



Does problem posing affect self-efficacy, task value, and performance in mathematical modelling?

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Abstract

Problem posing is a promising teaching method for enhancing motivation and performance in mathematics and more specifically in mathematical modelling. Hence, the goals of our study were twofold: (1) to examine the effects of problem posing on modelling performance, self-efficacy, and task values in solving modelling problems, and (2) to analyze whether problem posing affects modelling performance via self-efficacy and task values. In a randomized control trial involving ninth- and tenth-grade students ($N=210$), participants were assigned to either a problem-posing and problem-solving group or to one of two problem-solving groups. Students in the problem-posing and problem-solving group received a booklet with descriptions of real-world situations and were prompted to pose and subsequently solve their own problems. Students in the two problem-solving groups received the same real-world situations with given problems and were asked to solve the problems. Before solving the problems, students in all groups reported their self-efficacy and task values. Prompting students to pose their own problems positively enhanced students' self-efficacy and partially improved their task values in solving modelling problems. Further, problem posing indirectly affected modelling performance via self-efficacy but not task values. However, problem posing had no total effect on modelling performance. The findings for self-efficacy and task values are in line with expectancy-value theories, adding new insights to the field by highlighting the importance of motivational constructs in problem-posing approaches and instructions aimed at fostering mathematical modelling.

Keywords Problem posing · Mathematical modelling · Motivation · Self-efficacy · Task values · Intervention · Real-world problems · Problem solving

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One of the central goals of a mathematics education is to enable students to apply their mathematical knowledge in their actual current and future lives (Niss & Blum, 2020, p. 28). This goal has been highlighted in recent calls for the future of math education (Cevikbas et al., 2022; Schukajlow et al., 2023a) and in many national curricula, such as in Germany (Kultusministerkonferenz (KMK), 2022) and the USA (National Governors Association Center for Best Practices, 2010). To achieve this goal, teachers ask students to solve modelling problems in mathematics classes. Working on modelling problems is intended to help students develop the specific competencies needed to solve problems in real life with the help of mathematics. However, research has shown that solving modelling problems is often demanding for students (Niss & Blum, 2020, pp. 94–95), and some studies have demonstrated that students' motivation to solve modelling problems tends to be lower than for other types of problems (Krawitz & Schukajlow, 2018). Consequently, there seems to be a need to help students develop modelling competencies and foster their motivation to work on modelling problems. Building on previous research on fostering students' modelling competencies (for an overview, see Cevikbas, 2022) and their motivation in modelling (e.g., Czoher et al., 2020; Krawitz et al., 2022; Schukajlow et al., 2019), we focused on prompting learners to pose their own modelling problems. We consider this approach promising for two reasons. First, problem posing and modelling are genuinely related to each other (English et al., 2005; Galbraith & Stillman, 2001; Hansen & Hana, 2015). In the real world, individuals often need to formulate problems themselves before solving them (Kilpatrick, 1987), and thus problem posing can affect modelling. Second, we view problem posing as a promising way to enhance motivation (Cai & Leikin, 2020). When learners engage in problem posing, they have choices in their problem posing processes, allowing them to establish their own goals (Bonotto & Santo, 2015; Silver, 1994) or create a problem that aligns with their own goals. According to motivational theories, such as Eccles' expectancy-value theory (Eccles & Wigfield, 2020), establishing one's own goals is important for enhancing self-efficacy and task values, which, in turn, are important factors that influence performance-related outcomes. Thus, prompting students to pose their own problems for real-world situations might improve their self-efficacy and task values in modelling, in turn potentially enhancing their performance in solving modelling tasks. Despite the importance of problem posing for motivation and modelling, it remains relatively underexplored in modelling research. Thus, in this study, we aimed to investigate how prompting students to pose their own problems for given real-world situations affects their self-efficacy, task values, and modelling performance. Further, this study was designed to uncover the mechanisms of action between problem posing, self-efficacy, task values, and modelling performance by proposing and testing a theoretical model.

1 Effects of problem-posing interventions on mathematical modelling, self-efficacy, and task values

1.1 Mathematical modelling

Mathematical modelling is the process of translating a problem from the real world into a mathematical world with the aim of solving the problem (Niss et al., 2007). Aligned with the cognitive perspective on mathematical modelling (Kaiser & Sriraman, 2006; Schukajlow et al., 2023a), several theoretical models (e.g., Blum & Leiss, 2007; Galbraith & Stillman, 2006; Verschaffel et al., 2000), so-called modelling cycles, depict modelling as

a cyclic process. Modelling cycles begin with the real-world situation and involve going back and forth into the mathematical world while passing through several cognitive activities. These activities include understanding the situation and structuring and simplifying the information in order to set up a mental model of the idealized situation called the real (or real-world) model. These activities are subsumed under the term premathematization (Niss & Blum, 2020, p. 14). The real model is a mental model anchored in the real world that includes simplifications of the initially complex real-world situation (for a detailed description and examples, see Krawitz et al., 2022). After the construction of the real model, this model can be transferred into a mathematical model, mathematical procedures can be applied, and the results can be interpreted and validated with respect to the real-world situation.

Each step in the modelling process is a potential barrier, and previous studies have shown that significant difficulties arise even when conducting premathematizing activities (Krawitz et al., 2022; Leiss et al., 2010). Students often carry out premathematizing activities superficially by using keyword strategies with the goal of applying mathematical procedures recently taught in class (Krawitz, 2020; Verschaffel et al., 2000). If a student is not able to master these activities, they will get stuck at the beginning of the problem-solving process, will work with inadequate models, and will not be able to solve the problem. Therefore, research is necessary to find out which instructions are successful in helping students cope with the demands of modelling, including instructions that focus on premathematizing activities. Instructions in modelling include comprehensive teaching interventions and brief instructions, such as providing prompts (for an overview see Cevikbas, 2022). Here, we specifically focused on the latter. Studies examining the effects of prompts show mixed results (Krawitz et al., 2022; Schukajlow et al., 2023b; Wijaya, 2017), suggesting that prompts tailored to the specific problem at hand positively influence solution performance, while general prompts tend to be less beneficial. However, specific prompts may only aid in solving the problem at hand and may contribute less to learning modelling compared to more general prompts that can be applied across various problems. Therefore, further research is needed to identify how to balance the specificity and the generality of the prompts and develop prompts that assist students in solving modelling problems. Prompting students to pose their own problems for given real-world situations could be one such general prompt because it might help students develop a deeper understanding of the real-world situation, thus leading to an overall improvement in modelling performance.

1.2 Modelling-related problem posing


Problem posing is considered to be a powerful teaching approach for enhancing the learning of mathematics (Cai & Leikin, 2020; Liljedahl & Cai, 2021). The number of studies focusing on problem posing has increased in recent years, but the number of studies, particularly intervention studies, has remained low in comparison with those on problem solving (Lee, 2020). According to Silver (1994), problem posing is defined as both the generation of new problems and the reformulation of given problems before, during, or after the problem-solving process. This definition subsumes two different cognitive activities under the term problem posing (generation and reformulation). Here, we focus on the generation of problems. The generation of problems can be stimulated by various problem posing tasks that can be categorized in different ways (Baumanns & Rott, 2021; Stoyanova & Ellerton, 1996). One possibility is to consider a problem-posing task as consisting of two elements: a problem situation and a problem-posing

prompt (Cai & Hwang, 2023). The problem situation provides context and data and can be intra-mathematical (e.g., geometrical figures, sequences, equations) or connected to the real world (e.g., descriptions of real-world situations, photographs, artefacts). The problem-posing prompt tells learners how to work with the problem situation. Similar to the given situation, the problem-posing prompt can refer to posing either intra-mathematical problems or real-world problems. Hence, four different forms of problem-posing tasks can be distinguished by combining intra-mathematical and real-world problem situations and problem-posing prompts (see Table 1).

In this article, we focus on the fourth type of problem-posing tasks called modelling-related problem-posing. In this type of task, the problem situation and the problem-posing prompt are both connected to the real world. The description of the Salt Mountain real-world situation presented in Fig. 1 serves as an example of a problem situation with a connection to the real world. Along with a prompt asking students to pose problems, this problem situation is a problem-posing task that can be categorized as a modelling-related problem-posing task. We labelled this task as a problem-posing task because students can pose modelling problems using information from the real-world situation (i.e., they can develop an authentic and open real-world problem; see Krawitz et al., 2024).

Examples of possible questions that can be posed for this problem situation are as follows: How many trips does the truck have to make to transport the salt mountain away? How long will it take to transport the salt mountain away? What is the height of the salt mountain?

Table 1 Types of problem-posing tasks

		Problem-posing prompt	
		Intra-mathematical	Connected to real world
Problem situation	Intra-mathematical	(1) Intra-mathematical problem posing Example (e.g., Liu et al., 2020): Situation:  Prompt: Pose a mathematical problem that refers to the number pyramid.	(2) Application-related problem posing Example (e.g., Tichá & Hošpesová, 2013): Situation: $\frac{1}{2}; \frac{1}{4}$ Prompt: Pose a word problem that refers to the fractions.
		(3) Mathematizing problem posing Example (e.g., Christou et al., 2005): Situation: Alex has 180 pencils, while Chris has 25 pencils more than Alex. Prompt: Write a question for the story so that the answer to the problem is 385.	(4) Modelling-related problem posing Example: Situation: Problem situation presented in Figure 1 Prompt: Pose a problem on the basis of this real-world situation.
	Connected to real world		

Salt Mountain

In the Middle Ages, salt was obtained from the evaporation of sea water. Nowadays, it is mainly obtained through mining. The salt (1.2 t per m³) is piled up into high salt mountains by means of 1.2 m wide conveyor belts. Later, it is transported away by trucks.

In the figure, you see a huge salt mountain. The length of its edge is $c = 20$ m, and its diameter is $d = 30$ m. This salt mountain weighs about 3,740 tons.

The technical data for one of the trucks that carry away the salt are:

Truck loading platform:	5.11 m x 2.3 m x 1.8 m
Power:	125 kw (175 HP)
Cubic capacity:	4249 cm ³
Maximum load capacity:	26.8 t



Fig. 1 The Salt Mountain problem situation (adapted from Schukajlow & Krug, 2014)

1.3 Effect of problem posing on modelling

To pose a problem for a real-world situation, learners need to understand and explore the situation and simplify and structure the given information (Hartmann et al., 2023). These activities are crucial for successfully solving modelling problems. Prompting students to pose their own problems can therefore stimulate students' engagement in modelling activities in a more comprehensive manner, thereby fostering their overall modelling performance.

Indications of the positive effect of problem posing on modelling have come from problem-solving research, where theoretical models (Baumanns & Rott, 2022; Cruz, 2006) aimed at describing the processes involved in problem posing have emphasized the close relationship between problem posing and problem solving. Findings from empirical studies have further confirmed the positive relationship between these constructs (Silver & Cai, 1996; Xie & Masingila, 2017) and demonstrated that problem posing in long-term interventions can improve problem-solving performance (Chen et al., 2015; Rudnitsky et al., 1995). For word problem solving, Rudnitsky et al. (1995) reported larger gains in the problem solving of primary school students who participated in a problem-posing intervention compared with their peers in a problem-solving intervention. However, the results of these studies cannot be directly transferred to the current approach because these studies focused on word problem solving and not on modelling and conducted problem-posing teaching interventions that lasted for longer periods of time (e.g., 18 days in the study by Rudnitsky et al. (1995), and 23 weeks with 90 min per week in the study by Chen et al. (2015)). Short-term interventions might be promising, as they can be easily implemented in teaching practice, and they can be scaled up by repeated use in the classroom. To the best of our knowledge, the effects of a short-term intervention (e.g., providing problem-posing prompts) have yet to be analyzed.

1.4 Self-efficacy and task values in mathematical modelling

There is a broad consensus that it is important to support students' motivation. Modern expectancy-value theories, such as the recent situative expectancy-value theory (Eccles & Wigfield, 2020), highlight the importance of learners' expectancies for success and

the subjective task values they attribute to academic tasks for their academic motivation, performance, and educational choices. Consequently, interventions aimed at fostering students' motivation should target both their expectancies for success and their task values. Expectancies for success are "individuals' beliefs about how well they will do on an upcoming task" (Eccles & Wigfield, 2020, p. 3). This construct is closely related to performance-related beliefs in other motivation theories, particularly the construct of self-efficacy in Bandura's (1997) social-cognitive theory (see Wigfield & Eccles, 2000, for a comparison). Self-efficacy is defined as "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (Bandura, 1997, p. 3).

Task values refer to the question, "Do I want to do this activity and why?" (Eccles et al., 1998, p. 1028). Eccles et al. (1983) identified four types of values that refer to the various reasons why a particular task can be perceived as appealing or not: the importance of doing well (attainment value), the joy that comes from engaging in the task (intrinsic value), the importance of a task for a future goal (utility value), and the negative aspects of engaging a task (perceived cost).

Several factors contribute to the development of self-efficacy and task values. One of the most important sources of self-efficacy is mastery experience, the experience of success in completing challenging tasks or overcoming difficulties (Butz & Usher, 2015; Usher & Pajares, 2009). In addition, goals—even if they are short-term—are centrally important for students' outcome expectancies and task values (Eccles & Wigfield, 2020; Wigfield & Cambria, 2010). Having personal learning goals (e.g., to develop a career plan, succeed in a course, or master a task) are hypothesized to predict self-efficacy and task values, which in turn influence performance-related behaviors.

In previous studies, students and also preservice teachers were found to report similar (Schukajlow et al., 2012) or even lower (Böswald & Schukajlow, 2022; Krawitz & Schukajlow, 2018) self-efficacy and task values for modelling problems compared with intra-mathematical problems. These findings contradict theoretical assumptions that solving modelling problems enhances motivation (Blum & Niss, 1991; Kaiser & Sriraman, 2006). One possible explanation for these unexpected findings is the use of simple problems in these studies, which can be solved in regular lessons by students at all achievement levels. The results might be different if both modelling and mathematical problems were more complex or if they were conducted with high-achieving students. Another possible explanation is that students may have lacked experience in modelling and thus did not have enough opportunities to build mastery experience. In addition, they may have perceived low self-efficacy and task values because the problems did not relate to their personal goals. Interventions aimed at enhancing self-efficacy and task values in modelling should address these issues. We propose that prompting students to pose their own problems is an intervention that has the potential to serve this purpose.

1.5 Effect of problem posing on self-efficacy and task values

Prompting students to pose their own problems might enhance self-efficacy and task values for two reasons. First, while posing problems, students engage in problem solving and modelling activities (Baumanns & Rott, 2022; Hartmann et al., 2023), which can lead to the development of mastery experience and can therefore enhance self-efficacy. Second, problem posing allows students to generate problems that are aligned with their abilities,

prior knowledge and interests (Bonotto & Santo, 2015; Silver, 1994). For example, students might decide to pose the problem “What is the height of the salt mountain?” when they feel able to calculate the height of the cone, have knowledge of the Pythagorean theorem, or are interested in the height of the salt mountain. In contrast, students pose the problem “How long will it take to transport the salt mountain away?” when they feel able to calculate the volume of the cone and each transport, know how to calculate the volume, or are interested in the overall time needed for transporting the salt mountain. Because of the possibility of choices in the process of problem posing, the posed problem aligns more strongly with personal goals, which, in turn, can positively influence self-efficacy and task values. Indeed, prior research indicates positive effects of choices on motivation. Students who were offered choices in a mathematical computer game reported higher competence in playing the game (Cordova & Lepper, 1996).

1.5.1 Self-efficacy

Few studies have addressed the question of whether interventions with a focus on problem posing would positively affect self-efficacy (Akay & Boz, 2010; Chen et al., 2015; Kaya et al., 2012). In Akay and Boz’s (2010) study, preservice teachers who participated in a problem-posing intervention embedded in an analysis course at a university reported higher self-efficacy in mathematics than their peers in a traditional course. Chen et al. (2015) carried out a problem-posing training program with a class of fifth-graders. Using a quasi-experimental design, they showed that problem posing had a positive effect on students’ problem-posing- and problem-solving-related beliefs—including their self-efficacy. Consequently, a positive effect of problem posing on self-efficacy can be expected.

1.5.2 Task values

Less research has focused on task values in the context of problem posing. Indications for benefits of problem posing on task values come from research on enjoyment and interest (Headrick et al., 2020; Voica et al., 2020), which are closely related to students’ task values (Eccles et al., 1983). Spontaneous problem posing was found to be positively related to students’ affect, including enjoyment and interest (Headrick et al., 2020). In addition, Voica et al. (2020) found that problem-posing activities conducted by preservice teachers improved their enjoyment, sense of autonomy, and control more than problem-solving activities did. Based on these studies, we expect a positive effect of problem posing on task values.

1.6 Effects of self-efficacy and task values on modelling and self-efficacy and task values as intervening variables

Previous studies have widely confirmed the positive relationship between self-efficacy and performance-related measures, such as test performance, final exams, or school grades (Pajares & Miller, 1994; Talsma et al., 2018; Zhu & Leung, 2011). In modelling research, self-efficacy has also been found to be positively related to performance. Specifically for

modelling, self-efficacy was found to predict modelling performance ($\beta = .17$) (Holenstein et al., 2022).

A positive relationship between task values and final exams in mathematics was also confirmed (Meyer et al., 2019). However, the results of this study indicate that, in comparison with self-efficacy, task values have less of an impact on performance-related measures, and the effect of task values vanishes when both predictors are included in a regression model. These findings are in line with theoretical assumptions that task values have more influence on academic choices (e.g., course enrollment) than on performance (Schukajlow et al., 2022; Wigfield & Cambria, 2010). Specifically, for modelling, to the best of our knowledge, no studies have investigated the relationship between task values and modelling performance.

As we expected effects of problem posing on self-efficacy and task values (see above), and we expected self-efficacy and task values to be positive related to modelling performance, we also expected that using problem posing to enhance self-efficacy and task values would lead to improved modelling performance.

2 Research questions and hypotheses

In this study, we aimed to investigate the effects of a problem-posing intervention on self-efficacy, task values, and modelling performance. For this purpose, we compared students in a problem-posing and problem-solving condition (PP&PS) with students in problem-solving-only conditions (PS1 and PS2). Our aims were to investigate the impact of prompting students to pose their own problems on their modelling performance and on their self-efficacy and task values as well as to determine whether self-efficacy and task values would mediate the effect on modelling performance to uncover the relationships between problem posing, self-efficacy, task values, and modelling performance. To address these aims, we hypothesized the path analytic mediation model presented in Fig. 2 on the basis of the theoretical considerations and empirical findings on problem posing, modelling, and expectancy-value theory Fig. 2.

Thus, we had the following research questions and hypotheses:

RQ1: Does prompting students to pose their own problems affect (a) modelling performance, (b) self-efficacy, and (c) task values?

Hypothesis 1: Students in the PP&PS condition will (a) outperform, (b) report higher self-efficacy, and (c) report higher task values than students in the problem-solving conditions.

RQ2: Does prompting students to pose problems affect modelling performance via (a) self-efficacy and (b) task values?

Hypothesis 2: The positive effect of prompting students to pose their own problems on modelling performance will be transmitted by (a) self-efficacy and (b) task values.

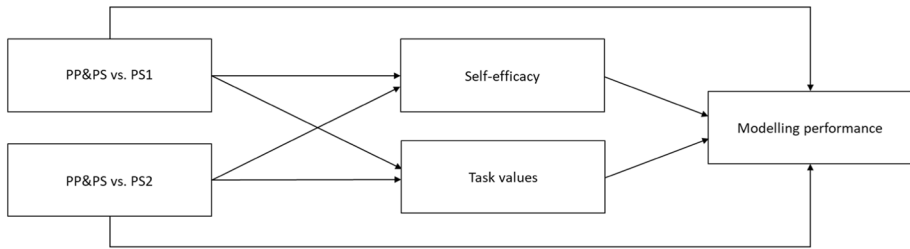


Fig. 2 Hypothesized path model. PP&PS, problem-posing and problem-solving condition; PS1, problem-solving condition 1; PS2, problem-solving condition 2

3 Method

3.1 Participants

The sample involved 210 ninth- and tenth-graders (PP&PS=127, PS1=41, PS2=42; 43.8% female adolescents; mean age $M=15.3$ years, $SD=0.69$) from three German schools including two middle-track schools (German Realschule and Gesamtschule; 66.4% of students) and one high-track school (German Gymnasium; 33.6% of students).

3.2 Design

Within each class, students were randomly assigned to one experimental (PP&PS) and two control conditions (PS1/PS2). Students in the PP&PS condition received a test with five written descriptions of real-world situations. The descriptions were adapted from modelling problems used in prior studies (e.g., Krawitz & Schukajlow, 2018). An example of one of the situations is depicted in Fig. 1. The students were requested to pose problems on the basis of the real-world situations and to subsequently solve their self-generated problems. After posing a problem, they answered a questionnaire on motivation and were asked to solve the problem. The request was embedded in the test, and the formulation was as follows:

In this booklet, you will find a number of different situations from the real world. Unlike most of the tasks you are familiar with, there is no mathematical problem for you to solve for these situations because today you will develop the problem yourself. First, read the description of the situation. Then try to pose a mathematical problem that is based on the given situation and can be solved by using information from these situations and write this problem down. Then you should solve your self-generated problem.

Students in the PS1 and PS2 conditions received test booklets with the same real-world situations as those in the PP&PS condition, but with given problems. Then they were asked to answer the questionnaire on motivation and subsequently solve the problems. The given problems were purposefully selected to ensure their equivalence to the self-developed problems. Problems were considered equivalent if they asked for the same quantity. For example, in the Salt Mountain problem situation, “What is the height of the salt mountain?” and “How high does the salt pile up?” were considered

equivalent, whereas “How much salt does the salt mountain consist of?” was considered different. By using two control conditions (PS1 and PS2), we ensured that more than 50% of the self-developed problems in the experimental condition were equivalent to problems in the control conditions. If we had taken one control group, the number of equivalent problems in the PP&PS condition and the control condition would have decreased to about 34%. As a result, the students’ modelling performance in the PP&PS condition and the control condition would have been much less comparable. The problems presented in the PS1 and PS2 conditions were determined in a prior problem-posing study in which ninth- and tenth-graders received the same five real-world situations and were asked to pose problems that referred to these situations (Hartmann et al., 2021). For the present study, we took the two most frequently self-generated problems for each situation and used them in the PS1 and PS2 test booklets. For the Salt Mountain problem situation (see Fig. 1), the two most frequently generated problems were “How many trips does the truck have to make to transport the salt mountain?” and “What is the height of the salt mountain?” In addition, we coded the self-generated problems from the PP&PS group with respect to their fit to the given problems in the PS groups. A self-generated problem in the PP&PS group was coded 1 when it was equivalent to the problem given in the PS1 group, 2 when it was equivalent to the problem in the PS2 group, and 0 when it was different from both (Interrater reliability $> .80$). More than half (57.5%) of the self-generated problems in the PP&PS group were equivalent to problems that were included in the tests given to the PS groups (33.52% PS1 and 23.98% PS2). Even if this is a large amount, a noticeable number of self-generated problems differed from the problems given to both control groups. Hence, we decided to conduct an additional analysis in which we included only solutions to problems that were equivalent to the PS groups’ problems. The results were very similar to the ones from our main analysis, and we therefore report only the findings from the main analysis here. Results from the additional analysis can be found in the 5.4.

3.3 Measures

3.3.1 Modelling performance

To measure modelling performance, students’ solutions to each of the five problems were scored. For this purpose, we analyzed whether the solution was based on a correct real-world model of the situation, based on an adequate mathematical model, included correct

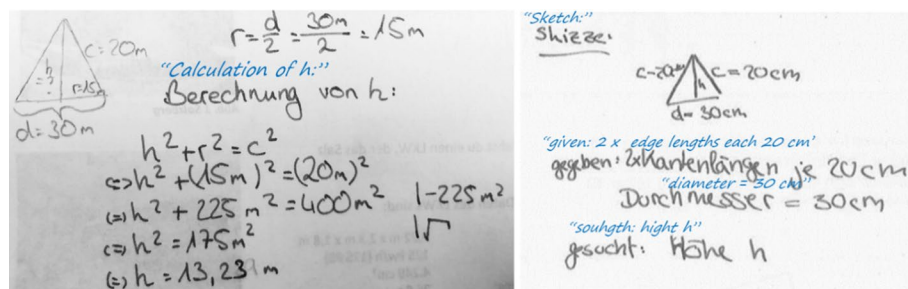


Fig. 3 Student solutions to the Salt Mountain problem situation

mathematical results, and comprised an adequate interpretation of results with respect to the real-world situation. One point was given for each property that was successfully mastered. Hence, students could achieve a maximum of 4 points for a solution. The interrater reliability (Cohen's $\kappa \geq .65$) and the internal consistency (Cronbach's $\alpha = .81$) were satisfactory. Figure 3 presents two solutions to the problem "What is the height of the salt mountain?". The left solution (scored 3 points) features a correct real-world model (assessed here by the drawing with accurately assigned measures), an appropriate mathematical model (setting up the equation using the Pythagorean theorem), and correct mathematical results. However, there is no visible interpretation of the results. The right solution (scored 1 point) features a correct real-world model but no mathematical model, no mathematical results and no interpretation.

3.3.2 Self-efficacy and task values in solving modelling problems

We used questionnaires to measure students' self-efficacy and task values. The self-efficacy and task values scales were adapted from previous studies (Krawitz & Schukajlow, 2018; Schukajlow et al., 2012). Before students solved each problem, they were asked to indicate on a 5-point Likert scale (1 = *not at all true*, 5 = *completely true*) the extents to which they agreed with the statements "I am confident I can solve my [this] problem" (self-efficacy; [PS1 and PS2 version in brackets]) and "I think it is important to be able to solve my [this] problem" (task values). In the PP&PS group, students ranked their self-efficacy and task values regarding their self-developed problems, and in the PS1 and PS2 groups, they ranked their self-efficacy and task values regarding the problems selected by the research team. Self-efficacy and value in solving the problems were aggregated for the five problems, leading to a scale on self-efficacy consisting of five items and to a scale on task values consisting of five items. The scale reliabilities (Cronbach's α) were .88 for self-efficacy and .91 for task values.

3.3.3 Complexity of problems

We included the complexity of the problems in the path model as a control variable, as the complexity of the problems is related to performance and self-efficacy (Hoffman, 2010). To assess the complexity of the problems, the self-generated and given problems were scored from 0 to 3 (Krawitz et al., 2024). The codes referred to problems that could be solved by taking information directly out of the text (coded 0), by applying a simple mathematical model (coded 1; e.g., only basic arithmetic operations required) or by applying a complex mathematical model (coded 2, e.g., models that were based on more advanced mathematical concepts, e.g., the Pythagorean Theorem or quadratic functions). Problems that required mathematics higher than the ninth-grade level were coded 3. The interrater reliability (κ ranged from .564 to .907; $M = 0.810$) indicated at least moderate agreement. The internal consistency ($\alpha = .61$) was rather low, indicating that the complexity of problems depended on the specific real-world situation and varied considerably within students.

3.4 Comparability of groups

To get an indication of the comparability of the three groups, we assessed students' interest in mathematics by using a well-evaluated questionnaire with three items (sample item: "I am interested in mathematics"; adapted from Frenzel et al., 2012). The scale's reliability was satisfactory ($\alpha = .80$). The three experimental conditions did not differ in interest

in mathematics ($M_{PP\&PS}=2.23$, $SD_{PP\&PS}=1.02$; $M_{PS1}=2.12$, $SD_{PS1}=0.82$; $M_{PS2}=2.11$, $SD_{PS2}=0.97$, $F(2, 206)=0.317$, $p=.729$) and in their grades in mathematics taken from their last school reports ($M_{PP\&PS}=2.78$, $SD_{PP\&PS}=0.99$; $M_{PS1}=2.64$, $SD_{PS1}=0.96$; $M_{PS2}=2.92$, $SD_{PS2}=0.90$, $F(2, 204)=0.827$, $p=.439$). These results indicate that the groups formed by randomization were comparable.

3.5 Dummy coding of the treatment variables

In intervention studies, dummy coding is often used to represent categorical variables, such as treatment groups or conditions, in statistical analyses. The purpose of dummy coding is to convert categorical variables into numerical variables, which can be used in statistical models. For the dummy variable PP&PS versus PS1, students from the PP&PS and PS2 conditions were coded 0, whereas students from the PS1 condition were coded 1. Similarly, for the PP&PS versus PS2 variable, students from the PP&PS and PS1 conditions were coded 0, and students from the PS2 condition were coded 1. However, to simplify the interpretation of results, the results are reported in reverse, that is, the positive regression values indicate superior performance of the PP&PS condition compared with PS1 or PS2, respectively.

3.6 Data analysis

Three path models were analyzed with Mplus 8.9. The first path model (see Fig. 2) was based on the complete data set. The reported p values were one-tailed because our expectations were directional. The fit indices for the path models are reported in Table 2 and indicated an acceptable model fit ($SRMR < .08$; $RMSEA < .06$; $TLI > .95$; $CFI > .95$; Hu & Bentler, 1999).

3.7 Missing values

In this study, the percentage of missing values was 13.4% for modelling performance, 2.8% for self-efficacy, and 2.8% for task values. Missing values in the data were estimated with the maximum likelihood algorithm (FIML) implemented in Mplus. This algorithm uses all the information from the covariance matrices to estimate the missing values. Seven

Table 2 Fit statistic for the path models

	χ^2	df	p	SRMR	RMSEA	CFI	TLI	R^2
Model 1	1.143	1	.285	.018	.026	.998	.980	.173
Model 2 (Same problems: PP&PS vs. PS1)	0.191	1	.662	.010	.000	1.000	1.000	.125
Model 3 (Same problems: PP&PS vs. PS2)	0.047	1	.828	.005	.000	1.000	1.000	.201

SRMR standardized root mean square residual, *RMSEA* root mean square error of approximation, *CFI* comparative fit index, *TLI* Tucker-Lewis Index, R^2 variance explained in modelling performance

students were excluded from the analysis because they had missing values on both self-efficacy and task values, so the data could not be estimated for them.

3.8 Descriptive statistics

Table 3 presents descriptive statistics. All correlations were in the expected direction. For example, modelling performance and self-efficacy were positively related.

4 Results

The estimates that were calculated to test the hypothesized path model are illustrated in Fig. 4.

Table 3 Means, standard deviations, and correlations for the study variables

Variable	PP&PS		PS1		PS2		Correlations					
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	1	2	3	4	5	6
1. Modelling	2.08	1.01	1.95	1.05	1.73	1.32	1	.39**	.15**	-.17	.00	.10
2. Self-efficacy	3.56	0.90	3.05	1.02	2.86	0.74		1	.41**	-.20**	.16**	.24**
3. Task values	3.43	1.04	3.15	1.20	3.34	0.95			1	.07	.10	.01
4. Complexity	1.65	0.36	1.61	0.00	2.00	0.00				1	.16**	.44**
5. PP&PS vs. PS1											1	.24**
6. PP&PS vs. PS2												1

PP&PS vs. PS1 and PP&PS vs. PS2 are dummy coded, see details in Sect. 3.2

* $p < .05$. ** $p < .01$, two-tailed

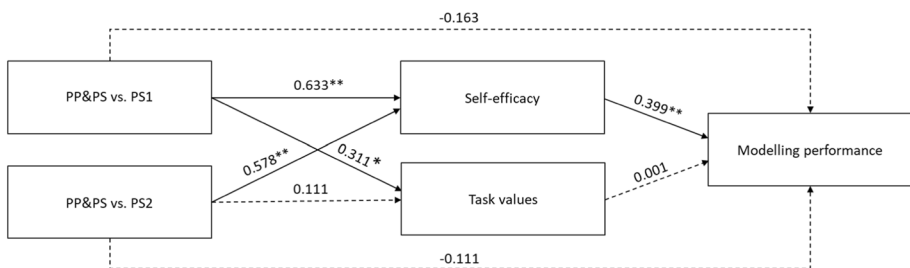


Fig. 4 Path model for testing the effects of problem posing on self-efficacy, task values, and modelling performance. Significant paths ($p < .05$; one-tailed) are presented as solid lines and nonsignificant paths as broken lines. The reported estimates of the effects of PP&PS versus PS1 and PP&PS versus PS2 (binary covariates) are standardized with respect to the dependent variable (STDY). Estimates of the effects of self-efficacy and task values (continuous covariates) are standardized with respect to both the covariate and the dependent variable (STDYX). STDY standardized coefficients can be interpreted as the predicted change in the (residualized) criterion measures (in standard deviation units) when the covariate changes by 1 unit, here from 0 (PS conditions) to 1 (PP&PS condition). For example, if the value for the treatment condition changes from 0 to 1, self-efficacy increases by $\beta \cdot SD_{se} = 0.633 \cdot SD_{se}$. STDYX is interpreted as the predicted change in standard deviation units when the covariate changes by 1 *SD*

4.1 Effects of problem posing on modelling performance, self-efficacy, and task values

We hypothesized that students in the PP&PS group, who were prompted to pose their own problems, would show better modelling performance than students in the PS1 or PS2 groups who solved given problems (Hypothesis 1a). This hypothesis was not supported. Students from the PP&PS group did not outperform students from the PS1 group ($\beta = .036$, $p = .310$) or the PS2 group ($\beta = .034$, $p = .366$) in their modelling performance.

For self-efficacy, our hypothesis that students from the PP&PS group would report higher self-efficacy (Hypothesis 1b) was supported. The problem-posing instructions had a strong positive effect on students' self-efficacy compared with the PS1 group ($\beta = .633$, $p < .001$) and the PS2 group (PP&PS vs. PS2: $\beta = .578$, $p < .001$; see Fig. 4).

For task values, our hypothesis that students from the PP&PS group would report higher task values (Hypothesis 1c) was partially supported. Prompting students to pose problems had a positive effect compared with the PS1 condition ($\beta = .311$, $p = .042$) but no effect compared with the PS2 condition ($\beta = .111$, $p = .267$).

4.2 Indirect effects of problem posing on modelling performance via self-efficacy and task values

We hypothesized that prompting students to pose their own problems would have a positive indirect effect on their modelling performance via an increased self-efficacy (Hypothesis 2a). This hypothesis was supported (PP&PS vs. PS1: $\beta = .253$, $p = .002$; PP&PS vs. PS2: $\beta = .231$, $p = .004$), meaning that students who were asked to pose their own problems reported higher self-efficacy in solving the problems than students who were given problems to solve, and the students with higher self-efficacy in turn showed better modelling performance. Thus, self-efficacy was found to be an intervening variable between problem-posing instructions and performance.

However, contrary to our expectations (Hypothesis 2b), no indirect effect was found for problem posing on modelling performance via task values (PP&PS vs. PS1: $\beta = .000$, $p = .500$; PP&PS vs. PS2: $\beta = .000$, $p = .500$). The reason for this finding is that task values and modelling performance were revealed to be unrelated to each other ($\beta = .001$, $p = .500$).

5 Discussion

Prompting students to pose their own problems has been acknowledged as a powerful tool for enhancing motivation and performance (Cai & Leikin, 2020). In response to the call for more research investigating the complex interplay between problem posing, motivation, and performance and to address the lack of intervention studies (Lee, 2020), our study aimed to investigate the impact of problem-posing instructions on performance and motivation, as well as their interrelationships, in the context of modelling-related problem posing. Our study was based on a randomized control trial comparing one problem-posing and problem-solving condition with two problem-solving-only conditions. The problems in the problem-solving conditions were derived from the analysis of solutions developed in a pilot study (Hartmann et al., 2021). Therefore, our study stands out from prior research on problem-posing instructions, where problem-solving tests with

problems that differed from learners' self-generated problems were used (Chen et al., 2015; Rudnitsky et al., 1995).

5.1 Empirical contributions

In our study, we found that prompting students to pose problems based on given situational descriptions did not improve their modelling performance. This research adds to prior research on the effects of prompts on modelling performance (Krawitz et al., 2022; Schukajlow et al., 2023b; Wijaya, 2017). Interestingly, our findings diverge from previous studies which demonstrated a positive effect of problem posing on problem-solving performance (Chen et al., 2015; Rudnitsky et al., 1995). One potential reason for this difference could be the longer duration of the interventions in those studies. Apparently, simply prompting students to pose their own problems is not sufficient for increasing performance. Future studies are needed to investigate whether teaching problem posing and integrating problem posing into mathematics lessons can have a positive impact on modelling. Another difference between our study and previous research is our focus on modelling-related problem posing. The role of different forms of problem posing and, specifically, the influence of prompts used in problem-posing tasks appear to be important factors that need further examination (Cai & Hwang, 2023). Another important research direction involves analyzing various problem-posing prompts, such as those inviting students to include additional information in their problems or indicating the types of problems they should develop.

A major finding of our study is that prompting students to pose their own problems increased students' self-efficacy. Students who posed their own problems reported higher self-efficacy in solving the problems than their peers who worked on given problems that referred to the same real-world situations. This finding is in line with previous studies (Akay & Boz, 2010; Chen et al., 2015), which implemented problem-posing interventions in mathematics classrooms and investigated their impact on problem-solving performance. Our study makes a new empirical contribution by focusing on modelling-related problem posing and demonstrating that prompting students to pose their own problems already enhances their self-efficacy.

Further, we examined the impact of problem posing on students' task values for modelling. The results were not entirely consistent, revealing advantages in problem posing compared with one problem-solving group but not the other. This result adds to prior studies that analyzed the effects of various interventions, such as increasing the relevance of the content by writing essays (Hulleman et al., 2010; Kosovich et al., 2019) or enhancing the value of modelling through an independence-oriented teaching method in the classroom (Durandt et al., 2022). Particularly for problem-posing interventions, prior research showed benefits for related affective constructs, such as enjoyment and interest (Headrick et al., 2020; Voica et al., 2020). Adding to this research, our results indicate that modelling-related problem posing could be beneficial for students' task values. However, future studies need to examine the potential effects of individual factors that might have been responsible for why problem posing was beneficial only for some, but not all, students' task values.

Another significant finding of our study is that problem posing had a positive effect on modelling performance via self-efficacy but not via task values. This finding means that prompting students to pose problems improved their self-efficacy in problem solving, which subsequently predicted their performance in solving modelling problems.

However, students' task values did not act as an intervening variable between problem-posing and modelling performance because of the missing relationship between task values and modelling performance. For the weak or missing relationships between task values and performance in modelling, see Schukajlow et al. (2022), and more generally for values and performance, see Meyer et al. (2019). These findings are important, as they unveil one mechanism by which problem posing enhances performance. To the best of our knowledge, no prior study has examined these specific mechanisms in problem-posing research.

5.2 Theoretical contributions

In our research, we investigated problem-posing instructions by exploring their potential benefits for a cognitive outcome (modelling performance in our study) and motivational outcomes (self-efficacy and task values in our study). This investigation was based on process models of problem posing with respect to real-world situations, as such models hypothesize that important modelling activities occur during the problem-posing process (Hartmann et al., 2023). Additionally, we drew upon expectancy-value theory, which emphasizes the importance of personal goals for self-efficacy and task values and, in addition, posits that self-efficacy and task values are central factors that have a substantial impact on students' academic performance (Eccles & Wigfield, 2020). On the basis of these theoretical foundations, we hypothesized, first, that prompting students to pose their own problems would have a positive impact on modelling performance, self-efficacy, and task values (Hypothesis 1) and, second, that the positive effect of prompting students to pose problems on performance would be mediated by self-efficacy and task values (Hypothesis 2).

The results of our study partially supported Hypothesis 1, as problem posing was found to have positive effects on self-efficacy and, to some extent, on task values. These findings provide support for the importance of goal-setting, particularly in relation to self-efficacy. However, the results concerning task values were mixed. A possible explanation for these unexpected findings is that, in our study, students were given descriptions of problem-posing situations. Because of the given problem-posing situations, the choices of potential problems that might align with students' individual goals were limited. In more open forms of problem posing (e.g., Bonotto, 2013), where students have more options for shaping the problem, they might have more opportunities to set their own goals or make choices during the problem-posing process to create problems that are closer to their personal goals, and the effect on task values might be stronger. These considerations are supported by research on the effects of students' choices and by research on the effects of teaching students how to solve open modelling problems on motivational outcomes and values of mathematics (Cordova & Lepper, 1996). However, the quality of self-generated problems was found to be higher when more specific problem-posing prompts were used (e.g., posing an easy, medium, and difficult problem) (Cai et al., 2023). Hence, exploring the benefits and pitfalls of openness in problem-posing tasks on motivation and performance appears to be an important direction for future research.

However, contrary to our expectations, problem posing did not positively affect modelling performance. One explanation might be that even though problem posing enhanced important modelling activities (e.g., understanding, structuring, and idealizing the given information), the learners' modelling performance did not improve because they may have

struggled with calculations or interpretation of results, activities that were addressed less by problem posing (Hartmann et al., 2023). In addition, problem posing has some costs, as posing a challenging but solvable problem requires effort. However, the results indicate that, despite the potential additional cost, posing problems did not lead to poorer modelling performance. To better understand the mechanisms of how problem posing improves performance, future studies should examine the cognitive and motivational processes induced by problem posing, including assessing the psychological cost of problem posing.

Our second hypothesis—that problem posing would have an indirect positive effect on performance—was supported for self-efficacy. As expected, problem posing enhanced self-efficacy, which in turn improved modelling performance, as proposed in theories on problem posing and in expectancy-value theories. However, for task values, no relationship between task values and modelling performance was found, and thus, there was also no indirect effect of problem posing on modelling performance via task values. One explanation for this finding is that task values for solving problems are of limited importance for performance in school settings but of high importance for educational choices, such as choosing to study mathematics at university after high school graduation (Meyer et al., 2019; Schukajlow et al., 2022; Wigfield & Cambria, 2010). In addition to cognitive processes (e.g., understanding and exploring the situation and simplifying and structuring the given information) (Hartmann et al., 2023), the enhancement of self-efficacy was found to be an important intervening factor between problem posing and students' performance. Therefore, one theoretical contribution of our findings is the recognition of the importance of incorporating motivational components into process models that aim to describe the integrated process of problem posing and solving, which is not present in models from the literature (Baumanns & Rott, 2022; Cruz, 2006; Hartmann et al., 2023).

5.3 Practical contributions

Along with previous research (e.g., Akay & Boz, 2010; Chen et al., 2015; Headrick et al., 2020), our findings suggest that prompting students to pose their own problems has a positive impact on their motivation. Consequently, we view problem posing as a potentially valuable teaching method that is aligned with other teaching methods in modelling (Durandt et al., 2022; Parhizgar & Liljedahl, 2019). In particular, teachers should consider incorporating problem posing when their aim is to foster students' motivational outcomes (e.g., self-efficacy). Furthermore, our finding that prompting students to pose their own problems did not improve their modelling performance indicates that prompting alone is not sufficient to trigger the cognitive processes necessary to improve performance. Hence, more comprehensive teaching interventions, in which students learn how to pose problems, seem to be necessary. Moreover, along with the existence of an indirect effect on performance via self-efficacy, the absence of a positive total effect on performance suggests that problem posing may also have certain costs and is challenging for the students. The problem-posing intervention affected modelling performance for students whose self-efficacy was high after problem posing. Consequently, teachers need to guide students' problem-posing processes, and teachers might pay specific attention to enhancing students' self-efficacy.

5.4 Strengths, limitations, and future directions

In our study, we were interested in the question of whether modelling performance is enhanced when students pose and solve their own modelling problems. Our research is

grounded in the cognitive perspective on mathematical modelling, which is one of several perspectives in modelling research (Kaiser & Sriraman, 2006; Schukajlow et al., 2023a). We carefully designed the study by identifying the most frequently self-generated problems to real-world situations in a prior study and assigning problems to two problem-solving groups on the basis of these problems. In addition, we conducted two analyses, one including the solutions to all self-generated problems and another including only the solutions to the problems that were identical to one of the control groups. However, the fact that the PP&PS group and the two problem-solving groups did not work on the exact same problems remains a limitation of our study.

Another limitation addresses the use of specific real-world situations as problem-posing prompts. Our findings may be influenced by the choice of real-world situations used in our study. There are some indications from prior research in physics and from algebraic word problems that motivation to solve a problem can be affected by the choice of real-world situations (Bernacki & Walkington, 2018; Lepik, 1990). Further research is needed to explore how the task variables of problem-posing tasks affect problem posing itself (Cai & Hwang, 2023). Using more open problem-posing tasks (e.g., tasks that allow students to find their own real-world situations) may potentially yield a stronger effect of problem posing on task values. In future studies, research should also clarify whether the results of this study hold for more complex modelling problems and for other types of mathematical problems, such as word problems and intramathematical problems. Furthermore, we used an open prompt in this study and did not include in the prompt typical characteristics of modelling problems such as openness and authenticity. The variation in the formulation of prompts is another important direction of future research on modelling-related problem-posing tasks.

6 Conclusions

This study represents an approach that connects problem posing with mathematical modelling. Our findings demonstrate that prompting students to pose their own problems to given real-world situations did not yield benefits for modelling performance. However, problem posing had a positive impact on students' self-efficacy in solving modelling problems and, to some extent, on their task values in modelling. Further, the positive effect of problem posing on self-efficacy transferred into better modelling performance. Therefore, problem posing enhances learners' self-efficacy, which in turn improves their modelling performance. Our study highlights the importance of motivational constructs as mediators between problem posing and performance. It also underlines the need to further explore the potential costs and challenges associated with problem posing, which may have decreased the positive effect of problem posing on modelling performance.

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Declarations

Conflict of interest The authors declare no competing interests.

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