Abstract

Algebra assumes a key role within mathematics. Symbolic algebraic language provides the foundation for the description of mathematical structures. With the help of variables general laws can be established and functional dependencies can be described. Algebra forms the basis of almost all mathematical activities at a formal level. The understanding of variables and the accurate manipulation of variables are essential for the understanding of advanced mathematics. Mathematic-didactical studies suggest that the difficulties encountered by students when learning algebra centre around a lack of understanding of the concept of a variable and the algebraic manipulation of variables.

This dissertation will provide a new perspective to the aforementioned difficulties using the historical didactical approach to reconstruct mathematical knowledge. Firstly, the historical development of algebra is analysed and secondly the introduction of algebra in current mathematical education is explored by using the classical method of textbook analysis.

The analysis of the historical development of algebra examines the underlying understanding of algebra and identifies epistemological obstacles in the evolution of the concept of variables. Symbolic algebraic language has developed over the centuries starting from a pure verbal expression, continuing with the use of letters for unknown quantities and ending with completely symbolic expressions. Acquisition of these concepts is each linked with different epistemological obstacles. However, these epistemological obstacles are rooted in the nature of the objects themselves and therefore cannot be avoided. The semantic context during the development of algebra provides insight to the ontological state of the algebraic objects and thus to the underlying understanding of algebra. As a paradigmatic model for the understanding of algebra during its historical development Euler's textbook „Vollständige Anleitung zur Algebra" is analysed. Euler's approach is based on measurable quantities. Mathematical statements are justified by empirical examples or by reference to an empirical subject area. Contrary to the modern understanding of algebra, which is formal and abstract, Euler's approach in his textbook is empirical.

The textbook analyses of the two textbook series, „Elemente der Mathematik" and „Schnittpunkt", serve to determine the understanding of algebra as it is learned from these books, through their introduction of concepts, the nature of the discussed objects, and their justifications of mathematical statements. The basic arithmetic operations of natural numbers are explained with the help of lengths joined one after another. As a result, a geometrical concept of the arithmetic operations is established. Algebraic terminology and educational contents are introduced by using of meaningful mathematical problems and practical applications, and are illustrated with the help of geometrical representations. Algebraic rules are also explained and justified in terms of empirical quantities. From the point of view of the textbooks' authors these representations serve to illustrate mathematical concepts, but from the student's perspective the representations are the objects of algebra themselves. Therefore, it is justified to say that the student's understanding of algebra by studying of these textbooks is foremost empirical. This contrasts with teachers‘ formal abstract mathematical knowledge. However, the textbooks‘ understanding is historically legitimated as demonstrated by Euler's Algebra, and is thus an adequate description of theory for students.