1$ firms with the largest capacity in period t instead of m, those largest firms set price $\bar{p}^*_{j,t}(\kappa_t, K_t)$ for the next $m-1$ instead of $m-2$ periods. It follows that there is an additional period of low prices, period $t+m-1$, compared to the situation without the merger.

From period $t+m$ onwards, there is one largest firm with capacity $\kappa_{t+m} = \kappa_t$ remaining, as would have been in the case without a merger. $\square$

Equilibrium prices decrease temporarily in case a merger creates an additional firm with the same capacity as the previously largest firm(s) in the market as those firms compete intensely as long as there are multiple firms with the same largest capacity. If there are two or more firms sharing the largest capacity initially, a merger that creates an additional firm with the largest capacity does not influence prices immediately, however, it extends the phase of lower prices due to intense competition between the large firms relative to a situation with a single largest firm. Our model thus supports the notion of competition authorities that so-called 'catch-up' mergers, where the number of firms with the largest capacity is increased through merging, are beneficial to consumer welfare. An example equilibrium price path of a catch-up merger where smaller firms have merged to match the capacity of the (single) largest firm ($\kappa_t = 4$) is given in Figure 3.4.

We next turn to the analysis of the profitability of horizontal mergers in our framework.

3.4.2 *MERGER PROFITABILITY*

In addition to price effects, a key question in the analysis of horizontal mergers is merger profitability. We analyze the profitability of mergers in terms of comparing the sums of future expected payoffs starting in the period of the merger, t. In our framework, the profitability of mergers is simple to analyze when the largest firm is not involved because market prices are not influenced by the redistribution of capacity among the small firms. A merger between $i \leq n-1$ firms does not change the future expected payoffs of the merging firms or any outsider firm as long as the largest firm is not involved in the merger and as long as the identity

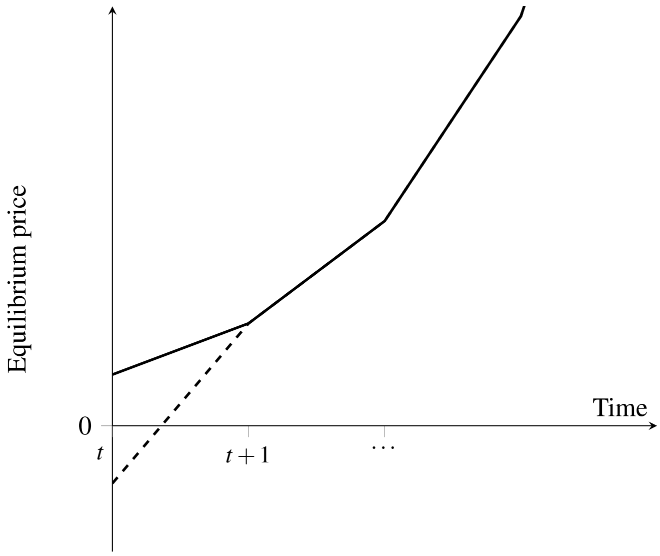


Figure 3.4: Relevant part of equilibrium price path with catch-up merger (dashed) and without merger (solid) with $\kappa_t = 4$, $K_t = 16$, $v = 0.5$, $\delta = 0.5$.

of the largest firm does not change. As the expected payoffs of any 'small' firm depend on the equilibrium prices determined by the largest firm, it follows that a merger without the involvement of the largest firm does not influence expected payoffs.

We next focus on mergers involving the largest firm(s). We first analyze the case where a merger alters the capacity of the largest firm in the market and as a direct consequence, equilibrium prices. We analyze merger profitability by comparing the sum of future expected payoffs of the merged firm with the aggregated sum of future expected payoffs of the involved firms pre-merger in period t. Two possible constellations arise. First is the case where the largest firm or one of the largest firms in the market merges with $j \leq n-2$ smaller rivals in some period t, resulting in an increase in capacity of the largest firm by $\sum_j k_{j,t}$ units. In the next proposition, we show that such a merger is always profitable for the merged firm as well as for all outsider firms.

Proposition 8. *A merger in period t between the firm with the largest capacity and any number of smaller rivals is always profitable for the merging firms as well as any outsider firm.*

Proof. A merger in period t between the largest firm (firm 1) and j smaller rivals is profitable for the merged firm if $V_1^*(\kappa_{a,t}, K_t) \geq \sum_j V_j^*(k_{j,t}, \kappa_{b,t}, K_t) + V_1^*(\kappa_{b,t}, K_t)$ where $\kappa_{b,t}$ is the capacity of the largest firm before the merger and $\kappa_{a,t} = \kappa_{b,t} + \sum_j k_{j,t}$ is the capacity of the largest firm after the merger. Using the equilibrium sums of future expected payoffs from Corollary 1 leads to

$$\sum_{j=K_t-\kappa_{a,t}}^{K_t-1} \delta^j v \geq \left(\sum_j k_{j,t}\right) \delta^{K_t-\kappa_{b,t}} v + \sum_{j=K_t-\kappa_{b,t}}^{K_t-1} \delta^j v$$

$$\Leftrightarrow \sum_{j=K_t-\kappa_{b,t}-\sum_j k_{j,t}}^{K_t-1} \delta^j - \sum_{j=K_t-\kappa_{b,t}}^{K_t-1} \delta^j \geq \left(\sum_j k_{j,t}\right) \delta^{K_t-\kappa_{b,t}}$$

$$\Leftrightarrow \sum_{j=K_t-\kappa_{b,t}-\sum_j k_{j,t}}^{K_t-\kappa_{b,t}-1} \delta^j \geq \left(\sum_j k_{j,t}\right) \delta^{K_t-\kappa_{b,t}}. \tag{3.4.1}$$

The sum in the LHS of inequality (3.4.1) as well as the right-hand side (RHS) have $\sum_j k_{j,t}$ elements each. The last element of the sum of the LHS is $\delta^{K_t-\kappa_{b,t}-1}$, which is also the smallest element of the LHS sum, while each element on the RHS is $\delta^{K_t-\kappa_{b,t}}$. Because $\delta^{K_t-\kappa_{b,t}-1} > \delta^{K_t-\kappa_{b,t}}$ holds, it follows that all other elements of the LHS sum are also larger than any element of the RHS. Thus, inequality (3.4.1) holds for all $k_{j,t}, \kappa_{b,t}$. Because the largest firm is involved in the merger, the equilibrium prices from period t onwards increase due to the merger. It follows that all outsider firms receive a larger sum of future expected profits post-merger. □

A merger between the largest firm and a number of smaller competitors is always profitable for the merged firm because of two positive effects on the sum of future expected profits induced by the merger. First, the additional units of capacity in control of the merged firm increase the probability of selling the first unit as the sole supplier. Secondly, on the equilibrium path each acquired unit of capacity is sold in a situation without competition, resulting in a higher selling price as compared to selling these newly acquired units under competition. Overall, this leads to an increase in the sum of future expected payoffs of the merged firm. The outsiders also profit as the larger probability of becoming the monopolist of the largest firm increases equilibrium prices from period t onwards.

In the second case, we analyze a merger where i firms with capacities $\sum_j k_{j,b,t}$ such that $\sum_j k_{j,b,t} > \kappa_{b,t}$ decide to merge. The merger creates a new largest firm with capacity $\kappa_{a,t} = \sum_j k_{j,b,t}$ while the previously largest firm has a capacity of $k_{2,a,t} = \kappa_{b,t}$. Observe that the previously largest firm is always the second largest firm after the merger. The merger is profitable for the merged firm if $V_1^*(\kappa_{a,t}, K_t) \geq \sum_j V_j^*(k_{j,t}, \kappa_{b,t}, K_t)$. We record our findings in the following proposition.

Proposition 9. *A merger between $i \in n$ firms such that the merged firm becomes the largest firm in the market is profitable for the merged firm if and only if*

$$\sum_{j=K_t-\kappa_{a,t}}^{K_t-\kappa_{b,t}-1} \left(\delta^j - \delta^{K_t-\kappa_{b,t}}\right) \geq \sum_{j=K_t-\kappa_{b,t}}^{K_t-1} \left(\delta^{K_t-\kappa_{b,t}} - \delta^j\right). \tag{3.4.2}$$

Proof. Substituting the sums of future expected payoffs from Corollary 1 into the profitability condition leads to

$$\sum_{j=K_t-\kappa_{a,t}}^{K_t-1} \delta^j v \geq \kappa_{a,t}\delta^{K_t-\kappa_{b,t}} v. \tag{3.4.3}$$

We can now split up the sum on the LHS of (3.4.3) into two parts, where the first part has $\kappa_{a,t} - \kappa_{b,t}$ terms and the second part has $\kappa_{b,t}$ terms. Similarly, the RHS can be split up such that, when expanded, the parts have $\kappa_{a,t} - \kappa_{b,t}$ and $\kappa_{b,t}$ terms, respectively. Rearranging then leads to

$$\begin{aligned}
&\sum_{j=K_t-\kappa_{a,t}}^{K_t-\kappa_{b,t}-1} \delta^j - (\kappa_{a,t} - \kappa_{b,t})\delta^{K_t-\kappa_{b,t}} \geq \kappa_{b,t}\delta^{K_t-\kappa_{b,t}} - \sum_{j=K_t-\kappa_{b,t}}^{K_t-1} \delta^j \\
\Leftrightarrow &\sum_{j=K_t-\kappa_{a,t}}^{K_t-\kappa_{b,t}-1} \left(\delta^j - \delta^{K_t-\kappa_{b,t}}\right) \geq \sum_{j=K_t-\kappa_{b,t}}^{K_t-1} \left(\delta^{K_t-\kappa_{b,t}} - \delta^j\right)
\end{aligned}$$

□

Whether the merger is profitable for the merged firm depends on a trade-off between higher prices it receives for each unit of capacity and a lower probability of sale of each unit of capacity post-merger. On the one hand, the merged firm sells each unit of its capacity at price v as opposed to $p^*(\cdot,\cdot)$. This is to the benefit of the merged firm. On the other hand, since it behaves less aggressively in periods with competition and sells its units when it is the sole supplier, the probability of sale decreases. This is to the detriment of the merged firm.

Because prices are strategic complements, due to a merger, the opportunity cost of winning of the merged firm increases, inducing less aggressive pricing behavior. This in turn allows the outsider firms to behave less aggressively as well, which benefits all firms. Conversely, the change in the probability of sale of the merged firm has a characteristic of substitutability in case a number of small firms merge to become the new largest firm in the market. Then, the merged firm sells its units when all competitors have exhausted their capacity while before the merger, sales occur at an earlier point in time. On the other hand, the previously largest firm sells its units at an earlier date after the merger. This can be to the detriment of the merging firms and depending on parameter values, can render the merger unprofitable.

The two opposing effects of a higher per-unit price and a lower probability of sale are shown in inequality (3.4.2). The merged firm sells each unit at the constant price v. Combined with the ex ante probability of each sale, which decreases over time due to increasing demand uncertainty, this results in decreasing expected per-unit profits of the merged firm. Instead, before the merger, the expected per-unit profit (i.e. the equilibrium price multiplied by the probability of sale) for each unit sold for the set of firms involved in the merger is constant. The LHS of (3.4.2) gives an expression for the (decreasing) gain of each of the $\kappa_a - \kappa_b$ units for which the merged firm receives a larger expected per-unit profit compared to the situation before the merger. This can be interpreted as the gain of the merger. The RHS of (3.4.2) on the other hand quantifies the loss of merging due to κ_b units for which the merged firm receives a lower expected per-unit profit than before the merger. An illustration of this trade-off is given in Figure 3.5.

Whether the gain due to increased prices dominates the loss due to the decreased probability of sale depends on the capacity of the merged firm, the capacity of the largest firm pre-merger, and on the probability of future demand.

An increase in $\kappa_{a,t}$, i.e., an increase in the share of industry capacity under control of the merged firm, increases the likelihood that the merger is profitable. The reason is that this increases the probability of sale of the merged firm, thus strengthening the positive effect of increased prices. A larger capacity of the

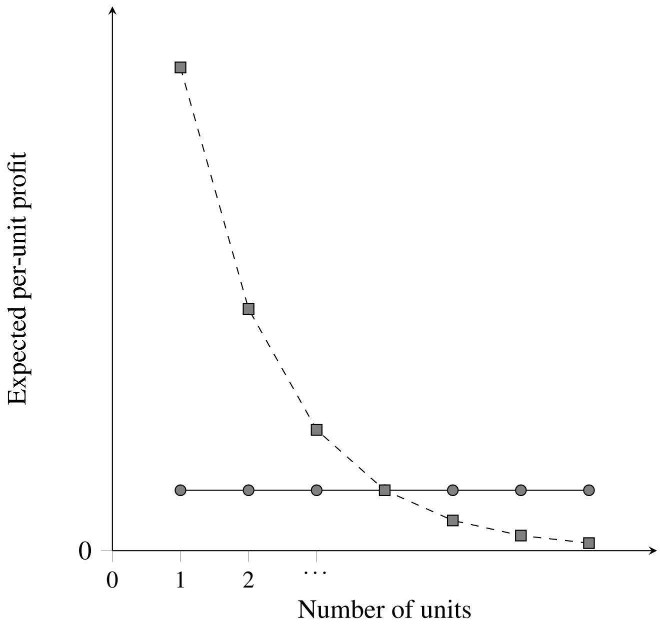


Figure 3.5: Trade-off of per-unit profits pre- and post-merger. Each circle (square) depicts the per-unit profit pre-merger (post-merger). The summed-up differences of per-unit profits in each period are given by inequality (3.4.2). Values used are $K_t = 16$, $\kappa_{b,t} = 4$, $\kappa_{a,t} = 7$, $v = 0.5$, $\delta = 0.5$.

largest firm before the merger ($\kappa_{b,t}$) decreases the likelihood that the merger is profitable as pre-merger prices increase while is has no effect on post-merger prices. The impact of the capacities of the largest firms before and after the merger on merger profitability suggest that mergers which increase symmetry in the industry are less likely to occur. Conversely, this can also be interpreted as a low likelihood of mergers in industries with one firm that is significantly larger than all rivals, i.e., in markets with a very asymmetric distribution of capacity.

After analyzing the merger incentives of the involved firms, we briefly analyze the effect of the merger on the sum of future expected profits of the outsider firms. As an immediate consequence of Proposition 9, all outsiders to a merger involving the largest firm except the previously largest firm always profit from it because the only effect on their overall expected payoffs is the increase in equilibrium price from period t onwards which is clearly to the firms' benefit.

The previously largest firm also profits from the merger. After the merger, it sells its units at lower prices p^* but with a larger probability of sale. This clearly leads to an increase in expected profits for the previously largest firm because by setting prohibitively high prices as long as a competitor is active and selling all its units as a monopolist, it could replicate its pre-merger expected profits.

3.5 DISCUSSION AND CONCLUSION

In this paper, we investigate the unilateral effects of horizontal mergers and merger profitability in a bidding market where firms are capacity-constrained and face an uncertain number of sequentially arriving consumers.

Any merger that does not involve the largest firm in the market or that does not change the identity of the largest firm or firms does not change equilibrium prices from the date of the merger onwards and hence does not have an adverse effect on consumer welfare. Conversely, any merger involving the largest firm or creating a new largest firm increases all future prices. This is because market prices are determined by the price the largest firm expects to be paid for the first unit of capacity it sells. A merger decreases prices temporarily if the merged firm has the same capacity as the previously largest firm.

Our results indicate the following implications on how to treat firms active in bidding markets from a competition policy perspective. If the merged firm is smaller than the largest firm after the merger, there are no adverse effects on market prices and thus our results indicate that such mergers do not pose a threat to consumer surplus. If the merged firm is similar in size to the previously largest firm in the market, we find that this has a positive effect on consumer surplus as prices decrease temporarily. Thus, our analysis suggests that 'catch-up' mergers should actively be supported by competition authorities. Finally, a merger that creates a new largest firm in the market or increases the capacity of the largest firm tends to lead to increased prices, suggesting that competition authorities should be especially careful in merger cases involving the largest firm(s) in an industry. Our analysis does not support the notion that mergers in bidding markets do not pose a threat for competition as long as many firms compete for each contract.

The largest firm in the market always profits from a merger with smaller rivals because all units of capacity gained through merging can be sold at a larger price and the overall probability of sale of the largest firm is increased by adding capacity. If a number of small firms merges in order to become the new largest firm, whether the merger is profitable depends on a comparison of the size of the new largest firm with the largest firm pre-merger. The merger is more likely to be profitable if the difference in capacity between both largest firms is large and if the previous largest firm is relatively small. Mergers are thus less likely to occur in markets with one dominant firm with a significant size advantage over its competitors. If the probability of further demand is small, mergers tend to be profitable. In both scenarios, the outsider firms always profit from a merger in the market. Due to the reduced competition, the outsiders are able to sell their units of capacity at higher prices.

The literature has identified three key characteristics of an ideal bidding market, which we incorporated in our model. We next discuss how these characteristics enter our analysis.

In a bidding market competition is winner-take-all such that contracts cannot be split between suppliers. We incorporate this into our model by assuming buyers' demand to be unity each period. Under the assumption that there are n firms and a single largest firm, this firm has the least aggressive pricing behavior because it expects to be the monopolist sooner and for a longer period of time than any rival. To relax this 'winner-take-all' assumption, suppose that a buyer arrives at the market in one period and demands two or more units, possibly splitting the demand between different suppliers. Intuitively, in this situation the largest firm has an even stronger incentive to price less aggressive than its smaller competitors because if more units are sold at the same time, the largest firm will become the monopolist with a larger probability. Thus, we argue that it is possible to introduce a larger per-period demand that can be split between sellers without altering the main intuition of the price equilibrium.

To model the significance of each contract up for auction, we assume uncertainty with respect to future demand. If we interpret increasing demand uncertainty (decreasing δ) as a measure of increasing significance of a contract, it is easy to see that different degrees of significance do not alter the main intuition.

Only in the extremes, the results will differ: Without demand uncertainty ($\delta = 1$), all suppliers will set the reservation (monopoly) price in each period as all capacity will be sold eventually. On the other extreme ($\delta = 0$), the model collapses to one-shot Bertrand competition as only a single costumer arrives.

The third characteristic of an ideal bidding market is that competition starts afresh each period. We model this by assuming that winning a contract does not influence a supplier's cost function and that buyers' preferences in any new auction are independent of the results of previous auctions.

In the current setting, smaller firms set more aggressive prices because they have a lower opportunity cost of winning and thus are, in terms of overall costs, more efficient than larger competitors. Depending on how the cost function changes after winning an auction, this effect can either be dampened or strengthened. As discussed in the literature review, two typical changes in the cost function after winning an auction can be considered. If costs increase in the number of previous auctions won (e.g., due to a larger backlog), winning an auction results in less aggressive pricing behavior by the previous winner. Assuming the previous winner is a smaller firm, and, if there is only one (larger) other competitor in the market, this might result in the larger firm winning the next auction, hence, changing our main implication of the equilibrium pricing strategies. Conversely, if winning an auction results in a downward shift of the cost function (e.g., due to a learning effect), this leads to more aggressive pricing by the previous winner and an increase in the probability of winning another contract, in line with our equilibrium prediction.

Our analysis suggests that the assumption that competition starts afresh each period may affect our results. Allowing firms' cost functions in future auctions to change conditional on winning an earlier contest may change the pricing equilibrium of our model depending on the direction of the dominating effect on the cost function. A similar effect could be expected if buyers' preferences depend on the results of previous auctions. For example, a buyer may be more inclined to buy from the winner of a previous auction due to anticipated experience effects.

One shortcoming of the current framework is the fact that sold capacity vanishes and does not return into possession of the supplier after some time. As a consequence, in equilibrium, it is not possible for the largest firm to be surpassed by other competitors. Therefore, a natural extension is the introduction of a backlog where firms' capacity is occupied for a limited number of periods and then might become available again. This would result in a infinitely repeated game with a stochastic link between periods. In the same vein, a (possibly probabilistic) link between previously won contracts (and hence reduced available capacity) and the costs in future auctions could be introduced. In connection with the introduction of returning capacity this feature would allow us to analyze the effects of efficiency gains due to learning as well as the possible influence of previously won auctions on costs and bidding behavior. While this may give rise to interesting new dynamics, we would still expect the result that equilibrium prices are driven by the firm with the largest capacity to hold in this richer setting.

CHAPTER 4

THE SCOPE FOR COLLUSION UNDER DIFFERENT PRICING SCHEMES

Abstract

We analyze and compare the incentives to collude under different pricing schemes in a differentiated-products market. We show that allowing firms to set two-part (nonlinear) tariffs as opposed to linear prices facilitates collusion at maximum prices independent of the degree of differentiation. However, compared to a situation where firms can only set fixed fees that are independent of the quantity purchased, collusion at maximum prices is less sustainable with two-part tariffs.

4.1 INTRODUCTION

In this paper, we analyze the impact of different pricing structures on firms' ability to collude. In particular, we are interested in nonlinear pricing, a common business practice in many industries. Examples include mobile telecommunications, media markets, amusement parks, gas, and electricity. In these industries, the pricing structures consist of at least two components: a fixed (entry) fee which is independent of the quantity demanded and a linear per-unit price.

Another prominent example of an industry where prices consist of multiple fixed and variable components is air cargo or air freight. In this market, shipping or freight rates can be considered flat fees within a certain weight segment and/or type of commodity. Different from that, (per-kilo and/or per-km) surcharges typically depend on the exact chargeable weight and/or distance and can be considered linear. Collusive agreements on a global scale have been revealed in the air cargo industry in recent years. In June 2008, the United States Department of Justice announced that major airlines have agreed to plead guilty and pay fines exceeding $500 million for fixing one or more components of total air cargo rates (U.S. Department of Justice, 2008). Similar judgements were made in other jurisdictions such as the European Union where fines amounted to a total of €799 million (European Commission, 2010), Canada (over $24 million in fines, see Canadian Competition Bureau, 2013), and New Zealand (NZ$42.5 million in fines, see Commerce Commission New Zealand, 2013).[1] Many major airlines were involved in the cartel in one or more jurisdictions including Air Canada, Air France-KLM, British Airways, Cathay Pacific, Cargolux, Emirates, Japan Airlines, LAN Chile, Martinair, SAS Cargo Group, Singapore Airlines, and Qantas. Lufthansa—the largest German carrier—was also involved in the cartel in the European Union but did not have to pay a fine because it reported the cartel to the European Commission (European Commission, 2010). However, rail operator Deutsche Bahn announced in December 2014 that it is suing Lufthansa, amongst other airlines, seeking damages amounting to €1.76 billion for the airlines' role in the cartel (Reuters, 2014). Furthermore, an investigation by the Swiss competition authority (COMCO) concluded that "airlines had agreed on freight rates, fuel surcharges, war risk surcharges, customs clearance surcharges for the U.S. and the commissioning of surcharges" and fined several airlines a total of CHF 11 million (Swiss Competition Commission, 2014, p. 1).[2]

We take the firm conduct observed in the air cargo price fixing case as a motivating example to address the question as to whether and how the possibility to coordinate on multiple fixed and linear price components instead of agreeing upon one (linear or fixed) collusive price only influences firms' ability to collude. To that end, building on Yin (2004) who models two-part tariff competition in duopoly, we analyze the incentives to collude under different pricing schemes in a differentiated-products setup à la Hotelling (1929) with elastic demand. Total demand is elastic as local demand—the demand of a single customer—decreases in price and in the distance to the respective firm (i.e., transport cost). We start by analyzing collusive incentives in a baseline setting where firms can set and

[1] In contrast to the US, the European Commission did not find sufficient evidence that airlines coordinated on freight rates but based their decision on the coordination of fuel surcharges only (European Commission, 2010).

[2] The sum of fines is relatively small in Switzerland because, as COMCO reports, Lufthansa and its subsidiary Swiss Air received full immunity because they reported the cartel. In addition, several other airlines received significant fine reductions because of their cooperation during the investigation.

coordinate on a single linear or fixed price only. Turning to two-part tariffs (i.e., setting a fixed fee in addition to a linear price), we find that the comparison regarding the sustainability of collusion crucially depends on the type of the single price (linear or fixed): the scope for collusion is largest for all values of the transport-cost parameter when firms are allowed to use fixed fees only whereas setting linear prices only results in the lowest incentives to collude. When setting two-part tariffs, collusion is easier to sustain compared to linear pricing but harder to sustain compared to fixed fees.

The main effect which renders collusion more attractive under nonlinear pricing compared to linear pricing is a relatively large profit from deviation under linear pricing. When a firm deviates in the linear-pricing scenario to increase market share, lowering its price has an additional positive effect on profits because it increases local demand. This effect is absent under nonlinear pricing because firms use the fixed part of the tariff to compete for market share. The result are relatively low profits from deviation.

Collusion is easiest to sustain in the fixed-fee scenario. This is also caused by relatively lower deviation profits under fixed fees relative to two-part tariffs. Under two-part tariffs, the deviating firm is able to fine-tune local demand using the linear part of the tariff especially when differentiation is large and optimal deviation does not entail covering the whole market, giving rise to larger profits from deviation and a lower incentive to collude.

Our result of intermediate incentives to collude under two-part tariffs is also relevant for the ongoing discussion of whether the simplification of tariff structures benefits customers. For example, the British Office of Gas and Electricity Markets (OFGEM) recently prohibited “complex multi-tier tariffs, where, for example, customers are initially charged a higher rate, which only falls if their consumption increases above certain levels” (Office of Gas and Electricity Markets, 2014, p. 1) in the markets for electricity and gas in the UK. The argument brought forward by regulators is that increasing transparency and enhancing comparability of prices through simplification of tariffs leads to an increase in customer surplus. Consistent with this argument, when interpreting two-part tariffs as complex and fixed fees as well as linear prices as simplified tariffs, our model predicts an increase in customer surplus in a static, competitive environment when firms are not allowed to offer two-part tariffs.

When considering dynamic effects of simplified tariff structures, however, our analysis points out that the effects of simplification on customer surplus are not clear cut at all. Prohibiting firms to set two-part tariffs but allowing them to charge simpler flat fees harms customers as it fosters collusion. However, the incentives to collude are reduced when firms may only set linear prices instead of two-part tariffs. As a policy implication, we stress that a careful approach to simplifying tariff structures is necessary to prevent pro-collusive effects even if short-term customer surplus increases.

Two-part tariffs or nonlinear pricing can be considered a form of second-degree price discrimination (see Varian, 1989) in the sense that all customers are offered the same schedule of price-quantity combinations. When customers are heterogeneous in their demand, they self-select different quantities and hence end up paying different per-unit prices. A typical example of two-part tariffs are quantity discounts which can take a large number of different forms (e.g., loyalty discounts, rebates).

Although price discrimination and nonlinear tariffs are important features of antitrust concerns, the literature on the impact of different pricing schemes on collusion is sparse.[3] To the best of our knowledge, this is the first study analyzing the incentives to collude when firms compete in two-part tariffs as opposed to linear prices. Concerning the relationship between third-degree price discrimination and collusion, Liu and Serfes (2007) investigate the impact of the availability of customer-specific information for market segmentation in a linear-city model on the feasibility to collude. A higher degree of market segmentation accompanied by a more diversified pricing structure is possible as the quality of customer information increases. Better information has opposing effects regarding the sustainability of collusion: on the one hand, it implies higher

[3]There is a large body of literature on the use of two-part tariffs in monopoly starting with Oi (1971). As far as the formal treatment of competition with nonlinear prices is concerned, Armstrong and Vickers (2001) as well as Rochet and Stole (2002) analyze nonlinear pricing in a setting with both horizontal and vertical heterogeneity whereas Laffont et al. (1998) focus on competition in two-part tariffs in the context of access pricing. We build our analysis on Yin (2004), who focuses on a duopoly model with horizontal product differentiation, in order to isolate the effect of different pricing schemes.

collusive profits and harsher punishment; on the other hand, deviation becomes more profitable. The authors show that the latter effect dominates, i.e., collusion is harder to sustain as the firms' ability to segment customers improves.

A related study to Liu and Serfes (2007) is Colombo (2010): the author allows for different degrees of product differentiation (i.e., firms are not located at the extremes of the linear city and hence are not maximally differentiated) and analyzes perfect (or first-degree) price discrimination. With perfect price discrimination, firms may set prices based on the exact location of a customer (so-called delivered pricing).[4] The author shows that collusion is easier to sustain the lower transport costs are[5] and that colluding on discriminatory prices is harder than on a uniform price.[6]

Both Liu and Serfes (2007) and Colombo (2010) find that third-degree and perfect price discrimination tend to reduce firms' incentives to collude. In contrast, our study suggests increased incentives to collude under second-degree price discrimination.

Fong and Liu (2011) analyze intertemporal price discrimination in an overlapping-generations model and show that, in comparison to uniform pricing, loyalty rewards of different forms facilitate collusion. Loyalty rewards are a form of quantity discounts and can also be interpreted as second-degree price discrimination. Customers live for two periods and demand at most one unit in each period. Firms can then price discriminate by allowing for a discount for repeat customers. Fong and Liu (2011) show that with loyalty discounts, deviating firms are unable to steal the industry profit for one period and hence, collusion is more likely to occur. This effect is further strengthened when firms can commit to offering discounts because the commitment limits firms' options when deciding upon the optimal deviation strategy. In contrast to our model, firms in Fong and Liu (2011)

[4] This is a special case of the analysis in Liu and Serfes (2007) with maximally differentiated firms. Further contributions investigating the implications of delivered pricing on collusion are, among others, Jorge and Pires (2008) and Miklós-Thal (2008).

[5] In the standard setting with unit demand and without the ability of firms to discriminate between customers, lower transport costs make it harder for firms to sustain collusion at maximum prices (Chang, 1991).

[6] Note that in his setup, firms always punish and deviate using discriminatory prices which is different from the present setup where punishment and deviation profits depend on the pricing instruments available to the firms.

can set linear prices only. If they can discriminate, they may set two different linear prices in a given period for first-time and repeat customers. In our model, firm can set both a linear and a fixed tariff component at the same point in time.

The rest of this paper is organized as follows. In section 4.2, we present the setup and derive demand functions. In section 4.3, we derive profits in the competitive, collusion, and deviation cases for linear prices (subsection 4.3.1), fixed fees (subsection 4.3.2), and two-part tariffs (subsection 4.3.3). The resulting critical discount factors are compared in subsection 4.4. The last section concludes.

4.2 THE MODEL

We consider a model of horizontal product differentiation à la Hotelling (1929) with two symmetric firms 1 and 2 which are located at the extremes of the linear city of unit length, i.e., at $L_1 = 0$ and $L_2 = 1$. Fixed and marginal costs are equal to zero.[7] Firms discount future profits by the common discount factor δ per period. We compare the incentives to collude in three different pricing scenarios in the following section:

(i) linear-price scenario (denoted by subscript L): firms compete in prices $p_{i,L}$ per unit purchased (see subsection 4.3.1);

(ii) fixed-fee scenario (denoted by subscript F): firms compete in fixed fees $f_{i,F}$, i.e., customers pay a flat (subscription) fee independent of actual usage (with $i \in \{1,2\}$) (see subsection 4.3.2); and

(iii) two-part tariffs (denoted by subscript T): firms compete in tariffs which are made up of a fixed component $f_{i,T}$ and a variable part $p_{i,T}$ charged per unit sold (see subsection 4.3.3).

Customers of mass one are uniformly distributed along the line. Each customer buys either from firm 1 or from firm 2. Building on Yin (2004) who models two-part tariff competition in duopoly, we allow individual demands to be

[7] We will relax the assumption of zero marginal costs below.

elastic.[8] A customer who is located at x and purchases q receives the following utility when buying from firm i:

$$U(x,q,p_i,f_i) = q - \frac{q^2}{2} - q\,(p_i + \tau|L_i - x|) - f_i \tag{4.2.1}$$

where τ is the transport-cost parameter. We note that the quantity demanded depends on transport cost. In the product differentiation interpretation of the model, this would mean that mismatch costs occur for each unit purchased. Then, $q\tau|L_i - x|$ represents the total disutility suffered by a customer with preferred product characteristics of x when consuming a product that is not ideal (and thus not located at x but at L_i). Note that the larger are q and/or $|L_i - x|$, the greater the disutility.

Customers maximize their utility when deciding on the quantity they want to purchase. This implies that a customer has the following local demand at firm i

$$\max_q U(x,q,p_i,f_i) \Rightarrow \frac{\partial U}{\partial q} = 1 - q - p_i - \tau|L_i - x| = 0$$
$$\Rightarrow q(x,p_i,f_i) = \begin{cases} 1 - p_i - \tau|L_i - x| \text{ if } U(x,q,p_i,f_i) \geq 0 \\ 0 \text{ else.} \end{cases} \tag{4.2.2}$$

Customer heterogeneity with respect to product preferences is also reflected in the individual demand which decreases as the difference in preferences and actual product characteristics grows.

Before analyzing the three different pricing regimes and their impact on collusion, a note on the measure for collusive stability seems in order. We will derive the critical discount factor for the different scenarios. Using grim-trigger strategies (see Friedman, 1971), we can compute critical discount factors according to the well-known formula

$$\delta \geq \bar{\delta} := \frac{\pi^d - \pi^c}{\pi^d - \pi^*} \tag{4.2.3}$$

where π^c, π^d, and π^* denote collusive profits, deviation profits and competitive (punishment) profits, respectively. All things equal, a lower (higher) punish-

[8] More precisely, we use the shipping model with linear demand which is discussed in section 3.2 in Yin (2004).

ment or deviation profit leads to a stabilization (destabilization) of the collusive agreement whereas the opposite is true for a change in the collusive profit.

Our focus here is on the situation where the market is covered, i.e., all customers along the line buy which is why the following is assumed to hold for customers' transport costs:

Assumption 1. *Transport costs are not too high:* $0 < \tau \leq 2/5 =: \bar{\tau}$.

The assumption guarantees that the whole market is served under any of the pricing scenarios to be considered. For larger transport costs, firms prefer not to serve the customers located around $1/2$. As a result, firms are local monopolists and the notion of collusion has no bite. Note that the assumption regarding transport costs is standard in the literature (see Yin, 2004).

We proceed with the derivation of the profits and the critical discount factors in the three scenarios.

4.3 ANALYSIS

4.3.1 LINEAR PRICING

We first consider firms' incentives to collude in a situation where they set linear prices. The results presented in this section are due to Rothschild (1997). We start by analyzing the customers' purchasing decision. Plugging the local demand specified in expression (4.2) into the utility expression in (4.2.1) implies that the indifferent customer located at $\tilde{x}$ is given by

$$U(\tilde{x}, p_i) = U(\tilde{x}, p_j) \Leftrightarrow \tilde{x}(p_i, p_j) = \frac{1}{2} - \frac{p_i - p_j}{2\tau}.$$

Consider the case where the indifferent customer $\tilde{x}$ is located in between both firms, i.e., $0 \leq \tilde{x} \leq 1$. Then, aggregate demand of firm i is given by

$$Q_i(p_i, p_j) = \int_0^{\tilde{x}(p_i, p_j)} (1 - \tau x - p_i)\, dx.$$

In the punishment stage, firms compete by simultaneously setting linear prices. The profit function of firm i is given by

$$\pi_i(p_i, p_j) = p_i Q_i(p_i, p_j).$$

We can derive the competitive equilibrium prices and profits in the standard way. Dropping subscripts, they are given by

$$p_L^* = \frac{2+3\tau-\sqrt{4-4\tau+13\tau^2}}{4}$$

and

$$\pi_L^* = \frac{\left(2+3\tau-\sqrt{4-4\tau+13\tau^2}\right)\left(2-4\tau+\sqrt{4-4\tau+13\tau^2}\right)}{32},$$

respectively. As in the Hotelling model with linear transport costs and unit demand, the model converges to Bertrand competition as differentiation vanishes ($\tau \to 0$). Equilibrium prices increase in τ in the relevant range of τ but the mark-up is smaller than in the model with unit demand. Larger differentiation dampens competition, giving firms an incentive to raise prices with both elastic and unit demand. Besides this competition effect, with elastic demand, mark-up is lower because firms have an additional incentive to lower prices to counter the reduction of individual demand caused by an increase in τ (elasticity effect).[9] An equivalent argument holds for the equilibrium profits.

If firms collude, they share the market equally and jointly set the optimal linear price in order to maximize industry profit which equals

$$p_L^c = \frac{1}{2} - \frac{\tau}{8}.$$

The resulting collusive profit of each firm is given by

$$\pi_L^c = \frac{(4-\tau)^2}{128}.$$

Given that the competitor sticks to the collusive agreement and sets the optimal collusive price p_L^c, we can derive the price set by a deviating firm i. When deriving this price, we have to distinguish between the cases where firm i serves (i) the whole market and (ii) shares the market with the other firm. For relatively large values of the transport-cost parameter, it is optimal to leave some market share to the competitor when deviating because covering the whole market would

[9] The separation of effects due to a change in product differentiation into competition and elasticity effects was first discussed by Mérel and Sexton (2010).

require a steep downward adjustment of the price. Below a cut-off value of τ, τ', the optimal deviation leads to a market share of 1 of the deviating firm.

Define $A := \sqrt{592 - 392\tau + 637\tau^2}$ and $\tau' := (4\sqrt{249} - 16)/233 \approx 0.2022$. The optimal deviating price and the resulting profit are then given by

$$p_L^d = \begin{cases} \frac{1}{2} - \frac{9\tau}{8} & \text{if } 0 < \tau \leq \tau' \\ \frac{40+14\tau-A}{72} & \text{if } \tau' < \tau \leq \bar{\tau} \end{cases}$$

and

$$\pi_L^d = \begin{cases} \frac{1}{4} - \frac{\tau}{4} - \frac{45\tau^2}{64} & \text{if } 0 < \tau \leq \tau' \\ \frac{(40+14\tau-A)(208+952\tau+20A+7A\tau-539\tau^2)}{497664\tau} & \text{if } \tau' < \tau \leq \bar{\tau}. \end{cases}$$

Using the profits in the collusive, deviating, and punishment phases and (4.2.3), the critical discount factor under linear pricing is given by

$$\bar{\delta}_L = \begin{cases} \frac{91\tau^2+24\tau-16}{28\tau\sqrt{4-4\tau+13\tau^2}-16+20\tau-5\tau^2} & \text{if } 0 < \tau \leq \tau' \\ \frac{637\tau^2A-15893\tau^3-392\tau A+12876\tau^2+592A-17520\tau-3520}{637\tau^2A-108864\tau^2\sqrt{13\tau^2-4\tau+4}+376795\tau^3+392\tau A-49332\tau^2+592A+44688\tau-3520} & \text{if } \tau' < \tau \leq \bar{\tau} \end{cases}$$

As first shown by Rothschild (1997), the critical discount factor first increases in the transport-cost parameter and then decreases.[10] We discuss the effects in more detail in the context of Proposition 12 below. We next turn to the case of fixed fees.

4.3.2 FIXED FEES

Fixed fees can be interpreted as a special case of two-part tariffs where the linear part p_i is set equal to zero. Then, local demand at firm i depends on the location of the customer only: $q(x) = 1 - \tau|L_i - x|$. In this case, the marginal customer is given by

$$U(\tilde{x}, f_i) = U\left(\tilde{x}, f_j\right) \Leftrightarrow \tilde{x}\left(f_i, f_j\right) = \frac{1}{2} - \frac{f_i - f_j}{\tau(2-\tau)}.$$

In the punishment stage, firms compete by simultaneously setting fixed fees. Firm i maximizes

$$\pi_{i,F}\left(f_i, f_j\right) = f_i \tilde{x}\left(f_i, f_j\right) \tag{4.3.1}$$

[10] For an in-depth analysis and explanation of the results, see Mérel and Sexton (2010) or Rasch and Herre (2013).

with respect to f_i. We have the following result:

Lemma 8. *In the punishment scenario with fixed fees, firms set an equilibrium fixed fee of*

$$f_F^* = \tau - \frac{\tau^2}{2}$$

and make a profit of

$$\pi_F^* = \frac{\tau}{2} - \frac{\tau^2}{4}. \tag{4.3.2}$$

Proof. Differentiating $\pi_{1,F}(f_1, f_2)$ with respect to f_1 yields

$$\frac{\partial \pi_{1,F}(f_1, f_2)}{\partial f_1} = \frac{2f_2 - 4f_1 + 2\tau - \tau^2}{2\tau(2-\tau)}.$$

The second-order condition is given by

$$\frac{\partial^2 \pi_{1,F}(f_1, f_2)}{\partial f_1^2} = -\frac{2}{\tau(2-\tau)} < 0.$$

Setting $\partial \pi_{1,F}(f_1, f_2)/\partial f_1 = 0$, using symmetry, solving for f_1, and re-substituting into $\pi_{1,F}(f_1, f_2)$ immediately leads to the lemma. $\square$

Fixed prices and profits increase in differentiation for τ as defined by Assumption 1. It is well-known that differentiation softens competition, resulting in an incentive to increase fees in order to extract more surplus from each customer because switching suppliers becomes more costly as differentiation increases. Profits are always larger under fixed fees compared to linear pricing. When a fixed fee is available, firms can use it to directly target consumer surplus, resulting in larger profits. Similar to linear pricing, as differentiation vanishes ($\tau \to 0$), competition becomes tougher and profits converge to zero.

In the optimal collusive agreement, each firm serves exactly half of the market and sets the fixed fee such that all surplus of the customer located at $1/2$ is extracted. We arrive at the following result:

Lemma 9. *With fixed fees, collusive prices and profits are given by*

$$f_F^c = \frac{(2-\tau)^2}{8}$$

and

$$\pi_F^c = \frac{(2-\tau)^2}{16}. \tag{4.3.3}$$

Proof. The customer at position $x = 1/2$ when buying from firm 1 has a utility of $1/2 - \tau/2 + \tau^2/8 - f_1$. The optimal f_1 follows immediately. The collusive profit is then given by $\pi^C_{i,F} = f^C_{i,F}/2$. Due to symmetry, the values for firm 2 are identical. □

In the fixed-fee scenario, firms can fully extract the surplus of the indifferent consumer in order to maximize joint profits. Extracting all surplus of the indifferent consumer is impossible under linear pricing, resulting in a larger collusive profit with fixed fees. Turning to the optimal deviating strategy given that the other firms sets the collusive fixed fee, we again have to distinguish between the cases where the deviator finds it profitable to serve the whole market or leaves some market share to the other firm. Define $\tau'' := 2/13$. We then find the following:

Lemma 10. *With fixed fees, deviation prices and profits are given by*

$$f^d_F = \begin{cases} \frac{1}{2} - \frac{3\tau}{2} + \frac{5\tau^2}{8} & \text{if } 0 < \tau \leq \tau'' \\ \frac{(2-\tau)(2+3\tau)}{16} & \text{if } \tau'' < \tau \leq \bar{\tau} \end{cases}$$

and

$$\pi^d_F = \begin{cases} \frac{1}{2} - \frac{3\tau}{2} + \frac{5\tau^2}{8} & \text{if } 0 < \tau \leq \tau'' \\ \frac{(2-\tau)(2+3\tau)^2}{256\tau} & \text{if } \tau'' < \tau \leq \bar{\tau}. \end{cases} \tag{4.3.4}$$

Proof. The optimal deviation fixed fee set by firm 1 when the other firm sets pricing according the collusive agreement is determined by maximizing the profit function $\pi_1(f_1, f^c_F)$ over f_1. The partial derivative is given by

$$\frac{\partial \pi_1(f_1, f^c_F)}{\partial f_1} = \frac{-16 f_1 - 3\tau^2 + 4\tau + 4}{8\tau(2-\tau)}.$$

Observe that the second-order condition is satisfied due to Assumption 1:

$$\frac{\partial^2 \pi_{1,F}(f_1, f^C_{2,F}; \tau)}{\partial f_1^2} = -\frac{2}{\tau(2-\tau)} < 0$$

Solving the first-order condition for f_1 leads to $f^{d,l}_F = (2-\tau)(2+3\tau)/16$. Given $f^{d,l}_F$, we have to ensure that the market share of the deviating firm does not exceed 1, i.e., $\tilde{x}\left(f_F^{d,l}, f_F^c\right) \leq 1 \Leftrightarrow \tau \geq 2/13$. Thus, for $\tau \geq 2/13$, the optimal deviation fixed fee is $f_F^{d,l}$. For $\tau \leq 2/13$, the optimal fixed fee is set such that the indifferent customer is at location $x = 1$ and receives a utility of zero: $1 = \tilde{x}(f_1, f_F^c) \Leftrightarrow f_1 = 1/2 - 3t/2 + 5t^2/8 =: f_F^{d,s}$. Plugging the respective fixed fees into the profit function (4.3.1) leads to the deviation profits. □

Using (4.2.3) and the respective profits we just derived, we can calculate the critical discount factor and analyze its slope.

Proposition 10. *When firms can set fixed fees only, the critical discount factor is given by*

$$\bar{\delta}_F = \begin{cases} \frac{9\tau-2}{14\tau-4} & \textit{if } 0 < \tau \leq \tau'' \\ \frac{2-5\tau}{11\tau+2} & \textit{if } \tau'' < \tau \leq \bar{\tau}, \end{cases}$$

where $\tau'' := 2/13$.
Furthermore, $\bar{\delta}_F$ *is decreasing in* τ.

Proof. The first part of the lemma follows from substituting expressions (4.3.2), (4.3.3), and (4.3.4) into inequality (4.2.3) and simplifying. For the second part, the derivative of $\bar{\delta}_F$ with respect to τ is given by

$$\frac{\partial \bar{\delta}_F}{\partial \tau} = \begin{cases} \frac{-2}{(7\tau-2)^2} & \text{if } 0 < \tau \leq \tau'' \\ \frac{-32}{(11\tau+2)^2} & \text{if } \tau'' < \tau \leq \bar{\tau}. \end{cases}$$

$\partial \bar{\delta}_F / \partial \tau < 0$ follows immediately. □

The observation that the critical discount factor decreases in the scope of product differentiation is similar to the case analyzed in Chang (1991) and Häckner (1996). In those contributions, it is shown that with unit demand, the effect of an increased level of product differentiation on deviation—which is less profitable as customers incur higher transport costs—outweighs the opposing, sustainability-decreasing effect on competitive profits which increase as firms enjoy a greater degree of market power. As the linear price is zero in the present setup, local demand is independent of any linear pricing component; hence, a similar result concerning the impact of product differentiation on the critical discount factor is obtained.

We next analyze the case where firms choose two-part tariffs.

4.3.3 TWO-PART TARIFFS

In a situation where firms set both linear prices and fixed fees, the marginal customer is located at

$$U\left(\tilde{x}, p_i, f_i\right) = U\left(\tilde{x}, p_j, f_j\right) \tag{4.3.5}$$

$$\Leftrightarrow \tilde{x}\left(p_i, f_i, p_j, f_j\right) = \frac{1}{2} - \frac{p_i - p_j}{2\tau} - \frac{f_i - f_j}{2\tau(2 - p_i - p_j - \tau)}. \tag{4.3.6}$$

If firms compete in the punishment stage, each of them maximizes

$$\pi_i\left(p_i, f_i, p_j, f_j\right) = p_i Q_i\left(p_i, f_i, p_j, f_j\right) + f_i \tilde{x}\left(p_i, f_i, p_j, f_j\right)$$

with respect to p_i and f_i. The first term of the profit function is the revenue generated by charging a variable price per unit purchased while the second term is the market share or the number of customers multiplied with the fixed fee. The results in the competitive stage are due to Yin (2004) for the situation with a fully covered market where the following prices and profits result:

$$p_T^* = \frac{\tau}{4},$$

$$f_T^* = \frac{3\tau}{4} - \frac{9\tau^2}{16},$$

and

$$\pi_T^* = \frac{\tau}{2} - \frac{11\tau^2}{32}. \tag{4.3.7}$$

In two-part-tariff competition, the main instrument used to compete for the indifferent customer is the fixed fee. The use of the linear price can be best thought of as a sequential procedure. Firms determine the optimal linear price as a function of market share and then compete for the indifferent customer and market share by setting fixed fees.

When setting the linear price for given market shares, firms have to balance two opposing effects caused by the fact that individual demands depend on both price *and* location. As a consequence, local demand decreases for each customer as the distance between the customer and the respective firm increases. Lowering the linear price leads to a larger surplus for all customers. In particular, it leads to a larger surplus of the marginal customer, i.e., the customer with the lowest demand at one firm, which can potentially be extracted via the fixed fee. On the other hand, lowering the linear price leads to a larger rent given to customers located closer to the firms. This gives firms an incentive to increase the linear price in order to extract additional surplus from these inframarginal customers. In the resulting compromise, firms set a linear price above marginal cost.

While the competitive profit in both the two-part-tariff and the fixed-fee scenario is larger compared to the profits under linear prices because of the ability of the firms to directly extract surplus via a fixed fee, it is lower under two-part tariffs compared to the fixed-fee scenario. With two-part tariffs, firms can gain additional profits for a given market share by setting a positive linear price. This option is not present in the fixed-fee scenario. Anticipating this source of additional income, firms behave more aggressively when competing for market share using the fixed fee. The result is a lower competitive profit under two-part tariff competition.

If firms collude, they share the market equally, and set a linear price to maximize overall customer surplus which they partly extract via the fixed fee. We then have the following result:

Lemma 11. *In the two-part-tariff scenario, the collusive prices and profits are given by*

$$p_T^c = p_T^* = \frac{\tau}{4},$$

$$f_T^c = \frac{(4-3\tau)^2}{32},$$

and

$$\pi_T^c = \frac{1}{4} - \frac{\tau}{4} + \frac{5\tau^2}{64}. \tag{4.3.8}$$

Proof. Writing the profit function of firm 1 (which is w.l.o.g.) as a function of p_1 yields

$$\pi_{1,T} = \frac{1}{4}\left(1 - 2p_1 - \tau + p_1^2 + p_1\tau + \frac{\tau^2}{4}\right) + p_1 \int_0^{\frac{1}{2}} (1 - p_1 - \tau x)dx.$$

The first-order condition is given by

$$\frac{\partial \pi_{1,T}}{\partial p_1} = -\frac{p_1}{2} + \frac{\tau}{8}.$$

Observe that the second-order condition is satisfied. Setting $\partial \pi_{1,T}/\partial p_1 = 0$ and solving for p_1 yields $p^C_{i,T}$. The optimal fixed fee is derived by substituting $p^C_{i,T}$ into $f_{1,T}$. The collusive profit follows immediately by substituting $p^C_{i,T}$ and $f_{1,T}$ into the profit function. □

Observe that the linear prices under competition and collusion are identical. This is because the firms' market shares are identical in both scenarios. Because the linear price can be thought of as a function of market share only, this results in the same linear price. The optimal fixed fee is larger without competition as firms are now able to extract all surplus of the indifferent consumer.

The largest collusive profits can be obtained with two-part tariffs while they are lowest under linear pricing. In contrast to linear pricing, the ability to use fixed fees allows firms to fully extract the indifferent consumer's surplus. The availability of a linear price to extract additional surplus of inframarginal consumers ranks the two-part-tariff scenario above the fixed-fee scenario.

Next assume that firm i's competitor follows the collusive agreement by setting p^c_T and f^c_T and define $B := \sqrt{61 - 83\tau + 28\tau^2}$ and $\tau''' := 14/19 - 2\sqrt{30}/19 \approx 0.1603$. The optimal deviation strategy of firm i and the resulting deviation profit are characterized as follows:

Lemma 12. *Under two-part tariffs, the optimal deviation from the collusive agreement yields the following prices and profits:*

$$p^D_{1,T} = \begin{cases} \frac{1}{2}\tau & \text{if } \tau \leq \tau''' \\ \frac{8}{3} - \frac{5}{3}\tau - \frac{1}{3}B & \text{else,} \end{cases}$$

$$f^d_T = \begin{cases} \frac{1}{2} - 2\tau + \frac{11}{8}\tau^2 & \text{if } \tau \leq \tau''' \\ \frac{197}{9} - \frac{268}{9}\tau + \frac{181}{18}\tau^2 - \frac{25}{9}B + \frac{35}{18}\tau B & \text{else,} \end{cases}$$

$$\pi^d_T = \begin{cases} \frac{1}{2} - \frac{3}{2}\tau + \frac{7}{8}\tau^2 & \text{if } \tau \leq \tau''' \\ \frac{1}{27\tau}(5\tau - 8 + B)\left(58 - 86\tau + 31\tau^2 - 8B + 5\tau B\right) & \text{if } \tau''' < \tau \leq \bar{\tau}. \end{cases} \tag{4.3.9}$$

Proof. For the proof it is helpful to rewrite the profit function of the deviating firm as a function of the linear prices of both firms, the fixed fee of the firm that sticks to collusion as well as the marginal customer. Maximization is then over p_i and $\tilde{x}$. The idea is that the deviating firm can choose the optimal market size and, given the market size, the optimal linear price. The profit function of firm i is then given by

$$\pi_T^d = \tilde{x}\left(f_j + \frac{(1-p_i-\tau\tilde{x})^2}{2} - \frac{(1-p_j-\tau(1-\tilde{x}))^2}{2}\right) + p_i\int_0^{\tilde{x}}(1-p_i-\tau y)dy$$

where the condition for the indifferent customer was solved for firm i's fixed fee and substituted. Substituting the collusive values of firm j leads to

$$\pi_T^d = \tilde{x}\left(\frac{(4-3\tau)^2}{32} + \frac{(1-p_i-\tau\tilde{x})^2}{2} - \frac{(1-\frac{\tau}{4}-\tau(1-\tilde{x}))^2}{2}\right) + p_i\int_0^{\tilde{x}}(1-p_i-\tau y)dy$$

Differentiating with respect to p_i gives

$$\frac{\partial \pi_T^d}{\partial p_i} = \frac{\tau\tilde{x}^2}{2} - p_i\tilde{x},$$

differentiating the profit function w.r.t. to $\tilde{x}$ leads to

$$\frac{\partial \pi_T^d}{\partial \tilde{x}} = \frac{1}{2} + \frac{\tau}{2} - \frac{\tau^2}{2} - \frac{p_i^2}{2} + \tilde{x}p_i\tau - 4\tau\tilde{x} + \frac{5\tau^2\tilde{x}}{2}.$$

Solving for p_i and $\tilde{x}$ leads to

$$p_i^{D,l} = \frac{8}{3} - \frac{5\tau}{3} - \frac{B}{3}$$

and

$$\tilde{x}^D = \frac{1}{\tau}\left(\frac{16}{3} - \frac{10\tau}{3} - \frac{2B}{3}\right),$$

where $B := \sqrt{61-83\tau+28\tau^2}$. Given these two critical values, we now check whether second order conditions are satisfied, i.e., if they constitute a maximum of the profit function. The Hessian of the profit function is given by

$$H(p_i,\tilde{x}) = \begin{bmatrix} -\tilde{x} & \tau\tilde{x}-p_i \\ \tau\tilde{x}-p_i & \tau p_i - 4\tau + \frac{5\tau^2}{2} \end{bmatrix}.$$

The determinant of the Hessian is given by

$$D(p_i,\tilde{x}) = \tau p_i\tilde{x} + 4\tau x - \frac{5\tau^2\tilde{x}}{2} - \tau^2\tilde{x}^2 - p_i^2.$$

It follows that the pair $p_i^{D,l}, \tilde{x}^D$ is a local maximum if $D(p_i^{D,l}, \tilde{x}^D) > 0$ and $\partial^2 \pi_T^d / \partial p_i^2 < 0$. Substituting the critical values into the determinant and simplifying gives

$$D(p_i^{D,l}, \tilde{x}^D) = -\frac{B(5\tau - 8 + B)}{3}.$$

Observe that $D(p_i^{D,l}, \tilde{x}^D)$ is a function of τ only, define it $D(p_i^{D,l}, \tilde{x}^D) := K(\tau)$. $K(\tau)$ has three roots, one at $\tau = 1/2 - \sqrt{5}/2 \approx -0.618$, a second one at $\tau = 83/56 + \sqrt{57}/56 \approx 1.617$ and a third one at $\tau = 83/56 - \sqrt{57}/56 \approx 1.347$. As none of these roots in the range of τ as defined by Assumption 1, it follows that $K(\tau)$ does not have any roots in the relevant range. Substituting a valid value into $K(\tau)$ leads to $K(0.1) \approx 0.537$. It follows that $D(p_i^{D,l}, \tilde{x}^D) > 0$. The second condition follows immediately, $\partial^2 \pi_T^d / \partial p_i^2 = -\tilde{x} < 0$ as $\tilde{x} \in [0,1]$.

The resulting optimal fixed fee $f_i^{D,l}$ follows from substituting the collusive values of firm j, $p_i^{D,l}$, and $\tilde{x}^D$ into equation (4.3.6) and solving for f_i. Resubstituting the optimal values into the profit function leads to the deviation profit for large τ. Clearly, $\tilde{x}^D$ cannot exceed 1. Solving $\tilde{x}^D \leq 1$ for τ leads to

$$\tau \leq \frac{14}{19} - \frac{2\sqrt{30}}{19} =: \tau'''.$$

For $\tau > \tau'''$, the optimal deviating values are given by $p_i^{D,l}$ and $f_i^{D,l}$. For $\tau \leq \tau'''$, the optimal linear price is derived by substituting $\tilde{x} = 1$ into the first-order condition and solving for p_i, leading to

$$p_i^{D,s} = \frac{\tau}{2}.$$

The optimal fixed fee is calculated by setting $\tilde{x} = 1$, substituting firm j's collusive values and the optimal linear price of firm i into equation (4.3.6) and solving for f_i which gives

$$f_i^{D,s} = \frac{1}{2} - 2\tau + \frac{11\tau^2}{8}.$$

The respective profit follows immediately. □

For low levels of differentiation, the deviating firm covers the whole market. This becomes costly as differentiation increases because customers close to the firm sticking to the collusive agreement are expensive to attract. If transport costs are sufficiently large, this becomes too costly such that some market share is covered by the non-deviating firm.

Given firms' profits in all scenarios under two-part-tariff pricing, we are now able to calculate the critical discount factor according to condition (4.2.3). We summarize our findings in the following lemma.

Proposition 11. *The discount factor under two-part tariffs for which collusion with grim-trigger strategies can be sustained is given by*

$$\bar{\delta_T} = \begin{cases} \frac{17\tau-4}{26\tau-8} & \text{if } \tau \leq \tau''' \\ \frac{1}{2}\frac{3584\tau^2 B+18745\tau^3-10624\tau B-83856\tau^2+7808B+124176\tau-60928}{1792\tau^2+9737\tau^3-5312\tau B-42576\tau^2+3904B+62304\tau-30464} & \text{if } \tau''' < \tau \leq \bar{\tau}, \end{cases}$$

where $\tau''' := 14/19 - 2\sqrt{30}/19$ *and* $B := \sqrt{61-83\tau+28\tau^2}$.
Furthermore, $\bar{\delta_T}$ *is decreasing in* τ.

Proof. The first part of the lemma follows from substituting expressions (4.3.7), (4.3.8), and (4.3.9) into inequality (4.2.3) and simplifying. For part two of the lemma, the derivative of $\bar{\delta_T}$ with respect to τ is given by

$$\frac{\partial \bar{\delta_T}}{\partial \tau} = \begin{cases} \frac{-8}{(13\tau-4)^2} & \text{if } \tau \leq \tau''' \\ \frac{1}{B(1792B+9737\tau^3-5312\tau B-42576\tau^2+3904B+62304\tau-30464)^2}(432(112980\tau^5 \\ +21318\tau^4 B - 672977\tau^4 - 95401\tau^3 B + 1539029\tau^3 + 149280\tau^2 B \\ -1653042\tau^2 - 91392\tau B + 795440\tau + 15232B - 119072)) & \text{if } \tau''' < \tau \leq \bar{\tau}, \end{cases}$$

For $\tau \leq \tau'''$, it follows immediately that $\partial\bar{\delta_T}/\partial\tau < 0$. For $\tau''' < \tau < \bar{\tau}$, first note that because $B > 0$ in the relevant range of τ, the first part of the expression is positive. Define the second part of the expression as

$$\begin{aligned} G(\tau) =& (432(112980\tau^5 + 21318\tau^4 B - 672977\tau^4 - 95401\tau^3 B + 1539029\tau^3 \\ &+ 149280\tau^2 B - 1653042\tau^2 - 91392\tau B + 795440\tau + 15232B \\ &- 119072)). \end{aligned}$$

$G(\tau)$ has one root at $\tau = 4/9$ which contradicts $\tau''' < \tau < \bar{\tau}$. Plugging in a smaller, positive value of τ, e.g., $\tau = 1/3$ gives $G(1/3) \approx -1231.62$. Because the function is continuous, it follows that $\partial\bar{\delta_T}/\partial\tau < 0$ if $\tau''' < \tau < \bar{\tau}$. □

Consider the impact of the level of product differentiation on the critical discount factor for two-part tariffs: as mentioned before, firms use the fixed part of the two-part tariff to compete for the indifferent customer. As a consequence, the impact of the product differentiation is similar to the case where firms only charge a fixed fee, i.e., an increase in differentiation increases the sustainability of collusion.

We are now in a position to compare profits and analyze the incentives to collude across all three scenarios.

4.4 COMPARISON OF PROFITS AND CRITICAL DISCOUNT FACTORS

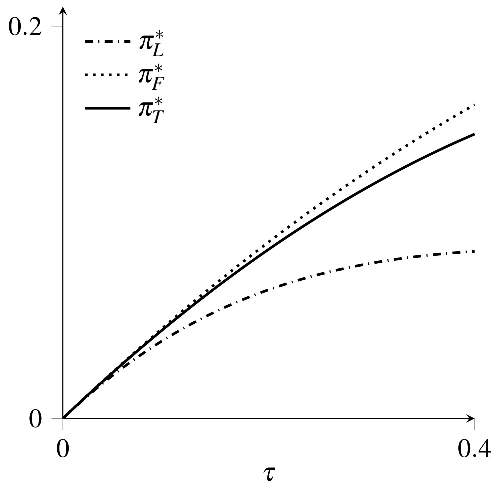


Figure 4.1: Comparison of competitive profits.

Consider the competitive profits illustrated in Figure 4.1. Punishment profits are larger with two-part tariffs compared to linear pricing although intuition would first point in the opposite direction: typically, competitive pressure is higher if firms have more instruments available, which should lead to lower profits. In the present case, however, firms use the fixed fee to compete for the indifferent customer (both under a fixed fee and a two-part tariff) without having to pay attention to (inframarginal) local demand. Once market shares are set, firms are monopolists when it comes to optimizing local demand (or customer surplus) through the linear price. Consumer surplus can then partly be appropriated through the fixed fee (see also Yin, 2004). Moreover, in the linear-pricing case, firms cannot directly target consumer surplus but have to use the linear component to compete for the indifferent customer. As a result, firms make a lower profit under linear pricing which means that in this scenario, punishment is harshest.

Competitive profits are lower with two-part tariffs compared to fixed fees. Under two-part tariffs, when competing for market shares via the fixed fee, firms anticipate that they can earn additional profits by adjusting the linear price once market shares are set. This additional source of income induces firms to compete more intensely for market shares by lowering the fixed fees, resulting in relatively lower profits under two-part tariffs.

The collusive per-firm profits in all three scenarios are displayed in Figure 4.2. When firms collude, the largest (industry) profit is obtained when two-part tariffs are used because in this scenario firms have most instruments available to extract customer surplus. Under linear pricing, firms are unable to extract all surplus from the indifferent customer and hence from any customer located closer to them because they can only set a linear price.[11] When firms are able to charge a fixed fee only, they can fully extract the indifferent customer's surplus but leave positive surplus to the other customers located closer to the firm(s). Because local demand is decreasing in the distance to the firms, these customers have a positive surplus even after paying the fixed fee. With two-part tariffs, the additional instrument of a linear price allows firms to further increase their profits by extracting additional surplus off those customers located in closer proximity to their locations. In conclusion, collusion is most profitable if firms can set two-part tariffs.

A comparison of the profits of a deviating firm yields qualitatively similar results (see Figure 4.3). Two-part tariffs lead to the largest profit because it allows the deviating firm to extract the largest amount of surplus from the customers. Note that collusive as well as deviating profits decrease in the transport-cost parameter because it decreases local demand for all customers. It follows that

[11] Under linear pricing, even the indifferent customer is left with a positive surplus. This is because the firms optimize against a linear demand function. Furthermore, it can be shown that optimal collusion involves serving the customer located at $1/2$ only if $\tau \leq 2/3$ which holds under Assumption 1. For larger values of τ, firms do not have an incentive to serve all customers in the market, leading to a number of customers close to $1/2$ being left out.

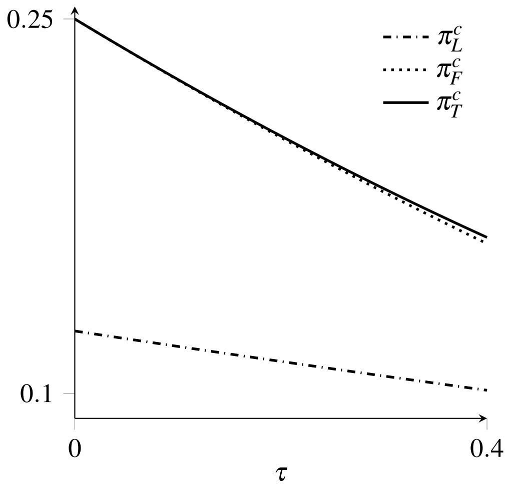


Figure 4.2: Comparison of collusive profits.

deviation is least profitable under linear pricing and most profitable under two-part tariffs.

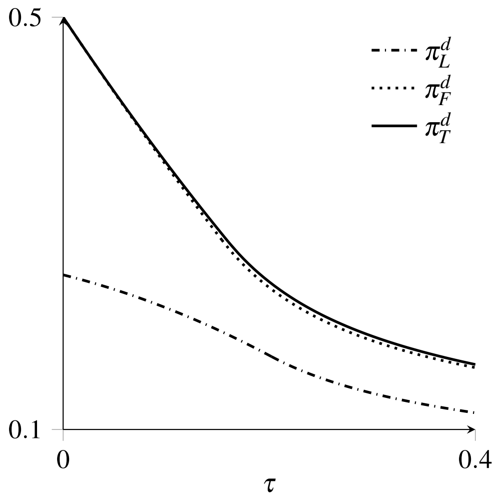


Figure 4.3: Comparison of deviation profits.

Overall, we find that there are opposing effects regarding the sustainability of collusion under linear pricing compared to fixed fees and two-part tariffs. On the one hand, incentives to collude are strongest under two-part tariffs because of the largest collusive profits; on the other hand, the largest incentives to deviate are also present under two-part tariffs because it is most profitable to deviate but also because punishment is least harsh. When comparing fixed fees and two-part tariffs, we find that in the former scenario, the collusive and deviation profits are smaller under fixed fees while competitive profits are larger, leading to ex ante unclear effects between the two nonlinear pricing schemes.

In the following proposition we present our main result which sheds light on how the different profits affect the discount factors in the three pricing scenarios considered. Interestingly, we find that the ranking of critical discount factors is the same for all values of τ.

Proposition 12. *For all values of* $\tau \in (0, \bar{\tau})$ *it holds that* $\bar{\delta}_F < \bar{\delta}_T < \bar{\delta}_L$.

Proof. We start by showing that $\bar{\delta}_T < \bar{\delta}_L$ holds for $0 < \tau \leq \bar{\tau}$. At $\tau = 0$, all three critical discount factors are equal to $1/2$. Since $\bar{\delta}_L$ is increasing at first and both $\bar{\delta}_F$ and $\bar{\delta}_T$ are decreasing, $\bar{\delta}_F < \bar{\delta}_T < \bar{\delta}_L$ holds for values of τ close to 0. For $\tau > 0$, we look for a solution to $\bar{\delta}_T = \bar{\delta}_L$ with respect to τ in the regions of τ defined by the respective deviation profits. For $0 < \tau < \tau'''$ we find that $\bar{\delta}_T = \bar{\delta}_L$ only holds for $\tau = 0$ which contradicts that $0 < \tau < \tau'''$. For the area of $\tau''' < \tau < \tau'$, $\bar{\delta}_T = \bar{\delta}_L$ holds for $\tau \approx 0.0829$ which violates $\tau''' < \tau$. For $\tau > \tau'$, the only solution to $\bar{\delta}_T = \bar{\delta}_L$ is $\tau = 0$. Since the discount factors are continuous functions, $\bar{\delta}_T < \bar{\delta}_L$ follows.

In the second step, we show in the same way that $\bar{\delta}_F < \bar{\delta}_T$ holds for $0 < \tau \leq \bar{\tau}$. In the region where $0 < \tau < \tau''$, $\bar{\delta}_F = \bar{\delta}_T$ only holds for $\tau = 0$ which contradicts $\tau > 0$. For $\tau'' < \tau < \tau'''$, the only solution to $\bar{\delta}_F = \bar{\delta}_T$ is at $\tau \approx 0.1355$ which is not in the range of $\tau'' < \tau < \tau'''$. For $\tau > \tau'''$, $\bar{\delta}_F = \bar{\delta}_T$ only holds for $\tau = 0$. Since the discount factors are continuous functions, $\bar{\delta}_F < \bar{\delta}_T$ follows.

As $\bar{\delta}_T < \bar{\delta}_L$ and $\bar{\delta}_F < \bar{\delta}_T$, it follows that $\bar{\delta}_F < \bar{\delta}_T < \bar{\delta}_L$. □

As shown in Proposition 12, firms have relatively larger incentives to collude if their pricing instruments include a fixed fee (i.e., in the fixed-fee and two-part-tariff scenarios) as compared to linear prices. A graphical illustration of the critical discount factors is given in Figure 4.4.

If a fixed fee is available, it is used as the main instrument to compete for the indifferent customer in the punishment case and as the main means to undercut the collusive prices when deviating. The additional linear price (if available) is used to optimize local demand for a given market share. This difference in behavior is the key ingredient to explaining our results. Because we find the effects underlying the incentives to collude in both the two-part-tariff and fixed-fee scenario to be qualitatively similar, it is instructive to first compare the incentives to collude under fixed-fee and linear pricing and then discuss the more subtle differences between the fixed-fee and two-part-tariff pricing regimes.

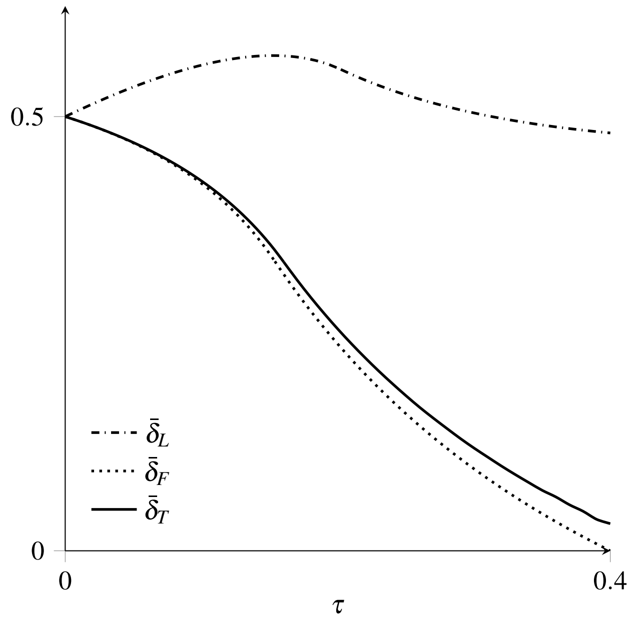


Figure 4.4: Comparison of the critical discount factors in the three pricing scenarios.

The main effect explaining our finding that collusion is easier to sustain if firms' pricing instruments include a fixed fee is the relatively unattractive profit from deviation compared to linear pricing. While deviation profits decrease in all pricing scenarios as differentiation increases because increasing market share becomes more costly, this effect is less pronounced under linear pricing. Under linear pricing, firms deviate from a collusive agreement by reducing the linear price. Besides increasing market share, this price cut has a positive effect on local demand, leading to an increase in deviation profits. This effect is not present in either nonlinear-pricing scenario because firms use the fixed fee as the pricing instrument to undercut their rival. As a result, deviation is relatively unattractive, increasing the incentives to collude with nonlinear pricing.

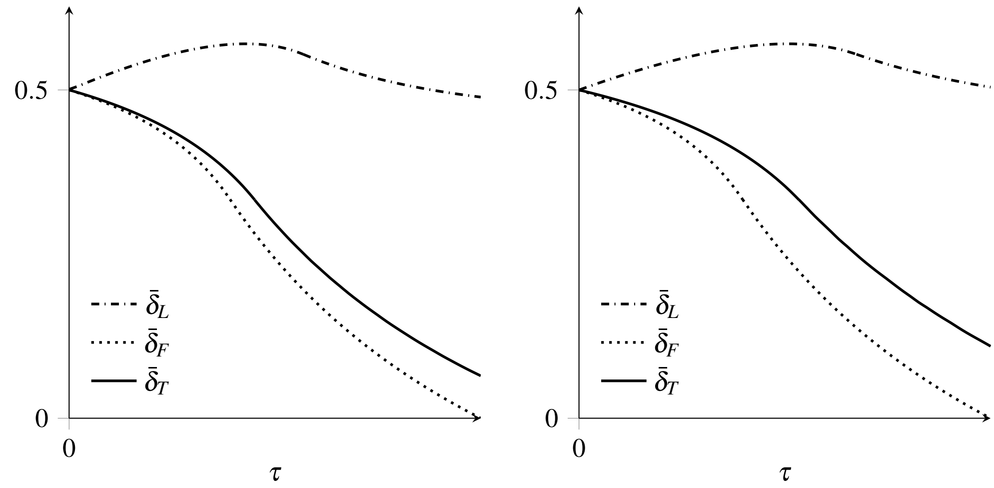
When comparing the incentives to collude under fixed fees and two-part tariffs, we find that incentives to collude are strongest under fixed fees. In the two-part-tariff scenario deviation is slightly more attractive because the deviating firm can use the linear price to fine-tune local demand when covering the whole market becomes too costly. This relatively stronger incentive to deviate under two-part tariffs overcompensates the harsher punishment and thus increased incentive to collude in this scenario.

Extension: Positive marginal costs

Finally, we note that our results continue to hold qualitatively when introducing a common positive marginal cost for the firms despite the inefficiency this causes in case of fixed fees because firms then make a loss with each unit sold. As marginal costs increase, collusion becomes easier to sustain under fixed fees and two-part tariffs for any given value of the transport-cost parameter. The reason is a lower utility for consumers located in the middle of the unit interval, leading to an increased incentive by firms to serve only consumers located in their proximity under non-linear pricing, i.e., to behave like local monopolists. The cases of $c = 1/10$ and $c = 1/5$ are displayed in Figure 4.5.

4.5 CONCLUSION

Motivated by the recent world-wide cartel case in the air cargo industry where firms coordinated on multiple price components, we analyze the influence of three different pricing regimes on firms' ability to collude. It is shown that full collusion is easiest to sustain with fixed fees. Compared to two-part tariffs, where incentives to collude are intermediate, this result is driven by smaller deviation profits under fixed fees. We have shown that this pro-collusive effect dominates the opposing effect of a softer punishment under fixed fees. Collusion at maximum prices can be sustained for the smallest range of the discount factor under linear pricing when compared to both nonlinear-pricing scenarios. This result is driven by the relatively large profit of deviation under linear pricing. This is caused by an



(a) Marginal cost $c = 1/10$. (b) Marginal cost $c = 1/5$.

Figure 4.5: Critical discount factors with positive marginal costs.

increase in local demand due to undercutting, an effect that is a lot weaker under nonlinear pricing.

The present analysis not only helps to get a better understanding of recent anti-trust cases but also has implications for competition policy in a more general context: customer protection agencies as well as policymakers often criticize firms' complex pricing schedules designed to price discriminate between customers. They demand that firms reduce the complexity of their pricing schemes in order to make decisions for customers easier and more transparent. This paper highlights that the implications of such changes are not clearcut: it is true that a smaller number of available contracts, i.e., less instruments to price discriminate among customers, may reduce prices customers have to pay in a static context. Indeed, when moving from two-part tariffs to linear prices or fixed fees, we find that consumer rents increase. However, there may be the undesired anti-competitive consequence that collusion turns out to be easier to sustain and firms end up generating higher supra-competitive profits in a dynamic setting when moving to a simpler pricing regime. As a consequence, it seems of importance for competition authorities to carefully assess how the simplification of pricing structures—through fixed or linear prices—is achieved.

An aspect which we have not analyzed is the strategic choice of the pricing schedule employed by firms. If firms decide on the tariff before they collude, they may use other pricing techniques when they choose to deviate. Furthermore, it may be interesting to investigate the effect of the number of firms on the incentives to collude under nonlinear pricing. We leave this as an open question for future research.

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