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Fields, Flows, and the Fabric of Spacetime
Probing Horizon-Scale Physics in
Active Galactic Nuclei

Saurabh
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Fields, Flows, and the Fabric of Spacetime Probing Horizon-Scale Physics in Active Galactic Nuclei

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Dedicated to the memory of

Dr. N. Rathnasree

(1964–2022)

Abstract

A century after its formulation, the general theory of relativity continues to pass every test the instruments can devise. It constitutes the paradigm within which modern gravitational astrophysics operates. Among its various theoretical predictions, one stands out as having been particularly difficult for the scientific community to assimilate at the time of its inception: black holes. Today, they occupy a central role in gravitational astrophysics, providing a unique laboratory for testing the interplay between strong-field gravity, plasma physics, and radiative processes. Active Galactic Nuclei (AGN) find themselves at the center of this investigation, thought to be powered by accretion onto supermassive compact object at the center—now, *ab initio*, this is a black hole. They are among the most energetically extreme environments the universe offers such as the M87* and Sagittarius A* (center of our Milky Way galaxy). The physics governing their emission is, however, deeply entangled—the observed polarization structure, the disk kinematics, and the spacetime curvature.

The EHT observations at 230 GHz of M87* revealed a polarized ring emission structures with substantial central brightness depression, meanwhile very-long baseline interferometric (VLBI) observations at 86 GHz with the Global Millimetre VLBI Array resolved both the ring and the extended jet emission of M87* simultaneously, identifying a connection between the horizon-scale structure and the large-scale outflow. At 230 GHz, however, earlier EHT campaigns lacked the intermediate baselines needed to probe emission at the scales bridging these two regimes. The 2021 EHT campaign closed this gap by introducing NOEMA and the Kitt Peak 12 m telescope. We imaged and analyzed the 2021 data using the regularized maximum likelihood method utilizing the `eht-imaging` pipeline through an extensive hyperparameter survey resulting in $\sim 600,000$ images. We additionally analyzed the closure phases on the new intermediate baselines which recovered a Gaussian component offset from the compact ring, centered at a projected separation of ≈ 5500 AU, identifying the location, with a flux density of ~ 60 mJy, an upper limit, of the jet base. The data do not favor more complex models, and the recovered flux implies that most of the missing large-scale emission in earlier EHT studies arises from scales beyond those probed here.

Interpreting these observations broadly requires connecting the observed polarized emission to the underlying plasma and field configurations through theoretical models. General relativistic magnetohydrodynamic (GRMHD) simulations provide the most physically self-consistent framework for this purpose. Despite their involvement and inference, however, it remains contentious. Both, astrophysics and gravity, are complicit in this act. Semi-analytic models of radiatively

inefficient accretion flows (RIAF) provides the understanding for disentangling their individual contributions. Within the black hole framework, we apply the semi-analytic RIAF model to the 2017 EHT polarimetric dataset of M87* across a systematic parameter grid, ray-tracing synthetic images through a range of magnetic field geometries and extracting their polarimetric signatures for direct comparison with the observations. The effect of disk thickness on the observables is found to be limited, while magnetic configurations dominated by toroidal and poloidal fields can be distinguished reliably. The flow dynamics, in particular the degree of radial inflow, significantly affects the EHT observables, pointing to the presence of a partially infalling sub-Keplerian flow in M87*.

Each of these investigations operates under the working hypothesis that the central object in M87* is a black hole. GR, however, also admits gravitational end-states that lack event horizons, arising from physically reasonable matter conditions departing from the idealized initial conditions. GR is a falsifiable theory, and the arrival of horizon-scale precision means that the assumption of an event horizon is no longer beyond observational reach. We adopted the Joshi-Malafarina-Narayan 1 (JMN-1) spacetime, a horizonless compact object arising from gravitational collapse with anisotropic pressure—an exact solution of Einstein’s field equations and the *best mimicker* of the black hole—and subjects it first to the semi-analytic RIAF imaging framework and then to the first three-dimensional GRMHD simulation of sustained accretion onto such an object. The semi-analytic model finds that JMN-1 produces synthetic synchrotron images indistinguishable from a black hole at current EHT resolution, with a central brightness excess as the sole diskriminant. The GRMHD simulation confirms that this is not a geometric coincidence: JMN-1 reaches a magnetically arrested state with an accretion rate $\sim (3 - 6) \times 10^{-6} M_{\odot} \text{yr}^{-1}$ comparable to the black hole case, and the presence of a smaller *inner shadow* is a key observational target for the near-future observations by the next-generation EHT (ngEHT) or the Black hole Explorer (BHEX).

Collectively, these findings map the boundary of what horizon-scale precision currently establishes and what it does not. We explored various problems dealing with the same underlying question: *what governs the behavior of matter, energy, and spacetime in the most extreme environments the universe produces*. As the next generation of arrays delivers dynamic reconstructions and multi-epoch movies of M87* and Sgr A*, the diskriminants identified here will move from theoretical targets to observational tests, and the nature of the central engine will become a question the data can finally answer.

Zusammenfassung

Ein Jahrhundert nach ihrer Formulierung besteht die Allgemeine Relativitätstheorie weiterhin jeden Test, den die Instrumente zu entwickeln vermögen. Sie bildet das Paradigma, innerhalb dessen die moderne gravitative Astrophysik operiert. Unter ihren vielfältigen theoretischen Vorhersagen sticht eine besonders hervor, die zur Zeit ihrer Einführung von der wissenschaftlichen Gemeinschaft nur schwer zu assimilieren war, Schwarze Löcher. Heute nehmen sie eine zentrale Rolle in der Gravitationsastrophysik ein und stellen ein einzigartiges Labor zur Untersuchung des Zusammenspiels von Starkfeldgravitation, Plasmaphysik und Strahlungsprozessen dar. Aktive Galaktische Kerne (AGN) stehen im Zentrum dieser Untersuchungen und gelten als durch Akkretion auf ein supermassereiches kompaktes Objekt im galaktischen Zentrum angetrieben, *ab initio* handelt es sich hierbei um ein Schwarzes Loch. Sie gehören zu den energetisch extremsten Umgebungen, die das Universum bietet, wie etwa M87* und Sagittarius A*, das Zentrum unserer Milchstraße. Die Physik, die ihre Emission bestimmt, ist jedoch tief miteinander verflochten, die beobachtete Polarisationsstruktur, die Kinematik der Akkretionsscheibe und die Raumzeitkrümmung lassen sich nicht unabhängig voneinander betrachten.

Eine umfassende Interpretation dieser Beobachtungen erfordert die Verknüpfung der beobachteten polarisierten Emission mit den zugrunde liegenden Plasma- und Feldkonfigurationen mithilfe theoretischer Modelle. Allgemein-relativistische magnetohydrodynamische (GRMHD-)Simulationen stellen hierfür den physikalisch konsistentesten Rahmen dar. Trotz ihrer breiten Anwendung und ihres interpretativen Einsatzes bleibt ihre Aussagekraft jedoch umstritten, wobei sowohl astrophysikalische als auch gravitative Unsicherheiten eine Rolle spielen. Globale semi-analytische Modelle radiativ ineffizienter Akkretionsflüsse liefern ein wichtiges Verständnis, um diese individuellen Beiträge voneinander zu trennen. Im Rahmen des Schwarze-Loch-Paradigmas wenden wir das semi-analytische RIAF-Modell auf den polarisimetrischen EHT-Datensatz von 2017 zu M87* über ein systematisches Parameter-Gitter an, ray-tracen synthetische Bilder für eine Vielzahl magnetischer Feldgeometrien und extrahieren deren polarisimetrische Signaturen zum direkten Vergleich mit den Beobachtungen. Der Einfluss der Scheibendicke auf die Observablen erweist sich als gering, während magnetische Konfigurationen, die von toroidalen oder poloidalen Feldern dominiert werden, zuverlässig unterschieden werden können. Die Flussdynamik, insbesondere der Grad des radialen Inflows, beeinflusst die EHT-Observablen signifikant und deutet auf das Vorhandensein eines teilweise einfallenden subkeplerschen Flusses in M87* hin.

Alle diese Untersuchungen operieren unter der Arbeitshypothese, dass das

zentrale Objekt in M87* ein Schwarzes Loch ist. Die Allgemeine Relativitätstheorie erlaubt jedoch auch gravitative Endzustände ohne Ereignishorizont, die aus physikalisch plausiblen Materiebedingungen hervorgehen und von idealisierten Anfangsbedingungen abweichen. Die Allgemeine Relativitätstheorie ist eine falsifizierbare Theorie, und die erreichte Präzision auf Horizontskalen bedeutet, dass die Annahme eines Ereignishorizonts nicht länger außerhalb der beobachtbaren Reichweite liegt. Wir wählten die Joshi-Malafarina-Narayan-1-(JMN-1-)Raumzeit, ein horizontloses kompaktes Objekt, das aus gravitativer Kollapsdynamik mit anisotropem Druck hervorgeht, eine exakte Lösung der Einstein'schen Feldgleichungen und der *beste Nachahmer* eines Schwarzen Lochs, und unterzogen sie zunächst dem semi-analytischen RIAF-Bildgebungsrahmen und anschließend der ersten dreidimensionalen GRMHD-Simulation einer nachhaltigen Akkretion auf ein solches Objekt. Das semi-analytische Modell zeigt, dass JMN-1 synthetische Synchrotronbilder erzeugt, die bei der aktuellen EHT-Auflösung nicht von denen eines Schwarzen Lochs zu unterscheiden sind, wobei ein zentraler Helligkeitsüberschuss das einzige Unterscheidungsmerkmal darstellt. Die GRMHD-Simulation bestätigt, dass es sich dabei nicht um einen geometrischen Zufall handelt. JMN-1 erreicht einen magnetisch arretierten Zustand mit einer Akkretionsrate von $\sim (3-6) \times 10^{-6} M_{\odot} \text{ yr}^{-1}$, die mit der des Schwarzen-Loch-Falls vergleichbar ist, und das Vorhandensein eines kleineren *inneren Schattens* stellt ein zentrales Beobachtungsziel für zukünftige Messungen mit dem Next-Generation EHT (ngEHT) oder dem Black Hole Explorer (BHEX) dar.

In ihrer Gesamtheit markieren diese Ergebnisse die Grenze dessen, was Präzision auf Horizontskalen derzeit festlegen kann und dessen, was sie nicht leisten kann. Wir untersuchten verschiedene Fragestellungen, die sich alle um dieselbe grundlegende Frage drehen, *was bestimmt das Verhalten von Materie, Energie und Raumzeit in den extremsten Umgebungen, die das Universum hervorbringt*. Mit der nächsten Generation von Interferometerarrays, die dynamische Rekonstruktionen und Mehr-Epochen-Filme von M87* und Sgr A* liefern werden, werden die hier identifizierten Diskriminanten von theoretischen Zielgrößen zu beobachtbaren Tests übergehen, und die Natur der zentralen Maschine wird schließlich zu einer Frage, die durch die Daten beantwortet werden kann.

Publications related to this thesis

1. Saurabh, Wielgus, M., Bambhaniya, P., de Gouveia Dal Pino, E. M., Lobanov, A. P. and Joshi, P. S. (2026), “*GRMHD accretion beyond the black hole paradigm: Light from within the shadow*,” submitted *Physical Review Letters*, [arXiv:2604.15430](#).
2. Saurabh, Wielgus, M., Tursunov, A., Lobanov, A. P. and Emami, R. (2026), “*Semi-analytic studies of the accretion disk and magnetic field geometry in M 87**,” *Astronomy & Astrophysics*, 705, A166, doi:10.1051/0004-6361/202556880.
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8. Saurabh, Bambhaniya, P. and Joshi, P. S. (2024), “*Imaging ultracompact objects with radiatively inefficient accretion flows*,” *Astronomy & Astrophysics*, 682, A113, doi:10.1051/0004-6361/202347941.

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1. Bernshteyn, V. et al. (2026), “*Ring Asymmetry and Spin in M87**,” *The Astrophysical Journal*, 1000 (2), 231, doi:10.3847/1538-4357/ae34af.

2. Emami, R. et al. (2026), “*Novel Polarimetric Analysis of Near-horizon Flaring Episodes in M87* in Millimeter Wavelength*,” *The Astrophysical Journal*, 998 (1), 129, doi:10.3847/1538-4357/ae38bc.
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List of Abbreviations

ADAF	Advection-Dominated Accretion Flow
AGN	Active Galactic Nucleus
ALMA	Atacama Large Millimeter/submillimeter Array
AMR	Adaptive Mesh Refinement
BH	Black Hole
BHEX	Black Hole Explorer
BZ	Blandford-Znajek
BP	Blandford-Payne
CLEAN	Iterative Deconvolution Algorithm
CP	Circular Polarization
DEC	Declination
eDF	Electron Distribution Function
EHT	Event Horizon Telescope
ELT	Extremely Large Telescope
EVPA	Electric Vector Position Angle
FOV	Field of View
GR	General Relativity
GRMHD	General Relativistic Magnetohydrodynamics
GRRT	General Relativistic Radiative Transfer
GMVA	Global Millimetre VLBI Array
ISCO	Innermost Stable Circular Orbit
JMN-1	Joshi-Malafarina-Narayan 1
JNW	Janis-Newman-Winicour
KHARMA	Kokkos-based High-Accuracy Relativistic Magnetohydrodynamics
LISA	Laser Interferometer Space Antenna
LP	Linear Polarization
MAD	Magnetically Arrested Disk
MHD	Magnetohydrodynamics
MKS	Modified Kerr-Schild (coordinates)
MRI	Magnetorotational Instability

ngEHT	next-generation Event Horizon Telescope
ngVLA	next-generation Very Large Array
NIR	Near-Infrared
NkS	Naked Singularity
PA	Position Angle
RA	Right Ascension
RIAF	Radiatively Inefficient Accretion Flow
RIME	Radio Interferometer Measurement Equation
RM	Rotation Measure
RML	Regularized Maximum Likelihood
RN	Reissner-Nordström
SANE	Standard and Normal Evolution
SED	Spectral Energy Distribution
Sgr A*	Sagittarius A*
SKA	Square Kilometre Array
SNR	Signal-to-Noise Ratio
TMT	Thirty Meter Telescope
TOV	Tolman-Oppenheimer-Volkoff
VLA	Very Large Array
VLBI	Very Long Baseline Interferometry
ZAMO	Zero Angular Momentum Observer

Units and Convention

This thesis employs two complementary unit systems depending on context: the Gaussian CGS (centimeter-gram-second) system for physical quantities and geometric units for theoretical expressions. This convention follows standard practice in gravitational physics and relativistic astrophysics. This section establishes the notation, unit systems, and physical conventions used throughout. The fundamental physical constants in SI units are:

Constant	Symbol	Value (SI)
Speed of light	c	$2.99792458 \times 10^8 \text{ m s}^{-1}$
Gravitational constant	G	$6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Reduced Planck constant	$\hbar = h/(2\pi)$	$1.054571817 \times 10^{-34} \text{ J s}$
Planck constant	h	$6.62607015 \times 10^{-34} \text{ J s}$
Elementary charge	e	$1.602176634 \times 10^{-19} \text{ C}$
Electron mass	m_e	$9.1093837015 \times 10^{-31} \text{ kg}$
Proton mass	m_p	$1.67262192369 \times 10^{-27} \text{ kg}$
Boltzmann constant	k_B	$1.380649 \times 10^{-23} \text{ J K}^{-1}$
Stefan-Boltzmann constant	σ	$5.670374419 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Permittivity of free space	ϵ_0	$8.8541878128 \times 10^{-12} \text{ F m}^{-1}$
Permeability of free space	μ_0	$1.25663706212 \times 10^{-6} \text{ H m}^{-1}$

In Gaussian CGS, there is no separate permittivity constant ϵ_0 or permeability constant μ_0 ; instead, electromagnetism is formulated without these factors explicitly.

Throughout this thesis, geometric units are employed in all theoretical expressions, particularly in Chapters 2 and 3 (except for the calculations related to radiation). In geometric units, we set:

$$c = G = \hbar = 1. \quad (1)$$

This convention dramatically simplifies the appearance of equations by eliminating ubiquitous factors of c , G , and $\hbar$. In geometric units, all quantities are expressed in units of length (or equivalently, inverse mass or inverse time).

Part I

Foundations

Chapter 1

Introduction

“What we observe is not nature itself, but nature exposed to our method of questioning.”

—Werner Heisenberg

Physics and Philosophy: The Revolution in Modern Science, 1958

1.1 Motivation

THE human quest to understand our place in the Universe has journeyed and evolved through distinct intellectual stages. It started from the stones of Göbekli Tepe¹, through the philosophical ‘*kosmos* (κόσμος)’ of the Greeks and crystallized by Einstein’s General Theory of Relativity, thereby opening a new arena for thought. The question then we ought to ask, before anything else, is this: *what kind of thought should we be engaging in?* What is the character of the intellectual lens through which we should interpret the Universe?

To answer this question, or at least partake in the very thought which led to it and motivates our work, we start with the Kuhn’s reflection of the *history of scientific advancements* (Kuhn 1962). There was a time when inquiry was directed almost entirely at organizing empirical observations. We asked whether the Sun or the Earth stood at the center of the cosmos. Later, with Newton’s law of gravitation, the question shifted. It was no longer merely about where celestial bodies were, but whether their motions obeyed a single, universal mechanics. The *heavens* and the Earth were unified under one mathematical law, $F = Gm_1m_2/r^2$.

Solidifying the idea that Galileo had initiated and meeting other objectives set by past scientific advancements, Newton’s work was staggeringly successful,

¹Archaeological site in upper Mesopotamia, modern-day Turkey, (c. 9500–8000 BCE), suggested to feature the world’s oldest known lunisolar calendar.

thus, later referred to by Kuhn as “*normal science*”² where scientists proved their ingenuity by “*puzzle solving*,” much like solving a problem of Sudoku. That, however, did not stop Newton’s period from encountering what he called “*paradigm shift*.”

A paradigm supplies the criteria by which problems are chosen and the rules by which valid solutions are recognized. Newton’s period could not address otherwise significant problems, thereby bringing about a paradigm shift, where scientists would legally address these problems. One example of such a problem is the very nature of space and time, left unexamined by Newton in his Scholium to the *Principia* (Rynasiewicz 2022)³ as absolute. Another example comes from Leibniz. He argued that if the Universe occupied one location in space rather than another, there could be no sufficient reason to prefer that location, making absolute spatial position a metaphysically idle notion (Alexander 1956). This was a genuine foundational objection, but *normal science* had no purchase on it. It could not be stated as a puzzle in Newtonian *framework*⁴, and so the Newtonian framework had no machinery to address it. It laid outside the paradigm, not because it was wrong, but because the paradigm’s own tools could not reach it.

And both these problems were taken up in the succeeding paradigm, where Einstein’s General Relativity (GR) addressed them by refining the concepts of space and time themselves: they ceased to be fixed, inert backgrounds and became dynamic geometric entities in the physical framework. This was not an advancement in the same line in that Newton’s concepts were mere reduction of Einstein’s concepts, but rather epistemically radical. For example, where gravity was explained in Newton’s paradigm as the force of attraction between two masses, it was in Einstein’s paradigm much like a geometric property of a four dimensional space-time manifold in which the distribution of mass-energy determined curvature via Einstein’s Field Equation, and freely falling bodies traced geodesics defined by that geometry.

After a prolonged period of *normal science*, we again drifted to ‘*revolutionary*’ period. To answer our question, *what kind of thought should we be engaging in*, we must answer where we now stand? Do we find ourselves in a period of *normal science* built on the relativistic paradigm, extending its precision and scope? Or are we awaiting a paradigm shift? Or perhaps somewhere in between, at a point where what we observe cannot be interpreted without the theoretical structure,

²“...research firmly based upon one or more past scientific achievements, achievements that some particular scientific community acknowledges for a time as supplying the foundation for its further practice”—Chapter 4, *The Structure of Scientific Revolutions*, 1962.

³Scholium to the Definitions in *Philosophiae Naturalis Principia Mathematica*.

⁴By *framework*, we emphasize on the mathematical/predictive structure of a presented paradigm.

as Leibniz once did, whether the foundations in a certain hierarchy we take for granted are the right ones?

Einsteinian view, in the Kuhnian sense, marks the paradigm within which modern relativistic astrophysics operates. GR has passed several tests across a century of research, from Eddington’s 1919 solar eclipse measurement of light deflection (Coles 2001) and the detection of gravitational waves from merging compact objects by LIGO/Virgo (LIGO Scientific Collaboration et al. 2016), to the astrometric tracking of stellar orbits at the Galactic center by GRAVITY (GRAVITY Collaboration et al. 2020) and the first horizon-scale images of M87* by the Event Horizon Telescope (Event Horizon Telescope Collaboration 2019). Given such a vast enterprise of what this paradigm has opened, spanning compact objects, gravitational waves, cosmology, and strong-field astrophysics with increasingly precise observation, specific theoretical commitments have achieved a *foundational status* within it, guiding entire bodies of research, models, and observational tests. Kuhn operates at the level of the paradigm as a whole, and since there is a hierarchy of fundamental-ness within the paradigm, he does not provide the provision to characterize how certain internal commitments are protected from refutation while their surrounding assumptions bear the brunt of observational tests. Lakatos (Lakatos 1970) addressed this gap by introducing the concept of *research programmes*.

According to Lakatos, research programmes have two elements: *hard core*, theoretical commitments that the community treats as foundational by methodological decision (*relative frames* as treated in Einsteinian view), and a surrounding protective belt of *auxiliary hypotheses* that can be progressively refined (less fundamental ideas; such as the structure of the Universe). The community protects this *hard core* from direct refutation, not because it is unfalsifiable in principle, but because the accumulated weight of evidence has made it the foundation from which all further research proceeds, which in our case is Einsteinian view⁵(Larvor 2021).

Further, this constitutes a sequence of individual falsifiable theories by this shared hard core. The commitment that astrophysical compact objects are uniquely

⁵It should be noted that the distinction here is acute (since hard core, standing alone, is dissociated with empirical consequences): it is between the central theses that there is no absolute reference frame (space and time, as in the Newtonian framework) but relative frame of reference (space-time, introduced by Hermann Minkowski, as in Einsteinian framework). The reason to choose this view, in broader sense, is to be consistent with what Kuhn’s *paradigm* and Lakatos’ *research programme* takes at their core, for its ultimate survival. The notion of GR, gravitation as the curvature of the four-dimensional spacetime manifold, and the mathematical framework, *Einstein-Hilbert action*, in principle, can be contested. For instance *teleparallel gravity*, another theory of gravity where the spacetime is essentially (always) flat, the gravitational effects arises due to the *torsion* of the spacetime. For a detailed discussion refer to Weatherall and Meskhidze 2024.

described by the Kerr metric has secured the status of a *hard core* within the *relativistic research programme*. Together with the auxiliary hypotheses surrounding it—the accretion models, magnetic field configurations—it constitutes what we refer to as the *black hole paradigm*⁶. Black holes are among the objects that most severely test the limits of the relativistic framework, having driven more development of physical theory and observational technique than any other class of object in relativistic astrophysics. Hence, it would not be unreasonable to call them as the ‘king of the cosmos’ (Cardoso et al. 2016).

It is within this historical and conceptual understanding of the central idea, this thesis works out on the question which was posited in the very starting, hence finding its motivation. Within the broader relativistic framework, the Kerr description is treated as the *hard core*—as the *astrophysical community* does—the *research programme* proceeds extending and testing the auxiliary hypotheses that surround it such as the accretion models that translate the Kerr commitment into observable predictions. Two chapters of this thesis take exactly this view, combining models such the magnetic field geometry, and the accretion structure to empirical affairs (Chapters 5 and 8).

The *gravity community*, however, takes a different position. It treats the Kerr description not as a *hard core* to be protected but as one solution among those GR admits, placing it in the role of a testable assumption within the broader relativistic framework. This view is further motivated by internal tensions that the auxiliary belt cannot absorb: the *information paradox* and the presence of Cauchy horizons leading to potential causality violations⁷ reside within the hard core prediction itself. GR, under physically reasonable matter conditions departing from the idealized assumptions of pressureless homogeneous collapse, admits gravitational end-states that lack event horizons with singularities⁸ (Christodoulou 1984; Joshi et al. 2011; Cardoso and Pani 2019). The horizonless alternatives explored in Chapters 6 and 7 are exact solutions of Einstein’s field equations, occupying a different region of the solution space that GR permits, not a region outside it. They constitute a competing research programme within the *hard core* (which is now GR), one that replaces the event horizon assumption while leaving GR’s field

⁶Paradigm here is not used in the strict Kuhnian sense, but in the Lakatosian sense of a research programme: a hard core theoretical commitment, the Kerr description of gravitationally collapsed objects, surrounded by a belt of auxiliary hypotheses whose combination produces falsifiable predictions.

⁷While there exist physical problems pertaining to the existence of the event horizon, philosophical issues also arise (see, for example, Romero 2015).

⁸Spacetime singularities are not merely extreme physical regions, but signal a breakdown of the theoretical framework itself. In GR, they are typically characterized by geodesic incompleteness rather than as well-defined points in spacetime. The theory ceases to provide a complete physical description, and singularities are often interpreted as indicating its limits rather than as physically realized entities (Curiel et al. 2025).

equations intact. Therefore, we see that whether this, a competing programme, is equally consistent with what the instruments now see or not?

1.2 The Black Hole Paradigm and Active Galactic Nuclei

The acceptance of black holes as real astrophysical objects, and their eventual identification as the engines powering Active Galactic Nuclei (AGN), did not follow a single line of reasoning. It emerged from a sustained and often contentious interplay between the theoretical predictions of GR and a sequence of unexpected astronomical discoveries spanning nearly a century. What began as a mathematical curiosity embedded in the Schwarzschild solution became, by the end of the twentieth century, the foundational assumption of an entire observational program.

1.2.1 Early and later history—Black holes

In 1784, John Michell proposed⁹ the existence of “dark stars,” (Figure 1.1 Michell 1784) objects so massive and compact that their escape velocity would exceed the speed of light, rendering them invisible to distant observers. Although speculative and rooted in classical mechanics, Michell’s argument represented the first conceptual step toward compact objects. This line of reasoning lay largely dormant for over two centuries, resurfacing only with the advent of a new gravity theory. The groundwork was laid shortly after Einstein his new theory of gravity. Karl Schwarzschild’s 1916 solution (Schwarzschild 1916) for a spherically symmetric mass introduced the concept of a critical radius (the Schwarzschild radius), a point of mathematical breakdown, known to be an event horizon. For decades, such solutions were treated as mathematical rather than physical descriptions, a skepticism exemplified by Arthur Eddington’s arguments against their formation.

A decisive conceptual shift occurred when advances in stellar astrophysics established firm physical limits on the stability of compact objects. In 1931, Subrahmanyan Chandrasekhar showed that white dwarfs, supported by electron degeneracy pressure, possess a maximum stable mass of approximately $\sim 1.4, M_{\odot}$, beyond which no equilibrium configuration can exist (Chandrasekhar 1931). This result implied that sufficiently massive stars cannot avoid further gravitational

⁹“On the means of discovering the distance, magnitude, & c. of the fixed stars, in consequence of the diminution of the velocity of their light, in case such a diminution should be found to take place in any of them, and such data should be procured from observations, as would be farther necessary for that purpose.”

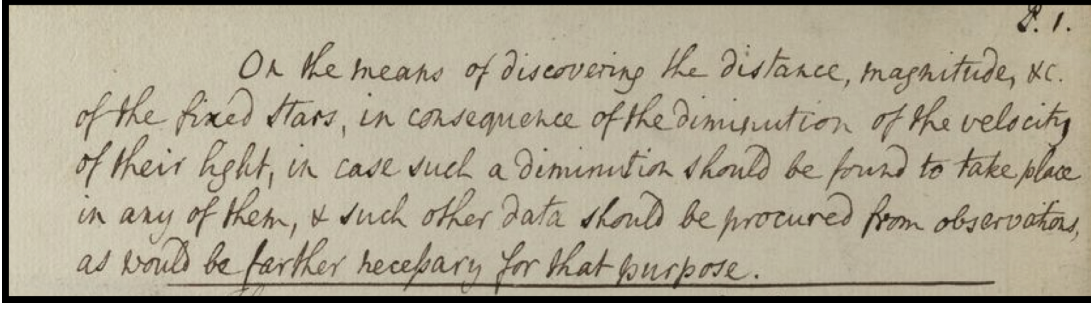


Figure 1.1: Michell’s paper read in 1783 and published in 1784 (credit - Royal Society of London)

collapse once nuclear fuel is exhausted. An analogous limitation arises for neutron stars, which are supported by neutron degeneracy pressure but are subject to a maximum mass set by the Tolman-Oppenheimer-Volkoff (TOV) limit. This limit, of order $\sim 2\text{--}3, M_{\odot}$, emerges from GR itself, as the pressure contributing to spacetime curvature ultimately enhances gravitational instability rather than preventing it (Tolman 1939; Oppenheimer and Volkoff 1939). Together, these stability bounds raised a fundamental question concerning the ultimate fate of stars exceeding all known degeneracy pressure limits. For stars whose mass exceeds this limit, no known form of degeneracy pressure can provide long-term support against self-gravity. Once nuclear fuel is exhausted and thermal pressure declines, continued gravitational collapse becomes unavoidable within classical physics. The evolutionary pathway of sufficiently massive stars is therefore qualitatively different from that of low-mass stars that can settle into stable compact remnants. As emphasized early on by Subrahmanyan Chandrasekhar in 1934 (Chandrasekhar 1934), “...one is left speculating on other possibilities.”. In this regime gravity becomes the dominant interaction governing the subsequent evolution, and the ultimate fate of the star must be determined within the framework of GR.

To address this question, Oppenheimer and Snyder 1939 (and independently by Datt 1938) analyzed the continued gravitational collapse of a spherically symmetric, pressureless dust cloud within GR. Their calculation showed that the collapsing matter contracts through the Schwarzschild radius $r = 2m$, which is not a true singularity of the geometry but a coordinate artifact. A distant observer never sees the stellar surface cross this radius, as photons emitted from it suffer a progressively divergent gravitational redshift. The matter itself, however, continues to collapse inward, and a genuine spacetime singularity forms at the center, where the density diverges. This was the first explicit relativistic model of complete gravitational collapse: a region of spacetime from which no signal can escape to infinity, surrounding a central singularity. The result, however, rested on

highly idealized assumptions: exact spherical symmetry and a pressureless dust equation of state. Whether realistic, asymmetric collapse with non-trivial internal pressure would produce the same outcome remained an open and contested question. In the Oppenheimer-Snyder model, all matter is directed precisely at a single central point, so the resulting density singularity could be viewed as an artifact of this exact focussing. A small departure from spherical symmetry might cause infalling material to miss the center, allowing the density to remain finite and potentially permitting the spacetime to continue without singular behavior (Lifshitz and Khalatnikov 1963). Indeed, the Russian school of Lifshitz and Khalatnikov initially claimed that singularities do not arise in generic solutions of the Einstein equations (Lifshitz and Khalatnikov 1963), a conclusion later shown to contain a significant error (Belinsky et al. 1970; Belinskii et al. 1972).

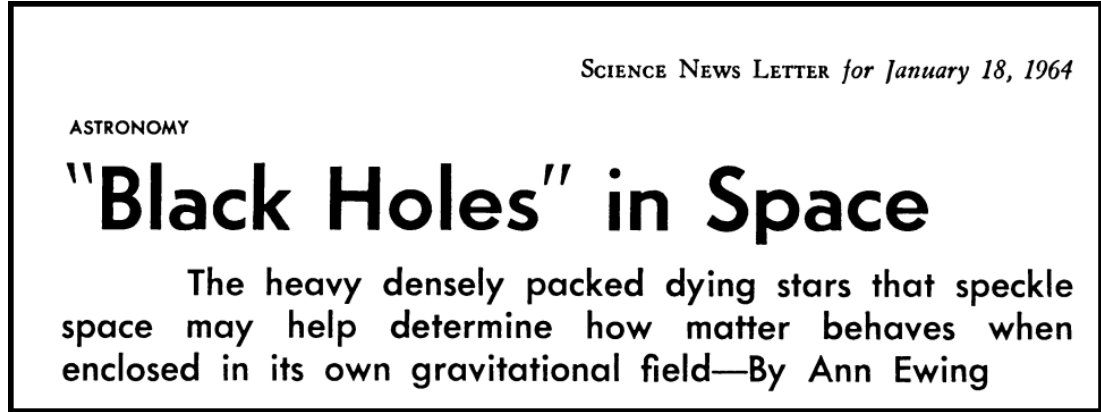


Figure 1.2: First in-print mention of the term ‘black hole’ by Ann Ewing on January 18, 1964 in the issue of *Science News Letter*.

Hawking, Penrose and Geroch resolved this debate by shifting the argument away from the detailed dynamics of any particular collapse model (Penrose 1965; Hawking and Ellis 1973). It was shown that a spacetime, under a rather general framework with certain reasonable physical assumptions, will admit singularities. This conclusion requires no symmetry assumption and is therefore generic, meaning spacetime singularities are a generic feature of the relativistic theory. These considerations also ensured the existence of singularities in modified theories of gravities (based on the framework of spacetime manifold). The *singularity theorems*, however, emphasizes that singularity-free evolution cannot happen (with reasonable energy-positivity condition) once the collapse reaches a certain point of no return—a *trapped surface*¹⁰. However, the requirement for a singularity

¹⁰*Trapped surface* can be characterized as a compact, spacelike 2-surface in the spacetime whose null normals converge into the future. And once a trapped surface forms, there’s no possibility of extending the spacetime indefinitely. In the OSD collapse, as soon as the collapsing matter shrinks within the event horizon, trapped surfaces surrounds immediately the region of spacetime.

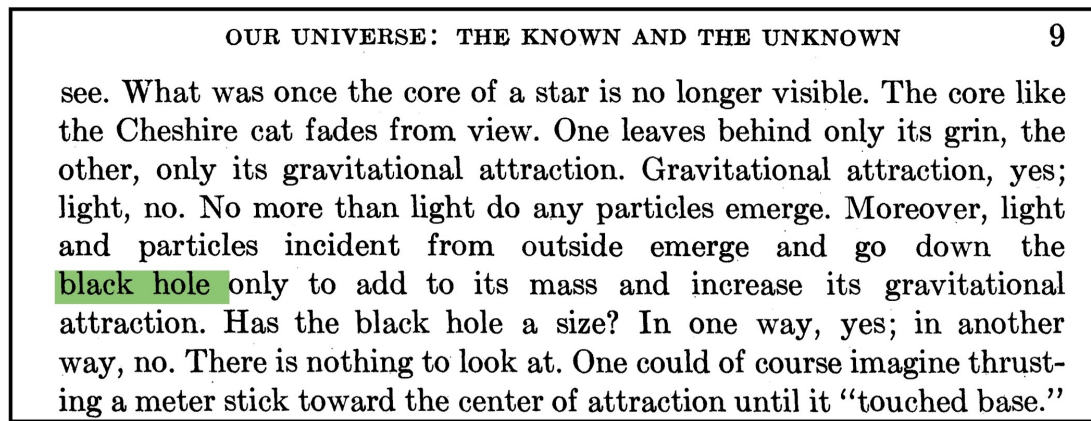


Figure 1.3: John Wheeler’s 1968 article in *American Scientist* (Wheeler 1968), in which he adopted and published the term ‘black hole,’ securing its universal usage in the scientific community.

to be present without an event horizon is a question of great puzzle with no formal proof. This possibility is conjectured as the “*cosmic censorship conjecture*” (Penrose 1969). The very nature of this question opens up a domain of *naked singularities*, and as we shall see in Chapter 2, singularities emerging in an end-state of collapse may not necessarily harbor an event horizon, and be, in fact, visible.

These completely collapsed gravitational objects, as they were initially called, acquired their now-standard name. The term appeared in print for the first time in the January 18, 1964 issue of *Science News Letter*, in a report by science journalist Ann Ewing on an American Association for the Advancement of Science meeting in Cleveland (Ewing 1964) (see Figure 1.2). The phrase had been circulating orally at least since the December 1963 Texas Symposium on Relativistic Astrophysics in Dallas, and may trace further to Robert Dicke’s informal use of the Black Hole of Calcutta analogy at a Princeton seminar around 1960 (Herdeiro and Lemos 2018). John Wheeler adopted the term at a 1967 lecture and published it in 1968 (Wheeler 1968) (see Figure 1.3), and his authority in the field secured its universal adoption.

These early theoretical developments collectively established the conceptual and mathematical foundations from which black holes would emerge as concrete predictions of relativistic gravity. While foundational, they remained theoretical constructs awaiting an observational catalyst.

1.2.2 AGNs and Radio Astronomy

The paradigm shift was driven by the dawn of radio astronomy by Karl Jansky in 1933 with the first reported radio emission from outside the solar system (Jansky 1933), and then subsequently in the 1950s and 1960s (see Shields 1999,

for a review). The discovery of incredibly luminous, point-like “quasi-stellar” radio sources (quasars) in mid 20th century, presented an acute energy problem. Their compact size, inferred from rapid variability, and their extreme brightness, demanded an engine far more efficient than nuclear fusion. A critical piece of evidence came in 1963 February 5, when Maarten Schmidt identified hydrogen emission lines in quasar 3C 273 spectra. Their extreme redshift provided conclusive proof that quasars were not local stellar objects but were, in fact, located at vast cosmological distances (Schmidt 1963), confirming their immense intrinsic luminosities. This transformed quasars from a radio anomaly into an observational crisis. Hoyle and Fowler 1963; Hoyle and Fowler 1963 proposed a supermassive object, with a mass on the order of 10^8 solar masses, as potential energy source for quasars, drawing from both gravitational and thermonuclear processes. They also proposed that “a magnetic field could be wound toroidally between the central star and a surrounding disk.” The only viable solution was the accretion of matter onto a supermassive black hole, a process capable of generating radiation with far greater efficiency than any known thermonuclear process.

Soon after the discovery of quasars, Salpeter and Zeldovich (Salpeter 1964; Zel’dovich 1964) independently proposed that their enormous luminosities could be powered by accretion onto supermassive black holes. They showed that matter spiraling inward to the innermost stable circular orbit of a non-rotating black hole at $r = 6GM/c^2$ ¹¹, can release up to $\sim 5.7\%$ of its rest-mass energy, an efficiency sufficient to account for the observed energy output of quasars with a plausible mass supply. Salpeter further argued that effective angular momentum transport—envisioned in terms of turbulent or viscous processes—would allow gas to accrete while the central black hole grows in mass. Despite these insights, the black hole model initially attracted limited attention. A crucial advance was made by Lynden-Bell in 1969, who argued that if quasars were powered by accretion, then remnant “collapsed bodies” (black holes) should be ubiquitous in galactic nuclei today (Lynden-Bell 1969). It was further suggested that such quiescent black holes could be detected through their dynamical influence on the mass-to-light ratios of galactic centers and explored the thermal and non-thermal emission expected from accretion disks, with dissipation driven by magnetic and turbulent processes.

¹¹Here G is Gravitational constant, c is the speed of light in vacuum, and M is the mass of the supermassive black hole

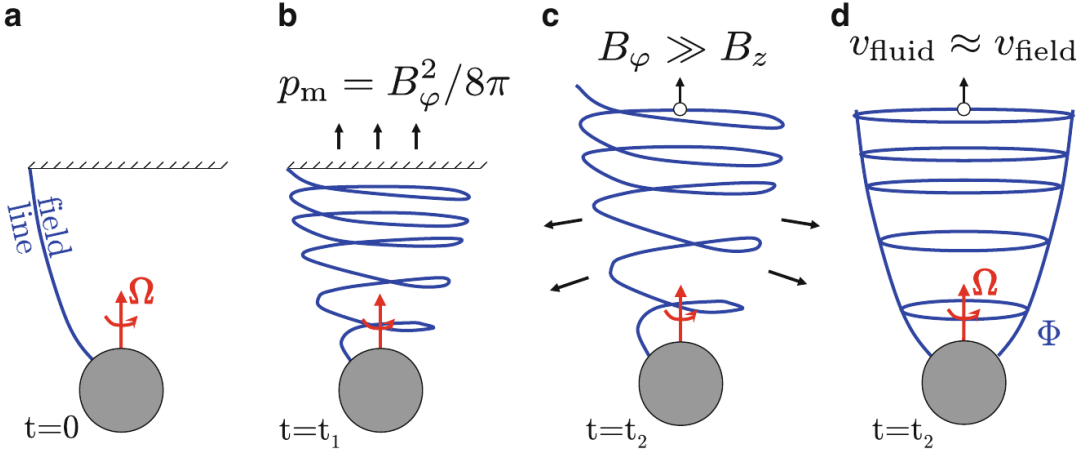


Figure 1.4: Schematic illustration of jet formation by magnetic fields. Image adapted from [Contopoulos et al. 2015](#)

1.3 Accretion and Magnetic Fields

The transport of angular momentum in accretion disks presented a longstanding theoretical difficulty. Conservation of angular momentum alone would lock infalling matter into near-Keplerian orbits; without an efficient outward angular momentum flux, mass cannot spiral inward on timescales consistent with the observed luminosities of AGN and X-ray binaries. Molecular viscosity falls orders of magnitude short of the required stress. For decades, no adequate physical mechanism was identified.

The resolution came in the 1990s with the recognition that magnetic fields, amplified by the magnetorotational instability (MRI), provide the missing angular momentum transport. In a differentially rotating disk, even a weak magnetic field threading the disk is dynamically unstable. The Lorentz tension along field lines couples radially separated fluid elements, transferring angular momentum outward and allowing mass to accrete inward ([Balbus and Hawley 1998](#)). The resulting turbulence produces viscous stresses sufficient to drive accretion at the rates observed in AGN and X-ray binaries ([Davis and Tchekhovskoy 2020](#)).

Beyond enabling accretion, magnetic fields drive relativistic jets. The tight collimation and extreme bulk velocities of AGN jets require a mechanism that extracts rotational energy from the black hole or accretion flow and converts it into directed outflow over large scales. As accretion proceeds, turbulent dynamo action and inward advection of field lines cause magnetic flux to accumulate near the black hole. Large-scale poloidal field components become anchored to the rotating black hole and disk; through the Blandford-Znajek mechanism, the black hole's rotation twists these field lines, extracting rotational energy and launching it outward as a magnetically collimated jet ([Blandford and Znajek](#)

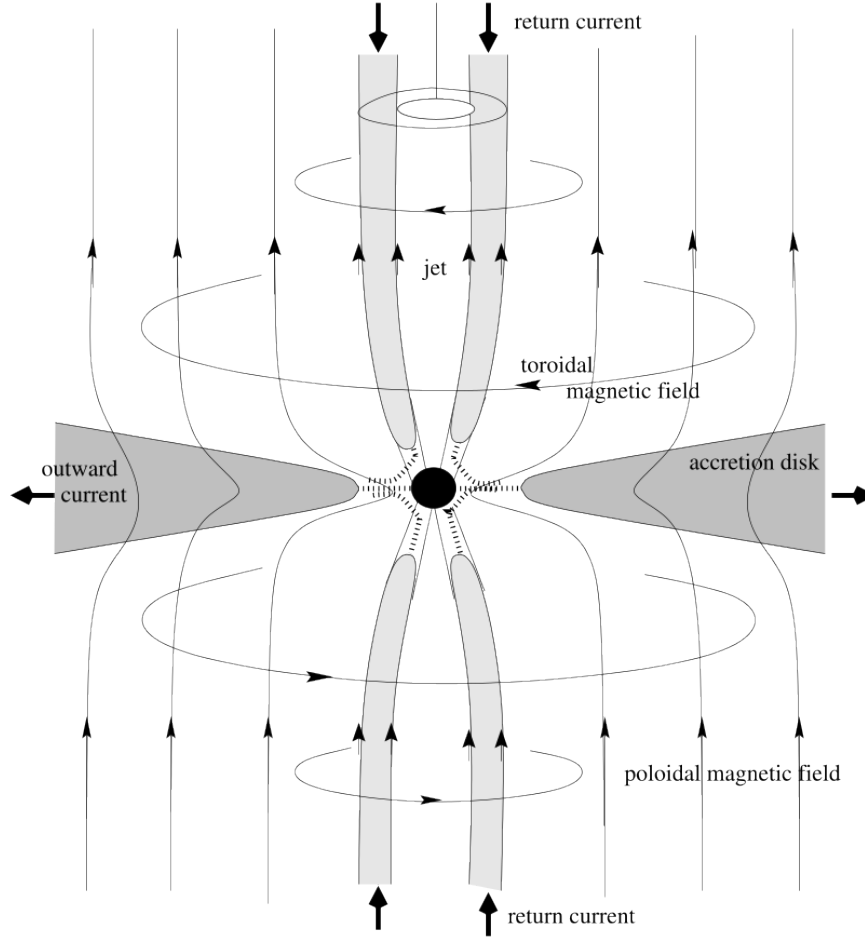


Figure 1.5: Schematic illustration of global geometries a magnetic fields and plasma flows around a black hole. Image adapted from [Kino et al. 2000](#)

1977; McKinney 2004). Each rotation progressively winds the initially poloidal field into a toroidal component, and as magnetic energy accumulates the jet is accelerated and collimated. Figure 1.4 illustrates this process schematically, and Figure 1.5 shows the global geometry of the dual magnetic field components, the inward accretion flow, and the jet launched along the rotation axis.

The observational case for this picture is complemented by high-resolution GRMHD simulations of magnetized accretion flows where they reproduce many features of observed AGN and jets (Narayan et al. 2012; Event Horizon Telescope Collaboration 2019). Polarimetric observations of M87* and Sgr A* reveal ordered magnetic field structures in the innermost accretion flow (Event Horizon Telescope Collaboration 2021; Lu 2023), consistent with the large-scale poloidal configurations in Figure 1.5. Jet power and collimation depend sensitively on field strength and geometry, with more organized fields producing narrower, faster jets (McKinney 2004). The energy budget of AGN therefore has two components: gravitational binding energy released as matter spirals inward, and rotational energy extracted from the black hole itself — both channeled by organized magnetic

fields.

A complementary mechanism operates at the level of the accretion disk itself. [Blandford and Payne 1982](#) examined the possibility that energy and angular momentum are removed magnetically from accretion disks by field lines that leave the disk surface and extend to large distances, driving a centrifugally driven outflow of gas. At large distances from the disk, the toroidal component of the magnetic field becomes important and collimates the outflow into a pair of anti-parallel jets moving perpendicular to the disk ([Blandford and Payne 1982](#)). In the context of AGN jet formation, the Blandford-Znajek and Blandford-Payne processes are not mutually exclusive. Jets are formed when the black hole spins and the accretion disk is strongly magnetized, and it is the magnetic stress associated with this disk wind that collimates the jet close to the black hole ([Blandford et al. 2019](#)).

1.4 The Event Horizon Telescope and Horizon-Scale Imaging

The Event Horizon Telescope (EHT) is a global millimeter-wavelength very long baseline interferometry (VLBI) array ([Figure 1.6](#)) that achieves an angular resolution of $\sim 20 \mu\text{as}$, sufficient to resolve the horizon-scale emission around the two primary targets: M87*, a giant elliptical galaxy at $D_s \approx 16.9 \text{ Mpc}$ harboring a black hole of mass $M \approx 6.5 \times 10^9 M_\odot$, and Sgr A*, the compact radio source at the Galactic center with $M \approx 4.15 \times 10^6 M_\odot$ at $d \approx 8.15 \text{ kpc}$ ([Event Horizon Telescope Collaboration 2019](#); [Event Horizon Telescope Collaboration 2022](#)). Both systems correspond to the largest apparent angular sizes of any known black hole candidates, with gravitational radii subtending $\theta_g = GM/c^2 D_s \approx 3.8 \mu\text{as}$ and $5.1 \mu\text{as}$ respectively.

Interpreting EHT observations requires accounting for the multi-component structure of the target sources. Multi-wavelength observations of M87* ([Figure 1.7](#)) reveal emission spanning from radio through X-rays and gamma-rays, arising from spatially distinct regions: the inner accretion disk, the black hole magnetosphere, the relativistic jet base, and the extended lobes. The EHT observations at 230 GHz probe the millimeter-wavelength synchrotron emission from the innermost accretion flow, and understanding what this component traces within the broader spectral energy distribution is essential for physical interpretation of the images.

In 2019, the EHT Collaboration released the first horizon-scale images of M87*, revealing a bright asymmetric ring of diameter $\sim 42 \mu\text{as}$ encircling a central brightness depression ([Event Horizon Telescope Collaboration 2019](#)). The

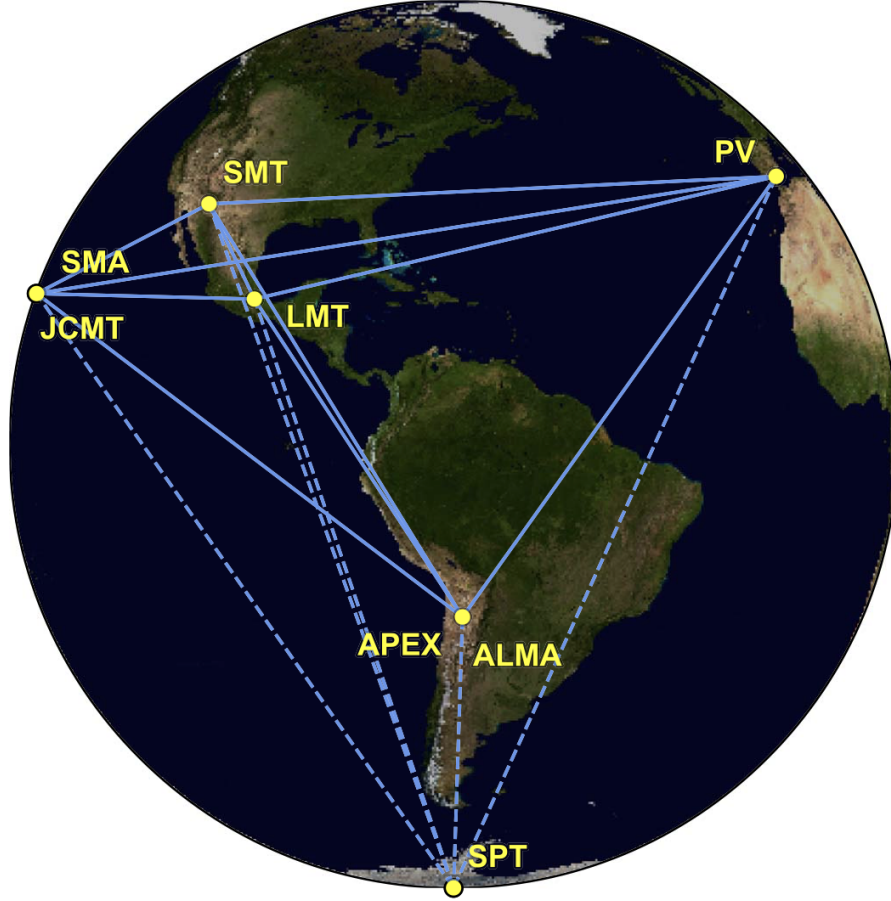


Figure 1.6: Global Event Horizon Telescope array. The EHT combines signals from millimeter-wavelength radio telescopes distributed across the Earth, including stations at the South Pole (SPT), Chile (ALMA, APEX, JCMT, LMT), Hawaii (SMA, JCMT), Arizona (SMT), Europe (PV), and other locations. The long baselines formed by these widely-separated stations (shown as connecting lines) provide the angular resolution necessary to image horizon-scale structures around supermassive black holes. The baseline lengths and geographic distribution determine the resolution, sensitivity, and imaging fidelity of EHT observations.

ring arises from synchrotron emission in the magnetized accretion flow, gravitationally lensed and Doppler-boosted by the black hole’s gravity and the jet-disk kinematics. The central depression—the black hole “shadow”—is broadly consistent with the photon capture cross-section of a Kerr black hole, placing competitive constraints on the mass and excluding significantly sub-Schwarzschild compact objects ([Event Horizon Telescope Collaboration 2019](#); [Vagnozzi et al. 2023](#)).

In 2022, the first images of Sgr A* followed, with the additional observational challenges of interstellar scattering and rapid intra-hour variability of the source ([Event Horizon Telescope Collaboration 2022](#)). These images again revealed a ring-like morphology consistent with the expected shadow of a Schwarzschild or mildly spinning Kerr black hole. Both sets of images were interpreted through

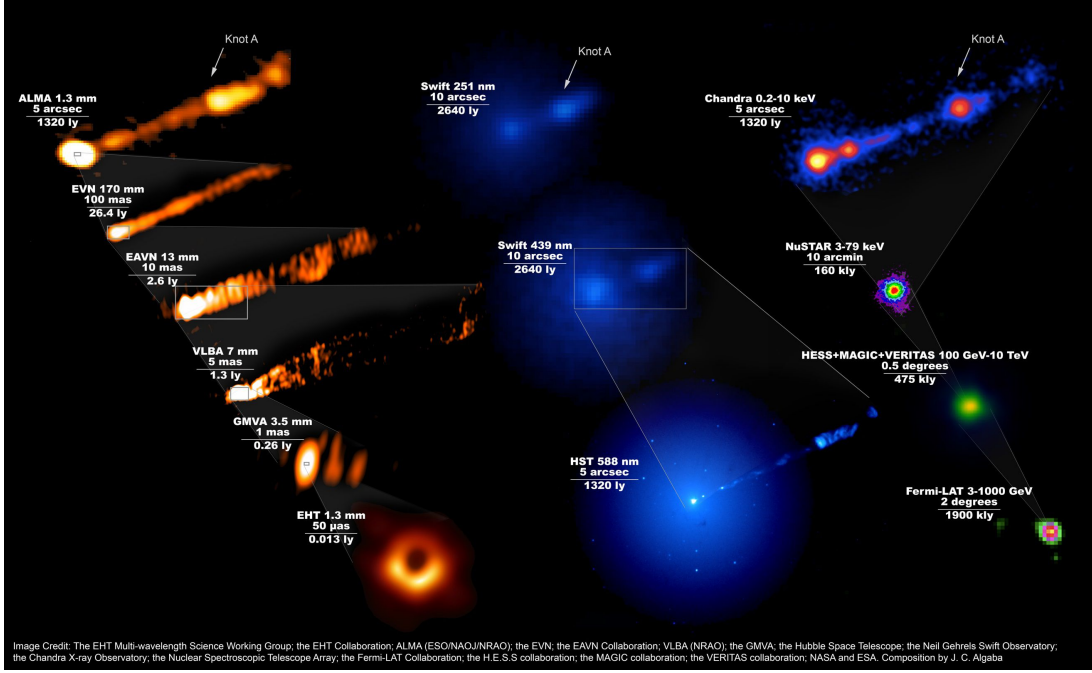


Figure 1.7: Multi-wavelength observations of M87*. Credit: EHT MWL science working group. Composition by J. C. Algaibya.

an extensive library of GRMHD simulations post-processed with general relativistic radiative transfer (GRRT) codes, spanning a range of black hole spins, magnetic field configurations, and electron heating prescriptions (Event Horizon Telescope Collaboration 2019; Event Horizon Telescope Collaboration 2022). The simulations fall into two magnetically distinct accretion states: the magnetically arrested disk (MAD), in which accumulated magnetic flux near the horizon periodically disrupts the accretion flow, and the standard and normal evolution (SANE) state, in which the magnetic field remains sub-dominant throughout (these states are discussed in detail in Chapter 3). For both M87* and Sgr A*, MAD models with moderate-to-high spin were broadly preferred by the data (Event Horizon Telescope Collaboration 2021; Event Horizon Telescope Collaboration 2022). Polarimetric EHT observations provided an additional discriminant: the resolved linear polarization structure of M87* is consistent with ordered poloidal magnetic fields near the horizon and has been used to constrain the rotation measure and hence the accretion rate (Event Horizon Telescope Collaboration 2021). Together, these results established horizon-scale VLBI as a quantitative tool for testing strong-field GR and constraining plasma physics in the immediate vicinity of supermassive black holes.

Ongoing and planned developments—including additional EHT stations, next-generation EHT (ngEHT) with improved coverage and sensitivity (Doeleman et al. 2023; Johnson et al. 2023), and proposed space-VLBI missions such as BHEX (Johnson et al. 2024)—will extend the achievable dynamic range and an-

gular resolution, enabling detection of the photon ring substructure and improving sensitivity to faint features interior to the bright emission ring. It is precisely in this regime—the sub-apparent-boundary region inside the photon ring—that alternative compact objects, including horizonless spacetimes, may leave their most distinctive observational imprints, as explored in this thesis.

1.5 The Current Paradigm and Beyond

1.5.1 Why Black Holes?

The black hole paradigm dominates astrophysics for reasons that are observationally compelling and theoretically grounded. The EHT measurements of M87* and Sgr A* confirm that the predicted shadow sizes and ring structures match Kerr geometry to remarkable precision ([Event Horizon Telescope Collaboration 2019](#); [Event Horizon Telescope Collaboration 2022](#)). Decades of astrometric monitoring of stellar orbits near the Galactic center have placed increasingly stringent constraints on the mass and spin of Sgr A*, all consistent with general relativity’s predictions ([GRAVITY Collaboration et al. 2020](#)). Across the electromagnetic spectrum — from millimeter wavelengths where the EHT observes, through X-rays and gamma-rays — the accretion luminosity and jet morphologies align with the picture outlined in Section 1.3: turbulent magnetized flows powered by the release of gravitational binding energy near a Kerr black hole. The paradigm works.

Theoretically, black holes are not optional endpoints of gravitational collapse; they are unavoidable. Once a massive star exhausts its nuclear fuel and loses thermal support, nothing known to physics can halt the infall. The Oppenheimer-Snyder calculation demonstrated within general relativity itself that continued collapse necessarily produces an event horizon ([Oppenheimer and Snyder 1939](#)). Penrose’s singularity theorems went further: under generic conditions on the matter, the formation of a spacetime singularity is not an exotic accident but the typical outcome of gravitational collapse ([Penrose 1965](#)). The Kerr metric, discovered in 1963, encodes this inevitability elegantly: parameterized by only mass and spin (by the no-hair theorem), it represents the unique stationary solution for a rotating black hole ([Kerr 1963](#)). Confronting GRMHD simulations of the accretion and magnetic physics against EHT data, the agreement between theory and observation is striking ([Narayan et al. 2012](#); [Event Horizon Telescope Collaboration 2019](#); [Event Horizon Telescope Collaboration 2021](#)).

Yet within this framework lies a problem that general relativity itself predicts but cannot resolve. At the center of the black hole, curvature invariants diverge

and the equations of general relativity break down — the theory predicts its own failure. When quantum effects are considered, a further incompatibility emerges. Hawking showed that black holes are not thermodynamically inert; they radiate, losing mass (Hawking 1974; Hawking 1975). The radiation produced during evaporation appears to carry no information about the initial state of the matter that collapsed, in apparent violation of the unitarity of quantum mechanical evolution. This is the information paradox.

These two issues point to a fundamental incompleteness in our understanding of gravitational collapse (Cardoso and Pani 2019). The singularity signals that general relativity has limits. The information paradox reveals an incompatibility between general relativity and quantum mechanics. Together, they demonstrate that the black hole paradigm, successful though it is at describing what we observe today, cannot be the complete or final description of nature.

1.5.2 Beyond Black Holes

The tensions identified above cannot be resolved within classical general relativity. If the singularities predicted by general relativity are real, then a theory that cannot handle them is incomplete. If information is truly destroyed in black hole evaporation, then quantum mechanics and gravity are fundamentally incompatible in their current formulations. One or both must be modified. Nature must provide something beyond the classical black hole.

Over the past decades, theoretical physics has responded to this necessity by constructing a diverse landscape of compact objects that could potentially replace classical black holes. These alternatives emerge from serious attempts to resolve the tensions outlined above, not from model-building unconstrained by physical motivation. In semiclassical approaches to quantum gravity, the singular interior is regulated or replaced by nonsingular structures. String theory proposes fuzzballs: horizonless ensembles of microstate geometries whose complexity extends to the would-be event horizon scale (Mathur 2005; Mayerson 2020). Modified theories of gravity — Einstein-dilaton-Gauss-Bonnet, scalar-tensor theories, and others — alter the gravitational equations themselves, producing modified or entirely new solutions in place of Kerr (Cardoso and Pani 2019). Regular black hole models avoid singularities through nonlinear electrodynamics or other mechanisms. More generally, one can consider horizonless objects with reflective or dissipative surfaces, capable of bouncing back infalling matter near the would-be event horizon (Cardoso et al. 2016). What unites these diverse proposals is their common goal: provide a completion of gravitational physics that avoids the pathologies — singularities and information loss — that plague classical black

holes.

The critical question is observational: how would we know if nature has chosen one of these alternatives over the classical black hole? Here emerges a profound subtlety. Nearly all known horizonless alternatives possess photon spheres — the circular orbits of massless particles that define the boundary of the shadow. The photon sphere emerges from the geometry near a compact object; it does not require an event horizon to exist (Cardoso and Pani 2019). Consequently, many horizonless alternatives produce shadows and photon rings visually indistinguishable from those of Kerr black holes (see for e.g. Perlick and Tsupko 2022). This creates what can be termed the “shadow degeneracy” (Cardoso and Pani 2019): static imaging of the photon ring cannot uniquely identify the nature of the compact object. A ring-like structure observed by the EHT could in principle originate from a classical black hole, a regular black hole, a fuzzball, or an exotic compact object with a reflective surface. The shadow alone cannot discriminate.

This limitation is both a conceptual challenge and an opportunity. Testing whether classical black holes remain adequate requires looking beyond the shadow itself. The accretion flow encodes metric information through its magnetic field structure, electron heating, and brightness distribution near the photon ring. Orbital dynamics offer a complementary window: the precession of stars near Sgr A* probes the metric’s structure in ways that shadow observations cannot (GRAVITY Collaboration et al. 2020). The photon ring — the sequence of increasingly faint lensed images surrounding the bright ring — records strong-field lensing in detail that future instruments will measure with high precision (Johnson et al. 2023). Each observable is sensitive to different aspects of the underlying geometry.

The task is therefore to develop theoretical predictions for alternative compact objects and to identify what signatures they would leave across these multiple observables. Constructing detailed models of how different spacetimes produce different accretion morphologies, orbital precessions, and lensing patterns equips us to assess whether observations are consistent with a classical black hole or whether the data require something different. This is pragmatic preparation: laying out the theoretical framework needed to interpret observations as they move into the regime where alternatives might leave detectable imprints.

1.6 Roadmap of the thesis

There is a natural order to this kind of work. One must first understand the spacetime, then the plasma in it, then the telescope that looks at both, before one can say anything meaningful about what M87* actually is or surrounded with. That is the order followed here. The thesis is organized in two parts: Part I

builds the foundations, Part II utilizes those foundations for several applications. Each chapter in Part I closes with a set of open questions that are not meant to be exhaustive, but rather to identify the specific problems this thesis takes up—the thread connecting the theoretical foundations to the work presented in Part II. What follows is a brief account of what each chapter contains, how they fit together, and how to navigate through this thesis.

- Chapter 2 establishes the spacetime geometry that underpins the rest of the thesis. The Schwarzschild and Kerr metrics are reviewed, with particular attention to the properties that have direct observational counterparts at horizon scales: the photon sphere, the shadow boundary, the ISCO, and the ergosphere. Alongside the canonical black hole solutions, the JMN-1 spacetime is introduced as a motivated alternative arising from gravitational collapse with anisotropic pressure in general relativity. Its geodesic and lensing properties are derived in parallel with the Kerr case. The open questions at the end of this chapter frame a thread that runs through the entire thesis: whether the compact object at the center of M87* is a Kerr black hole, and whether an alternative spacetime with the same photon sphere can produce observationally distinguishable signatures.
- Chapter 3 builds the astrophysical layer on top of the spacetime geometry, treating accretion at three levels of physical completeness. The Novikov-Thorne thin disk establishes the baseline. The semi-analytic RIAF model introduces the key free parameters: magnetic field topology, disk scale height, and sub-Keplerian velocity fraction. GRMHD simulations then provide a self-consistent treatment of the plasma dynamics and magnetic field evolution, including the MAD and SANE accretion states. In each case, the polarized radiative transfer formalism is developed and connected to synthetic Stokes maps through backward ray-tracing with `ipole`. The resulting image grids form the reference against which EHT observations are interpreted throughout Part II. The open questions identify three degeneracies that observations must break: between field topology and disk geometry, between accretion states, and between black hole and horizonless spacetimes.
- Chapter 4 completes the foundations by developing the observational machinery. Beginning from the van Cittert-Zernike theorem and the two-element interferometer, the chapter builds up to the full measurement equation, closure quantities, and the image reconstruction methods used by the EHT, including the `eht-imaging` framework that is a central computational

tool of this thesis. The EHT observations of M87* across 2009 to 2021 are reviewed, the physical interpretation of the 2017 images is developed in terms of the models of Chapter 3, and the open questions identify where the confrontation between models and data is most pressing.

The four paper chapters of Part II each address one or more of the questions raised in Part I.

- Chapter 5 takes up the magnetic field geometry and spin degeneracy identified in Chapters 3 and 4. A systematic semi-analytic RIAF parameter grid spanning field topology, disk scale height, velocity profile, and spin is compared against the full 2017 EHT polarimetric dataset of M87*, providing independent constraints on each parameter.
- Chapters 6 and 7 together address the compact object question of Chapter 2. Chapter 6 applies the RIAF imaging framework to the JMN-1 space-time, establishing that null singularity configurations produce images consistent with the EHT observations at current resolution, with the central brightness region as the only distinguishing feature. Chapter 7 replaces the semi-analytic prescription with a full three-dimensional GRMHD simulation of accretion onto JMN-1, confirming both the viability of the horizonless accretion flow and the central brightness excess as a target for next-generation instruments. These two chapters represent a progression in physical completeness, from prescribed to self-consistent accretion.
- Chapter 8 addresses the jet-launching question raised in Chapters 2 and 4. The 2021 EHT M87* data are analyzed with `eht-imaging` to place the first direct constraints on the jet base structure at sub-milliarcsecond angular scales. The imaging contribution of this thesis also forms one of the three independent regularized maximum likelihood pipelines in the companion multi-epoch variability analysis of the EHT collaboration (Event Horizon Telescope Collaboration 2025).

Chapter 2

Theory of Compact Objects

THE study of compact objects constitutes the most direct observational probe of strong-field gravity. In the environments near black holes and other ultra-compact objects, spacetime curvature is extreme, post-Newtonian approximations break down entirely, and the full nonlinear structure of general relativity governs the physics. The theoretical framework developed in this chapter — from the exact vacuum solutions of Einstein’s equations to the geodesic structure of black hole spacetimes and their horizonless alternatives — forms the foundation for the accretion, radiative transfer, and imaging calculations presented in subsequent chapters.

2.1 Foundations of General Relativity

Einstein’s general relativity fundamentally re-conceptualized gravity not as a force but as a geometric property of spacetime itself. The central postulate is captured by Wheeler’s memorable formulation:

*“Spacetime tells matter how to move,
matter tells spacetime how to curve.”*

The geometry of spacetime in GR is captured in the metric tensor $g_{\mu\nu}$, whose dynamics are governed by the Einstein field equations:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (2.1)$$

where $R_{\mu\nu}$ is the Ricci tensor, $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar, $G_{\mu\nu}$ is the Einstein tensor, and $T_{\mu\nu}$ is the stress-energy tensor. We adopt geometrized units $G = c = 1$ throughout this thesis, unless otherwise indicated. For vacuum spacetimes surrounding a compact object, $T_{\mu\nu} = 0$, and the field equations reduce to $R_{\mu\nu} = 0$ (Ricci flatness). The line element $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ defines the proper interval

between events. We employ the metric signature $(-, +, +, +)$, so that timelike, null, and spacelike intervals satisfy $ds^2 < 0$, $ds^2 = 0$, and $ds^2 > 0$, respectively.

2.2 The Schwarzschild Geometry

The Schwarzschild solution is the unique static, spherically symmetric, asymptotically flat vacuum solution of Einstein's field equations, $R_{\mu\nu} = 0$, first obtained by Schwarzschild in 1916. In Schwarzschild coordinates (t, r, θ, ϕ) the metric takes the form

$$ds^2 = -f(r) dt^2 + g(r) dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (2.2)$$

where

$$f(r) = 1 - \frac{2M}{r}, \quad g(r) = \frac{1}{f(r)}, \quad . \quad (2.3)$$

Here M denotes the Arnowitt–Deser–Misner (ADM) mass measured at spatial infinity. The spacetime admits a timelike Killing vector field associated with time-translation invariance and three rotational Killing vectors; staticity implies that the timelike Killing vector is hypersurface orthogonal, reflecting the absence of rotation and frame dragging. A fundamental consequence of these symmetries is Birkhoff's theorem: any spherically symmetric vacuum solution of Einstein's equations is locally isometric to the Schwarzschild spacetime, irrespective of the dynamical behavior of the interior matter distribution ([Misner et al. 1973](#)). The exterior gravitational field of a spherically symmetric body is therefore determined entirely by the single parameter M .

2.2.1 Global structure

The Schwarzschild geometry exhibits two distinguished hypersurfaces. The surface $r = r_S$ corresponds to the vanishing of $f(r)$ and the divergence of g_{rr} in Schwarzschild coordinates, but this divergence is a coordinate artifact: the Kretschmann invariant,

$$\mathcal{K} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{48M^2}{r^6}, \quad (2.4)$$

remains finite there. Ingoing Eddington–Finkelstein coordinates and the maximal Kruskal–Szekeres extension both extend smoothly through $r = r_S$, revealing it as a null hypersurface — the event horizon — that acts as a causal boundary separating the exterior asymptotically flat region from an interior domain from which no future-directed causal curve reaches future null infinity. The Kruskal–Szekeres extension further reveals the global causal structure: the interior con-

tains a spacelike singularity at $r = 0$ that terminates all future-directed timelike and null worldlines. At $r = 0$ the Kretschmann invariant diverges as r^{-6} , indicating a genuine breakdown of the classical geometric description rather than a coordinate artifact.

As $r \rightarrow \infty$ the metric approaches Minkowski spacetime,

$$g_{\mu\nu} \rightarrow \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1), \quad (2.5)$$

establishing asymptotic flatness. The leading correction, $g_{tt} \simeq -\left(1 - \frac{2M}{r}\right)$, recovers the Newtonian gravitational potential and provides the physical interpretation of M as the total gravitational mass measured by distant observers through Keplerian orbital dynamics. The Schwarzschild geometry — a one-parameter vacuum solution with a regular null horizon and a spacelike curvature singularity — is the natural starting point for the rotating generalization considered in the following section.

2.3 The Kerr Geometry

The Schwarzschild solution is restricted to non-rotating sources. Astrophysical black holes are, in general, rotating objects. The Schwarzschild solution, restricted to zero spin, is therefore an idealization, and the physically relevant vacuum solution is the one that includes angular momentum. The unique stationary, axisymmetric, asymptotically flat vacuum solution of Einstein's field equations describing a rotating black hole was found by Kerr (Kerr 1963). In Boyer–Lindquist coordinates (t, r, θ, ϕ) (Boyer and Lindquist 1967) the line element takes the form

$$\begin{aligned} ds^2 = & -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4Mar \sin^2\theta}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 \\ & + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2Ma^2r \sin^2\theta}{\Sigma}\right) \sin^2\theta d\phi^2, \end{aligned} \quad (2.6)$$

where the two fundamental combinations are

$$\Delta \equiv r^2 - 2Mr + a^2, \quad \Sigma \equiv r^2 + a^2 \cos^2\theta. \quad (2.7)$$

Here M is the ADM mass and $a \equiv J/M$ is the specific angular momentum, with J the total angular momentum of the source; the dimensionless spin parameter $a^* \equiv a/M$ satisfies $|a^*| \leq 1$. The metric is independent of t and ϕ , reflecting stationarity and axisymmetry, but the off-diagonal component $g_{t\phi} \neq 0$ breaks the time-reflection symmetry of the Schwarzschild metric and encodes the rotational dragging of inertial frames. Setting $a = 0$ recovers the Schwarzschild metric (2.2).

By the no-hair theorem (Israel 1967; Carter 1971; Hawking 1972), the Kerr family is the unique stationary, axisymmetric, asymptotically flat vacuum solution of the Einstein equations, fully characterized by the two parameters (M, a) . The theorem extends to the Einstein–Maxwell case: the Kerr–Newman metric (Newman et al. 1965), which additionally carries electric charge Q , is the unique stationary, axisymmetric, asymptotically flat solution of the coupled Einstein–Maxwell equations, characterized by (M, a, Q) . Astrophysical black holes are expected to carry negligible net charge, as any macroscopic charge would be rapidly neutralized by selective accretion of opposite-sign particles from the surrounding plasma (Thorne et al. 1986). The effective Q/M ratio for an astrophysical black hole is estimated at $\lesssim 10^{-18}$ (Eardley and Press 1974), so the two-parameter Kerr metric suffices for all observational purposes considered in this thesis.

2.3.1 Global structure

The coordinate singularities of the Kerr metric arise from the vanishing of Δ and of Σ . Setting $\Delta = 0$ yields two roots,

$$r_{\pm} = M \pm \sqrt{M^2 - a^2}, \quad (2.8)$$

which are the outer (r_+) and inner (r_-) event horizons, provided $|a| \leq M$. Both are null hypersurfaces; r_+ is the causal boundary of the black hole, separating the exterior region accessible to distant observers from an interior domain from which no future-directed causal curve escapes. The surface gravity of the outer horizon is

$$\kappa = \frac{r_+ - r_-}{2(r_+^2 + a^2)} = \frac{\sqrt{M^2 - a^2}}{2Mr_+}, \quad (2.9)$$

and the angular velocity of the horizon,

$$\Omega_H = \frac{a}{r_+^2 + a^2}, \quad (2.10)$$

determines the rotational coupling between the horizon and infalling matter or electromagnetic fields (Misner et al. 1973). The stationarity Killing vector $\xi^\mu = (\partial_t)^\mu$ is timelike at large r but becomes null on the stationary limit surface, where $g_{tt} = 0$,

$$r_{\text{SLS}}(\theta) = M + \sqrt{M^2 - a^2 \cos^2 \theta}. \quad (2.11)$$

Since $r_{\text{SLS}} \geq r_+$ at all θ , with equality only at the poles, there exists a region between r_+ and r_{SLS} where ξ^μ is spacelike: the *ergosphere* (Penrose 1969; Bardeen et al. 1972). Within the ergosphere no static observer can exist; all worldlines must co-rotate with the black hole to some degree. The ergosphere is the locus of the

Blandford–Znajek mechanism (Blandford and Znajek 1977) for electromagnetic energy extraction and of the Penrose process for particle energy extraction, both of which rely on the existence of negative-energy orbits in this region. The vanishing of Σ requires simultaneously $r = 0$ and $\theta = \pi/2$, corresponding in Boyer–Lindquist coordinates to a ring of coordinate radius a in the equatorial plane — the Kerr *ring singularity*. The Kretschmann invariant,

$$\mathcal{K} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{48M^2(r^2 - a^2 \cos^2\theta)[\Sigma^2 - 16a^2r^2 \cos^2\theta]}{\Sigma^6}, \quad (2.12)$$

diverges as $\Sigma \rightarrow 0$, confirming it as a genuine curvature singularity (Misner et al. 1973). The ring topology is a direct consequence of rotation: the $a = 0$ limit collapses the ring to the point singularity of Schwarzschild. The maximal analytic extension of the Kerr spacetime (Boyer and Lindquist 1967; Hawking and Ellis 1973) reveals that the ring singularity is timelike rather than spacelike — in contrast to the spacelike singularity of Schwarzschild — so that geodesics need not terminate there. The extended manifold contains regions that admit closed timelike curves; these causality-violating features are expected to be removed by any realistic matter distribution or perturbation in the interior, and are not physical properties of astrophysical black holes. For the extremal Kerr black hole, $a = M$, the two horizons coincide at $r_+ = r_- = M$ and the surface gravity vanishes ($\kappa = 0$). The sub-extremal case $|a| < M$ encompasses almost all astrophysically relevant systems; observational estimates of a^* in stellar-mass and supermassive black holes span a broad range of the interval $[0, 1)$ (Bambi 2017).

As $r \rightarrow \infty$ the Kerr metric reduces to Minkowski spacetime, $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$, and the leading corrections take the form

$$g_{tt} \simeq -\left(1 - \frac{2M}{r}\right), \quad g_{t\phi} \simeq -\frac{2aM \sin^2\theta}{r}, \quad (2.13)$$

recovering the Newtonian gravitational potential in g_{tt} and, in $g_{t\phi}$, the gravitomagnetic potential whose curl gives the Lense–Thirring precession of gyroscopes (Lense and Thirring 1918). The total angular momentum $J = aM$ is measured by distant observers through the multipole expansion of the metric. The quadrupole moment of the Kerr spacetime is $\mathcal{M}_2 = -a^2M$, and all mass multipole moments are uniquely determined by M and a alone (Hansen 1974) — the strong-field expression of the no-hair theorem, in contrast to realistic stellar exteriors where higher multipoles are not a unique function of (M, J) . The Kerr geometry thus represents the two-parameter family of stationary vacuum black hole solutions.

2.4 Alternatives Solutions

The Schwarzschild and Kerr solutions occupy a central position in classical general relativity: under broad assumptions of stationarity, asymptotic flatness, and the validity of the Einstein equations in vacuum, they are the standard description of the exterior geometry of astrophysical black holes. This “black hole paradigm” is not the only logical possibility admitted by the Einstein equations, nor is it uniquely selected by current observational data. Over the past decades, a variety of horizonless ultra-compact configurations have been proposed as alternatives, motivated by quantum-gravity considerations, attempts to avoid spacetime singularities, or phenomenological models of gravitational collapse beyond the idealized Kerr scenario (e.g. [Cardoso and Pani 2019](#)). In the contemporary literature these are grouped under the labels “black hole mimickers” or “exotic compact objects” (ECOs) ([Cardoso and Pani 2019](#)). For the purposes of this thesis, it is useful to classify them by two structural properties of the spacetime: (i) the presence or absence of an event horizon, and (ii) the presence or absence of a curvature singularity visible to distant observers.

- A first broad category consists of *horizonless, nonsingular compact objects*. In these models the classical event horizon is replaced by a material or effective surface at some radius $r = R_{\text{surf}} > 2M$, and the spacetime remains regular everywhere. Examples include gravastars, boson stars, and other ultra-compact configurations supported by exotic equations of state or quantum-motivated stresses ([Mazur and Mottola 2023](#); [Visser and Wiltshire 2004](#); [Schunck and Mielke 2003](#); [Cardoso and Pani 2019](#)). The exterior geometry is typically indistinguishable from Schwarzschild or Kerr down to radii marginally larger than the would-be horizon, so that all classical weak-field tests are automatically satisfied. The defining structural feature is the absence of a one-way causal boundary: null and timelike geodesics that reach the surface can, in principle, return to infinity, and the spacetime can be made geodesically complete.
- A second category comprises *horizonless spacetimes with a central curvature singularity*, commonly referred to as *naked singularities*. The precise notion of nakedness is causal: a singularity is globally naked if it lies within the causal past of future null infinity, $J^-(\mathscr{I}^+)$, meaning there exist future-directed non-spacelike geodesics that originate arbitrarily close to the singularity and reach $\mathscr{I}^+$ in finite affine parameter. Equivalently, the singularity is visible to distant observers in the sense that no event horizon — defined as the boundary $\partial J^-(\mathscr{I}^+)$ — forms to act as a one-way causal barrier. This

is precisely the condition whose generic violation is asserted by the weak cosmic censorship conjecture (Penrose 1969; Joshi 2007): that for physically reasonable matter and generic initial data in asymptotically flat spacetimes, gravitational collapse produces singularities that are always hidden behind event horizons. Naked singularities arise within GR itself as exact solutions when the parameters of known black hole families are extended beyond their horizon-forming ranges. The overcharged Reissner–Nordström solution, with charge $Q > M$, has no real roots of $\Delta = 1 - 2M/r + Q^2/r^2$ and therefore no event horizon; the central singularity at $r = 0$ is globally naked and the spacetime fails to satisfy the condition $Q \leq M$ required for horizon formation (Misner et al. 1973). Similarly, the Kerr metric with $|a| > M$ — the so-called Kerr superspinar (Carter 1968) — has $\Delta = r^2 - 2Mr + a^2 > 0$ for all r , so again no horizons exist and the ring singularity at $\Sigma = 0$ is exposed. Both solutions are exact vacuum (or electrovacuum) solutions of the Einstein equations; their nakedness is a direct consequence of exceeding the extremal bound. Whether such configurations can be reached through physically realistic collapse or accretion — and whether they are stable — remains an open question (Cardoso and Pani 2019), but they demonstrate that the Einstein equations themselves do not prohibit horizonless compact objects.

Beyond these parameter extensions of known black hole solutions, naked singularities can also arise as the end states of gravitational collapse from regular initial data when the matter distribution departs from the idealized assumptions of the Oppenheimer–Snyder model. The Joshi–Malafarina–Narayan (JMN) family of solutions (Joshi et al. 2011) provides an explicit realization of this class: it describes the end state of spherically symmetric gravitational collapse of anisotropic matter in which trapped surfaces never form, the central singularity is not covered by an event horizon, and the exterior region is exactly Schwarzschild. For the JMN-1 member of this family, the nakedness condition reduces to only a specified parameter space, under which null geodesics from the central region reach $\mathcal{I}^+$ in finite affine parameter, as discussed in detail in Section 2.4.1. From a theoretical standpoint, both classes are strictly horizonless. Their internal structure and causal properties differ, however, in an essential way. In nonsingular ECO models all geodesics can be extended to complete curves, and any effective surface is implemented at finite curvature and finite redshift. In naked singularity models such as JMN-1, geodesics terminate at a genuine curvature singularity in finite affine parameter, and families of null geodesics emerge from the singularity to reach distant observers. This behavior places such space-

times in direct conflict with the weak cosmic censorship conjecture, and makes them concrete test cases for strong-field gravity and horizon-scale observations.

2.4.1 The JMN-1 spacetime

Among the horizonless compact-object models considered as alternatives to astrophysical black holes, the Joshi–Malafarina–Narayan (JMN) family occupies a special place because it arises as the end state of explicit gravitational collapse scenarios from regular initial data, rather than being introduced as a stationary metric ansatz (Joshi et al. 2011). We summarize here the structure and basic properties of the first member of this family, JMN-1, which serves as the concrete example of a spherically symmetric horizonless spacetime.

The JMN-1 spacetime describes a static, spherically symmetric configuration in which an anisotropic fluid interior is matched smoothly at a finite matching radius $r = R_b$ to an exterior Schwarzschild metric. In the interior region $0 < r \leq R_b$, the line element is

$$ds^2 = -(1 - M_0) \left(\frac{r}{R_b} \right)^{\frac{M_0}{1-M_0}} dt^2 + \frac{dr^2}{1 - M_0} + r^2 d\Omega^2, \quad (2.14)$$

where $0 < M_0 < 1$ is a dimensionless parameter controlling the compactness and matter distribution, and $d\Omega^2$ is the metric on the unit two-sphere. The interior is filled with an anisotropic fluid whose energy density ρ , radial pressure p_r , and tangential pressure p_θ are

$$\rho(r) = \frac{M_0}{r^2}, \quad p_r(r) = 0, \quad p_\theta(r) = \frac{M_0}{4(1 - M_0)} \frac{1}{r^2}, \quad (2.15)$$

so the matter has vanishing radial pressure and non-zero tangential pressure. The weak and dominant energy conditions are satisfied throughout the interior, and the tangential sound speed remains subluminal for $M_0 \leq 4/5$ (Joshi et al. 2011). Outside the matching radius the spacetime is exactly Schwarzschild,

$$ds^2 = - \left(1 - \frac{M_0 R_b}{r} \right) dt^2 + \left(1 - \frac{M_0 R_b}{r} \right)^{-1} dr^2 + r^2 d\Omega^2, \quad r \geq R_b, \quad (2.16)$$

with total mass

$$M = \frac{1}{2} M_0 R_b. \quad (2.17)$$

The matching across $r = R_b$ is performed via the Darmois–Israel junction conditions: continuity of the induced metric and continuity of the extrinsic curvature. Since $p_r = 0$ at the boundary, no surface layer of stress-energy is required. The

metric functions satisfy (from general equation (2.2)):

$$f_{\text{int}}(R_b) = 1 - M_0 = f_{\text{ext}}(R_b), \quad g_{\text{int}}(R_b) = \frac{1}{1 - M_0} = g_{\text{ext}}(R_b), \quad (2.18)$$

and their radial derivatives agree, ensuring a smooth junction without thin shells. The interior stress-energy tensor satisfies the anisotropic Tolman–Oppenheimer–Volkoff equation,

$$p'_r(r) = -\frac{[\rho(r) + p_r(r)][m(r) + 4\pi r^3 p_r(r)]}{r[r - 2m(r)]} + \frac{2}{r}[p_t(r) - p_r(r)], \quad (2.19)$$

which is satisfied identically upon substituting (2.15) and $m(r) = \frac{1}{2}M_0 r$. JMN-1 is therefore a consistent static equilibrium configuration of anisotropic matter joined to a Schwarzschild exterior.

Since $g^{rr} = 1 - M_0 > 0$ everywhere in the interior for $0 < M_0 < 1$, the hypersurface $r = 2M$ is not null inside the matter distribution; no event horizon exists anywhere in the spacetime. The Misner–Sharp mass function satisfies

$$\frac{2m(r)}{r} = M_0 < 1, \quad 0 < r \leq R_b, \quad (2.20)$$

so trapped surfaces never form during the collapse leading to this configuration. At $r = 0$ curvature invariants diverge, indicating a genuine curvature singularity. The question of whether this singularity is naked is a question about geodesic completeness and causal structure. For $M_0 < 2/3$, future-directed radial null geodesics originating arbitrarily close to $r = 0$ reach the exterior Schwarzschild region and future null infinity $\mathscr{I}^+$ in finite affine parameter. The coordinate time elapsed for a radial null geodesic to propagate from radius r to R_b is

$$t(R_b) - t(r) = \int_r^{R_b} \frac{dr'}{(1 - M_0)(r'/R_b)^{\frac{M_0}{2(1-M_0)}}}, \quad (2.21)$$

which remains finite as $r \rightarrow 0$ for $M_0 < 2/3$. The singularity is therefore globally naked in this regime: it lies within $J^-(\mathscr{I}^+)$ and is visible to distant observers. For $M_0 \geq 2/3$, the integral (2.21) diverges as $r \rightarrow 0$, so photons emitted arbitrarily close to the singularity require arbitrarily large coordinate time to reach R_b as measured by a distant observer; however, the affine parameter along these geodesics remains finite. The spacetime is geodesically incomplete at $r = 0$ in both regimes, and no event horizon covers the singularity in either case.

2.5 Geodesic motion in the Schwarzschild space-time

The Schwarzschild geometry provides the simplest realization of strong-field gravity and serves as a useful reference for understanding orbital dynamics and light propagation near compact objects. Before specializing to the Schwarzschild case, we briefly recall the general framework of geodesic motion in a curved spacetime.

In a curved spacetime with metric $g_{\mu\nu}$, the worldline $x^\mu(\lambda)$ of a freely falling particle — massive or massless — is a geodesic, satisfying

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\nu\rho}^\mu \frac{dx^\nu}{d\lambda} \frac{dx^\rho}{d\lambda} = 0, \quad (2.22)$$

where λ is an affine parameter and $\Gamma_{\nu\rho}^\mu$ are the Christoffel symbols,

$$\Gamma_{\nu\rho}^\mu = \frac{1}{2} g^{\mu\sigma} (\partial_\nu g_{\sigma\rho} + \partial_\rho g_{\sigma\nu} - \partial_\sigma g_{\nu\rho}). \quad (2.23)$$

For a massive particle λ is its proper time τ , with normalization $g_{\mu\nu} u^\mu u^\nu = -1$ where $u^\mu = dx^\mu/d\tau$ is the four-velocity. For photons $g_{\mu\nu} k^\mu k^\nu = 0$ where $k^\mu = dx^\mu/d\lambda$ is the wave four-vector. Both cases are unified by

$$g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = -\varepsilon, \quad (2.24)$$

where $\varepsilon = 1$ for massive particles and $\varepsilon = 0$ for photons. When the metric admits Killing vectors $\xi_{(i)}^\mu$ satisfying $\nabla_{(\mu} \xi_{\nu)} = 0$, each generates a conserved quantity along geodesics,

$$\mathcal{C}_{(i)} = g_{\mu\nu} \xi_{(i)}^\mu \frac{dx^\nu}{d\lambda} = \text{const.} \quad (2.25)$$

In practice, the conserved energy and angular momentum associated with stationarity and axisymmetry reduce the geodesic equations to a first-order system, bypassing direct integration of (2.22).

Owing to spherical symmetry, geodesic motion may always be confined to the equatorial plane $\theta = \pi/2$ without loss of generality. The timelike and axial Killing vectors yield the conserved energy and angular momentum,

$$E = f(r)\dot{t}, \quad L = r^2\dot{\phi}, \quad (2.26)$$

where $f(r) = 1 - 2M/r$ and overdots denote differentiation with respect to the affine parameter. Substituting into (2.24), the radial motion takes the effective potential form

$$\dot{r}^2 + \mathcal{V}_{\text{eff}}(r) = E^2, \quad (2.27)$$

which provides a direct geometric interpretation of orbital dynamics.

2.5.1 Timelike geodesics

For massive particles ($\varepsilon = 1$) the effective potential is

$$\mathcal{V}_{\text{eff}}^{\text{timelike}}(r) = \left(1 - \frac{2M}{r}\right) \left(1 + \frac{L^2}{r^2}\right). \quad (2.28)$$

Figure 2.1 shows this potential for representative angular momenta; the depth and location of the minimum shift with L . At large radii the potential recovers the Newtonian limit. At small r the purely relativistic $-2ML^2/r^3$ term dominates, providing a strong attractive correction absent in Newtonian gravity and responsible for the destabilization of circular orbits below a minimum radius. Circular orbits correspond to extrema of $\mathcal{V}_{\text{eff}}$ and exist only for $r > 3M$. Stability further requires the extremum to be a minimum, leading to the innermost stable circular orbit (ISCO),

$$r_{\text{ISCO}} = 6M. \quad (2.29)$$

The transition from stable to unstable motion is evident in Fig. 2.1 from the disappearance of the local minimum at $r = 6M$. Inside this radius matter plunges toward the horizon rather than remaining in a bound orbit. In geometrically thin accretion disks the ISCO sets the inner edge of the disk and determines the maximum radiative efficiency (Section 3.1),

$$\eta \simeq 1 - E_{\text{ISCO}} = 1 - \frac{2\sqrt{2}}{3} \approx 5.7\%. \quad (2.30)$$

2.5.2 Null geodesics

For photons ($\varepsilon = 0$) the rest-mass term drops and the trajectory depends only on the impact parameter $b = L/E$. The effective potential governing null motion is

$$\mathcal{V}_{\text{eff}}^{\text{null}}(r) = \left(1 - \frac{2M}{r}\right) \frac{b^2}{r^2}. \quad (2.31)$$

As shown in Fig. 2.2, this potential possesses a single unstable maximum at the photon sphere $r_{\text{ph}} = 3M$. Photons with $b < b_{\text{crit}} = 3\sqrt{3}M$ are captured by the black hole after a finite azimuthal evolution; those with $b > b_{\text{crit}}$ are deflected and escape to infinity. The photon sphere acts as an unstable separatrix in phase space separating these two outcomes. Trajectories are usefully organized by the

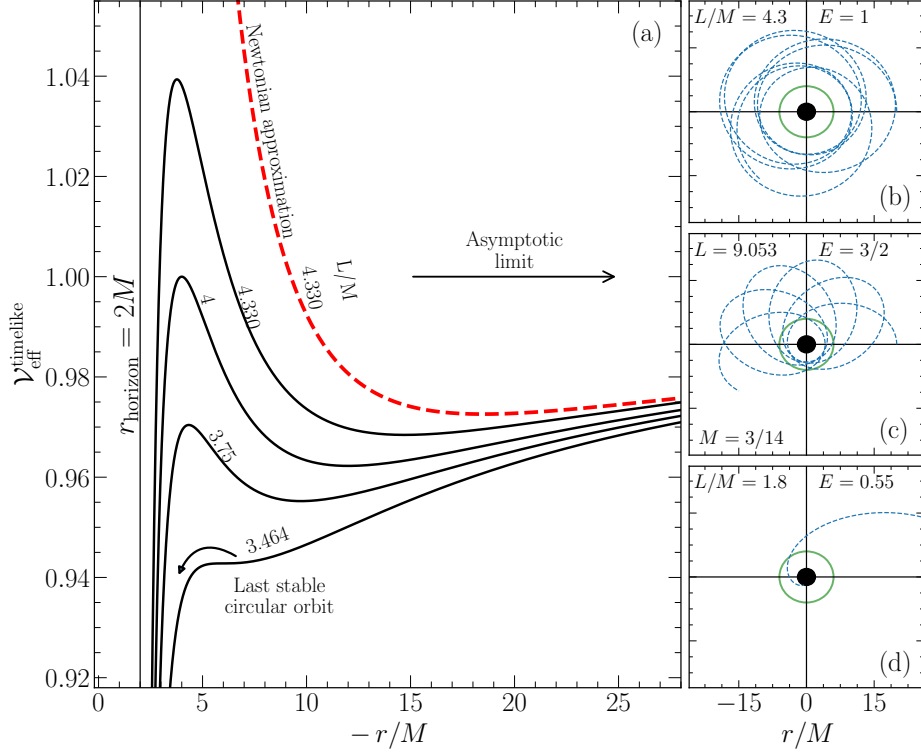


Figure 2.1: Effective potential for timelike geodesics in Schwarzschild spacetime, plotted as a function of radius for representative angular momenta. Stable circular orbits correspond to minima of $V_{\text{eff}}^{\text{timelike}}$; the innermost stable circular orbit occurs where the local minimum disappears, at $r = 6M$.

fractional number of azimuthal orbits $n = \Delta\phi/2\pi$ accumulated outside the horizon, as illustrated in the right panels of Fig. 2.2. Photons with modest deflection reaching the observer directly form the direct image ($n \lesssim 0.75$). As b approaches b_{crit} from above, the deflection grows and photons execute one or more partial revolutions, forming the lensed component ($0.75 \lesssim n \lesssim 1.25$) in a narrow band of impact parameters. The critical geodesic at $b = b_{\text{crit}}$ asymptotically approaches the unstable orbit at $r = 3M$, spending arbitrarily long affine time near the potential maximum. The azimuthal divergence is logarithmic,

$$\Delta\phi \sim -\ln |b - b_{\text{crit}}|, \quad (2.32)$$

a feature of unstable circular null orbits in spherically symmetric GR spacetimes. Exponentially small variations in b therefore produce large differences in orbital winding. Photons completing more than one full revolution ($n \gtrsim 1.25$) populate the photon ring regime: a hierarchical sequence of images accumulated exponentially close to the critical curve, each additional half-orbit corresponding to a higher image order with impact parameters approaching b_{crit} geometrically. The associated images are progressively narrower and increasingly demagnified,

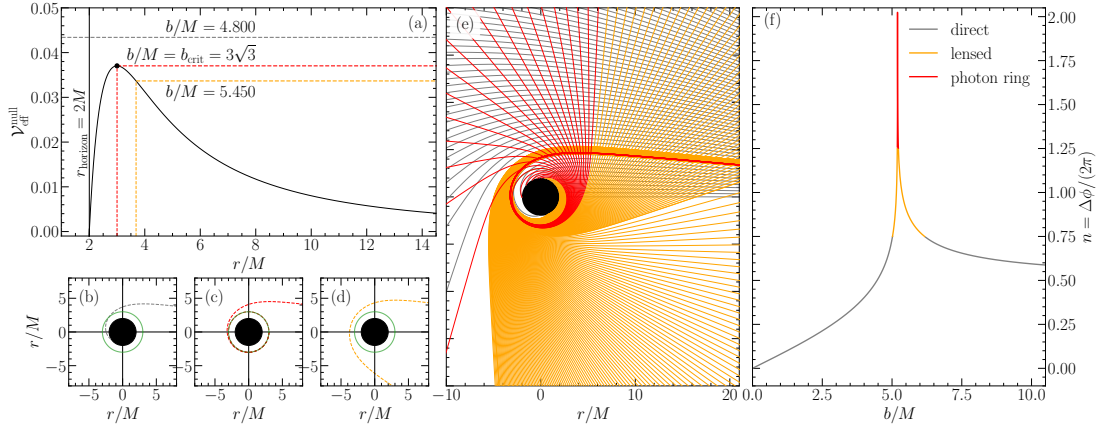


Figure 2.2: Effective potential governing null geodesics in Schwarzschild spacetime as a function of radius. The single maximum at $r = 3M$ is the unstable photon sphere that separates captured trajectories from those that escape to infinity; the corresponding critical impact parameter is $b_{\text{crit}} = 3\sqrt{3}M$. The right panels show representative null trajectories organized by the fractional winding number $n = \Delta\phi/2\pi$: direct ($n \lesssim 0.75$), lensed ($0.75 \lesssim n \lesssim 1.25$), and photon ring ($n \gtrsim 1.25$) regimes. Inspired and reproduced from [Gralla et al. 2020](#)

yet sharply localized in angle. Since these rays repeatedly sample the spacetime near r_{ph} , their observable properties are governed primarily by the local geometry near the unstable photon orbit rather than by the spatial distribution of the emitting plasma. This makes higher-order photon rings a particularly clean probe of strong-field gravity.

2.6 Geodesic Motion in the Kerr Spacetime

The geodesic structure of the Kerr spacetime differs qualitatively from that of the Schwarzschild geometry as a consequence of rotation and the associated breaking of spherical symmetry. While the Schwarzschild solution admits planar motion due to its full rotational invariance, generic trajectories in Kerr are intrinsically three-dimensional. Nevertheless, geodesic motion remains completely integrable owing to a hidden symmetry of the Kerr metric: the existence of a second-rank Killing tensor, discovered by Carter ([Carter 1968](#)), which yields a fourth conserved quantity beyond energy, axial angular momentum, and the normalization condition. In addition to the conserved energy and axial angular momentum associated with the stationarity and axisymmetry Killing vectors,

$$E = -g_{\mu\nu}(\partial_t)^\mu u^\nu, \quad L_z = g_{\mu\nu}(\partial_\phi)^\mu u^\nu, \quad (2.33)$$

the separability of the Hamilton–Jacobi equation yields the Carter constant $\mathcal{Q}$ ([Carter 1968](#)), which governs the latitudinal extent of orbits and has no Newto-

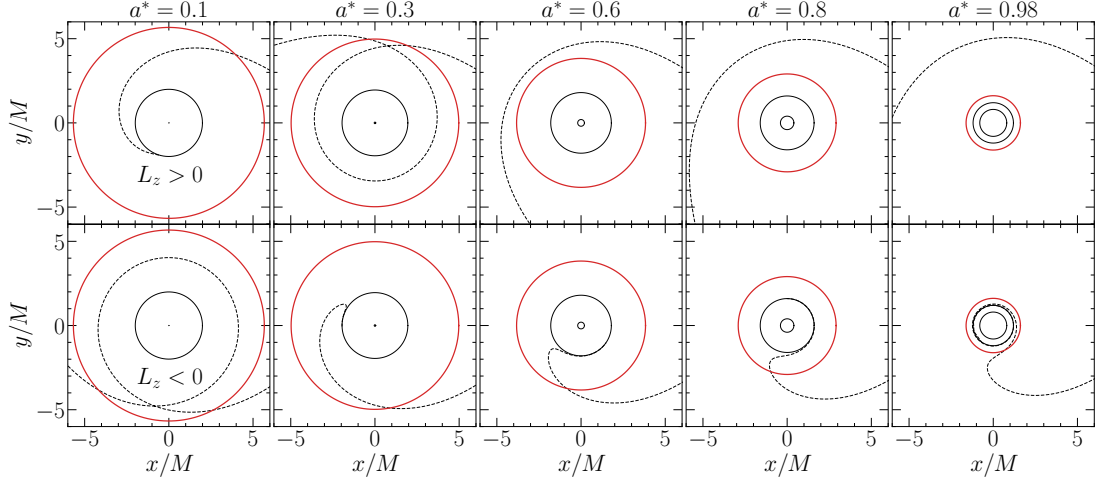


Figure 2.3: Equatorial particle trajectories in the Kerr spacetime for five values of the dimensionless spin parameter $a^* = a/M$, shown for prograde ($L_z > 0$, upper row) and retrograde ($L_z < 0$, lower row) orbits. In each panel the red circle marks the ISCO r_{ISCO} , the dashed black curve the marginally bound orbit r_{mb} , and the solid black circle the outer event horizon r_+ . With increasing a^* the prograde ISCO contracts monotonically toward r_+ , consistent with $r_{\text{ISCO}}^+ \rightarrow M$ in the extremal limit, while the retrograde ISCO expands outward, reflecting the growing energetic penalty of orbiting against the sense of frame-dragging.

nian analog. The presence of the off-diagonal component $g_{t\phi}$ couples the timelike and azimuthal evolution, breaking the prograde–retrograde degeneracy present in Schwarzschild. For orbits confined to the equatorial plane $\theta = \pi/2$ the Carter constant vanishes, $\mathcal{Q} = 0$, and the metric simplifies to

$$\Delta|_{\theta=\pi/2} = r^2 - 2Mr + a^2, \quad \Sigma|_{\theta=\pi/2} = r^2. \quad (2.34)$$

Using the normalization condition (2.24) and the conservation laws, radial motion in the equatorial plane takes the effective potential form

$$\dot{r}^2 + \mathcal{V}_{\text{eff}}^{\text{Kerr}}(r) = E^2, \quad (2.35)$$

where the effective potential now depends on both L_z and the spin parameter a .

2.6.1 Timelike geodesics

For massive particles ($\varepsilon = 1$) confined to the equatorial plane, the Kerr effective potential is

$$V_{\text{eff}}^{\text{timelike}}(r; L_z, a_*) = 1 - \frac{2M}{r} + \frac{L_z^2 - a_*^2(E^2 - 1)}{r^2} - \frac{2M(L_z - a_*)^2}{r^3}. \quad (2.36)$$

The three terms correspond respectively to the Newtonian gravitational potential, a centrifugal term modified by the spin–energy coupling, and a purely relativistic correction absent in both Newtonian gravity and Schwarzschild. The spin parameter a enters through the frame-dragging coupling between E and L_z in the last term, breaking the degeneracy between prograde ($L_z > 0$) and retrograde ($L_z < 0$) orbits. Circular orbits require

$$V_{\text{eff}}^{\text{timelike}}(r) = E^2, \quad \frac{dV_{\text{eff}}^{\text{timelike}}(r)}{dr} = 0, \quad (2.37)$$

while stability further demands $d^2V_{\text{eff}}^{\text{timelike}}/dr^2 \leq 0$. The boundary of stability defines the ISCO, whose radius is given analytically by [Bardeen et al. 1972](#),

$$r_{\text{ISCO}}^{\pm} = M \left\{ 3 + Z_2 \mp [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2} \right\}, \quad (2.38)$$

$$Z_1 = 1 + (1 - a_{\star}^2)^{1/3} [(1 + a_{\star})^{1/3} + (1 - a_{\star})^{1/3}], \quad Z_2 = (3a_{\star}^2 + Z_1^2)^{1/2}, \quad (2.39)$$

where $a_{\star} = a/M$ and the upper (lower) sign denotes prograde (retrograde) orbits. For $a^* = 0$ both roots reduce to $r_{\text{ISCO}} = 6M$, recovering the Schwarzschild result. As $a^* \rightarrow 1$, the prograde ISCO contracts to $r_{\text{ISCO}}^+ \rightarrow M$ while the retrograde ISCO expands to $r_{\text{ISCO}}^- \rightarrow 9M$. Figure 2.3 illustrates the prograde and retrograde equatorial trajectories for five representative spin values; the contraction of all prograde characteristic radii toward r_+ with increasing a^* and the corresponding outward migration of retrograde radii are direct consequences of frame-dragging. The binding energy per unit mass at the ISCO sets the maximum radiative efficiency of thin-disk accretion,

$$\eta_{\text{max}}(a^*) = 1 - E_{\text{ISCO}}(a^*) = 1 - \frac{r^{3/2} - 2r^{1/2} \pm a_{\star}}{r^{3/4} \sqrt{r^{3/2} - 3r^{1/2} \pm 2a_{\star}}}, \quad (2.40)$$

rising from $\eta \simeq 5.7\%$ at $a_{\star} = 0$ to $\eta \simeq 42\%$ in the extremal limit ([Bardeen et al. 1972](#)), a strong spin dependence that underpins observational methods for measuring black hole spin via continuum fitting and X-ray reflection spectroscopy ([Bambi 2021](#)). Although in the case Novikov–Thorne accretion disk (as we shall see in Section 3.1.1 of Chapter 3), not all radiation emitted by the disk can escape to infinity, and a part of it, after leaving the disk, is captured by the black hole, which reduces the total efficiency $\eta = 1 - E_{\text{ISCO}} - \zeta_{\text{E}}$. ζ_{E} accounts for the radiation captured by the black hole ([Thorne 1974](#)).

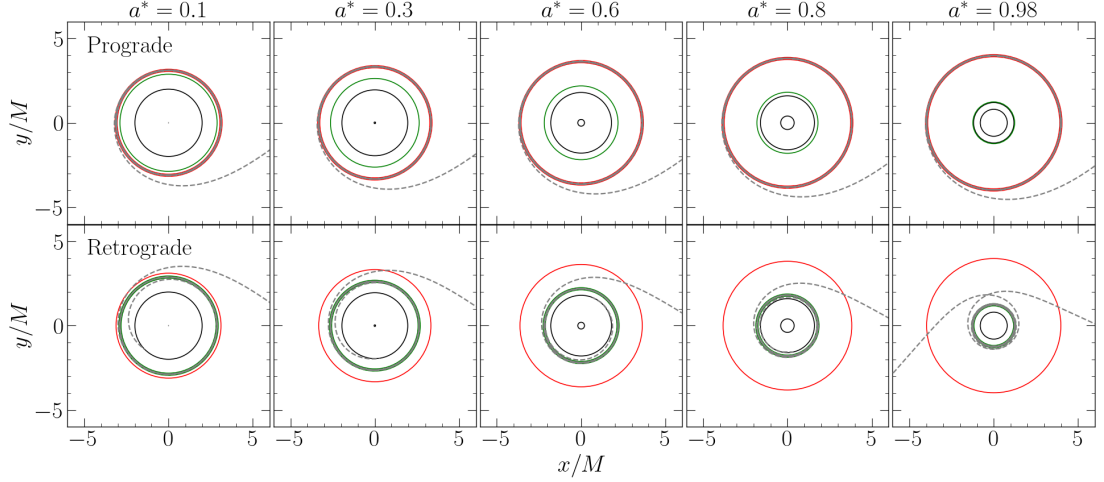


Figure 2.4: Critical null geodesics and photon orbits in the equatorial plane of Kerr black holes for several spin parameters (left to right), shown for prograde (upper row) and retrograde (lower row) configurations. In each panel thin black circles denote the outer and inner event horizons $r_{\pm}$; the red circle marks the prograde photon orbit r_{ph}^+ and the green circle the retrograde photon orbit r_{ph}^- . The dashed gray curve is a null geodesic integrated near the dominant photon orbit. With increasing a^* the prograde orbit contracts toward r_+ while the retrograde orbit expands, widening the photon shell.

2.6.2 Null geodesics

For photons ($\varepsilon = 0$), the trajectory depends only on the two dimensionless impact parameters

$$\xi \equiv \frac{L_z}{E}, \quad \eta \equiv \frac{\mathcal{Q}}{E^2}, \quad (2.41)$$

and the radial potential governing equatorial null geodesics is

$$\mathcal{R}(r) = [(r^2 + a^2) - a\xi]^2 - \Delta [(\xi - a)^2 + \eta]. \quad (2.42)$$

For equatorial orbits ($\eta = 0$) this reduces to

$$\mathcal{R}(r)|_{\eta=0} = [(r^2 + a^2) - a\xi]^2 - \Delta (\xi - a)^2, \quad (2.43)$$

and photon trajectories are accessible where $\mathcal{R}(r) \geq 0$. Unlike the Schwarzschild case, the off-diagonal component $g_{t\phi}$ distinguishes co-rotating ($\xi > 0$) from counter-rotating ($\xi < 0$) photons, so the critical impact parameter separating capture from escape differs for each sense of circulation. In the Schwarzschild geometry there is a single unstable circular photon orbit at $r = 3M$. The Kerr metric admits a richer structure: a continuous family of unstable spherical photon orbits at constant Boyer–Lindquist radius r but oscillating in θ , parameterized by (ξ, η) and occupying a finite radial range known as the *photon shell* (Teo 2003). The radial

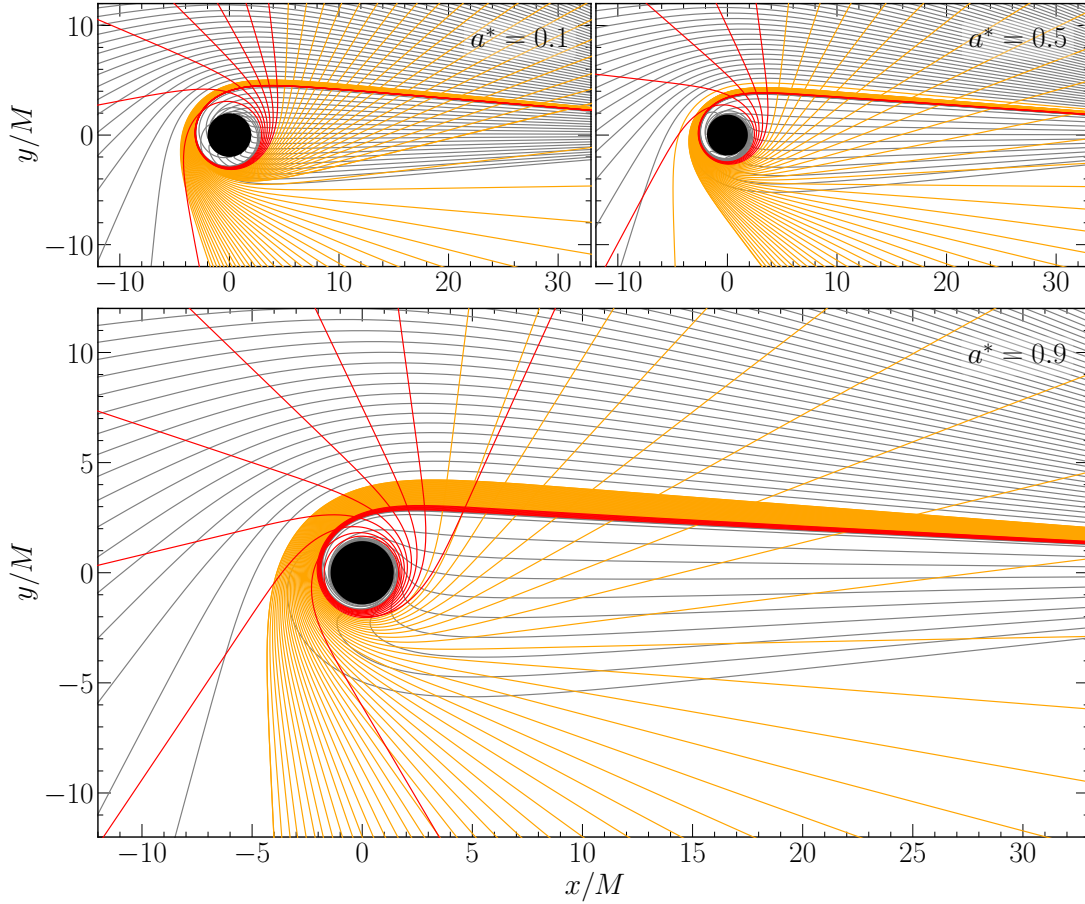


Figure 2.5: Equatorial null geodesics for Kerr black holes with three values of a^* (left to right), projected onto the Boyer–Lindquist equatorial plane. Orange: captured trajectories. Gray: escaping trajectories. Red: critical null geodesics asymptotically approaching the unstable photon orbit. The growing prograde–retrograde asymmetry of the capture cross-section with increasing spin is a direct consequence of frame-dragging.

boundaries of the photon shell are the prograde and retrograde equatorial photon orbits, found by imposing $\mathcal{R}(r) = 0$ and $\mathcal{R}'(r) = 0$ simultaneously with $\eta = 0$ (Bardeen et al. 1972),

$$r_{\text{ph}}^{\pm} = 2M \left\{ 1 + \cos \left[\frac{2}{3} \arccos(\mp a^*) \right] \right\}. \quad (2.44)$$

For $a^* = 0$ both roots reduce to $r_{\text{ph}} = 3M$; as $a^* \rightarrow 1$, the prograde orbit shrinks to $r_{\text{ph}}^+ \rightarrow M$ while the retrograde orbit grows to $r_{\text{ph}}^- \rightarrow 4M$. At each radius $r \in [r_{\text{ph}}^+, r_{\text{ph}}^-]$ there exists a spherical photon orbit with critical impact parameters

$$\xi(r) = \frac{r^3 - 3Mr^2 + a^2(r + M)}{a(r - M)}, \quad (2.45)$$

$$\eta(r) = \frac{r^3 [4a^2M - r(r - 3M)^2]}{a^2(r - M)^2}. \quad (2.46)$$

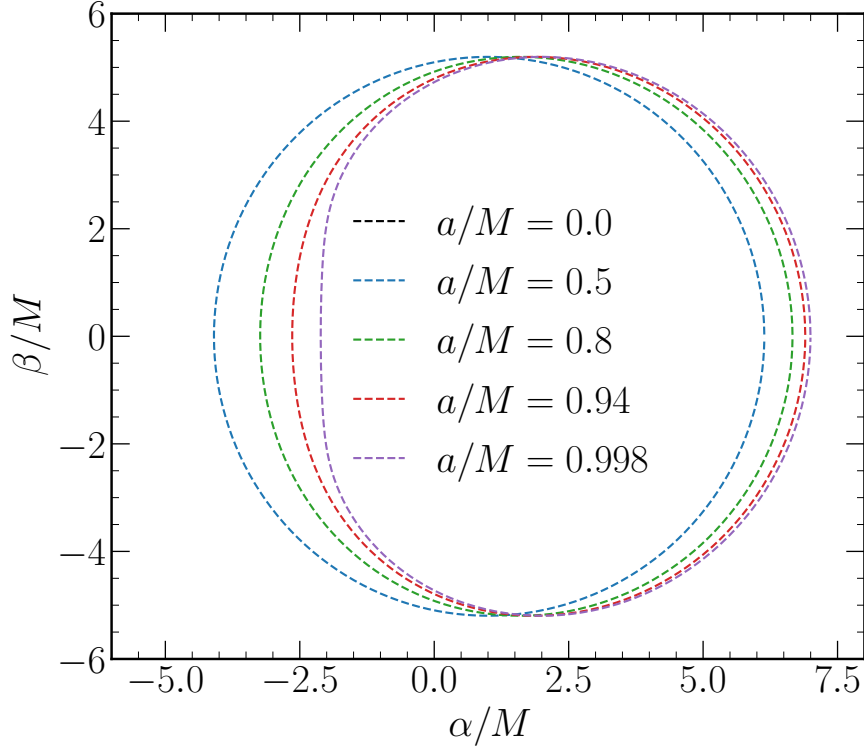


Figure 2.6: Shadow contours in the observer’s celestial plane $(\alpha/M, \beta/M)$ for a Kerr black hole viewed equatorially ($\theta_{\text{obs}} = \pi/2$), for several values of a^* . At $a^* = 0$ the shadow is a circle of radius $3\sqrt{3}M$. With increasing spin the contour shifts laterally and the prograde boundary develops a progressively *D*-shaped then pear-like indentation, while the overall angular size remains approximately constant.

All orbits within the shell are unstable: inward perturbations lead to capture, outward perturbations to escape. The azimuthal winding angle diverges logarithmically as the impact parameter approaches the critical value, producing the same hierarchical sequence of photon ring images as in Schwarzschild, now asymmetric between the prograde and retrograde sides (Bardeen 1973; Event Horizon Telescope Collaboration 2019; Event Horizon Telescope Collaboration 2022). Figure 2.4 shows the prograde and retrograde equatorial photon orbits for several spin values; the contraction of r_{ph}^+ toward the horizon and the expansion of r_{ph}^- with increasing a^* directly reflect the widening of the photon shell. The full equatorial null geodesic structure for three values of a^* is shown in Figure 2.5; the growing prograde–retrograde asymmetry of the capture cross-section with spin is a direct consequence of frame-dragging.

2.6.3 The Black Hole Shadow Boundary

A distant observer viewing a Kerr black hole sees a dark region — the *shadow* — bounded by the set of impact parameters that leads to photon capture. The

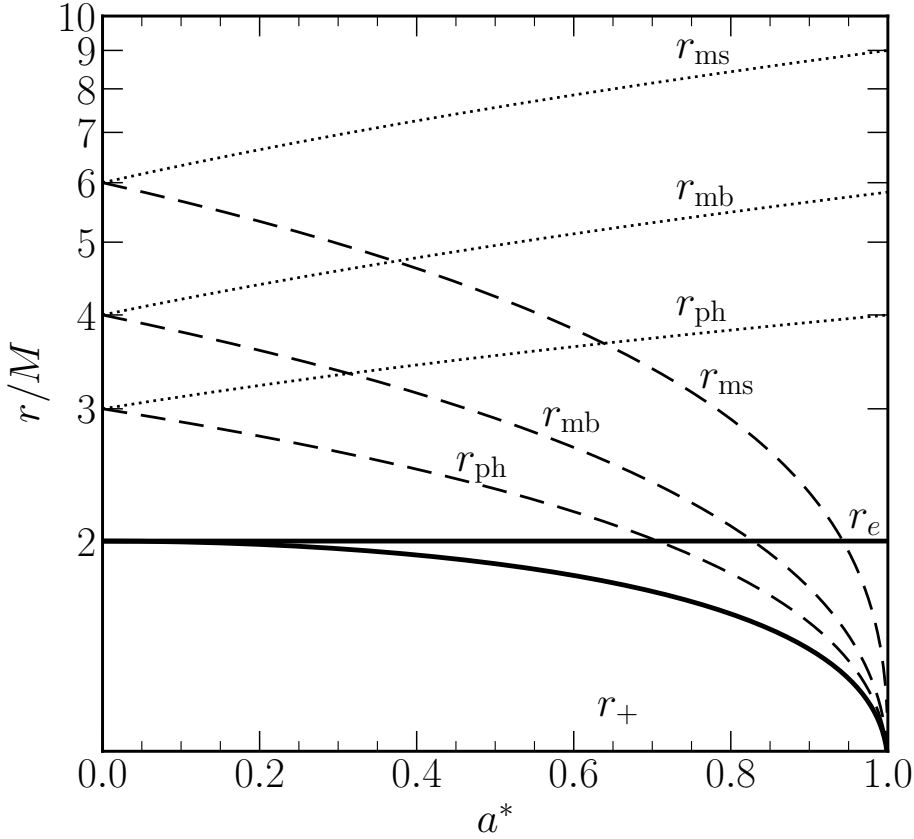


Figure 2.7: Characteristic radii of the Kerr geometry as functions of the dimensionless spin parameter $a^* = a/M$: prograde ISCO r_{ISCO}^+ (solid blue), retrograde ISCO r_{ISCO}^- (solid red), prograde photon orbit r_{ph}^+ (dashed blue), retrograde photon orbit r_{ph}^- (dashed red), marginally bound orbit r_{mb} (dot-dashed), and the event horizon r_+ (shaded gray), all in units of M . As $a^* \rightarrow 1$ the prograde ISCO, prograde photon orbit, and event horizon converge to $r = M$. The hierarchy $r_{\text{ph}} < r_{\text{mb}} < r_{\text{ISCO}}$ is preserved for both prograde and retrograde orbits at all spins.

shadow edge is traced by critical trajectories that, integrated backward from the observer, asymptotically approach unstable spherical photon orbits of the shell; it is therefore a direct projection of the photon shell onto the observer's sky (Bardeen 1973; Cunha and Herdeiro 2018). For an asymptotically distant observer at inclination θ_{obs} to the spin axis, the shadow boundary is expressed in celestial coordinates (α, β) as

$$\alpha = -\frac{\xi}{\sin \theta_{\text{obs}}}, \quad \beta = \pm \sqrt{\eta + a^2 \cos^2 \theta_{\text{obs}} - \xi^2 \cot^2 \theta_{\text{obs}}}. \quad (2.47)$$

Substituting the critical values $\xi(r)$ and $\eta(r)$ from equations (2.45)–(2.46) and sweeping $r \in [r_{\text{ph}}^+, r_{\text{ph}}^-]$ traces the closed shadow contour. Figure 2.6 shows these contours for several values of a^* at equatorial inclination. For $a^* = 0$ the shadow is a perfect circle of apparent radius $3\sqrt{3}M$. With increasing spin the prograde side

($\alpha > 0$) contracts inward as r_{ph}^+ approaches the horizon, while the retrograde side ($\alpha < 0$) extends outward following r_{ph}^- , producing the characteristic D -shaped and, near extremality, pear-like contour. The vertical extent of the shadow remains approximately constant, being controlled predominantly by r_{ph}^- , while the overall apparent size changes only modestly with a^* at fixed M/d_{obs} — consistent with the EHT observations of M87* and Sgr A* (Event Horizon Telescope Collaboration 2019; Event Horizon Telescope Collaboration 2022). The shape asymmetry carries the spin information.

The shadow encodes the mass-to-distance ratio M/d_{obs} in its overall angular size independently of the astrophysical emission model, while departures from circularity constrain both a^* and the inclination (Hioki and Maeda 2009). Deviations of the observed morphology from Kerr predictions would signal either a non-Kerr geometry or the presence of matter near the photon shell, making shadow measurements a direct test of the no-hair theorem in the strong-field regime (Cunha and Herdeiro 2018; Event Horizon Telescope Collaboration 2019; Event Horizon Telescope Collaboration 2022).

2.7 Geodesic Motion in the JMN-1 Spacetime

The JMN-1 spacetime, introduced in Section 2.4.1, provides an analytically tractable setting in which to examine how lensing and orbital dynamics depart from the Schwarzschild paradigm. It consists of an anisotropic fluid interior characterized by the dimensionless mass parameter $M_0 \in (0, 1)$, joined at a boundary radius R_b to an exterior Schwarzschild geometry with ADM mass M , with the junction fixing $R_b = 2M/(M_0)$ (Joshi et al. 2011; Joshi et al. 2014). The optical and dynamical character of the spacetime is controlled by how R_b compares to the Schwarzschild photon sphere radius $r_{\text{ph}} = 3M$. When $R_b < 3M$ (equivalently $M_0 > 2/3$), the photon sphere lies entirely in the exterior Schwarzschild region and the object produces a shadow observationally degenerate with a Schwarzschild black hole. When $R_b > 3M$ (equivalently $M_0 < 2/3$), the photon sphere disappears and the central singularity is causally exposed to null infinity, producing a qualitatively different image morphology. These two regimes have sharply different observational signatures, which we now discuss.

2.7.1 Timelike geodesics and orbital precession

Timelike geodesics in JMN-1 provide an independent observational channel through astrometric monitoring of stellar orbits near Sgr A*. The geodesic equations have been integrated through the matched metric by Bambhaniya et al. 2020, and

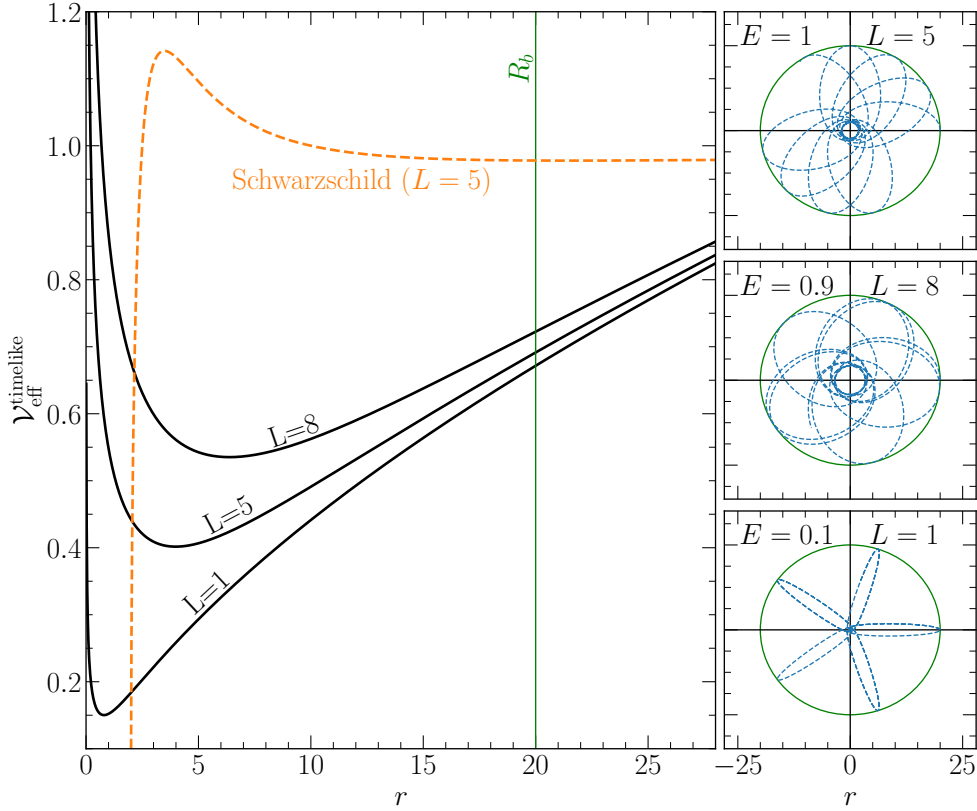


Figure 2.8: Left panel: effective potential $\mathcal{V}_{\text{eff}}$ for timelike geodesics in JMN-1 spacetime as a function of radius for representative angular momenta. The orange dashed curve shows the Schwarzschild reference case with $L = 5M$; black curves show the JMN-1 results. The vertical green line marks the matching radius R_b , separating the interior JMN-1 metric from the exterior Schwarzschild geometry. The interior matter distribution modifies the effective potential relative to Schwarzschild, producing qualitatively different orbital dynamics. Right panels: representative particle trajectories in the equatorial plane for different energy and angular momentum values. Solid green circles mark R_b ; dashed blue curves show the orbits.

representative trajectories are shown in Fig. 2.8. The orbital shape and precession depend sensitively on how deeply particles probe the interior matter distribution, since the anisotropic stress modifies the effective potential relative to Schwarzschild in the interior region $r < R_b$. The central result is that the perihelion precession of bound timelike orbits in JMN-1 can be *retrograde* — the orbit ellipse precesses opposite to the direction of the particle’s angular motion — for $M_0 \lesssim 1/3$. This behavior is absent from every vacuum GR solution, including Schwarzschild and Kerr, and arises because the interior anisotropic matter produces an effective radial force weaker than the Keplerian $1/r^2$ fall-off at intermediate radii (Bambhaniya et al. 2021). For $1/3 \lesssim M_0 < 2/3$ the precession remains prograde but differs in magnitude from the Schwarzschild prediction. The GRAVITY Collaboration measurement of the prograde Schwarzschild precession of the

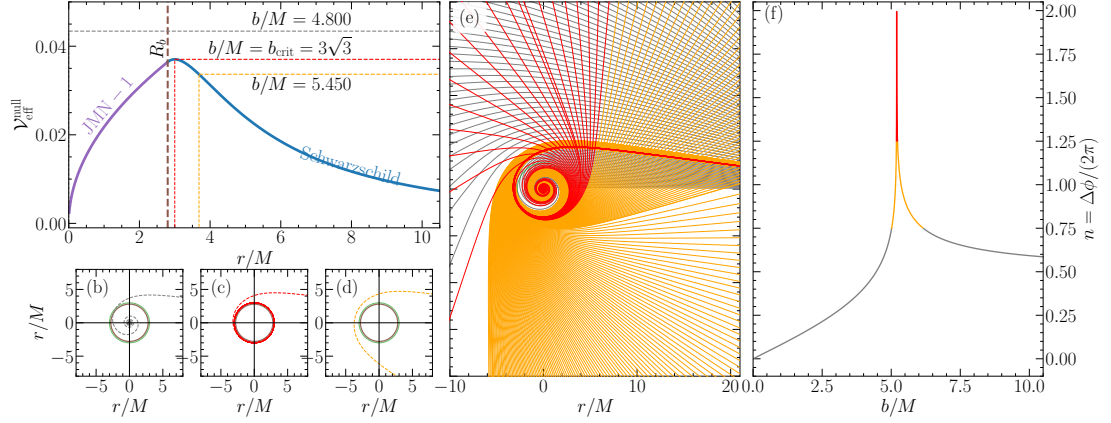


Figure 2.9: Null geodesic structure of JMN-1 spacetime with $R_b = 2.8 r_g$ and $M_0 > 2/3$. Panel (a): the null effective potential $\mathcal{V}_{\text{eff}}^{\text{null}}$ shares the same maximum at $r_{\text{ph}} = 3M$ with $b_{\text{crit}} = 3\sqrt{3}M$ as the Schwarzschild black hole. Panels (b)–(e): representative null geodesics showing captured, asymptoting, and escaping trajectories. Panel (f): the winding number diverges logarithmically at b_{crit} , identical to the Schwarzschild case. All photons forming the bright ring sample only the exterior Schwarzschild geometry.

S2 star at $f_{\text{SP}} = 1.10 \pm 0.19$ in units of the GR prediction (GRAVITY Collaboration et al. 2020) rules out the retrograde regime $M_0 \lesssim 1/3$ at the S2 semi-major axis $a \approx 1440$ AU. The window $1/3 \lesssim M_0 < 2/3$ is constrained in magnitude but not fully excluded by current data. The range $M_0 \geq 2/3$ is lensing-degenerate with Schwarzschild and cannot be ruled out by precession or shadow observations alone, directly motivating the accretion-based studies of Chapter 3. Near-future improvements from GRAVITY+ and complementary tracking of short-period stars will tighten these constraints to the percent level; a detection of retrograde precession in any S-star would constitute a model-independent signature of non-vacuum geometry at the Galactic center with no counterpart in vacuum GR.

2.7.2 Gravitational lensing and shadow boundary

The lensing properties of JMN-1 were first examined in the strong-field limit by Sahu et al. 2012 and extended to shadow formation by Shaikh et al. 2019. When $M_0 \geq 2/3$ (equivalently $R_b \leq 3M$), the photon sphere sits at $r_{\text{ph}} = 3M$ in the exterior Schwarzschild geometry. As shown in Fig. 2.9, the null effective potential admits a maximum at $b_{\text{crit}} = 3\sqrt{3}M$, identical to the Schwarzschild case. Every null geodesic forming the bright photon ring has its turning point in the exterior region and never enters the JMN-1 interior; the deflection angle is computed entirely in Schwarzschild, yielding the familiar strong-field logarithmic divergence (Bozza 2002) with identical coefficients. The shadow boundary subtends an an-

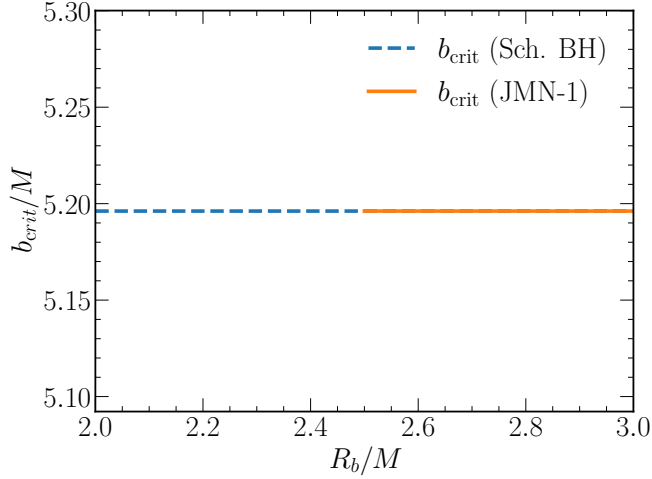


Figure 2.10: Critical impact parameter b_{crit} (shadow radius) as a function of matching radius R_b for the JMN-1 spacetime in the range $R_b \in [2.5M, 3M]$. The blue dashed line shows the Schwarzschild reference value $b_{\text{crit}} = 3\sqrt{3}M$ and the orange curve shows the JMN-1 result. Photon sphere exists only within $2.5 \leq R_b \leq 3M$, at $\frac{3}{2}M_0 R_b$, producing the shadow boundary coinciding with the Schwarzschild value.

gle $\theta_{\text{sh}} = 3\sqrt{3}M/D$ at distance D , indistinguishable from a Schwarzschild black hole, as shown in Fig. 2.10. This lensing degeneracy confines JMN-1 to the observationally viable window $M_0 \in [2/3, 1)$ but cannot exclude it through shadow observations alone.

In contrast, when $M_0 < 2/3$ (equivalently $R_b > 3M$), no photon sphere exists in the exterior and null geodesics penetrate the interior. The deflection angle remains finite but orbiting rays can wind multiple times around the singularity, generating relativistic Einstein rings with some lying inside the Schwarzschild critical angle $b_c = 3\sqrt{3}M$ (Sahu et al. 2012). The absence of a photon sphere results in a full-moon image — a uniformly illuminated disk without a dark central depression — rather than the bright ring observed around M87* and Sgr A* (Shaikh et al. 2019). This morphology is immediately distinguishable at EHT resolution and definitively excludes the $M_0 < 2/3$ regime. Shadow observations therefore constrain JMN-1 to the degenerate window $2/3 \lesssim M_0 < 1$, but cannot discriminate within it through lensing alone. Breaking this degeneracy requires probing the accretion flow and its emission properties — the subject of the following chapters.

2.8 Open Questions

The spacetime properties derived in the preceding sections—the photon sphere, the ISCO, the shadow boundary, the ergosphere—are purely geometric quantities

determined by the mass and spin of the central object alone. Once astrophysical emission is introduced, however, these clean geometric signatures become entangled with the plasma distribution, the magnetic field configuration, and the velocity structure of the accreting gas. This entanglement is the source of the principal degeneracies that arise when interpreting horizon-scale images, as developed in Chapter 3, and it is precisely what makes the following questions non-trivial.

Q. Is the central compact object in AGNs a black hole? The no-hair theorem guarantees that any stationary, axisymmetric, vacuum solution to the Einstein equations is fully characterized by mass and spin alone¹ (Israel 1967; Carter 1971; Newman et al. 1965). The Kerr metric describes the spacetime outside a stationary, uncharged black hole with extraordinary precision, and general relativity has passed every experimental test in the weak-field and slow-motion regime of the solar system (Will 2014), in the strong-field regime of binary pulsars (Hulse and Taylor 1975; Kramer et al. 2021), and through the detection of gravitational waves from compact binary mergers (LIGO Scientific Collaboration et al. 2016). However, exotic compact objects including boson stars, gravastars, wormholes, and naked singularities can produce ring-like images at current EHT resolution that are morphologically indistinguishable from a Kerr black hole surrounded by an accretion flow (Vagnozzi et al. 2023; Kocherlakota et al. 2023). As seen in the previous sections, especially Section 2.4.1, the timelike and null geodesic structure is very similar to that of the Schwarzschild case. Whether the observed ring size, brightness asymmetry, and polarimetric structure are consistent only with a Kerr black hole, or whether other compact objects produce equally viable images when surrounded by a realistic RIAF, as example of it is tested in Saurabh et al. 2024. The question connects directly to the accretion flow structure discussed in Chapter 3, since the RIAF emission geometry is the same in all cases and only the underlying spacetime differs. Additionally, given the accessibility and availability of tools to perform state-of-the-art simulations, it is feasible to understand effects, which may otherwise be not possible through simple analytic solutions. This further motivates to combine various sources (from Chapter 3) to take the problem further and perform complex tasks ($\rightarrow$ realistic scenario) in Chapter 7.

Q. What is the spin of the compact object in M87*? The Kerr spin parameter a_* determines the ISCO radius, the photon shell extent, the shadow asymmetry, and the energy available for extraction via the Blandford-Znajek

¹In the Einstein-Maxwell theory, the Kerr-Newman family adds a third parameter, the electric charge Q . See the discussion in Section 2.3.

mechanism (Blandford and Znajek 1977). Current EHT total intensity and polarimetric data are consistent with a wide range of spins from mildly retrograde to rapidly prograde. Measuring $a_\star$ more precisely requires either resolving the photon ring shape, whose asymmetry is a direct function of $a_\star$ and the observer inclination (Bardeen 1973), or jointly measuring the photon ring and inner shadow whose relative size and centroid offset break the mass-spin degeneracy (Chael et al. 2021). The polarimetric degeneracy between spin, magnetic field geometry, and disk kinematics is examined in Saurabh et al. 2026, as discussed further in the open questions of Chapter 3.

Chapter 3

Theory of Accretion Flows

 ACCRETION, the gravitational capture and infall of matter onto a compact object, is the dominant energy-release mechanism in active galactic nuclei (AGN), X-ray binaries, and gamma-ray bursts (Lynden-Bell 1969; Frank et al. 2002). The gravitational potential energy released per unit infalling mass near a black hole of mass M is of order $\Delta\Phi \sim GM/r$. Evaluated at the innermost stable circular orbit, this yields radiative efficiencies $\eta \sim 6$ to 42% depending on spin (Bardeen et al. 1972; Novikov and Thorne 1973), far exceeding the $\sim 0.7\%$ efficiency of hydrogen fusion. The structure, geometry, and thermodynamics of the accretion flow therefore control the broadband spectral energy distribution (SED), the jet power, and the horizon-scale morphology accessible to millimeter-wavelength VLBI.

Accretion flows can be broadly classified by two properties. The first is geometry, quantified by the disk scale height H/r . When $H/r \ll 1$, pressure gradients are dynamically negligible, the orbits are nearly Keplerian, and the disk is a cold, flat structure confined to the equatorial plane (Shakura and Sunyaev 1973; Novikov and Thorne 1973). When $H/r \sim 1$, thermal or magnetic pressure contributes comparably to the gravitational restoring force. The second property is the vertically integrated optical depth τ . At $\tau \gg 1$, radiation thermalizes before escaping and the local emission is blackbody, with effective temperature $T_{\text{eff}} \propto F^{1/4}$, where F is the radiative flux. At $\tau \ll 1$, the plasma is transparent to its own radiation and the emergent spectrum is set by local microphysics — synchrotron, bremsstrahlung, and inverse Compton scattering — rather than thermodynamic equilibrium.

The dominant regime is set by the dimensionless accretion rate $\dot{m} \equiv \dot{M}c^2/L_{\text{Edd}}$. A natural reference scale is the Eddington luminosity L_{Edd} , at which outward radiation pressure on electrons balances the inward gravitational force on protons

for a spherically symmetric geometry (Frank et al. 2002)

$$L_{\text{Edd}} = \frac{4\pi GMm_p c}{\sigma_T} = 1.25 \times 10^{38} \frac{M}{M_\odot} \text{ erg s}^{-1}, \quad (3.1)$$

where m_p is the proton mass and $\sigma_T = 6.65 \times 10^{-25} \text{ cm}^2$ is the Thomson cross-section. The corresponding Eddington accretion rate is

$$\dot{M}_{\text{Edd}} = \frac{L_{\text{Edd}}}{\eta c^2} = \frac{4\pi GMm_p}{\eta \sigma_T c}, \quad (3.2)$$

where η is the assumed radiative efficiency. At near- or super-Eddington rates, radiation pressure supports a geometrically thick, optically thick flow (Abramowicz et al. 1988). At intermediate sub-Eddington rates, efficient radiative cooling produces a geometrically thin, optically thick disk (Shakura and Sunyaev 1973). At $\dot{m} \lesssim \text{a few} \times 10^{-2}$, Coulomb coupling between ions and electrons is too slow for thermal equilibration on the inflow timescale (Shapiro et al. 1976; Narayan and Yi 1994). The ions retain temperatures near the virial value, the flow becomes geometrically thick and optically thin, and most of the viscous heat is advected across the horizon rather than radiated (Narayan and Yi 1994; Narayan and Yi 1995; Yuan and Narayan 2014).

modeling the emission from accreting compact objects requires different theoretical frameworks depending on the physical regime the flow occupies. In the thin-disk limit, the problem is well posed analytically: the flow is cold, nearly Keplerian, and radiatively efficient, and conservation laws in the Kerr metric determine the emitted flux as a function of radius without ambiguity (Novikov and Thorne 1973; Page and Thorne 1974). At low accretion rates the flow is hot, geometrically thick, and radiatively inefficient; a self-consistent analytic solution remains tractable through self-similar methods (Narayan and Yi 1994; Narayan and Yi 1995), but the electron temperature must be prescribed independently to compute the observable emission (Yuan and Narayan 2014). At the highest level of physical fidelity, GRMHD simulations evolve the density, velocity, and magnetic field self-consistently in the background spacetime (Gammie et al. 2003; Porth 2019), with the MRI-driven turbulence (Balbus and Hawley 1991), large-scale field geometry, and jet formation emerging from the dynamics rather than being imposed.

3.1 Thin disk Accretion

When the accreting gas can cool efficiently on a timescale shorter than the infall timescale, it settles into a geometrically thin, optically thick disk confined to

the equatorial plane. Vertical pressure support is negligible, the flow is nearly Keplerian, and each annulus radiates locally as a blackbody at an effective temperature set by the viscously dissipated flux. This is the thin-disk regime, and its relativistic formulation in the Kerr spacetime is the foundation for understanding radiatively efficient accretion around compact objects.

3.1.1 The Novikov–Thorne Model

The relativistic thin-disk equations were derived by [Novikov and Thorne 1973](#) and cast in a fully covariant form by [Page and Thorne 1974](#). The Novikov–Thorne (NT) model assumes stationarity, axisymmetry, and equatorial confinement with $H/r \ll 1$, and replaces Newtonian gravity with geodesic circular orbits in the Kerr metric, thereby incorporating gravitational redshift, frame-dragging, and the innermost stable circular orbit (ISCO). It marks the innermost radius at which stable geodesic circular motion is possible, and sets the inner truncation radius of the disk; matter interior to it plunges toward the horizon without contributing further to the radiative output.

The disk structure follows from integrating the conservation equations for rest-mass, angular momentum, and energy vertically over the disk thickness. For a steady-state flow with a spatially constant accretion rate $\dot{M}$, these take the form ([Page and Thorne 1974](#))

$$\dot{M} = -2\pi r \Sigma u^r = \text{const}, \quad (3.3)$$

$$\dot{M} L = 2\pi r W_\phi^r + \dot{M} L_{\text{ISCO}}, \quad (3.4)$$

$$\dot{M} E = 2\pi r u_t W_\phi^r + F(r) \cdot 2\pi r, \quad (3.5)$$

where Σ is the surface density ($\Sigma(r) = \int_{-H}^H \langle \rho \rangle dz$), u^μ is the four-velocity of material on circular geodesics, $L = u_\phi$ and $E = -u_t$ are the conserved specific angular momentum and energy per unit rest mass, W_ϕ^r is the vertically integrated viscous stress, and $F(r)$ is the radiative flux emitted from one face of the disk. Eliminating W_ϕ^r between equations (3.4) and (3.5) yields

$$F(r) = \frac{\dot{M}}{4\pi M^2} \frac{-\partial_r \Omega}{(E - \Omega L)^2} \int_{r_{\text{ISCO}}}^r (E - \Omega L) \partial_{r'} L dr', \quad (3.6)$$

where $\Omega = u^\phi/u^t$ is the Keplerian angular velocity of material on circular geodesic

orbits in the Kerr metric¹,

$$\Omega = \frac{M^{1/2}}{r^{3/2} + aM^{1/2}}, \quad (3.7)$$

and the specific orbital energy and angular momentum are (Bardeen et al. 1972)

$$E(r) = \frac{r^2 - 2Mr \pm aM^{1/2}r^{1/2}}{r(r^2 - 3Mr \pm 2aM^{1/2}r^{1/2})^{1/2}}, \quad (3.8)$$

$$L(r) = \pm \frac{M^{1/2}(r^2 \mp 2aM^{1/2}r^{1/2} + a^2)}{r(r^2 - 3Mr \pm 2aM^{1/2}r^{1/2})^{1/2}}, \quad (3.9)$$

with the upper and lower signs corresponding to prograde and retrograde orbits respectively. The integral in equation (3.6) vanishes at $r = r_{\text{ISCO}}$ by construction and increases monotonically outward, so $F(r)$ rises from zero at the inner edge, peaks at $r \approx (1.3\text{--}1.5)r_{\text{ISCO}}$ depending on spin, and falls as r^{-3} at large radii. This radial profile and the corresponding effective temperature,

$$T_{\text{eff}}(r) = \left(\frac{F(r)}{\sigma_{\text{SB}}} \right)^{1/4}, \quad (3.10)$$

are shown in Figure 3.1 for four values of the spin parameter. The disk radiates as a superposition of blackbodies at different radii, producing the characteristic multi-temperature disk spectrum whose peak shifts toward higher frequencies and smaller radii as the spin increases.

Integrating over both faces of the disk gives the total luminosity,

$$L_{\text{disk}} = 2 \int_{r_{\text{ISCO}}}^{\infty} F(r) 2\pi r \, dr = \eta_{\text{NT}} \dot{M} c^2, \quad \eta_{\text{NT}} = 1 - E_{\text{ISCO}}, \quad (3.11)$$

where E_{ISCO} is the specific orbital energy at the ISCO from equation (3.8). For a Schwarzschild black hole $\eta_{\text{NT}} \approx 5.7\%$, rising to $\eta_{\text{NT}} \approx 42\%$ for a maximally spinning prograde Kerr hole (Bardeen et al. 1972). This steep dependence on spin, which enters entirely through the shift of r_{ISCO} with a_* , is the physical basis of black hole spin measurement via continuum-fitting spectroscopy (McClintock et al. 2006; McClintock et al. 2014): the observed soft X-ray spectrum is compared against the multi-temperature emission profile to constrain r_{ISCO} and thereby a_* .

¹It is important to usage of the spin parameter a and the dimensionless spin parameter $a_* = a/M$ normalized by mass M . Refer to Chapter 2 for a discussion.

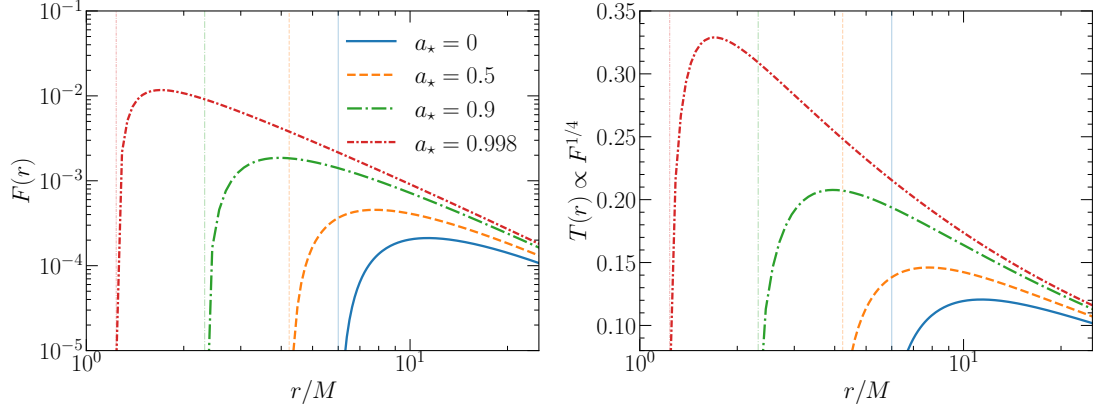


Figure 3.1: Novikov–Thorne radiative flux $F(r)$ (left) and effective temperature $T(r) \propto F^{1/4}$ (right) for a geometrically thin, optically thick accretion disk around a Schwarzschild black hole ($a = 0$) and three prograde spin values, in normalized units. Vertical lines mark the ISCO radius for each spin. The flux vanishes at r_{ISCO} by the zero-torque inner boundary condition and peaks at $r \approx (1.3\text{--}1.5) r_{\text{ISCO}}$. As spin increases, the ISCO moves inward and both the peak flux and temperature rise, reflecting the deeper gravitational potential available to prograde circular orbits.

3.2 Radiatively Inefficient Accretion Flows

At accretion rates well below the Eddington limit, $\dot{m} \lesssim \text{a few} \times 10^{-2}$, the Coulomb coupling timescale between ions and electrons exceeds the infall timescale, and the two species decouple thermally. Viscous dissipation and compressive work heat the ions toward the virial temperature,

$$T_i \simeq \frac{GMm_p}{3k_B r} \sim \frac{10^{12}}{r} \text{ K}, \quad (3.12)$$

while electrons remain substantially cooler, $T_e \sim 10^9\text{--}10^{11}$ K, losing energy primarily through synchrotron emission and inverse Compton scattering (Narayan and Yi 1994; Yuan and Narayan 2014). The ion thermal energy is not transferred to the electrons on a dynamically relevant timescale. Instead, it is advected inward with the flow. Such systems are termed advection-dominated accretion flows (ADAFs) (Narayan and Yi 1994; Narayan and Yi 1995). The broader designation, radiatively inefficient accretion flows (RIAFs), encompasses all solutions of this class in which the low radiative output arises not only from advection but also from energy losses through outflows or convection (Yuan and Narayan 2014).

The two-temperature structure arises from plasma microphysics, not from any imposed assumption. Ions remain non-relativistic throughout the flow (with adiabatic index $\hat{\gamma}_i = 5/3$) and are heated by compression as $T_i \propto \rho^{\hat{\gamma}_i - 1} = \rho^{2/3}$, while electrons become relativistic ($k_B T_e > m_e c^2$) in the inner region ($\hat{\gamma}_e \rightarrow 4/3$) and

heat by compression only as $T_e \propto \rho^{1/3}$. This disparity in compressional heating rates, compounded by the low densities that suppress Coulomb energy transfer from ions to electrons, establishes $T_i/T_e > 1$ throughout $r \lesssim 10^3 R_S$. With most of the ion thermal energy advected into the black hole rather than radiated, the bolometric luminosity satisfies $L \sim 0.1 \delta \dot{m} L_{\text{Edd}}$ in the deep ADAF regime, where δ is the fraction of viscous dissipation transferred directly to electrons (Yuan and Narayan 2014). The threshold accretion rate below which the RIAF solution is sustained is $\dot{m} \lesssim \dot{m}_{\text{crit}} \approx (0.1\text{--}0.3) \alpha^2$ (Narayan and Yi 1995; Yuan and Narayan 2014), where α is the Shakura-Sunyaev viscosity parameter and the numerical range reflects current estimates of the electron heating fraction $\delta \sim 0.1\text{--}0.5$. This suppression of radiative efficiency accounts for the observed luminosities of $10^{-9}\text{--}10^{-4} L_{\text{Edd}}$ in low-luminosity AGN and quiescent galactic nuclei (Yuan and Narayan 2014), including M87* and Sgr A*, whose central black hole masses span $10^6\text{--}10^{10} M_\odot$ (Event Horizon Telescope Collaboration 2019; GRAVITY Collaboration et al. 2019).

The primary geometrical ambiguity at horizon scales is whether the innermost RIAF is quasi-spherical or disk-like. These configurations are the limiting cases of the same solution. Depending on the angular momentum content of the accreting plasma and the accumulated poloidal magnetic flux, the flow settles into an extended thick disk or a more spherical inflow, and the two geometries produce qualitatively distinct image morphologies (Pu and Broderick 2018; Vincent et al. 2022). A disk-like geometry concentrates emission near the equatorial plane and produces a compact crescent on the approaching side; the combination of direct emission and lensed emission from material on the far side of the black hole generates a more irregular, less uniform brightness distribution (Pu and Broderick 2018). A quasi-spherical inflow populates the funnel region with emitting plasma, producing a larger crescent structure that can encompass the shadow with a comparatively smoother brightness profile. discriminating between these configurations is a primary observational goal of Event Horizon Telescope (EHT) imaging, and it requires a theoretical framework that can span the range of possible accretion geometries with a small number of physically interpretable parameters.

3.2.1 Semi-Analytic RIAF Model

The self-similar ADAF solution (Narayan and Yi 1995) provides the scaling relations for the bulk flow, but the electron temperature and magnetic field, the quantities that govern the millimeter-wavelength synchrotron emission, are not determined by the single-fluid equations alone. A standard approach, introduced

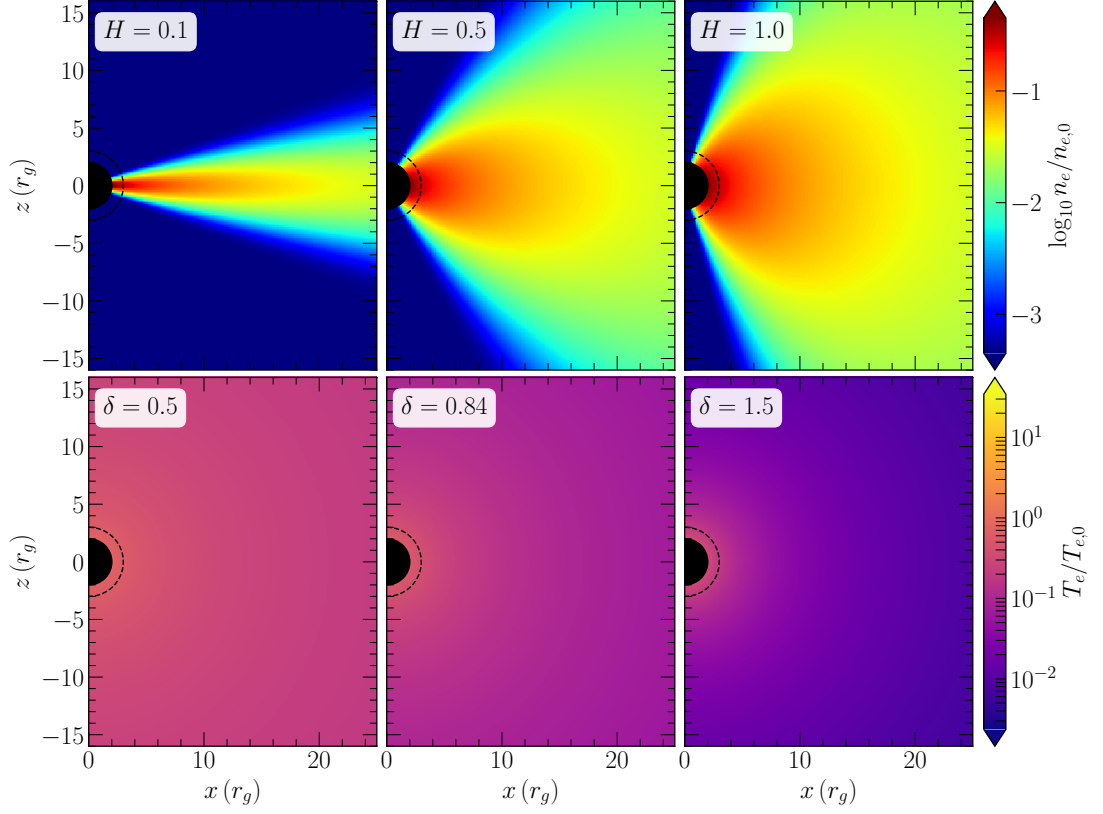


Figure 3.2: Representative two-dimensional poloidal maps of the electron number density n_e (top row) and electron temperature T_e (bottom row) for the semi-analytic RIAF model ($a_* = 0$), shown for three values of the scale height parameter H and three values of the radial temperature index δ . The filled circle marks the horizon at $r = 2M$ and the dashed circle the photon sphere at $r_{\text{ph}} = 3r_g$. Increasing H transitions the geometry from a disk-like to a quasi-spherical configuration, while δ controls how steeply the electron temperature rises toward the center.

for horizon-scale imaging of Sgr A* by [Broderick et al. 2009](#) and systematically developed by [Pu and Broderick 2018](#), is to prescribe the electron density, temperature, and field strength as power-law functions of radius with a Gaussian vertical stratification. This semi-analytic prescription has free parameters which map directly onto the physical properties of the flow, making it tractable for systematic parameter surveys. The electron number density and temperature can be written as

$$n_e = n_{e,\text{th}}^0 r^{-\alpha} e^{-\beta}, \quad (3.13)$$

$$T_e = T_e^0 r^{-\delta}, \quad (3.14)$$

where the Gaussian vertical stratification is present in

$$\beta \equiv \frac{z^2}{2\sigma^2}, \quad z \equiv r \cos \theta, \quad (3.15)$$

and the disk scale height parameter is

$$H \equiv \frac{\sigma}{x}, \quad x \equiv r \sin \theta. \quad (3.16)$$

The parameter H directly controls the disk width of the emission region, interpolating between a thin disk ($H \ll 1$) and a quasi-spherical flow ($H \sim 1$). The indices α and δ govern the radial steepness of the density and temperature profiles, and the normalizations $n_{e,\text{th}}^0$ and T_e^0 are fixed by matching the observed flux density and spectral constraints of the target. Representative two-dimensional profiles of number density and temperature profiles are shown in Figure 3.2.

Beyond the bulk geometry of the flow, the internal magnetic field configuration constitutes an independent degree of freedom with direct consequences for the emitted radiation and its polarization. In a magnetized RIAF, the gas and the magnetic pressure are in sub-equipartition to each other, therefore magnetic field strength can be set locally by specifying the ratio of gas to magnetic pressure, $\beta_p \equiv p_{\text{gas}}/p_{\text{mag}}$,

$$B = \left(\frac{8\pi n_e k_B T_e}{\beta_p} \right)^{1/2}, \quad (3.17)$$

where $p_{\text{gas}} = n_e k_B T_e$ and $p_{\text{mag}} = B^2/8\pi$. Synchrotron emission, which is the dominant radiative process at millimeter wavelengths in both M87* and Sgr A*, is produced by electrons gyrating along magnetic field lines. The emission coefficient at a given frequency depends on the pitch angle ϑ_B between the local wavevector k^μ and the magnetic four-vector b^μ , specifically as $\vartheta_B = \cos^{-1}(k_\mu b^\mu)$, and so the spatial distribution of the field orientation directly controls where and how strongly the plasma emits. The electric-vector position angle (EVPA) of the linearly polarized component rotates with respect to the sky-plane projection of the local field, and Faraday rotation subsequently modifies the observed polarization as the radiation propagates through the magnetized medium. Different large-scale field topologies, toroidal, poloidal, or mixed, therefore imprint qualitatively different polarization patterns on the image, even when the total intensity morphology is similar. This sensitivity means that the field geometry cannot be treated as an internal nuisance parameter to be marginalized over; it must be examined explicitly. Conventionally, this has been set to toroidal magnetic field (Pu and Broderick 2018), with $\beta_p = 10$. This has been extended to other magnetic fields in the context of hotspot (a localized hot Gaussian blob in a circular orbit) around Sgr A* (Vos et al. 2022). We follow a more systematic study of the impact of various magnetic fields and direct constraints on M87* (Chapter 5, Saurabh et al. 2026)

The velocity field is specified in Boyer-Lindquist coordinates following the

parameterization of [Tiede et al. 2020](#). Assuming axisymmetry and equatorial motion, the four-velocity takes the form

$$u^\mu = (u^t, u^r, 0, u^t\Omega), \quad \Omega \equiv \frac{u^\phi}{u^t}, \quad (3.18)$$

where Ω is the angular velocity as measured by a distant observer. The normalization condition $g_{\mu\nu}u^\mu u^\nu = -1$ determines u^t once u^r and Ω are specified,

$$u^t = \sqrt{\frac{1 + g_{rr}(u^r)^2}{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi}}}, \quad (3.19)$$

which requires $g_{tt} + 2\Omega g_{t\phi} + \Omega^2 g_{\phi\phi} < 0$. The radial and angular components are then prescribed as a linear combination of Keplerian and free-fall values,

$$u^r = u_K^r + \alpha(u_{\text{ff}}^r - u_K^r), \quad (3.20)$$

$$\Omega = \Omega_K + (1 - \kappa)(\Omega_{\text{ff}} - \Omega_K), \quad (3.21)$$

where $\alpha, \kappa \in [0, 1]$ are two free parameters. Outside the innermost stable circular orbit (ISCO) at $r_{\text{ISCO}}(a_*)$, the Keplerian radial component vanishes, $u_K^r = 0$, and Ω_K is the circular geodesic angular velocity in the Kerr metric. Inside the ISCO, $u_K^r \neq 0$ and is determined by matching the energy and specific angular momentum to their ISCO values. The free-fall components u_{ff}^r and Ω_{ff} are those of a radially infalling geodesic with the zero angular momentum.

The parameter pair (α, κ) spans a physically motivated range of accretion dynamics. Here, Keplerian motion corresponds to $(\alpha, \kappa) = (0, 1)$ and free-fall to $(\alpha, \kappa) = (1, 0)$. Fractional values yield a mixed prescription intermediate between these limits, more appropriate for a realistic astrophysical flow in M87*, where a sub-Keplerian azimuthal velocity profile and inward advection are expected ([Takahashi and Mineshige 2011](#)). These three limiting cases, Keplerian, sub-Keplerian, and free-fall, are the velocity models explored systematically in Chapter 5 for M87*.

3.3 General Relativistic Magnetohydrodynamics

The semi-analytic framework of Section 3.2 prescribes the density, temperature, and magnetic field structure of the accretion flow, but the level of turbulent stress that actually drives angular momentum transport, the geometry of the large-scale poloidal flux that accumulates near the horizon, and the conditions under which

a relativistic jet forms cannot be imposed from the outside. They are outcomes of the magnetized plasma dynamics. The magnetorotational instability (MRI; [Balbus and Hawley 1991](#)) grows from an arbitrarily weak seed field in a differentially rotating, weakly magnetized plasma, saturates nonlinearly, and sustains a turbulent stress whose effective magnitude depends on the net magnetic flux, the field geometry, and the disk thickness ([Yuan and Narayan 2014](#); [Abramowicz et al. 2026](#)). The secular accumulation of large-scale poloidal flux at the horizon, which distinguishes the magnetically arrested from the standard accretion state, follows from the competition between inward advection and outward diffusion of field lines, a balance that has no representation in any α -disk model. The power of a relativistic jet, driven by the Blandford–Znajek mechanism ([Blandford and Znajek 1977](#)), scales with the square of the ordered flux threading the horizon and with the black hole spin, and neither quantity is predictable without evolving the full magnetized flow. General relativistic magnetohydrodynamics (GRMHD) addresses each of these by solving the conservation equations of ideal MHD self-consistently on a fixed curved spacetime background, evolving the plasma density, velocity, and magnetic field from prescribed initial conditions with no symmetry imposed on the outcome. The nonlinear MRI, the large-scale field geometry, magnetically driven winds, and jet formation all emerge from the dynamics.

3.3.1 The ideal GRMHD equations

The equations of ideal GRMHD govern the evolution of a magnetized, perfectly conducting plasma in a fixed curved spacetime. On a background spacetime metric $g_{\mu\nu}$, the system expresses conservation of rest-mass plasma density, energy-momentum, and magnetic flux,

$$\nabla_\mu(\rho u^\mu) = 0, \quad (3.22)$$

$$\nabla_\nu T^{\mu\nu} = 0, \quad (3.23)$$

$$\nabla_\nu(b^\mu u^\nu - b^\nu u^\mu) = 0. \quad (3.24)$$

Here ρ is the rest-mass plasma density and u^μ the fluid four-velocity, normalized by

$$u^\mu u_\mu = -1. \quad (3.25)$$

The stress-energy tensor $T^{\mu\nu}$ receives contributions from both the fluid and the electromagnetic field,

$$T^{\mu\nu} = (\rho + u_{\text{int}} + p + b^2) u^\mu u^\nu + \left(p + \frac{b^2}{2}\right) g^{\mu\nu} - b^\mu b^\nu, \quad (3.26)$$

where u_{int} is the internal energy density, p the gas pressure, and $b^2 \equiv b^\mu b_\mu$. The quantity b^μ is the magnetic four-vector, constructed from the dual Faraday tensor ${}^*F^{\mu\nu}$ and the four-velocity by

$$b^\mu = u_\nu {}^*F^{\nu\mu}, \quad b^\mu u_\mu = 0, \quad (3.27)$$

where the orthogonality condition $b^\mu u_\mu = 0$ follows directly from the antisymmetry of ${}^*F^{\mu\nu}$. The system is closed by equation of state,

$$p = (\hat{\gamma} - 1) u_{\text{int}}, \quad (3.28)$$

with adiabatic index $\hat{\gamma}$.

The condition $b^\mu u_\mu = 0$ is equivalent to demanding that the electric field vanishes in the fluid rest frame, $E_{(\text{fluid})}^\mu = F_{\mu\nu} u^\nu = 0$. This is the ideal MHD assumption of perfect conductivity, and its direct consequence is the frozen-flux theorem in its covariant form, meaning the magnetic flux through any material surface is constant along the flow.

The spatial magnetic field (B^i) as measured by the Eulerian observer can be used to define the magnetic four-vector b^μ by (Gammie et al. 2003):

$$B^i = -{}^*F^{it} / \sqrt{-g} \quad (3.29)$$

$$b^t = \frac{B^i u_i}{u^t}, \quad (3.30)$$

$$b^i = \frac{B^i + b^t u^i}{u^t}, \quad (3.31)$$

where $g \equiv \det(g_{\mu\nu})$ is the metric determinant. The comoving magnetic energy density is then

$$b^2 = b^\mu b_\mu = \frac{B^i B^j g_{ij} + (B^i u_i)^2}{(u^t)^2 \alpha^2}, \quad (3.32)$$

where $\alpha = (-g^{tt})^{-1/2}$ is the lapse function. The plasma magnetization is characterized by $\sigma \equiv b^2/\rho$, with $\sigma \ll 1$ corresponding to gas-pressure-dominated flow and $\sigma \gg 1$ defining the strongly magnetized regime relevant to the jet funnel.

The spatial magnetic field must satisfy the divergence-free constraint at all times,

$$\partial_i (\sqrt{-g} B^i) = 0. \quad (3.33)$$

This can be enforced via constrained transport, evolving the field on a staggered grid such that the discrete version of Equation (3.33) is satisfied to machine precision at every timestep (Gammie et al. 2003). The induction equation in

coordinate form is

$$\partial_t(\sqrt{-g} B^i) + \partial_j(\sqrt{-g} [v^j B^i - v^i B^j]) = 0. \quad (3.34)$$

The source-free structure of this equation follows from the ideal MHD condition, since with no resistivity field lines are purely advected by the flow with no local creation or destruction of flux.

For numerical integration it is essential to cast the system in explicitly conservative form. Writing the equations in a coordinate basis, Equations (3.22)–(3.24) become (Gammie et al. 2003)

$$\partial_t(\sqrt{-g} \mathbf{U}) + \partial_i(\sqrt{-g} \mathbf{F}^i) = \sqrt{-g} \mathbf{S}, \quad (3.35)$$

where the vectors of conserved variables $\mathbf{U}$, fluxes $\mathbf{F}^i$, and source terms $\mathbf{S}$ are respectively

$$\mathbf{U} = \begin{pmatrix} \rho u^t \\ T^t_t + \rho u^t \\ T^t_j \\ B^k \end{pmatrix}, \quad \mathbf{F}^i = \begin{pmatrix} \rho u^i \\ T^i_t + \rho u^i \\ T^i_j \\ b^i u^k / u^t - b^k u^i / u^t \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ T^\mu_\nu \Gamma^\nu_{\mu t} \\ T^\mu_\nu \Gamma^\nu_{\mu j} \\ 0 \end{pmatrix}, \quad (3.36)$$

where $\Gamma^\nu_{\mu\lambda}$ are the Christoffel symbols of $g_{\mu\nu}$. The source vector is non-zero only for the energy-momentum equations, encoding the exchange of energy and momentum between the fluid and the curved spacetime geometry (Gammie et al. 2003). The mass conservation and induction equations are source-free. The scheme advances a set of primitive variables,

$$\mathbf{P} = (\rho, u_{\text{int}}, v^i, B^i), \quad (3.37)$$

where $v^i \equiv u^i / u^t$ is the coordinate three-velocity and B^i is the contravariant coordinate magnetic field. The conserved variables $\mathbf{U}$ follow algebraically from $\mathbf{P}$, while the inverse mapping is non-analytic and is solved numerically at each timestep via Newton–Raphson iteration (Gammie et al. 2003; Noble et al. 2006).

The ideal MHD approximation has known limitations in the context of black hole accretion. It excludes resistivity and therefore magnetic reconnection, which operates at current sheets where the frozen-flux condition breaks down and is likely the key mechanism for non-thermal particle acceleration and flaring in Sgr A* and M87* (Yuan and Narayan 2014). The single-fluid treatment imposes a common temperature on ions and electrons. The accreting plasma here is of sufficiently low density where the electron-proton Coulomb collision time significantly

larger than the dynamical timescale, developing a two-temperature scenario: protons and electrons with different temperatures and distribution functions. This implies a separate temperature prescription for the electrons (Section 3.2). In addition to this, radiative cooling can be neglected in regimes of low accretion rates² due to collisional timescales of electron-electron and proton-proton being longer than the dynamical time, and radiation has no significant effect on the flow structure (Mahadevan and Quataert 1997).

At higher accretion rates, approaching the Eddington limit, radiative cooling modifies the disk geometry and must be incorporated through radiation GRMHD, a substantially more computationally demanding framework. The spacetime is treated as fixed, so the metric remains Kerr throughout and there is no back-reaction from the accreting plasma. Within these constraints, ideal GRMHD on a Kerr background self-consistently captures MRI growth and saturation, the resulting turbulent angular momentum transport, large-scale magnetic flux evolution, magnetically driven outflows, and relativistic jet formation, none of which are accessible to above mentioned semi-analytic models.

3.3.2 Initial Conditions and Numerical Setup

Any GRMHD simulation of black hole accretion requires an initial state that satisfies the equations of motion exactly at $t = 0$, so that the subsequent evolution is driven by physical instabilities rather than by numerical transients from an inconsistent starting configuration. The most widely adopted choice is an axisymmetric, pressure-supported, differentially rotating torus of gas in exact hydrodynamic equilibrium in a background metric, threaded by a dynamically weak seed magnetic field. This class of initial conditions, of which the solution of Fishbone and Moncrief 1976 is the standard representative, provides a controlled mass reservoir whose equilibrium structure is known analytically, permits the magnetic field to be introduced as a perturbation that leaves the pressure and density distributions unmodified at $t = 0$, and ensures the fastest-growing MRI mode is resolved on the numerical grid before accretion begins. Alternative equilibrium configurations exist, including solutions with more complex angular momentum distributions (Noble et al. 2006) and tilted disks misaligned with the black hole spin axis (Fragile et al. 2007).

We start by a torus in hydrostatic equilibrium following Fishbone and Moncrief 1976. For a fluid in purely circular motion, $u^r = u^\theta = 0$, the relativistic Euler

²This is a typical assumption in ideal GRMHD simulations where no radiative losses are considered for the sources such as M87*.

equation reduces to the integrability condition

$$\frac{\nabla_\nu p}{\rho + u_{\text{int}} + p} = -\nabla_\nu \ln u_t - \frac{\ell \nabla_\nu \Omega}{1 - \ell \Omega}, \quad (3.38)$$

where $\ell \equiv -u_\phi/u_t$ is the specific angular momentum and $\Omega \equiv u^\phi/u^t$ is the coordinate angular velocity. Equation (3.38) can be integrated to yield a gravitational-centrifugal potential

$$W(r, \theta) \equiv \ln(-u_t) - \int_0^\Omega \frac{\ell \, d\Omega'}{1 - \ell \Omega'}, \quad (3.39)$$

and torus equilibrium requires W to be constant on equi-pressure surfaces, with the inner torus edge defined by $W_{\text{in}} \equiv W(r_{\text{in}}, \pi/2)$. For the standard constant- ℓ case this reduces to

$$\ln(-u_t) - \ln(-u_{t,\text{in}}) = \int_0^p \frac{dp'}{\rho + u_{\text{int}} + p'}, \quad (3.40)$$

which, combined with the Γ -law equation of state $p = (\hat{\gamma} - 1)u_{\text{int}}$, fully determines the density and pressure distribution, the inner torus edge r_{in} , and the radius of the pressure maximum r_{max} .

The torus at $t = 0$ is entirely gas-pressure dominated. A magnetic field is introduced as a weak perturbation, specified through a vector potential A_k so that the coordinate field

$$B^i = \frac{\varepsilon^{ijk}}{\sqrt{-g}} \partial_j A_k \quad (3.41)$$

satisfies the divergence-free constraint $\partial_i(\sqrt{-g} B^i) = 0$ exactly, the double partial derivative acting on the antisymmetric symbol ε^{ijk} vanishing identically (Gammie et al. 2003). The choice of A_k is not unique and has direct physical consequences for the late-time accretion state. A single large-scale poloidal loop, obtained by setting $A_\phi \propto \max(\rho/\rho_{\text{max}} - \rho_{\text{cut}}, 0)$ for some density threshold ρ_{cut} , carries a net poloidal flux of definite sign and favors the ordered accumulation of magnetic flux at the horizon (Gammie et al. 2003; Tchekhovskoy et al. 2011; McKinney et al. 2012). Multiple alternating poloidal loops, constructed by modulating the sign of A_ϕ across the torus, carry zero net flux; the flux of opposite polarity reconnects before reaching the horizon and the disk remains weakly magnetized, producing a SANE-like state (Narayan et al. 2012). Purely toroidal configurations, in which only the meridional components of A_k are non-zero, have also been explored and produce results qualitatively similar to the multi-loop case, though with somewhat different saturation levels for the Maxwell stress (Beckwith et al. 2008).

3.3.3 Ion and Electron Thermodynamics in Single-Fluid GRMHD

Standard ideal GRMHD evolves a single magnetized fluid with the equation of state $p = (\hat{\gamma} - 1) u_{\text{int}}$, where u_{int} is the total internal energy density and $\hat{\gamma}$ is the adiabatic index. The adiabatic index of an ideal monatomic gas is determined by the number of active translational degrees of freedom $f = 3$ and the relativistic state of the particles. In the non-relativistic limit, energy is quadratic in momentum and equipartition contributes $\frac{1}{2}kT$ per degree of freedom, giving $\hat{\gamma} = 1 + 2/f = 5/3$. In the ultra-relativistic limit, energy is linear in momentum with kT per degree of freedom, giving $\hat{\gamma} = 1 + 1/f = 4/3$. The intermediate value $\hat{\gamma} = 13/9$ corresponds to a single-temperature hydrogen plasma in which one species is non-relativistic and the other is relativistic, and is recovered by weighting the two contributions by their pressure fractions. In practice, simulations of low-luminosity black hole accretion have used all three values, and these choices produce measurably different flow structures, particularly in the temperature profile near the horizon (Gammie 2025). Understanding which value is physically appropriate requires examining the underlying two-temperature plasma thermodynamics, and the relevant cases are summarized in Table 3.1.

The near-horizon plasma in a RIAF is collisionless: due to sufficiently low density and low accretion rates, the Coulomb mean free path exceeds the system scale, and the ion-electron relaxation time is longer than the ion-ion or electron-electron relaxation times by a factor of $(m_i/m_e)^{1/2}$ each (Spitzer 1956). The two species therefore thermalize internally but not with each other, reaching a two-temperature state $T_i \neq T_e$ (Shapiro et al. 1976; Mahadevan and Quataert 1997). Near the horizon, ions remain non-relativistic ($\Theta_i \ll 1$, $\gamma_i \approx 5/3$) while electrons are relativistic ($\Theta_e \gg 1$, $\gamma_e \approx 4/3$), because the electron rest mass is roughly 1836 times smaller than the proton mass, placing the two species in the top and bottom rows of Table 3.1 respectively (Gammie 2025). The equilibrium value $\hat{\gamma} = 13/9$ for hydrogen implicitly assumes efficient ion-electron energy exchange and is therefore not appropriate for the collisionless EHT sources (Shapiro 1973; Gammie 2025). For decoupled species with $R \gg Z$, the composite index reduces to $\hat{\gamma} \approx 5/3$, because ion pressure dominates regardless of the softer electron equation of state. Two small corrections push the physically correct value fractionally below $5/3$: finite Θ_i contributes $-(2/3)\Theta_i$ and the relativistic electrons contribute $-Z/(3R)$, both negligible when $R \gg 1$. The physically better-motivated value is $\hat{\gamma}$ slightly less than $5/3$ ($5/3 - \epsilon$), and models with $\hat{\gamma} = 5/3$ produce higher near-horizon temperatures than those with $\hat{\gamma} = 4/3$, with the two choices yielding measurably different flow structures (Gammie 2025).

Table 3.1: Adiabatic index $\hat{\gamma}$ for ideal gas models relevant to low-luminosity black hole accretion from [Gammie 2025](#). $\Theta_s \equiv kT_s/m_s c^2$ is the dimensionless temperature of species s , $R \equiv T_i/T_e$ is the ion-to-electron temperature ratio, and $Z = n_e/n_i$ is the electron-to-ion number density ratio.

Species	Regime	Coupling assumption	$\hat{\gamma}$ (formula)	$\hat{\gamma}$ (value)
Single species	$\Theta \ll 1$ (non-relativistic)	–	$1 + 2/f$	$f = 3, 5/3 \approx 1.667$
Single species	$\Theta \gg 1$ (ultra-relativistic)	–	$1 + 1/f$	$f = 3, 4/3 \approx 1.333$
Ion + e [−] (H, $Z = 1$)	$\Theta_i \ll 1, \Theta_e \gg 1$	Full equilibrium ($T_i = T_e$)	$\approx (5 + 8Z)/(3 + 6Z)$	$13/9 \approx 1.444$
Ion + e [−] (He, $Z = 2$)	$\Theta_i \ll 1, \Theta_e \gg 1$	Full equilibrium ($T_i = T_e$)	$\approx (5 + 8Z)/(3 + 6Z)$	$21/15 = 1.400$
Ion + e [−]	$\Theta_i \ll 1, \Theta_e \gg 1$	Decoupled, $R \gg Z$	$(\gamma_i R + Z\gamma_e)/(R + Z)$	$\approx 5/3 \approx 1.667$
Ion + e [−]	$\Theta_i \ll 1, \Theta_e \gg 1$	Decoupled with corrections	$5/3 - (2/3)\Theta_i - Z/(3R)$	$\lesssim 5/3$

3.3.4 Horizon-Penetrating Coordinates

Boyer–Lindquist (BL) coordinates carry a coordinate singularity at the event horizon, where the metric component $g_{rr} = \Sigma/\Delta$ diverges at the outermost root of $\Delta(r) = 0$. GRMHD codes evolve fluid and field variables continuously across this surface, so the metric must be expressed in coordinates that are regular there. The appropriate construction replaces the BL time coordinate with one aligned to the ingoing principal null congruence (PNC) of the spacetime, whose generators cross the horizon in finite affine parameter by construction. Rather than performing this transformation separately for a specific spacetime, we follow the general solution-generating procedure of [Azreg-Aïnou 2014](#); [Kocherlakota et al. 2023](#), which takes any static, spherically symmetric seed metric and produces a broad family of stationary, axisymmetric spacetimes. The key modification to the original Newman–Janis algorithm ([Newman and Janis 1965](#)) is the replacement of an ad hoc reverse-complexification step by a regularity condition on the coordinate transformation.

We start with a seed metric describing a static, spherically symmetric line element,

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)g(r)} + R^2(r) d\Omega^2, \quad (3.42)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, and $f(r), g(r), R(r)$ are three free functions characterizing the geometry. The areal radius $R(r)$ satisfies $4\pi R^2(r) = \mathcal{A}$ for two-spheres at coordinate radius r , with $R(r) = r$ in the simplest models. The event horizon is located at the outermost root of $f(r) = 0$, since $g^{rr} \propto f(r)g(r)$. The function $g(r)$ can always be set to unity by a radial reparameterization, but retaining it allows f and R to take analytically convenient forms simultaneously.

The stationary, axisymmetric generalization of equation (3.42) derived via the Azreg-Aïnou (AA) method ([Azreg-Aïnou 2014](#)) takes the following form in BL coordinates $x^\mu = (t, r, \theta, \phi)$:

$$ds_{\text{BL}}^2 = -\left(1 - \frac{2F}{\Sigma}\right) dt^2 - \frac{4aF \sin^2\theta}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\Pi \sin^2\theta}{\Sigma} d\phi^2, \quad (3.43)$$

where a is the spin parameter. Defining the two radial auxiliary combinations

$$A(r) \equiv R^2 g, \quad B(r) \equiv f g R^2, \quad (3.44)$$

the metric functions are

$$\begin{aligned} F(r) &= \frac{1}{2}(A - B), \\ \Delta(r) &= B + a^2, \\ \Sigma(r, \theta) &= A + a^2 \cos^2 \theta, \\ \Pi(r, \theta) &= (A + a^2)^2 - \Delta a^2 \sin^2 \theta. \end{aligned} \quad (3.45)$$

A conformal factor $X(r, \theta)$ introduced in the solution-generating step is fixed by the field equations of the underlying theory. For axially rotating fluid matter in GR, demanding $G_{(\hat{r})(\hat{\theta})} = 0$ in the local orthonormal frame of the circulating matter has $X = \Sigma$ as the unique solution consistent with asymptotic flatness (Azreg-Aïnou 2014; Kocherlakota et al. 2023). This choice is adopted throughout the present work. Under it, $g_{rr} = \Sigma/\Delta$ diverges at $\Delta = 0$, the coordinate singularity to be resolved.

The construction of the horizon-penetrating coordinates proceeds through the following steps.

1. **Identify the principal null congruence:** The PNC consists of null geodesics whose tangent vectors $\pm \ell^\mu$ satisfy $\dot{\theta} = 0$, i.e. they move in the equatorial sense and do not oscillate in θ . The two families (ingoing and outgoing) have tangent vectors of the form

$$\pm \ell^\mu = \left(\frac{dt}{d\lambda}, \pm \frac{dr}{d\lambda}, 0, \frac{d\phi}{d\lambda} \right), \quad (3.46)$$

where λ is an affine parameter. The existence of the PNC for the AA metric is guaranteed by its separability structure.

2. **Separate the null geodesic equations:** The Killing symmetries ∂_t and ∂_ϕ yield conserved specific energy $\mathcal{E} = -u_t$ and specific angular momentum $\mathcal{L} = u_\phi$. With the choice $X = \Sigma$, the geodesic equations separate completely into

$$\Sigma \frac{dt}{d\lambda} = \frac{(A + a^2)[(A + a^2)\mathcal{E} - a\mathcal{L}]}{\Delta} - a(a\mathcal{E} \sin^2 \theta - \mathcal{L}), \quad (3.47)$$

$$\Sigma \frac{d\phi}{d\lambda} = \frac{a[(A + a^2)\mathcal{E} - a\mathcal{L}]}{\Delta} - \left(a\mathcal{E} - \frac{\mathcal{L}}{\sin^2 \theta} \right), \quad (3.48)$$

$$\Sigma^2 \left(\frac{dr}{d\lambda} \right)^2 = \mathcal{R}(r) \equiv [(A + a^2)\mathcal{E} - a\mathcal{L}]^2 - \Delta[\mathcal{C} + (a\mathcal{E} - \mathcal{L})^2], \quad (3.49)$$

$$\Sigma^2 \left(\frac{d\theta}{d\lambda} \right)^2 = \Theta(\theta) \equiv \mathcal{C} - \left(\frac{\mathcal{L}^2}{\sin^2 \theta} - a^2 \mathcal{E}^2 \right) \cos^2 \theta, \quad (3.50)$$

where $\mathcal{C}$ is the Carter constant (Kocherlakota et al. 2023). The azimuthal impact parameter is $\xi \equiv \mathcal{L}/\mathcal{E}$ and the separation constant is $\eta^2 \equiv \mathcal{C}/\mathcal{E}^2$.

3. **Extract the PNC from the geodesic equations:** The PNC corresponds to geodesics with $\dot{\theta} = \ddot{\theta} = 0$ for all θ . Setting $\Theta(\theta) = 0$ identically in equation (3.50) and requiring the condition to hold for every value of θ forces $\mathcal{C} = 0$ and $\xi = a \sin^2 \theta$, i.e. $\eta = 0$. Substituting these values into equations (3.47)–(3.49) gives the PNC co-vector

$$\pm \ell_\mu = \mathcal{E} \left(-1, \pm \frac{\Sigma}{\Delta}, 0, a \sin^2 \theta \right), \quad (3.51)$$

so that the radial component $\ell_r = \pm \Sigma/\Delta$ diverges at the horizon $\Delta = 0$. Normalizing to $\mathcal{E} = 1$, the ingoing co-vector is

$$\ell_\mu^{(-)} = \left[-1, -\frac{\Sigma}{\Delta}, 0, a \sin^2 \theta \right]. \quad (3.52)$$

4. **Define the horizon-penetrating coordinates:** The spherical-polar ingoing Kerr–Schild (siKS) coordinates $(\tau, r, \theta, \phi_{\text{KS}})$ are defined by requiring the ingoing PNC co-vector to take the regular form

$$\ell_\mu^{(-)} = [-1, 1, 0, a \sin^2 \theta], \quad (3.53)$$

in the new system. The coordinate differentials that achieve this regularization are

$$d\tau = dt_{\text{BL}} + \frac{\Sigma}{\Delta} dr_{\text{BL}}, \quad (3.54)$$

$$dr_{\text{KS}} = dr_{\text{BL}}, \quad (3.55)$$

$$d\theta_{\text{KS}} = d\theta_{\text{BL}}, \quad (3.56)$$

$$d\phi_{\text{KS}} = d\phi_{\text{BL}} + \frac{a}{\Delta} dr_{\text{BL}}, \quad (3.57)$$

which integrate to

$$\tau = t + \int \frac{\Sigma}{\Delta} dr, \quad \phi_{\text{KS}} = \phi_{\text{BL}} + \int \frac{a}{\Delta} dr. \quad (3.58)$$

In the limit $a \rightarrow 0$, τ reduces to the Eddington–Finkelstein advanced time coordinate (Eddington 1924; Finkelstein 1958).

5. **Construct the Jacobian:** The full Jacobian $\Lambda^\mu{}_\nu \equiv \partial x^\mu_{\text{KS}} / \partial x^\nu_{\text{BL}}$ of the

transformation is read off from equations (3.54)–(3.55),

$$\Lambda^\mu{}_\nu = \begin{pmatrix} 1 & \Sigma/\Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & a/\Delta & 0 & 1 \end{pmatrix}, \quad (3.59)$$

which is manifestly non-singular for all r outside the curvature singularity $\Sigma = 0$.

6. Transform the metric: Applying the Jacobian (3.59) to equation (3.43) via $g_{\mu\nu}^{\text{KS}} = \Lambda^\alpha{}_\mu g_{\alpha\beta}^{\text{BL}} \Lambda^\beta{}_\nu$ yields

$$g_{\mu\nu}^{\text{KS}} = \frac{X}{\Sigma} \begin{pmatrix} -\left(1 - \frac{2F}{\Sigma}\right) & \frac{2F}{\Sigma} & 0 & -\frac{2Fa \sin^2\theta}{\Sigma} \\ \frac{2F}{\Sigma} & 1 + \frac{2F}{\Sigma} & 0 & -\left(1 + \frac{2F}{\Sigma}\right) a \sin^2\theta \\ 0 & 0 & \Sigma & 0 \\ -\frac{2Fa \sin^2\theta}{\Sigma} & -\left(1 + \frac{2F}{\Sigma}\right) a \sin^2\theta & 0 & \frac{\Pi \sin^2\theta}{\Sigma} \end{pmatrix}. \quad (3.60)$$

The inverse metric in siKS coordinates is obtained by inverting equation (3.60).

The contravariant components are

$$g_{\text{KS}}^{\mu\nu} = \frac{\Sigma}{X} \begin{pmatrix} -\left(1 + \frac{2F}{\Sigma}\right) & \frac{2F}{\Sigma} & 0 & 0 \\ \frac{2F}{\Sigma} & \frac{\Delta}{\Sigma} & 0 & \frac{a}{\Sigma} \\ 0 & 0 & \frac{1}{\Sigma} & 0 \\ 0 & \frac{a}{\Sigma} & 0 & \frac{1}{\Sigma \sin^2\theta} \end{pmatrix}, \quad (3.61)$$

All covariant components $g_{\mu\nu}^{\text{KS}}$ are now finite at the horizon $\Delta = 0$; the only remaining singularity is the curvature singularity at $\Sigma = 0$.

Two limiting cases verify the construction. Setting $a = 0$ with Schwarzschild seed functions $f = 1 - 2M/r$, $g = 1$, $R^2 = r^2$ gives $F = Mr$ and $\Sigma = r^2$, so equation (3.60) reduces to the Schwarzschild metric in ingoing Eddington–Finkelstein coordinates. For nonzero a with the same seed, it reproduces the Kerr metric in the horizon-penetrating spherical Kerr–Schild form used canonically in GRMHD codes (Gammie et al. 2003; McKinney and Gammie 2004).

3.3.5 Modified Coordinates

The siKS coordinates, while horizon-penetrating, distribute grid cells uniformly in both r and θ . This is numerically inefficient because the physics of accretion is concentrated near the equatorial plane and the dynamical range in radius spans several orders of magnitude from the horizon to the outer boundary of the simulation domain. Practical GRMHD codes therefore introduce a further coordinate transformation from the siKS coordinates (τ, r, θ, ϕ) to a set of internal computational coordinates $x^\mu = (x^0, x^1, x^2, x^3)$ that concentrate numerical resolution where it is most needed. The metric in these computational coordinates is obtained via the Jacobian $\partial \bar{x}^\mu / \partial x^\nu$, which enters the covariant components of the metric as $g_{\mu\nu} = \bar{g}_{\bar{\alpha}\bar{\beta}} (\partial \bar{x}^{\bar{\alpha}} / \partial x^\mu) (\partial \bar{x}^{\bar{\beta}} / \partial x^\nu)$.

The Modified Kerr–Schild (MKS) coordinates, introduced in the original implementation of HARM (Gammie et al. 2003), define

$$x^0 = \tau, \quad (3.62)$$

$$x^1 = \ln r, \quad (3.63)$$

$$x^3 = \phi, \quad (3.64)$$

with the angular coordinate $x^2 \in [0, 1]$ mapped to the polar angle via

$$\theta(x^2) = \pi x^2 + \frac{1-h}{2} \sin(2\pi x^2), \quad (3.65)$$

where $h \in (0, 1]$ is a free parameter controlling the degree of concentration toward the equatorial plane. In the limit $h \rightarrow 1$, Eq. (3.65) reduces to the uniform mapping $\theta = \pi x^2$, and for $h < 1$ cells are progressively concentrated toward $\theta = \pi/2$. The logarithmic radial coordinate (3.63) ensures that the radial cell width $\Delta r = r \Delta x^1$ grows linearly with r , placing more cells near the horizon where gradients are steepest. The non-zero Jacobian components are

$$\frac{\partial r}{\partial x^1} = r, \quad \frac{\partial \theta}{\partial x^2} = \pi [1 + (1-h) \cos(2\pi x^2)], \quad (3.66)$$

with all other off-diagonal spatial components vanishing.

3.3.6 Resolving the MRI: Quality factors

Whether a GRMHD simulation produces physically meaningful turbulent transport of angular momentum depends entirely on whether the magnetorotational instability is resolved on the numerical grid. The MRI is a linear instability of magnetized differentially rotating flows (Balbus and Hawley 1991), amplifying a

seed magnetic field exponentially on the orbital timescale and, once nonlinear, driving MHD turbulence whose Maxwell stresses transport angular momentum outward and sustain accretion. If the fastest-growing MRI mode is under-resolved, turbulence is suppressed, the effective stress collapses, and the simulation does not represent the physical accretion flow regardless of how accurately the conservation equations are integrated (Hawley et al. 2011).

The standard resolution diagnostic is the MRI quality factor (Sano et al. 2004; Noble et al. 2010) which later developed into a convergence framework (Hawley et al. 2011; Hawley et al. 2013). For each coordinate direction i , the quality factor measures how many grid cells resolve the fastest-growing MRI wavelength in that direction,

$$Q_i \equiv \frac{\lambda_i^{\text{MRI}}}{\Delta x_i}, \quad (3.67)$$

where Δx_i is the proper cell size and λ_i^{MRI} is the MRI wavelength associated with the magnetic field component B_i in the fluid frame. For a vertical (poloidal) field the dominant MRI wavelength is

$$\lambda_\theta^{\text{MRI}} = \frac{2\pi v_{A,\theta}}{\Omega}, \quad v_{A,\theta} \equiv \frac{|b^\theta|}{\sqrt{\rho + u_{\text{int}} + p + b^2}}, \quad (3.68)$$

where $v_{A,\theta}$ is the θ -component of the relativistic Alfvén speed and $\Omega = u^\phi/u^t$ is the local orbital angular velocity (Noble et al. 2010). Analogous definitions hold for Q_r and Q_ϕ . In practice the poloidal quality factor Q_θ is the most constraining, because the fastest-growing MRI mode is the channel mode, which is vertically oriented and requires vertical field to develop.

From a combination of local shearing-box experiments and global GRMHD convergence studies, minimum thresholds for adequately resolved MRI turbulence have been established (Hawley et al. 2011; Hawley et al. 2013). A simulation is considered marginally resolved if

$$Q_\theta \gtrsim 10, \quad Q_\phi \gtrsim 20, \quad (3.69)$$

with the more conservative requirement $Q_\theta \gtrsim 15$ – 20 and $Q_\phi \gtrsim 25$ for quantitatively reliable stress levels (Hawley et al. 2013). The threshold $Q_\theta \geq 6$ represents an absolute floor below which turbulence cannot be sustained (Sano et al. 2004). These conditions must be satisfied not merely at the initial time but throughout the quasi-steady phase of the simulation, since the field strength evolves and $\lambda_\theta^{\text{MRI}}$ changes accordingly.

3.3.7 Magnetically Regulated disk States

The quasi-steady configuration reached at late simulation times depends on how much large-scale poloidal flux the disk has advected to the horizon over the course of the evolution, which is set by the initial field topology chosen at $t = 0$ (Section 3.3). When the net poloidal flux is small, the disk accretes with modest magnetic pressure at the horizon and the turbulent structure visible in the right-most column of Figure 3.3 persists without dramatic change. When the net flux is large, as in the single-loop configuration, the field that accumulates near the horizon eventually becomes strong enough to resist the inflowing gas and accretion proceeds in a qualitatively different manner. These two outcomes are referred to as the standard and normal evolution (SANE) and the magnetically arrested disk (MAD) states respectively (Narayan et al. 2012; Tchekhovskoy et al. 2011), and together they span the range of disk behaviors that enter the current EHT image libraries (Event Horizon Telescope Collaboration 2019; Event Horizon Telescope Collaboration 2022). A dimensionless measure of the accumulated flux can be calculated by a proxy function

$$\phi_{\text{BH}} \equiv \frac{\Phi_{\text{BH}}}{\sqrt{\dot{M}_{\text{BH}}}}, \quad \Phi_{\text{BH}} \equiv \frac{1}{2} \int |B^r| \sqrt{-g} d\theta d\phi, \quad (3.70)$$

where the integral is evaluated at the horizon and $\dot{M}_{\text{BH}}$ is the mass accretion rate (Tchekhovskoy et al. 2011).

In the SANE state (Narayan et al. 2012) the magnetic field throughout the disk body is subthermal, with plasma $\beta \sim 10$ –100, and angular momentum transport is dominated by MRI-driven Maxwell stresses. The Maxwell stress is consistently larger than the Reynolds stress by a factor of several, and the toroidal field component dominates in the disk interior. The flow separates into a dense turbulent disk body near the equatorial plane, a hot magnetized corona above and below it, and an evacuated magnetically dominated funnel along the rotation axis (Gammie et al. 2003; McKinney and Gammie 2004). The funnel sustains a relativistic jet, but the available magnetic flux is modest, so the Blandford–Znajek (BZ) jet power (Blandford and Znajek 1977),

$$P_{\text{BZ}} = \frac{\kappa}{4\pi c} \Phi_{\text{BH}}^2 \Omega_H^2, \quad (3.71)$$

where κ is a dimensionless numerical factor³, and $\Omega_H = a_* c / 2R_H$ is the angular velocity of the horizon, remains well below the total accretion energy budget and scales steeply with spin (Tchekhovskoy et al. 2011). The SANE disk structure does

³ $\kappa = 0.053$ for a split monopole geometry and $\kappa = 0.044$ for a parabolic geometry

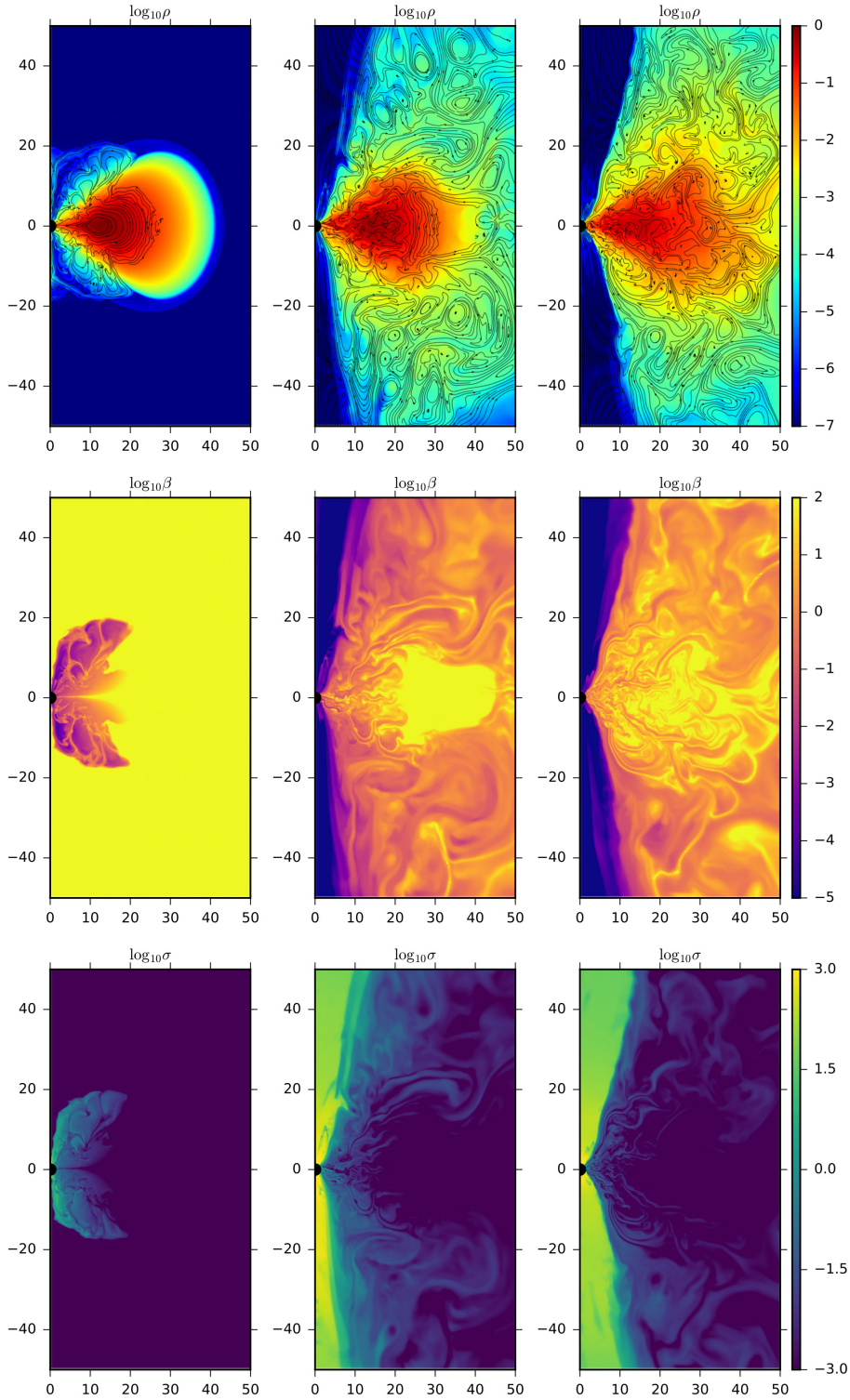


Figure 3.3: Evolution of the magnetized torus at resolution 1024×512 at times $t/M \in \{300, 1000, 2000\}$. Top row shows logarithmic rest-frame density $\log_{10} \rho$, middle row shows logarithmic plasma $\beta = p_{\text{gas}}/p_{\text{mag}}$, and the bottom row shows the logarithm of the magnetization parameter $\sigma = b^2/\rho$. Magnetic field lines are traced in the first panel as black contours. The MRI develops and saturates by $t/M \sim 1000$, by which point a strongly magnetized, evacuated funnel has formed along the rotation axis, reaching values of $\beta < 10^{-5}$ and $\sigma \approx 10^3$ (Porth et al. 2017).

not depend strongly on the initial field topology once a quasi-steady turbulent state is reached, so SANE simulations with different initial field configurations converge to similar time-averaged properties.

The MAD state (Narayan et al. 2003; Igumenshchev 2008; Tchekhovskoy et al. 2011) arises when large-scale poloidal flux has built up at the horizon to the point where the magnetic pressure is comparable to the ram pressure of the inflowing gas. In the Gaussian system the saturation flux is approximately

$$\Phi_{\text{MAD}} \approx 50 \dot{M}_{\text{BH}}^{1/2} R_g c^{1/2}, \quad (3.72)$$

where $R_g = GM/c^2$ (Tchekhovskoy et al. 2011). Beyond this threshold the ordered field disrupts axisymmetric inflow and forces gas to move inward in streams and blobs through a magnetic Rayleigh–Taylor interchange instability, a behavior first identified in three-dimensional pseudo-Newtonian MHD simulations by Narayan et al. 2003 and subsequently confirmed in full GRMHD (Tchekhovskoy et al. 2011; McKinney et al. 2012). The resulting accretion is highly time-variable, as illustrated in Figure 3.4: episodes of ordered infall are interrupted by magnetic flux eruptions that temporarily expel gas outward, producing the large-amplitude fluctuations in $\dot{M}$ and ϕ_{BH} visible in panels (e) and (f) of that figure. The most striking consequence of the MAD state is the jet power. With Φ_{BH} at its saturation value and appearing squared in equation (3.71), the Blandford–Znajek power at maximum spin can exceed the total rest-mass energy flux of the accreting gas, giving a jet efficiency $\eta_{\text{jet}} = P_{\text{jet}}/(\dot{M}_{\text{BH}} c^2) > 1$ (Tchekhovskoy et al. 2011; McKinney et al. 2012). This is possible because the ordered magnetic field threading the horizon taps the rotational energy of the black hole directly through the BZ mechanism, so the total energy output of the jet is not limited to the gravitational binding energy released by accretion (Yuan and Narayan 2014). Although at first sight this might appear to violate energy conservation, there is no inconsistency since the surplus energy is extracted electromagnetically from the spinning black hole itself, and the efficiency exceeds unity precisely when the spin energy reservoir is large and the horizon flux is near saturation (Tchekhovskoy et al. 2011; McKinney et al. 2012; Yuan and Narayan 2014). Substituting equation (3.72) into equation (3.71) gives the approximate relation

$$P_{\text{jet}} \approx 2.5 \left(\frac{a_\star}{1 + \sqrt{1 - a_\star^2}} \right)^2 \left(\frac{\Phi_{\text{BH}}}{\Phi_{\text{MAD}}} \right)^2 \dot{M}_{\text{BH}} c^2, \quad (3.73)$$

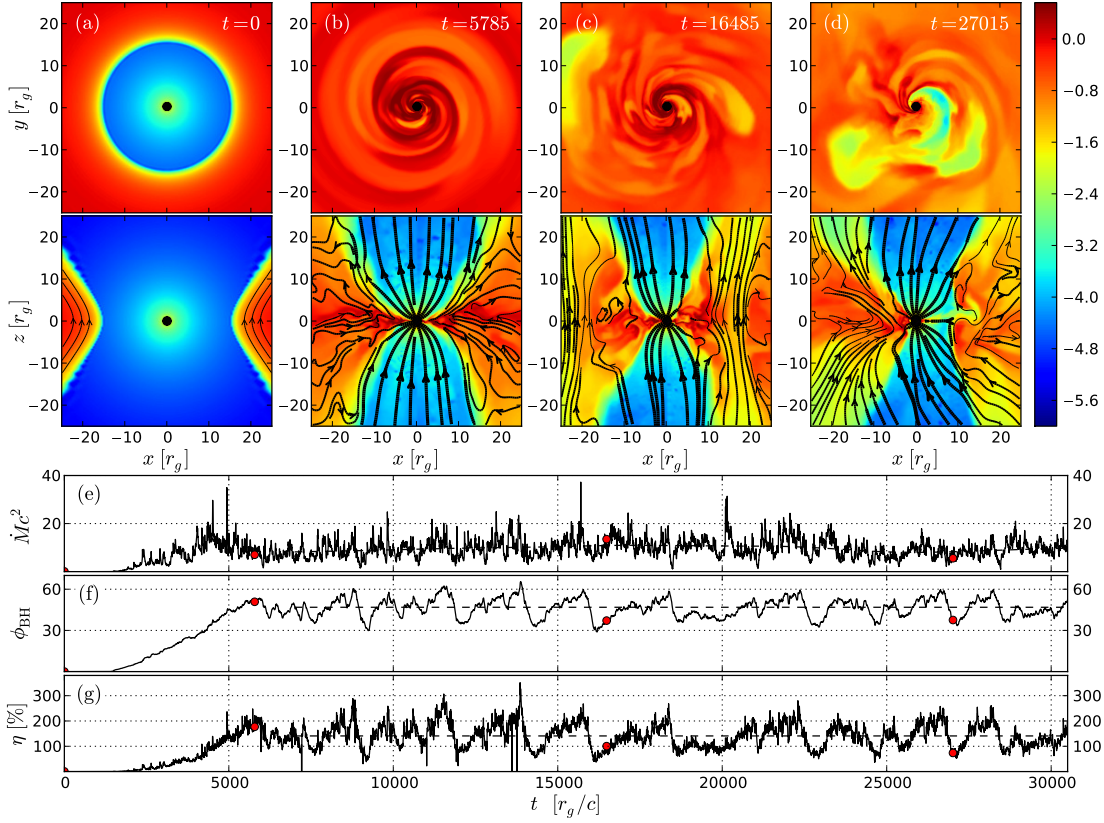


Figure 3.4: GRMHD simulation of a MAD accretion flow around a black hole with spin $a = 0.99$ (Tchekhovskoy et al. 2011). Top two rows: equatorial ($z = 0$, top) and meridional ($y = 0$, bottom) snapshots of the logarithmic rest-frame density $\log_{10} \rho c^2$ at $t \in \{0, 5785, 16485, 27015\} r_g/c$; field lines are shown as black streamlines. Bottom panels: time evolution of the rest-mass accretion rate $\dot{M} c^2$ (panel e), dimensionless magnetic flux ϕ_{BH} threading the horizon (panel f), and energy outflow efficiency η (panel g). The flux grows until $t \approx 6000 r_g/c$, after which accretion becomes magnetically arrested; the efficiency subsequently fluctuates around $\eta \sim 100\text{--}140\%$, confirming that the jet extracts energy from the black hole spin at a rate exceeding the rest-mass energy flux of the accreting gas.

with the more accurate polynomial calibration

$$P_{\text{jet}} \approx 0.65 a_{\star}^2 (1 + 0.85 a_{\star}^2) \left(\frac{\Phi_{\text{BH}}}{\Phi_{\text{MAD}}} \right)^2 \dot{M}_{\text{BH}} c^2 \quad (3.74)$$

fit against simulation data (Yuan and Narayan 2014). The most favorable configuration is when the magnetic flux has reached the MAD limit $\Phi_{\text{BH}} \rightarrow \Phi_{\text{MAD}}$ and the spin is maximum $a_{\star} \rightarrow 1$, in which case the BZ jet power can comfortably exceed $\dot{M}_{\text{BH}} c^2$ (Tchekhovskoy et al. 2011; McKinney et al. 2012). The saturated magnetic flux level and jet power in the MAD regime depend strongly on the black hole spin: the prograde case saturates at much higher relative magnetic flux and produces more powerful and wider jets than retrograde cases, and the

jet width itself scales as $w \propto z^k$ with power-law index $k \sim 0.27\text{--}0.42$ over heights $z = 5\text{--}100 r_g$ (Mizuno 2022). In MAD simulations, dynamically significant magnetic fields facilitate the outward transport of angular momentum and energy through the jet, with the electromagnetic component dominating the total angular momentum and energy current in the polar region, while fluid stresses drive inward transport within the disk (Dhruv et al. 2025).

The strong coupling between the horizon magnetic flux and the black hole spin in the MAD state has a direct consequence for the spin evolution. In prograde MAD simulations, magnetic field lines anchored at the horizon extract angular momentum from the black hole at a rate that overwhelms the inward supply of angular momentum from the disk, producing net spin-down (Dhruv et al. 2025). This spin-down is stronger for more rapidly spinning black holes, because the stronger radial magnetic fields anchored at the event horizon extract angular momentum more efficiently (Dhruv et al. 2025). As a result, the spin equilibrium in MAD simulations is significantly lower than the thin-disk value of $a_{\text{eq}} \approx 0.94$; long-duration GRMHD simulations find $a_{\text{eq}} \lesssim 0.1$ for MAD accretion (Narayan et al. 2022; Lowell et al. 2024; Dhruv et al. 2025). In SANE simulations, by contrast, the magnetic fields are dynamically less important in the angular momentum budget and the models follow the thin-disk expectation closely, achieving spin equilibrium near $a_{\text{eq}} \approx 0.94$ (Dhruv et al. 2025). Quantitatively, the v3 library jet power statistics confirm that the MAD $a_\star = +15/16$ model reaches $P_{\text{jet}}/(\dot{M}_{\text{BH}}c^2) \approx 1.51$, with the electromagnetic fraction of the jet power $P_{\text{jet,em}}/P_{\text{jet}} \approx 0.71$, while the non-spinning MAD model produces a negligible jet power of $\approx 6 \times 10^{-5}$, confirming that significant spin is required for a powerful jet in the MAD state (Dhruv et al. 2025). MAD disks are also consistently hotter than SANE disks within $r \lesssim 10 r_g$, likely due to compressional heating associated with the magnetically dominated, narrower accretion channels that constrain the infalling gas in the MAD regime.

Beyond the jet, the inner disk in the MAD state shows intermittent magnetic flux eruptions that can temporarily suppress accretion and produce rapid variability, associated with magnetic reconnection that provides particle heating and acceleration, and that contains sufficient energy to power the infrared flares observed in Sgr A* (Event Horizon Telescope Collaboration 2022; Mizuno 2022). The first systematic code-comparison study of GRMHD simulations, carried out by Porth 2019 across multiple independent codes on identical initial conditions, found good agreement in the time-averaged structure of SANE accretion flows, and agreement between codes improves as resolution increases, underpinning the physical picture presented here. Together, broadly, the SANE and MAD states define the two poles of current understanding the accretion problem in supermas-

sive black holes.

3.4 Radiative Processes in Accretion Flows

The SED of a RIAF reflects the interplay of these three processes across a huge range in frequency. Figure 3.5 illustrates the component structure for Sgr A*. At radio frequencies the emission rises with frequency as the flow becomes progressively less self-absorbed at shorter wavelengths, produced largely by synchrotron from non-thermal electrons distributed over a range of radii. The submillimeter bump, peaking near $\nu \sim 10^{12}$ Hz, is thermal synchrotron from hot electrons in the innermost region of the flow at $r \lesssim 10 r_g$ (Yuan et al. 2003; Yuan and Narayan 2014). This peak marks the transition from optically thick to optically thin synchrotron; below it the spectrum follows the Rayleigh–Jeans tail of the self-absorbed source function, while above it emission is optically thin and falls steeply. The same thermal electrons Compton upscatter these synchrotron photons, producing a hard continuum extending to $k_B T_e \sim 10\text{--}100$ keV that appears as a series of Compton humps at $\nu \sim 10^{17}\text{--}10^{20}$ Hz (Yuan and Narayan 2014). The relative importance of this component increases steeply with accretion rate; at the lowest accretion rates it is subdominant and bremsstrahlung from the outer flow near the Bondi radius contributes at hard X-ray energies instead (Yuan et al. 2003).

For M87* the same qualitative decomposition applies, though the larger black hole mass of $\sim 6.5 \times 10^9 M_\odot$ shifts the synchrotron peak to lower frequencies. The broadband SED of M87* shown in Figure 3.6, assembled from quasi-simultaneous observations during the 2017 EHT campaign, spans more than seventeen decades in frequency from centimeter radio to TeV γ -rays (EHT MWL Science Working Group and Algaba 2021). The radio through millimeter portion is dominated by synchrotron from the compact core and inner jet, with the EHT placing the 230 GHz flux within $R \leq 7.3 r_g$ of the black hole and thereby isolating it from the larger-scale jet contribution that dominates at lower frequencies. At γ -ray energies, EHT MWL Science Working Group and Algaba 2021 found that the simultaneous emission cannot arise from inverse Compton scattering in the same compact region producing the EHT 230 GHz signal, requiring either strongly particle-dominated regions in the inner jet or direct synchrotron from accelerated protons. The 230 GHz emission probed by the EHT therefore isolates the near-horizon plasma where thermal synchrotron is the dominant process and the physics described in the remainder of this section applies directly.

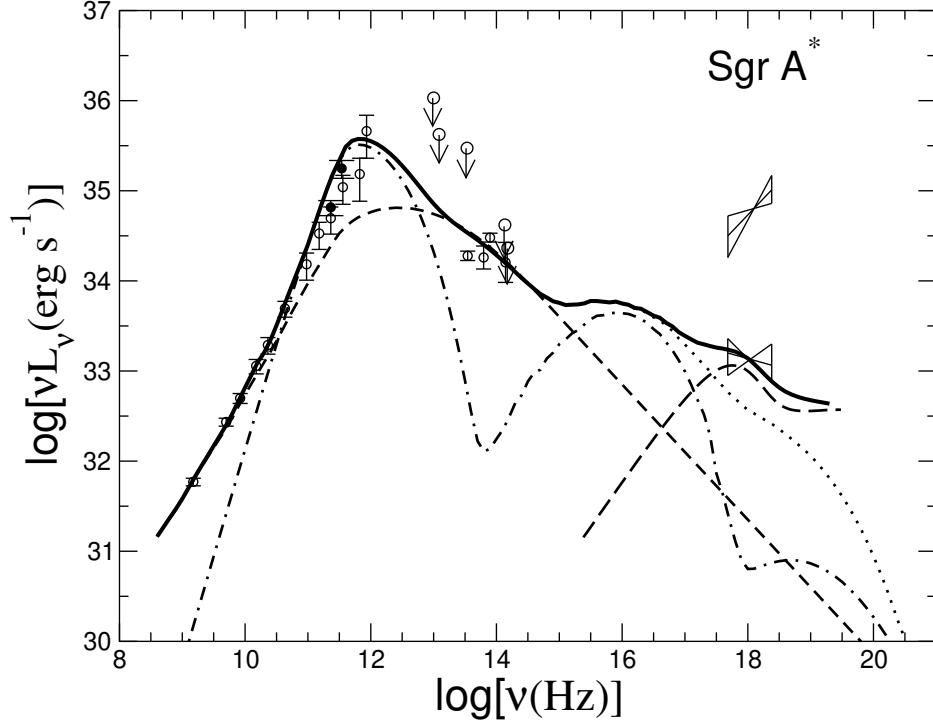


Figure 3.5: Spectral energy distribution of Sgr A* in quiescence from [Yuan and Narayan 2014](#). The total model spectrum (thick solid line) separates into thermal synchrotron and its inverse Compton humps (dot-dashed line), non-thermal synchrotron from accelerated electrons (short-dashed line), and bremsstrahlung from the outer flow (long-dashed line). The submillimeter bump at $r \lesssim 10 r_g$ is the primary target of EHT observations at 230 GHz.

3.4.1 Synchrotron Radiation

Synchrotron radiation is emitted by a relativistic electron accelerated by the Lorentz force as it spirals around a magnetic field line, as illustrated in Figure 3.7. The non-relativistic gyration frequency, known as the cyclotron frequency, is ([Rybicki and Lightman 1979](#))

$$\nu_B = \frac{eB}{2\pi m_e c}. \quad (3.75)$$

For an ultra-relativistic electron with Lorentz factor $\gamma \gg 1$, radiation is beamed into a forward cone of half-angle $\sim \gamma^{-1}$ and the observer receives a brief pulse once per gyration. The power spectrum of these pulses is broad, peaked near the critical frequency

$$\nu_c = \frac{3}{2} \gamma^2 \nu_B \sin \alpha, \quad (3.76)$$

where α is the pitch angle between the electron velocity and $\mathbf{B}$. The total power radiated by a single ultra-relativistic electron, averaged over an isotropic pitch-

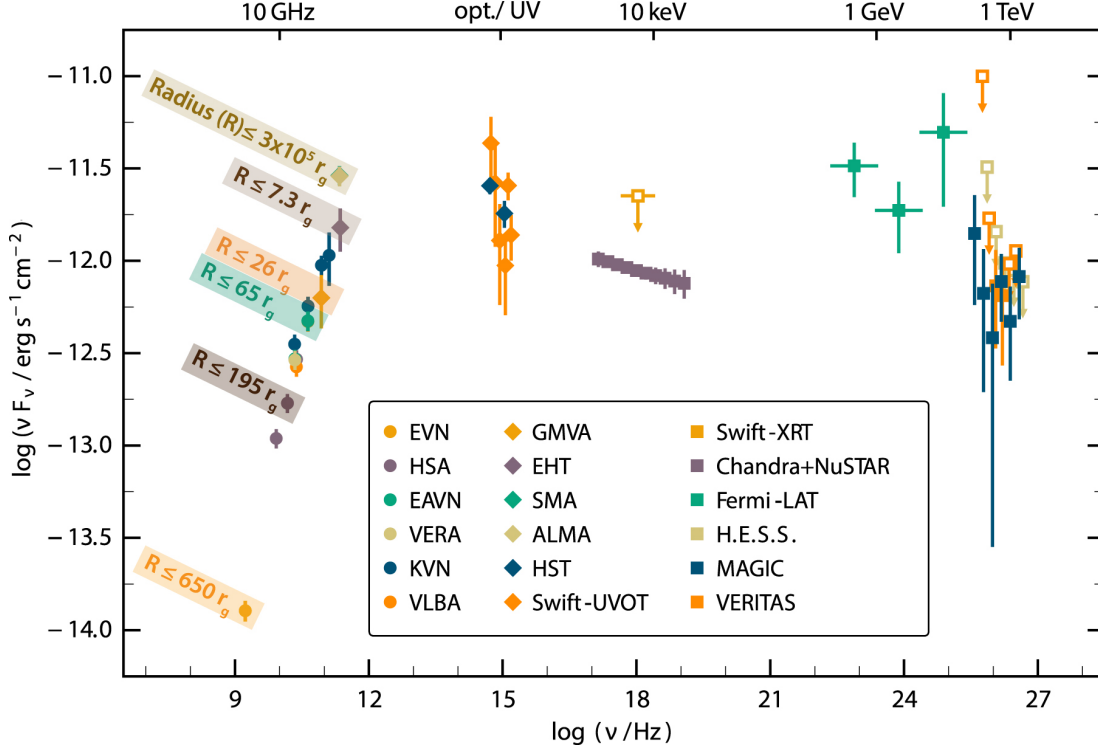


Figure 3.6: Quasi-simultaneous broadband SED of M87* during the 2017 EHT campaign, from [EHT MWL Science Working Group and Algaba 2021](#). Different instruments and angular scales are indicated by color and marker style. Upper limits on emission size from VLBI observations at representative radio frequencies are labeled.

angle distribution, is ([Rybicki and Lightman 1979](#))

$$\langle P_{\text{syn}} \rangle = \frac{4}{3} \sigma_T c \beta^2 \gamma^2 U_B, \quad (3.77)$$

where $\sigma_T = 6.65 \times 10^{-25} \text{ cm}^2$ is the Thomson cross-section, $\beta = v/c$, and $U_B = B^2/8\pi$ is the magnetic energy density. For $\beta \approx 1$ the power scales as $\langle P_{\text{syn}} \rangle \propto \gamma^2 U_B$.

The energy of the electron is $E = \gamma m_e c^2$, so the synchrotron cooling timescale, defined as $t_{\text{syn}} = E/\langle P_{\text{syn}} \rangle$, is

$$t_{\text{syn}} = \frac{\gamma m_e c^2}{(4/3) \sigma_T c \beta^2 \gamma^2 U_B} = \frac{6\pi m_e c}{\sigma_T B^2 \gamma} \approx \frac{7.75 \times 10^8}{B^2 \gamma} \text{ s} = \frac{24.6}{B^2 \gamma} \text{ yr}, \quad (3.78)$$

where B is in Gauss ([Ghisellini 2013](#)). The cooling time decreases steeply with both field strength and Lorentz factor. An electron with $\gamma = 10^3$ in the accretion flow of a radio-loud AGN where $B \sim 10^3 \text{ G}$ cools in $t_{\text{syn}} \sim 0.75 \text{ s}$, whereas the same electron in the diffuse radio lobes where $B \sim 10^{-5} \text{ G}$ takes $t_{\text{syn}} \sim 10^8 \text{ yr}$ ([Ghisellini 2013](#)). For the hot, tenuous plasmas characteristic of sub-Eddington accretion flows, equation (3.78) shows that synchrotron cooling is efficient enough

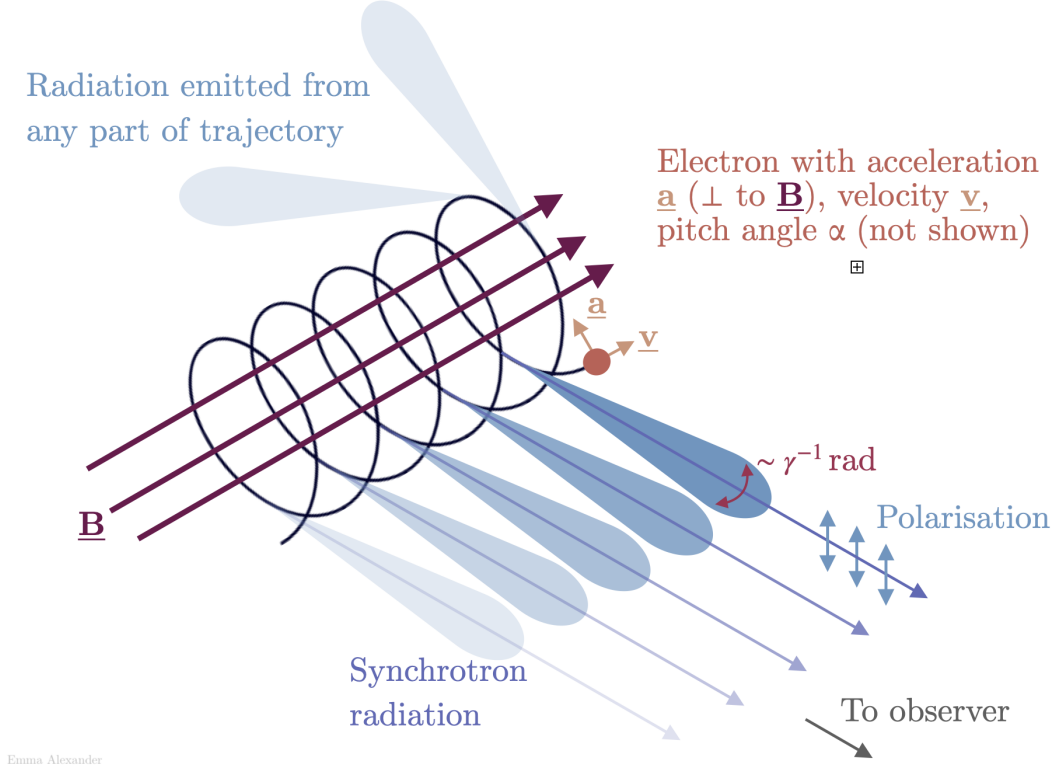


Figure 3.7: Schematic of the synchrotron radiation mechanism. An electron spirals around a magnetic field $\mathbf{B}$ with velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ perpendicular to $\mathbf{B}$, characterized by pitch angle α . Radiation is emitted along the helical trajectory and beamed into a cone of half-angle $\sim \gamma^{-1}$ in the direction of motion. The emission is linearly polarized perpendicular to the plane containing $\mathbf{v}$ and $\mathbf{B}$. Credit: Emma L. Alexander.

to dominate the electron energy budget when B and γ are both large, making synchrotron the primary radiative mechanism (Yuan and Narayan 2014).

3.4.2 General Relativistic Radiative Transfer

The radiation field measured by a distant observer from an accreting black hole is obtained by combining photon propagation in curved spacetime with the local interaction of radiation and plasma. This requires solving the radiative transfer equation along null geodesics that connect the emitting region to the observer. The geodesic construction is described in Section 3.5.1, and here we focus on the evolution of radiation along those trajectories.

In a covariant formulation, the radiative transfer equation is written in terms of the Lorentz invariant intensity $\mathcal{I} = I_\nu/\nu^3$. Along a null geodesic parameterized by the affine parameter λ , it satisfies (Lindquist 1966; Misner et al. 1973)

$$\frac{d\mathcal{I}}{d\lambda} = \frac{j_\nu}{\nu^2} - \nu\alpha_\nu\mathcal{I}, \quad (3.79)$$

where j_ν and α_ν are the emissivity and absorptivity evaluated in the comoving frame of the plasma, and ν denotes the photon frequency measured in that frame. In the absence of emission and absorption, $\mathcal{I}$ remains constant along the geodesic, reflecting Liouville's theorem in curved spacetime.

The mapping between the emitter frame and the observer frame is governed by the photon four momentum k^μ and the four velocities of the emitter and observer. The frequency shift is given by

$$g = \frac{\nu_{\text{obs}}}{\nu_e} = \frac{k_\mu u_{\text{obs}}^\mu}{k_\nu u_e^\nu}, \quad (3.80)$$

which incorporates gravitational redshift, Doppler boosting, and aberration in a unified manner. The observed specific intensity is related to the emitted one through $I_\nu^{\text{obs}} = g^3 I_\nu^e$, a direct consequence of the invariance of $\mathcal{I}$.

For numerical integration it is often convenient to express the transfer equation in terms of the optical depth. Defining $d\tau_\nu = \alpha_\nu \nu d\lambda$, equation (3.79) can be written as

$$\frac{d\mathcal{I}}{d\tau_\nu} = \mathcal{S}_\nu - \mathcal{I}, \quad (3.81)$$

where $\mathcal{S}_\nu = j_\nu/(\nu^3 \alpha_\nu)$ is the invariant source function. The formal solution along a geodesic segment then follows as an exponential attenuation of incoming intensity combined with local emission.

The emission and absorption coefficients depend on the plasma properties supplied by the underlying accretion flow model. For the systems considered in this thesis, the dominant radiative process at millimeter wavelengths is synchrotron radiation from a thermal electron population. The electron distribution is described by a Maxwell-Jüttner distribution characterized by the dimensionless temperature $\Theta_e = k_B T_e / m_e c^2$. The emissivity is obtained by integrating the single electron synchrotron power over this distribution, which in general requires numerical evaluation. In practice, accurate fitting formulas are employed to capture both the mildly relativistic and ultra relativistic regimes relevant for low luminosity accretion flows (Pandya et al. 2016). The absorptivity is determined consistently through Kirchhoff's law, ensuring that the source function approaches the Planck spectrum in the optically thick limit.

3.4.3 Polarized Radiative Transfer

Synchrotron radiation from a magnetized accretion flow is intrinsically polarized. In the geometric-optics regime relevant to millimeter-wavelength observations, the photon wavelength is much shorter than the characteristic curvature and plasma

scales, so the radiation field can be treated as a bundle of null rays carrying a slowly varying amplitude and polarization state. The natural starting point is the Wentzel–Kramers–Brillouin (WKB) expansion of Maxwell’s equations in curved spacetime, from which the null geodesic equation and the transport law for the polarization vector follow directly (Misner et al. 1973; Gammie and Leung 2012).

Geometric optics and the polarization vector

Consider the electromagnetic four-potential in the WKB form

$$A^\mu = \Re[(a^\mu + \epsilon b^\mu + \dots) e^{i\theta/\epsilon}], \quad (3.82)$$

where θ is a rapidly varying phase, a^μ is the leading-order complex amplitude, and $\epsilon \sim \lambda_{\text{em}}/L$ is the ratio of the photon wavelength to the background length scale. The wave covector is

$$k_\mu \equiv \nabla_\mu \theta. \quad (3.83)$$

In vacuum, Maxwell’s equations are

$$\nabla_\mu F^{\mu\nu} = 0, \quad \nabla_{[\alpha} F_{\beta\gamma]} = 0. \quad (3.84)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Substituting equation (3.82) into equation (3.84) and collecting terms in powers of ϵ yields the geometric-optics hierarchy (Misner et al. 1973; Gammie and Leung 2012). At order ϵ^{-2} one obtains the eikonal equation,

$$k^\mu k_\mu = 0, \quad (3.85)$$

so the wave fronts are null. Taking the covariant derivative of equation (3.85) gives

$$k^\nu \nabla_\nu k_\mu = \frac{1}{2} \nabla_\mu (k^\nu k_\nu) = 0, \quad (3.86)$$

which is the null geodesic equation for affinely parametrized rays.

At order ϵ^{-1} , Maxwell’s equations give the transport equation for the complex amplitude,

$$k^\nu \nabla_\nu a^\mu + \frac{1}{2} a^\mu \nabla_\nu k^\nu = 0, \quad (3.87)$$

together with the transversality condition

$$k_\mu a^\mu = 0. \quad (3.88)$$

It is convenient to split the amplitude into a scalar factor and a normalized

polarization vector f^μ ,

$$a^\mu = \mathcal{A} f^\mu, \quad f_\mu \bar{f}^\mu = 1, \quad k_\mu f^\mu = 0, \quad (3.89)$$

where $\mathcal{A}$ is the scalar amplitude and $\bar{f}^\mu$ denotes the complex conjugate. Substituting equation (3.89) into equation (3.87) separates the scalar amplitude transport from the polarization transport,

$$k^\nu \nabla_\nu \mathcal{A} = -\frac{1}{2} \mathcal{A} \nabla_\nu k^\nu, \quad (3.90)$$

$$k^\nu \nabla_\nu f^\mu = 0. \quad (3.91)$$

Equation (3.91) states that the polarization vector is parallel transported along the null ray. In affine parameter form, with $k^\mu = dx^\mu/d\lambda$, this becomes

$$\frac{df^\mu}{d\lambda} = -\Gamma_{\nu\sigma}^\mu k^\nu f^\sigma, \quad (3.92)$$

which is the form used when the polarization state is propagated directly along geodesics.

Covariant polarized radiative transfer

Once the geometric-optics limit has been established, the radiation field is no longer described by a single wave packet but by an ensemble of incoherent packets with slowly varying amplitudes. The convenient covariant object is the polarization tensor, or coherency matrix,

$$N_{\mu\nu} \equiv \langle a_\mu a_\nu^* \rangle, \quad (3.93)$$

where the angle brackets denote an ensemble average over wave packets (Gammie and Leung 2012). The tensor $N_{\mu\nu}$ carries the full polarization content of the field, but only its transverse components are physical. The observable part is obtained by projecting onto the two-dimensional screen orthogonal to both the photon wavevector k^μ and the observer four-velocity u^μ .

At leading order in the WKB expansion,

$$F^{\mu\nu} = i (k^\mu a^\nu - k^\nu a^\mu), \quad (3.94)$$

and the electric field measured by an observer with four-velocity u^μ is

$$E^\mu \equiv u_\nu F^{\mu\nu}. \quad (3.95)$$

Using $\omega \equiv -k_\mu u^\mu$ and the screen projection operator $s^\mu{}_\nu$, this becomes

$$E^\mu = -i\omega s^\mu{}_\nu a^\nu, \quad (3.96)$$

with

$$\begin{aligned} s^{\mu\nu} &= g^{\mu\nu} + u^\mu u^\nu - e_{(K)}^\mu e_{(K)}^\nu, \\ e_{(K)}^\mu &= \frac{k^\mu}{\omega} - u^\mu, \end{aligned} \quad (3.97)$$

which shows that the measurable field depends only on the transverse part of the amplitude. The corresponding polarization tensor is

$$P^{\mu\nu} \equiv \langle E^\mu E^{*\nu} \rangle = \omega^2 \tilde{N}^{\mu\nu}, \quad \tilde{N}^{\mu\nu} \equiv s^\mu{}_\alpha s^\nu{}_\beta N^{\alpha\beta}, \quad (3.98)$$

so $P^{\mu\nu}$ is simply the screen-projected version of $N_{\mu\nu}$. Its non-zero components form a 2×2 Hermitian matrix in any orthonormal basis on the screen, and this matrix is the direct relativistic analog of the usual coherency matrix.

If $\{e_{(1)}^\mu, e_{(2)}^\mu\}$ is an orthonormal basis spanning the screen, then the projected tensor $P_{(a)(b)}$ contains the Stokes parameters in the standard way,

$$I = P_{(1)(1)} + P_{(2)(2)}, \quad (3.99)$$

$$Q = P_{(1)(1)} - P_{(2)(2)}, \quad (3.100)$$

$$U = P_{(1)(2)} + P_{(2)(1)}, \quad (3.101)$$

$$V = i(P_{(1)(2)} - P_{(2)(1)}). \quad (3.102)$$

The covariant transfer equation then evolves $N_{\mu\nu}$ along the null geodesic in a plasma as,

$$k^\alpha \nabla_\alpha N_{\mu\nu} = J_{\mu\nu} + H_{\mu\nu\kappa\lambda} N^{\kappa\lambda}, \quad (3.103)$$

where $J_{\mu\nu}$ represents the polarized emissivity and $H_{\mu\nu\kappa\lambda}$ represents the absorption, Faraday rotation, and Faraday conversion ([Gammie and Leung 2012](#)).

The form of equation (3.103) makes the separation between spacetime propagation and local plasma interaction explicit. In practice, the integration can be carried out in different but equivalent ways, depending on the choice of dependent variables and basis along the ray. One may evolve the full tensor $N_{\mu\nu}$ in a coordinate basis, project it onto a chosen tetrad at each step, or restrict the evolution to its screen-projected components, which correspond directly to the Stokes parameters. An alternative is to construct a parallel-transported polarization basis along the geodesic, in which the geometric evolution is absorbed into the basis and only the plasma terms contribute to the transfer. These ap-

proaches differ primarily in numerical implementation and stability, particularly in strongly magnetized or turbulent flows where the local polarization frame can vary rapidly along the line of sight. The following section adopts the ray-tracing framework, where null geodesics are integrated through the spacetime and the polarized transfer equation is solved simultaneously along each ray.

3.5 Synthetic Images of Accreting Black Holes

The radiation field seen by a distant observer is obtained by solving the general relativistic radiative transfer problem along null geodesics connecting the emitting plasma to the observer, effectively introducing ‘general relativistic ray-tracing and radiative transfer (GRRT)’. For a given accretion model, this amounts to mapping each point on the observer’s image plane to a unique photon trajectory in the spacetime, evaluating the local emission and absorption along that trajectory, and transporting the radiation field to infinity.

The three accretion models considered in this chapter produce images with the same underlying structure, namely a bright emission region surrounding the black hole shadow and bounded by the photon ring, but differ in their detailed morphology, asymmetry, and vertical extent of emission. The comparison between models therefore isolates the role of plasma dynamics and magnetic field geometry at fixed spacetime.

3.5.1 Backward Ray-Tracing

The GRRT problem is solved using backward ray-tracing. A virtual camera is placed at large distance from the black hole, and each pixel on the observer’s screen defines a direction on the sky. A null geodesic is then integrated backward in affine parameter from the observer into the spacetime. The radiative transfer equation is solved simultaneously along this trajectory. This construction ensures that only photons that reach the observer are followed.

The photon trajectory $x^\mu(\lambda)$ satisfies the null geodesic equation

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\nu\sigma}^\mu \frac{dx^\nu}{d\lambda} \frac{dx^\sigma}{d\lambda} = 0, \quad g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0. \quad (3.104)$$

In the Kerr spacetime, these equations are separable (Carter 1968). The conserved quantities E , L_z , and $\mathcal{C}$ define the observer-plane impact parameters

$$\tilde{\alpha} = -\frac{L_z}{E}, \quad \tilde{\beta} = \frac{\sqrt{\mathcal{C}}}{E}, \quad (3.105)$$

which map directly to Cartesian coordinates on the image plane (Bardeen et al. 1972). In non-separable spacetimes, the system in equation (3.104) is integrated directly from the metric connection. As discussed in Section 3.3.4, the AA spacetime with $X = \Sigma$ admits a Carter constant for null geodesics, so the same parameterization applies.

Along each geodesic, the radiation field is evolved according to the polarized transfer equation introduced in Section 3.4.3. The mapping between emitted and observed frequency is set by the redshift factor in equation (3.80). In the absence of absorption, the Lorentz-invariant quantity I_ν/ν^3 is conserved, so that $I_\nu^{\text{obs}} = g^3 I_\nu^{\text{em}}$. In the general case, the full Stokes vector is evolved along the ray, with local emissivities, absorptivities, and Faraday coefficients evaluated in the comoving frame and transformed to the integration frame.

The polarization state is transported consistently with the spacetime geometry. In practice, this is achieved either by parallel-transporting a polarization basis along the ray or by evolving the coherency tensor and projecting onto the local screen. The observed EVPA is obtained from the final projection onto the observer’s image plane.

All calculations in this thesis are performed with *ipole* (Mościbrodzka and Gammie 2018; Prather et al. 2023), which integrates the geodesic and polarized transfer equations simultaneously in arbitrary axisymmetric stationary spacetimes. At each step along the ray, the plasma properties are evaluated in the fluid rest frame and the transfer equation is solved assuming constant coefficients over the step. The synchrotron emissivities, absorptivities, and Faraday coefficients are computed using the fitting formulas of Pandya et al. 2016 for a thermal electron distribution. The code has been benchmarked against independent GRRT implementations and analytic solutions, demonstrating sub-percent agreement in total intensity and polarized structure (Gold 2020; Prather et al. 2023).

The framework described here provides a direct mapping from plasma properties in the accretion flow to observable images, which is used in the following sections to compare the thin-disk, semi-analytic RIAF, and GRMHD models.

3.5.2 Polarimetric Observables

The polarization state of radiation at a given frequency and sky position is described by the Stokes parameters (I, Q, U, V) . The total intensity is given by I , while Q and U carries linear polarization with respect to a chosen basis on the image plane, and V describes circular polarization. These satisfy $I^2 \geq Q^2 + U^2 + V^2$, with equality for fully polarized radiation.

The linear polarization can be written in complex form as

$$\mathcal{P} = Q + iU, \quad (3.106)$$

which directly represents both its magnitude and orientation. The fractional linear polarization and the electric vector position angle (EVPA) are then

$$|\mathcal{P}| = \sqrt{Q^2 + U^2}, \quad (3.107)$$

$$\chi = \frac{1}{2} \arg(\mathcal{P}) = \frac{1}{2} \arctan\left(\frac{U}{Q}\right), \quad (3.108)$$

where the factor of one half reflects the invariance of the polarization direction under a rotation by π . The fractional circular polarization is given by

$$\mathcal{V} = \frac{V}{I}. \quad (3.109)$$

The EVPA is measured on the observer's sky relative to a fixed reference direction.

For a resolved image, let A denote the field of view solid angle on the observer's screen. The quantities with a tilde are the Stokes brightnesses, that is, the flux density per unit solid angle. The total intensity and the complex linear polarization integrated over the image are then

$$I_{\text{tot}} = \int_A \tilde{I} \, dA, \quad (3.110)$$

$$\mathcal{P}_{\text{tot}} = \int_A \tilde{\mathcal{P}} \, dA = \int_A (\tilde{Q} + i\tilde{U}) \, dA, \quad (3.111)$$

where I_{tot} and $\mathcal{P}_{\text{tot}}$ are measured in Jansky (Jy), while tilde over a quantity refers to the flux density per solid angle. The net linear polarization fraction, which corresponds to the coherently summed polarization over the full image, is

$$|m|_{\text{net}} = \frac{|\mathcal{P}_{\text{tot}}|}{I_{\text{tot}}} = \frac{1}{I_{\text{tot}}} \left| \int_A \tilde{\mathcal{P}} \, dA \right|. \quad (3.112)$$

A complementary measure is the resolved linear polarization fraction,

$$\langle |m| \rangle = \frac{1}{I_{\text{tot}}} \int_A |\tilde{\mathcal{P}}| \, dA, \quad (3.113)$$

which weights the local polarized intensity without phase cancellation across the image. The difference between $|m|_{\text{net}}$ and $\langle |m| \rangle$ therefore measures how coherently the linear polarization is ordered over the field of view.

In analogy with the linear polarization, the total circularly polarized flux den-

sity is obtained by integrating the corresponding brightness over the image,

$$V_{\text{tot}} = \int_{\mathcal{A}} \tilde{V} \, dA, \quad (3.114)$$

and the net circular polarization fraction is defined as

$$m_{V,\text{net}} = \frac{V_{\text{tot}}}{I_{\text{tot}}}. \quad (3.115)$$

Since V is a signed quantity, this measure retains information about the handedness of the circular polarization.

A resolved measure of the circular polarization can be defined by taking the magnitude of the local contribution,

$$\langle |m_V| \rangle = \frac{1}{I_{\text{tot}}} \int_{\mathcal{A}} |\tilde{V}| \, dA, \quad (3.116)$$

which removes cancellation between regions of opposite sign. In the models considered here, circular polarization is generated primarily through Faraday conversion of linearly polarized emission, and therefore remains sub-dominant compared to the linear component across the image except in cases where turbulent magnetic fields introduce additional structure, as in GRMHD simulations.

Together, $|m|_{\text{net}}$, $\langle |m| \rangle$, $m_{V,\text{net}}$, and $\langle |m_V| \rangle$ provide a consistent set of observables that distinguish between coherent large-scale polarization structure and local polarization strength, while retaining sensitivity to propagation effects within the accretion flow.

3.5.3 The Novikov–Thorne Thin disk

The NT thin disk occupies the equatorial plane, radiates as a local blackbody at the effective temperature given by equation (3.10), and carries no emission above or below the midplane. Figure 3.8 presents a representative grid of synthetic polarized images spanning five spin values and five observer inclinations. The background greyscale represents the total intensity, and the EVPA tick marks are overlaid with their color indicating the local fractional linear polarization $|m| = |\tilde{P}|/\tilde{I}$.

For face-on inclination ($\theta_i = 0^\circ$) the image is nearly axisymmetric for all spin values. The emitting disk appears as a bright annulus whose inner boundary traces the shadow, bounded by the critical curve on the observer’s sky. At any finite image resolution, only the $n = 0$ direct image and the $n = 1$ lensed image contribute appreciably to the total flux in the ring region. Spin modifies the face-on morphology in two ways: it shifts the inner edge of the emitting disk,

which terminates at the ISCO, and it breaks the axisymmetry of the critical curve through frame-dragging. For a Schwarzschild black hole $r_{\text{ISCO}} = 6 M$; increasing the prograde spin to $a_\star = 0.99$ contracts the ISCO to $r_{\text{ISCO}} \approx 1.45 M$ (Bardeen et al. 1972), placing the inner disk edge close to the horizon and producing a tighter, more luminous ring. The progression is visible across the rows of Figure 3.8: both the shadow and the photon ring shrink with increasing $a_\star$, and the peak intensity in the innermost disk region rises, consistent with the deeper gravitational potential accessible to prograde circular orbits at small radii (cf. equation (3.11) and Figure 3.1).

As inclination increases, Doppler boosting and relativistic aberration break the left-right symmetry and produce the characteristic crescent morphology analyzed by Luminet 1979. The approaching limb brightens and the receding limb dims. At $\theta_i = 75^\circ$ and $\theta_i = 90^\circ$ the direct image of the near disk face dominates, while the secondary image of the far disk face is gravitationally lensed over the shadow and compressed into a narrow arc along the projected equatorial plane. The Doppler asymmetry strengthens with spin at fixed inclination, reflecting the higher orbital velocity at the ISCO and the steeper gravitational redshift gradient at smaller radii.

The face-on EVPA panels display a symmetric pattern with tick marks oriented radially, the signature of a predominantly toroidal field whose sky-plane projection is tangential at every point on the disk. The tick colors at $\theta_i = 0^\circ$ are uniformly blue across all spin values, confirming that the bulk disk polarization fraction is small throughout: $|m| \lesssim 0.02$ over most of the emitting region. The net linear polarization integrated over the image vanishes exactly at $\theta_i = 0^\circ$ by axisymmetry. The exception is the photon ring, where strongly curved photon trajectories rotate the polarization basis vector by a large angle during parallel transport to the observer, producing a ring of elevated $|m|$ visible in the tick colors even though the disk emission is locally thermal and nearly unpolarized. This photon ring contribution is responsible for the higher values toward the red end of the colorbar, which are spatially confined to the innermost annulus and become progressively narrower as $a_\star$ increases, tracking the inward contraction of the ISCO. As inclination increases, the cancellation of orthogonal polarization components is incomplete across the asymmetric crescent, and a net linearly polarized flux accumulates. The Doppler asymmetry between the two limbs concentrates the polarized flux on the approaching side, visible as the preferential brightening of tick colors on that side at high inclinations.

Overall, the NT disk provides a controlled reference morphology for evaluating more complete accretion models. The shadow is sharply bounded because the funnel above and below the equatorial plane is evacuated, and the photon ring is

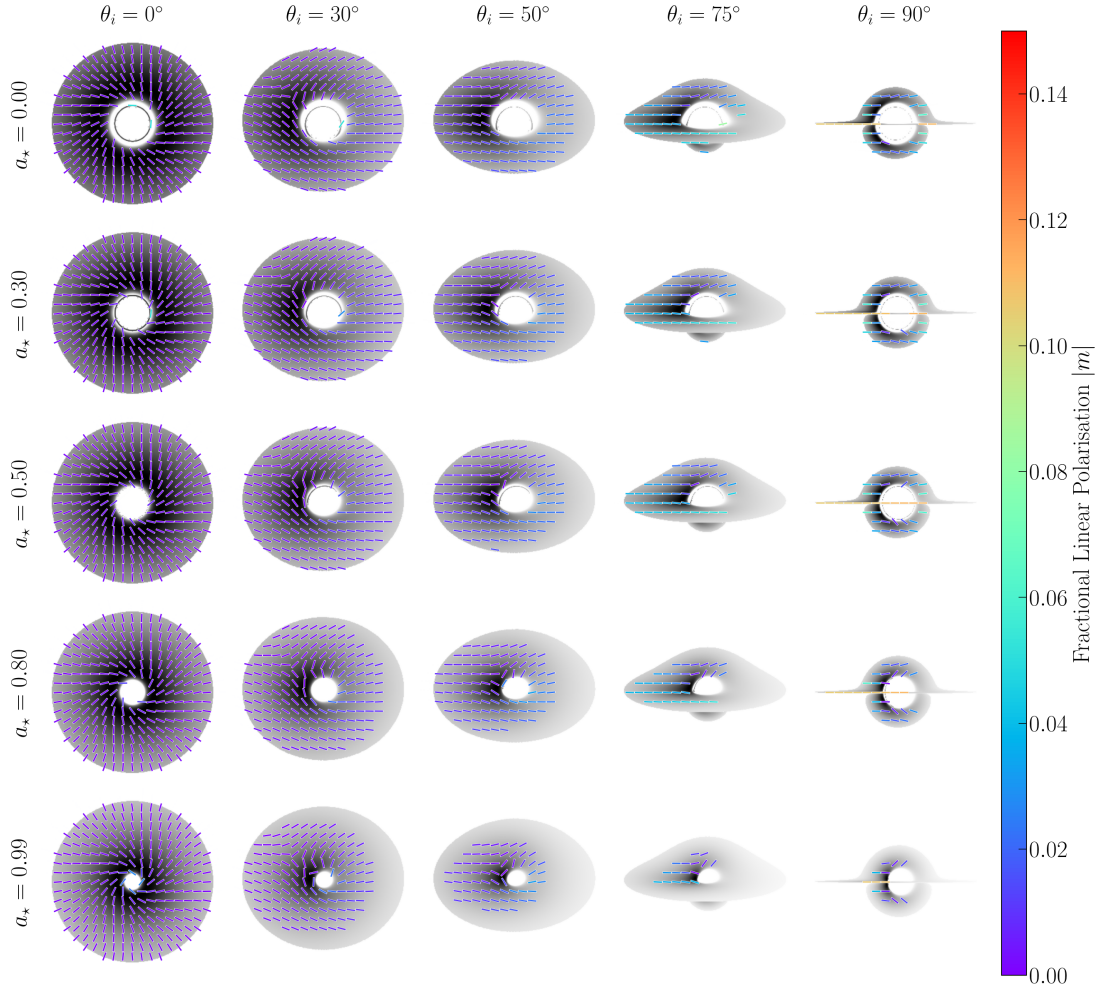


Figure 3.8: Raytraced polarized images of the Novikov–Thorne thin disk around a Kerr black hole, computed with `ipole`, for five spin values a_* (rows, top to bottom) and five observer inclinations θ_i (columns, left to right). The background greyscale shows the normalized total intensity. EVPA tick marks are overlaid with color encoding the local fractional linear polarization.

uncontaminated by off-equatorial emission. Any flux appearing inside or near the photon ring in a RIAF or GRMHD model that exceeds the NT template directly reflects the presence of emitting material above the disk midplane.

3.5.4 The Semi-Analytic RIAF

The semi-analytic RIAF model described in Section 3.2.1 has three free inputs that shape the synthetic image: the geometric distribution of emitting plasma, the velocity field, and the magnetic field configuration. The electron density and temperature follow the power-law profiles of equations (3.13) and (3.14), with the vertical extent controlled by the scale height H . Unlike the NT disk, which places all emission in the equatorial plane, the RIAF fills the funnel region with optically thin plasma continuously above and below the midplane. The azimuthal

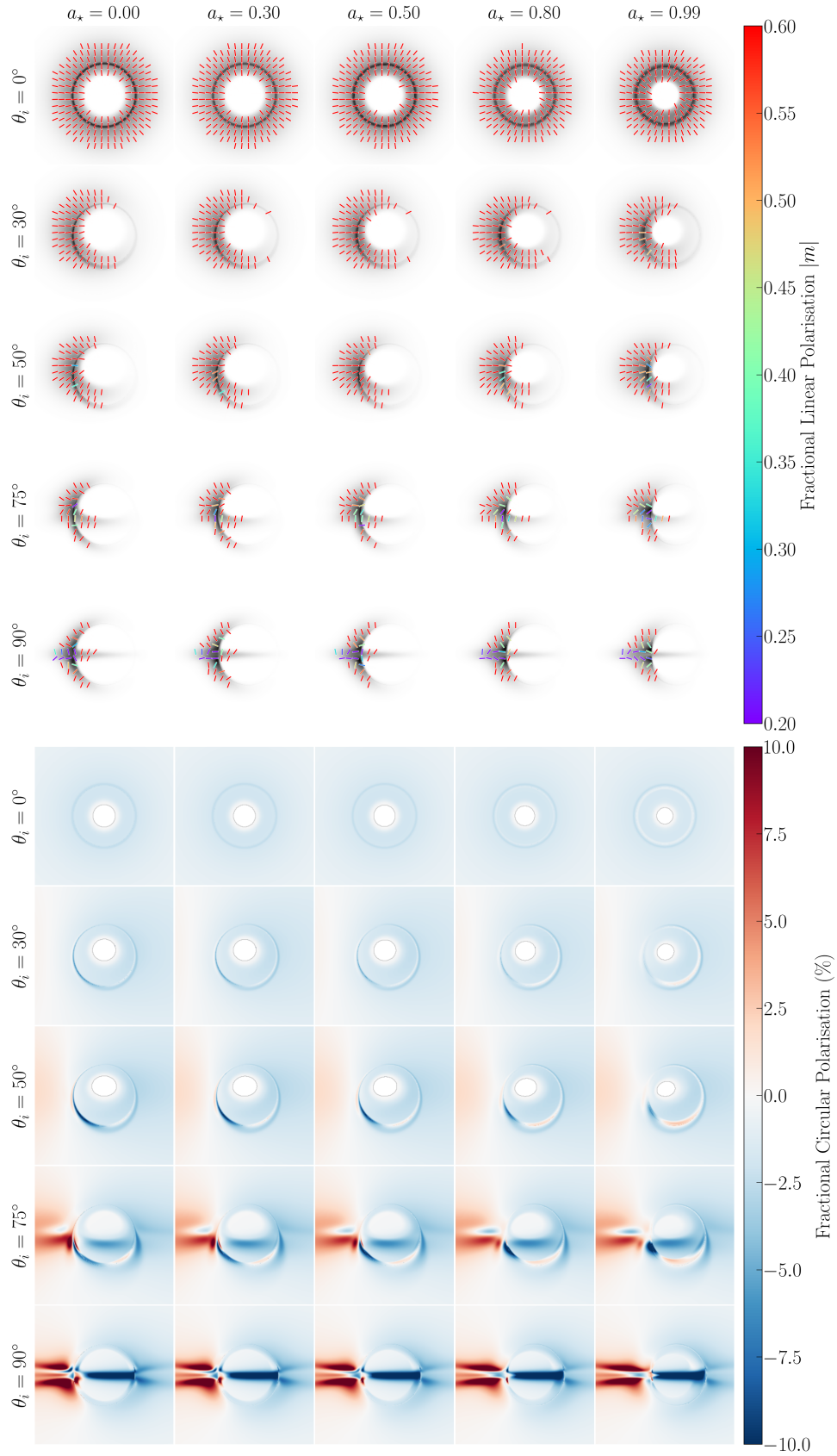


Figure 3.9: Same as Figure 3.8 but for RIAF model with fractional circular polarization $m_V = \tilde{V}/\tilde{I}$ in the bottom block.

velocity fraction κ sets the orbital contribution to the four-velocity, ranging from pure Keplerian rotation to radial free-fall, while the magnetic field is prescribed independently as a toroidal, poloidal, or tangled configuration with strength given by β_p (equation (3.17)). Radiative transfer is computed using a thermal Maxwell-Jüttner eDF throughout. The synchrotron emissivities and absorptivities for all four Stokes parameters are taken from the fitting formulas of Pandya et al. 2016, and the Faraday rotation and conversion coefficients follow Pandya et al. 2018, evaluated within `ipole` (Mościbrodzka and Gammie 2018; Prather et al. 2023) as described in Section 3.5.1.

The total intensity structure across Figure 3.9 broadly follows the inclination trends established for the NT disk, with a nearly symmetric ring at low inclination giving way to a Doppler-brightened crescent as θ_i increases. Off-equatorial plasma fills the funnel region, producing a more gradual inward intensity falloff and partially filling the central brightness depression relative to the NT case.

The fractional linear polarization throughout the RIAF is substantially higher than for the NT disk, which is expected for optically thin thermal synchrotron emission at 1.3 mm: without the opacity that suppresses LP in the thermal disk, locally ordered field geometries produce intrinsic polarization fractions approaching the theoretical maximum (Rybicki and Lightman 1979). At face-on inclination the EVPA pattern is radially oriented, the standard signature of a toroidal magnetic field (Broderick et al. 2009). Net image-integrated LP vanishes at $\theta_i = 0^\circ$ by symmetry and grows as the Doppler asymmetry between the approaching and receding sides breaks the bilateral cancellation, consistent with results from earlier semi-analytic RIAF modeling (Broderick et al. 2009; Pu and Broderick 2018). At near-edge-on inclinations, the larger Faraday depth along near-equatorial lines of sight reduces the polarization fraction in the projected disk midplane, as has been seen in comparable models (Mościbrodzka et al. 2016).

The fractional linear polarization is substantially higher than for the NT disk, which is expected for optically thin thermal synchrotron at 1.3 mm. Without the optical depth that suppresses LP in the thermal disk, locally ordered field geometries can approach the theoretical synchrotron maximum (Rybicki and Lightman 1979). At face-on inclination the EVPA pattern is radially oriented, the standard signature of a toroidal magnetic field (Broderick et al. 2009). The net image-integrated LP vanishes at $\theta_i = 0^\circ$ by symmetry and grows as the Doppler asymmetry between the approaching and receding sides breaks the bilateral cancellation, consistent with earlier semi-analytic RIAF modeling (Broderick et al. 2009; Pu and Broderick 2018). At near-edge-on inclinations, the larger Faraday depth along near-equatorial lines of sight reduces the polarization fraction in the projected disk midplane, as has been seen in comparable models (Mościbrodzka

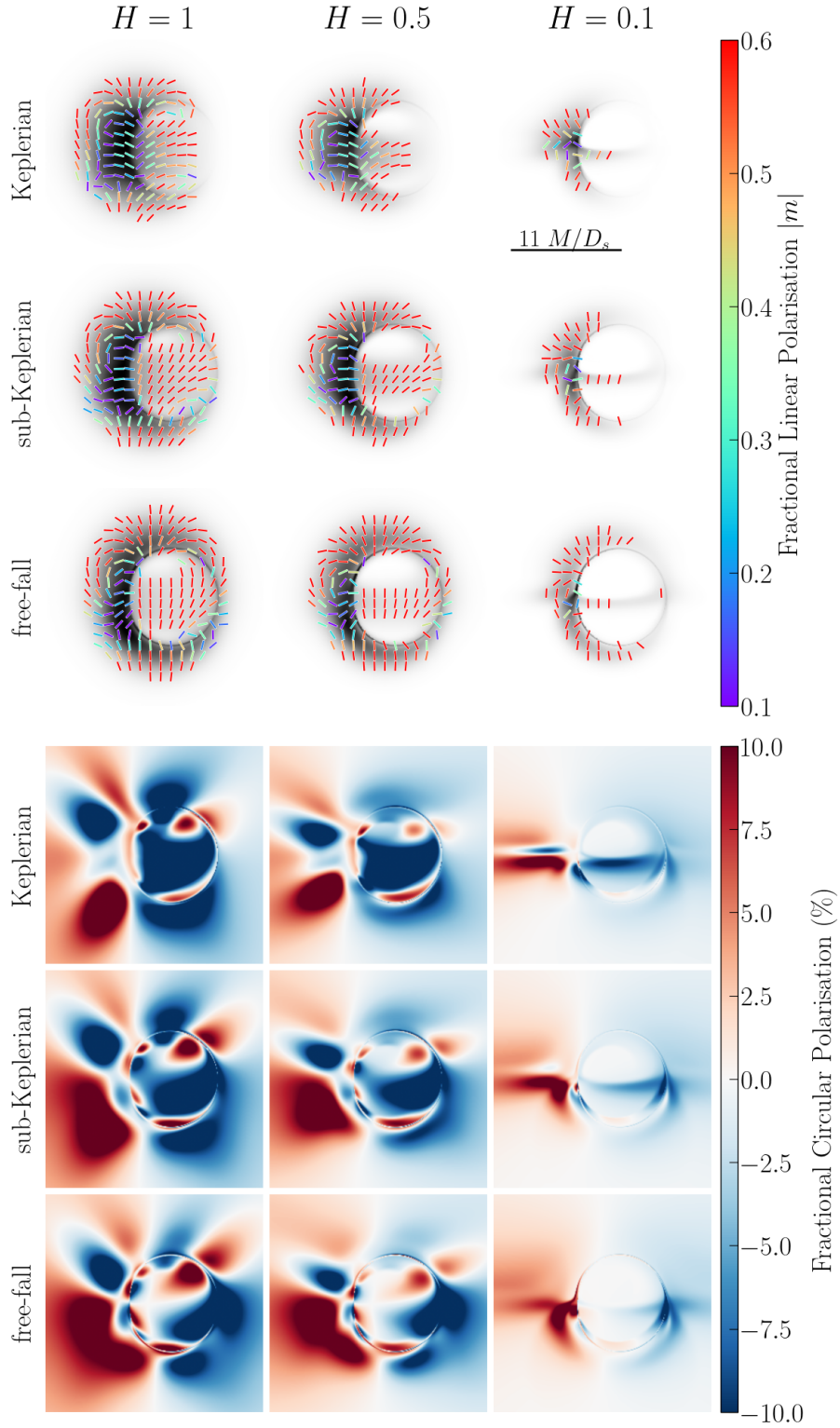


Figure 3.10: Same as Figure 3.9 but varying H (columns) and velocity model (rows: Keplerian, sub-Keplerian, free-fall), at fixed $a_\star = 0.94$ and $\theta_i = 80^\circ$.

et al. 2016).

The m_V panels show essentially no signal at low inclinations, growing in amplitude and developing an antisymmetric spatial structure with a sign reversal across the disk midplane from $\theta_i \approx 50^\circ$ onward. This arises geometrically since a toroidal field acquires a significant line-of-sight component only at high inclination, where $B_{\parallel}$ changes sign between the upper and lower hemispheres. Local m_V values reach several percent at $\theta_i \geq 75^\circ$, while the net image-integrated CP remains small due to near-bilateral cancellation. The dominant generation mechanism is Faraday conversion (Broderick and Blandford 2004; Pandya et al. 2018), the ρ_Q coupling in equation (3.103).

Figure 3.10 shows how the images change with H and κ at $a_\star = 0.94$ and $\theta_i = 80^\circ$. The disk height has the strongest effect. At $H = 1$ the flow is nearly spherical with only a shallow shadow, while at $H = 0.1$ emission concentrates near the midplane and a narrow lensed arc recovers the thin-disk geometry, consistent with the models of Pu and Broderick 2018. Decreasing κ reduces the Doppler asymmetry between limbs, as the growing radial velocity component enters the frequency shift g in equation (3.80) and shifts the apparent photon ring position (Broderick et al. 2009). The EVPA orientations remain broadly consistent with the toroidal field geometry across all velocity models, though the distribution would change substantially for different field topologies (Vos et al. 2022; Saurabh et al. 2026).

The m_V maps are most complex at $H = 1$, where contributions from plasma at widely separated latitudes along quasi-spherical lines of sight produce multiple sign changes. As H decreases the pattern collapses toward the antisymmetric two-lobe structure seen at high inclination in Figure 3.9. The velocity model modulates this further. Radial infall in the free-fall case modifies both the Faraday depth and the projected line-of-sight field direction, producing a more extended CP distribution than the Keplerian case. Both the $|m|$ and m_V morphologies are therefore sensitive to the disk geometry and the flow kinematics, and these were used in Saurabh et al. 2026 to constrain the disk scale height, velocity profile, and dominant field configuration in M87*.

Since H , κ , β_p , and the field orientation are independent RIAF model parameters, each can be varied to isolate its contribution to the Stokes maps. GRMHD does not permit this, as the field geometry, magnetization, and velocity structure are all coupled MHD outputs. The RIAF framework is therefore useful precisely where GRMHD is less transparent, in attributing specific image features to individual physical parameters, and the two approaches are compared directly in Section 3.3.

3.5.5 GRMHD Simulations

The fluid snapshots used here are drawn from the v3 Illinois KHARMA GRMHD library described in [Dhruv et al. 2025](#), which was used in the EHT analysis of Sgr A* ([Event Horizon Telescope Collaboration 2022](#)). Each simulation is initialized from a Fishbone–Moncrief torus ([Fishbone and Moncrief 1976](#)) parameterized by an inner disk radius r_{in} and a pressure-maximum radius r_{max} , with the internal energy perturbed to seed the magnetorotational instability ([Balbus and Hawley 1991](#)). The magnetic field configuration is set by a prescribed axisymmetric vector potential $A_\phi(r, \theta)$. For MAD simulations,

$$A_\phi = \max \left[\frac{\rho}{\rho_{\text{max}}} \left(\frac{r}{r_{\text{in}}} \right) \sin^3 \theta e^{-r/400} - 0.2, 0 \right], \quad (3.117)$$

which concentrates flux near the inner disk edge; for SANE,

$$A_\phi = \max \left[\frac{\rho}{\rho_{\text{max}}} - 0.2, 0 \right]. \quad (3.118)$$

In both cases the field is normalized so that the maximum ratio of gas to magnetic pressure is ~ 100 initially. The library covers five black hole spin values $a_\star = 0, \pm 1/2, \pm 15/16$ for each of the MAD and SANE states, with all simulations run to $t = 30\,000\,t_g$ ($t_g \equiv GM/c^3$) using $\hat{\gamma} = 4/3$ to ensure consistency with the EHT Sgr A* model library ([Event Horizon Telescope Collaboration 2022](#)). The full parameter set is listed in Table 3.2.

Here, MAD simulations show greater accretion rate variability than SANE at the same spin, with variability increasing with $a_\star > 0$ in both states. MAD disks are hotter than SANE disks within $r \lesssim 10\,r_g$, and produce wider jets with outflow efficiencies exceeding unity for $a_\star = +15/16$. The angular momentum and energy transport in MAD is dominated by the electromagnetic component through the jet, whereas in SANE the fluid sector drives inward transport with the MRI facilitating outward angular momentum flux in the disk ([Dhruv et al. 2025](#)).

GRMHD simulations treat the accreting plasma as a single ideal fluid, so the electron temperature is not a direct output and must be prescribed in post-processing. We follow [Mościbrodzka et al. 2016](#), in which the ion-to-electron temperature ratio depends on the local plasma magnetization:

$$\frac{T_i}{T_e} = R_{\text{high}} \frac{\beta_p^2}{1 + \beta_p^2} + R_{\text{low}} \frac{1}{1 + \beta_p^2}, \quad (3.119)$$

where $\beta_p = P_{\text{gas}}/P_{\text{mag}}$ and $R_{\text{low}} = 1$ throughout ([Mościbrodzka et al. 2016](#); [Wong](#)

Table 3.2: v3 GRMHD simulation suite (Dhruv et al. 2025). r_{in} and r_{max} are the inner and pressure-maximum radii of the initial Fishbone–Moncrief torus; r_{out} is the outer radial boundary; all radii in units of $r_g \equiv GM/c^2$. Resolution is $N_r \times N_\theta \times N_\phi$.

Flux	$a_\star$	$r_{\text{in}} (r_g)$	$r_{\text{max}} (r_g)$	$r_{\text{out}} (r_g)$	$\hat{\gamma}$	Resolution	Duration (t_g)
MAD	−0.94	20	41	1000	4/3	$288 \times 128 \times 128$	30,000
MAD	−0.5	20	41	1000	4/3	$288 \times 128 \times 128$	30,000
MAD	0	20	41	1000	4/3	$288 \times 128 \times 128$	30,000
MAD	0.5	20	41	1000	4/3	$288 \times 128 \times 128$	30,000
MAD	+0.94	20	41	1000	4/3	$288 \times 128 \times 128$	30,000
SANE	−0.94	10	20	1000	4/3	$288 \times 128 \times 128$	30,000
SANE	−0.5	10	20	1000	4/3	$288 \times 128 \times 128$	30,000
SANE	0	10	20	1000	4/3	$288 \times 128 \times 128$	30,000
SANE	0.5	10	20	1000	4/3	$288 \times 128 \times 128$	30,000
SANE	+0.94	10	20	1000	4/3	$288 \times 128 \times 128$	30,000

et al. 2022). In strongly magnetized regions ($\beta_p \ll 1$), $T_i/T_e \rightarrow 1$; in weakly magnetized regions ($\beta_p \gg 1$), $T_i/T_e \rightarrow R_{\text{high}}$. The parameter R_{high} therefore controls how strongly disk emission contributes at millimeter wavelengths: at low R_{high} disk and jet contribute comparably, while at high R_{high} disk electrons are cool and emission concentrates in the jet sheath and low- β_p corona (Mościbrodzka et al. 2016; Event Horizon Telescope Collaboration 2019). Each snapshot is ray-traced with `ipole` (Mościbrodzka and Gammie 2018; Prather et al. 2023) using the transfer coefficients of Pandya et al. 2016 and Pandya et al. 2018, post-processed following the PATOKA pipeline (Wong et al. 2022), with the mass unit adjusted to match the observed 1.3 mm flux of Sgr A* (Event Horizon Telescope Collaboration 2022).

Figure 3.11 shows five frames from a MAD simulation at $R_{\text{high}} = 160$ and $\theta_i = 70^\circ$, each separated by $500 t_g$ within $T \in [15\,000, 17\,000] t_g$. In the total intensity images, the bright arcs and filamentary spirals appear and dissolve on dynamical timescales driven by magnetic flux eruption events, a behavior that is characteristic of the MAD state (Narayan et al. 2022; Dhruv et al. 2025). The Stokes $\mathcal{Q}$ and $\mathcal{U}$ maps vary substantially between frames, with rapid sign changes reflecting the turbulent field configuration at snapshot resolution. Stokes $\mathcal{V}$, which is considerably weaker than the linear Stokes parameters, varies on similar timescales. Since individual snapshots are not representative of the time-averaged structure, all model-observation comparisons throughout this chapter

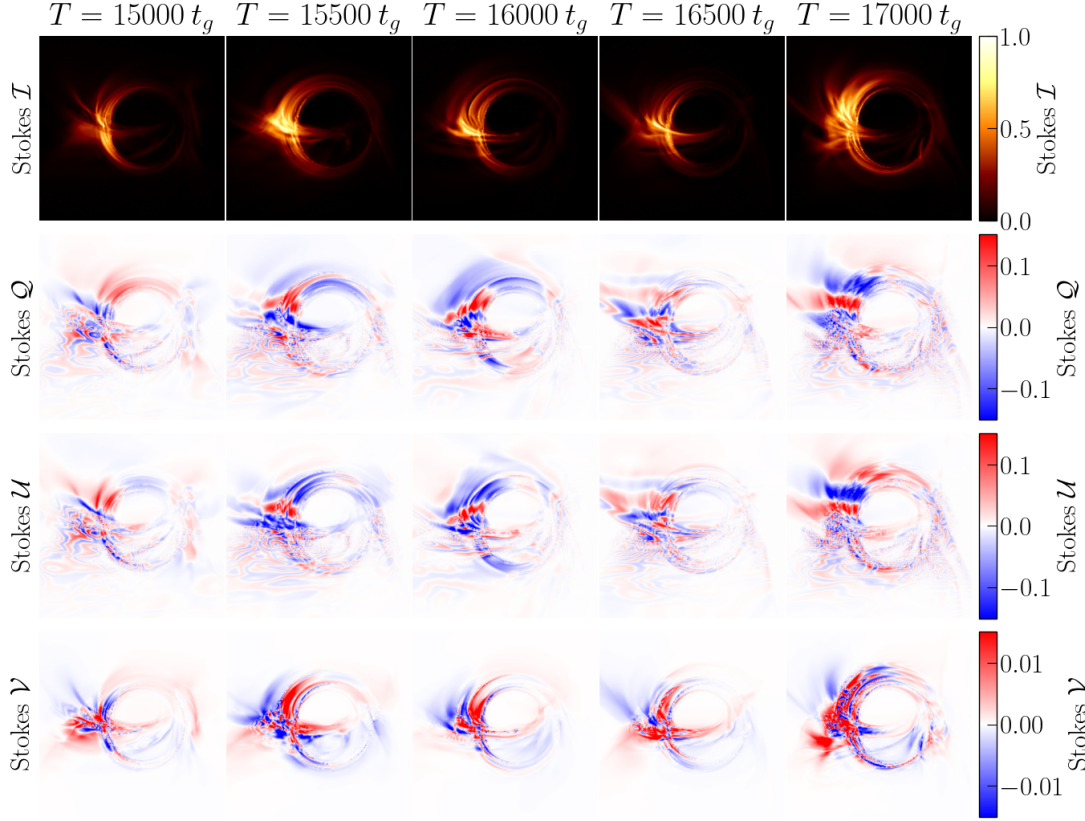


Figure 3.11: Five consecutive snapshots of a MAD GRMHD model at $R_{\text{high}} = 160$ and $\theta_i = 70^\circ$, separated by $500 t_g$. Rows show normalized Stokes $\mathcal{I}$, $\mathcal{Q}$, $\mathcal{U}$, and $\mathcal{V}$ respectively.

use images averaged over the full equilibrium window. Figures 3.12 and 3.13 show time-averaged polarimetric images across the R_{high} -spin grid at fixed low inclination. For MAD, the total intensity is ring-dominated throughout in which the ring size and brightness asymmetry vary modestly with spin, as expected from the changing ISCO location. The LP maps are relatively ordered with moderate fractional values, broadly consistent with the coherent large-scale field that characterizes the magnetically arrested state (Event Horizon Telescope Collaboration 2021). In the CP maps, there is a clear reversal of the dominant sign of m_V between retrograde and prograde spin. This trend persists across all R_{high} values, and arises because the field handedness threading the horizon is opposite in the two cases (Joshi et al. 2024). The non-spinning model shows a comparatively axisymmetric CP pattern. For SANE (Figure 3.13), at low R_{high} the total intensity is noticeably more extended with jet-like features outside the ring, owing to the weaker magnetic pressure and less evacuated funnel. As R_{high} increases the emission progressively concentrates toward the ring, mirroring the MAD trend. The LP maps are considerably more disordered than in MAD throughout the grid, a consequence of the absence of large-scale coherent flux (Event Horizon

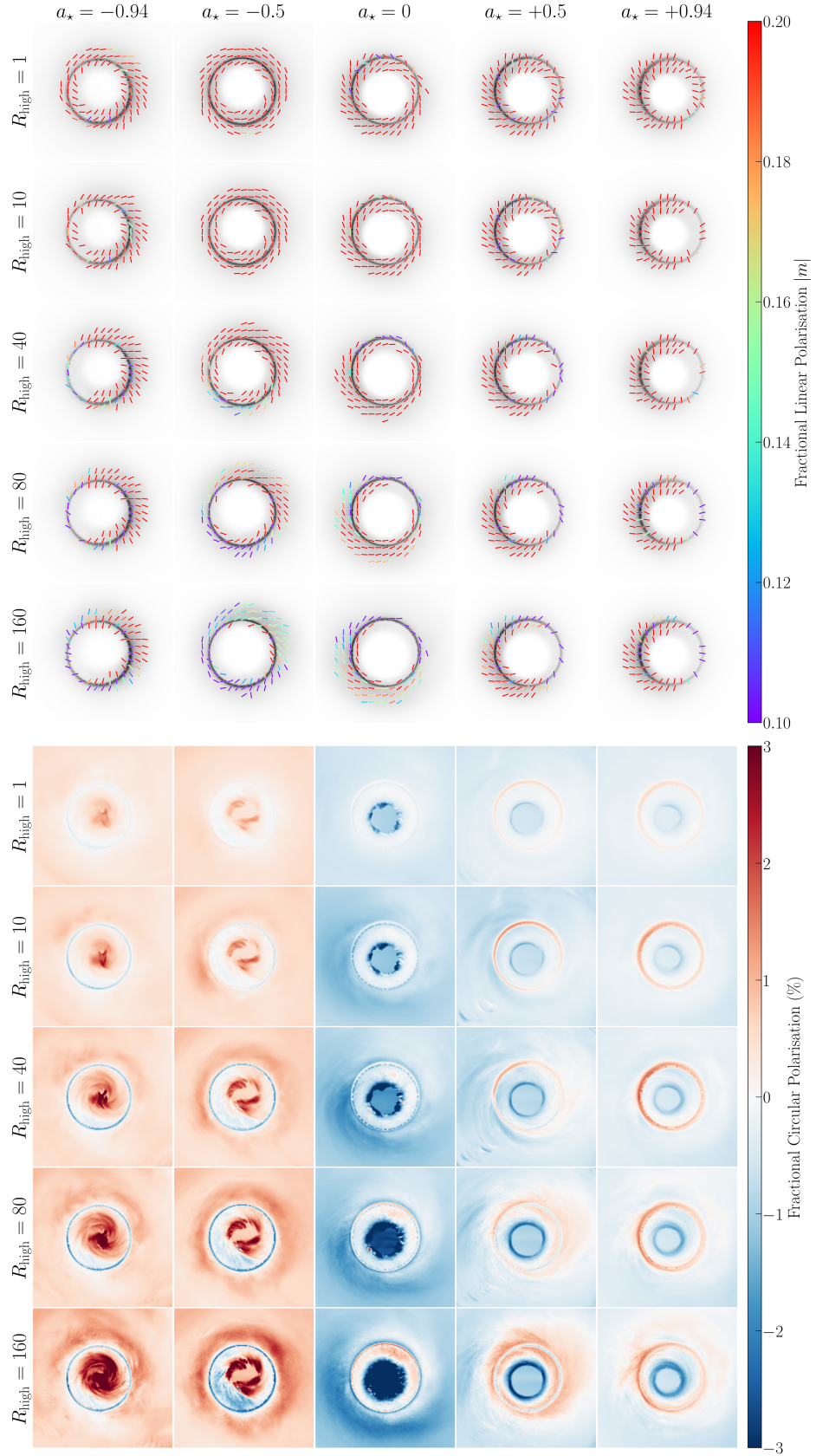


Figure 3.12: Time-averaged full polarimetric images from MAD GRMHD simulations. Top block: normalized total intensity (greyscale) with EVPA ticks colored by $|m|$. Bottom block: fractional circular polarization m_V . Columns are spin a_* ; rows are R_{high} .

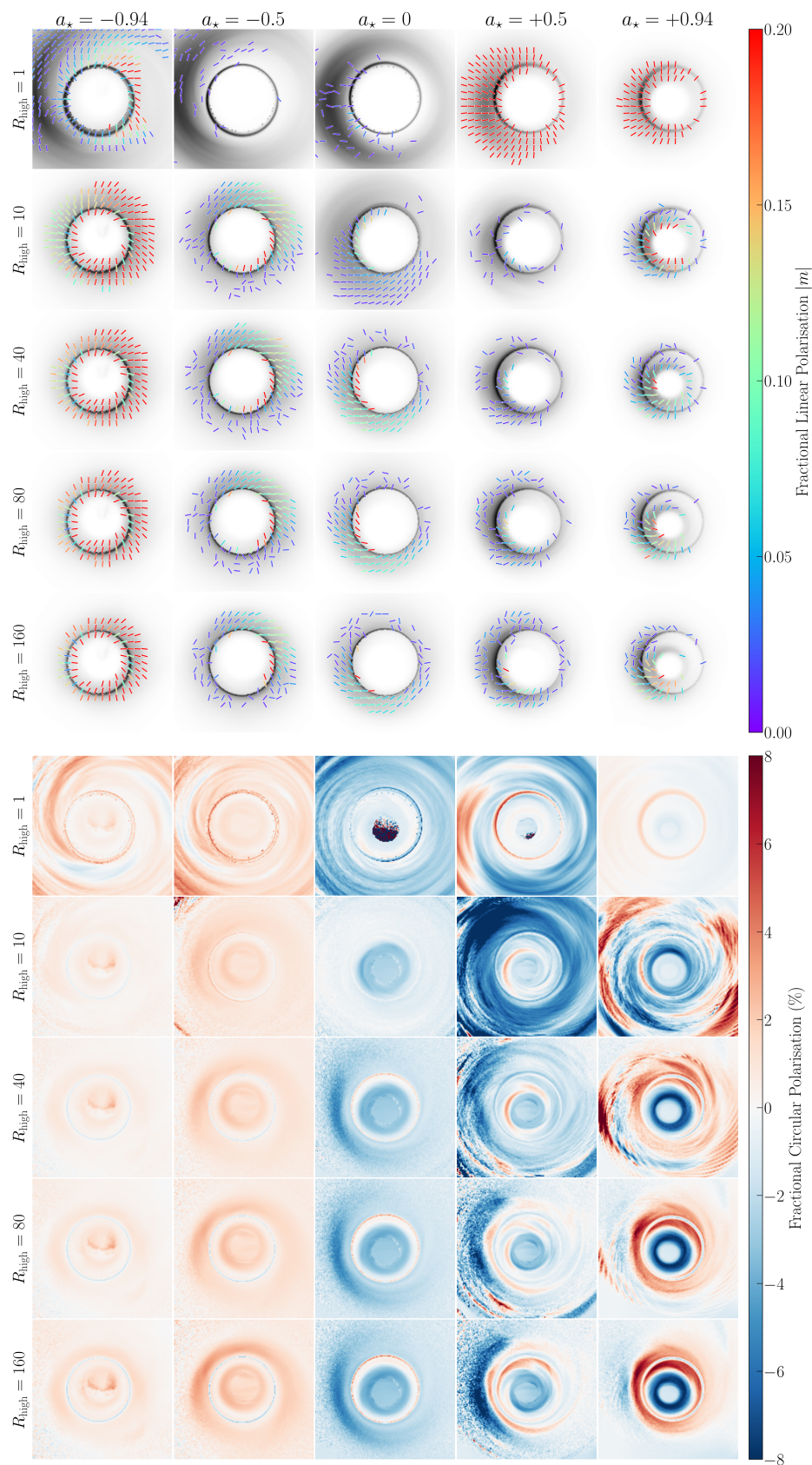


Figure 3.13: Same as Figure 3.12 but for SANE simulations.

Telescope Collaboration 2022). The CP maps show qualitatively richer structure and substantially larger amplitudes than in MAD at equivalent spin and R_{high} . This difference traces back to the Faraday conversion depth where the turbulent, weakly magnetized SANE disk generates significantly more ρ_Q coupling along typical lines of sight than the more strongly magnetized MAD flow (Joshi et al. 2024; Ricarte et al. 2021).

Figures 3.14 and 3.15 show the spin and inclination dependence at fixed $R_{\text{high}} = 160$. The total intensity morphology broadly follows the trends discussed in Sections 3.5.3 and 3.5.4 showcasing a roughly symmetric ring at low inclination giving way to a Doppler-brightened crescent as θ_i increases. Off-equatorial emission from the disk wind and the magnetically supported funnel walls adds flux outside the photon ring, something that is absent in the NT disk by construction.

In the LP maps, MAD shows a comparatively regular EVPA structure at low inclinations, where the organized large-scale field projects more cleanly onto the sky. As θ_i increases, a growing line-of-sight column through the inner flow depolarizes the emission via internal Faraday rotation, consistent with the EHT M87* polarimetric analysis (Event Horizon Telescope Collaboration 2021). SANE shows considerably more LP disorder across inclinations, driven by the more turbulent field structure. This contrast in the resolved LP morphology is among the main arguments for preferring MAD in EHT model comparisons for both M87* and Sgr A* (Event Horizon Telescope Collaboration 2021; Event Horizon Telescope Collaboration 2022)⁴.

The CP maps in both states share a common qualitative behavior that regions of opposite sign that largely cancel on image integration. At low inclinations the CP signal is weak, since the line-of-sight component of a predominantly toroidal field is small. From $\theta_i \approx 50^\circ$ onward an antisymmetric two-lobe structure develops as $B_{\parallel}$ changes sign between hemispheres (Ricarte et al. 2021). The main difference between the two states is amplitude. SANE produces considerably larger local m_V values, which again reflects the higher Faraday conversion depth in the less organized disk (Joshi et al. 2024). On image integration $|m_{V,\text{net}}|$ stays small in both states, with SANE models reaching larger values than MAD overall. ALMA measurements of Sgr A* at 1.3 mm find $|m_{V,\text{net}}| \approx 1\%$ (Joshi et al. 2024), which the majority of MAD models are consistent with, while the high-Faraday-depth end of the SANE parameter space is harder to reconcile with this constraint.

Among the three accretion models in this chapter, GRMHD is physically the most complete. The magnetic field geometry, velocity structure, and plasma density are MHD outputs rather than being prescribed by hand, and the electron

⁴Additionally, the presence of large scale jet across the spectrum, assuming the BZ process to be in play, adds to this constraint

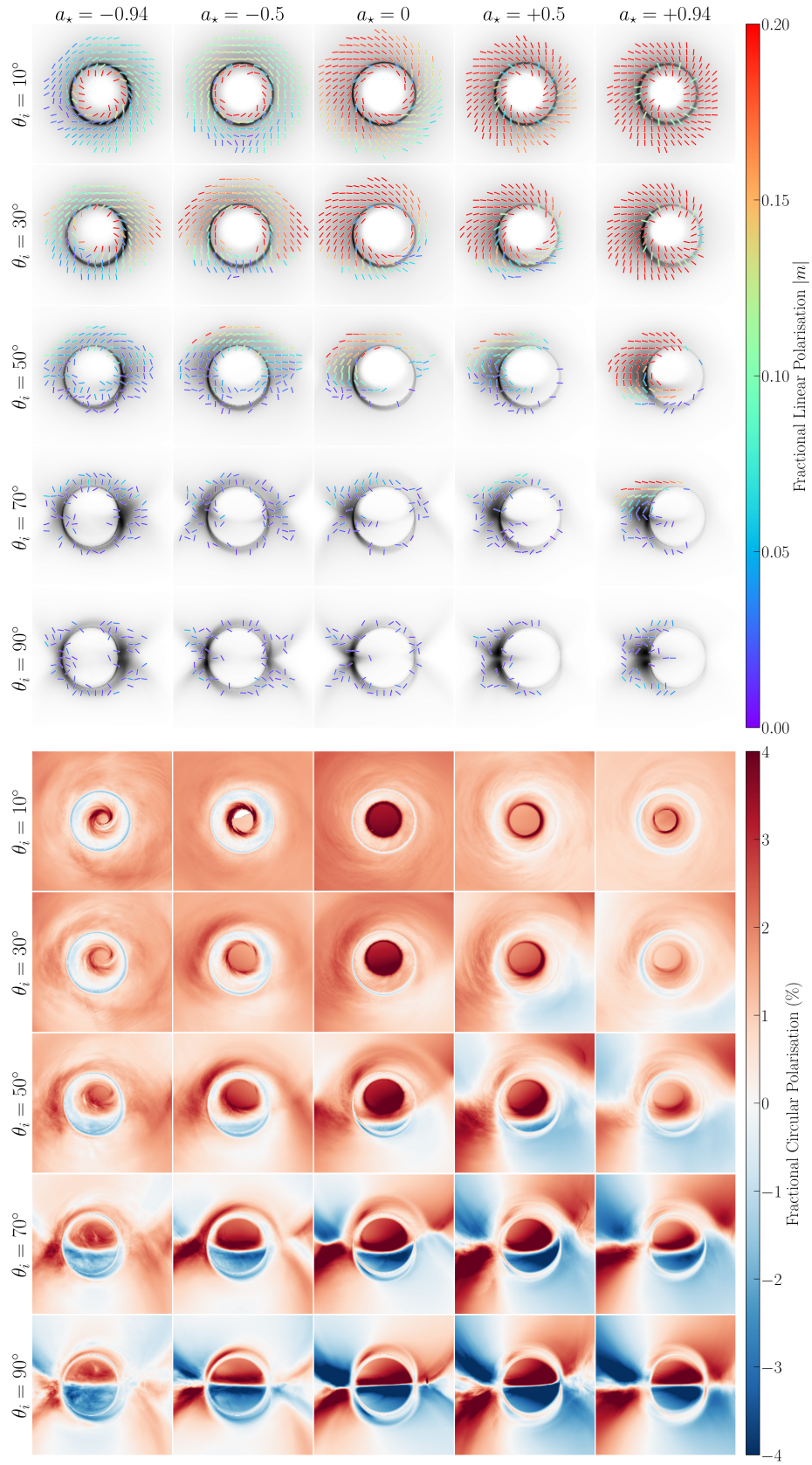


Figure 3.14: Same as Figure 3.12 but varying a_* (columns) and θ_i (rows) at fixed $R_{\text{high}} = 160$, for MAD simulations.

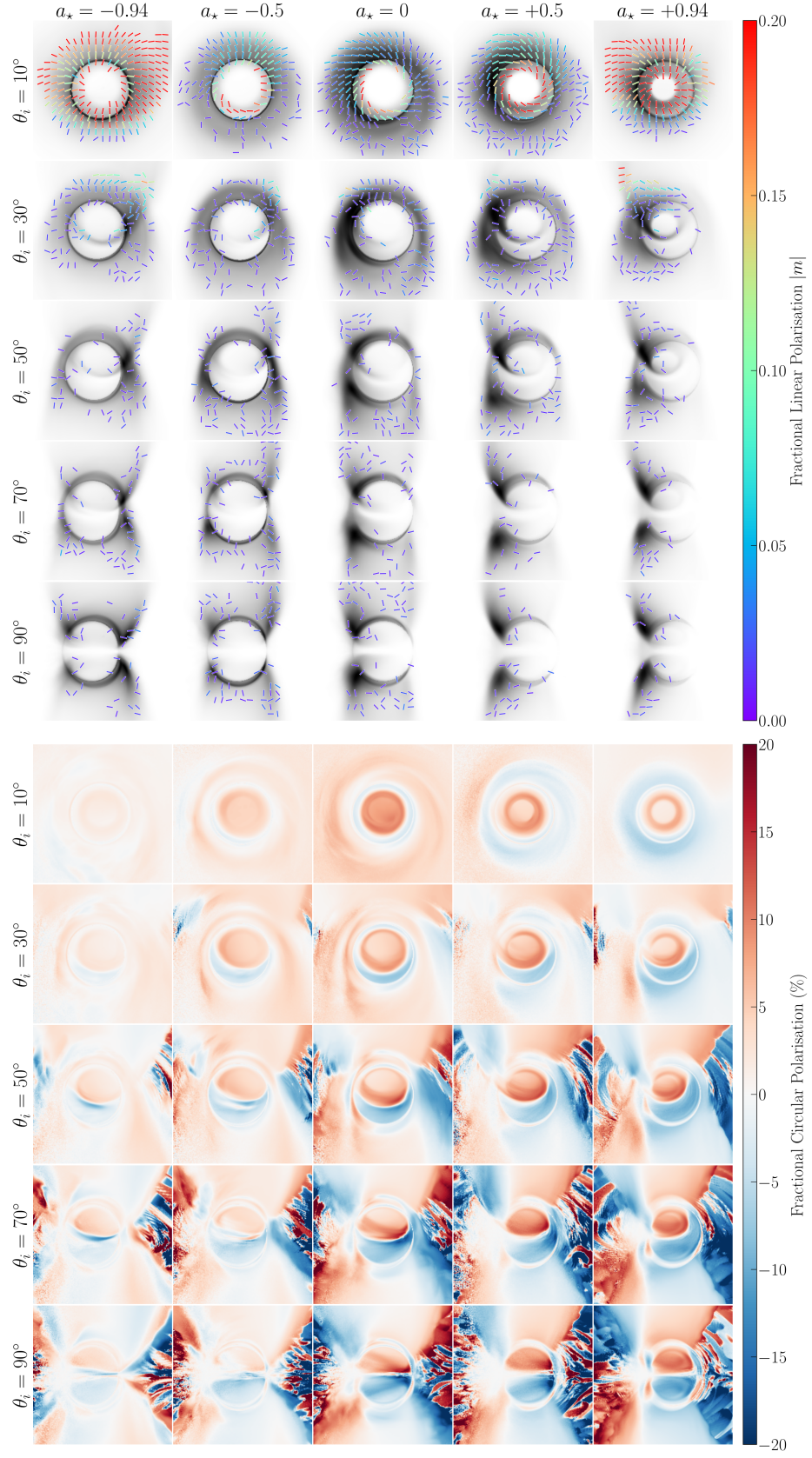


Figure 3.15: Same as Figure 3.14 but for SANE simulations.

temperature is tied to the local magnetization through equation (3.119). EHT model comparisons for M87* and Sgr A* generally favor MAD with cool disk electrons ($R_{\text{high}} \gtrsim 40$) and moderate inclination (Event Horizon Telescope Collaboration 2021; Event Horizon Telescope Collaboration 2022), and the spin is less well-constrained though prograde or mildly retrograde configurations are preferred. Unlike the RIAF model where H , κ , and the field geometry can each be changed independently, here R_{high} , spin, and magnetic flux state all shape the images together, making the interpretation less transparent.

3.6 Image Features and Their Physical Origins

Several terms are used to describe the images discussed in the previous sections which carry different meanings across the GR and astrophysical communities. The following definitions clarify each term, organized by whether it is a property of the spacetime geometry or a feature that appears in an observed image.

Black-hole-frame quantities

- **The photon sphere and photon shell:** For a Schwarzschild black hole, the photon sphere is the two-sphere at $r = 3M$ on which null geodesics can execute unstable circular orbits (Perlick and Tsupko 2022). For Kerr, this generalizes to a three-dimensional photon shell whose radial extent depends on the orbital inclination and azimuthal direction (Teo 2003; Perlick and Tsupko 2022). These are local geometric objects, defined entirely by the spacetime metric at that location, independent of any emission or observer distance. Photons that approach the photon sphere asymptotically from a large distance do so with a specific impact parameter $b_c = 3\sqrt{3}M$ for Schwarzschild, which is a global geodesic quantity defined with respect to an observer at infinity and serves as the boundary between captured and escaping trajectories (Perlick and Tsupko 2022).

Observer-plane quantities

- **The critical curve/apparent boundary:** The image of the photon sphere or photon shell on the observer's screen is the critical curve (Gralla et al. 2019). For Schwarzschild it is a circle of apparent radius b_c , while for Kerr it becomes a slightly asymmetric closed curve whose detailed shape depends on spin and observer inclination (Bardeen 1973; Perlick and Tsupko 2022). The critical curve is the mathematical limit that all photon ring sub-rings approach as the subring order $n \rightarrow \infty$. It is never itself observed as

a brightness feature in a finite observation. This is what Bardeen’s original calculation describes, and it is the ‘shadow/apparent boundary’ as calculated by Bardeen (Bardeen 1973).

- **The image brightness depression:** When a black hole is surrounded by optically thin emitting plasma, the dark central region in the resulting image is the brightness depression. Its boundary does not in general coincide with the critical curve. Falcke et al. 2000 first pointed out that this depression has an apparent size approximately set by b_c , making it a proxy for the black hole mass, and this is the “shadow” as used throughout the EHT literature. However, Gralla et al. 2019 showed through analytical calculations that the size, depth, and shape of the brightness depression depend on the emission profile, velocity field, and optical depth distribution, not on the spacetime metric alone. There is therefore no general reason to equate the brightness depression boundary with the critical curve, and in realistic accretion models the two can differ substantially. This distinction is central to interpreting EHT images of M87*.
- **Direct emission, the lensing ring, and photon ring/subrings:** Emission reaching the observer can be decomposed by the number of equatorial plane crossings n along the photon trajectory (Gralla et al. 2019; Johnson et al. 2020). Direct emission ($n = 0$) consists of photons that travel to the observer without completing a half-orbit. It carries the dominant flux and is strongly emission-model dependent. Photons completing roughly half an orbit before escaping produce a moderately lensed image of the back of the disk superposed on the direct emission, which Gralla et al. 2019 call the lensing ring. It sits at a radius roughly 5% larger than the critical curve, has a width of $\sim 0.5\text{--}1 M$, and can be a factor of 2–3 brighter than the surrounding direct emission. Photons completing at least one full half-orbit form the photon ring proper, which is an infinite sequence of self-similar subrings indexed by $n \geq 1$ (Johnson et al. 2020). Successive subrings are exponentially demagnified by a factor $e^{-\gamma}$, where γ depends on black hole spin and observer inclination. The notation used here follows Gralla et al. 2019 and Johnson et al. 2020: $n = 0$ is direct emission, the lensing ring corresponds to $n \approx 1$ secondary images, and the photon ring subrings are $n \geq 1$. Some other authors (for e.g., Vincent et al. 2022) use the term photon ring to include the lensing ring as well; where this ambiguity is relevant it will be noted explicitly. At 2017 EHT resolution, the direct emission, lensing ring, and $n = 1$ subring are all blended into a single unresolved bright ring.

- **The inner shadow:** For equatorial disk emission extending to the event horizon, as in MAD GRMHD models where 230 GHz synchrotron emission is concentrated near the midplane, the directly lensed image of the equatorial horizon produces a small dark region interior to the photon ring (Chael et al. 2021). This is the inner shadow. Its boundary traces the projected image of the equatorial horizon rather than the critical curve, making it a more direct probe of the event horizon than the photon ring alone. In spherical accretion models, where emission fills the funnel at all latitudes (the large scale height $H \rightarrow 1$ limit of Section 3.5.4), the central brightness depression extends out to approximately the critical curve and the inner shadow is not separately distinguishable. As the disk thins toward the equatorial plane ($H < 1$, or the MAD equatorial case), the inner shadow becomes a spatially distinct dark region, separated from the photon ring by a bright annulus (Chael et al. 2021).

The terms used in the rest of this thesis are as follows. The *critical curve* refers to the image of the photon shell, a purely geometric quantity set by b_c . The *brightness depression* refers to the observed central dark region. Throughout this thesis, n counts the number of half-orbits completed by the photon before reaching the observer, following Johnson et al. 2020. Direct emission is $n = 0$. The lensing ring of Gralla et al. 2019 corresponds to $n = 1$ secondary images. Johnson et al. group $n \geq 1$ together under the umbrella term photon ring, whereas Gralla et al. reserve photon ring for $n \geq 2$ and treat the $n = 1$ lensing ring separately. Both conventions appear in the literature; here we adopt the Johnson et al. convention and use photon ring to mean $n \geq 1$, with the $n = 1$ subring being the astrophysically accessible one. The *photon ring* then refers to subrings with $n \geq 1$ and the *inner shadow* is the brightness depression interior to the photon ring in equatorial emission models.

3.7 Open Questions

The synthetic images presented in this chapter demonstrate that the polarimetric observables of horizon-scale accretion flows carry physical information. The EVPA pattern tracks the magnetic field geometry, the fractional LP reflects the Faraday depth of the emitting plasma, and the CP morphology carries both the field handedness and the disk inclination. The model grids of Sections 3.5.4 and 3.5.5 show that varying the scale height H , the velocity profile κ , the field topology, and the accretion state each produce distinguishable image-level signatures. The challenge is that several of these parameters are degenerate in their effect

on the images, and others are fundamentally not accessible to the fluid-level description used throughout this chapter. Real observations are needed to break these degeneracies, and the following questions identify where that confrontation between models and data is most pressing.

Q. What is the dominant magnetic field topology of the inner disk, and can it be constrained from polarimetric observations?

As shown in Section 3.5.4, toroidal, poloidal, and tangled field geometries produce distinct EVPA patterns and LP fractions in the synthetic images (Broderick et al. 2009; Vos et al. 2022), and the approximately azimuthal EVPA observed by the EHT in M87* favors an ordered large-scale field (Event Horizon Telescope Collaboration 2021). However, the dominant field geometry at horizon scales cannot be uniquely determined from a single-epoch polarimetric image because the same EVPA morphology can arise from different combinations of field configuration, disk scale height, and velocity profile (Pu and Broderick 2018). The semi-analytic framework of Section 3.2.1, which permits independent variation of each parameter, may provide a route to breaking this degeneracy, and its application to M87* is taken up in Saurabh et al. 2026.

Q. What are the electron temperatures in the innermost accretion flow, and how are they set? In a collisionless relativistic plasma the electron-ion Coulomb coupling time exceeds the accretion timescale, leaving ions and electrons thermally decoupled with different temperatures (Narayan and Yi 1994). The R_{high} prescription of equation (3.119) is a parametric stand-in for this physics as it assigns hotter electrons to magnetized regions (low β) and cooler electrons to the disk midplane (high β). Particle-in-cell simulations of relativistic magnetic reconnection suggest electrons are heated preferentially in current sheets (Rowan et al. 2017), but no consensus exists on the heating partition in the geometrically thick, magnetically dominated flows relevant to M87* and Sgr A*. Two-temperature GRMHD simulations, which evolve separate fluid equations for ions and electrons with a prescribed heating partition (Rowan et al. 2017), and radiative GRMHD simulations, which couple the radiation field self-consistently to the two-temperature fluid (Ressler et al. 2015; Ryan et al. 2018), represent physically more complete approaches, but both still depend on a subgrid model for the fraction of dissipated energy channeled into electrons. EHT model comparisons consistently favor cool disk electrons with $R_{\text{high}} \gtrsim 40$ (Event Horizon Telescope Collaboration 2021; Event Horizon Telescope Collaboration 2022).

Q. How does the emission morphology depend on the disk scale height, and does this produce a distinct inner shadow? In the quasi-spherical $H \rightarrow 1$ limit, emission fills the funnel at all latitudes and the central brightness depression extends to approximately the critical curve, resembling the classical Falcke shadow (Falcke et al. 2000). As H decreases toward the equatorial thin-disk regime, the inner shadow becomes a spatially distinct dark region separated from the photon ring by a bright annulus (Chael et al. 2021), as illustrated in Section 3.5.4. The MAD GRMHD simulations of Section 3.5.5 are firmly in the thin-disk regime, and the inner shadow is visible at native resolution but completely washed out at 2017 EHT resolution (Section 4.8). Detecting it requires both higher angular resolution and higher dynamic range than the current EHT provides (Johnson et al. 2023; Chael et al. 2021), but it encodes direct geometric information about the equatorial horizon that the photon ring alone cannot provide.

Q. Can a horizonless compact object sustain a radiatively inefficient accretion flow? Advective cooling is fundamental to the energy balance of both M87* and Sgr A*, where the mass accretion rate is far below the Eddington limit and viscously generated heat cannot be radiated away locally (Narayan and Yi 1994). In a black hole spacetime, this excess energy disappears behind the event horizon, maintaining the thermodynamic structure of the RIAF. For a horizonless compact object, however, the absence of a horizon means energy must either accumulate in the system or be removed via outflows, fundamentally altering the accretion dynamics and observable signatures. GRMHD simulations of non-Kerr black hole spacetimes have been carried out (Chatterjee et al. 2023; Chatterjee et al. 2025; Uniyal et al. 2026), and more recently for naked singularity backgrounds (Uniyal et al. 2025; Dihingia et al. 2025), but in most of these cases matter is expelled from the innermost region rather than accreted, preventing the formation of a sustained RIAF. The JMN-1 spacetime (Joshi et al. 2011) is a particularly challenging black hole mimicker arising from gravitational collapse with anisotropic pressure in GR: for a compactness parameter $R_b < 3r_g$ it possesses a null singularity, admits the Schwarzschild photon sphere at $r_\gamma = 3r_g$, and produces an observable shadow consistent with EHT constraints (Event Horizon Telescope Collaboration 2022; Saurabh et al. 2024). Whether such an object can host an advection-dominated, radiatively inefficient accretion flow with energy balance comparable to a black hole is the question taken up in Chapter 6, where the first GRMHD simulation of sustained accretion onto a JMN-1 null singularity is presented.

Chapter 4

Very long baseline interferometry and Event Horizon Telescope

THE theoretical models developed in Chapter 3 predict that the shadow of a supermassive compact objects (M87* and Sgr A*) subtends angular scales of tens of microarcseconds on the sky. Accessing those scales observationally requires unprecedented resolution that cannot be achieved by any single telescope. Radio interferometry addresses this obstacle this by correlating signals from widely separated antennas, synthesizing an effective aperture whose diameter is set by the largest antenna separation rather than the size of any individual dish. At its practical limit, with antennas distributed across continents, this technique — very long baseline interferometry (VLBI) — achieves angular resolutions in the microarcsecond regime at millimeter wavelengths. This chapter develops the physical and mathematical foundations of interferometric imaging, from the geometry of a two-element correlator through the measurement equation that governs what is recorded, to the image reconstruction methods used to recover a sky brightness distribution from an incomplete and noise-corrupted set of Fourier measurements. The Event Horizon Telescope (EHT), the specific realization of VLBI that produced the first resolved images of M87* and Sagittarius A* (Sgr A*), is then described in detail, including the multi-epoch imaging campaign from 2017 to 2021 that forms part of the work presented in Chapter 5.

4.1 The Resolution Requirement

The primary obstacle to imaging a black hole shadow is angular resolution. For a circular aperture of diameter D illuminated at wavelength λ , the Rayleigh criterion sets the minimum resolvable angular separation between two point sources

at (Rayleigh 1879; Thompson et al. 2017)

$$\theta_{\min} \approx 1.22 \frac{\lambda}{D}. \quad (4.1)$$

The half-power beamwidth of the main diffraction lobe, which sets the effective resolution in practice, is $\theta_{\text{HPBW}} \approx \lambda/D$ (Thompson et al. 2017). For a Schwarzschild black hole the critical impact parameter is $b_c = 3\sqrt{3}GM/c^2$, so the angular diameter of the shadow is (Event Horizon Telescope Collaboration 2019)

$$\theta_{\text{sh}} = \frac{6\sqrt{3}GM}{c^2 D_s}, \quad (4.2)$$

where M is the black hole mass and D_s is the distance. For the two primary EHT targets this gives $\theta_{\text{sh}} \approx 40 \mu\text{as}$ for M87* ($M \approx 6.5 \times 10^9 M_\odot$, $D_s \approx 16.8 \text{ Mpc}$) and $\theta_{\text{sh}} \approx 50 \mu\text{as}$ for Sgr A* ($M \approx 4.0 \times 10^6 M_\odot$, $D_s \approx 8.15 \text{ kpc}$) (Event Horizon Telescope Collaboration 2019; Event Horizon Telescope Collaboration 2022). Resolving a $50 \mu\text{as}$ feature at $\lambda = 1.3 \text{ mm}$ requires, from equation (4.1), a telescope diameter $D_s \gtrsim \lambda/\theta \approx 10^4 \text{ km}$, comparable to the diameter of the Earth.

The next obstacle to resolve is the choice of the observing wavelength, which is not arbitrary; it is mainly because of the atmospheric opacity. At centimeter and longer wavelengths the angular broadening from interstellar electron scattering scales as λ^2 (Event Horizon Telescope Collaboration 2019), and for Sgr A* this scattering smears the image to several hundred microarcseconds at $\lambda = 7 \text{ mm}$, entirely obscuring the shadow regardless of baseline length. At $\lambda = 1.3 \text{ mm}$ the scattering broadening falls to $\sim 22 \mu\text{as}$, comparable to but smaller than the shadow (Event Horizon Telescope Collaboration 2022). At the same time, the synchrotron emission from the optically thin accretion flow and jet base peaks in the millimeter to sub-millimeter range for the electron temperatures and magnetic field strengths characteristic of M87* and Sgr A* (Section 3.4.1), making 1.3 mm a suitable wavelength for horizon-scale imaging (Event Horizon Telescope Collaboration 2019).

An interferometric array overcomes the resolution constraint by using the separation between antennas, the baseline, rather than the diameter of any individual dish as the effective aperture. For a baseline of length b , the fringe spacing on the sky is $\theta_{\text{fringe}} \approx \lambda/b$ (Thompson et al. 2017), giving the interferometer a resolution equivalent to a filled aperture of diameter b . Earth-spanning baselines at 1.3 mm then provide $\theta_{\text{fringe}} \approx 20 \mu\text{as}$, sufficient to resolve both targets.

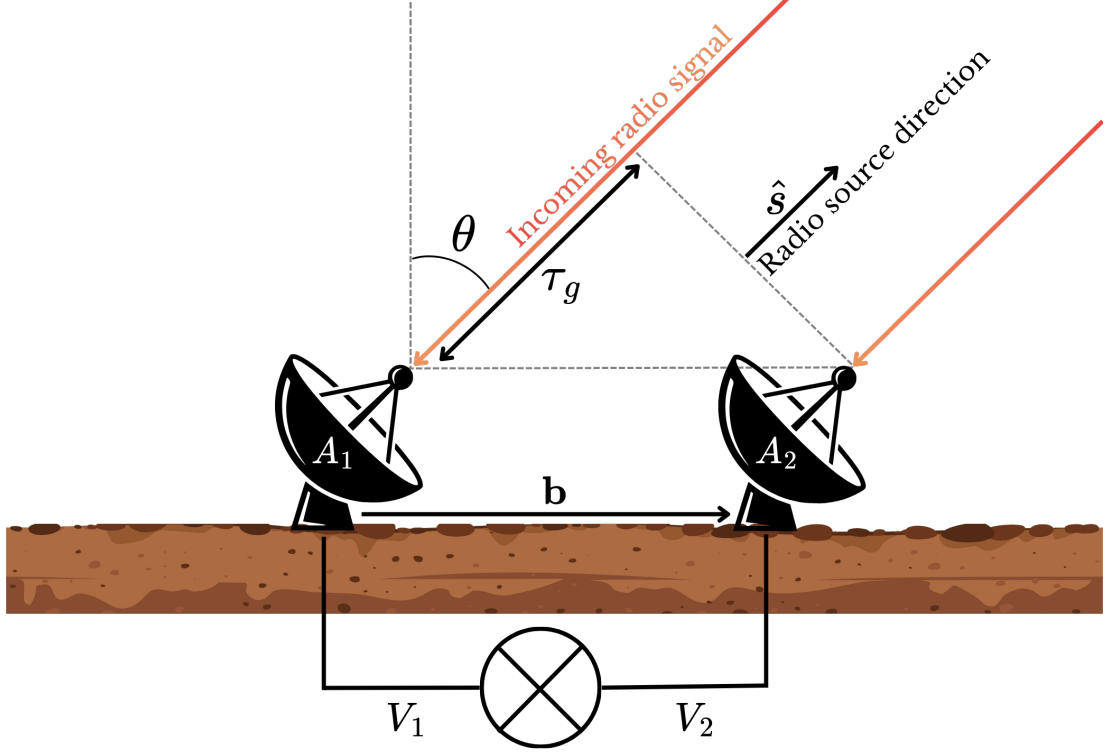


Figure 4.1: Geometry of the two-element interferometer. Antennas A_1 and A_2 are separated by the baseline vector $\mathbf{b}$. A plane wave from a distant source in direction $\hat{\mathbf{s}}$ arrives at A_1 with a geometric time delay τ_g relative to A_2 .

4.2 The Two-Element Interferometer

Consider two antennas separated by the baseline vector $\mathbf{b}$, as in Figure 4.1. A plane wave from direction $\hat{\mathbf{s}}$ reaches one antenna before the other, with a geometric time delay set by the projection of $\mathbf{b}$ onto the line of sight,

$$\tau_g = \frac{\mathbf{b} \cdot \hat{\mathbf{s}}}{c}. \quad (4.3)$$

The voltage outputs V_1 and V_2 , mixed to an intermediate frequency with a shared local oscillator, are multiplied and time-averaged in the correlator. For a monochromatic signal at frequency ν the correlator output is proportional to $\cos(2\pi\nu\tau_g)$, a fringe pattern in τ_g (Thompson et al. 2017).

As the Earth rotates, τ_g changes continuously, producing a fringe rate $\dot{\phi} = 2\pi\nu\dot{\tau}_g$ that the correlator tracks to maintain coherence (Thompson et al. 2017). Coherent averaging is limited in both time and bandwidth. Averaging over an interval Δt attenuates sources offset from the phase center by $\delta\theta \gtrsim 1/(2\pi\nu\dot{\tau}_g\Delta t)$, which is time-average smearing, and averaging over bandwidth $\Delta\nu$ introduces a sinc envelope restricting the usable delay range to $|\tau_g| \lesssim 1/\Delta\nu$, which is band-

width smearing (Thompson et al. 2017).

The noise on a given baseline depends on the properties of both contributing stations. For antenna p with system temperature $T_{\text{sys},p}$ and effective collecting area $A_{\text{eff},p}$, the system equivalent flux density (SEFD) is (Thompson et al. 2017)

$$\text{SEFD}_p = \frac{2k_B T_{\text{sys},p}}{A_{\text{eff},p}}, \quad (4.4)$$

where k_B is the Boltzmann constant. The thermal noise on the correlated output of baseline pq , integrated over bandwidth $\Delta\nu$ and time Δt , is

$$\sigma_{pq} = \frac{1}{\eta_c} \sqrt{\frac{\text{SEFD}_p \cdot \text{SEFD}_q}{2 \Delta\nu \Delta t}}, \quad (4.5)$$

where $\eta_c \leq 1$ is the correlator efficiency, accounting for quantization losses when the voltage signals are digitized before correlation (Thompson et al. 2017). The signal-to-noise ratio scales as $(\Delta\nu \Delta t)^{1/2}$ and is set by the less sensitive of the two stations on a given baseline.

4.3 Spatial Coherence and the UV Plane

The connection between the correlator output and the sky brightness follows from the van Cittert-Zernike theorem (Thompson et al. 2017). For an extended, spatially incoherent source with specific intensity $I_\nu(l, m)$, where (l, m) are direction cosines measured east and north from a reference position on the sky and $n = \sqrt{1 - l^2 - m^2}$ is the third direction cosine along the line of sight, the complex visibility measured on a baseline with components $(u, v, w) = (b_X, b_Y, b_Z)/\lambda$ is

$$\mathcal{V}(u, v, w) = \iint \frac{I_\nu(l, m)}{n} e^{-2\pi i(ul+vm+w(n-1))} dl dm, \quad (4.6)$$

where the w -term encodes the non-coplanarity of the array (Thompson et al. 2017). For a sufficiently small field of view, $l^2 + m^2 \ll 1$, the third direction cosine satisfies $n \approx 1$, the w -dependent phase factor vanishes, and Equation (4.6) reduces to a two-dimensional Fourier transform

$$\mathcal{V}(u, v) = \iint I_\nu(l, m) e^{-2\pi i(ul+vm)} dl dm, \quad (4.7)$$

where $(u, v) = (b_X, b_Y)/\lambda$ are the east and north components of the projected baseline in units of the observing wavelength. This approximation holds for all EHT targets, whose angular extents are of order microarcseconds to milliarcseconds. An interferometer therefore measures not the image directly but a sparse

sampling of its two-dimensional Fourier modes. Due to rotation of the Earth, the projected baseline produces an extended track in the (u, v) plane (see Figure 4.2). The full set of arcs over an observation constitutes the UV coverage, which is the area of the sampled Fourier components in the visibility domain (Thompson et al. 2017). This sampling directly determines the point spread function of the array (Figure 4.2). The visibility function of simple source geometries has analytic forms that are worth noting. A point source of flux S at the phase center gives $\mathcal{V}(u, v) = S$ at all baselines and cannot be resolved. A thin ring of angular radius ρ_0 and total flux S produces

$$\mathcal{V}(u, v) = S J_0 \left(2\pi\rho_0\sqrt{u^2 + v^2} \right), \quad (4.8)$$

where J_0 is the zeroth-order Bessel function; the amplitude passes through zero at $b/\lambda \approx 0.38/\rho_0$ and recovers a secondary maximum at longer baselines (Thompson et al. 2017). A circular Gaussian of FWHM θ_{FWHM} gives

$$\mathcal{V}(u, v) = S \exp[-\pi^2\theta_{\text{FWHM}}^2(u^2 + v^2)/(4 \ln 2)], \quad (4.9)$$

which falls monotonically (Thompson et al. 2017). The visibility amplitude as a function of baseline length therefore carries qualitative information about source morphology (see Figure 4.4). A compact source maintains high amplitude to long baselines; an extended source falls off rapidly; and a ring-like source produces the oscillating pattern of Equation (4.8). For a crescent-like accretion flow the visibility amplitude depends on both baseline length and orientation, and carries information on the ring radius, the brightness asymmetry, and the position angle.

Taking the inverse Fourier transform of the incompletely sampled visibility set yields the dirty image, the convolution of the true brightness with the dirty beam (Thompson et al. 2017),

$$I_{\nu}^{\text{dirty}}(\mathbf{l}) = I_{\nu}(\mathbf{l}) * B(\mathbf{l}), \quad (4.10)$$

where $B(\mathbf{l})$ is the Fourier transform of the sampling function $S(\mathbf{u})$, which is unity at measured (u, v) points and zero elsewhere. Recovering I_{ν} from I_{ν}^{dirty} is the image reconstruction problem treated in Section 4.6.

4.4 The Measurement Equation and Calibration

Equation (4.7) describes an ideal measurement equation without noise and calibration errors. Every real antenna modifies the incoming signal through its gain, feed response, parallactic angle rotation, and atmospheric phase, and a full polari-

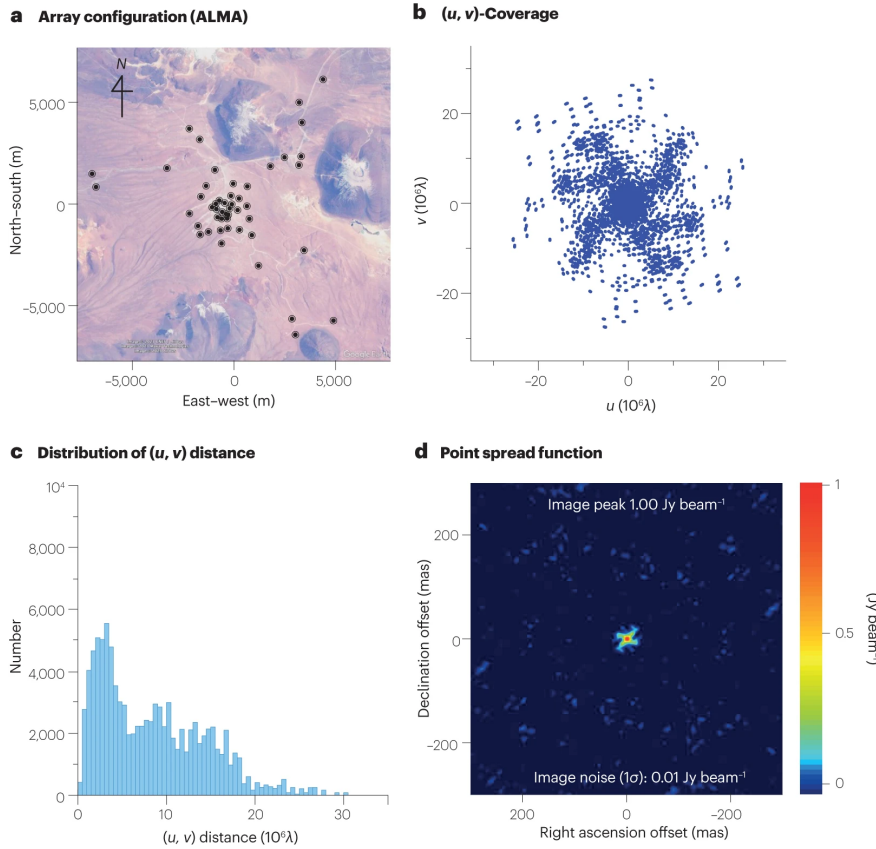


Figure 4.2: Synthesis imaging with an interferometric array. (a) Antenna distribution of the ALMA array. (b) Corresponding (u, v) coverage obtained through Earth rotation. (c) Distribution of sampled baseline lengths. (d) Resulting point spread function, illustrating how incomplete Fourier sampling maps into image-domain structure. Image adapted from [Asaki et al. 2023](#).

metric treatment must account for all of these effects. The Radio Interferometric Measurement Equation (RIME) [Hamaker et al. 1996](#); [Smirnov 2011](#), provides a neat mathematical framework for calibration and imaging, under the assumption that all transformation in the signal chain is linear.

The starting point is the electromagnetic waves from the source measured at the antenna. For quasi-monochromatic radiation propagating along the z -axis, the field at the aperture can be described as a two-component complex vector in a linear polarization basis,

$$\mathbf{e} = \begin{pmatrix} e_x \\ e_y \end{pmatrix}, \quad (4.11)$$

where e_x and e_y carry the amplitude and phase along each transverse direction. Every effect along the signal path from the sky to the correlator that can be approximated as a linear transformation is represented by a 2×2 complex Jones matrix ([Jones 1941](#); [Smirnov 2011](#)). The output voltage vector of antenna p is

then

$$\mathbf{v}_p = \begin{pmatrix} v_{pa} \\ v_{pb} \end{pmatrix} = \mathbf{J}_n \mathbf{J}_{n-1} \cdots \mathbf{J}_1 \mathbf{e} \equiv \mathbf{J}_p \mathbf{e}, \quad (4.12)$$

where a and b label the two feed channels and $\mathbf{J}_p$ is the total Jones matrix for station p incorporating multiple effects along the path of propagation of the signal.

The correlator forms the outer product of the voltage vectors from antennas p and q . The time-averaged result is the 2×2 visibility coherency matrix (Smirnov 2011),

$$\mathbf{V}_{pq} = 2 \langle \mathbf{v}_p \mathbf{v}_q^\dagger \rangle = \mathbf{J}_p \mathbf{B} \mathbf{J}_q^\dagger, \quad (4.13)$$

where $\dagger$ denotes the Hermitian conjugate. The sky brightness coherency matrix $\mathbf{B}$ connects to the Stokes parameters. In a linear feed basis

$$\mathbf{B} = \begin{pmatrix} I + Q & U + iV \\ U - iV & I - Q \end{pmatrix}, \quad (4.14)$$

and in the circular feed basis,

$$\mathbf{B} = \begin{pmatrix} I + V & Q + iU \\ Q - iU & I - V \end{pmatrix}. \quad (4.15)$$

The total Jones matrix $\mathbf{J}_p$ is a product of matrices, one per physical effect in the signal chain, in the reverse order of the actual signal path,

$$\mathbf{J}_p = \mathbf{G}_p \mathbf{D}_p \mathbf{P}_p, \quad (4.16)$$

where in the circular basis

$$\mathbf{G}_p = \begin{pmatrix} g_p^r & 0 \\ 0 & g_p^l \end{pmatrix}, \quad \mathbf{D}_p = \begin{pmatrix} 1 & D_p^r \\ D_p^l & 1 \end{pmatrix}, \quad \mathbf{P}_p = \begin{pmatrix} e^{-i\phi_p} & 0 \\ 0 & e^{+i\phi_p} \end{pmatrix}. \quad (4.17)$$

Here g_p^r and g_p^l are the direction-independent antenna-based complex gains of the right- and left-circular feeds respectively, D_p^r and D_p^l are the polarization leakage terms (D-terms) describing the leaking signal between the two feed channels, and ϕ_p is the field rotation angle. The field rotation angle accounts for the rotation of the sky relative to the antenna coordinate frame as the source transits and depends on the antenna mount type. For an alt-azimuth mount with Cassegrain focus, ϕ_p equals the parallactic angle ψ_{par} of the source at that station. For Nasmyth-focus mounts, the elevation angle enters as well (Smirnov 2011). In practice $\mathbf{G}_p$ absorbs clock offsets, atmospheric phase, and residual amplitude calibration errors in addition to the electronic gains.

For an extended source the RIME generalizes equation (4.13) to an integral

over the sky,

$$\mathbf{V}_{pq} = \mathbf{J}_p \left(\iint \mathbf{B}(l, m) e^{-2\pi i(ul+vm)} dl dm \right) \mathbf{J}_q^\dagger, \quad (4.18)$$

under the small-field approximation $n \approx 1$.

For total intensity imaging with antenna-based gain corruption, the RIME often can be approximated to the gain model

$$V_{pq}^{\text{obs}} = G_p G_q^* V_{pq}^{\text{true}}, \quad (4.19)$$

where $G_p = |g_p| \exp(i\phi_p)$. Recovering V^{true} from V^{obs} requires solving for $2N$ complex gain factors with N is the degrees of freedom for gain amplitude. Self-calibration iterates between solving for gains given a current image model and updating the image given the new gains (Thompson et al. 2017), but always requires a starting model.

4.5 Very Long Baseline Interferometry

As discussed in Section 4.1, resolving horizon-scale structure at millimeter wavelengths requires baselines of order 10^4 km, comparable to the diameter of the Earth. Connected-element arrays such as the Very Large Array (VLA) and the Atacama Large Millimeter/submillimeter Array (ALMA), in which all antennas are phase-locked to a common frequency in real time over fiber or coaxial links, are physically limited to baselines of at most a few tens of kilometers. Achieving the angular resolution required to resolve structure on microarcsecond scales is therefore not possible with such facilities. Very Long Baseline Interferometry (VLBI) overcomes this limitation by recording the voltage output at each antenna independently against a local clock, allowing baselines to extend over thousands of kilometers. The time reference is provided by a local hydrogen maser whose frequency stability is $\delta\nu/\nu \sim 10^{-15}$ over timescales of hundreds of seconds which is sufficient for coherent fringe detection at millimeter wavelengths (Thompson et al. 2017). Intercontinental baselines of $\sim 10\,000$ km at $\lambda = 1.3$ mm give fringe spacings of $\theta \approx \lambda/b \sim 20 - 25 \mu\text{as}$, sufficient to resolve the shadows of both M87* and Sgr A*. As an example, the UV coverage from by EHT 2017 observing campaign as the Earth rotates is shown in Figure 4.6. The recorded data streams are transported physically or electronically to a central correlator, where software correlates all baseline pairs using a delay model computed from the station positions, Earth orientation parameters, and source coordinates. The independence of station clocks introduces phase errors that standard connected-array calibration cannot remove which requires additional steps called global fringe fitting



Figure 4.3: 2018 GMVA array configuration for the M87* observation. Credit: NAOJ

(Schwab and Cotton 1983). At millimeter wavelengths, tropospheric phase corruption becomes significant since this corruption effect is linearly proportional to the frequency. The column of precipitable water vapor above each station imposes phase fluctuations varying on timescales of seconds, uncorrelated between stations, and reaching many radians at 230 GHz in poor conditions. The coherence time t_{coh} limits the viable integration interval in equation (4.5): for $\Delta t > t_{\text{coh}}$ the visibility phase rotates during integration and the detected amplitude falls into the thermal noise floor. Under typical conditions at millimeter wavelengths t_{coh} is of order ten seconds, much shorter than the hours available to connected-element arrays at centimeter wavelengths (Thompson et al. 2017).

Several VLBI arrays operate across the centimeter, millimeter, and sub-millimeter bands. The Very Long Baseline Array (VLBA) uses ten 25 m dishes across the continental United States and Hawaii, giving baselines from ~ 200 to ~ 8600 km (Napier et al. 1994). The European VLBI Network (EVN) coordinates telescopes across Europe, Asia, and South Africa at centimeter wavelengths (Schilizzi 2000). The Global mm-VLBI Array (GMVA; Figure 4.3) extends to 86 GHz by combining various stations including the VLBA, reaching angular resolutions of $\sim 50 \mu\text{as}$ (Krichbaum et al. 2006). The RadioAstron space antenna (Kardashev et al. 2013) demonstrated fringe detections below $10 \mu\text{as}$ on baselines exceeding 350 000 km, extending VLBI into Earth orbit. The East Asian VLBI Network (EAVN) provides coverage at 22 and 43 GHz (Janssen et al. 2020). The EHT (Figure 4.6)

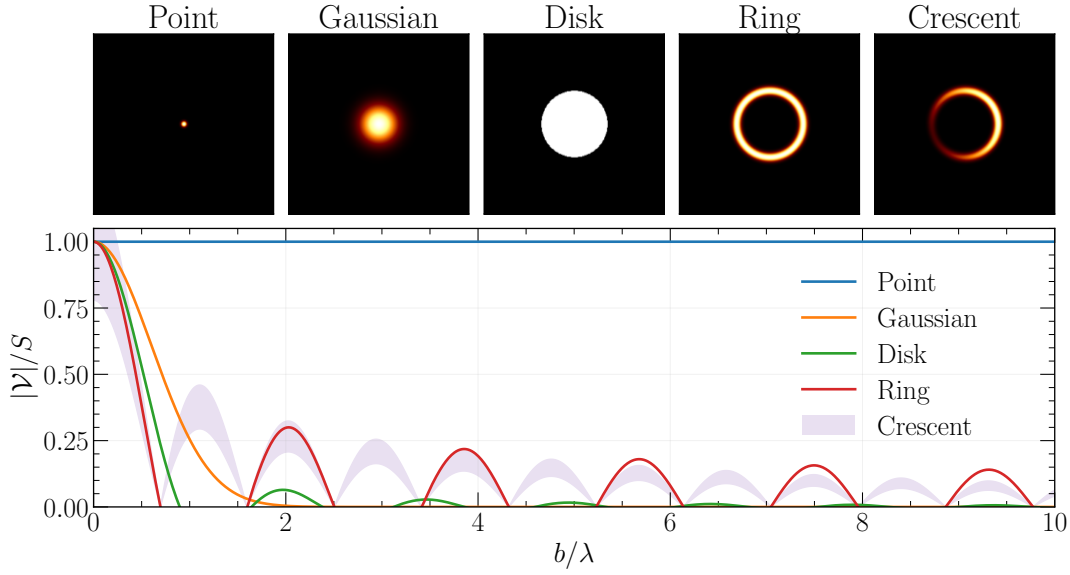


Figure 4.4: Visibility amplitudes as a function of baseline length for representative source structures. The point source remains unresolved, the Gaussian falls monotonically, the uniform disk shows a first null with weak oscillations, and the thin ring exhibits the characteristic Bessel oscillations. The crescent produces a spread in amplitudes due to its intrinsic asymmetry and orientation dependence.

pushes VLBI to 230 GHz with Earth-spanning baselines, achieving angular resolutions approaching $15\text{--}20\ \mu\text{as}$ (Event Horizon Telescope Collaboration 2019). The next-generation EHT (ngEHT) plans to substantially increase the station count and add 345 GHz capability to enable dynamical imaging of Sgr A* on its orbital timescale (Johnson et al. 2023).

4.5.1 Closure Quantities

The gain model of equation (4.19) shows that every measured visibility is corrupted by the product of two station-based complex gains. With N stations there are $N(N-1)/2$ baselines but only N independent gain factors, so the gains can be solved by taking advantage of the redundancy, under the assumption that the source structure is point-like. However, specifically in mm-VLBI, even the calibrator sources are resolved and our phases are highly corrupted. Closure-only imaging can be an alternative to circumvent this problem.

Consider three stations p, q, r , as illustrated in Figure 4.5. The measured visibility phase on each baseline contains a station-based phase error,

$$\begin{aligned}\psi_{pq}^{\text{obs}} &= \psi_{pq}^{\text{true}} + \phi_p - \phi_q, \\ \psi_{qr}^{\text{obs}} &= \psi_{qr}^{\text{true}} + \phi_q - \phi_r, \\ \psi_{rp}^{\text{obs}} &= \psi_{rp}^{\text{true}} + \phi_r - \phi_p.\end{aligned}$$

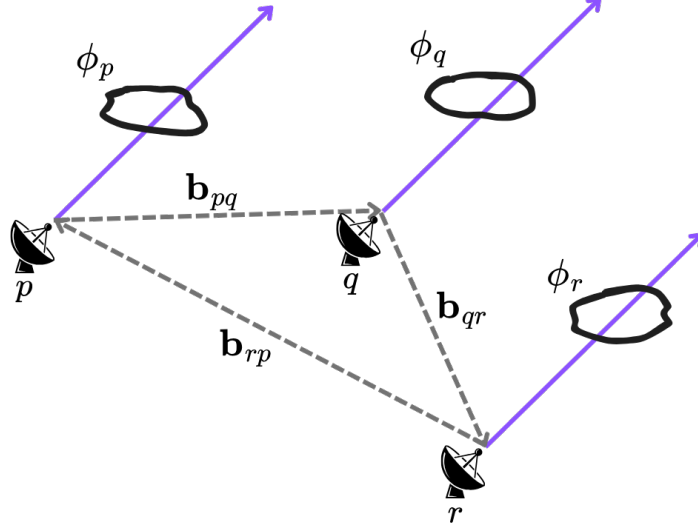


Figure 4.5: Illustration of the closure phase triangle formed by three stations p , q , and r with baselines $\mathbf{b}_{pq}$, $\mathbf{b}_{qr}$, and $\mathbf{b}_{rp}$. Each station carries an independent phase error due to atmospheric cloudlets that cause excess phase shifts of ϕ_p , ϕ_q , ϕ_r (arrows).

Summing the three measured phases around the triangle, every station phase appears once with a positive and once with a negative sign, so they cancel exactly. The closure phase for triangle pqr is (Jennison 1958; Thompson et al. 2017)

$$\Psi_{pqr} = \psi_{pq}^{\text{obs}} + \psi_{qr}^{\text{obs}} + \psi_{rp}^{\text{obs}} = \psi_{pq}^{\text{true}} + \psi_{qr}^{\text{true}} + \psi_{rp}^{\text{true}}. \quad (4.20)$$

This is a direct observable of the source structure, requiring no phase calibration whatsoever (Thompson et al. 2017). Its physical content is also clear that a point-symmetric brightness distribution produces zero closure phase on every triangle, while a non-zero value signals asymmetry in the source. An array of N stations produces $\binom{N}{3}$ triangle closures, of which $(N-1)(N-2)/2$ are independent, since any triangle can be written as a linear combination of others sharing a common baseline. The same cancellation applies to amplitude gains. The measured visibility amplitude on baseline pq is $|\mathcal{V}_{pq}^{\text{obs}}| = |g_p||g_q||\mathcal{V}_{pq}^{\text{true}}|$. Consider four stations p , q , r , s . Forming the ratio

$$\mathcal{A}_{pqrs} = \frac{|\mathcal{V}_{pq}||\mathcal{V}_{rs}|}{|\mathcal{V}_{pr}||\mathcal{V}_{qs}|}, \quad (4.21)$$

each station gain appears in exactly one numerator factor and one denominator factor, so all four cancel (Twiss et al. 1960). The closure amplitude is therefore insensitive to station-based flux antenna gains. An array of N stations provides $(N-1)(N-2)/2$ independent closure amplitudes.

Together, closure phases and closure amplitudes contain source information without the absolute flux and absolute phase information in the visibility dataset.

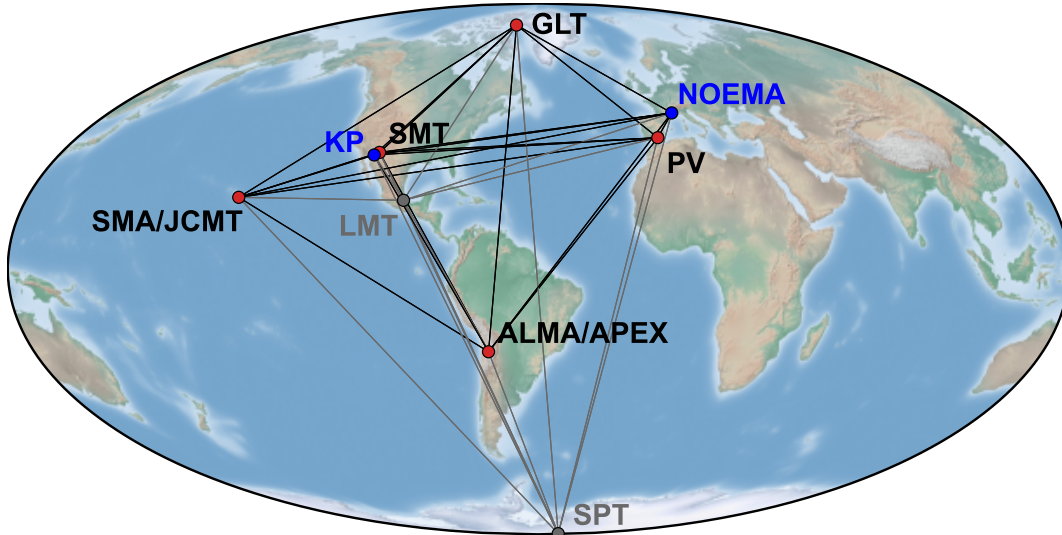


Figure 4.6: EHT array from 2021 observing campaign. Image from [Event Horizon Telescope Collaboration 2025](#)

4.6 Image Reconstruction

An interferometric array measures a finite, non-uniform sampling of the Fourier transform of the sky brightness. Recovering I_ν from this sparse set of noisy measurements is underdetermined as there are infinite number of brightness distributions (images) are consistent with the same data, so additional constraints must be imposed to reconstruct a reliable image ([Thompson et al. 2017](#)). Different algorithms impose different prior assumptions, either explicitly or through the form of the inversion such as implicit priors (regularizers).

4.6.1 The CLEAN Algorithm

The dirty image of equation (4.10) is the convolution of the true sky brightness with the dirty beam, and recovering I_ν from I_ν^{dirty} is therefore a deconvolution problem. The CLEAN algorithm ([Högbom 1974](#)) is the classical solution to this, modeling the sky brightness as a superposition of point sources and iteratively identifying and subtracting them from the dirty image. Starting from I_ν^{dirty} , the algorithm locates the brightest pixel, subtracts a fraction (the loop gain, typically ~ 0.1) of the dirty beam centered on that pixel, and records the corresponding point component. This repeats until the residuals reach the noise level. The final image is the accumulated point-source model convolved with a Gaussian restoring beam fitted to the main lobe of the dirty beam, with the residual image added back ([Högbom 1974](#)). Multi-scale CLEAN ([Cornwell 2008](#)) extends this to handle extended emission. For the sparse UV coverage of a few VLBI stations, CLEAN

often requires user interaction, such as weighting scheme and CLEAN windows. It may produce imaging artifacts and limit reproducibility.

4.6.2 Regularized Maximum Likelihood Imaging

The de-facto imaging algorithm CLEAN has been successfully employed due to its simplicity and scalability. However, CLEAN suffers from a number of limitations, such as sub-optimal resolution and difficulty to describe diffuse emission. Recently, novel imaging approaches have been investigated. One of most popular methods is RML imaging to treat image reconstruction as a constrained optimization problem, which is the basis of regularized maximum likelihood (RML) methods ([Event Horizon Telescope Collaboration 2019](#)).

The central idea is that the true image $I(\mathbf{l})$ must simultaneously satisfy two competing requirements: data fidelity term and regularizers. It should reproduce the measured visibility data as closely as possible, and regularizers encode our prior knowledge about what a physically reasonable image looks like. These two requirements are combined into a single objective function,

$$\mathcal{L}(I) = \sum_k \alpha_k \chi_k^2(I) - \sum_r \beta_r \mathcal{R}_r(I), \quad (4.22)$$

where χ_k^2 measures the misfit to data product k (which can be visibility amplitudes, closure phases, closure amplitudes, or the full complex visibilities), $\mathcal{R}_r$ is a regularization functional encoding prior information about the image, and α_k, β_r are positive hyperparameters that control the relative weight of each term ([Event Horizon Telescope Collaboration 2019](#)). The reconstructed image is obtained by minimizing $\mathcal{L}$.

The choice of regularizer $\mathcal{R}_r$ determines what prior assumptions are imposed. Maximum entropy method (MEM) regularization ([Cornwell and Evans 1985](#)) maximizes the Shannon entropy of the image relative to a default model, favoring smooth and compact solutions. Total variation (TV) penalizes the integrated absolute image gradient and promotes piecewise smooth images with sharp boundaries, while total squared variation (TSV) penalizes the squared gradient and gives smoother solutions than the TSV. Different regularizers are suited to different source morphologies, and in practice multiple terms are often combined¹. When the data terms χ_k^2 are built from closure quantities rather than raw visibil-

¹The weighting parameters are determined through a hyperparameter survey conducted on synthetic data with known ground truth. Fiducial parameters are selected to perform well across a variety of source structures, identifying regions of imaging parameter space that consistently produce faithful reconstructions and distinguishing which image features are robust across parameter choices from those that vary with specific assumptions ([Event Horizon Telescope Collaboration 2019](#)).

ities, the reconstruction is insensitive to station-based gain errors by construction (Chael et al. 2018), which is particularly important for VLBI imaging.

4.6.3 The eht-imaging Framework

The `eht-imaging`² library (Chael et al. 2016; Chael et al. 2018) is an open-source Python implementation of RML imaging that has been widely used for reconstructing images from sparse VLBI data. It implements the objective function of equation (4.22) using closure phases and log-closure amplitudes, and complex visibilities as the primary data constraints. MEM, TV, and TSV regularization terms can be combined in any linear combination, giving flexibility to adapt the prior to different expected source morphologies.

A practical challenge in RML imaging is that the hyperparameters α_k and β_r must be set before the reconstruction, yet their optimal values are not known in advance. The `eht-imaging` library addresses this by surveying a grid of hyperparameter combinations, running reconstructions for each, and scoring the resulting images against synthetic data tests on known ground-truth source models. For the image reconstructions used in EHT analysis, the final image is a mean over the ensemble of top-scoring combinations, which reduces sensitivity to any particular hyperparameter choice (Event Horizon Telescope Collaboration 2019).

Beyond RML method, Bayesian imaging reconstructs posterior images by solving Bayes’ theorem such `resolve` (Junklewitz et al. 2016; Arras et al. 2022), `Themis` (Broderick et al. 2020), and `comrade` (Tiede 2022). The SMILI framework (Akiyama et al. 2017) implements RML imaging using ℓ_1 regularization, which promotes images with compact support by penalizing the sum of absolute pixel values and is well suited to sources with isolated compact components. The DIFMAP/CLEAN pipeline (Event Horizon Telescope Collaboration 2019) provides a traditional self-calibration-based reconstruction as a cross-check. Agreement among multiple independent pipelines on the reconstructed morphology, ring diameter, brightness asymmetry, and closure phases is required before any feature is attributed to the source rather than to the reconstruction (Event Horizon Telescope Collaboration 2019; Event Horizon Telescope Collaboration 2022). The `eht-imaging` library has been used in all published EHT imaging results for M87* and Sgr A* (Event Horizon Telescope Collaboration 2019; Event Horizon Telescope Collaboration 2021; Event Horizon Telescope Collaboration 2022; Event Horizon Telescope Collaboration 2025) and for imaging other AGN sources: Cygnus A (Janssen et al. 2020), OJ287 (Zhao 2022), and 3C279 (Fuentes 2023).

²<https://github.com/achael/eht-imaging>

4.7 The EHT Array Configuration and Instrumentation

The Event Horizon Telescope is a intercontinental VLBI array at millimeter and sub-millimeter wavelengths built to resolve horizon-scale structure around nearby supermassive black holes. In 2017 it comprised eight stations: ALMA and APEX in Chile, the Large Millimeter Telescope (LMT) in Mexico, the IRAM 30 m telescope (PV) in Spain, the SMT in Arizona, the JCMT and SMA on Mauna Kea, and the SPT in Antarctica ([Event Horizon Telescope Collaboration 2019](#)). Observations were at 228 GHz in dual circular polarization with 4 GHz recorded bandwidth at 32 Gbps per station using the Mark 6 VLBI recorder ([Event Horizon Telescope Collaboration 2019](#)). ALMA contributed as a phased array, coherently summing signals from N_{ant} dishes to give an effective collecting area of $N_{\text{ant}}A_0$ and an SEFD reduced by a factor N_{ant} relative to a single antenna, making it the most sensitive station in the array ([Event Horizon Telescope Collaboration 2019](#)). The longest baselines, set by the Hawaii-to-Spain and Hawaii-to-South-Pole pairs, reach $\approx 8.7 \text{ G}\lambda$ at 230 GHz, corresponding to a fringe spacing of $\sim 23 \mu\text{as}$ ([Event Horizon Telescope Collaboration 2019](#)). Data were correlated independently at MPIfR Bonn and MIT Haystack ([Event Horizon Telescope Collaboration 2019](#)).

4.7.1 Early Observations of M87*: 2009–2013

The first horizon-scale 230 GHz VLBI detections of M87* were reported by [Doeleman et al. 2012](#) from 2009 April. A three-station array comprising the JCMT, ARO/SMT, and CARMA detected fringes on projected baselines up to $3.5 \text{ G}\lambda$, constraining the intrinsic source size to $\lesssim 5.5 r_s$ ($r_s = 2GM/c^2$) and the minimum brightness temperature to $T_B \gtrsim 10^{10} \text{ K}$ ([Doeleman et al. 2012](#)). This brightness temperature exceeds what thermal disk emission can produce at these scales and points to compact synchrotron emission within the innermost gravitational radii. Subsequent 230 GHz observations in 2011 and 2012 expanded the array incrementally, and [Akiyama et al. 2015](#) applied sparse-modeling reconstruction to the combined 2009–2011 dataset, recovering images consistent with an asymmetric ring of diameter $\approx 40 \mu\text{as}$. This provided the first morphological indication of a compact ring-like structure at horizon scales consistent with the expected angular size of the photon orbit, anticipating the full imaging result of 2017.

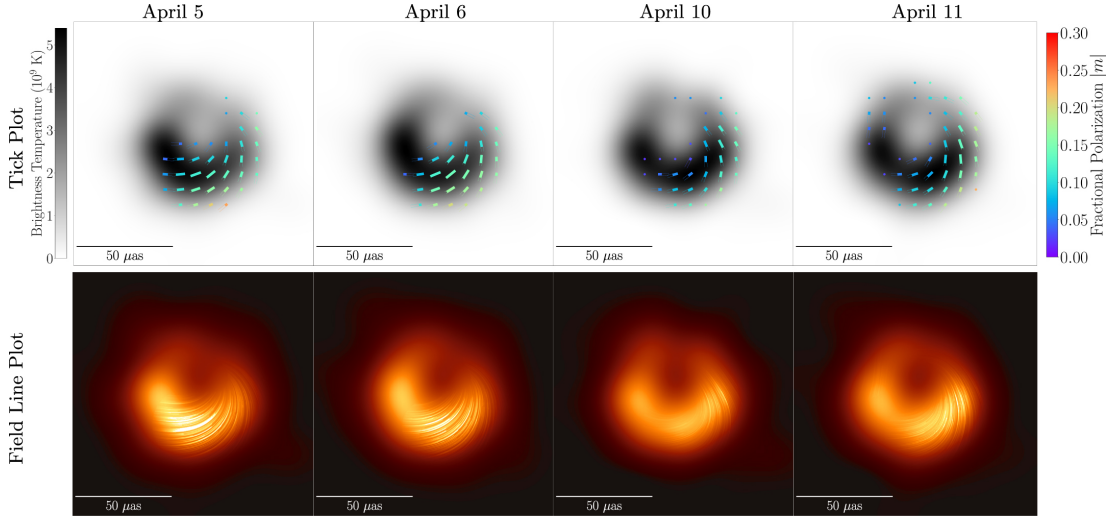


Figure 4.7: Polarimetric images of M87* from the four 2017 EHT observing days. Top row: total intensity (greyscale, in units of brightness temperature) with EVPA tick marks colored by fractional linear polarization $|m|$. Bottom row: corresponding field line maps showing the inferred magnetic field orientation, with total intensity shown in color. Image from [Event Horizon Telescope Collaboration 2021](#).

4.7.2 First Horizon-Scale Images: M87*, 2017

The 2017 EHT campaign observed M87* over four nights in April with the eight-station array as described in Section 4.7. The results were published as a six-paper series ([Event Horizon Telescope Collaboration 2019](#); [Event Horizon Telescope Collaboration 2019](#); [Event Horizon Telescope Collaboration 2019](#); [Event Horizon Telescope Collaboration 2019](#); [Event Horizon Telescope Collaboration 2019](#); [Event Horizon Telescope Collaboration 2019](#)). Three independent imaging pipelines each recovered images consistent with a bright asymmetric ring of angular diameter $42 \pm 3 \mu\text{as}$, with the emission concentrated on the south-west side corresponding to the Doppler-boostered approaching side of the disk ([Event Horizon Telescope Collaboration 2019](#)). The ring surrounds a central brightness depression with a flux ratio of $\gtrsim 10:1$ relative to the ring peak, and the closure phase structure and brightness asymmetry were used to constrain physical models, favoring MAD GRMHD models with moderate inclination ([Event Horizon Telescope Collaboration 2019](#)). The ring diameter, set by the photon orbit of the central black hole was used to infer $M = (6.5 \pm 0.7) \times 10^9 M_\odot$, broadly consistent with stellar-dynamical measurements ([Gebhardt et al. 2011](#); [Event Horizon Telescope Collaboration 2019](#)).

Figure 4.7 shows the polarimetric images of M87* from the four 2017 EHT observing nights ([Event Horizon Telescope Collaboration 2021](#); [Event Horizon Telescope Collaboration 2021](#)). The resolved LP images show that only a por-

tion of the ring is significantly polarized, with the emission concentrated in the south-west arc and an approximately azimuthal EVPA pattern across the polarized region (Event Horizon Telescope Collaboration 2021). The local fractional LP reaches a maximum of $\sim 15\%$ in the south-west arc (Event Horizon Telescope Collaboration 2021), while the net image-integrated LP is substantially lower due to partial cancellation arising from the azimuthal EVPA pattern. The ratio of the net to resolved LP fraction implies significant internal Faraday rotation, which scrambles the polarimetric signal on sub-resolution scales and partially cancels the coherent LP when averaged over the image (Event Horizon Telescope Collaboration 2021). The EVPA structure is characterized by the β_2 coefficient, a Fourier mode of the polarization field on the ring that is sensitive to the magnetic field geometry and the black hole spin (Event Horizon Telescope Collaboration 2021). MAD accretion states with dynamically important magnetic fields reproduce the observed EVPA pattern and LP fraction better than SANE models (Event Horizon Telescope Collaboration 2021). Analyzing the right and left circular polarization measurements provided a “firm evidence for a weak, astrophysical (noninstrumental) circular polarization”, on horizon scales (Event Horizon Telescope Collaboration 2023). An upper limit of $\langle |v| \rangle < 3.7\%$ was established on the unsigned resolved circular polarization fraction at the horizon scale of $20 \mu\text{as}$. The rotation measure toward M87*, measured from the EVPA frequency dependence across the 4 GHz observing band, yields $|\text{RM}| \approx 10^5 \text{ rad m}^{-2}$ and constrains the accretion rate and the electron density in the Faraday-rotating plasma between the emitting region and the observer (Event Horizon Telescope Collaboration 2021).

The first analysis of CP using the 2017 EHT data was presented in (Event Horizon Telescope Collaboration 2023). There was weak evidence for circularly polarized emission on event-horizon scales from interferometric closure quantities, with an upper limit on the resolved CP fraction of $\langle |v| \rangle < 3.7\%$ after blurring to a $20 \mu\text{as}$ beam.

4.7.3 Multi-Epoch Monitoring: 2009–2017 and 2017–2021

Wielgus et al. 2020 analyzed M87* observations spanning 2009 to 2017 and found the ring diameter consistent with $\sim 40 \mu\text{as}$ across all epochs despite significant year-to-year variation in the azimuthal brightness distribution and total compact flux. The azimuthal position of the brightness maximum changed between epochs at the level of tens of degrees, rotating from south-east in 2009 to south in 2017, which is broadly consistent with a turbulent accretion flow whose instantaneous brightness distribution fluctuates around a time-averaged ring set by the photon

orbit (Wielgus et al. 2020). The diameter stability over eight years, despite this morphological variability, is consistent with the photon orbit interpretation of the compact ring structure.

Event Horizon Telescope Collaboration 2025 reported and analyzed three fully polarimetric epochs in 2017, 2018, and 2021, producing the first multi-year set of resolved polarimetric EHT images of M87*, shown in Figure 4.8. The 2021 observations included two new stations, the 12 m Kitt Peak Telescope and NOEMA, which substantially improved UV coverage and enabled the first EHT constraints on emission outside the ring at scales $\lesssim 1$ mas (Event Horizon Telescope Collaboration 2025). All three epochs show a persistent ring of diameter $43.9 \pm 0.6 \mu\text{as}$, with the azimuthal brightness distribution varying from year to year as expected for a stochastic accretion flow. The peak of the brightness maximum shifted approximately 30° anti-clockwise between 2017 and 2018, broadly consistent with predictions from numerical simulations of M87*. The 2018 and 2021 images are remarkably similar despite a gamma-ray flare in M87 quasi-contemporaneous with the 2018 observations, suggesting the compact millimeter emission on horizon scales is largely decoupled from high-energy activity at larger scales. The local LP fraction peaked at $\sim 15\%$ in 2017 and dropped to $\sim 5\%$ in both 2018 and 2021, with the EVPA helicity reversing between 2017 and 2021, which may reflect evolution in the magnetized accretion flow or in an external Faraday screen (Event Horizon Telescope Collaboration 2025).

The imaging analysis for this paper employed three independent regularized maximum likelihood methods: `eht-imaging` (Chael et al. 2016; Chael et al. 2018), DoG-HIT (Event Horizon Telescope Collaboration 2025), and a multi-objective evolutionary algorithm (MOEA/D), applied to both total intensity and polarization data. For the 2017 and 2018 data the HOPS calibration pipeline was used, while the 2021 data was utilized using the rPICARD pipeline due to NOEMA’s instrumental phase jumps. The fiducial images presented in Event Horizon Telescope Collaboration 2025 are averages over all methods. A blinded parameter recovery test on synthetic data produced from MAD GRMHD snapshots demonstrated reconstruction cross-correlations exceeding 0.975 with the ground truth for all methods and all epochs, confirming the fidelity of the multi-epoch analysis. The contribution from the present thesis to this imaging effort, specifically the application of `eht-imaging` to the multi-epoch polarimetric dataset, is described in detail in Chapter 5.

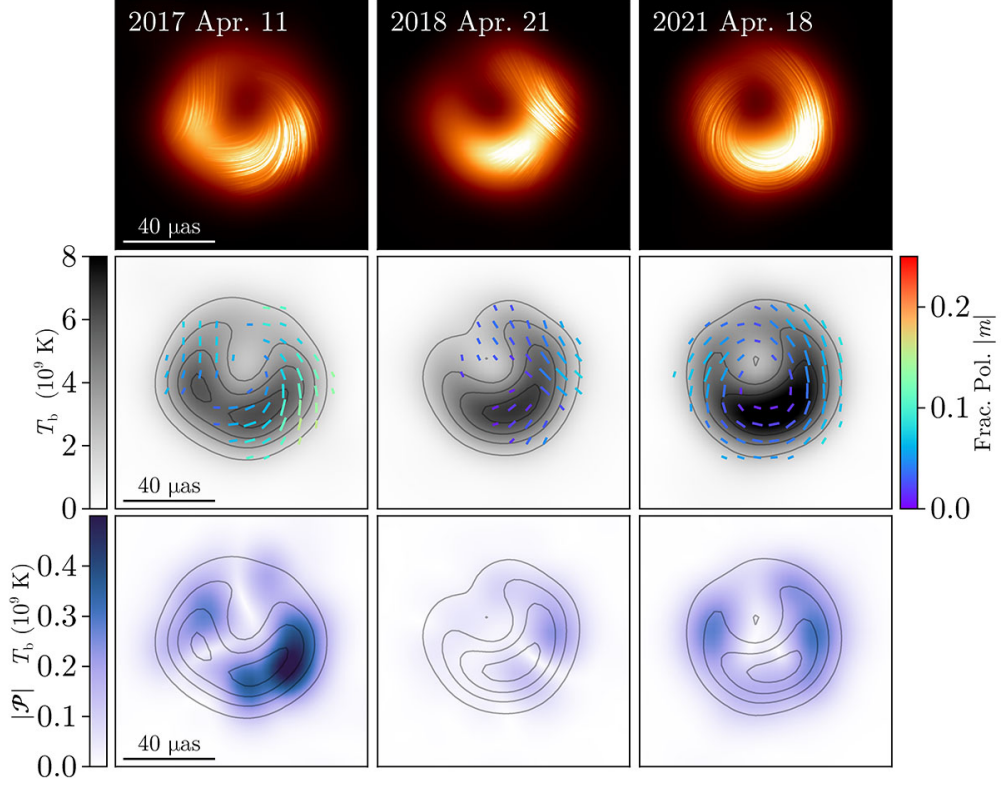


Figure 4.8: Fiducial multi-epoch images of M87* from the 2017, 2018, and 2021 EHT campaigns. Each epoch uses a single representative observing day (2017 April 11, 2018 April 21, and 2021 April 18). Top row: total intensity overlaid with polarization field lines indicating the projected magnetic field orientation. Middle row: total intensity (greyscale) with EVPA tick marks colored by fractional linear polarization $|m|$, contours at 22.5%, 45%, 67.5%, and 90% of peak. Bottom row: linearly polarized brightness $|P|$. Adapted from [Event Horizon Telescope Collaboration 2025](#).

4.8 Physical Interpretation of the M87* Images

In Chapter 3, Section 3.6 we have explicitly defined the terms, we are now ready to understand and interpret the M87* images. The bright ring in the 2017 EHT images of M87* (first image in Figure 4.9) appears as the unresolved superposition of the direct emission from the accretion flow, lensing ring, and $n = 1$ photon ring subring, all blended by the $\sim 15 \mu\text{as}$ beam. Despite this blending, the ring diameter of $42 \pm 3 \mu\text{as}$ is consistent with the critical curve diameter $b_c/D_s = 3\sqrt{3}GM/c^2D_s$, making it a reliable proxy for the black hole mass and distance ratio. The dominant flux in the ring comes from the $n = 0$ direct emission and the $n = 1$ lensing ring, with the photon ring subrings ($n \geq 1$) contributing only ~ 10 – 20% of the total ring flux ([Johnson et al. 2020](#)).

Figure 4.9 shows the relationship between the ring and the features defined above. In the native-resolution MAD simulation, the photon ring and inner shadow

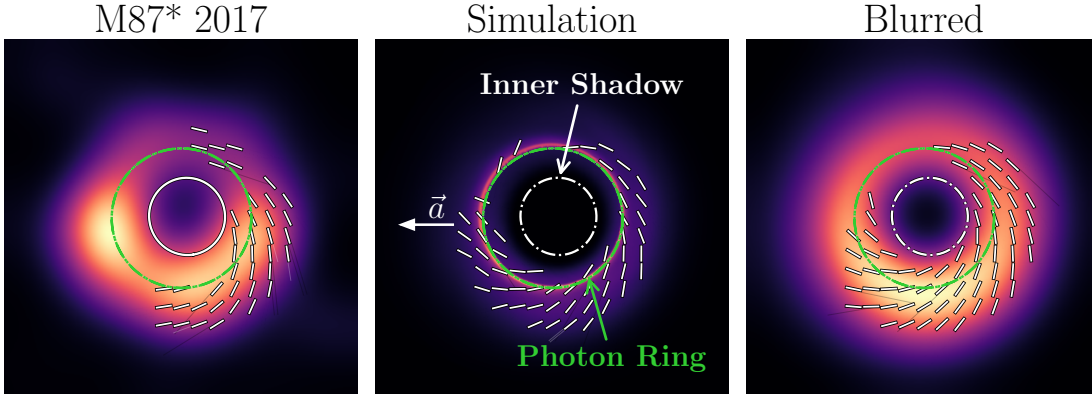


Figure 4.9: Comparison of the observed M87* ring with a MAD GRMHD simulation. The left panel shows the reconstructed 2017 EHT image of M87*, and the center panel shows a time-averaged MAD simulation at $a_\star = 0$, $R_{\text{high}} = 50$, and $\theta_i = 163^\circ$ at native resolution, with the photon ring (green dashed) and inner shadow (white dashed) marked and the spin vector direction $\vec{a}$ indicated. The right panel shows the same simulation blurred with the EHT beam of $\approx 15 \mu\text{as}$ FWHM.

are spatially distinct where a bright narrow annulus surrounds a smaller dark region bounded by the lensed image of the equatorial horizon. The coherent large-scale magnetic field of the MAD state concentrates the emission into a narrow equatorial sheet, placing the emission morphology firmly in the thin-disk regime and producing the inner shadow visible in the native-resolution simulation (Chael et al. 2021). When the simulation is convolved with the 2017 EHT beam (right panel), the inner shadow is completely washed out and the result is morphologically indistinguishable from the observed M87* ring. This directly connects to the imaging results of Chapter 3: in the semi-analytic RIAF images, decreasing the scale height H from unity toward $H = 0.1$ collapsed the emission toward the equatorial plane and progressively sharpened the inner shadow, separating it from the photon ring, while the $H = 1$ quasi-spherical case produced a single broad brightness depression extending to the critical curve. The MAD GRMHD model is firmly at the thin-disk end of this sequence (Chael et al. 2021).

GR also predicts an infinite sequence of photon ring subrings at $n \geq 1$, each exponentially demagnified by $e^{-\gamma}$ relative to the previous one (Johnson et al. 2020). Resolving the $n = 1$ subring from the $n = 0$ direct emission requires baselines well beyond Earth diameter. Both the $n = 1$ subring and the inner shadow are primary science targets for the ngEHT and space-VLBI concepts (Johnson et al. 2023; Chael et al. 2021).

The accretion physics presented in the polarimetric images connects directly to the model library of Chapter 3. The azimuthal EVPA pattern and net LP fraction of $\sim 15\%$ are reproduced only by MAD models with dynamically important

coherent magnetic fields; SANE models predict lower, more disordered LP (Event Horizon Telescope Collaboration 2021). The suppression of net LP relative to local resolved values arises from internal Faraday rotation scrambling the polarimetric signal on sub-resolution scales, consistent with the Faraday rotation discussed in Section 3.4.3. The rotation measure $|\text{RM}| \approx 10^5 \text{ rad m}^{-2}$ constrains the accretion rate $\dot{M}$ to $\sim (3 - 20) \times 10^{-4} M_{\odot} \text{ yr}^{-1}$ (Event Horizon Telescope Collaboration 2021), firmly in the RIAF-ADAF regime.

4.9 Open Questions

The chapters so far have built up from the spacetime geometry to the accretion physics to the synthetic observables, and this chapter has then taken those models and confronted them with actual EHT data. That confrontation has been productive where the ring diameter constrains the black hole mass, the EVPA structure largely disfavors SANE accretion, the polarization properties constrains the accretion rate, and the multi-epoch monitoring establishes the stability of the photon orbit across eight years of observations. Yet the very act of comparing models to data at this level of detail exposes how much remains unresolved. Some of the following questions could not even have been posed before the EHT images existed, meanwhile others have been open for decades and the images have sharpened rather than answered them. All of them motivate the work presented in Part II.

Q. Can EHT data constrain the spin of M87*? As established in Section 2.8, measuring a_* requires either resolving the photon ring shape (Bardeen 1973; Gralla et al. 2019) or detecting the inner shadow jointly with the photon ring (Chael et al. 2021). Neither is accessible at 2017 EHT resolution. The EVPA structure carries spin information through the β_2 coefficient (Event Horizon Telescope Collaboration 2021), but this signal is degenerate with the magnetic field geometry and Faraday effects as discussed in Section 3.7. The extent to which the multi-epoch polarimetric dataset can nevertheless constrain a_* through the full semi-analytic model grid is examined in Saurabh et al. 2026.

Q. What drives the year-to-year variability in the polarization of M87*? The fractional LP dropped from $\sim 15\%$ in 2017 to $\sim 5\%$ in 2018 and 2021, and the EVPA helicity reversed between 2017 and 2021 (Section 4.7.3). Whether this reflects a physical change in the magnetized accretion flow — such as the flux eruption dynamics of the MAD state (Narayan et al. 2022) — or a structural change in an external Faraday

screen, is not established. The multi-epoch imaging analysis of [Event Horizon Telescope Collaboration 2025](#), to which this thesis contributes through the `eht-imaging` pipeline (Section 4.6.3), addresses this directly through resolved polarimetric images across the three epochs.

- Q. What is the structure of the M87* jet base at horizon scales, and where is it launched?** The disk-jet transition region on scales $\lesssim 1$ mas was first observationally constrained with the improved 2021 EHT coverage (Section 4.7.3). How the jet is collimated, where it is launched relative to the photon ring, and whether the inner-jet emission is separable from the disk contribution at 230 GHz are all unresolved. This connects directly to the jet-launching problem of Section 2.8: determining whether the Blandford-Znajek mechanism ([Blandford and Znajek 1977](#)) or disk-driven winds ([Blandford and Payne 1982](#)) dominate at horizon scales requires resolving the spatial structure of the emission near the base of the jet. Constraints from the 2021 dataset are presented in [Saurabh et al. 2026](#).
- Q. Can the inner shadow and photon ring substructure ever be resolved with VLBI?** At current EHT resolution and dynamic range, the inner shadow is completely washed out and the photon ring subrings are blended into the observed bright ring, as shown in Section 4.8. Detecting the inner shadow requires both higher angular resolution and higher imaging dynamic range ([Chael et al. 2021](#)). Resolving the $n = 1$ photon ring subring from the $n = 0$ direct emission requires baselines well beyond Earth diameter ([Johnson et al. 2020](#)). Both are primary science targets for the ngEHT and space-VLBI concepts ([Johnson et al. 2023](#)), and would provide the most direct observational tests of the compact object question posed in Section 2.8.

Part II

Applications

Chapter 5

Semi-analytic Studies of the Accretion disk and Magnetic Field Geometry in M87*

This chapter is based on the following publication:

Saurabh, M. Wielgus, A. Tursunov, A. P. Lobanov, and R. Emami, “Semi-analytic studies of the accretion disk and magnetic field geometry in M87*,” *Astronomy & Astrophysics*, 2025 ([Saurabh et al. 2026](#)).

The 2017 EHT polarimetric images of M87*, discussed in Section 4.7.2, show a resolved ring with an approximately azimuthal EVPA pattern and a local LP fraction reaching $\sim 15\%$ in the south-west arc. The model comparisons of [Event Horizon Telescope Collaboration 2021](#); [Event Horizon Telescope Collaboration 2021](#) found that MAD GRMHD models broadly reproduce these features, while SANE models are generally disfavored at 230 GHz. Within the MAD family, however, the allowed parameter space remains broad. The magnetic field topology, the disk scale height, and the velocity profile each leave distinct imprints on the Stokes maps, but disentangling their individual contributions from a single observational epoch is non-trivial. This degeneracy, identified in Section 3.7, cannot be resolved from within GRMHD because the field geometry, the flow structure, and the disk geometry all emerge together as self-consistent outputs of the nonlinear MHD evolution and cannot be adjusted independently. The semi-analytic RIAF model of Section 3.2.1 is better suited for this problem precisely because each of these parameters enters as a free input and can be varied in isolation.

We compute polarimetric images at 230 GHz over a grid of toroidal, poloidal, and mixed helical field configurations, with $H \in [0.1, 1]$, $\kappa \in [0, 1]$, for several spin values. Radiative transfer is performed using `ipole` ([Mościbrodzka and Gammie](#)

2018; Prather et al. 2023), which solves the full polarized radiative transfer equation along backwards-traced null geodesics in the Kerr spacetime, computing all four Stokes components simultaneously. Radiative transfer coefficients are evaluated from the thermal fitting formulae of Pandya et al. 2016. These are the same prescriptions described in Section 3.5.1 and used throughout the synthetic image grids of Section 3.5.5.

A helical or mixed poloidal-toroidal field configuration, combined with moderate sub-Keplerian velocities and a geometrically thick disk ($H \gtrsim 0.5$), produces Stokes maps broadly consistent with the 2017 EHT polarimetric observations. Purely toroidal field configurations produce radial EVPA patterns that are inconsistent with the observed azimuthal structure. Purely poloidal field models reproduce azimuthal EVPA patterns, but they overpredict the LP fraction for most H and κ combinations unless the Faraday depth is high. The β_2 coefficient constrains the field helicity and EVPA ordering on the ring (Event Horizon Telescope Collaboration 2021), carrying some spin-dependent signal that is nonetheless partially degenerate with the field configuration, as anticipated in Section 4.9.

To introduce custom magnetic fields in `ipole`, modifications were required in the accretion model scripts. As discussed in the paper, the magnetic field geometry can be found by prescribing the vector potential, which consistently implies divergence free fields $\nabla \cdot \vec{B} = 0$. I then developed the RIAF model parameter grid, performed all polarimetric ray-tracing calculations, produced all figures, and led the analysis. Maciek Wielgus provided significant input on the interpretation of the results in the context of M87*, and important/relevant discussions by other collaborators of the paper. Arman Tursunov provided with an additional input with a particular magnetic field sensitive to the spin of the black hole. All authors contributed to the writing and revision of the manuscript.

Semi-analytic studies of the accretion disk and magnetic field geometry in M87*

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ABSTRACT

Context. Magnetic fields play a pivotal role in the dynamics of black hole accretion flows and in the formation of relativistic jets. Observations by the Event Horizon Telescope (EHT) provided unprecedented insights into accretion structures near black holes. Interpreting these observations requires a theoretical framework that links polarized emission to the underlying system properties and magnetic field geometries.

Aims. We investigated how the system properties, in particular, the magnetic field geometry in the region of the event horizon scale, affect the structure of the observable synchrotron emission in M87*. Specifically, we characterized the sensitivity of observables used by the EHT to black hole spin, plasma dynamics, accretion disk thickness, and magnetic field geometry.

Methods. We adopted a semi-analytic radiatively inefficient accretion flow model in Kerr spacetime. We varied the magnetic field geometry, black hole spin, accretion disk dynamics, and geometric thickness of the disk. We performed general relativistic ray-tracing with a full polarized radiative transfer to obtain synthetic images of M87*. We extracted EHT observables, such as disk diameter, asymmetry, and polarimetric metrics from synthetic models. We also considered a number of general relativistic magnetohydrodynamics simulations and compared them with the semi-analytical models.

Results. The effect of the disk thickness on the observables is limited. On the other hand, magnetic configurations dominated by the toroidal and poloidal fields can be distinguished reliably. The flow dynamics, in particular, radial inflow, also significantly affects the EHT observables.

Conclusions. The M87* system is most consistent with a flow dominated by the poloidal magnetic field with partially radial inflow. While the spin remains elusive, moderate or high positive values are preferred.

Key words. accretion, accretion disks – black hole physics – gravitation – magnetic fields – polarization – radiative transfer

1. Introduction

It is unclear how magnetic fields govern the behavior of matter near black holes (BHs), as is the role they play in the formation of powerful jets and accretion flows in these extreme environments. This question remains central to our understanding of BH astrophysics. The interplay between magnetic fields and accretion flows significantly affects the immediate vicinity of BHs and extends to broader implications for galaxy dynamics and the evolution of supermassive BHs in active galactic nuclei (AGN; Blandford & Znajek 1977; Fabian 2012). At the event horizon scale, magnetic fields are thought to be crucial for angular momentum transport and energy dissipation within accretion flows (Balbus & Hawley 1991), thereby affecting the accretion rate and playing a key role in launching and collimating relativistic jets (Kovalev et al. 2005; Beckwith et al. 2008). These processes can be modeled with the equations of magnetohydrodynamics (MHD; e.g., Gammie et al. 2003; McKinney et al. 2012), with the structure and evolution of mag-

netic fields being key to deciphering the observed properties of BH systems, including their polarized emission.

It was not until the recent radio observations of the centers of galaxies Messier 87 (M87*) and the Milky Way, Sagittarius A* (Sgr A*), by the Event Horizon Telescope (EHT; EHTC 2019a,d, 2022a,b) and the near-infrared (NIR) observations by the GRAVITY instrument (GRAVITY Collaboration 2018) that new avenues were opened to probe horizon-scale properties of BHs. EHT observations of M87* and Sgr A* revealed compact bright ring-like emission structures with substantial central brightness depression, providing compelling evidence for the existence of supermassive compact objects in their cores (EHTC 2019a, 2022a). Subsequent analyses of the EHT observations yielded further insights into the physics of these systems, particularly through the study of polarized emission structures. They constrained the magnetic field geometry in the core region (EHTC 2021a,b, 2024a,b). These observations revealed dynamically important magnetic fields that are expected to play a crucial role in launching and collimating relativistic jets.

In order to explain the observations and model the behavior of magnetized plasma in the strong gravitational field near BHs, general relativistic magnetohydrodynamics (GRMHD) simulations are employed (Gammie et al. 2003; McKinney & Gammie 2004). These simulations have become an indispensable tool

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for investigating the intricate interplay between magnetic fields (McKinney et al. 2012; Gold et al. 2017; Ripperda et al. 2022), accretion flows (Porth et al. 2019; Narayan et al. 2022), and spacetime geometry (Mizuno et al. 2018; Chatterjee et al. 2023), and their combined effects (EHTC 2019a; Chatterjee et al. 2020; Nathanail et al. 2022).

While GRMHD simulations offer the most physically self-consistent framework for studying BH accretion, the inference based on these models remains contentious (e.g., Gralla 2021; Wielgus 2021). Despite its successes as an effective model, the set of equations solved by GRMHD is inadequate to describe collisionless plasma, assuming highly uncertain initial conditions. Furthermore, most of the results are obtained within the ideal MHD framework, while the important role of nonideal MHD effects such as magnetic reconnection in turbulent accretion flows near the event horizon is recognized (e.g., Ripperda et al. 2022). Global stationary semi-analytic approaches offer a complementary perspective by enabling parametric studies of accretion flows and magnetic field geometries. They allow us to investigate the effect of the prescribed disk properties and magnetic field geometry on the observable properties, including polarization of the emitted radiation (Narayan & Yi 1995; Pu & Broderick 2018). This approach constitutes a bridge between toy models that are useful for building intuition (e.g., Narayan et al. 2021) and computationally exhaustive GRMHD, where the parameters of the accretion disk and magnetic field follow the evolution of the MHD equations and cannot be modified explicitly.

A low-luminosity AGN system of M87* falls into the category of radiatively inefficient accretion flows (RIAFs; Yuan et al. 2003) with a geometrically thick and optically thin accretion disk. Unlike optically thick and geometrically thin disks (Shakura & Sunyaev 1973), which efficiently cool down by blackbody radiation, RIAFs are characterized by low number densities and an extremely high temperature of ions that is decoupled from that of the radiating electrons. Thus, radiative cooling is inefficient, and excess energy is removed from the system through advection. A stationary semi-analytic RIAF model framework for M87* was proposed (e.g., Broderick & Loeb 2009; Vincent et al. 2021). We systematically study the effect of the magnetic field configuration, BH spin, as well as geometric thickness and dynamics of the accretion disk on the observational appearance of RIAF models. We compare the RIAF results with GRMHD simulations, as well as with the characteristic observables extracted from the EHT data, including polarimetric quantities.

2. Model images of a black hole

We set up a stationary axisymmetric RIAF model in Kerr spacetime and subsequently performed numerical general relativistic ray-tracing using the open-source code *ipole*¹ (Mościbrodzka & Gammie 2018). We assumed that the observed emission originates near the equatorial plane in the accretion flow, and we thus neglected potential contributions from the jet base. While the jet plays a major role for the source morphology and its energetic output at lower radio frequencies (e.g., Walker et al. 2018), the compact ring-like morphology of the 230 GHz EHT images of M87* was found to be more consistent with the emission dominated by the accretion flow (e.g., EHTC 2019e, 2021b; Emami et al. 2023). This is particularly true for the favored strongly magnetized models (see EHTC 2019e and

Sect. 2.4). While the extended jet in M87* has been detected at 230 GHz (Goddi et al. 2021), observations support its subdominant role on compact scales at this frequency. The emission component corresponding to the jet on scales of ~ 100 Schwarzschild radii has been weakly constrained only most recently in the EHT datasets (EHTC 2025a; Saurabh et al. 2025). Nonetheless, even if the emission is dominated by the accretion disk, the foreground jet may still act as a Faraday screen, modifying some polarimetric observables (Goddi et al. 2021; Wielgus et al. 2024). To capture these effects in the semi-analytic framework, a more complicated two-component disk + jet model would be necessary (see, e.g., preliminary work of Vincent et al. 2019). We provide relevant details of the disk model we used below.

2.1. RIAF setup

We adopted a simple semi-analytic RIAF model of a radiatively inefficient accretion flow (e.g., Broderick et al. 2016; Pu & Broderick 2018; Vos et al. 2022). In this framework, we prescribed a stationary model of the accretion flow instead of evolving conservative MHD equations. In particular, we assumed that the number density of electrons n_e and electron temperature T_e behaves as power laws of the spherical radius,

$$n_e = n_{e,0} \left(\frac{r}{r_g} \right)^{-\delta} \exp \left[-\frac{1}{2} (H \tan \theta)^{-2} \right], \quad (1)$$

$$T_e = T_{e,0} \left(\frac{r}{r_g} \right)^{-\gamma}, \quad (2)$$

for a gravitational radius $r_g = GM_{\text{BH}}/c^2$, and we used the standard Boyer-Lindquist coordinate system (t, r, θ, ϕ) in Kerr spacetime of a rotating BH, parameterized with a spin (angular momentum) parameter a (Kerr 1963). We also defined a dimensionless spin parameter $a_* = a/M$. Parameter H (dimensionless as well) controls the geometric thickness of the disk (Pu & Broderick 2018; Vincent et al. 2022). We took $H = 0.5$ as a fiducial geometrically thick model and additionally tested $H = 0.1$ and $H = 0.3$. There are some observational handles on γ in RIAF sources based on radiative spectra (e.g., Broderick et al. 2011) and VLBI brightness temperatures (Wielgus et al. 2024) of Sgr A*, and we therefore adopted a value of $\gamma = 0.84$ for our fiducial model, which is consistent with observational constraints. Our choices for all fiducial model parameters are given in Table 1.

To model the four-velocity of the accretion flow, we followed Pu et al. (2016), Tiede et al. (2020), and many others and interpolated between Keplerian orbital motion and geodesic free fall, hence

$$u^r = u_K^r + \kappa_{\text{ff}}(u_{\text{ff}}^r - u_K^r), \quad (3)$$

$$\Omega = \Omega_K + (1 - \kappa_K)(\Omega_{\text{ff}} - \Omega_K), \quad (4)$$

where (u_K^r, Ω_K) and $(u_{\text{ff}}^r, \Omega_{\text{ff}})$ represent the radial four-velocity component u^r and angular velocity $\Omega = u^\phi/u^t$ for Keplerian and for free-fall motion with zero angular momentum, respectively. In the Keplerian case, fluid motion inside the innermost stable circular orbit (ISCO) is modeled as a radial inspiral, following from the conservation of the ISCO values of energy $E = -u_t$ and angular momentum $L = u_\phi$. We set $u^\theta = 0$ and obtained u^t from the four-velocity normalization. We assumed $u^r < 0$, that is, we did not allow for outflows (hence, no emission from the jets). Under the assumed parameterization, Keplerian motion corresponds to $(\kappa_{\text{ff}}, \kappa_K) = (0, 1)$, and free fall corresponds to

¹ <https://github.com/AFD-Illinois/ipole>

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Table 1. Fiducial accretion flow model parameters.

Parameter	Value	Parameter Description
M_{BH}	$6.4 \times 10^9 M_\odot$	Black hole mass
D_s	$16.9 \times 10^6 \text{ pc}$	Distance to the source
θ_i	163°	Inclination angle
δ	1.5	n_e power law index
γ	0.84	T_e power law index
H	0.5	Geometric thickness
κ_K	0.5	Keplerian parameter
κ_{ff}	0.5	Radial infall parameter
B_0	5 G	Magnetic field strength
$n_{e,0}$	$\approx 10^4 \text{ cm}^{-3}$	Number density
$T_{e,0}$	$\approx 10^{11} \text{ K}$	Temperature of electrons
ν_{obs}	230 GHz	Observing frequency
p_0	-100	Combined field parameter
FOV	200 μs	Field of view
$N_X \times N_Y$	200 $\times$ 200 px	Number of pixels

$(\kappa_{\text{ff}}, \kappa_K) = (1, 0)$. Fractional values of these parameters result in a mixed nongeodesic prescription that is more appropriate for a model of a realistic astrophysical flow in M87*, where a sub-Keplerian azimuthal velocity profile and advection are expected (e.g., Takahashi & Mineshige 2011). We considered $(\kappa_{\text{ff}}, \kappa_K) = (0, 1)$, $(1, 0)$ and $(0.5, 0.5)$, with the latter used as the fiducial RIAF model.

2.2. Magnetic field geometry

We considered several geometries of the magnetic field, embedded in the accretion flow described in Sect. 2.1. These are typically defined through vector potential A , thus enforcing the vanishing divergence of the magnetic field $\nabla_i B^i = 0$. In our convention, Latin indices correspond to three spatial dimensions, while Greek ones span over all four spacetime dimensions. We assumed axisymmetric configuration, with the only nonzero component A_ϕ , that is, $A_r = A_\theta = 0$. The frame-dragging effect twists magnetic field lines and inevitably induces an electric field. As shown by Komissarov (2004, 2022), this induced electric field cannot be completely screened in the vicinity of a rotating BH. In particular, it is unscreened within the ergosphere. This is a general feature of this class of semi-analytic models, and we only focused on the magnetic field effect on the observables here. We considered the following seven geometries, illustrated in Fig. 1:

- Radial (split monopole, e.g., Blandford & Znajek 1977)

$$A_{\phi,R} = 1 - |\cos \theta|, \quad (5)$$

- vertical (e.g., Vos et al. 2022)

$$A_{\phi,V} = r \sin \theta, \quad (6)$$

- dipole (McKinney & Blandford 2009)

$$A_{\phi,D} = \frac{1}{2}((r + r_0)^\nu \mathcal{F}_- + 2\mathcal{F}_+(1 - \ln(\mathcal{F}_+))), \quad (7)$$

where $\mathcal{F}_- = 1 - \cos^\mu \theta$, $\mathcal{F}_+ = 1 + \cos^\mu \theta$, $\nu = 3/4$, $\mu = 4$, and $r_0 = 4r_g$,

- quadrupole (McKinney & Blandford 2009)

$$A_{\phi,Q} = A_{\phi,D} \cos \theta, \quad (8)$$

- parabolic (e.g., Tchekhovskoy et al. 2010; Kološ et al. 2023)

$$A_{\phi,P} = r^k(1 - |\cos \theta|) = r^k A_{\phi,R}, \quad (9)$$

where we set $k = 0.75$ and for $k = 0$, this expression reduces to the radial magnetic field (split monopole).

- combined field (Kenzhebayeva et al. 2024). This solution combines two fundamental field configurations, namely the vertical homogeneous solution of Wald (1974) and the radial (split-monopole), resulting in a paraboloidal field that is a self-consistent solution to the Maxwell equations in Kerr spacetime,

$$A_{\phi,C} = g_{\phi\phi} + 2ag_{t\phi} + p_0 \frac{(r^2 + a^2)|\cos \theta|}{r^2 + a^2 \cos^2 \theta}, \quad (10)$$

where p_0 is the ratio of the radial to vertical magnetic field components.

- Finally, for a toroidal configuration, it is straightforward to directly define a divergence-free magnetic field as

$$B^r = B^\theta = B^t = 0, \quad B^\phi = \frac{1}{\sqrt{-g}}, \quad (11)$$

where g is the Kerr metric determinant.

For the configurations defined with a vector potential, with $A_r = A_\theta = 0$, the laboratory frame (static observer's frame) magnetic field components can be defined simply as

$$B^r = \frac{\partial_\theta A_\phi}{\sqrt{-g}}, \quad B^\theta = -\frac{\partial_r A_\phi}{\sqrt{-g}}, \quad B^\phi = B^t = 0. \quad (12)$$

The asymptotic behavior of the magnetic field strength following these prescriptions is $|B| \propto r^{-1}$ for the vertical and toroidal configurations, $|B| \propto r^{k-2}$ for the parabolic field, and $|B| \propto r^{-2}$ for the case of a radial field, dipole and quadrupole. The combined field asymptotes to a constant vertical magnetic field. Furthermore, the dipole field is predominantly vertical in the equatorial plane, and the quadrupole is predominantly radial in the equatorial plane, which is relevant for the signatures of a geometrically thin disk.

Subsequently, we normalized the magnetic fields by a constant factor, so that the magnetic field strength at the equatorial ring ($r = 3r_g$, $\theta = \pi/2$) was equal to $B_0 = 5 \text{ G}$, which is comparable to the EHT estimates (EHTC 2021b). In ideal magnetohydrodynamics the laboratory frame magnetic field B^μ is related to the fluid-frame magnetic field four-vector b^μ as (Gammie et al. 2003)

$$b^t = B^t u_t, \quad b^i = \frac{B^i + b^i u^i}{u^t}, \quad (13)$$

for the fluid four-velocity u^μ , which we have defined in Sect. 2.1. The magnitude of b^μ and its angle with respect to the momentum of the emitted photon k^μ were then used to compute the intrinsic synchrotron emission, along with other radiative transfer parameters. While the magnetic fields that we defined are divergence-free by construction, we did not enforce the resulting configuration of (b^μ, u^μ) to be a stationary solution of the full induction equation in Kerr metric.

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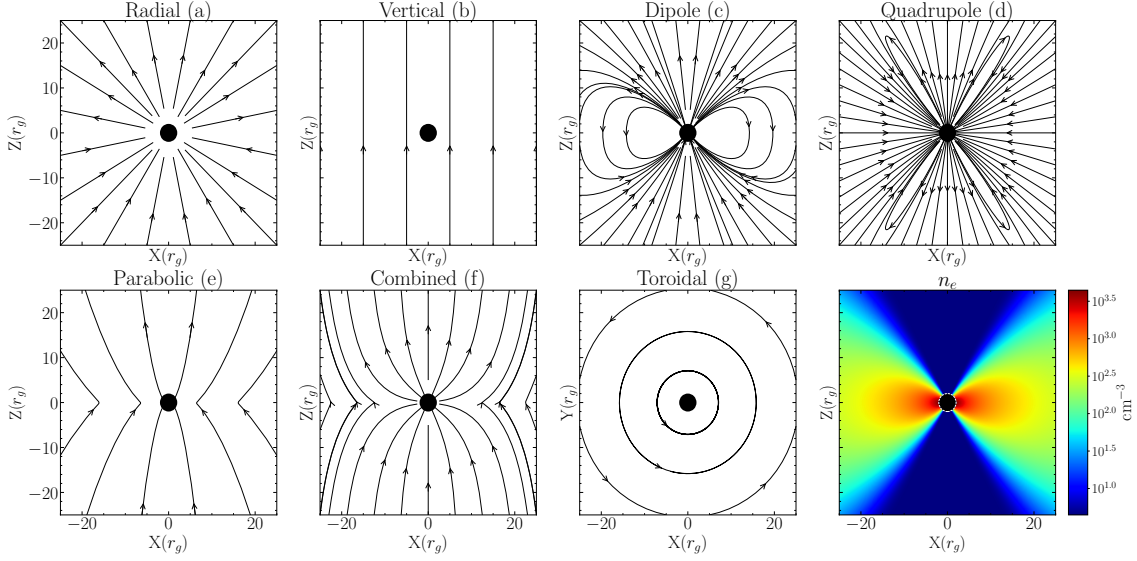


Fig. 1. Panels a–g: Schematic representation of the magnetic field configurations discussed in Sect. 2. Last panel: Example number density map for an RIAF model with a default disk thickness parameter $H = 0.5$.

2.3. Emission and synthetic images

In order to perform the radiative transfer and ray-trace the synthetic images of the source, we followed the numerical setup described by Mościbrodzka & Gammie (2018). At radio frequencies, the emission from RIAF systems is dominated by the synchrotron radiation produced by hot electrons gyrating in the magnetic field (Yuan & Narayan 2014). We assumed a relativistic thermal (Maxwell-Jüttner) energy distribution of electrons and followed the fitting formulas of Pandya et al. (2016) to compute the necessary coefficients of the fully polarized radiative transfer, including emission, absorption, and Faraday effects.

For the mass and distance of M 87*, we assumed $M_{\text{BH}} = 6.4 \times 10^9 M_{\odot}$ and $D_s = 16.9 \text{ Mpc}$, respectively (Gebhardt et al. 2011; EHTC 2019f, 2025b). We set the observing frequency to $\nu_{\text{obs}} = 230 \text{ GHz}$, which is within the observing frequency band of the EHT (EHTC 2019b,c). The inclination (viewing angle) with respect to the jet axis θ_i was estimated observationally (Mertens et al. 2016; Walker et al. 2018), with the ambiguity between θ_i and $180^\circ - \theta_i$ following from the overall symmetry of the system. The symmetry is broken by the orientation of the rotation, however. We adopted $\theta_i = 17^\circ$ or $\theta_i = 163^\circ = 180^\circ - 17^\circ$, assuming that the jet axis, angular momentum vector of the BH, and that of the accretion flow are all aligned along the same axis. The sign of the BH spin a_* informs us whether the accretion flow and BH angular momenta are parallel (prograde rotation, $a_* > 0$) or antiparallel (retrograde accretion, $a_* < 0$). For our RIAF model, which lacks a jet, we identified θ_i with the angle calculated with respect to the angular momentum vector of the accretion disk rather than that calculated with respect to the angular momentum of the BH (which is either parallel or antiparallel). An additional prior comes from the EHT results EHTC (2019a), which constrain the brighter side of the observed ring to be located in the bottom side of the image. EHTC (2019e) argued that reproducing this orientation within the GRMHD library of images requires that the angular momentum vector of the BH is pointed away from the observer’s screen (see also comments in Sect. 4.2). Thus, we followed the convention of the EHT and took $\theta_i = 163^\circ$ for the BH spins $a_* \geq 0$ and $\theta_i = 17^\circ$ for $a_* < 0$,

which guarantees that the BH spin vector points away from the observer in all cases. Keeping $\theta_i = 163^\circ$ for all BH spin values would result in a bright side of the ring flipped to the upper part of the image for $a_* < 0$ GRMHD models, which is inconsistent with the EHT results.

We ray-traced images in the field of view (FOV) of $200 \mu\text{as}$, corresponding to about $40 r_g/D_s$. The emission in the wider FOV in our idealized RIAF disk model is negligible, similarly as in the images reconstructed by the EHT (EHTC 2019d, 2024c). For each considered magnetic field model, accretion flow parameters κ_{ff} , κ_K , and disk thickness parameter H , we tuned the number density scale $n_{e,0}$ and temperature scale $T_{e,0}$ to match the typically observed compact scale emission in M 87* of $I_{\text{tot}} = 0.5 \text{ Jy}$ at 230 GHz (EHTC 2019d; Wielgus et al. 2020; EHTC 2024c) within 10% for the model with dimensionless spin $a_* = -0.94$. We kept the same normalization for other spin values, which led to a flux density deviation from 0.5 Jy of up to a factor of 2. In all cases, our $n_{e,0}$ was within an order of magnitude from 10^4 cm^{-3} , and $T_{e,0}$ was within an order of magnitude from 10^{11} K . This is reasonably consistent with the EHT estimates (EHTC 2021b).

2.4. GRMHD images

For comparison purposes, we also considered time-averaged GRMHD images from the EHT 230 GHz images library (EHTC 2021b; Wong et al. 2022; Dhruv et al. 2025). In these simulations, plasma and magnetic field co-evolve following the conservative MHD equations in a curved Kerr spacetime, reaching a quasi-stationary turbulent state of accretion. We considered ten simulations, computed for five different values of BH spin and for two different states of magnetization, a magnetically arrested disk (MAD), characterized by strong, coherent magnetic fields (Narayan et al. 2003), and a weak, turbulent magnetic field solution, “standard and normal evolution” (SANE; Narayan et al. 2012). Each of these two states predicts distinct dynamical outcomes and observable phenomena (Tchekhovskoy et al. 2011; Narayan et al. 2012). The models differ primarily in the strength and configuration of the magnetic fields threading the accretion

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Table 2. EHT constraints on M87* (EHTC 2019a, 2021a,b, 2024c).

Parameter	Lower Bound	Upper Bound
α	10.3	11.7
$\mathcal{I}_r$	0.55	0.75
$ m _{\text{net}}$	1%	3.7%
$\langle m \rangle$	5.7%	10.7%
$ \beta_2 $	0.04	0.07
$\angle \beta_2$	-163°	-127°
EVPA	0°	-80°

flow, leading to distinct predictions for accretion rates, jet power, and observable emission characteristics. In the MAD regime, the magnetic flux threading the BH accumulates until it reaches a saturation point, which results in a magnetically dominated inner accretion flow. The accretion flow becomes highly nonaxisymmetric, with low-density magnetic flux bundles alternating with high-density streams of gas (Igumenshchev 2008), and the magnetic field lines exhibit a characteristic split-monopole configuration in the inner regions (Tchekhovskoy et al. 2011). In contrast, the SANE model represents a weakly magnetized accretion flow in which the accretion flow maintains a more axisymmetric structure than in MAD (Narayan et al. 2012). In this case, the magnetic field lines exhibit a more tangled and turbulent configuration (Sądowski et al. 2014). The location of the compact scale emission observed by the EHT in M87* was discussed in EHTC (2019e). All MAD models considered in the EHT library were found to be dominated by the near-equatorial emission from the accretion flow. Emission from the jet base only manifests itself in some of the SANE models under assumptions resulting in a relatively low temperature of the electrons in the accretion disk.

For all models we considered, the electron energy distribution was relativistic thermal, which is a common assumption for modeling emission at frequencies ν_{obs} relevant for this work (EHTC 2019e). While the electron temperature T_e in our RIAF models is governed by a radial power law given by Eq. (2), however, the GRMHD models prescribe T_e as a function of the ion temperature T_i and the plasma β_p parameter, a local ratio of gas pressure to magnetic pressure. These two quantities are constrained within a GRMHD simulation, and the local ion-to-electron temperature ratio is then evaluated as a postprocessing step with a parameter R_h , following Mościbrodzka et al. (2016)

$$\frac{T_i}{T_e} = \frac{R_h \beta_p^2 + 1}{1 + \beta_p^2}. \quad (14)$$

The results of EHTC (2019e, 2021b, 2023), and Joshi et al. (2024), where a range of values $1 \leq R_h \leq 160$ were investigated, indicate a preference for a high value of R_h . In particular, most SANE models are inconsistent with a rather conservative lower limit on the observed jet power with only high negative BH spin or high positive BH spin, and $R_h \geq 80$ surviving this constraint (EHTC 2019e). Hence, for comparisons with our RIAF models, we adopted a fixed value of $R_h = 160$, resulting in relatively cold electrons in the accretion disk region, and as a consequence, in stronger Faraday effects (Quataert & Gruzinov 2000) and associated depolarization EHTC (2021b, 2024b). Another consequence of larger R_h is that in order to reproduce the observed total intensity with colder electrons, a higher mass-accretion rate is required, which further increases the Faraday depth and generally enhances the jet power.

3. Method

To compare our synthetic images of the RIAF model and the EHT observations of M87* (EHTC 2019a, 2021a), we defined quantifiable image metrics (not to be confused with the metric tensor) characterizing the source appearance in observations and in model images. We discuss the angular diameter of the ring and its brightness asymmetry as total intensity image metrics. Furthermore, we considered polarimetric consistency metrics following the method of EHTC (2021b). The summary of the EHT constraints for the image metrics we considered is given in Table 2.

3.1. Angular diameter of the ring

We extracted the size of the ring from the images using the REX module in the eht-imaging² library (Chael et al. 2016), following the procedures of EHTC (2019d). The REX module allowed us to calculate the radial profiles of the intensity distribution, which can be used to determine the center and diameter of the ring. The diameter $\mathcal{D}$ was estimated by the radial distance of the pixel with the peak intensity in the images. We blurred the synthetic RIAF images with a $15 \mu\text{s}$ circular Gaussian before extracting the profiles, in order to mimic the effective resolution of the EHT. Since the observable angular diameter scales with the uncertain mass over distance estimate $\theta_g = r_g/D_s$, equal to $3.74 \mu\text{s}$ for our fiducial parameters, we report the following normalized parameter (EHTC 2019f):

$$\alpha = \frac{\mathcal{D}}{\theta_g} = \frac{\mathcal{D}}{3.74 \mu\text{s}}. \quad (15)$$

The EHTC reported a measurement of parameter α for M87* to be $\alpha = 11.0 \pm 0.7$, corresponding to a diameter of $42 \pm 3 \mu\text{s}$ under the assumed r_g/D_s (EHTC 2019a, 2024c). This is remarkably close to the theoretical prediction for the geometric BH shadow, in the sense of a trace of the photon shell (e.g., Bardeen et al. 1972) on the distant observer's screen (the "apparent boundary" of Bardeen 1973 or the "critical curve" of Gralla et al. 2019). This prediction, which is dictated exclusively by GR through the null geodesic structure of spacetime, gives $\alpha = 6\sqrt{3} \approx 10.4$ for a Schwarzschild BH.

3.2. Brightness asymmetry

To quantify the brightness asymmetry in the total intensity image, we considered the ratio of the observed flux density between two halves of the image, separated by the line of the projected spin axis as indicated by Fig. 2,

$$\mathcal{I}_r = \frac{\mathcal{I}_2}{\mathcal{I}_1}. \quad (16)$$

We used REX in order to define the center of the ring. We used the position angle of the forward jet, constrained by the lower-frequency radio observations to be $\theta_{\text{PA}} = 288^\circ$ (Walker et al. 2018), as a proxy for the BH spin axis (see Fig. 2 and comments in Sect. 2.3). We measured $\mathcal{I}_r$ in the 2017 EHT images of M87* (EHTC 2019d) to be 0.7 ± 0.1 and in the 2018 EHT images of M87* (EHTC 2024c) to be 0.6 ± 0.1 , and we therefore took 0.65 ± 0.10 as the measurement of $\mathcal{I}_r$ to be compared to the models.

² <https://github.com/achael/eht-imaging>

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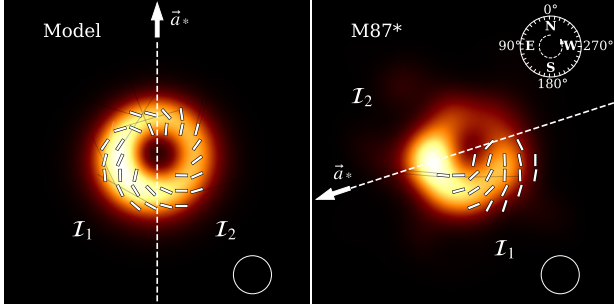


Fig. 2. Left: Example of an RIAF model with $a_* > 0$ ray-traced at an inclination of 163° , blurred with a $15 \mu\text{s}$ circular Gaussian, indicated with a circle in the bottom right corner, before the 108° counterclockwise rotation to match it with the observed appearance of the M87*. The ticks indicate the direction of polarization, vector $\vec{a}_*$ shows the observer's screen projection of the BH spin vector pointing away from the observer (into the screen). Right: Image of M87* reconstructed from the EHT 2017 observations (EHTC 2019d). The axis shows the observed position angle of the jet, with the forward jet appearing toward the west at about 288° (Walker et al. 2018). The arrow shows the expected direction of the BH spin projection, pointing away from the observer. The $15 \mu\text{s}$ effective resolution of the image is indicated with a circle in the bottom right corner. Polarization ticks are shown in the region where $I > 0.1 I_{\text{max}}$ and $|\mathcal{P}| = \sqrt{Q^2 + U^2} > 0.2 |\mathcal{P}|_{\text{max}}$, as shown in Fig. 1 of EHTC (2021b).

3.3. Polarimetric observables

We followed EHTC (2021a,b) to define the metrics quantifying the polarization properties in the resolved and unresolved images. We denote the flux densities integrated over the field of view solid angle A as I_{tot} for the total intensity,

$$I_{\text{tot}} = \int_A \tilde{I} dA \quad (17)$$

and $\mathcal{P}_{\text{tot}}$ for the linear polarization,

$$\mathcal{P}_{\text{tot}} = \int_A \tilde{\mathcal{P}} dA = \int_A (\tilde{Q} + i\tilde{U}) dA. \quad (18)$$

I_{tot} and $\mathcal{P}_{\text{tot}}$ are measured in jansky (Jy), while quantities with a tilde refer to the flux density per solid angle (brightness). $\mathcal{I}$, $\mathcal{Q}$, $\mathcal{U}$, $\mathcal{V}$ are the four Stokes parameters, describing the observed state of polarized emission. $\mathcal{P}_{\text{tot}}$ is a complex number defining the strength and orientation of the polarized emission. The net polarization fraction, corresponding to a coherent average of $\tilde{\mathcal{P}}$ over the field of view and thus measurable from unresolved observations, is defined as

$$|m|_{\text{net}} = \frac{|\mathcal{P}_{\text{tot}}|}{I_{\text{tot}}} = \frac{1}{I_{\text{tot}}} \left| \int_A \tilde{\mathcal{P}} dA \right|. \quad (19)$$

For resolved EHT images, we also defined the incoherently averaged linear polarization fraction

$$\langle |m| \rangle = \frac{1}{I_{\text{tot}}} \int_A |\tilde{\mathcal{P}}| dA. \quad (20)$$

This metric depends on the effective image resolution, and following EHTC (2021a), we computed it on images blurred with a $15 \mu\text{s}$ circular Gaussian to mimic the effective EHT resolution. Furthermore, we considered the second mode of the azimuthal

(in the ϕ angle direction) Fourier decomposition of the polarized image, denoted as β_2 (Palumbo et al. 2020),

$$\beta_2 = \frac{1}{I_{\text{tot}}} \int_A \tilde{\mathcal{P}} e^{-2i\phi} dA. \quad (21)$$

The β_2 metric exploits the approximate polar symmetry of the BH image observed at low inclination θ_i . The argument of this complex number characterizes the angle between the EVPA pattern and the BH image ring, while its absolute value informs us about how rotationally symmetric the EVPA pattern is.

4. Results

We present a grid of RIAF model images in Fig. 3. They were calculated for the fiducial parameters given in Table 1 and incorporate the various magnetic field geometries discussed in Sect. 2.2 and different values of the BH spin a_* . All images exhibit a general ring morphology, with a prominent narrow photon ring (Johnson et al. 2020; Pagnat et al. 2022), formed by photons executing at least half loop around the BH on their way to the observer's screen. The reconstructed EHT images, achieving an effective resolution of $\sim 15 \mu\text{s}$, generally blend this feature with the diffused direct emission (see Fig. 2). The morphology of RIAF images is very similar to that found in time-averaged GRMHD images shown in Fig. 4. One outstanding feature is the second smaller ring that can be seen in SANE $R_h = 160$, $a_* = 0.94$, which is the model for which the jet base emission dominates the disk emission (EHTC 2019e). A similar feature appears in the RIAF models for a dipole magnetic field configuration, where it is connected to the enhanced emission in the off-equatorial high-density region of magnetic field lines at intermediate values of θ (see panel (c) of Fig. 1). Another apparent feature in the GRMHD images is the depolarization of the northeast part of the ring seen in many of the images. This effect has been attributed to the Faraday rotation scrambling the polarization along light rays that spend a longer time in the accreting plasma (EHTC 2021b). The depolarization does not affect our fiducial RIAF images, which generally remain Faraday-thin. This is discussed further in Sect. 5.

EHTC (2021b) and Narayan et al. (2021) discussed the relation between the magnetic field geometry and the observable EVPA pattern of the accretion system modeled as a thin emission ring. Following a simple argument of perpendicularity between polarization vector $\mathbf{P}$, magnetic field $\mathbf{B}$, and wave vector $\mathbf{k}$ for synchrotron emission, expressed with a $\mathbf{P} \propto \mathbf{k} \times \mathbf{B}$ relation, they concluded that toroidal magnetic fields produce a radial EVPA pattern and poloidal magnetic fields generally produce azimuthal EVPA patterns. With the images shown in Fig. 3, we confirm that these conclusions hold well for a more realistic spatially extended global emission models and geometries, as long as the Faraday depth is low.

4.1. Angular diameter of the ring

The ring size α parameter estimates for the fiducial velocity model are given in the left panel of Fig. 5. There is a small bias, with most models producing a slightly lower α than the observational constraints of the EHT. This can easily be explained through errors on the assumed a priori $\theta_g = r_g/D_s$, however, which are about 10%. The ring diameter depends weakly on the BH spin seen in the RIAF and in GRMHD simulations, with low spin models forming slightly larger rings. The effect

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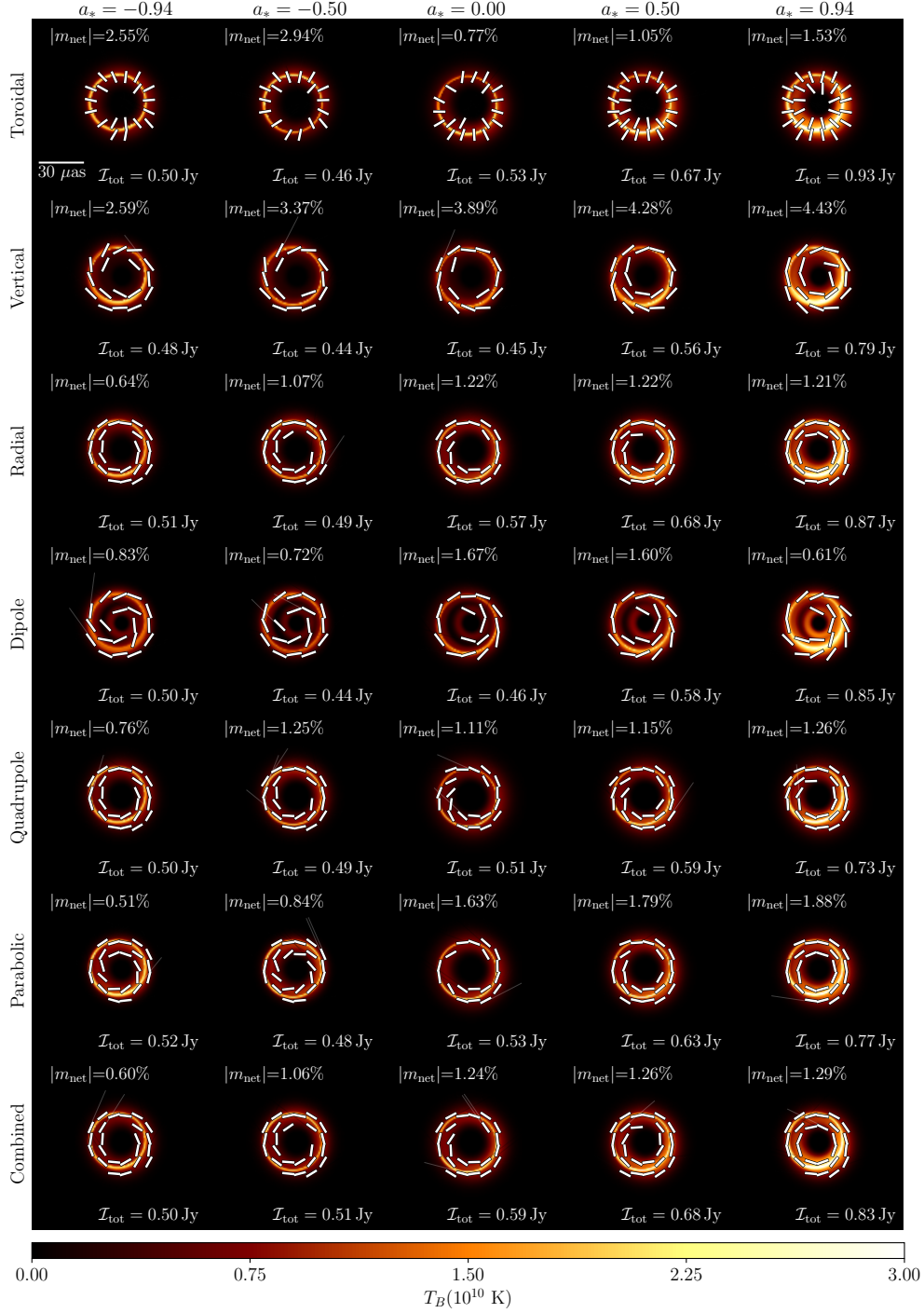


Fig. 3. Ray-traced images of the fiducial RIAF model (Table 1) with various magnetic field geometries and different BH spin a_* values, shown in units of brightness temperature. The conventions for the inclination angle θ_i and the position angle θ_{PA} are described in Sect. 2.3. The total flux density I_{tot} and the net fractional polarization $|m_{\text{net}}|$ are indicated. The convention for showing EVPA ticks is the same as in Fig. 2.

is related to the BH event horizon and BH shadow, in the sense of a trace of the photon shell (e.g., Bardeen et al. 1972) on the distant observer’s screen (“apparent boundary” of Bardeen 1973 or the “critical curve” of Gralla et al. 2019) that decreases with the increase in $|a_*|$.

It is notable that the sign of a_* only plays a secondary role for the image size. The magnetic field configuration, on the other hand, does not significantly affect the measured diameter for the fiducial RIAF model. The high-spin SANE GRMHD model results in a significantly lower measurement of the diameter as

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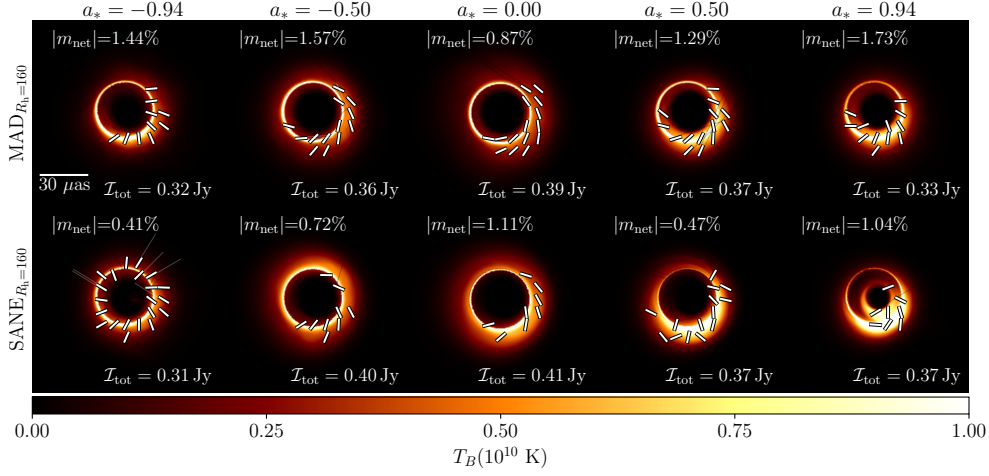


Fig. 4. Time-averaged ray traced GRMHD images from the EHT image library (EHTC 2019e, 2021b; Wong et al. 2022), representing different BH spin values a_* and MAD or SANE states of magnetization. The conventions for the inclination angle θ_i and the position angle θ_{PA} are described in Sect. 2.3. The convention for showing EVPA ticks is the same as in Fig. 2.

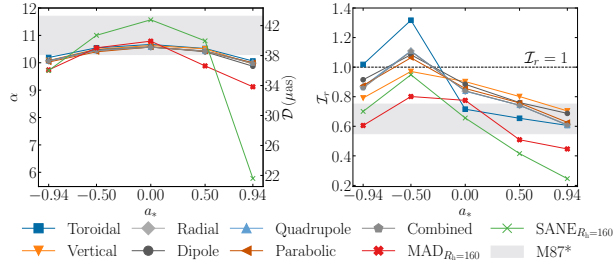


Fig. 5. Left: Diameter of the ring $\mathcal{D}$ and normalized diameter α for the fiducial RIAF model with various magnetic field configurations and for GRMHD images as a function of BH spin a_* . Right: Same for the brightness ratio parameter $\mathcal{I}_r$. The gray bands indicate constraints derived from the EHT observations of M87*.

a consequence of the jet base emission. This demonstrates that models dominated by jet emission are discouraged under r_g/D_s priors and given the EHT observations (EHTC 2019e, 2021b, and Sect. 2 of this paper). Furthermore, in the first row of Fig. 7, we inspect the effect of the accretion flow velocity model on the normalized diameter α . While the fiducial sub-Keplerian and free-fall four-velocity models are close to the EHT constraints as well as to the GR predictions of the geometric BH shadow, Keplerian models with a high spin produce significantly smaller observable rings, with a size decreasing with a_* . In these models, the accretion flow velocity remains purely azimuthal above the ISCO, while below the ISCO, the growing radial inflow velocity component in the plunging region results in beaming radiation toward the BH. This reduces the observed brightness and adds to the effects of gravitational redshift. Thus, most of the observed emission in the images of Keplerian models corresponds to the region close to ISCO. The ISCO radius is very sensitive to (signed) BH spin, changing from $9M$ for $a_* = -1$ to $1M$ for $a_* = 1$ (Bardeen et al. 1972). At the same time, the radius of the image of ISCO changes from about $10 \theta_g$ to about $2 \theta_g$ (see, e.g., Fig. 4 of Wielgus 2021). This means that for high positive BH spins, the primary image of ISCO manifests itself as a bright compact emission region contained within the geometric shadow (which itself is weakly dependent on spin). This biases the ring size measurements toward lower values. A similar behavior of

Keplerian models has been reported by Vincent et al. (2022). As a consequence, high positive spin models can only be consistent with the θ_g prior for a non-Keplerian four-velocity, with significant inflow already above the ISCO. This effect underscores the importance of the astrophysical setup and that the measured ring diameter does not always approximately track the size of the geometric shadow (see also Gralla et al. 2019; Wielgus 2021; Urso et al. 2025). Finally, the geometric thickness of the disk does not affect the diameter strongly, as demonstrated in Fig. 8.

4.2. Brightness asymmetry

The emission forming the 230 GHz M87* ring is expected to originate from the extremely compact region near the BH event horizon. The EHT observations and their interpretation confirm that most of the observed intensity originates at around $4\text{--}5 r_g$ (EHTC 2019e). In this region, the effect of the BH spin on the photon trajectories is significant. Hence, the brightness asymmetry $\mathcal{I}_r$ of the M87* image essentially originates from the interplay of two effects: special relativistic Doppler boosting associated with the fluid motion, and general relativistic effects related to the BH spin. These effects deform the null geodesics and affect the length of the path that is within the emission region. EHTC (2019e) pointed out that for the GRMHD models in the EHT library, the BH spin position controls the direction of the image asymmetry, which is the origin of the convention for the image inclination dependent on the BH spin (see Sect. 2.3). Following this convention, if the $\mathcal{I}_r$ estimates presented in the right panel of Fig. 5 are smaller than $\mathcal{I}_r = 1$, then the brighter side of the image is the bottom one ($\mathcal{I}_1$ defined in Fig. 2). This is consistent with the EHT observations. In particular, for a negative BH spin, $\mathcal{I}_r < 1$ implies that the location of the brighter side is decided by the BH spin and is opposite to what would be observed if the BH did not spin (in which case, the Doppler boost would be the only cause of the brightness asymmetry). For most of our RIAF models, we found $\mathcal{I}_r < 1$ for a negative BH spin. Nonetheless, some Keplerian velocity models with negative BH spin indicate $\mathcal{I}_r > 1$ and might be brought into consistency with the observations by the image position angle rotation (see Fig. 7). The Keplerian models are also generally most asymmetric, and their brightness asymmetry is driven predominantly by the Doppler effect. Many of these Keplerian models can be rejected based on the excessive amount of asymmetry in comparison to the EHT images. On the

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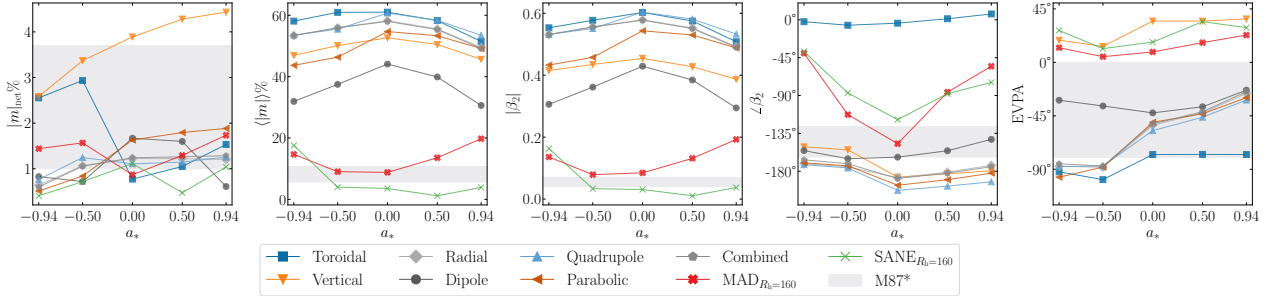


Fig. 6. Variation in the net fractional polarization $|m|_{\text{net}}$, resolved fractional polarization $\langle m \rangle$, amplitude and phase of the β_2 parameter, and the net EVPA for the fiducial RIAF model with various magnetic field configurations and for GRMHD images as a function of BH spin a_* . The gray bands indicate constraints derived from the EHT observations of M87*.

other hand, far less asymmetry is observed in RIAF images corresponding to radial infall, for which no Doppler effect is associated with the azimuthal motion (Fig. 7). As a consequence, the zero angular momentum radial inflow RIAF models are too symmetric and thus inconsistent with the observations for all magnetic field configurations and all BH spins.

With the exception of the negative spin MAD models, GRMHD images appear to be more asymmetric than the RIAF images with the fiducial sub-Keplerian velocity pattern (see Fig. 5). To obtain the amount of asymmetry similar to the positive spin GRMHD within RIAF framework, we require a more strongly azimuthal velocity pattern and a more azimuthally aligned velocity pattern in conjunction with a toroidal magnetic field (Fig. 7). A mixture of toroidal and poloidal magnetic fields is present in the GRMHD models, while the RIAF models we considered are either purely toroidal or purely poloidal.

The dependence of I_r on the BH spin a_* in Figs. 5–7 shows that the brightness asymmetry generally increases with growing positive spin because the Doppler and spin effects act in the same direction in this case. For $a_* < 0$, the effects related to BH spin and the Doppler boosting act against one another and in most cases, produce a too symmetric image that is inconsistent with the observational constraints. Hence, the brightness asymmetry constrains the BH spin in a model-dependent way, with a preference toward high positive values of a_* in fiducial RIAF models and moderate positive values for GRMHD. The models with a low negative BH spin are disfavored in both frameworks. Similarly as in case of the ring diameter, the accretion disk thickness H has a very limited effect on the brightness asymmetry (Fig. 8).

4.3. Polarimetric quantities

For a low inclination angle θ_i , a polarization pattern from an axisymmetric magnetic field averages out to a low value of $|m|_{\text{net}}$. Many models, both GRMHD and RIAF, survive this observational constraint (see the first panel of Fig. 6). On the other hand, the resolved polarization fraction $\langle m \rangle$ is always too large in our RIAF models, significantly larger than observations and GRMHD models. Many SANE models are even too depolarized, and only some of the MAD models for low or mildly negative BH spin match the observational constraints. The reason for the large $\langle m \rangle$ in our RIAF models is likely the low contribution from Faraday effects, which causes significant depolarization in GRMHD models (particularly for a high assumed value of R_h) and, presumably, in the EHT observations (EHTC 2021b). For similar physical reasons, the absolute value of β_2 is high for the circularly symmetric patterns in RIAF models, but much lower in GRMHD images as well as in the EHT observations. Keplerian models generally produce the least symmetric images (Sect. 4.2) and yield

the lowest $|\beta_2|$. Nonetheless, the argument of β_2 appears to be a powerful test of the magnetic field geometry, particularly for a distinction between the dominance of the azimuthal magnetic field component, yielding $\arg \beta_2 \approx 0^\circ$, and various poloidal magnetic field configurations, for which $\arg \beta_2 \approx 180^\circ$. These findings also agree with the results obtained by Emami et al. (2023). GRMHD models reach values in between, which is expected from a mixed magnetic field configuration (see the fourth panel of Fig. 6). Keplerian RIAF models with poloidal magnetic fields exhibit a somewhat similar behavior (Fig. 7). Unlike GRMHD, these RIAF models do not obey the fluid four-velocity and magnetic field relation, constrained by the ideal MHD induction equation. The observed β_2 angle is also in the intermediate range ($-163^\circ, -127^\circ$), indicating either a mixed magnetic field geometry or an impact from Faraday rotation. For vertical magnetic field configurations, we recovered the trends discussed by Palumbo (2025) and proposed as a test of a BH spin. While the trends are there, however, the values obtained with a global RIAF model differ from the predictions of a thin ring model used by Palumbo (2025), and a different magnetic field configuration may shift the values further or even reverse the trend direction. Finally, net EVPA, another quantity sensitive to the magnetic field geometry, was only weakly constrained by EHTC (2021a), but broadly corresponds to the direction parallel to the jet ($\theta_{\text{PA}} = -72^\circ$). We reproduced the observed EVPA with a number of RIAF models of various magnetic field configurations. The GRMHD models we considered, however, produce small positive net EVPA and are inconsistent with the observations. In a very simplistic (although commonly used) framework, vertical magnetic fields (parallel to the jet axis) would produce a perpendicular net EVPA at about 18° , while azimuthal magnetic fields (perpendicular to the jet axis) would produce a net EVPA around -72° . This heuristic works reasonably well for infalling and sub-Keplerian RIAF models with an azimuthal or vertical magnetic field structure (last panel of Fig. 6), but the higher azimuthal velocity and more complicated magnetic field structure affect this simple picture in a nontrivial way, however (last row of Fig. 7). Furthermore, we ignored the potential effect of Faraday rotation. For the M87* rotation measure $\sim 10^5 \text{ rad m}^{-2}$ (Goddi et al. 2021), the argument of β_2 can be affected by up to about 20° and EVPA by up to about 10° .

4.4. Effect of the geometric disk thickness

The RIAF flows are expected to be very hot, and as a consequence, to be geometrically thick. Models often make a simplifying assumption of equatorial emission (e.g., Gralla et al. 2019; Pagnat et al. 2022; Urso et al. 2025). We tested the effect of geometric thickness on a number of observables. We summarize the results in Fig. 8. In most cases, the geometric thickness

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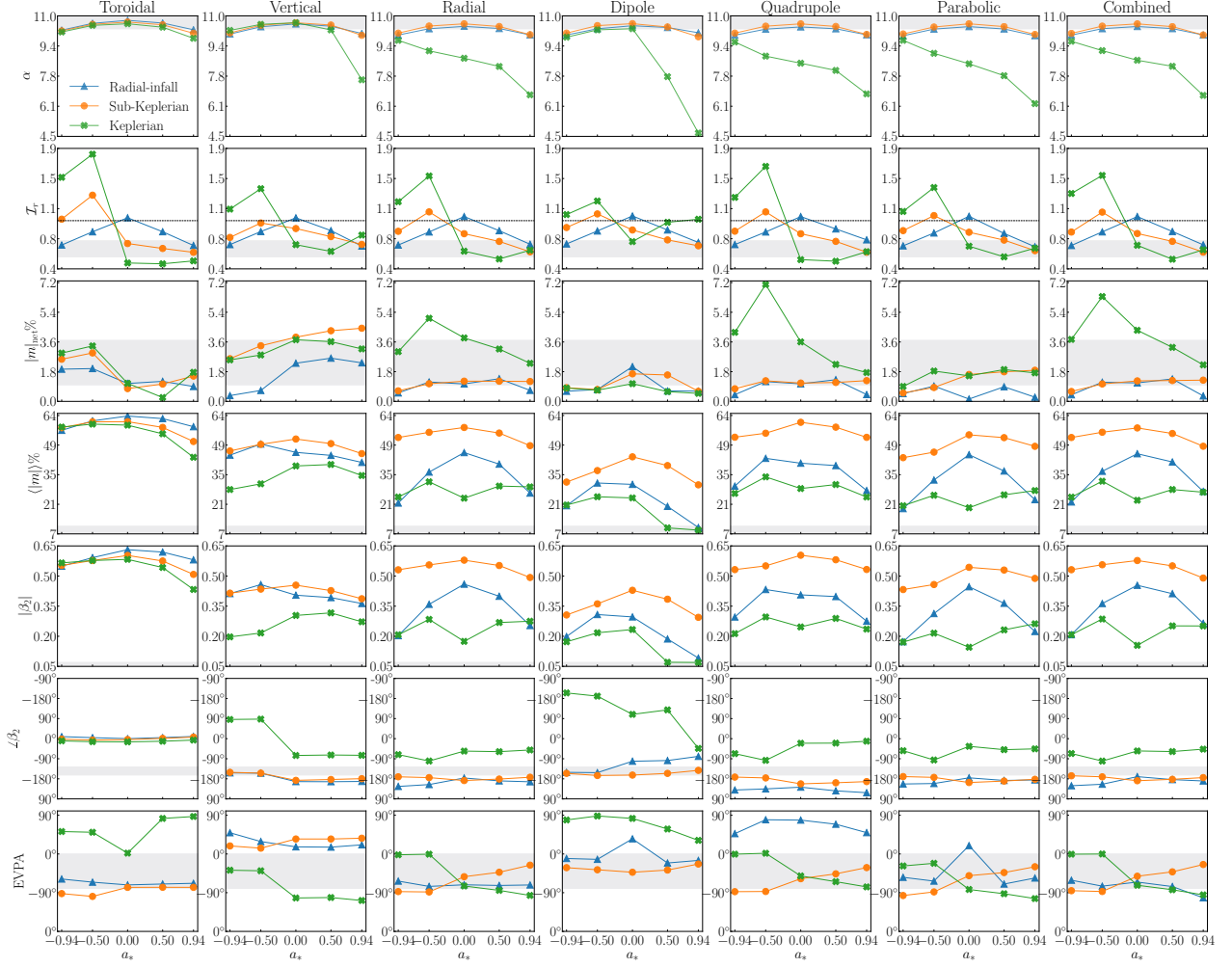


Fig. 7. Dependence of the image metrics discussed in Sect. 3 on the magnetic field geometry and BH spin a_* for the fiducial RIAF model. Three models of plasma four-velocity pattern are considered (see Sect. 2.1). The gray bands indicate the constraints derived from the EHT observations of M87*.

does not affect the results significantly. Hence, as long as the disk (and not the off-equatorial jet base) dominates the emission, the equatorial emission assumption provides a reasonable approximation of a more realistic thick disk. One exception is the dipolar magnetic field configuration, for which the geometrically thin disk is mostly threaded by a vertical magnetic field, but as the thickness increases, the emission region extends more into the off-equatorial radial magnetic field region (see Fig. 1). The effect is particularly strong for polarimetric observables, which are more sensitive to the magnetic field geometry.

5. Discussion and conclusions

We used a semi-analytic geometric RIAF disk model of BH accretion to study the effect of the system parameters on the seven observables, polarimetric ones in particular, that are used for evaluating theoretical models of M87* (EHTC 2019e, 2021b). We considered five values of BH spin, seven distinct models of magnetic field geometry, three models of the plasma velocity pattern, and three values of the geometric thickness of

the accretion flow. We compared our ray-traced synthetic images with the EHT observations as well as with a subset of GRMHD simulations from the EHT M87* library computed at the observing frequency of 230 GHz.

We aimed to verify the simplifying assumptions that matter for the observables, and to which degree results obtained under simplifying assumptions can be interpreted quantitatively. We observed that the heuristic of radial polarization pattern implying azimuthal magnetic field structure and azimuthal polarization pattern implying poloidal magnetic fields generally holds for global accretion models, particularly for flows with non-negligible inflow, as expected in RIAFs. The geometric thickness of the disk does not affect the observables significantly in most cases, which might be an argument for the validity of conclusions obtained with a simplifying assumption of the equatorial emission. On the other hand, magnetic field geometry, plasma velocity profile, and BH spin all leave a strong imprint on the resulting images.

While we adjusted parameters to match the observed M87* flux density of ~ 0.5 Jy, our models remained Faraday thin, which is not the case for GRMHD simulations. The observations also

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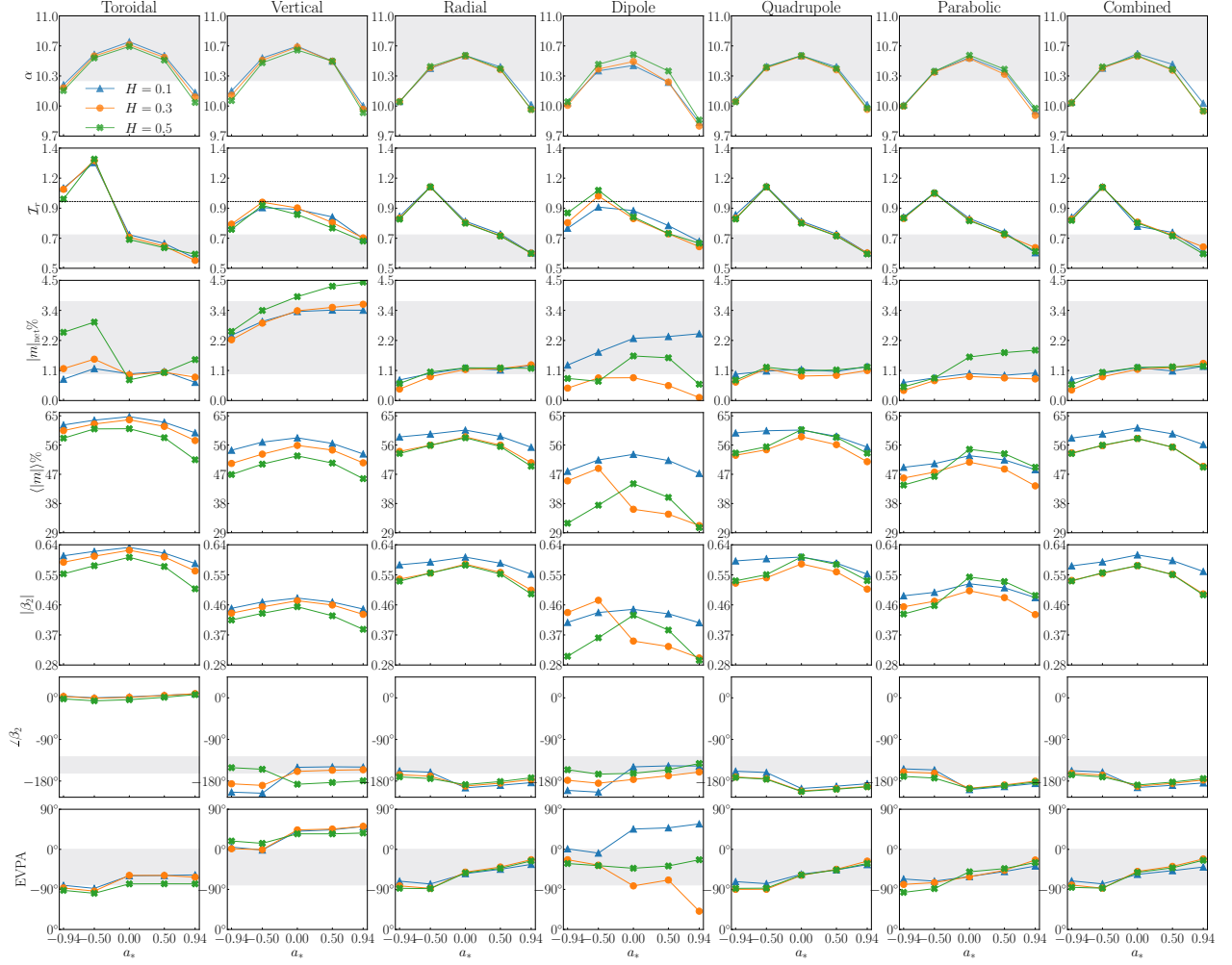


Fig. 8. Same as Fig. 7, but comparing three values of the geometric thickness H , as defined in Sect. 2.1.

indicate an appreciable Faraday depth through partial depolarization (EHTC 2021a,b). We further inspected this discrepancy by comparing our RIAF model setup to the GRMHD results in the EHT library, discussed by Dhruv et al. (2025). We found significant differences between the assumed RIAF structure discussed in Sect. 2.1 and predictions of GRMHD, such as a much steeper decrease in the gas temperature with radius seen in GRMHD. As a consequence, lower temperatures of the electrons in the emitting region might be a reason for the more pronounced Faraday effects seen in GRMHD (Quataert & Gruzinov 2000). Attempts to reproduce the appreciable Faraday depth by roughly matching the RIAF model properties to those of the averaged GRMHD were unsuccessful, however, which led us to the tentative conclusion that either nonaxisymmetric turbulent flow structure or off-equatorial jet base region plasma cause the observed depolarization.

As a consequence of the lack of depolarization, our RIAF models are all very symmetric, resulting in a large amplitude of β_2 and resolved polarization degree $\langle m \rangle$. This is inconsistent with the observations. There is a caveat, related to the fact that we did not explore the effect of the inclination angle. The uncertainties of the observational estimate of the inclination are

not smaller than about 5° – 10° , which leaves some space for the fine-tuning. A higher inclination would generally reduce the image symmetry, bringing measurements of $|\beta_2|$ down. For the other remaining observables, most of the RIAF models violate at least some of those. The argument of β_2 appears to be particularly constraining. One model that fares reasonably well against these constraints is the sub-Keplerian flow with a dipolar magnetic field configuration and a spin of $a_* = 0.5$. This solution visually disagrees with the EHT image reconstruction, however, with a presence of a second ring (see the discussion in Sect. 4). More broadly, our results (particularly the argument of β_2) indicate dominance of poloidal magnetic fields in M87*, and the relatively high image brightness asymmetry discourages zero or mildly negative BH spins. It is apparent that the EHT observations have a strong constraining power over the accretion models. The model that is most consistent with the data would necessarily combine poloidal and toroidal magnetic field components, as well as azimuthal and radial velocity patterns.

Our analysis demonstrated that the horizon-scale observables depend in a systematic and physically interpretable way on the magnetic field geometry, plasma velocity structure, and black hole spin. Our main conclusions are summarized below.

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- The magnetic field geometry (in particular the distinction between the poloidal and toroidal field) strongly affects the structure of the polarized image and the EVPA pattern.
- The plasma velocity profile affects multiple image properties, most notably, the asymmetry (which is highest for the Keplerian model), but also the polarimetric observables.
- The black hole spin predominantly affects the images by increasing the brightness asymmetry. There are spin-dependent trends in the EVPA structure, but the effects are degenerate with respect to the magnetic field geometry and the velocity profile.
- The geometric thickness plays a secondary role under the assumption that the emission is dominated by the accretion disk.

We discussed the effect of RIAF model parameters on the EHT observables mostly qualitatively. Some of the limitations of this work include fixed assumptions on the power-law structure of the flow and a lack of mixed toroidal-poloidal magnetic field configurations. Fitting a more general RIAF model directly to the EHT data would constitute an interesting follow-up work. The feasibility of a similar approach was recently demonstrated by SaraerToosi & Broderick (2025). Within the RIAF framework, accretion disk configurations in non-Kerr spacetimes might also be tested without resorting to computationally exhaustive covariant MHD numerical simulations (see Vincent et al. 2021 or the recent developments from Yin et al. 2025).

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Chapter 6

Imaging Ultracompact Objects with RIAFs

This chapter is based on the following publication where I am the first author:

Saurabh, P. Bambhaniya, and P. S. Joshi, “Imaging ultracompact objects with radiatively inefficient accretion flows,” *Astronomy & Astrophysics*, 682, A113, 2024 ([Saurabh et al. 2024](#)).

The JMN-1 spacetime, introduced in Section 2.4.1 of Chapter 2, is a black hole mimicker arising from gravitational collapse with anisotropic pressure in general relativity ([Joshi et al. 2011](#)). The compactness parameter R_b controls the nature of the central singularity. For $R_b < 3r_g$ the singularity is null, and the photon sphere at $r_\gamma = 3r_g$ is identical to that of a Schwarzschild black hole of the same mass, as established in Section 2.7. The shadow boundary and photon ring location are therefore indistinguishable from the Schwarzschild case based on geodesic geometry alone ([Saurabh et al. 2024](#); [Vagnozzi et al. 2023](#)), and even the higher-order photon ring substructure proposed as a future test of the Kerr metric ([Johnson et al. 2020](#)) would yield results consistent with Schwarzschild for this object since the exterior spacetime is unchanged. We refer to such configurations as ultracompact objects, using the presence of a photon sphere as the defining criterion rather than whether the singularity is naked in the strict sense of geodesic completeness. This chapter is directly motivated by the question raised in Section 2.8: whether the accretion-related observables at 230 GHz break the degeneracy that pure geodesic considerations cannot.

For $R_b > 3r_g$ (timelike singularity), the effective potential develops a centrifugal barrier for particles with any non-zero angular momentum, as shown in the effective potential comparison of Section 2.4.1. This prevents matter from reaching the central singularity, and such configurations are not consistent with the observed ring-dominated morphology of M87*. We therefore focus on $R_b < 3r_g$,

and in particular on the null singularity case $R_b = 2.8 r_g$ which is the most compact physically motivated configuration.

The semi-analytic RIAF model of Section 3.2.1 in Chapter 3 is used to describe the accretion environment. Radiative transfer is performed with `ipole` (Mościbrodzka and Gammie 2018; Prather et al. 2023), which was extended to support the JMN-1 interior metric. Since the spacetime is identical to Schwarzschild for $r \geq R_b$, no modification to the ray-tracing is required in the exterior. For $r < R_b$, the JMN-1 metric tensor components g_{tt} and g_{rr} from Section 2.4.1 are implemented directly. The code computes all geometric quantities, including the Christoffel symbols required for geodesic integration and the parallel transport of the polarisation vector, numerically from the metric at each integration step. The matching condition at $r = R_b$, where the metric is continuous but the connection coefficients are discontinuous, is handled numerically. The synchrotron emissivities and Faraday coefficients follow Pandya et al. 2016 and Pandya et al. 2018 throughout, consistent with the black hole case.

For $R_b = 2.8 r_g$, the synthetic images at EHT resolution are broadly consistent with the 2017 EHT observations of M87*. The ring diameter, brightness asymmetry, and overall morphology are all consistent with the Schwarzschild reference case. This follows from the shared geodesic structure: photons reaching the observer from the photon ring have trajectories that remain in the Schwarzschild exterior and never enter $r < R_b$, so the exterior metric controls their lensing. The difference between the two spacetimes appears only in the central image region. For a black hole, the line-of-sight column through the accretion disk terminates at the event horizon, producing the inner shadow of Chael et al. 2021 discussed in Section 4.8. For JMN-1, there is no event horizon, and the emission region extends inward all the way to the singularity, producing non-negligible brightness in the region that would be dark in a black hole image. This central excess is visible in logarithmic-scale images at native resolution but falls entirely below the dynamic range of current EHT reconstructions when convolved to the $\sim 15 \mu\text{as}$ EHT beam. The two spacetimes are therefore observationally degenerate at current EHT sensitivity, and distinguishing them requires both higher dynamic range and improved imaging fidelity. For $R_b = 5.0 r_g$, the emission morphology differs substantially from the EHT observations, ruling out this class of configurations. These results establish that only null singularity configurations with $R_b < 3 r_g$ are viable black hole mimickers at current EHT resolution, and that the inner brightness region is the observable to target with future instruments.

I developed the semi-analytic RIAF imaging framework for the JMN-1 background. This also required to extend `ipole` to handle the non-Kerr metric. To implement a non-Kerr spacetime in `ipole`, requires changing in multiple scripts,

and various places wherever the metric is used. This also required modifying the accretion model files for the accretion flow, for proper integration limits, and velocity profiles. To handle everything self-consistently, which simultaneously requires calculating various spacetime geometric quantities, multiple modifications were made to the code in the geometry. The method starts by implementing a spacetime in an appropriate coordinate system such as Boyer-Lindquist or Kerr-Schild. Owing to the accessibility of a Kerr-Schild like coordinate system transformation for axisymmetric metrics (see Section 3.3.4, Chapter 3 for a detail discussion), this part is implemented for general spacetimes of the form

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)g(r)} + R^2(r) d\Omega^2, \quad (6.1)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, and $f(r)$, $g(r)$, $R(r)$ are three free functions characterizing the geometry. The resulting KS-like form of the spacetime is then found including the spin, and is directly implemented along with the BL form in `ipole`. For transformations between KS-like $\rightarrow$ BL or BL $\rightarrow$ KS-like form, a Jacobian is also implemented giving a proper handling of the quantities, and their transformations.

I then performed all ray-traced polarimetric calculations, produced all figures, and led the analysis and interpretation. All authors contributed to the writing and revision of the manuscript.

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**Astronomy
&
Astrophysics**

Imaging ultracompact objects with radiatively inefficient accretion flows

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ABSTRACT

Context. Recent Event Horizon Telescope observations of M 87* and Sgr A* strongly suggest the presence of a supermassive black hole at their respective cores. We use the semi-analytic radiatively inefficient accretion flows (RIAF) model to investigate the resulting images of the Joshi–Malafarina–Narayan (JMN-1) naked singularity and the Schwarzschild black hole.

Aims. We chose the JMN-1 naked singularity model and compared the synchrotron images with the Schwarzschild solution to search for any distinct features that can distinguish the two objects and to find an alternative to the solution with a black hole.

Methods. We performed general relativistic ray-tracing and radiative transfer simulations using the *Brahma* code to generate synchrotron-emission images using the thermal distribution function for emissivity and absorptivity. We investigated effects in the images by varying the inclination angle, the disk width, and the frequency.

Results. The shadow images simulated with the JMN-1 model closely resemble those generated by the Schwarzschild black hole. The disparities between the two images are very small. We conducted simulations using various plasma parameters, but the resulting images remained largely consistent for both scenarios. This similarity is evident in the horizontal cross-sectional brightness profiles of the two scenarios. Notably, the JMN-1 model exhibits a slightly higher intensity than the Schwarzschild black hole.

Conclusions. We conclude that JMN-1 is a viable substitute for the black hole scenario. This conclusion is not solely grounded in the fact that the two scenarios are indistinguishable from their respective shadow observations, but also in the consideration that JMN-1 emerges as an end state of a continual gravitational collapse. This paradigm not only allows for constraints on spacetime, but also provides a good probe for the nature of the central compact object.

Key words. accretion, accretion disks – black hole physics – radiative transfer

1. Introduction

In 1939, Oppenheimer, Snyder, and Datt (Oppenheimer & Snyder 1939) derived the first dynamical collapse solution for a spherically symmetric and homogeneous dust cloud. This was later known as OSD collapse. Their groundbreaking work demonstrated that a spherically symmetric and homogeneous dust collapse inevitably leads to the formation of a Schwarzschild black hole (BH). This result aligns with Birkhoff’s theorem, which states that the Schwarzschild spacetime represents the most general vacuum solution of Einstein’s equations. However, the OSD collapse model relies on several idealized assumptions, such as the homogeneous density of the collapsing star, the neglect of gas pressure, and no rotation, to simplify the complex Einstein equations (Joshi & Malafarina 2011). Although this model unveils the development of an event horizon during the collapse, which prevents the escape of matter, particles, or light to distant observers, it necessitates the exploration of more realistic scenarios that go beyond these idealistic assumptions. By considering the physical complexities and incorporating factors such as non-

homogeneous density profiles and gas pressure, we can expand our understanding of gravitational collapse and its implications.

Large astrophysical bodies typically do not possess a homogeneous density profile throughout their structure. Instead, their density can vary, with a higher density at the core and decreasing values toward the surface. When more physically realistic scenarios are considered that introduce inhomogeneity into the matter distribution profile and account for nonzero pressures, the dynamical collapse can result in the formation of a strong-curvature singularity within the framework of general relativity (Joshi & Malafarina 2011; Joshi & Dwivedi 1993; Joshi et al. 2011; Janis et al. 1968; Mosani et al. 2020, 2021). The final states of these collapses depend on the initial conditions of the collapsing matter.

The general theory of relativity predicts that spacetime singularities necessarily form when sufficiently massive objects collapse under their own gravity. However, the simultaneous formation, or absence, of an event horizon is not enforced. A singularity that forms without an event horizon is known as a naked or cosmic singularity. Naked singularities are intriguing, horizonless compact objects characterized by ultrahigh density regions, where physical quantities diverge arbitrarily. This ultradense compact region can be described by models of various astrophysical compact objects, for instance, a BH,

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a naked singularity (NS), or a boson star (Joshi & Malafarina 2011; Joshi & Dwivedi 1993; Joshi et al. 2011; Janis et al. 1968; Schunck & Mielke 2003). These compact objects have fascinating physical and geometrical properties, and it would be observationally significant to distinguish them.

The cosmic censorship conjecture (CCC) proposed by Penrose (Penrose 1965) posits that strong horizonless singularities are prohibited. However, several research studies have revealed the potential formation of naked singularities during continuous gravitational collapses of inhomogeneous matter clouds. In a seminal work, Joshi, Malafarina, and Narayan (JMN; Joshi et al. 2011) demonstrated that nonzero tangential pressure can prevent the formation of trapped surfaces around the high-density core of a collapsing matter cloud, leading to the emergence of a central NS in extended comoving time. This NS, occurring in physically realistic astrophysical scenarios, holds significant observational consequences. It gives rise to captivating phenomena such as distorted shadows, intense gravitational lensing, and orbital precession of nearby stars (Vagnozzi et al. 2023; Sahu et al. 2012; Bambhaniya et al. 2022a).

On the other hand, black holes have astonishing features as well. As its distinct signature, a black hole possesses an event horizon, which is a one-way membrane in spacetime through which things can fall in, but not escape, and even light cannot escape. Almost every galaxy is thought to have a supermassive black hole at its core (Kormendy & Ho 2013). There is no conclusive proof at this time for the existence of an event horizon, however. The Event Horizon Telescope (EHT) captured the first shadow image in 2019 in the galaxy M87 (Event Horizon Telescope Collaboration 2019), and more recently, in the center of our own galaxy, Sgr A* (Event Horizon Telescope Collaboration 2022a). While these results are indeed exciting, they should be interpreted with caution. For instance, Mizuno et al. (2018) and Röder et al. (2023) performed general relativistic magnetohydrodynamics (GRMHD) simulations of a dilated BH and compared it with the Kerr BH. They concluded that it is difficult to distinguish them based on the shadow images alone. Additionally, many other such works indicated the presence of such “mimickers”. Therefore, the shadow images of M87* and the Milky Way galactic centers observed by the EHT do not confirm the existence of an event horizon and hence a supermassive black hole (SMBH). Other compact objects can also cast similar shadows. The shadows cast by compact objects such as black holes, naked singularities, grava-stars, and wormholes have been extensively studied in Gralla et al. (2019), Vagnozzi & Visinelli (2019), Bambhaniya et al. (2021, 2022b), Gylchev et al. (2019), Atamurotov et al. (2015), Ohgami & Sakai (2015), Stuchlík & Schee (2019), Sakai et al. (2014), Solanki et al. (2022), Younsi et al. (2023).

It is generally thought that the shadows arise due to the presence of a photon sphere. Recently, however, Joshi et al. (2020) have introduced a new spherically symmetric NS solution of the Einstein field equation that lacks a photon sphere, but still casts a shadow. The general criteria for a shadow to occur in the absence of a photon sphere are then determined for null and time-like naked singularities, where both types of singularities satisfy all the energy conditions (Dey et al. 2021). Moreover, another model of a NS that was proposed in 1968 by Janis–Newman–Winicour (JNW) can cast a shadow similar to that of a Schwarzschild black hole (Gylchev et al. 2020; Sau et al. 2020). The JNW NS is a minimally coupled massless scalar field solution of the Einstein field equations. Sau et al. (2020) showed that as the scalar field charge q increases, the radius

of the photon sphere r_{ph} increases while the shadow radius decreases.

Shaikh et al. (2019) investigated the shadows of JMN-1 NS spacetimes and compared the results to the shadow of a Schwarzschild black hole. For the range of the characteristic parameter $0 < M_0 < 2/3$, there is no photon sphere, and hence no shadow is formed in JMN-1 NS, but an intriguing, full moon image is formed. However, JMN-1 can cast a shadow for $2/3 < M_0 < 1$, similar to that of a Schwarzschild black hole.

The rotating black holes or naked singularities describe the observational data for Sgr A* much better than the static solutions. For rotating spacetimes, the shadow is slightly asymmetric along the spin axis, which will be smaller in diameter than in the Schwarzschild case. The inclusion of spin in the black holes and naked singularities will give the prolate contour shape of the shadow (Bambhaniya et al. 2021), although for some parameter space that includes spin and scalar field charge, naked singularities will give an arc-shaped shadow (Solanki et al. 2022). However, for simplicity, we restricted ourselves to static spherically symmetric metrics here, that is, we neglected the effect of spin. The reason for this choice is twofold. First, the effect of spin on the shadow radius is small for the case of a Kerr BH (see, e.g., Vagnozzi et al. 2023). Second, and perhaps most importantly, there is currently no clear consensus on the spin and inclination angle of Sgr A*. The EHT images are in principle consistent with high spin and low inclination angle, but they are far from being inconsistent with low spin and high inclination angle (Event Horizon Telescope Collaboration 2022a,b). To be precise, however, this is strictly speaking only true for the Kerr metric, which was assumed to derive these results. Independent works based on radio, infrared, and X-ray emission as well as millimeter VLBI, excluded extremal spin ($1 - a_* \ll 1$), but were unable to place strong constraints otherwise (Broderick et al. 2009, 2011; Kato et al. 2010). Estimates based on semi-analytical models, magnetohydrodynamics simulations, or flare emissions have reported constraints across a wide range of spin values (Huang et al. 2012; Shcherbakov et al. 2012). One of the most recent dynamical estimates of the Sgr A* spin was reported in Fragione & Loeb (2020), based on the impact on the orbits of the S stars of frame-dragging precession, which would tend to erase the orbital planes in which the S stars formed and are found today: Observations of the alignment of the orbital planes of the S stars today require the spin of Sgr A* to be very low, $a_* \lesssim 0.1$ (see also Fragione & Loeb 2022). Some of these estimates rely on an assumed metric, although it is worth pointing out that the result of Fragione & Loeb (2020) does not. Constraints on the inclination angle are even more uncertain. The inconsistency for different estimates of the Sgr A* spin prompts us to choose a conservative approach in which we neglect the effect of spin, while taking the very recent estimate of Fragione & Loeb (2020) as an indication that the spin may be low: for $a_* \lesssim 0.1$. Additionally, the effect of spin on the shadow radius is negligible at all inclination angles. Moreover, we lack a complete rotating solution of JMN-1 naked singularity so far. Therefore, we restricted ourselves to static spherically symmetric metrics here.

We used a semi-analytic RIAF model to investigate the resulting images of JMN-1 and the Schwarzschild black hole surrounded by accretion flows. Motivated by RIAF, we integrated the system of geodesics and its radiative transfer equation to compute the theoretically observed intensity.

The paper is organized as follows. In Sect. 2, we briefly introduce the JMN-1 model. We describe the semi-analytic RIAF model in Sect. 3. In Sect. 4, we describe the ray-tracing

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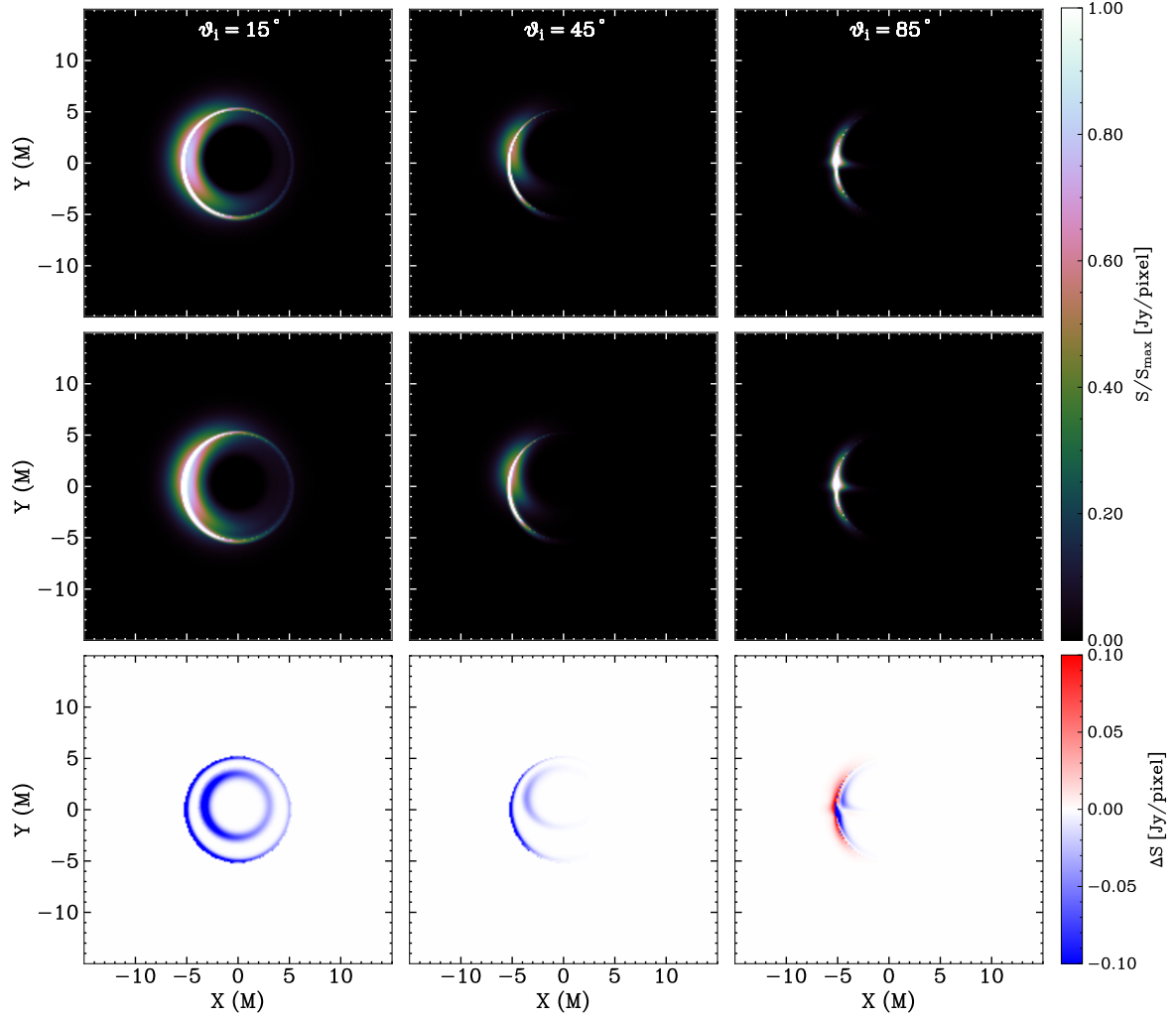


Fig. 1. Model images for the Schwarzschild BH (first row) and the JMN-1 model (second row) for varied inclination angles (θ_{obs}). The third row corresponds to the various differences in images in the second row from the first row.

formalism and the initial conditions we used to generate accretion images. In Sect. 5, we summarize our results. Finally, the discussion and conclusion are given in Sect. 6.

2. JMN-1 naked singularity

We briefly review the solution that was proposed in the literature for the JMN-1 NS. The spacetime metric for a static and spherically symmetric object in the generalized form can be written as

$$ds^2 = -g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + g_{\theta\theta}(r)d\theta^2 + g_{\phi\phi}(r)d\phi^2, \quad (1)$$

where $g_{\mu\nu}$ are the metric tensor components, all of which are only functions of r . The JMN-1 naked singularity can be formed as an end state of gravitational collapse with zero radial pressure and nonzero tangential pressures. These equilibrium configurations of the collapsing fluid within the general theory of relativity were extensively studied in Joshi et al. (2011). The JMN-1 naked singularity spacetime is described by the following metric tensor

components:

$$g_{tt}^{\text{jmn}} = (1 - M_0) \left(\frac{r}{R_b} \right)^{M_0/(1-M_0)}, \quad (2)$$

$$g_{rr}^{\text{jmn}} = (1 - M_0)^{-1}, \quad (3)$$

$$g_{\theta\theta}^{\text{jmn}} = r^2, \quad (4)$$

$$g_{\phi\phi}^{\text{jmn}} = r^2 \sin^2 \theta, \quad (5)$$

where M_0 and R_b are positive constants. Here, R_b represents the radius of the distributed matter around the central singularity, and M_0 should be within the range $0 < M_0 < 1$. The JMN-1 spacetime contains a curvature singularity at $r = 0$. The stress-energy tensor of the JMN-1 spacetime gives the energy density ρ and pressures p as

$$\rho = \frac{M_0}{r^2}, \quad p_r = 0, \quad p_\theta = \frac{M_0}{4(1 - M_0)}\rho. \quad (6)$$

The JMN-1 naked singularity is a nonvacuum solution of the Einstein field equations that satisfies all energy conditions. An

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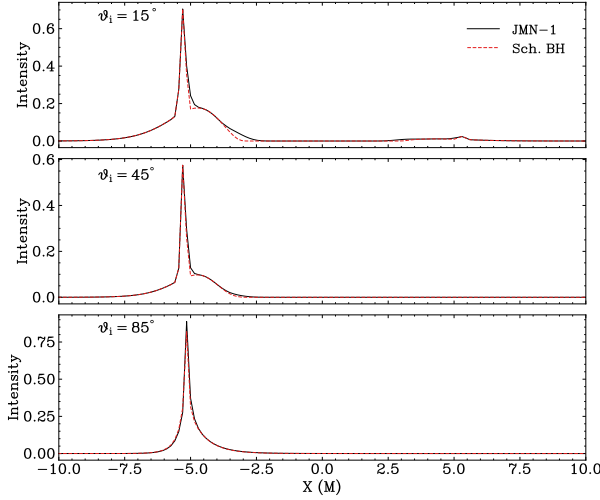


Fig. 2. Cross-sectional horizontal intensity for the model images of Fig. 1 with varied inclination angles (θ_{obs}).

equilibrium configuration of a massive anisotropic matter fluid can be achieved as an end state of dynamical gravitational collapse with nonzero tangential pressure. The matter source can be considered as baryonic and/or dark matter.

The spacetime metric is modeled by considering a high-density compact region in a vacuum, which means that the spacetime configuration should be asymptotically flat. Therefore, if any spacetime metric is not asymptotically flat, we must match that interior spacetime to an asymptotically flat exterior spacetime with a particular radius. The latter is the case for the JMN-1 naked singularity. We can smoothly match this interior spacetime to exterior Schwarzschild spacetime at $r = R_b$ as

$$ds^2 = -\left(1 - \frac{M_0 R_b}{r}\right) dt^2 + \left(1 - \frac{M_0 R_b}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (7)$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ and $M = \frac{1}{2} M_0 R_b$ is the total mass of the compact object. The extrinsic curvatures of JMN-1 and Schwarzschild spacetimes are automatically smoothly matched at $r = R_b$ because the JMN-1 spacetime has zero radial pressure (Bambhaniya et al. 2019). The complete solution for the JMN-1 metric in the interior $r < R_b$ is given by

$$ds^2 = -(1 - M_0) \left(\frac{r}{R_b}\right)^{\frac{M_0}{(1-M_0)}} dt^2 + \frac{dr^2}{(1 - M_0)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \quad (8)$$

In order to specify the nature of the central singularity, we note that the outgoing radial null geodesics in the spacetime above are given by

$$\frac{dr}{dt} = (1 - M_0) \left(\frac{r}{R_b}\right)^{M_0/2(1-M_0)}. \quad (9)$$

It is then easy to confirm that light rays escape from the singularity (for all values of $M_0 < 2/3$). From the above equation, which gives

$$t(r) = \frac{2R_b^{\frac{M_0}{2(1-M_0)}}}{2 - 3M_0} r^{\frac{2-3M_0}{2(1-M_0)}}, \quad (10)$$

we immediately see that the comoving time required by a photon to reach the boundary is $t_b = \frac{2R_b}{(2-3M_0)} < +\infty$. It follows that there are future directed null geodesics in the spacetime that reach the boundary of the cloud, and which in the past terminated at the singularity, thus showing this to be a naked singularity.

3. Radiative transfer

In this section, we describe the prescription for the semi-analytic radiative inefficient accretion flows (RIAF) model. We closely follow Pu & Broderick (2018) for some basic definitions of the parameters. The covariant form of general relativistic radiative transfer is expressed as

$$\frac{d\mathcal{I}}{d\tau_\nu} = -\mathcal{I} + \frac{\eta}{\chi}, \quad (11)$$

where $\mathcal{I}$ is the Lorentz-invariant intensity and is related to the specific intensity via $\mathcal{I} = I_\nu/\nu^3 = I_{\nu_0}/\nu_0^3$, where the subscript 0 denotes quantities in the local rest frame. τ_ν is defined as the optical depth. χ and η are the invariant absorption coefficient and emission coefficient at frequency ν . These coefficients are defined by the specific emissivity and absorptivity. For this work, we used the thermal electron distribution function (eDF; Pandya et al. 2016).

The number density and temperature of electrons can be written as a hybrid combination of radial and exponential functions given by

$$n_{\text{th}} = n_{\text{e,th}} r^{-\alpha} e^{-z^2/2h^2}, \quad (12)$$

$$T_{\text{th}} = T_{\text{e,th}} r^{-\beta}, \quad (13)$$

where $z = r \cos \theta$, and h is the disk width. $n_{\text{e,th}}$ and $T_{\text{e,th}}$ are the respective normalization parameters. The magnetic field strength of the toroidal field is then given by

$$\frac{B^2}{8\pi} = \frac{1}{10} n_{\text{th}} \frac{m_p c^2}{6r} [r_g], \quad (14)$$

where $r_g = GM/c^2$, and m_p is the mass of proton.

We modeled the four-velocity of the plasma accreted onto the central compact object by following Younsi et al. (2023). The four-velocity can be written as

$$u^\mu = (u^t, u^r, 0, \Omega u^t), \quad (15)$$

where (Ω) is the angular velocity. The radial component of the four-velocity can be written in parameterized form,

$$u^r = -\eta_r \left(\frac{r}{R_{\text{ISCO}}}\right)^{-n_r}, \quad (16)$$

where $\eta_r = 0.1$ and $n_r = 1.5$ are the free parameters. We kept their values fixed for the current analyses.

Because no stable circular orbits are possible beyond the innermost stable circular orbit (ISCO) radius (r_{ISCO}), the particles will plunge with the constant energy (E_{ISCO}) and the constant angular momentum (L_{ISCO}) acquired at the ISCO radius. In this way, we can model the motion of particles for $r < r_{\text{ISCO}}$.

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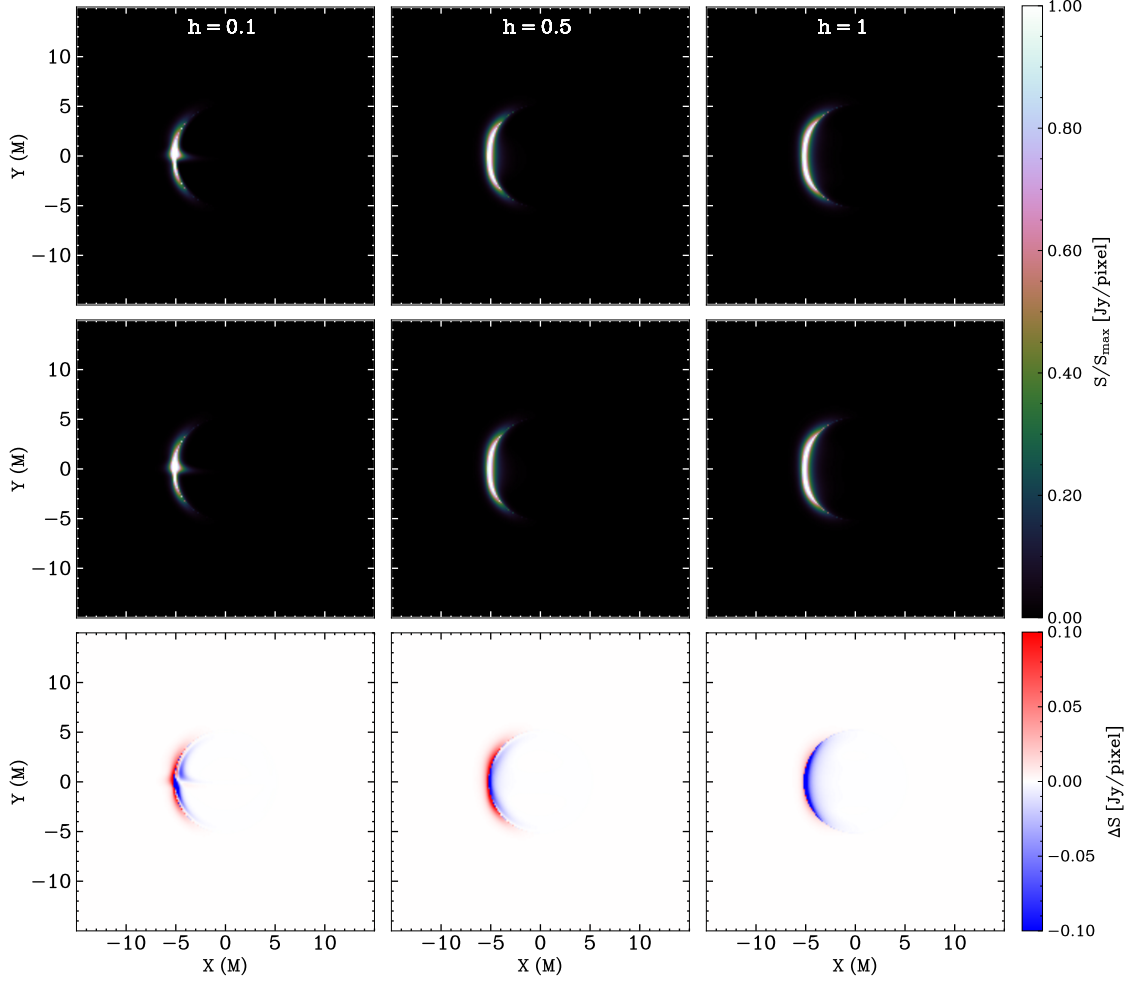


Fig. 3. Model images for the Schwarzschild BH (first row) and the JMN-1 model (second row) for a varied disk width (h). The third row corresponds to the difference in the images in the second row from the first row.

4. Ray-tracing formalism

In order to calculate the shadow image of a BH or a naked singularity, the geodesic equations in the considered background spacetime must first be solved. To this end, we used the *Brahma* code (in prep.). We briefly describe the approach for ray-traced images of the models described in Sect. 3. We solve a system of six differential equations ($\dot{t}, \dot{r}, \dot{\theta}, \dot{\phi}, \dot{p}_r, \dot{p}_\theta$).

Geodesic equations of motion. For a given metric $g_{\alpha\beta}$, the Lagrangian and the Hamiltonian can be written as

$$2\mathcal{L} = g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta, \quad (17)$$

$$2\mathcal{H} = g^{\alpha\beta} p_\alpha p_\beta, \quad (18)$$

where an overdot denotes differentiation with respect to the affine parameter, λ . The corresponding equations of motion are given by

$$\dot{x}^\alpha = \frac{\partial \mathcal{H}}{\partial p_\alpha}, \quad (19)$$

$$\dot{p}_\alpha = -\frac{\partial \mathcal{H}}{\partial x^\alpha}. \quad (20)$$

From the Lagrangian, the covariant four-momenta of a geodesic may be written as

$$p_\alpha = \frac{\partial \mathcal{L}}{\partial \dot{x}^\alpha}. \quad (21)$$

Using the conservation of energy and angular momentum, this can be broken down to write $p_t = -E$ and $p_\phi = L$, where E is the total energy of the particle, and L is the angular momentum in the direction of ϕ . To solve the system of an ordinary differential equation (ODE), initial conditions are required. Different authors have assumed different initial conditions based on the type of work and ODEs that were to be solved. In *Brahma*, we used the formalism described in [Younsi et al. \(2016\)](#). The observer is placed far away from the compact object ($r_{\text{obs}} = 10^3 M$), where the spacetime is assumed to be flat (asymptotic flatness).

5. Results

We used the formalism described in Sect. 4 to trace the photons along geodesics and solved for the observed intensity. In principle, the model consists of various parameters that can be

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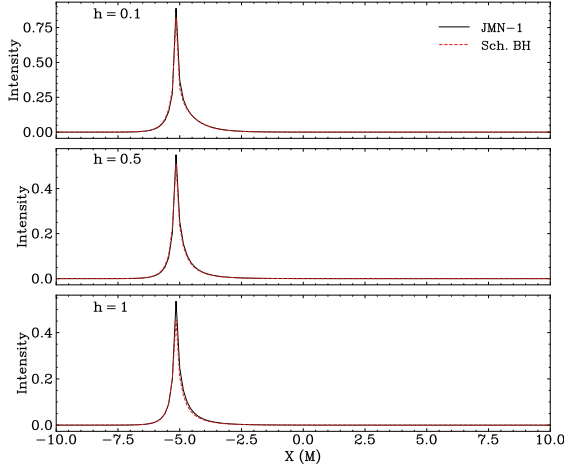


Fig. 4. Cross-sectional horizontal intensity for the model images of Fig. 3 with a varied disk width (h).

altered to study specific properties of the accretion flows. In our simulations, we currently restricted ourselves to a specific set of parameters $\Theta_p = (\theta_{\text{obs}}, \nu_{\text{obs}}, h)$. We studied the impact of these parameters in the shadow images of the Schwarzschild BH and the JMN-1 naked singularity. The default values for the parameters were $M_0 = 0.7$, $\nu_{\text{obs}} = 230$ GHz, $h = 0.1$, $n_{\text{e,th}} \approx 10^5 \text{ cm}^{-3}$ and $T_{\text{e,th}} \approx 10^{10}$ K.

– *Inclination angle.* The first row in Fig. 1 corresponds to the Schwarzschild BH, the second row corresponds to the JMN-1, and the third row represents the image difference by subtracting second-row images from the first-row images. The first, second, and third columns represent $\theta_{\text{obs}} = 15^\circ$, 45° , and 85° , respectively. Although the Schwarzschild BH and JMN-1 model images may look identical in terms of shadow features and emission, slight differences are visible in the third row. Figure 2 represents the intensity along the horizontal cross section of the images, and the overall profile remains similar throughout the image except for minor changes.

– *Disk width.* Fig. 3 is the same as Fig. 1, except for the change in the disk width h . The first, second, and third columns correspond to $h = 0.1$, 0.5 , and 1 , respectively. Higher and lower values of h allow for a thicker and thinner flow geometry of the accreting matter, which would correspond to a transformation from a spherical-like flow into a disk-like flow. The inclination angle was set to 85° . For $h = 0.1$, a distinct disk-like geometry is visible, but this does not hold for the other cases. Because we used a thermal eDF, the images are different from those presented in Pu & Broderick (2018). For different values of h , the images are very similar for the Schwarzschild BH and the JMN-1 model, and the corresponding horizontal cross section is shown in Fig. 4. The profiles for both spacetimes are again similar here, with JMN-1 having a slightly higher intensity for $h = 0.1$.

– *Frequency.* Currently, the EHT operates at 230 GHz, and there are plans for including other frequency bands as part of EHT and ngEHT. We therefore also performed simulations for different frequencies, and the results are plotted in Fig. 5 for 84 GHz, 230 GHz, and 345 GHz. No significant changes are observed for the two spacetimes relative to each other when the observed frequency is changed. As the frequency is increased, however, the emission becomes thinner, which agrees with previous results. The horizontal cross-sectional intensity as plotted

in Fig. 6 retains a similar profile as in previous cases for different values of the disk width and inclination angle.

6. Conclusion and discussion

We have modeled radiative inefficient accretion flows around the JMN-1 cosmic singularity and calculated the resulting shadow images. Because JMN-1 causes a shadow of the same size as the Schwarzschild BH, it is worthwhile to investigate various effects from the change in spacetime on the accretion flows and the resulting images. We first defined the geodesic equations for both photons and particles. We then defined our semi-analytic RIAF model in Sect. 3. Thereafter, we solved the system geodesic equations with the ray-tracing formalism as described in Sect. 4 simultaneously with the radiative transfer equation to calculate the observed intensity. In order to compare the JMN-1 naked singularity with the Schwarzschild BH, we carried out simulations to generate images for various values of the parameters: for the inclination angle (θ_{obs}), the disk width (h), and the observed frequency (ν_{obs}). The results from the simulations are presented in Sect. 5.

The visual representations of the Schwarzschild BH and JMN-1 model exhibit striking similarities. Subtle differences lie in the slightly heightened intensities observed for the JMN-1 model. Even though we varied several parameters associated with the accretion disk, the overall characteristics of the disk remained relatively unchanged. This reflects the results of previous studies (Shaikh et al. 2019; Shaikh & Joshi 2019). The results we obtained here contribute to an expanded understanding of a realistic model for the accretion disk, which is astrophysically relevant in particular in the context of AGNs. Following the fact that JMN-1 emerges as the end state of gravitational collapse and acts as galactic spacetime, this opens up an intriguing avenue in the study of AGNs in which the central engine powering the galaxy is a visible cosmic singularity. This analysis concludes that if horizon-scale observations of the central supermassive compact object produce a shadow-like structure, it is impossible to distinguish between a BH and a naked singularity, and by extension, this is valid for the recent studies of M 87* and Sgr A*.

However, many other paths can be pursued further in order to test and investigate the robustness of the model. For example, we can generate synthetic VLBI observations from the ray-traced images presented in this paper to directly compare our results to observational data.

- For instance, Shaikh et al. (2019) have considered a spherically accreting system and studied the corresponding shadow images. Bondi accretion flow was also used to study the shadow and spectral properties in MHD simulations. This treatment can also be applied to our model to study the spectra, which can then be used to study specific sources such as Sgr A*. The spectral signatures will be studied in an accompanying paper.
- We only studied the horizon-scale accretion-flow images of the models, and in GRMHD simulations, black holes produce highly energetic jets that were variously observed. Studies of jets and accretion flows in GRMHD simulations (Kocherlakota et al. 2023) will help us to conclude whether the Universe contains horizonless objects.

Using the models described in this work, it will be possible to unveil the true nature of the objects in M 87* and Sgr A*. From the studies so far, it would appear that naked singularities present a possible alternative to the standard picture of a black hole within the framework of nonmodified general relativity.

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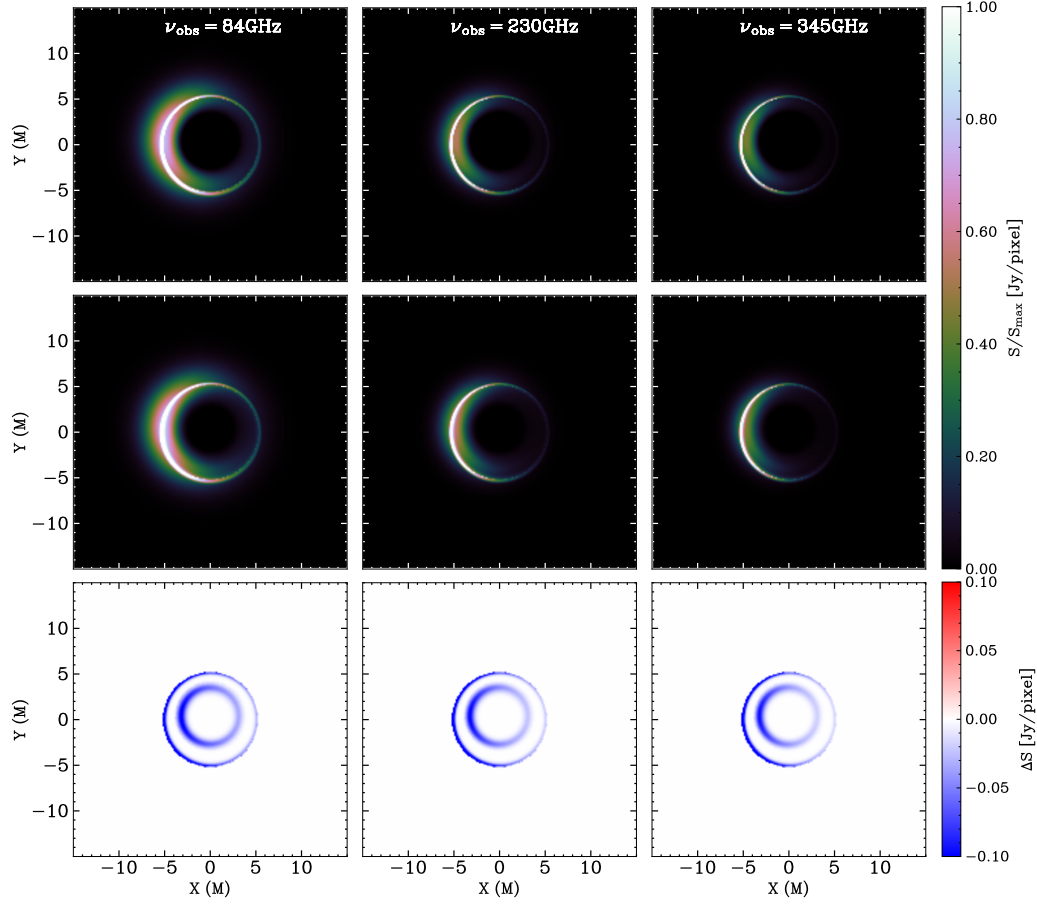


Fig. 5. Model images for the Schwarzschild BH (first row) and the JMN-1 model (second row) for varied observed frequencies (ν_{obs}). The third row corresponds to the difference in the images in the second row from the first row.

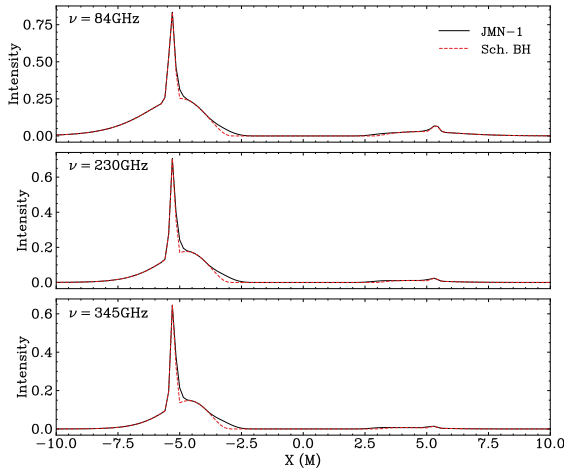


Fig. 6. Cross-sectional horizontal intensity for the model images of Fig. 5 with varied observed frequencies (ν_{obs}).

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Chapter 7

GRMHD accretion beyond the Black hole paradigm

This chapter is based on the paper currently under review where I am the first author:

Saurabh, M. Wielgus, P. Bambhaniya, E. M. de Gouveia Dal Pino, A. P. Lobanov, and P. S. Joshi, “GRMHD accretion beyond the black hole paradigm: Light from within the shadow,” submitted *Physical Review Letters*, arXiv:.

Chapter 6 showed, using a semi-analytic accretion model, that JMN-1 with $R_b = 2.8 r_g$ produces 230 GHz images consistent with the 2017 EHT observations of M87* at current resolution. The key discriminant identified there is a non-negligible brightness in the central image region, arising from emission below the photon ring in the absence of an event horizon [Shaikh et al. 2019](#). In that analysis, however, the accretion flow was prescribed semi-analytically and the plasma dynamics were not self-consistently evolved. Whether the JMN-1 spacetime can actually sustain a magnetised, radiatively inefficient accretion flow at the mass accretion rates inferred for M87* is a separate and non-trivial question. The open question raised in Section 3.7 bears directly on this that without a horizon, the excess thermal energy of the accreting plasma cannot disappear behind a causal boundary, and in many horizonless singularity spacetimes an infinite potential barrier prevents matter from reaching the central object entirely ([Uniyal et al. 2025](#); [Dhingia et al. 2025](#)). This chapter presents the first GRMHD simulation of accretion onto a JMN-1 null singularity in which matter reaches the central object and a sustained accretion state develops.

Simulations are carried out using KHARMA ([Prather 2024](#); [Trott et al. 2021](#); [Carter Edwards et al. 2014](#)), a GPU-accelerated GRMHD code implementing the HARM conservative scheme ([Gammie et al. 2003](#)). KHARMA was modified to

support the JMN-1 spacetime by implementing the metric tensor components $r < R_b$ from Section 2.4.1. It handles the matching at $r = R_b$ where the connection is discontinuous, and adapting the horizon-penetrating coordinate system to the non-singular JMN-1 interior. A parallel Schwarzschild black hole simulation is run with identical initial conditions for direct comparison. Both simulations begin from a Fishbone-Moncrief torus (Fishbone and Moncrief 1976) threaded by a single-loop poloidal magnetic field. The magnetorotational instability (Balbus and Hawley 1991) develops in the differentially rotating torus, driving turbulent angular momentum transport and facilitating accretion, and both systems evolve toward the magnetically arrested disk (MAD) state (Narayan et al. 2003; Tchekhovskoy et al. 2011).

The JMN-1 simulation reaches and sustains a MAD state, with a time-averaged accretion rate of $(3.0 \pm 0.5) \times 10^{-6} \dot{M}_{\text{Edd}}$ over the equilibrium window $t \in (8-10) \times 10^3 t_g$. This is fully consistent with the Schwarzschild reference simulation and with the MAD Kerr black hole models of the EHT M87* library (Event Horizon Telescope Collaboration 2019). The equatorial flow structure, the density and magnetic field profiles, and the plasma β distribution all closely resemble the Schwarzschild case for $r > R_b$, as expected from the continuous metric matching. Inside R_b , the flow continues inward to the singularity rather than being expelled. The funnel region develops an outflowing velocity pattern in JMN-1, in contrast to the Schwarzschild case where the funnel shows inflow near the horizon. This suggests that the central null singularity acts as an efficient energy sink analogous to a black hole event horizon, maintaining the thermodynamic structure of the RIAF discussed in Section 3.2.

Synthetic polarimetric images at 230 GHz are computed using `ipole` (Mościbrodzka and Gammie 2018; Prather et al. 2023) with parameters appropriate for M87* with inclination $\theta_i = 163^\circ$, and $R_{\text{high}} = 50$ (Event Horizon Telescope Collaboration 2019; Event Horizon Telescope Collaboration 2021). The images use the JMN-1 extension of `ipole` developed in Chapter 6. Both snapshot and time-averaged total intensity and polarimetric images are produced. In linear brightness scale, the Schwarzschild and JMN-1 time-averaged images are remarkably similar and both are broadly consistent with the EHT M87* observations. The photon ring location and the overall ring morphology match, as expected. In logarithmic scale, the JMN-1 images show a clear excess of emission in the central region relative to the black hole case. For the black hole, the inner shadow (Chael et al. 2021) sets a well-defined lower boundary for the emission, as discussed in Section 4.8. In JMN-1, emission extends inward below this boundary because no event horizon conceals the innermost region. At roughly half the inner shadow radius, the brightness reaches a few percent of the peak ring brightness.

This signature is below the dynamic range of current EHT image reconstructions, but falls within the projected imaging capabilities of the next-generation EHT ([Johnson et al. 2023](#)), making it a concrete target for near-future observations.

I modified `KHARMA` to support the JMN-1 spacetime which required modifications in several places. Helpful discussions with Ben Prather lead to a successful setup of `KHARMA` for non-Kerr spacetimes. Similar to handling of non-Kerr spacetime (formulation) in `ipole` is used for GRMHD, and then used `ipole` for calculating/handling non-Kerr spacetime which was modified for Chapter 6. I then set up and ran the GRMHD simulations, performed all `ipole` ray-tracing calculations, produced all figures, and led the analysis and interpretation. Maciek Wielgus helped immensely in the interpretation of GRMHD quantities (including Elisabete), with the calculations of inner shadow in analytic model and Parth and Prof. Joshi for theoretical understanding of JMN-1 spacetime. All authors contributed to writing of the manuscript.

GRMHD accretion beyond the black hole paradigm: Light from within the shadow

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We present the first three-dimensional general relativistic magnetohydrodynamic simulation of sustained accretion onto a horizonless singularity in which matter reaches the central object rather than being accumulated outside of it or expelled in outflows. We consider a Joshi–Malafarina–Narayan (JMN-1) spacetime, a well-motivated black hole mimicker that arises from gravitational collapse with anisotropic pressure in general relativity, and adopt a compactness parameter for which the central singularity is null. We find that the system evolves into a sustained magnetically arrested disk state. For parameters appropriate to the low-luminosity active galactic nucleus M87*, we obtain an accretion rate of $\sim (3.0 \pm 0.5) \times 10^{-6} \dot{M}_{\text{Edd}}$, in full agreement with estimates based on black hole models and, in particular, comparable to that of our reference Schwarzschild black hole simulation. Synthetic ray-traced images at 230 GHz, computed using polarized general relativistic radiative transfer, are broadly consistent with the Event Horizon Telescope observations of M87*. We identify a key observational discriminant between a black hole and JMN-1: the presence of detectable brightness inside of the “observable shadow” of JMN-1. This emission originates very close to the central singularity, in a region that would be hidden behind the event horizon in a black hole spacetime. Although this signature is beyond the reach of current observations, it falls within the projected imaging dynamic range of next-generation radio interferometric instruments, offering a robust test of the black hole paradigm.

Introduction—Black holes (BHs) are predicted by the vacuum equations of general relativity (GR). The fundamental solution, Schwarzschild metric, describes a spherically symmetric static compact object possessing an *event horizon*, a one way causal boundary cloaking the central singularity. This solution, as well as its rotating generalization Kerr metric [1], carries theoretical pathologies which remain unresolved. In particular, no consensus resolution of the information paradox [2] exists despite decades of effort and the proposed solutions introduce further inconsistencies at the semi-classical level [3]. The weak cosmic censorship conjecture [4], which posits that singularities are always cloaked by horizons, remains unproven and GR does admit collapse solutions terminating in horizonless configurations [5, 6]. Recent observational breakthroughs, including gravitational waves detection [7], stellar orbits monitoring at the Galactic Center [8], and horizon-scale imaging of M87* and Sagittarius A* by the Event Horizon Telescope (EHT) [9, 10] have established broad observational consistency with the black hole solutions. Therefore, despite the fundamental theoretical tensions, the predictive success of GR in the strong-field regime consolidated the black hole paradigm as the standard framework for modeling compact astrophysical objects across the mass spectrum.

With the EHT and its continuous developments [11–13] we are gaining an unprecedentedly detailed insight into the interactions between compact supermassive objects and their immediate environments of accreting plasma and ejected outflows. Thus, an observational window to test the black hole paradigm is opening and it motivates extensive studies of horizonless ultra-compact objects. BH alternatives ex-

plored in this context include wormholes [14–16], boson stars [15, 17], and horizonless singularities [18–21], each representing a qualitatively distinct topology resolving the horizon problem. By considering such alternatives, and then confronting them with the observational data, the black hole paradigm can be placed on a stronger empirical footing [22].

In this Letter we study the JMN-1 spacetime [23], a notable example of a horizonless compact object and a strong black hole mimicker [24]. We consider JMN-1 because, unlike many alternatives, it arises as a physically motivated solution of gravitational collapse in GR, providing a well-defined setting to test whether the event horizon is observationally required. Using numerical general relativistic magnetohydrodynamic (GRMHD) simulations we compare accretion in JMN-1 and Schwarzschild spacetimes and find remarkably similar properties. We then propose a viable observational strategy to distinguish the two with future radio-interferometric observations.

Background spacetime—The horizonless JMN-1 spacetime arises as an equilibrium end state of inhomogeneous gravitational collapse in GR from regular initial conditions with nonzero tangential pressure and vanishing radial pressure [23], unlike the idealized Oppenheimer–Snyder–Datt dust (pressureless and homogeneous) collapse that inevitably forms a BH [26, 27]. JMN-1 corresponds to a non-vacuum GR solution satisfying all standard energy conditions, forming a curvature singularity at $r = 0$ and remaining free of closed timelike curves, thereby ensuring that causality is preserved [23, 28]. JMN-1 is linearly stable against axial perturbations [29], which, along with its dynamical stability, shows that the solution represents a genuine equilibrium configuration rather

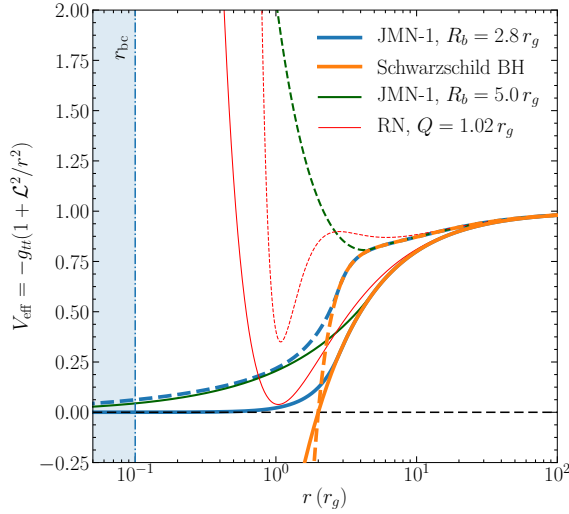


Figure 1. Comparison of the effective potential $V_{\text{eff}}(r)$ for the metrics compared in this paper, Schwarzschild and JMN-1 with $R_b = 2.8 r_g$, as well as for the Reissner-Nordström (RN) naked singularity spacetime simulated in [19, 25] and the static seed metric JMN-1 with $R_b = 5.0 r_g$ of [20]. We show the potential $V_{\text{eff}}(r)$ for zero angular momentum massive particles $\mathcal{L} = u_\phi = 0$ with continuous lines, and for $\mathcal{L} = 3 r_g$ with dashed lines. RN naked singularity spacetimes (charge parameter $Q > r_g$) always indicate an infinite potential barrier preventing infall of particles. In JMN-1 spacetimes for $R_b > 3 r_g$ the centrifugal potential barrier is present for all particles with $\mathcal{L} \neq 0$, rendering efficient accretion impossible. There is no potential barrier present in JMN-1 with $R_b < 3 r_g$. The location of the inner boundary of our numerical simulations of JMN-1 r_{bc} is shown with a blue vertical line.

than a transient collapse stage.

Physically, JMN-1 appears as a spherically symmetric object of radius R_b , a compactness parameter of the solution, consisting of gravitating and otherwise non-interacting matter. A requirement of subluminal speed of sound implies $R_b > 2.5 r_g$, where $r_g = MG/c^2$ for a total (ADM sense) mass M [23]. The metric inside the object is matched continuously to the vacuum Schwarzschild solution for $r \geq R_b$. The relevant JMN-1 metric tensor coefficients for $r \leq R_b$ are

$$g_{tt} = -\left(1 - \frac{2r_g}{R_b}\right)\left(\frac{r}{R_b}\right)^{\frac{2r_g}{R_b-2r_g}}, \quad (1)$$

$$g_{rr} = \left(1 - \frac{2r_g}{R_b}\right)^{-1}. \quad (2)$$

For sufficiently compact configurations with $R_b < 3 r_g$ JMN-1 spacetime admits Schwarzschild's photon sphere located at $r_\gamma = 3 r_g$. Hence, such an object exhibits gravitational lensing structure similar to that of the Schwarzschild geometry [30, 31] and as a consequence the observable shadow of JMN-1 is consistent with the current EHT observational constraints [32, 33]. Furthermore, even inspecting higher order

images lensed around the photon sphere (the photon ring image structure), which is often proposed as a robust future test of the metric [34–36], would necessarily yield results consistent with a Schwarzschild spacetime. To address this particularly challenging BH mimicker scenario, for our numerical considerations we fix $R_b = 2.8 r_g$.

A somewhat less appreciated aspect of the studies of BH alternatives is whether they allow for efficient advection of energy. Both EHT horizon-scale targets correspond to low mass accretion rate, radiatively inefficient accretion flows, for which advective cooling plays a fundamental role for maintaining the energy balance [37]. In case of BHs, excess energy disappears behind the event horizon. For horizonless objects the energy may be removed by outflows, or may accumulate in the system. Either possibility qualitatively impacts the accreting system structure, its secular evolution, and the expected observational signatures. In many horizonless singularity spacetimes an infinite potential barrier prevents accretion. This is illustrated with several examples in Fig. 1, explaining the absence of inflow all the way to the central singularity in recent numerical studies [19, 20, 25]. A qualitative difference between the variants of JMN-1 spacetime in Fig. 1 is related to the change of the g_{tt} power-law index in Eq. 1, also associated with the transition of the singularity type from a naked timelike for $R_b > 3 r_g$ to a null type for $R_b < 3 r_g$ [38].

Numerical simulations— Numerical GRMHD constitutes the most physically complete framework for global simulations of compact accreting systems currently available at an acceptable computational cost. It allows us to model the interactions between matter and magnetic fields, fully accounting for special relativity as well as for the background curved spacetime, while preserving the fundamental laws of baryon number, energy, and momentum conservation. Magnetorotational instability (MRI) [39] develops natively in GRMHD simulations, enabling turbulent transport of angular momentum and facilitating accretion. An extensive library of numerical simulations and ray-traced images was assembled by the EHT collaboration to aid the theoretical interpretations of the M87* and Sagittarius A* observations, with remarkable successes [40–43]. For the vast majority of the existing global MHD simulations of relativistic accretion, Kerr spacetime background has been assumed, with several notable exceptions [17, 20, 44–46]. In particular, simulations of accretion in horizonless singularity spacetimes constitute a very recent development [19, 20, 25, 46, 47], with only [20, 46] including magnetic fields and fully covariant formulation. Our simulation of JMN-1 with $R_b = 2.8 r_g$ is the first one demonstrating efficient accretion onto the central singularity in a horizonless spacetime, in contrast to the other simulations, in which matter is expelled from the innermost region. Thus, such a background spacetime can be expected to mimic well the flow of energy around an astrophysical BH. The details of our numerical setup using KHARMA [48], modified to enable non-Kerr simulations, are given in the End Matter.

In Fig. 2 we compare snapshots from GRMHD simulations of Schwarzschild BH and JMN-1, started from identical initial

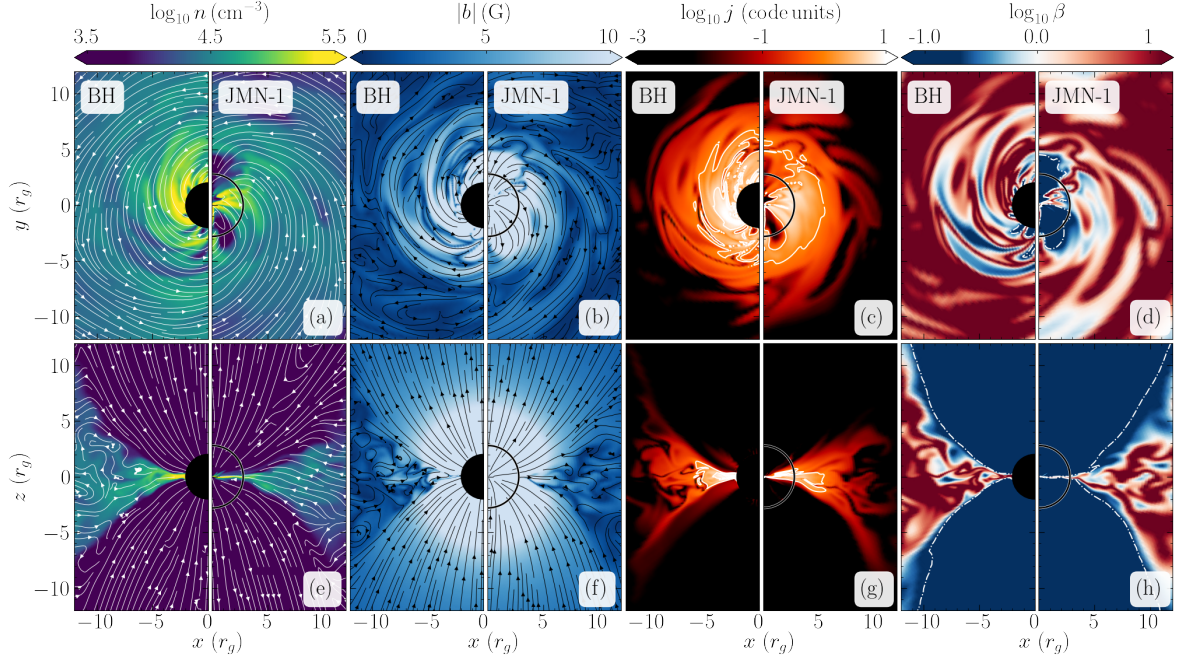


Figure 2. Equatorial ($x - y$, top row) and meridional ($x - z$, bottom row) slices through a GRMHD simulation snapshot ($t = 8000 t_g$ for $t_g = r_g/c$). For each column the left panel shows Schwarzschild BH with a black filled semi-circle $r_h = 2 r_g$ indicating the event horizon. The right panels show JMN-1 with a semi-circle marking the matching radius $R_b = 2.8 r_g$. Maps of number density n (panels a and e), magnetic field strength $|b|$ (panels b and f), emissivity proxy j (panels c and g), and gas to magnetic pressure ratio β (panels d and h) are shown. Solid white contours represent velocity flow lines in the number density n panels (a) and (e), while black contours represent magnetic field lines in the magnetic field panels (b) and (f). The white contour in panels (c) and (g) indicates 50% of the emissivity peak. The white dashed contour in panels (d) and (h) represents the magnetic to fluid energy density ratio $\sigma = 1$.

conditions, demonstrating broad consistency for radii $r > R_b$ across the relevant quantities. At $r = R_b$, the metric is continuous and smooth in the sense of extrinsic curvature [30], but neither the metric connection nor the Ricci or Kretschmann curvature scalars are continuous there, as expected at an interface between vacuum and a self-gravitating material medium. Nonetheless, the numerical solution is well-behaved across the $r = R_b$ sphere, with weak shocks formation visible only through close inspection of quantities such as the gas pressure. These mild shocks are captured well by finite volume conservative numerical schemes such as those implemented in Kharma. The flow is turbulent in both spacetimes and both solutions approach a magnetically arrested disk (MAD) state [49]. Strongly magnetized regions of low density, characteristic to MAD accretion, develop and appear more prominent in the JMN-1 simulation. This is particularly visible in a plot of gas to magnetic pressure ratio $\beta = p_{\text{gas}}/p_{\text{mag}}$ (panel d) and that of number density n (panel a). Magnetic field geometry is very similar in both cases (panel f), typical to a MAD state [50]. In its innermost part, the accretion disk is geometrically thinner for JMN-1 than for a BH, although both are thin in that region (panel e). We use the emission proxy j (panels c and g; this quantity represents the scaling of the high-frequency

limit of the thermal synchrotron emissivity, following Section 5.1 of [51]) to demonstrate that most emission is expected to originate in the innermost region and close to the equatorial plane. Furthermore, in the case of JMN-1 significant amount of radiation originates very close to the singularity – the analogous region is hidden by the event horizon in a BH spacetime. In the funnel region of JMN-1 outflowing velocity pattern develops, contrary to the BH case with inflow in the BH vicinity (panel e). However, both number density and radial velocity in the funnel region are very small in both simulations, so there is very little momentum transport differentiating the two solutions.

We convert number density n and the fluid frame magnetic field strength $|b|$ from the scale-invariant GRMHD code units to physical units by fixing M87* parameters [9, 52, 53]. As a result, both simulations produce n and $|b|$ (panels a, e and b, f of Fig. 2) consistent with the constraints from [42]. While the mass accretion rate is reduced for JMN-1, the suppression factor is only $\lesssim 2$. Importantly, both simulations result in mass accretion rates $\dot{M} \sim (4 - 8) \times 10^{-4} M_{\odot}/\text{yr}$, or $\dot{M} \sim (3 - 5) \times 10^{-6} \dot{M}_{\text{Edd}}$, where Eddington mass accretion rate $\dot{M}_{\text{Edd}}$ is defined through Eddington luminosity L_{Edd} as $\dot{M}_{\text{Edd}} = L_{\text{Edd}}/0.1c^2$. The reported values of $\dot{M}$ were aver-

aged over $t \in (8 - 10) \times 10^3 t_g$, see Fig. 4 in the End Matter. These findings are very well consistent with the MAD Kerr BH models of M87* in the EHT library, which themselves vary between one another by a significantly larger factor [40].

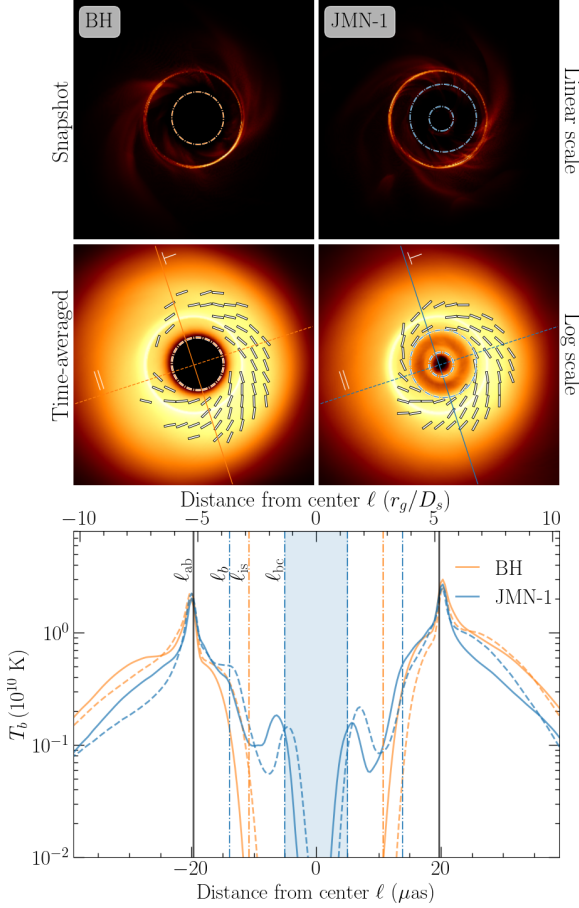


Figure 3. *Top row*: snapshot images ($t = 8000 t_g$) of a GRMHD accretion flow around a BH (left) and JMN-1 object (right), plotted in a linear colormap scale. The orange oval shape in the BH panel indicates the shape of the inner shadow ℓ_{is} , while the blue oval shapes in the JMN-1 panel correspond to ℓ_{bc} the image of the inner boundary of the simulation in the equatorial plane, that is, an equatorial circle $r_{bc} = 0.1 r_g$ (smaller one) and ℓ_b the image of the equatorial circle $r = R_b = 2.8 r_g$ (larger one). *Middle row*: time-averaged images shown with a logarithmic brightness scale. Orientation of the polarization vectors is shown with ticks. Characteristic oval shapes and accretion-disk-spin-parallel ($\parallel$) and accretion-disk-spin-perpendicular ($\perp$) directions are indicated. *Bottom panel*: spin-parallel (dashed line) and spin-perpendicular (continuous line) cross sections through time-averaged images, comparing brightness temperature T_b as a function of distance from the image center ℓ . The vertical lines give the location of the apparent boundary ℓ_{ab} , inner shadow ℓ_{is} of a BH, image of the JMN-1 matching radius ℓ_b , and image of the inner boundary of the simulation domain ℓ_{bc} . These locations are exact for the spin-perpendicular slices (and for the face-on images).

Simulated images— We used *ipole* [54] to perform a full polarized relativistic radiative transfer calculations and to compute synthetic images of the two objects at 230 GHz, the observing frequency of the EHT. The images are strongly dominated by the synchrotron emission from relativistic electrons interacting with the magnetic field. We assumed characteristic parameters of M87*, namely, a distance $D_s = 16.9 \text{ Mpc}$, mass $M = 6.5 \times 10^9 M_\odot$ and inclination angle $\theta = 163^\circ$ [9, 52, 53]. For the examples shown in Fig. 3 we additionally assumed the electron temperature parameter $R_{\text{high}} = 50$ [40, 55], adjusted the plasma density scale to match the EHT measurements of 230 GHz flux density within 0.5–1.0 Jy range [56–58], and matched the projected spin axis of the accretion disk with the projected jet axis, defined by the observed forward jet position angle of 72° clockwise from the north direction. In Fig. 3 we show snapshots and time-averaged images of the two objects. The colors represent brightness temperature, and the ticks in the second row panels additionally show the geometry of the linearly polarized emission. When viewed with the linear brightness scale, the two images are remarkably similar to one another, as well as to those from the M87* EHT imaging library [40, 42].

The prominent feature in the linear scale images is the location of the brightness peak – the *photon ring* [34, 35, 59]. This strong-lensing feature approximates the location of the *apparent boundary*, which is the trace of the photon sphere r_γ in the image plane. The apparent boundary is defined exclusively through spacetime geometry and it is located at the image-domain radius of $\ell_{ab} = r_\gamma [-g_{tt}(r_\gamma)]^{-1/2} = 3\sqrt{3} r_g/D_s$ for Schwarzschild spacetime, e.g., [60]. The photon ring appearance is closely reproduced in the JMN-1 image, as the two spacetimes share identical structure above $r = R_b = 2.8 r_g < r_\gamma$. Given the limited instrumental resolution, a robust observational detection of this sharp image feature currently remains elusive as it blends with the direct emission that spreads more flux over larger region of the image [34, 61].

Inspecting the spin-parallel and spin-perpendicular image profiles shown in the bottom panel of Fig. 3 we see that the centroid of the brightness distribution does shift inward for the JMN-1 image. When integrated over the entire image ($\int \ell d\Omega / \int I d\Omega$ for brightness I and distance from the image center related to the photon’s impact parameter $\ell = -k_\phi/k_t$), we find the difference of $\sim 10\%$. This difference is not large enough to be readily distinguished from the impact of astrophysical setup, including uncertain plasma physics parameters such as R_{high} [40, 42] and from the mass and distance prior uncertainties [52].

First row of Fig. 3 presents the snapshot images in linear color scale and second row shows the time-averaged images of total intensity in the logarithmic color scale. In the second row images the presence of dim emission inside the apparent boundary ℓ_{ab} is clearly visible. For a Schwarzschild BH image, under equatorial emission approximation, the brightness only extends inward to the *inner shadow* [62], located at $\ell_{is} \approx 2.85 r_g/D_s$ for a face-on observer. In that case the emission observed within the apparent boundary ($\ell_{is} < \ell < \ell_{ab}$)

corresponds to photons emitted between the event horizon and the photon sphere ($r_h < r < r_\gamma$). For JMN-1 the emission region extends all the way to singularity, as the event horizon is absent. Our study is limited by the inner boundary condition of the GRMHD simulation at $r_{bc} = 0.1 r_g$, the image of which is located at $\ell_{bc} \approx 1.32 r_g/D_s$ for the equatorial emission, face-on approximation, see Fig. 3 and the End Matter. The detailed structure of the image is sensitive to the accretion flow properties and the gravitational redshift of the spacetime. In the JMN-1 images from our GRMHD simulations we see a mild rise of brightness near ℓ_{bc} , as gas density, temperature, and magnetic field strength all grow toward the singularity and so is the expected emissivity. While the appearance of this feature may be affected by the inner boundary condition treatment, our absorbing boundary condition should not artificially increase the emission in the simulation domain. In our spot-check tests we noticed that the feature depends also on the R_{high} parameter choice. Nonetheless, in all considered cases there is a dim but non-negligible brightness in JMN-1 images in the central image region, significantly more than in the BH images. In the End Matter we discuss lensing and redshift structure of JMN-1 spacetime. We also demonstrate the differences in the image structure between BH and JMN-1 for the analytic model of an equatorial accretion flow, free of the limitations related to the boundary condition treatment in GRMHD simulations.

Summary and discussion—We presented the first GRMHD simulation of accretion onto a JMN-1 null singularity horizonless object, demonstrating a sustained MAD state with an accretion rate comparable to that of a BH with the same mass. Hence, a singularity in such a spacetime acts as an efficient energy sink, similar to a BH’s event horizon, allowing for an advection-dominated, radiatively inefficient accretion flow to form, as expected in objects such as M87* or Sagittarius A*. This property is uncommon among horizonless alternatives to BHs, and renders such an object a particularly challenging BH mimicker.

Both Schwarzschild and JMN-1 spacetimes produce images consistent with current EHT observations of M87*, yet differ structurally in the central image region. BH images indicate deep central brightness depression that can be understood approximately in terms of the inner shadow of the equatorial emission, related to the size of the event horizon. In the absence of the event horizon, intensity in the JMN-1 images extends inward below the inner shadow, and the effective inner boundary depends on detailed accretion flow properties. In our GRMHD example the image brightness at about half of the radius of the inner shadow reaches a few percent of the peak brightness. While confirming or excluding such a feature with the current EHT observations is impossible because of the low dynamic range of the image reconstructions, it should become possible with the planned EHT array upgrades within the next few years [11, 12, 61], enabling a test of the BH paradigm.

In this work we only discussed comparison between static spacetimes, Schwarzschild and JMN-1. Further studies of

spinning models are necessary. Based on results of [62] we expect Kerr characterization to be qualitatively similar to our findings, despite more expected foreground emission in the jet. For the JMN-1, finding geometry of the valid spinning generalization is not straightforward. An algorithmic solution was proposed by [63], but it is not guaranteed to obey GR equations with consistent equation of state, or at least with an admissible stress-energy tensor content. The geometry produced by [63] was subsequently used for the GRMHD simulations of [20], but only in the naked timelike singularity regime $R_b > 3 r_g$.

This pilot study motivates detailed further investigations of BH and JMN-1 observables, including polarimetric ones, quantifying differences between these distinct topologies for a larger space of model parameters. Our results highlight the importance of high image dynamic range, rather than extreme resolution, for the future tests of gravity. These developments will guide the implementation of the proposed BH paradigm test in the context of upcoming radio-interferometric instruments such as the next-generation EHT [11].

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End Matter

Model setup—We used KHARMA¹ [48] (Kokkos-based High-Accuracy Relativistic Magnetohydrodynamics) to evolve ideal GRMHD equations. It is a performance-portable, C++ implementation based on i harm3D, leveraging the Parthenon Adaptive Mesh Refinement Framework and the Kokkos programming model [64, 65] to run efficiently on CPUs and GPUs. It uses a second-order predictor–corrector scheme to step forward in time.

We implemented the JMN-1 metric in KHARMA in Kerr-Schild like coordinates, following the approach described in [66] for non-vacuum spacetimes. Specifically, we used modified Kerr-Schild (MKS) coordinates (x^0, x^1, x^2, x^3) , with a res-

¹ publicly available at <https://github.com/AFD-Illinois/kharma>

olution of $288 \times 128 \times 128$, consistent with the standard resolution adopted in the EHT GRMHD model libraries [41, 51]. The radial coordinate is exponential, $r = \exp(x^1)$, spanning from r_{bc} to $r_{\text{out}} = 1000 r_g$, increasing the density of grid zones at small radii. The polar coordinate is modified according to $\theta = \pi x^2 + \frac{1}{2}(1-h)\sin(2\pi x^2)$, with $h = 0.3$, concentrating grid zones toward the midplane ($\theta = \pi/2$) to better capture accretion disk physics, while widening zones near the poles. For JMN-1 the inner boundary is set at $r_{\text{bc}} = 0.1 r_g$. Standard outflow boundary conditions are imposed at both the inner and outer radial boundaries. In the polar (θ) direction, transmitting boundary conditions across the coordinate poles are applied. For JMN-1 the inner radial boundary excises the curvature singularity from the domain. Although no event horizon is present, outflow/absorbing conditions remain physically appropriate since the accretion disk inflow becomes strongly supersonic near the inner boundary, preventing causal communication with the exterior flow and thereby minimizing artificial reflections. The boundary condition is also well motivated by the spacetime potential structure, see Fig. 1.

For both simulations we initialize a standard torus in hydrodynamic equilibrium [67] with inner edge at $r_{\text{in}} = 10 r_g$ and pressure maximum at $r_{\text{max}} = 20 r_g$. We assume adiabatic index $\gamma = 5/3$. Small perturbations ($\pm 2\%$ randomization) are introduced in the fluid's thermal energy to trigger MRI and incite accretion. The torus is threaded by a poloidal magnetic field loop, defined through a vector potential

$$A_\phi = \max \left[\left(\frac{\rho}{\rho_{\text{max}}} \right) \left(\frac{r}{r_{\text{in}}} \sin \theta \right)^3 \exp \left(-\frac{r}{400} \right) - 0.2, 0 \right], \quad (3)$$

with $A_r = A_\theta = 0$ and plasma density maximum ρ_{max} . The initial plasma beta, $\beta = p_{\text{gas}}/p_{\text{mag}}$, has a minimum value of 100. This setup enables sufficient magnetic flux to be advected toward the central object for the MAD state to develop. A complete description of the implementation of an equivalent torus can be found in Appendix A of [68].

As a diagnostic, we compute mass accretion rate $\dot{M}$, and magnetic flux Φ_B , through a sphere of radius r ,

$$\dot{M} = \int_0^{2\pi} \int_0^\pi \rho u^r \sqrt{|\det(g)|} d\theta d\phi, \quad (4)$$

$$\Phi_B = \frac{1}{2} \int_0^{2\pi} \int_0^\pi |B^r| \sqrt{|\det(g)|} d\theta d\phi, \quad (5)$$

where ρ is the fluid-frame gas density, u^r is the radial component of the 4-velocity, B^r the radial magnetic field component, and $\det(g)$ is the determinant of the metric $g_{\mu\nu}$. The level of magnetic saturation is characterized by the normalized magnetic flux $\phi = \Phi_B \dot{M}^{-0.5}$. The temporal evolution of $\dot{M}$ and ϕ are shown in Fig. 4. Both simulations reach a sustained MAD state characterized by $\phi_c \sim 15$. The accretion rate exhibits strong variability typical of MAD flows, while remaining comparable between the black hole and JMN-1 cases, and reaches a quasi-stationary state around $t = 6500 t_g$. We also calculated the MRI quality factors $Q_{r,\theta,\phi}$, which

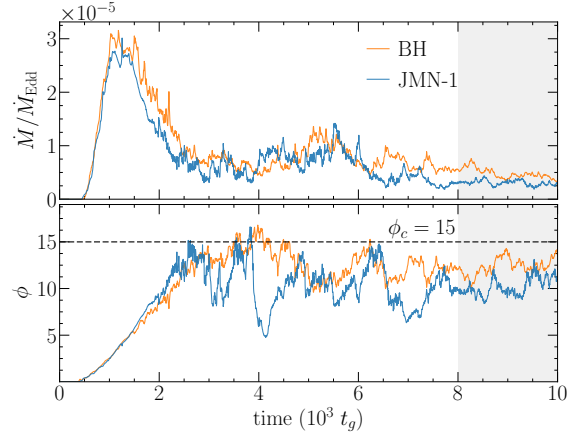


Figure 4. Evolution of mass accretion rate $\dot{M}$ normalized by the Eddington rate, and of normalized magnetic flux ϕ calculated at $r_h = 2 r_g$ for BH and at $r_{\text{bc}} = 0.1 r_g$ for JMN-1. For analyses we used the part of the simulation corresponding to a gray-shaded time range $t \in (8 - 10) \times 10^3 t_g$. The time-averaged $\dot{M}/\dot{M}_{\text{Edd}}$ values are $\sim (4.6 \pm 0.9) \times 10^{-6}$ for BH and $(3.0 \pm 0.5) \times 10^{-6}$ for JMN-1. The time-averaged values of ϕ are 12.4 ± 1.0 and 10.2 ± 0.9 for BH and JMN-1, respectively.

specify the number of grid cells resolving the fastest growing MRI mode. We obtained mean values $\langle Q_{r,\theta,\phi} \rangle^{\text{Sch}} \gtrsim (30, 20, 6)$, $\langle Q_{r,\theta,\phi} \rangle^{\text{JMN-1}} \gtrsim (100, 20, 20)$, exceeding the minimum requirements [51].

Image generation—We generated synthetic images by post-processing GRMHD simulation snapshots using *ipole*² [54, 69]. The code numerically solves general relativistic polarized radiative transfer equations in a chosen background spacetime. The 230 GHz emission is synchrotron-dominated and we assume relativistic thermal spectral energy distribution of electrons [40]. In the considered type of accretion flow electrons are thermally decoupled from ions [37]. As GRMHD evolves a single fluid temperature approximating the temperature of massive ions, we assign the electron temperature using a prescription [40, 55] with an R_{high} parameter (fixed $R_{\text{low}} = 1$). The field-of-view of images shown in Fig. 3 was set to $160 \mu\text{as}$ ($42 r_g/D_s$) with (300×300) px² resolution.

Lensing and redshift in JMN-1 spacetime—We consider a mapping between the location in the image plane ℓ and the equatorial emission radius r_{em} under face-on viewing angle approximation, that is, a *transfer function* [70]. In the first panel of Fig. 5 we show transfer functions corresponding to the direct image $n = 0$ and the secondary image $n = 1$, the first component of the photon ring. We compare results for a Schwarzschild BH and for a JMN-1 horizonless spacetime with $R_b = 2.8 r_g$. These are obtained by numerically integrating the photon deflection angle in each spacetime for assumed

² publicly available at <https://github.com/AFD-Illinois/ipole>

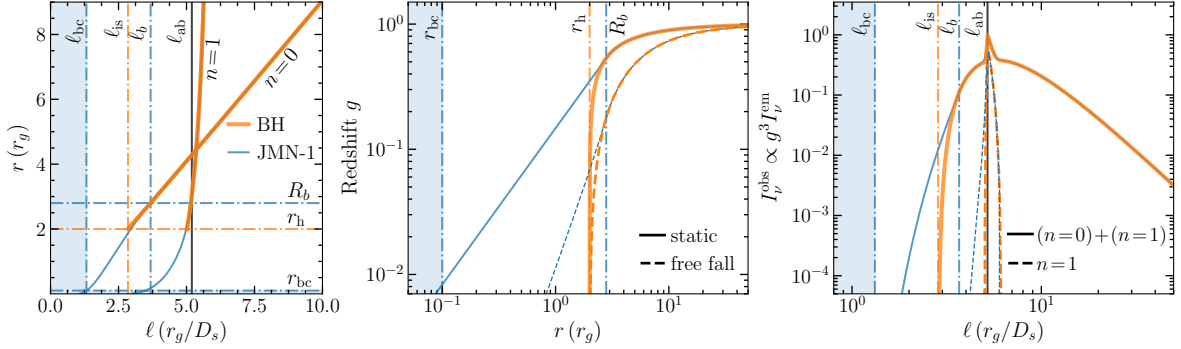


Figure 5. *Left*: transfer functions connecting photon's radial location in the image plane ℓ with the equatorial emission radius for the primary $n = 0$ and secondary $n = 1$ images for Schwarzschild BH (orange) and for JMN-1 with $R_b = 2.8 r_g$ (blue). *Middle*: gravitational redshift as a function of the emission radius in the two spacetimes for a static emitter (continuous lines) and a free-falling one (dashed lines). *Right*: A model of the observable brightness distribution $I_v^{\text{obs}}(\ell)$ in the image, originating from equatorial free-fall emission, assuming profile of emission $I_v^{\text{em}} \propto r^{-3}$. Profiles of secondary images are shown with dashed lines.

impact parameter ℓ

$$\Delta\phi(r_{\text{em}}; \ell) = \int_{\infty}^{r_{\text{em}}} \frac{k^\phi}{k^r} d\lambda, \quad (6)$$

where k^μ is the photon four-momentum, $k^\mu k_\mu = k^\theta = 0$ and $\ell = -k_\phi/k_t$ until reaching $\Delta\phi(r_{\text{em}}; \ell) = \pi/2$ for $n = 0$ and $\Delta\phi(r_{\text{em}}; \ell) = 3\pi/2$ for $n = 1$. The JMN-1 transfer function is identical with the BH one for the photons that remain in the Schwarzschild geometry, $r_{\text{em}} > R_b$. However, the emission that reaches the distant observer is not limited to $r_{\text{em}} > r_h$, and instead corresponds to all $r_{\text{em}} > 0$. The lensing very close to the singularity is extreme and a very small emission region $r_{\text{em}} < 0.002 r_g$ is mapped to a relatively large region of the image with $\ell < 1 r_g/D_s$. We expect strong gravitational redshift to limit the detectability of the emission from the vicinity of the singularity. This is demonstrated in the second panel of Fig. 5, where the redshift is calculated for static and free-fall emitters. We define the redshift g as the ratio between observed and emitted frequencies, calculated as

$$g = \frac{\nu_{\text{obs}}}{\nu_{\text{em}}} = \frac{u_{\text{obs}}^\beta k_\beta}{u_{\text{em}}^\alpha k_\alpha} \quad (7)$$

for the four-velocity of the emitter u_{em}^α and that of the observer at infinity $u_{\text{obs}}^\beta = (1, 0, 0, 0)$. For the static emitter $g = (-g_{tt})^{0.5}$ and we use the $n = 0$ transfer functions for the (ℓ, r_{em}) mapping in the free-fall case. Full free-fall redshift formulas are given in [30]. While the static redshift near the Schwarzschild horizon behaves like $g \propto (r - r_h)^{0.5}$, following Eq. 1 we see $g \propto r^{1.25}$ dependence in the vicinity of the central singularity of our JMN-1. As a consequence, the impact of redshift is significantly stronger for the material approaching the center of JMN-1 object. Hence, the location of the matching radius R_b is discernible in the log-scale image in Fig. 3 at an image domain distance ℓ_b , with reduced brightness for $\ell < \ell_b$. Using transfer functions and redshifts, we construct a simple model

of the observable image brightness profile corresponding to a free-fall equatorial emission, see the last panel of Fig. 5. Following the invariance of I_v/ν^3 we compute the observable brightness distribution as $I_v^{\text{obs}} \propto g^3 I_v^{\text{em}}$. For illustration we assume the intrinsic emission profile $I_v^{\text{em}} \propto r^{-3}$. While divergent at $r = 0$, the divergence of emission is mitigated by the strong JMN-1 redshift. We ignored effects of spectral index, disk thickness, and opacity, and we computed brightness distribution as a sum of primary and secondary images, neglecting higher order images $n \geq 2$. The image appears as a ring of approximately ℓ_{ab} radius both in BH and JMN-1 case. However, there is a clear difference regarding the inner edge of the brightness distribution – it necessarily terminates at the inner shadow ℓ_{is} for a BH, but extends further inward for JMN-1, limited by an interplay of the intrinsic emission profile and redshift, thus demonstrating the same general result as our GRMHD simulations.

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Chapter 8

Probing the Jet Base of M87* with EHT Observations

This chapter is based on the following publication where I am the first author:

Saurabh, Hendrik Müller, and Sebastiano D. von Fellenberg et al.,
“Probing the jet base emission of M87* with the 2021 Event Horizon
Telescope observations,” *Astronomy & Astrophysics*, 2026 ([Saurabh et
al. 2026](#)).

This paper is an official EHT project complementing/part of the multi-epoch variability analysis of [Event Horizon Telescope Collaboration 2025](#), with which it shares the 2021 dataset. As an official EHT publication, the author list includes the general author list (GAL), that is, the full EHT collaboration, in addition to the principal author list.

The 2021 EHT campaign added the 12 m Kitt Peak Telescope and NOEMA to the array, substantially improving the UV coverage relative to 2017. Among the new capabilities this brings is sensitivity to emission between the compact photon ring at $\sim 40 \mu\text{as}$ and the large-scale jet at $\lesssim 1 \text{ mas}$, a range largely inaccessible in previous EHT campaigns. The disk-jet transition region at horizon scales is one of the central open questions identified in Sections 2.8 and 4.9. Whether 230 GHz emission at sub-milliarcsecond scales traces the accretion disk, the jet sheath, or a compact jet-base component has direct implications for where the jet is launched relative to the ring structure, a question that the accretion flow modeling cannot answer by itself.

Imaging is performed using `eht-imaging` ([Chael et al. 2016](#); [Chael et al. 2018](#)), the regularised maximum likelihood framework described in Section 4.6.3 in Chapter 4, which has been used in every EHT imaging analysis of M87* and Sgr A* to date ([Event Horizon Telescope Collaboration 2019](#); [Event Horizon Telescope](#)

Collaboration 2021; Event Horizon Telescope Collaboration 2022; Event Horizon Telescope Collaboration 2025). The 2021 data were calibrated with the rPI-CARD pipeline to handle the phase jumps characteristic of NOEMA baselines, with the same calibration solutions as Event Horizon Telescope Collaboration 2025. A hyperparameter survey was carried out on synthetic data generated from MAD GRMHD models that passed the EHT 2017 and 2018 model comparisons, with computations performed on the Max Planck Computing and Data Facility (MPCDF) cluster. The survey selected regulariser weight combinations consistent with prior EHT imaging studies. These were then applied to the calibrated 2021 visibilities, targeting both the compact ring and any extended emission component at $\lesssim 1$ mas.

Beyond this paper, the `eht-imaging` analysis contributed as one of the three RML imaging pipelines in Event Horizon Telescope Collaboration 2025, alongside DoG-HIT (Event Horizon Telescope Collaboration 2025) and MOEA/D. The fiducial multi-epoch images of Event Horizon Telescope Collaboration 2025 are produced by averaging across all imaging methods and blurring to a common resolution. For the 2021 epoch, the hyperparameter survey and parameter selection for the `eht-imaging` pipeline used in that analysis were also performed on the MPCDF cluster as part of this thesis work. The blinded validation on synthetic data confirmed total intensity cross-correlations exceeding 0.975 with the ground truth for all methods across all epochs (Event Horizon Telescope Collaboration 2025).

The 2021 data provide the first direct EHT constraints on jet-base emission at sub-milliarcsecond scales. Evidence for a compact emission component spatially distinct from the ring structure is found, and its morphology and flux are characterized. Its location relative to the photon ring constrains the disk-jet coupling geometry at horizon scales. These constraints are also relevant to the polarimetric variability discussed in Section 4.7.3: structural changes in the jet-base emission on year timescales could contribute to the LP fraction drop and EVPA helicity reversal observed between 2017 and 2021.

I performed the `eht-imaging` hyperparameter survey on the MPCDF cluster, applied the pipeline to the 2021 M87* dataset, carried out the jet-base emission modeling, and led the analysis and interpretation. Sebastiano mentored this project with Hendrik Müller performing additional analysis on the interpretation and comparison with the M87* GMVA image. I also contributed the `eht-imaging` pipeline and parameter survey to the Event Horizon Telescope Collaboration 2025 analysis. All authors contributed to the writing and revision of the manuscript.

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Probing jet base emission of M87* with the 2021 Event Horizon Telescope observations

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ABSTRACT

We investigate the presence and spatial characteristics of the jet base emission in M87* at 230 GHz, enabled by the significantly enhanced (u,v) coverage in the 2021 Event Horizon Telescope (EHT) observations. The integration of the 12-m Kitt Peak Telescope (USA) and NOEMA (France) stations into the array introduces two critical intermediate-length baselines to SMT (USA) and IRAM 30-m (Spain), providing sensitivity to emission structures at spatial scales of $\sim 250 \mu\text{as}$ and $\sim 2500 \mu\text{as}$ ($\sim 0.02 \text{ pc}$ and $\sim 0.2 \text{ pc}$). Without these new baselines, previous EHT observations of the source in 2017 and 2018 lacked the capability to constrain emission on large scales, where a “missing flux” of order $\sim 1 \text{ Jy}$ is expected to reside. To probe these scales, we analyzed closure phases—robust against station-based gain calibration errors—and model the jet base emission using a simple Gaussian component offset from the compact ring emission at spatial separations $> 100 \mu\text{as}$. Our analysis revealed a Gaussian feature centered at ($\Delta\text{RA} \approx 320 \mu\text{as}$, $\Delta\text{Dec.} \approx 60 \mu\text{as}$), projected separation of $\approx 5500 \text{ AU}$, with an estimated flux density of only $\sim 60 \text{ mJy}$, implying that most of the missing flux identified in previous EHT studies had to originate from different, larger scales. Brighter emission at the relevant spatial scales is firmly ruled out, and the data do not favor more complex models. This component aligns with the inferred position of the large-scale jet and is therefore physically consistent with the emission of the jet base. While our findings point to detectable jet base emission at 230 GHz, the limited coverage provided by only two intermediate baselines limits our ability to robustly reconstruct its morphology. Consequently, we treated the recovered Gaussian as an upper limit on the jet base flux density. Future EHT observations with expanded intermediate baseline coverage will be essential to constrain the structure and nature of this component with higher precision.

Key words. accretion, accretion disks – black hole physics – gravitation – relativistic processes – galaxies: individual: M 87* – galaxies: jets

1. Introduction

The giant elliptical galaxy Messier 87 (M87) serves as a cornerstone for understanding active galactic nuclei (AGNe), primarily due to its prominent relativistic jet powered by a supermassive black hole (SMBH) M87* at its center. This large-scale jet is visible across the electromagnetic spectrum, including the infrared (Röder et al. 2025), optical (Perlman et al. 2011), X-ray (Marshall et al. 2002), and gamma-ray regime (Abramowski et al. 2012). In the radio regime, the jet in M87 and its dynamics have been monitored at multiple wavelengths and spatial scales (e.g. Walker et al. 2018; Lister et al. 2018; Kim et al. 2018, 2023; Cui et al. 2023; Lu et al. 2023). At milli-arcsecond scales, the jet displays hints of triple-peaked helical dynamics (Asada et al. 2016; Hada 2017; Nikonov et al. 2023), likely connected to precession (Cui et al. 2023). The envelope of the jet limb shows a quasi-parabolic profile over a wide range of spatial scales (Asada & Nakamura 2012; Hada et al. 2013; Nakamura & Asada 2013; Nakamura et al. 2018; Walker et al.

2018; Lister et al. 2018), which appears to persist down to just a few gravitational radii, as observed in the very long baseline interferometry (VLBI) observations from the Global Millimetre VLBI Array (GMVA) (Kim et al. 2018; Lu et al. 2023; Kim et al. 2025) and space VLBI (Kim et al. 2023).

The Event Horizon Telescope (EHT) captured the first resolved image of the M87 nucleus at event horizon scales (EHTC 2019a,b,c,d,e; Akiyama 2019f), revealing a ring with a central brightness depression interpreted as the shadow of the SMBH. Subsequent EHT studies examined this feature in linearly and circularly polarized light (EHTC 2021a,b, 2023) and found the ring to be persistent between 2017 and 2018 (EHTC 2024b). Wielgus et al. (2020) used model-fitting on proto-EHT data at 230 GHz to study the ring evolution in the years prior to the first resolved images.

A recent polarimetric analysis of EHT data spanning three epochs – 2017, 2018, and 2021 – studied the dynamics across years of M87* (EHTC 2025). It was found that the ring stays remarkably stable in total intensity (i.e., the ring diameter

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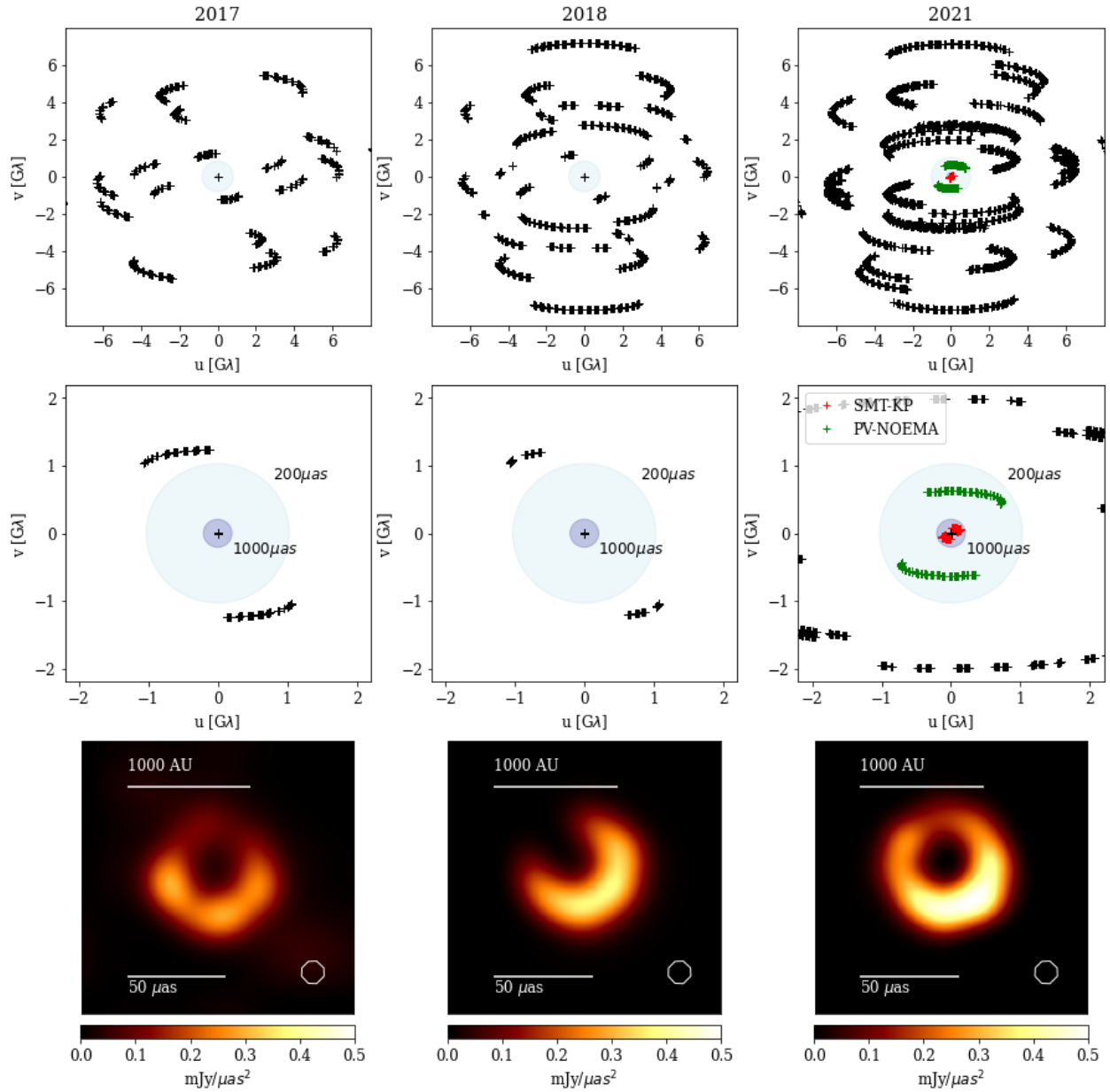


Fig. 1. (u,v) coverage of the EHT and representative ring image obtained by DoG-HIT (EHTC 2025) in 2017, 2018, and 2021. Upper panels: (u,v) with the SMT-KP (red) and PV-NOEMA baselines (green) highlighted. These are used to constrain the jet base emission. Middle panels: a zoomed-in view of the (u,v) coverage, highlighting the short and intermediate baselines. We also highlight the (u,v) coordinates related to spatial scales of $1000 \mu\text{as}$ and $200 \mu\text{as}$, corresponding to the expected range for extended emission. Bottom panels: DoG-HIT image of the central ring obtained from these data (EHTC 2025).

and thickness), with varying brightness asymmetry over the years; but the linear polarization fractions and polarization patterns vary drastically between the years. Although the milliarcsecond-scale jet has been studied extensively over the years (Walker et al. 2018; Cui et al. 2023), its direct connection to the jet base and the inner accretion flow is not well constrained (e.g., Hada et al. 2024). A major step forward was made through 86 GHz GMVA imaging (Lu et al. 2023), which revealed a ring-like structure with northern and southern components linking to a collimated, edge-brightened jet base. However, a central ridge-

line present in the images may be an artifact from the deconvolution procedure (Kim et al. 2025). In this work, we aim to build on these findings and constrain the resolved, extended emission associated with the jet base at 230 GHz using the 2021 EHT observations.

In general, interferometric observations can only constrain spatial scales corresponding to baseline lengths present in a given array (e.g., Thompson et al. 2017). Very long baselines (thousands of kilometers) resolve the finest angular scales, such as the $\sim 40 \mu\text{as}$ ring. So-called trivial intra-site baselines (a few

hundred meters) are sensitive to arcsecond-scale structure but provide no resolution near the black hole (Georgiev et al. 2025). Intermediate-length or short baselines (a few hundred to a few thousand kilometers) probe angular scales of hundreds to thousands of microarcseconds, making them particularly important for studying the connection between the accretion flow and the jet base.

While the presence of the large-scale jet at 230 GHz is evident in Atacama Large Millimeter/submillimeter Array (ALMA) observations (Goddi et al. 2021), detecting and imaging this emission with VLBI at 230 GHz is challenging for 2017 and 2018 EHT datasets lacking short or intermediate length baselines. This challenge results in a missing flux–flux density on the order of ~ 1.0 – 1.4 Jy detected on trivial baselines from co-located stations (ALMA/APEX, JCMT/SMA) – that compact ring-only models (flux density ~ 0.5 – 1.0 Jy depending on the imaging approach) cannot fully explain (EHTC 2019d, 2024b). Although a robust detection was impossible due to the lack of short and intermediate length baselines, some hints of extended emission, particularly to the southwest of the ring, in the pre-2021 EHT data were reported in Broderick et al. (2022) and EHTC (2024b) and through independent inspections of the residual maps in Carilli & Thyagarajan (2022) and Arras et al. (2022).

The EHTC’s data analysis methodology applied various tools to deal with the missing flux issue, validated across datasets (EHTC 2019d, 2024b, 2025). All methods have been explicitly validated using synthetic data generated with the 2021 coverage¹, in order to assess any potential systematic biases in the reconstructions. We refer to the synthetic data tests in EHTC (2025) for more details.

The regularized maximum likelihood (RML) methods typically deal with the missing flux problem by rescaling the flux density at intra-site baseline spacings. DoG–HIT attempts to recover the image from closure quantities only (in arbitrary units) and scales the whole image structure globally to a flux density that minimizes the χ^2 of the amplitudes. CLEAN methods add a large-scale Gaussian to account for the missing large-scale flux density. Comrade and THEMIS apply similar strategies, but typically with more degrees of freedom for the large-scale component. The Hybrid-THEMIS framework (Broderick et al. 2020b) incorporates the model-fitting of a narrow ring, allowing more emission to be placed as jet base emission (Broderick et al. 2022).

The 2021 EHT observations offer the best (u, v) -coverage to date, providing a new opportunity to detect the faint, extended emission associated with the jet. In particular, this is aided by the addition of two new stations; the Northern Extended Millimeter Array (NOEMA) in France and the Kitt Peak 12-m telescope (KP) in the USA. These stations add intermediate baselines of ~ 1100 km and ~ 100 km to the IRAM 30-m telescope on Pico Veleta (PV) in Spain and the Submillimeter Telescope (SMT) in the USA, respectively. The corresponding angular scales of these two baseline pairs are ~ 250 μ as and ~ 2500 μ as, i.e., these baselines are in principle sensitive to extended emission that could not be detected by the EHT in previous years. However, each baseline only probes a narrow range of spatial frequencies. Due to the limited number of baselines sensitive to intermediate angular scales, the (u, v) coverage is too sparse to directly image the faint, extended structure. It would require several intermediate baselines with resolving power of a few hundred μ as to robustly image the jet base region.

¹ That is, images of a variety of simple geometric structures, physically motivated structures, as well as simulations of accretion flow (EHTC 2025).

As a consequence, much like in the case of other very sparse VLBI data sets (e.g., Wielgus et al. 2020), we resort to a geometric model-fitting approach to constrain the presence of extended emission in the 2021 EHT data. For this purpose, we used closure phases, which are the sum of the visibility phases on a closed loop of three baselines (a triangle). This quantity is powerful because it is immune to station-dependent atmospheric and instrumental phase errors, making it a robust observable in VLBI (Thompson et al. 2017; Blackburn et al. 2020). Therefore, we use closure triangles of the baselines of interest, formed with the most sensitive station in the array, ALMA. On the relatively short baselines that make up these triangles, the compact ~ 40 μ as ring is only marginally resolved. While these baselines individually have limited resolving power for such small-scale structures, closure phases remain sensitive to asymmetries in the overall brightness distribution. For a point source or a symmetric ring, the closure phases on these triangles are expected to be zero. However, when we compare the observed closure phases with those predicted by image reconstructions, we find non-zero residuals. This indicates that the observed phase structure cannot be fully explained by the ring alone – even when intrinsic asymmetries in the ring are included – and instead points to the presence of additional, extended and asymmetric emission on angular scales probed by the intermediate baselines.

To test this interpretation, we model the extended emission using an offset Gaussian component in addition to the compact ring. Varying the position of the Gaussian component, we found that the most plausible location for this excess emission is to the South-West of the ring (i.e., the direction of the large-scale jet), consistent with the conclusions reached by, for e.g., Broderick et al. (2022).

The analysis presented in this manuscript builds on the success of previous EHT studies, explicitly fitting a large-scale component, and does not question the validity of the recovered compact-scale images. As a complement to the compact-scale studies, we present a deeper analysis of emission features of the jet base at intermediate spatial scales (~ 250 – 2500 μ as).

2. EHT observations of M87* in 2021

In this section, we summarize the main data properties and results, with details provided in the companion paper (EHTC 2025). The data were correlated with DiFX (Deller et al. 2007, 2011) using the output bands mode, a spectral window width of 58 MHz, and a bandwidth of 1856 MHz per band. For this analysis, we used band 3 and band 4 data, centered on 227.1 GHz and 228.1 GHz, respectively. The data were converted from linear to circular polarization feed basis using POLCONVERT (Martí-Vidal et al. 2016). Fringe-fitting was done using rPICARD (Janssen et al. 2019, 2022; von Fellenberg et al. 2025). The 2021 EHT observing campaign (April 9–19) consisted of M87* observations on April 9, 13, 14, 17 and 18. Since April 14 and 17 were relatively shorter tracks and NOEMA did not participate in April 13, we used only April 18. In 2021, NOEMA and KP joined the array for the first time, providing intermediate-length baselines that may be sensitive to the extended jet base emission (see Figure 1).

In EHTC (2025), the data were analyzed by seven teams using different imaging algorithms. The robustness of the reconstruction was demonstrated via cross-validation. Two teams used Difmap (Shepherd 1997), in combination with the AIPS task LPCAL (Leppanen et al. 1995) and GPCAL (Park et al. 2021, 2023a,b). The other teams used the RML methods DoG–HIT (Müller & Lobanov 2022, 2023a,b), ehtim

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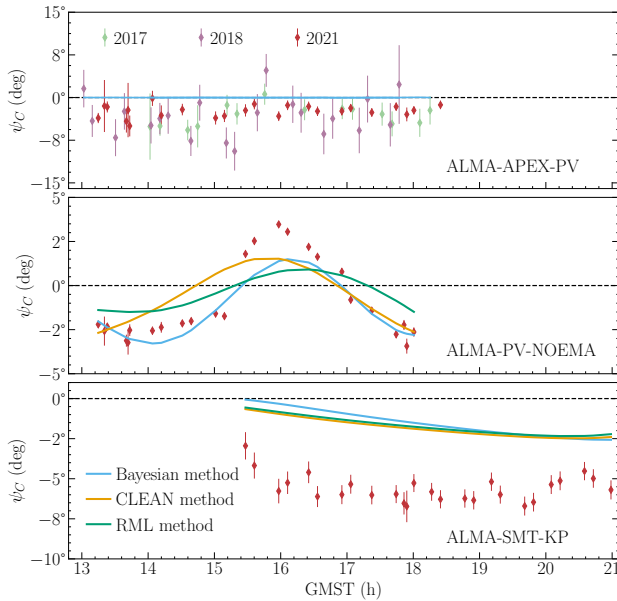


Fig. 2. ALMA-APEX-PV (top) closure phases of M87* as observed in 2017, 2018 and 2021. The ALMA-PV-NOEMA (middle) and ALMA-SMT-KP (bottom) closure phases on M87* observed on April 18, 2021 (figure is adapted from [EHTC 2025](#)) are shown together with ring-image models obtained with different imaging algorithms (see text for details). The top panel shows an example of a trivial closure phase triangle, where two stations are co-located, see [Georgiev et al. \(2025\)](#) for details. The image models are discrepant with the observed data by a few degrees, which may be attributable to intermediate-scale emission observed on the PV-NOEMA and SMT-KP baselines.

(Chael et al. 2016, 2018), and MOEA/D (Müller et al. 2023; Mus et al. 2024a,b), as well as the Bayesian imaging techniques THEMIS (Broderick et al. 2020a) and Comrade (Tiede 2022). Although these imaging algorithms rely on different assumptions, they converge on a consistent recovered image structure on event horizon scales ([EHTC 2025](#)). We show a representative image from this analysis obtained by DoG-HIT in the bottom panels of Figure 1. The 2021 observations of M87* are well represented by a ring structure with an asymmetry oriented to the southwest. Although the total-intensity images are very consistent across years and methods, there is some significant evolution of the polarized structure over the years (for more details, we refer the reader to [EHTC 2025](#)). In this work, we focused on total intensity and leave the discussion of polarization of extended components for future work.

3. Jet base emission

3.1. Bump and offset in closure phases

We show the (u,v) coverage of the EHT in 2017, 2018, and 2021 in Figure 1 and highlight two new baselines introduced in 2021. The PV-NOEMA baseline is ~ 1100 km in length, corresponding to on-sky spatial scales $\sim 250 \mu\text{as}$; the SMT-KP baseline is ~ 100 km in length, corresponding to $\sim 2500 \mu\text{as}$ scales. These intermediate baselines are sensitive to jet base emission, if it exists and is bright and compact enough to be detectable. However, each of these spatial scales is probed by a single baseline with a limited track length. Because of that, self-calibration techniques may inadvertently introduce or suppress extended

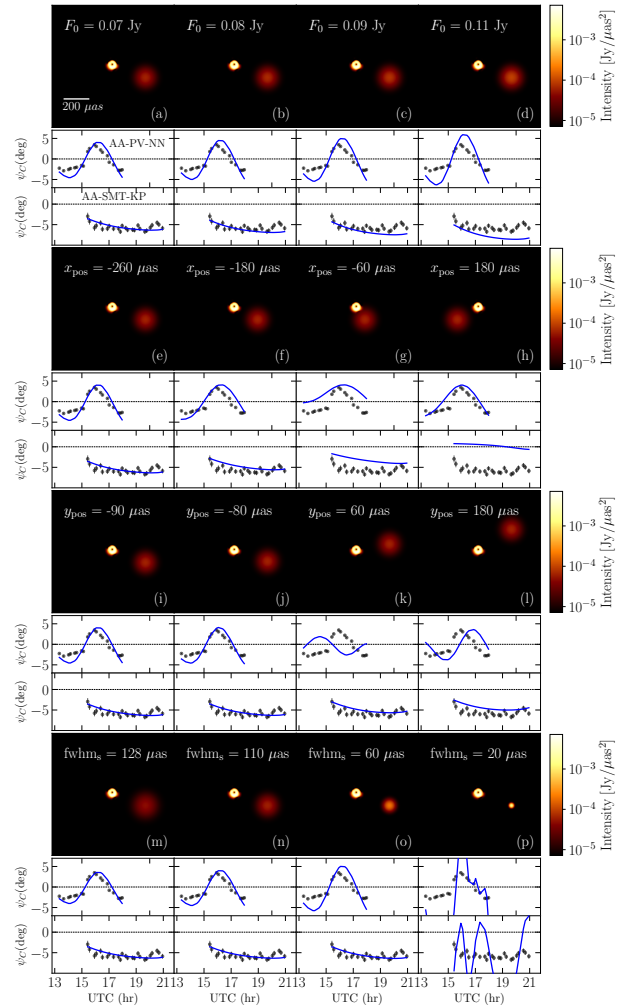


Fig. 3. Symmetric Gaussian component with varying parameters over the reconstructed DoG-HIT image as represented in Figure 1 represented in log scale. Different rows correspond to changes in the values of different parameters (see text for details). The solid lines represents the corresponding complete model (ring + Gaussian) closure phases on the respective triangles.

emission if calibrated against a ring-only image or central point sources (e.g., Carilli & Thyagarajan 2022). To overcome this problem, we analyze data that were not self-calibrated, and use closure phases that are insensitive to station-based gain errors (e.g., Blackburn et al. 2020) to probe the jet base emission at scales accessible with SMT-KP and PV-NOEMA.

Three stations are required to construct closure phases. Because no other intermediate-length baselines are available, the closure triangles formed necessarily also probe small-scale structures. However, we can choose triangles that are particularly useful, i.e., show low variations during the track (indicating that the long baseline contribution is small or canceled), are long in duration, and have high signal-to-noise ratio (S/N). The high-sensitivity station ALMA satisfies all these requirements for both baselines of interest. We thus focused on the ALMA-SMT-KP and ALMA-PV-NOEMA triangles in this analysis.

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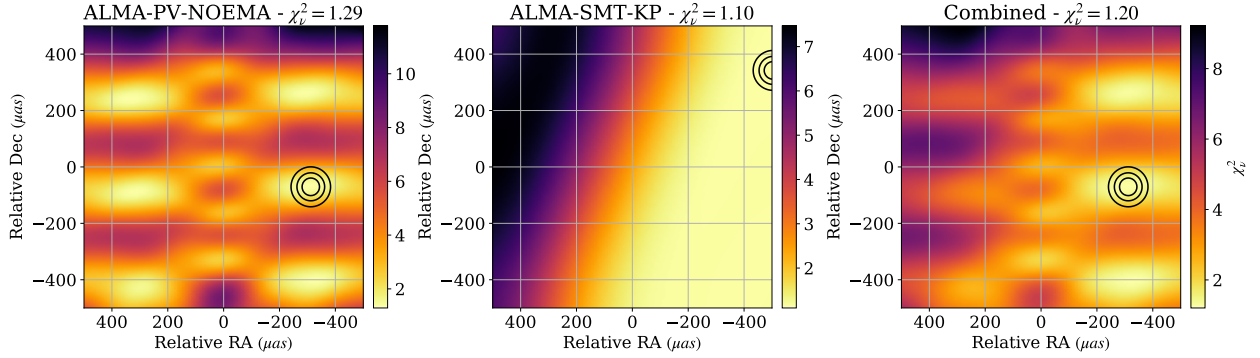


Fig. 4. χ^2_v achieved with the Gaussian model for a fixed width and brightness ratio, but varying position. The left and middle panels show the χ^2_v for the ALMA-PV-NOEMA and the ALMA-SMT-KP triangles, respectively, and the right panel shows the combined χ^2_v . The contours indicate the position of the best-fit model.

Since the small-scale information of the long ALMA baselines in those triangles cannot be disentangled from the intermediate-scale information, the jet base information has to be derived from the difference between the data and the ring-only image. In Figure 2 we show the closure phases on various triangles. The top panel displays an example triangle containing the ALMA-APEX baseline across the three years. The middle and bottom panels display the triangles of interest obtained for the small-scale ring images for different methods—one representative image is chosen for each framework—*Di fmap* (CLEAN), *DoG-HIT* (RML) and *Comrade* (Bayesian). None of the imaging methods (marked by solid lines) can fully recover the closure phases with the compact ring image alone on these triangles. However, we note that the different imaging algorithms applied and validated various strategies to mitigate the effect of jet base emission on the ring images. These include the addition of large-scale Gaussian components that are not used to create the fits in Figure 2. Figure 2 displays the image fits without these additional factors, focusing solely on the compact ring feature.

In particular, the ALMA-PV-NOEMA triangle features a ‘bump’ that is not well captured by the ring images, and, similarly, ALMA-SMT-KP has a visible ‘offset’. Due to the high sensitivity on those triangles, the difference is significant despite its small magnitude.

One plausible explanation for this residual of a few degrees in the closure phase is a faint but measurable emission on intermediate spatial scales sensed by PV-NOEMA and SMT-KP.

3.2. Model

In the following, we motivate a simple model to explain the observed closure phase residuals on the two selected triangles. The triangle that includes the longer baseline PV-NOEMA features a bump with a period of a few hours. Conversely, the ALMA-SMT-KP triangle does not have a beating but a simple offset from the ring-image models. This suggests that the relevant spatial scales may be on the order of, or slightly larger than, the PV-NOEMA scale, as any larger structure would introduce a beating on the ALMA-SMT-KP triangle as well. Moreover, it needs to be located close to the image center. Such a closure phase signature is characteristic of a “binary model”; in our case it consists of the ring, which is not resolved by either of the two baselines, and an extra component.

Based on these simple considerations, we propose the following source model: the compact ring emission as seen on long

baselines, with an additional, faint, Gaussian component located at scales 100–2000 μas from the center of the ring emission. This model gives four additional degrees of freedom, the x and y positions, the brightness F_0 , and the full width at half maximum (fwhm) of the Gaussian component. The model is selected to be as simple as possible to restrict the number of degrees of freedom, which is a necessary assumption for the model fitting due to the limited information on jet base emission present in the data. Because the PV-NOEMA and SMT-KP baselines are sensitive to two different spatial scales, they are not necessarily described by the same model component. Thus the model’s complexity can be trivially extended by allowing for an elliptical Gaussian component or even a second Gaussian component.

Deriving from these simple initial considerations, we can set approximate limits for the expected parameters of the additional Gaussian component. The size should be such that it is probed by the PV-NOEMA and SMT-KP baselines, creating a beating on the former within just two hours. Hence, it does not describe extended emission but most likely jet base emission with spatial scales similar to the ring diameter.

We refer to detailed analysis of the total flux density of the ring feature in EHT observations presented in EHTC (2024b). Although for the arcsecond scale structure in M87* typically flux densities between 1.0 Jy and 2.0 Jy were measured (Bower et al. 2015; Wielgus et al. 2020), for the milliarcsecond and microarcsecond structure usually flux densities between 0.5 and 1.0 Jy have been reported (Doeleman et al. 2012; Akiyama et al. 2015; Kim et al. 2018; Wielgus et al. 2020; Lu et al. 2023), consistent with the values found by the EHT (EHTC 2019d, 2024b, 2025). The discrepancy between the ring image and the flux density detected with ALMA alone of about 1 Jy is due to the existence of an (as of yet) undetectable large-scale jet. The additional Gaussian component fitted in this manuscript is likely to explain some, but not necessarily all, of the missing flux. This sets an approximate upper limit of 0.5 Jy for the flux of the Gaussian component, preferably less.

4. Localization of the emission region

4.1. Impact of the model parameters

In the following analysis, we examine the impact of jet base emission when modeled as a Gaussian component. We base our analysis on a *DoG-HIT* reconstruction: since the algorithm separates small and large scale emission naturally, it is optimized to fit to the closure phases and closure amplitudes in the 2021 data.

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However, we note that the black hole shadow images obtained by various algorithms have similar characteristics on the ALMA-PV-NOEMA and ALMA-SMT-KP triangles with respect to the bump and the offset, see Figure 2.

Figure 3 shows the ring image obtained by DoG-HIT with an additional Gaussian added on top, varying some of the parameters described in Section 3.2. The upper part of the panels display the recovered central region with an additional Gaussian component in logarithmic scale. Panels e, f, h, and i show the observed closure phases (black markers) and model closure phases (solid lines) on the ALMA-PV-NOEMA triangle (AA-PV-NN) and ALMA-SMT-KP triangle (AA-MG-KP).

The addition of a Gaussian component creates a periodic modulation – a beating – on the PV-NOEMA baseline. There are locations (primarily to the east and west of the ring, see Figure 3) where the beating is broadly aligned with the data, and locations where it misaligns and worsens the fit to the closure phase (primarily to the north). On the ALMA-SMT-KP triangle, the component produces a closure phase offset, which matches the data for some locations (good fits primarily to the West). The brightness of the component scales the amplitude of the beating and offset on the respective triangles. An overly bright Gaussian component overestimates the beating and the correction of the offset, while a too dim component underestimates the necessary corrections (first row of Figure 3). Finally, an overly compact component causes large-magnitude swings in the closure phase, while for a larger feature it is too smooth (fourth row in Figure 3).

These findings are summarized in Figure 4. In the left panel we show an example of reduced χ^2 (χ^2_ν) of the ALMA-PV-NOEMA triangle as a function of the location of the Gaussian component with a fixed flux density of 60 mJy and a size of 180 μ as. Locations 200 μ as to the west or the east of the ring are preferred. Similarly, the middle panel shows the χ^2_ν for the ALMA-SMT-KP triangle, with the preferred location to the southwest. In the right panel we show the combined χ^2_ν and a preferred region to the southwest, roughly 300 μ as to the West and 100 μ as to the South of the ring. However, the presence of multiple regions with low χ^2_ν indicates that the current model remains weakly constrained and degenerate.

4.2. Symmetric Gaussian component

To obtain the maximum posterior position for a Gaussian component, we use the Markov chain Monte Carlo (MCMC) sampler EMCEE (Foreman-Mackey et al. 2013). We adopt uniform priors for the flux ($F_0 \in [0, 0.2]$ Jy), the location ($x, y \in [-500, 500]$ μ as), and the width ($\text{fwhm}_s \in [50, 250]$ μ as). The maximum posterior parameters are given in Table 1. The corner plots are shown in Appendix A (Figure A.1). The best fit component has a flux density of ~ 60 mJy and is located towards the Southwestern portion of the ring.

We show the obtained ring image as well as the ring image with an additional Gaussian component in Figure 5. The respective fits of the Gaussian components to the ALMA-PV-NOEMA and ALMA-SMT-KP closure phases are shown in the panels e, f, h, and i. We highlighted notable improvements in the fitting quality of the closure phases visible from panel h and i. Despite the simplicity of the model, the χ^2_ν is 1.29 for the bump (compared to 2.73 without the jet base emission), and 1.12 for the offset (compared to 2.52). The component has almost negligible impact on all other triangles; the χ^2_ν over all triangles improves from 1.5 to 1.39. The χ^2_ν does not increase for any triangle, see Appendix B for further details. In Figure B.2, we show the impact of the

Table 1. Parameter estimates for the different Gaussian model fits described in Section 4.2, Section 4.3 and Section 4.4.

Parameter	Symmetric	Asymmetric	Double
F_0 (mJy)	70^{+10}_{-10}	60^{+20}_{-20}	50^{+30}_{-30}
x_1 (μ as)	-304^{+38}_{-38}	-300^{+37}_{-37}	-297^{+46}_{-46}
y_1 (μ as)	-68^{+12}_{-12}	-63^{+12}_{-12}	-68^{+18}_{-18}
fwhm_1 (μ as)	169^{+27}_{-27}	180^{+35}_{-35}	150^{+56}_{-56}
$\mathcal{R}_1$	–	$0.7^{+0.3}_{-0.3}$	$0.6^{+0.5}_{-0.5}$
Φ_1 ($^\circ$)	–	14^{+119}_{-119}	23^{+101}_{-101}
F_2 (mJy)	–	–	20^{+20}_{-20}
x_2 (μ as)	–	–	-291^{+96}_{-96}
y_2 (μ as)	–	–	117^{+126}_{-126}
fwhm_2 (μ as)	–	–	150^{+56}_{-56}
$\mathcal{R}_2$	–	–	$0.5^{+0.3}_{-0.3}$
Φ_2 ($^\circ$)	–	–	1^{+120}_{-120}

additional Gaussian component on the amplitudes for different baselines. Amplitudes on baselines longer than $\sim 2G\lambda$ stay mostly unaffected by the additional Gaussian component. This is reflected by the fit to the self-calibrated amplitudes shown in panel g and j in Figure 5 and the left panel in Figure B.2. The effect of the Gaussian component can be completely absorbed into the amplitude self-calibration.

Finally, synthetic data tests were performed in EHTC (2025). These tests also include the validation of a ring image with an extended jet component. The results of this test for all seven imaging techniques are described in Appendix D3 of that paper. We reprint the main result of this validation in Appendix B, Figure B.1. We note that all techniques were able to recover the ring emission correctly with exceptionally high cross-correlation > 0.995 for the total-intensity 2021 data.

We conclude that the addition of the Gaussian model does not worsen the otherwise excellent fit to the data obtained in EHTC (2025) with the black hole shadow images only. Vice versa, the Gaussian component only leaves a significant effect on the ALMA-PV-NOEMA and ALMA-SMT-KP triangles, exhibiting a small effect on the overall fitting statistics. Hence, based on the success of the fitting in the aforementioned study, and the synthetic data tests that were performed in EHTC (2025), we have no reason to question the validity of the ring emission presented in EHTC (2025).

4.3. Asymmetric Gaussian component

To explore the possibility of asymmetry of the jet base emission we extend the model by two additional parameters: the ratio of fwhm_{maj} and fwhm_{min} ($\mathcal{R}$) and the rotation angle of the asymmetric Gaussian component Φ , see Appendix C for additional details. We again use uniform priors ($\mathcal{R} \in [0, 1]$, and $\Phi \in [-180^\circ, 180^\circ]$). Neither the flux density nor the position change significantly; however, the model prefers to be symmetric, and hence the rotation angle is poorly constrained. The χ^2_ν values are 1.28 for the bump and 1.10 for the offset, i.e., there is no significant improvement to the fit. The best-fit values are provided in Table 1, we show the best-fit model in the left panel of Figure 6.

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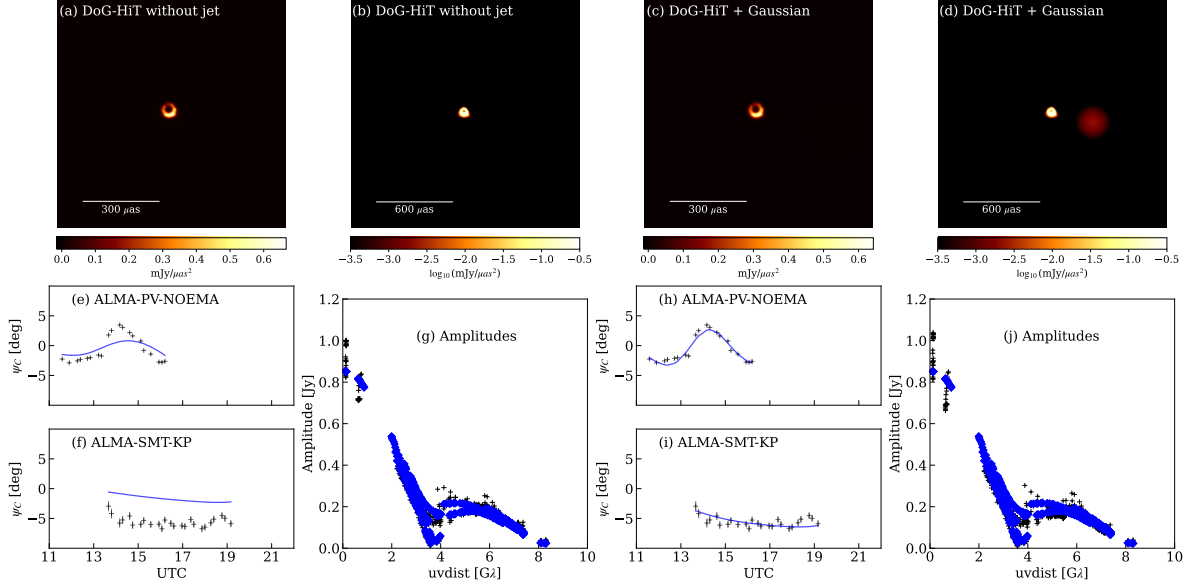


Fig. 5. Ring-only DoG-HIT image (panel a: linear scale, panel b: logarithmic scale) compared to the DoG-HIT ring image with an additional Gaussian (panels c and d). In the bottom row, we show the fit to the bump and offset in closure phases and the fit to the amplitudes for DoG-HIT (panels e,f,g) and for DoG-HIT with the symmetric Gaussian component added (panels h,i,j).

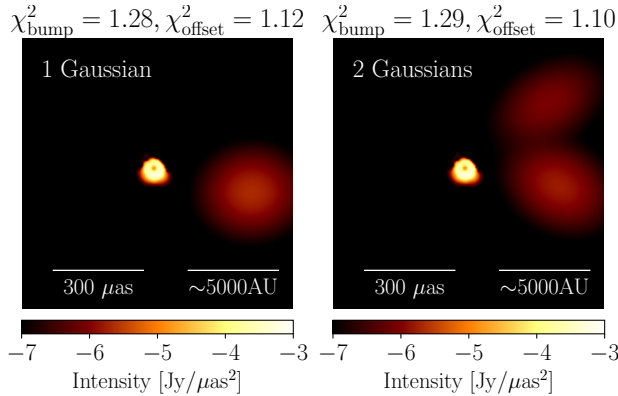


Fig. 6. Images for the best fits with an asymmetric Gaussian component (left) and two Gaussian components (right) in log scale.

4.4. Two Gaussians

For completeness, we explore the possibility of a secondary Gaussian component in the fit. However, given the complexity of the model and limitations of the data, we constrain the second component in the positive y direction, that is, we allow the y position to vary only within $[0, 500 \mu\text{as}]$. In this way, we test the presence of the second component in the northern region, but allow all parameters to vary (12 parameters). The flux density of the first southern Gaussian is $F_1 = 50^{+30}_{-20}$ mJy. The second Gaussian has a flux density of $F_2 = 20^{+20}_{-10}$ mJy. The first component is smaller in size compared to the previous modeling, but the positions do not vary significantly. The second, northern, component is relatively fainter. Since, the rotation angle for this component is poorly constrained, it prefers to be symmetric as well. The χ^2_ν values are 1.29 for the bump and 1.10 for the offset. Thus, there

is no statistical evidence for the inclusion of a secondary Gaussian component.

The Gaussian fit parameters are provided in Table 1, and the corner plot is provided in Appendix A. The third image in Figure 6 (right panel) represents the best-fit model consisting of two Gaussian components.

5. Alternative fitting scenarios

Due to the limited number of data points on the two chosen closure triangles, multiple plausible emission features may exist. In the previous section, we demonstrated that adding a simple Gaussian component to the compact emission provides a good fit to the data; the emission is most likely located southwest of the ring emission.

In this section, we discuss alternative models that may fit the data and discuss their validity. This list is not complete and focuses on some physically-motivated emission structures instead. Hence, we cannot rule out that alternative emission scenarios exist.

5.1. GMVA jet structure

At 86 GHz, Lu et al. (2023) imaged a ring-like feature together with the innermost jet in M87* with the GMVA complemented by the ALMA and the Greenland Telescope (GLT). Here, we discuss whether this jet structure would be able to fit the respective triangles. To this end, we need to extract the jet component of the GMVA image from the ring feature observed at 86 GHz, and stack this jet feature to the ring feature observed at 230 GHz.

Specifically, Kim et al. (2025) presented a reanalysis of this data set, among others, with DoG-HIT. In DoG-HIT, the image structure is represented in multiple spatial scales by wavelets Ψ_i :

$$I = \sum_i \Psi_i * \omega_i. \quad (1)$$

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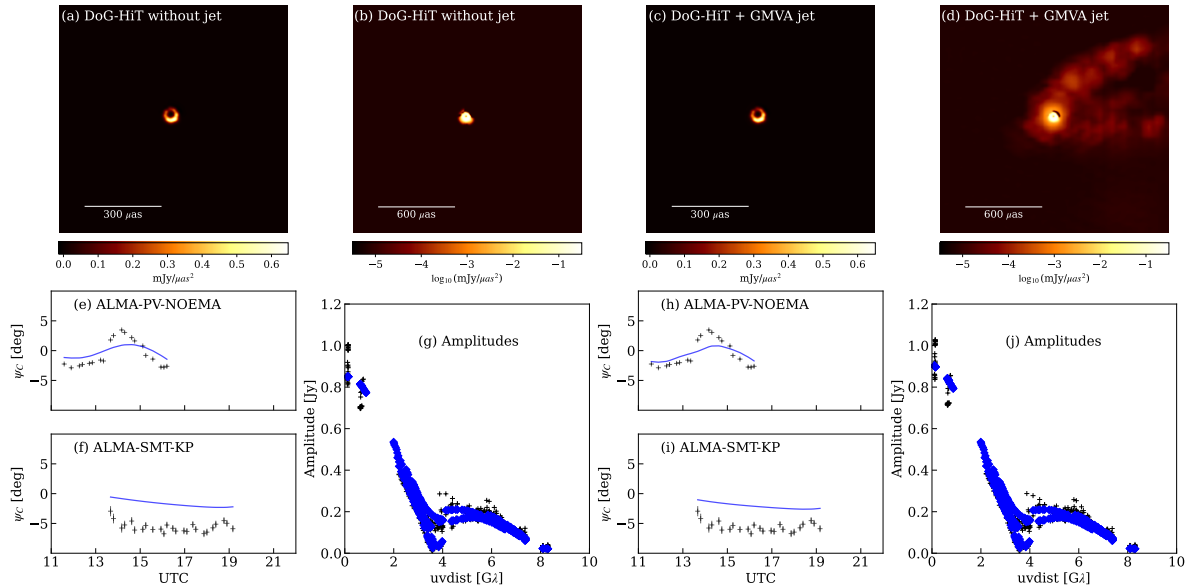


Fig. 7. Ring-only DoG-HIT image (first panel, top row: linear scale, second-panel first row: logarithmic scale) compared to the DoG-HIT ring image stacked with the jet observed with the GMVA (third and fourth panel, top row). In the bottom row, we show the fit to the bump and offset in closure phases and the fit to the amplitudes for DoG-HIT (left panels) and for DoG-HIT with the GMVA jet added (right panels).

The smallest-scale wavelets describe the ring (the small-scale feature), and the largest spatial scales describe the jet emission. In this way, the image is naturally decomposed into the compact ring structure and the jet base emission across different spatial scales. We use this natural decomposition to study whether the jet observed by the GMVA provides a good fit: we filter out the ‘ring-components’ in the 86 GHz image by setting all small-scale wavelets to zero, then we add the 230 GHz EHT ring image and the jet emission at 86 GHz seen by the GMVA. Motivated by the findings in Section 4.1, the GMVA jet image is rescaled to add a flux density between 0.0 Jy and 0.2 Jy to match the 230 GHz ring image. We grid-search for the best flux density in this interval and find the best χ^2 at an additional flux density of 0.06 Jy, consistent with the findings in Section 4.2. The results are shown in Figure 7. We note that we can absorb the large-scale jet component completely in the amplitude gains, providing a good fit to the self-calibrated amplitudes in both cases. The fit to the offset and the bump is improved by including the jet component. However, our model of the 86 GHz jet is not able to provide a good fit, performing worse than the simpler fiducial model presented in Section 4.2. This is likely a limitation of our attempt to scale the 86 GHz image, rather than an indication that the 86 GHz and 230 GHz emission structures are inherently different.

5.2. Imaging with weakened assumption on compactness

Next, we check whether we can find an alternative way of fitting the data, including the offset and the bump utilizing the DoG-HIT imaging algorithm. DoG-HIT represents the recovered image by a dictionary of wavelet functions Ψ_i and images at spatial sub-bands as described by Eq. 1. DoG-HIT utilizes a sparsity-constraining approach, effectively defining multiresolution support, i.e., a set of statistically-significant wavelet coefficients to describe the image. As demonstrated in Müller & Lobanov (2023a), the multiresolution support encodes two constraints: the

location of the emission and the spatial scales needed to explain the emission at these locations. The latter relates to the (u, v) coverage, i.e., which spatial scales are measured.

This way, DoG-HIT allows for three alternative approaches to fit the bump and the offset. In the first approach, we investigate whether large spatial scales can account for the observed deviations without a constraint on the emission location or its spatial scale. In the second approach, we focus on localized emission in the ring. In the third approach, we constrain the location and spatial scales. For each approach, we take the ring-only image from DoG-HIT and try to improve the fit quality to the closure phases only, using a gradient descent approach with a small step size.

5.2.1. Fit without spatial scale and localization constraints

First, we do this fit without any mask on the spatial scales or the location of the emission, allowing every pixel to vary (labeled unmasked Figure 8, right column). The unmasked image fits the bump and also shows a much-improved fit to the offset. Moreover, improving the fits in the direction of the closure phases does not violate the match to the amplitudes. The recovered ring image shows no significant variation. However, comparing the unmasked image with the original image, which was masked in spatial scale and localization (Figure 8, left column), on a logarithmic scale, we observe that the improved precision has been achieved by fitting a ‘waffle-like pattern’ associated with the PV-NOEMA and SMT-KP baselines, which is nonphysical.

5.2.2. Fit with localization constraint

Here, we add a constraint on the emission location by performing the same gradient descent fit to the closure phases but only allowing the pixels on the ring to vary, defined by the 1% intensity contour in the DoG-HIT images. The resulting reconstructions and model closure phases are shown in Figure 9.

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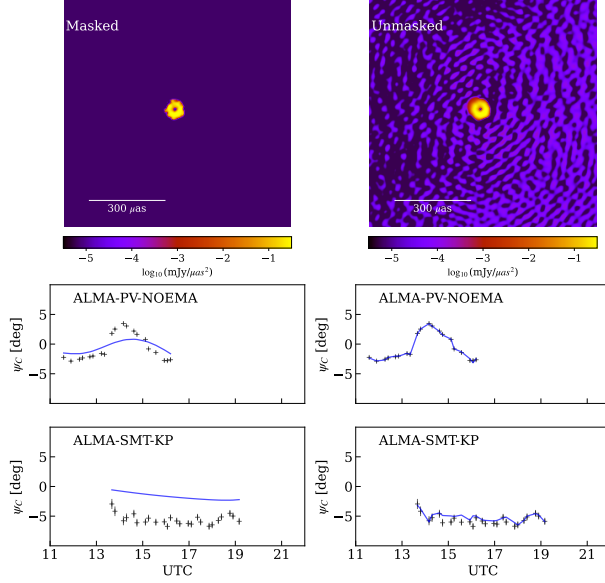


Fig. 8. Reconstruction of the ring and potential jet base emission, when restricted to spatial scales determined by DoG-HIT in the compact emission region (left panels, called masked), and completely unmasked (right panels, called unmasked).

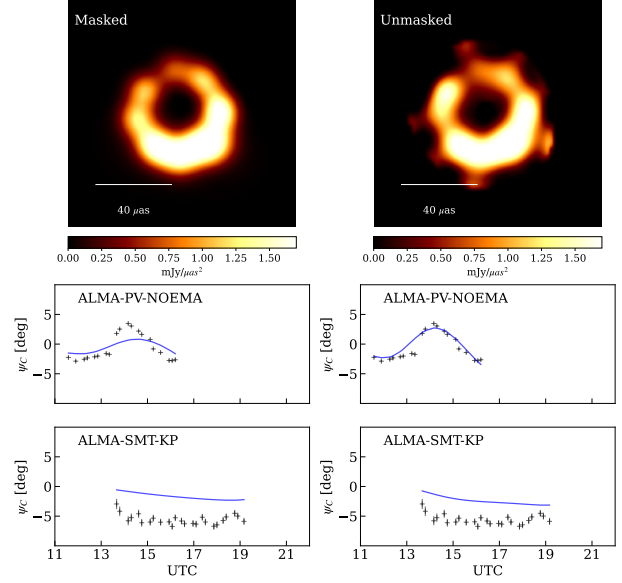


Fig. 9. Reconstruction of the ring and potential jet base emission when restricted to spatial scales determined by DoG-HIT in the compact emission region (left panels, called masked), and when restricted to the ring emission, but flexible in spatial scale (right panels, called unmasked).

In this case, we can fit the bump reasonably well but have issues fitting the offset. A closer inspection of the structures that fit the bump in the top right panel of Figure 9 indicates that this fitting is achieved by asymmetries in the ring itself through a number of components. Each component exhibits structure significantly smaller than the resolution limit of roughly $20 \mu\text{as}$.

5.2.3. Fit with localization and spatial constraints

Finally, we constrain the emission both by its localization and its spatial scale, as imprinted in the wavelet approach of DoG-HIT. The respective fitting quality is shown in the left columns of Figure 8 and Figure 9. Even if we fit for the ALMA-PV-NOEMA and ALMA-KP-SMT closure phases alone with a gradient-descent approach, restricted by the localization and spatial constraints imprinted by the wavelet dictionary, we remain limited in our potential to fit the bump or the offset.

This suggests that the model mismatch results from limiting the imaging to ring-scale emission only. This further supports the view that the bump and the offset are signatures of jet base emission, rather than artifacts from the imaging procedure being imprecise in its representation of the ring.

5.2.4. Data corruptions and systematic errors

Both the bump and offset are formally significant relative to a 0° closure phase, assuming the thermal errorbars are reliable. However, it is likely that the thermal errorbars underestimate the true error, as they do not account for non-closing errors such as polarization leakage (e.g., Broderick & Pesce 2020). Therefore, imaging methods add a fractional systematic Gaussian uncertainty to the errors of $\sim 2\%$ of the visibility amplitudes (EHTC 2019c). In Appendix D we demonstrate that leakage cannot account for the bump or the offset. Nevertheless, we cannot rule out the possibil-

ity that other unknown sources of systematic error may corrupt the data.

5.3. Impact on previous EHT observations

The information about jet base emission has been derived primarily from the KP-SMT and PV-NOEMA baselines in this manuscript. These baselines are unique to the 2021 EHT data, as this was the first year in which the Kitt Peak telescope and NOEMA participated in EHT observations (EHTC 2025). In 2017 and 2018, LMT formed a baseline with the SMT of $\sim 1.3 \text{ G}\lambda$ (compared to $0.8 \text{ G}\lambda$ for PV-NOEMA in 2021), making it the shortest baseline between two stations that are not co-located (ALMA/APEX and SMA/JCMT) during these years. However, LMT did not participate in the 2021 observations. Here, we examine whether the jet base component derived in this manuscript – based on the enhanced coverage of the EHT in 2021 – would have a significant impact on data taken in 2017 and 2018. In other words, does our knowledge of an extended component influence the black hole shadow reconstructions presented in EHTC (2019a, 2024b, 2025)?

In Figure 10, we show the closure phases observed in 2017 (left panel) and 2018 (right panel) with the LMT-SMT baseline (black data points), and their respective fit to the DoG-HIT model of that year presented in EHTC (2025) (red line). The closure phases undergo a swing of almost 80° over the full duration of the observing track. This swing is completely explained by the ring models recovered by DoG-HIT. There is no additional feature visible that would motivate the fitting of an additional component in these years from the LMT-SMT baseline. In addition, we show with a blue dotted line the closure phases of the DoG-HIT model with the additional Gaussian component obtained from our analysis in 2021. In the bottom panels, we show the difference between the predicted closure phases with and without the additional Gaussian component compared to the

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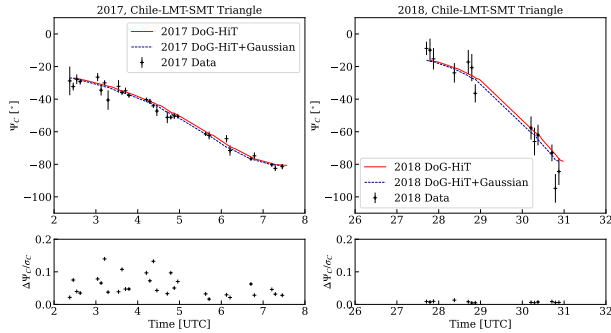


Fig. 10. Closure triangle formed by Chilean stations (ALMA/APEX), LMT, and SMT in 2017 (left panel) and 2018 (right panel). The LMT-SMT baseline is the shortest baseline between non-co-located stations in EHT observations of 2017 and 2018. Black markers: observed closure phases. Red line: fit to the DoG-HIT reconstruction in 2017 (left panel) and 2018 (right panel). Dark blue line: fit to the DoG-HIT reconstruction (ring) with an additional Gaussian model component determined from 2021 data as it would appear in 2017 (left panel) and 2018 (right panel). The line has been offset by a time offset of 0.1 hours to make it visible. In the bottom panels, we show the significance of the difference between the DoG-HIT model and the DoG-HIT model with additional Gaussian component, measured by the difference in Ψ_c divided by the closure phase error. In both years, the effect of the additional Gaussian component on the Chile-LMT-SMT closure phases is an order of magnitude smaller than the respective noise.

noise level. The difference in closure phase introduced by our model on the LMT-SMT baseline is negligible. We conclude that, due to the lack of short baselines, the data obtained in 2017 and 2018 by the EHT are fully fitted by ring-only emission. Additionally, the Gaussian component identified in this work is not detectable on any baseline in the EHT longer than the PV-NOEMA baseline, and thus was not detectable/had no measurable impact on observations conducted in 2017 and 2018. Moreover, the component fitted here is relatively faint, only 60 mJy. Any significantly brighter component would introduce effects in ALMA-PV-NOEMA and ALMA-KP-SMT triangles larger than a few degrees, setting a robust upper limit for the amount of resolved jet emission in EHT observations to date.

6. Discussion

Broderick et al. (2022) found a component roughly $50 \mu\text{as}$ to the south and $50 \mu\text{as}$ to the West of the black hole shadow by modeling a narrow ring component to describe the black hole ring feature in the 2017 EHT observations (EHTC 2019a). This result is consistent with similar findings reported in Arras et al. (2022), Carilli & Thyagarajan (2022), although a robust detection could not be claimed by either study due to the lack of short and intermediate baselines in the 2017 EHT observations.

Broderick et al. (2022) demonstrated that the feature aligns well with the southern arm of the edge brightened jet observed at 86 GHz (Kim et al. 2018) when taking into account the expected core shift between 230 and 86 GHz. Recently Lu et al. (2023) observed the innermost jet in M87* at 86 GHz with the GMVA expanded by ALMA. They found a ring-like feature with an edge-brightened jet base. In Figure 11 we present the EHT results (green-colored contours) on top of the GMVA image (in viridis colormap). To this end, we use the intensity map reported by Kim et al. 2025 (a reanalysis of results reported in Lu et al. 2023) due to its improved resolution. The images

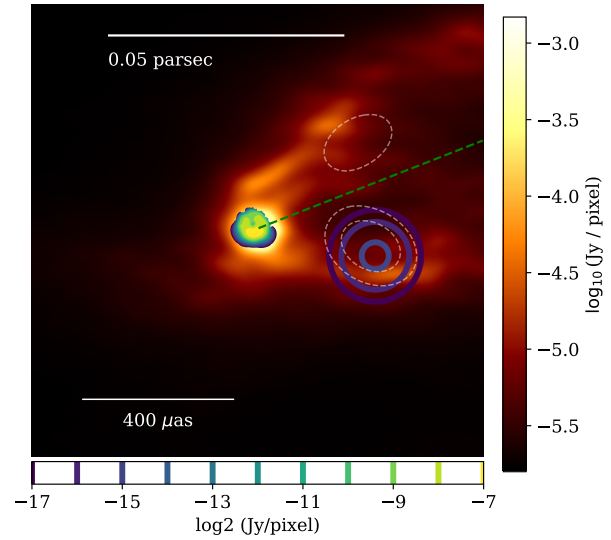


Fig. 11. Fiducial ring structure of the M87* black hole shadow (green contour) with a single Gaussian component (solid blue contours) corresponding to the best-fit parameters found in Section 4.2 (Table 1, first set) and double Gaussian components (dashed contours) corresponding to the best-fit parameters found in Section 4.4 (Table 1, second set), overplotted on the jet in M87* observed at 86 GHz (heatmap) with the GMVA (Lu et al. 2023; Kim et al. 2025). The green dotted line shows the jet orientation.

are stacked by aligning the ring features observed at 230 and 86 GHz. By visual inspection, the jet base Gaussian component studied here aligns to first order with the southern arm of the edge-brightened jet observed with the GMVA. The second Gaussian component in the two-Gaussian model would align with the northern arm, but most likely represents a degeneracy in the posterior.

The jet collimation profile at sub-milliarsecond scales is parabolic. Kim et al. (2018) found a jet width profile $W \propto z^{0.5}$ (where z is the projected distance from the core), also in agreement with observations at milliarsecond scales (Hada et al. 2013, 2016; Hada 2017). This parabolic shape is consistent with a jet driven either by black hole spin via the Blandford-Znajek mechanism (Blandford & Znajek 1977; Nakamura et al. 2018) or by the differential rotation of the accretion flow via the Blandford-Payne mechanism (Blandford & Payne 1982). The preference for a southwestern component in our model over a northwestern component is consistent with a rapidly rotating jet structure near the black hole (Broderick et al. 2022), however the scope for interpretation remains limited due to the limited amount of information. Lu et al. (2023) reported a deviation from the parabolic shape near the core ($\leq 0.2 \text{ mas}$). This profile flattening may be a powerful indicator of the jet launching mechanism, potentially requiring an additional emission component, for example by non-thermal electrons in gravitationally unbound, non-relativistic winds (Nakamura et al. 2018; Park et al. 2019; Lu et al. 2023). While this is an anticipated observable for the EHT and its successors to connect the jet dynamics to the black hole and study jet launching mechanisms (EHTC 2024a), we note that the jet base component found in this study is too far out to make reliable conclusions about the physics of the jet within the first ten Schwarzschild radii.

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7. Conclusions

We used the improved intermediate baseline coverage of the 2021 EHT campaign to search for resolved jet base emission around M87*. We found evidence for a faint offset component on angular scales accessible to the PV-NOEMA and KP-SMT baselines. Fitting a simple geometric model – a single offset Gaussian – yields a flux density of ~ 60 mJy at a location $\approx (-60, -320)$ μ as relative to the ring center. The Gaussian component has $\text{fwhm} \approx 180$ μ as.

Introducing the Gaussian component into the source model reduces the closure phase residuals on the ALMA-PV-NOEMA and ALMA-KP-SMT triangles. For the ALMA-PV-NOEMA triangle the χ^2_ν falls from 2.73 to 1.29, and for the ALMA-KP-SMT triangle from 2.52 to 1.12; the χ^2_ν for all triangles improves from 1.50 to 1.39; and it is improved on every single triangle, albeit the improvement is insignificant for most. This demonstrates that a compact, faint asymmetry at intermediate scales is an economical explanation for the observed closure phase structure in the 2021 data.

The recovered component's position and scale are consistent with the southern/southwestern arm of the edge-brightened jet seen in the 86 GHz GMVA image. Stacking the component over the GMVA jet indicates a preference for an additional flux density of ~ 60 mJy to match the EHT closure phases. However, the data do not statistically favor more complex models (an elliptical Gaussian component or a two-component component model) over the single Gaussian model.

We emphasize two important caveats. First, while our modeling improves the data-minus-model residuals, the limited number of intermediate baselines and the single-baseline sensitivity at the relevant spatial frequencies restrict a robust morphological reconstruction: other mathematically viable structures, including imaging artifacts consistent with the measured (u, v) sampling, cannot be fully excluded. Second, single-baseline or small-triangle measurements are susceptible to non-closing systematics. While we rule out polarization leakage as the origin of the signal and have performed imaging and synthetic data checks, unknown systematic errors cannot be completely ruled out. For these reasons, we conservatively treat the recovered Gaussian flux density as an upper limit on resolved jet base emission at spatial scales probed by the PV-NOEMA and KP-SMT baselines.

Finally, the presence of this faint component does not conflict with earlier EHT reconstructions from 2017, 2018, and 2021. The arrays in earlier years lacked the intermediate baselines required to detect emission at the spatial scales we probe here, so ring-only models adequately describe the earlier epochs. Excellent fitting quality, synthetic data tests, and the negligible effect of the Gaussian model on baselines longer than PV-NOEMA also indicate the robustness of the ring-only images obtained in 2021. The detection (or upper limit) reported here therefore complements earlier results and constrains where the EHT missing flux cannot reside: resolved, brighter emission ~ 100 – 300 μ as southeast and northeast relative to the ring is firmly ruled out by these closure triangles.

In summary, the 2021 EHT data provide the first constraints at 230 GHz on faint, asymmetric emission at intermediate scales near M87*, locating a plausible component southwest of the ring with a flux density < 0.1 Jy. Although this result is robust under the assumptions and tests performed, definitive confirmation and more precise constraints will require future EHT observations with higher sensitivity and improved intermediate-baseline coverage via additional stations and expanded frequency range.

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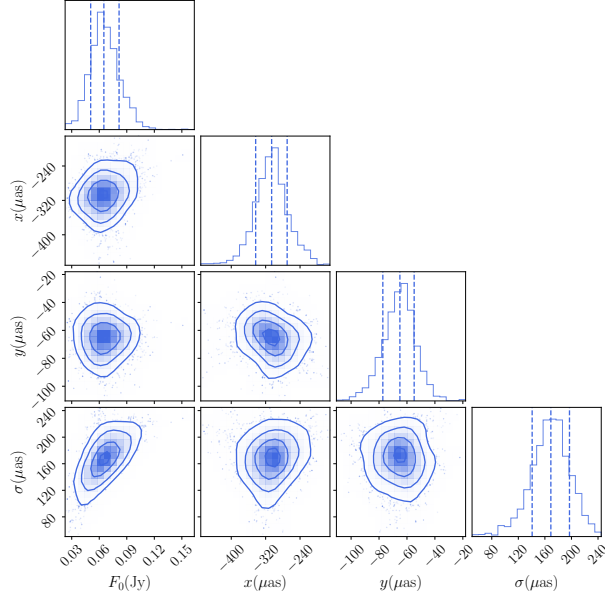


Fig. A.1. Corner plot showcasing our estimates for the location (x, y in units of μas), flux (F_0 in units of Jy), and width (σ in units of μas) of the Gaussian toy model.

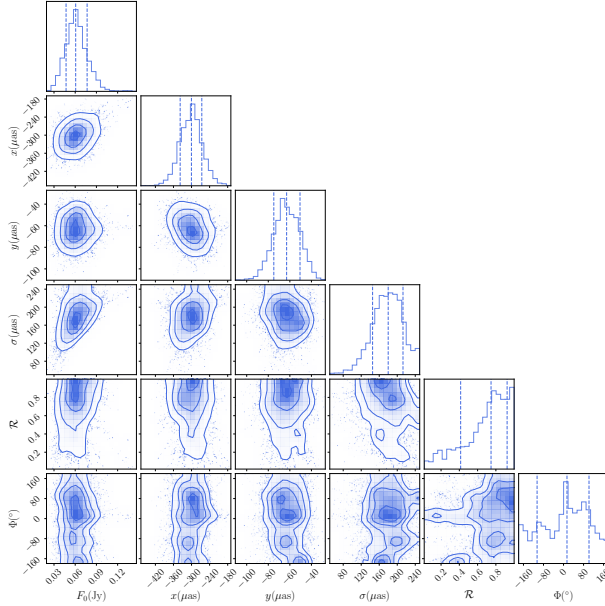


Fig. A.2. Same as Figure A.1 but with two additional parameters: R , the ratio of fwhm_{maj} and fwhm_{min} , and Φ , the rotation angle, of the asymmetric Gaussian toy model. See Section 4.3 for more details.

Appendix A: Corner plots

In the following, we show the MCMC corner plots of the three model fits that we performed. Figure A.1 displays the corner plot of the four-parameter Gaussian model, Figure A.2 displays the corner plot of the six-parameter Gaussian model, and Figure A.3 displays the fit with a twelve-parameter double Gaussian model.

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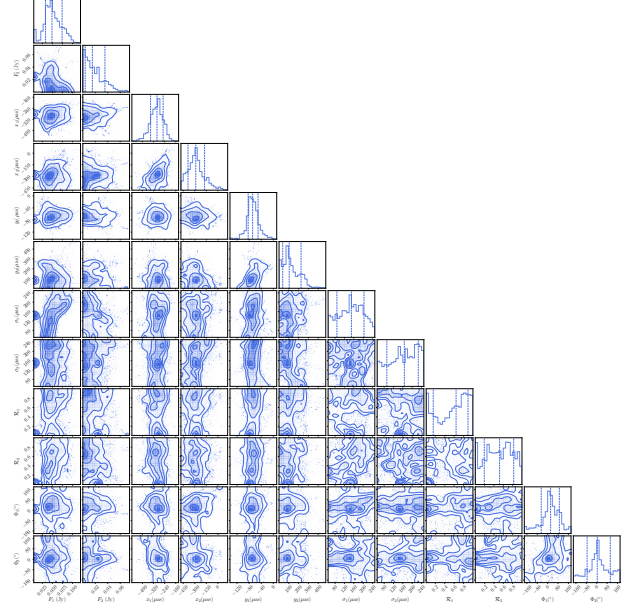


Fig. A.3. Same as Figure A.1 but for two asymmetric Gaussian toy models. See Section 4.4 for more details.

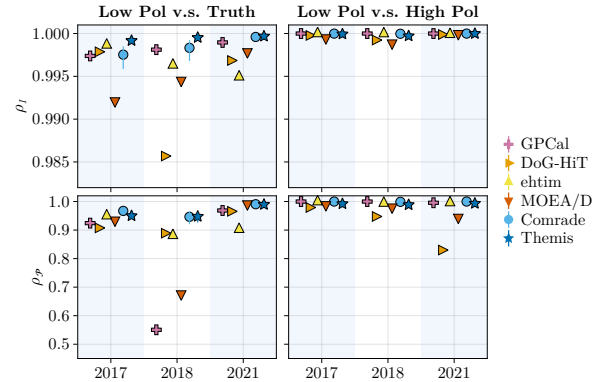


Fig. B.1. Figure D3 of EHTC (2025). Cross-correlation between the recovered ring model and synthetic ground truth model for seven different imaging techniques in 2017, 2018, and 2021. The synthetic data included a large-scale jet component. Upper panels show the cross-correlation in total intensity, lower panels in polarization. Despite the synthetic data containing a large scale jet component, the black hole shadow has been recovered for all methods and all years with exceptional accuracy, achieving cross-correlations > 0.995 in total intensity in 2021.

Appendix B: Effect on visibility amplitude

In Figure B.1 we show the evaluation of the synthetic data test performed with an extended jet component in EHTC (2025). In Figure B.2 we show the amplitudes of the ring model, the fiducial model, and the difference between the two models as a function of the projected baseline length.

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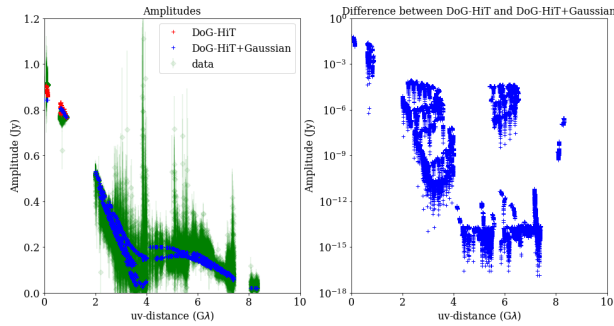


Fig. B.2. Left panel: comparison of the predicted amplitudes for the DoG-HIT ring-only image (red markers), the DoG-HIT image with additional Gaussian component (blue markers), and the self-calibrated visibility amplitudes (green markers) as a function of projected baseline length. Right panel: the difference between the predicted visibilities for the DoG-HIT model and DoG-HIT with the additional Gaussian component as a function of projected baseline length. The plot demonstrates that the additional Gaussian component has no sizeable effect on baselines longer than PV-NOEMA.

Appendix C: Parameter exploration for 6 parameter Gaussian model

We demonstrate the effect of adding two additional degrees of freedom to the model with an asymmetric Gaussian component that can be rotated around its axis. In Figure B.3 we show that these parameters can be constrained in the extreme case of a very asymmetric Gaussian component. However, these parameters are poorly constrained.

Appendix D: Impact of non-closing errors

Because the phase signature of the offset Gaussian component(s) is small, we cannot completely rule out systematic effects as an origin. Here, we assess the impact of one known non-closing error: polarization leakage. Polarization leakage causes signal dispersion from one correlation of polarization handedness (e.g., RR) to another (e.g., LL), and is baseline-based in nature (e.g., Müller 2024). One data-driven test to rule out leakage as the cause of the bump and offset is to compare the phase-residuals of the $RR - LL$ closure phases on the respective triangles (Figure ??). Under the assumptions that there is no significant leakage and that circular polarization is $\rightarrow 0$, the difference between the parallel hands closure phases is zero. While there is a small offset on the ALMA-PV-NOEMA triangle, neither the bump nor the offset can be explained by a difference between the parallel hands, and we thus conclude that the signal is not caused by polarization leakage. The difference in the ALMA-PV-NOEMA triangle may be caused by either intrinsic circular polarization of the source or by leakage, and we thus place a limit on this type of non-closing error of $\sim 1^\circ$.

Appendix E: Acknowledgments

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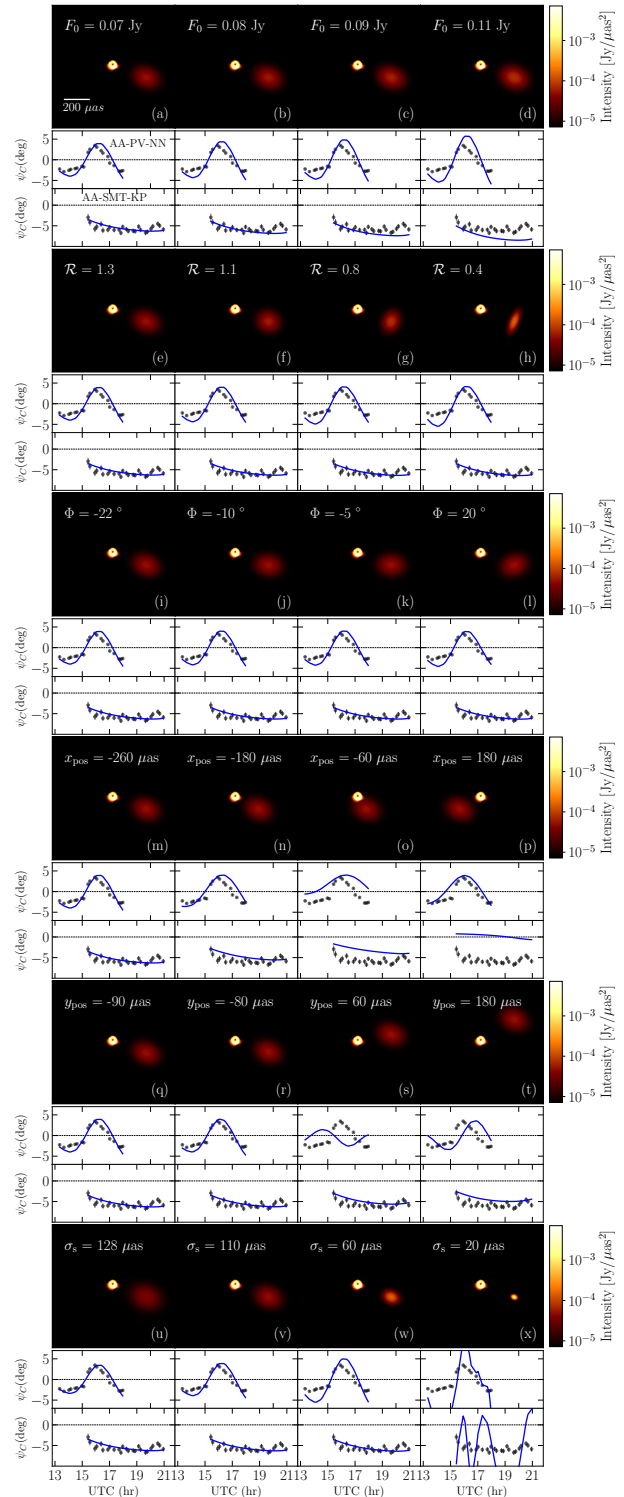


Fig. B.3. Same as Figure 3, but with asymmetry and position angle as additional parameters.

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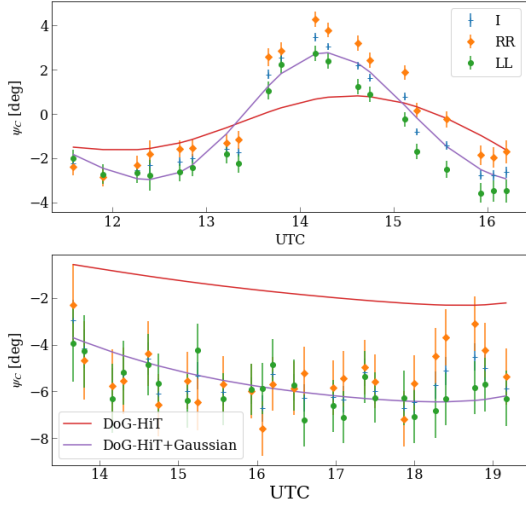


Fig. D.1. Comparison of the *RR* (orange) and *LL* (green) closure phases on the ALMA-NOEMA-PV (top) and ALMA-KP-SMT triangles (bottom). The red line indicates the DoG-HIT ring model, the magenta line indicates the DoG-HIT model with an additional Gaussian component.

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106-2112-M-001-011, 106-2119-M-001-013, 106-2119-M-001-027, 106-2923-M-001-005, 107-2119-M-001-017, 107-2119-M-001-020, 107-2119-M-001-041, 107-2119-M-110-005, 107-2923-M-001-009, 108-2112-M-001-048, 108-2112-M-001-051, 108-2923-M-001-002, 109-2112-M-001-025, 109-2124-M-001-005, 109-2923-M-001-001, 110-2112-M-001-033, 110-2124-M-001-007 and 110-2923-M-001-001); the National Science and Technology Council (NSTC) of Taiwan (111-2124-M-001-005, 112-2124-M-001-014, 112-2112-M-003-010-MY3, and 113-2124-M-001-008); the Ministry of Education (MoE) of Taiwan Yushan Young Scholar Program; the Physics Division, National Center for Theoretical Sciences of Taiwan; the National Aeronautics and Space Administration (NASA, Fermi Guest Investigator grant 80NSSC23K1508, NASA Astrophysics Theory Program grant 80NSSC20K0527, NASA NuSTAR award 80NSSC20K0645); NASA Hubble Fellowship Program Einstein Fellowship; NASA Hubble Fellowship grants HST-HF2-51431.001-A, HST-HF2-51482.001-A, HST-HF2-51539.001-A, HST-HF2-51552.001A awarded by the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., for NASA, under contract NAS5-26555; the National Institute of Natural Sciences (NINS) of Japan; the National Key Research and Development Program of China (grant 2016YFA0400704, 2017YFA0402703, 2016YFA0400702); the National Science and Technology Council (NSTC, grants NSTC 111-2112-M-001 -041, NSTC 111-2124-M-001-005, NSTC 112-2124-M-001-014); the US National Science Foundation (NSF, grants AST-0096454, AST-0352953, AST-0521233, AST-0705062, AST-0905844, AST-0922984, AST-1126433, OIA-1126433, AST-1140030, DGE-1144085, AST-1207704, AST-1207730, AST-1207752, MRI-1228509, OPP-1248097, AST-1310896, AST-1440254, AST-1555365, AST-1614868, AST-1615796, AST-1715061, AST-1716327, AST-1726637, OISE-1743747, AST-1743747, AST-1816420, AST-1935980, AST-1952099, AST-2034306, AST-2205908, AST-2307887, AST-2407810); NSF Astronomy and Astrophysics Postdoctoral Fellowship (AST-1903847); the Natural Science Foundation of China (grants 11650110427, 10625314, 11721303, 11725312, 11873028, 11933007, 11991052, 11991053, 12192220, 12192223, 12273022, 12325302, 12303021); the Natural Sciences and Engineering Research Council of Canada (NSERC); the National Research Foundation of Korea (the Global PhD Fellowship Grant: grants NRF-2015H1A2A1033752; the Korea Research Fellowship Program: NRF-2015H1D3A1066561; Brain Pool Program: RS-2024-00407499; Basic Research Support Grant 2019R1F1A1059721, 2021R1A6A3A01086420, 2022R1C1C1005255, RS-2022-NR071771, 2022R1F1A1075115); the POSCO Science Fellowship of the POSCO TJ Park Foundation; NOIRLab, which is managed by the Association of Universities for Research in Astronomy (AURA) under a cooperative agreement with the National Science Foundation; Onsala Space Observatory (OSO) national infrastructure, for the provisioning of its facilities/observational support (OSO receives funding through the Swedish Research Council under grant 2017-00648); the Perimeter Institute for Theoretical Physics (research at Perimeter Institute is supported by the Government of Canada through the Department of Innovation, Science and Economic Development and by the Province of Ontario through the Ministry of Research, Innovation and Science); the Portuguese Foundation for Science and Technology (FCT) grants (Individual CEEC program – 5th edition, CIDMA through the FCT Multi-Annual Financing Program for R&D Units UID/04106,

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Chapter 9

Conclusion and Next Steps

The Event Horizon Telescope observations of M87* at 230 GHz and complementary GMVA campaigns at 86 GHz now resolve emission on scales where the spacetime curvature, the plasma physics, and the magnetic field geometry are deeply entangled. Interpreting what these observations reveal about the central object and its immediate environment requires forward models that can connect the observed intensity and polarization structure to the underlying accretion flow, field configuration, and spacetime. This thesis employs a range of modeling tools, from semi-analytic models of radiatively inefficient accretion flows through three-dimensional GRMHD simulations. General relativistic ray-traced radiative transfer (GRRT) and interferometric data analysis is then applied to M87* across multiple observing frequencies and epochs; where the foundational framework is explained in Part I. Two complementary lines of exploration run through the chapters in Part-II. The first operates within the black hole paradigm, extracting constraints on the magnetic field geometry, disk dynamics, and jet-base emission from EHT data. The second probes the boundaries of that paradigm by constructing and simulating accretion onto a *physically well-motivated* compact object.

9.1 Summary

Each chapter in Part I closed with a set of open questions identifying the specific problems this thesis takes up. In the following, Part I is summarized through those questions, and Part II is organized by the exploration of those questions.

Part I: Foundations

- Chapter 2 reviewed and derived the theoretical foundations of motion of light and matter in curved spacetimes. The geodesic and lensing proper-

ties of the Schwarzschild, Kerr, and JMN-1 spacetimes were developed in parallel. Unstable circular photon orbits define the photon sphere (or photon shell in the case of rotating spacetime), and the set of null geodesics asymptotically captured by these orbits delineates the shadow boundary as seen by a distant observer. The innermost stable circular orbit (ISCO) sets the inner edge of the accretion flow for timelike geodesics, and its radius depends sensitively on the spin parameter a_* in the Kerr geometry. The JMN-1 spacetime, for the parameter range $M_0 \in [2/3, 1)$, shares the photon orbit radius with a Schwarzschild black hole of the same mass, producing a shadow boundary indistinguishable from Schwarzschild through lensing alone. This complete degeneracy cannot be broken by shadow observations at current EHT resolution and requires instead probing the accretion flow and its emission properties. Two open questions closed this chapter, taken up directly in Part II: *Is there a reasonable alternative of the black hole and can these be the central compact object in AGNs?* (addressed in Chapters 6 and 7), and *What is the spin of the compact object in M87*?* (addressed in Chapter 5).

- Chapter 3 built the astrophysical layer on top of the spacetime geometry which requires theoretical framework for accretion depending on the physical regime the flow occupies: the Novikov-Thorne thin disk, the semi-analytic RIAF model, and full GRMHD simulations. The polarized synchrotron radiative transfer formalism was developed and connected to synthetic polarized images through backward ray-tracing with `ipole`. Three degeneracies were identified that observations must break: between magnetic field topology and disk geometry, between MAD and SANE accretion states, and between black hole and horizonless spacetimes when both are surrounded by the same RIAF emission geometry.
- Chapter 4 developed the observational machinery from the van Cittert-Zernike theorem through to the full measurement equation, closure quantities, and the `eht-imaging` RML framework. The multi-epoch EHT observations of M87* from 2009 to 2021 were reviewed, and the open questions identified where the comparison between models and data is most pressing: the disk-jet transition region at horizon scales remained unprobed by the 2017 EHT campaign, which lacked the intermediate baselines needed to detect emission between the compact ring at $\sim 40 \mu\text{as}$ and the large-scale jet at $\lesssim 1 \text{ mas}$.

Part II: Research Results

The four research chapters each take up one or more of the questions raised in Part I:

- **Chapter 5** takes up the magnetic field geometry and spin degeneracy identified in Chapters 2 and 3. A seven-parameter semi-analytic RIAF model in Kerr spacetime was compared against the 2017 EHT total intensity and polarimetric data of M87*. Poloidal magnetic field configurations with partial radial inflow are strongly preferred over purely toroidal fields, and the two topologies produce distinct linear polarization morphologies distinguishable at EHT resolution. The geometric thickness of the accretion flow has a limited effect on the 230 GHz image morphology, indicating that the ring diameter and asymmetry are governed primarily by the spacetime geometry and the radial emission profile rather than the vertical disk structure. The spin-field topology degeneracy, however, persists: different combinations of spin magnitude and field geometry can reproduce the observed ring diameter, flux, and polarization fraction simultaneously. Breaking this degeneracy requires additional constraints from polarimetric imaging at multiple frequencies or from independent jet-power estimates.
- **Chapters 6 and 7** together address the compact object question of Chapter 2, progressing from prescribed to self-consistent accretion. Chapter 6 applied the RIAF imaging framework to the JMN-1 spacetime. At the angular resolution and sensitivity of the current EHT, the JMN-1 and Schwarzschild B images are indistinguishable: both produce a ring of diameter $\sim 42 \mu\text{as}$ with a brightness asymmetry set by Doppler beaming, consistent with the published M87* images. The sole discriminant is a central brightness excess in the JMN-1 images, produced by photons that traverse the region interior to the would-be horizon and escape to infinity. This excess is unresolved by the current EHT array, and resolving it requires baselines exceeding $\sim 15 G\lambda$, corresponding to the angular resolution anticipated from the ngEHT or space VLBI missions.

Chapter 7 replaced the semi-analytic prescription with the first three-dimensional GRMHD simulation of sustained accretion onto a JMN-1 null singularity, using *KHARMA* modified for non-Kerr spacetimes. The configuration $R_b = 2.8 r_g$ was chosen for its status as the *best mimicker*: its photon sphere coincides with Schwarzschild, yielding the same shadow/apparent boundary as well. The simulation, initiated with a FM torus and a magnetic field configuration, reaches a sustained MAD state with normalized magnetic flux

$\phi_c \sim 15$ and a time-averaged accretion rate $\dot{M} \approx (3.0 \pm 0.5) \times 10^{-6} \dot{M}_{\text{Edd}}$, suppressed by at most a factor $\lesssim 2$ relative to the Schwarzschild reference and fully consistent with the MAD Kerr models in the EHT library. The JMN-1 singularity acts as an efficient energy sink analogous to the event horizon, permitting the advection-dominated, radiatively inefficient flow expected in M87*. Polarized ray-traced images at 230 GHz computed with `ipole` reproduce the ring morphology and brightness asymmetry of the EHT observations in linear scale. In logarithmic scale, however, brightness extends inward below the inner shadow $\ell_{\text{is}} \approx 2.85 r_g / D_s$, reaching a few percent of the peak at roughly half the inner shadow radius, a level within the projected dynamic range of the ngEHT and BHEX. Any future detection or non-detection of this feature therefore tests the *hard core* (the black hole paradigm), the presence or absence of a horizon directly.

- **Chapter 8** addresses the jet-launching and disk-jet transition question raised in Chapters 2 and 4. The 2021 EHT observations of M87*, which include the new KP-SMT and PV-NOEMA baselines, were analyzed to search for extended emission beyond the compact ring. A hyperparameter survey of RML imaging with `eht-imaging` was performed for producing polarized images of M87* EHT 2017-2021 datasets. Closure phase analysis on the NOEMA-KP-SMT triangle reveals a systematic deviation from the signature of a symmetric ring, consistent with the presence of a faint emission component offset from the ring center. Modeling this excess as a Gaussian component yields a flux density of ~ 60 mJy at a projected separation of $\sim 320 \mu\text{as}$ (~ 5500 AU) from the core, aligned with the position angle of the large-scale jet observed at 86 GHz with the GMVA. This provides an upper limit on the 230 GHz flux density of the jet base within the inner $\sim 200 \mu\text{as}$, constraining the electron density, temperature, and bulk Lorentz factor of the emitting jet sheath at horizon scales.

Together, these results trace the boundary of what horizon-scale precision currently delivers and what it does not. This further illustrated what kind of observations will be required to for achieving precision in answering the questions.

This thesis opened with a question drawn from the history of scientific thought: *what kind of thought should we be engaging in?* The work presented here has operated in both modes that question implies. The first is the essential work of *normal science* within the relativistic paradigm: constraining the magnetic field topology and flow kinematics in M87* with increasing precision, locating the jet base at horizon scales, and extending the EHT’s imaging methodology to new baseline configurations and reconstruction strategies. The second is the critical

work of probing the limits of the paradigm itself: taking a competing research programme within GR, one that replaces the event horizon with a horizonless singularity while leaving Einstein’s field equations intact, and subjecting it to the same observational and computational scrutiny applied to the BH solution. The two modes are, as Kuhn and Lakatos recognized in different ways, complementary aspects of a mature scientific enterprise. A paradigm tested only at its center, where its predictions are most secure, learns nothing about its own boundaries. The instruments now exist to reach those boundaries. What this thesis has done is prepare the discriminants, the observational targets, and the computational tools needed for the data to answer the question the paradigm cannot answer about itself.

9.2 Current and Future Work

Two ongoing investigations extend the lines of inquiry developed in this thesis into new observational and theoretical territory.

Polarized hotspots as probes of gravitational physics

Near-infrared observations of Sgr A* with the GRAVITY instrument have revealed polarized flares consistent with compact plasma features, hotspots, orbiting the central object at radii of $\sim 6\text{--}12 r_g$ on timescales of $\sim 30\text{--}70$ minutes (GRAVITY Collaboration 2018; Gravity Collaboration 2023). Complementary millimeter-wavelength polarimetric monitoring with ALMA has detected $\mathcal{Q} - \mathcal{U}$ loops, periodic patterns in the Stokes $\mathcal{Q} - \mathcal{U}$ plane, simultaneous with these orbital signatures and with a consistent period (Wielgus et al. 2022). These loops encode the phase-resolved evolution of the linear polarization vector as a hotspot completes its orbit, and their morphology is shaped by the combined effects of relativistic beaming, gravitational lensing, and the parallel transport of the polarization vector along curved null geodesics (Vincent et al. 2024).

This dependence on photon propagation makes Q-U loops a direct probe of the background spacetime geometry, independent of the emission model details that dominate total intensity observations. In ongoing work (Saurabh et al., in prep.), we perform the first systematic comparison of time-dependent, polarized hotspot emission across a suite of static, spherically symmetric metrics spanning charged black holes (Reissner-Nordström, GMGHS), regular black holes (Bardeen, Hayward), and naked singularities (JMN-1, JNW). Using fully general relativistic polarized radiative transfer with the `ipole` code, we extract Stokes- I light curves, linear and circular polarization profiles, flux-weighted astrometric centroid tracks,

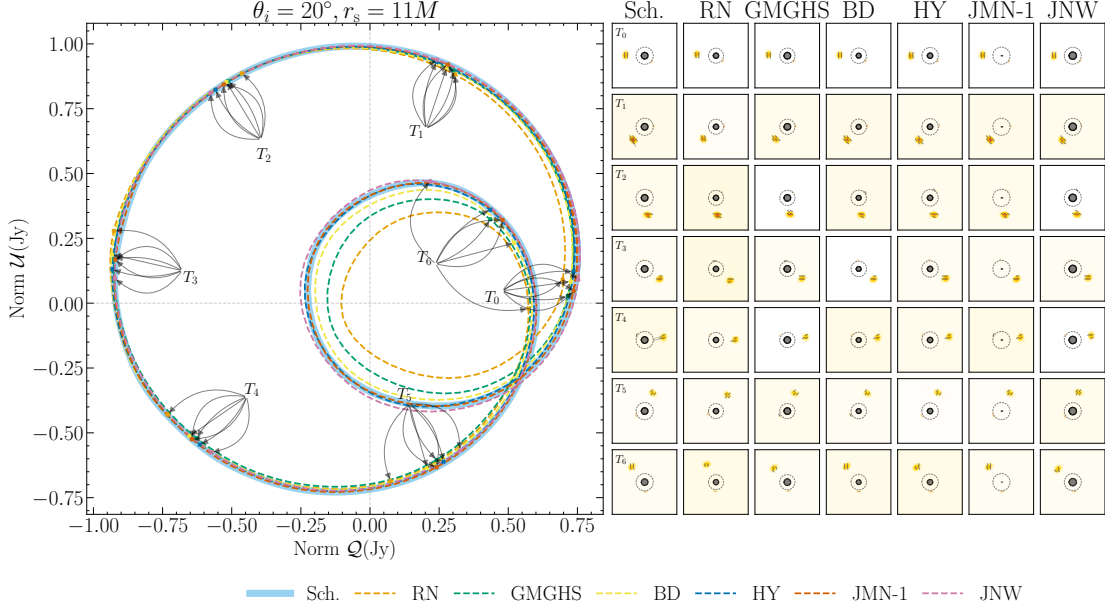


Figure 9.1: Normalized $\mathcal{Q} - \mathcal{U}$ loops for polarized hotspot emission orbiting at $r_{\text{hs}} = 11M$ with observer inclination $\theta_o = 20^\circ$, computed across a suite of static, spherically symmetric spacetimes using general relativistic polarized radiative transfer with `ipole`. All black hole solutions (Schwarzschild, Reissner-Nordström, GMGHS, Bardeen, Hayward) and the JMN-1 naked singularity form a tightly overlapping family, while the JNW naked singularity produces an elongated, off-center loop enclosing $\sim 30\text{--}50\%$ more area, offering a testable polarimetric discriminant with current ALMA and future GRAVITY+ monitoring of Sgr A* flares. Figure from Saurabh et al. (in prep.).

and Normalized $\mathcal{Q} - \mathcal{U}$ loops for each spacetime across a range of orbital radii and observer inclinations (see Figure 9.1).

The results indicate that, for compact orbital radii and low to moderate inclinations, the $\mathcal{Q} - \mathcal{U}$ loop morphologies of all black hole solutions and the JMN-1 horizonless singularity are mutually degenerate, forming a tightly overlapping family. Certain naked singularity spacetimes, however, depart from this family, producing elongated and displaced loops with measurably larger enclosed area. The astrometric centroid tracks show corresponding phase lags and positional offsets that fall within the projected precision of GRAVITY+. These polarimetric and astrometric anomalies are testable with current or near-future sub-mm polarimetry and infrared interferometry, offering an independent and complementary avenue to the imaging-based tests pursued in Part II of this thesis.

Multi-frequency VLBI constraints on the M87 jet base

The jet-base detection reported in Chapter 8 provides a single-epoch, single-frequency constraint on the innermost jet emission. How this emission connects to the accretion flow and how it evolves across observing frequencies remain open

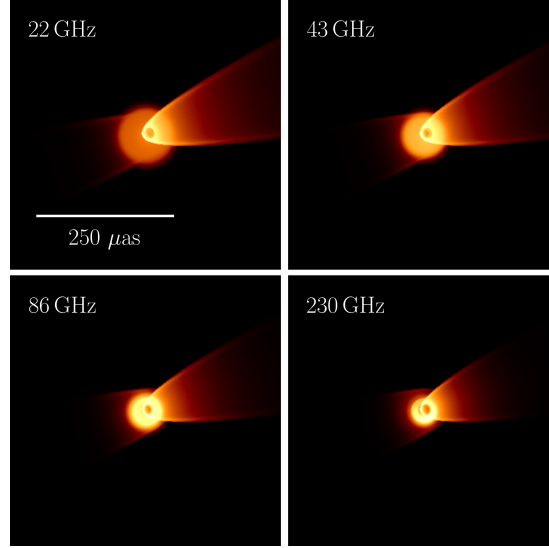


Figure 9.2: Synthetic synchrotron images of the fiducial semi-analytic accretion-jet model of M87* at 22, 43, 86, and 230 GHz (left to right), generated with `ipole` in Kerr spacetime. Lower frequencies probe the extended, parabolically collimated jet sheath on scales of ~ 0.2 – 1 mas, while 230 GHz isolates the compact ring and the innermost jet base within $\sim 100 \mu\text{as}$. The combined multi-frequency constraints collapse the seven-dimensional parameter space to a compact region favoring hot, dilute, mildly relativistic jet sheaths with parabolic collimation. Figure from Müller, Saurabh et al. (in prep.).

questions. In a companion study (Müller, Saurabh et al., in prep.), we address this by performing a seven-dimensional parameter sweep of the semi-analytic accretion-jet model described in Chapter 5, now extended to include a parabolically collimated jet sheath with independent density, temperature, gradient, and bulk Lorentz factor parameters. Synthetic synchrotron images are generated at 22, 43, 86, and 230 GHz using `ipole` and compared simultaneously against transverse jet-width measurements from cm-mm VLBI, the EHT shadow observables (ring diameter, flux, asymmetry), multi-epoch upper limits on unresolved jet emission on intermediate EHT baselines, and the jet-counterjet flux ratio reported at 22–86 GHz (see Figure 9.2).

The intersection of all constraints collapses the seven-dimensional parameter space to a compact cluster favoring parabolically collimated, hot, dilute jet sheaths with mildly relativistic bulk motion. No single observable would yield this conclusion in isolation: the jet geometry is pinned by cm-mm widths, the thermodynamics by mm-flux limits, the kinematics by the counterjet ratio, and the compact structure by EHT shadow properties. The convergence of independent constraints toward a single region suggests that current multi-frequency VLBI data already capture the dominant physical ingredients of the M87 jet base.

9.3 Some Philosophical Reflections

The discriminants and tools prepared in this thesis will meet their sharpest tests in the coming decade. The ngEHT will add ~ 10 new stations to the array, delivering the short-baseline coverage and imaging dynamic range needed to detect or exclude several black hole alternatives (Johnson et al. 2023; Doeleman et al. 2023). The Black Hole Explorer (BHEX), a space VLBI mission operating at 230 and 345 GHz, will resolve the photon ring substructure and probe the inner shadow on baselines inaccessible from the ground (Johnson et al. 2024). GRAVITY+ will improve astrometric precision on Sgr A* flares by a factor of several, bringing the $\mathcal{Q}$ - $\mathcal{U}$ loop morphology into the regime where charged and regular black hole deviations become testable (GRAVITY+ Collaboration, Abuter, et al. 2022). At longer wavelengths, the Square Kilometre Array (SKA; Dewdney et al. 2009) and the next-generation Very Large Array (ngVLA; Murphy 2018) will map jet collimation and magnetic field structure from parsec scales down to the jet launching region. The Laser Interferometer Space Antenna (LISA; Amaro-Seoane 2017) will open the millihertz gravitational wave band, probing the spacetime geometry of massive compact objects through inspiral waveforms that carry independent information about the presence or absence of an event horizon (Cardoso et al. 2016). In the optical and infrared, the Extremely Large Telescope (ELT; Gilmozzi and Spyromilio 2007) and the Thirty Meter Telescope (TMT; Skidmore and TMT International Science Development Teams 2015) will track stellar orbits at the Galactic center with precisions that constrain the extended mass distribution and test for deviations from the Kerr quadrupole moment.

Each of these instruments probes a different aspect of the same underlying question this thesis has taken up. Their combined reach will transform the discriminants identified here from theoretical targets into observational verdicts. The data, however, do not interpret themselves. Every measurement of a shadow diameter, every upper limit on inner-shadow brightness, every $\mathcal{Q}$ - $\mathcal{U}$ loop extracted from a flare, arrives embedded in a web of modeling choices, prior assumptions, and theoretical commitments that determine what it is taken to mean. The question of whether our interpretive structures are adequate to what the instruments will deliver is not a question the instruments can answer. It belongs to a different register of inquiry.

The motivation of this thesis (Section 1.1) traced a recurring pattern in the history of physics. The moments of sharpest advance were not moments of better measurement alone. They were moments at which the terms of the inquiry itself changed. The Leibniz-Clarke correspondence questioned whether space is a thing or a relation. Riemann generalized geometry beyond Euclid. Mach stripped

away absolute motion. Einstein made spacetime dynamical. In each case, the conceptual shift preceded or accompanied the technical work that gave it predictive power. The same pattern extends beyond gravity. Heisenberg’s 1925 paper, the starting point of matrix mechanics, opens not with a calculation but with a philosophical declaration: the aim is to construct a theory “*based exclusively on relations between quantities that are in principle observable*” (Heisenberg 1925). That sentence came before the mathematics. The commitment to observability, drawn from the intellectual atmosphere of Göttingen and the Vienna Circle, gave Heisenberg the license to abandon the classical trajectory, something no amount of spectroscopic data would have compelled on its own¹.

Einstein was unusually direct about this dependence. His route to general relativity drew not on new astronomical data but on a decade-long struggle with the internal contradictions between Newtonian gravity and special relativity, guided by the relational thinking of Leibniz, the empiricist critique of Mach, and pervasive influence by Schopenhauer (Howard 1997). Late in life, he put the point plainly (Einstein 1944):

“A knowledge of the historic and philosophical background gives that kind of independence from prejudices of his generation from which most scientists are suffering. This independence created by philosophical insight is—in my opinion—the mark of distinction between a mere artisan or specialist and a real seeker after truth.”

A physicist who treats the reigning methodology as self-evident and the foundational commitments as closed has already given up the possibility of noticing when those commitments need to change. But the opposite error is equally costly: treating all unfalsified possibilities as equally worthy of pursuit, as though a Kuhnian revolution might lurk behind any untested hypothesis, leads to arbitrary exploration of unbounded possibility space with negligible chance of success. The history of the discipline points to a narrower path between these two failures. The most radical conceptual shifts that actually worked were strictly motivated, almost forced, either by the overwhelming weight of new data or by a careful analysis of internal contradictions within existing, successful theories (Rovelli 2018). Kepler did not invent ellipses from speculation; the data of Mars forced them upon him. Copernicus did not rely on new observations; he traced unexplained coincidences in Ptolemaic astronomy to their geometric origin. Einstein’s general

¹Einstein, who had drawn on a similar positivist impulse to reach special relativity, later distanced himself from strict positivism, remarking to Heisenberg that “it is the theory which decides what we can observe” (Heisenberg 1971). The circularity is not a defect. It is what makes the relationship between philosophy and physics productive: each constrains and reshapes the other.

relativity was not a leap into the unknown; it was, as has been rightly observed, “insightful conservatism”. It was this realization that the principle of relativity, already successful for uniform motion, demanded extension to accelerated frames, and that the resulting theory had to account for the long-known coincidence between inertial and gravitational mass. In each case, science advanced through continuity with what was already known, not through discontinuity from it.

The work of this thesis required a specific instance of this kind of examination. The question was: does the commitment to the Kerr metric in modeling M87* function as an empirically tested result, or as an organizing assumption that shapes what we look for and how we interpret what we find? Although, as it was mentioned, it was due to the success of the theory and its application to the observations, which led them as *foundational*. The Lakatosian framework mentioned in Chapter 1 provided the vocabulary to answer: it functions as the hard core of a research programme, surrounded by a protective belt of accretion models, field configurations, electron distribution functions, and imaging assumptions. Refining the belt is *normal science* (Chapters 5 and 8). Testing the core requires constructing a competing programme that shares enough structure with the original to be testable by the same data (Chapters 6 and 7). The fact that the formation and evolution of *supermassive* compact object at the center of AGNs is indeed an open question, leads to these explorations. Without the Lakatosian distinction, the JMN-1 results would read as one more comparison between exotic compact object images and EHT data. With it, the central brightness excess acquires an important meaning: it is a prediction that the protective belt cannot reproduce, and its detection or non-detection therefore bears on the hard core, the presence or absence of a horizon, rather than on the auxiliary assumptions.

This is not an abstract methodological point. It has immediate practical consequences for how horizon-scale data are analyzed. The choice of imaging algorithm, the regularizers imposed on the reconstruction, the spacetime assumed in the forward model, the electron distribution function adopted in the radiative transfer, each of these encodes assumptions that constrain what the data can say (Galison et al. 2023). The question of what constitutes a detection of an *event horizon* depends on all of them. If the forward model assumes Kerr, then consistency with Kerr is the only possible outcome, leading to *theory-ladenness* (Doboszewski and Elder 2025). The capacity to ask whether an alternative spacetime could equally well reproduce the data requires building the alternative into the analysis pipeline from the start; the approach taken in Part II of this thesis. These are philosophical questions in a precise, operative sense: *they determine what counts as evidence*.

The study of the cosmos, as the motivation observed, has always oscillated be-

tween two impulses: the need to comprehend our place within the Universe, and the drive to construct a predictive mathematical model of it. The most productive episodes in the history of physics are those in which the two impulses reinforced each other: when the mathematical model raised questions about its own foundations that only a broader, more reflective mode of inquiry could address. We are at such a moment now. The instruments exist to reach the boundaries of the black hole paradigm. The simulations and the ray-tracing codes can generate the predictions. The imaging pipelines can extract the observables. What remains, and what no instrument can provide, is the clarity to recognize what the data mean when they arrive, and the willingness to follow them even if they lead away from what the paradigm has taught us to expect. The cosmos does not organize its answers according to our theoretical commitments. It answers the questions we have the conceptual apparatus to ask. The quality of that apparatus will determine whether we hear what it says.

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तद्विद्धि प्रणिपातेन परिप्रश्नेन सेवया।
उपदेक्ष्यन्ति ते ज्ञानं ज्ञानिनस्तत्त्वदर्शिनः॥ ४.३४॥

*tad viddhi praṇipātena paripraśnena sevayā
upadekṣhyanti te jñāna jñāninas tattva-darśinaḥ (4.34)*

This verse from the Bhagavad Gita (4.34) speaks of approaching those who see the truth with humility, thoughtful questions, and a spirit of service; in return, they impart knowledge and insight. It captures, in just a few words, what a PhD truly is: not a solitary achievement, but a journey made possible by teachers, mentors, colleagues, friends, and family who share their time, patience, and understanding. It feels only appropriate to begin these acknowledgments by recognizing that whatever I have learned in these years has been received, not achieved alone.

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I would like to express my sincere gratitude to Prof. Zensus, for providing me the freedom to work independently while also creating an environment in which one could think, ask, and provide invaluable opportunities for constant learning and growth.

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Now I would like to dedicate the rest of my appreciation to those without whom, perhaps, I wouldn't have made it this far. To my dearest friends, old and new, who made the working atmosphere warm, lively, and human. Aditya, Aleksie, Cecilia, Georgios, Ivalu, Jan, Jong-Seo, Lena, Petra, Teresa, Vlad, all who contributed in their own ways to a very memorable period of my life in Germany. The office was often the place where research, coffee, inside jokes, cheeky buddies (Aleksei a.k.a my *zayabal*, and Jong-Seo who also proofread various sections), philosophical discussions (with Georgios), constant teasing (of/with Lena), random hugs (by Teresa), and some unhinged conversations (with Cecilia) all blended into one. Where random life updates slowly converged—and, before long, became my only reason to go to the office. I am deeply grateful that what we shared was not just research but laughter, arguments, and conversations that had nothing to do with astronomy—the best company I could have asked for this period.

A special thanks goes to my friendly neighbor Sarwar for providing me with new names, being a genuinely good friend, endless cooking together and memorable trips; to Anahat for every sweet gesture and always looking out for me. I am also thankful to Arshia, Camilla, Rohit, to Asmita for such memorable trips, teasing and wonderful cooking, and Johanna, who has been such an amazing neighbor and, in many ways, an elder sister. Together, they turned hallways and kitchens into places of conversation and comfort and made life in Germany feel far less distant from home. I hope to take more trips with you *beautiful people*, and in the process make more fond memories.

To my friends back in India, my gratitude is no less deep and never forgotten. Although I consider myself an agnostic, for lack of a better word I would still put it this way: I am *blessed* to have a friend circle with whom I can share, both, my thoughts and my silence. Despite my constant insistence on advising others not to have a *philosopher* friend, the irony is not lost on me. I am particularly grateful to Divyansh, whom I rarely get the chance to thank properly for being a true friend and for helping with proofreading the philosophical sections. Along the same lines, Kshitij, for being there since my Master's and for all the dynamic moments since then, and being, perhaps what one would call, a *partner in crime*; Srushti, whom I admire deeply, although it is a pity we rarely find the time to spend together; Samruddhi, for being such a kind and sweet *gujju* friend, often only a video call away; Priyamvada, for always being there for me and for sending Rakhi every year since I moved to Bonn, which I hope continues to reach me; Bhasha, for being such a patient listener; and Tejaswi for fun conversations.

Calls with them have helped more than I can properly express. Time zones

and distance meant that messages were sometimes answered late at night or in the middle of their working day, yet they always showed up—with reassurance, with nonsense, and with the kind of familiar chaos that makes you feel like you never really left. Living abroad is easier when there are people back home who remain present in the most ordinary and dependable ways.

My gratitude goes further to Parth and Siddharth for giving me a trio, where we have shared, on most occasions, the sentiment which can be well resonated with this particular dialogue from *Harry Potter and the Half Blood Prince*—“Why Is It When Something Happens, It’s Always You Three?”. And the answer from our side be exactly this: *Believe me! We’ve been asking ourselves the exact same thing for the past six years*, but perhaps because our lives have been intertwined in more ways than we understand ourselves, it is inevitable. I am and shall always be grateful to have Parth as a close friend, brother, mentor, colleague, and collaborator who was there with me, when no one else was, and in the process constantly motivating me. We have, indeed, come long way from *Changa*.

I am deeply grateful to my parents and my family for their constant love, patience, and support. They have stood by me through the uncertainty and stress that accompany a PhD, and their faith in me never wavered, even when mine did. They continue to miss me, and I them. Their quiet sacrifice is what carried me here and continues to carry me forward.

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The list of people does not end here. Pardon me if you do not find yourself in here, but if you are reading this, you are indeed part of my life, of my journey. If this thesis has any measure of completion, it is because of the people around me. Whatever I have learned in these years, I learned in conversation, in collaboration, and in the company of those who gave their time, their thought, and often their patience. It would then not be inaccurate to say that these people contributed 99.99999% of what I am today, and hence what proceeded was an attempt to acknowledge them, knowing that any such attempt will fall short. I may have written the pages, but I did not write the journey alone because

जो लिखी है ये दास्तान, एक सिलसिला सा बन गया,
हर मोड़ पे कोई मिलता रहा, ये सफ़र आसान सा बन गया।
जो रौशनी मुझे मिली, वो सब उन्हीं की देन थी,
दिखाए जिन्होंने रास्ते, हर नक्श रहनुमा सा बन गया।
जो कुछ भी सीख पाया हूँ, वो आप ही की सोहबत से,
मेरे हुनर का हर इक रंग, नया आसमाँ सा बन गया।
जो नाम अगर है, तो उनकी ही दुआओं का असर है ये,
वर्ना मैं अपने आप में, बस एक गुमाँ सा बन गया।
ये पन्ने मैंने लिख तो दिए, मगर आवाज़ें उनकी हैं,
मेरी क़लम का हर लफ़्ज़, उनका तरजुमाँ सा बन गया।
अगर कुछ है मेरी पहचान, तो उनकी मेहरबानी है,
ये सिलसिला जो चल पड़ा, एक कारवाँ सा बन गया।

*The tale I wrote unfolded thus—an unbroken chain it came to be,
At every bend, a soul I met, the road, easily came to be.
Whatever light has reached my eyes—it was their gift alone,
They showed the paths I had to walk, marking a guide it came to be.
Whatever I have come to learn—was learned within their company,
Each shade & color of my craft, a new and open sky it came to be.
If there is any name to me—it is the echo of their prayers,
Without them, of my very self, a passing whisper I came to be.
These pages, yes, I wrote them—but every voice in them was theirs,
Each word my pen set to page, was their interpretation came to be.
If I am known at all today—it is their kindness, all of it,
This chain that started with one step, at last, a caravan came to be.*

Cover image: Number density of accreting plasma onto a horizonless singularity, from a GRMHD simulation snapshot (Saurabh et al., 2026).