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SURVEY PAPER

A Systematic Review of International Perspectives on Mathematical Modelling: Modelling Goals and Task Characteristics

Janina Krawitz[1] · Stanislaw Schukajlow[2] · Xinrong Yang[3] · Vince Geiger[4]

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Abstract
Mathematical modelling is a dynamic research field. This article presents a systematic literature review of recent developments in mathematical modelling from an international perspective. In identifying relevant modelling perspectives, we draw on the goals of modelling and the characteristics of modelling tasks as our theoretical foundation and analyzed recent modelling research in terms of (1) the attributes of studies, including geographical distribution, participants, methodological approaches, and conceptual frameworks; (2) the goals of modelling, characteristics of tasks, and perspectives on modelling; and (3) the relationships between modelling perspectives and study attributes. Focusing on studies concerning students from early childhood to secondary school, we identified 4045 initial publications, from which we selected and analyzed 108 peer-reviewed journal articles and ICTMA book chapters published between January 2020 and April 2024. The analysis revealed significant interest in mathematical modelling across a range of perspectives and, in particular, an emphasis on formative goals related to the development of modelling competence. Authenticity and cognitive richness also emerged as key characteristics of modelling tasks. Additionally, we noted that the educational modelling perspective defined by a combination of formative goals and cognitively rich tasks emerged as a prominent focal point within recent modelling research. A key finding of the review was that further research is needed on under-explored nuances of each modelling perspective, such as upper-secondary students within the educational perspective. Our findings also highlighted the need for theory development to connect the various conceptual frameworks in modelling.

1 Introduction

Educational systems worldwide acknowledge the importance of mathematical modelling, as evidenced by its incorporation into curricula internationally. The importance assigned to modelling is due to its role in supporting problem solving related to personal, work, and civic life and as a key capability that underpins science, technology, engineering, and mathematics (STEM) careers (Siller et al., 2024). Although there is consensus regarding the importance of addressing modelling in curricula and in mathematics teaching and learning practice within schools, there is no universal understanding of modelling. Moreover, there are diverse purposes and goals for the teaching and learning of modelling and the essential characteristics of the tasks necessary for achieving these goals remains a matter of contention – issues that require further research to bring coherence to the field.

The importance of mathematical modelling is also reflected in the increasing number of special issues of peer-reviewed journals and literature reviews related to modelling research. Previous literature reviews on mathematical modelling education have been based on: contributions presented at conferences (English et al., 2016), identification of themes emergent from the content of articles in special issues of journals (Carreira & Blum, 2021), modelling competencies (Cevikbas et al., 2022), the role of digital technologies (Cevikbas et al., 2023), cognitive perspectives (Schukajlow et al., 2023a), approaches to instruction (Borromeo Ferri, 2018), and broad overviews of modelling research (Niss & Blum, 2020). These reviews were often

✉ J. Krawitz

[1] Institute of Mathematics Education, University of Cologne, Cologne, Germany

[2] Institute of Mathematics, Education, and Computer Science Education, University of Münster, Münster, Germany

[3] Macau University, Macau, China

[4] Australian Catholic University, Brisbane, Australia

broad syntheses of research rather than systematic reviews. While such syntheses have merit, they are dependent on the authors' knowledge, which is likely to be stronger in some areas than others, and the reasons for inclusion are not always transparent. Systematic reviews, on the other hand, make use of transparent processes and provide detailed descriptions of the methodological approaches used. The aim of such reviews is to provide more complete coverage of a field and to ensure replicability of results. This article adopts an approach consistent with that of a systematic review.

Diverse approaches have been used in modelling research to distinguish between and describe different perspectives on modelling (e.g., Frejd & Bergsten, 2018; Geiger, 2013; Mousoulides et al., 2007). In this article, we adopted two conceptual lenses to analyze international perspectives on modelling. Kaiser and Sriraman (2006) described modelling perspectives according to their epistemological foundations and geographical origins. Historically, perspectives on modelling have encompassed a broad range of global research drawn from various scientific traditions—often originating in specific countries or regions with shared languages—but it is uncertain how perspectives on mathematical modelling are represented in the current research landscape. Blum (2015) more formally anchored these perspectives in goals of modelling and characteristics of modelling tasks, with specific combinations defining a specific perspective, thus further developing Kaiser's and Sriraman's conceptualization. To identify perspectives, we use Blum's (2015) formal classification. We apply Kaiser and Sriraman's (2006) typology to examine and discuss perspectives in relation to other factors, such as their geographical distribution. We focused modelling research conducted on early childhood to secondary school students. To focus on the most recent modelling research we, included studies published between January 2020 and April 2024 that investigated the teaching/learning of modelling across school levels. The overarching objective of this literature review is to provide insight into various perspectives on mathematical modelling and their alignment with different goals and characteristics of modelling tasks. The subgoals were as follows:

1. To provide an overview of recent modelling research.
2. To identify the frequency of goal/task combinations, confirming their alignment with perspectives described in the literature or recognize those that differ.
3. To analyze whether the perspectives differ in terms of geographical distribution, participants, methodological approaches, and conceptual frameworks.
4. To identify new or less-explored areas of recent modelling research.

2 Theoretical background and current research landscape

2.1 Modelling Research: Geographical Distribution, Participants, Methodological Approaches, and Frameworks

Mathematical tasks with a connection to reality are defined as modelling tasks when their solutions require substantial translation processes between the real-world and mathematics (Schukajlow et al., 2023a). The solution process for modelling tasks is comprised of the development of models and the selection and application of mathematical procedures relevant to the description of the real-world situation. Mathematical modelling is closely related to other notions related to solving problems in the real world, such as numeracy or mathematical literacy. The different ways of thinking about the application of mathematics emerged as responses to diverse mathematical demands in society over time and were thus developed to serve different goals, leading to different traditions in research (Geiger et al., 2022). Modelling emerged from the problem-solving contexts in engineering which is typically described as a cyclic process involving the translation of real-world situations into a mathematical problem, the solution of this mathematical problem, the translation of this idealized mathematical solution into the original problem context to ascertain its fit, and the revision of all these steps if the solution requires further refinement. Numeracy, however, evolved from concerns in society about the capacity of citizens to use mathematics to respond to problems in personal, civic and work life. The concept of numeracy has evolved over time to include critical capabilities, such as those needed to detect mis/dis-information that makes use of mathematics as evidence. Literature written from a numeracy perspective does not generally include descriptions of specified processes that lead to a solution, but rather identifies the capabilities employed in addressing a situation. This means that to engage in numeracy activity, aspects generally associated with modelling may or may not be used as the chosen approach is situated in the particulars of a problem or circumstance. For example, it might not be necessary to formally identify assumptions or validate solutions in certain circumstances. Numeracy activities are often socially constructed, while modelling has tended to take a more cognitive direction. However, over time, some of the differences between numeracy activities and modelling have become less distinct, with both approaches being used to address similar issues. Thus, research regarding these different subfields should be seen as complementary, since both approaches aim to use mathematics to address real-world problems, despite their different origins (see for example Geiger & Schmid, 2024).

An important line of research related to mathematical modelling is the solving of word problems. Word problems are commonly defined as verbal descriptions of problem situations presented in a scholastic context. They involve one or more questions, the answers to which can be determined by applying mathematical operations to the numerical data provided in the problem statement or derived from it (Verschaffel et al., 2020). Situations described in word problems usually involve highly specific conditions and can therefore be solved through the application of straightforward mathematical operations and procedures (although the so-called problematic word problems (p-problems) are exceptions). Thus, word problems do not require students to simplify situations, make realistic assumptions, or structure and validate solutions (Kaiser, 2017; Niss et al., 2007), as is the case for modelling problems, leading some scholars to see word problems and modelling tasks as conceptually distinct. Other researchers have viewed word problems as specific (albeit potentially oversimplified) forms of modelling tasks (Verschaffel et al., 2020) and have included them in the broader field of modelling research. In this study, we adopted a broad conceptualization of modelling to capture various modelling perspectives. However, we did only include studies when the authors framed their research under modelling. Accordingly, we searched for studies that claimed to analyze modelling and did not explicitly include word problems or numeracy problems in the search terms.

Previous literature reviews have emphasized the diversity of modelling education research, noting contributions across all continents (Preciado Babb et al., 2023), but they have highlighted the significant influence of certain countries/regions on modelling discourse. For instance, Preciado Babb et al. (2023) reported that Germany, the United States, and Australia were the top three countries contributing to the field, collectively accounting for about half of the total number of publications. In a further review relating to secondary school mathematical modelling, Stohlmann et al. (2016) found that studies generally originated from the same three countries. This contrasted with studies related to primary school students, which came predominantly from Australia and the United States, with German studies being far less frequent (Stohlmann & Albarracín, 2016).

Although mathematical modelling is of interest to researchers across all educational levels (e.g., Geiger & Frejd, 2015; Schukajlow et al., 2018), most studies have been conducted on secondary school students (e.g., 45% in the literature review on modelling competencies by Cevikbas et al. (2022) and 33% in the literature review by Preciado Babb et al. (2023)). That said, different countries/regions tend to focus on different groups of participants in modelling research. For example, in Brazil, there is a strong focus on the postsecondary level (24.3%), particularly on teachers (18.9%) (Preciado Babb et al., 2023).

Methodological approaches also differ in the literature. Schukajlow et al. (2018), for example, reported a significant lack of quantitative studies, particularly concerning the International Conference on the Teaching of Mathematical Modelling and Applications (ICTMA) books, with only 7% of researchers applying quantitative methods, compared to 92% applying qualitative methods, and 1% using mixed-methods approaches. Cevikbas et al. (2022), in a further literature review of modelling competencies, reported quite different findings, with most researchers (32%) using quantitative methods, followed closely by qualitative approaches (27%), with only a few employing alternative approaches, such as theoretical, overview/survey, or design-based methods. These differences may stem from the different foci (modelling research vs. research on modelling competencies) in these literature reviews, highlighting the importance of understanding the context of such reviews.

Studies on mathematical modelling have often been based on distinct frameworks (e.g., Geiger et al., 2022; Lesh & Doerr, 2003; Niss & Blum, 2020). In this review, we focused on conceptual frameworks that considered analytical reconstructions of the modelling process (Niss & Blum, 2020, p. 14), including modelling cycles. Modelling cycles are based on activities that involve understanding real-world situations, making realistic assumptions, simplifying initially complex real-world problems, developing mathematical models, interpreting mathematical results in light of the initial real-world problems, and validating the results against real-world situations (Borromeo Ferri, 2006; Kaiser, 2017). These activities are generally depicted as an iterative cycle that can be employed to develop models of varying complexity and adjust initial assumptions to achieve solutions that best fit specific situations with the necessary degree of precision. There are, however, different versions of the modelling cycle, which vary in what they emphasize in the relationship between the real world and mathematics, and which key activities need to be employed to address specific problems. Geiger and Frejd (2015) observed this diversity in approaches to modelling, noting increasing diversity over time regarding the conceptual frameworks used to anchor papers and a strong emphasis on modelling cycles and modelling competencies, often without a proper theoretical grounding.

In summary, the findings of previous literature reviews raise questions about the geographical distribution of studies, the origin of participants, and the diverse methodological approaches and conceptual frameworks used in international modelling research.

2.2 Modelling Goals

Julie (2019) described two types of modelling goals, modelling as content and modelling as a vehicle. Modelling as content focuses on teaching modelling approaches to develop students' capabilities to solve real-world problems. Modelling as a vehicle means engaging in the modelling process to learn mathematical content. According to Niss et al. (2007), both approaches are required for effective learning to take place.

The purposes and goals of modelling have been explored by other scholars (Galbraith, 2012; Kaiser & Sriraman, 2006). Blum (2015) has distinguished between four main modelling goals:

- pragmatic goals
- formative goals
- cultural goals
- psychological goals

In a broad sense, pragmatic goals align with modelling as content, whereas the other goals align with modelling as a vehicle. *Pragmatic goals* emphasize the importance of teaching modelling in schools to enable learners to use mathematics to understand and solve real-world problems. From this perspective, learning should mirror the way modelers, in practice, work without prescriptive mathematical knowledge or procedures (Julie, 2002). *Formative goals* highlight the role of modelling in developing competencies, particularly those related to modelling, as well as others, such as mathematical problem-solving and mathematical reasoning. *Cultural goals* include cultural emancipatory goals and cultural mathematical goals. Cultural emancipatory goals emphasize the role of modelling in supporting students' abilities to understand and critically evaluate real-world phenomena. Cultural mathematical goals focus on theory-oriented objectives within mathematics. They highlight that modelling is part of mathematics, and teaching modelling contributes to a comprehensive understanding of mathematics as a science. *Psychological goals* comprise goals related to motivation and sense-making, and those related to mathematical learning. Psychological goals related to motivation and sense-making are concerned with how modelling real-world connections can enhance students' interest in and perceived relevance of learning mathematics. Psychological goals related to mathematical learning focus on using real-world connections through modelling to structure knowledge and facilitate the comprehension and retention of mathematical concepts over time. As far as we could determine, previous literature reviews have not considered the goals researchers assigned to modelling.

Kaiser and Sriraman (2006) differentiated between pragmatic-utilitarian goals (corresponding to pragmatic goals), subject-related goals (corresponding to psychological goals related to mathematical learning), psychological goals (corresponding to psychological goals related to motivation and sense-making), pedagogical goals (including formative goals and cultural emancipatory goals), and theory-oriented goals (corresponding to cultural mathematical goals) (see Table 10 in the Appendix). This classification of modelling goals presented in the literature highlights the multifaceted nature of modelling, reinforcing its significance in both mathematical learning and real-world contexts.

2.3 Characteristics of Modelling Tasks

Different characteristics can be distinguished in modelling tasks at both the theoretical (e.g., Galbraith & Stillman, 2006) and empirical levels (e.g., Czocher, 2017; Krawitz et al., 2024). Blum (2015), for example, identified five task characteristics that are necessary for achieving the goals of modelling:

- authenticity
- cognitive richness
- epistemological richness
- motivational or illustrative
- mathematical richness

Authentic tasks are based on real-world contexts, such as those associated with genuine social or environmental issues or phenomena (Vos, 2011). Tasks can originate from contexts such as industry, economics, health, and science. These tasks must be constructed in a way that appears credible to experts working in a relevant area, even if somewhat simplified (Niss & Blum, 2020; Palm, 2008). *Cognitive richness* relates to the need for modelling problems to engage students in cognitive processes. Cognitively rich tasks should make significant cognitive demands, such as when tasks are open-ended (Schukajlow et al., 2023b). *Epistemological richness* characterizes problems that are designed to provide insight into the nature of mathematics as a science. Real-world connections are less important in this category, with some scholars arguing that some types of intramathematical tasks meet the description of modelling. For example, graphs can be considered as models of linear equations, with solutions to some problems represented by their intersection(s) (Garcia et al., 2006). That said, most researchers in the field recognize that the defining characteristic of mathematical modelling is the connection between mathematics and the real world (Borromeo Ferri, 2006; Kaiser, 2017). The *motivational or illustrative* characteristics of modelling problems provoke students' interest, motivation, positive emotions, or provide meaning for mathematics education due to their real-world connections. Dressed-up word problems can also serve this purpose. *Mathematical richness* relates to the possibility of developing conceptual depth for specific mathematical knowledge or ideas. In one of the limited number of studies on task characteristics, Cevikbas et al. (2022) identified characteristics that relate to two of the presented categories: authenticity (assessed via terms such as "authentic" and "realistic") and epistemological richness (assessed in terms of "intramathematical modelling"). We found no other reports of task characteristics in previous literature reviews concerning mathematical modelling.

2.4 Modelling Perspectives as a Combination of Goals and Task Characteristics

Goals for teaching and learning modelling, along with the characteristics of modelling tasks, form the basis of the following six modelling perspectives (Blum, 2015):

- applied modelling (pragmatic goals, authenticity)
- educational modelling (formative goals, cognitive richness)
- sociocritical modelling (cultural goals, authenticity)
- epistemological modelling (cultural goals, epistemological richness)
- pedagogical modelling (psychological goals, motivational or illustrative)
- conceptual modelling (psychological goals, mathematical richness)

Each of these perspectives relates to specific conceptual frameworks.

Applied modelling is connected to pragmatic goals as it is related to solving practical problems in the real world. The use of authentic modelling tasks is considered essential for achieving this goal. Studies that take an applied modelling perspective often use conceptual frameworks based on the modelling cycle, encompassing the processes of mathematising, working mathematically, interpreting, validating, and, in some cases, the reporting solutions (e.g., Haines et al., 2000).

Educational modelling is aimed at improving students' modelling competencies. Cognitive richness is a crucial task characteristic of studies conducted from this perspective. Educational modelling can also focus on fostering specific subcompetencies, making nuanced conceptual frameworks particularly valuable. For instance, pre-mathematizing processes, which involve learners' understanding and processing of real-world situations, involve distinguishing between a real situation, a situation model (a mental model of the real situation), and a real model (a simplified and structured version of the situation) (e.g., the cycle from Blum & Leiss, 2007). Additionally, metacognitive competencies are considered especially important (Stillman, 2011), with the modelling cycle, in its simplified form, serving as a metacognitive tool that fosters students' modelling competencies and provides support for monitoring their solution processes (Durandt et al., 2022).

Sociocritical modelling is a perspective based on cultural emancipatory goals and the use of authentic examples. It emphasizes the role of modelling in the service of critical or emancipatory thinking and action. In this sense, modelling can be used to initiate, support, or challenge claims and arguments made by individuals, institutions, governments, industries, and individuals or groups within society at large (Geiger, 2024). The use of authentic tasks is considered particularly important in achieving these aims. Maaß et al. (2023) extended Blum and Leiss's (2007) modelling cycle by adding steps for discussing potentially contradictory results, drawing conclusions, and making decisions to establish a framework for research from this perspective. However, some scholars have questioned the use of modelling cycles as conceptual frameworks for research from this perspective (Barbosa, 2006), and others, such as Skovsmose and Greer (2012), have argued that research on modelling should include critiques of models when considering the role of mathematics in society.

Epistemological modelling emphasizes cultural mathematical goals and considers epistemologically rich tasks as key to effective implementation. In contrast to other perspectives, frameworks used in epistemological modelling do not necessarily begin the modelling process with a real-world situation and may even omit the distinction between the real world and mathematics (Arrieta & Díaz, 2015).

Pedagogical modelling focuses on psychological goals and emphasizes the use of motivating or illustrative tasks. The authenticity of the situation is less important and may even concern an imagined world (Lesh & Doerr, 2003). Conceptual frameworks aligned with pedagogical modelling place less emphasis on differentiating cognitive processes (e.g., differentiating between understanding and simplifying) than other perspectives (Borromeo Ferri, 2006). Instead, the focus is on the process of model development through sequences of model transformations, aiming to engage students in thinking about models, working with them, and applying them to different contexts (Lesh & Doerr, 2003).

Conceptual modelling is connected to educational modelling, because it also emphasizes psychological goals but focuses more on mathematical learning content; therefore, the use of mathematically rich tasks is particularly important.

Kaiser and Sriraman's (2006) typology, the origin of the notion of 'modelling perspectives', describes six modelling perspectives that align with those presented above, although the researchers used different terminology. The researchers distinguished between realistic modelling (corresponding to applied modelling), educational modelling, which they further divided into educational–didactical modelling (corresponding to educational modelling) and educational–conceptual modelling (corresponding to conceptual modelling), sociocritical modelling, epistemological modelling, and contextual modelling (corresponding to pedagogical modelling). Kaiser and Sriraman (2006) emphasized that these perspectives align with different modelling goals but allow a single goal to contribute to multiple perspectives. Pedagogical goals, for example, play a role in educational–didactical modelling, educational–conceptual modelling, and sociocritical modelling (see Table 10 in the Appendix). Thus, rather than serving as strict classification criteria, as in

Table 1 Search strings for data collection

Database	Search string
Scopus	TITLE (model*) AND TITLE-ABS (math* AND model*) AND PUBYEAR > 2019 AND (LIMIT-TO (SRCTYPE, "j")) AND (LIMIT-TO (SUBJAREA, "PSYC") OR LIMIT-TO (SUBJAREA, "SOCI")) AND (LIMIT-TO (LANGUAGE, "English"))
WoS	(TI=(model*) AND (AB=(model* AND math*) OR TI=(model*AND math*))) AND (LA==("ENGLISH") AND PY==("2024" OR "2023" OR "2022" OR "2021" OR "2020") AND SJ==("PSYCHOLOGY" OR "EDUCATION EDUCATIONAL RESEARCH"))
ERIC	title:model* AND ((+abstract:model* math*) OR (+title:model* math*))
	Limit search to publications since 2020

Blum's (2015) formal characterization, the modelling goals in this framework are used to describe (rather than define) a perspective. In addition, Kaiser and Sriraman (2006) emphasized the significance of task characteristics, such as the use of authentic tasks, although they did not provide a classification for different task characteristics. Instead, they based their typology on the epistemological roots of various perspectives. For instance, they grounded realistic modelling in Anglo-Saxon pragmatism (e.g., Pollak, 1969), based educational modelling on scientific–humanistic approaches (e.g., De Lange, 1987), and drew epistemological modelling from anthropological theory (Chevallard, 2007). Sociocritical modelling is closely related to the ethnomathematical approach (D'Ambrosio, 1985; Orey & Rosa, 2021), which examines the role of mathematics in society (Barbosa, 2006), whereas contextual modelling originates from the American problem-solving debate and the theory of model-eliciting activities (Lesh & Doerr, 2003).

In addition to the six modelling perspectives, the cognitive modelling perspective is concerned with understanding and describing the cognitive and metacognitive activities involved in the modelling process (Kaiser & Sriraman, 2006; Schukajlow et al., 2023a; Stillman, 2011). Some researchers have considered it a meta-perspective (Schukajlow et al., 2023a). Unlike other perspectives, it is not tied to specific learning and teaching goals, and it does not emphasize any specific characteristics of modelling tasks. Thus, we excluded this perspective from our literature review.

Since the evolution of modelling perspectives is rooted in various scientific traditions, often originating in specific countries or geographies with shared languages, the distribution of current perspectives on modelling tends to mirror the regional traditions from which they emerged. In today's increasingly globalized scientific landscape, it remains an open question whether these historical roots continue to shape the geographical distribution of modelling perspectives or new trends and perspectives may have been established. For instance, through a literature review on modelling perspectives, Preciado Babb et al. (2023) identified a global emphasis on educational modelling, as well as regional foci (e.g., *pedagogical modelling* in the United States and *sociocritical modelling* in Brazil), although the researchers did not differentiate between the goals and characteristics of modelling tasks. Consequently, the connections between modelling perspectives and modelling goals and characteristics appear to be under-researched.

3 Research Questions (RQs)

The following RQs guided our systematic literature review of mathematical modelling research:

RQ1: What are the characteristics of recent modelling research in terms of geographical distribution, participants, methodological approaches, and conceptual frameworks?

RQ2: a) What goals for teaching and learning modelling, b) what characteristics of modelling tasks are described in recent research, and c) which perspectives on modelling emerge from these goals and tasks?

RQ3: How do the identified perspectives on modelling in recent research differ with respect to geographical distribution, participants, methodological approaches, and conceptual frameworks?

4 Methodology of the Systematic Literature Review

4.1 Data Collection

We conducted the systematic literature review in accordance with the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines (Page et al., 2021). We collected data from Web of Science (WoS) and Scopus—the most comprehensive scientific databases (Chadegani et al., 2013)—and from Education Resources Information Center (ERIC), which is a particularly important for the field of education. To identify relevant studies, we used search strings to receive articles with "model*" in the title and "math*" and "model*" in the title or abstract (Table 1). We focused on recent modelling research and therefore limited the search to journal articles in the social sciences or psychology, published since 2020, and written in English (see IC3 and IC5 in Table 2). Given the vast amount of research on modelling, we chose to limit our literature re-

Table 2 Inclusion and exclusion criteria

Inclusion criteria (IC)	Exclusion criteria (EC)
IC1: Primary research focus on mathematical modelling in the context of mathematics education	EC1: Primary research focus not on mathematical modelling in the context of mathematics education
IC2: Focus of the study on learners from early childhood education up to secondary school	EC2: Focus not on learners from early childhood education up to secondary school (e.g., other educational levels or teachers)
IC3: Studies published in English	EC3: Studies published in languages other than English
IC4: Studies published in journal articles indexed in Scopus, WoS, ERIC, or the ICTMA books	EC4: Studies not published as journal articles, not indexed in any database included in IC3, and not published in the ICMTA books.
IC5: Studies published since 2020	EC5: Studies published earlier than 2020

view to learners from early childhood education through to upper secondary school (see IC2 in Table 2). We conducted the search on April 22, 2024.

4.2 Inclusion and Exclusion Criteria

We established inclusion criteria (IC) and exclusion criteria (EC) to determine eligible studies for our literature review, as listed in Table 2.

4.3 Selection Process

The search yielded 4045 records, from which we eliminated 725 duplicates were eliminated, resulting in 3320 remaining records. We screened the abstracts, titles, and keywords of these records, which resulted in the further exclusion of 3043 records due to EC1. We retrieved the remaining 277 reports. For 10 reports, no full text was available, resulting in 267 reports. In addition, we decided to include studies from the ICTMA books, given ICTMA's significant importance for research on mathematical modelling. At the time of the search, two ICTMA books had been published since 2020 (Leung et al., 2021; Stillman et al., 2020), comprising 99 chapters. We assessed the journal articles from the digital databases, together with the ICTMA chapters, for eligibility. EC1 led to the exclusion of 6 journal articles and one ICTMA chapter, EC2 led to the exclusion of 182 journal articles and 62 ICTMA chapters, and EC3 led to the exclusion of 7 journal articles, resulting in *a total of 108 papers* (72 journal articles and 36 ICTMA chapters) included in the literature review. The flowchart presented in Fig. 1 illustrates the selection process.

4.4 Data Analysis

As the unit of analysis, we selected single research items (either journal articles or chapters from the ICTMA books) and developed a coding scheme to categorize the publications included in the literature review.

Coding of geographical distribution, participants, methodological approaches, and conceptual frameworks To answer RQ1, we coded the study characteristics, such as geographical distribution (including continents and countries/regions), participants, methodological approaches (including research approaches and study designs), and frameworks (Table 3).

Coding of modelling goals, characteristics of modelling tasks, and self-reported perspectives To systematically analyze the modelling perspectives for RQ2, we deductively coded the *modelling goals* and *characteristics of modelling tasks* (see descriptions of categories and examples in Table 4 and the electronic supplementary material). We assigned modelling perspectives according to the identified goals of modelling and characteristics of modelling tasks. In addition, we inductively coded the authors' self-reported perspectives.

The first author conducted the coding. For the more challenging categories of *modelling goals* and *characteristics of modelling tasks*, 19% of the publications ($n = 21$) were double-coded by both the first and second authors, with satisfactory interrater agreement (Cohen's κ of 0.61 for *modelling goals* and 0.81 for *characteristics of modelling tasks*). Discrepancies were discussed and resolved until a consensus was reached.

5 Results

5.1 Analysis of Geographical Distribution, Participants, Methodological Approaches, and Frameworks

In summary, authors from 6 continents and 33 countries/regions contributed to the 108 publications. Figure 2 depicts the distribution of publications according to the continents of the authors' affiliated institutions.

The largest numbers of publications were written by authors with affiliations in Europe ($n = 59$, 55%), followed by Asia ($n = 29$, 27%) and North America ($n = 20$, 19%). The distribution of countries/regions is presented in Table 5. Most publications were written by authors with affiliations

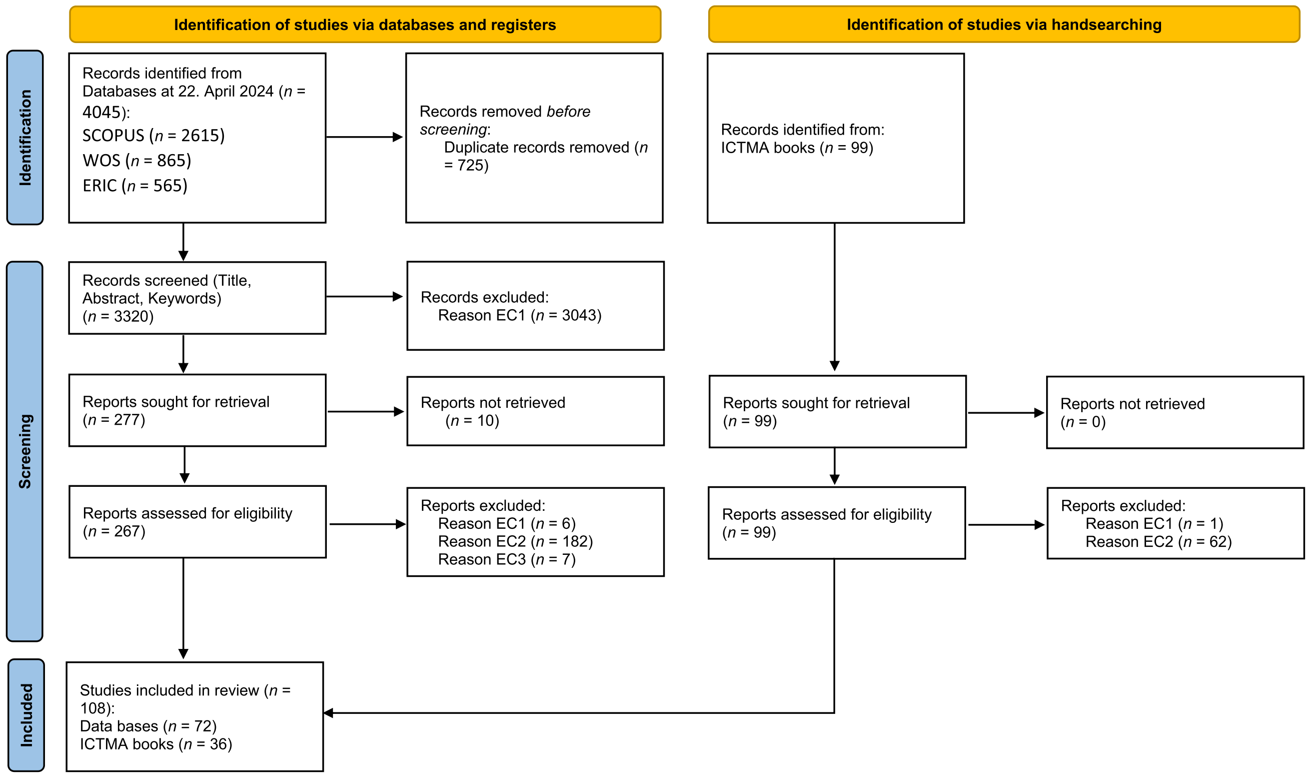

Fig. 1 Flowchart of the selection process

Table 3 Coding scheme for geographical distribution, participants, methodological approaches, and conceptual frameworks

Category	Subcategories	Short description
Year	2020 to 2024	Year of publication
Country/region	Countries/regions of the participating authors' affiliated institutions	If several authors from one country/region participated, the country/region was counted only once. If authors from different countries/regions contributed to a publication, each country/region was credited with one publication. No distinction was made based on the authors' positions (e.g., first or second author).
Participants	Preschool Year levels 1–6 ("Elementary school") Year levels 7–10 ("Lower secondary school") Year levels 11–13 ("Upper secondary school")	Since there is no international consensus on year levels (e.g., year levels assigned to high school), we based our coding on the participants' actual year level. If multiple populations were targeted, each population was coded separately, yielding 127 populations across all publications.
Research approaches	Qualitative Quantitative Mixed methods	If both qualitative and quantitative methods were used, the study was assigned to the mixed-methods category
Study design	Case study Design-based research Descriptive (no intervention, e.g., correlational studies) Quasi-experimental Experimental	Study designs differentiated based on Jha (2024). If multiple studies with different study designs were reported, each design was coded separately, yielding 118 study designs across all publications.
Conceptual frameworks	See electronic supplementary material for the results of the inductive coding	Inductive coding of conceptual frameworks by coding the references to frameworks as used by the authors in the studies.

Table 4 Short version of the coding scheme for modelling goals, characteristics of modelling tasks, and self-reported perspectives

Category	Subcategories	Short description
Modelling goals	Pragmatic goals	Understanding real world situations
	Formative goals	Fostering competencies
	Cultural goals (emancipatory)	Supporting critical thinking about mathematics in society
	Cultural goals (mathematics)	Structuring both mathematical and nonmathematical fields
	Psychological goals (motivational and sense-making)	Motivating and providing a sense of purpose
	Psychological goals (learning)	Enhancing the learning of mathematical content
	No or other goals mentioned	No goal mentioned or a goal different to the above goals mentioned
	Mixed goals	Different goals presented without emphasizing which one was the most important.
Characteristics of modelling tasks	Authenticity	Genuine real-world contexts
	Cognitive richness	Challenging students' modelling activities
	Epistemological richness	Shedding light on mathematics as a scientific discipline.
	Motivational or illustrative	Motivation or illustration purposes (could also be dressed-up word problems)
	Mathematical richness	Mathematical richness to make mathematical topics more comprehensible
	No modelling task presented or mixed characteristics	No example presented, no important task characteristics mentioned, or various characteristics mentioned without highlighting which one was the most important or the primary focus.
Self-reported perspectives	See Sect. 5.2 for the results of the inductive coding	Inductive coding of the modelling perspectives that authors attributed to their own research.

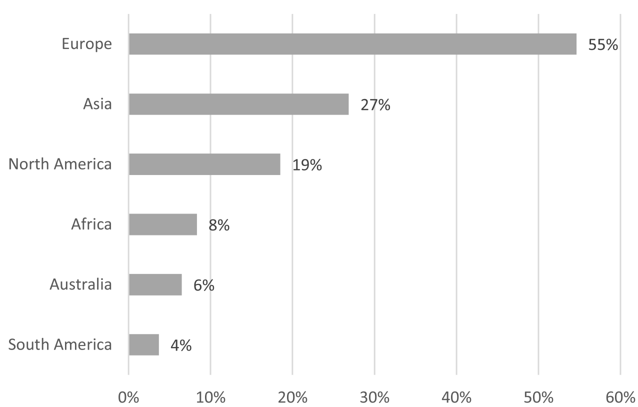

Fig. 2 Distribution of publications based on the continents of affiliation of at least one author (a single publication could have authors from multiple continents)

in Germany ($n = 29$, 27%) and the United States ($n = 19$, 18%) (Table 5).

Considering the participants, the results showed that more than half the studies focused on lower secondary school students (Year levels 7–10; $n = 70$, 65%), followed by studies focusing on upper secondary school students (Year levels 11–13; $n = 29$, 27%), and a similar number focusing on elementary school students (Year levels 1–6; $n = 26$, 24%). Very few studies ($n = 2$, 2%) focused on early childhood education.

Table 5 Frequencies and percentages of publications based on the countries/regions of affiliation of at least one author

Country/region	*n*
Germany	29 (27%)
USA	19 (18%)
Turkey	9 (8%)
Australia	8 (7%)
South Africa	7 (6%)
Spain	6 (5%)
Norway	5 (4%)
China; Japan	4 each (3 each)
Brazil, Portugal, Sweden, Switzerland	3 each (2 each)
Denmark, Indonesia, Israel, Morocco, South Korea, Chinese Taipei, Vietnam	2 each (2 each)
Austria, Cananda, Costa Rica, Cyprus, France, Greece, Italy, Korea, Mexico, the Netherlands, New Zealand, Palestine, Thailand	1 each (1 each)

Note. The total exceeds 100% because multiple countries/regions could contribute to a single publication.

To analyze the methodological approaches, we coded whether the researchers used qualitative, quantitative, or mixed methods. The majority of publications fell under the qualitative research approach ($n = 63$, 58%), with fewer

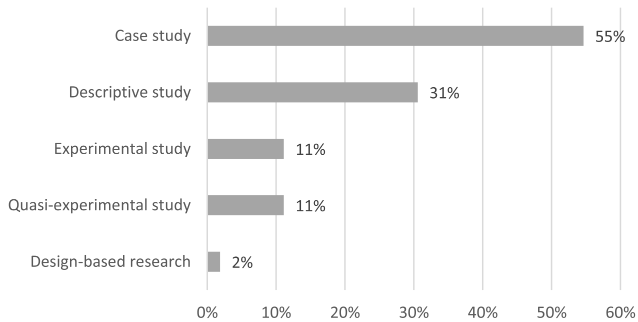

Fig. 3 Distribution of publications by study design (a single publication could include multiple studies with different designs)

publications using quantitative methods ($n = 27$, 25%) or mixed methods ($n = 18$, 17%).

Additionally, the analysis of the study designs (Fig. 3) showed that about half of all studies were case studies ($n =$ 59, 55%). Descriptive studies comprised the second-largest category ($n = 33$, 31%), followed by quasi-experimental studies ($n = 12$, 11%), experimental studies ($n = 12$, 11%), and a small number of design-based studies ($n = 2$, 2%).

In addition, we captured the conceptual frameworks referred to in the publications. In total, we identified 23 different conceptual frameworks. Blum and Leiss's (2007) modelling cycle was the most frequently cited framework ($n =$ 18, 17%), but a large number of other frameworks highlighted the diversity in the field. Nearly half of the publications ($n = 52$, 48%) could not be assigned to one conceptual framework category because several frameworks were addressed without highlighting which was primarily used ($n = 24$, 22%), no conceptual framework was mentioned ($n = 23$, 21%), or frameworks from other fields of research were applied to modelling ($n = 5$, 5%). A full list of the conceptual frameworks is given in the electronic supplementary material.

5.2 Analysis of Modelling Goals, Characteristics of Modelling Tasks, and Perspectives

All *modelling goals* described in Sect. 2.2 were identified in the publications (Table 6), with *formative goals* appearing more frequently than others. *Formative goals* primarily focused on fostering modelling competence, although some studies also aimed to enhance other competencies or skills, such as creativity (Özbek & Cho, 2023). In contrast to the other types of goals, *cultural goals* (both emancipatory and those related to mathematics) were mentioned only rarely (Table 6).

Furthermore, we found that a significant number of publications either did not specify any goals or referred to other modelling goals that did not align with those outlined in Sect. 2.2. Some of these other modelling goals were related to general or practical teaching objectives that were not exclusively focused on learning mathematics. For instance, some modelling goals aimed to address the diversity of learners in class by including modelling tasks (Ay & Ostkirchen, 2021) or to promote learning across domains (Huffman & Mentzer, 2021).

Table 6 Frequencies and percentages of modelling goals

Modelling goals	*n*	Example
Pragmatic	16 (15%)	Jablonski (2023)
Formative	24 (22%)	Beckschulte (2020)
Cultural emancipatory	5 (5%)	Edelen et al. (2020)
Cultural mathematics	3 (3%)	Dickes et al. (2020)
Psychological (motivational and sense-making)	16 (15%)	Jung and Brady (2023)
Psychological (learning)	11 (10%)	Kafetzopoulos and Psycharis (2022)
No or other	20 (19%)	Huffman and Mentzer (2021)
Mixed	13 (12%)	Albarracín et al. (2023)

Table 7 Frequencies and percentages of characteristics of modelling tasks

Characteristics of modelling tasks	*n*	Example
Authenticity	30 (28%)	Vos and Frejd (2020)
Cognitive richness	29 (27%)	Beckschulte (2020)
Epistemological richness	4 (4%)	Yvain-Prébiski (2021)
Motivating or illustrative	15 (14%)	Ärlebäck and Frejd (2021)
Mathematical richness	11 (10%)	Kafetzopoulos and Psycharis (2022)
No or mixed	19 (18%)	Chamberlin and Parks (2020)

Regarding the *characteristics of modelling tasks* (Table 7), *authenticity* and *cognitive richness* were most frequently identified as the primary task characteristics. *Epistemological richness* was only rarely highlighted as an important task characteristic.

Furthermore, to address the question of which *modelling perspectives* (RQ2c) were most frequently addressed in recent modelling research, we analyzed the co-occurrence of *modelling goals* and *characteristics of modelling tasks* (Table 8).

The cells highlighted in dark gray represent the six modelling perspectives that emerged from the studies (based on *goals* and *tasks*). In total, 52 publications (48%) matched one of the modelling perspectives described by Blum (2015) and Kaiser and Sriraman (2006). The *applied modelling* perspective (pragmatic goals, authenticity) was noted in 8 publications (7%) (e.g., Jablonski, 2023), the *educational modelling* perspective (formative goals, cognitive richness) in 17 publications (16%) (e.g., Lu & Kaiser, 2022), the *sociocrit-*

Table 8 Co-occurrence of modelling goals and characteristics of modelling tasks

	Characteristics of modelling tasks						
	Aut	Cog	Ep	Mot	Mat	No	Total
Goals							
Pragmatic	8	4	1	0	1	2	16
Formative	1	17	0	1	0	5	24
Cultural emancipatory	5	0	0	0	0	0	5
Cultural mathematics	0	0	3	0	0	0	3
Psychological motivation	1	1	0	13	0	1	16
Psychological learning	4	0	0	1	6	0	11
No or other	7	4	0	0	1	8	20
Mixed	4	3	0	0	3	3	13
Total	30	29	4	15	11	19	108

Note: Aut = Authenticity, Cog = Cognitive richness, Ep = Epistemological richness; Mot = Motivating or illustrative; Mat = Mathematical richness; No = No modelling task presented or mixed characteristics

ical modelling perspective (cultural emancipatory goals, authenticity) pedagogical in 5 publications (5%) (e.g., Edelen et al., 2020), *epistemological modelling* (cultural mathematics, epistemological richness) in 3 publications (3%) (e.g., Dickes et al., 2020), the *pedagogical modelling* (psychological goals related to motivation or illustration) in 13 publications (12%) (e.g., Jung & Brady, 2023), and the *conceptual modelling* perspective (psychological goals related to learning, mathematical richness) in 6 publications (6%) (e.g., Kafetzopoulos & Psycharis, 2022). Thus, all perspectives were represented, with *educational modelling* and *pedagogical modelling* being more dominant than the others.

In addition, two new combinations of *modelling goals* and *characteristics of modelling tasks* appeared (highlighted in light gray in Table 8), indicating an inductive enrichment of modelling perspectives. *Pragmatic goals* co-occurred in 4 publications (4%), with *cognitive richness* as the primary task characteristic, and *psychological learning goals* co-occurred in 4 publications (4%), with *authenticity* as the primary task characteristic (e.g., Brunet-Biarnes & Albarracín, 2024).

The combination of *pragmatic goals* with *cognitive richness* as a characteristic of modelling tasks can be seen as a blend of *applied* and *educational perspectives*. This approach emphasizes the need to address the specific cognitive demands of modelling in the classroom by focusing on cognitively rich tasks, rather than emphasizing the importance of authentic tasks to prepare students for solving problems in the real world. We labeled this perspective "practical modelling". For example, Cohen-Nissan and Kohen (2023) examined a five-day online summer camp for ninth graders designed to introduce students to mathematics within the tech industry. Although the primary goals aligned with pragmatic modelling goals—preparing students for future citizenship, careers, and scientific issues in life—cognitive richness was the defining characteristic of the modelling tasks. These tasks were designed to develop specific subcompetencies (e.g., understanding real-world situations or identifying mathematical models) and aimed to provide students with a comprehensive understanding of the modelling process.

Table 9 Numbers and percentages of self-reported perspectives and their alignment with Blum's (2015) terminology

Self-reported perspective	*n*	Related to perspective (Blum, 2015)
Realistic modelling	4 (4%)	Applied modelling
Educational modelling	2 (2%)	Educational modelling
Ethnomodelling	2 (2%)	Sociocritical modelling
Sociocritical modelling	2 (2%)	Sociocritical modelling
Contextual modelling	1 (1%)	Pedagogical modelling
Models and Modeling	10 (9%)	Pedagogical modelling
Cognitive modelling	5 (5%)	Cognitive modelling

The combination of *psychological learning goals* and *authenticity* suggests a perspective holding that the authenticity of tasks is crucial for achieving the goal of enhancing the learning of mathematical content through modelling. We labeled this perspective "experiential modelling". For example, Brunet-Biarnes and Albarracín (2024) stated: "We chose mathematical modelling activities to promote students' conceptual learning of mathematics," which we coded as a psychological learning goal. The authors also emphasized the importance of authenticity, arguing that tasks should be familiar and drawn from students' everyday lives to support the learning of mathematical concepts: "During the mathematics learning process, students must have the opportunity to develop and apply mathematical concepts and procedures in situations close to their own reality so that they understand the abstract concepts that describe them" (Brunet-Biarnes & Albarracín, 2024).

In addition to the perspectives resulting from *modelling goals* and the *characteristics of modelling tasks* according to our analysis, we captured perspectives explicitly named in these publications by the authors themselves as *self-reported perspectives* (Table 9). We found self-reported perspectives in 26 of the 108 analyzed publications.

The most frequently reported perspective was the Models and Modeling perspective (e.g., Aguilar, 2021) based on the theoretical Modelling Eliciting Activities (MEA) approach (Lesh & Doerr, 2003), related to the pedagogical perspective in our typology.

5.3 Analysis of the Relationship Between Study Attributes and Modelling Perspectives

Regarding geographical distribution, the results showed that some countries/regions contributed to various perspectives, while others contributed primarily to a single perspective. The United States had publications assigned to four perspectives—applied, educational, epistemological, and pedagogical modelling, with most focusing on pedagogical modelling ($n = 4$, 31% of publications with this perspective, 21% of total publications from this country/region). Australia contributed to applied educational, and conceptual modelling, with a particular emphasis on conceptual modelling alongside Japan ($n = 2$, 33% of publications in this perspective, 25% of total publications from this country/region). Publications from Germany predominantly focused on educational modelling ($n = 8$, 47% of publications with this perspective, 28% of total publications from this country/region), whereas publications from Brazil primarily contributed to sociocritical modelling ($n = 2$, 40% of publications with this perspective, 67% of publications from this country/region). An overview of the geographical distribution of perspectives is shown in Table 11 in the appendix.

The distribution of participant groups across the modelling perspectives showed that the general focus on lower secondary school students applied only to educational modelling ($n = 10$, 56% of publications with this perspective, 36% of publications on lower secondary students) and pedagogical modelling ($n = 7$, 44% of publications with this perspective, 25% of publications on lower secondary students). The sociocritical perspective had a relatively strong focus on upper secondary school students ($n = 3$, 50% of publications with this perspective, 21% of publications on upper secondary students), considering, for example, indexical discourse analysis for business profitability problems (Lewis & Manouchehri, 2020). Applied modelling ($n = 4$, 44% of publications with this perspective, 24% of publications on elementary school students) and conceptual modelling ($n = 3$, 50% of publications with this perspective, 18% of publications on lower secondary students) had a relatively strong focus on elementary school students. For example, in a study conducted according to the conceptual perspective, English (2023) examined how elementary-grade students engaged in modelling within STEM learning and developed key conceptual understandings of statistical concepts, such as variation and covariation. An overview of the distribution of participants across perspectives is presented in Table 12 in the Appendix.

The distribution of methodological approaches showed that studies focusing on educational modelling differed from other modelling perspectives. Regarding educational modelling, a more balanced distribution of research approaches was evident among the publications exhibiting this perspective: qualitative ($n = 7$, 41%), quantitative ($n = 7$, 41%), and mixed methods ($n = 3$, 18%). In contrast, other perspectives primarily focused on qualitative research. Compared to other perspectives, educational modelling included more descriptive studies ($n = 11$, 61% of publications with this perspective, 65% of all publications using descriptive study designs) and experimental studies ($n = 3$, 17% of publications with this perspective, 75% of all publications using experimental study designs), with fewer case studies ($n = 3$, 17% of publications with this perspective, 10% of all case study publications). Table 13 in the Appendix provides an overview of methodological approaches by perspective.

Analyzing the relationships between modelling perspectives and conceptual frameworks revealed that the perspective of *educational modelling* authors most often referred to multiple conceptual frameworks without specifying a primary focus ($n = 7$, 41% of educational modelling publications). The second most common reference was to Blum and Leiss's (2007) modelling cycle ($n = 3$, 18% of educational modelling publications). Regarding pedagogical modelling multiple conceptual frameworks ($n = 4$, 31% of pedagogical modelling publications) were referenced as often as Lesh and Doerr's (2003) conceptual framework ($n = 4$, 31% of pedagogical modelling publications). Regarding the other perspectives, due to the small number of publications per perspective and conceptual framework, no clear connection could be identified.

6 Discussion and Future Directions

The number of journal articles published in the past four years reflects a strong interest in modelling research comparable to that of other key competence-related topics in mathematics education, such as proof and proving (Stylianides et al., 2024). The geographical distribution of these publications across continents and countries revealed contributions from countries/regions worldwide, while also highlighting a particularly strong interest from researchers in Europe, and confirming the findings of earlier literature reviews (Cevikbas, 2022; Preciado Babb et al., 2023; Stohlmann et al., 2016).

Most studies were small-scale and were conducted using qualitative research approaches. For instance, the number of experimental studies was relatively low compared to case studies and descriptive studies. Given the challenge of establishing credible knowledge in education, this review suggests that larger-scale studies are needed, but this would require a more balanced approach to the use of qualitative and quantitative approaches in line with a previous literature review (Schukajlow et al., 2018).

Most of the studies were conducted using qualitative research approaches. In contrast to the high number of case studies and descriptive studies, the number of experimental studies was relatively low. Given the importance of diverse research methodologies, this result highlights the need for more controlled studies on mathematical modelling in line with a previous literature review (Schukajlow et al., 2018).

Formative goals for the teaching/learning of modelling were most frequently mentioned, indicating that the focus of many researchers is on fostering students' competencies.

Although modelling competence was a major focus (e.g., Beckschulte, 2020), other competencies or capabilities, such as creativity (Özbek & Cho, 2023), were also identified. The finding that other competencies were also addressed suggests that modelling research contributes to discourses in other fields, such as creativity or STEM education research. In terms of the characteristics of the modelling tasks, authenticity and cognitive richness were most frequently identified. This aligns with previous literature reviews of modelling competencies (Cevikbas et al., 2022), which have indicated that the characteristics of being realistic or authentic were frequently mentioned by authors when discussing tasks.

Our findings also showed that all the modelling perspectives identified by Kaiser and Sriraman (2006) and Blum (2015) are evident in recent research even though mostly not explicitly acknowledged by the authors themselves. Educational modelling, followed by pedagogical modelling, was the most frequently observed perspective, whereas epistemological modelling was rarely evident. This finding suggests that the theoretical basis of the epistemological modelling perspective needs further examination to understand why it is rarely empirically represented. One possible explanation for this observation is that the combination of the characteristics of cultural goals and epistemological richness may not capture this perspective effectively and may require retheorization. Two additional combinations were identified in our review, in addition to those of goals and characteristics: pragmatic goals with cognitively rich problems (titled practical modelling); and psychological learning goals with authentic problems (titled experiential modelling). This finding suggests that nuances of already theorized perspectives exist. The practical modelling perspective emphasizes the importance of cognitive richness when designing tasks that require students to solve problems in the real world (e.g., Cohen-Nissan & Kohen, 2023). Experiential modelling, by contrast, highlights authenticity as a key characteristic of tasks aimed at supporting students' understanding and acquisition of mathematical knowledge through connections to personal real-world experiences (e.g., Brunet-Biarnes & Albarracín, 2024). These findings highlight the importance of exploring new perspectives in modelling, in this case by analyzing the role of cognitive richness in achieving pragmatic goals and task authenticity in attaining psychological learning goals. This means that future research should seek to elaborate further on how the characteristics of tasks contribute to the achievement of specific modelling goals.

An analysis of self-reported perspectives (i.e., the perspectives to which authors explicitly referred in their contributions) revealed a different picture from the analysis based on modelling goals and tasks, with the Models and Modelling perspective (Lesh & Doerr, 2003, related to pedagogical modelling) being the most frequently referenced. An explanation for this finding is that the theoretical foundation of this perspective is stronger compared to others, including an explicit description of its background. In contrast, researchers working on other perspectives less often explicitly referred to a perspective. These findings suggest that further theorization is needed regarding perspectives to explore the unique features of each perspective (see, for example, (Schukajlow et al., 2023a,b)).

The findings related to geographical distribution revealed an emphasis on particular perspectives in different countries/regions, such as Germany's emphasis on educational modelling and Brazil's emphasis on sociocritical modelling. The geographical origins of some perspectives were also evident. For example, the epistemological modelling perspective, which emerged in Romance-language-speaking regions (Kaiser & Sriraman, 2006), was reflected in publications from France and Canada. However, the distribution also revealed a more globalized picture, with countries such as the United States and Australia contributing to various modelling perspectives.

The distribution of groups of participants (e.g., across sectors in education) revealed notable gaps. It is not surprising that only limited modelling research has focused on early childhood education, as few international curricula include expectations that this age group will engage with modelling activities. More surprisingly, no studies on the upper secondary sector have focused on the applied and conceptual modelling perspectives. In contrast, studies related to sociocritical modelling were found relatively often for this population. These findings extend the notion of local trends identified by Preciado Babb et al. (2023) by linking specific perspectives to school levels. This issue requires further research to establish why different aspects of modelling are unevenly emphasized across upper secondary mathematics education.

Additionally, the distribution of the research approaches and study designs identified in this review revealed differences between the perspectives. This might explain the differences between the findings in previous literature reviews (Cevikbas et al., 2022; Schukajlow et al., 2018). Educational modelling has a stronger focus on quantitative research and experimental studies than other perspectives. This may be due to the specific interest of this perspective in identifying interventions that can improve modelling competencies. However, other perspectives could also benefit from more quantitative research, particularly based on experimental and quasi-experimental studies, as these study designs would provide complementary insights into additional opportunities for modelling research.

Additionally, differences between perspectives were evident in the distribution of research approaches and study designs. This aligns with the variations in findings between research on modelling competencies (Cevikbas, 2022) and a broader literature review on modelling (Schukajlow et al., 2018). We found that educational modelling had a stronger focus on quantitative research and experimental studies compared to the other perspectives. This may be due to the specific focus of this perspective on improving modelling competencies—leading to the implementation of interventions designed to achieve this goal. We can only speculate on why other studies appeared to employ quantitative approaches less often. This may well be aligned with the particular research questions being addressed. Future research might gain further insights into these perspectives by employing more balanced (qualitative, quantitative, and mixed methods) research methodologies and conducting more experimental studies to complement current approaches.

Our results also showed that studies aligned with the educational modelling perspective, identified by goal and task characteristics, most often drew on Blum and Leiss's (2007) modelling cycle as conceptual framework. In contrast, pedagogical modelling appeared to most often refer to the conceptual framework by Lesh and Doerr (2003), validating the theoretical background of this perspective. However, the authors often referred to multiple conceptual frameworks in their investigations, indicating, in the best-case scenario, an awareness of the diversity of conceptual lenses that might be used in modelling research, and in the worst case, a lack of understanding about the important alignment of research questions with both conceptual frameworks and methodological approaches. The choice of a conceptual framework and a methodological approach should be guided by the specific focus of the respective study (Kaiser, 2017). It is possible that the diversity of conceptual frameworks used in modelling research reflects the different foci of investigative work, indicating the theoretical richness of the field (Geiger & Frejd, 2015). This positive message must be tempered, however, by our finding that, despite some clustering, most conceptual frameworks were referenced in only single publications, which may indicate a low consensus regarding the application of theory.

While this diversity of perspectives may support rich discussions based on different perspectives on modelling, it also suggests that it is challenging to bring these perspectives together and focus on the "big" problems in the field. Such an endeavor should begin by developing a mutual awareness and understanding of conceptual differences and the reasons behind these differences. Although the source of this diversity is probably the cultural and historical differences among conceptual frameworks, many conceptual frameworks share similarities, including key modelling activities, such as mathematizing, and differ mainly in specific activities or in slight variations in the way activities are labeled. This might explain why we found numerous authors referring to multiple frameworks, especially when analyzing specific modelling activities for mathematizing (e.g., Stillman & Brown, 2014). It is obvious that research studies must be anchored in theory, and this can be achieved by employing one or more frameworks, although the use of multiple frameworks increases the risk of theoretical clutter, making the coherence of findings harder to establish. This observation, as an outcome of our review of the literature, suggests that there is merit in researchers investigating what different conceptual frameworks, as they are used in modelling, have in common and in what ways they differ. A deeper understanding of these commonalities and differences is an important first step in developing a coherent theoretical perspective that extends across the field, perhaps leading to an overarching framework that connects different conceptual frameworks while highlighting the unique features of each. Over time, future researchers might link this overarching framework to more general theories of teaching and learning—an important connection if curriculum developers and policymakers are to see the value of modelling within mathematics teaching and learning.

7 Strengths and Limitations

In this systematic literature review, we followed the PRISMA guidelines to ensure broad coverage of the field of modelling and the replicability of the results. However, this approach also has limitations, since it relies on the selection of search terms that do not always capture all publications aligned with the theme of a literature review and the coverage of relevant topics in available databases. One important limitation of this literature review is, that we did not explicitly addressed research related to aspects of modelling, such as studies on numeracy, mathematical literacy, or word problems. Some studies on these aspects might have been important for modelling even if the authors use other terms. Integrating these and other related research areas by including the respective search terms in future reviews will significantly enrich the findings. Furthermore, adopting a more dynamic approach to conducting literature reviews, such as those based on open science (Angerer et al., 2024), could enhance communication among researchers, provide deeper insights into the review process, and promote the advancement of scientific discourse.

In our systematic literature review, we included publications on learners from early childhood education through upper secondary school. This decision allowed us to maintain a broad scope while keeping the number of publications manageable. As a result, the literature review emphasized the learning of mathematical modelling rather than the teaching of it. However, the groups of participants vary across different perspectives (Preciado Babb et al., 2023), and a different distribution of perspectives might emerge if all educational levels, including university students and teachers, were considered. Future research should explore whether perspectives on modelling differ between learning and teaching contexts. Additionally, our analysis included only studies published in English. The results—particularly regarding geographical distribution—might differ if studies published in other languages were also analyzed.

Modelling perspectives were identified in our review based on the goals of modelling and the characteristics of modelling tasks. This allowed us to analyze these perspectives in relation to others, such as various modelling cycles and geographical distributions. However, other characteristics could be used to identify modelling perspectives. For example, Blum (2015) suggested to include the type of modelling cycle as a third component alongside the goals and the tasks, and Kaiser and Sriraman (2006) recommended considering the epistemological background and geographical origins when identifying perspectives.

Additionally, we limited the scope of our literature review to studies published between January 2020 and April 2024. We did this to capture the most recent research on modelling and to keep the number of publications manageable, particularly given the extensive coding of modelling goals and task characteristics required. However, we recommend that future literature reviews should focus on single perspectives without restricting the timeframe, allowing for an analysis of the trends and developments of single perspectives over a longer period.

8 Contributions to the Special Issue on International Perspectives on Mathematical Modelling

We analyzed the contributions published in this special issue regarding the goals of modelling and the characteristics of modelling tasks. This analysis resulted in the identification of various perspectives on modelling. In this section, we present the perspectives and themes addressed in the articles.

The *applied modelling* perspective is absent from the contributions to this special issue. *Educational modelling* is addressed in six articles. Based on this perspective, teaching approaches were explored, including method-integrative and teacher-directive approaches (Lindl et al., in press) as well as a teaching approach that included using a solution plan (Vogelsanger-Holenstein et al., 2025). Additionally, obstacles faced by teachers when implementing modelling were examined (Yang et al., 2025). Other contributions of the educational perspective focused on the relationship between modelling and creativity (Lu et al., 2025) or metacognition (Vorhölter, 2025; Zhu et al., 2025). The *sociocritical modelling* perspective is addressed in three contributions, focusing on the notion of fairness in modelling (Ärlebäck & Frejd, 2025), examining various enablers of modelling (Geiger et al., 2025), and analysing the contributions of African mathematical knowledge to the development of a pedagogical action for ethnomodelling (Oray et al., 2025). In contrast, the *epistemological modelling* perspective is not represented in the contributions. Three contributions addressed the *pedagogical modelling* perspective, focusing on assumption-making when solving Fermi problems (Albarracín & Ärlebäck, 2025), openness in respect to students' motivation (Wiehe et al., 2025), and the spatial modelling of young learners (English, 2025). The perspective of *conceptual modelling* was identified for Roan et al. (2025) who examined task classification systems for multiplication tasks. The *practical modelling* perspective, newly identified in our study, is addressed by Kohen (2025) with a focus on STEM education. Ten contributions could not be assigned to any specific perspective, as they do not explicitly describe a modelling goal, do not detail the characteristics of modelling tasks, or employ combinations of goals and task characteristics that do not align with the previously identified modelling perspectives. Among these contributions, the following topics are prominent: problem posing (Ferrando et al., 2025; Sevinc, 2025; Leikin et al., 2025); modelling with technology, including video problems (Da Rocha Canedo et al., in press); explanatory videos (Cevikbas et al., 2025); solving modelling problems with the use of digital tools (Quarder et al., 2025); the role of analogies in knowledge transfer (Saeki et al., 2025); teaching tactics to support modelling (Stillman & Brown, 2025); and preservice teachers' self-efficacy (Ikeda et al., 2025).

The distinct perspectives and topics addressed by the contributors to this special issue highlight the diversity of modelling research and its connections to broader educational and societal goals.

9 Conclusion

Based on our systematic literature review, we conclude that modelling research is a thriving field with significant diversity in goals, tasks, and international perspectives. This literature review revealed that all perspectives on modelling suggested by Blum (2015) and Kaiser and Sriraman (2006) are represented—albeit with varying degrees of prominence—in recent discourse on mathematical modelling. Additionally, we identified two new perspectives—practical modelling and experiential modelling—which emerged from the combination of pragmatic goals with cognitively rich tasks and psychological learning goals with authentic tasks.

The need to synthesize research emphasizes the importance of conducting systematic literature reviews on each perspective. Empirical work is essential for each modelling perspective, with particular emphasis on under-researched populations, such as upper secondary students, based on the educational perspective and early childhood populations across all perspectives. Important theoretical work is needed to advance theory regarding perspectives on mathematical modelling and to develop an integrative framework that connects the various conceptual frameworks of modelling.

Appendix

Table 10 Perspectives and goals according to the typologies by Blum (2015) and Kaiser and Sriraman (2006)

Perspectives (*Goals*) according to Blum (2015)	Perspectives (*Goals*) according to Kaiser and Sriraman (2006)
Applied modelling (*Pragmatic goals*)	Realistic modelling (*Pragmatic-utilitarian goals*)
Educational modelling (*Formative goals*)	Educational - didactical modelling (*Pedagogical and subject-related goals*)
Socio-critical modelling (*Cultural emancipatory goals*)	Socio-critical modelling (*Pedagogical goals*)
Epistemological modelling (*Cultural mathematical goals*)	Theoretical modelling (*Theory-oriented goals*)
Pedagogical modelling (*Psychological goals, related to motivation and sense-making*)	Contextual modelling (*Subject-related and psychological goals*)
Conceptual modelling (*Psychological goals, related to mathematical learning*)	Educational - conceptual modelling (*Pedagogical and subject-related goals*)

Table 11 Perspectives and countries/regions of authors' affiliations

Perspective	Countries/regions	Frequency	Total number of publications
Applied	Australia, Germany	2 each	8
	Cyprus; Norway; Spain, Sweden; Chinese Taipei; Thailand; USA; Vietnam	1 each	
Educational	Germany	8	17
	Australia	4	
	China; USA	2 each	
	Austria; Japan; Netherlands; South Africa; Switzerland; Chinese Taipei	1 each	
Socio-critical	Brazil; USA	2 each	5
	Norway	1	
Epistemological	USA	2	3
	Canada; France	1 each	
Pedagogical	USA	4	13
	Sweden; Turkey	2 each	
	Italy; Japan; Mexico; New Zealand; South Africa; Spain	1 each	
Conceptual	Australia; Japan	2 each	6
	Greece; Portugal	1 each	

Table 12 Distribution of participant groups among modelling perspectives

	Learners' population				
	Preschool	Elementary school	Lower secondary school	Upper secondary school	Total
Perspective					
Applied	0	4	5	0	9
Educational	0	3	10	5	18
Socio-critical	0	1	2	3	6
Epistemological	0	2	2	2	6
Pedagogical	1	4	7	4	16
Conceptual	1	3	2	0	6
Total	2	17	28	14	61

Note. In total, 61 participant groups were analyzed across 52 publications, as some studies focused on multiple populations.

Table 13 Distribution of study designs among modelling perspectives

	Study design					
	Case study	Design based research	Descriptive study	Quasi-experimental study	Experimental study	Total
Perspective						
Applied	5	0	1	1	1	8
Educational	3	0	11	1	3	18
Socio-critical	4	0	1	0	0	5
Epistemological	2	0	1	0	0	3
Pedagogical	11	1	2	0	0	14
Conceptual	6	0	1	0	0	7
Total	31	1	17	2	4	55

Note. In total, 55 study designs were used across 52 publications, as some puplications reported multiple studies featuring different study designs.

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References

Aguilar, J. J. (2021). Modeling through model-eliciting activities: An analysis of models, elements, and strategies in high school. The cases of students with different level of achievement. *Mathematics Teaching-Research Journal*, *13*(1), 52–70.

Albarracín, L., & Ärlebäck, J. B. (2025). Exploring the role of assumptions in mathematical modeling teacher training using Fermi problems. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01677-0.

Albarracín, L., Segura, C., Ferrando, I., & Gorgorió, N. (2023). Supporting mathematical modelling by upscaling real context in a sequence of tasks. *Teaching Mathematics and Its Applications*, *41*(3), 183–197. https://doi.org/10.1093/teamat/hrab027.

Angerer, E., Burke, M. C., Dirks, S., Bakker, A., Bilinski Torres, A., & Pieters, T. (2024). The living library: A process-based tool for open literature review, probing the boundaries of open science. *SN Social Sciences*, *4*(9), 167. https://doi.org/10.1007/s43545-024-00964-z.

Ärlebäck, J. B., & Frejd, P. (2021). The Red Book Activity—a Model Eliciting Activity to Introduce and Initiate a Section on Statistics Focusing on Variability and Sampling. *Mathematical Modelling Education in East and West*, 595–605. https://doi.org/10.1007/978-3-030-66996-6_50.

Ärlebäck, J. B., & Frejd, P. (2025). Exploring Swedish secondary students' mathematical models of fairness in ranking real-life, multi-faceted heptathlon outcomes. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01678-z.

Arrieta, J., & Díaz, L. (2015). A modeling perspective from socioepistemology. *Revista Latinoamericana De Investigación En Matemática Educativa*, *18*(1), 19–48. https://doi.org/10.12802/relime.13.1811.

Ay, I., & Ostkirchen, F. (2021). Influence of social background on mathematical modelling—the DiMo+ project. In F. K. S. Leung, G. Stillman, G. Kaiser, & K. Wong (Eds.), *International perspectives on the teaching and learning of mathematical modelling ser. mathematical modelling education in East and West* (1st ed., pp. 93–102). Springer.

Barbosa, J. C. (2006). Mathematical modelling in classroom: A socio-critical and discursive perspective. *ZDM – Mathematics Education*, *38*(3), 293–301. https://doi.org/10.1007/BF02652812.

Beckschulte, C. (2020). Mathematical Modelling with a Solution Plan: An Intervention Study About the Development of Grade 9 Students' Modelling Competencies. *Mathematical Modelling Education and Sense-Making*, 129–138. https://doi.org/10.1007/978-3-030-37673-4_12.

Blum, W. (2015). Quality teaching of mathematical modelling: What do we know, what can we do? In J. S. Cho (Ed.), *Proceedings of the 12th international congress on mathematical education* (pp. 73–96). Springer.

Blum, W., & Leiss, D. (2007). How do students and teachers deal with mathematical modeling problems? The example of sugerloaf. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling: Education, engineering and economics - ICTMA12* (pp. 222–231). Horwood.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM – Mathematics Education*, *38*(2), 86–95. https://doi.org/10.1007/BF02655883.

Borromeo Ferri, R. (2018). *Learning how to teach mathematical modeling in school and teacher education*. Springer. https://doi.org/10.1007/978-3-319-68072-9.

Brunet-Biarnes, M., & Albarracín, L. (2024). Exploring the negotiation processes when developing a mathematical model to solve a Fermi problem in groups. *Mathematics Education Research Journal*, *36*(1), 177–198. https://doi.org/10.1007/s13394-022-00435-9.

Carreira, S., & Blum, W. (2021). Mathematical modelling in the teaching and learning of mathematics: Part 1. *Quadrante: Revista De*

Investigação Em Educação Matemática, *30*(1), 1–7. https://doi.org/10.48489/quadrante.24926.

Cevikbas, M. (2022). Fostering mathematical modelling competencies: A systematic literature review. In N. Buchholtz, B. Schwarz, & K. Vorhölter (Eds.), *Initiationen mathematikdidaktischer Forschung: Festschrift zum 70. Geburtstag von Gabriele Kaiser* (pp. 51–73). Springer. https://doi.org/10.1007/978-3-658-36766-4_3.

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2022). A systematic literature review of the current discussion on mathematical modelling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering. *Educational Studies in Mathematics*, *109*(2), 205–236. https://doi.org/10.1007/s10649-021-10104-6.

Cevikbas, M., Greefrath, G., & Siller, H.-S. (2023). Advantages and challenges of using digital technologies in mathematical modelling – a descriptive systematic literature review. *Frontiers in Education*, *8*, 1142556. https://doi.org/10.3389/feduc.2023.1142556.

Cevikbas, M., Mießeler, D., & Kaiser, G. (2025). Pre-service mathematics teachers' experiences and insights into the benefits and challenges of using explanatory videos in flipped modelling education. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01650-x.

Chadegani, A. A., Salehi, H., Yunus, M. M., Farhadi, H., Fooladi, M., Farhadi, M., & Ebrahim, N. A. (2013). A Comparison between Two Main Academic Literature Collections: Web of Science and Scopus Databases. *Asian Social Science*, *9*(5). https://doi.org/10.5539/ass.v9n5p18.

Chamberlin, S. A., & Parks, K. (2020). A comparison of student affect after engaging in a mathematical modeling activity. *International Journal of Education in Mathematics, Science and Technology*, *8*(3), 177–189. https://doi.org/10.46328/IJEMST.V8I3.721.

Chevallard, Y. (2007). Readjusting didactics to a changing epistemology. *European Educational Research Journal*, *6*(2), 131–134. https://doi.org/10.2304/eerj.2007.6.2.131.

Cohen-Nissan, O., & Kohen, Z. (2023). Secondary school students' competencies and motivation to engage in mathematical modelling tasks in a virtual learning environment. *Frontiers in Education*, *8*. https://doi.org/10.3389/feduc.2023.1140364.

Czocher, J. A. (2017). Mathematical modeling cycles as a task design heuristic. *The Mathematics Enthusiast*, *14*(1–3), 129–140.

Da Rocha Canedo, N., Borba, M. C., & Villa-Ochoa, J. A. (in press). The role of video in a mathematical modelling teacher education course. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01681-4.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics*, *5*(1), 44–48.

Dickes, A. C., Farris, A. V., & Sengupta, P. (2020). Sociomathematical norms for integrating coding and modeling with elementary science: A dialogical approach. *Journal of Science Education and Technology*, *29*(1), 35–52. https://doi.org/10.1007/s10956-019-09795-7.

Durandt, R., Blum, W., & Lindl, A. (2022). Fostering mathematical modelling competency of first year South African engineering students: What influence does the teaching design have? *Educational Studies in Mathematics*, *109*(2), 361–381. https://doi.org/10.1007/s10649-021-10068-7.

Edelen, D., Bush, S. B., Simpson, H., Cook, K. L., & Abassian, A. (2020). Moving toward shared realities through empathy in mathematical modeling: An ecological systems theory approach. *School Science and Mathematics*, *120*(3), 144–152. https://doi.org/10.1111/ssm.12395.

English, L. (2023). Multidisciplinary modelling in a sixth-grade tsunami investigation. *International Journal of Science and Mathematics Education*, *21*(S1), 41–65. https://doi.org/10.1007/s10763-022-10303-4.

English, L. (2025). Spatial modelling in second grade: Creating crab colonies. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01667-2.

English, L., Ärlebäck, J. B., & Mousoulides, N. (2016). Reflections on progress in mathematical modelling research. In Á. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education: The journey continues* (pp. 383–413). Sense Publishers.

Ferrando, I., Barquero, B., & Segura, C. (2025). Does training matter? Effect of training strategies on how pre-service teachers pose and assess modelling problems. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01658-3viv.

Frejd, P., & Bergsten, C. (2018). Professional modellers' conceptions of the notion of mathematical modelling: Ideas for education. *ZDM – Mathematics Education*, *50*(1), 117–127. https://doi.org/10.1007/s11858-018-0928-2.

Galbraith, P. (2012). Models of modelling: Genres, purposes or perspectives. *Journal of Mathematical Modelling and Application*, *1*(5), 3–16.

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *ZDM – Mathematics Education*, *38*(2), 143–162. https://doi.org/10.1007/BF02655886.

Garcia, F. J., Pérez, J. G., Higueras, L. R., & Casabó, M. B. (2006). Mathematical modelling as a tool for the connection of school mathematics. *ZDM – Mathematics Education*, *38*(3), 226–246. https://doi.org/10.1007/BF02652807.

Geiger, V. (2013). Strässer's didactic tetrahedron as a basis for theorising mathematical modelling activity within social contexts. In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (pp. 107–116). Springer. https://doi.org/10.1007/978-94-007-6540-5_9.

Geiger, V. (2024). The need for a critical orientation to mathematical modelling in times of disruption. In H.-S. Siller, V. Geiger, & G. Kaiser (Eds.), *Researching mathematical modelling education in disruptive times* (pp. 15–34). Springer. https://doi.org/10.1007/978-3-031-53322-8_2.

Geiger, V., & Frejd, P. (2015). A reflection on mathematical modelling and applications as a field of research: Theoretical orientation and diversity. In G. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modelling in education research and practice: Cultural, social and cognitive influences* (pp. 161–171). Springer.

Geiger, V., & Schmid, M. (2024). A critical turn in numeracy education and practice. *Frontiers in Education*, *9*, 1363566. https://doi.org/10.3389/feduc.2024.1363566.

Geiger, V., Galbraith, P., Niss, M., & Delzoppo, C. (2022). Developing a task design and implementation framework for fostering mathematical modelling competencies. *Educational Studies in Mathematics*, *109*(2), 313–336. https://doi.org/10.1007/s10649-021-10039-y.

Geiger, V., Galbraith, P., Niss, M., & Schmid, M. (2025). Identifying and describing generic, specific, and catalytic enablers of mathematical modelling. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01653-8.

Haines, C., Crouch, R., & Davis, J. (2000). *Mathematical modelling skills: A research instrument* (Technical Report No 55). University of Hertfordshire, Department of Mathematics.

Huffman, T. J., & Mentzer, N. (2021). The impact of modeling-eliciting activities on high school student design performance. *International Journal of Technology and Design Education*, *31*(2), 255–280. https://doi.org/10.1007/s10798-019-09557-x.

Ikeda, T., Borromeo Ferri, R., & Kawakami, T. (2025). Factors influencing Japanese pre-service teachers' self-efficacy in teaching mathematical modelling. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01666-3.

Jablonski, S. (2023). Is it all about the setting? — a comparison of mathematical modelling with real objects and their representation. *Educational Studies in Mathematics*, *113*(2), 307–330. https://doi.org/10.1007/s10649-023-10215-2.

Jha, A. (2024). Qualitative and quantitative research design. In A. Jha (Ed.), *Social research methodology: Qualitative and quantitative designs* (1st ed., pp. 161–200). Routledge. https://doi.org/10.4324/9781032624860-9.

Julie, C. (2002). Making relevance relevant in mathematics teacher education. In *Proceedings of the 2nd international conference on the teaching of mathematics (at the undergraduate level)* (pp. 1–8). Wiley.

Julie, C. (2019). Practicing teachers and the development of mathematIcal modellIng competencies through mathematIcal modellIng as content. In *Caught in the act: Reflections on continuing professional development of mathematics teachers in a collabrative partnership* (1st ed., pp. 95–118). African Sun Media.

Jung, H., & Brady, C. (2023). Modeling actions foregrounded in whole-class modeling discourse: A case study of a model-eliciting activity and a three-act task. *Mathematical Thinking and Learning*. Scopus. https://doi.org/10.1080/10986065.2023.2180849.

Kafetzopoulos, G. I., & Psycharis, G. (2022). Conceptualization of function as a covariational relationship between two quantities through modeling tasks. *Journal of Mathematical Behavior*, *67*. https://doi.org/10.1016/j.jmathb.2022.100993.

Kaiser, G. (2017). The teaching and learning of mathematical modeling. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 267–291). National Council of Teachers of Mathematics.

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM – Mathematics Education*, *38*(3), 302–310. https://doi.org/10.1007/BF02652813.

Kohen, Z. (2025). Structured mathematical modelling in an authentic scientific-engineering context. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01654-7.

Krawitz, J., Hartmann, L., & Schukajlow, S. (2024). Do task variables of self-generated problems influence interest? Authenticity, openness, complexity, and students' interest in solving self-generated modelling problems. *The Journal of Mathematical Behavior*, *73*, 101129. https://doi.org/10.1016/j.jmathb.2024.101129.

Lange, J. de. (1987). *Mathematics, insight and meaning*. Rijksuniversiteit.

Leikin, R., Boriskovsky, M., Ovodenko, R., & Miskin, M. (2025). Problem posing or mathematical modeling? The process of expert instructional design. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01668-1.

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism* (pp. 3–33). Routledge.

Leung, F. K. S., Stillman, G., Kaiser, G., & Wong, K. (Eds.) (2021). *International perspectives on the teaching and learning of mathematical modelling ser. mathematical modelling education in East and West* (1st ed.). Springer. https://doi.org/10.1007/978-3-030-66996-6.

Lewis, S. T., & Manouchehri, A. (2020). Mapping Shifting Discourses in Classroom Talk Through Indexical Discourse Analysis: From the World of Mathematics to the World of Work. *Mathematical Modelling Education and Sense-Making*, 85–94. https://doi.org/10.1007/978-3-030-37673-4_8.

Lindl, A., Durandt, R., & Blum, W. (in press). Fostering Mathematical Modelling Competency in Different Learning Environments and Educational Contexts – an Exploratory Comparative Analysis of four Intervention Studies. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01680-5.

Lu, X., & Kaiser, G. (2022). Can mathematical modelling work as a creativity-demanding activity? An empirical study in China. *ZDM - Mathematics Education*, *54*(1), 67–81. https://doi.org/10.1007/s11858-021-01316-4.

Lu, X., Kaiser, G., Zhu, Y., Ma, H., & Yan, Y. (2025). Mathematical creativity in modelling: Further development of the construct, its measurement, and its empirical implementation. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01652-9.

Maaß, K., Zehetmeier, S., Weihberger, A., & Flößer, K. (2023). Analysing mathematical modelling tasks in light of citizenship education using the COVID-19 pandemic as a case study. *ZDM – Mathematics Education*, *55*(1), 133–145. https://doi.org/10.1007/s11858-022-01440-9.

Mousoulides, N., Sriraman, B., & Christou, C. (2007). From problem solving to modelling–the emergence of models and modeling perspectives. *Nordic Studies in Mathematics Education*, *12*(1), 23–47.

Niss, M., & Blum, W. (2020). *The learning and teaching of mathematical modelling*. Taylor & Francis Group.

Niss, M., Blum, W., & Galbraith, P. L. (2007). Introduction. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 1–32). Springer.

Oray, D. C., Rosa, M., & da Silva, J. B. N. (2025). The ZIGZAG: Contributions of African Mathematical Knowledge on Gold Mines to Develop a Pedagogical Action for Ethnomodelling through the Elaboration of Ethnomodels. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01691-2.

Orey, D.C., & Rosa, M. (2021). From ethnomathematics to ethnomodelling. *Journal of Mathematics and Culture*, *15*, 148–168.

Özbek, G., & Cho, S. (2023). Effects of mathematical modelling based project production and management program on gifted students' mathematical modelling and reflective thinking for real-life problem solving. *Gifted Education International*, *39*(3), 318–336. https://doi.org/10.1177/02614294221118005.

Page, M. J., McKenzie, J. E., Bossuyt, P. M., Boutron, I., Hoffmann, T. C., Mulrow, C. D., Shamseer, L., Tetzlaff, J. M., Akl, E. A., Brennan, S. E., Chou, R., Glanville, J., Grimshaw, J. M., Hróbjartsson, A., Lalu, M. M., Li, T., Loder, E. W., Mayo-Wilson, E., McDonald, S., Moher, D., et al. (2021). The PRISMA 2020 statement: An updated guideline for reporting systematic reviews. *Campbell Systematic Reviews*, *10*(1), 89. https://doi.org/10.1186/s13643-021-01626-4.

Palm, T. (2008). Impact of authenticity on sense making in word problem solving. *Educational Studies in Mathematics*, *67*(1), 37–58. https://doi.org/10.1007/s10649-007-9083-3.

Pollak, H. (1969). How can we teach applications of mathematics? *Educational Studies in Mathematics*, *2*(2–3), 393–404.

Preciado Babb, A. P., Acuña, F. P., Rocha, Y. A. O., & Rojas, A. S. (2023). Diversity of perspectives on mathematical modelling: A review of the international landscape. In G. Greefrath, S. Carreira, & G. Stillman (Eds.), *International perspectives on the teaching and learning of mathematical modelling. Advancing and consolidating mathematical modelling: Research from ICME-14* (1st ed., pp. 43–57). Springer.

Quarder, J., Greefrath, G., Gerber, S., & Siller, H.-S. (2025). Pedagogical content knowledge for simulations and mathematical modelling with digital tools: A quasi-experimental study with pre-service mathematics teachers. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01673-4.

Roan, E., Kularajan, S., & Czocher, J. (2025). Reconsidering task classification systems for situations that can be modelled with multiplication. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01657-4.

Saeki, A., Kawakami, T., & Lamb, J. (2025). The role of children's analogies in the transfer of their knowledge in mathematical modelling processes. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01675-2.

Schukajlow, S., Kaiser, G., & Stillman, G. (2018). Empirical research on teaching and learning of mathematical modelling: A survey on the current state-of-the-art. *ZDM – Mathematics Education*, *50*, 5–18. https://doi.org/10.1007/s11858-018-0933-5.

Schukajlow, S., Kaiser, G., & Stillman, G. (2023a). Modeling from a cognitive perspective: Theoretical considerations and empirical contributions. *Mathematical Thinking and Learning*, *25*(3), 259–269. https://doi.org/10.1080/10986065.2021.2012631.

Schukajlow, S., Krawitz, J., Kanefke, J., Blum, W., & Rakoczy, K. (2023b). Open modelling problems: Cognitive barriers and instructional prompts. *Educational Studies in Mathematics*, *114*, 417–438. https://doi.org/10.1007/s10649-023-10265-6.

Sevinc, S. (2025). Reflective problem posing instructions for preservice mathematics teachers to elicit characteristics of the modelling problems: Context vis-a-vis content. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01672-5.

Siller, H.-S., Günster, S. M., & Geiger, V. (2024). Mathematics as a central focus in STEM – theoretical and practical insights from a special study program within pre-service (prospective) teacher education. In Y. Li, Z. Zeng, & N. Song (Eds.), *Advances in STEM education. Disciplinary and interdisciplinary education in STEM* (pp. 317–343). Springer.

Skovsmose, O., & Greer, B. (2012). Introduction: Seeing the cage? The emergence of critical mathematics education. In O. Skovsmose & B. Greer (Eds.), *Opening the cage: Critique and politics of mathematics education* (pp. 1–19). Springer.

Stillman, G. (2011). Applying metacognitive knowledge and strategies in applications and modelling tasks at secondary school. In G. Kaiser, W. Blum, R. B. Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling: ICTMA14*, Springer.

Stillman, G., & Brown, J. (2014). Evidence of implemented anticipation in mathematising by beginning modellers. *Mathematics Education Research Journal*, *26*(4), 763–789. https://doi.org/10.1007/s13394-014-0119-6.

Stillman, G., & Brown, J. (2025). Teaching tactics supporting beginner modelling in upper secondary. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01676-1.

Stillman, G., Kaiser, G., & Lampen, C. (Eds.) (2020). *Mathematical modelling education and sense-making*. Springer.

Stohlmann, M. S., & Albarracín, L. (2016). What is known about elementary grades mathematical modelling. *Education Research International*, *2016*(1), 5240683. https://doi.org/10.1155/2016/5240683.

Stohlmann, M. S., DeVaul, L., Allen, C., Adkins, A., Ito, T., Lockett, D., & Wong, N. (2016). What is known about secondary grades mathematical modelling–a review. *Journal of Mathematics Research*, *8*(5), 12–28. https://doi.org/10.5539/jmr.v8n5p12.

Stylianides, G. J., Stylianides, A. J., & Moutsios-Rentzos, A. (2024). Proof and proving in school and university mathematics education research: A systematic review. *ZDM. Zentralblatt für Didaktik der Mathematik*, *56*(1), 47–59. https://doi.org/10.1007/s11858-023-01518-y.

Verschaffel, L., Schukajlow, S., Star, J., & van Dooren, W. (2020). Word problems in mathematics education: A survey. *ZDM – Mathematics Education*, *52*(1), 1–16. https://doi.org/10.1007/s11858-020-01130-4.

Vogelsanger-Holenstein, M., Schukajlow, S., & Bruckmaier, G. (2025). Mathematical modelling and self-efficacy: Immediate and long-lasting effects of teaching mathematical modelling with a solution plan. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01674-3.

Vorhölter, K. K. (2025). Metacognitive behaviour when working on modelling problems in small groups. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01661-8.

Vos, P. (2011). What is 'authentic' in the teaching and learning of mathematical modelling? In G. Kaiser, W. Blum, R. B. Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling: ICTMA14* (pp. 713–722). Springer.

Vos, P., & Frejd, P. (2020). The object-tool duality in mathematical modelling. A framework to analyze students' appropriation of Sankey diagrams to model dynamic processes. *Avances De Investigacion En Educacion Matematica 17*, 52–66. https://doi.org/10.35763/AIEM.V0I17.305.

Wiehe, K., Schukajlow, S., Krawitz, J., & Rakoczy, K. (2025). Openness in mathematical modelling: Do experiences of competence and autonomy mediate the effects of an intervention on modelling problems on task values and cost? *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01670-7.

Yang, X., Li, X., Deng, Z., & Kaiser, G. (2025). Obstacles to in-service Chinese high school mathematics teachers' implementation of mathematical modelling in classrooms: an empirical investigation of teachers' perspectives. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01660-9.

Yvain-Prébiski, S. (2021). Didactical Adaptation of Professional Practice of Modelling: A Case Study. *Mathematical Modelling Education in East and West*, 305–315. https://doi.org/10.1007/978-3-030-66996-6_26.

Zhu, Y., Huang, J., & Xu, B. (2025). Metacognitive activities in high school students' mathematical modelling: Findings from an epistemic network analysis. *ZDM – Mathematics Education*, *57*(2/3). https://doi.org/10.1007/s11858-025-01664-5.