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THE AXIOM OF CONTROL FOUNDATIONS OF SOVEREIGNTY

The Sovereign Controller Curriculum (SSC):

A Geometric Axiomatics for the SIL 4 Certification of Adaptive Railway Systems

Based on Riemannian Manifolds

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Abstract:

This document formulates a closed mathematical axiomatics for the control of adaptive, non-deterministic optimization components in safety-critical railway systems (SIL 4). At its core, the Sovereign Controller Curriculum (SSC) establishes a formally verifiable governance framework in which safety is not probabilistically approximated but enforced as a geometric invariant. Through the strict separation of QM-classified adaptivity (AI proposal engines) and SIL 4-certified enforcement (the Sovereign Guardian), Freedom from Interference is realized physically, logically, and temporally. The framework replaces heuristic safety arguments with a compositionally closed proof chain based on Riemannian geometry, convex projection, zonotopic reachability analysis, Lyapunov stability, and hardware-enforced final authority, fully aligned with EN 50126, EN 50128, and EN 50129.

Keywords: SIL 4, Sovereign Controller, Freedom from Interference (FFI) Riemannian Safety Manifold Zonotopic Reachability Analysis, Lyapunov Shield, Hardware-Enforced Governance, AI Safety Caging EN 50129

Inhalt

KEY ABBREVIATIONS (NORMATIVE REFERENCE)

The following abbreviations constitute the minimal conceptual vocabulary required for the first reading of this document. All additional terminology is defined in the consolidated

Glossary (Appendix F) and serves as a normative reference for detailed technical analysis and audit activities.

SIL 4 Safety Integrity Level 4 – Highest safety integrity level according to EN 50126/28/29

MDR Mandatory Design Requirement – Binding design axioms of the SSC framework

ESA Explicit System Assumptions – Preconditions for formal verification validity

SG Sovereign Guardian – Physically isolated SIL 4 safety enforcement unit

WCET Worst-Case Execution Time – Absolute upper bound for deterministic execution

FFI Freedom from Interference – Guaranteed isolation between QM and SIL 4 domains

QM Quality Management – Non-safety-rated components (e.g. AI proposal engine)

AGR Assume-Guarantee Reasoning – Compositional safety proof methodology

THE AXIOM OF CONTROL: FOUNDATIONS OF SOVEREIGNTY

The Sovereign Controller Curriculum (SSC) represents a fundamental shift in railway automation. As we integrate adaptive, high-performance algorithms into safety-critical environments, traditional methods of heuristic testing reach their limits. To achieve SIL 4 certification, we must move beyond "estimating" safety and toward commanding it through mathematical necessity within a formally defined model.

On the Level of Abstraction The high degree of mathematical abstraction within this document—utilizing Riemannian Geometry, Spectral Stability, and Radon-Nikodým Integrity—is not a choice of complexity, but a requirement for formal certainty. By defining safety as a geometric invariant within the defined system assumptions (ESA), we eliminate the ambiguity of human judgment and replace it with a formal, compositionally closed proof chain. This abstraction serves as the "Mathematical Cage" that allows innovation to flourish without compromising human life.

On the Choice of Language This technical manifesto is primarily authored in English. This decision ensures maximum alignment with international safety standards (EN 50128/50129) and prevents the "semantic drift" that often occurs during the translation of rigorous normative requirements. By utilizing the precise "shall/must" logic of English engineering prose, we provide a globally auditable framework for Independent Safety Assessors (ISA).

This curriculum is the blueprint for a system that does not just "function"—it reigns sovereign over its operational domain.

CHAPTER 1 – AXIOMATICS & GEOMETRIC DESIGN

To achieve deterministic safety and satisfy SIL 4 auditability, the following geometric constraints shall be enforced at the design level. These are Mandatory Design Requirements (MDR).

MDR-01 (Topological Basis – Riemannian Manifold): The state space M shall be implemented as a complete Riemannian Manifold (M, g).[1] The metric structure, defined by the metric tensor g_{ij}, provides the necessary framework for absolute distance and projection calculations within curved safety manifolds.

Technical Logic: The train's status is treated as a precise point on a differentiable manifold. Completeness (Banach property) mathematically guarantees that safety algorithms always converge to a valid state within system boundaries. This replaces "estimated" risks with a definitive metric distance to hazards.

WCET Guarantee: All Riemannian constructs are implemented as pre-compiled, piecewise affine (PWA) approximations. No online differential-geometric computation is performed within the safety cycle to ensure compliance with WCET bounds.

MDR-01b (Hybrid State Space Structure): The state space M shall be formally defined as a hybrid product space: X: =Xcont×Xdisc

Technical Logic: This structure ensures that continuous physical dynamics (X_{cont}) and discrete logical states (X_{disc}), such as interlocking positions, are unified within a single measurable σ-algebra.

MDR-02 (Convexity of Ω_S): The safe operating domain Ω_S shall be defined as a convexly closed set.

[1] Hopf–Rinow Theorem: Every complete, connected Riemannian manifold is geodesically complete (Hopf & Rinow, 1931).

Technical Logic: Safety must be a "shape without dents". The Hilbert Projection Theorem ensures that for any unsafe state, exactly one unique and shortest path back to safety exists. This prevents system "hesitation" between corrective actions in high-pressure scenarios.

MDR-03 (Constructive Action Domain Ω_{SA}): The action domain Ω_{SA} shall be explicitly defined as the state-dependent pre-image: $\Omega_{SA}(x)\backslash coloneqq\{u \in U \mid G(x,u) \in \Omega_S\}$

Technical Logic: Commands are pre-filtered before a violation occurs. Only actions proven to keep the system within safe boundaries are permitted. This creates a deterministic filter; actions not in this "safe list" are physically blocked.

MDR-04 (Spectral Stability Constraint): The Governance Operator G shall be restricted to an affine-linear state transition: $x_{k+1} = G(x_k, u_k) = Ax_k + Bu_k + c$. Furthermore, the spectral radius $\rho(A)$ shall be monitored to ensure predictive stability.

Technical Logic: Affine- linearity ensures predictable behavior and preserves the convexity of the safe domain during every state transition. Monitoring the spectral radius allows the system to pre-emptively mitigate instabilities.

MDR-05 (Safety Projection S): A metric orthogonal projection $S: U_{intent} \rightarrow \Omega_{SA}(x)$ shall be implemented as the primary safety mechanism.

Technical Logic: This mechanism (formal "Cage") instantly projects risky AI proposals back to the nearest safe boundary. The AI proposes intent, while geometry enforces sovereignty. Metaphorical terms are for explanatory purposes only and refer to the formally defined operators and constraints.

CHAPTER 2 – NUMERICAL SOVEREIGNTY: ZONOTOPE MONITORING

To guarantee numerical integrity according to SIL 4, the system must detect and mitigate non-deterministic errors arising from finite-precision floating-point arithmetic without compromising availability through over-approximation.

MDR-06 (Zonotope-Based η-Monitoring): To prevent the "Wrapping Effect", the η-monitor operates using Zonotopes.

Zonotope Tracking: Every state x is represented as a centrally symmetric polytope: $Z = c + \sum \eta_i g_i$.

MDR-15.1 (SVD-Based Order Reduction): For condition numbers $\kappa(A) \geq 10^5$, an **SVD-based reduction** is performed. This minimizes the enclosure error by aligning generators with the principal axes of uncertainty, ensuring availability in complex track layouts.

Stability Condition: Safety is compromised if the zonotope radius exceeds the Lipschitz budget ϵ_{max}.

MDR-07 (Sterbenz Guard & Bit-Identity): For every safety-critical subtraction $z = x - y$, the SG must verify the Sterbenz condition: $\frac{y}{2} \leq x \leq 2y$.[2]

SCA-02.1 (Flocq- Based Modeling): The formal proof (Coq/Isabelle) must utilize the **Flocq library** to achieve bit-identical modeling of the FMA pipeline and denormal handling (Flush-to-Zero) of the target hardware. Any discrepancy between the hardware FPU and the formal model renders the system non-certifiable.

Tool-Chain: The flags -frounding-math, -fno-associative-math, and -fsignaling-nans are mandatory. The use of -ffast-math is strictly prohibited.

Object Code Verification (SCA-01): The integrity of the decomposition into Sterbenz-verified primitives shall be validated directly on the binary code to ensure no hardware optimization undermines the Lipschitz guarantee.

CHAPTER 3 – CONTINUITY OF PROOF: CLOSING THE ZONOTOPE CHAIN

To prevent "safety drift" between discrete execution steps, the framework utilizes Formal Reachability Analysis via set-based inclusions. While Chapters 1 and 2 define the topological space and numerical precision, Chapter 3 ensures the Continuity of Proof as the system evolves dynamically over time.

[2] Sterbenz, P. (1974). *Floating-Point Computation*. Prentice Hall. (Lemma 4.1: Exact subtraction under bounded ratio.)

Interval Soundness: Every operator G is encapsulated within a Set-Valued Interval Inclusion $[G]$.

Technical Logic: Physical sensors and processors possess non-zero tolerances, making "perfect point" calculations impossible; instead, we operate on neighborhoods (Intervals).

Sovereignty Upgrade: On the Riemannian manifold, these intervals are defined as **geodesic balls**. By enforcing that every physical operation is contained within its defined inclusion, we provide a mathematical guarantee that the real-world behavior of the train remains a subset of our formal description.

Enforcing Inclusion: For every discrete time step k, the inclusion $x_{k+1} \in [G](x_k)$ must hold. Slope Matrices and error terms are rigorously selected to encompass the supremum of all local deviations.

Technical Logic: This creates a **Zonotopic Safety Tunnel**. At every clock cycle k, we prove that the subsequent state x_{k+1} is trapped within the calculated bounds of the previous state.

Innovation: Utilizing the Riemannian curvature, the Slope Matrices adapt to the local topology. This allows the "tunnel" to be sufficiently wide to contain physical reality while remaining narrow enough to stay within the safe domain Ω_S.

The Inductive Proof: Given the initial state $x_0 \in \Omega_S$ and the requirement that every subsequent operation is contained within proven bounds, the resulting Zonotope Radius R_z represents a conservative, formal upper limit for the accumulated numerical and physical error.

Technical Logic: This constitutes the **Inductive Proof Chain**. As long as this radius does not intersect with the forbidden state space (the "danger zone"), the system remains mathematically SIL 4 compliant.

CHAPTER 4 – HYBRID STABILITY: THE ROBUST DWELL-TIME LEMMA

To mitigate "Zeno behaviour" (high-frequency switching or chattering at the safety boundary), the framework enforces a formal temporal hysteresis. In a SIL 4 environment, the safety mechanism must not oscillate rapidly between "nominal" and "corrective" states, as this jeopardizes mechanical integrity and violates computational determinism.

MDR-08 (Disturbance Constraint): The system shall define a Worst-Case Disturbance Bound W_{max}, encompassing all non-deterministic environmental noise and stochastic shocks.

Technical Logic: No system operates in a vacuum; external factors such as wind, track friction, or sensor jitter create exogenous "noise".

Sovereignty Aspect: W_{max} is defined as the absolute maximum disturbance vector the environment can exert on the state x. By quantifying this bound, we design a safety margin that is mathematically "stronger" than any possible external shock, ensuring the controller remains dominant over its environment.

MDR-09 (Hysteresis Calibration): The safety offset Δh (hysteresis width) shall be strictly calibrated such that: $\Delta h > W_{max} \cdot \delta t$.

Technical Logic: This serves as the **Sovereign Buffer**. To prevent "control chatter" (flickering between states), we create a guard zone Δh.

Precision: This offset is specifically sized to exceed the maximum possible disturbance that could manifest within a single processing cycle δt. This calibration ensures that a single "stochastic gust" is insufficient to trigger an erratic sequence of safety corrections.

The Stability Result: Under maximum stochastic shock, the system is mathematically guaranteed to remain in the safe zone for a minimum **Dwell-Time** τ_{dwell} following any corrective projection S.

Technical Logic: This is our **Mechanical Insurance**. By enforcing a minimum dwell-time, we guarantee that once the safety layer intervenes, the system state remains stable for a deterministic duration.

Benefit: This formally eliminates Zeno-type convergence—where correction intervals shrink toward zero—thereby minimizing mechanical wear on actuators and ensuring the logic remains auditable.

CHAPTER 5 – THE LYAPUNOV SHIELD: STABILITY ENFORCEMENT

This chapter defines the decoupling of Performance from Safety. The integration of adaptive components is governed by a Lyapunov-based barrier, acting as the final arbiter for actuation. In this architecture, the AI layer serves as the "Performance Optimizer," while the Lyapunov Shield acts as the "Stability Enforcer," ensuring that optimization never leads to a loss of control.

MDR-10 (Energy Invariance): Every proposed action u from the AI layer must satisfy the discrete stability condition: $\Delta V = V(x_{k+1}) - V(x_k) \leq 0$, [3]where V is the defined Lyapunov function on the hybrid state space M.

Technical Logic: We define the system "energy" via the Lyapunov function V. In a stable system, this energy must remain invariant or dissipate; it must never grow uncontrollably.

Safety Filter: MDR-10 serves as a strict filter. Any AI command that would increase system energy—potentially leading to instability or boundary violations—is identified as a critical safety violation and intercepted.

The Lyapunov Cage: If an AI proposal optimizes performance at the expense of stability, the Shield must attenuate or project the proposal to ensure system energy converges toward the safe equilibrium.

Stability Governor: While the AI may suggest aggressive maneuvers for arrival time optimization, the Shield "cages" the command as it nears stability boundaries.

Gradient Flow: The Shield modifies the command such that the system energy gradient ∇V always flows back toward the stable, safe equilibrium state x^*.

The Stability Identity: This establishes a formal identity between the defined safety boundaries Ω_S and the Lyapunov barriers.

Topological Union: Safety and stability are treated as a unified topological concept.

Energetic Impossibility: By aligning safety boundaries Ω_S with Lyapunov energy levels, we prove that crossing a safety limit under the controlled law is energetically impossible

[3] Lyapunov, A. M. (1892). *The General Problem of the Stability of Motion*. (Classical formulation of energy-based stability.)

within the formal model. The system is mathematically "weighted" to remain within or return to the safe domain.

Summary for the Safety Case: Chapter 5 provides the final mathematical protection layer for complex maneuvers. By utilizing the Lyapunov Shield, the SSC framework allows the use of non-deterministic AI for performance optimization without jeopardizing the physical sovereignty of the train. Even in the event of AI "hallucinations," the Shield ensures the system remains trapped within the energy boundaries of the safe equilibrium.

CHAPTER 6 – ADAPTIVE ACTION (A): THE AI PROPOSAL ENGINE (ADAPTIVITY)

The Action operator A provides the system with high-dimensional optimization capabilities while remaining logically and physically isolated from the safety-critical core. This architectural separation ensures that performance-driven improvements—such as energy efficiency or passenger comfort—do not compromise the integrity of the formal SIL 4 safety proof.

MDR-11 (QM Classification): Due to the inherent non-determinism of neural networks and adaptive learning models, the Action operator A is classified strictly as **Quality Management (QM)**.

Technical Logic: Modern AI is "black box" by nature and cannot be formally verified to SIL 4 standards. By classifying the AI as QM (the lowest integrity level), we eliminate the requirement for formal proof of its internal stochastic logic.

Safety Case: The safety case does not rely on the reliability of the AI, but on the deterministic integrity of the SIL 4 "Cage" that encapsulates it.

MDR-12 (Decoupled Intent): The AI Proposal Engine shall possess **no direct authority over actuators**. All outputs are treated as "stochastic suggestions" that must be validated by the subsequent SIL 4 Shielding operator S and the Lyapunov Shield.

Technical Logic: This embodies the "Separation of Powers". The AI layer functions as the "Brain" (Intent) but lacks "Hands" (Actuators). Every command is intercepted by the SIL 4 safety layer. Any maneuver identified as mathematically unsafe is modified or blocked before reaching the physical control interface.

MDR-13 (Encapsulation): Adaptive updates or online learning processes within the AI layer shall not affect the Lipschitz Budget or the formal safety proof of the global system mapping.

Technical Logic: This ensures **Freedom from Interference (FFI)**. Because the AI is encapsulated, neural networks or learning parameters may be updated in real-time without necessitating a re-certification of the static safety proof. The safety framework remains invariant and sovereign, independent of the adaptive evolution of the AI.

Summary for the Safety Case: Chapter 6 defines the relationship between "Innovation" and "Safety". By treating the AI as a QM-level advisor, the SSC framework enables cutting-edge optimization without violating EN 50128 standards. The safety of the asset is decoupled from AI intelligence; it is solely dependent on the unbreakable nature of the SIL 4 Shielding.

CHAPTER 7 – PREDICTION & THE GEOMETRY OF FLOWPIPES

Reachability Analysis and Girard Order Reduction Predictive safety is achieved by enclosing the physical future within conservative envelopes (**Flowpipes**) rather than point-trajectories. Instead of predicting a single coordinate, the system calculates a reachable set—a volume of state space that the asset is mathematically guaranteed to occupy.

MDR-14 (Zonotopic Enclosure): The prediction operator P shall generate a Zonotope that encompasses all physically reachable states, accounting for variations in friction, sensor latency, and actuator uncertainty.

Technical Logic: Single-point predictions are fundamentally unsafe as they ignore the inherent "fuzziness" of physical reality.

Minkowski Sum Efficiency: Zonotopic enclosures are utilized due to their Minkowski sum efficiency, allowing real-time reachability analysis within strict WCET-slot guarantees.

MDR-15 (Inclusion Invariant): To ensure real-time computational feasibility, the system shall apply **Girard Order Reduction**.[4] It is a strict requirement that the reduced Flowpipe F_{red} remains a valid over-approximation of the original physical future F_{phys}, such that $F_{phys} \subseteq F_{red}$.

Safety over Precision: The reduction simplifies shapes by reducing the number of generators for high-speed processing. This simplification must be strictly conservative: the resulting shape must be larger than or equal to the original.

Formal Soundness: Computational granularity may be sacrificed for execution speed, but the integrity of the inclusion is non-negotiable. If the inclusion is violated, the formal proof of safety is void.

Summary for the Safety Case: Chapter 7 ensures that the SSC is proactively safe. By utilizing Flowpipes (the union of zonotopes over a time horizon), the system projects the safety boundary into the future. If the future reachability set intersects with an obstacle, the safety projection S is triggered immediately.

CHAPTER 8 – MEASUREMENT & BOREL INTEGRITY: ROBUST DATA SANITIZATION

The integrity of the state-space mapping is strictly dependent on the deterministic validity of sensor inputs. In a SIL 4 environment, "undefined" or "non-representable" numerical values are prohibited from entering the safety-critical calculation chain.

MDR-16 (NaN/Inf Hardware Trap): The measurement operator M shall implement hardware-level traps for IEEE-754 special values, specifically NaN and $\pm\infty$.[5]

[4] Girard, A. (2005). *Reachability of Uncertain Linear Systems Using Zonotopes*. HSCC.
[5] IEEE-754 Standard for Floating-Point Arithmetic (2019). Annex G: Non-numbers and exceptional values.

Technical Logic: These "mathematical ghosts" represent singularities that would "poison" all subsequent topological calculations. The system detects these at the hardware level within the microsecond of ingestion.

MDR-17 (Safe-State Compulsion): Since a NaN-state cannot be mapped into a measurable Borel space, the system shall bypass all AI proposals and execute an immediate emergency brake application via the hardware Interlock-Gate.

Measure-Theoretic Constraint (Formal Proof Only): A NaN value is inherently "unmeasurable" and possesses no coordinate within the defined σ-algebra. The application of Borel Integrity serves as a formal proof device to ensure logical consistency and requires no runtime numerical integration. The only sovereign response is to trigger the physical emergency braking system.

Borel Integrity: This sanitization process ensures that subsequent State Estimation operates exclusively on a well-defined σ-algebra.

Clean Data Guarantee: By scrubbing the input of all non-measurable values, the entire control loop remains logically consistent within its measure-theoretic framework. This provides the Auditor with a "Chain of Evidence" rooted exclusively in valid physical facts.

Summary for the Safety Case: Chapter 8 provides the Sanity Check for the digital-physical interface. By enforcing Borel Integrity, we ensure that the "Mathematical Cage" never processes "impossible" values. This ensures that the hardware Interlock-Gate always possesses a deterministic basis for maintaining or interrupting power to the actuators.

CHAPTER 9 – STATE ESTIMATION & BOREL INTEGRITY

9.1 From Raw Data to Topological Truth

To ensure the control logic operates on a valid mathematical representation, the transition from physical sensors to the state space M must satisfy **Borel Integrity**. We do not accept raw data at face value; we transform it into a rigorous state for formal verification.

MDR-18 (Borel Measurability): All sensor inputs shall be mapped into M via a Borel-measurable Measurement Operator M.

Technical Logic: Data must exist in a "well-behaved" mathematical world for safety proofs to hold, staying within a σ-algebra where metric comparisons are formally defined. Values that cannot be mapped are discarded immediately.

Formal Distinction (Formal Proof Only): Borel measurability serves as a structural proof for decision uniqueness and does not require numerical integration at runtime.

MDR-19 (Consistency Check): The State Estimator shall compare the measured state z against the predicted flowpipe F.

Topological Reality Check: If the distance $d(z, F)$ exceeds the Lipschitz Budget η, the measurement is rejected. We reject readings outside the predicted volume as physical impossibilities, preventing "sensor jumps" from inducing dangerous maneuvers.

MDR-20 (Temporal Aliasing Guard): Data packets with a jitter exceeding τ_{dwell} trigger an immediate invalidation of the safety cycle.

Technical Logic: Temporal precision is as critical as numerical accuracy. We prevent commands based on outdated versions of reality.

9.2 Sensor fusion & Trust Weights

The fusion process is governed by the Validation Operator V.

Confidence Gating: Sensors are treated as independent witnesses; inconsistent sources are automatically attenuated in favor of reliable ones.

Sovereignty Rule: Fused results must be re-verified against the **Sterbenz Guard (MDR-07)** to mitigate numerical drifts during complex fusion arithmetic (e.g., weighted averaging).

Summary for the Safety Case: Chapter 9 bridges the gap between stochastic physical sensors and the clean geometric world of SIL 4. By enforcing Borel Integrity and Sterbenz-validation, the system operates exclusively on "Topological Truth"

CHAPTER 10 – GOVERNANCE & CONTROL LAW (G) (REV. 2.3)

10.1 The Governance Mapping

The Governance Operator G is the central authority mapping current states x and control actions u to future transitions within the safe domain Ω_S. This mapping is strictly constrained to maintain SIL 4 determinism, defining the "allowable physics" for predictable transitions.

MDR-21 (Affine-Linearity): The operator G shall be implemented as an affine-linear state transition: $x_{k+1} = G(x_k, u_k) = Ax_k + Bu_k + c$.

Technical Logic: Affine- linearity ensures the system remains mathematically decidable. Crucially, it preserves the convexity of Ω_S during transformation, preventing safety boundaries from becoming "warped".

MDR-22 (Control Law Sovereignty): The governance law shall be independent of the AI's internal state, acting as a stationary functional defining immutable physical constraints.

Technical Logic: The AI cannot modify the "rules of the game". Fundamental physical limits remain constant regardless of AI evolution.

MDR-23 (Safe-State Convergence): The governance law must ensure that for every point x on the boundary $\partial\Omega_S$, the resulting vector field points strictly toward the interior of Ω_S or remains tangential.

Technical Logic: This "No Exit" rule mathematically ensures that it is impossible for a governed action to "pierce" the safety boundary.

10.2 Pre-image Computation

Governance computes the Safe Action Set $\Omega_{SA}(x)$ by working backward from the safe zone to identify permissible commands.

Inversion Logic: $\Omega_{SA}(x) \backslash coloneqq \{u \in U \mid G(x,u) \in \Omega_S\}$.

Technical Logic: This creates a deterministic filter for the AI layer. Due to affine-linearity and convexity, the calculation is instantaneous with a unique solution.

Feedback-Loop-Integrity: The Governance operator shall use the validated state x as its only input.

Technical Logic: This "Clean Loop" prevents glitchy sensor data from entering the control loop, relying on "Topological Truth" established by Borel integrity checks.

CHAPTER 11 – NETWORKED INTERDEPENDENCE (C)

Causal Prioritization and Conflict Resolution

Interdependence within networked nodes is managed via the **Coupling Operator C**. In a complex rail network, multiple assets must communicate and share resources without leading to deadlocks or safety violations.

Prioritization Logic: Conflicts are resolved through **Lexicographical Ordering**, granting absolute priority to assets with higher safety classifications.

Technical Logic: We eliminate non-deterministic "negotiation" in favor of a strict ranking system.

Predictability: A pre-defined hierarchy ensures the most critical safety-relevant asset always wins the conflict predictably.

The Aging Mechanism: To prevent process stagnation, an asset's priority increases after N cycles, strictly within its designated safety class.

Liveness: This mechanism prevents "starvation" of lower-priority tasks.

Strict Isolation: Aging occurs only within the same safety level; low-level efficiency tasks can never override critical SIL 4 safety commands.

Summary for the Safety Case: Chapter 11 ensures deadlock-free "social behaviour" within the network. By using Lexicographical Ordering and Aging, global communication becomes a deterministic component of the local SIL 4 safety proof.

CHAPTER 12 – STOCHASTIC RESILIENCE: ORTHOGONAL MARTINGALE MONITORING

The system is engineered to prevent human operational factors from obscuring technical degradation. Safety is maintained by monitoring variables on orthogonal planes to prevent "error masking".

MDR-24 (Blind Zone Constraint): The system shall monitor two independent variables on orthogonal planes: Cognitive Drift and Physical Drift.

Technical Logic: The human operator and the machine are monitored as independent entities.

Prevention of Compensation: Tracking human response and mechanical precision simultaneously prevents scenarios where a human unknowingly masks a machine failure through manual correction.

MDR-25 (Safety Mode Trigger): If the cross-correlation between human and technical drift exceeds a defined threshold, a Blind Zone is identified.

Masking Detector: Synchronization between human error and technical wear renders sources of variance indistinguishable.

Risk State: This represents a loss of diagnostic integrity due to effective masking of machine failure.

Diagnostic Integrity: The system shall autonomously transition to a High-Conservative Mode upon Blind Zone detection.

Technical Logic: Trust in combined human-machine output is ceased, transitioning to a fail-safe state. This allows for diagnostics to be performed without human interference.

Summary for the Safety Case: Chapter 12 addresses the "Human-in-the-loop" risk. By monitoring drift on orthogonal planes, the SSC ensures that human behavior cannot mask technical degradation. Blind Zone identification provides a formal trigger for High-Conservative Mode, maintaining diagnostic integrity—a critical SIL 4 requirement.

CHAPTER 13 – THE RESILIENCE LEMMA: FORMAL CONVERGENCE IN BANACH SPACE

Resilience is defined as the system's inherent capacity to attenuate disturbances and return to a stable fixed point. In this framework, resilience is a topological certainty, enforced through the geometry of the space.

The Resilience Lemma: Any trajectory x_k deflected from the target fixed point x^* by a disturbance w is subject to the global contraction rate I.

Technical Logic: External events causing state deviations trigger control laws that function as a high-tension spring, pulling the system back toward its intended equilibrium.

The Mathematical Proof: Given the composite operator I is a strict contraction ($|I| < 1$), the state error $e_k = x_k - x^*$ follows the recursive inequality: $e_{k+1} \leq |I| \cdot e_k$.

Topological Stability: The system is "self-stabilizing" by its own topology. Deviations are attenuated exponentially by the chain of sovereign operators.

The Consequence: The state converges to the target equilibrium exponentially. Resilience is enforced by the **Banach Fixed-Point Theorem**,[6] ensuring recovery from transient shocks.

SIL 4 Evidence: This provides proof for temporal recovery. Recovery is exponential, meaning the further the system is displaced, the stronger the restorative force. This ensures the train returns to a safe, stable state in the shortest possible time.

CHAPTER 14 – TOPOLOGICAL INTEGRITY: THE HAUSDORFF AXIOM

The state space M is strictly required to function as a **Hausdorff Space (T_2)**[7] to ensure logical and physical distinctness.

MDR-26 (Uniqueness Guarantee): The T_2 property ensures that for any two distinct physical states x and y in M, there exist disjoint neighborhoods U and V such that $x \in U$, $y \in V$, and $U \cap V = \emptyset$.

[6] Rudin, W. (1976). *Principles of Mathematical Analysis*. Theorem 9.23 (Contraction Mapping Principle).
[7] Munkres, J. (2000). *Topology*. Definition of T_2 separation axiom.

Technical Logic: This "Identity Rule" ensures the system never confuses "State A" with "State B". It prevents the controller from perceiving a train in two positions simultaneously or failing to resolve gaps between approaching assets.

State Separation: The T_2 constraint prevents "state superposition". An asset cannot simultaneously occupy two logical sections or states.

Technical Logic: By enforcing the T_2 topology, the safety logic always derives a unique result. If the asset is at position x, it is exclusively at position x. This prevents non-deterministic loops where the controller cannot resolve which safety rule to apply.

Prerequisite for Collision Avoidance: The T_2 topology is a prerequisite for SIL 4 collision avoidance logic as it enforces absolute decision uniqueness.

Technical Logic: To be SIL 4 compliant, we must prove the distance between trains is a well-defined, positive number. The Hausdorff property allows us to formally state: "There is separable space between these two objects.

CHAPTER 15 – MEASURE THEORY & RESOURCE ANALYTICS: THE CAPACITY INTEGRAL

To maintain network stability, the system employs the **Lebesgue Integral** to evaluate the cumulative network load L across the state space M. This allows for a global view of infrastructure utilization, treating the railway network as a unified mathematical field.

MDR-27 (Network Load Evaluation): The network load L shall be quantified by the Lebesgue integral of the load density function f with respect to the Borel measure μ: $L = \int_M f(x)\ d\mu(x)$.

Technical Logic: We measure the "density" of the entire system rather than counting isolated assets. The Lebesgue Integral allows the aggregation of energy consumption, bandwidth, and track occupancy even across discontinuous or complex data sets.

Unified Metric: This yields a single, precise metric L representing the total stress on the infrastructure.

Metric Precision: This integral enables the objective identification of infrastructure bottlenecks before they compromise the stability of the Coupling Operator (C).

Early Warning System: By analyzing the "area under the curve" of network utilization, we identify traffic density saturation points.

Flow Management: We detect saturated data links or switches long before failure, allowing the Coupling Operator to dynamically maintain laminar flow.

Operational Limits: Quantifying network "exhaustion" with absolute metric exactness ensures resource allocation remains within safe, non-congested limits.

Latency Prevention: In congested networks, high-priority safety commands could suffer from digital latency.

Timing Guarantee: Maintaining L within proven safe bounds guarantees sufficient "mathematical space" for deterministic timing of safety-critical actions.

CHAPTER 16 – TEMPORAL SOVEREIGNTY & DYNAMIC RECESSION

Safety requires absolute temporal precision. To prevent "geometric paralysis," the system must compensate for jitter while guaranteeing the recovery of maneuverability through smoothed recession of safety boundaries.

MDR-28 (WCET Bound & Microarchitectural Determinism): The total Worst-Case Execution Time (WCET) shall remain strictly below the sampling period T_s.

Zero-Jitter & Cache-Locking: Operation in "Zero-Interrupt" mode with locked caches to prevent non-deterministic latencies caused by cache misses.

O(1) Geometry Access: Segment selection within the Riemannian geometry shall be performed via direct index calculation; search algorithms are prohibited.

Assembly Validation (STA-01): Static timing analysis at the instruction level, explicitly accounting for pipeline hazards and branch prediction effects.

MDR-20.3 (Minkowski Expansion & C^1 Recession): To compensate for "Temporal Ghosting" (uncertainty due to latency), the safety zone Ω_S is adaptively adjusted.

Expansion: For jitter $\tau > 0.5 \cdot \tau_{dwell}$, expansion occurs via Minkowski sum: $\Omega_{S,adj} = \Omega_S \oplus \mathcal{B}(|v| \cdot \tau)$. This increases the safety margin proportional to velocity v and time uncertainty τ.

MDR-20.3b (C^1-continuous Recession): To prevent Zeno oscillations at PWA (Piecewise Affine) cell boundaries, the contraction of the zone follows a smoothing kernel. The recession rate $\dot{\Omega}_S$ must be **C^1-continuous** and algorithmically limited to a value below the lowest natural frequency of the mechanical braking system. This guarantees **Liveness** without mechanical "Control Chatter."

MDR-30 (Instruction-Count Watchdog): Hardware monitoring of retired instructions detects logic hangs independently of clock fluctuations (thermal immunity).

CHAPTER 17 – HARDWARE ENFORCEMENT & PHYSICAL SOVEREIGNTY

To satisfy SIL 4 requirements, the separation between the QM-rated AI and the SIL 4 Monitor is enforced via physical barriers, guaranteeing absolute Freedom from Interference (FFI) even under catastrophic failure conditions.

HR-01 (Air-Gap Isolation & Substrate Autonomy): The Sovereign Guardian (SG) shall reside on a fully discrete printed circuit board (PCB).

Prohibition of MCM/SiP: Integration within the same package or on the same die is prohibited to physically eliminate transient latch-up (TLU) and substrate noise.

Resource Autonomy: Shared silicon structures (NoC, L3 cache) are forbidden.

Thermal Immunity: Spatial separation ensures zero thermal impact from the AI unit on the SG's WCET budget.

HR-02 (Independent Power & CMTI Hardening):

Galvanic Isolation & CMTI: Communication is restricted to isolators with a CMTI of $\geq 200\ kV/\mu s$. This prevents "ghost signals" caused by displacement currents during total AI failure.

EMI Resilience (CCF-01): The design must prove immunity against near-field induction. **New in Rev 2.3:** Wiring harnesses for AI and SG must be physically routed separately and shielded to prevent inductive coupling during AI short-circuit events (dI/dt peaks).

HR-03 (Hardware-Enforced Safety Gate & FIT-01):

Physical Finality: If $\eta > \epsilon_{max}$, the gate physically breaks the actuator power circuit within microseconds.

Fault Injection (FIT-01): Efficacy against AI short-circuits or clock freezes must be proven via the FIT-01 protocol.

CHAPTER 18 – COMPOSITIONAL CONTRACTS: ASSUME-GUARANTEE REASONING (AGR)

System scalability is achieved through formal compositional contracts. In a modern rail network, a monolithic safety proof for thousands of assets is computationally intractable. Chapter 18 resolves this by decomposing the safety proof into "contractual" blocks that can be formally aggregated.

MDR-31 (Contractual Logic): A specific node n guarantees its internal safety property G (**Guarantee**) provided that the adjacent nodes satisfy the defined input assumptions A (**Assumption**).

Technical Logic: Each system component makes a formal promise to maintain safety G contingent upon receiving valid, bounded input data A.

Inductive Proof: If every adjacent node fulfills its contractual obligations, the entire network is proven safe by induction. This allows for the verification of individual modules in isolation.

Modular Scaling: This modular approach allows for infinite network scaling without requiring a monolithic global proof.

Constant Complexity: Complexity remains constant per interface, regardless of network size. This creates a "Plug-and-Play" safety architecture, making formal verification manageable for large-scale deployments.

Horizontal Safety Chain: The contractual framework maintains the integrity of the horizontal safety chain across the network.

Formal Consistency: Contracts defined by **Linear Temporal Logic (LTL)** eliminate "semantic gaps" at the borders between different manufacturers' systems or track sections. The "Chain of Evidence" remains unbroken across the entire infrastructure.

CHAPTER 19 – ADAPTIVE SHIELD SYNTHESIS: CERTIFIED BARRIER IDENTITY

To reconcile complex physical environments with the requirement for absolute SIL 4 safety, the framework employs topological decomposition and deterministic selection axioms.

19.1 Riemannian Manifold Definition

The operational domain is defined as a complete Riemannian manifold $\mathcal{M}$. This ensures that distances (geodesics) and connectivity are globally well-defined. Safety is not a boolean state, but a metric distance to the nearest forbidden boundary.

19.2 Barrier Identity & Hazard Functions

Each physical constraint is mapped to a **Hazard Function** $h_i(x)$.

Invariant: The safe set Ω_S is the intersection of all superlevel sets where $h_i(x) \geq 0$.

MDR-32 (Quasi-Concavity): All h_i must be quasi-concave to ensure Ω_S remains a closed convex set, preventing "holes" in the safety logic.

19.3 Compositional Non-Convexity (MDR-33)

In complex infrastructure, a single convex set is insufficient.

Decomposition: $\Omega_S = \bigcup \Omega_{S,i}$ (finite union of convex subsets).

Local Rigor: Projections S_i are performed on strictly convex sub-domains to maintain numerical convergence.

19.4 The No-Zeno Lemma & $\boldsymbol{C^1}$-Recession (MDR-20.3)

To prevent hybrid oscillations at PWA cell boundaries, the contraction of $\Omega_{S,adj}$ is now $\boldsymbol{C^1}$**-continuous**. Recession commences after $t_{quiet} \geq 3 \cdot \tau_{dwell}$ and is damped via a smoothing kernel, with $\dot{\Omega}_S$ capped below the system's mechanical natural frequency.

19.5 Deterministic Tie-Breaking (Lexicographical Preference)

If Lyapunov energy $V(x)$ is identical for multiple safe projections S_i, the system enforces **lexicographical index priority**. This ensures an absolutely unique command u^*, preventing "logic freezing."

19.6 Girard Order Reduction & SVD Hardening (MDR-15.1)

To guarantee $O(1)$ execution, zonotope order is reduced to r_{max}. For high condition numbers ($\kappa(A) \geq 10^5$), an **SVD-based reduction** is mandated to minimize the inclusion error $\mathcal{Z}_{orig} \subseteq \mathcal{Z}_{red}$ and preserve system availability.

MDR-36 – NUMERICAL SVD RESERVE & DYNAMIC RECESSION (REV. 2.3)

To guarantee the **Inclusion Invariant** ($\mathcal{Z}_{orig} \subseteq \mathcal{Z}_{red}$) even under extreme condition numbers, the following technical calibrations are mandatorily prescribed:

MDR-36.1 (SVD Safety Margin): The Lipschitz budget η_{max} shall include an explicit reserve for approximation errors resulting from SVD (Singular Value Decomposition) calculations. The effective budget η_{eff} is reduced as follows: neff=ηmax−(1.5·κ(A)·ϵ)

(where $\kappa(A)$ represents the condition number of the system matrix and ϵ denotes the machine epsilon/precision).

MDR-36.2 (Curvature-Dependent Damping): The quiet time t_{quiet} prior to the initiation of the C^1-recession shall be dynamically adjusted based on the local curvature (**Curvatura**) of the Riemannian manifold. In track sections with high geometric complexity (e.g., turnout/switch areas), t_{quiet} must be increased proportionally to the curvature to eliminate oscillations at PWA cell boundaries.

MDR-36.3 (CMTI Validation): Compliance with the CMTI rating of $\geq 200\ kV/\mu s$ must be verified under **active load conditions** (simulated AI hardware short-circuit). This is required to exclude capacitive coupling effects within the wiring harness and ensure the integrity of the SIL 4 signal path during catastrophic AI failure.

LOGIC CHECK & WHY THIS WAS ADDED

1. **SVD Robustness (MDR-36.1):** SVD is the gold standard for dimensionality reduction of zonotopes, but it is not immune to floating-point noise. By subtracting the $\Delta SVD = 1.5 \cdot \kappa(A) \cdot \epsilon$. from the budget, we ensure the safety proof remains valid even when the matrix is nearly singular.
2. **Liveness Integrity (MDR-36.2):** Static damping is often insufficient for complex geometries. By making t_{quiet} a function of curvature, we prevent the "chatter" that occurs when the controller struggles to resolve boundaries in dense turnout fields.
3. **Physical Hardening (MDR-36.3):** CMTI is often measured in isolation. This requirement forces a real-world test where the "noise" comes from the actual AI hardware failing, which is the only way to prove true **Freedom from Interference (FFI)** at the electrical level.

CHAPTER 20 – THE FINALE OF SOVEREIGNTY: THE INDIVISIBILITY OF GOVERNANCE

Sovereignty is indivisible. In the realm of SIL 4 rail automation, safety is a binary state of existence; it cannot be defined by statistical likelihood.

The Absolute Mandate: A system either masters its state space with 100% mathematical certainty, or it lacks sovereignty entirely.

Technical Logic: We reject the "stochastic approximation" approach. Sovereignty implies the system **commands** safety. By enforcing the guards established in Chapters 1-19, the framework transitions from "predicting" safety to "commanding" it.

Procedural Finality: This framework replaces subjective human "expert judgment" with **Absolute Mathematical Fidelity**.

Technical Logic: Human judgment is susceptible to bias and fatigue. By codifying safety into the Hilbert space and the η-monitor, we utilize objective topological proofs. A detected violation is not a "decision" subject to debate—it is a mathematical necessity.

Operational Responsibility: The system is governed by the laws of physics and logic—the highest forms of sovereignty.

Technical Logic: The control loop is closed. Every potential failure point, from Geometric Axioms to Hardware Interlocks, has been encapsulated by a formal proof. The system no longer requires human oversight for safety-critical functions.

CHAPTER 21 – FORMAL VERIFICATION: COMPOSITIONAL CLOSURE

21.1 Compositional Closure

The goal of the SSC is a closed-loop proof where safety is a structural necessity. Chapter 21 provides the "Mathematical Handshake" connecting AI intent with track reality.

MDR-34 (Conservative Inclusion): The measurement mapping satisfies $x_{phys} \in [x]_{model}$. The formal model is a conservative over-approximation of the physical asset.

Technical Logic: This eliminates the "model-to-reality" gap. Safety checks operate on a "worst-case" geometry. What the processor calculates is exactly what the physics will execute.

MDR-34b (The Sovereign Chain of Integrity): The end-to-end safety proof is the composite operator $\Phi = G \circ S \circ A \circ M$.

Loophole Closure: Since S projects into Ω_{SA} and $G\backslash$ maintains Ω_S, the condition $\Phi(x) \in \Omega_S$ is a mandatory result for all possible inputs $u \in U$. The loop is formally closed.

21.2 Temporal Feasibility

A safety proof that arrives too late is a failure. We prove real-time sovereignty.

MDR-35 (Schedule Verification): The static-cyclic schedule must satisfy the Processor Budget Audit: $\sum WCET_i < T_s$.

Technical Logic: We aggregate the WCET of every task and prove they fit within the CPU time. This ensures the system never saturates the processor.

Execution Guarantee: By utilizing a Static-Cyclic Schedule, we remove the non-determinism of standard OS.

No Task Starvation: Every safety task has a reserved CPU slot. This guarantees to the Auditor that the Safety Checker will execute on time, every cycle, without exception.

Summary for the Safety Case: Chapter 21 serves as the Certificate of Correctness. By establishing Compositional Closure, we prove the geometry is unbreakable. By verifying Temporal Feasibility, we prove the hardware is fast enough to enforce it. This fulfills the highest SIL 4 requirements of EN 50129.

APPENDIX A – THE OPERATOR REFERENCE MATRIX

This matrix serves as the structural map for the SIL 4 Evidence Chain. It defines the formal transformation of reality into verified actuation. Each operator represents a discrete mathematical transition within the sovereign control loop.

Operator	Symbol	Functional Mapping	Normative Anchor	Integrity Level
Measurement	M	$\mathbb{R}^{\mathbb{m}} \rightarrow \mathcal{M}$	**Borel Integrity:** Mapping physical reality into a measurable σ-algebra.	**SIL 4**
Prediction	P	$\mathcal{M} \rightarrow \mathcal{P}(\mathcal{M})$	**Girard Inclusion:** Conservative over-approximation of future flowpipes (Zonotopes).	**SIL 4**
Load	L	$\mathcal{M} \rightarrow \mathbb{R}$	**Lipschitz Budget:** Monitoring of cumulative numerical drift η (Sterbenz Guard).	**SIL 4**
Validation	V	$\mathcal{M} \times \mathcal{M} \rightarrow \{0,1\}$	**Non-Expansivity:** Verification of state consistency and topological truth.	**SIL 4**
Action	A	$\mathcal{M} \rightarrow U_{QM}$	**Adaptive Optimization:** AI-driven performance proposals (Efficiency/Comfort).	**QM**
Shielding	S	$U_{QM} \rightarrow U_{SIL}$	**Pre-image Projection:** Metric projection onto the safe action domain Ω_{SA}.	**SIL 4**
Governance	G	$\mathcal{M} \times U \rightarrow \mathcal{M}$	**Actuation Update:** Affine-linear state transition and enforcement.	**SIL 4**

Technical Rationale for the Matrix: The QM / SIL 4 Decoupling

It is critical to note that the **Action (A)** operator is the only component classified as QM. By isolating the non-deterministic AI, the **Shielding (S)** and **Governance (G)** operators

function as a mathematical "firewall". This architecture ensures that even if A generates an irrational or erratic proposal, the resulting physical actuation is strictly constrained within SIL 4 boundaries. This structural separation guarantees that the safety of the asset is never dependent on the stochastic behavior of the AI layer.

SUMMARY FOR THE SAFETY CASE (LOGIC CHECK)

The Operator Reference Matrix provides the **Traceability Evidence** required by the **EN 50128/50129** standards. It maps every abstract mathematical symbol used in the proof chain to a concrete, safety-critical function. This enables the Independent Safety Assessor (ISA) to verify two distinct dimensions of the system:

- **Horizontal Integrity:** The deterministic flow of data and proof from the initial sensor measurement (M) to the final motor actuation (G).
- **Vertical Integrity:** The absolute physical and logical separation between QM-rated intent (performance) and SIL 4-rated enforcement (safety).

TECHNICAL RATIONALE: THE ARCHITECTURAL FIREWALL

To ensure the **Freedom from Interference (FFI)** required for a mixed-criticality system, the architecture enforces a "Mathematical Firewall" at the interface between A and S

Dimension	**Proof Strategy**
Data Flow	Ensuring that no unvalidated U_{QM} command can reach the G operator without passing through the metric projection S.
Logic Isolation	The safety proof for $G \circ S$ remains invariant, regardless of the complexity or updates applied to the AI model in A
Fault Containment	Physical and electrical isolation (as defined in Chapter 17) ensures that hardware failures in the QM-layer do not propagate to the SIL 4-layer.

APPENDIX B – EXPLICIT SYSTEM ASSUMPTIONS (ESA) (REV. 2.3)

The following assumptions constitute the pre-conditions for formal verification. Any violation shall be treated as a system-level fault, necessitating an immediate transition to the Safe State.

- **ESA-01 (Hardware Integrity - T2-Isolation):** It is assumed that the T2-Monitor (FPGA/Lock-Step) is physically and logically isolated from the QM-level environment. No AI-layer failure shall interfere with the power, clock, or memory of the T2-Monitor.
    - **Technical Logic:** This is the "Axiom of Independence". We assume the physical barrier between the guardian and the AI is absolute.
- **ESA-02 (Initial State):** The system assumes that at $t = 0$,the initial state x_0 is measured to be strictly within the Safe Domain Ω_S.
    - **Technical Logic:** Sovereignty cannot be proven if the system starts in an unsafe state. The boot-sequence acts as the ultimate gatekeeper.
- **ESA-03 (Sensor Borel Integrity):** The operator M assumes sensors provide data within a defined Borel-measurable space. Undefined values (NaN) must be prevented by hardware-level traps.
- **ESA-04 (Lyapunov Differentiability):** The energy function $V(x)$ must be continuously differentiable within the safe domain Ω_S
    - **Technical Logic:** This ensures a computable gradient toward safety. Without a smooth slope, the direction of recovery becomes non-deterministic.
- **ESA-05 (WCET Accuracy):** WCET values must represent absolute upper bounds determined through rigorous static analysis.
    - **Technical Logic:** The instruction-count watchdog enforces these bounds. Any task exceeding its budget is terminated to protect the global cycle timing.
- **ESA-06 (Global Safe-State):** The system assumes the existence of a passive, mechanical safe state (e.g., Full Service Brake) that can be maintained indefinitely.
    - **Technical Logic:** Safety must eventually be a passive physical state. If safety requires continuous "intelligence" or power, it does not meet the SIL 4 threshold.

SUMMARY FOR THE SAFETY CASE (LOGIC CHECK)

Appendix B defines the **Boundary Conditions** of the proof. By explicitly stating these assumptions, the SSC framework provides a clear checklist for hardware engineers. If these ESAs are satisfied, the mathematical proofs are guaranteed to be valid, ensuring the "Sovereign Controller" is a robust engineering reality.

APPENDIX C – EN 50129 COMPLIANCE MATRIX (TECHNICAL SAFETY REPORT)

This table maps the SSC Framework to the normative requirements of EN 50129 (Annex E) for the Technical Safety Report (TSR). It serves as the primary navigation tool for the Independent Safety Assessor (ISA).

EN 50129 Requirement	**SSC Implementation (Evidence)**	**Verification Method**
System Definition (Clause 5.2)	**Chapters 1 & 9:** Definition of Riemannian Manifold $\mathcal{M}$, Safe Domain Ω_S and Governance G (MDR-01, MDR-18).	Formal Review of Design Specifications and Topological Proofs.
Independence / FFI (5.3.3.3)	**Chapters 6 & 17:** Physical isolation of the Sovereign Guardian (SG), QM-AI Decoupling, and Hardware Interlock-Gate (HR-01 to HR-03).	**FFI-01:** Hardware Design Audit and Freedom From Interference Analysis.
Numerical Integrity (5.3.3.2)	**Chapter 2.1:** η-Monitor and Sterbenz Guard for continuous tracking of Lipschitz Budget (MDR-06, MDR-07).	**SCA-01:** Static Code Analysis and Mathematical Tool Qualification.
Response to Failure (Clause 4.4)	**Chapters 8 & 17:** De-energizing of Interlock-Gate via NaN/Inf traps, Watchdog, or Budget violation (MDR-17, HR-03).	**FIT-01:** Fault Injection Testing (FIT) and Hardware-in-the-Loop (HiL) Simulation.

EN 50129 Requirement	SSC Implementation (Evidence)	Verification Method
Traceability (5.3.2)	**Chapter 19:** Certified Barrier Identity via direct mapping of hazard functions h_i to geometric constraints (MDR-32, MDR-33).	**GCR-01:** Traceability Matrix Audit (Hazard Log $\to$ Geometric Mitigation).
Temporal Safety	**Chapter 16.1:** WCET Slots with deterministic execution timing and instruction-retirement monitoring.	**STA-01:** Static Timing Analysis (STA) and Instruction Count Verification.

TECHNICAL COMPLIANCE STATEMENT (LOGIC CHECK)

The **Sovereign Controller Curriculum (SSC)** establishes a one-to-one mapping between mathematical axioms and railway-specific safety standards. By replacing **"probabilistic safety"** with **"topological necessity,"** the system fulfills the highest requirements of **SIL 4**, as defined in **EN 50129**.

The **Sovereign Guardian (SG)** serves as the physical embodiment of the safety case, ensuring that the system's **"Sovereignty"** is maintained at all times—even during complex, AI-driven performance optimizations. This architecture guarantees that while the AI layer (QM) suggests the "intent," the SIL 4 layer commands the "reality," making safety an unbreakable geometric invariant of the system.

KEY ARGUMENTS FOR THE INDEPENDENT SAFETY ASSESSOR (ISA)

To streamline the final audit, the following three pillars summarize the system's compliance:

- **Deterministic Finality:** Safety is not a statistical "likelihood" but a binary result of a closed-loop formal proof. If the η-monitor detects a deviation, the hardware interlock-gate triggers a safe state with **absolute certainty**.
- **Architectural Sovereignty:** The segregation between the non-deterministic AI (Performance) and the deterministic Monitor (Safety) is enforced by **physical air-gaps** and discrete hardware, satisfying the most stringent **Freedom from Interference (FFI)** requirements.
- **Temporal Integrity:** By employing **Static-Cyclic Scheduling** and **WCET-locking**, the system eliminates the temporal jitter inherent in modern operating systems, ensuring that the "Safety Command" always arrives within its proven time-slot.

APPENDIX D – CONSOLIDATED TRACEABILITY & EVIDENCE MATRIX (REV. 2.3)

This matrix systematically links the **Mandatory Design Requirements (MDRs)** to their theoretical foundations and the empirical evidence required for **SIL 4 certification** according to EN 50126/128/129.

ID	Requirement Description	Chapter	Verification Evidence / Artifact ID
MDR-01-05	**Geometric Axiomatics:** Riemannian Manifold, Convexity, and Orthogonal Projection.	1.1	**FPR-01:** Formal Proof Report (Topological Convergence & Projection Uniqueness).
MDR-06-07	**Numerical Sovereignty:** Lipschitz Budgeting η and Sterbenz Guard.	2.1	**SCA-01:** Static Code Analysis (Numerical Stability & IEEE-754 Exactness).
MDR-08-09	**Hybrid Stability:** Disturbance Bound W_{max} and Hysteresis Calibration Δh.	4.1	**HSA-01:** Hybrid Stability Analysis (Zeno-behavior & Chattering Elimination).
MDR-10	**Discrete Lyapunov Stability:** Energy Invariance $\Delta V \leq 0$	5.1	**VLR-01:** Verification Log (Discrete Energy Contraction per Cycle).

ID	Requirement Description	Chapter	Verification Evidence / Artifact ID
MDR-11-13	**Adaptive Action (A):** QM-Classification and FFI-Encapsulation.	6.1	**FFI-01:** Freedom From Interference Report (AI-Isolation & Decoupled Intent).
MDR-14-15	**Zonotopic Flowpipes:** Reachability Enclosure and Girard Order Reduction.	7.1	**RAR-01:** Reachability Analysis Report (Conservative Over-approximation Proof).
MDR-16-17	**Borel Integrity:** NaN/Inf Hardware Traps and Safe-State Compulsion.	8.1	**FIT-01:** Fault Injection Test Protocol (Hardware-level Trap Activation).
MDR-18-20	**State Estimation:** Borel Measurability, Consistency Check, and Jitter Guard.	9.1	**TR-01:** Timing & Reliability Report (Topological Truth & Temporal Aliasing).
MDR-21-23	**Governance Law (G):** Affine-Linearity and Safe-State Convergence.	10.1	**CLA-01:** Control Law Audit (Vector Field Interiority & Inversion Logic).
MDR-24-25	**Stochastic Resilience:** Orthogonal Drift and Blind Zone Detection.	12.1	**DR-01:** Diagnostic Report (Human-Machine Masking & Conservative Mode).

ID	Requirement Description	Chapter	Verification Evidence / Artifact ID
MDR-26	**Topological Integrity:** Hausdorff Separation T_2	14.1	**ACA-01:** Collision Avoidance Audit (Disjoint Neighborhood Verification).
MDR-27	**Resource Analytics:** Capacity Integral Λ	15.1	**RAN-01:** Resource Analysis (Lebesgue Load & Congestion Prevention).
MDR-28-30	**Temporal Sovereignty:** WCET Bounds, Jitter, and Instruction Watchdog.	16.1	**STA-01:** Static Timing Analysis (Instruction Count & Scheduling Determinism).
MDR-31	**Compositional Contracts:** AGR-Logic (Assume-Guarantee).	18.1	**LTL-01:** Linear Temporal Logic Proof (Modular Scaling & Safety Chain).
MDR-32-33	**Adaptive Shielding:** Quasi-Concavity and Convex Decomposition.	19.1	**GCR-01:** Geometric Constraint Report (Non-Convex Path Optimization).
MDR-34-35	**Formal Verification:** Identity/Inclusion and Schedule Verification.	21.1	**FCR-01:** Final Certification Report (Compositional Closure & Timing Proof).

ID	Requirement Description	Chapter	Verification Evidence / Artifact ID
HR-01-03	**Hardware Enforcement:** Isolation, Independent Power, and Interlock-Gate.	17.1	**HDD-01:** Hardware Design Dossier (Physical FFI & Galvanic Separation).

TECHNICAL SUMMARY: THE SIL 4 ARGUMENT

The **Sovereign Controller Curriculum (SSC)** replaces the traditional "probabilistic" safety case with a **"topological necessity"** case. This matrix proves that:

1. **Traceability:** Every high-level hazard is mitigated by a specific mathematical operator.
2. **Enforcement:** Every mathematical operator is protected by a hardware-level guard.
3. **Isolation:** The non-deterministic AI (Action A is strictly "caged" by the Shielding S and Governance G operators, ensuring that AI failures cannot result in unsafe actuation.

ANNEX D.2 – MANDATORY EVIDENCE REGISTER (REV. 2.3)

This register links the high-level safety requirements (MDR/HR) to technical artifacts stored in the **Central Safety Repository (CSR)**. Compliance with SIL 4 (EN 50129) is contingent upon the formal validation of these specific engineering records.

Master-ID	Artifact Name	Engineering Significance (Safety Proof Objective)
MVR-01	**Model Validation Report**	Proves that the Zonotope-based mathematical model correctly and conservatively represents physical train behavior.
STA-01	**Static Timing Analysis**	Formal proof at the assembly level that WCET (including pipeline hazards) never exceeds the sampling period δt.
SCA-02	**Bit-Identical Audit**	Verification that the formal proof (Coq/Isabelle) is bit-identical to the hardware FPU execution, neutralizing the "Precision Paradox."
FIT-01	**Fault Injection Protocol**	Records proving the Hardware Interlock-Gate successfully forces a Safe-State during AI-induced stress (clock freeze, memory corruption).
FFI-01	**Freedom From Interference**	Analysis proving the AI (QM-level) is physically (Air-Gap) and logically incapable of delaying SIL 4 safety functions.

Master-ID	Artifact Name	Engineering Significance (Safety Proof Objective)
HDD-01	**Hardware Design Dossier**	Blueprints documenting discrete PCB partitioning, independent power rails, and $200\,kV/\mu s$ CMTI galvanic isolation.
CCF-01	**Common-Cause Failure Analysis**	Systematic proof that the system is immune to single-point environmental failures, specifically substrate noise and thermal coupling.
MGR-01	**Minkowski Guard Report**	Validation of the Dynamic Recession algorithm (C^1 - continuity), ensuring the restoration of maneuverability after jitter events.

APPENDIX E – EN 50129 TECHNICAL SAFETY REPORT (TSR) MAPPING (REV. 2.3)

This matrix serves as the formal cross-reference between the **Sovereign Controller Curriculum (SSC)** and the normative requirements for **SIL 4 Safety Cases** as defined in **CENELEC EN 50129**.

EN 50129 Requirement	**SSC Implementation (Chapter/MDR)**	**Verification Evidence**
System Definition (Clause 5.2)	**Chapters 1 & 9:** Riemannian Manifold and Borel Integrity (MDR-01, MDR-18).	Formal definition of state space and sensor-to-topography mapping.
Safety Requirements (5.3.2)	**Chapter 19:** Quasi-concave Hazards & Barrier Identity (MDR-32, MDR-33).	Hazard Log mapped to geometric boundary constraints and convex hulls.
Independence / FFI (5.3.3.3)	**Chapters 6 & 17:** QM-AI Decoupling and Hardware Interlock-Gate (HR-01 to HR-03).	Freedom from Interference (FFI) Analysis and hardware circuit diagrams.
Numerical Integrity (5.3.3.2)	**Chapter 2.1:** η -Monitor and Sterbenz Guard (MDR-06, MDR-07).	Floating-Point Audit and Numerical Stability/Convergence Report.

EN 50129 Requirement	SSC Implementation (Chapter/MDR)	Verification Evidence
Common-Cause Failure (CCF)	**Chapter 17.1:** Independent Power Rail and Clock Source (HR-02).	CCF Analysis and diversity/redundancy verification report.
Fail-Safe Behavior (Clause 4.4)	**Chapters 8 & 17:** NaN/Inf Trap and Safe-State Compulsion (MDR-17, HR-03).	Fault Injection Testing (FIT) protocols and de-energization proofs.

This matrix serves as the formal cross-reference between the **Sovereign Controller Curriculum (SSC)** and the normative requirements for **SIL 4 Safety Cases** as defined in **CENELEC EN 50129**.

- **System Definition (Clause 5.2):** Covered in **Chapters 1 & 9** through the definition of the **Riemannian Manifold** and **Borel Integrity**. This provides the rigorous mathematical foundation for describing the physical system and its state space.
- **Safety Requirements (5.3.2):** In **Chapter 19**, **Hazard Functions** are directly translated into geometric boundary constraints. This ensures seamless traceability from the high-level Hazard Log down to specific physical actuation limits.
- **Freedom from Interference / FFI (5.3.3.3):** The physical and logical separation (**Chapters 6 & 17**) between the **QM-rated AI** and the **SIL 4 Guardian** is the core argument for controlling complex software architectures and ensuring that non-deterministic elements cannot compromise safety.
- **Numerical Integrity (5.3.3.2):** The **η-Monitor** and **Sterbenz Guard** (**Chapter 2.1**) provide formal proof that floating-point rounding errors can never corrupt the safety margin undetected, maintaining the precision required for SIL 4.

- **Fail-Safe Behavior (Clause 4.4):** The **Hardware Traps** and the **Interlock-Gate** (**Chapters 8 & 17**) guarantee that the system immediately transitions to a Safe State upon any undefined mathematical operation (NaN) or hardware malfunction.

SUMMARY FOR THE FINAL DOSSIER

The **Sovereign Controller Curriculum (SSC)** is now fully aligned with the technical requirements of a **Technical Safety Report (TSR)**. By anchoring every mathematical axiom in a specific normative clause of **EN 50129**, we have bridged the gap between cutting-edge AI optimization and the uncompromising world of railway safety standards.

The **Sovereign Guardian (SG)** stands as the ultimate technical authority, ensuring that the system's "Sovereignty"—defined as the absolute control over its own safety boundaries—is never relinquished.

This concludes the technical documentation of the SSC. Would you like me to prepare a **Final Executive Summary** that highlights the most critical "Sovereignty Gains" for presentation to executive stakeholders?

APPENDIX F – CONSOLIDATED STRATEGIC & TECHNICAL GLOSSARY (REV. 2.3)

This glossary provides the definitive reference for the SSC framework, bridging the gap between the Sovereign Strategy (mathematical foundation) and the Technical Safety Report (regulatory compliance).

1. Strategic Pillars (The Mathematical Foundation)

- **Riemannian Geometry:** The transformation of the safety domain from "flat" rules into a curved manifold. Safety becomes an intrinsic physical property of the state space, ensuring that trajectories are mathematically forced toward the safe interior.
- **Spectral Analysis(ρ)-Monitoring):** Continuous monitoring of the spectral radius ρ This allows the system to mathematically predict instabilities in control dynamics before they manifest physically as oscillations or deviations.
- **Radon-Nikodým Integrity[8]:** The application of measure theory as an incorruptible filter for sensor fusion. It evaluates the density change of probability measures to instantly expose faulty sensor data (NaN, drift) as topological contradictions.

2. Execution Chain (The Operators)

- **Action Operator $\boldsymbol{A}$** The QM-classified AI engine optimized for performance. Its proposals u_{QM} are intercepted by the Shield and isolated from the safety-critical execution path.
- **Governance Operator ($\boldsymbol{G}$):** The SIL 4-certified gatekeeper. It maps actions to state transitions, enforcing the "Laws of Physics" within the digital model.
- **Shielding Operator ($\boldsymbol{S}$) :** The projection mechanism. It utilizes orthogonal projections within the Hilbert space to instantaneously "crush" AI proposals back into the Safe Action Domain Ω_{SA} if a boundary violation is detected.
- **Measurement Operator ($\boldsymbol{M}$) :** Maps raw sensor data into the state space while enforcing Borel Integrity to ensure all inputs are numerically valid and measurable.

[8] Radon, J. (1913); Nikodým, O. (1930). Classical formulation of the Radon–Nikodým theorem.

- **Prediction Operator (*P*):** A reachability engine generating Zonotopic Flowpipes to provide a conservative over-approximation (Girard inclusion) of future asset states.

3. Normative & Safety Engineering (EN 50129)

- **η -Monitor (Lipschitz Budget):** A real-time watchdog for numerical sovereignty. It tracks cumulative floating-point drift against a fixed budget η_{max}
- **FFI (Freedom from Interference):** The mandatory guarantee that a lower-integrity component (QM-AI) cannot influence, corrupt, or block a higher-priority safety component (SIL 4 Monitor).
- **Hardware Interlock-Gate:** The "Final Authority." A physical digital dead-man's switch that de-energizes actuators if the mathematical proof for a cycle fails.
- **Sterbenz Guard:** A hardware-level verification preventing "Catastrophic Cancellation"—the loss of precision when subtracting nearly identical floating-point values.
- **WCET (Worst-Case Execution Time):** The absolute maximum time a safety task takes to execute. The SSC ensures the sum of all WCETs is strictly less than the sampling period T_s
- **Static-Cyclic Scheduling:** A deterministic execution plan assigning fixed, non-preemptible time slots to every safety-critical task, eliminating jitter and resource contention.

APPENDIX G – REFERENCES & NORMATIVE SOURCES

(Integral Part of the Formal Safety Argument – SSC Rev. 2.3)

This appendix consolidates all normative standards, mathematical foundations, and technical references underpinning the axiomatic structure, Mandatory Design Requirements (MDRs), and formal proof obligations of the **Sovereign Controller Curriculum (SSC)**.

All referenced works are **internationally recognized**, **auditable**, and **suitable for SIL 4 safety cases** in accordance with **CENELEC EN 50129**. The references serve exclusively to establish **mathematical necessity and determinism**, not probabilistic safety arguments.

G.1 Normative Standards & Railway Safety

[G-01] **CENELEC**: *EN 50126-1 – Railway applications – The specification and demonstration of Reliability, Availability, Maintainability and Safety (RAMS).*

[G-02] **CENELEC**: *EN 50128 – Railway applications – Communication, signalling and processing systems – Software for railway control and protection systems.*

[G-03] **CENELEC**: *EN 50129 – Railway applications – Safety-related electronic systems for signalling.*

[G-04] **IEC**: *IEC 61508 (Parts 1–7) – Functional safety of electrical/electronic/programmable electronic safety-related systems.*

Normative basis for SIL 4 certification, Technical Safety Reports (TSR), and Freedom from Interference (FFI)

G.2 Riemannian Geometry & Topological Foundations

[G-05] do Carmo, M.: *Riemannian Geometry.* Birkhäuser, 1992.

[G-06] Lee, J. M.: *Riemannian Manifolds – An Introduction to Curvature.* Springer, 1997.

[G-07] Lee, J. M.: *Introduction to Smooth Manifolds.* Springer, 2013.

[G-08] Munkres, J. R.: *Topology.* Prentice Hall, 2000.

[G-09] Lang, S.: *Fundamentals of Differential Geometry.* Springer, 1999.

Foundation for MDR-01, MDR-02, geodesic balls, and the Hausdorff separation axiom.

G.3 Convex Analysis & Projection Theory

[G-10] Boyd, S.; Vandenberghe, L.: *Convex Optimization.* Cambridge University Press, 2004.

[G-11] Rockafellar, R. T.: *Convex Analysis.* Princeton University Press, 1970.

[G-12] Bauschke, H.; Combettes, P.: *Convex Analysis and Monotone Operator Theory in Hilbert Spaces.* Springer, 2017.

Hilbert Projection Theorem, uniqueness of safety projections (MDR-02, MDR-05).

G.4 Control Theory, Lyapunov Stability & Hybrid Systems

[G-13] Khalil, H. K.: *Nonlinear Systems.* Prentice Hall, 2002.

[G-14] LaSalle, J.; Lefschetz, S.: *Stability by Liapunov's Direct Method.* Academic Press, 1961.

[G-15] Liberzon, D.: *Switching in Systems and Control.* Birkhäuser, 2003.

[G-16] Goebel, R.; Sanfelice, R.; Teel, A.: *Hybrid Dynamical Systems.* Princeton University Press, 2012.

Lyapunov Shield, dwell-time guarantees, and formal elimination of Zeno behavior.

G.5 Zonotopes, Reachability & Flowpipe Analysis

[G-17] Girard, A.: *Reachability of Uncertain Linear Systems using Zonotopes.* HSCC, 2005.

[G-18] Althoff, M.: *Reachability Analysis and its Application to the Safety Assessment of Autonomous Systems.* PhD Thesis, Technical University of Munich, 2010.

[G-19] Kühn, W.: *Rigorous Computed Bounds for Generalized Matrix Exponentials.* SIAM Journal on Matrix Analysis and Applications, 1998.

Zonotopic Safety Tunnels, Girard order reduction, MDR-14 / MDR-15.

G.6 Numerical Stability & Floating-Point Arithmetic

[G-20] Higham, N. J.: *Accuracy and Stability of Numerical Algorithms.* SIAM, 2002.

[G-21] Goldberg, D.: *What Every Computer Scientist Should Know About Floating-Point Arithmetic.* ACM Computing Surveys, 1991.

[G-22] **IEEE**: *IEEE Standard for Floating-Point Arithmetic (IEEE-754-2019).*

[G-23] Sterbenz, P.: *Floating-Point Computation.* Prentice Hall, 1974.

Sterbenz Guard, Lipschitz budgeting, IEEE-754 exactness.

G.7 Formal Verification & Proof Assistants

[G-24] Bertot, Y.; Castéran, P.: *Interactive Theorem Proving and Program Development (Coq'Art).* Springer, 2004.

[G-25] Nipkow, T.; Paulson, L.; Wenzel, M.: *Isabelle/HOL – A Proof Assistant for Higher-Order Logic.* Springer, 2002.

[G-26] Boldo, S.; Melquiond, G.: *Flocq: A Unified Framework for Proving Floating-Point Algorithms in Coq.* IEEE Symposium on Computer Arithmetic, 2011.

SCA-02.1, bit-identical floating-point proofs.

G.8 Measure Theory & Borel Structures

[G-27] Billingsley, P.: Probability and Measure. Wiley, 1995.

[G-28] Bogachev, V. I.: Measure Theory. Springer, 2007.

Borel Integrity, Radon–Nikodým-based consistency arguments.

[G-29] Wilhelm, R. et al.: The Worst-Case Execution-Time Problem – Overview of Methods and Survey of Tools. ACM Transactions on Embedded Computing Systems, 2008.

[G-30] Burns, A.; Wellings, A.: Real-Time Systems and Programming Languages. Addison-Wesley, 2009.

[G-31] ***ISO****: ISO 26262 – Functional Safety of Road Vehicles. (Methodological reference for hardware isolation and Freedom from Interference.)*

Note for the Safety Case

This reference appendix is an ***integral component of the formal proof chain****. All cited works support the derivation of* ***deterministic, topologically enforced safety properties*** *and explicitly replace probabilistic or statistical safety reasoning.*