Life Cycle Economics, Health, and Inequality

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Contents

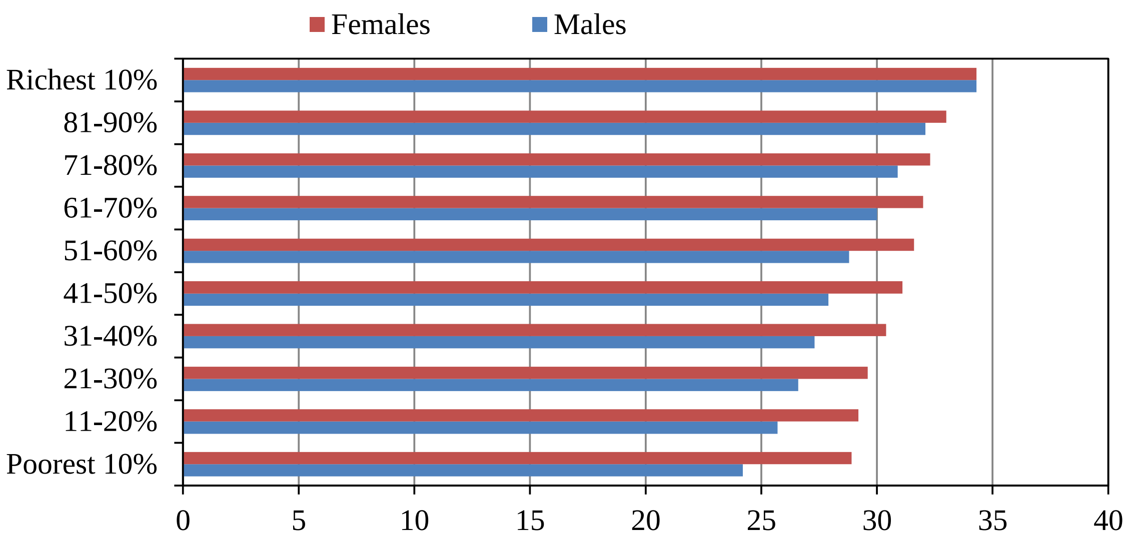
List of Figures

List of Tables

Introduction

This thesis consists of three self-contained chapters. Although the first two chapters study different questions and contribute to distinct branches of the literature, they are related with respect to the research question. Both chapters contribute to the field of health economics, reflecting my research interest in a better understanding of the interdependence between the economic situation of individuals and their health. This interest is motivated by the well-documented correlation between health and income. Exemplifying this correlation, Figure 1 shows the average additional life expectancy at age 55 across earnings deciles. The richest 10% have a 10 years longer life expectancy than the poorest 10%.

Figure 1: Average Additional Life Expectancy at Age 55, by Mid-career Income



Source: Bosworth and Burke (2014), own illustration.

However, correlations without an understanding of causation are no sufficient guide for policy. Building on this insight, Chapter 1 and 2 investigate potential mechanisms to identify the link between health and the economic sphere. Chapter 1 studies recuperation time and shows that fear of unemployment drives a wedge between the long-run health of rich and poor workers. Chapter 2 investigates whether stress caused by a low economic performance compared to a self-determined circle of acquaintances may pose a threat to health.

Chapter 3 deviates from the first two chapters in its nature as it contributes to the literature of computational economics. More precisely, it develops a method which deals with the computational challenges of heterogeneous agent models that require a higher dimensional state space. The method can be applied to large-scale overlapping generations as well as other models with heterogeneous agents and could be applied in Chapter 2.

Chapter 1 The first chapter studies the relationships among sick leave, income and unemployment. It investigates these relationships under the generous German sick leave regulation of 100% wage replacement, whereby workers do not bear any direct costs from sick leave. Using information from the German Socioeconomic Panel (SOEP), I identify three stylized facts of sick leave in Germany. First, the number of sick leave days shows a strong pro-cyclical pattern, i.e., workers are on average less absent from work in times of high unemployment. Second, the average use of sick leave is hump-shaped over income quintiles. Workers in the medium income quintile have on average almost 10% more days of sick leave than workers in the bottom income quintile. This figure is noteworthy because average health monotonically increases with income. Assuming sick leave is driven only by health would therefore lead to a decrease in the number of days of sick leave between bottom and medium income quintiles, and this is not observed. Third, the number of days of sick leave is a strong predictor of future unemployment. Taking three additional days of sick leave increases the risk of becoming unemployed by a factor of 1.1. Using micro-evidence, I develop a dynamic structural model with heterogeneous agents that rationalizes these facts. I argue that in the absence of direct costs of sick leave, the fear of future unemployment is the main driving force restraining sick leave. The model, calibrated to the German labor market, is able to reproduce the (non-targeted) stylized facts of sick leave days in Germany. The calibrated model allows me to conduct counterfactual policy analysis. For this purpose, I contrast the benchmark economy, for instance, against an economy with no unemployment benefits. An immediate implication is that the average worker reduces her recuperation time by 1.2 days a year, and a worker in the bottom income quintile reduces it by more than 1.7 days a year.

Chapter 2 The second chapter of this thesis investigates the relationship between an individual's health and relative economic performance. Using a unique dataset with explicit information on social circles, I find robust and significant positive effects of relative performance on self-reported health and negative effects on detrimental health behavior such as smoking and obesity. People that consider themselves poorer than their circle of acquaintances are significantly less likely to report good health. I further show that this effect exhibits asymmetries, i.e., being worse off than one's circle of acquaintances has a strong negative effect, whereas being better off exhibits only a mild positive effect. Furthermore, groups with lower absolute income are more strongly affected by the relative performance effect than are groups with high absolute income. I also document that the standard approach of artificially constructing reference groups based on the characteristics of the respondents yields weaker and insignificant results.

Chapter 3 The third chapter of this thesis is a slightly modified version of a joint paper with Alexander Ludwig. It contributes to the computational methods to solve higher dimensional households problems. It investigates extensions of the method of endogenous gridpoints (ENDGM) introduced by Carroll (2006) to higher dimensions with more than one continuous endogenous state variable. We compare three different categories of algorithms: (i) the conventional method with exogenous grids (EXOGM), (ii) the pure method of endogenous gridpoints (ENDGM) and (iii) a hybrid method (HYBGM). ENDGM comes along with Delaunay interpolation on irregular grids. Comparison of methods is done by evaluating speed and accuracy. We find that HYBGM and ENDGM both dominate EXOGM. In an infinite horizon model, ENDGM also always dominates HYBGM. In a finite horizon model, the choice between HYBGM and ENDGM depends on the number of gridpoints in each dimension. With less than 150 gridpoints in each dimension ENDGM is faster than HYBGM, and vice versa. For a standard choice of 25 to 50 gridpoints in each dimension, ENDGM is 1.4 to 1.5 times faster than HYBGM.

1

Unemployment, Sick Leave and Health – Sick, Poor and Forced to Work?

1.1 Introduction

Absence from work due to sickness poses a major threat to the economic situation of households. On the one hand, workers who take sick leave face direct opportunity costs arising from a reduction of working time for which they would otherwise be paid. In most countries, these costs are partially insured by paid sick leave schemes. The extent of this insurance coverage varies greatly across industrialized countries, cf. Scheil-Adlung and Sandner (2010).[1] On the other hand, workers have to take into account the indirect costs of sick leave that stem from reductions in future expected earnings. The layoff and promotion decisions of employers depend on workers' past days of sick leave, cf. Markussen (2012) and further evidence below. To fully understand the role of sickness absence, it is therefore important to distinguish between the two types of costs.

In this chapter, I identify the costs of sick leave stemming from lower employment prospects, and I analyze their economic implications. To do so, I focus on Germany, which features a very generous sick leave system that almost completely rules out direct opportunity costs of sick leave. In cases of work absence due to sickness, every (full-time, part-time or temporary) employee is eligible for six weeks of 100% wage replacement.[2]

[1] An extreme case among developed countries is the US, where there is no statutory paid sick leave, a situation US President Barack Obama recently urged to change in his State of the Union in 2015. Paid sick leave may be provided by employers on a contractual basis. According to the 2015 State of the Union, the number of workers without any sick leave scheme amounts to 43 million.

[2] This generosity is not without a price. The expenditures of paid sick leave, which are borne by employers, amounted to almost 40 billion € in 2013, or more than 1.5% of GDP, according to the German Federal Ministry of Labour and Social Affairs, cf. Bundeministerium für Arbeit und Soziales (2014). This number does not include the contribution to the social insurance system, which amounted to 6.9 billion €. For more information on the German regulations on paid sick leave, see Appendix 1.A.

The first objective of this chapter is to identify and document patterns of sick leave utilization over business cycles and income groups in Germany. The second objective is to rationalize these empirical findings within a theoretical framework and to highlight the main mechanism: the decision to trade off utility-enhancing health against expected future earnings due to increased risk of job loss. The third objective is to analyze the distributional effects of indirect costs of sick leave and to evaluate counterfactual policies within a structural model calibrated to the German labor market.

For the first objective, I employ data from the German Sozio-oekonomische Panel (SOEP), a nationally representative longitudinal dataset. With respect to aggregated data, I find three remarkable patterns of sick leave utilization in Germany. First, claims of sick leave exhibit a strong pro-cyclical pattern; i.e., workers are on average less absent from work in times of high unemployment. The correlation coefficient of average days of sick leave and the German unemployment rate is minus 0.6383. Second, the average number of days of sick leave in Germany displays a marked hump-shaped pattern across income quintiles. Workers in the medium income quintile have on average almost 10% more days of sick leave than workers in the bottom income quintile. This figure is noteworthy because average health monotonically increases with income.[3] Assuming sick leave is driven only by health would therefore lead to a decrease in the number of days of sick leave between bottom and medium income quintiles, and this is not observed. Third, the variance of sick leave differs greatly between income quintiles. On the one hand, employees in the bottom quintile have the highest probability of not missing any day in a year. On the other hand, they also have the highest probability of missing more than two weeks. Top income employees miss a small number of days but do so at a higher frequency.

Regarding the second objective, the mechanism that rationalizes these three stylized facts of sick leave is that workers who become sick face the decision either to stay at home and recover or to go to work sick. Staying at home restores utility-enhancing health but simultaneously increases the risk of job loss. Exploiting the panel structure of the SOEP, I show that sick leave is one key predictor of future unemployment. Taking three additional days of sick leave increases the risk of becoming unemployed by a factor of 1.1. Going to work sick preserves expected future earnings, but a perpetual neglect of recuperation diminishes long-run health prospects.[4] In times of high unemployment rates, work-

[3] A finding well documented in the health economics literature, cf. Smith (1999).

[4] Time requirements for recovery in the health economics literature go back to the very beginning, c.f. the seminal paper of Grossman (1972). Clinical, experimental, and empirical evidence in support of this idea can be found in the bio-medical science, public health, psycho-biology, bio-sociology, and empirical health literature, which shows a negative effect of neglect of re-

ers face both higher overall firing rates and lower reemployment probabilities. Resulting higher marginal costs of unemployment shift the trade-off toward presenteeism and drive the cyclical pattern. Workers facing financial constraints, i.e., low skilled workers, are less able to smooth consumption over periods of unemployment and have an overall higher risk to become unemployed. Therefore, they are particularly compelled to go to work when sick. Consequently, optimal sick leave utilization differs across income groups. The rich constantly use fewer days absent from work to conserve their health. Poorer workers reduce their number of days of missing work to keep their job; however, this practice comes with the cost of a perpetual worsening of their health. In the long run, lower health increases the number of sick leave days for the poor compared to the rich. This outcome of the mechanism is affirmed by the finding of a widening sick leave gap between bottom and top income quintiles over the life cycle. Beginning at almost the same level of sick leave at age 20, workers in the bottom income quintile have 40% more days of sick leave at the end of their working life than those in the top income quintile.

To quantify the distributional effects of sick leave, the third objective, I develop a heterogeneous agent model with an endogenous health state. A shock process mimics acute sickness. Additionally, I implement central characteristics of the German labor market and worker protection system in my model.[5] The government imposes a flat income tax on agents. The collected revenues are used to finance (i) expenditures due to sick leave payments, (ii) unemployment benefits, including means-tested welfare, and (iii) a retirement system.

To implement my quantitative analysis empirically, I first estimate and calibrate the model using SOEP data to match key statistics on sick leave, health status and unemployment. The estimated model is able to successfully explain the targeted features of the data in the estimation (e.g., the distribution of days of sick leave utilized by low income workers). It is also capable of reproducing other (non-targeted) dimensions, such as the hump-shaped pattern of average days of sick leave across income quintiles, the income gradient in health, and the cyclicality of claims of sick leave. I then use the parameterized version of the model as a laboratory to evaluate the consequences of different policy options. For this purpose, I contrast the benchmark economy, for instance, against an economy with no unemployment benefits. An immediate implication is that the average worker reduces her recuperation time by 1.2 days a year, and a worker in the bottom income quintile reduces it by more than 1.7 days a year.

cuperation time, cf. Kivimäki et al. (2005), Bergström et al. (2009).

[5] German workers have universal health insurance that covers medical expenditures. For this purpose, I omit medical expenditures in my model.

Related Literature There is a sizable body of literature that documents a positive relationship between workers' economic situation and their average claims of sick leave, cf. Leigh (1985), Pfeifer (2013). Arai and Thoursie (2005) and Askildsen, Bratberg, and Nilsen (2005) show that this pro-cyclical variation in sickness absence is caused by established workers reducing sick time rather than the absence behavior of marginal workers entering or leaving the working population in various states of the business cycle. Another growing strand of literature studies the effect of sick leave on individuals' future labor market outcomes. Hesselius (2007), Markussen (2012), and Chadi and Goerke (2015) show that an increase in the sick leave rate lowers the probability of being employed in the future. Andersen (2010) also finds that a high number of days of sick leave not only affects employment status but also decreases post-sick leave earnings. I contribute to both strains of the literature by showing that these findings also hold for Germany. I add to the literature by merging the findings of sick leave, and I expand the discussion with a cross-sectional dimension.[6]

The structural model I present in this chapter is part of a broad and growing body of literature that incorporates endogenous health into dynamic life-cycle models. Important related contributions include Grossman (1972), Ehrlich and Chuma (1990), Hall and Jones (2007), Ales, Hosseini, and Jones (2012), Halliday, He, and Zhang (2012), Ozkan (2014), Cole, Kim, and Krueger (2014). A small body of literature allows for interactions between endogenous health, employment and productivity. In a recent paper, Laun (2013) analyzes optimal insurance against unemployment and disability in a private information economy with endogenous health and search efforts. Little research has been conducted on such dynamic models distinguishing between long run health and the onset of acute illnesses. Gilleskie (1998) predicts changes in sickness-related absenteeism that arise with improvements in access to health care through more complete health insurance and sick leave coverage in the US. The paper, however, focuses only on the direct costs of work absence and does not take into account the risk of unemployment. It also falls short of providing a link to the endogenous health literature. To the best of my knowledge, this is the first study to incorporate endogenous health and acute sickness into a heterogeneous agent life-cycle model.

The rest of the chapter is organized as follows. In Section 1.2, I discuss the main data source, the methodology and the empirical findings. Then, I introduce

[6]There is also considerable literature exploiting reforms of sick pay provision in Scandinavian countries (cf. Henrekson and Persson (2004), Johansson and Palme (2005), Dale-Olsen (2014)) and, more recently, Germany (cf. Ziebarth and Karlsson (2010), Ziebarth (2013), Ziebarth and Karlsson (2014), Puhani and Sonderhof (2010)). They found that an increase in generosity in the sick leave system induces a higher number of days missed at work, indicating a moral hazard.

a full structural model in Section 1.3. In Section 1.4, I discuss the estimation of the model and the model's fit to the data. Section 1.5 presents counter-factual policy experiments using the model. Finally, I conclude in Section 1.6.

1.2 Empirical Facts on Sick Leave Days

The purpose of this section is to carve out stylized facts of sick leave in Germany and to motivate the key modeling assumptions of the structural model in Section 1.3. After discussing the data source and the methodology in Section 1.2.1, I present in Section 1.2.2 findings on aggregated data that show that taking sick leave is an endogenous choice of workers. Then, in Section 1.2.3, I show results based on a panel analysis that underline the importance of this choice for the employment prospects of workers.

1.2.1 Data and Methodology

Description of the Survey

My empirical analysis is based on the German Sozio-oekonomische Panel, a nationally representative longitudinal dataset. Starting in 1984 and conducted annually, it comprises 30 waves of household data. It oversamples foreigners, immigrants, and East Germans to allow for more precise estimates for population subgroups that may be of particular policy interest.[7] The SOEP provides detailed information about demographic (e.g., sex, age), socioeconomic (e.g., educational level, marital status) and economic characteristics. The respondents report their current monthly income and their household income in the current and the previous year.[8] The employment history contains the current employment status (e.g., full time, part time), the point in time of the layoff in the previous year, the length off the unemployment spell and information about the time worked for the same firm. Information about health since 1990 is requested. In addition to self-reported health, the SOEP contains information about the number of doctor visits and hospital stays.[9] Further detailed information about the characteristics of the SOEP is provided in Wagner, Frick, and Schupp (2007).

[7] I include all sub-samples of SOEP with the appropriate cross-sectional weights.

[8] All monetary variables are deflated with the consumer price index contained in the SOEP using 2005 as the base year.

[9] It also includes an SF-12 indicator of physical health. This measure combines several self-reported indicators; see Nuebling et al. (2007) for further information. This measure is available only every other year since 2002 and is of limited use for the panel analysis.

The key variable for the purpose of this chapter is the number of working days missed due to sickness. The SOEP asks respondents to state whether they missed any day due to sickness in the previous year and, if so, how many days they missed. Puhani and Sonderhof (2010) show that, though self-reported, the SOEP adequately depicts the true number of days of absent from work. The SOEP also contains information about the number of spells that last longer than six weeks. However, the survey does not ask about occasions on which respondents go sick to work.

The only information that is used and not contained in the SOEP is the unemployment rate of Germany. I use official data from the federal employment agency, cf. Bundesagentur für Arbeit (2014).

Determination of the Sample

For the following empirical results, not all observations of the SOEP are used. Because I am interested in the sick leave utilization of workers, I focus only on the working age population. I drop all observations of respondents younger than 18 years and older than 65 years (the official German retirement age). I restrict the sample to respondents who report working in the current or the previous year and those who report being unemployed. I exclude people doing military service, people working in a sheltered workshop, and unemployed people not looking for work. The number of sick leave days is reported in absolute numbers and not in relative fractions of work time. The probability and the intensity of annual sick leave are biased when respondents work only a fraction of the year. To control for this bias, I exclude respondents who report fewer than five days or 35 hours of work a week. I also drop all respondents with a monthly income of less than 500 €.[10]

As for the time period, I use waves 1994 to 2013, corresponding to information about sick leave from 1993 to 2012. Waves 1984, 1990 and 1993 do not contain information about sick leave. Waves 1991 and 1992 capture the unique economic situation of German reunification in 1990 and the liberalization of a state-owned socialist economy. I drop both periods because the income distribution and employment situation changed dramatically.[11] Waves 1985-1989 could potentially be used in the analysis of pro-cyclicality. I drop them for various reasons. First, these waves do not contain information about health and cannot be used in cross-

[10] Altering the limits of days or hours of work a week or the minimum income does not change the qualitative results of the following empirical analysis.

[11] The following waves are also affected, but the effect is strongly mitigated over time. The classification in income quintiles is particularly disturbed because the income scale was lower in East Germany.

sectional or panel regressions. Second, the unemployment rate varied in these years only between 7.9% and 8.1%. Hence, not much variation can be used to determine the cyclicality of sick leave. Third, I want to use an uninterrupted sample period for the time series analysis.

For the benchmark results, I exclude civil servants and self-employed workers from the sample. Self-employed workers do not receive paid sick leave; it is provided by the employer. Civil servants have paid sick leave but are not eligible for layoffs. Hence, they are not affected or are less affected by the indirect effect of fear of job loss.[12]

The days of sick leave have a highly skewed distribution with many observations at the 0 boundary and few observations at the highest value of 365. Of the observations, 95% percent report fewer than 42 days a year, and only slightly more than one percent report more than 120 days. Hence, many results, e.g., the average number of sick leave days, are prone to be driven by only a few observations. To control for outliers, I exclude in the benchmark results all observations that have one or more spells of sick leave that last longer than six weeks. Of the remaining sample, I cut off the highest two percentages, i.e., workers with more than 30 days of sick leave a year.

After sample selection, the sample used for the benchmark results consists of 100,526 observations.[13] The sample includes 20 waves, and each wave has at least 3,910 observations.[14]

Empirical Approach

In Section 1.2.2, I run cross-sectional regressions of sick leave on various regressors, where I pool the observations of all waves in one sample. The regression equations for OLS and Logit estimation are

$$S_i = \alpha + \beta\, log(W_i) + \beta\, H_i + \beta\, \mathcal{U}_i + X_i\theta + \varepsilon_i \tag{1.1}$$

$$Logit\left[S_i^{ext} = 1\right] = \mathcal{P}\{\alpha + \beta\, log(W_i) + \beta\, H_i + \beta\, \mathcal{U}_i + X_i\theta + \varepsilon_i\} \tag{1.2}$$

[12] The results of both groups are an additional argument for the proposed mechanism. The cyclical pattern is either not existent for the self-employed or much weaker for the civil servants, a result also found by Pfeifer (2013). Additionally, the income gradient in sick leave does not exist for both groups. Unfortunately, both groups vary from the rest of the sample in various respects (e.g., income, age, education). Therefore, they cannot be used as an adequate control group.

[13] Further details on the sample selection can be found in Appendix 1.B.

[14] The size of the waves increases over time. There were refreshments of the SOEP in 1998, 2000, 2002 and 2006.

where S_i is a countable variable used to denote days of sick leave, whereas S_i^{ext} is a dummy variable used to denote either missing any day in a year at work $\left(S_i^{ext} = 1\right)$ or not $\left(S_i^{ext} = 0\right)$. W_i is the monthly income of the respondent in the previous year.[15] H_i is self-reported health, $\mathcal{U}_i$ the German unemployment rate, $X_{i,t}$ is a set of control variables (e.g., sex, age, years of education, marital status, number of children living in the household, year dummies), and ε_i is the random error term.

In Section 1.2.3, I employ a logistic panel regression. I estimate the effect of the number of sick leave days in the previous period on the current probability of unemployment $\left[I_{i,t}^U = 1\right]$. I restrict the sample in this section to people who were employed at least six months in the previous year. The panel structure of the SOEP additionally allows me to use a fixed-effects model. The fixed effect will incorporate all unobserved characteristics of the agent. The regression equations are

$$Logit\left[I_{i,t}^U = 1\right] = \mathcal{P}\{\alpha + \beta\, S_{i,t-1} + X_{i,t-1}\theta + \epsilon_{i,t}\} \tag{1.3}$$

$$Logit\left[I_{i,t}^U = 1\right] = \mathcal{P}\{\alpha + \beta\, S_{i,t-1} + C_{i,t-1}\theta + a_i + \epsilon_{i,t}\} \tag{1.4}$$

where $C_{i,t}$ is a set of control variables that vary over time. It contains health, age and the log income in the previous year. Days of sick leave in the previous period are denoted by $S_{i,t-1}$. The a_is represent the individual specific and time-invariant fixed-effect component, and $\epsilon_{i,t}$ is the random error term.

1.2.2 Stylized Facts on Aggregated Data

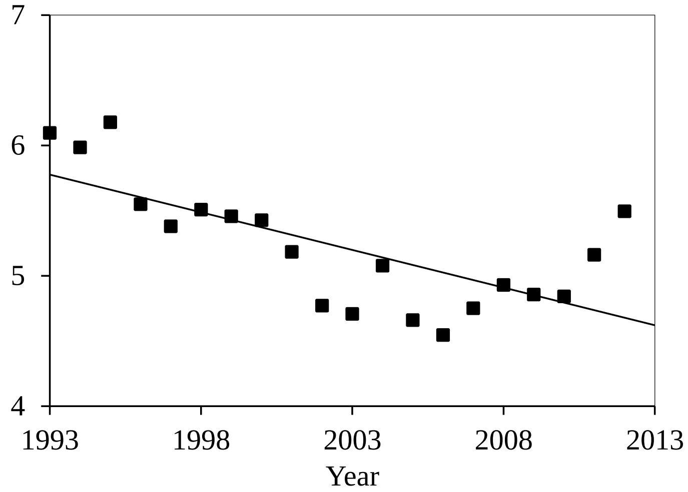
Time Series

Figure 1.2 shows for the benchmark sample the average annual number of sick leave days per worker in the observed time period and a fitted linear trend. The average number of sick leave days varies between 4.5 and 6.2 days. An obvious first finding about the days of sick leave in Germany is the long-term decline. In the last 19 years, the average claims of sick leave have dropped by almost one day, or 20% relative to 1993.

A second characteristic is the strong pro-cyclical pattern of the average claims of sick leave in Germany once the time series is de-trended. Figure 1.3 depicts the absolute deviation of average days of sick leave from the linear trend (dashed blue line) and the unemployment rate for Germany (solid black line). The average number of days of sick leave is high when the German unemployment rate is low

[15] I use other measures of income that are also included in the SOEP such as gross income or household net income. The qualitative results do not change.

Figure 1.2: Average Days of Sick Leave per Worker 1993-2012



Notes: Dots: Average annual claims of sick leave per worker in benchmark sample. Solid line: Fitted linear trend, slope: -0.0578.

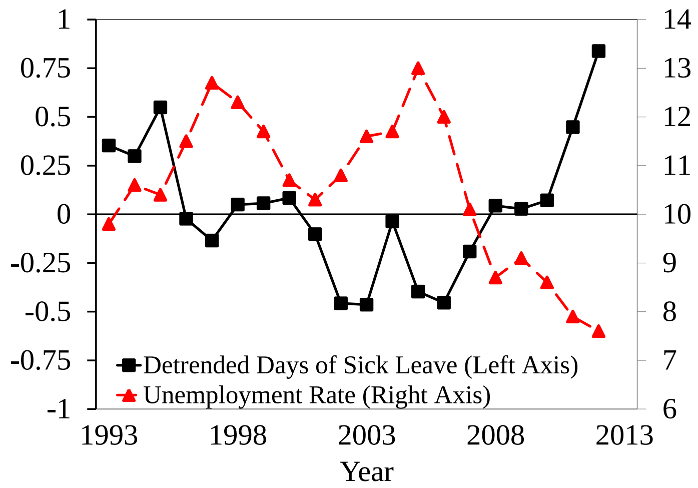
and vice versa. The correlation between the de-trended time series of days of sick leave and the German unemployment rate for the benchmark sample is minus 0.6383.[16]

To control for a composition effect, i.e., the absence behavior of the marginal worker, I construct a sub-sample consisting of workers who report never being unemployed for at least five consecutive years. This sub-sample shows, on the one hand, a lower number of average days of sick leave compared to the benchmark sample. On the other hand, the cyclical pattern of the always-employed sample is still distinctively negative, with a correlation of minus 0.6576.[17] The remaining correlation supports the idea of an incentive effect. In times of low reemployment, workers reduce their days of sick leave to avoid unemployment. The incentive effect implicitly assumes that absence from work is not mechanically tied to the incidence of sickness. Workers are free to decide whether to go to work sick or stay at home and recover. This is a key assumption of the structural model in Section 1.3.

[16] In Appendix 1.C, I provide additional robustness checks using other measure of central tendencies, e.g., the number of days of sick leave for the median worker.

[17] Other composition effects would occur if specific occupation groups or sectors that exhibit many days of sick leave, e.g., the construction sector, are hit stronger by business cycles than others. In Appendix 1.C, I show that the general pattern of pro-cyclical behavior holds for all occupation and sectors.

Figure 1.3: De-trended Average Days of Sick Leave and Unemployment Rate



Notes: Dashed line (left axis): Absolute deviation of average annual claims of days of sick leave per worker from linear trend in benchmark sample. Solid line (right axis): German unemployment rate.

Cross Section

This incentive effect, i.e., the economic trade-off between recuperation time on the one hand and an increased layoff probability on the other hand, can also be observed in cross-sectional analyses. For each wave and age, I classify all respondents into one of five income quintiles based on their monthly earnings in the previous year.[18] Figure 1.4 plots the average claims of sick leave (solid line) for each income quintile. It additionally shows the average self-reported health (dashed line) for each quintile. Workers in the top income quintile miss the fewest days at work due to sickness. Workers in the medium income quintile claim on average 10% more days of sick leave than workers in the bottom income quintile.[19] Conversely, the health profile across income quintiles is monotonically increasing. The poorest workers have the lowest average health, whereas the top income quintile shows the highest average health.[20]

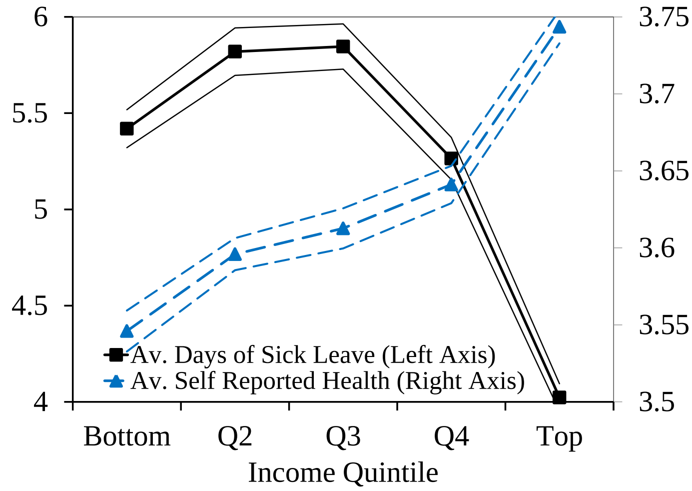
[18] Controlling for age in the quintile classification is important; otherwise, older people are more likely to be in the top income quintile. Because older respondents are, on average, less healthy, the relationship between income and days of sick leave would be biased.

[19] These patterns also hold for each age bin separately and for both sexes; see Figure 1.15 and Figure 1.16 in Appendix 1.C. The patterns also exist for the median number of days of sick leave and other cut-off levels for days of sick leave.

[20] A simple probit regression of health (good or bad) on income controlling for age and sex confirms this pattern and yields a highly significant positive effect for log income. This income

Health and days of sick leave are naturally related; i.e., workers in generally bad health conditions are more likely to become sick and stay at home. Consequently, the observed differences in health could explain the small use of sick leave in the top income quintile compared to the rest of the workforce. Rich people are, on average, in better health condition; they are therefore less sick and require fewer days at home to recover. However, the same rationale is puzzling on the other side of the income distribution. The workers who are most unhealthy use fewer days at home to recover than medium income workers, who enjoy, on average, better health. This finding shows that sick leave is not mechanically tied to health, and the absence behavior of workers must have a second economic determinant.

Figure 1.4: Average Days of Sick Leave and Health across Income Quintiles



Notes: Solid lines (left axis): Average days of sick leave of workers and 95% bootstrap confidence interval. Income quintiles are based on the monthly gross income of the respondent. Dashed lines (right axis): Average self-reported health and 95% bootstrap confidence interval. Health is reported on an ordinal five-point scale, where 0 denotes "bad" health and 4 denotes "very good" health.

This graphical inspection is confirmed by estimating Equation (1.1) using days of sick leave as the dependent variable. Both income coefficients, for log income and for log income squared, are highly significant and suggest a hump-shaped relationship of income and days of sick leave. Health has the assumed protective effect against days of sick leave. Other coefficients confirm former results; see Table 1.1. There is a long-run negative trend in claims of sick leave of minus 0.0488 days per year. More importantly, related to the cyclicality of days of sick

gradient in health is well established in the literature; see Smith (1999) leave, the coefficient of the unemployment rate is negative and highly significant. This indicates that during periods of high unemployment, the average days of sick leave are reduced.

Table 1.1: OLS and Logit Regressions of Days of Sick Leave on Income

Days of Sick Leave	(1)	(2)	(3)
Log Income	10.0384***	0.0496***	-1.7516***
Log Income Squared	-0.6633***	–	–
Health	-1.4626***	-0.0805***	-1.2924***
Wave	-0.0488***	0.0035***	-0.1383***
Unemployment Rate	-0.1599***	-0.0063**	-0.1878***
Other Controls	Yes	Yes	Yes
Observations	51,179	51,179	28,216

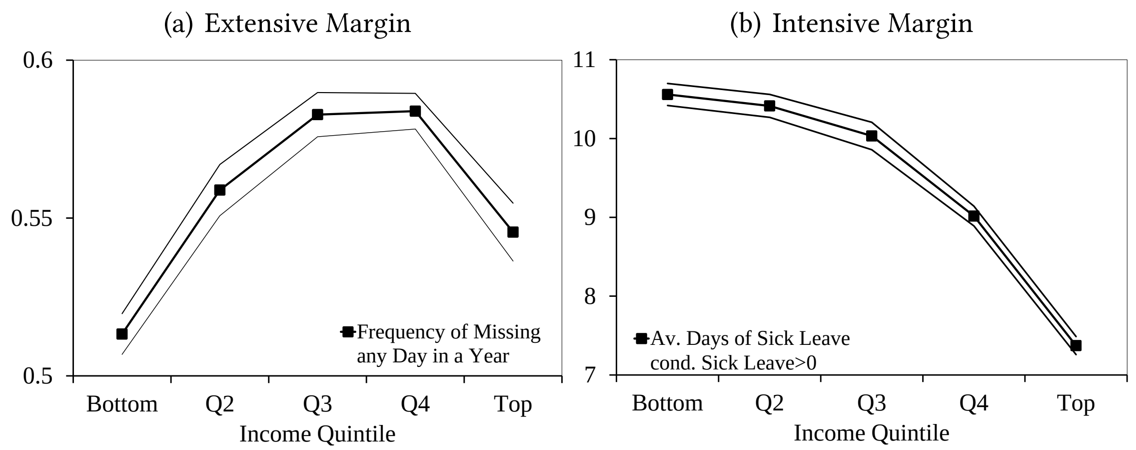
Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
All regressions are based on the benchmark sample. Other controls include sex, age, age^2, years of education, marital status, number of children and dummies for the industrial sector. Robust standard errors are clustered on the personal level. Entries in Column (2) show marginal effects at the means.

Further insights into the characteristics of claims of sick leave are provided by distinguishing between the extensive margin, i.e., missing any day in a year or not, and the intensive margin, i.e., conditional on missing at least one day a year, the number of days the respondent is absent from work.

The left panel of Figure 1.5 shows the extensive margin of sick leave. Workers in the bottom income quintile exhibit the lowest probability of missing any day in a year. The higher the income quintile the higher is the probability to miss at least one day. Only at the very top of the income distribution does the extensive margin decrease. This pattern is confirmed by estimating Equation (1.2) using the extensive margin as a dependent variable. The results are presented as marginal effects at the means in the Column 2 of Table 1.1. The estimate for log income is highly significant and confirms the positive relationship between income and the probability of missing any day. Further results show an unsurprising protective effect of self-reported health against missing any day, and that cyclicality is also present in the extensive margin.

The right panel of Figure 1.5 displays the average days of sick leave conditional on missing at least one day (intensive margin). In contrast to the extensive margin, the intensive margin is monotonically decreasing across income quintiles. The decline in conditional averages originates from different distributions of claims of sick leave. The upper income quintiles have a higher probability of experiencing few days of sick leave (1 up to 14 days). Workers in the lower income quintile have a higher probability of claiming many days of sick leave (more than 14 days) once they miss one day.[21] Column 3 of Table 1.1 presents results of estimating Equation (1.1) using the intensive margin as the dependent variable. The results confirm that a high income has a protective effect against many days of sick leave. As the income increases, the number of days of sick leave a year conditional on being sick decreases. Health has again a protective effect against days of sick leave. The coefficient of the unemployment rate also confirms the cyclical pattern for the intensive margin.

Figure 1.5: Composition of Sick Leave across Income Quintiles



Notes: Left panel: Frequency of absence at least one day a year from work (extensive margin) of workers and 95% bootstrap confidence interval separated for income quintiles. Right panel: Average days of sick leave conditional on being at least one day absent from work (intensive margin) and 95% bootstrap confidence interval separated for income quintiles.

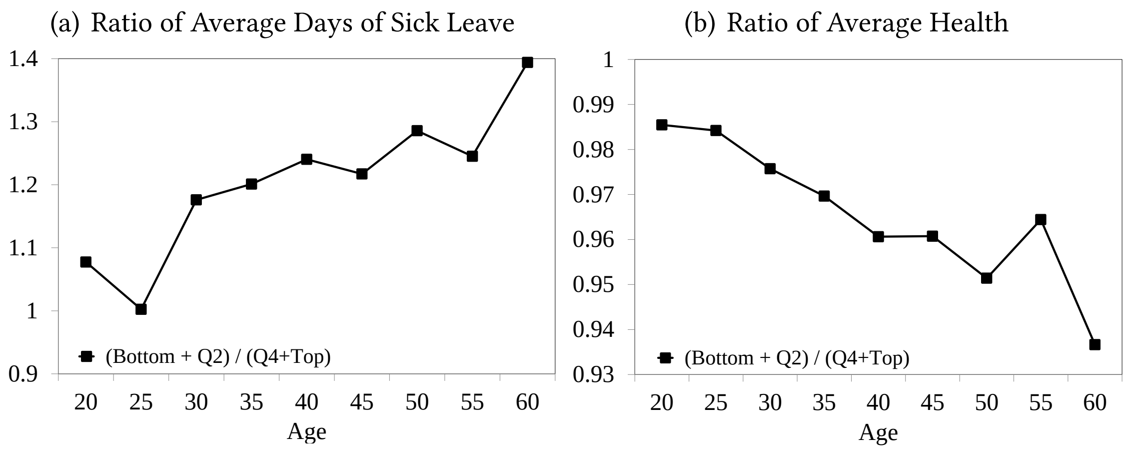
Summarizing, there is a remarkable difference in the utilization of sick leave between income groups. The top income quintile has the lowest average days of sick leave but a high probability of missing a day. The medium income quintile has the highest average days of sick leave. The bottom income quintile has the worst health but only a moderate number of days of sick leave. Furthermore, the lowest income group has the highest probability of both not missing any day and missing more than 14 days compared to other income groups.

[21]The density functions of days of sick leave for the bottom and top income quintiles are shown in Figure 1.14 in Appendix 1.C.

Life Cycle

The left panel of Figure 1.6 presents the ratio of the average days of sick leave of the bottom two to the top two income quintiles. It shows that the average number of days of sick leave is almost the same across income quintiles for workers in their twenties. Over the life cycle, the gap between the bottom and top income groups widens, and shortly before retirement, low income workers have almost 40% more days of sick leave than their high-income peers. The right panel of Figure 1.6 presents the same ratio but for average health. Here, I similarly find that the gap between rich and poor workers widens over the life cycle. At the beginning of working life, the average health of both groups is almost the same. At retirement, the average health of the bottom two income groups is considerably lower than average health of the top two income groups.

Figure 1.6: Ratio of Bottom vs. Top of Sick Leave and Health over Life Cycle



Notes: Left Panel: Ratio of average days of sick leave of the bottom two income quintiles (Bottom & Q2) to the top two income quintiles (Q2 & Top) over 5-year age bins. Right Panel: Ratio of average self-reported health of the bottom two income quintiles to the top two income quintiles over 5-year age bins.

These empirical findings are further evidence for the proposed mechanism that poor people reduce their recuperation time to retain their jobs and hazard the consequence of a reduction in overall health over time.

1.2.3 Micro Evidence Using Panel Data

The panel structure of SOEP allows me to carve out further features of sick leave in Germany. A first finding is that days of sick leave are persistent. People who claim sick leave in the last period also have higher claims of sick leave in the current period. Including lagged days of sick leave in estimating Equation (1.1) returns a positive and significant estimate. All other results remain qualitatively unchanged.

More important is the relationship of days of sick leave and the employment prospect of workers. Table 1.2 shows descriptive statistics of workers who report continuous employment in the entire previous year. In Column (2) and (3), this sample is separated by employment status in the current period. Workers who switched from employment to unemployment are not surprisingly poorer, less educated and less healthy than respondents who stayed employed. They are also slightly younger. Regarding the days of sick leave, respondents who are unemployed in the current period missed in the previous year on average one day more from work than workers who stayed employed.

Table 1.2: Summary Statistics of Workers Employed in t-1

Employed in t-1	Whole Sample	Employed in t	Unemployed in t
Days of Sick Leave in t-1	4.83	4.81	5.85
Days of Sick Leave in t-2	4.46	4.44	5.31
Age	41.1	41.1	40.2
Income in t-1	2,934€	2,952€	1,977€
Years of Education	12.3	12.4	11.5
Health in t-1	2.65	2.66	2.46
Male	64%	64%	52%
Observations	102,125	99,025	3,100

Notes: Descriptive statistics for the sample used in the panel logit model. Only workers who report employment the entire previous year. Unemployed people in t are classified as people who report at least one month of unemployment in t.

Table 1.3 shows the odds ratios of estimating Equation (1.3). In Column 1 are results of a regression that includes only controls contained in the structural model. These coefficients will later be used in the calibration of the model. Column 2 contains results of regressing Equation (1.3), additionally including other control variables, e.g., health, age, and education. In both columns, income in the previous period has an odds ratio smaller than 1; i.e., the highest probability of becoming unemployed is among workers in the bottom income quintile. To be in good health, on the other hand, has a protective effect against the risk of unemployment. Naturally, the risk of unemployment is large in times of a high overall unemployment rate. The key results here are the coefficients for the days of sick leave. In both columns, days of sick leave show odds ratios that are greater than 1 and highly significant. Three additional days of sick leave lead to an increase in the probability of being unemployed in the next period by a factor of 1.1; e.g., a 10% layoff probability would then be 11%.

To account for unobserved heterogeneity in workers, I estimate Equation (1.4) including a fixed-effect component (Column 3). The effect of sick leave on the probability of becoming unemployed is qualitatively not affected and is still highly significant. The results for the effect of income and the unemployment rate on the risk of unemployment remain unchanged. Health still has the same qualitative sign but becomes insignificant in the model with fixed effects.

In the relationship between sick leave and employment exists the problem of reverse causality. Workers who know that they will lose their job could be tempted to take sick leave because they do not fear retaliation. Note that the period length is one year, and therefore, this effect should not be present in the preceding period. Estimating Equation (1.4) using days of sick leave in t-2 instead of t-1 still yields an odds ratio that is significantly greater than 1 (Column 4).[22]

Table 1.3: Panel Results for Sick Leave and Unemployment

Unemployed in t	(1)	(2)	(3)	(4)
Days of Sick Leave in t-1	1.0328***	1.0287***	1.0270***	–
Days of Sick Leave in t-2	–	–	–	1.0149**
Income Quintile in t-1	0.6132***	0.6132***	0.9129**	0.9519
Unemployment Rate in t-1	1.1626***	1.0798***	1.2376***	1.2542***
Health in t-1	–	0.8873***	0.9248	0.9242
Other Controls	No	Yes	Yes	Yes
Fixed Effects	No	No	Yes	Yes
Observations	60,052	60,052	7,578	4,868

Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
Unemployed in t is defined as reporting unemployment for at least one month in the current year. Other controls include age, sex, years of education, number of children, marital status, and year dummies. All columns report odds ratios at the population average. Robust standard errors are clustered.

[22] This specification, however, has the disadvantage of reducing the sample size and potentially biasing the results. In looking at the effect of days of sick leave in t-2 on the employment status in t, I have to exclude all respondents who became unemployed in period t-1. Many days of sick leave in t-2, however, increases the likelihood of unemployment in t-1 and therefore downward biases the results seen in Column 4.

1.3 Structural Model

In this section, I describe a dynamic stochastic model of work absence decisions over the life cycle. It captures both the standard consumption-saving decisions and sequential decision-making behavior of employed individuals regarding their health. I will later use this model to evaluate the consequences of economic inequalities for the utilization of sick leave and health as well as the consequences emanating from different policies.

1.3.1 Household's Problem

The economy is populated by overlapping generations of a continuum of agents that live up to a maximum age of J^T.[23]

Health, Acute Sickness, and Sick Leave One important feature of this chapter is the distinction between the general health status, H_t, the event of acute sickness, S_t, and the time an agent missed work, l_t.

Health status, H_t, reflects the overall constitution of agents. It is a persistent state that adjusts only gradually. Agents start their economic life in a certain health state, H_0. At the end of each period, agents may drop into the next lower health state with probability Π^W, ascend into the next higher health state with probability Π^B, or stay in the same health state with probability $1 - \Pi^B - \Pi^W$.

In contrast, acute sickness, S_t, has a transitory notion and mimics the contraction of an illness or an injury, e.g., the flu or back pain. At the beginning of each period, individuals face the risk of either staying well, S_t^0, or contracting one of m types of acute illnesses, S_t^m, which vary in severity with $S_t^m < S_t^{m+1}$.

Health and acute sickness are interdependent. On the one hand, the probability of contracting an illness of type m, $\Omega_m\left(H_t, j\right)$, depends on the overall health status and the age, j, of an individual. As the general health status decreases, the probability both to become ill (extensive margin) and to contract a more severe illness (intensive margin) increases. The severity of the contracted sickness, on the other hand, affects the health transition probabilities, with $\Pi^B(S_t^m) < \Pi^B(S_t^{m+1})$ and $\Pi^W(S_t^m) > \Pi^W(S_t^{m+1})$.

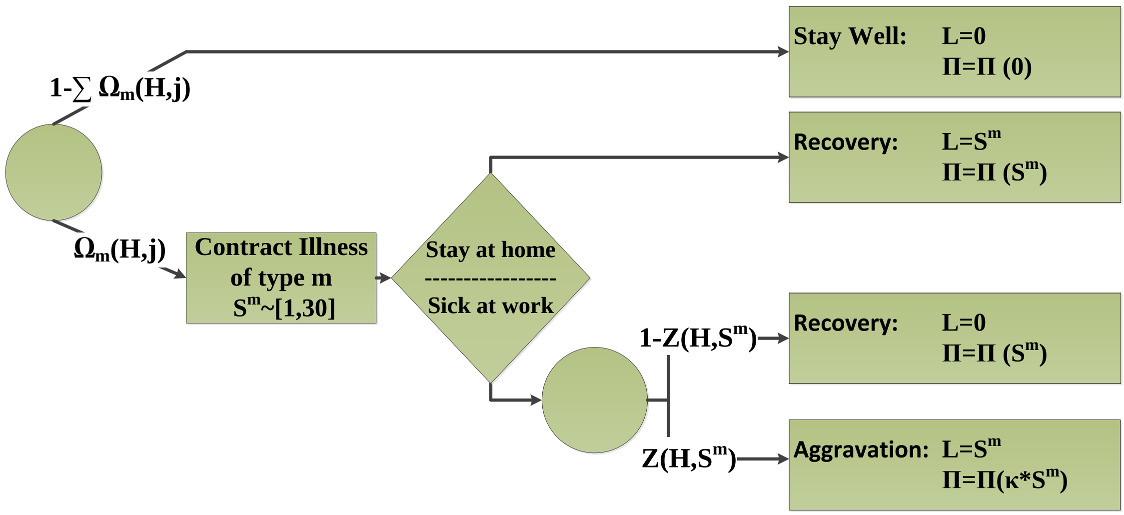
The decision to take sick leave, l_t, depends essentially on the presence of an acute sickness shock. When staying well, $S_t = S_t^0$, there is no reason to be absent from work, $l_t = S_t^0 = 0$. Only when becoming ill do individuals have to decide whether to take sick leave. By being absent from work, I assume for simplicity

[23] Unlike most of the health economics literature, I abstract for simplification from survival rates. Making the survival rate health dependent would further strengthen the mechanism because rich people would have an additional incentive to invest in health.

of the model that individuals are required to take a fixed amount of days of sick leave according to the severity of their illness, $l_t = S_t^m > 0$. By going to work sick, workers face two possible consequences. First, with the health state dependent probability $1 - Z(H_t, S_t^m)$, workers are fortunate and recover without taking recuperation time, i.e., $l_t = 0$. The health transition probabilities are the same as if they had stayed at home. Second, with probability $Z(H_t, S_t^m)$, the sickness aggravates; e.g., a cold becomes the flu. In this case, workers are forced to stay at home, $l_t = S_t^m$. In addition, an aggravation of sickness leads to lower health prospects; i.e., the sickness factor in health transition is multiplied by a factor of κ.

Note that the decision of going to work sick or not is made simultaneously with the consumption-saving decision at the beginning of the period. Workers cannot change their consumption if their sickness aggravates. The model also ignores preventive treatment, so individuals cannot improve their overall health by taking sick leave without being sick.

Figure 1.7: Events and Decisions - Sickness, Sick Leave and Sickness Aggravation



Notes: Circles denote events, rectangles denote realizations, diamonds denote decisions.

Preferences Individuals value consumption, c_t, and general health, H_t, over their life cycle according to a standard time-separable utility function

$$\mathbb{E}\left\{\sum_{j=0}^{J} \beta^j u(c_t, H_t)\right\}$$

where β is the raw time discount factor, and expectations are taken over a stochastic employment, health and sickness history. Days of sick leave do not enter the utility function directly but rather yield utility from an increased probability of being in a higher health state in the future.[24]

Labor Income and Employment Status There are two sources of heterogeneity that affect an agent's labor productivity, $\Gamma_{j,k}$, in this model. First, the labor productivity differs according to the age of an agent. Second, each household belongs to a particular group, k, that shares the same productivity. Differences in groups stand in for differences in education or ability, characteristics that are fixed at entry into the labor market and affect a group's relative income.[25] It is important to note that neither the general health status nor acute sickness directly affect labor productivity.[26] Agents get paid only for the time they work; i.e., working time is reduced by sick leave l_t.[27] The wage rate is w. Labor income $y_{j,k,t}$ is then given by

$$y_{j,k,t} = \Gamma_{j,k} w(1 - l_t)$$

In addition to labor productivity and days of sick leave, the labor income of agents depends crucially on their employment status $\mathcal{I}_t$. At the end of each period, agents may be dismissed with probability $1 - \Phi^e$. Central to this chapter is that workers are able to influence their layoff probability by reducing absence from work, and they take this into their optimization reasoning.[28] The probability of

[24]Including both sick leave and overall health in the utility function would not alter the qualitative predictions of the model; the main trade-off between consumption and health/sick leave remains. Augmenting the utility function by an additional argument, however, significantly complicates the identification of preferences parameters.

[25]By focusing on innate differences across workers instead of idiosyncratic skill shocks as the main source of heterogeneity I follow Keane and Wolpin (1997), Storesletten, Telmer, and Yaron (2004) and Huggett, Ventura, and Yaron (2011). They estimate that innate heterogeneity is a major contributor to the observed variations of lifetime earnings, contributing 50-90%.

[26]Health-dependent labor productivity would reinforce the inequality between ability groups because low-productivity workers are less healthy. For simplicity of the model, I refrain from this additional channel.

[27]The maximum amount of working days per year in this economy is set to 250.

[28]For simplification reasons, I refrain from modeling the demand side of the labor market. There are various theoretical explanations for the described linkage from an employer's perspective. First, health is important for the productivity of a worker. However, employers might be unable to directly observe health and instead use days of sick leave as a signal for health. Table 1.1 shows the high correlation of sick leave and health. Second, related to the literature on shirking, employers can only imperfectly monitor workers in terms of work absence. As the numbers of days of sick leave increases, so does the probability of being discovered when shirking. Third, due to the structure of the German sick leave system, days of sick leave are

staying employed, Φ^e, depends, in addition to days of sick leave, on the skill type of workers and the current unemployment rate, $\sqcap_t$.

$$\Phi^e = \Phi^e\left(l, k, \mathcal{U}\right)$$

The probability of finding a new job when unemployed, Φ^u, is again determined by workers' skill type and the current unemployment rate. Unemployed workers do not take sick leave.

$$\Phi^u = \Phi^u\left(k, \mathcal{U}\right)$$

Unemployment Rate The evolution of the unemployment rate, $\mathcal{U}_t$, is exogenous (i.e., I do not model general equilibrium effects). I assume that the unemployment rate in the model follows an AR(1) process approximated by a 5-state Markov process.

1.3.2 Government Policies

The government imposes a flat income tax, τ. The collected revenues are used for three main purposes:

(i) to finance unemployment benefits b^U. The unemployment insurance (Arbeitslosengeld I) replaces a constant fraction of the previous net income and therefore depends on the age and skill of an individual. When this unemployment insurance falls below some consumption floor c^W, workers might become eligible for government provided welfare (Arbeitslosengeld II). This welfare is means tested; i.e., before workers receive welfare, they have to run down their assets to a certain amount, a^W. Unemployment benefits are then given by

$$b^U_{j,k,i} = \begin{cases} c^W & \text{if } \rho^U(1-\tau)\Gamma_{j,k}w < c^W \quad \cap \quad a_i \leq a^W \\ \rho^U(1-\tau)\Gamma_{j,k}w & \text{else} \end{cases}$$

where ρ^U is the unemployment insurance net replacement rate, and a_i are the assets of worker i.

(ii) to finance paid sick leave b^S. For the time absent from work, l_t, workers get reimbursed by the government with payments depending on their regular labor

costly for employers. Days of sick leave are persistent over time. A simple test for autocorrelation yields a coefficient of 0.34. Therefore, the incentive for employers is high to get rid of workers with many days of sick leave to decrease these costs. In all these cases, a higher number of sick leave days leads to a higher layoff probability.

income.[29] Sick leave benefits are given by

$$b^S_{j,k} = \rho^S(1-\tau)\Gamma_{j,k}w$$

where ρ^S is the sick leave net replacement rate.

(iii) to finance retirement benefits, b^R. Workers receive these skill-dependent retirement benefits after their fixed retirement age J^R.[30] Retirements benefits are given by

$$b^R_{j,k} = \rho^R(1-\tau)\Gamma_{j^R,k}w$$

where ρ^R is the retirement system net replacement rate.

1.3.3 Individual's Dynamic Program

I model the decisions to miss work during an episode of acute illness as the choices of workers solving a discrete choice stochastic dynamic programming problem. At each discrete period, the forward-looking individual chooses whether to miss work based on expected utility maximization. Individuals can accumulate assets, a, at a constant interest rate R. They are not allowed to borrow. They allocate their total resources between consumption, c, and asset holdings a_{t+1} for the next period.

At the beginning of period t, individuals are indexed by their age j, their skill group k, their asset holdings a, their health status H, their realization of acute sickness S, and their employment status $\mathcal{I}$. To simplify the analysis, I assume that the factor prices are exogenous. Each individual starts her life in a specific health state H_0 and employment state $\mathcal{I}_0$ and is endowed with initial assets a_0. Thus, her maximization problem reads as

$$\begin{aligned} V(j,k,a_t,H_t,S_t,\mathcal{I}_t) = \max_{c_t,l_t,a_{t+1}} \quad & u\left(c_t, H_t\right) \\ + \beta \sum_{H_{t+1}} \Pi(j,k,H_t,S_t,l_t) & \sum_{S_{t+1}} \Omega(j,H_{t+1}) \sum_{\mathcal{I}_{t+1}} \Phi(k,l_t,\mathcal{U}_t) \\ & V(j+1,k,a_{t+1},H_{t+1},S_{t+1},\mathcal{I}_{t+1}) \end{aligned} \tag{1.5}$$

[29] This reimbursement is actually provided not by the government but by the employer. Because I do not model the firm site, I make this shortcut.

[30] To reduce the state space in the quantitative approach, these retirement benefits do not depend on the history of idiosyncratic employment shocks. This is a deviation from the actual German system. Lower pension benefits from increased periods of unemployment would strengthen the underlying mechanism of the model because, again, more days of sick leave reduce future expected earnings.

subject to the constraints

$$a_{t+1} + c_t = \begin{cases} (1-\tau)\left(\Gamma_{j,k} w\,(1-l_t) + l_t b^S_{j,k}\right) + R a_t & \text{if} \quad j < J^R \cap \mathcal{I}_t = 1 \\ b^U_{j,k} + R a_t & \text{if} \quad j < J^R \cap \mathcal{I}_t = 0 \\ b^R_{j,k} + R a_t & \text{if} \quad j \geqslant J^R \end{cases}$$

$$a_{t+1} \geq 0$$

1.4 Quantitative Analyses

In this section, I begin by discussing the specification of the model parameters. Then, in Section 1.4.2, I present simulation results and their counterparts in the data to evaluate the model's performance. These results contain cross-sectional distribution and lifetime profiles of days of sick leave and health and the average days of sick leave for a time series of unemployment rates.

1.4.1 Parameter Estimation and Calibration

To fully determine the structural model, I need to choose parameters that govern the employment status, incidence of sickness, health transitions, preferences, and policy settings. The determination of the model parameters proceeds in three steps. First, I fix a subset of parameters exogenously. Second, parts of the model parameters can be estimated from SOEP data directly. These include parameters governing labor productivity Γ; probabilities to keep employment Φ^e, to find a new job, Φ^u, and to contract acute sicknesses Ω; health transition probabilities Π; and the Markov process of the overall unemployment rate. Third (and given the parameters obtained in steps 1 and 2), the remaining parameters (i.e., governing sickness aggravation Z and κ) are determined through a method of moments estimation. I now describe these three steps in detail.

A Priori Chosen Parameters

In the first step, I choose parameters from the literature or set them according to the data. All values of the a priori chosen parameters are shown in Table 1.11 in Appendix 1.D.

General Settings The model period is one year. Workers start their economic life at age 20, retire at age 65 and live until age 80. Because I do not model childhood of a household explicitly, I denote its 20th year of life by j=0, its retirement age by J^R=45 and the terminal age of life by J^T=60. After retiring, the problem of agents is reduced to a consumption-saving decision.

I use self-reported health status as an empirical counterpart from the SOEP for the model's health state. Thus, H_t takes one of five values: 0-"bad", 1-"not so good", 2-"satisfactory", 3-"good", or 4-"very good".

I assume that the interest rate, R, is determined exogenously by world factors in an open-economy equilibrium, and following Siegel (2002), I set R=1.0402. The wage rate is normalized to w=1.

Preferences I choose for the instantaneous utility function the specification of Finkelstein, Luttmer, and Notowidigdo (2013). They estimate using household panel data a health state-dependent utility function with the functional form:

$$u\left(c_t, H_t\right) = \left(1 + \psi\left(4 - H_t\right)\right) \frac{c_t^{1-\sigma}}{1-\sigma}$$

where σ determines the inter-temporal elasticity of substitution, and ψ captures state dependence because it allows the state of health to affect the marginal utility of consumption.

Finkelstein, Luttmer, and Notowidigdo (2013) estimate that a one standard deviation increase in the number of chronic diseases is associated with an 11% decline in the marginal utility of consumption relative to this marginal utility when the individual has no chronic disease. This implies that ψ=0.112.[31] I choose σ=2 to obtain an inter-temporal elasticity of substitution of 0.5, which is a value widely used in the literature (e.g., Fernandez-Villaverde and Krueger (2007)). Consistent with values commonly used in the quantitative macroeconomics literature, I choose a time discount factor of β=0.96 per annum.

Policy Parameters For the benchmark calibration, I choose the current institutional setting for Germany. It shuts down two direct effects of income on health. First, Germany has universal health care coverage, so individuals do not have to pay for standard medical expenditures, e.g., doctor visits. Second, I set the benchmark paid sick leave coverage to 100% of the current wage, ρ^S=1, ruling out any reduction in the current income of workers due to sick leave; this step is important to isolate the indirect effects of sick leave via the risk of unemployment. I set the unemployment insurance net replacement rate to ρ^U=0.60. This is the current German setting for the first 12-24 months in Germany. The means-tested welfare

[31] I deviate from Finkelstein, Luttmer, and Notowidigdo (2013) by substituting the number of chronic diseases (0-5) by the ordinal five health-state variables used in this model.

provision, c^W, I set to 0.25 ($\triangleq$ 700 €(2005)), which represents the German basic tax-free allowance. The amount of assets of an individual that are protected without losing eligibility of welfare, a^W, I set to 0.44 ($\triangleq$ 1220 €(2005)).[32] Retirement benefits are set to ρ^R=0.50 of the former net labor income of a worker. I set the tax rate τ to 33%.

Parameters Estimated Directly from the Data

In a second step, I estimate a part of the model parameters directly from the SOEP data. Detailed results for the estimated parameters are shown in Table 1.12 - 1.13 in Appendix 1.D.

Γ – Labor Productivity Using SOEP data on labor income, I compute age-dependent productivity for five different income quintiles. For each age (20-65) and wave separately, I split the sample into five income quintiles. Within each age-income cell, I take the median income as the labor productivity $\Gamma_{j,k}$ of workers in that quintile. Figure 1.8 shows the resulting income profiles over the life cycle. It exhibits the well-known increase in income at younger ages and the flattening out at older ages. The distance between the bottom and the top income quintiles widens over age. I normalize the life-cycle profiles using the income third quintile at age 40 as a basis (1$\triangleq$ 2768 €(2005)).[33]

Φ – Employment Transition For each combination of days of sick leave, skill group, and unemployment rate, the model predicts a certain probability of retaining the job Φ^e, which is given by

$$\widehat{\phi^e} = 6.775832 - 0.0331853 * l + \sum_k \beta_k^e * k^{Dummy} - 0.1506627 * \mathcal{U}$$

$$\Phi^e\left(l, k, \mathcal{U}\right) = \frac{e^{\widehat{\phi^e}}}{1 + e^{\widehat{\phi^e}}}$$

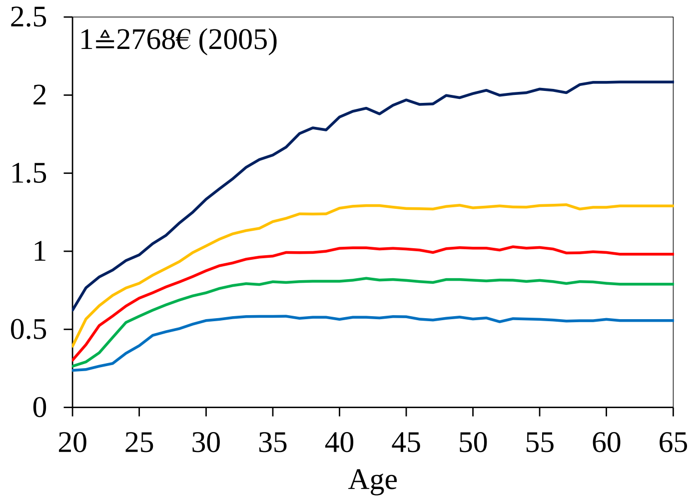
The used parameters are obtained by a panel logit regression using SOEP data. The resulting odds ratios of this estimation are shown in Section 1.2.3.

The probability of finding new employment when unemployed in the last pe-

[32] The German unemployment insurance system underwent a major reform in 2005, which might limit the historical comparison between the model predictions and the data.

[33] These results are broadly in line with other findings on the German income structure, cf. Hujer et al. (2001).

Figure 1.8: Life Cycle Profile of Labor Income by Income Quintiles



Notes: Labor income over the life cycle for five quintiles computed for the benchmark sample. Quintiles are defined for each age and wave separately.

riod, Φ^u, is computed the same way except using days of sick leave.

$$\widehat{\phi^u} = 4.347449 + \sum_k \beta_k^u * k^{Dummy} - 0.199108 * \mathcal{U}$$

$$\Phi^u(k, \mathcal{U}) = \frac{e^{\widehat{\phi^u}}}{1 + e^{\widehat{\phi^u}}}$$

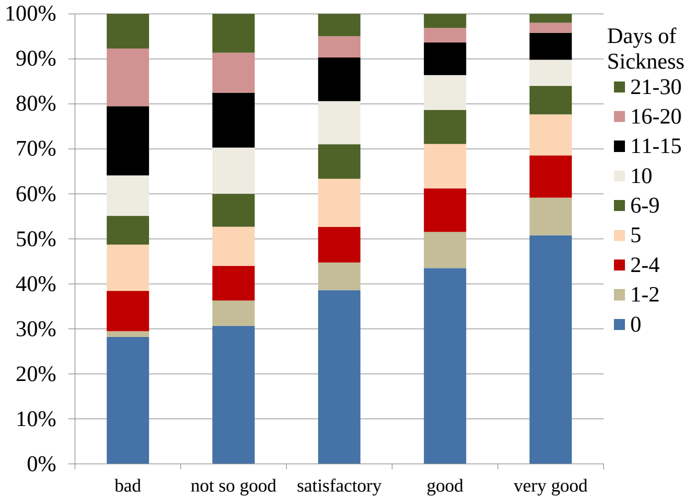
Ω – Incidence of Sickness One shortcoming of the dataset is that it contains only days missed at work, l, and not the incidence of sickness, S. To compute the probabilities of sickness, I make the identifying assumption that workers with a high productivity always stay at home to recover, and therefore, observed sick leave is equal to the occurrence of a sickness, $S = l$.[34]

The incident of sickness S depends on the current health state H of an agent. Individuals with lower health have both a higher probability of contracting a sickness and a higher probability that the sickness is more severe. In addition to health, the number of days of sick leave also seems to depend on the age of work-

[34] In the benchmark calibration, I use only the second to the top income quintiles. The empirical analysis and the later simulation results suggest that the top income quintile is an outlier. In a robustness check, I include all observations in the estimation of the probabilities of sickness. The qualitative patterns remain the same.

ers.[35] To take this trend into account, I estimate incident probabilities conditional on the health status and the age group of workers. To ensure sufficient observations for each sickness state, I restrict the number of sickness shock realizations to $m = 9$ states, $S \in$ [0, 1-2, 3-4, 5, 6-9, 10, 11-15, 16-20, 21-30]. Figure 1.9 shows the frequency of days of sick leave conditional on health for the top two income quintiles for the specific age group 40-50.[36]

Figure 1.9: Frequency of Days of Sick Leave Conditional on Health and Age



Notes: Frequency of days of sick leave conditional on self-reported health state. The sample consists of the fourth income quintiles for the age group 40-50.

II – Health Transition To estimate the effect of acute sickness on the general health status, it is important to differentiate between sickness and sick leave. Taking sick leave should be beneficial to health, whereas an acute sickness itself should harm overall health. As in the estimation of Ω, it is therefore necessary to restrict the sample to those for whom observed sick days are equal to sickness, i.e., the high-productivity workers.

To estimate health transition probabilities, I define a new variable

$$\Delta_{i,t,t-2} = \begin{cases} 1 & H_{i,t} > H_{i,t-2} \\ 0 & H_{i,t} = H_{i,t-2} \\ -1 & H_{i,t} < H_{i,t-2} \end{cases}$$

[35] This age dependency of sick leave can be observed in Figure 1.15 in Appendix 1.D. During early working life, workers use on average more days of sick leave than later in their working life.

[36] The results for all age-groups are shown in Table 1.13 in Appendix 1.D.

which marks a change, either better or worse, in the health status of an individual between period t-2 and t.[37] To assess the transition probabilities of health, I employ an ordered logistic panel regression. The regression equation is

$$OLogit\left[\Delta_{i,t,t-2}\right] = \mathcal{F}\{\alpha + \beta_1\, S_{i,t-1} + H^{Dum}_{i,t-2}\,\beta_2 + \beta_3\, k_i + \beta_4\, j_{i,t-1} + \epsilon_{i,t}\}$$

where days of sick leave are denoted by $S_{i,t-1}$, $H^{dum}_{i,t-2}$ are dummies for each health state, k_i is the income quintile (skill type), and $j_{i,t-1}$ the age of the respondent. $\epsilon_{i,t}$ is a random error term.

Table 1.4 shows the results for the specification used for calibration (Column 1) and a regression in which numerous additional controls are included (Column 2). In both specifications, sickness has a negative effect on the probability of an improvement in health and consequently a positive effect on the probability of a deterioration in health. Age has a detrimental effect on health, reflecting a worsening health status over the life cycle. Being in a high income quintile has, in contrast to age, a protective effect against health deterioration. Including years of education makes the income effect insignificant, reflecting the strong correlation between income and education.

Table 1.4: Ordered Logit Results for Changes of Health

$\Delta_{t,t-2}$	(1)	(2)
Days of Sick Leave (t-1)	-0.0459***	-0.0414***
Age (t-1)	-0.0292***	-0.0296***
Income Quintile (t-1)	Yes	Yes
Health State Dummies(t-2)	Yes	Yes
Other Controls	No	Yes
Observations	40,460	34,935

Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
Other controls include sex, years of education, number of children, marital status, recession, and year dummies. All columns report the regression coefficient.

[37] I choose a lag of two periods to compute health transitions. Doing so is necessary because the question about health refers to the point in time at which the interview is conducted (distributed over the year), whereas the question about days of sick leave refers to the time span of the whole previous year. This might lead to a situation in which the acute sickness occurs later in the year than the respondent answers the health question. In a robustness check, I look at a one lag health transition and find qualitatively similar results.

$\mathcal{U}$ – Unemployment Rate I estimate the unemployment rate with an AR(1) process using the time span from 1994 to 2012 (with ρ=0.9152). This continuous AR(1) process is then approximated by a finite state Markov process applying Tauchen's method, cf. Tauchen (1986). I restrict the number of unemployment rate states to n=5. Doing so leads to an unemployment rate state grid $\mathcal{U} \in$ [7.5%, 9%, 10.5%, 12%, 13.5%] and a transition matrix, Ξ, given by

$$\Xi = \left\{ \begin{array}{ccccc} 0.7939 & 0.2056 & 0.0005 & 0 & 0 \\ 0.0733 & 0.7752 & 0.1512 & 0.0002 & 0 \\ 0.0001 & 0.1072 & 0.7853 & 0.1072 & 0.0001 \\ 0 & 0.0002 & 0.1512 & 0.7753 & 0.0733 \\ 0 & 0 & 0.0005 & 0.2055 & 0.7939 \end{array} \right\}$$

Parameters Calibrated Within the Model

In the final step, I use my model to find parameters that govern the probability and severity of sickness aggravation. All estimated parameters and the fit of the targeted moments to the data can be found in Table 1.16 in Appendix 1.D.

Sickness Aggravation Important for the decision to stay at home to recover or go to work sick is the aggravation probability, Z, and the aggravation factor κ.

For the aggravation probability, Z, I make two assumptions. On the one hand, it depends negatively on the overall health constitution, i.e., that people in a bad health state are much more likely to face an aggravation than people in very good health. On the other hand, the aggravation probability depends positively on the severity of the sickness; i.e., being hit by a bad illness makes it very unlikely not to recover without taking sick leave. The functional form for Z is given by

$$\begin{aligned} \widehat{Z} =& (H-1)S^{\zeta} \\ Z(H,S) =& 2\frac{e^{-\widehat{Z}}}{1+e^{-\widehat{Z}}} \end{aligned}$$

where $0 \leq Z \leq 1$ and $\zeta \leq 0$. Parameters ζ and κ are calibrated to minimize the distance between the distribution of days of sick leave of the bottom income quintile found in the data and predicted by the model.

1.4.2 Model Fit and Benchmark Results

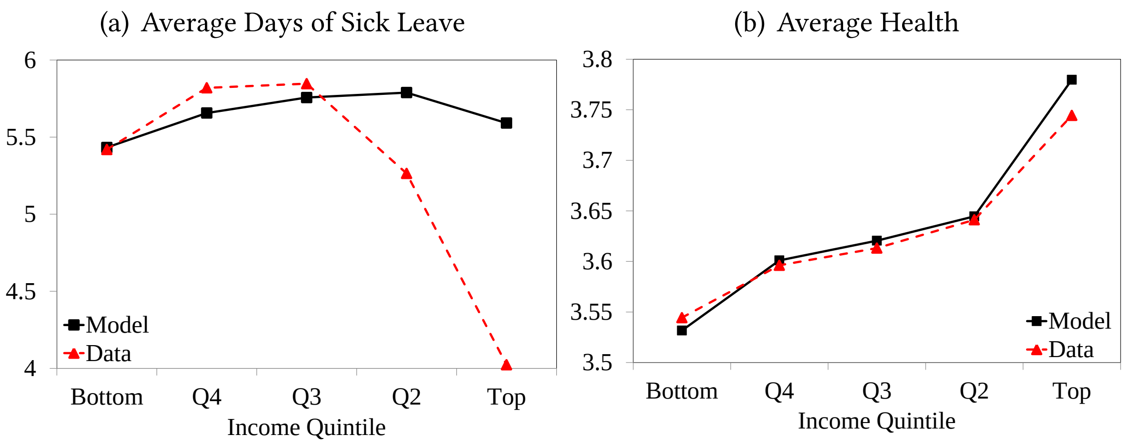
In this section, I examine the fit of the model to the non-targeted data moments. Moments of the model are computed by aggregating over the state distribution of the population. The initial states for health, H_0, and employment, $\mathcal{I}_0$, are drawn from distributions conditional on the skill type that match the data; see Table 1.14 - 1.15 in Appendix 1.D. All individuals start their lives with no initial assets, a_0=0. For cross-sectional and life-cycle patterns, I fix the general unemployment rate constant at $\mathcal{U}$=10.8%.

Cross Section The left panel of Figure 1.10 plots the simulated average days of sick leave across income quintiles (the solid black line) and the data counterpart (the dashed red line). The model is able to endogenously generate the key hump-shaped profile of average days of sick leave, particularly the increase in days of sick leave between the bottom and the medium income quintiles. This increase is driven by the indirect cost effect, i.e., that low income people reduce their recuperation time to reduce their layoff probability. Note that the model is also able to reproduce the decrease in the average days of sick leave for high-income individuals compared to medium-income workers, although the difference is not as large as that observed in the data.[38] The right panel of Figure 1.10 shows the simulated average health for each income quintile and the data counterpart. The cross-sectional pattern is very close to the data moments and reflects the income gradient in health. Only in the very top income quintile does the model predict a higher average health than is found in the data.

Figure 1.11 shows the decomposition of the average days of sick leave into the extensive (left panel) and intensive (right panel) margins. The fit of the model prediction to the data for the probability of missing any day in a year is good. The model can account for the sharp increase in the probability of missing any day across the income quintiles. Again, the top income quintile has the worst fit. Looking at the intensive margins (the number of days missed conditional on being absent for at least one day), the model can generate the monotonic decrease over income quintiles. However, particularly for the top two income quintiles, the simulated level of the intensive margins exhibits a poor fit to the data moments.

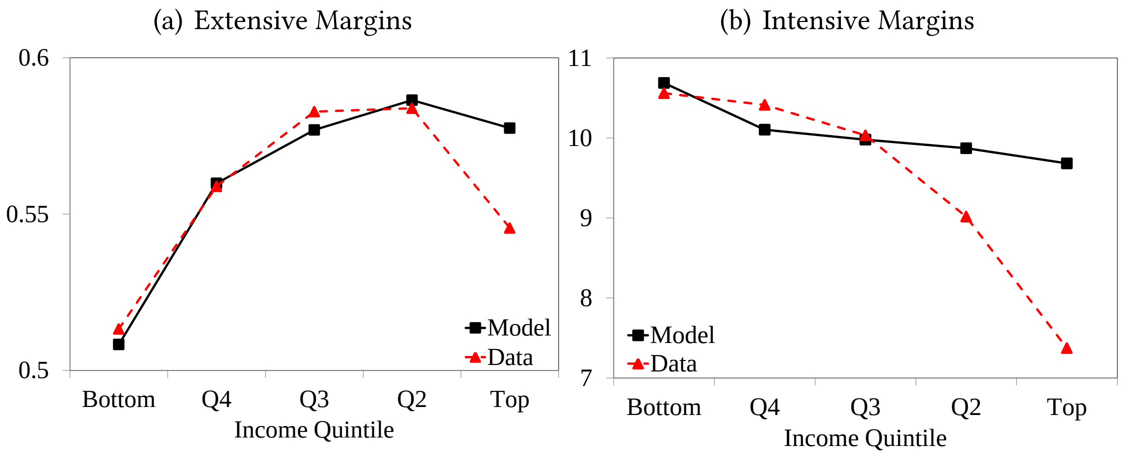
[38] The not-so-good fit of the model for the top income quintile hints that characteristics of very high income jobs, e.g., high responsibility, might also have an effect on the utilization of sick leave. Additionally, a higher work flexibility (e.g., a home office) allows for fewer days of sick leave compared to workers on an assembly line.

Figure 1.10: Health and Days of Sick Leave across Income Quintiles



Notes: The dashed red lines show moments taken from the SOEP data. The solid black lines show averages from simulating the structural model using the parameters explained in Section 1.4.1.

Figure 1.11: Extensive and Intensive Margins of Days of Sick Leave across Income Quintiles

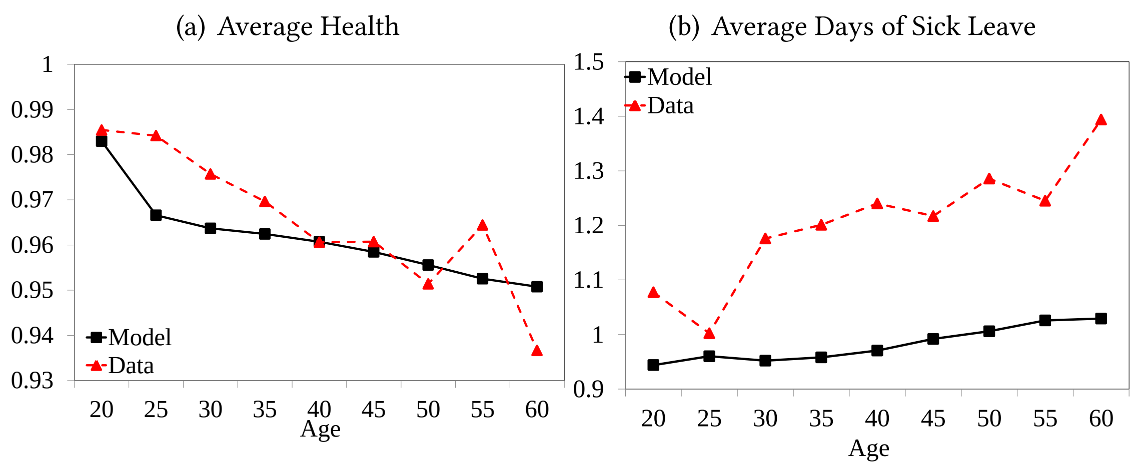


Notes: The dashed red lines show moments taken from the SOEP data. The solid black lines show averages from simulating the structural model using the parameters explained in Section 1.4.1.

Life Cycle In addition to cross-sectional moments, the model can replicate the widening gap in average health across the income quintiles over the life cycle. The left panel of Figure 1.12 shows the ratio of the two bottom to the two top income quintiles for average health. At the beginning of working life at age 20, average health is almost identical between the bottom and the top.[39] Over the life course, the simulated health ratio decreases (the health gap widens). At the end of working life at age 60, the model predicts a clear income gradient in health, as observed in the data.

The right panel of Figure 1.12 shows the ratio for the average days of sick leave of the bottom to top income groups. The simulated model is able to reproduce the increase in the ratio, i.e., the low-income workers used more days of sick leave compared to their high-income peers as they advanced in age. However, the match of the level of this ratio is poor, which, as mentioned, is due to the fact that the very top income quintile has a low utilization that cannot be explained only by their overall good health.

Figure 1.12: Inequalities in Days of Sick Leave and Health over Life Cycle



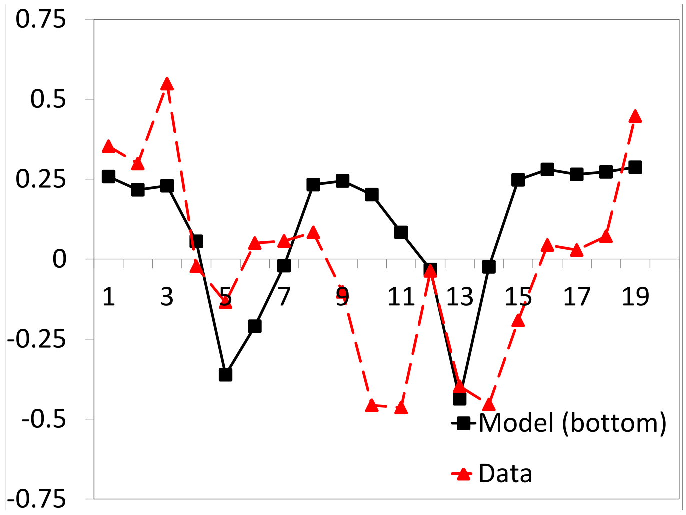
Notes: The dashed red lines show moments taken from the SOEP data. The solid black lines show averages from simulating the structural model using the parameters explained in Section 1.4.1.

Business Cycle The model is able to represent the cyclical behavior of the average claims of sick leave in Germany. Figure 1.13 shows the simulated average days of sick leave for all workers. The unemployment is set according to the Ger-

[39] The prediction of the model at age 20 is matched to the data by construction; I set the initial values of the model according to the data.

man unemployment rate in the time span from 1994 to 2012. When the unemployment rate is high, the average days of sick leave are reduced and vice versa. The magnitude is, however, less distinctive than that observed in the data. The model is particularly able to reproduce the drop in days of sick leave in times of high unemployment. The strong increases in the days of sick leave in times of low unemployment are not well matched.

Figure 1.13: Cyclicality of Average Days of Sick Leave for German Business Cycle



Notes: The dashed red lines show moments taken from the SOEP data. The solid black lines show averages from simulating the structural model using the parameters explained in Section 1.4.1.

1.4.3 Effects of Indirect Costs of Sick Leave

To illustrate the quantitative dimension of the indirect costs of sick leave, I simulate a counterfactual model in which the layoff probability, Φ, is independent of workers' past days of sick leave. Table 1.5 shows the central sick leave moments for the population average and bottom income quintile workers. The average number of days of sick leave for all workers is increased by 4% compared to the benchmark economy. This difference in work absence is primarily driven by low skilled workers. The bottom income group would increase their average number of days of sick leave by more than 11%. A similar pattern is shown for the extensive margin of sick leave, where the bottom income is 10% higher than the benchmark case. The drop in the intensive margin follows from the fact that more people stay at home and reduce the conditional average of sick leave.

Table 1.5: Quantitative Effect of Indirect Costs of Sick Leave

	Benchmark Case	Sick Leave Independent Layoff	Difference
Average sick leave days	5.64	5.84	+4%
Average sick leave days (Bottom)	5.43	6.02	+11%
Extensive Margin	0.56	0.59	+3%
Extensive Margin (Bottom)	0.50	0.60	+10%
Intensive Margin	10.06	9.90	-2%
Intensive Margin (Bottom)	10.69	10.06	-6%

Notes: Results are shown for the benchmark economy and a model without a sick-leave-dependent layoff probability.

1.5 Policy Evaluation

The determination of policy-invariant structural parameters allows for the introduction and evaluation of different counterfactual policies that affect the financial constraints of a worker's decision-making problem. These policies include variations in the paid sick leave coverage and the benefit structure of unemployed workers.

1.5.1 Paid Sick Leave Coverage

Having identified the importance of sick leave for households, the model allows for the analysis of the effect of introducing direct costs of sick leave, i.e., a reduction in the replacement rate of paid sick leave. Table 1.6 shows the average days of sick leave of the whole work force and of workers in the bottom income quintile for different sick leave replacement rates. Introducing the additional costs of being absent from work naturally leads to a decrease in the average days of sick leave. A reduction from 100% (benchmark economy) to only 80% of the current income reduces the days an average worker is absent from work by more than one day or 19% relative to the benchmark economy. These numbers are broadly in line with those of Ziebarth and Karlsson (2010), who estimated the reduction of sick leave for such a policy change as 12% using a natural experiment. Reducing the replacement rate further to zero (as in the US) would lower the average days of sick leave to 3.29 days or only 58% of the days of sick leave of the benchmark economy.

Although both direct and indirect costs of sick leave reduce the number of days missed at work, it is important to note that these reductions are borne by different groups of the income distribution. Although indirect costs primarily affect the bottom income quintile (see results above), Table 1.6 shows that direct costs of sick leave also have a strong effect on the higher income groups. The reduction of paid sick leave to 80% reduces the days of sick leave of the average worker by 20%. Bottom income quintile workers reduce their days of sick leave by only 17%. The underlying reason for this finding is that the sick leave replacement rate is a fraction of income, and therefore, high-income agents lose more income in absolute terms than their low-income peers.

Table 1.6: Variation of Income Replacement Rate of Paid Sick Leave

	All Workers		Bottom Income Quintile	
Policy Change	Days of Sick Leave	Difference to Benchmark	Days of Sick Leave	Difference to Benchmark
b^S=100% (benchmark)	5.64	–	5.43	–
b^S= 99%	5.61	-0.5%	5.38	-0.9%
b^S= 90%	5.27	-6.5%	4.73	-12.9%
b^S= 80%	4.52	-19.8%	4.51	-16.9%
b^S= 50%	4.07	-27.8%	3.89	-28.3%
b^S= 0%	3.29	-41.7%	3.08	-43.2%

Notes: Simulations of structural model using the parameters explained in Section 1.4.1, varying the income replacement rate of sick leave.

1.5.2 Unemployment Benefits

Corresponding to the replacement rate b^S for direct costs of sick leave, unemployment benefits are a main determinant of the indirect costs. Altering the financial situation for workers who are laid off changes the incentives to go to work sick; e.g., full unemployment insurance would completely eliminate the indirect costs of sick leave. Table 1.7 shows the average days of sick leave of all workers and of workers in the bottom income quintile for various changes in the unemployment benefit structure, i.e., the unemployment insurance net replacement rate b^U, the level of means tested welfare c^W, and the amount of protected assets a household is allowed to save before losing welfare eligibility a^W.

A 10% increase in the unemployment insurance net replacement rate leads to a 2% increase in the days of sick leave for all workers. This increase is strongest among low-income workers, whose days of sick leave would increase by more than 5%. Similarly, a 10% reduction in the unemployment benefit level would reduce sick leave by 1%. Important to note here is that due to the unemployment benefit structure in Germany, when most low-income workers become unemployed, they will fall directly into means-tested welfare and are therefore not affected by this reduction.

Table 1.7: Variation of Unemployment Benefits Structure

Policy Change	All Workers		Bottom Income Quintile	
	Days of Sick Leave	Difference to Benchmark	Days of Sick Leave	Difference to Benchmark
b^U=70%	5.75	+2.0%	5.71	+5.1%
b^U=60% (benchmark)	5.64	–	5.43	–
b^U=50%	5.58	-1.1%	5.38	-0.9%
c^W= 770€	5.68	+0.7%	5.58	+2.8%
c^W= 700€ (benchmark)	5.64	–	5.43	–
c^W= 630€	5.61	-0.7%	5.29	-2.6%
c^W= 0€	5.56	-1.4%	5.10	-9.4%
b^U= 0% & c^W= 0€	4.41	-21.8%	3.70	-31.9%
a^W=5000€	5.68	+0.7%	5.61	+3.3%
a^W=1200€ (benchmark)	5.64	–	5.43	–
a^W= 0€	5.58	-1.1%	5.16	-5.0%

Notes: Simulations of structural model using the parameters explained in Section 1.4.1, varying the income replacement rate of unemployment benefits and changing the means-tested welfare consumption floor and the asset exemption.

Reducing means-tested welfare has, on average, a weaker effect on the population average but a much stronger effect on low-income workers. Reducing the level of welfare by 10% reduces the days of sick leave by almost 3%, whereas increasing the level of welfare by 10% increases the days missed at work by almost 3%. Taken to the extreme, setting both the unemployment benefits and the welfare to zero reduces by more than 1.2 days the days missed at work for all workers and by more than 1.7 days for poor workers in absolute terms; in relative terms, these reductions are 22% for all workers and 32% for poor workers.

In addition to the level of welfare benefits, it is also important to look at the amount of assets that unemployed people are allowed to save without reducing their welfare. These protected assets enable workers to smooth their consumption over an unemployment period and therefore ease the negative effect of unemployment. Setting a^W to zero has the same effect as setting c^W to 450€. Both reduce the days of sick leave by 5%.

1.6 Conclusion

In this chapter, I have studied disparities in the utilization of sick leave across income quintiles, over the life course and during business cycles. Using data from the SOEP, I have found first that in times of high unemployment, days of sick leave are, on average, low. Second, low-income workers use surprisingly few days of sick leave taking into account their general low overall health state. Furthermore, I have documented that the effect of the number of days of sick leave on future employment is empirically relevant and serves as the economic rationale behind these stylized facts of aggregated days of sick leave. Based on this finding, I have developed and estimated a life-cycle model that includes an endogenous health state. I have estimated this model intensively using micro panel data and have shown that it is able to replicate data moments on the aggregated number of sick leave days in many dimensions, e.g., the hump-shaped pattern across income quintiles. I have found that the costs of sick leave stemming from reductions in future expected earnings primarily affect the lowest income quintile, whereas the costs stemming from reductions in current income (i.e., a reduction in paid sick leave coverage) also affect higher income quintiles. Furthermore, my results from counterfactual policy experiments suggest that changing the unemployment benefits (particularly means-tested welfare) would lead to sizeable changes in the number of sick leave days in Germany.

Appendix 1.A German Sick Leave Policy

Compulsory sick pay with 100% wage replacement was established in 1930 for white-collar employees and in 1969 for blue-collar workers. The current regulation of sick leave (*Entgeltfortzahlung im Krankheitsfall*) in Germany is determined in the *Entgeltfortzahlungsgesetz*. According to the law, employees eligible for paid sick leave are those (also including part-time and temporary workers) that fulfill the following conditions:

- The employment has to be in place for four weeks.
- The worker has to be incapable of working.
- The incapability has to be a consequence of an illness.
- The illness is not a result of a gross negligence.

Sick pay has to be provided by the employer from the first day (no grace period) up to six weeks. After six weeks, sick pay is provided by the health insurance with a reduced wage replacement rate of 80%. The claim of full wage replacement, however, renews if the worker contracts a different illness, or more than 6 months has elapsed in which the worker was sick with the same illness. Workers receive the average earnings that they would have earned if they had not been sick. These earnings include the fixed salary potential commissions. Furthermore, if workers become sick while they are on vacation, holiday entitlement is not reduced. Workers absent from work have to inform the employer immediately about their incapability to work. On the fourth day of a sick spell, workers have to send a sick certificate issued by a health practitioner.

Monitoring the worker is highly restricted. The German Federal Labor Court decided that the observation of employees by their employer is illegal without concrete evidence supporting the suspicion of fraud. (Court decision 19. February 2015 - 8 AZR 1007/13). The employer can only request that the employee be reexamined by the practitioner of the Medical Service of the Health Funds (Medizinischer Dienst der Krankenversicherung).

Between October 1996 and December 1998, there was a temporary change in the law. The main changes were a reduction of wage replacement from 100% to 80%. However, this reduction applied to only a fraction of the German workforce, as collective labor agreements between unions and firms mostly kept 100% wage replacement. Empirical research on this law discontinuity is conducted by Ziebarth and Karlsson (2010) and Ziebarth (2013).

Appendix 1.B Sample Selection

Table 1.8 shows the descriptive statistics before and after sample selection. I use in these statistics weighted samples. Weights are provided by the SOEP to match the German micro-census. The final sample is younger due to a focus on the working age population. The higher percentage of men in the sample can be explained by their higher participation rate in the labor force. The average number of reported days of sick leave is lower as the upper tail of the sick day distribution is cut off.

Table 1.8: Descriptive Statistics for Sample Selection

	Whole Sample	Benchmark Sample
Male	46%	63%
Age	49	41
Years of Education	11.7	12.1
Health	2.32	2.60
Income	2,359€	2,814€
Unemployed	6.80%	5.40
Sick Leave	9.73	5.26
Observations	393,245	100,526

Notes: Descriptive statistics before and after sample selection. A benchmark sample used in the cross-sectional and panel analysis.

Appendix 1.C Robustness Check of Empirical Part

1.C.1 Different Measures of Sick Leave

Workers' number of sick leave days has a skewed distribution. Table 1.9 provides results for the correlation of the unemployment rate and different measures of sick leave days. First, it shows the result for the median worker. The second column shows the correlation with the extensive margin, i.e., whether the respondent has missed a day or more. In the last three columns, different cut-off levels for the maximum days of sick leave are used. All results are negative and in the same range as the benchmark result. The pro-cyclical pattern is extremely robust.

Table 1.9: Days of Sick Leave and Unemployment - Different Sick Leave Measures

Correlation	Median	Ext. Marg.	Max 120	Max 60	Max 40
Unemployment Rate	-0.5605	-0.4960	-0.7033	-0.6951	-0.6347

Notes: Times series correlation of different measures of days of sick leave and unemployment rate. First, the days of sick leave of the median respondent; second, the cyclical behavior of the extensive margin. The last three columns represent the correlation of the mean with different cut-off levels for the maximum number of sick days.

1.C.2 Composition Effects

A potential different explanation for the cyclicality of days of sick leave could arise if sectors (e.g., the construction sector) with a high usual number of days of sick leave are more prone to business cycles than the rest of the economy. To ensure that the general effect is not driven by this reason, I check whether the exclusion of different sectors alter the general finding. Table 1.10 shows that the exclusion of the construction sector does not alter the benchmark result. The correlation coefficient is reduced only slightly to -.7188.

I also check whether this cyclical behavior is different for different occupation types. The SOEP provides the ISCO88 classification and uses the white/blue collar distinction as in the European working conditions surveys. Table 1.10 shows that for both subgroups, the pro-cyclicality of days of sick leave holds.

Table 1.10: Days of Sick Leave and Unemployment - Different Sectors and Occupations

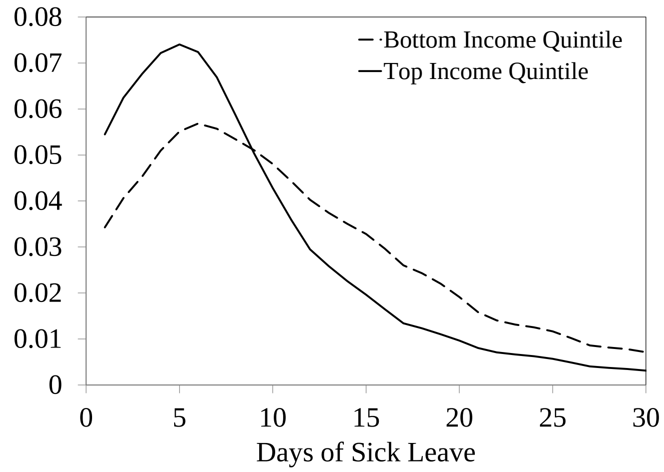
Correlation	No Construction	Blue Collar	White Collar	Never Unemployed
Unemployment Rate	-0.7032	-0.6289	-0.6272	-0.6576

Notes: Time-series correlation of average days of sick leave and unemployment rate for selected subgroups.

1.C.3 Density Function of Days of Sick Leave

Figure 1.14 shows the density functions for the top and the bottom income quintiles conditional on missing any day in a year. The frequency of a low number of sick leave days is higher for the top income quintile, whereas the frequency of a high number of days of sick leave is higher for the bottom income quintile. Combined with the differences in the extensive margin, this shows that workers in the bottom income quintile have either no or many days of sick leave, whereas workers in the top income quintile have few days of sick leave but at a higher frequency.

Figure 1.14: Density Function of Days of Sick Leave for Bottom and Top Income Quintiles



Notes: Density function of days of sick leave for bottom and top income quintiles. Graphs exclude the probability of having no days of sick leave.

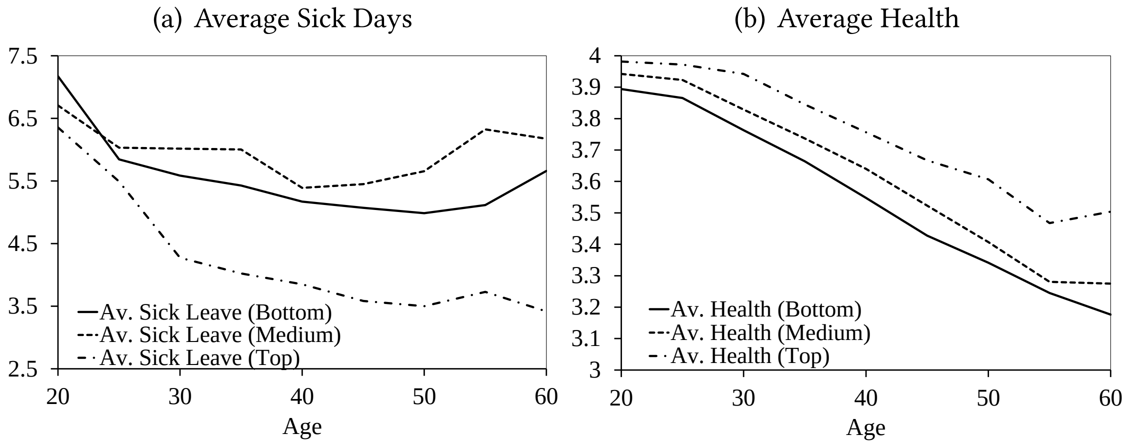
1.C.4 Age Profiles of Days of Sick Leave and Health

The left panel of Figure 1.15 confirms that the observed hump-shaped income profile also holds within all but one age group (20-25). It is noteworthy that there is no significant increase in days of sick leave with age, which is driven in part by the exclusion of a very high number of sick leave days (>30). The right panel shows that the income gradient in health increases with age.

1.C.5 Controlling for Gender in Sick Day Profiles over Income Quintiles

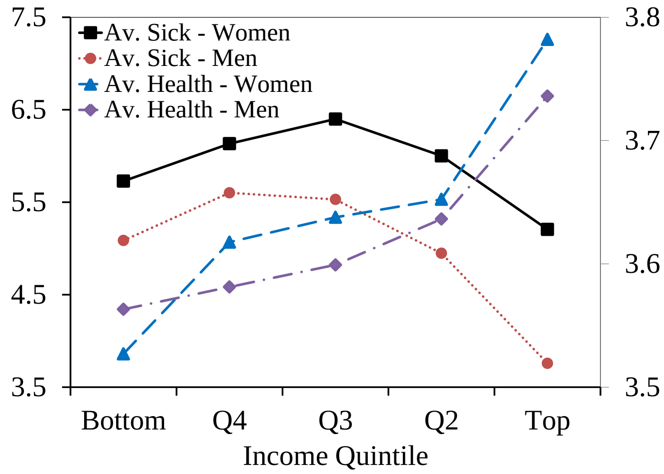
Figure 1.16 shows the average annual claims of days of sick leave and the average self-reported health separated by gender. Both groups show a hump-shaped profile in the average number of sick leave days across income quintiles. On average, women miss more days at work due to sickness than men. Health increases monotonically with income for both groups, with better average health for woman than for men.

Figure 1.15: Days of sick leave and Health over Life Cycle by Income Quintiles



Notes: Left Panel: Average self-reported health of bottom (solid), medium (dashed) and top (dotted) income quintiles over the life cycle. Right Panel: Average number of sick leave days of the bottom (solid), medium (dashed) and top (dotted) income quintiles over the life cycle. Age bins: 18-22, 23-27, 28-32, 33-37, 38-42, 43-47, 48-52, 53-57, 58-62.

Figure 1.16: Average Days of Sick Leave and Health over Income Quintiles - Gender



Notes: Dashed and dotted-dashed lines (right axis): Average self-reported health separated by gender. Health is reported on an ordinal five-point scale, where 1 denotes "bad" health, and 5 denotes "very good" health. Solid and dotted lines (left axis): Average days of sick leave of workers separated by income quintile and gender.

Appendix 1.D Parameters of Structural Model

Table 1.11: Fixed Parameters

Parameter	Description	Value
	Economy	
J^T	Life Span	60
J^R	Retirement Age	45
R	Interest Rate	1.04
w	Wage Rate	1
	Preferences	
β	Time Discount Factor	0.9659
σ	Inter-temporal Elasticity of Substitution	2
ψ_0	Health Weight (level)	0.011
ψ_1	Health Weight (marginal)	0.19
	Policy	
ρ^U	Unemployment Benefit (ALGI)	60%
c^W	Consumption Floor Welfare (ALGII)	0.25$\triangleq$700€
a^W	Asset Limit for Welfare (ALGII)	0.44$\triangleq$1225€
ρ^S	Sick Leave Replacement Rate	100%
ρ^R	Retirement Benefit	50%
τ	Tax Rate	33%

Notes: Parameters used in the structural model and taken from the literature or chosen to match the German labor market.

Table 1.12: Labor Productivity of Income Quintile over Life Cycle - Γ

	K_1	K_2	K_3	K_4	K_5
J_{20}	0.224826037	0.328405423	0.456578007	0.587303446	0.732877732
J_{21}	0.250005566	0.409269009	0.560519145	0.685354277	0.850657669
J_{22}	0.311148441	0.509774966	0.622307323	0.724721362	0.884406497
J_{23}	0.350360594	0.552927485	0.656849717	0.754174695	0.924644522
J_{24}	0.425638928	0.598862981	0.700761760	0.809373763	0.979296359
J_{25}	0.453304887	0.633908213	0.749711757	0.848907970	1.043437128
J_{26}	0.486555339	0.666411652	0.784269193	0.909526063	1.105854970
J_{27}	0.505469995	0.689869703	0.819220306	0.957395861	1.196744849
J_{28}	0.526567728	0.729833055	0.848928302	1.009396469	1.256001953
J_{29}	0.534257324	0.749711757	0.892406923	1.043371438	1.333614431
J_{30}	0.552043113	0.775403025	0.923040554	1.094749559	1.398805958
J_{31}	0.588814584	0.811998956	0.964703602	1.141444947	1.461369255
J_{32}	0.581567748	0.813786468	0.967081217	1.143405056	1.507091996
J_{33}	0.590600670	0.827264526	0.997438473	1.182487624	1.598022908
J_{34}	0.610923651	0.844675221	1.0	1.195602273	1.638440613
J_{35}	0.605390496	0.850657669	1.019549932	1.236543287	1.696821211
J_{36}	0.587309058	0.841163678	1.035278366	1.258038975	1.730506925
J_{37}	0.595460368	0.850998814	1.034804554	1.262927700	1.803819125
J_{38}	0.588814584	0.851471889	1.046094062	1.291190800	1.845452180
J_{39}	0.598372424	0.854338956	1.050974415	1.293505670	1.844733642
J_{40}	0.596219571	0.856083687	1.053135456	1.324056682	1.900608466
J_{41}	0.596219571	0.850717011	1.053102151	1.303667321	1.891957676
J_{42}	0.598372424	0.854338956	1.051454666	1.293505670	1.899733341
J_{43}	0.587912961	0.852549696	1.051454666	1.299490326	1.910960195
J_{44}	0.597444858	0.842776110	1.030489463	1.275986504	1.913979709
J_{45}	0.601222195	0.844669241	1.028788249	1.275986504	1.913979709
J_{46}	0.595460368	0.845894251	1.033453225	1.261745563	1.913979709
J_{47}	0.588814584	0.842776110	1.027148308	1.265043937	1.910960195
J_{48}	0.590662450	0.850657669	1.035278366	1.275986504	1.921742678
J_{49}	0.598372424	0.862337174	1.043399683	1.262927700	1.934141825
J_{50}	0.591030413	0.851508322	1.035278366	1.257473805	1.974699741
J_{51}	0.592397659	0.842936838	1.035278366	1.275231717	1.986756456
J_{52}	0.575729560	0.854603003	1.034199823	1.284230276	1.958125531
J_{53}	0.574158805	0.840822441	1.031918443	1.264579325	1.971260512
J_{54}	0.571549022	0.837721413	1.018490066	1.275348100	1.955603380
J_{55}	0.565836095	0.838220617	1.033453225	1.278164842	1.987566444
J_{56}	0.560519145	0.834560214	1.020789203	1.263601985	2.009223412
J_{57}	0.570354465	0.818330276	0.993699316	1.259025884	1.919270208
J_{58}	0.566708552	0.827973127	0.995205761	1.232760613	1.976734648
J_{59}	0.562409792	0.835161909	0.991687686	1.213715589	1.943266795
J_{60}	0.573288049	0.824682942	0.992062779	1.222188079	2.009503467
J_{61}	0.567814006	0.857883160	1.031918443	1.254869129	2.060114657
J_{62}	0.575595834	0.849653188	1.050129374	1.262336586	2.084111243
J_{63}	0.609652318	0.891687833	1.095787160	1.333614431	2.155842844
J_{64}	0.611094040	0.807900162	0.993699316	1.217998479	2.135917358

Notes: Parameters directly estimated from the SOEP.

Table 1.13: Incident Probability of Sickness Conditional on Age & Health - Ω

J	S_m	H_1	H_2	H_3	H_4	H_5
	0	0.11111	0.16842	0.28280	0.35153	0.42068
	1-2	0	0.05263	0.04977	0.05870	0.08275
	3-4	0.22222	0.03157	0.07239	0.09556	0.10000
	5	0	0.12631	0.14932	0.11058	0.11379
20-29	6-9	0.11111	0.10526	0.10633	0.09965	0.07241
	10	0.11111	0.15789	0.10633	0.10784	0.06896
	11-15	0.11111	0.12631	0.10859	0.09419	0.08793
	16-20	0.22222	0.08421	0.06108	0.04641	0.02931
	21-30	0.11111	0.14736	0.06334	0.03549	0.02413
	0	0.38461	0.26499	0.32163	0.37864	0.49342
	1-2	0	0.07500	0.06140	0.10097	0.08717
	3-4	0	0.10000	0.08966	0.10368	0.10361
	5	0.07692	0.09000	0.12280	0.11378	0.09046
30-39	6-9	0.07692	0.07500	0.08187	0.07495	0.08881
	10	0	0.10499	0.11306	0.08854	0.05756
	11-15	0.07692	0.11999	0.10428	0.07184	0.04934
	16-20	0.23076	0.09000	0.05360	0.03495	0.01480
	21-30	0.15384	0.07999	0.05165	0.03262	0.01480
	0	0.44444	0.30662	0.41076	0.47238	0.57420
	1-2	0	0.06629	0.07349	0.08451	0.09245
	3-4	0.11111	0.10220	0.08727	0.09916	0.07785
	5	0.05555	0.07458	0.09908	0.08577	0.08272
40-49	6-9	0.05555	0.05524	0.06364	0.07531	0.02676
	10	0.05555	0.09944	0.08858	0.06569	0.05109
	11-15	0.16666	0.12154	0.09186	0.06108	0.05839
	16-20	0.05555	0.09944	0.04396	0.02928	0.02433
	21-30	0.05555	0.07458	0.04133	0.02677	0.01216
	0	0.30769	0.33756	0.43397	0.52779	0.65730
	1-2	0	0.04568	0.05489	0.06657	0.04494
	3-4	0.11538	0.06852	0.05934	0.07892	0.09550
	5	0.03846	0.09644	0.09272	0.09334	0.06741
50-65	6-9	0.07692	0.08883	0.08160	0.05422	0.04494
	10	0.19230	0.08883	0.08234	0.05628	0.01685
	11-15	0.15384	0.10659	0.09421	0.07275	0.02808
	16-20	0.07692	0.08375	0.04747	0.02059	0.02808
	21-30	0.03846	0.08375	0.05341	0.02951	0.01685

Notes: Parameters directly estimated from the SOEP.

Table 1.14: Initial Distribution of Health States Conditional on Income Quintile - H_0

	H_1	H_2	H_3	H_4	H_5
K_1	0.53%	5.60%	20.68%	56.28%	17.23%
K_2	0.33%	4.54%	19.50%	57.13%	18.83%
K_3	0.37%	4.88%	18.88%	57.69%	18.55%
K_4	0.45%	4.36%	17.97%	58.66%	19.01%
K_5	0.25%	3.37%	17.21%	57.61%	21.81%

Notes: Parameters estimated directly from the SOEP.

Table 1.15: Initial Distribution of Unemployment Conditional on Income Quintile - $\mathcal{I}_0$

	K_1	K_2	K_3	K_4	K_5
$\mathcal{I}_0 = 0$	8%	5%	2.5%	1.5%	0.4%
$\mathcal{I}_0 = 1$	92%	95%	97.5%	98.5%	99.6%

Notes: Parameters estimated directly from the SOEP.

Table 1.16: Calibrated Parameters - Sickness Aggravation

Param.	Statistics	Data	Model
	Probability of $l \in$\{0\} for K_1	0.4874	0.4885
	Probability of $l \in$\{1-2\} for K_1	0.0449	0.0315
ζ=0.1135	Probability of $l \in$\{3-4\} for K_1	0.0632	0.0613
	Probability of $l \in$\{5\} for K_1	0.0751	0.0845
	Probability of $l \in$\{6-9\} for K_1	0.0633	0.0721
	Probability of $l \in$\{10\} for K_1	0.0760	0.0824
κ=4.021	Probability of $l \in$\{11-15\} for K_1	0.0920	0.0883
	Probability of $l \in$\{16-20\} for K_1	0.0462	0.0436
	Probability of $l \in$\{21-30\} for K_1	0.0517	0.0475

Notes: Calibrated parameter values and match of model to data.

2
Social Comparison and Health – Does Envy Make You Sick?

2.1 Introduction

Health is highly related to economic performance. Wealthy people experience a longer and healthier life than poor people, cf. Deaton and Paxson (2001). A straightforward interpretation regards this income gradient in health as an individual -level phenomenon. The relationships of income or wealth with health operate through individuals capacities to purchase medical goods and services. Without disputing the effect of absolute economic resources on health, part of the literature argues that this gradient also possesses a social component. Individuals with greater wealth enjoy better health not only because of some process affecting the individual in isolation but also because of the individuals position in a social hierarchy, cf. Cutler, Lleras-Muney, and Vogl (2011).

This theory posits that a low relative economic position is associated with feelings of inferiority, depression, and isolation and triggers chronic stress and anxiousness. Recent medical studies, e.g., by Blackburn and Epel (2012), show that chronic stress places the body permanently in a "fight or flight" situation. It channels bodily energy to the physiological processes essential for producing rapid responses to an immediate threat and puts recreational processes on hold. Whereas this "fight or flight" behavior might be beneficial in the short run, it increases the general vulnerability of the body in the long run, cf. McEwen (1998), Seeman et al. (2001). Finally, stress and anxiety are often linked to behaviors that are detrimental to health such as smoking, alcohol drinking and a diet that leads to obesity. Elstad (1998) provides a good overview of other mechanisms of relative deprivation on health.

Contrary to absolute measures of economic performance, relative measures necessarily require a reference point. It is crucial to carefully select this reference point. The dominant approach used in the literature to determine this socioeconomic reference group is to artificially assign it according to sorting assumptions such as demographic characteristics (e.g., gender, race, education, age) and geographic proximity (e.g., country, state, zip code). Studies employing this approach generally indicate that there is an effect, cf. Eibner, Sturm, and Gresenz (2004), Kondo et al. (2008), Subramanyam et al. (2009), McLaughlin et al. (2012), Cuesta, Cottini, and Herrarte (2012). Others find that the effect is, if anything, weak and not significant, cf. Jones and Wildman (2008). For a good survey of the existing literature, see Adjaye-Gbewonyo and Kawachi (2012).

Constructing artificial reference groups based on demographic characteristics does not stem from deep sociological understanding. To a large extent, such an approach is taken because links between people are hard to identify empirically, cf. Soetevent (2006). Clark and Senik (2010) show that reference groups tend to be localized and are mostly limited to family, friends, neighbors and work colleagues. Their study also reveals that the personal reference point differs based on personal characteristics.[1] As reference groups are heterogeneous and likely not solely determined by age and education, a more natural approach is to directly ask households to evaluate their own performance within their social circle.

This chapter follows such an approach. Specifically, I use unique information from the Dutch National Bank Household Survey (DNBHS) that includes such questions and circumvents the need for artificially constructing social circles. Respondents have to answer by referring to a self-determined circle of acquaintances explained as *"people with whom [they] associate frequently, such as friends, neighbors, acquaintances, or maybe people at work"*. This open definition does not restrict the circle of acquaintances to a specific group but, in contrast to the traditional approach, accounts for the results of Clark and Senik (2010). In addition to the unique features of households' subjective relative economic performance, this central bank questionnaire contains rich information on absolute income and assets. This is needed to carefully distinguish between absolute and relative economic performance. As the survey provides a panel data structure over 20 years, it allows for fixed effects analysis, which is largely absent from the existing literature.

I find a robust and significant positive effect of relative performance on self-reported health and negative effects on detrimental health behavior such as smoking and obesity. These findings are based on subjective relative performance information, and I control for both demographic characteristics and absolute economic performance. People that consider themselves poorer than their circle of acquaintances are significantly less likely to report good health. I further

[1] Men rely less on comparisons with family members than do women. Employees in more professional occupations rely more on comparisons with their colleagues than do those in elementary occupations.

show that this effect exhibits asymmetries, i.e., being worse off than one's circle of acquaintances has a strong negative effect, whereas being better off exhibits only a mild positive effect. Furthermore, lower absolute income groups are more strongly affected by the relative performance effect than are high absolute income groups. I also show that exploiting the same data set but pursuing the traditional approach of using a reference group based on demographic characteristics yields weaker and insignificant results.

Related Literature There are a few papers that follow this direct approach to determining the reference group, but they are limited to either a fraction of the population or to non-developed countries. Pham-Kanter (2009) uses the National Social Life, Health, and Aging Project (NSHAP) data set that reports the income positions of people older than 55 within their self-defined social networks. She examines whether there is an association between relative position and health in the US. She finds significant results for lower rank deprivation with self-reported health and cardiovascular disease. Mangyo and Park (2011) analyze a nationally representative data set from China and find support for the relative deprivation hypothesis. They suggest that relatives and classmates are salient reference groups for urban residents and that neighbors are important for rural residents. Using a representative data set from a developed country (with universal health insurance) significantly advances this literature.

My analysis proceeds as follows. Section 2.2 presents the data set and highlights important features that this chapter exploits. Section 2.3 describes the results of a cross-sectional nonlinear model, whereas section 2.4 provides the results from a dynamic nonlinear panel model. In section 2.5, I compare these results with the traditional approach using an artificially constructed reference group, and in section 2.6, I discuss endogeneity problems. I conclude in section 2.7.

2.2 Data Set and Estimation Methods

The Dutch National Bank Household Survey (DNBHS) is an online household survey beginning in 1993. The DNBHS covers work, pensions, housing, mortgages, income, assets, debts, health, economic and psychological concepts and other variables. It thus allows the study of the health consequences of both absolute material resources and perceptions of relative economic status. The initial survey was administered to approximately 2,790 Dutch households over-sampled from the top 10% of the income distribution and weighted to be representative of the Dutch-speaking population. Since then, households have been re-interviewed annually, with new households added each year to counteract the non-negligible attrition and maintain the representativeness of the cross-sectional sample.[2] The household survey underwent a major overhaul in 2001, resulting in a sample of 1,861 households.

2.2.1 Measures of Absolute Economic Performance

The most concerning issue in an analysis of relative economic performance is to separate the effect of relative performance from the effect resulting from naturally related effects of absolute economic performance. Absolute economic performance has both a positive association with relative economic performance and a positive impact on health. Assuming that there were be no connection between relative performance and health, a simple OLS regression omitting measures of absolute economic performance would falsely report a positive coefficient for a relative measure. Therefore, neglecting absolute measures biases the effect of relative performance upwards.

To address this problem, I include both absolute household income and absolute household net wealth as control variables. The DNBHS includes detailed questions on the sources of income that respondents may have. These sources of income serve as the basis for computing total gross income at a personal level. The DNBHS also provides rich information on personal assets and liabilities. I construct a proxy for total wealth consisting of real and financial assets and liabilities (including mortgages). Both income and assets are adjusted for inflation using OECD price deflators. Personal income, assets and liabilities are predominantly reported by males, and few households report assets for different members. To increase the number of observations, I aggregate household incomes and net assets. I further adjust for household size using the Luxembourg Income Study approach of dividing assets or income by the square root of the number of household members, (cf. Buhmann et al. (1988)) and ascribe it to each member of the household. I allow for nonlinear effects of household income and net wealth (all of which have skewed distributions) by means of a log transformation and an inverse hyperbolic sine (IHS) transformation, respectively.[3] The advantage of this near-logarithmic IHS transformation is that it is defined for zero and negative values (see also Pence (2006)). The qualitative results of relative position are robust to alternative specifications of the aforementioned covariates (e.g., dummies

[2]The DNBHS is based on the CentERpanel, which is largely representative of the Dutch population, exceptions are under-representation of those with moderate education, single households and people living in a highly urbanized area, cf. Teppa and Vis (2012). A comparison with Netherlands Official Statistics is provided in appendix 2.A.

[3]The functional form of the hyperbolic inverse sine is $log[x + [x^2 + 1]^{1/2}]$, where x denotes assets.

denoting absolute economic performance quartiles).[4]

2.2.2 Measures of Relative Economic Performance

The key features of the DNBHS for this analysis are questions that the respondent has to answer by referring to a self-determined circle of acquaintances.[5] Previous to the questions, the DNBHS defines this circle of acquaintances as *"people with whom [the respondents] associate frequently, such as friends, neighbors, acquaintances, or maybe people at work"*. This phrasing of the question allows the composition of reference groups to differ across respondents. The DNBHS also asks the respondents to report various characteristics of their acquaintances. In addition to the (perceived) average household income of their circle of acquaintances, they provide information on the age category to which most of their acquaintances belong, average education, average household size, the most prevalent type of employment (e.g., employed, self-employed or no paid work) and the average hours of work per week, distinguished by gender.

Direct Answers to Relative Performance among Acquaintances. The central variables that are used in subsequent sections are derived from direct questions on the respondents' relative performance with respect to that of their acquaintances. To answer these questions, the respondents have to indicate the extent to which they agree or disagree with certain statements on a seven-point ordinal scale from "strongly disagree"to "strongly agree". The exact statements of the questionnaire are provided in Table 2.1. The statements cover how respondents perceive their relative financial situation, their relative asset holding, or their ability to spend more than their acquaintances. The answers to these statements are correlated but not identical. Taking the average of the seven-point scale answers of *Assets*, *Spending*, and *Financial*, I construct the additional measure *Combined*.[6]

The answers to these statements are not based on objectively calculated incomes and assets of the reference group, as would be the case in the traditional

[4]I experiment with different categories of assets, as there are many missing observations in the assets section (which leads to a decline in the total number of observations), but the results are insensitive to these variations.

[5]This unique feature is also used in other studies that investigate social effects. Georgarakos, Haliassos, and Pasini (2012) find considerable effects of relative economic performance on borrowing and on indebtedness, suggesting a link to financial distress.

[6]Appendix 2.B contains the coefficients of correlation between all three statements and *Combined*. Importantly, *Spending* is less related to the two other statements. This might be due to the reverse formulation of the statement.

Table 2.1: Statements of Relative Economic Performance

Assets	I think my household has more assets than others in my environment.
Spending	Other people in my environment have more money to spend than I do.
Financial	If I compare myself with my friends, I think in general I am financially better off.

Notes: Spending has an opposing formulation. In later results, I transform this variable such that the coefficients have consistent signs.

approach. Rather, they reflect the respondents' subjective perceptions of their environment. This is particularly valuable because the main mechanism connecting relative economic performance and health is assumed to operate via the perception of inferiority. Actual differences in absolute economic performance computed in the standard approach might differ from the perception and are a less accurate match for the proposed link.

A potential problem with subjective relative measures is that too many people consider themselves to be average. If the answers exhibit only little variation, regression results will be less significant. The last column of Table 2.2 refutes this suspicion, showing that the responses to *Financial* have a reasonable distribution. More than 31.7% of the respondents report performing below their acquaintances, while 27.3% report performing better. A second concern arises from the natural relationship between relative and absolute measures, e.g., given a constant reference point, increasing absolute income improves the relative income position. Perfect correlation between the two variables would prohibit distinguishing between the associated effects. Table 2.2 provides evidence that this is not an issue in this study. While 17.5% of the lowest income quintile feel themselves in a relatively better position than their milieu, 46.6% feel that they are worse off. On the opposite side of the income scale, 17.8% believe that others in their milieu do better, whereas only 44.1% of the respondents think that they are better off than other people in their environment.[7]

[7] Appendix 2.C contains a cross-tabulation of wealth and the perception of relative position that shows similar results. The results also hold for different classifications of absolute economic performance and for other measures of relative economic performance.

Table 2.2: Absolute Income vs. Relative Perception

Compared to Other I'm Financially Better Off	Absolute Income Quintiles 1st	2nd	3rd	4th	5th	Total
Totally Disagree	11.63	7.17	4.64	3.71	2.45	5.92
2	15.26	11.64	8.50	7.50	4.59	9.50
3	19.80	18.91	18.34	13.91	10.78	16.35
4	35.85	42.49	43.81	43.80	38.01	40.79
5	10.44	12.27	16.45	18.48	23.55	16.24
6	4.89	5.96	6.79	10.15	16.38	8.83
Totally Agree	2.12	1.57	1.46	2.45	4.24	2.37
Total	100.0	100.0	100.0	100.0	100.0	100.0

Notes: Cross tabulation of absolute adjusted household income quintiles and perception of relative performance among circle of acquaintances. Entries are in percentages.

Indirect Answer to Relative Performance Although I argue that it is more plausible to use subjective perceptions, I compute two additional measures of relative economic performance that rely on more quantitative measures. To do so, I use responses to the following DNBHS question: "If you think of your circle of acquaintances, how much do you think is the average total net income per year of those households?"

The first indirect measure of relative economic performance is a binary variable denoting whether a household has a higher income than its acquaintances. The second indirect measure is the difference between the natural logarithms of household income and the reported income of its acquaintances.[8]

$$Ind\_IncDist_i = \begin{cases} 1 \text{ if } Inc_i > IncAcq_i \\ 0 \text{ if } Inc_i < IncAcq_i \end{cases} \tag{2.1}$$

$$Log\_IncDist_i = log(Inc_i) - log(IncAcq_i) \tag{2.2}$$

As the DNBHS provides no information on the asset holdings of the circle of acquaintances, these measures are restricted to household income only.

[8]The answer to the question concerning the net income of the circle of acquaintances is reported in brackets. To avoid the difficulty of comparing income brackets to continuous household income, I use a question that directly asks the respondents to classify own income in the same brackets. I use the midpoints of these brackets as household income and adjust it using reported average household size.

2.2.3 Measures of Health and Health Behavior

In the DNBHS, the respondents report the standard survey measure of self- reported health (SRH) on a five-point ordinal scale ("excellent", "good", "fair", "not so good", "poor"). In the benchmark case, I collapse this multidimensional answer to a binary variable, either being in good health ("excellent ", "good") or in poor health ("fair", "not so good", "poor"). In addition to capturing an individual's subjective well-being, poor self-reported health has been shown to be a robust predictor of mortality and correlates highly with other objective health indicators, especially in the context of the working age population and developed countries, cf. Miilunpalo et al. (1997), Idler and Benyamini (1997).

In addition to self-reported health, this chapter investigates the relationship between relative economic performance and various behaviors that are related to health. This proxies for the mechanism connecting relative position with health explained above. People with low relative standing may compensate for the resulting unhappiness with short-term pleasant but unhealthy activities. Smoking is often considered to reduce acute stress symptoms, heavy and permanent alcohol consumption may be employed to drown one's frustrations, and eating for comfort mitigates bad mood, cf. Wilkinson (2000). Given these considerations, relative economic performance has negative long-term effects on health. The DNBHS contains information on three health-related behaviors. Smoking is reported in various intensities. I assume smoking to be harmful when the respondent reports smoking every day. Drinking alcohol is reported only in terms of whether one drinks more than four alcoholic drinks alcohol per day. The survey also includes information on a respondent's height and weight, which allows me to compute the body mass index (BMI).[9] I classify someone as obese when the BMI exceeds 30, following the criteria used by the World Health Organization.

2.2.4 Data Selection

Not all of the observations in the DNBHS are applicable to the following analysis. First, as I am interested in a respondent's economic performance, I focus only on adults and drop all observations of respondents younger than 18. Second, all observations that have at least one non-response among health or control variables are excluded. Third, as with many surveys that include questions on financial status, the greatest constraints on sample size are the response rates for the income and asset questions. The response rate in the DNBHS for the questions on absolute income, absolute assets and questions regarding the circle of acquaintances

[9] The BMI is computed as BMI=weight[kg]/(height[m])^2.

is 61%, leading to a final sample size of 19,811 observations.[10]

If the non-responding households differ from the responding households in a relevant characteristic, this could bias the results. Table 2.3 presents the summary statistics for the complete DNBHS, the sample excluding non-responses to health and control variables and the final sample excluding also non-respondents to absolute economic performance and questions concerning relative economic performance. The sample that answered the control and health questions and the sample that also answered the income, asset and relative performance questions are quite similar in terms of demographic characteristics. The final sample includes more males and is slightly older. This is also reflected in the fact that the final sample includes fewer students and more retirees. Fortunately, there are only small differences between the two samples with respect to economic characteristics. Respondents who do not answer one of the economic questions are on average only a somewhat less wealthy and have a lower income. The average evaluation of one's relative position does not differ between the two samples. The summary statistics for the health variables also signal that there is no systematic bias from non-responders. In both samples, approximately 80% report good health and 20% poor health. Health behavior is not affected.

2.2.5 Model Estimation

Estimating the association between relative economic performance and health requires addressing the problem of omitted variables. I have already discussed this for the most obvious case of absolute economic performance, but there are other variables that might bias my results. To address biases from omitted variables, two general strategies can be applied: (a) introduce the omitted measure explicitly into the analysis and estimate the adjusted degree of association between relative position and health, and (b) estimate "fixed effects"models. Fixed effect models difference out effects of persistent characteristics (both measurable and not) of households, cf. Daly et al. (1998). In this section, I follow both strategies and in the first part incorporate several control variables in a cross-sectional estimation of a reduced-form nonlinear model. In second part, I exploit the DNBHS panel structure and use a nonlinear dynamic model with fixed effects. For both approaches, I use data from 15 waves of the DNBHS, i.e., 1995-2007/2009/2011.

[10] Unfortunately, many of the questions central to relative economic performance, which are key variables for this chapter, are not included in the 1993, 1994, 2008, 2010, and 2012 waves.

Table 2.3: Descriptive Statistics of Complete DNBHS and Final Sample

Sample	Complete Sample	Responded to Health & Control	Final Sample
Variable		Mean or Proportion	
Age	45.9	48.0	49.6
% Male	50.5	54.0	55.7
Household Size	2.8	2.7	2.6
% Urban	60.7	60.9	60.4
% Less than High School	15.3	14.5	11.6
% High School	39.1	37.0	37.0
% College	43.8	46.7	49.9
% Employed	54.7	53.6	56.8
% Unemployed	1.8	1.8	1.7
% Retired	13.4	15.7	17.5
% Students	7.0	4.3	1.5
% Others	23.1	24.6	22.6
Net Assets	199,350	200,683	202,602
	(686,468)	(723,604)	(293,218)
Gross Income	30,287	30,300	33,226
	(35,849)	(36,441)	(39,734)
% Fin. Better Off than Others	26.3	26.5	27.4
% Fin. Worse Off than Others	33.1	32.9	31.8
Self-Reported Health			
% Excellent		17.4	16.6
% Good		62.2	63.7
% Fair		16.4	15.8
% Not so Good		3.4	3.3
% Poor		0.6	0.6
% Smoking (Every Day)		22.0	20.7
% Alcohol (Drinks/Day>4)		7.2	7.5
Body Mass Index (BMI)		25.2	25.3
		(4.3)	(4.2)
% Obese (BMI>30)		10.0	10.0
Observations	57,656	32,486	19,766

Notes: Summary statistics of the pooled 15 waves of DNBHS 1995-2007/2009/2011 and only respondents with age$\geq$18. The base year for the deflation of assets and income is 2010. Standard deviation in parentheses

Cross-Sectional Model In the first approach, I consider the observations of all 15 waves as one huge cross-sectional data set. The cross-sectional results are estimated by a nonlinear probit model with following regression equation

$$Pr[Health_i = H] = \Phi\{\alpha + \beta \; RP_i + X_i\theta + \epsilon_i\} \tag{2.3}$$

where $Health_i$ is a binary health variable of respondent i, i.e., being in good or poor health, smoking or not smoking, and RP_i is a relative economic performance measure (either direct or indirect). X_i represents a set of explanatory variables that may affect health, including the natural logarithm of adjusted household income, the inverse hyperbolic sine of adjusted net household assets and other control variables. These control variables are age, age^2, gender, educational attainment, degree of urbanity, labor market status dummies and general variables such as year dummies. The β and the vector θ are parameters to be estimated. The error term ϵ_i is individual specific, is assumed to be uncorrelated with X_i and across individuals and is assumed to be drawn from a distribution with mean zero and constant variance. I cluster standard errors at the individual level to account for correlations of individual health over time.

In my models, I do not control for possible reverse causality running from health to absolute and relative economic performance, and hence I may overestimate the impact of income. I interpret the models in reduced form.

Panel Models In the second approach, I exploit the panel structure of the data set. I estimate two dynamic logit models, one with fixed effects and one without a fixed effect.

$$Pr[Health_{i,t} = H] = \Phi\{\alpha + \beta \; RP_i + X_{i,t}\theta + \epsilon_{i,t}\} \tag{2.4}$$

$$Pr[Health_{i,t} = H] = \Phi\{\alpha + \beta \; RP_i + X_{i,t}\theta + u_i + \epsilon_{i,t}\} \tag{2.5}$$

where $Health_{i,t}$ is health of individual i at time t. $X_{i,t}$ consists of the same control variables as above except for those that do not vary over time, as these are incorporated in the fixed effect. The u_is represent the individual-specific and time-invariant fixed effect component.

I additionally estimate a panel ordered logit and a panel OLS with a fixed effects using the five-point ordinal scale of health condition as the dependent variable.

2.3 Cross-Sectional Estimation

2.3.1 Self-Reported Health

First, I run separate probit regressions of equation (2.3) for all direct variables *Assets*, *Spending*, *Financial* and the constructed measure *Combined.* As my interest lies in the role of absolute and relative economic performance, I focus solely on these results.[11] The main result of the cross-sectional estimation are shown in Table 2.4. The first two rows of Table 2.4 show the standard result that absolute economic performance (household income and household net wealth) is positively associated with self-reported health. All regression coefficients are positive and highly significant. The probit estimates in the last four rows show that all relative performance measures are also positively associated with self-reported health. The regression coefficients of *Assets*, *Spending*, *Financial* and *Combined* are all positive and highly significant, with the strongest association being observed for the combined measure. This is clear evidence that relative economic performance has a significant affect on reporting good health.[12]

Table 2.4: Probit Regression Coefficients for Direct Measures

Self-Reported Health	I	II	III	IV
Log Income	0.1120***	0.1077***	0.1096***	0.0980***
IHS Assets	0.0057**	0.0057**	0.0053**	0.0036
Assets	0.0537***			
Spending		0.0666***		
Financial			0.0616***	
Combined				0.1117***

Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
All regressions include age, age^2, gender, degree of urbanization of place of residence, education dummies, dummies for employment status and year dummies. Clustered robust standard errors.

To facilitate interpretation of the quantitative results of these nonlinear estimations, Table 2.5 presents the marginal effects of relative economic performance

[11] The estimations also yield standard results, i.e., the probability of good health is decreasing in age, and education has a protective effect on health. They are omitted for clarity, but full results are available from the author upon request.

[12] In Appendix 2.D, I provide results of an ordered probit regression. Therefore, I use the original five-point health variable instead of the collapsed binary variable as the dependent variable. This estimation produces similar results.

measures for the median respondent. The marginal effects indicate how the odds of reporting good health are changed by varying an independent variable by one degree.[13] [14] A strong perception that one is better off than one's circle of acquaintances is highly related to the probability of reporting good or excellent health. Increasing the subjective perception of own relative performance by one degree would increase the probability of reporting good health by up to 3.07%.[15]

Table 2.5: Marginal Effects for Median Respondent

Self-Reported Health	Assets	Spending	Financial	Combined
Marginal Effect	0.0145***	0.0188***	0.0169***	0.0307***

Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
Marginal effects at median of independent variables included in probit estimation.

Non-Linearity of Relative Performance Effect The effect of relative performance on self-reported health seems to be highly nonlinear. I estimate Equation (2.3) using dummies for either being below or above the relevant peer group instead of the continuous measure. The resulting estimates exhibit strong asymmetries in the impact of relative economic performance. The regression coefficient for being in a low position is -0.1891, whereas the coefficient of being in a better position is 0.0634. The absolute magnitude of the negative effect of being in a relatively poor position is higher than the absolute magnitude of the positive effect of being in a relatively high position. In terms of marginal effects, this means that feeling deprived compared with one's circle of acquaintances reduces one's probability of being in good health by 5.2%. Being in an economic situation that is advantageous compared with one's acquaintances increases the likelihood of being in good health by only 1.7%. This result also holds for a finer distinction of being in the lower or upper position, but due to few observations at the boundary, the estimates are statistically insignificant.[16]

[13]The median respondent in the sample used here is a 49-year-old male who is employed, has a high school degree, an adjusted income of 30,662€, and net assets of 134,951€. He neither agrees nor disagrees with the relative economic performance questions.

[14]I also computed the average marginal effect instead of the marginal effect of the median worker, but the two sets of results are similar. For more a detailed discussion on which is the appropriate measure, see Long and Freese (2006).

[15]These results seem sizable, but they are not inconsistent with other studies on socioeconomic status and mortality. For instance, Marmot et al. (1991), find that British civil servants from the lowest socioeconomic class were three times more likely to die than their high-status counterparts.

[16]The results of this estimation are contained in appendix 2.E.

Gender, Age, Absolute Economic Performance Differences In the previous estimations, I control for various demographic characteristics using dummy variables. This approach does not take into account that the relationship between relative economic performance and health might differ across demographic groups. To determine whether the results are stronger for specific groups, I include in the probit estimation interaction terms of the relative economic performance measure and the respective sub-sample group. Table 2.6 presents the estimates of relative economic performance and the interaction terms.[17]

Column 2 reports the result of incorporating interaction terms for different age groups (young working (25-45), old working (45-65) and retired (>65)). For all three age groups, the relative economic performance effect remains positive and significant. The interaction terms are nearly zero and insignificant. There are no notable differences in the relative economic performance effect across different age groups.

The third column reports the results for differences in the effect between men and women. For each gender, relative economic performance has highly significant effects. It seems that the effect is stronger for the males than for females. This difference, however, is not statistically significant.

I further examine whether the effect is the same for different absolute economic performance groups. I divide the full sample into three different income groups (poor, medium, rich) and include interaction terms. For each income group, the estimate of relative economic performance remains positive and significant. The magnitude of this effect, however, differs remarkably across the groups. The health of the top income group is much more weakly affected by relative performance measures than is the bottom or medium income group. The coefficient of the top income group is reduced to nearly one-third of the estimate of the medium and bottom income groups. The difference between the bottom and medium income groups is not statistically significant, but the findings suggest that the effect of relative economic performance is strongest for the bottom income group.[18]

Indirect Measure of Relative Income Analogous to the previous section, I run probit regressions of Equation (2.3) for both indirect relative economic performance measures *Ind_IncDist* and *Log_IncDist*. The estimates in Table 2.7 generate qualitatively identical results to those obtained for the direct relative performance measures. Both *Ind_IncDist* and *Log_IncDist* are statistically not significant. Par-

[17] I do not report the marginal effects for the sub-sample regressions. Each sub-sample has a different median respondent, and a comparison across groups cannot be reasonably made.

[18] This pattern in absolute economic performance is not significant when groups are constructed by placing respondents into low, medium and high net wealth categories.

Table 2.6: Probit Regression Including Interaction Terms

Self-Reported Health	Age	Gender	Abs. Income	Abs. Wealth
Rel. Income	0.1206***	0.0963***	0.1225***	0.1111***
Rel. Income * Young	0.0082			
Rel. Income * Retired	-0.0358			
Rel. Income * Male		0.0263		
Rel. Income * Poor			0.0294	
Rel. Income * Rich			-0.0897***	
Rel. Income * Poor				0.04934
Rel. Income * Rich				-0.07401**

Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
Probit regression coefficients. Older working, female and medium income group are the omitted reference groups in the respective regressions. Clustered robust standard errors.

tially, this is because the sample size in this regression is much smaller (8,009 observations) than in Table 2.4. The income of the circle of acquaintances is reported in income bands, and to match this information to the income of the respondents, I had to use a different variable for income that has more non-respondents.

2.3.2 Health-Related Behavior

The results for behaviors that are considered detrimental to health are mixed. The marginal effects of this estimation for the median respondent are displayed in Table 2.8.

The estimates in the second column display the estimates for absolute and relative economic performance on the probability of daily smoking. They suggest that smoking and one's economic situation are linked. For both absolute economic measures, the coefficient is negative and highly significant. One can see that poor people are more likely to smoke than rich people. This is result is in line with the rest of the literature, cf. Auld (2005). The estimate for relative economic performance is also negative but not significant.

Unlike smoking, the entries in the third column show inconclusive results regarding the relationship between absolute economic performance and alcohol. This relationship is negative and highly significant for net assets. However, indi-

Table 2.7: Probit Regression Coefficient for Indirect Relative Performance Measures

Self-Reported Health	I	II
Log Income	0.1120***	0.1134***
IHS Assets	0.0055	0.0057
Ind_IncDist	0.0786**	
Log_IncDist		0.0901***

Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
All regressions include age, age^2, gender, degree of urbanization of place of residence, education dummies, dummies for employment status and year dummies. Clustered robust standard errors.

viduals with high income seem more prone to consume alcohol. Both results can also be found in the literature, cf. Ettner (1996). The marginal effect for relative economic performance is is positive and significant. Alcohol is more common among people that are financially better off than among their acquaintances.

The clearest evidence for a relationship between economic variables, absolute and relative, and health behavior is found for the probability of being obese. The estimates in the fourth column signal that income and net assets reduce the prevalence of obesity. A good relative economic performance measure is also negatively related to the probability of reporting obesity. An improved perception of one's relative position by one step is associated with a greater than one percent decrease in the probability of being obese.

Table 2.8: Marginal Effects for Health-Related Behavior

Health Behavior	Smoking	Alcohol	Obesity
Log Income	-.0138**	0.0046	-0.0136**
IHS Assets	-.0056***	-0.0016***	-0.0045***
Combined	-.0072	0.0076**	-0.0092**

Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
Marginal effects for the median respondent from probit estimation. All regressions include age, age^2, gender, degree of urbanization of place of residence, education dummies, dummies for employment status and year dummies. Clustered standard errors.

2.4 Dynamic Logit Model

By exploiting the advantages of the data set's panel structure, I can estimate a dynamic logit model with fixed effects. These fixed effects should account for possible time-invariant characteristics of the household, as a respondent's self-esteem could affect both his self-reported health and his relative social position.

In Table 2.9, I report the baseline results estimated for the dynamic logistic model with and without fixed effects, the panel ordered logit model panel and the panel OLS with fixed effect estimation for the relative performance measure *Combined.* In all four regressions, the coefficients for relative economic performance have the expected positive sign. In all but the logit with fixed effects, the effect is statistically significant. These results support the evidence obtained in the cross-sectional estimations that the relative economic comparisons controlling for unobserved factors are associated with health. The coefficients of absolute economic performance are inconclusive. Whereas the coefficient on absolute income has a positive sign and is significant in two of four models, the coefficient on wealth is insignificant and varies in sign.

Table 2.9: Panel Results for Self-Reported Health

Self-Reported Health	Logit - RE	Logit - FE	Ordered Logit	OLS - FE
Log Income	0.1212***	0.0244	0.1039***	0.0033
IHS Assets	0.0098	-0.0092	0.0050	-0.0015
Combined	0.2510***	0.0111	0.2140***	0.0119**

Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
All regressions include age, dummies for employment status and year dummies. Clustered robust standard errors.

2.5 Comparison with the Traditional Approach

In the following, I show why the results of the previous sections improve on prior studies employing artificially constructed peer groups. First, I illustrate that the self-determined peer group significantly deviates from the demographic and socioeconomic characteristics of the respondents. Second, I show that running the same regression using the economic performance relative to the artificial reference group yields weaker or even insignificant results.

2.5.1 Differences of Respondents from the Circle of Acquaintances

Table 2.10 shows the demographic and socioeconomic characteristics of respondents and the characteristics of their self-determined circle of acquaintances. Most people with no high school degree (86%) have a peer group that possess at least a high school diploma. Just over half of the people with a high school degree report that individuals in their circle of acquaintances have the same educational level. The only group for which the characteristics of the respondents and their peer group mostly coincide are college graduates.

Dependent employees are the largest group in the sample. In this group, nearly all (92%) report having a circle of acquaintances who also work as dependent employees. A more surprising observation is that in the other employment categories, only a fraction of the respondents state that their peer group has the same employment status. Only less than one-third of self-employed people report that their circle of acquaintances is mostly self-employed. Even more astonishing is that only 9% of unemployed people report that most people in their environment people are unemployed. The response options to the question regarding the employment status of the circle of acquaintances do not include retirement, but it is nevertheless remarkable that nearly two-thirds of retirees indicate that their peer group is predominantly working. Thus, similar to education, there is a mismatch between respondents and their peer group with respect to employment status.

The third important category on which most traditional approaches base their reference group is age. The last part of Table 2.10 shows the age of the respondents and the answers to the question of into what age bin most of their acquaintances fall. A clear pattern is that younger people are more likely to have a peer group that is older and less likely to have a group that is younger. At older ages, this pattern is reversed. On average, only 44% of the respondents indicate that their peer group falls into the same age bin as they do. More people report that their peer group is younger than people that state that their peer group is older. This difference may hint at a misperception of the true circle of acquaintances. For the research question addressed in this chapter, however, only the perception of the respondent is important, as the primary mechanism for the effect of relative deprivation runs through subjective feelings and need not reflect the objective situation.

To summarize, constructing a peer group based on the demographic and socioeconomic characteristics of respondents seems not to acknowledge the fact that people have diverse peer groups.[19]

[19] Further information on the peer group reveals that the size of the household of the respondent

Table 2.10: Comparison of Respondent with the Circle of Acquaintances

Respondents	Circle of Acquaintances		
		Education	
	No High School (%)	High School (%)	College (%)
No high School	14.13	44.49	41.39
High school	12.89	51.62	35.49
College	1.39	21.28	77.34
		Employment Status	
	Employed (%)	Self-employed (%)	No Paid Work (%)
Employed	92.44	5.73	1.82
Self-employed	66.28	29.70	4.03
Unemployed	80.56	10.80	8.64
Retired	50.97	12.35	36.68
		Age	
	Younger (%)	Same Age Group (%)	Older (%)
16-20	0.00	70.64	29.36
21-25	3.29	65.13	31.58
26-30	6.61	58.02	35.36
31-35	14.13	66.20	19.67
36-40	27.89	55.14	16.97
41-45	37.77	50.09	12.14
46-50	47.86	42.63	9.51
51-55	53.43	38.59	7.98
56-60	58.87	33.89	7.24
61-65	61.52	33.23	5.25
66-71	70.61	26.69	2.70
older than 71	68.87	31.13	0.00
Total	44.02	44.40	11.57

Notes: Demographic characteristics of respondents and the reported average characteristics of the circle of acquaintances.

2.5.2 Results of the Traditional Approach

As shown, the circle of acquaintances and an artificial reference group based on respondents' characteristics do not appear to coincide. Nevertheless, it is interesting to determine whether the approaches produce different results concerning the effect of one's relative income position on health. For this purpose, I generate the same two indirect measures as in Section 2.3 ((2.1) and (2.2)) but replace average income of acquaintances with the average income of the constructed reference group that is based on age, educational level, and sex. Additionally, I compute both indirect measures using net asset holdings of the respondents and their artificial reference group.

For all four measures, I run separate probit regressions of equation (2.3). The results are shown in Table 2.11. For both measures of absolute economic performance, the results exhibit positive and (mostly) highly significant estimates for all regressions. The results for relative economic performance reflect inconclusive and weak results. The indicator variables (2.1) display the same sign as in the previous section, i.e., having a higher income or wealth than one's reference group seems to increase average health. However, neither coefficient is statistically significant. Columns III and IV show the results for measure (2.2). Here, the extent of the difference between the absolute economic performance of the respondents and their reference group is also taken into account. Only the coefficient for relative income is statistically significant.[20] The relative wealth coefficient is insignificant and even exhibits a reversed sign. Using the traditional approach, I do not find a relationship between relative performance measures and self-reported health, which is contrary to my previous findings.

2.6 Discussion

The results shown in the previous section suggest that there is a strong relationship between relative economic performance and self-reported health status and various health-related behaviors. Some aspects of the setting, however, limit a causal interpretation of this link.

and the size of the circle of acquaintances is only the same in 50% of the observations.

[20] Note that individual absolute economic performance is used to construct indirect measures (2.2). This results in a high correlation between the two variables and might limit a separate interpretation of the results of absolute and relative measures of economic performance.

Table 2.11: Probit Regression Coefficients for Artificial Reference Group

	Ind_Dist_Trad		Log_Dist_Trad	
	I	II	III	IV
Log Income	0.0436***	0.0251***	0.0158	0.0208***
IHS Assets	0.0015***	0.0018***	0.0016***	0.0167**
Relative Income	0.0119		0.0401*	
Relative Assets		0.0107		-0.0063

Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
All regressions include age, age^2, gender, degree of urbanization of place of residence, education dummies, dummies for employment status and year dummies. Robust standard errors clustered at the group level.

Endogeneity Problem of Circle of Acquaintances A potential challenge to the presented results is presented the fact that the self-determined circle of acquaintances is affected by the individual's health and might change over time. People aware of the negative impact of wealthier friends on their health might select their circle of acquaintances, or change their milieu, to feel more comfortable. According to this logic, everyone would select a circle of acquaintances such that his own performance would be equal to or better than theirs. This is clearly ruled out by the strong dispersion of perceptions shown in Table 2.2. Although some of one's acquaintances are chosen endogenously, this is likely not the case for others such as sisters or work colleagues.

Relative Performance and Causality This analysis carefully attempted to ensure that the correlation between health and relative economic performance is not due to omitted variables. What is still an open question is in which direction the mechanism operates, i.e., whether health is affected by relative position or relative performance is affected by health (without directly being affected by absolute performance). In general, it is not possible to interpret the above estimates as unambiguously causal. Pham-Kanter (2009) notes that these non-causal estimates might be helpful because they suggest an upper bound for the causal effects operating through relative position. That is, one could hypothetically assume that there was no reverse causality or omitted variables and interpret the estimate as giving us the largest possible causal effect of relative position on health. [21]

[21] Although reverse causality might be at work in the overall effect, it is less clear how this reverse causality could explain the asymmetries found in the results.

2.7 Conclusion

Using subjective relative performance measures and controlling for demographic characteristics and absolute economic performance, this chapter reports a significant positive effect of relative position on self-reported health and a negative effect on detrimental health behaviors such as smoking and obesity. It further indicates that the effect on self-reported health exhibits asymmetries, i.e., lower income groups are more strongly affected than are high income groups and that being worse off than one's circle of acquaintances has a strong effect, whereas being better off shows only mild positive effects. In this chapter, I also show that compared with the traditional approach of using a reference group constructed based on demographic characteristics, the approach based on self-reported groups generates stronger results.

This chapter suggests that social comparisons based on economic performance are both an independent risk factor, in addition to absolute economic performance, as well as a conciliating mechanism to explain the association between income inequality and health. This result might imply that a reduction of inequality within peer groups could improve average health in the population.

Appendix 2.A Comparison of DNBHS to the Dutch Census

Table 2.12 compares the DNBHS to the official census for the Netherlands taken from the Centraal Bureau voor de Statistiek (CBS). It shows that the DNBHS matches this census in important demographic characteristics such as age, gender and education and in economic data such as unemployment.

Table 2.12: Comparison of DNBHS to the Dutch Census

Variable	CBS	DNBHS
Average Age	39	38
% Male	49	51
% Urban	45	63
% Less high school	9	8
% Master, Doctoral	9	10
% Unemployed	5	3

Notes: Summary statistics of the pooled 15 waves of DNBHS 1995-2007/2009/2011 compared to official CBS data.

Appendix 2.B Correlation Direct Measures

Table 2.13 shows the correlation among the various direct relative performance measures. Most of the correlations are strong but not perfect. In particular, *Spending* has a weaker connection to the other variables, potentially resulting from of its reverse formulation.

Table 2.13: Correlation among Statements

	Assets	Spending	Financial	Combined
Assets	1			
Spending	0.19	1		
Financial	0.64	0.25	1	
Combined	0.81	0.64	0.83	1

Notes: Correlation among the several measures of relative position.

Appendix 2.C Relative vs. Absolute Performance

Table 2.14 shows the distribution of the responses to *Financial* for quintiles of absolute wealth. As for income in all columns, there is a range of perceptions concerning whether the households are better or worse off than their circles of acquaintances.

Table 2.14: Absolute Wealth vs. Relative Perception

Compared to others I'm	Absolute Net Wealth Quintiles					
financially better off	1st	2nd	3rd	4th	5th	Total
Totally disagree	13.07	5.43	4.01	4.12	2.98	5.92
2	17.69	10.78	7.75	6.54	4.75	9.50
3	20.64	18.25	16.25	14.95	11.66	16.35
4	32.27	42.10	43.83	43.12	42.63	40.79
5	10.37	14.66	16.50	18.96	20.70	16.24
6	4.44	6.71	9.34	9.85	13.83	8.83
Totally agree	1.51	2.07	2.32	2.47	3.46	2.37
Total	100.0	100.0	100.0	100.0	100.0	100.0

Notes: Cross tabulation of absolute adjusted household net wealth quintiles and perception of relative position among acquaintances. Entries are in percentages.

Table 2.15 shows the distribution of the responses to *Financial* for different income bands.

Appendix 2.D Ordered Logit

In Table 2.16, I report the margins estimated from an ordered logit for all relative performance measures on health condition, which is an ordinal measure ranging from 1 *excellent* to 5 *poor*. The results are in line with the results of the binary probit model and indicate that an increased relative position is associated with a better self-assessed health condition including for a finer distinction of health. All results are statistically significant.

Table 2.15: Absolute Income vs Relative Position - Income bands

Comp. to others I'm fin. better off	Absolute Income Bands <7	 7-14	 14-20	 20-30	 30-40	 40-50	 >50	 Total
Totally disagree	8.49	15.09	8.95	5.58	4.08	2.60	2.48	5.92
2	11.37	16.77	14.62	10.09	7.39	6.70	3.84	9.50
3	20.85	18.44	19.80	18.81	15.40	12.07	10.42	16.35
4	37.98	34.79	38.84	43.49	44.02	42.70	36.63	40.79
5	11.98	8.44	11.58	14.31	17.79	20.59	24.68	16.24
6	6.29	4.61	4.76	6.31	9.05	12.44	17.34	8.83
Totally agree	3.03	1.86	1.40	2.05	2.26	2.89	4.61	2.37
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Notes: Cross-tabulation of absolute adjusted household income bands and perception of relative position among acquaintances. Income bands measured in 1000 €. Entries in percentages.

Table 2.16: Self-Reported Health Condition - Ordered Probit

Self-reported Health Condition	(1)	(2)	(3)	(4)
Log Income	0.0007**	0.0009***	0.0009***	0.0010***
IHS assets	0.0001**	0.0001***	0.0001***	0.0001**
Combined	0.0018***			
Assets		0.0009***		
Spending			0.0012***	
Financial				0.0011***

Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
All regressions include age, age^2, gender, education dummies, degree of urbanization of place of residence, and dummies for employment status. All results are from ordered probit regressions. Clustered robust standard errors.

Appendix 2.E Asymmetric Effect

Table 2.17 reveals the asymmetry of the general effect for a finer distinction of the relative economic performance measure. In absolute terms, the marginal effects of being worse off than one's peer group are much stronger than the effect when one is better off than one's friends at the extreme, and this insignificance might be due to the limited number of observations for these cases.

Table 2.17: Self-Reported Health - Asymmetric Effect

Compared to other better off	Totally Agree	(2)	(3)	(5)	(6)	Totally disagree
Marginal Effect	-0.054***	-0.080***	-0.062***	0.025***	0.029**	0.001

Notes: *** Significant at α=0.01, ** Significant at α=0.05, * Significant at α=0.1.
All regressions include age, age^2, gender, education dummies, degree of urbanization of place of residence, and dummies for employment status. Clustered robust standard errors.

3

Endogenous Grids in Higher Dimensions – Delaunay Interpolation and Hybrid Methods

3.1 Introduction

Dynamic models of equilibrium in discrete time are workhorse models in Economics. However, most of these models do not have an analytic closed form solution and equilibria have to be approximated numerically. To this purpose, numerous procedures have been developed in the literature, cf. Judd (1998), Miranda and Fackler (2004). If the problem is differentiable, a popular approach is to use first-order methods, i.e., to iterate on first-order conditions. An important contribution to this literature is Carroll (2006) who introduces the method of endogenous gridpoints (ENDGM). In comparison to the method of exogenous gridpoints (EXOGM), ENDGM greatly enhances computational speed because part of the problem can be computed in closed form.

This chapter investigates extensions of Carroll's ENDGM to dynamic problems with more than one continuous endogenous state variable. The key insight of ENDGM is that the choice of the variable on which to define the grid is subject to the user in any dynamic problem. A smart choice may then lead to closed form solutions of first-order conditions, greatly enhancing speed of computations. We here introduce this general idea by first considering the standard implementation of ENDGM in a one-dimensional problem, i.e., in a setup with one endogenous state variable. To this purpose, we introduce some minimal notation, otherwise keeping the presentation as informal as possible. A more in depth treatment is contained in Section 3.3.

We base the exposition on a consumption-savings problem, as in our application. In a standard exogenous grid method (EXOGM), one solves in each time period (or iteration) for each grid point on grid $\mathcal{G}^a$ of today's state variable a (=assets) some non-linear problem. The solution is given by the associated control variable c (=consumption) and next period's endogenous state variable assets, a'. Solution of this equation also requires interpolation on some function(s) f on a' because generally $a' \notin \mathcal{G}^a$—e.g., f could be the derivative of the value function or, depending on the nature of the problem, the value function itself. Given a, c, a', the additional control savings, s, can be computed. To summarize, the mapping in EXOGM is $a \to (c, a') \to s$ where the mapping $a \to (c, a')$ requires, among other numerical operations, solving a non-linear equation and interpolation. Also observe that, for some regular grid $\mathcal{G}^a$—think, for simplicity, of equally spaced grid points—the "endogenous" grid of a' is generally irregular because the spacing between grid points is a result of the entire mathematical operation.

The trick of ENDGM in such a setup is to revert the mapping, i.e., $s \to (a', c) \to a$. Instead of working on an exogenous grid for a, this is achieved by defining a grid on savings, $\mathcal{G}^s$. Depending on the nature of the problem it is then possible to solve for c (and a') analytically. This is the crucial step: The speed advantage of ENDGM relative to EXOGM is achieved because the mapping $s \to (a', c)$ has a closed form solution. For given contemporaneous variables s, c, and next period's a' one can then endogenously compute today's endogenous state a. Again observe that, for some regular grid $\mathcal{G}^s$, the "endogenous" grid of a is generally irregular. In subsequent iterations, it is necessary to interpolate on such an irregular grid. In one dimension this does not cause any specific problems.

In this chapter we highlight, however, that this irregularity of endogenous grids is the source of a problem specific to ENDGM in higher dimensions. We emphasize that this drawback is not related to the solution of the system of equations per se but results from the endogenously computed states. As we show, the resulting state grid is generally not rectangular, i.e., gridpoints are irregularly distributed in the space. In consequence, even linear interpolation is much more costly than for conventional rectangular grids.

This is easiest to understand again by example. Consider two endogenous state variables a and h, where h is human capital, as in our application. Accordingly, (a', h') are next period's endogenous state variables. Control variables are consumption c, as before, as well as investment in human capital, i. In addition, consider the endogenous controls s (=savings, as before) and current period gross holdings of human capital, z, where z is some function of the human capital stock, h, and the flow investment into human capital, i. Corresponding to the one-dimensional problem the mapping in EXOGM is $(a, h) \to (c, i, a', h') \to (s, z)$ where the mapping $(a, h) \to (c, i, a', h')$ requires solution of a system of two non-linear equations. In ENDGM, the mapping is again reversed, i.e., $(s, z) \to (a', h', c, i) \to (a, h)$. Depending on the nature of the problem, the mapping $(s, z) \to (a', h', c, i)$ has a closed form solution. As in the application in one dimension, the endogenous grid formed of a, h is irregular. In subsequent iterations one has to interpolate on such an irregular grid. While such an interpolation is unproblematic in one dimension, this irregularity severely complicates location of points for interpolation in higher dimensions.

This exposition clarifies that there exists a fundamental trade-off between EXOGM and ENDGM in higher dimensions. On the one hand, EXOGM requires the use of numerical routines throughout whereas ENDGM computes solutions to first-order conditions in closed form. On the other hand, interpolation in EXOGM is on regular grids and therefore simple. Interpolation in ENDGM on irregular grids is much more complex.

We solve this complex interpolation by Delaunay triangulation, cf. Delaunay (1934). Delaunay interpolation, originally coming from the field of geometry. It was only recently introduced to the field by Brumm and Grill (2014). Broer, Kapicka, and Klein (2013) is the only other (unpublished) paper in Economics we are aware of that applies the method. Our contribution is to investigate its performance in combination with ENDGM.

In addition to EXOGM and ENDGM, we consider a third algorithm, a hybrid method of exogenous gridpoints in one dimension and endogenous gridpoints in the other (HYBGM).[1] Consequently, the endogenously computed grid is only irregular in one dimension. This is a so-called rectilinear grid. Interpolation on a rectilinear grid is easy, just as in the one-dimensional problem. The trade-off between HYBGM and ENDGM is therefore between numerically more costly routines, e.g. Broyden's method, in some dimensions vis-à-vis analytical solutions in all dimensions but a more complex interpolation.

To analyze and to compare these methods we use a simple human capital model. As we already discussed above, this model features two endogenous state variables, financial assets and human capital. Evaluation of methods in this two dimensional setup is done by comparing speed and accuracy of the different approaches.

Our main finding is that HYBGM and ENDGM both dominate EXOGM. They are both substantially faster. In our infinite horizon application, ENDGM also dominates HYBGM. In our finite horizon application, the choice between HYBGM and ENDGM depends on the number of gridpoints in each dimension. For a relatively low number of gridpoints, ENDGM is advantageous and vice versa for HYBGM. We also discuss limitations of ENDGM and HYBGM which are both only applicable to specific problems at hand.

To the best of our knowledge ENDGM in higher dimensions is not yet fully understood. This chapter is an important contribution to fill this gap. Related

[1] This is similar to the approach of Hintermaier and Koeniger (2010), also see below.

work by Krueger and Ludwig (2007) and Barillas and Fernandez-Villaverde (2007) extends ENDGM to problems with two control variables but just one endogenous state variable. Hintermaier and Koeniger (2010) use ENDGM in a durable goods model with two endogenous state variables. The main difference of their approach to ours is that ENDGM is only applied in one dimension. Their method still requires solving a nonlinear equation and is thereby very similar to our HYBGM.[2] Our contribution is to implement ENDGM in two dimensions.

Other related literature extends ENDGM to a class of dynamic programming problems with both discrete and continuous choices in which the value function is non-smooth and non-concave, cf. Fella (2014) and Iskhakov et al. (2014).

Our analysis proceeds as follows. Section 2 presents the simple human capital setting on which we base the evaluation of methods. Section 3 introduces the main features of the methods under evaluation, the method of exogenous gridpoints, the pure method of endogenous gridpoints and the hybrid method. Section 4 presents results according to speed and accuracy of all three methods. Section 5 concludes. Additional material is contained in an appendix.

3.2 General Framework

We develop a consumption and savings model which allows us to illustrate and to compare three approaches to solve dynamic models with two endogenous states using first-order methods. In addition to assets there is a second endogenous state variable, a human or health capital stock (we will use both interpretations interchangeably). Human capital can be accumulated over time and is produced with a nonlinear production function. For expositional purposes we keep the model very simple. For example, despite the degenerate risk of survival, we ignore any stochasticity to the effect that, e.g., wage processes are fully deterministic. Of course, the underlying trade-off between solution methods will also hold in more complex problems.

3.2.1 A Simple Human Capital Model

A risk averse agent with maximum time horizon T, $T = \infty$ possible, derives utility from consumption, c_t, in each period, with standard additive separable life

[2] One difference to our version of HYBGM is that we solve this non-linear equation with a univariate solver whereas Hintermaier and Koeniger (2010) use interpolation techniques that are generally less accurate. This is essentially analogous to applying a bisection method for one iteration only.

time utility

$$U = \sum_{t=1}^{T} \beta^{t-1} s\left(h_t\right) u\left(c_t\right),$$

where $\beta \in (0,1)$ is the discount factor. The instantaneous utility function $u\left(c_t\right)$ as well as the probability to survive to the next period $s\left(h_t\right)$ are assumed to be strictly increasing and concave in their respective arguments. Income of the agent, y_t, consists of labor income which depends on the amount of accumulated human capital, h_t, hence

$$y_t = wh_t,$$

where w is the wage rate.

In each period the household faces the decision to consume, c_t, to invest savings, s_t, in a risk-free financial asset, a_t, which earns (gross) interest R and to invest an amount i_t into human capital, h_t. Human capital depreciates at constant rate δ and is produced by the production function $f\left(i\right)$. We assume that $f_i > 0$, $f_{ii} < 0$ and that the Inada conditions are satisfied, i.e., $\lim_{i_t \to 0} f_i = \infty$ and $\lim_{i_t \to \infty} f_i = 0$.[3] The human capital accumulation equation is accordingly given by

$$h_{t+1} = (1-\delta)\left(h_t + f\left(i_t\right)\right), \tag{3.1}$$

where h_0 is given.

Financial markets are imperfect and households are not allowed to hold negative financial assets. The dynamic budget constraint writes as

$$a_{t+1} = R(a_t + wh_t - c_t - i_t) \geq 0,$$

where a_0 is given.

Recursive Formulation of the Household Problem The recursive formulation of the household problem is as follows:

$$V_t(a_t, h_t) = \max_{c_t, i_t, a_{t+1}, h_{t+1}} \left\{u(c_t) + \beta s\left(h_{t+1}\right) V_{t+1}(a_{t+1}, h_{t+1})\right\}$$

[3] These conditions are crucial because otherwise it could turn out to be optimal to invest in only one asset. The other asset would be redundant and our problem would collapse to a problem in one dimension.

subject to the constraints

$$\begin{aligned}
a_{t+1} &= R\left(a_t + wh_t - c_t - i_t\right)\\
h_{t+1} &= (1-\delta)\left(h_t + f\left(i_t\right)\right)\\
a_{t+1} &\geq 0\\
h_{t+1} &> 0. \qquad (3.2)
\end{aligned}$$

Assumptions on Functional Forms For our numerical approach we assume that instantaneous utility has the CRRA property with coefficient of relative risk aversion denoted by $\theta > 0$:

$$u\left(c_t\right) = \frac{c_t^{1-\theta} - 1}{1-\theta}.$$

The human capital production function is

$$f\left(i_t\right) = \frac{1}{\alpha} i_t^{\alpha}$$

for curvature parameter $\alpha \in (0,1)$. As to the functional form of the per-period survival probability we follow Hall and Jones (2007) and assume that

$$s\left(h_t\right) = 1 - \phi \frac{1}{1+h_t},$$

for $\phi \in (0,1]$.

We assume that the value function is strictly concave and unique maximizers are continuous policy functions, cf. Stokey and Lucas (1989). It is well-known that strict concavity of the value function may be violated in models with endogenous human capital formation (value functions may have concave and convex regions). Hence, first-order conditions are generally necessary but not sufficient. In applications, one way to accommodate this is to use first-order methods at the calibration stage of the model (where speed is an issue). Upon convergence, one can then test for uniqueness by checking for alternative solutions by use of global methods. To focus our analysis we do not further address these aspects here.[4]

[4] We checked ex-post if value functions are globally concave which they are for the parameter space considered here. A crucial parameter is α as it governs the curvature of the human capital production function. If we were to choose a higher degree of curvature (lower α) than non-concavities may arise. These results are available upon request.

Solution The optimal solution is fully characterized by the following set of first-order conditions and constraints:

$$c_t^{-\theta} = \beta R \left(1 - \phi \frac{1}{1+h_{t+1}}\right) V_{t+1_a}\left(a_{t+1}, h_{t+1}\right) \tag{3.3a}$$

$$\gamma i_t^{-(1-\alpha)} = \frac{R}{(1-\delta)} \frac{V_{t+1_a}\left(a_{t+1}, h_{t+1}\right)}{\frac{\phi}{(1+h_{t+1}-\phi)(1+h_{t+1})} V_{t+1}\left(a_{t+1}, h_{t+1}\right) + V_{t+1_h}\left(a_{t+1}, h_{t+1}\right)} \tag{3.3b}$$

$$a_{t+1} = R\left(a_t + w h_t - c_t - i_t\right) \tag{3.3c}$$

$$h_{t+1} = (1-\delta)\left(h_t + f\left(i_t\right)\right) \tag{3.3d}$$

$$a_{t+1} \geq 0. \tag{3.3e}$$

V_{t_a} and V_{t_h} are derivatives of the value function with respect to financial assets and human capital, respectively. The first equation relates today's consumption to consumption of tomorrow, whereas the second equation relates costs and gains of investing in human capital. Notice that constraint (3.2) can be dropped because of the lower Inada condition of the human capital investment function $f(i)$. Searching for the solution of this model amounts to finding the four optimal policies for consumption, $c_t(\cdot,\cdot)$, investment in human capital, $i_t(\cdot,\cdot)$, next period's financial assets, $a_{t+1}(\cdot,\cdot)$, and next period's human capital, $h_{t+1}(\cdot,\cdot)$, as functions of the two endogenous state variables, financial assets, a_t, and human capital, h_t, that solve equation system (3.3).

The envelope conditions are:

$$V_{t_a}\left(a_t, h_t\right) = u_c = c_t^{-\theta} \tag{3.4a}$$

$$V_{t_h}\left(a_t, h_t\right) = \left(w + \frac{1}{f_i}\right) u_c = \left(w + \frac{1}{\gamma i_t^{-(1-\alpha)}}\right) c_t^{-\theta}. \tag{3.4b}$$

Using (3.3a) together with (3.4a) gives the standard Euler equation of consumption.[5]

3.2.2 Calibration

We choose the same parametrization of the model for all solution methods described in Section 3.3. The coefficient of relative risk aversion is set to θ=0.5 to assure a positive value of life. We set the time preference rate to ρ=0.04. In order to provide sufficient incentives to save in the finite horizon setting without

[5] For derivation of (3.3) and the Envelope conditions see Appendix 3.A.

introducing risk we set an interest rate of R-1=0.05. In the infinite horizon setting we set an interest rate of R-1=0.03 which is smaller than ρ in order to assure that financial assets are bounded. For the depreciation rate of human capital we take δ=0.05. The curvature parameter of the human capital production function is α=0.35. The wage rate w is set to 0.1. The survival rate parameter is ϕ=0.5.

3.3 Solution Methods

The main idea of all methods is to exploit the FOCs (3.3a) and (3.3b) to compute optimal policies at discrete points that constitute a mesh in the state space. All three methods use the recursive nature of the problem. Correspondingly, in the finite horizon version, the model is solved backwards from the last to the first period ($t = T, T-1, \ldots, 0$). In the infinite horizon implementation the iteration continues until convergence on policy functions.

Differences between methods arise because of different solution procedures to the multi-dimensional nonlinear equation system (3.3) and different interpolation methods, respectively. To provide a preview: The first algorithm (EXOGM) applies a multi-dimensional Quasi-Newton method. Standard interpolation methods are used. The second algorithm (ENDGM) uses the method of endogenous gridpoints and thereby solves the system of equations (3.3) analytically. It is accompanied by Delaunay interpolation. The third algorithm (HYBGM) combines the former two, i.e., it applies the method of endogenous gridpoints (and closed form solutions) in one dimension and uses a one-dimensional Quasi-Newton method in the other dimension. As EXOGM, HYBGM comes along with a standard interpolation procedure.

3.3.1 Multi-Dimensional Root-Finding with Regular Interpolation (EXOGM)

The most direct approach to solve (3.3) is to insert the constraints into the FOCs and to rely on a numerical multi-dimensional root-finding routine. Multi- dimensional solvers are necessary because c and i show up on both sides of the respective non-linear equations in (3.3). In our application we use a Quasi-Newton method, more specifically Broyden's method, cf. Press et al. (1996).

The implementation steps of EXOGM are as follows:

1. To initialize EXOGM predefine two grids, one for financial assets a, $\mathcal{G}^a = \left\{a^1, a^2, ..., a^K\right\}$ and one for human capital h, $\mathcal{G}^h = \left\{h^1, h^2, ..., h^J\right\}$ and construct $\mathcal{G}^{a,h} = \mathcal{G}^a \otimes \mathcal{G}^h$.

2. In period T, savings and investment in human capital are zero as both assets are useless in period $T+1$[6] and income is completely consumed for all $\left(a^k, h^j\right) \in \mathcal{G}^{a,h}$:

$$
\begin{aligned}
c_T(\cdot,\cdot) &= a_T^k + w h_T^j \\
i_T(\cdot,\cdot) &= 0.
\end{aligned}
$$

   Using the above in equations (3.4a) and (3.4b) the value function and its derivatives with respect to a and h in T are

$$
\begin{aligned}
V_T\left(a_T^k, h_T^j\right) &= \frac{1}{1-\theta}\left(c_T^{k,j}\right)^{1-\theta} \\
V_{T_a}\left(a_T^k, h_T^j\right) &= \left(c_T^{k,j}\right)^{-\theta} \\
V_{T_h}\left(a_T^k, h_T^j\right) &= \left(w + \frac{1}{\gamma}\left(i_T^{k,j}\right)^{1-\alpha}\right)\left(c_T^{k,j}\right)^{-\theta} = w\left(c_T^{k,j}\right)^{-\theta}.
\end{aligned}
$$

3. Iterate backwards on $t = T-1, \dots, 0$. In each t for each $\left(a_t^k, h_t^j\right) \in \mathcal{G}^{a,h}$:

   a) Given (suitably interpolated values of) V_{t+1}, V_{t+1_a} and V_{t+1_h}, solve the two-dimensional equation system

$$
\begin{aligned}
\left(c_t^{k,j}\right)^{-\theta} &= \beta R \left(1 - \phi \frac{1}{1 + (1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}\left(i_t^{k,j}\right)^{\alpha}\right)}\right) \\
&\quad V_{t+1_a}\left(\overbrace{R\left(a_t^k + w h_t^j - c_t^{k,j} - i_t^{k,j}\right)}^{a_{t+1}^{k,j}}, \overbrace{(1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}\left(i_t^{k,j}\right)^{\alpha}\right)}^{h_{t+1}^{k,j}}\right) \\
\gamma\left(i_t^{k,j}\right)^{-(1-\alpha)} &= \\
&\frac{R}{(1-\delta)} \frac{V_{t+1_a}\left(a_{t+1}^{k,j}, h_{t+1}^{k,j}\right)}{\frac{\phi}{\left(1+h_{t+1}^{k,j}-\phi\right)\left(1+h_{t+1}^{k,j}\right)} V_{t+1}\left(a_{t+1}^{k,j}, h_{t+1}^{k,j}\right) + V_{t+1_h}\left(a_{t+1}^{k,j}, h_{t+1}^{k,j}\right)}
\end{aligned}
$$

   for $c_t^{k,j}$ and $i_t^{k,j}$ using Broyden's method.

[6]This rationale does not imply that h must be zero in period $T+1$ because human capital is—in contrast to financial assets—inalienable.

If $c_t^{k,j} + i_t^{k,j} > a_t^k + wh_t^j$ (binding borrowing constraint) recompute $i_t^{k,j}$ by solving

$$\left(a_t^k + wh_t^j - i_t^{k,j}\right)^{-\theta} - \frac{1}{\left((1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}(i_t^{k,j})^\alpha\right)\right)^2} \cdot V_{t+1_a}\left(0, (1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}(i_t^{k,j})^\alpha\right)\right)\beta(1-\delta)\gamma(i_t^{k,j})^{-(1-\alpha)} - \left(1 - \frac{1}{\left((1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}(i_t^{k,j})^\alpha\right)\right)}\right) \cdot V_{t+1_h}\left(0, (1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}(i_t^{k,j})^\alpha\right)\right)\beta(1-\delta)\gamma(i_t^{k,j})^{-(1-\alpha)} = 0$$

for $i_t^{k,j}$. Next, re-compute $c_t^{k,j} = a_t^k + wh_t^j - i_t^{k,j}$.

b) Save/Update both the value function and its derivatives

$$V_t\left(a_t^k, h_t^j\right) = \frac{1}{1-\theta}\left(c_t^{k,j}\right)^{1-\theta} + \beta\left(1 - \phi\frac{1}{1 + h_{t+1}^{k,j}}\right)V_{t+1}(a_{t+1}^{k,j}, h_{t+1}^{k,j})$$

$$V_{t+1_a}\left(a_t^k, h_t^j\right) = \left(c_t^{k,j}\right)^{-\theta}$$

$$V_{t+1_h}\left(a_t^k, h_t^j\right) = \left(w + \frac{1}{\gamma}\left(i_t^{k,j}\right)^{1-\alpha}\right)\left(c_t^{k,j}\right)^{-\theta}.$$

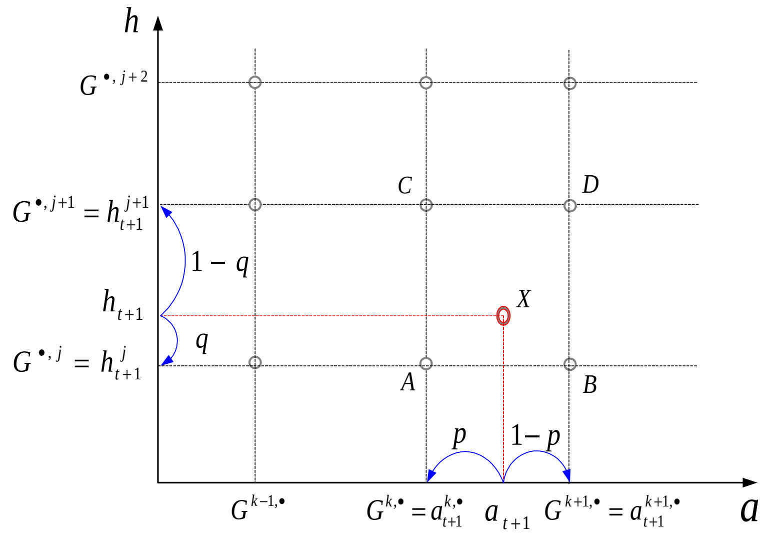
Since EXOGM requires to apply the solver for each point in $\mathcal{G}^{a,h}$, this procedure entails solving the multidimensional equation system $[K \cdot J]$ times in each $t = T-1, \ldots, 0$. Depending on the stopping criterion in the numerical routine this could be either quite costly in terms of computing time or the computed solutions suffer under low accuracy. An additional shortcoming of EXOGM compared to ENDGM and HYBGM is that the region where the borrowing constraint is binding is not determined.[7] In consequence, policy functions are imprecise at the kink. This may also cause convergence problems. Furthermore, numerical methods often require fine tuning so that stability of numerical routines is ascertained. We initially encountered several such instability problems which we managed to fix by setting options of the solver accordingly.[8]

[7] In principle, this could be accommodated by an additional rootfinder to detect the kink—i.e., the a, h-combination at which the borrowing constraint just becomes unbinding—and to add in additional grid points there. We do not extend the method along this dimension. A naive extension along these lines would further slow down EXOGM. However, see Brumm and Grill (2014) for a sophisticated application.

[8] An alternative would be to avoid multivariate solvers and to instead use fixed point iterations with nested univariate solvers. However, this would further slow down EXOGM.

Interpolation on a Rectilinear Grid Step 3a requires evaluation of both the value function V_{t+1} and its derivatives, V_{t+1_a} and V_{t+1_h}. As, in general, $(a^{k,j}_{t+1}, h^{k,j}_{t+1}) \notin \mathcal{G}^{a,h}$ we have to interpolate these functions. We apply bilinear interpolation. Precisely, we determine interpolation nodes by the concept "grid square", cf. Press et al. (1996). In order to apply this procedure it is necessary to have a rectilinear grid, i.e., the state space has to be tessellated by rectangles.[9] In this case all gridpoints in row $\mathcal{G}^{\bullet,j}$ have the same value of h^j, and all gridpoints in column $\mathcal{G}^{k,\bullet}$ have the same value of a^k. The problem of locating a point in a multi-dimensional grid is split up into several problems of locating the point in one dimension. Within each dimension and a total number of N points in the point set closest neighbors in the grid are identified in about $\log_2 N$ trials using bisection methods. Figure 3.1 shows the location of interpolation nodes $[A; B; C; D]$ for point X in a two-dimensional rectilinear grid.

Figure 3.1: Rectilinear Grid



Notes: Interpolation on rectilinear grids. In any row locate the two columns ($G^{k,\bullet}$ and $G^{k+1,\bullet}$) that form the most narrow bracket of a_{t+1}. In any column locate the two rows ($G^{\bullet,j}$ and $G^{\bullet,j+1}$) that form the most narrow bracket of h_{t+1}. Interpolation nodes: (k,j); $(k+1,j)$; $(k,j+1)$; $(k+1,j+1)$.

In EXOGM, $\mathcal{G}^a \otimes \mathcal{G}^h$ is predetermined as a rectilinear grid (in every iteration). After locating the nodes, bi-linear interpolation of any function of F—in our case the value function in t as well as its first derivatives with respect to a and h—at point X requires computing $F(X) = \varphi_A F(A) + \varphi_B F(B) + \varphi_C F(C) + \varphi_D F(D)$

[9] Notice that these rectangles do not necessarily have to be congruent to each other.

with the four basis functions φ where $\varphi_A = p \cdot q$, $\varphi_B = (1-p) \cdot q$, $\varphi_C = p \cdot (1-q)$ and $\varphi_D = (1-p) \cdot (1-q)$ with $p = \frac{a_X - a_A}{a_B - a_A}$ and $q = \frac{h_X - h_A}{h_C - h_A}$, cf. Judd (1998).

3.3.2 Analytical Solution with Delaunay Interpolation (ENDGM)

The above setting has a straightforward economic interpretation. Given an exogenous state today (a_t, h_t) compute the endogenous state variables (a_{t+1}, h_{t+1}). The main idea of ENDGM is to redefine exogenous and endogenous objects in the numerical solution: the grid of contemporaneous control variables is taken as exogenous whereas the grid of today's state variables is determined endogenously.

In our two-dimension setup, implementation of the method requires definition of two endogenous control variables on which to base the exogenous grids. To this purpose define by

$$s_t \equiv a_t + wh_t - c_t - i_t = \frac{a_{t+1}}{R} \tag{3.5a}$$

$$z_t \equiv h_t + f(i_t) = \frac{h_{t+1}}{1-\delta} \tag{3.5b}$$

return adjusted stock of physical and human capital, respectively. Our implementation of the method defines grids on (s_t, z_t) and maps from (s_t, z_t) to (a_{t+1}, h_{t+1}) by $a_{t+1} = Rs_t$ and $h_{t+1} = (1-\delta) z_t$.[10] Next, the system of FOCs can be solved analytically to determine the corresponding set of contemporaneous controls, (c_t, i_t). Finally, we use the budget constraint and the law of motion for human capital to get the corresponding endogenous state variables, (a_t, h_t). Precisely, the implementation steps are as follows:

1. To initialize ENDGM predefine two grids, one for gross savings s, $\mathcal{G}^s \equiv \left\{s^n, s^{n+1}, ..., s^K\right\}$ and one for gross investment in human capital z, $\mathcal{G}^z \equiv \left\{z^1, z^2, ..., z^J\right\}$ as defined in (3.5) and form $\mathcal{G}^{s,z} = \mathcal{G}^s \otimes \mathcal{G}^z$.

2. Define $\mathcal{G}^{a,h} = \mathcal{G}^a \otimes \mathcal{G}^h$ for T. Compared to $\mathcal{G}^s$, the grid $\mathcal{G}^a$ includes n additional gridpoints. These gridpoints represent the region in which the borrowing constraint is binding (see step 3e). In period T, as in EXOGM,

$$\begin{aligned} c_T(\cdot,\cdot) &= a_T^{k,j} + wh_T^{k,j} \\ i_T(\cdot,\cdot) &= 0 \end{aligned}$$

[10] In deterministic model such as ours, this mapping is of course deterministic. We could therefore directly work on a grid of (a_{t+1}, h_{t+1}). However, this would generally not be possible in a stochastic model because the realizations of (a_{t+1}, h_{t+1}) depend on the realizations of shocks in period $t+1$. For sake of generality, we therefore define the grid on (s_t, z_t).

for all $\left(a^{k,j}, h^{k,j}\right) \in \mathcal{G}^{a,h}$ and

$$\begin{aligned}
V_T\left(a_T^{k,j}, h_T^{k,j}\right) &= \frac{1}{1-\theta}\left(c_T^{k,j}\right)^{1-\theta} \\
V_{T_a}\left(a_T^{k,j}, h_T^{k,j}\right) &= \left(c_T^{k,j}\right)^{-\theta} \\
V_{T_h}\left(a_T^{k,j}, h_T^{k,j}\right) &= \left(w + \frac{1}{\gamma}\left(i_T^{k,j}\right)^{1-\alpha}\right)\left(c_T^{k,j}\right)^{-\theta} = w\left(c_T^{k,j}\right)^{-\theta}.
\end{aligned}$$

3. Iterate backwards from $t = T-1, ..., 0$. In each t, for each $\left(s^k, z^j\right) \in \mathcal{G}^{s,z}$:
   a) Compute a_{t+1}^k and h_{t+1}^j:
   $$\begin{aligned}
   a_{t+1}^k &= Rs^k, \\
   h_{t+1}^j &= (1-\delta)\, z^j.
   \end{aligned}$$
   b) Given V_{t+1}, V_{t+1_a} and V_{t+1_h} interpolate the value function and its derivatives at $\left(a_{t+1}^k, h_{t+1}^j\right)$ to get (interpolated values of) $V_{t+1}\left(a_{t+1}^k, h_{t+1}^j\right)$, $V_{t+1_a}\left(a_{t+1}^k, h_{t+1}^j\right)$ and $V_{t+1_h}\left(a_{t+1}^k, h_{t+1}^j\right)$ using Delaunay interpolation (see below).
   c) Compute $c_t^{k,j}$ and $i_t^{k,j}$:
   $$\begin{aligned}
   c_t^{k,j} &= \left(\beta R\left(1 - \phi\frac{1}{1+(1-\delta)\, z^j}\right) V_{t+1_a}\left(\overbrace{Rs^k}^{a_{t+1}^k}, \overbrace{(1-\delta)\, z^j}^{h_{t+1}^j}\right)\right)^{-\frac{1}{\theta}}, \\
   i_t^{k,j} &= \frac{1}{\gamma}\left(\frac{R}{(1-\delta)}\right)^{-\frac{1}{1-\alpha}} \cdot \\
   &\left(\frac{V_{t+1_a}\left(a_{t+1}^k, h_{t+1}^j\right)}{\frac{\phi}{\left(1+h_{t+1}^j-\phi\right)\left(1+h_{t+1}^j\right)} V_{t+1}\left(a_{t+1}^k, h_{t+1}^j\right) + V_{t+1_h}\left(a_{t+1}^k, h_{t+1}^j\right)}\right)^{-\frac{1}{1-\alpha}}
   \end{aligned}$$
   d) Compute $a_t^{k,j}$ and $h_t^{k,j}$:
   $$\begin{aligned}
   h_t^{k,j} &= z^j - \frac{\gamma}{\alpha}\left(i_t^{k,j}\right)^{\alpha} \\
   a_t^{k,j} &= s^k - wh_t^{k,j} + c_t^{k,j} + i_t^{k,j}.
   \end{aligned}$$

e) If $a_t^{n+1,j} > 0$, define for each j an auxiliary grid $\mathcal{G}^{aux} \equiv \{a^1, a^2, ..., a^n\}$ between 0 and $a_t^{n+1,j}$.[11] In this region the borrowing constraint is binding. Compute $i_t^{k,j}$ by solving

$$\left(a_t^k + w\left(\frac{h_{t+1}^j}{1-\delta} - \frac{\gamma}{\alpha}(i_t^{k,j})^\alpha\right) - i_t^{k,j}\right)^{-\theta}$$
$$- \frac{1}{\left(h_{t+1}^j\right)^2} V_{t+1_a}\left(0, h_{t+1}^j\right)\beta\,(1-\delta)\,\gamma(i_t^{k,j})^{-(1-\alpha)}$$
$$- \left(1 - \frac{1}{h_{t+1}^j}\right) V_{t+1_h}\left(0, h_{t+1}^j\right)\beta\,(1-\delta)\,\gamma(i_t^{k,j})^{-(1-\alpha)} = 0$$

using a non-linear solver. Then compute

$$c_t^{k,j} = a_t^k + w\left(\frac{h_{t+1}^j}{1-\delta} - \frac{\gamma}{\alpha}(i_t^{k,j})^\alpha\right) - i_t^{k,j}.$$

f) Save/Update both the value function and its derivatives

$$V_t\left(a_t^{k,j}, h_t^{k,j}\right) = \frac{1}{1-\theta}\left(c_t^{k,j}\right)^{1-\theta} + \beta\left(1 - \phi\frac{1}{1+h_{t+1}^j}\right) V_{t+1}\left(a_{t+1}^k, h_{t+1}^j\right)$$
$$V_{t_a}\left(a_t^{k,j}, h_t^{k,j}\right) = \left(c_t^{k,j}\right)^{-\theta}$$
$$V_{t_h}\left(a_t^{k,j}, h_t^{k,j}\right) = \left(w + \frac{1}{\gamma}\left(i_T^{k,j}\right)^{1-\alpha}\right)\left(c_t^{k,j}\right)^{-\theta}.$$

The clear advantage of ENDGM compared to EXOGM becomes obvious in step 3c. By conditioning on the grid of s_t and z_t the system of FOCs can be solved for c_t and i_t analytically and hence no numerical root-finder is needed. Furthermore, ENDGM provides, by construction, an exact determination of the range of the borrowing constraint and produces higher accuracy of the solution than EXOGM in this region. However, in contrast to the standard one-dimensional problem considered by Carroll (2006), the policy function itself does not have a closed form solution in this range, see step 3e.[12]

[11] If $a_t^{n+1,j} \leq 0$, the borrowing constraint is not binding and we add in some artificial numbers for the solution here. Observe that the method can be further improved by working with t- (or iteration-) dependent grids, an approach we do not adopt here.

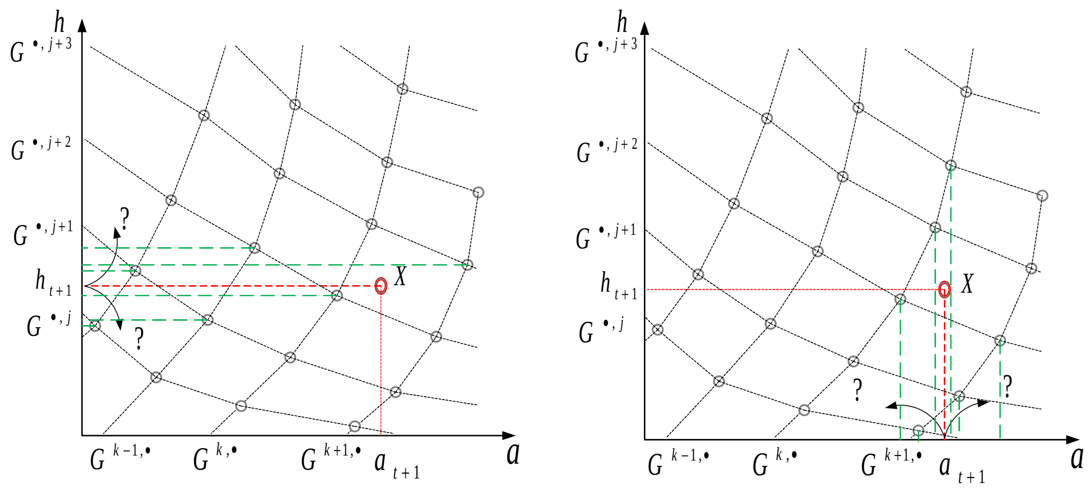
[12] In a standard consumption-savings model with only one endogenous continuous state variable the policy function is computed by linearly interpolating between the policy at zero saving and the origin, cf. Carroll (2006).

Remark 3.1. In contrast to EXOGM, ENDGM is not a general method. Suppose we were to adopt a general Ben-Porath human capital function, cf. Ben-Porath (1967), in which the level of human capital directly affects the productivity of human capital investments, i.e., we replace $f(i)$ in equation (3.1) with $f(h, i)$. ENDGM is no longer applicable in such a formulation. This exemplifies that an application of ENDGM often requires specific modeling assumptions.

Delaunay Interpolation In EXOGM the grid is rectilinear by construction whereas in ENDGM the endogenously computed grid $\mathcal{G}^{a,h}$ is not. This constitutes the main drawback of ENDGM because location of interpolation nodes is not obvious. As illustrated in Figure 3.2, separating the multi-dimensional problem into several one-dimensional problems is not possible. In each row not just the value of a changes but also the value of h so that the concept of bi-linear interpolation in a square grid is not applicable. ENDGM hence generates a situation where neighboring points in the state space do not need to be neighboring elements in the grid matrix.

Figure 3.2: Irregular Grid

(a) Panel (a) (b) Panel (b)



Notes: Interpolation on irregular grids. Multidimensional interpolation cannot be separated into several one-dimensional interpolations as the values of a and h change in each column or row.

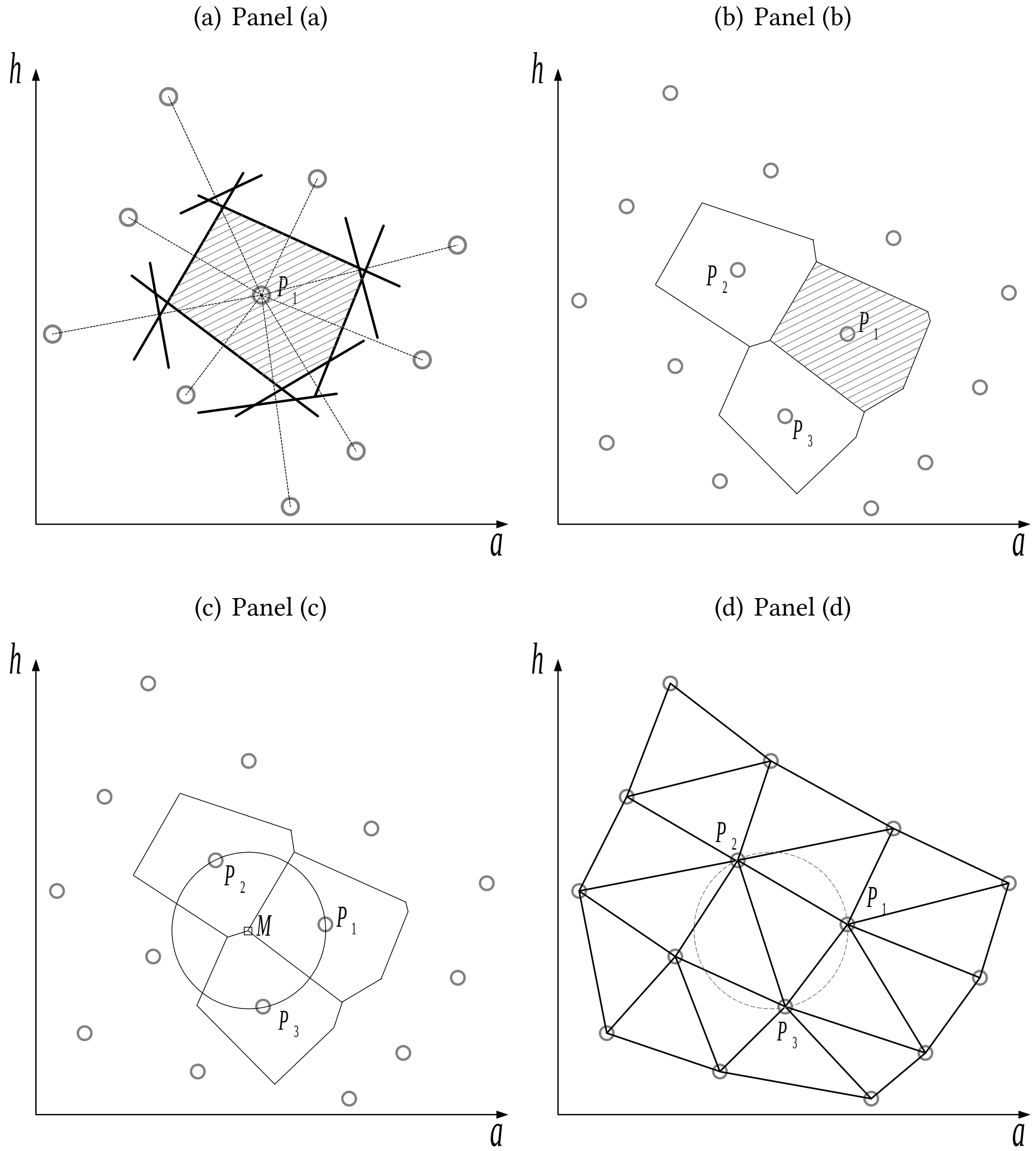
The most common approach adopted in other scientific fields such as geometry or geography to locate neighboring points in an irregular grid is the concept of Delaunay triangulation and its related geometric construct, the Voronoi diagram. We explain the geometric construction of the Voronoi diagram by use of

Figure 3.3. The Voronoi diagram (polygon)—shown in Panel (a) of Figure 3.3—is the region of the state space consisting of all points closer to gridpoint P_1 than to any other gridpoint. The Voronoi diagram is obtained from the perpendicular bisectors of the lines connecting neighboring points. Voronoi diagrams for all points form a tessellation of the space, cf. Panel (a). Edges of the Voronoi diagram are all the points in the plane that are equidistant to the two nearest gridpoints, cf. Panel (b). The Voronoi vertices are the points equidistant to three gridpoints, i.e., they are the center of circumcircles including the three neighboring gridpoints, cf. Panel (c). Connecting these gridpoints constitutes the unique triangulation known as the Delaunay triangulation as displayed in Panel (d), cf. Baker (1999). The vertices of a triangle are the nearest neighbors of all points contained in that triangle. These concepts can also be generalized to more than two dimensions.

The computational implementation of a Delaunay triangulation is done by the so-called randomized incremental algorithm, illustrated in Figure 3.4. It is incremental in the sense that it adds points to the triangulation one at a time to maintain a Delaunay triangulation at each stage. It is randomized in that points are added in a random order which guarantees $O(N\ log\ N)$ expected time for the algorithm where N is the total number of points in the point set, cf. Press et al. (2007). To construct the Delaunay triangulation for a given point set we initially have to add three "fictitious" points $[\Theta_1, \Theta_2, \Theta_3]$, forming a large starting triangle which encloses all "real" points, cf. Panel (a) of Figure 3.4. This is necessary in order to ensure that added points lie within an existing triangle. These "fictitious" points are deleted once the triangulation is complete. In each following step of Delaunay triangulation a point from the point set is added to the existing triangulation and connected to the vertices of the enclosing triangle. We illustrate this step in Panel (b) of the figure. Consider the existing triangle P_1, P_2, P_3 and a new point from the point set, P_5, which is not yet connected to other points. Connecting P_5 to P_1, P_2 and P_3, respectively, gives rise to three new triangles. Next, it is checked whether the newly created triangles are "legal", i.e., whether the circumcircle of any triangle does not contain any other point of the point set.[13] In our example, we first visit triangle P_2, P_3, P_5 in Panel (c). As shown in the figure, the circumcircle contains point P_4. Hence, triangle P_2, P_3, P_5 is not legal. Therefore, flip the edge opposite of P_5 connecting P_5 with P_4. This operation creates two new triangles, P_3, P_4, P_5 and P_2, P_4, P_5, cf. Panel (d) of the figure, which must be checked for legality. In our example, triangle P_3, P_4, P_5 is legal because the cir-

[13]This principle is derived from the definition that a triangulation fulfills the Delaunay property if and only if the circumcircle of any triangle does not contain a point in its interior, cf. Berg et al. (2008).

Figure 3.3: The Voronoi Diagram



(a) Panel (a)
(b) Panel (b)
(c) Panel (c)
(d) Panel (d)

Notes: Panel (a): Generating the Voronoi polygon: Edges are perpendicular bisectors of lines connecting neighboring points. Panel (b): Several Voronoi tiles in mesh grid. Panel (c): Circle with center at vertex includes three closest points. Panel (d): Delaunay Triangulation: Vertices are nearest neighbors of all points within triangle.

cumcircle does not contain other existing points from the point set. The process is recursive and never wanders away from any point P (point P_5 in our example). The only edges that can be made illegal by inserting a point P are edges opposite P (in triangles with P as a vertex).[14]

At interpolation, to locate a (query) point X in a given planar triangular mesh we adopt a procedure referred to as visibility walk, illustrated in Figure 3.5. The search starts from an initial guess of a triangle, Δ_1. Then, it is tested if the line supporting the first edge e separates Δ_1 from the query point X which reduces to a single operation test. If this is the case, the next triangle being visited is the neighbor of Δ_1 through e, Δ_2. Otherwise the second edge is tested in the same way. In case the test for the second edge also fails then the third edge is tested. The failure of this third test means that the goal has been reached. In Figure 3.5, this would be the case at triangle Δ_X which contains X.[15] Devillers, Pion, and Teillaud (2001) find that performance of the visibility walk is better than other possible algorithms. The location step for the visibility walk takes only $O \log(N)$ operations, cf. Press et al. (2007). The starting triangle may be arbitrary. However, an informed choice may radically shorten the length of the walk. We accommodate this by initializing the search with our solutions to gridpoints visited previously.

After locating the triangle we compute the normalized barycentric coordinates (weights) of the query point X with respect to the vertices (A, B, C) of the triangle Δ_X,

$$
\begin{aligned}
\varphi_A &= \frac{(a_X - a_C)(h_B - h_C) + (a_C - a_B)(h_X - h_C)}{(a_A - a_C)(h_B - h_C) + (a_C - a_B)(h_A - h_C)} \\
\varphi_B &= \frac{(a_X - a_C)(h_C - h_A) + (a_A - a_C)(h_X - h_C)}{(a_A - a_C)(h_B - h_C) + (a_C - a_B)(h_A - h_C)} \\
\varphi_C &= 1 - \varphi_A - \varphi_B.
\end{aligned}
$$

Finally, the interpolated value of any function F at point X is given as the weighted average of the respective function values at the vertices,

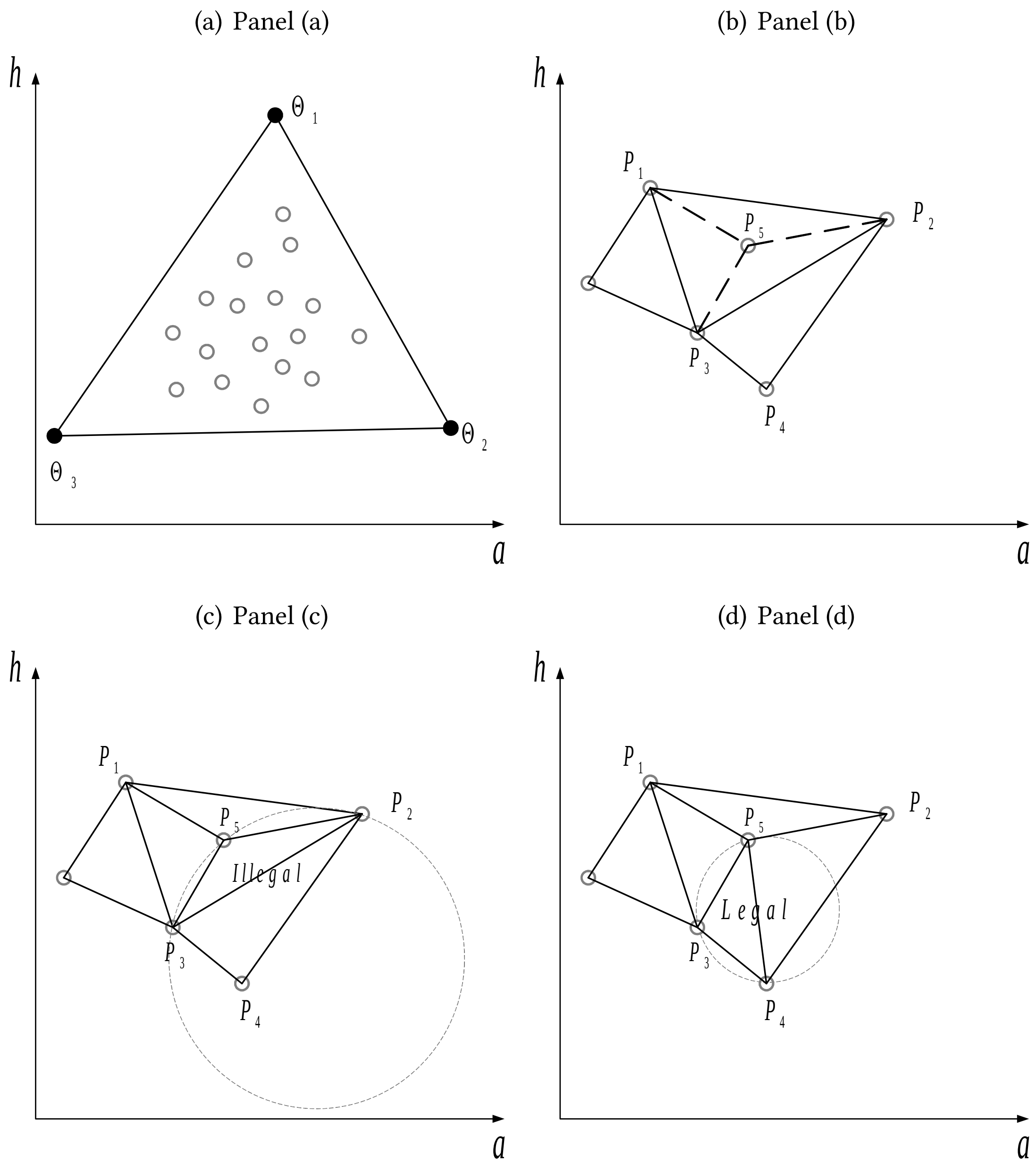
$$F(X) = \varphi_A F(A) + \varphi_B F(B) + \varphi_C F(C).$$

In our code we also incorporate the option of a multi-linear interpolation used

[14]This procedure is described in Press et al. (2007). We use the numerical package geompack3 based on Joe (1991) for both the Delaunay triangulation and the "visibility walk", described next.

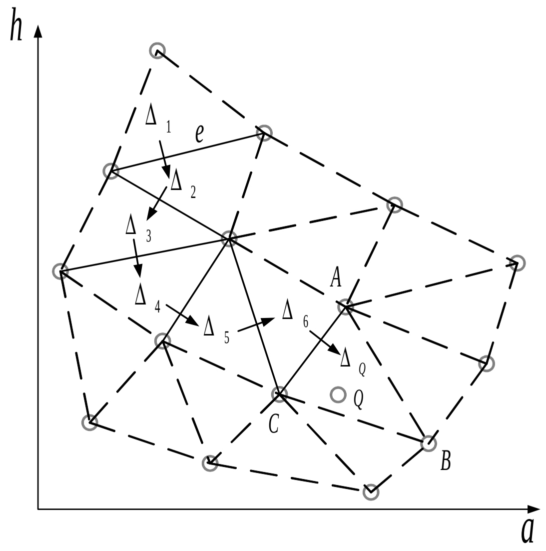
[15]In non-Delaunay triangulations, the visibility walk may fall into a cycle, whereas in Delaunay triangulations the visibility walk always terminates, cf. Devillers, Pion, and Teillaud (2001).

Figure 3.4: Incremental Algorithm



Notes: Panel (a): Three "fictional" points added to constitute the first triangle which includes all "real" points of the point set. Panel (b): Point added to existing Delaunay Triangulation and connected to vertices of enclosing triangle. Panel (c): Circumcircle contains a point and the triangle is therefore illegal. Panel (d): Circumcircle does not contain any point and is therefore legal.

Figure 3.5: Visibility Walk



Notes: Visibility walk in Delaunay triangulation - Locate triangle Δ_X containing X with initial guess Δ_1. If the line supporting e separates Δ from X, which reduces to a single orientation test, then the next visited triangle is the neighbor of Δ through e.

by Broer, Kapicka, and Klein (2013). This alternative interpolation method is very useful in applications in which existing triangles are visited frequently. In our specific applications, this is, however, not the case so that the method does not have an advantage over the simple interpolation method we use. We therefore do not apply it when generating our results below.[16]

[16] The basic idea of multilinear interpolation is as follows: We can write

$$\begin{bmatrix} a_X \\ h_X \end{bmatrix} = \begin{bmatrix} a_A \\ h_A \end{bmatrix} + s \begin{bmatrix} a_B - a_A \\ h_B - h_A \end{bmatrix} + t \begin{bmatrix} a_C - a_B \\ h_C - h_B \end{bmatrix} = \begin{bmatrix} a_A \\ h_A \end{bmatrix} + A \begin{bmatrix} s \\ t \end{bmatrix}, \text{ for } A = \begin{bmatrix} a_B - a_A & a_C - a_B \\ h_B - h_A & h_C - h_B \end{bmatrix}$$

and some scalars s and t. Given (a_X, h_X), the solution for (s, t) is accordingly given by

$$\begin{bmatrix} s \\ t \end{bmatrix} = A^{-1} \begin{bmatrix} a_X - a_A \\ h_X - h_A \end{bmatrix} = A^{-1} \begin{bmatrix} a_X \\ h_X \end{bmatrix} - b, \text{ where } b = A^{-1} \begin{bmatrix} a_A \\ h_A \end{bmatrix}. \tag{3.6}$$

The value of function $F(X)$ then follows as

$$F(X) = F(A) + s\,(F(B) - F(A)) + t\,(F(C) - F(B)). \tag{3.7}$$

Matrix A^{-1} and vector b must only be computed once when triangle Δ_X is visited for the first time and can accordingly be stored. In subsequent visits of Δ_X one can compute, for any point $\tilde{X} \in \Delta_X$, the scalars $(\tilde{s}, \tilde{t})$ directly from equation (3.6) and the interpolated value from equation (3.7).

3.3.3 One-Dimensional Root-Finding with Hybrid Interpolation (HYBGM)

We next consider a hybrid method (HYBGM) which combines EXOGM and ENDGM. Specifically, we use ENDGM in one dimension of the problem only. Hence, we define one of the two state variables on an "endogenous" grid, whereas the other is on an "exogenous" grid. The algorithm proceeds in three steps. In the first step, conditioning on control variable s_t and period t endogenous state h_t, we compute next period's endogenous state variable a_{t+1} and exploit one of the two FOCs to derive the value of one period t control variable—in this setup investment in human capital, i_t. In this step a one-dimensional solver is required. To preserve comparability with the previously described methods we choose Broyden's method.[17] In the second step, control i_t is used to get the value of the second period $t+1$ endogenous state variable, h_{t+1} from the budget constraint. Exploiting the second FOC we can then compute the second control variable, c_t. In the third step, we compute the corresponding endogenous state variable a_t from the budget constraint. The implementation steps are as follows:

1. To initialize HYBGM predefine two grids, one for gross savings s, $\mathcal{G}^s \equiv \left\{s^1, s^2, ..., s^K\right\}$ and one for human capital h, $\mathcal{G}^h \equiv \left\{h^1, h^2, ..., h^J\right\}$ and form $\mathcal{G}^{s,h} = \mathcal{G}^s \otimes \mathcal{G}^h$
2. In period T, define an initial guess for $\mathcal{G}^{a,h} = \mathcal{G}^a \otimes \mathcal{G}^h$. $\mathcal{G}^a$ includes n additional gridpoints compared to $\mathcal{G}^s$. These gridpoints represent the region in which the borrowing constraint is binding (see step 3d). Compute

$$
\begin{aligned}
c_T(\cdot,\cdot) &= a_T^{k,j} + wh_T^j \\
i_T(\cdot,\cdot) &= 0
\end{aligned}
$$

for all $\left(a_T^{k,j}, h_T^j\right) \in \mathcal{G}^{a,h}$ and

$$
\begin{aligned}
V_T\left(a_T^{k,j}, h_T^j\right) &= \frac{1}{1-\theta}\left(c_T^{k,j}\right)^{1-\theta} \\
V_{T_a}\left(a_T^{k,j}, h_T^j\right) &= \left(c_T^{k,j}\right)^{-\theta} \\
V_{T_h}\left(a_T^{k,j}, h_T^j\right) &= \left(w + \frac{1}{\gamma}\left(i_T^{k,j}\right)^{1-\alpha}\right)\left(c_T^{k,j}\right)^{-\theta}.
\end{aligned}
$$

3. Iterate backwards on $t = T-1, ..., 0$. In each t, for each $\left(s^k, h^j\right) \in \mathcal{G}^{s,h}$:
   a) Compute $a_{t+1}^k = Rs^k$.

[17] Using Brent's method instead turns out to slow down speed of HYBGM.

b) Given (suitably interpolated values of) V_{t+1}, V_{t+1_a} and V_{t+1_a}, solve the one-dimensional equation system for $i_t^{k,j}$

$$i_t^{k,j} = \frac{1}{\gamma}\left(\frac{R}{(1-\delta)}\right)^{-\frac{1}{1-\alpha}} \cdot \left(\frac{V_{t+1_a}\left(\overbrace{Rs^k}^{a_{t+1}^k}, \overbrace{(1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}\left(i_t^{k,j}\right)^{\alpha}\right)}^{h_{t+1}^{k,j}}\right)}{\frac{\phi}{\left(1+h_{t+1}^{k,j}-\phi\right)\left(1+h_{t+1}^{k,j}\right)}V_{t+1}\left(a_{t+1}^k, h_{t+1}^{k,j}\right) + V_{t+1_h}\left(a_{t+1}^k, h_{t+1}^{k,j}\right)}\right)^{-\frac{1}{1-\alpha}}$$

using Broyden's method. This includes several computations of $h_{t+1}^{k,j} = (1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}\left(i_t^{k,j}\right)^{\alpha}\right)$ and hybrid interpolations —described below— on V_{t+1}, $V_{t+1_a}$and V_{t+1_h}.

c) Compute $c_t^{k,j}$ as

$$c_t^{k,j} = \left(\beta R\left(1-\phi\frac{1}{1+h_{t+1}^{k,j}}\right)V_{t+1_a}\left(Rs^k, h_{t+1}^{k,j}\right)\right)^{-\frac{1}{\theta}}.$$

d) If $a_t^{n+1,j} > 0$, define for each j an auxiliary grid $\mathcal{G}^{aux} \equiv \left\{a^1, a^2, ..., a^n\right\}$ between 0 and $a_t^{n+1,j}$. In this region the borrowing constraint is binding. Compute $i_t^{k,j}$ by solving

$$\begin{aligned}\left(a_t^k + wh_t^j - i_t^{k,j}\right)^{-\theta} &- \frac{1}{\left((1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}(i_t^{k,j})^{\alpha}\right)\right)^2}\cdot \\ &V_{t+1_a}\left(0, (1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}(i_t^{k,j})^{\alpha}\right)\right)\beta(1-\delta)\gamma(i_t^{k,j})^{-(1-\alpha)} \\ -&\left(1-\frac{1}{(1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}(i_t^{k,j})^{\alpha}\right)}\right)V_{t+1_h}\left(0, (1-\delta)\left(h_t^j + \frac{\gamma}{\alpha}(i_t^{k,j})^{\alpha}\right)\right)\cdot \\ &\beta(1-\delta)\gamma(i_t^{k,j})^{-(1-\alpha)} = 0\end{aligned}$$

for $i_t^{k,j}$. Next, compute $c_t^{k,j} = a_t^k + wh_t^j - i_t^{k,j}$.

e) Compute $a_t^{k,j}$ from the budget constraint, hence

$$a_t^{k,j} = s^k - wh_t^j + c_t^{k,j} + i_t^{k,j}.$$

f) Save/Update both the value function and its derivatives

$$
\begin{aligned}
V_t\left(a_t^{k,j}, h_t^j\right) &= \frac{1}{1-\theta}\left(c_t^{k,j}\right)^{1-\theta} + \beta\left(1 - \phi\frac{1}{1+h_{t+1}^{k,j}}\right) V_{t+1}(a_{t+1}^k, h_{t+1}^{k,j}) \\
V_{t_a}\left(a_t^{k,j}, h_t^j\right) &= \left(c_t^{k,j}\right)^{-\theta} \\
V_{t_h}\left(a_t^{k,j}, h_t^j\right) &= \left(w + \frac{1}{\gamma}\left(i_T^{k,j}\right)^{1-\alpha}\right)\left(c_t^{k,j}\right)^{-\theta}.
\end{aligned}
$$

As EXOGM, HYBGM requires to run a numerical solver $[K \cdot J]$ times in each $t = T-1, \dots, 0$. However, computational burden is alleviated by reducing complexity of the equation system. Furthermore, as in ENDGM, it is possible to exactly determine the range of the borrowing constraint. In contrast to ENDGM in two dimensions, there is no need for a complex interpolation method.

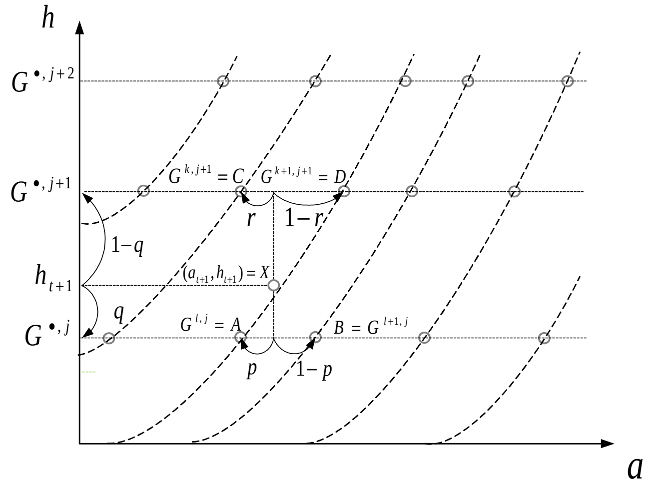
Remark 3.2. As ENDGM, HYBGM is not a general method. Suppose that consumption has an additional effect on human (or health capital). Consider for example an application where health capital is negatively affected by the consumption of junk food. Correspondingly rewrite (3.1) to

$$
h_{t+1} = (1-\delta)\left(h_t + f(i_t) - g\left(c_t\right)\right)
$$

to the effect that both controls c_t and i_t appear on both sides of the equation system even after applying the reformulation of endogenous states. This renders HYBGM inapplicable.

Hybrid Interpolation Hybrid interpolation, illustrated in Figure 3.6, is defined on a curvilinear grid where one dimension is being held constant. To locate any query point X hybrid interpolation proceeds in three steps. First, in the dimension of the exogenous grid (current state h_t) find the most narrow bracket of h_{t+1} and compute the weights according to the relative distance to these gridpoints. Second, in both rows, find those gridpoints that form the most narrow bracket of a_{t+1} and compute the according weights. Third, interpolation of any function of F at point X requires computing $F(X) = \varphi_A F(A) + \varphi_B F(B) + \varphi_C F(C) + \varphi_D F(D)$ with the four basis functions φ where $\varphi_A = p \cdot q$, $\varphi_B = (1-p) \cdot q$, $\varphi_C = r \cdot (1-q)$ and $\varphi_D = (1-r) \cdot (1-q)$ with $p = \frac{a_X - a_A}{a_B - a_A}$, $r = \frac{a_X - a_C}{a_D - a_C}$ and $q = \frac{h_X - h_C}{h_C - h_A}$. Thus, HYBGM reduces complexity of the problem without involving advanced interpolation procedures.

Figure 3.6: Hybrid Interpolation



Notes: Hybrid Interpolation. First, in the exogenous dimension, locate the two rows $G^{\bullet,j}$ and $G^{\bullet,j+1}$ that form the most narrow bracket of h_{t+1}. Second, locate in these two rows the gridpoints that form the most narrow bracket of a_{t+1}. Interpolation nodes: (k, j); $(k, j+1)$; $(l, j+1)$; $(l+1, j+1)$.

3.4 Results

We present results separately for the finite and infinite horizon versions of our model. Throughout, we use triple exponential grids for a, h, s, z, respectively. We set the range of grid $\mathcal{G}_s$ to $[0, 500]$ and of $\mathcal{G}_z$ to [1,500]. The according grids $\mathcal{G}_a$ and $\mathcal{G}_h$ are adjusted to cover the corresponding range of the state space.[18]

3.4.1 Error Evaluation

In both the finite and the infinite horizon version of the model, evaluation of accuracy of the solution is done by applying normalized Euler equation errors, cf. Judd (1992), as has become standard in the literature, cf., e.g., Santos (2000) and Barillas and Fernandez-Villaverde (2007). In our approach we get the Euler equation errors e_1 and e_2 by using the respective envelope conditions and combine them

[18] Also observe, by construction, there is only one occasionally binding constraint in our model. This would be different in a situation with durable consumption goods as in Hintermaier and Koeniger (2010). As ENDGM is a very efficient way in dealing with occasionally binding constraints such an alternative model may further improve the relative performance of ENDGM.

with the FOCs to get:

$$e_{1,t} = 1 - \frac{\left(Rs(h_{t+1})\beta\,(c_{t+1})^{-\theta}\right)^{-\frac{1}{\theta}}}{c_t}, \tag{3.8a}$$

$$e_{2,t} = 1 - \frac{\left(\frac{R}{(1-\delta)}\left(\frac{s_h(h_{t+1})V_{t+1}}{s(h_{t+1})(c_{t+1})^{-\theta}} + w + \frac{1}{\gamma} i_{t+1}^{1-\alpha}\right)^{-1}\right)^{-\frac{1}{\alpha}}}{i_t}. \tag{3.8b}$$

These errors are dimension free quantities. Equation (3.8a) expresses the optimization error as a fraction of current consumption. An error of $e_{1,t}$=10^{-3}, for instance, means that the household makes a \$1 mistake for each \$1000 spent, cf. Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006). These errors are expressed in units of base 10-logarithm which means that -4 is an error of 0.0001.

3.4.2 Finite Horizon

We iterate over T=100 time periods. Computational speed of the respective algorithms is measured in seconds. To compare all three methods in terms of accuracy we simulate 100 life-cycles profiles and evaluate Euler equation errors accordingly. Initial assets a_0 are set in the range $[10, 100]$ whereas initial human capital h_0 is drawn from the range [50,100]. For each simulation and each age we compute $e_{1,t}$ and $e_{2,t}$ from equation (3.8).[19] We next compute average and maximum errors across all simulations and ages. These are provided in Table 3.1. Both are of similar magnitudes across algorithms. To evaluate the relative performance of the different algorithms, we can therefore further concentrate on comparison of speed only.

Table 3.1 shows computing times for EXOGM, ENDGM and HYBGM for different numbers of gridpoints. We report absolute computing time as well as relative speed, i.e., relative to the ENDGM method. As our model is (on purpose) very stylized, absolute computing times are low across all models. However, relative speed is the relevant measuring rod because absolute speed scales up in the complexity of the model's specification, e.g., in fully stochastic models, applications in general equilibrium or estimation of models with structural methods. With regard to this relative comparison, observe from Panel (a) of Figure 3.7 that EXOGM is outperformed by both ENDGM and HYBGM.

Panel (b) of Figure 3.7 shows that ENDGM has a relative advantage in comparison to HYBGM in solving the model with a relatively small number of gridpoints.

[19] Euler equation errors are not computed if the borrowing constraint is binding.

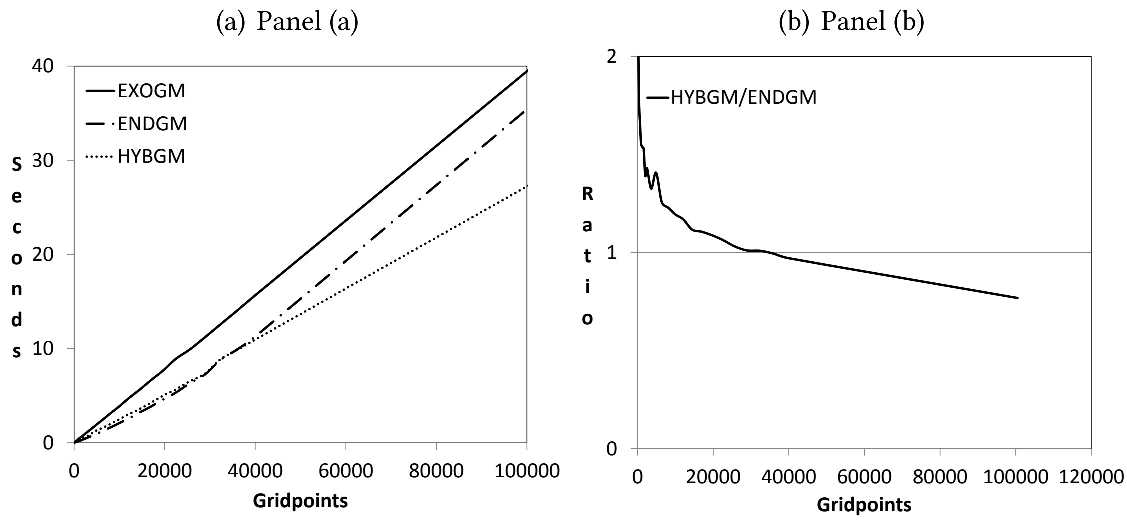
Table 3.1: Finite Horizon Model: Performance Results

	Speed		Euler Equation Error	
Number of Gridpoints for (a, h)	Seconds	Relative to ENDGM	Maximum for c ; i	Average for c ; i
ENDGM				
(25,25)	0.094	–	-2.56; -2.17	-3.70; -2.94
(50,50)	0.437	–	-2.92; -2.60	-4.36; -3.53
(100,100)	2.090	–	-3.37; -3.07	-4.91; -4.05
(200,200)	11.278	–	-3.84; -3.47	-5.44; -4.51
HYBGM				
(25,25	0.156	1.7	-2.62; -2.25	-3.88; -2.90
(50,50)	0.624	1.4	-2.99; -2.71	-4.43; -3.52
(100,100)	2.496	1.2	-3.43; -3.10	-5.00; -3.98
(200,200)	10.218	0.9	-4.16; -3.52	-5.54; -4.45
EXOGM				
(25,25)	0.234	2.5	-2.60; -2.24	-3.89; -2.90
(50,50)	0.982	2.3	-2.95; -2.71	-4.42; -3.52
(100,100)	3.868	1.9	-3.42; -3.10	-4.99; -3.98
(200,200)	15.663	1.4	-4.18; -3.52	-5.54; -4.45

Notes: Computing time for T=100 and resulting maximum and average Euler equation errors. Computing time is reported in seconds and absolute errors in units of base-10 logarithms.

At a grid size of 25^2, ENDGM is about 1.7 times faster than HYBGM. For solving the model with a higher number of gridpoints, however, HYBGM is advantageous. At a grid size of 300^2, HYBGM is about 1.3 times faster than ENDGM. In our setting the break-even point between both algorithms is at a number of 180^2 gridpoints and a computing time of 8.8s. As can be seen from Table 3.1, for a standard choice of 25 to 50 gridpoints in each dimension, ENDGM is $\frac{0.624}{0.437} \approx 1.4$ to $\frac{0.156}{0.094} \approx 1.7$ times faster than HYBGM and $\frac{0.982}{0.437} \approx 2.3$ to $\frac{0.234}{0.094} \approx 2.5$ times faster than EXOGM.

Figure 3.7: Finite Horizon Model: Speed



Notes: Panel (a): Computing time as a function of gridpoints in seconds (with equally many gridpoints in both dimensions). Solid line: computing time of EXOGM; dotted line: computing time of HYBGM; dashed-dotted line: computing time of ENDGM. Panel (b): Ratio of computing time of ENDGM to HYBGM as a function of gridpoints (with equally many gridpoints in both dimensions).

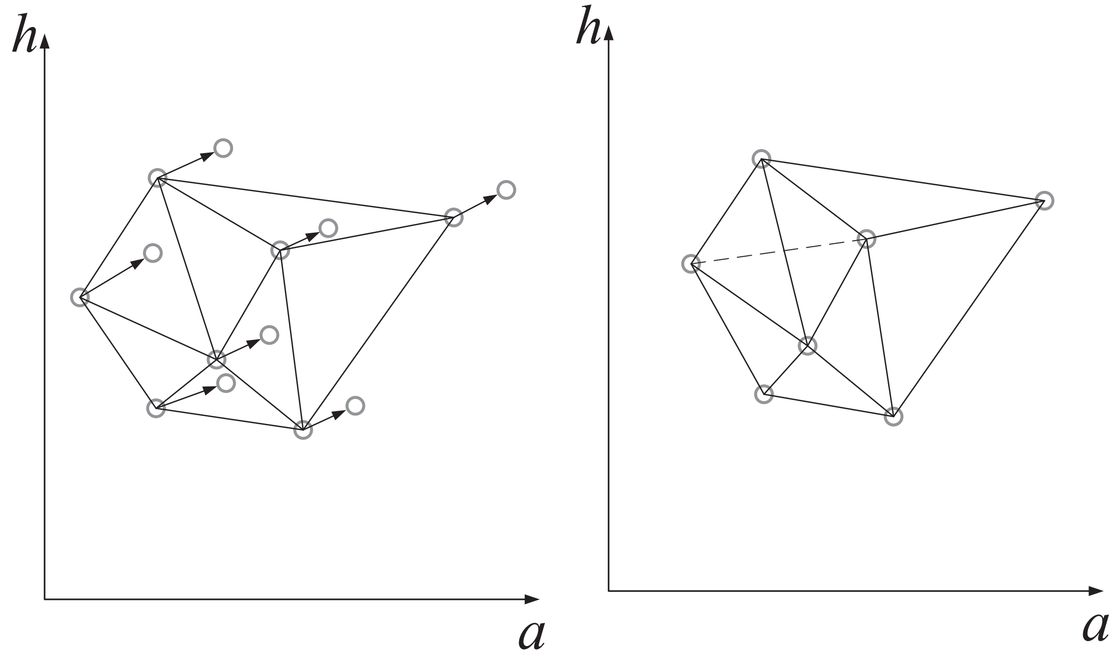
3.4.3 Infinite horizon

To compare the algorithms in the infinite horizon setting, we make the same initial guesses for derivatives V_{0_a} and V_{0_h} and iterate until convergence on policy functions subject to convergence criterion $\varepsilon = 10^{-6}$ in terms of the maximum absolute distance of policy functions. In the infinite horizon setting, speed of ENDGM can be increased if the Delaunay Triangulation is not constructed every iteration. Instead, we hold the triangulation pattern fixed after a certain number of iterations—50 in our case. We call this modification of the algorithm "Approximate Delaunay". Figure 3.8 illustrates this. Panel (a) of the figure shows how endogenous grid-points move in the (a, h) space from one iteration to the next. Panel (b) shows the new triangulation, holding constant the respective triangles from Panel (a). However, this triangulation is not Delaunay because edge P_1-P_2 becomes illegal.

In "Approximate Delaunay" it is necessary to ensure that the endogenously computed gridpoints form a convex hull. This might be violated without further adjustments. For example, in our illustration in panel (b) of Figure 3.8 violation of convexity would occur if point P_3 is shifted even further to the right. In such cases we redo the entire Delaunay tessellation.

Figure 3.8: Infinite Horizon Model: Approximate Delaunay

(a) Panel (a) (b) Panel (b)



Notes: Panel (a): In each iteration of ENDGM the gridpoints are relocated. Distance and direction of this movement is different for each gridpoint. Panel (b): The resulting grid might not be Delaunay - Edge between P_1 and P_2 becomes illegal and must be flipped to P_3 and P_4. *Approximate Delaunay* keeps the old triangulation in order to save computing time, accepting a less accurate interpolation.

To compute Euler equation errors we simulate the model for various different initial conditions of financial assets and health capital over 50 periods. We set initial assets a_0 in the range of [100,400] and the health capital stock in the range of [40,80]. We compute $e_{1,t}$ and $e_{2,t}$ from equation (3.8) for the first 50 periods.

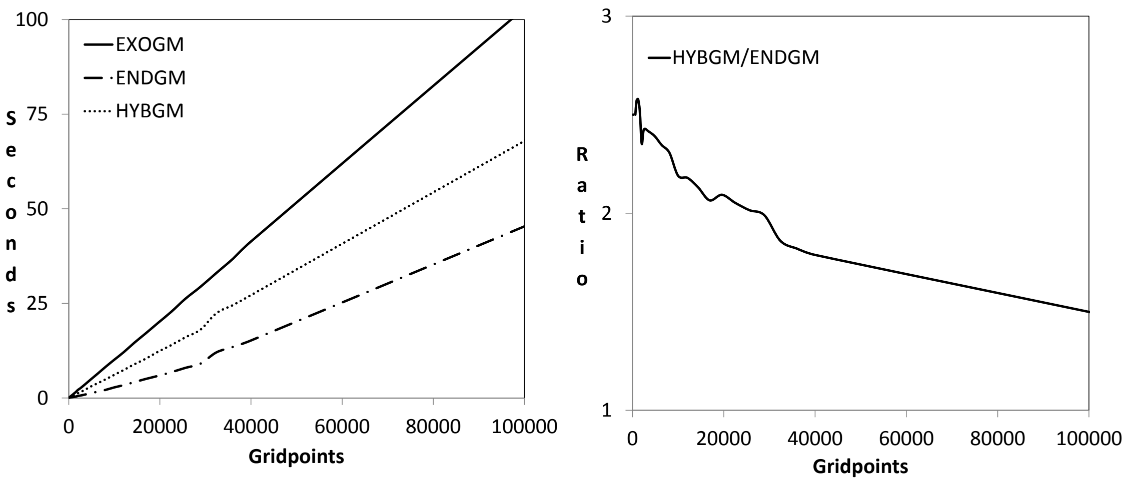
Average and maximum errors are provided in Table 3.2.

As in the finite horizon setting, average Euler equation errors are of similar magnitudes across algorithms—which we also achieve by appropriate settings of the respective numerical routines—so that we can again further concentrate on a comparison of speed only.[20]

We find that ENDGM is the fastest method for all numbers of gridpoints considered. In this respect our findings differ from the finite horizon version of the model in which the speed advantage of ENDGM relative to HYBGM was found to depend on the number of gridpoints. The reason for this difference is the use of the variant "Approximate Delaunay" in the infinite horizon model, as described above. As in the finite horizon model, the comparative advantage of ENDGM decreases in the number of gridpoints. Both, ENDGM and HYBGM, again clearly dominate EXOGM. For a standard choice of 25 to 50 gridpoints in each dimension, ENDGM is $\frac{1.153}{0.624} \approx 2.4$ to $\frac{0.390}{0.156} \approx 2.5$ times faster than HYBGM and $\frac{2.527}{0.624} \approx 4.0$ to $\frac{0.640}{0.156} \approx 4.1$ times faster than EXOGM, cf. Table 3.2.

Figure 3.9: Infinite Horizon Model: Speed

(a) Panel (a) (b) Panel (b)



Notes: Panel (a): Computing time to convergence of policy functions (criterion ε=10^{-6}) as a function of gridpoints (with equally many gridpoints in both dimensions). Solid line: computing time of EXOGM; dotted line: computing time of HYBGM; dashed-dotted line: computing time of ENDGM. Panel (b): Ratio of computing time to convergence of ENDGM and HYBGM as a function of gridpoints (with equally many gridpoints in both dimensions).

[20] The maximum Euler equation errors are considerably higher for EXOGM. They occur in the simulations just before the depletion of all financial assets. This is due to the fact that we do not determine explicitly the region where the borrowing constraint becomes binding and accordingly have no gridpoints located there.

Table 3.2: Infinite Horizon Model: Performance Results

	Speed		Euler Equation Error	
Number of Gridpoints for (a, h)	Seconds	Relative to ENDGM	Maximum for c ; i	Average for c ; i
ENDGM				
(25,25)	0.156	–	-2.09; -2.10	-2.87; -2.87
(50,50)	0.624	–	-2.37; -2.40	-3.61; -3.52
(100,100)	2.792	–	-2.84; -2.91	-4.17; -4.15
(200,200)	15.194	–	-3.14; -3.24	-4.80; -4.66
HYBGM				
(25,25)	0.390	2.5	-2.16; -2.10	-2.92; -2.97
(50,50)	1.513	2.4	-2.49; -2.58	-3.73; -3.66
(100,100)	6.115	2.2	-2.91; -2.98	-4.29; -4.23
(200,200)	27.175	1.8	-3.19; -3.29	-4.91; -4.80
EXOGM				
(25,25)	0.640	4.1	-1.53; -1.64	-2.80; -2.87
(50,50)	2.527	4.0	-1.81; -1.92	-4.17; -4.52
(100,100)	10.109	3.6	-2.44; -2,55	-4.17; -4.15
(200,200)	41.371	2.7	-2.40; -2.51	-4.69; -4.66

Notes: Computing time to convergence of policy functions (criterion ε=10^{-6}) and resulting maximum and average Euler equation errors. Computing time is reported in seconds and absolute errors in units of base-10 logarithms.

3.5 Conclusion

We compare three numerical methods—the standard exogenous grid method (EXOGM), Carroll's method of endogenous gridpoints (ENDGM), cf. Carroll (2006), and a hybrid method (HYBGM), cf. also Hintermaier and Koeniger (2010)—to solve dynamic models with two continuous state variables and occasionally binding borrowing constraints. To illustrate and to evaluate these methods we develop a life-cycle consumption-savings model with endogenous human capital formation. Evaluation of methods is based on speed and accuracy in both a finite and an infinite horizon setting. We show that applying ENDGM gives rise to irregular grids. We emphasize that this leads to a trade-off: On the one hand, closed form solutions in ENDGM greatly simplify the problem relative to standard EXOGM. On the other hand, interpolation becomes more costly due to the irregularity of grids. We apply Delaunay methods to interpolate on these irregular grids.

Despite this more complex interpolation, we find that ENDGM outperforms EXOGM in both the finite as well as the infinite horizon version of the model. In the infinite horizon model, ENDGM also always dominates HYBGM. For a standard choice of 25 to 50 gridpoints in each dimension, ENDGM is 2.4 to 2.5 times faster than HYBGM and 4.0 to 4.1 times faster than EXOGM. As the number of gridpoints increases, interpolation on irregular grids becomes increasingly costly to the effect that the relative speed advantage of ENDGM decreases. This becomes more apparent in the finite horizon model. Here, ENDGM dominates HYBGM for small to medium sized problems whereas HYBGM dominates for a large number of gridpoints. For a standard choice of 25 to 50 gridpoints in each dimension, ENDGM is 1.4 to 1.7 times faster than HYBGM and 2.3 to 2.5 times faster than EXOGM.

Two additional remarks on ENDGM and HYBGM are in order. First, within the class of problems solvable with first-order methods, neither of the two is a general method. Both are applicable only to specific problems at hand. This requires restrictions on the model's specification and on functional forms. Second, as HYBGM uses analytical solutions in only one dimension and standard numerical methods in others, its relative advantage can be expected to decrease in the dimensionality of the problem. For example, in a three dimensional problem, as HYBGM can only use analytical solutions in one dimension, HYBGM requires to solve a two-dimensional problem numerically. On the other hand, however, complexity of interpolation in ENDGM will also increase. As we restrict attention to two dimensional problems in this chapter, we cannot address how this trade-off is ultimately resolved. We leave such extensions to higher dimensions for future research.

Appendix 3.A Derivation of FOC

The dynamic version of the household problem reads as

$$V_t(a_t, h_t) = \max_{c_t, i_t, a_{t+1}, h_{t+1}} \{u(c_t) + \beta s\,(h_{t+1})\,V_{t+1}(a_{t+1}, h_{t+1})\}$$

subject to

$$\begin{aligned} a_{t+1} &= R\,(a_t + wh_t - c_t - i_t) \\ h_{t+1} &= (1-\delta)\,(h_t + f\,(i_t)) \\ a_{t+1} &\geq 0. \end{aligned}$$

Assigning multiplier μ to the borrowing constraint, the two first order conditions with respect to c_t and i_t are:

$$\begin{aligned} &\frac{\partial V_t\,(a_t, h_t)}{\partial c_t} = u_c - \beta s\,(h_{t+1})\,V_{t+1_a} R - R\mu \stackrel{!}{=} 0 \\ &\Leftrightarrow u_c - \beta s\,(h_{t+1})\,RV_{t+1_a} = R\mu, \end{aligned} \tag{3.9}$$

$$\begin{aligned} &\frac{\partial V_t\,(a_t, h_t)}{\partial i_t} = s_h\,(h_{t+1})\,(1-\delta)\,f_i \beta V_{t+1} + s\,(h_{t+1})\,\beta\,(V_{t+1_a}\,(-R) + V_{t+1_h}\,(1-\delta)\,f_i) \\ &- R\mu \stackrel{!}{=} 0 \\ &\Leftrightarrow \\ &s_h\,(h_{t+1})\,(1-\delta)\,f_i \beta V_{t+1} + s\,(h_{t+1})\,\beta\,(V_{t+1_a}\,(-R) + V_{t+1_h}\,(1-\delta)\,f_i) = R\mu \end{aligned} \tag{3.10}$$

and $a_{t+1} \geq 0$, $\mu \geq 0$ and $a_{t+1}\mu = 0$.

In order to compute optimal policies we need to distinguish two cases.

Case 1: Interior Solution

In the first case the borrowing constraint is not binding so that μ=0. This reduces the system of equations to

$$\begin{aligned} u_c - \beta s\,(h_{t+1})\,RV_{t+1_a} &= 0 \\ s_h\,(h_{t+1})\,(1-\delta)\,f_i \beta V_{t+1} + s\,(h_{t+1})\,\beta\,(V_{t+1_a}\,(-R) + V_{t+1_h}\,(1-\delta)\,f_i) &= 0. \end{aligned}$$

Rearranging gives

$$u_c = \beta s\left(h_{t+1}\right)V_{t+1_a}R$$
$$f_i = \frac{R}{(1-\delta)}\frac{s\left(h_{t+1}\right)V_{t+1_a}}{s_h\left(h_{t+1}\right)V_{t+1}+s\left(h_{t+1}\right)V_{t+1_h}}.$$

Case 2: Corner Solution—Binding Borrowing Constraint

In the second case the borrowing constraint is binding so that a'=0 and μ >0. From (3.9) and (3.10) it then follows that

$$u_c = s_h\left(h_{t+1}\right)\beta V_{t+1} + s\left(h_{t+1}\right)\beta V_{t+1_h}\left(1-\delta\right)f_i \tag{3.11}$$

and

$$u_c = \beta\left(1-\delta\right)f_i\left(s_h\left(h_{t+1}\right)V_{t+1}+s\left(h_{t+1}\right)V_{t+1_h}\right)$$

$$a_{t+1} = 0 \Leftrightarrow c_t = a_t + wh_t - i_t.$$

Making use of our assumptions on functional forms, equation (3.11) reduces in EXOGM and HYBGM to

$$\begin{aligned}&\left(a_t+wh_t-i_t\right)^{-\theta} - \\ &\qquad \frac{1}{\left(\left(1-\delta\right)\left(h_t+\frac{\gamma}{\alpha}i_t^{\alpha}\right)\right)^2}V_{t+1}\left[0,\left(1-\delta\right)\left(h_t+\frac{\gamma}{\alpha}i_t^{\alpha}\right)\right)\beta\left(1-\delta\right)\gamma i_t^{-(1-\alpha)} \\ &-\left(1-\frac{1}{\left(1-\delta\right)\left(h_t+\frac{\gamma}{\alpha}i_t^{\alpha}\right)}\right)V_{t+1_h}\left(0,\left(1-\delta\right)\left(h_t+\frac{\gamma}{\alpha}i_t^{\alpha}\right)\right)\beta\left(1-\delta\right)\gamma i_t^{-(1-\alpha)} = 0\end{aligned}$$

and in ENDGM to

$$\begin{aligned}&\left(a_t+w\left(\frac{h_{t+1}}{1-\delta}-\frac{1}{\gamma}i_t^{1-\alpha}\right)-i_t\right)^{-\theta} - \frac{1}{\left(h_{t+1}\right)^2}\beta V_{t+1}\left(0,h_{t+1}\right)\left(1-\delta\right)\gamma i_t^{-(1-\alpha)} \\ &\qquad -\left(1-\frac{1}{1+h_{t+1}}\right)\beta V_{t+1_h}\left(0,h_{t+1}\right)\left(1-\delta\right)\gamma i_t^{-(1-\alpha)} = 0.\end{aligned}$$

Observe that this equation is not linear in i_t. We therefore need to use a numerical routine in the region where the borrowing constraint is binding also for ENDGM, cf. our discussion in the main text in Subsection 3.3.2.

In both cases—i.e., for interior solutions and for binding borrowing constraints—the envelope conditions are

$$
\begin{aligned}
\frac{\partial V_t\left(a_t, h_t\right)}{\partial a_t} &\equiv V_{t_a}=\beta V_{t+1_a} R+R \mu=u_c \\
\frac{\partial V_t\left(a_t, h_t\right)}{\partial h_t} &\equiv V_{t_h} \\
&=\beta s_h\left(h_{t+1}\right) V_{t+1}(a_{t+1}, h_{t+1})\left(1-\delta\right)+\beta s\left(h_{t+1}\right) V_{t_a}(a_{t+1}, h_{t+1}) w R+ \\
&\quad\ \beta s\left(h_{t+1}\right) V_{t+1_h}(a_{t+1}, h_{t+1})\left(1-\delta\right)+R \mu \\
&=\left(w+\frac{1}{f_i}\right) u_c .
\end{aligned}
$$

Bibliography

Adjaye-Gbewonyo, K. and I. Kawachi (2012). "Use of the Yitzhaki Index as a test of relative deprivation for health outcomes: A review of recent literature". *Social Science & Medicine* 75, 129 –137.

Ales, L., R. Hosseini, and L. E. Jones (2012). *Is There "Too Much" Inequality in Health Spending Across Income Groups?* NBER Working Papers. National Bureau of Economic Research, Inc.

Andersen, S. H. (2010). "The cost of sickness: On the effect of the duration of sick leave on post-sick leave earnings". *Social Science & Medicine* 70, 1581–1589.

Arai, M. and P. S. Thoursie (2005). "Incentives and selection in cyclical absenteeism". *Labour Economics* 12, 269–280.

Aruoba, S. B., J. Fernandez-Villaverde, and J. F. Rubio-Ramirez (2006). "Comparing solution methods for dynamic equilibrium economies". *Journal of Economic Dynamics and Control* 30, 2477–2508.

Askildsen, J. E., E. Bratberg, and O. A. Nilsen (2005). "Unemployment, labor force composition and sickness absence: a panel data study". *Health Economics* 14, 1087–1101.

Auld, M. C. (2005). "Smoking, Drinking, and Income". English. *The Journal of Human Resources* 40, pp. 505–518.

Baker, T. J. (1999). "Delaunay - Voronoi Methods". In: *Handbook of Grid Generation.* CRC Press. Chap. 16.

Barillas, F. and J. Fernandez-Villaverde (2007). "A generalization of the endogenous grid method". *Journal of Economic Dynamics and Control* 31, 2698–2712.

Ben-Porath, Y. (1967). "The Production of Human Capital and the Life Cycle of Earnings". *Journal of Political Economy* 75, 352.

Berg, M. de et al. (2008). *Computational Geometry: Algorithms and Applications.* 3rd. Springer.

Bergström, G. et al. (2009). "Does sickness presenteeism have an impact on future general health?" English. *International Archives of Occupational and Environmental Health* 82, 1179–1190.

Blackburn, E. H. and E. S. Epel (2012). "Telomeres and adversity: Too toxic to ignore". *Nature* 490, 169–171.

Bosworth, B. P. and K. Burke (2014). *Differential Mortality and Retirement Benefits in the Health and Retirement Study.* Working Papers, Center for Retirement Research at Boston College. Center for Retirement Research.

Broer, T., M. Kapicka, and P. Klein (2013). *Consumption risk sharing with private information when earnings are persistent.* Tech. rep. Simon Fraser University.

Brumm, J. and M. Grill (2014). “Computing equilibria in dynamic models with occasionally binding constraints”. *Journal of Economic Dynamics and Control* 38, 142 –160.

Buhmann, B. et al. (1988). “Equivalence Scales, Well-Being, Inequality, and Poverty: Sensitivity Estimates across Ten Countries Using the Luxembourg Income Study (LIS) Database”. *Review of Income and Wealth* 34, 115–42.

Bundeministerium für Arbeit und Soziales (2014). *Sozialbudget 2013*. Tech. rep. Bundesministerium für Arbeit und Soziales.

Bundesagentur für Arbeit (2014). *Arbeitslosigkeit im Zeitverlauf*. Tech. rep. Bundesagentur für Arbeit.

Carroll, C. D. (2006). “The method of endogenous gridpoints for solving dynamic stochastic optimization problems”. *Economics Letters* 91, 312–320.

Chadi, A. and L. Goerke (2015). *Missing at Work - Sickness-related Absence and Subsequent Job Mobility*. IAAEU Discussion Papers. Institute of Labour Law and Industrial Relations in the European Union (IAAEU).

Clark, A. E. and C. Senik (2010). “Who Compares to Whom? The Anatomy of Income Comparisons in Europe”. *The Economic Journal* 120, 573–594.

Cole, H. L., S. Kim, and D. Krueger (2014). *Analyzing the Effects of Insuring Health Risks: On the Trade-off between Short Run Insurance Benefits vs. Long Run Incentive Costs*. PIER Working Paper Archive. Penn Institute for Economic Research, Department of Economics, University of Pennsylvania.

Cuesta, M. B., E. Cottini, and A. A. Herrarte (2012). *Socioeconomic Gradient in Health: How Important is Material Deprivation?* Tech. rep. AIAS, Amsterdam Institute for Advanced Labour Studies.

Cutler, D. M., A. Lleras-Muney, and T. Vogl (2011). “Socioeconomic Status and Health: Dimensions and Mechanisms”. In: *The Oxford Handbook of Health Economics*. Oxford University Press.

Dale-Olsen, H. (2014). “Sickness Absence, Sick Leave Pay, and Pay Schemes”. *Labour* 28, 40–63.

Daly, M. C. et al. (1998). “Macro-to-micro links in the relation between income inequality and mortality.” *The Milbank quarterly* 76, 315–339.

Deaton, A. S. and C. Paxson (2001). “Mortality, Education, Income, and Inequality among American Cohorts”. In: *Themes in the Economics of Aging*. National Bureau of Economic Research, Inc.

Delaunay, B. N. (1934). “Sur la sphère vide”. *Bulletin of Academy of Sciences of the USSR*, 793–800.

Devillers, O., S. Pion, and M. Teillaud (2001). “Walking in a triangulation”. In: *Proceedings of the seventeenth annual symposium on Computational geometry*. Medford, Massachusetts, United States.

Ehrlich, I. and H. Chuma (1990). "A Model of the Demand for Longevity and the Value of Life Extension". *Journal of Political Economy* 98, 761–82.

Eibner, C., R. Sturm, and C. R. Gresenz (2004). "Does Relative Deprivation Predict the Need for Mental Health Services?" *The Journal of Mental Health Policy and Economics* 7, 167–175.

Elstad, J. I. (1998). "The Psycho-social Perspective on Social Inequalities in Health". *Sociology of Health & Illness* 20, 598–618.

Ettner, S. L. (1996). "New evidence on the relationship between income and health". *Journal of Health Economics* 15, 67–85.

Fella, G. (2014). "A generalized endogenous grid method for non-smooth and non-concave problems". *Review of Economic Dynamics* 17, 329–344.

Fernandez-Villaverde, J. and D. Krueger (2007). "Consumption over the Life Cycle: Facts from Consumer Expenditure Survey Data". *The Review of Economics and Statistics* 89, 552–565.

Finkelstein, A., E. F. P. Luttmer, and M. J. Notowidigdo (2013). "What good is wealth without health? The effect of health on the marginal utility of consumption". *Journal of the European Economic Association* 11, 221–258.

Georgarakos, D., M. Haliassos, and G. Pasini (2012). *Household debt and social interactions.* Tech. rep. Center for Financial Studies (CFS).

Gilleskie, D. B. (1998). "A Dynamic Stochastic Model of Medical Care Use and Work Absence". *Econometrica* 66, 1–46.

Grossman, M. (1972). "On the Concept of Health Capital and the Demand for Health". *Journal of Political Economy* 80, 223–55.

Hall, R. E. and C. I. Jones (2007). "The Value of Life and the Rise in Health Spending". *The Quarterly Journal of Economics* 122, 39–72.

Halliday, T., H. He, and H. Zhang (2012). *Health Investment over the Life-Cycle.* Working Papers. University of Hawaii at Manoa, Department of Economics.

Henrekson, M. and M. Persson (2004). "The Effects on Sick Leave of Changes in the Sickness Insurance System". *Journal of Labor Economics* 22, 87–114.

Hesselius, P. (2007). "Does sickness absence increase the risk of unemployment?" *The Journal of Socio-Economics* 36, 288–310.

Hintermaier, T. and W. Koeniger (2010). "The method of endogenous gridpoints with occasionally binding constraints among endogenous variables". *Journal of Economic Dynamics and Control* 34, 2074–2088.

Huggett, M., G. Ventura, and A. Yaron (2011). "Sources of Lifetime Inequality". *American Economic Review* 101, 2923–54.

Hujer, R. et al. (2001). "Testing for uniform wage trends in West-Germany: A cohort analysis using quantile regressions for censored data". *Empirical Economics* 26, 41–86.

Idler, E. L. and Y. Benyamini (1997). "Self-Rated Health and Mortality: A Review of Twenty-Seven Community Studies". English. *Journal of Health and Social Behavior* 38, pp. 21–37.

Iskhakov, F. et al. (2014). *Estimating Discrete-Continuous Choice Models: Endogenous Grid Method with Taste Shocks.* Tech. rep.

Joe, B (1991). "GEOMPACK: A software package for the generation of meshes using geometric algorithms". *Advances in Engineering Software and Workstations* 13, 325–331.

Johansson, P. and M. Palme (2005). "Moral hazard and sickness insurance". *Journal of Public Economics* 89, 1879 –1890.

Jones, A. M. and J. Wildman (2008). "Health, income and relative deprivation: Evidence from the BHPS". *Journal of Health Economics* 27, 308 –324.

Judd, K. L. (1992). "Projection methods for solving aggregate growth models". *Journal of Economic Theory* 58, 410–452.

——— (1998). *Numerical Methods in Economics.* 1st ed. Vol. 1. The MIT Press.

Keane, M. P. and K. I. Wolpin (1997). "The Career Decisions of Young Men". *Journal of Political Economy* 105, 473–522.

Kivimäki, M. et al. (2005). "Working While Ill as a Risk Factor for Serious Coronary Events: The Whitehall II Study." *American Journal of Public Health* 95(1), 98 –102.

Kondo, N. et al. (2008). "Do social comparisons explain the association between income inequality and health?: Relative deprivation and perceived health among male and female Japanese individuals". *Social Science & Medicine* 67, 982 –987.

Krueger, D. and A. Ludwig (2007). "On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare". *Journal of Monetary Economics* 54, 49–87.

Laun, T. (2013). *Optimal Social Insurance with Endogenous Health.* SSE/EFI Working Paper Series in Economics and Finance. Stockholm School of Economics.

Leigh, J. P. (1985). "The effects of unemployment and the business cycle on absenteeism". *Journal of Economics and Business* 37, 159–170.

Long, J. and J. Freese (2006). *Regression models for categorical dependent variables using Stata.* StataCorp LP.

Mangyo, E. and A. Park (2011). "Relative Deprivation and Health: Which Reference Groups Matter?" *Journal of Human Resources* 46, 459–481.

Markussen, S. (2012). "The individual cost of sick leave". *Journal of Population Economics* 25, 1287–1306.

Marmot, M. et al. (1991). "Health inequalities among British civil servants: the Whitehall II study". *The Lancet* 337, 1387 –1393.

McEwen, B. S. (1998). "Protective and Damaging Effects of Stress Mediators". *New England Journal of Medicine* 338, 171–179. eprint: http://dx.doi.org/10.1056/NEJM199801153380307.

McLaughlin, K. A. et al. (2012). "Socioeconomic Status and Adolescent Mental Disorders". *American Journal of Public Health* 102, 1742–1750.

Miilunpalo, S. et al. (1997). "Self-rated health status as a health measure: The predictive value of self-reported health status on the use of physician services and on mortality in the working-age population". *Journal of Clinical Epidemiology* 50, 517 –528.

Miranda, M. J. and P. L. Fackler (2004). *Applied Computational Economics and Finance.* 1st ed. Vol. 1. The MIT Press.

Nuebling, M. et al. (2007). "Computation of Standard Values for Physical and Mental Health Scale Scores Using the SOEP Version of SF12v2". *Schmollers Jahrbuch : Journal of Applied Social Science Studies / Zeitschrift für Wirtschafts- und Sozialwissenschaften* 127, 171–182.

Ozkan, S. (2014). *Preventive vs. Curative Medicine: A Macroeeconomic Analysis of Health Care over the Life Cycle.* Tech. rep. University of Toronto.

Pence, K. M. (2006). "The Role of Wealth Transformations: An Application to Estimating the Effect of Tax Incentives on Saving". *The B.E. Journal of Economic Analysis & Policy* 5, 1–26.

Pfeifer, C. (2013). "Cyclical Absenteeism Among Private Sector, Public Sector And Self-Employed Workers". *Health Economics* 22, 366–370.

Pham-Kanter, G. (2009). "Social comparisons and health: Can having richer friends and neighbors make you sick?" *Social Science & Medicine* 69, 335 –344.

Press, W. et al. (1996). *Numerical Recipes in Fortran 90: Volume 2, Volume 2 of Fortran Numerical Recipes: The Art of Parallel Scientific Computing*. Fortran Numerical Recipes , Vol 2. Cambridge University Press.

Press, W. et al. (2007). *Numerical Recipes 3rd Edition: The Art of Scientific Computing*. Numerical Recipes: The Art of Scientific Computing. Cambridge University Press.

Puhani, P. A. and K. Sonderhof (2010). "The effects of a sick pay reform on absence and on health-related outcomes". *Journal of Health Economics* 29, 285–302.

Santos, M. S. (2000). "Accuracy of Numerical Solutions Using the Euler Equation Residuals". English. *Econometrica* 68, pp. 1377–1402.

Scheil-Adlung, X. and L. Sandner (2010). *The Case for Paid Sick Leave.* World Health Report Background Paper. World Health Organization.

Seeman, T. E. et al. (2001). "Allostatic Load as a Marker of Cumulative Biological Risk: MacArthur Studies of Successful Aging". English. *Proceedings of the*

National Academy of Sciences of the United States of America 98, pp. 4770–4775.
Siegel, J. (2002). *Stocks for the Long Run*. McGraw-Hill Education.
Smith, J. P. (1999). "Healthy Bodies and Thick Wallets: The Dual Relation between Health and Economic Status". English. *The Journal of Economic Perspectives* 13, pp. 145–166.
Soetevent, A. R. (2006). "Empirics of the Identification of Social Interactions; An Evaluation of the Approaches and Their Results". *Journal of Economic Surveys* 20, 193–228.
Stokey, N. L. and R. E. Lucas (1989). *Recursive Methods in Economic Dynamics*. Harvard University Press.
Storesletten, K., C. I. Telmer, and A. Yaron (2004). "Consumption and risk sharing over the life cycle". *Journal of Monetary Economics* 51, 609–633.
Subramanyam, M. et al. (2009). "Relative deprivation in income and self-rated health in the United States". *Social Science & Medicine* 69, 327 –334.
Tauchen, G. (1986). "Finite state markov-chain approximations to univariate and vector autoregressions". *Economics Letters* 20, 177–181.
Teppa, F. and C. Vis (2012). *The CentERpanel and the DNB Household Survey: Methodological Aspects*. DNB Occasional Studies. Netherlands Central Bank, Research Department.
Wagner, G. G., J. R. Frick, and J. Schupp (2007). "The German Socio-Economic Panel Study (SOEP) - Scope, Evolution and Enhancements". *Schmollers Jahrbuch : Journal of Applied Social Science Studies / Zeitschrift für Wirtschafts- und Sozialwissenschaften* 127, 139–169.
Wilkinson, R. (2000). *Mind the Gap: Hierarchies, Health and Human Evolution*. Darwinism Today. Weidenfeld & Nicolson.
Ziebarth, N. R. (2013). "Long-term absenteeism and moral hazard - Evidence from a natural experiment". *Labour Economics* 24, 277–292.
Ziebarth, N. R. and M. Karlsson (2010). "A natural experiment on sick pay cuts, sickness absence, and labor costs". *Journal of Public Economics* 94, 1108–1122.
——— (2014). "The effects of expanding the generosity of the statutory sickness insurance system". *Journal of Applied Econometrics* 29, 208 –230.

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