Article

Number Line Strategies of Students with Mathematical Learning Difficulties and Students with General Learning Difficulties: Findings Through Eye Tracking

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Abstract

In many countries, the number line (NL) is an important tool in mathematics education to develop an understanding of numbers. However, students may have difficulties using the NL. To provide students showing NL difficulties with appropriate support, more research is needed on how these students interact with NLs. In this study, we investigated how three different groups of students located numbers on an NL: 20 fifth-grade students with general learning difficulties (LD) from a special school, and 60 fifth-grade students with mathematical learning difficulties (MD) from a school of general education, compared to 55 fifth-grade students without MD/LD. We analyzed students' strategies based on qualitative analysis of eye-tracking videos and students' error rates. We found that all students were generally able to solve the tasks correctly. Analyses of students' strategies showed that the types of strategies used for locating numbers on an NL did not differ between students with LD, with MD, and without MD/LD. Differences were found in the frequency with which certain strategies were used, particularly for numbers between the midpoint and endpoint of the NL—indicating differences in the mathematical development regarding the flexible use of NL strategies between students with LD, with MD, and without MD/LD.

Keywords: arithmetic; eye tracking; general learning difficulties; mathematical learning difficulties; number line

1. Introduction

Mathematical skills are an important factor for coping with everyday life and are also necessary in various professions (e.g., Jansen et al., 2016). However, there are students who have difficulties acquiring these necessary mathematical skills (e.g., Moser Opitz et al., 2017). To be able to adequately support students with difficulties in learning mathematics, research regarding their mathematical learning is needed. In this paper, we focus on two different groups of students who often struggle with mathematics—students with mathematical learning difficulties (MD), and students with general learning difficulties (LD). Students with MD typically show difficulties in basic mathematical skills at the primary school level (e.g., Moser Opitz et al., 2017), which can persist into secondary school or even beyond school years. In recent years, mathematics education research has increasingly focused on

students with MD. Another group of students often struggling with learning mathematics is students with LD, who represent the largest group of students with special educational needs in many countries (e.g., Pullen, 2016). Students with LD typically show significant and persistent difficulties in various school subjects, also including mathematics (e.g., Heimlich et al., 2016). However, there has been little research on the mathematical learning of students with LD.

An important tool for developing and promoting mathematical skills in primary school in many countries is the number line (NL) (Freudenthal, 1973, 1999; Schulz & Wartha, 2021). NLs are often used to support the development of number concepts in children (e.g., Teppo & van den Heuvel-Panhuizen, 2014). Research has shown that students' performance in locating numbers on NLs correlates with their general mathematical performance (Schneider et al., 2018). It is assumed that the general mathematical performance of students and their performance in locating numbers on the NL influence each other (Friso-van den Bos et al., 2015), highlighting the potential of NLs to influence mathematical learning. However, the NL is a complex matter—especially for students with difficulties in learning mathematics (Scherer & Moser Opitz, 2010). Students with MD have been found, for example, to locate numbers on an NL less accurately than students without MD (e.g., Landerl et al., 2017).

For students who struggle with learning mathematics to benefit from the use of NLs, particularly in developing a profound understanding of numbers, it is essential to gain a deeper understanding of how they interact with NLs. A valuable method to explore these interactions, especially to investigate students' strategies for locating numbers on an NL, is eye tracking (ET) (e.g., van't Noordende et al., 2016). ET has been shown to be useful for investigating strategies of students who have difficulties with verbalizing their strategies (Schindler & Lilienthal, 2018). Therefore, ET is a particularly valuable method for students with MD and students with LD who may have difficulties with mathematics, language, and metacognitive reflection (Heimlich, 2016). Previous ET research has predominantly focused on locating numbers on empty NLs (where only the starting point and the endpoint are marked and labeled). There is little research utilizing marked NLs (containing more hatch marks for numbers), which are typically used in mathematics education to develop number concepts. The few previous ET studies on marked NLs (Simon & Schindler, 2022) investigated relatively small samples of students with and without MD, and no students with LD (Simon et al., 2023). For larger groups of students with MD, and students with LD, to the best of our knowledge, it is currently not known how these students interact with marked NLs used in mathematics teaching.

In this paper, we study how fifth-grade students with LD, with MD, and without MD/LD (in total 135 students) interact with marked NLs. We investigate whether these groups of students differ in locating numbers on a marked NL. Beyond analyzing students' responses, we use ET to examine students' NL strategies.

1.1. Number Line

The NL is considered as an important mathematical tool (Freudenthal, 1973, 1999). It can be described as "a representation of numbers on a straight line where points represent . . . numbers and the distance between points matches the arithmetic difference between the corresponding numbers" (Heeffer, 2011, p. 865). In Western cultures, the NL is usually displayed horizontally, with the higher numbers on the right side (Bartolini Bussi, 2015; Heeffer, 2011). It represents the basic idea of the number series and is often used to develop and deepen an ordinal understanding of numbers (e.g., Schulz & Wartha, 2021). On an NL, every natural number has a unique position—even if this is not always visibly marked—and natural numbers are equidistant to each other (Schulz & Wartha, 2021). In addition to developing an ordinal understanding of numbers, the NL is essential for

building a relational understanding of numbers, that is, for interpreting numbers in relation to other numbers (e.g., 28 is between 20 and 30, closer to 30). The NL can help students to "understand the relative magnitude and position of numbers" (Frykholm, 2010, p. 5) and to build and understand relationships between numbers (Teppo & van den Heuvel-Panhuizen, 2014; Tikhomirova et al., 2022). With its linear spatial representation of numbers and number ranges, the NL can also be interpreted as a "measurement model" (Gaidoschik, 2024; see also Fuson, 1984). According to this interpretation, number relationships among natural numbers are represented as relationships of measured quantities (lengths). For example, the length of the NL segment between two hatch marks labelled with 0 and 5 corresponds to the length of the segment between hatch marks 1 and 6, or 2 and 7. Likewise, a number relationship such as 50 being half of 100 is reflected spatially as half the distance on the NL from 0 to 100 (Gaidoschik, 2024).

The NL is an important tool in mathematics education as it offers particular opportunities: It can be used across several school levels, has a high degree of abstraction, and allows flexibility in terms of types of numbers (e.g., whole numbers or integers), and number ranges (e.g., 0–10 or 0–100) (Nuraydin et al., 2023; Schulz & Wartha, 2021; Teppo & van den Heuvel-Panhuizen, 2014). There are several types of NLs that differ in the amount and type of information displayed about numbers. On an empty NL (also called proportional NL, e.g., Teppo & van den Heuvel-Panhuizen, 2014), for example, only the starting point and endpoint are marked as reference points and labelled with numbers (Teppo & van den Heuvel-Panhuizen, 2014, p. 48). Marked NLs (also called structured NLs, Diezmann & Lowrie, 2007; or filled NLs, Teppo & van den Heuvel-Panhuizen, 2014) contain additional hatch marks, ranging from fully marked with hatch marks and labelled with numbers (e.g., for whole numbers) to partially marked and labelled. Commonly used reference points, such as the midpoint or quarters of the NL, are often visually emphasized with thicker or longer hatch marks (e.g., Schulz & Wartha, 2021).

NL estimation tasks—in which numbers are located on *empty* NLs—are a prominent type of task in cognitive neuroscience. In mathematics education, however, these tasks currently attract little attention in mathematics education research (Gaidoschik, 2024). In this paper, we address locating numbers on an NL from a mathematics education perspective and use a *marked* NL that is a common tool in teaching mathematics. We focus on how students locate numbers on a marked NL, that is, on students' use of strategies. NL strategies may differ with respect to the use of different reference points: For example, it was found that children's use of NL strategies develops from relying solely on the starting point of the NL to also using the endpoint of the NL (e.g., Peeters et al., 2017). The midpoint of the NL also represents an important reference point for locating numbers (e.g., Ashcraft & Moore, 2012; Petitto, 1990). Research indicates that the use of reference points other than the starting point can positively influence NL performance (Newman & Berger, 1984; Peeters et al., 2016, 2017). However, the ability to use these reference points is related to children's experience with numbers (Peeters et al., 2017). For example, to use the midpoint of the NL effectively, children need to understand that it corresponds to the "numerical midpoint" of the number range depicted on the NL (e.g., Ashcraft & Moore, 2012). Depending on the children's prior knowledge of numbers and their familiarity with the given number range, students may need reference points not only to be marked, but also labelled with numbers to make use of them (e.g., Peeters et al., 2017; Schulz & Wartha, 2021). In addition to using different reference points, children can locate numbers on an NL by using "sequential strategies" (Petitto, 1990), that is, by counting from a reference point such as the starting point (e.g., Newman & Berger, 1984; Scherer & Moser Opitz, 2010). Over the course of primary school, there is a qualitative change in children's NL strategies from counting in steps of one from the starting point of the NL to counting from other reference points

and in alternative counting intervals such as steps of ten (e.g., Newman & Berger, 1984; Petitto, 1990).

Students' performance in locating numbers on the NL and their general mathematical performance were found to influence each other (Friso-van den Bos et al., 2015), which underlines the potential of the NL to influence mathematical learning. However, since NLs have a multitude of changeable elements and must be interpreted and filled with mathematical meaning anew each time, the NL is a complex matter. This makes the NL often difficult to understand, especially for students who struggle with learning mathematics (Scherer & Moser Opitz, 2010).

1.2. Students with Mathematical Learning Difficulties

To this day, there is no common understanding or definition of difficulties in learning mathematics (e.g., Moser Opitz et al., 2017). In this paper, we follow a broad understanding of *mathematical learning difficulties* (MD) from a mathematics education perspective. By students with MD, we mean students who show difficulties in learning basic mathematical skills (Gaidoschik et al., 2021; Moser Opitz et al., 2017). These difficulties can be seen as a discrepancy between the school's expectations and students' current mathematical skills (Heyd-Metzuyanim, 2013; Schmidt, 2016). Over the course of their schooling, these difficulties can contribute to long-term challenges in acquiring other important mathematical skills, leading students to perform below their peers (e.g., Gaidoschik et al., 2021; Moser Opitz et al., 2017).

Students with MD typically show difficulties of varying degrees of intensity and severity in three central areas of basic arithmetic in primary school (e.g., Gaidoschik et al., 2021). These areas include understanding natural numbers, understanding the decimal place value system, and understanding arithmetic operations. Difficulties typically experienced by students with MD at the primary level are also experienced by students at the secondary level (e.g., Gaidoschik et al., 2021; Schindler & Schindler, 2022). For example, students with MD in fifth grade usually do not show a comprehensive (often primarily ordinal) understanding of numbers, and relationships between numbers are not or only partially recognized (e.g., Gaidoschik et al., 2021). This leads students with MD to often rely on immature counting strategies and to have difficulties developing different and more advanced strategies (e.g., Moser Opitz et al., 2018; Verschaffel et al., 2007). Students with MD were also found to have difficulties in NL tasks: For example, students with MD tend to show less accurate estimations of numbers on NLs compared to students without MD (e.g., Geary et al., 2008; Landerl et al., 2017).

1.3. Students with General Learning Difficulties

In this paper, we use the term students with general *learning difficulties* (LD) for students who have special educational needs in the area of learning, that is, significant, extensive, and persistent difficulties in academic learning (§4(2), AO-SF NRW, 2022; transl. by the first author; see also OECD, 2007). This definition is similar to international definitions. In these definitions, LD is often referred to when children have persistent and significant difficulties in acquiring academic skills not caused by "other disabilities such as sensory impairments" (Lloyd et al., 2007, p. 159) or "disorder of intellectual development" (WHO, 2019; see also APA, 2013). However, in contrast to other understandings of LD, in which LD are also understood as temporary difficulties or difficulties that only occur in a single school subject (Heimlich et al., 2016), this paper refers to LD for students who typically encounter persistent difficulties in all core subjects in school, including mathematics (Grünke & Cavendish, 2016). Students with LD show persistent difficulties with reading, writing, mathematics, and "learning to learn" (i.e., in metacognitive abilities) (Heimlich,

2016; Pullen, 2016). Among all students with special educational needs, students with LD represent the largest group (Büttner & Hasselhorn, 2011; KMK, 2022; Pullen, 2016; Tzouriadou, 2020). Within this group of students with LD, there is considerable heterogeneity with respect to academic learning. Although they all show persistent challenges across core academic areas, the nature and causes of these difficulties vary widely (e.g., Heimlich, 2016). LD must be understood as expressions of complex and often adverse learning and life situations, as they often arise from a multifaceted interplay of endogenous and exogenous factors. These factors include socio-economic circumstances, school's performance requirements and available learning resources, as well as students' learning opportunities (Dudley-Marling, 2004; Heimlich, 2016; Heimlich et al., 2016; Okoli et al., 2022). Additional contributing factors may include emotional or behavioral issues, such as attention or concentration problems (e.g., Gabriel & Börnert-Ringleb, 2023; Heimlich, 2016).

Students with LD often show difficulties in developing basic mathematical skills (e.g., Schindler & Schindler, 2022; Heimlich, 2016). Research on the mathematical learning of students with LD, as defined in this paper, shows that these students often enter primary school with a limited understanding of basic arithmetic concepts (Moser Opitz, 2008). These difficulties tend to increase over time. For example, Werner et al. (2019) found that sixth-grade students with LD lacked mathematical skills expected of students in fourth grade. Additionally, analyses of PISA results showed that about 90% of 15-year-old students with LD perform at or below the lowest competence level assessed by PISA (Gebhardt et al., 2015; Müller et al., 2017). Even by the end of schooling, many students with LD continue to struggle with primary school level mathematics skills and often do not meet the mathematical requirements needed for employment (Gebhardt et al., 2013, 2014; Lehmann & Hoffmann, 2009; Lutz et al., 2023). However, it is assumed that students with LD do not differ fundamentally from students without LD in their learning process (e.g., Hecht et al., 2011; Heimlich, 2016). Previous research indicates that difficulties in mathematics tend to occur in the same mathematical domains among students with different performance prerequisites and learning profiles (e.g., Hanich et al., 2001; van der Sluis et al., 2004). Nevertheless, the extent of difficulties may vary (Hanich et al., 2001; Jordan et al., 2002), meaning that difficulties of students with LD may be of greater extent compared to students with MD.

Most research on students with LD has focused on outcomes, such as performance on standardized mathematics tests. These studies often summarize results across multiple mathematical domains, providing limited insight into the specific domains and processes. Given that students with LD rarely overcome their difficulties in mathematics without specific support (e.g., Heimlich, 2016), further research on their mathematical learning is crucial to effectively address their needs.

1.4. Eye Tracking and Conceptual Framework

ET is the recording of a person's eye movements (Duchowski, 2017; Holmqvist et al., 2011). It has become a commonly used method in mathematics education research in recent years (Schindler et al., 2025b; Strohmaier et al., 2020). ET studies have been conducted covering a broad range of different topics in all relevant areas of the mathematics curriculum (Schindler et al., 2025b). The majority of studies were conducted in the area of arithmetic (e.g., in the area of quantity comparison; see Pitta-Pantazi et al., 2024). Other studies examined, for example, how students read mathematical task and how this relates to mathematization processes (e.g., Dröse et al., 2021; Spagnolo et al., 2021).

By using ET, researchers can capture eye movements related to foveal vision—that is, the area of sharpest visual acuity on the retina. There are different theoretical perspectives and assumptions regarding how a person's eye movements can be interpreted and

what a person's gaze—that is, the direction and focus of the eyes—may indicate. These perspectives can be understood as background theories (Bikner-Ahsbahs, 2025; Schindler et al., 2025a).

In this paper, we conceptualize eye movements as an indicator of *attention processes* (e.g., Duchowski, 2017). When the eyes move directly to the stimulus to which attention is being paid, this is referred to as overt visual attention (Carrasco, 2011; Duchowski, 2017; Posner, 1980). Accordingly, ET allows us to investigate what information students attend to when solving mathematical tasks (e.g., Andrá et al., 2015). However, it is important to note that covert attention—processing of stimuli in peripheral (extrafoveal) vision—cannot be captured by ET (Klein & Ettinger, 2019). Therefore, interpretations of gaze behavior must consider the limitations of ET and remain sensitive to the mathematical domain and the specific characteristics of the tasks (Schindler et al., 2025a, 2025b). In the context of NL tasks, students' visual attention to different parts of the NL and to different structural elements (hatch marks) reflects students' orientation on the NL. These gazes provide insight into which structures students use for locating numbers on the NL—revealing their use of NL strategies. Furthermore, they can indicate students' conceptual understanding of numbers and number ranges as reflected in their use of strategies.

Previous ET studies in the domain of NL tasks (e.g., Schneider et al., 2008; Sullivan et al., 2011; van der Weijden et al., 2018) have shown that people's gazes when solving NL tasks show overt attention to different parts of the NL. These gaze patterns reflect different NL strategies in both empty NL tasks (e.g., Schneider et al., 2008; van't Noordende et al., 2016), and marked NL tasks (Simon & Schindler, 2022; Simon et al., 2023). A few studies in this domain also examine NL strategies of people with MD (e.g., van Viersen et al., 2013; van't Noordende et al., 2016) and ET research indicates that students' inaccuracy in locating numbers on an empty NL may stem from difficulties in flexibly using strategies (e.g., van't Noordende et al., 2016). The previous studies examining students' strategy use on marked NLs (Simon & Schindler, 2022; Simon et al., 2023) showed differences in the use of NL strategies between students with and without MD: Specifically, students with MD used counting strategies more often compared to students without MD, and they showed less use of the nearest reference points. In contrast, students without MD more often used strategies requiring fewer gazes to locate numbers on the NL. The use of fewer gazes can be interpreted as an indicator of processing efficiency and expertise in NL tasks (for this interpretation of gazes in other research areas, see Holmqvist et al., 2011). With increasing expertise, "a more task efficient selection of gaze positions" arises (Holmqvist et al., 2011, p. 396). In the context of NL tasks, fewer gazes can mean that students consider the magnitude of the number to be located when orienting themselves on the NL (Reinert et al., 2015). For example, when locating the number 90 on the NL up to 100, children might use the number proximity and orient themselves backwards from 100. The use of many gazes, on the other hand, can be indicative of students' difficulties in conceptual understanding of numbers and number ranges. For example, students can only make use of near reference points—such as the marked but unlabeled midpoint of an NL—if they know the numerical value corresponding to this position (e.g., Peeters et al., 2017). Another factor that may influence students' performance and strategy use in NL tasks are the demands placed on working memory (e.g., Namkung & Fuchs, 2016; van der Weijden et al., 2018). Working memory can be described as a limited-capacity cognitive system responsible for both maintaining and processing information (e.g., Baddeley, 2012) (for a detailed description of working memory theories and models, see Baddeley, 2012). Working memory demands are particularly relevant for students with LD or with MD, as research indicates that they may have lower working memory capacities than students without MD/LD (e.g., Heimlich, 2016; Peng & Fuchs, 2016; Winkel & Zipperle, 2023). Consequently,

they may rely on less demanding strategies. Furthermore, the "perceived difficulty" of NL tasks (e.g., Nicchiotti & Spagnolo, 2024) may also influence students' strategy use. A central component of perceived task difficulty is students' self-perception as mathematics learners, including any negative beliefs about their mathematical skills. Students with negative experiences in mathematics teaching and with regard to their mathematical skills may rely on safer, less efficient strategies even if they have sufficient numerical knowledge.

Research gap: The findings described above have been derived from studies with relatively small sample sizes and have focused on students with and without MD. To the best of our knowledge, there is so far no research on strategies in marked NL tasks for students with LD. Given the importance of mathematical skills for coping with everyday life and succeeding in life (e.g., Jansen et al., 2016; NCTM, 2000), and the role that NLs can play in developing these skills (e.g., Woods et al., 2018), it is important to investigate students' interaction with marked NLs for larger groups of students with MD and for students with LD in general. Such research is essential to better understand how these students interact with marked NLs and to understand how they can benefit from using marked NLs, in order to provide them with more appropriate support.

1.5. Aim and Research Question

The aim of this study is to investigate if and how fifth graders with LD, with MD, and without MD/LD differ in locating numbers on a marked NL. For this purpose, we conducted an ET study. The potential of ET to provide insights into learners' strategies was shown in several studies in mathematics education, and also in particular for NL tasks (e.g., Simon & Schindler, 2022; van't Noordende et al., 2016). In our study, we analyze ET videos qualitatively and investigate strategies for locating numbers on a marked NL. In addition, we analyze error rates. We ask the following research question: *Do fifth-grade students with LD, with MD and without MD/LD differ in locating numbers on a marked number line, and how?*

2. Materials and Methods

2.1. Participants

The study involved students from a special school for LD and students from an inclusive comprehensive school at the beginning of fifth grade. In North Rhine-Westphalia (where the study was conducted), the beginning of fifth grade corresponds to the transition from primary to secondary school, which typically takes place after four years of primary education. The students from the special school for LD were officially diagnosed with special educational needs in the area of learning (AO-SF NRW, §4(2))—in the following referred to as students with LD. We conducted a standardized paper-and-pencil arithmetic test as a class test (HRT; Haffner et al., 2005) in order to assess students' mathematical performance prior to the ET study. We only used the first part of HRT, which focuses on arithmetic operations (addition, subtraction, multiplication and division), completion tasks (e.g., 6 + _ = 7) and number comparison tasks (e.g., 11 _ 12 (correct answer: <)). In the following, according to Haffner et al. (2005), students with a percentile rank ≤10 are referred to as students with MD. Students with a percentile rank >25 are referred to as students without MD. The sample considered for the present study consisted of 20 students with LD from three different classes of a special school, and 115 students from six different classes of an inclusive comprehensive school (60 students with MD, and 55 students without MD/LD), that is, 135 students in total. Information on each group is shown in Table 1.

Table 1. Participants of the study.

	LD (*n* = 20)	MD (*n* = 60)	Without MD/LD (*n* = 55)
Participant information			
Age: mean (SD)	11.7 (0.7)	10.8 (0.6)	10.6 (0.6)
Mathematical abilities			
HRT: mean t-value (SD)	24.95 (5.91)	33.68 (2.77)	48.84 (4.98)
Mathematical learning difficulties: *n* (%)	20 (100.00)	60 (100.00)	0 (0.00)

Note. For the HRT, the smallest t-value presented in the norm table is 16. Students who scored lower are each considered with the smallest values presented in the norm tables.

2.2. Eye Tracking Device

For the recording of students' eye movements, we used the Tobii Pro X3-120 (Tobii AB, Danderyd, Sweden). This screen-based eye tracker allows for binocular tracking of eye movements at a sampling rate of 120 Hz. It is very unobtrusive for participants, as it is attached to the bottom of a screen on which the stimuli are displayed. Students sat about 60 cm away from screen. The average ET accuracy in our study was 0.8°, which corresponds to an error of about 0.8 cm on the screen.

2.3. Tasks

We used two different NLs ranging from 0 to 10 (0–10) and 0 to 100 (0–100). Although the NL is one of the most popular tools in primary mathematics education, students in fifth grade can still experience difficulties with the NL (e.g., Rodriguez et al., 2001; Schulz & Wartha, 2021). As this was also evident during the piloting of our NL tasks, and since we were addressing students with LD and students with MD, we selected NL tasks with a relatively low level of difficulty, using NLs ranging from 0 to 10 and from 0 to 100. Additionally, this ensured that all students—including those with LD—were familiar with NLs within these ranges of numbers. Students' familiarity with numbers is a crucial factor in NL task performance (e.g., Ebersbach et al., 2015). In addition to the two different number ranges, we used two different types of tasks. In the *position-to-number-task*, students were shown a position (red cross) on the NL and asked to name the corresponding number (similar to Gomez et al., 2017). In the *number-to-position-task*, students were presented a symbolic number (written in the upper left corner of the screen) and asked to place the number on the NL (common task type in ET studies using NL tasks, e.g., Schneider et al., 2008).

Both NLs had labelled starting and endpoints as well as hatch marks. For the NL 0–10, each number was marked with a hatch mark (see Figure 1, left), so that this NL allowed for orientation in units of 1. For the NL 0–100, the distance between adjacent marks had a unit of 10 (see Figure 1, right). We used this intermediate form between a fully marked and an empty NL (Schulz & Wartha, 2021) not to represent all the information (i.e., every single number) but to give students an aid to orientation (steps of 10) on the NL. To help children quickly orient themselves on the NL, we highlighted the midpoint of the NL with a longer hatch mark.
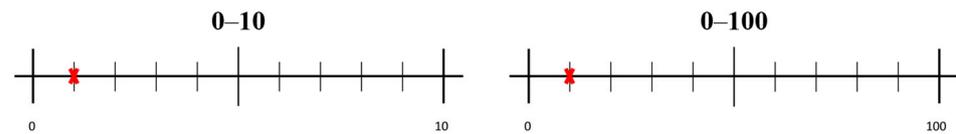
Figure 1. Marked number lines (position-to-number-task).

We used three tasks for each combination of types of task (position-to-number-task or number-to-position-task) and range of numbers (0–10 or 0–100) of the NL. This resulted in a total of 12 tasks that were tested in this study. For the specific tasks we used, see Table 2.

Table 2. Marked number line tasks.

	Position-to-Number-Task		Number-to-Position-Task	
Number range	0–10	0–100	0–10	0–100
Numbers	9, 3, 6	80, 40, 60	7, 4, 8	70, 30, 90

Note. Listed numbers in the order they were presented to the students.

2.4. Procedure

Students were tested individually in a quiet room at their school. The tasks were arranged in a random order (see Table 2) that was the same for each student. Before each task, the students were told to fixate a star at the upper left corner of the screen. This ensured a clear transition from one task to another, so that for all tasks, gazes started from the same place. Students were instructed to work on the tasks quickly and correctly. They did not receive feedback during the tasks. We recorded students' oral responses with an audio-recorder.

Our study covered a total of 1620 tasks (135 students × 12 tasks). Due to student-related data loss, where students did not answer the task, or technology-related data loss, where the eye tracker did not function properly and no valid recordings of students' eye movements were available, 25 tasks (1.54%) were excluded, resulting in a final analysis of 1595 tasks.

2.5. Qualitative Data Analysis

To analyze students' strategies, we used gaze-overlaid videos provided by the software Tobii Pro Lab. These videos show the students' gazes represented as a semitransparent dot, wandering around. Analysis of gaze-overlaid videos allows for an in-depth investigation of all eye movements (e.g., Schindler et al., 2020), which has previously been shown to be beneficial for the analysis of student NL strategies (Simon & Schindler, 2022). The analyses conducted in this paper build on the analyses of Simon and Schindler (2022): We categorized students' gazes deductively, based on an afore inductively developed category system by the first author of this paper (published in Simon & Schindler, 2022). This category system was developed on the NL 0–100 used in this study and could be transferred to the NL 0–10. Interrater reliability for the category system (between codes of the first and the last author of the paper) was $\kappa = 0.83$, which can be considered almost perfect (Landis & Koch, 1977) (see Simon & Schindler, 2022). The category system (see Figure 2) includes six different categories of strategies for locating numbers on the marked NL. For the visualization of gaze patterns, we use gaze plots provided by the software Tobii Pro Lab, although we used gaze-overlaid videos for the analysis of student NL strategies.
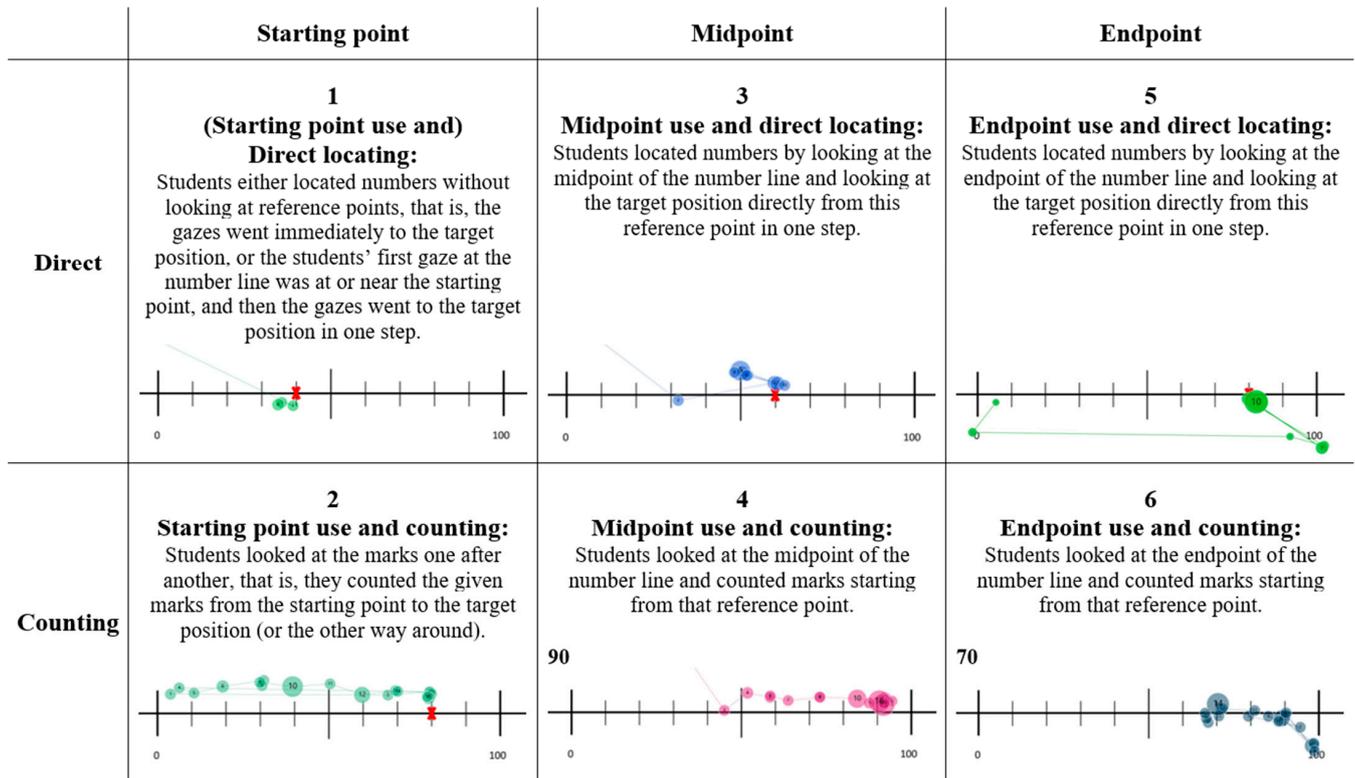
Figure 2. Categories of student strategies. Note. For reasons of space, the numbers that students were asked to locate are shown close above the NL. In the study, the numbers were displayed in the upper left corner of the screen and the distance between numbers and NL was greater.

Table 3 shows the observed strategies for the individual tasks. For numbers whose position is only one hatch mark away from a reference point (i.e., the numbers 4, 6, 40, and 60 when using the midpoint, and the numbers 9, and 90 when using the endpoint), it is not possible to distinguish between counting and direct strategies. In these cases, the students' gazes are categorized as *direct locating*, either from the *midpoint* or from the *endpoint*. Accordingly, counting strategies are not possible in these cases, which is represented by the black cells in Table 3. Gray cells represent possible but unobserved strategies.

Table 3. Observed strategies.

		Numbers					
		3 30	4 40	6 60	7 70	8 80	9 90
Starting point	Direct	x	x	x	x	x	x
	Counting	x	x	x	x	x	x
Midpoint	Direct	x	x	x	x	x	x
	Counting	x	███████	███████	x	x	x
Endpoint	Direct	▒▒▒▒▒▒▒	▒▒▒▒▒▒▒	x	x	x	x
	Counting	▒▒▒▒▒▒▒	▒▒▒▒▒▒▒	x	x	x	███████

Note. "x" marks the observed strategies for the respective tasks.

2.6. Statistical Analysis

For the statistical analyses, we used the software IBM SPSS 29. For comparing error rates of students with LD, students with MD and students without MD/LD, we used Kruskal–Wallis tests as non-parametric tests due to not normally distributed error rates (Shapiro–Wilk tests: $p < 0.05$). For comparing students' strategy use, we used chi-squared

tests and calculated effect sizes using Cramér's *V*. According to Cohen (1988), Cramér's *V* can be interpreted as follows: *V* = 0.10 is a small effect, *V* = 0.30 is a medium effect, and *V* ≥ 0.50 is a large effect (for different interpretations of Cramér's *V* corresponding to the different degrees of freedom for chi-squared tests, see Cohen, 1988). Bonferroni–Holm adjusted *p*-values were used for tests with multiple comparisons.

3. Results

3.1. Error Rates

The total number of errors was relatively low. Out of 1595 tasks, the students made 40 errors (2.51%) overall. While the error rate of students with LD was 4.72% (11 errors/233 tasks), students with MD and without MD/LD had slightly lower error rates (MD: 2.12%, i.e., 15 errors/707 tasks; without MD/LD: 2.14%, i.e., 14 errors/655 tasks). Kruskal–Wallis test showed no significant difference in error rates of students with LD, with MD and without MD/LD for all tasks together: H (2) = 2.79, *p* = 0.496 (Figure 3).
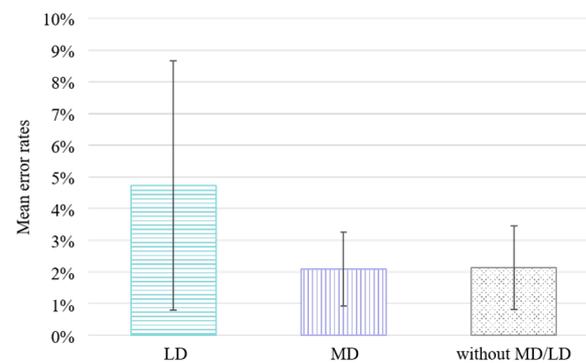

Figure 3. Students' error rates for all tasks together[1]. Note. Vertical bars denote 95% Confidence Interval.

3.2. Students' Strategy Use

Chi-squared tests showed significant differences in the strategy use for all tasks together: χ^2 (10) = 47.07, *p* < 0.001, *V* = 0.12 (see Figure 4). Cell tests showed that students with LD (χ^2 (1) = 18.52, *p* < 0.001, *V* = 0.14) and students with MD (χ^2 (1) = 13.28, *p* = 0.002, *V* = 0.10) used *direct locating* significantly less often than students without MD/LD. Cell tests also showed that students with LD (χ^2 (1) = 28.69, *p* < 0.001, *V* = 0.18) and students with MD (χ^2 (1) = 12.37, *p* = 0.002, *V* = 0.10) showed *starting point use and counting* significantly more often than students without MD/LD. Comparing students with LD and students with MD, it was found that students with LD used strategy *starting point use and counting* significantly more often (χ^2 (1) = 6.91, *p* = 0.009, *V* = 0.09).

A comparison of the summarized non-counting strategies (i.e., direct strategies 1, 3, 5) and the summarized counting strategies (i.e., counting strategies 2, 4, 6) also showed significant differences in the strategy use for all tasks together (χ^2 (2) = 15.01, *p* = 0.002, *V* = 0.10) (Figure 5). Cell tests showed that students without MD/LD used *counting strategies* significantly less often and *direct strategies* significantly more often than students with MD (χ^2 (1) = 7.05, *p* = 0.016, *V* = 0.07), and students with LD (χ^2 (1) = 13.11, *p* = 0.002, *V* = 0.12).

A descriptive analysis[2] of the NL strategies used by students at the task level—focusing on numbers between the starting point and the midpoint of the NL (left side in the diagrams in Figure 6)—revealed a general tendency among students to use strategy *starting point use and counting*. This strategy was particularly used by students with LD and students with MD (in about 55% of the tasks). Students without MD/LD also showed a tendency to use

strategy *starting point use and counting* for these numbers, but they tended to use strategy *direct locating* more often (about 49%).
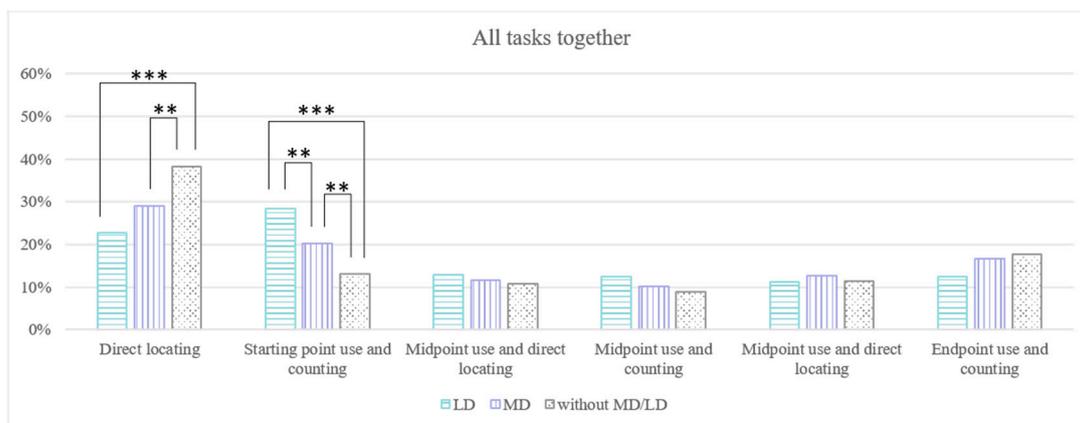

Figure 4. Students' strategy use for all tasks together. Note. Significant group differences are marked with ** $p < 0.01$, *** $p < 0.001$.
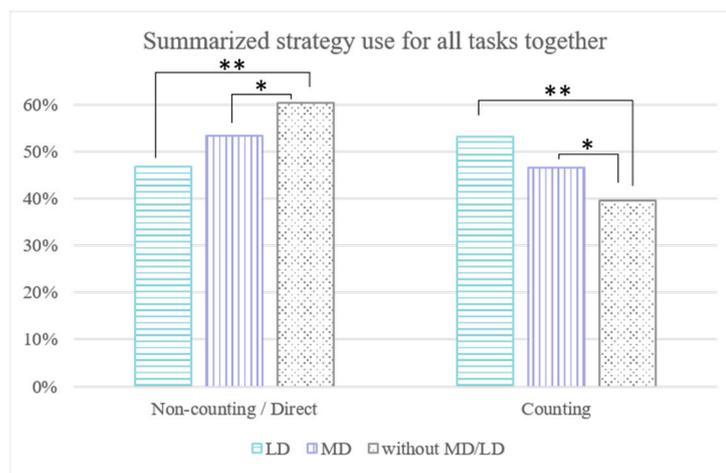

Figure 5. Summarized counting and non-counting strategies for all tasks together. Note. Significant group differences are marked with * $p < 0.05$, ** $p < 0.01$.

For numbers located further away from the starting point, that is, for numbers between the midpoint and the endpoint of the NL (right side in the diagrams in Figure 6), a general decrease in the use of strategy *starting point use and counting* was observed. For students with LD, the frequency of using this strategy declined to about 19%. However, they still tended to use this strategy more often than students with MD (about 3%) or students without MD/LD (less than 1%). In contrast, students with MD and students without MD/LD often tended to use the *endpoint* of the NL (with MD: about 44%; without MD/LD: about 43%). Students without MD/LD also tended to use the strategy *direct locating* more often (about 33%). In addition, it was found that students with LD tended to use the *midpoint* of the NL more often (about 31%).
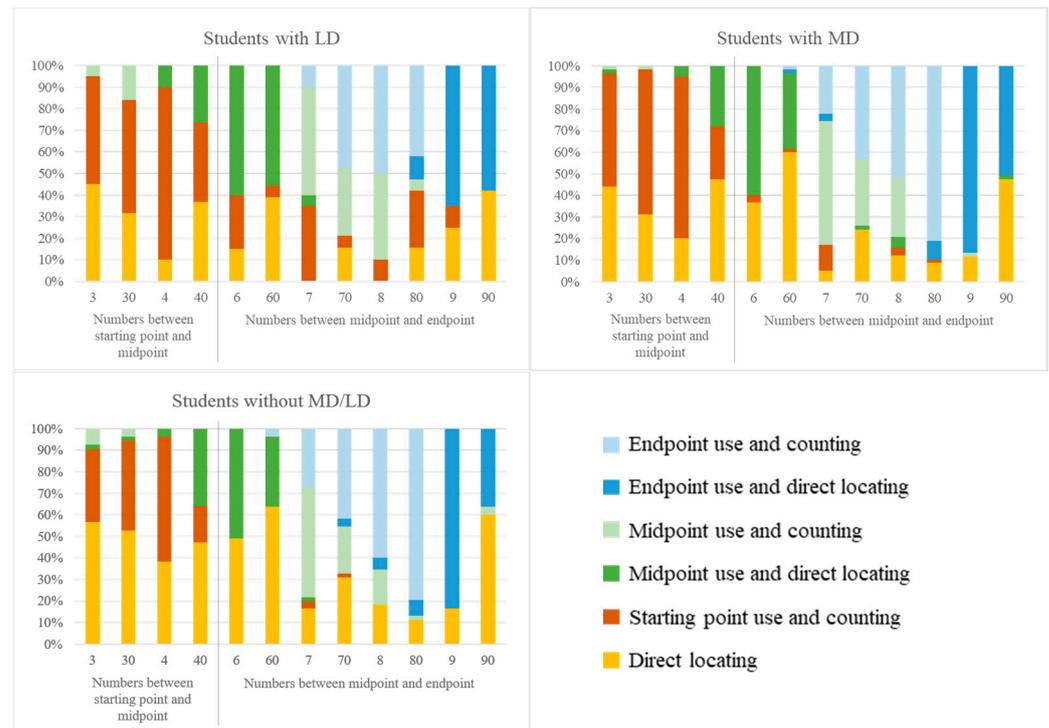
Figure 6. Students' strategy use at the task level—separated for the different student groups.

4. Discussion

The aim of this study was to investigate if and how students with LD, students with MD, and students without MD/LD differ in locating numbers on a marked NL. We analyzed students' strategies for locating numbers on a marked NL qualitatively based on ET videos and we analyzed error rates. We used data from 135 students (20 students with LD, 60 students with MD, and 55 students without MD/LD).

Empirical findings. The total number of *errors* was relatively low. Even though students with LD tended to make slightly more mistakes than students with MD and without MD/LD, there was no significant difference between the error rates of students with LD, students with MD, and students without MD/LD. This is likely due to the fact that the tasks involved number ranges up to 10 and 100, which represent familiar number ranges for all students. Another reason was the design of the NL. Research indicates that increasing the number of hatch marks on an NL positively affects the estimation accuracy in locating numbers (e.g., Peeters et al., 2017). For our NL, each number that had to be located could be assigned to a hatch mark on the NL. This allowed students to locate the numbers exactly, without having to estimate their positions as in tasks on the empty NL (e.g., van't Noordende et al., 2016).

Regarding students' *use of strategies*, we found that all groups of students used the same types of NL strategies, that is, students with LD, with MD, and without MD/LD did not differ fundamentally in the types of strategies that they used. This supports the assumption that students with LD do not have a divergent learning process in mathematics compared to students without LD (e.g., Hecht et al., 2011). Differences between the students were found in the frequency with which certain strategies were used. Comparing students' strategy use for all tasks together, we found that students without MD/LD used *non-counting/direct strategies* more often and *counting strategies* less often than students with MD and students with LD. This is in line with previous studies showing that students without a comprehensive understanding of numbers and relationships between numbers often rely

on counting strategies and have difficulties developing other, more advanced strategies (e.g., Moser Opitz et al., 2018; Verschaffel et al., 2007).

With regard to the six different NL strategies (Figure 2) in particular, we found that students with LD and students with MD used more *starting point use and counting* and less *direct locating* than students without MD/LD. This is in line with previous findings on the use of NL strategies of students with MD compared to students without MD (Simon & Schindler, 2022; van Viersen et al., 2013; van't Noordende et al., 2016). Furthermore, it was found that students with LD used the strategy *starting point use and counting* more often than students with MD. This connects to findings that students with LD often lag behind students without LD in mathematics (e.g., Moser Opitz, 2008) and that students with LD show difficulties with mathematical content to a greater extent than students with MD (Jordan et al., 2002).

Comparisons of students' NL strategies at the task level showed similar within-group trends (see Figure 6): All groups of students tended to show more *starting point use and counting* for numbers between the starting point and the midpoint of the NL than for numbers between the midpoint and the endpoint of the NL. Between-group comparisons for numbers between the starting point and the midpoint of the NL showed that students with LD and students with MD tended to use *starting point use and counting* similarly often, whereas students without MD/LD tended to use *direct locating* more often. For numbers further away from the starting point (i.e., numbers between the midpoint and the endpoint of the NL), students' *starting point use and counting* generally decreased. However, students with LD continued to use this strategy more often than students with MD and students without MD/LD, who tended to use the *endpoint* of the NL more often. In contrast, students with LD tended to use the *midpoint* more often. Similar trends were found in a previous study on the use of NL strategies of children with MD compared to children without MD (Simon et al., 2023). In a study of van der Weijden et al. (2018), differences in the use of reference points were shown in adults with MD compared to adults without MD: Adults with MD showed a tendency to use a reference point that was smaller than the number that had to be located, while adults without MD more often used a reference point that was larger than the number. Following van der Weijden et al. (2018), it can be assumed that the use of a reference point that was smaller than the target number by students with LD and students with MD and the related forward orientation (counting forward) compared to students without MD/LD may be related to working memory demands: Counting forward is less demanding than counting backward (e.g., Baroody, 1984). This may have influenced strategy use, particularly for students with LD and MD, who may have lower working memory capacities (e.g., Heimlich, 2016; Peng & Fuchs, 2016; Winkel & Zipperle, 2023). Furthermore, the use of reference points on the NL is dependent on the students' prior knowledge of numbers and number relationships (e.g., Peeters et al., 2017; Schulz & Wartha, 2021). Only students with a profound understanding of numbers and sufficient familiarity with the number range can use the unlabeled midpoint of the NL (e.g., Peeters et al., 2017). Another possible factor influencing the students' use of strategies might be the students' perceived difficulty of the tasks (e.g., Nicchiotti & Spagnolo, 2024). Students with LD and students with MD might perceive the NL tasks as more difficult than students without MD/LD and therefore rely on safe strategies and show less flexible use of strategies. The NL tasks used in this study involved small number ranges (up to 10 and 100) that fifth graders are generally familiar with. Nevertheless, students with LD and students with MD often relied on counting strategies. This suggests that even in familiar contexts, there are differences in the use of NL strategies.

In general, students without MD/LD used the strategy *direct orientation* more often than students with LD and students with MD. This strategy involves only few gazes and peripheral vision for orientation in order to locate numbers on the NL. This gaze behavior of students without MD/LD compared to students with LD and students with MD is in line with the association of fewer gazes with expertise in other ET studies (see Holmqvist et al., 2011, p. 383). In the domain of mathematics, it has also been shown that increasing expertise is associated with increasing extrafoveal analyses (through peripheral vision) of mathematical tasks—for example, in the identification of geometric shapes (Chumachenko et al., 2024; Simon et al., 2025), or in pattern extension tasks (Pitta-Pantazi et al., 2024).

Although the effect sizes for differences in strategy use between the groups of students were small, they consistently pointed to meaningful and theoretically coherent patterns. Students with LD and students with MD tended to rely more on less efficient, sequential strategies. In contrast, students without LD/MD more frequently used direct strategies—commonly regarded as more advanced and flexible. For larger numbers between the midpoint and endpoint of the NL, this pattern was also evident between students with LD and students with MD: Students with LD continued to use counting strategies, while students with MD did so less frequently. Moreover, students' use of reference points (i.e., midpoint use and endpoint use) aligned well with findings from previous research. While the effect sizes were small, the consistency of the results—across groups, tasks, and in alignment with theoretical backgrounds and prior findings—strengthens the significance of the observed differences. Such subtle but consistent patterns may accumulate over time to influence long-term mathematical development. In particular, differences in NL strategy use between students with MD, with LD, and without MD/LD may contribute to widening achievement gaps. Students who rely on less efficient sequential/counting strategies may progress more slowly in developing conceptual understanding of numbers and number ranges, such as building profound and flexible number representations and relationships between numbers. This in turn can limit the development of other mathematical concepts, such as understanding arithmetic operations and developing computational fluency.

Pedagogical implications. In this study, even small differences in strategy use reflect important differences in how students interact with NLs. They point to differences in the development of the students with LD, with MD, and without MD/LD in how they use number relationships and how flexibly they use NL strategies. These findings highlight the need for targeted support, which could be built on these insights. For educational practice, our results indicate that supporting the development of a sufficient understanding of numbers, also in the small number ranges used in this study, seems necessary for fifth-grade students with LD and MD. Although the students could locate the numbers correctly, they (especially students with LD) showed a comparatively high use of counting strategies. It is important to overcome their frequent counting from the starting point of the NL and to foster relational interpretations of numbers on the NL, especially for numbers between the midpoint and the endpoint of the NL.

Developing support to familiarize students with LD and students with MD with different ways of locating numbers on a marked NL should be aimed for. The flexible use of reference points should be supported, encouraging the development of concepts for numbers in general. In support sessions, various features of different NLs could be changed, compared, and discussed. Possible modifications of NLs include adjusting the length of the NL and selecting different numerical sections (e.g., by choosing starting points other than 0). Students should be encouraged to describe relationships of natural numbers on the NL and to explain relations of target numbers to given structuring features to support the relational interpretation of numbers. Conversations about different strategies to locate numbers on the NL could be encouraged by asking questions such as: "How do you find

out which number belongs to the midpoint of the NL?" or "How can you find out how far apart two given numbers are?" (e.g., Schulz & Wartha, 2021). A possible activity for fostering the above-mentioned implications could be activities asking "How do I locate numbers quickly?", where children can explore strategies for locating numbers on the NL and discuss them with other children. The strategies developed with the children should be practiced and consolidated in various exercises. At the beginning, many numbers could be marked and labeled on the NL, and important structuring features such as the midpoint of the NL could be highlighted. Over time, the labeling and highlighting can be gradually reduced. If children are already familiar with the number ranges shown on the NL and know about advantageous NL strategies, they may still rely on supposedly safe counting strategies from the starting point of the NL. Tasks in which children can see the NL only for a few seconds—forcing them to rely on the nearest reference point for quick orientation rather than counting—might help strengthen students' confidence in using other NL strategies.

Limitations and implications for future studies. The present study has some limitations from which suggestions for future work can be derived. One limitation of this study is the relatively small number of tasks. Further research should investigate if our results can be generalized to a larger set of tasks. Since we have chosen tasks in small ranges of numbers, it would also be interesting to investigate if the differences found in this study are even more apparent in more advanced tasks. There are also limitations related to the groups of students with MD and with LD. The classification of students with MD was based on their results in a standardized mathematics test. It is important to note that this test was carried out at only a single point in time. Furthermore, the group of students with LD is heterogeneous (e.g., Heimlich, 2016), which is reflected in diverse individual learning profiles and varying degrees of mathematical knowledge and skills. Students with LD can vary greatly in their mathematical skills—for example, in their understanding of number magnitude or flexibility in strategy use. This heterogeneity was also evident in the variation in error rates within the LD group in this study. The resulting within-group variance posed a limitation for the statistical analyses, making it more difficult to detect statistically significant differences. Furthermore, the heterogeneity of students with LD was evident in the observed strategy use in the NL tasks. Some students relied mostly on counting strategies, while others used more advanced strategies, albeit sometimes inconsistently, or did not use the strategy *starting point use and counting* in any task. This variability may have led to an increased within-group variance. Moreover, the relatively small sample size of students with LD increases the impact of individual differences, which may further limit the generalizability of the findings. In general, it must be acknowledged that students with LD do not form a homogeneous group in terms of their mathematical learning. When interpreting the results of this study, it is important to consider that various factors, including teaching, have an influence on LD. No specific data were available regarding the previous mathematics teaching of the participating students. Mathematics teaching at special schools for LD is traditionally highly structured and often focuses on smaller steps than mathematics teaching at schools of general education (e.g., Moser Opitz, 2013). Future studies should aim for a more representative sample, including a larger number of students with LD, also from inclusive schools. It should be examined whether the findings of this study can be confirmed when students with LD are taught together with their peers without LD in an inclusive school setting.

Methodological implications. Whereas the analyses of error rates in marked NL tasks revealed no significant differences between the different groups of students, the analysis of students' strategy use based on ET videos revealed significant differences between students with LD, students with MD, and students without MD/LD. From a product-oriented view

(focusing on error rates), it can be stated that students with LD and MD were generally able to correctly solve the NL tasks in the small number ranges used in this study. The use of ET and the qualitative analysis of the student strategies in NL tasks (process-oriented view), in contrast, revealed differences between the groups of students. This approach provided detailed insights into how students who struggle with mathematics deal with the NL. Hence, this study is in line with other studies that have shown what potential the analysis of strategies based on student gazes can have in mathematics education research, also for NL tasks (e.g., Simon & Schindler, 2021).

Notes

[1] For better comparability of the error rates, we visualize the mean error rates, even though the Kruskal–Wallis test is a non-parametric test and is based on rank-sums.

[2] Due to the relatively small sample size of students with LD, we refrained from further statistical analyses at the task level.

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