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To my wife

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Chapter 1

Introduction

How do monetary policy and financial regulation affect economies populated by heterogeneous households under financial frictions? The existing theoretical literature that analyzes the impact of these policy interventions on household saving and consumption typically applies relatively stylized frameworks concerning the role of household heterogeneity and financial frictions. While there is a growing branch of the literature examining monetary policy effects in heterogeneous agent economies under incomplete markets, these studies are typically silent on the role of financial frictions for the monetary transmission (e.g. Debortoli and Gali (2018), Kaplan et al. (2018) or Auclert (2019)). In contrast, studies on the effects of financial regulation on household behavior emphasize the importance of these frictions while the focus is typically on relatively stylized specifications of household heterogeneity (e.g. Bianchi (2011), Bianchi and Mendoza (2018) or Jeanne and Korinek (2019)).

As an important contribution to this literature, my doctoral thesis consists of three self-contained chapters that examine how *heterogeneous households* interact in *quantitative models* of household saving and consumption under *financial frictions* and how monetary policy and financial regulation affect these economies. This thesis finds that heterogeneous household economies with financial frictions give rise to a novel monetary transmission channel as well as to important ways financial interventions affect these economies that are neglected in the existing literature.

Chapter 2 addresses the effects of monetary policy in this type of framework. The research in this chapter is joint work with Andreas Schabert and provides a novel mechanism of monetary policy non-neutrality. The monetary transmission to the real economy is one of the most discussed research areas in economics.[1] In monetary policy analyses, the commonly used benchmark models typically disregard the role of household heterogeneity (see e.g. Woodford (2003)). Empirical studies, however, show that the

[1]See for example Woodford (2003).

sensitivity of consumption to interest rate changes in these types of models is typically too small to be reconciled with empirical evidence (e.g. Campbell and Mankiw (1989), Yogo (2004) or Canzoneri et al. (2007)). In recent years, therefore, a growing literature has started to re-analyze the effects of monetary policy in models with heterogeneous agents under incomplete markets (e.g. Debortoli and Gali (2018), Kaplan et al. (2018) or Auclert (2019)). These studies show that the transmission of monetary policy to the real economy depends to relatively large extent on the way agents' heterogeneity is modelled. This heterogeneity, for example, affects the monetary transmission by changing the relative magnitude of direct and indirect (general equilibrium) effects of monetary policy (e.g. Kaplan et al. (2018)) or by generating important redistributive effects across different income groups (e.g. Auclert (2019)).

Based on broad empirical evidence, the vast majority of studies on monetary policy effects on household saving and consumption considers nominal rigidities in goods and labor markets as the main sources of monetary non-neutrality.[2] In contrast, the role of financial frictions in this regard has received much less attention in the literature, even though their existence is undisputed.[3] Financial frictions, for example, can rationalize the existence of borrowing constraints whose role for household decisions is emphasized by numerous empirical and theoretical studies.[4] While these financial constraints in models with incomplete markets are crucial drivers of household behavior as well as important sources of inefficiencies, the literature typically disregards the interaction of these constraints with monetary policy.[5] However, if monetary policy is able to influence borrowing constraints, it might exert important redistributive effects that would complement the above mentioned studies on monetary policy in economies with heterogeneous agents. Therefore, chapter 2 addresses this interaction of monetary policy and borrowing constraints in a heterogeneous household economy calibrated to match distributional targets based on US data and thereby works out a novel mechanism of monetary non-neutrality which is based on borrowing constraints related to current income. This type of financial constraint induces redistributive effects of monetary policy among heterogeneous households that exist even under flexible goods prices.

The central elements of this analysis are that only nominal debt is available and that outstanding debt is limited by current income. In principal, this type of constraint can be rationalized by the inability of borrowers to commit to repayment. Under a repudiation-proof debt contract, outstanding debt is then restricted by current income. Such limits for unsecured debt, for which broad empirical evidence exists (see e.g. Jappelli (1990) or Lian and Ma (2019)), do not account for expected price changes until maturity, implying that monetary policy can alter the real terms of borrowing. In our analysis, we introduce this

[2]This is true for monetary policy analyses that neglect the role of household heterogeneity as Woodford (2003) or Christiano et al. (2005) as well as for studies that acknowledge this heterogeneity as Kaplan et al. (2018).

[3]See for example the extensive survey of the role of financial frictions in macroeconomics in Brunnermeier et al. (2013).

[4]See e.g. Diaz and Luengo-Prado (2010), Saiz (2010), Mian and Suffi (2011) or Guerrieri and Iacoviello (2017)).

[5]Note that there does exist a borrowing constraint in Kaplan et al. (2018). This non-microfounded constraint, however, induces no additional monetary transmission channels in their study.

type of borrowing constraint into a Huggett (1993)-type heterogeneous agent economy with idiosyncratic endowment and non-state contingent bonds, calibrate this model to US data and examine the effects of an exogenously given change in the inflation rate on saving and consumption of heterogeneous households.

Given this borrowing constraint, a reduction in inflation tends to increase the maximum amount of debt that can be issued, while it also raises the beginning-of-period stock of debt to be repaid. The impact of inflation depends on the probability of borrowers to be unconstrained at maturity which especially depends on the persistence of the endowment process. The lower this probability is, the smaller is the beneficial effect of lower inflation for borrowers. The effect of monetary policy on the debt limit is opposed to the classic debt deflation effect when borrowers are initially indebted. While a reduction in the inflation rate increases the debt limit, the real value of initial debt obligations rises, too. The overall effect is therefore ex-ante ambiguous and depends on the initial debt/wealth position as well as the willingness to borrow. The study shows that lower inflation particularly benefits agents with low initial debt by relaxing effective borrowing constraints, whereas highly indebted borrowers suffer from the dominant debt deflation effect. For our benchmark calibration, we find that aggregate welfare losses due to the inflation reduction via the (conventional) effects of initial debt deflation are reduced by 83% via the effects induced by the borrowing constraint. If the persistence of the endowment process is low enough, a reduction of the inflation rate can even enhance aggregate welfare.

Chapter 3 and 4 shift the focus towards the analysis of financial regulation in an economy with an empirically relevant specification of borrowing constraints and household heterogeneity. Research on financial regulation in macroeconomic frameworks intensified in the aftermath of the last financial crisis. The severe consequences of the disruptions in financial markets convinced most economists to complement the microprudential approach to financial regulation by a macroprudential one that focusses on the financial system as a whole (see e.g. Hanson et al. (2011)). The literature on macroprudential financial regulation especially addresses the interaction between de-leveraging and asset prices induced by price-dependent collateral constraints. Given that the scope to borrow against collateral crucially depends on the price of (pledgeable) assets, borrowers tend to de-leverage in states where asset prices fall, giving rise to a financial amplification mechanism (see e.g. Kiyotaki and Moore (1997)). Given that (borrowing) agents do not internalize the impact of their behavior on prices, they might tend to overborrow. This pecuniary externality with regard to the collateral price provides a straightforward rationale for a macroprudential approach to financial regulation as for example shown by Lorenzoni (2008), Bianchi (2011), Bianchi and Mendoza (2018) or Jeanne and Korinek (2019). These studies then typically find that the pecuniary externality regarding the collateral price can be corrected by an ex-ante policy that constrains or dis-incentivizes borrowing, such as a reduction in the loan-to-value ratio or a

Pigouvian tax on borrowing.

While these studies focus on the role of financial frictions - especially of price-dependent borrowing constraints - the analyzed frameworks abstain from empirically relevant specifications of household heterogeneity. They usually employ infinite-horizon small open economy models (based on Mendoza (2010)), where a representative domestic agent borrows from abroad, or three-period closed economy models with distinct types of agents, who either borrow or lend (e.g. Lorenzoni (2008) or Jeanne and Korinek (2019)). The focus of these analyses is on the effects of collateral externalities that result from endogenous collateral prices in borrowing limits. Distributive externalities, however, which are a second type of pecuniary externalities that generally matters for (in-)efficiency in economies under financial frictions (see e.g. Davila and Korinek (2018)), are irrelevant in these studies. Distributive effects arise when the relative price at which agents trade goods or assets changes. This price adjustment then redistributes funds among buyers and sellers and thereby influences relative demand. The studies on financial regulation mentioned above, however, focus on models in which only a representative (domestic) borrower is able to hold the collateral asset. In these frameworks, therefore, changes in collateral prices do not induce distributive effects across lenders and borrowers. Furthermore, given that the real interest rate on bonds is fixed in these studies, distributive effects via changes in the real rate also do not exist.

While these studies analyze policy interventions in relatively stylized frameworks, my thesis examines financial regulation in an economy with both - an empirically relevant specification of borrowing constraints as well as of household heterogeneity.[6] The framework is a Huggett (1993)-type heterogeneous agent economy extended by an occasionally binding collateral constraint and by a durable good, that has a dual role as a non-financial (collateral) asset and as a utility providing consumption good.[7] The model is calibrated to match several aggregate and distributional targets based on US data following Diaz and Luengo-Prado (2010). In this framework, changes in market prices induce not only collateral effects but also distributive effects via adjustments in collateral prices and the real interest rate.

The magnitude of pecuniary externalities and - as a consequence - the role of policy interventions in this type of model especially depend on the following two aspects. Firstly, it depends on the sensitivity of market prices. The higher the price sensitivity is, the more important are pecuniary externalities. A key driver of the sensitivity of the collateral price is the elasticity of aggregate supply of the collateral asset with respect to collateral price changes. The lower this elasticity of supply is, the larger is the price sensitivity. In the model applied to analyze financial interventions in this thesis - as in the studies

[6] Note that the empirical literature shows the existence of a pronounced observed and unobserved heterogeneity among households (e.g. Krueger et al. (2016a) or Hai and Heckman (2017)) and that theoretical studies find that this heterogeneity is important for the evolution of aggregate quantities and prices (e.g. Krusell and Smith (1998)) as well as for normative questions (e.g. Kaplan and Violante (2014)).

[7] Since the 1980s, household debt secured by durable consumption goods (like vehicles or especially residential real estate) has accounted for more than 90% of US household debt in the United States (see Hintermaier and Koeniger (2016)).

mentioned above - the supply of the collateral asset is fixed such that the price sensitivity is relatively strong. Secondly, the impact of financial regulation depends on the relative magnitude of collateral and distributive effects of price changes which can have opposing effects that are not equally distributed among households. In the model in this thesis - in contrast to the studies mentioned above - the role of both types of price effects for policy interventions is examined.

Before chapter 4 analyzes the effect of financial regulation on household saving and consumption, chapter 3 at first examines the role of the price sensitivity of the collateral asset and the relative magnitude of collateral and distributive effects on household decisions in this type of framework *absent* any policy interventions. To do so, this study examines the relative magnitude of direct effects and indirect ones arising from changes in the collateral price on household saving and consumption induced by a shock on household preferences for the durable good. The direct effects of the shock are those that operate in the absence of any changes in market prices. In general equilibrium, additional indirect effects on households' behaviour arise from changes in market prices that emanates from the direct effects.

The magnitude of the shock is set to induce an increase in the price of durables in the impact period which is close to the impulse response of the housing price under a shock on housing preferences in the estimated model in Guerrieri and Iacoviello (2017). The motivation to analyze shocks on preferences for the durable consumption good is based on quantitative studies which find that house price movements can be explained to a large extent by housing preference shocks (see e.g. Liu et al. (2013), Berger et al. (2017) or Guerrieri and Iacoviello (2017)). The focus of this study is on the fluctuations of the collateral price - as in the studies mentioned above - while the real interest rate is time-invariant.

At first, the relative magnitude of direct and indirect effects is analyzed in a model version with a fixed aggregate supply of collateral. Empirical studies, however, find that growth rates in prices of housing, which is typically the dominant component of household wealth in the US (see e.g. Li and Yao (2007)), are not homogeneous across areas in the US in the 1980s and from 2002 to 2006 and that this heterogeneity is driven to a relatively large extent by heterogeneous supply elasticities of housing (see e.g. Glaeser et al. (2008), Mian and Sufi (2009), Saiz (2010) or Mian et al. (2013)). Areas with a relatively inelastic housing supply, due to e.g. geographical limitations, experienced relatively high growth rates in house prices in the 1980s and from 2002 to 2006 whereas areas with a relatively elastic supply had lower growth rates.[8] Given the high share of housing in wealth of the household sector, household borrowing and consumption depends to a relatively large degree on the sensitivity of housing prices and therefore on the elasticity of housing supply (see e.g. Mian and Sufi (2011, 2013)). For example, those areas

[8]Saiz (2010) constructs an objective index measuring the possibility to expand new housing in metro areas. If land-topology in a metro area is such that expansion from the center is restricted - for example by hills, oceans or lakes - this area gets a low housing supply elasticity score.

that experienced relatively high growth rates in house prices due to a relatively inelastic housing supply typically also experienced larger increases in home-equity borrowing and thereby also in consumption. These results imply that the model version with a fixed supply of durables generates an upper bound for the importance of indirect price effects. Therefore, the relative magnitude is re-analyzed in a model version with a more elastic supply in which the importance of price effects is reduced.

The analysis finds that in the considered type of model changes in the price of the collateral asset are not only a key driver of household behavior under a *fixed* aggregate supply of collateral but also when the price sensitivity is relatively strongly reduced by increasing the elasticity of aggregate collateral supply. This result suggests that the large scope for financial regulation under fixed collateral supply - derived in chapter 4 as well as in the studies above - should also exist under less extreme assumptions concerning the supply elasticity. Furthermore, the study finds that households' saving and consumption behavior are especially driven by distributive effects and only to lower extent by collateral ones. This result is in contrast to the importance of collateral effects in the studies on financial regulation with relatively stylized specifications of household heterogeneity. While these studies focus on the collateral price effect, the results from chapter 3 suggest that policy interventions, that address pecuniary externalities, should also take into account distributive effects of price changes. This finding i.a. motivates the analysis of financial regulation in a heterogeneous household economy in chapter 4 where - in contrast to the existing studies o financial regulation - both price effects - collateral and distributive effects - emerge.

Chapter 4 is joint work with Joost Röttger and Andreas Schabert and examines financial regulation and corrective policies in a heterogeneous agent economy similar to one analyzed in chapter 3. In contrast to chapter 3 and also in contrast to the literature on financial regulation mentioned above, the real interest rate in this framework is time-variant such that changes in two market prices - the price of durables and the real interest rate - are important for household saving and consumption. As a novel contribution, we analyze the impact of financial regulation in a framework in which household decisions are not only affected by collateral effects but also by distributive effects via changes in the collateral price and in the real interest rate.

In the first part of chapter 4, we solve for Pigouvian-type taxes/subsidies in debt and asset markets that can correct the different pecuniary externalities and thereby implement a constrained efficient allocation in a simplified three-period model. The framework analyzed in our study is an extension of the model in Davila and Korinek (2018) by adding a price dependent borrowing constraint in the initial period. The analysis then shows that without further information on preferences and the distributions of bonds and durables it is *in general* unclear whether the implementation of a constrained efficient allocation requires taxes or subsidies on debt and durables.

In the second part of chapter 4, we therefore analyze the effects of *given* policy interventions in a quantitative model calibrated to US data. While existing studies, as explained above, focus on collateral effects, this analysis reveals that the welfare consequences of policy interventions in credit and asset markets mainly depend on distributive effects of price changes, i.e. distributive effects of collateral price changes and of changes in the real interest rate.

The analysis finds that a loan-to-value reduction - an instrument typically suggested in the literature on financial regulation - benefits only few unconstrained borrowers and reduces social welfare. A Pigouvian-type debt-tax/savings-subsidy, however, raises collateral prices and lowers interest rates, which stimulates borrowing and generates welfare gains for almost all income groups. The analysis suggests that in heterogeneous agent economies especially interventions in savers' decisions are beneficial while these instruments are typically ineffective in the macroprudential studies mentioned above. Overall, collateral effects are of minor importance in this framework and interest rates rather than asset price responses are decisive for welfare effects of corrective policies.

Chapter 2

Monetary Policy, Financial Constraints, and Redistribution

This chapter is based on Loenser and Schabert (2019).

2.1 Introduction

Based on broad empirical evidence, the vast majority of studies on monetary policy effects considers nominal rigidities in goods and labor markets as the main sources of monetary non-neutrality. In contrast, the role of financial frictions in this regard has received much less attention in the literature, even though their existence is undisputed. Debt is typically issued in nominal terms and in a non state-contingent way, such that changes in the price level can alter real payoffs. This transmission channel of unexpected monetary policy (the so-called Fisher debt deflation channel) is well-established and has been examined in several studies.[1] In this paper, we examine a novel channel of monetary transmission via financial constraints expressed in nominal terms.[2] Thereby, monetary policy exerts redistributive effects, which complements other recently-studied general equilibrium effects of monetary policy on heterogeneous agents (see Kaplan et al. (2018) or Auclert (2019)).[3]

The central elements of our analysis are that only nominal debt is available and that outstanding debt is limited by current income. The latter assumption is motivated by empirical evidence provided by numerous studies that current income or earnings serve as a relevant limit for unsecured debt (see e.g.

[1]For example, Doepke et al. (2015) or Auclert (2019) are recent contributions to this literature. They further provide comprehensive overviews over studies on distributional effects of monetary policy.

[2]Gariga et al. (2017) examine the transmission of monetary policy via nominal rigidities induced by fixed-rate and adjustable-rate mortgage contracts.

[3]Kaplan et al. (2018) show that indirect (general equilibrium) effects via labor income of heterogeneous households can outweight direct effects of monetary policy, in particular, via intertemporal substitution.

Japelli and Pagano (1989), Jappelli (1990), Duca and Rosenthal (1993), Del Río and Young (2006), Choi et al. (2018), Dettling and Hsu (2018), Drechsel (2019) or Lian and Ma (2019)) and by various theoretical studies that focus on current – rather than on future – factors that limit debt. In particular, studies on fire sales consider the impact of asset sales on their current period value (see e.g. Stein (2012), Woodford (2016), Davila and Korinek (2018)), inducing debt deleveraging by tightening the limit for end-of-period debt within the same period. Moreover, studies that rationalize macroprudential regulation by pecuniary externalities consider borrowing limits that restrict end-of-period debt by current period income valued at current relative prices (see e.g. Bianchi (2011), Benigno et al. (2016), Korinek (2018) or Schmitt-Grohe and Uribe (2019)).[4] This type of constraints can principally be rationalized by the inability of borrowers to commit to repayment. If debt can be renegotiated after issuance, borrowers – who cannot commit – might make a take-it-of-leave-it offer to reduce the value of debt. Suppose that lenders who reject this offer, can seize borrowers' wealth. When assets are not available, as considered in this paper, an offer will thus be accepted when the repayment value of debt does not exceed borrowers' available income. Under a repudiation-proof debt contract, outstanding debt is then restricted by current income.[5]

We explore implications for monetary policy and its redistributive effects under fully flexible goods prices when repayment of unsecured debt is constrained by current income. Apparently, the debt limit in terms of commodities at maturity can then be affected by price level changes and thereby by monetary policy. To make this argument more transparent, consider a nominal repayment S_{t+1} that is contracted in t at the period t price Q_t and due in $t+1$. Suppose that it is limited at issuance by current income, $S_{t+1} \leq P_t y_t$, where y_t denotes an exogenous real income and P_t the price level in period t. Then, real debt repayment in terms of commodities in period $t+1$, $x_{t+1} = S_{t+1}/P_{t+1}$, has to satisfy $x_{t+1} \leq y_t/\pi_{t+1}$, where π_{t+1} denotes the inflation rate $\pi_{t+1} = P_{t+1}/P_t$. Thus, a change in the inflation rate alters the effective debt limit, i.e. the maximum debt in terms of commodities at maturity.[6]

To understand the macroeconomic effects of monetary policy under a debt limit based on current income, consider for example an unexpected permanent increase in the inflation rate. On the one hand, a higher inflation rate implies the debt limit to shrink in terms of commodities at maturity. Given that this reduction in the value of debt at maturity is internalized by lenders, they also demand a lower debt price Q_t at issuance, which tends to reduce the maximum amount of funds that can be borrowed. On the other hand, there is a beneficial effect of the reduced debt repayment value in terms of commodities at

[4]Other theoretical studies, which consider current income as debt limits are, for example, Laibson et al. (2003) or Mendoza (2006).

[5]Note that such a constraint relates total outstanding debt to income and therefore differs from payment-to-income ratios that are relevant for mortgage (see Corbae and Quintin (2015) or Greenwald (2018)).

[6]If debt limits instead account for expected future price changes, debt limits would be specified in terms of commodities at maturity, implying that monetary policy does not affect the effective tightness of the borrowing constraints. Then, monetary policy matters just due to the (conventional) effects of initial debt deflation.

maturity, which is in fact identical to the conventional debt deflation effect in the initial period. Hence, the increase in inflation tends to reduce the maximum amount of debt that can be issued (*debt limit effect*) as well as the stock of debt to be repaid (*debt deflation effect*). The beneficial debt deflation effect is opposed to the impact on the effective debt limit. Thus, the effect of higher inflation on borrowers' overall consumption possibilities and welfare is ex-ante ambiguous, and particularly depends on the likelihood that the borrowing constraint is binding and on the borrowers' initial debt level. Moreover, when borrowing decreases due to a tighter effective debt limit under a higher inflation rate, the real interest rate and thus the real cost of borrowing tend to fall.[7]

To assess the overall impact of changes in the inflation rate, we examine two distinct models. We first consider the highly stylized case of a stationary equilibrium of an economy where agents permanently differ by their degree of patience (as for example studied by Kiyotaki and Moore (1997)). Relatively impatient agents tend to frontload consumption and are willing to borrow from more patient agents up to the maximum amount. A higher inflation rate then leads to the two effects described above: Debt repayment as well as the amount of newly issued debt are reduced. In this economy, where agents never switch types (borrower/lender), the beneficial debt deflation effect dominates the debt limit effect, such that borrowers are better off with higher inflation rates. In contrast, if the borrowing limit were exogenously tightened, say, by an exogenous reduction of the fraction of seizable income, borrowers' welfare would tend to decrease. The apparent reason is that this impulse lacks the beneficial effect from a reduction of the initial debt burden, while it reduces initial and future consumption possibilities of borrowers due to a tighter effective debt limit.

For the main part of our analysis, we focus on a second – less stylized – framework and apply an incomplete market model (see Huggett (1993)). Agents differ with regard to their random individual income, while they are equally impatient.[8] When an agent draws a very low realization of income, he is willing to borrow up to the debt limit. The adverse (beneficial) effect of a higher (lower) inflation rate that tends to lower (raise) the effective debt limit and, correspondingly, the maximum amount of borrowed funds at issuance might then outweigh the beneficial (adverse) debt deflation effect. This is actually the case when the probability of drawing again a low realization of individual income at maturity is small enough, such that the marginal valuation of funds at issuance is sufficiently higher than the expected marginal valuation of funds at maturity. Ex-ante, a borrower tends to prefer a lower inflation rate and

[7]Notably, this pecuniary externality (non-trivially) applies also for the opposite case of lower inflation, which increases the maximum debt repayment: Given that agents do not internalize how their demand for funds affects the real interest rate, a lower inflation rate can cause an increase in debt due to borrowers exploiting the higher debt limit, which tends to increase the real cost of borrowing. A related externality with regard to the real interest rate is discussed by Smith (2009).

[8]This set-up closely relates to Auclert's (2019) incomplete market model, which he uses to examine redistribution of monetary policy. In contrast to our model, the borrowing constraint in his model limits issued debt rather than outstanding debt, such that the changes in inflation does not alter the effective debt limit.

thus a higher effective debt limit even with the higher debt repayment, if he has a relatively high valuation of funds when debt is issued.[9]

We examine two versions of the incomplete market model with idiosyncratic risk. For the first version, we assume that preferences are linear-quadratic and that income shocks ensure that the borrowing constraint always binds for borrowers, facilitating aggregation and allowing for the derivation of analytical results. Under these assumptions, the competitive equilibrium of the heterogenous agents economy can be characterized in terms of a representative borrower and a representative lender. For this economy, we show analytically that a reduction in the inflation rate can enhance welfare of the representative borrower if the autocorrelation of income shocks is sufficiently low, which tends to raise the gain from the debt limit effect relative to the debt deflation effect. The reason is that a constrained borrower is under a lower autocorrelation of individual income less likely to be constrained at maturity, such that the expected marginal utility of consumption at maturity is lower than the marginal utility of consumption at issuance. With this favorable effect for constrained agents, monetary policy can in principle enhance aggregate welfare by lowering inflation.

To quantitatively assess the effects of changes in the inflation rate, we apply a second version of the model, imposing less restrictive assumptions. Specifically, we consider a standard CRRA utility function, long-term debt, and a more realistic income process, such that the borrowing constraint is not permanently binding. Given that this version cannot be solved analytically, we calibrate the model to match characteristics of US postwar data and solve it numerically. The calibration is based on an inflation rate of 2%. We then assume that the central bank reduces the average inflation rate to -2%. We find that borrowers with a high initial debt position suffer most from lower inflation, given that the debt deflation effect is dominant for them. In contrast, borrowers who are initially less indebted gain from lower inflation due to a dominant debt limit effect. Apparently, a household with positive wealth benefits from both effects when the central bank reduces the inflation rate: Initial real wealth increases as well as debt limits in future periods in which they might be constrained. For our benchmark calibration, we find that aggregate welfare losses due to the inflation reduction via the (conventional) effects of initial debt deflation are reduced by 83% via the effects induced by the borrowing constraint.[10]

To assess the sensitivity of these results, we vary the maturity of debt, examine an equally-sized increase (instead of a reduction) in inflation, and we re-calibrate the model for an alternative income process with lower autocorrelation. Firstly, a reduction of debt maturity leads to an almost proportional

[9] Studies on monetary policy in incomplete market economies with zero debt and fixed borrowing limits typically find effects of higher inflation rates that are beneficial for borrowers (see e.g. Akyol (2004), Algan and Ragot (2010), or Kryvtsov et al. (2011)).

[10] As a measure for aggregate welfare, we apply agents' ex-ante expected lifetime utility, which relates to an utilitarian welfare measure.

reduction of the welfare effects. As described by Doepke and Schneider (2006), debt deflation effects of non-transitory inflation changes increase with the maturity of nominal debt. Likewise, fixing nominal payments for longer terms is crucial for the effects of monetary policy via nominal rigidities induced by fixed-rate mortgage contracts (see Gariga et al. (2017)). The debt limit effect is also enhanced with higher maturities, which – like a lower autocorrelation of income – increase the likelihood that borrowers are unconstrained at maturity. Secondly, we find that an increase in inflation leads to almost symmetric effects compared to an equally-sized inflation reduction. These effects are slightly less pronounced, given that the distortionary effects of the borrowing constraint are reduced under higher inflation rates. Finally, we also consider a lower autocorrelation for the income process, as suggested by Guvenen (2007) for the US and by Floden and Line (2001) for Sweden. Re-calibrating the model for Guvenen's (2007) estimates, we find that aggregate welfare (slightly) increases for a reduction in the inflation rate, consistent with the analytical results derived for the simplified version of the model.

In Section 2, we examine the redistributive effects of monetary policy in a stylized model with two agents which are characterized by different degrees of impatience. In Section 3, we apply a model where heterogeneity of agents, instead, originates from idiosyncratic income shocks, and examine the inflation effects analytically as well as numerically. Section 4 concludes.

2.2 A model with patient and impatient agents

Before we examine financial frictions for monetary policy effects in a Huggett (1993) type model (see Section 2.3), we analyze the effects in a more stylized model. We assume that two types of agents differ with regard to their degree of patience induced by different discount factors (as in Kiyotaki and Moore (1997)). The patient agents with the higher discount factor will permanently be lenders and the impatient agents with the lower discount factor will permanently be borrowers. The persistence of agents' types will be the main difference between this model and the model in Section 2.3, where agents might switch roles in the credit market depending on their particular income draws and their endogenous wealth positions.

2.2.1 The set-up

There is a continuum of infinitely lived agents of mass two, who have equal income from an exogenous labor supply, consume and trade one-period nominal non-state contingent discount bonds at the issuance price $1/R_t$ $(= Q_t)$, paying one unit of currency in period t. For simplicity, we neglect uncertainty and disregard holdings of fiat money, which can be interpreted as the limit case of a cashless economy, while we assume that money only serves as a unit of account (see also Sheedy (2014) or Auclert (2019)).

Households maximize the present value of utilities $\sum_{t=0}^{\infty}(\beta_i)^t u(c_{i,t})$ where $c_{i,t}$ is consumption of agent i and $i = l$ $(i = b)$ is the index of lenders (borrower), who constitute half of the population. The parameter β_i is the discount factor of agent i and satisfies $\beta_b < \beta_l < 1$. The utility function is identical for all agents and satisfies $u' > 0$ and $u'' < 0$. Agent i's budget constraint in nominal terms is given by

$$P_t c_{i,t} = -(S_{i,t+1}/R_t) + S_{i,t} + P_t y_{i,t}, \tag{2.1}$$

where P_t denotes the price level, $S_{i,t}$ denotes nominal debt with $S_{l,t} > 0$ and $S_{b,t} < 0$. The endowment $y_{i,t}$ will be identical for all agents, $y_{i,t} = y_t$. In each period, agents first trade in the asset market before they enter the goods market.

As the central element of our analysis, we consider that debt is restricted by current income, for which several studies found empirical support.[11] To rationalize this observation, we consider that agents cannot commit to repay debt. We assume that debt can be renegotiated after issuance. Borrowers might then make a take-it-of-leave-it offer to reduce the value of outstanding debt. Lenders who reject this offer, can take borrowers to court and can seize their available income up to a fraction $\gamma < 1$ (due to imperfections in legal enforcement). Hence, a repudiation-proof debt contract restricts debt repayment to $\gamma P_t y_{i,t}$, leading to the following borrowing constraint[12]

$$-S_{i,t+1} \leq \gamma P_t y_{i,t}. \tag{2.2}$$

In real terms, i.e. in terms of period t commodities, the budget and borrowing constraints are given by $c_{i,t} = -s_{i,t+1}/R_t + s_{i,t}/\pi_t + y_t$ and $-s_{i,t+1} \leq \gamma y_{i,t}$, where $\pi_t := P_t/P_{t-1}$ denotes the inflation rate and $s_{i,t+1} := S_{i,t+1}/P_t$ the real value of wealth at the end of the period t, which is a predetermined state variable in $t+1$. Accordingly, real end-of-period debt $s_{i,t+1}$ is constrained by a fraction of real income in period t, $y_{i,t}$. Yet, when debt matures, prices might have changed, such that the real value $s_{i,t+1}$ has to be adjusted by the inflation rate to account for real debt burden in terms of commodities at maturity, i.e. $s_{i,t+1}/\pi_{t+1} = S_{i,t+1}/P_{t+1}$. Accordingly, the borrowing constraint $-S_{i,t+1}/P_{t+1} \leq \gamma y_{i,t}/\pi_{t+1}$ shows that a higher inflation rate reduces the limit for debt repayment in terms of commodities at maturity $t+1$. Maximizing lifetime utility subject to the budget- and borrowing constraints, leads to the borrowers' and lenders' first order conditions given by

[11] Examples are Japelli and Pagano (1989), Jappelli (1990), Duca and Rosenthal (1993), Del Río and Young (2006), Choi et al. (2018), Dettling and Hsu (2018). Lian and Ma (2019) and Drechsel (2019) further provide evidence that firms' borrowing is constrained by their earnings.

[12] Studies where borrowing is also constrained by the current value of income, are for example Laibson et al. (2003), Mendoza (2006), Bianchi (2011), Benigno et al., (2016), Korinek (2018), Schmitt-Grohe and Uribe (2019).

$$u'(c_{b,t})/R_t = \beta_b u'(c_{b,t+1})\pi_{t+1}^{-1} + \zeta_{b,t}, \tag{2.3}$$

$$u'(c_{l,t})/R_t = \beta_l u'(c_{l,t+1})\pi_{t+1}^{-1}, \tag{2.4}$$

where $\zeta_{b,t}$ denotes the multiplier on the borrowing constraint (2.2), which is irrelevant for lenders. Further, the associated complementary slackness condition, $\zeta_{b,t}(\gamma y_{i,t} + s_{b,t+1}) \geq 0$, holds.

In this cashless economy, the central bank can control the nominal interest rate via a channel system. Given that changes in the nominal interest rate will affect the (expected) inflation rate, we will assume, for convenience, that the central bank controls the inflation rate by setting the interest rate in order to meet specific inflation targets, as for example in Sheedy (2014). Given that there is no aggregate uncertainty, we will focus on constant inflation targets, $\pi > 0$. Notably, the inflation choice might imply values for the nominal interest rate for which the zero lower bound, $R_t \geq 1$, is binding.

The equilibrium is then a set of sequences $\{c_{b,t}, c_{l,t}, s_{b,t+1}, s_{l,t+1}, R_t, \zeta_{b,t} \geq 0\}_{t=0}^{\infty}$ for a given constant inflation rate $\pi > 0$ and a given constant endowment $y_{i,t} = y > 0$ satisfying (2.3) and (2.4), $c_{b,t} = -(s_{b,t+1}/R_t) + (s_{b,t}/\pi) + y$, $c_{b,t} + c_{l,t} = 2y$, $-s_{b,t+1} \leq \gamma y$, $\zeta_{b,t}(\gamma y_{i,t} + s_{b,t+1}) \geq 0$, and $-s_{b,t} = s_{l,t}$, given $-s_{b,0} = s_{l,0}$. The real interest rate satisfies (2.4) and will be strictly positive in a long-run equilibrium, i.e. it equals the inverse of the lenders' discount factor, $R/\pi = 1/\beta_l > 1$. Given that $\beta_b < \beta_l$, borrowers will be constrained in a long-run equilibrium, $\zeta_b > 0$ (see 2.3).

2.2.2 Results

We now examine the effects of a permanent change in the inflation rate in this simple economy. Specifically, we consider an unanticipated permanent inflation shock in period $t = 0$, where borrowers are endowed with beginning-of-period wealth $s_{b,0} = S_{b,0}/P_{-1}$. Suppose that the latter is sufficiently close to its steady state value, such that the economy will be in the steady state in period $t \geq 1$. Using the steady state real interest rate, $R/\pi = 1/\beta_l$, and that borrowers are always constrained, $s_{b,t} = -\gamma y$, the borrowers' budget constraint, $c_{b,t} = -s_{b,t+1}R_t^{-1} + s_{b,t}\pi_t^{-1} + y$, implies initial consumption and steady state consumption (in $t \geq 1$) to satisfy

$$c_{b,0} = \underbrace{[s_{b,0}/\pi]}_{A.)\text{ initial debt deflation effect}} + \underbrace{\gamma\,[y/R_0(c_{l,0}, c_{l,1}, \pi)]}_{B.)\text{ initial debt limit effect}} + y, \tag{2.5}$$

$$c_{b,t} = \underbrace{-\underbrace{[\gamma y/\pi]}_{C.)\text{ debt deflation effect}} + \underbrace{[\gamma y/\pi]\beta_l}_{D.)\text{ debt limit effect}}}_{<0} + y, \ \forall t \geq 1, \tag{2.6}$$

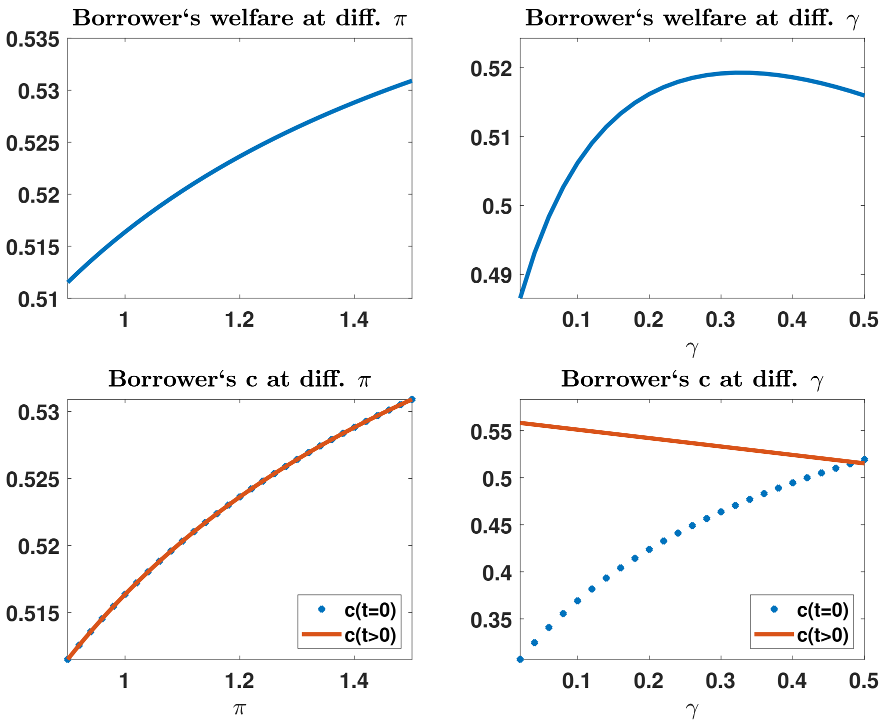
where the beginning of period stock of debt, $s_{b,0} < 0$, is given. Consider an unexpected permanent increase in the inflation rate in period 0. This tends to increase borrowers' consumption in period 0 according to the *initial debt deflation effect* (see A. in 2.5), which is independent of the borrowing constraint. At the same time, higher inflation tends to reduce $c_{b,0}$ due to the *initial debt limit effect* (see B.). Specifically, a higher inflation rate causes lenders' to demand a higher nominal interest rate according to their credit supply schedule (2.4), such that the amount of funds raised at issuance $\gamma y/R_0$ decreases. From $t = 1$ onwards, the economy is in the steady state, where the real interest satisfies $R/\pi = 1/\beta_l > 1$. As in $t = 0$, the *debt deflation effect* (C.) tends to raise and the *debt limit effect* (D.) tends to reduce borrowers' consumption under higher inflation (see 2.6). In contrast to the initial period, the latter effect is unambiguously weaker than the former, as debt is rolled over at a constant positive interest rate. Hence, borrowers' consumption strictly increases with higher inflation for $t \geq 1$. Two aspects should be noted here.

Firstly, a higher inflation rate would have no further effect on borrowers' consumption than the initial debt deflation effect (A.), if the borrowing limit were specified terms of commodities in period $t + 1$. If borrowing were instead limited by $-S_{i,t+1} \leq \gamma P_{t+1} y_t \Longleftrightarrow -s_{i,t+1}/\pi_{t+1} \leq \gamma y_t$, borrowers' consumption in $t \geq 1$ (in which the economy is in a steady state) would be given by $c_b = -\gamma y\,(1 - \beta_l) + y$. Monetary policy would then be neutral in the steady state, since the debt limit is not affected by price changes. Secondly, a permanent reduction of the fraction of seizable income γ starting in period $t = 0$, which for example might be imposed by regulation, is actually not equivalent to an increase in the inflation, which can be seen from (2.5). The reason is that a change of γ in $t = 0$ cannot affect real initial wealth $s_{b,0}$ (for which γ would already have to be changed in $t = -1$).

For demonstrative purposes, we provide quantitative results for the effects of inflation. To abstract from transitional dynamics, we assume that borrowers are initially endowed with the steady state stock of debt, $s_{b,0} = -\gamma y$. Notably, the latter assumption implies that the borrower will be in a steady state in all periods $t \geq 0$ with $s_{b,t} = -\gamma y$ regardless of the inflation rate. Figure 2.1 shows the steady state effect of the inflation rate π and the parameter γ on borrower's consumption and welfare for the period $t = 0$ and $t > 0$. The corresponding effects on lenders are shown in Figure 2.9 in Appendix C.2.

We compute welfare of borrowers by $v_b = \sum_{t=0}^{\infty} \beta_b^t u(c_{b,t}) = u(-\gamma(y/\pi)\,(1 - \beta_l) + y)/(1 - \beta_b)$ and display consumption equivalents $CE_b = u^{-1}\left((1 - \beta_b)v_b\right)$. The chosen parameter values are $y = 0.56$, $\beta_l = 0.82$, $\beta_l = 0.84$ and $\gamma = 0.487$ with a CRRA utility function $u(c_i) = c_i^{1-\sigma}/(1 - \sigma)$ and $\sigma = 2$ (see Section 2.3.3 for a discussion of the parameter values). The first column shows the effects of a change in the inflation rate. Consumption and welfare of borrowers unambiguously increase with the inflation rate, in accordance with the effects described above. The second column of Figure 2.1 displays the effects of changes in the fraction γ at a constant inflation rate π.

Figure 2.1: Consumption and welfare (in consumption units) of relatively impatient borrowers



A reduction of γ has a positive impact on borrowers' consumption in $t > 0$ by lowering debt (see solid line), which qualitatively accords to the impact of a higher inflation rate. In contrast to the latter, a lower value γ has an adverse effect on borrowers' initial consumption, since it simply implies a more restricted access to external funds under a given initial debt level $s_{b,0}$ (see 2.5). For $\gamma < 0.32$, borrowers' welfare monotonically decreases with a tighter borrowing constraint induced by a lower fraction γ. For larger values of γ that are nevertheless associated with a binding borrowing constraint, we find that borrowers' welfare can increase under a reduction of γ. The reason is that adverse effects of an increased borrowing on (higher) interest rate and future consumption possibilities are not internalized by agents (see Gottardi and Kubler (2015) for a similar finding).

2.3 A model with idiosyncratic risk

In this Section, we examine the effects of inflation in a Hugget-type model, where agents have the identical discount factor. Idiosyncratic endowment shocks induce agents to borrow/lend, while there is no aggregate risk. As in the model presented in the previous section, only non-state-contingent nominal debt is available such that agents cannot share risk. This model can in general not be solved analytically, given that agents might have different histories of $y_{i,t}$-draws and their decisions depend on their beginning-of-period wealth $s_{i,t}$. We will therefore apply some simplifying assumptions in the first part of the analysis. Specifically, we consider a constant borrowing limit, a linear-quadratic utility function and we assume that the borrowing constraint binds for agents who draw a low income level, which facilitates aggregation and derivation of analytical results. In the second part, we calibrate a more realistic version of the model to assess the inflation effects in a quantitative way. There, we examine a less stylized framework, which will be calibrated for US data.

2.3.1 The set-up

Consider an economy with infinitely lived and infinitely many households i of mass two. These households share the same utility function, but might differ with regard to a random idiosyncratic income. Preferences of a household i are given by

$$E_i \sum_{t=0}^{\infty} \beta^t u(c_{i,t}), \tag{2.7}$$

where E_i denotes an expectations operator and $c_{i,t}$ consumption of household i. As before, the utility function $u(c_{i,t})$ is assumed to satisfy $u' > 0$ and $u'' < 0$. Note that in the subsequent analysis, we will examine the model for two different types of utility functions. First, we apply a linear-quadratic (LQ) utility function, which facilitates aggregation and the derivation of analytical results. Second, we use a standard CRRA utility function for a numerical analysis.

Real income $y_{i,t} = Y_{i,t}/P_t$ is identically and independently distributed over all households, but might be serially correlated over time. We consider a finite set of n possible realizations of the random variable y, y_1,... y_n, where $y_i < y_{i+1}$ and with transition probabilities $p_{k,l}$ from state k to state l and a positive unconditional mean $Ey_i = \overline{y} > 0$. Households who draw an income y_i tend to borrow from households who draw $y_j > y_i$. Shocks are realized at the beginning of each period, before the asset market opens. Once, these shocks are realized, households enter the asset market where they repay debt and can borrow/lend funds from/to other households.

To allow for a more realistic debt maturity, we introduce long-term debt contracts that mature probabilistically (see for example Chatterjee and Eyigungor (2012)). We assume that each unit of outstanding debt matures in the subsequent period with a constant probability θ. Given that a unit bond issued in period $t-k$ leads to the same payoff as an unit bond issued in $t-k'$ with $k' > k > 1$, it is sufficient to keep track of the total number of bonds. Bond units are infinitesimally small, such that for s_{t+1} bond units outstanding at the beginning of period $t+1$ real payment obligations are $\theta s_{t+1}\pi_{t+1}^{-1}$ with certainty. Let Q_t be the issuance price of a unit bond in period t. The budget constraint for a household i in income state $y_{i,t}$ for $i = 1, ..., n$ and wealth state $s_{i,t}$ is

$$P_t c_{i,t} + Q_t \left(S_{i,t+1} - (1-\theta) S_{i,t} \right) = \theta S_{i,t} + P_t y_{i,t}. \tag{2.8}$$

For one-period debt, $\theta = 1$, the budget constraint reduces to (2.1), where $R_t = 1/Q_t$. As borrowers cannot commit to repay, the borrowing constraint restricts total outstanding debt $-S_{i,t+1}$ (irrespective of maturity) to a fraction of current income (2.2). To disclose the main mechanism, we will further apply a simplified borrowing constraint for the derivation of analytical results in the first part of the analysis: $-s_{i,t+1} \leq b$, where the constant b can be interpreted as referring to mean income, $b = \gamma \overline{y}$ (see Section 2.3.2).

Households aim at maximizing lifetime utility (2.7) subject to (2.2) and (2.8) taking prices as given. The first order conditions for a household i in income state $y_{i,t} = y_j$ for $j = 1, ..., n$ and wealth state $s_{i,t} = s_t$ is

$$u'_{i,t} Q_t = \beta E_{i,t} \left[\left(Q_{t+1}(1-\theta) + \theta \right) u'_{i,t+1} / \pi_{t+1} \right] + \zeta_{i,t}, \tag{2.9}$$

where $\zeta_{i,t} \geq 0$ denotes the multiplier on (2.2). Further, the budget constraint (2.8) is binding and the complementary slackness conditions for (2.2), $0 = \zeta_{i,t}(\gamma y_j + s_{i,t+1})$, and $\zeta_{i,t} \geq 0$, hold. Notably, the first order condition (2.9) for one-period debt ($\theta = 1$) simplifies to $u'_{i,t}/R_t = \beta E_{i,t}[u'_{i,t+1}\pi_{t+1}^{-1}] + \zeta_{i,t}$.

In equilibrium, prices adjust such that plans are realized and markets clear. A competitive equilibrium is a set of sequences $\{c_{i,t}, s_{i,t+1}, Q_t, \zeta_{i,t}\}_{t=0}^{\infty}$ satisfying (2.9), $-s_{t+1} \leq \gamma y_j$, $c_t + Q_t \left(s_{t+1} - (1-\theta) s_t \pi_t^{-1} \right) = \theta s_t \pi_t^{-1} + y_{i,t}$, $y_t = \Sigma_i y_{i,t} = \Sigma_i c_{i,t}$ and $\Sigma_i s_{i,t+1} = 0$, and the complementary slackness conditions for a given inflation rate π_t and given $s_{i,0}$. The first best allocation $\{c_{i,t}^*\}_{t=0}^{\infty}$ evidently satisfies $u'(c_{i,t}^*) = u'(c_{j,t}^*)$ for all agents $i \neq j$, which we will consider as a benchmark case.

2.3.2 A version with two representative agents

In this subsection, we apply a simple version of the model and analytically examine the main effects of changes in the inflation rate. We consider two realizations for income, y_1 and y_2, with symmetric transition probabilities, and we consider one-period debt, $\theta = 1$. To derive analytical results, we further impose a linear-quadratic utility function.

Assumption 1 *Households' preferences satisfy* $u(c_{i,t}) = (\delta c_{i,t} - c_{i,t}^2)$, *where* $\delta \geq \Sigma_i y_i$.

When preferences satisfy Assumption 1, the marginal utilities are linear in individual consumption, which greatly facilitates aggregation over individual household choices. We further consider a constant borrowing limit b and restrict our attention to the case where the variance of the preference shocks is sufficiently large such that the borrowing constraint will always be binding for agents drawing y_1. To achieve this, we apply a relatively large income difference $y_2 - y_1$ compared to the parameter b governing the tightness of the borrowing constraint.

Assumption 2 *The borrowing constraint is given by $-s_{i,t+1} \leq b$. Idiosyncratic income satisfies $y_{i,t} \in \{y_2, y_1\}$, where $p_{12} = p_{21}$, $p_{11} = p_{22} > 0$, and $(y_2 - y_1)/b$ is sufficiently large such that $\zeta_{j,t} > 0$ for all households j drawing y_1.*

Hence, borrowers' end-of-period wealth positions equals $-b$. Accordingly, lenders, which are of the same mass as borrowers, have a wealth position equal to (minus) the debt level of borrowers (b). As for the model with different degrees of patience, we analyze the effects of inflation on agents initially endowed with $s_{i,0} = -b$ or $s_{i,0} = b$ and $\Sigma_i s_{i,0} = 0$ to abstract from transitional dynamics. Under Assumptions 1 and 2, we can analytically aggregate over individual choices of agents. We separately analyze two types of agents, borrowers drawing y_1 and potential lenders drawing y_2. The choices of the former are characterized by the conditions $(\delta - 2c_{(b,i),t})/R_t = (\beta/\pi)\left[p_{11}(\delta - 2c_{(b,i),t+1}) + p_{12}(\delta - 2c_{(l,i),t+1})\right] + \zeta_{(b,i),t}$, $-s_{(b,i),t+1} \leq b$, and $c_{(b,i),t} = -s_{(b,i),t+1}R_t^{-1} + s_{(b,i),t}\pi^{-1} + y_1$, where $\zeta_{(b,i),t} \geq 0$ and $\zeta_{(b,i),t}\left(s_{(b,i),t+1} + b\right) = 0$. Given that all conditions are linear in the choice variables for $\zeta_{(b,i),t} > 0$, we can easily aggregate. Let $c_{b,t} = \Sigma_{b,i} c_{(b,i),t}$, $\zeta_{b,t} = \Sigma_{b,i}\zeta_{(b,i),t}$ and $s_{b,t+1} = \Sigma_{b,i} s_{(b,i),t+1}$. Then, we get the following set of conditions describing the behavior of a <u>representative borrower</u>:

$$(\delta - 2c_{b,t})/R_t = (\beta/\pi)\left[p_{11}(\delta - 2c_{b,t+1}) + p_{12}(\delta - 2c_{l,t+1})\right] + \zeta_{b,t}, \tag{2.10}$$

$$-s_{b,t+1} = b, \tag{2.11}$$

$$c_{b,t} = -(s_{b,t+1}/R_t) - p_{11}(b/\pi) + p_{21}(b/\pi) + y_1, \tag{2.12}$$

and $\zeta_{b,t} > 0$. Note that we used that beginning of period wealth either equals b or $-b$, depending on whether the current borrower was a lender or a borrower in the previous period. Using the law of large numbers, a fraction of p_{11} (p_{21}) of previous borrowers (lenders) draw y_1 in the current period. Thus, current period initial wealth of the representative borrower equals the weighted average $-p_{11}(b/\pi) + p_{21}(b/\pi)$. Apparently, the same arguments apply for all agents drawing y_2, such that we can proceed analogously and get the following conditions describing the behavior of a <u>representative lender</u>:

$$(\delta - 2c_{l,t})/R_t = (\beta/\pi)\left[p_{21}(\delta - 2c_{b,t+1}) + p_{22}(\delta - 2c_{l,t+1})\right], \tag{2.13}$$

$$c_{l,t} = -(s_{l,t+1}/R_t) - p_{12}(b/\pi) + p_{22}(b/\pi) + y_2. \tag{2.14}$$

Hence, we can characterize a competitive equilibrium in terms of a representative borrower and lender.

Proposition 1 *Under Assumptions 1 and 2, a competitive equilibrium with one-period debt ($\theta = 1$) can be characterized as a set of sequences $\{c_{b,t},\, c_{l,t},\, s_{b,t+1},\, R_t,\, \zeta_{b,t} > 0\}_{t=0}^{\infty}$ satisfying (2.10), (2.11), (2.13),*

$$c_{b,t} - c_{l,t} = -(2s_{b,t+1}/R_t) - (p_{11} - p_{21})\,(b/\pi) + (p_{12} - p_{22})\,(b/\pi) + y_1 - y_2, \tag{2.15}$$

$$c_{b,t} + c_{l,t} = y_1 + y_2, \tag{2.16}$$

for a given inflation rate $\pi > 0$.

We now examine how monetary policy affects the allocation and aggregate welfare in the representative agents economy given in Proposition 1. Specifically, we analyze the effects of the inflation rate on borrowers' consumption and on aggregate welfare, measured as ex-ante expected lifetime utility. This can be interpreted as measuring welfare of an unborn agent facing the equally likely possibility of being a representative borrower or a representative lender, which is equivalent to a utilitarian welfare measure. Under Assumption 2, (2.10) and (2.13) imply that consumption of the representative borrower is strictly smaller than consumption of the representative lender due to the binding borrowing constraint. Combining (2.10) and (2.13) and using $\zeta_{b,t} > 0$, shows that the marginal utility of the representative borrower is strictly larger than the marginal utility of the representative lender, $(\delta - 2c_{b,t}) > (\delta - 2c_{l,t})$. As long as this inequality holds, a redistribution of consumption from the latter to the former increases aggregate welfare. It can be shown that this can be induced by reducing the inflation rate if the serial correlation of endowment shocks is not too high. Then, monetary policy can, in principle, also implement first best as long as the zero lower bound is respected.

Proposition 2 *Consider a competitive equilibrium as given in Proposition 1. A reduction in the inflation rate raises borrowers' consumption and enhances aggregate welfare if $p_{12} > (1 - \beta)/2$. Monetary policy is then able to implement first best if $1 \leq 2b[1 + (p_{21} - p_{11})/\beta]/(y_2 - y_1)$.*

Proof. See Appendix A.2. ■

According to Proposition 2, monetary policy should choose a low inflation rate to maximize aggregate welfare if the probability of changing income types is sufficiently large, i.e. $p_{21} = p_{12} > (1 - \beta)/2$. The reason for this result is that inflation exerts the previously discussed opposing effects, i.e. debt deflation and debt limit (see Section 2.2), on borrowers. Under the borrowing constraint, $-s_b \leq b$ (as assumed here), the amount of funds that can be issued b/R and the repayment b/π decrease with the inflation rate. Thus, monetary policy is non-neutral, while its overall impact on borrowers depends on the subjective valuation of funds at issuance and at maturity, which depends on the marginal utility of consumption.

Consider for example a household who draws y_1 today and borrows funds up to the borrowing constraint. If the probability of being unconstrained at maturity (for y_2) is positive, its expected marginal utility then tends to be lower than today. This household would gain from a proportional reduction in the nominal interest rate, which raises the amount of funds that can be borrowed today, even if its is accompanied by a proportional reduction in the inflation rate, which tends to raise real debt repayment. Thus, under a sufficiently large probability of drawing a high income shock and being unconstrained at maturity the debt limit effect dominates the debt deflation effect, such that monetary policy should lower rather than raise inflation to benefit borrowers. This result is consistent with the findings in Section 2.2, where borrowers are permanently constrained and therefore gain from higher rather than lower inflation. Though, the condition for lower inflation to enhance welfare, i.e. $p_{21} = p_{12} > (1-\beta)/2$, seems to be fairly week (given that discount factors are typically close to one), it remains to assess whether the arguments made are of quantitative relevance.

2.3.3 A calibrated version

The previous analysis has shown that monetary policy can enhance aggregate welfare by reducing inflation and the nominal interest rate, when borrowing agents are less likely to be constrained at maturity. Yet, this analysis has been conducted under simplifying assumptions on preferences, the debt limit, maturity, and shocks. Here, we examine a less stylized framework, which will be calibrated for US data. For this, we omit the Assumptions 1 and 2. We apply a conventional CRRA period utility function for households $i \in [0,1]$ $u(c_{i,t}) := c_{i,t}^{1-\sigma}/(1-\sigma)$, where $\sigma > 0$. We further use the borrowing constraint (2.2) and we do not restrict the analysis to the case where the borrowing constraint is always binding for borrowers. As a consequence, individual wealth/debt of agents can vary over time depending on the individual history of income shocks. The realizations of these shocks are now assumed to satisfy $y_{i,t} \in \{y_1, y_2, ..., y_n\}$, where $0 < y_j < y_{j+1}$ for $j = 1, ..., n-1$, and to follow a first-order Markov process with transition matrix $\mathcal{P}$. The elements are $\mathcal{P}_{k,l} := p_{k,l}$ for $k, l = 1, ..., n$, where $p_{k,l}$ is the probability to switch from state k in $t-1$ to state l in period t.

We examine the effects of the following policy experiment. Initially, the economy is in the stationary equilibrium induced by the benchmark inflation rate of 2%. We then introduce an unexpected permanent reduction in the inflation rate to -2% in period 0 and assess the effects on the allocation and agents' welfare. After the change in inflation, the economy leaves the stationary equilibrium induced by an inflation rate of 2% and converges to the new one under the lower rate of -2%. Therefore, we first calculate the stationary equilibrium for both inflation rates and then the transition path from the old to the new stationary equilibrium.

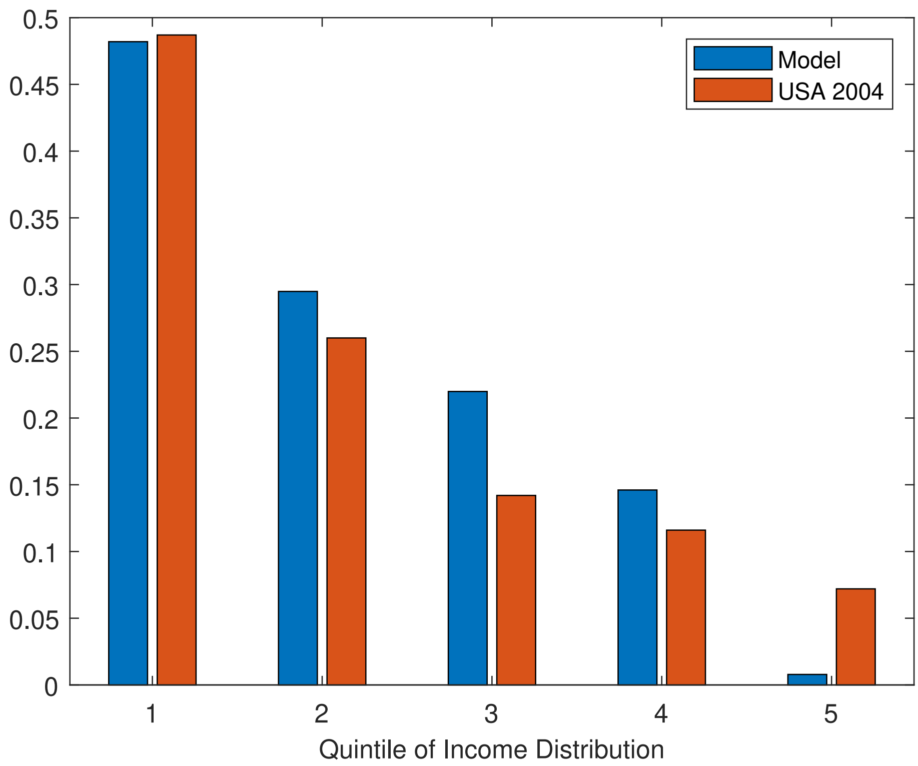
Let λ be a distribution of agents, where $\lambda(s, y)$ is the measure of agents with wealth s and income y. The stationary equilibrium then consists of a price Q, constant policy functions $c(s, y)$ and $s'(s, y)$ and a distribution $\lambda(s, y)$ consistent with a particular inflation rate such that 1) decision rules solve the individual optimization problem, 2) markets clear $\sum_{s,y} \lambda(s, y)c(s, y) = \sum_{s,y} \lambda(s, y)y$ and $\sum_{s,y} \lambda(s, y)s'(s, y) = 0$, and 3) $\lambda(s, y)$ is time invariant (see Appendix B.2.3). Having constructed the stationary equilibria, we calculate the transition path from the old to the new stationary equilibrium (see Appendix B.2.4). Note that the policy functions, wealth distribution and nominal interest rate are not constant over the transition period, but then converge to the time invariant functions and values of the stationary equilibrium under the new inflation rate of -2%.

Calibration To solve the model numerically, we need to assign values for the degree of relative risk aversion σ, the seizable fraction of income γ, debt maturity $1/\theta$, the subjective discount factor β, and the moments of the idiosyncratic income process. The length of a period is assumed to equal 1 year. For σ, we apply the value 2 in accordance with many related studies. As the empirical counterpart of debt, we apply installment loans, where we disregard loans for vehicles and housing. The reason is that the latter typically serve as collateral, while debt is not collateralized in our model. We apply US postwar data for installment loans and after tax income in 2004 taken from the CBO and the Survey of Consumer Finances (see Appendix B.2.1). Based on these data, we set γ equal to 0.49 to match the ratio of debt to income in the first income quintile, and $1/\theta$ equal to 2 to match the average maturity. While empirical interest rates on installment loans are relatively high, we calibrate the model, i.e. we set $\beta = 0.83$, to get an annual real rate of return $r^*_{t+1} = [Q_{t+1}(1-\theta) + \theta] / (Q_t\pi)$ of 4%. This value relates to a risk free rate, which is more suited for our model specification, since it does not account for default (risk).

For the income process, we assume that log individual annual income follows an AR1 process, $\ln(y_{i,t}) = \rho \ln(y_{i,t-1}) + \epsilon_{i,t}$, with $\epsilon_{i,t}$ i.i. normally distributed with mean 0, variance σ^2_ϵ. We apply Tauchen and Hussey's (1991) algorithm for the five states of the log-labour-income process. This leads to the transition matrix $\mathcal{P}$ given in Appendix B.2.2 and a stationary distribution with 20% of the population in each income state, given by $y_1 = 0.49$, $y_2 = 0.76$, $y_3 = 1$, $y_4 = 1.31$ and $y_5 = 2.04$. For the benchmark parametrization, we use Floden and Linde's (2001) estimates for the autocorrelation coefficient and the variance, $\rho = 0.9136$ and $\sigma^2_\epsilon = 0.0426$. For an alternative specification, we use Guvenen's (2007) income process estimates, providing a lower autocorrelation and a lower variance, $\rho = 0.821$ and $\sigma^2_\epsilon = 0.029$. We compute the solution of the model applying an endogenous grid point method to calculate the stationary equilibrium (see Appendix B.2.3). Under these parameter values, the nominal interest always satisfies the zero lower bound.

To see how the model implied distribution of debt for a benchmark inflation rate of 2% relates to its empirical counterpart, Figure 2.2 shows the ratios of debt to income for the five income states and the empirical counterparts of 2004.[13] The model is actually able to fit the ratios of debt to income for the income quintiles reasonably well. Yet, the model underestimates the value for the highest income. The reason is that households in the highest income quintile have a relatively low incentive to borrow in our model, as they tend to save for consumption smoothing.

Figure 2.2: Ratios of debt to income for US income quintiles



Note: The debt to income ratios in the model (data) are given by the blue (red) bars.

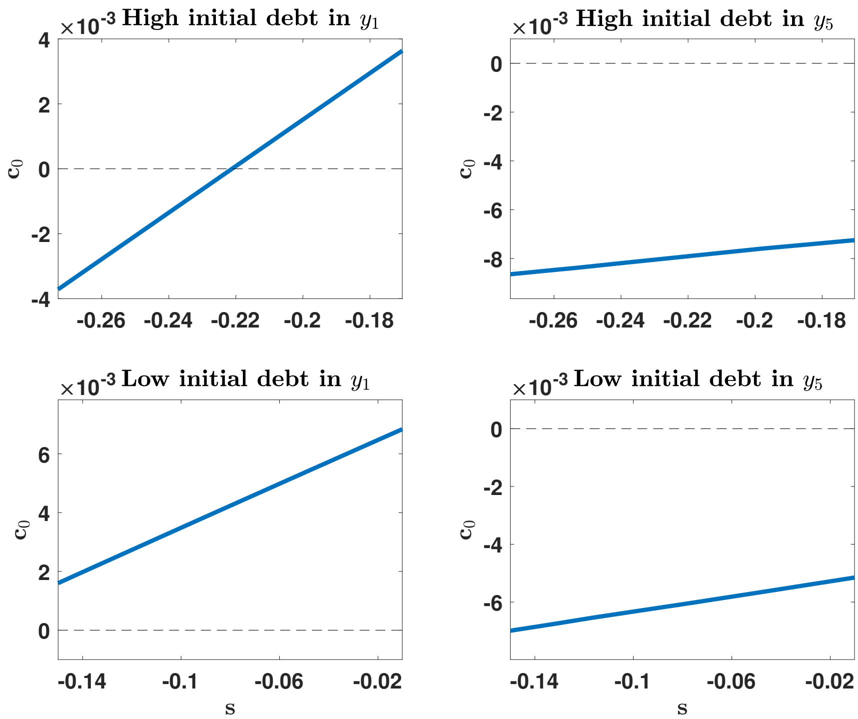
How does inflation affect agents' choices? We first examine the effects of a change in the inflation rate on consumption and saving/borrowing. Assume that the distribution of predetermined wealth s_0 is initially given by the stationary distribution induced by an inflation rate of 2%. Then, monetary policy unexpectedly decreases the inflation rate to -2% and holds it constant at -2% thereafter. The economy leaves the old stationary equilibrium under 2% inflation and converges to the new one under -2%.[14] The

[13]The ratios of debt to income for the different income quintiles are calculated by using average installment loans w.o. vehicle installment loans of households with holdings in income quintiles in 2004 from SCF 2004 (for debt) and average after tax income in income quintiles in 2004 from CBPP (for income).

[14]To calculate the transition path we first compute the old and the new stationary equilibrium (see Appendix B.2.3). We assume that the economy reaches the new stationary equilibrium after T periods and then calculate the path for the rate of return on debt such that the corresponding policy functions imply a path for the wealth distribution that converges to the wealth distribution of the new stationary equilibrium after T periods (see Appendix B.2.4).

reduction in inflation has no impact on the distribution of predetermined wealth s_0. Yet, the initial debt deflation effect raises the real value of initial wealth in terms of current period commodities s_0/π. Via the debt deflation effect, the lower inflation rate also tends to raise all future debt repayments in terms of commodities at maturity for non-matured initial debt $(1-\theta)^t s_0/\pi$ and for borrowers who are constrained at issuance. Via the debt limit effect, lower inflation raises the issuance price of debt and the maximum amount of funds that can be borrowed. Whether lower inflation is beneficial or not for a borrower who is constrained at issuance depends – inter alia – on the likelihood to be again constrained at maturity. Remember that the debt deflation effect has dominated the overall welfare result in the model with different degrees of patience, where borrowers are constrained in all periods (see Section 2.2). In contrast, the debt limit effect can dominate in the economy with idiosyncratic shocks if borrowers who are constrained at issuance have to repay higher debt obligations while being unconstrained at maturity with a positive probability (see Proposition 2).

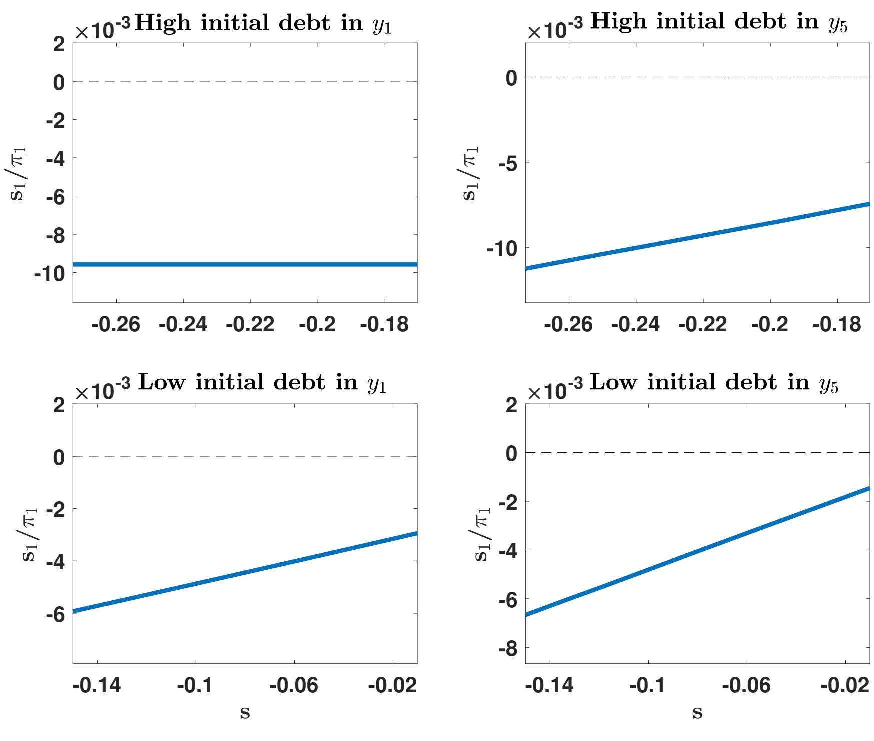
Figure 2.3: Change in the policy functions for consumption c in period 0 for an inflation reduction from 2% to -2%



To unveil the effects on the allocation and on aggregate welfare, we first examine policy functions for the given wealth distribution in period 0. Specifically, we compute the policy functions for consumption $c(s, y)$ and for beginning-of-period wealth $s'(s, y)/\pi$ for different income states of the economy under an inflation rate of 2% and of the economy under a lower inflation rate of -2%. The lower inflation rate reduces the nominal interest rate,[15] implying an increase in the effective limit for borrowed funds. The changes in the period-0 policy functions for consumption and beginning-of-period wealth for a reduction in the inflation rate from 2% to -2% are shown in Figure 2.3 and 2.4. For convenience, we focus on the incomes states y_1 and y_5 and on initially indebted agents ($s_0 < 0$), while corresponding policy functions that also include agents with positive initial wealth are shown in Figure 2.10 in Appendix C.2.

Intuitively, the reduction in the inflation rate increases the effective value of initial debt $-s_0/\pi$ (wealth s_0/π) and thereby tends to decrease (increase) consumption. The changes in the policy functions in Figure 2.3 show that borrowers in the income state y_1 with relatively high initial debt (see upper left panel) decrease consumption in the initial period due to the debt deflation effect. However, the initial debt deflation effect is not dominant for all initially indebted households. Firstly, constrained borrowers with relatively low initial debt tend to raise consumption by increasing borrowing (i.e., by reducing s'/π, see Figure 2.4), indicating that the debt limit effect dominates the initial debt deflation effect.

Figure 2.4: Change in the policy functions for beginning-of-period wealth s'/π in period 0 for an inflation reduction from 2% to -2%



Secondly, consumption under low inflation is also higher for unconstrained borrowers with low initial debt in y_1 (see bottom left panel in Figure 2.3), as these households, who have a relatively high probability to be constrained in future periods, can potentially increase borrowing due to higher effective debt limits

[15] The net nominal interest rate falls from 6% to 2.115% in period 0 and then converges to 2.125%.

in the future. Put differently, their precautionary savings motive is less pronounced due to an improved access to external funds.[16] For the highest income state y_5, for which consumption and beginning-of-period wealth are shown in the right hand columns of Figure 2.3 and 2.4, borrowers are not constrained and the debt deflation effect dominates, such that they reduce consumption. Finally, it should be noted that the policy functions under a stationary wealth distribution for an inflation rate of -2% are virtually identical with the policy functions in period 0 (see Figure 2.11 in Appendix C.2). Hence, the effects for $t = 0$ also apply for the subsequent periods $t \geq 1$.

Who gains from lower inflation? The policy functions presented above have shown changes in consumption and savings due to lower inflation in the initial period in which the shock realizes. To disclose how inflation affects agents' welfare, we calculate the change in expected lifetime utility given by $v(s, y) = E_0 \sum_{t=0}^{\infty} \beta^t c_t(s_t, y_t)^{1-\sigma}/(1-\sigma)$ given $s_0 = s$ and $y_0 = y$. Denote by $v_\pi(s, y)$ the expected lifetime utility of a household with income y and wealth s for a specific inflation rate π. Hence, a reduction in the inflation rate from 2% to -2% increases expected lifetime utility of a household in the initial state (s, y) if $v_{-2}(s, y) - v_2(s, y) > 0$. To quantify the welfare consequences of the change in the inflation rate for a household of type (s, y) we express the differences in units of consumption. Therefore, we calculate the percentage change in consumption in the stationary equilibrium with an inflation rate of 2%, in each date and state, for the household of type (s, y) to be indifferent between an inflation rate of 2% and a permanent reduction in the inflation rate to -2%. The gain g of the inflation reduction is then implicitly given by $v_2(s, y; g) = v_{-2}(s, y)$ with $v(s, y; g) = E_0 \sum_{t=0}^{\infty} \beta^t \left((1+g)c_t(s_t, y_t)\right)^{1-\sigma}/(1-\sigma)$.

The solid black lines in the left hand column of Figure 2.5 show the gain $g(s, y)$ for the different income states. Furthermore, the figure splits $g(s, y)$ into the contribution of the effects of initial debt deflation (ID, see dotted lines), which are independent of the borrowing constraint, and of $g(s, y)$ without the effects of initial debt deflation, which captures the monetary non-neutrality due to the borrowing constraint (BC, see dashed line). Notably, effects of initial debt deflation as well as effects due to the borrowing constraint are more persistent under longer-term debt than under one-period debt (see also Figure 2.6).[17] Let $\widetilde{g}(s, y)$ denote the contribution of $g(s, y)$ without the effects of initial debt deflation, implicitly defined by $v_2(s, y; \widetilde{g}) = v_{-2}(\tilde{s}, y)$ where $\tilde{s}$ is given by $\tilde{s}/0.98 = s/1.02$,[18] such that the effects of initial debt deflation are shut down. The borrowing constraint effects $\widetilde{g}(s, y)$ are then given by the debt

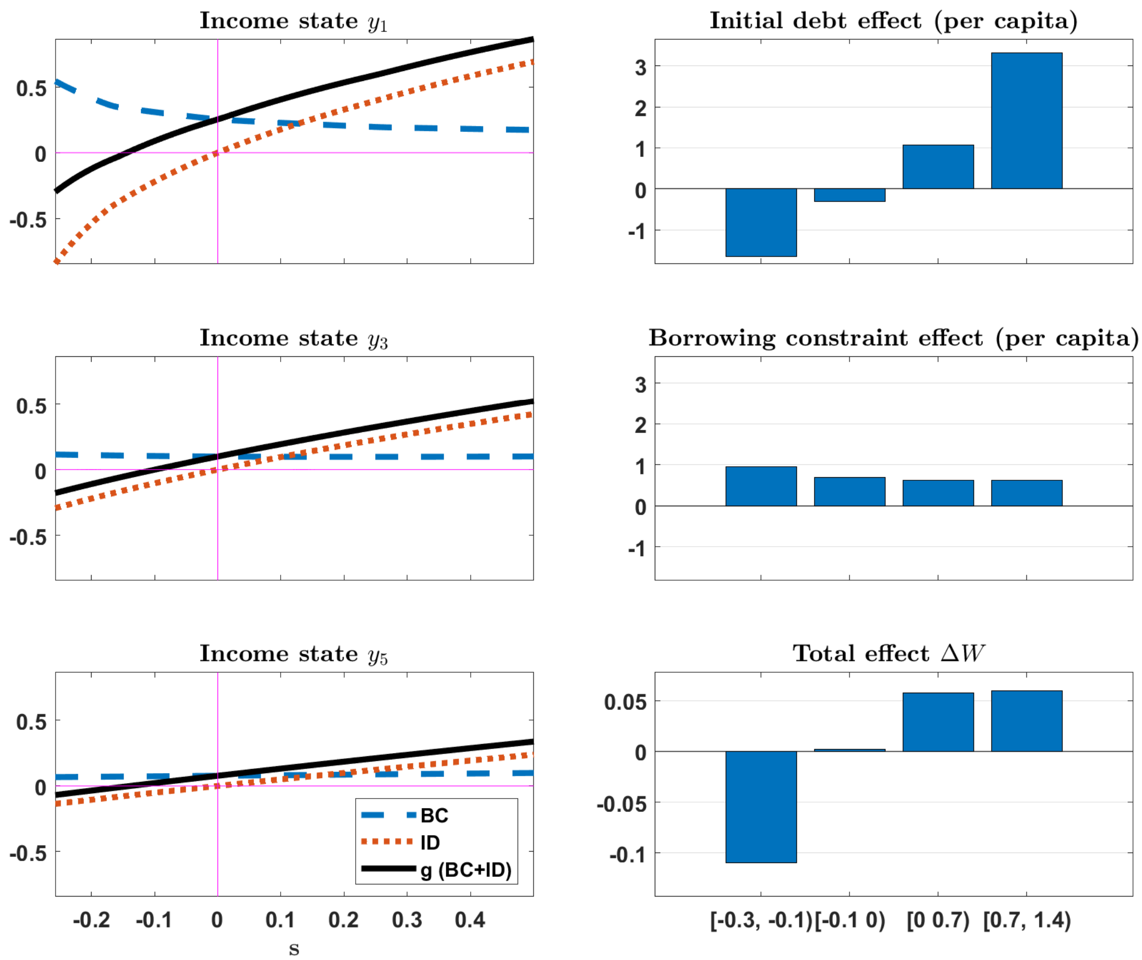
[16]Lower inflation further tends to increase consumption more for positive initial wealth levels s_0 (see first row of Figure 2.10 in Appendix C.2).

[17]For example, the fraction $1-\theta$ of initial debt that has not matured contributes to the effects of debt deflation in $t = 1$.

[18]Put differently, the effect $v_{-2}(\tilde{s}, y) - v_2(s, y)$ is the difference in expected lifetime utility between a household who lives in an economy with an inflation rate of 2% and has a real value of beginning of period wealth $s/1.02$ and another households who lives in an economy with a permanent reduction in the inflation rate to -2% and has a real value of beginning of period wealth $\tilde{s}/0.98(= s/1.02)$.

limit effects under a lower inflation rate as well as the deflation effects on debt issued in $t \geq 0$,[19] while the contribution of the effects of initial debt deflation are the residual to $g(s, y)$.

Figure 2.5: Individual welfare effects for three income states and welfare aggregated for four wealth sets



Note: In the right column, the first row shows ID (per capita), the second row BC (per capita) and the third row ΔW_{s_x}.

Apparently, the welfare contribution of the effects of initial debt deflation are negative (positive) for households with initial debt (positive wealth). The borrowing constraint effects $\widetilde{g}(s, y)$ tend to increase expected lifetime utility, in particular, of constrained borrowers and households with a high probability to be constrained in future periods by increasing the borrowing limit. However, the borrowing constraint effects tend to increase expected lifetime utility also of wealthier agents due to the increase in the effective debt limit. In total, agents with relatively high initial debt (especially the constrained borrowers) suffer due to dominant effects of initial debt deflation (see also Figure 2.3). Agents with positive wealth benefit

[19]These effects correspond to the effects $B.$), $C.$), and $D.$) in (2.5) and (2.6).

from the reduction in the inflation rate due to both a higher real wealth in the initial period and higher borrowing limits in future periods in which they might be constrained. Importantly, agents with relatively low initial debt, i.e. $s > -0.14$ for y_1, $s > -0.1$ for y_3, and $s > -0.13$ for y_5, also benefit from the lower inflation rate (see Figure 2.5). This is due to the beneficial debt limit effect which allows to increase borrowing in future periods, where these agents might be constrained. In these cases, the borrowing constraint effects dominate the effects of initial debt deflation.

What are the aggregate effects of a reduction in the inflation rate? In the previous analysis, we have shown how individual agents' welfare is affected by a reduction in the inflation rate. Here, we assess the effect of inflation on aggregate welfare measured by agents' ex-ante expected lifetime utility. Hence, we examine welfare of agents who are randomly placed into the cross-sectional distribution over individual characteristics in an economy with an inflation rate of either 2% or -2%.[20] As defined above, $v_\pi(s, y)$ is the expected lifetime utility of household of type (s, y) for the inflation rate π and $g(s, y)$ measures by how much this household prefers to be assigned to an economy with an inflation rate of -2% compared to 2% in consumption terms, $g(s, y) = \left(\frac{v_{-2}(s,y)}{v_2(s,y)}\right)^{\frac{1}{1-\sigma}} - 1$. The change in aggregate welfare measured by ex-ante expected lifetime utility is then given by $\Delta W = \left(\frac{\Sigma_{s,y}\lambda_2(s,y)v_{-2}(s,y)}{\Sigma_{s,y}\lambda_2(s,y)v_2(s,y)}\right)^{\frac{1}{1-\sigma}} - 1$, where λ_2 is the wealth distribution before inflation is changed.[21]

The right hand column of Figure 2.5 shows the welfare effects in percentages of consumption units aggregated over agents within four wealth sets, $s_I \in [-0.3, -0.1)$, $s_{II} \in [-0.1, 0)$, $s_{III} \in [0, 0.7)$, and $s_{IV} \in [0.7, 1.4]$. The right hand panel in the first row displays the *per capita* welfare effects that are solely induced by the effects of initial debt deflation.[22] Apparently, agents with a high debt position suffer more from a reduction in the inflation rate, while agents with positive savings gain from the lower inflation rate. The right hand panel in the second row shows that welfare within all wealth sets is positively affected by the borrowing constraint effects. The increased debt limit under lower inflation is thereby most beneficial for indebted agents. Comparing the effects of initial debt deflation with the borrowing constraint effects, indicates that the total welfare effect is positive for less indebted agents with $s \in [-0.1, 0)$. These agents do not face a binding borrowing constraint. Yet, they assign a positive probability of being constrained in the future such that a relaxation of the effective debt limit is beneficial for them. For them, the borrowing constraint effects outweighs the effects of initial debt deflation. For highly indebted agents (s_I) the latter effect dominates the former, while lenders unambiguously gain from the inflation reduction.

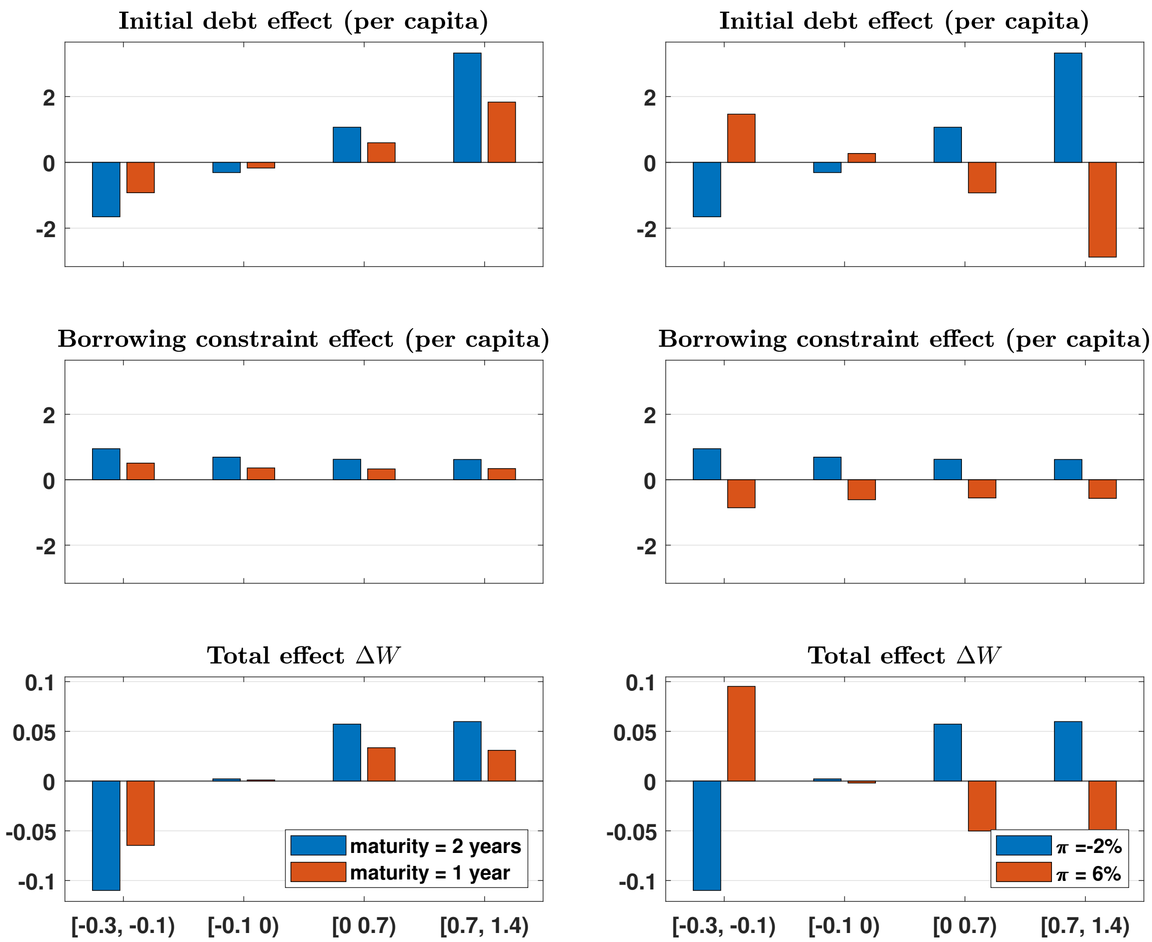
[20] Given the law of large numbers, such that the probability of drawing a specific individual state equals the mass of agents with this specific individual state, this measure relates to a utilitarian welfare measure.

[21] Notably, the distribution of initial real wealth s_0 is not affected by the change in inflation, in contrast to the distribution of real wealth in the subsequent periods.

[22] Specifically, we compute the per capital welfare effects for each wealth set, i.e. $[\sum_{s_x,y} \lambda_2(s, y)g(s, y)]/\sum_{s_x,y} \lambda_2(s, y)$ for $s_x \in \{s_I, s_{II}, s_{III}, s_{IV}\}$ and proceed as described above to separate the effects of initial debt deflation from the borrowing constraint effects.

The right hand panel in the last row of Figure 2.5 shows the welfare effects within the four wealth sets, ΔW_{s_x}. Computing the contribution to the total welfare effects over the entire population, shows that the aggregate welfare falls due to the effects of initial debt deflation by $\Delta W(ID) = -0.283\%$ and increases due to the borrowing constraint effects by $\Delta W(BC) = 0.234\%$. Hence, the decline of aggregate welfare due effects of initial debt deflation is reduced by 83% via the novel borrowing constraint effect, such that the total aggregate welfare effect is just slightly negative, $\Delta W = -0.049\%$.

Figure 2.6: Welfare aggregated for four wealth sets with different maturities (left column) and different inflation rates (right column)



To assess the sensitivity of these results, we compute corresponding results for a shorter maturity, for an increase instead for a reduction in the inflation rate by 4%, and for a lower autocorrelation of idiosyncratic income. Notably, the wealth distribution is not unaffected by these experiments, from which we abstract in the following discussion, for convenience. Reducing the debt maturity $1/\theta$ from 2 to

1 periods essentially reduces all effects in a proportional way (see left hand column of Figure 2.6), keeping their relative magnitudes unchanged. As in related studies (see Doepke and Schneider (2006)), effects of initial debt deflation induced by non-transitory inflation changes are more persistent and amplified under longer-term nominal debt.[23] At the same time, the borrowing constraint effects also increase with higher maturities, as they increase the likelihood of borrowers to be unconstrained at maturity (similar to a lower autocorrelation of income). Increasing the inflation rate by 4% to 6% leads to welfare effects that are qualitatively symmetric to the effects of the inflation reduction to -2% (see right hand column of Figure 2.6). Yet, the size of all effects under higher inflation are smaller ($\Delta W_{6\%} = 0.04\%$) compared to the effects under an equally-sized inflation reduction. On the one hand, a higher inflation rate reduces the effective debt limit. On the other hand, an increase in inflation reduces the value of beginning-of-period debt $-s/\pi$. In total, the distortion induced by the borrowing constraint decreases with the inflation rate, such that the welfare effects of initial debt deflation as well as of the borrowing constraint are smaller under higher inflation rates.[24]

Notably, the specification and parametrization of the idiosyncratic income process is not undisputed. Guvenen (2007) for example suggests an income process which leads to much lower estimates for the autocorrelation of idiosyncratic income. To assess the impact of these estimates, we adjust the income process including the income states and we re-calibrate relevant parameters. We therefore apply Guvenen's (2007) estimates $\rho = 0.821$ and $\sigma_\epsilon = 0.029$, and we set $\beta = 0.8968$ and $\gamma = 0.51$ to match the previously described targets. For this alternatively calibrated model specification, Figure 2.7 shows individual and aggregate welfare effects, which are comparable to the benchmark specification. Here, the separate welfare effects due to initial debt deflation and the borrowing constraint are $\Delta W(ID) = -0.108\%$ and $\Delta W(BC) = 0.126\%$, respectively. Apparently, the reduction in the inflation rate now leads to a (small) positive aggregate welfare effect, $\Delta W = 0.018\%$, consistent with the results summarized in Proposition 2.

Finally, we examine the time path of aggregate beginning-of-period debt $-s/\pi$ and the real rate of return r^* in response to the inflation rate reduction (see Figure 2.8). When the inflation rate is reduced, the wealth distribution is initially consistent with an inflation rate of 2%. When the inflation rate is then reduced to -2%, the effective debt limit is raised, such that agents' access to external funds is less constrained and the aggregate credit volume increases on impact. From then onwards, the economy converges to a new stationary wealth distribution with a debt level that settles on an intermediate level.

[23]Similarly, monetary policy exert more persistent effects when nominal payments are fixed for longer terms as under mortgage contracts (see Gariga et a. (2017)).

[24]This is indicated by the average value of the multiplier on the borrowing constraint ζ within the lowest wealth state s_I, which monotonically decreases from an inflation rate of -2%, to 2% and 6%. The values for $\overline{\zeta}_\pi = \Sigma_{s_I,y}\lambda_\pi(s,y)\zeta_\pi(s,y)/\left(\Sigma_{s_I,y}\lambda_\pi(s,y)\right)]$ are $\overline{\zeta}_{-2} = 1.048$, $\overline{\zeta}_2 = 1.014$, and $\overline{\zeta}_6 = 0.984$.

Figure 2.7: Individual welfare effects for three income states and welfare aggregated for four wealth sets under a calibration with lower autocorrelation of income

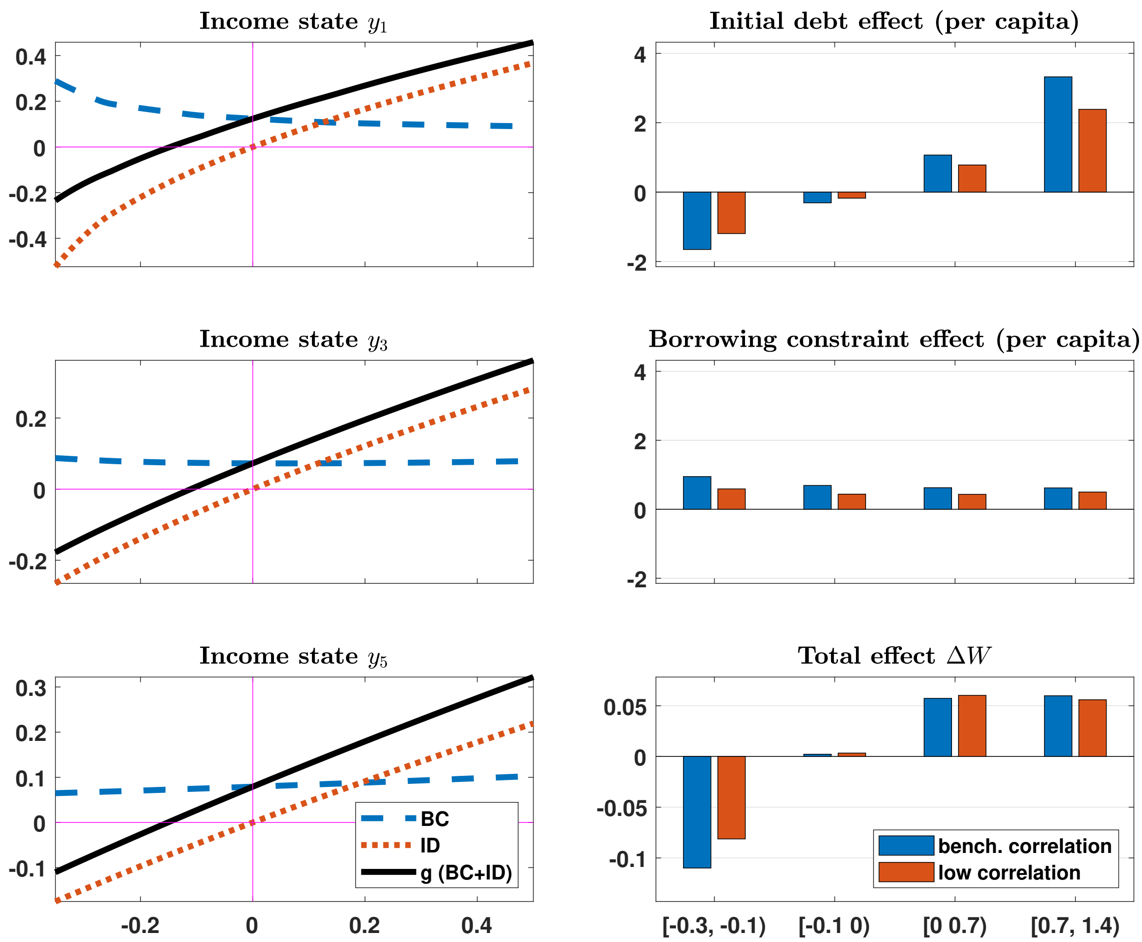
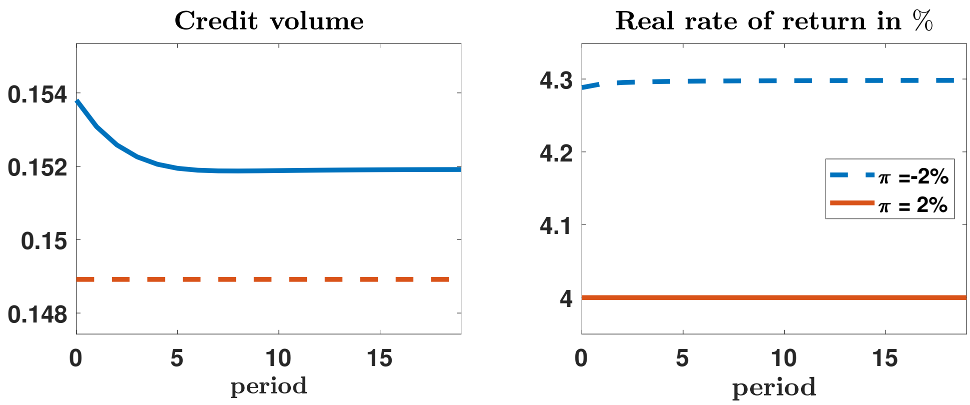


Figure 2.8: Paths of aggregate debt $-s/\pi$, and the real rate of return r^*



Given that aggregate debt $-s/\pi$ is higher under a lower inflation rate, market clearing requires a higher real rate of return r^*, which under our benchmark calibration increases from 4% and converges to 4.3% (see right panel of Figure 2.8). This uninternalized change in the real rate of return tends to reduce the overall welfare impact of the borrowing constraint effects.

2.4 Conclusion

We analyze how financial frictions contribute to redistributive effects of monetary policy. We explore a novel mechanism of monetary non-neutrality, which is based on borrowing constraints related to current income. Such limits for unsecured debt, for which broad empirical evidence exists, do not account for expected price changes until maturity, implying that monetary policy can alter the real terms of borrowing. A reduction in inflation tends to increase the maximum amount of debt that can be issued, while it also raises the beginning-of-period stock of debt to be repaid. The impact of inflation depends on the probability of borrowers to be unconstrained at maturity. The lower this probability is, the smaller is the beneficial effect of lower inflation for borrowers. The debt limit effect is opposed to debt deflation effects when borrowers are initially indebted. The overall effect is therefore ex-ante ambiguous and depends on the initial debt/wealth position as well as the willingness to borrow. We show that lower inflation particularly benefits agents with low initial debt by relaxing effective borrowing constraints, whereas highly indebted borrowers suffer from the dominant debt deflation effect. A reduction of the inflation rate can nonetheless enhance aggregate welfare, specifically, when the autocorrelation of idiosyncratic income is relatively low.

Appendix

A.2 Proof of Proposition 2

We start by establishing the first claim of the proposition. Under a constant inflation rate, the equilibrium exhibits no time variation, such that we can neglect time indices. Substituting out the interest rate with (2.13), which can – by using (2.16) – be rewritten as $1/R = (\beta/\pi)\,[p_{21}\frac{\delta-2c_b}{\delta+2c_b-2(y_1+y_2)} + p_{22}]$, in the borrower's budget constraint (2.12), implying

$$c_b = (b/\pi)\left[\beta p_{21}\frac{\delta-2c_b}{\delta+2c_b-2(y_1+y_2)} + \beta p_{22} + p_{21} - p_{11}\right] + y_1, \tag{2.17}$$

where the fraction on the RHS is strictly decreasing in c_b. Thus, a lower inflation rate increases c_b if the term in the squared brackets in (2.17) is positive, i.e.

$$\beta\left\{p_{21}\frac{\delta-2c_b}{\delta+2c_b-2(y_1+y_2)} + p_{22}\right\} + 2p_{21} - 1 > 0, \tag{2.18}$$

where we used $p_{21}+p_{11}=1$. The term in the curly brackets in (2.18) is larger than one under a binding borrowing constraint, since $p_{21}+p_{22}=1$ and the marginal utility of the representative borrower is larger than the marginal utility of the representative lender implying $\frac{\delta-2c_1}{\delta+2c_1-2(y_1+y_2)} > 1$. Thus, $\beta+2p_{21}-1>0$ is sufficient to satisfy the inequality (2.18). In this case, a lower inflation rate increases c_b. Given that $(\delta-2c_{b,t}) > (\delta-2c_{l,t}) \Leftrightarrow c_b < c_l$, an increase in c_b and thus a decrease in c_l by the same amount causes a reduction of the gap between the marginal utility of the representative borrower and the marginal utility of the representative lender. Hence, aggregate welfare, measured as $(1-\beta)^{-1}[(\delta c_b - c_b^2) + (\delta c_l - c_l^2)]$, unambiguously increases if $p_{12} > (1-\beta)/2$.

To establish the claim regarding first best, we use that $c_{b,t} = c_{l,t}$ holds under first best. Then, (2.15) implies

$$-s = R(y_2 - y_1 + 2(b/\pi)(p_{11} - p_{21}))/2 \leq b, \tag{2.19}$$

where the inequality is due to the non-binding borrowing constraint under first best. Under first best, (2.13) further implies $R/\pi = 1/\beta$. Substituting out inflation with the latter in (2.19), gives $R \leq 2b\frac{1+(p_{21}-p_{11})/\beta}{y_2-y_1}$, which together with the ZLB imply $1 \leq 2b\frac{1+(p_{21}-p_{11})/\beta}{y_2-y_1}$ for monetary policy to be able to implement first best. If however $1 > 2b\frac{1+(p_{21}-p_{11})/\beta}{y_2-y_1}$ monetary policy cannot implement first best due to the ZLB.

B.2 Appendix to the calibrated model

B.2.1 Data on household debt

The ratios of debt to income for different income quintiles are calculated as follows. For income we use average after tax income in the income quintiles in 2004 (in 2004 dollars) taken from CBO (www.cbo.gov/sites/default/files/109th-congress-2005-2006/reports/ EffectiveTaxRates2006.pdf) which we denote by Av. ATI. (see second column of Table 2.1).

Income Quintile	Av. ATI	Av. IL w.o. VIL	debt to income
Q1	14.7k	7.16k	0.49
Q2	32.7k	8.52k	0.26
Q3	48.4k	6.90k	0.14
Q4	67.7k	7.88k	0.12
Q5	155.2k	11.13k	0.07

Table 2.1: Average after tax income, average value of these debt holdings, and debt-to-income for income quintiles in 2004 (in 2004 dollars)

For debt we use the following component of installment loans taken from the SCF 2016 (where dollar variables are inflation-adjusted to 2004 dollars): All installment loans (which exclude loans secured by residential property) minus vehicle installment loans. For every income quintile, we then use the average value of these debt holdings of those households who hold this type of debt. We then denote this type of debt by Av. IL w.o. VIL (see third column of Table 2.1) and it is calculated by "Av. IL w.o. VIL" = "Av. IL" - "Av. VIL" * "% w. VIL" / "% w. IL", where Av. IL denotes the average value of all installment holdings of households who hold this type of debt in a given income quintile, Av. VIL is the average value only of vehicle installment loans, % w. IL denotes the fraction of households who have an installment loan in a given income quintile, and % w. VIL is the fraction of households who hold only vehicle installment loans. The debt to income ratios we use (see Figure 2.2) are then given by the ratio of average after tax income and installment loans net of vehicle loans (see fourth column of Table 2.1).

B.2.2 Transition matrix

The transition matrix of idiosyncratic income with the conditional probabilities $\mathcal{P}(a_l|a_k)$ is given by

$$\mathcal{P} = \begin{pmatrix} 0.767 & 0.207 & 0.025 & 0.001 & 10^{-6} \\ 0.207 & 0.496 & 0.253 & 0.043 & 0.001 \\ 0.025 & 0.253 & 0.446 & 0.253 & 0.0245 \\ 0.001 & 0.043 & 0.253 & 0.496 & 0.207 \\ 10^{-6} & 0.001 & 0.025 & 0.207 & 0.767 \end{pmatrix}.$$

B.2.3 Calculation of the stationary equilibrium under a given inflation rate

Under a given inflation rate π, we calculate the decision rules and the time-invariant distribution at a given issuance price of a unit bond $Q = Q'$ by using an endogenous grid point method (see Carroll (2006)) combined with time iteration and we calculate the stationary equilibrium issuance price by a bisection method as follows:.

I. For the bisection method we need (i) a value for the issuance price denoted by Q^l, i.e. $Q = Q' = Q^l$, at which $\sum_{s,y} \lambda(s,y)s'(s,y) > 0$ and (ii) a value for the issuance price denoted by Q^h, i.e. $Q = Q' = Q^h$, at which $\sum_{s,y} \lambda(s,y)s'(s,y) < 0$. The stationary equilibrium issuance price satisfying $\sum_{s,y} \lambda(s,y)s'(s,y) = 0$ is then in the interval $\left(Q^l, Q^h\right)$. To find a value that satisfies the condition in (i) we choose a relatively low value for the issuance price, calculate steps III-IV and check the condition $\sum_{s,y} \lambda(s,y)s'(s,y) > 0$. If this condition is not satisfied, we repeat steps III-IV with lower issuance prices until we have found a value that satisfies the condition in (i). Proceed analogously for Q^h.

II. Calculate a guess for the stationary equilibrium issuance price Q^0 by $Q^0 = 0.5\left(Q^l + Q^h\right)$.

III. Calculate for Q^0 the consumption policy function $c(s,y)$ and the wealth policy function $s'(s,y)$ with an endogenous grid point method combined with time iteration neglecting market clearing for loans (see below).

IV. Given the wealth policy function $s'(s,y)$, compute the implied stationary distribution $\lambda(s,y)$ (see below).

V. Check market clearing for loans. Choose a pararmeter $\epsilon > 0$, which is relatively small. If $|\sum_{s,y} \lambda(s,y)s'(s,y)| < \epsilon$, stop: $Q = Q^0$ is the equilibrium issuance price. If $\sum_{s,y} \lambda(s,y)s'(s,y) > \epsilon$, set $Q^l = Q^0$ and go back to step II. If $\sum_{s,y} \lambda(s,y)s'(s,y) < \epsilon$, set $Q^h = Q^0$ and go back to step II.

The endogenous grid point method combined with time iteration for a given issuance price $Q = Q' = Q^0$ is computed as follows:

1. Discretize next period wealth space $s' = \left\{s'_1, s'_2, ..., s'_{y_4}, ..., s'_{y_3}, ..., s'_{y_2}, ..., s'_{y_1}, ..., s'_m\right\}$, $s'_i < s'_{i+1}$ with $s'_1 = s'_{y_5} = -\gamma y_5$ and $s'_{y_i} = -\gamma y_i$. Thus, the discretized 2-dimensional state space is given by $\{s'_1, s'_2, ..., s'_m\} \times \{y'_1, y'_2, ..., y'_n\}$, where y'_k, $k = 1, ..., n$, are the possible income states. Choose a stopping rule parameter $\epsilon^{egm} > 0$. Note that the calculation of a stationary equilibrium in I-V requires a bounded wealth space where the maximum value denoted by s_{max} satisfies $s'(s_{max}, y) \leq s_{max}$ for all y under the wealth policy function $s'(s,y)$ calculated by the endogenous grid point

method for a given issuance price Q^0. The highest value s_m in our wealth space is a guess for a state that satisfies this condition. We check this condition after having calculated the policy functions at a given issuance price Q^0 (see 5).

2. Make a guess for next period's consumption policy function $(c')^0\,(s'_i, y'_k)$, where $k \in \{1, ..n\}$ and the guess is computed by $(c')^0\,(s'_i, y'_k) = -Q^0\,(s'_i - (1-\theta)s'_i/\pi) + \theta s'_i/\pi + y'_k$, at all states in the discretized state space.

3. Calculate a guess for current period's consumption policy function $c^0(s_i, y_k)$ (using two auxiliary functions $\hat{c}(s'_i, y_k)$ and $\hat{s}(s'_i, y_k)$):

   - Use $(c')^0\,(s'_i, y'_k)$ to compute a guess for current period consumption using $\hat{c}(s'_i, y_k)$ for future period wealth s'_i and some current period income y_k by using the Euler equation:

     $$\hat{c}(s'_i, y_k) = \\ \left(\frac{1-\theta+\theta/Q^0}{\pi/\beta}\left(p_{k1}\,(c')^0\,(s'_i, y'_1)^{-\sigma} + p_{k2}\,(c')^0\,(s'_i, y'_2)^{-\sigma} + ... + p_{kn}\,(c')^0\,(s'_i, y'_n)^{-\sigma}\right)\right)^{-1/\sigma}$$

     where $s'_i \geq s'_{y_k}$ at today's income state y_k due to the borrowing constraint.

   - Use the budget constraint and the auxiliary function $\hat{c}(s'_i, y_k)$ to compute current period wealth $\hat{s}$ for s'_i and y_k:

     $$\hat{s}(s'_i, y_k) = \left(\hat{c}(s'_i, y_k) + Q^0 s'_i - y_k\right)\pi/\left(Q^0(1-\theta)+\theta\right)$$

   - Calculate current period's consumption policy function at $(s_i, y_k) \in \{s_1, s_2, ..., s_m\} \times \{y_1, y_2, ..., y_n\}$ where the grid for today's wealth states is the next period's grid, i.e. $s_i = s'_i$, as follows:

     – The beginning-of-period wealth $\hat{s}(s', y_k)$ for $s' = -\gamma y_k$ is the highest wealth position in the discretized wealth space at which a household with income y_k borrows the maximum amount.

     – At $s_i \leq \hat{s}(-\gamma y_k, y_k)$, a household with the same income y_k but with beginning-of-period wealth s_i that is smaller or equal to $\hat{s}(-\gamma y_k, y_k)$ is borrowing constrained as well. The current period's consumption policy function at (s_i, y_k) is then computed by

     $$c^0(s_i, y_k) = Q^0\,(\gamma y_k + (1-\theta)s_i/\pi) + \theta s_i/\pi + y_k$$

     and end-of-period wealth is given by

     $$(s')^0\,(s_i, y_k) = -\gamma y_k.$$

– At $s_i > \hat{s}(-\gamma y_k, y_k)$, the borrowing constraint is not binding at beginning-of-period wealth s_i and income y_k in the current period. The current period's consumption policy function c^0 at (s_i, y_k) is then calculated using the implicit definition $\widetilde{c}^0(\hat{s}(s_i', y_k), y_k) = \hat{c}^0(s_i', y_k)$ where s_i' is today's choice for future beginning-of-period wealth when today's income is y_k while $\hat{s}(s_i', y_k)$ is today's beginning-of-period wealth which under current income y_k leads to this choice of s_i'. Then, $c^0(s_i, y_k)$ is computed by a linear interpolation of $\widetilde{c}^0(\hat{s}, y)$ at (s, y), where s takes on-grid values. The wealth policy function at (s_i, y_k) is then computed by using the budget constraint

$$(s')^0(s_i, y_k) = -\left(c^0(s_i, y_k) - \left(Q^0(1-\theta) + \theta\right) s_i/\pi - y_k\right)/Q^0.$$

- IF $||(c')^0(s' = s, y' = y) - c^0(s, y)|| < \epsilon^{egm}(1 + ||c^0(s, y)||)$, stop. Under the current guess for the issuance price Q^0, the policy function for consumption is then given by $c(s, y) = c^0(s, y)$ and the policy function for wealth is given by $s'(s, y) = (s')^0(s, y)$
ELSE $(c')^0 = c^0$ and start again step 3.

4. IF $s'(s_m, y) \leq s_m$ for all y, stop.
ELSE choose a higher value s_m and go back to step 1.

The stationary distribution for given policy functions is computed by calculating the normalized eigenvalue of the Markov transition matrix:

1. We add further wealth states to get a finer grid than the one used for the calculation of the policy functions (from 5 to 100 thousand grid points for s) and we calculate the wealth policy function values for the new states.

2. Calculate the transition probability of being in the state (s_j, y_l) in the next period if the current state is (s_i, y_k) and denote it by $P((s_i, y_k), (s_j, y_l))$. This probability is computed by $P((s_i, y_k), (s_j, y_l)) = \mathcal{P}(y_l|y_k) * I(s'(s_i, y_k) = s_j)$, where $I(s'(s_i, y_k) = s_j) = 1$ if $s'(s_i, y_k) = s_j$ and 0 otherwise. The Markov transition matrix is then given by the transition probabilities $P((s_i, y_k), (s_j, y_l))$ for all combinations of states.

3. Compute the eigenvector of the transition matrix associated with the largest eigenvalue (which is one). The stationary distribution on the grid is then given by the normalization of this eigenvector.

B.2.4 Calculation of the transition path to the new stationary equilibrium

At the beginning of period 0 the economy is in the stationary equilibrium under an inflation rate of 2% with the beginning-of-period distribution of wealth s induced by this inflation rate. In period 0 then the inflation rate unexpectedly and permanently changes to -2%. The economy then leaves the old stationary equilibrium in period 0 and converges to the new stationary equilibrium under an inflation rate of -2%. The transition path is computed as follows (see e.g. Rios-Rull (1999)):

- Calculate the stationary equilibria for the two inflation rates of 2% and -2% as described above and denote the respective stationary distributions by $\lambda_{2\%}$ and $\lambda_{-2\%}$.
- The beginning-of-period distribution in period t of the transition path is denoted λ_t. In period 0, this distribution is given by $\lambda_0 = \lambda_{2\%}$. The beginning-of-period distribution after the economy has converged into the new stationary equilibrium is denoted λ_∞ and given by $\lambda_\infty = \lambda_{-2\%}$.
- Calculate the transition path:

  1. Assume that the transition into the new stationary equilibrium takes T periods. This implies $\lambda_T = \lambda_\infty$.
  2. Find two price paths $Q^{l,1} = \{Q_t^{l,1}\}_{t=0}^T$ and $Q^{h,1} = \{Q_t^{h,1}\}_{t=0}^T$ with $Q_t^{l,1} < Q_t^{h,1}$ for all $t \leq T$ that satisfy (i) $|\sum_{s,y} \lambda_t^Q(s,y) s_{t+1}^Q(s,y)| > 0$ at $Q_t^{l,1}$ and (ii) $|\sum_{s,y} \lambda_t^Q(s,y) s_{t+1}^Q(s,y)| < 0$ at $Q_t^{h,1}$ for all $t \leq T$ where λ_t^Q and $s_{t+1}^Q(s,y)$ denote the distribution and wealth policy function in period t of the transition path under a given price path. To find a price path that satisfies (i), we choose a path with relatively low values for the issuance prices, calculate steps 4-5 and check the condition (i) for all $t \leq T$. If this condition is not satisfied, we repeat steps 4-5 with a lower price path until we have found a path that satisfies the condition (i). Proceed analogously for Q_t^h. Choose stopping rule parameters $\epsilon^s > 0$ and $\epsilon^\lambda > 0$.
  3. Denote the current iteration step by i. Calculate a price path $\{\tilde{Q}_t\}_{t=0}^T$ with $\tilde{Q}_T$ given by the value of the stationary equilibrium induced by the inflation rate of -2% and $\tilde{Q}_t = 0.5(Q_t^{l,1} + Q_t^{h,1})$ for $t < T$. In iteration step i, a guess for the equilibrium sequence of issuance prices of the transition path $\{\hat{Q}_t^i\}_{t=0}^T$ is then calculated by:
     - If $i = 1$, $\{\hat{Q}_t^i\}_{t=0}^T = \{\tilde{Q}_t\}_{t=0}^T$
     - If $i > 1$, $\{\hat{Q}_t^i\}_{t=0}^T = \varphi\{\hat{Q}_t^{i-1}\}_{t=0}^T + (1-\varphi)\{\tilde{Q}_t\}_{t=0}^T$ with $\varphi \in [0,1)$
  4. Since we know $c_T(s,y)$, which is given by the policy function of the new stationary equilibrium, and have a guess $\{\hat{Q}_t^i\}_{t=0}^T$, we can solve backwards for the policy functions in period $t =$

$0, ..., T-1$ of the transition path at the given price path $\{\hat{Q}_t^i\}_{t=0}^T$. We denote these policy functions by $\{c_t^Q(s,y), s_{t+1}^Q(s,y)\}_{t=0}^T$ where $c_T^Q(s,y)$ and $s_{T+1}^Q(s,y)$ are the policy functions of the new stationary equilibrium.

5. Use the policy functions $\{s_{t+1}^Q(s,y)\}_{t=0}^{T-1}$ and λ_0 to iterate the distribution forward to get a path for the distribution at the given price path $\{\hat{Q}_t^i\}_{t=0}^T$. We denote this path for the distribution by $\{\lambda_t^Q\}_{t=0}^T$ with $\lambda_0^Q = \lambda_0$.

6. Use $\{\lambda_t^Q\}_{t=0}^T$ to compute $\hat{A}_t = \sum_{s,y} \lambda_t^Q(s,y) s_{t+1}^Q(s,y)$ for $t = 0, ..., T$. Check for debt market clearance: If
$$\max_{0 \leq t \leq T} \left| \hat{A}_t \right| < \epsilon^s$$
go on. If not, set $Q_t^{l,i+1} = \varpi Q_t^{l,i} + (1-\varpi)\hat{Q}_t^i$ with $\varpi \in [0,1)$ in periods in which $\hat{A}_t > \epsilon^s$ and $Q_t^{h,i+1} = \varpi Q_t^{h,i} + (1-\varpi)\hat{Q}_t^i$ in periods in which $\hat{A}_t < \epsilon^s$ and go back to step 3.

7. Check for $\left\| \lambda_T^Q - \lambda_T \right\| < \epsilon^\lambda$. If yes, the transition converges smoothly into the new stationary equilibrium, $\{Q_t\}_{t=0}^T = \{\hat{Q}_t^i\}_{t=0}^T$ is the equilibrium price path and the equilibrium policy functions are given by $\{c_t, s_{t+1}\}_{t=0}^T = \{c_t^Q, s_{t+1}^Q\}_{t=0}^T$. If not, go back to step 1 and start again with a higher T.

8. After having calculated the transition path for the policy functions and wealth distribution, we calculate the transition path for the value functions $\{v_t(s,y)\}_{t=0}^T$. Denote the value function in the stationary equilibrium induced by an inflation rate $\pi = -2\%$ ($\pi = 2\%$) by $\overline{v}_{-2}$ ($v_{2\%}$). The value function in period T is then given by $v_T = \overline{v}_{-2}$. We solve for the value functions in periods $t = 0, ..., T-1$ backwards from period T on by
$$v_t(s_i, y_k) = u(c_t(s_i, y_k)) + \beta \sum_{l=1}^{5} p_{k,l} v_{t+1}\left(s_{t+1}(s_i, y_k), y_l'\right)$$
using v_T and policy functions c_t and s_{t+1} where y_l' for $l = 1, ..., 5$ denotes the possible income states in the next period $t+1$.

- Note that $v_{-2}(s,y)$ is the expected lifetime utility in period 0 of a household with income y and beginning of period 0 wealth s who has just been hit by the change in the inflation rate to -2%. This lifetime utility takes into account all the transition dynamics which the household is going to live through while $v_2(s,y)$ gives the expected lifetime utility in period 0 of a household with the same income y and beginning of period 0 wealth s but who lives in an economy under an unchanged inflation rate of $\pi = 2\%$. If $v_{-2}(s,y) > (<) v_2(s,y)$, a household in state (s,y) in period 0 benefits (looses) under the reduction in the inflation rate.

C.2 Additional figures

Figure 2.9: Consumption and welfare (in consumption units) of relatively patient lenders

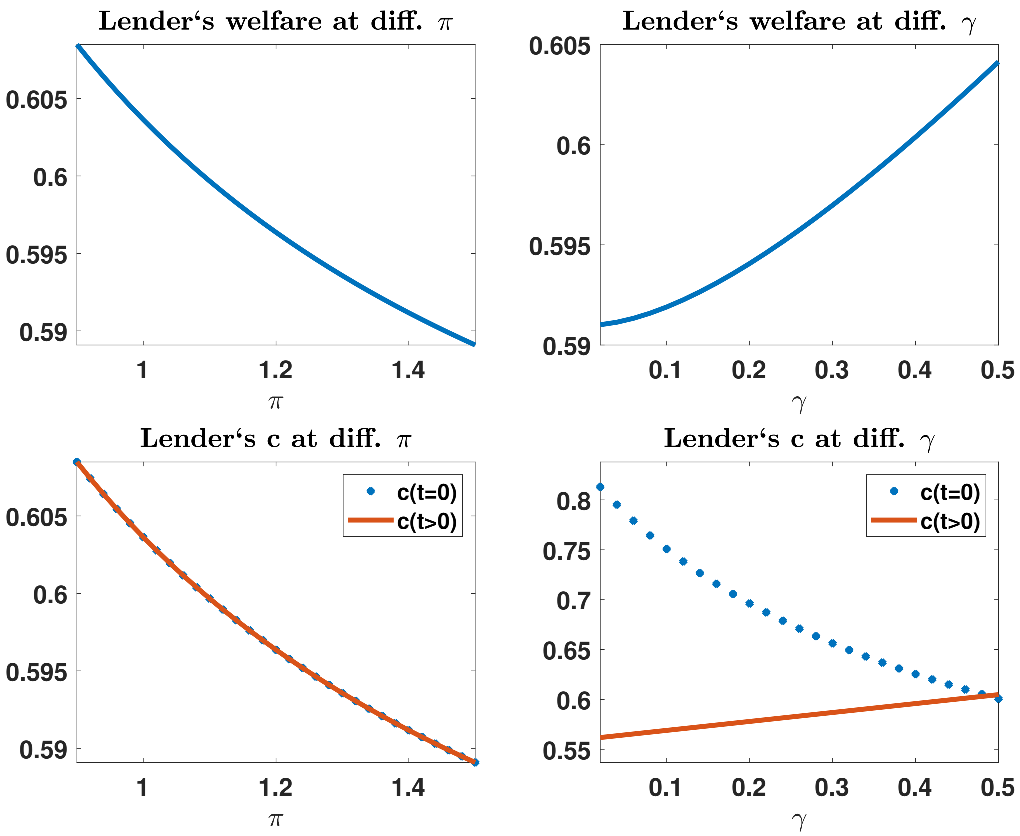


Figure 2.10: Policy functions for consumption and savings in period 0

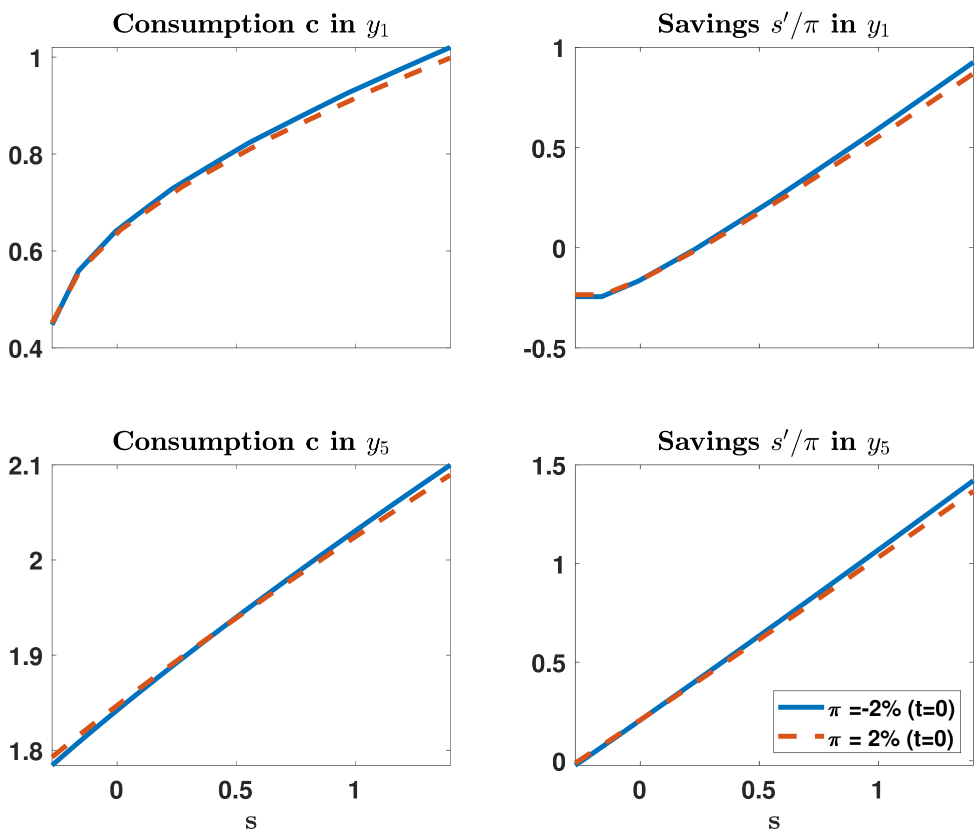
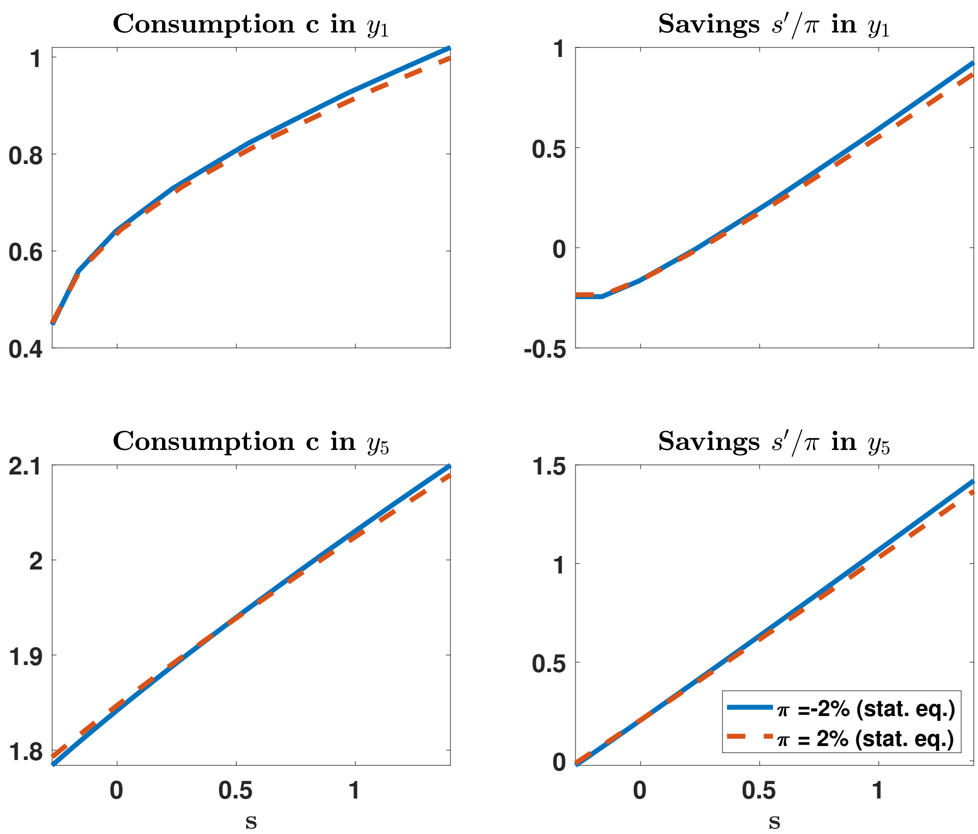


Figure 2.11: Policy functions for consumption and savings for stationary wealth distributions



Chapter 3

A quantitative analysis of general equilibrium effects in a heterogeneous household economy

This chapter is based on Loenser (2020).

3.1 Introduction

How important are general equilibrium effects via price changes in heterogeneous household economies under incomplete markets? Unexpected changes in economic fundamentals influence households' saving and consumption behaviour through both direct and indirect effects. Direct effects are those that operate in the absence of any changes in market prices. If, for example, preferences for a specific good or asset rise, households tend to increase their relative demand for it and reduce it for others. In general equilibrium, additional indirect effects on households' saving and consumption behaviour arise from changes in market prices that emanates from the direct effects. The relative magnitude of these direct and indirect effects is determined especially by the following aspects. Firstly, it depends on how strongly households' saving and consumption respond to - changes in economic fundamentals at given market prices and to - changes in market prices given economic fundamentals. Secondly, the magnitude depends on the sensitivity of market prices. In the following, this paper examines the relative contribution of direct- and price induced indirect effects on household decisions and the role of the price sensitivity for this relative contribution in a macroeconomic framework with collateralized loans and an empirically relevant specification of household heterogeneity.

There is already a quantitative literature that emphasizes the role of price fluctuations on household saving and consumption in models with collateralized loans. These studies typically find a relatively large quantitative impact of changes in the price of collateral on borrowers' decisions via adjustments in borrowing limits (e.g. Guerrieri and Iacoviello (2017) or Bianchi and Mendoza (2018)). In these models, the direct effects of shocks on borrowers' saving and consumption induce relatively large changes in collateral prices that strongly reinforce the effect on household decisions. While these studies emphasize the role of price changes via collateral effects, the above-mentioned aspects that are important for the relative magnitude of direct and indirect effects are typically modelled in a relatively stylized form.

Firstly, these studies analyze household saving and consumption under a relatively stylized specification of household heterogeneity. The way this heterogeneity is modelled, however, has important implications for the impact of changes in market prices on household decisions and thereby for the relative magnitude of direct and indirect effects. In Guerrieri and Iacoviello (2017), for example, there are two groups of households separated by a time-invariant difference in patience where low-patient households are borrowers restricted by an occasionally binding collateral constraint whereas high-patient households are unconstrained lenders who are never directly affected by this constraint. The collateral asset is given by housing which can be also used as a utility providing durable consumption good. While this study is focussed on collateral effects of price changes on borrowers, distributive effects across the different groups of households are not analyzed at all.[1] Distributive effects of price changes, however, can mitigate or reinforce the impact of collateral effects on constrained borrowers and can also affect unconstrained households by redistributing funds among net-buyers and net-sellers.

Bianchi and Mendoza (2018) apply a small open economy model populated by a domestic representative borrower and an exogenously given foreign lender where only the borrower is able to hold the collateral asset. In this type of model, distributive effects are completely shut down because there is no trade in the collateral asset.[2] Models, however, that allow for a *less stylized* specification of household heterogeneity possibly generate distributive effects that are relatively important for household saving and consumption.[3] Therefore, as a novel contribution, my paper divides the indirect effects of price changes into collateral and distributive effects to analyze their relative magnitude on household decisions in a quantitative model that allows for an empirically relevant specification of household heterogeneity.

[1] Distributive effects arise when the relative price at which agents trade goods or assets changes. This price adjustment then redistributes funds among buyers and sellers and thereby influences relative demand (see e.g Davila and Korinek (2018)).

[2] Distributive effects via changes in the price of bonds are also absent due to a fixed real interest rate.

[3] Diaz and Luengo-Prado (2010), for example, build a model that is able to match empirical wealth and debt distributions of the US household sector in which households are heterogeneous due to (not-perfectly insurable) idiosyncratic shocks concerning their efficiency of labor supply. The price of housing in their model, however, is assumed to be time-invariant.

Secondly, these studies assume a fixed supply of collateral assets which generates a relatively large sensitivity of collateral prices. In contrast to these papers, there are quantitative studies that analyze saving and consumption of the US household sector in models with a completely elastic supply of the collateral asset (see e.g. Diaz and Luengo-Prado (2010)). In this case, the collateral price is time-invariant. The elasticity of asset supply with respect to asset price changes has an important impact on the magnitude of indirect price effects. The lower this elasticity is, the higher are price fluctuations when asset demand changes and the higher is the importance of indirect effects. Empirical studies, for example, show that areas in the US with a relatively inelastic housing supply, due to e.g. geographical limitations, experienced relatively high growth rates in house prices in the 1980s and from 2002 to 2006 whereas areas with a relatively elastic supply had lower growth rates (see e.g. Glaeser et al. (2008), Mian and Sufi (2009), Saiz (2010) or Mian et al. (2013)).[4] Given that housing is typically the dominant component of household collateral and wealth in the US (see e.g. Li and Yao (2007) and Hintermaier and Koeniger (2016)[5]), household borrowing and consumption depends to a relatively large degree on the sensitivity of housing prices and therefore on the elasticity of housing supply (see e.g. Mian and Sufi (2011) and Mian et al. (2013)). For example, those areas that experienced relatively high growth rates in house prices due to a relatively inelastic housing supply typically also experienced larger increases in home-equity borrowing and thereby also in consumption.

These empirical studies suggest that the results of theoretical studies on household decisions under collateralized loans - positive as well as normative ones - depend on the modelled supply elasticity of collateral assets. From a positive perspective, models with a fixed (completely elastic) supply generate an upper (lower) bound for the sensitivity of household decisions. From a normative perspective, models with a fixed (completely elastic) supply induce a relatively large (small) role of financial regulation by emphasizing (disregarding) the role of a pecuniary externality that is induced by endogenous collateral prices.[6] The higher the sensitivity of endogenous prices is, the more this pecuniary externality matters for efficiency under financial frictions.[7] Therefore, as a novel contribution, my paper examines the role of variations in the supply elasticity of an collateral asset for the relative magnitude of direct and indirect effects in a heterogeneous household economy with incomplete markets. The focus of this analysis is on the positive implications of supply elasticities for household decisions while a normative analysis in a quantitative framework is left to future research.

[4]Saiz (2010) constructs an objective index measuring the possibility to expand new housing in metro areas. If land-topology in a metro area is such that expansion from the center is restricted - for example by hills, oceans or lakes - this area gets a low housing supply elasticity score.

[5]Since the 1980s, household debt secured by durable consumption goods (like vehicles or especially residential real estate) has accounted for more than 90% of US household debt in the United States (see Hintermaier and Koeniger (2016)).

[6]This pecuniary externality with regard to the collateral constraint provides a straightforward rationale for macroprudential financial regulation, as for example shown by Lorenzoni (2008), Bianchi (2011) or Jeanne and Korinek (2019).

[7]See Davila and Korinek (2018) for an analysis of the (in-)efficiency of pecuniary externalities in a three-period model with financial frictions.

The analysis in this paper addresses the following three questions:

- What is the relative magnitude of direct effects and total indirect ones arising from changes in market prices in a heterogeneous household economy with an empirically relevant specification of household debt and wealth?
- What is the relative importance of collateral and distributive effects?
- How important is the supply elasticity of the collateral asset for the relative magnitude of direct and indirect effects?

To address these points, the paper develops a heterogeneous agents model à la Huggett (1993) extended by a durable good, that has a dual role as a non-financial (collateral) asset and as a utility providing consumption good, by a collateral constraint as well as by a production sector. This study defines the empirical counterpart of durable consumption as residential housing and vehicles, given - that these two categories account for the majority of collateral used for household credit (see Hintermaier and Koeniger (2016)) and - that especially house price fluctuations are an important driver of households' borrowing and consumption decisions by affecting collateral values (see e.g. Case et al. (2005), Campbell and Cocco (2007), Glaeser et al. (2008), Saiz (2010), Mian and Sufi (2011), Abdallah and Lastrapes (2012), Mian et al. (2013) or Guerrieri and Iacoviello (2017)). The model is an adapted version of the framework analyzed in Diaz and Luengo-Prado (2010) that is able to match empirical wealth and asset distributions of the US household sector. The major difference to their model is that the aggregate supply of the collateral asset does not need to be completely elastic such that the collateral price can be time-variant. The focus of my analysis is then on the direct and indirect effects induced by an unexpected change in the preferences for the durable good. This approach is motivated by quantitative studies which find that house price movements can be explained to a large extent by housing preference shocks (see e.g. Liu et al. (2013), Berger et al. (2017) or Guerrieri and Iacoviello (2017)).

To address the first question, the total effects of the preference shock on households' decisions are calculated in a version of the model with a fixed supply of durables as in Guerrieri and Iacoviello (2017) or Bianchi and Mendoza (2018) and then decomposed into direct and indirect effects. Note that in this version under a fixed supply the price sensitivity and thereby the relative importance of indirect effects are at the maximum. The magnitude of the unexpected rise in preferences is set in a way to get an increase in the durables price of ca. 1.9% on impact, which is close to the impulse response of the housing price under a shock on housing preferences in the estimated model in Guerrieri and Iacoviello (2017).[8] In this experiment, indirect effects are both quantitatively and qualitatively more important for household saving

[8] See figure A.2 in the online appendix of their paper.

and consumption than the direct ones. While the direct effects on *constrained borrowers* at given prices are an *increase* in demand for durables and a *reduction* in non-durables goods in the impact period, the total effect is an *increase* in both, in demand for durables by ca. $2.68\% - 3.89\%$ but also in non-durable consumption by ca. $1.03\% - 1.36\%$. Furthermore, the total increase in borrowing of these households is ca. 8-12 times larger than the direct effect. *Savers* with a relatively high income also *increase* demand for durables and *reduce* non-durable consumption as a direct effect in the impact period, while the total effect is a *reduction* in durables by ca. $0.19\% - 0.26\%$ and an *increase* in non-durable goods by ca. $0.13\% - 0.20\%$.[9]

The total indirect effects of price changes on household decisions in the impact period induced by the preference shock can be decomposed into collateral and distributive effects. Under the persistent shock on preferences the price path of durables persistently changes. This then implies that decisions of (forward-looking) households in the impact period are not only affected by those collateral and distributive effects that are induced by price changes in the *current* period but also by those effects that are induced by price changes in *future* periods. An increase in the price path, for example, not only affects household decisions in the impact period by raising the *current* collateral value but also by increasing *future* ones. These different types of indirect effects at different points in time can have opposing effects on households in the impact period or can reinforce each other. To address their relative magnitude the total indirect effects are decomposed into current collateral- and distributive effects and future ones. While savers are mainly affected by distributive effects, the study finds that this type of indirect price effects is also relatively important for constrained borrowers. Comparing only the magnitude of current collateral and distributive effects, for example, the analysis shows that household decisions are mainly driven by the latter effects and *not* by the former ones. This importance of distributive effects contrasts with the quantitative studies that model a more *stylized* specification of household heterogeneity as in Guerrieri and Iacoviello (2017) or Bianchi and Mendoza (2018).

The relatively high importance of indirect price effects is, however, the result of the relatively high price sensitivity induced by an inelastic aggregate supply of durables. The model version with a fixed supply generates an upper bound for the importance of indirect price effects for household decisions. Therefore, the impact of the same preference shock analyzed before is also examined in a version of the model in which the aggregate supply of durables is more elastic. In this version, an increase in demand raises both the price and the supply of durables while the rise in the durables price and therefore the magnitude of indirect effects are smaller compared to the version under a fixed supply. The analysis then finds that even under a relatively high elasticity of aggregate supply *indirect price effects* mainly drive

[9] Note that *not all* constrained borrowers have the same beginning-of-period wealth. Changes in household decisions are therefore *not* equally distributed. The same is true for savers.

household saving and consumption. The total effects on constrained borrowers, for example, are still mainly driven by indirect ones if the supply elasticity is so high that the price of durables only increases by ca. 0.2% on impact. Only under a lower price sensitivity - induced by a higher supply elasticity - direct effects (and not indirect price effects) mainly drive household decisions.

The focus of this paper is on the importance of fluctuations in the price of the non-financial collateral asset (durables) while the price of the financial asset (the real interest rate on bonds) is time-invariant due to a specific form of the technology in the firm sector. There are two reasons for this approach. First, the price of the non-financial collateral asset is also central in the literature mentioned above. Second, the indirect price effects can be completely ascribed to the price of one asset while an additional endogenous price makes the individual effects of each price hard to pin down.[10]

The remainder is structured as follows. Sections 2 develops the heterogeneous agent model. Section 3 presents the calibration and the results of the quantitative analysis and section 4 concludes.

3.2 The model economy

The model analysed in this paper is a heterogeneous agents economy à la Huggett (1993) extended by a durable good, that has a dual role as a non-financial (collateral) asset and as a utility providing consumption good, as well as by a collateral constraint. The framework is an adapted version of the model presented in Diaz and Luengo-Prado (2010). As the major difference to their model, the elasticity of aggregate supply of the collateral good is not assumed to be completely elastic such that the collateral price is *not* time-invariant.[11]

Households Consider an economy with infinitely lived and infinitely many households i of mass one. In each period t, a household $i \in [0,1]$ derives utility from consumption of a non-durable good, $c_{i,t}$, and a durable good, $d_{i,t}$, as given by function $u(c_{i,t}, d_{i,t})$ which is increasing and concave with respect to both arguments. Expected lifetime utility of household i is given by

$$E \sum_{t=0}^{\infty} \beta^t u(c_{i,t}, d_{i,t}), \tag{3.1}$$

where E denotes an expectations operator and β is the discount factor. Each period, households receive a shock to the efficiency units of labor $e_{i,t} \in EL = \{e_1, ..., e_n\}$ with $e_1 < ... < e_n$ which is Markov with

[10] A further justification to treat the real interest rate as an exogenous object is based on Mendoza and Quadrini (2010) who document that ca. 50% of the increase in net credit in the US since the mid-1980s was financed by foreign capital inflows, and in 2010 more than 50% of US treasury bills was owned by foreign agents (see Bianchi and Mendoza (2018)).

[11] Further notable differences are (i) that the empirical counterpart of durable consumption is not only defined as residential housing but adds vehicles as well, given that these two categories account for the majority of collateral used for household credit and (ii) that there are no transaction costs in trading the durable good.

transition matrix $P_{e,e'}$. Total labor supply denoted by L is time invariant and given by $L = \int_0^1 e_{i,t} di$ $\forall t \geq 0$.

In this economy, financial markets are incomplete and the only financial asset is a non-state-contingent one-period bond. A bond issued in period t trades at unit price and promises the payment of $1 + r_t$ units of the non-durable good, which serves as the numeraire in the model, in period $t+1$. It is further assumed that there is a financial friction that gives rise to a price-dependent borrowing constraint for households. Specifically, assume that borrowers cannot commit to repay debt and that debt can be renegotiated after issuance but still within period t. Borrowers are allowed to make a take-it-or-leave-it offer to reduce the value of debt. If the lender rejects the offer, he can seize a fraction γ of the borrower's assets (durable goods), which he can sell at the competitive market price q_t. Offers are therefore accepted when the repayment value of debt at least equals the current value of seizable assets. Without loss of generality, it is assumed that default and renegotiation never happen in equilibrium. Hence, when debt is issued, an individual borrower i has to take into account that the amount of debt $-b_{i,t}$ is constrained by

$$- b_{i,t}(1 + r_t) \leq \gamma q_t d_{i,t}, \tag{3.2}$$

where $d_{i,t}$ denotes the amount of the asset (durable good) held during the debt contract. This type of borrowing constraint featuring the price of the asset for the period of issuance q_t is common in quantitative studies with collateralized debt (see e.g. Favilukis et al. (2017), Lorenzoni and Guerrieri (2017) or Berger et al. (2017)), and it is consistent with empirical evidence (see Cloyne et al. (2019)). A given household can borrow (lend) by issuing (buying) bonds to (from) other households or a representative financial institution (see below). Under no-arbitrage the interest rate on bonds issued by households and the one issued by the financial institution are equal.

The budget constraint of a household i for period t is given by

$$c_{i,t} + q_t(d_{i,t} - d_{i,t-1}) + b_{i,t} = b_{i,t-1}(1 + r_{t-1}) + w_t e_{i,t}, \tag{3.3}$$

given $d_{i,-1}$ and $b_{i,-1}(1+r_{-1})$ where w_t denotes the real wage rate which is equal for all households.[12] The non-durable consumption good is in endogenous aggregate supply and can be produced in two different ways. Firstly, there is a representative firm that rents capital and demands labor from households to produce the non-durable good similar to Diaz and Luengo-Prado (2010) (see the non-durable goods producing firm below). Secondly, there is representative firm that can transform durables and non-durables into each other. The durable consumption good is only produced by the latter firm and aggregate

[12] Households are also owners of the financial institution and the different non-financial firms. However, because these firms earns zero profits in equilibrium, the ownership does not show up in the budget constraint.

supply of this good needs not to be fixed. In contrast to Diaz and Luengo-Prado (2010), however, the transforming firm is not restricted to transform durables and non-durables into each other on a *one-to-one* basis. This implies that, contrary to Diaz and Luengo-Prado (2010), the price of durables is not restricted to be fixed - as explained in the following.

Households can change their stock of durables in the following two ways. The first possibility is by trading with other households at the price of durables given by q_t. Secondly, there exists, as explained above, a representative firm that is able to transform durable and non-durable goods into each other. For simplification, this firm is not assumed to be profit maximizing but is just a passive part of the economy. More precisely, households can hand in non-durable (durable) goods to the firm to get back durable (non-durable) goods. Net-demand of durables of the household sector is given by $\Delta d_t := \int_0^1 d_{i,t} - d_{i,t-1} di$. This net-demand is satisfied by the transforming firm. The transformation technology is given by the function T. If net-demand is positive in period t, i.e. $\Delta d_t > 0$, households have to hand-in $T(\Delta d_t)$ units of the non-durable good in the same period. If net-demand is negative, i.e. $\Delta d_t < 0$, households get back $T(\Delta d_t)$ units of the non-durable good. If net-demand is zero, i.e. $\Delta d_t = 0$, households only trade with each other and do not use the firm. The transformation technology satisfies $T(0) = 0$, $0 < T'(\Delta d_t) < \infty$ and $T''(\Delta d_t) \geq 0$.[13] In equilibrium then, the no-arbitrage condition $q_t = T'(\Delta d_t)$ must be satisfied.[14]

In the quantitative analysis in section 3.3, the transformation is modelled in two different ways. In the benchmark version analyzed in section 3.3.2, it is infinitely costly to transform durable and non-durable goods into each other.[15] In this case, aggregate supply of the durable good is time-invariant as in Guerrieri and Iacoviello (2017) or Bianchi and Mendoza (2018) and given by the initial endowment $\bar{d} > 0$ with $\bar{d} = \int_0^1 d_{i,-1} di$, i.e. $\int_0^1 d_{i,t} di = \bar{d}\ \forall t \geq 0$. The price of durables is then endogenously determined in the market for durables in which only households trade with each other.

In the second version analyzed in section 3.3.4, the cost of transformation is finite such that aggregate supply of durables is not fixed. As explained above, the price of durables is then given by $q_t = T'(\Delta d_t)$. This implies that the price sensitivity depends on the functional form of the transformation technology. If, for example, durable and non-durable goods can be transformed into each other at a constant rate, i.e. $T''(\Delta d_t) = 0$, the price of durables is time-invariant as in Diaz and Luengo-Prado (2010). If the rate of transformation is, however, not constant (but still positive), the price of durables is not fixed and changes with aggregate demand. Note that in the second version the price sensitivity is smaller compared to the

[13] Just assume that this firm is a machine that produces Δd_t units of the durable good, if $T(\Delta d_t)$ units of the non-durable good are handed in, and vice versa. This machine has fallen from heaven, does not belong to anybody and needs no maintenance.

[14] Assume that net-demand of durables of the household sector is Δd_t and that now a household wants to increase durables by a relatively small amount $\partial d > 0$. If this household uses the firm, $T'(\Delta d_t)\partial d$ units of the non-durable good have to be handed-in. If this household instead buys the amount of durables from other households, $q_t \partial d$ units of the non-durable good have to be paid. Under no-arbitrage, we then get $q_t = T'(\Delta d_t)$.

[15] In other words, the machine explained in the footnote above does not work anymore from period 0 on.

model version with a fixed supply of durables.[16]

Non-durable goods producing firm There is a representative firm that rents capital K_{t-1} from a financial institution at the end of period $t-1$ and demands labor L_t from households at the beginning of period t to produce non-durable goods according to the production function F. For simplicity, this study abstains from uncertainty in the production process. After production has taken place, the firm passes the complete amount of capital back to the financial institution. The firm solves the problem

$$max_{\{K_t,L_t\}_{t=0}^{\infty}} \; E\sum_{t=0}^{\infty} F(K_{t-1},L_t) - r^f_{t-1}K_{t-1} - w_tL_t \tag{3.4}$$

given $r^f_{-1}K_{-1}$ where the wage per efficiency unit of labor, w_t, and the rental price of capital, r^f_t, are taken as given. In equilibrium, $w_t = F_2(K_{t-1},L)$, $r^f_t = F_1(K_t,L)$ and the firm earns zero profits where L denotes the total labor supply. For simplicity, it is assumed that the firm is perfectly committed to return the complete amount of rented capital to the financial institution and to pay the complete contracted rental price in every period.

Financial institution There is a representative risk-neutral financial institution that borrows funds B_t from saving households in period t by issuing bonds at the real interest rate r_t and then invests these funds in two different ways. Firstly, it lends the amount S_t to borrowing households at the real interest rate r_t, where borrowing of a given household is constrained by (4.1).[17] Secondly, the financial institution is able to transform the remaining funds $B_t - S_t$ into physical capital K_{t+1} on a one-to-one basis, i.e. $K_{t+1} = B_t - S_t$, that can be rented to the non-durable goods producing firm at the non-state-contingent rental price r^f_t as in Diaz and Luengo-Prado (2010). The financial institution solves the following problem

$$max_{\{B_t,S_t,K_{t+1}\}_{t=0}^{\infty}} E\sum_{t=0}^{\infty} B_t - S_t - K_{t+1} + K_t\left(1+r^f_{t-1}\right) + (S_{t-1}-B_{t-1})(1+r_{t-1}) \tag{3.5}$$
$$s.t.\; K_{t+1} + S_t = B_t$$

under given $K_0(1+r^f_{-1})$ and $(S_{-1}-B_{-1})(1+r_{-1})$. The financial institution takes the interest rates as given and earns zero profits in equilibrium. The no-arbitrage condition $r_t = r^f_t$ must be satisfied in equilibrium for the problem to be well defined. After the firm has returned the capital, the financial institution transforms it back into non-durables on a one-to-one basis and uses these funds to repay debt to households. For simplicity, assume that the financial firm is perfectly committed to repay its debt.

[16] An equilibrium under a fixed supply as well as under an elastic supply of durables is defined in A.3.

[17] Under no-arbitrage, the real interest rate on bonds issued by the financial institution and the one on bonds issued by households are the same.

3.3 Quantitative analysis

In this section, the relative magnitude of direct effects and indirect ones arising from changes in market prices on household saving and consumption is analyzed in a calibrated version of the heterogeneous agent model. In the benchmark version of the model, the aggregate supply of the durable good is fixed as in Guerrieri and Iacoviello (2017) or Bianchi and Mendoza (2018). The focus of this analysis is on the impact of a shock affecting preferences for durables. The reason to do so is that quantitative studies typically suggest that house price volatilities in the US are to a large extent driven by housing preference shocks (e.g. Liu et al. (2013), Berger et al. (2017) or Guerrieri and Iacoviello (2017)). In section 3.3.1, functional forms and parameters are presented. Section 3.3.2 examines the relative magnitude of direct and total indirect effects induced by a positive shock on preferences for durables in the benchmark model with a fixed supply of durables. Section 3.3.3 decomposes the total indirect price effects into collateral and distributive effects to works out their relative importance for household decisions. In section 3.3.4, the role of the price sensitivity for the relative magnitude of direct and indirect effects is analyzed in a version of the model with an incompletely elastic supply of durables.

3.3.1 Functional forms and parameters

Since the model needs to be solved numerically, functional forms and parameters have to be specified.[18] The model is calibrated by choosing suited parameter values from related studies and by targeting selected statistics of the income, wealth, and durable distribution observed for the United States, similar to Diaz and Luengo-Prado (2010), based on data from the Survey of Consumer Finances 2016 for the year 1998. The parameter values are summarized in Table (3.1).

Parameter	Value	Target
α_K	1.04	Real interest rate 4%
β	0.90	Debt to durables
γ	0.80	Empirical LTV ratio
σ_c	2.00	Standard value
σ_d	2.00	Standard value
$\pi_{R,S} \times 100$	0.05	Gini coefficient income
$\pi_{S,R} \times 100$	0.70	Gini coefficient wealth
$\bar{d}$	0.12	Relative durables distribution
ϕ_d	0.10	$q\bar{d}/c$

Table 3.1: Model parameters in chapter 3

The empirical counterpart of durable consumption in this model is not defined only as residential housing but adds vehicles as well, given that these two categories account for the majority of collateral

[18]Details about the numerical solution procedure can be found in Appendix B.3.

used for household credit. For the household utility function, a additive separable specification is assumed as in Diaz and Luengo-Prado (2010) or Guerrieri and Iacoviello (2017) given by

$$u(c_t, d_t) = \frac{c_t^{1-\sigma_c}}{1-\sigma_c} + \phi^d \frac{d_t^{1-\sigma_d}}{1-\sigma_d} \tag{3.6}$$

where $\sigma_c > 0$ and $\sigma_d > 0$ are the inverse intertemporal elasticities of substitution and $\phi^d > 0$ captures preferences for durables. For σ_c and σ_d, a standard value of two is chosen. The discount factor β is 0.9 to closely match the average ratio of secured debt to durable goods in the first quartile of the wealth distribution.[19] The production function is assumed to be additive separable in capital and labor and is given by

$$F(K_{t-1}, L_t) = \alpha_K K_{t-1} + \alpha_L L_t \tag{3.7}$$

where $\alpha_K > 0$ and $\alpha_L > 0$ denote the marginal product of capital and labor. Under this production function the real interest rate and the wage rate are time-invariant and given by $r_t = \alpha_K - 1$ and $w_t = \alpha_L$.[20] The parameter α_K is set to 1.04 to get a real interest rate of 4% while α_L is set to one such that the individual labor supply state $e_{i,t}$ is identical to individual labor income $w_t e_{i,t}$. The fraction of seizable collateral γ is set at 0.8, implying an empirically plausible loan-to-value ratio of 80% (see Diaz and Luengo-Prado (2010)).

The support for the efficiency units of labor EL and the associated transition probabilities π are chosen to match the Gini coefficients for income $w_t e_{i,t}$ (0.43) and (net-)wealth $b + qd$ (0.8). As is well known in the literature (see e.g. Di Nardi et al. (2015)), without additional assumptions, a standard Bewley-Aiyagari-Huggett-type incomplete-markets model fails to match important features of the wealth distribution, the concentration of wealth at the top in particular. To address this shortcoming, Diaz and Luengo-Prado (2010) is followed and assumed that individual efficiency of labor supply follows a log-normal AR(1) process,

$$\ln e_{i,t} = \rho \ln e_{i,t-1} + \sigma \varepsilon_{i,t},$$

with autocorrelation $\rho = 0.9895$ and standard deviation $\sigma = 0.1257$ and, additionally, that there is also a small probability $\pi_{R,S}$ of transitioning to a relatively high efficiency state that results in a "superstar" income, which is left with probability $\pi_{S,R}$. Note that, due to a time-invariant wage rate equal for all households, the relation of individual labor supply to individual labor income is time-invariant and equal for all households. In the following, w.l.o.g., the exogenous individual states are given by individual

[19] The average ratio of secured debt, given by home-secured debt and vehicle installment loans, to durable goods, given by housing and vehicles, in the first quartile of the wealth distribution for the US household sector in 1998 is about 0.74 (see Survery of Consumer Finances).

[20] Under a constant interest- and wage rate, indirect effects can be completely assigned to changes in the durables price.

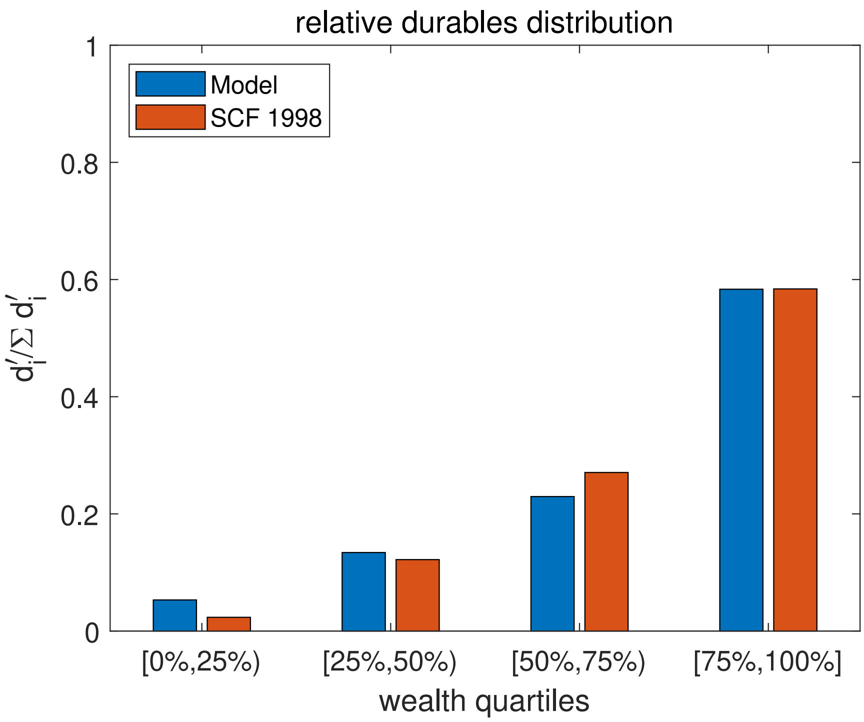


Figure 3.1: Relative durable holdings for different wealth quartiles (data vs model)

labor income $y_{i,t}$ with $y_{i,t} := e_{i,t}w_t$, and not by individual labor supply $e_{i,t}$.[21] While the AR(1) process provides a good fit for most of the population, it cannot suitably account for the top 1% of the labor income distribution. While the "regular" income states y_1 to y_6 are obtained by discretizing the AR(1) process via the method proposed by Tauchen and Hussey (1991), the superstar income value y_7 is set to match the empirical ratio $y_7/y_6 = 6$ and the transition probabilities are $\pi_{R,S} = 0.0005$ and $\pi_{S,R} = 0.0075$. Combining these values with the transition probabilities for the regular income states, obtained by discretizing the AR(1) process, yields the transition probabilities $\pi\left(y_{i,t+1}|y_{i,t}\right)$. The aggregate supply of the durable good $\bar{d} = 0.12$ is chosen to provide a reasonable fit for the durables distribution in the benchmark version, as given by Figure 3.1. Figure 3.2 shows the distribution of net-wealth for the model and the data. Lastly, the preference parameter for durables is set to $\phi^d = 0.1$ to get $q\bar{d}/c \approx 1.6$ which is roughly in line with Diaz and Luengo-Prado (2010).[22]

3.3.2 The relative magnitude of direct and indirect effects

This section examines the relative importance of direct and indirect effects on household saving and consumption induced by a shock on preferences for durables. As in Guerrieri and Iacoviello (2017) or

[21] The income states and the corresponding transition matrix is shown in B.3.1.

[22] In Diaz and Luengo-Prado (2010), this ratio is only 1.4. However, the numerator in their study does not include vehicles.

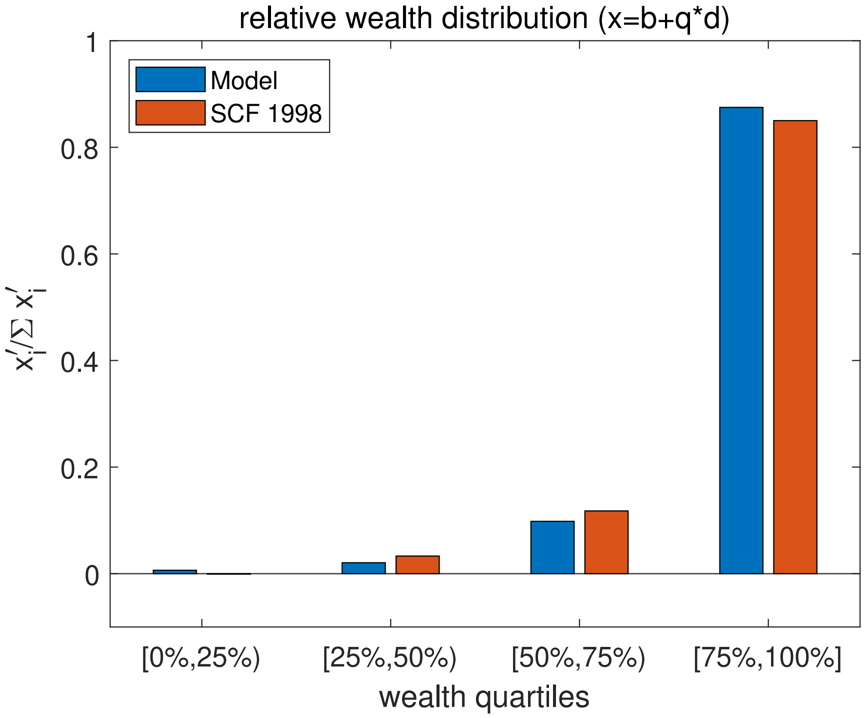


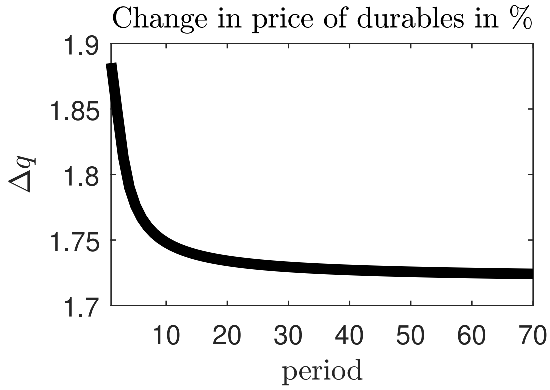
Figure 3.2: Relative net-wealth for different wealth quartiles (data vs model)

Bianchi and Mendoza (2018), the total effects of the considered shock are calculated under a fixed supply of collateral. In this case, the price sensitivity of durables is at its maximum, while the importance of this sensitivity for household decisions is addressed in 3.3.4. The total effects of the preference shock are analyzed in the following experiment. Assume that the economy is in a stationary equilibrium in period $t-1$ with $q_{t-1} = \tilde{q}$ where $\tilde{q}$ denotes the corresponding stationary price of durables. In period t, however, the parameter controlling the preferences for durables ϕ^d unexpectedly increases permanently by 2% from 0.100 to 0.102.[23] As a result, the economy leaves the old stationary equilibrium and converges over time to a new one induced by the higher preference parameter for durables.[24] During this transition path, the current and future prices of durables endogenously change, i.e. $q_{t+j} \neq \tilde{q}\ \forall j \geq 0$, while aggregate supply remains constantly. Figure 3.3 shows the endogenous transition path for the price of durables given as percentage deviations from the price in the old stationary equilibrium. Under fixed supply, a larger demand for durables induced by higher preferences increases the price of durables. The price increases on impact by ca. 1.9%, then converges down to a value that is still higher than in the old stationary equilibrium. Note that the real interest rate and the wage rate are time-invariant in this analysis due to the assumed firm technology.

In the following, we compare household saving and consumption in the impact period t of the above

[23]In this experiment, the increase in the price of durables in the impact period is relatively close to the impulse response of the housing price path under a shock on housing preferences in the estimated model of Guerrieri and Iacoviello (2017).

Figure 3.3: Transition path for the durables price after a positive shock on preferences for durables



Note: The path of the durables price is given as percentage deviation from the price in the old stationary equilibrium.

described experiment to household choices in a scenario in which the preference parameter does *not* adjust.[25] In the former case, the preference parameter increases to $\phi^d = 0.102$ from period t on and the endogenous price path of durables is as depicted in figure 3.3. In the latter case, the preference parameter remains at $\phi^d = 0.100$ and the price of durables is $q_{t+j} = \tilde{q}\ \forall j \geq 0$. The changes in household behaviour across these two cases are then the *total effects* of the considered shock and can be decomposed into *direct effects* induced by the change in preferences at given prices and *indirect effects* induced by the endogenous change in the price of durables that emanates from the direct effects.

The *direct effects* can be determined by the following experiment. Consider the same preference shock as explained above whereas now the price of durables is assumed to remain constantly, i.e. $q_{t+j} = \tilde{q}\ \forall j \geq 0$. The direct effects are then given by the changes in households' decisions in the impact period t of this experiment, i.e $\phi^d = 0.102$ and $q_{t+j} = \tilde{q}\ \forall j \geq 0$, compared to household choices in the scenario in which the preference parameter does not adjust, i.e. $\phi^d = 0.100$ and $q_{t+j} = \tilde{q}\ \forall j \geq 0$. In this experiment, household saving and consumption are completely driven by the adjustment in preferences. The *indirect effects* on household decisions are given by the differences between the total effects and the direct ones. These differences in saving and consumption are then driven by the adjustment in the price path of durables.

The following analysis is focussed on the behaviour of two different groups of households. The first group consists of households in the lowest income state. These households are most directly affected by current and future collateral constraints. Households in the second group are in the highest income state and therefore least directly affected by these constraints. To understand the different effects of the

See figure A.2 in the online appendix of their paper.

[24]see Appendix A

[25]If the preference parameter does not change, the economy remains in the old stationary equilibrium forever.

preference shock on these two groups, it is first necessary to understand the saving and consumption behaviour of these households in the old stationary equilibrium at the stationary price of durables $\tilde{q}$ as explained in the following. Households in the lowest income state tend to have a relatively small realization of non-durable goods compared to their beginning-of-period holdings of durables. Given that these goods are not perfect substitutes, households are net-sellers of durables and borrow to increase non-durable consumption. If the income realization, however, is low enough, the collateral constraint starts binding. Households in the highest income state, on the other hand, have a relatively high realization of non-durable goods compared to beginning-of-period holdings of durables. These households are then net-buyers of durables and the main savers in bonds in the economy. In the following, the total-, direct- and indirect effects of the preference shock on these two groups of households are analyzed in detail.

Direct effects First, the direct effects of the increase in the preference parameter on household saving and consumption in the impact period t are analyzed. The dotted lines in figure 3.4 show these effects as percentage changes in non-durable consumption (first row), durable consumption (second row) and bond saving (last row) compared to the choices in the scenario in which the preference parameter does *not* adjust. The left column depicts the effects on constrained households in the lowest income state, while the right column figures decisions of savers in the highest income state. Consider a given household i in the impact period t with beginning-of-period bonds $b_{i,t-1}$ and durables $d_{i,t-1}$. The abscissas in figure 3.4 then show the beginning-of-period wealth of this household in the scenario in which the preference parameter does not adjust, which is given by $\tilde{x}_{i,t} := b_{i,t-1}(1 + \tilde{r}) + \tilde{q}d_{i,t-1}$ where $\tilde{r}$ denotes the time-invariant real interest rate.[26] Note that this figure shows the changes for low-income households (left column) only at values of beginning-of-period wealth at which households are borrowing constrained under unchanged preferences ($\tilde{x_t} \in [0.003\ 0.0055]$).[27]

What are the direct effects of an increase in the preference parameter for durables on household saving and consumption in the impact period t? If the preference parameter for durables increases under an unchanged price of durables, relative preferences for non-durable consumption decrease and durables become more attractive as a saving device. Therefore, both groups of households (savers and constrained households) *reduce* non-durable consumption (see (i) and (iv)) as well as saving in bonds (see (iii) and (vi)) to *increase* holdings of durables (see (ii) and (v)). Constrained borrowers increase durables by ca. $0.47\% - 0.59\%$, decrease non-durable consumption by ca. 0.18% and reduce bond saving by ca. $0.47\% - 0.59\%$. Savers increase durables by ca. 0.93%, decrease non-durable consumption by ca. 0.06%

[26] Obviously, beginning-of-period wealth of this household in the impact period when the preference parameter *changes* but the price of durables remains *constantly* is identical to beginning-of-period wealth under *unchanged* preferences. In general equilibrium, however, the price of durables increases in the impact period and thereby raises beginning-of-period wealth.

[27] Note that if preferences *change*, some of these household get unconstrained.

and reduce bond saving by ca. $0.24\% - 1.1\%$.

Indirect effects The dashed-dotted lines in figure 3.4 show the indirect effects induced by the increase in the price path of durables to the one depicted in figure 3.3. Analog to the direct effects, the indirect effects are given by the percentage changes in non-durable consumption (first row), durable consumption (second row) and bond saving (last row) compared to the old stationary equilibrium. The indirect effects on household decisions are calculated by subtracting the direct effects from the total ones. These indirect effects can be interpreted as changes in household decisions in period t when the price path of durables, $\{q_{t+j}\}_{j\geq 0}$, exogenously increases from $\tilde{q}$ to the one shown in figure 3.3 while the preference parameter remains constantly at the value of the old stationary equilibrium.[28] Consider again a household i with beginning-of-period bonds $b_{i,t-1}$ and durables $d_{i,t-1}$ in the impact period t. Due to the change in the price path of durables, the beginning-of-period wealth in the impact period is now given by $x_{i,t} = b_{i,t-1}(1+\tilde{r}) + q_t d_{i,t-1}$ with $q_t > \tilde{q}$ compared to $\tilde{x}_{i,t} = b_{i,t-1}(1+\tilde{r}) + \tilde{q} d_{i,t-1}$ in the old stationary equilibrium such that $x_{i,t} > \tilde{x}_{i,t}$.[29]

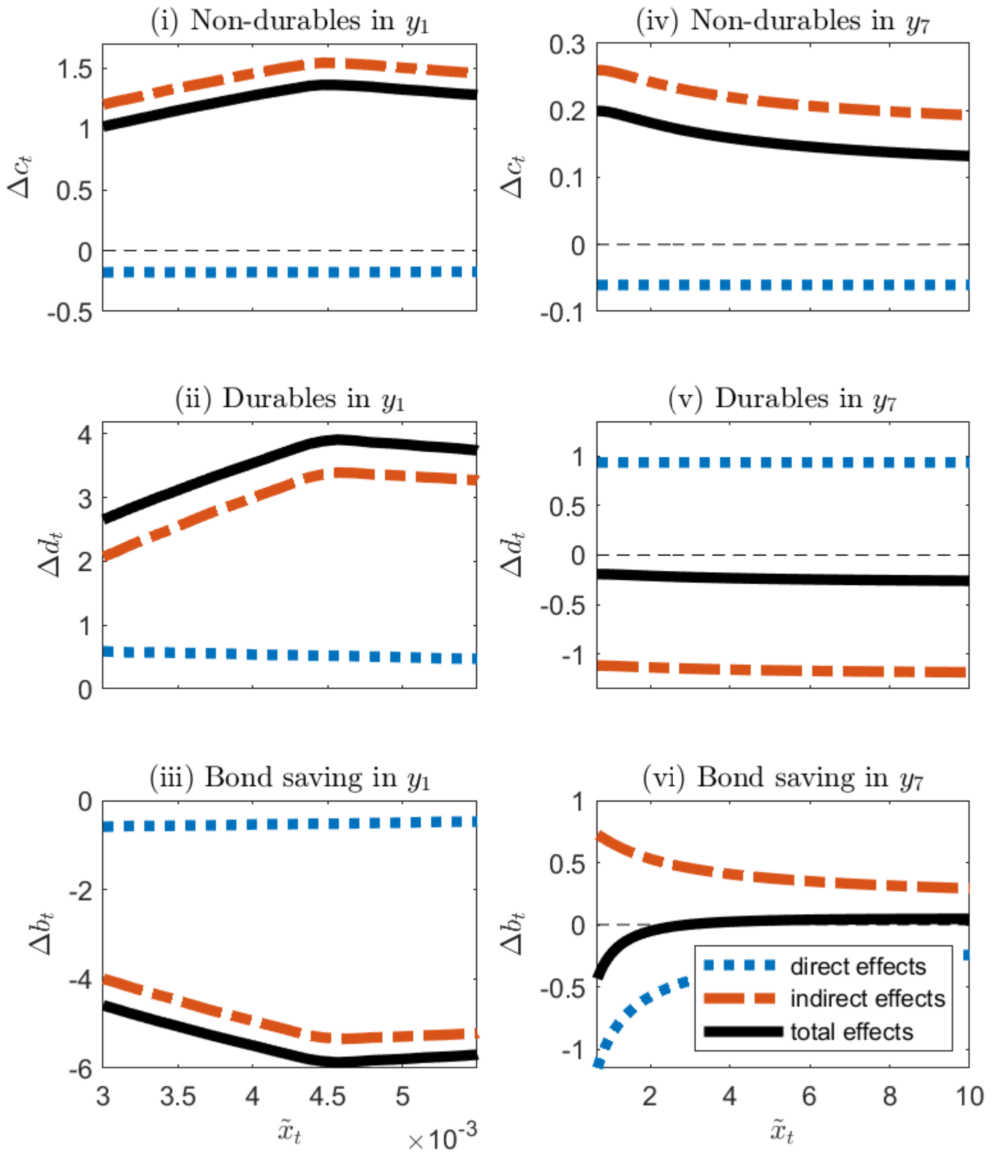
What are the *indirect effects* of a rise in preferences for durables on households' decisions in the impact period t? First, the decisions of *savers* in the highest income state (right column), who are not borrowing constrained in the impact period and have a relatively low probability to be constrained in future ones, are analyzed. The indirect effects are qualitatively different from to the direct ones. If the price path of durables increases as shown in figure 3.3 while the preference parameter is assumed to be constant, these households *decrease* demand for durables by ca. $1.12\% - 1.18\%$ to *increase* non-durable consumption by ca. $0.19\% - 0.26\%$ and saving in bonds by ca. $0.29\% - 0.71\%$. Non-durable consumption increases because these goods get relatively cheaper in the impact period under a higher relative price of durables. Saving in bonds rises because the return on bonds relative to the return on durables goes up since q_{t+j}/q_t decreases for all $j > 0$ while the real interest rate is fixed. In the next section, these indirect effects on savers' decisions are analyzed in more detail.

Next, the decisions of *constrained borrowers* (left column), who are directly affected by current and future collateral effects, are analyzed. In contrast to unconstrained savers, these households *increase* demand for both consumption goods. They raise durables by ca. $2.10\% - 3.27\%$ as well as non-durable consumption by ca. $1.21\% - 1.54\%$ and finance this higher demand for both goods by increasing borrowing

[28] Note that in this case the market for durables is not cleared.

[29] To get the true indirect (as well as total) effects, it is necessary to compare household decisions at given beginning-of-period bonds $b_{i,t-1}$ and durables $d_{i,t-1}$ and *not* at given beginning-of-period wealth. The reason is that the increase in the durables price changes beginning-of-period wealth in the impact period. The abscissas in figure 3.4 only show the beginning-of-period wealth of household i in the *old stationary equilibrium* under the old price of durables, i.e. $\tilde{x}_{i,t} = b_{i,t-1}(1+\tilde{r}) + \tilde{q} d_{i,t-1}$. In doing so, it is possible to plot changes in decisions of a household with beginning-of-period bonds $b_{i,t-1}$ and durables $d_{i,t-1}$ in a two-dimensional plot, while the impact of the current price on households' current decision via its effect on beginning-of-period wealth is still fully taken into account. Therefore, the figure indeed shows the true indirect (and total) effects.

Figure 3.4: Percentage changes in individual decisions under a positive durables preference shock



Note: This figure shows the decisions of constrained borrowers in the lowest income state (lhs) and savers in the highest income state (rhs).

by ca. $4.03\% - 5.34\%$. There are two reasons why constrained borrowers increase durables in the impact period although the relative price of this good rises in that period. Firstly, the higher collateral price in the impact period increases the collateral value of durables. Secondly, the higher future durables prices increase the future value of holdings of durables bought in the impact period. Although constrained borrowers increase holdings of durables, they are also able to increase non-durable consumption because the higher collateral price in the current period raises the borrowing limit for a given amount of durables. In the next section, these indirect effects on borrowers' saving and consumption are analyzed in more detail.

Total effects What are the *total effects* on household decisions in the impact period t? These effects are shown by the solid lines in figure 3.4. At first, the decisions of *savers* are analyzed. The total effects on these households are qualitatively different from the direct ones. If preferences for durables increase *and* thereby induce the change in the price path depicted in figure 3.3, savers *decrease* durables by ca. $0.19\% - 0.26\%$ and raise non-consumption by ca. $0.13\% - 0.20\%$. These results imply that the indirect effects in the form of price changes are qualitatively more important for savers' consumption than the direct ones. Especially the adjustment in non-durable consumption is driven to a relatively large extent by the change in the price path of durables. For *constrained borrowers* the total effects are qualitatively (see non-durable consumption) and quantitatively (see durable consumption and borrowing) different from the direct ones. If preferences for durables increase *and* thereby induce the change in the price path depicted in figure 3.3, these households *increase* durable holdings by ca. $2.68\% - 3.89\%$, non-durable consumption by ca. $1.03\% - 1.36\%$ and borrowing by ca. $4.62\% - 5.87\%$. These results imply that also the total effects on constrained borrowers are mainly driven by the indirect price effects. While the total adjustment in non-durable consumption qualitatively changes compared to the direct effect, ca. $78\% - 87\%$ of the total increase in durable holdings and ca. $92\% - 96\%$ of the total increase in borrowing are driven by these price effects.

Summarizing, this analysis has shown that the total effects on household saving and consumption under the considered shock on preferences are mainly driven by indirect price effects. Changes in market prices are not only quantitatively- but also qualitatively important for household decisions. In this section, however, the aggregate supply of durables has been fixed such that the sensitivity of the durables price and therefore the relative importance of indirect price effects have been at the maximum. While this extreme assumption concerning the elasticity of aggregate durables supply is shared by previous quantitative studies as Guerrieri and Iacoviello (2017) or Bianchi and Mendoza (2018), the empirical literature shows that elasticities of housing supply are *not* perfectly inelastic in many areas in the US (see e.g. Glaeser et al. (2008), Mian and Sufi (2009), Saiz (2010) or Mian et al. (2013)). In section 3.3.4, therefore, the magnitude of direct and indirect effects is re-examined under higher elasticities of aggregate durables supply. While the magnitude of indirect price effects is closely connected to the durables supply elasticity, the analysis there furthermore shows that the total effects of the preference shock are still driven by indirect effects even under relatively high supply elasticities. Before the impact of the supply elasticity is analyzed in 3.3.4, section 3.3.3 decomposes the indirect price effects under fixed supply to analyze the relative magnitude of collateral and distributive effects on household decisions.

3.3.3 Decomposition of indirect effects

The last section has shown that the total effects of the preference shock on household saving and consumption are mainly driven by indirect effects that arise from changes in the price path of durables. These indirect effects on household decisions in the impact period can be decomposed into collateral and distributive effects. Collateral effects appear when a change in the price of durables affects the collateral value and thereby the borrowing limit. Distributive effects emerge when a change in the price of durables redistributes funds across buyers and sellers and thereby changes relative demand for durables, non-durables and bonds. The behaviour of forward-looking households in the impact period is not only influenced by the change in the durables price in the impact period but also by adjustments in future prices. This implies that the indirect effects on household decisions in the impact period can be decomposed into - collateral and distributive effects that arise from the price change in the *impact period* and - collateral and distributive effects that arise from price changes in *future periods*.

In the following, the former effects are called *current* collateral and distributive effects, the latter ones are *future* collateral and distributive effects and the impact period is called the *current* period. The different types of indirect effects can have opposing impacts on household saving and consumption in the current period or can reinforce each other and their relative magnitude especially depends on the shape of the price path. In this section, the indirect effects of the change in the durables price path shown by figure 3.3 are decomposed into current collateral and distributive effects and into future ones to analyze their relative magnitude on household decisions in the current period.

How are these different types of indirect effects determined? The *current distributive effects* are calculated in an experiment in which (i) only the current price of durables increases as shown by figure 3.3 while the future prices do not change and (ii) the price in the borrowing limit is fixed at the value of the old stationary equilibrium in all periods.[30] In this experiment, changes in households' current decisions are completely driven by the distributive effect that arises from the price change in the current

[30] In this section, the preference parameter does not change and remains at $\phi^d = 0.100$.

period.

The *future distributive effects* are calculated in an experiment in which (i) only the future prices of durables increase as shown by figure 3.3 while the current price does not change and (ii) the price in the borrowing limit is fixed at the value of the old stationary equilibrium in all periods. In this case, households' current decisions are completely driven by the distributive effect that arises from the price changes in the future periods.

To get the *current collateral effects* the following experiment is conducted. Assume as in the experiment for the current distributive effects that only the current price of durables increases as shown by figure 3.3 while the future prices do not change. In contrast to the experiment for the current distributive effects, however, the price in the borrowing limit is now endogenously determined in all periods. The current collateral effects are then given by the differences in households' current decisions in this experiment compared to the experiment with a constant price in the borrowing limit used to generate the current distributive effects. The *future collateral effects* are determined in an analog way.

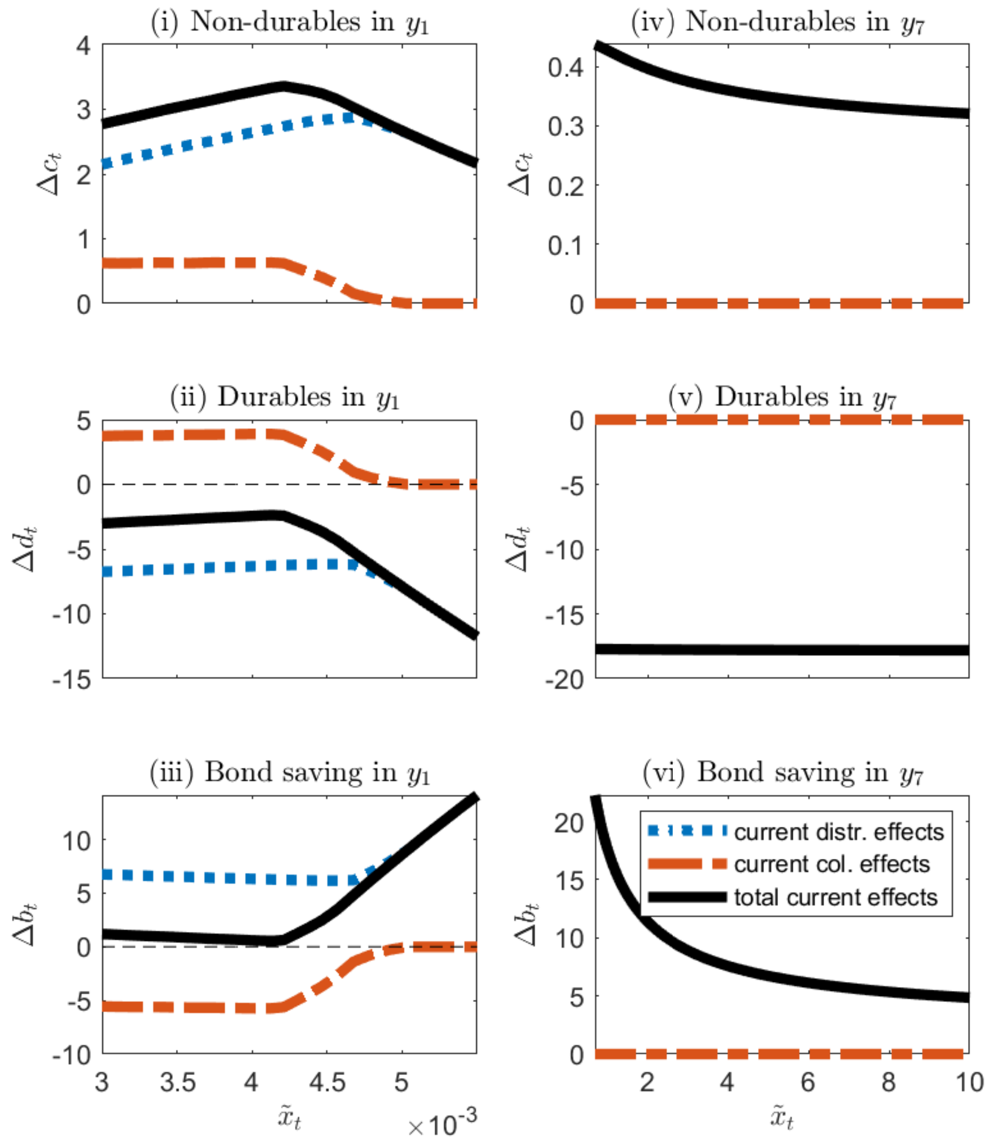
Current indirect effects How do constrained households react when the price of durables only increases in the *current* period? Figure 3.5, which is equivalently structured as figure 3.4, shows the different indirect effects of an increase in the *current* price of durables on decisions of constrained borrowers (left column) and savers (right column). The dotted lines depict the impact of the current distributive effects, the dashed-dotted lines show the impact of the current collateral effects and the solid lines figure the total current indirect effects.

Firstly, the current distributive effects are explained. If the price in the current period increases while the collateral value remains constantly in all periods, constrained borrowers as well as savers tend to *decrease* demand for durables (see (ii) and (v)) and *increase* non-durable consumption (see (i) and (iv)) as well as saving in bonds (see (iii) and (vi)).[31] The reason is that non-durable consumption gets relatively cheaper when the relative price of durables goes up. Furthermore, when the price of durables only increases in the current period, the return on bonds (which is fixed) relative to the return on saving in durables (which is given by q_{t+1}/q_t) rises.

Next, the current collateral effects are explained. Obviously, only constrained borrowers are affected by this type of indirect effects. If the collateral value of durable goods in the current period increases while distributive effects are shut down, constrained borrowers *raise* their holdings of durables. Furthermore, the rise in the collateral price increases the collateral value of the old stock of durables and thereby enables these households to *increase* non-durable consumption as well.

[31] Note that savers are not significantly affected by the collateral effects. This implies that for these households the lines for the total effects are identical to the lines for the distributive effects.

Figure 3.5: Percentage changes in individual decisions under an increase in the current price of durables



Note: This figure shows the decisions of constrained borrowers in the lowest income state (lhs) and savers in the highest income state (rhs).

Table 3.2: Summary of the qualitative impact of different indirect effects on households' current decisions

	DE	CE	$DE+CE$	DE'	CE'	$DE'+CE'$
non-durables (constrained borrowers)	↑	↑	↑	↓	↑	↓
durables (constrained borrowers)	↓	↑	↓	↑	↓	↑
bond saving (constrained borrowers)	↑	↓	↑	↓	↑	↓
non-durables (savers)	↑	→	↑	↓	→	↓
durables (savers)	↓	→	↓	↑	→	↑
bond saving (savers)	↑	→	↑	↓	→	↓

Note: The table shows the current distributive effects (DE), current collateral effects (CE), total current indirect effects ($DE+CE$), future distributive effects (DE'), future collateral effects (CE') and total future indirect effects ($DE'+CE'$).

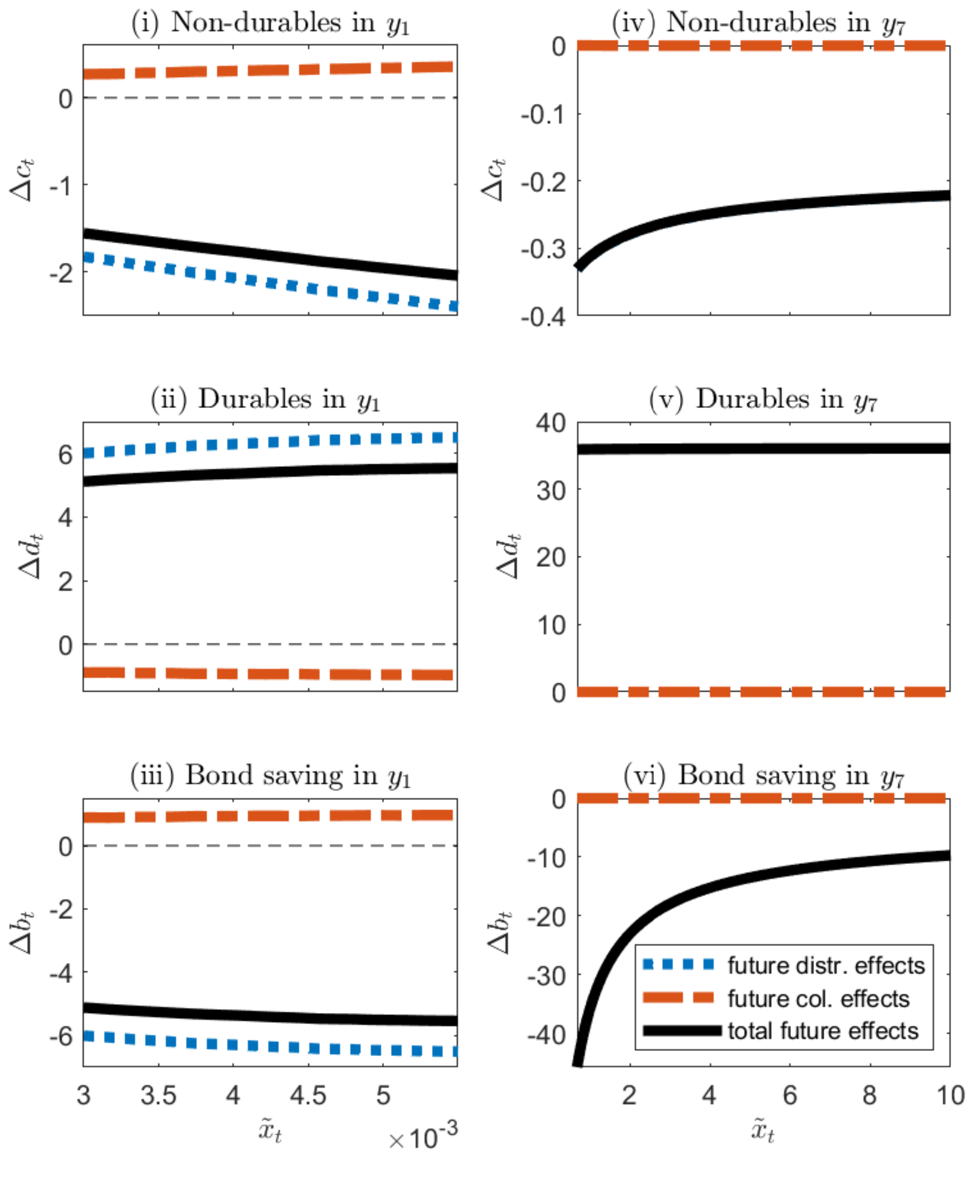
What are the total effects of the price increase in the current period? While savers' behaviour is driven by distributive effects, even the decisions of constrained borrowers are mainly determined by this type of indirect effects. The current distributive effects drive the qualitative adjustments in durable consumption and borrowing such that constrained borrowers *decrease* demand for durables and borrowing although the collateral value goes up under a higher current price of durables. This result is in sharp contrast to the quantitative studies that focus only on collateral effects (e.g. Guerrieri and Iacoviello (2017) or Bianchi and Mendoza (2018)).[32] In these models, an exogenously given rise in the price of durables in the current period would *increase* debt issuance of constrained borrowers and completely drive changes in their non-durable consumption due to a dominating collateral effect. In a model with a less stylized specification of household heterogeneity, however, the distributive effects invert the impact on borrowing of constrained households. Furthermore, current collateral effects maximally explain 23% of the increase in non-durable consumption if distributive ones are incorporated. These results show that distributive effects are relatively important for household saving and consumption in a model with a less stylized specification of household heterogeneity. The second, third and fourth column of table 3.2 summarize the qualitative impact of the current distributive, collateral and total indirect effects on saving and consumption of constrained borrowers and savers.

Future indirect effects How do constrained borrowers and savers react in the current period when the price of durables only increases in *future* periods? Figure 3.6 shows the different indirect effects of an increase in the *future* price path of durables on households' current decisions. The dotted lines depict the impact of the future distributive effects, the dashed-dotted lines show the impact of the future collateral effects and the solid lines figure the total future indirect effects.

Firstly, the future distributive effects are explained. If (only) the future price path of durables increases while the collateral value remains constantly in all periods, constrained borrowers and savers *increase*

[32]Note that the relative magnitude of collateral and distributive effects changes in models with transaction costs in the market for durables.

Figure 3.6: Percentage changes in individual decisions under an increase in the future price path of durables



Note: This figure shows the decisions of constrained borrowers in the lowest income state (lhs) and savers in the highest income state (rhs).

demand for durables (see (ii) and (v)) in the current period and *decrease* non-durable consumption (see (i) and (iv)) as well as saving in bonds (see (iii) and (vi)). The reason is that higher future prices of durables increase the future value of the current end-of-period stock of durables which raises current demand for these goods. Furthermore, when the price of durables only increases in future periods, the return on bonds (which is fixed) relative to the return on saving in durables (which is given by q_{t+1}/q_t) decreases.

Next, the future collateral effects are analyzed. While savers with a relatively high income are not affected by these effects, constrained borrowers *reduce* durable consumption in the current period to *increase* non-durable goods and saving in bonds when higher future prices increase future collateral values absent any distributive effects. The reason is the following. Borrowers who are constrained in the current period have a relatively high probability to be constrained in future periods as well. If future collateral values increase, these households reduce precautionary savings to increase non-durable consumption in the current period. Note that savings available in the next period are given by next period's beginning-of-period wealth, $d_t q_{t+1} + b_t(1+\tilde{r})$. Given that for constrained borrowers this expression is $d_t(q_{t+1} - \tilde{q}\gamma)$ with $q_{t+1} - \tilde{q}\gamma > 0$ under a constant current collateral value, these households decrease savings by reducing current holdings of durables by more than the future price of durables increases, i.e. $d_t(q_{t+1} - \tilde{q}\gamma)$ goes down. This reduction in holdings of durables then also implies a decrease in current debt issuance of these constrained households.[33]

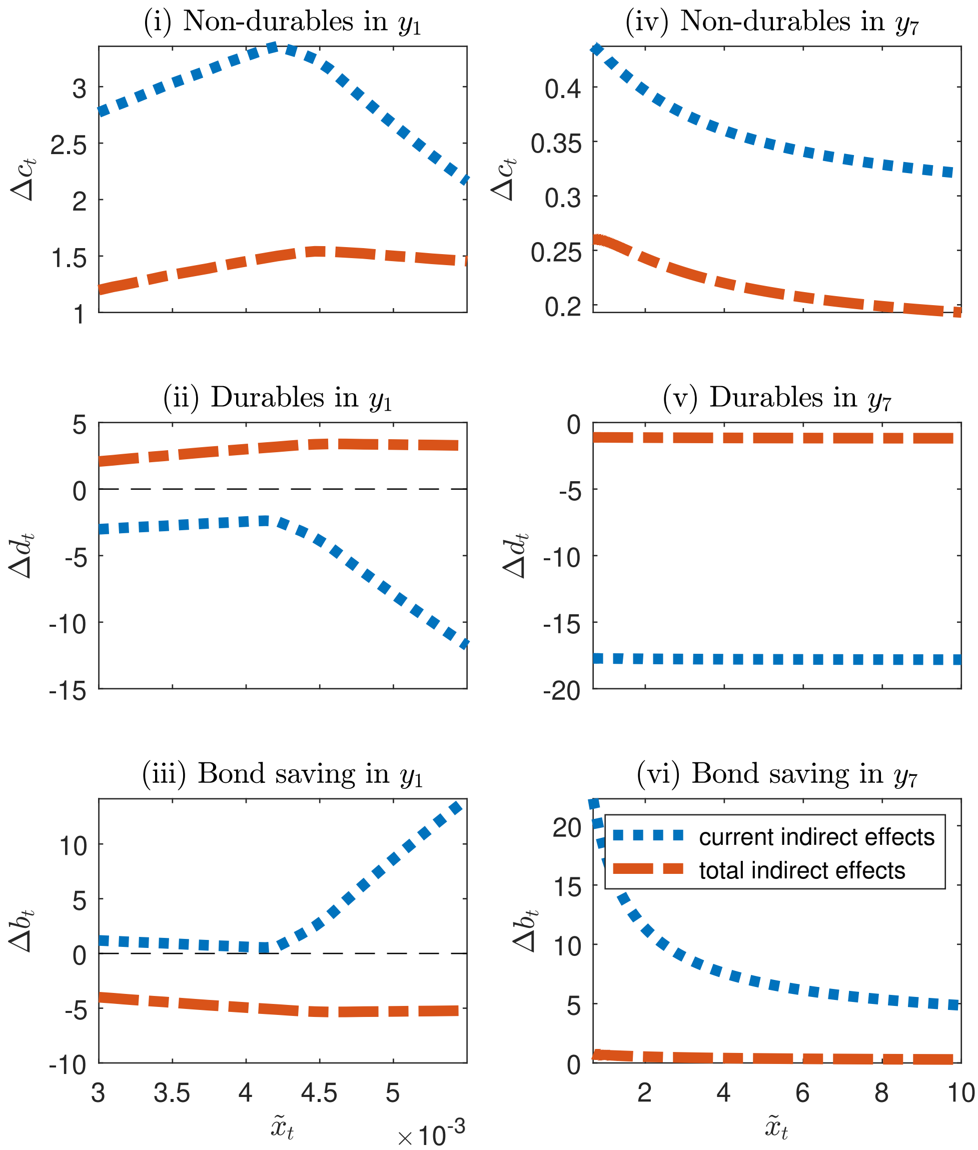
The total future indirect effects on household decisions in the current period are, however, driven by the distributive effects - not by collateral ones - such that constrained borrowers *increase* durable consumption as well as borrowing and *reduce* non-durable consumption when (only) the future price path of durables increases. The fourth, fifth and sixth column of table 3.2 summarize the qualitative impact of the different future price effects on saving and consumption of constrained borrowers and savers.

Total indirect effects As mentioned above, household behavior in the current period is not only affected by current price changes but also by changes in future prices. In the following, therefore, the relative magnitude of total current and total future indirect effects on households' current decisions is analyzed. Figure 3.7 compares the total current indirect price effects on household saving and consumption (explained above) to the total indirect price effects (explained in section 3.3.2) given by total current *and* future indirect effects. The total current indirect effects are shown by the dotted lines, while the total indirect effects are given by the dashed-dotted lines.

While a higher current price of durables relatively strongly reduces savers' demand for durable goods

[33] Note that if constrained households increase borrowing at a given collateral price under a loan-to-value ratio smaller than one, these additional funds are in fact used to partially finance durable consumption.

Figure 3.7: Percentage changes in individual decisions under an increase only in the current price of durables (dotted lines) and under an increase in both current and future prices of durables (dashed-dotted lines)



Note: This figure shows the decisions of constrained borrowers in the lowest income state (lhs) and savers in the highest income state (rhs).

and increases non-durable consumption as well as saving in bonds, the increase in future prices quantitatively mitigate the total adjustment in savers' saving and consumption decisions (see right column). Borrowing and durable consumption of constrained borrowers are even qualitatively driven by future price increases (see left column). While a higher current price *decreases* borrowing and demand for durables, the overall effect is an *increase* in debt issuance and durable consumption. These results show that the persistence of price changes is both quantitatively and qualitatively important for the relative magnitude of direct and indirect effects on households' saving and consumption decisions.

Summarizing, this section has shown that in a model with a less stylized specification of household heterogeneity changes in the price of durables drive household decisions mainly via distributive effects - not via collateral effects. While this is obviously the case for savers who are not directly affected by collateral constraints, it is also true for constrained borrowers. This result implies that the way of modelling household heterogeneity has important implications for positive analyses of household saving and consumption decisions. Furthermore, the importance of distributive effects in this study challenges the relevance of the normative results derived in the existing macroprudential literature for frameworks with an empirically relevant specification of household heterogeneity. In these studies, collateral effects completely drive household decisions while distributive effects are absent.

While the analysis so far was performed under a relatively large sensitivity of the durables price due to a fixed aggregate supply, the next section analyzes the importance of indirect price effects under higher elasticities of aggregate durables supply.

3.3.4 The importance of the supply elasticity

Section 3.3.2 has shown that the total effects of the preference shock under a *fixed supply* of durables are mainly driven by indirect price effects. In that analysis, the sensitivity of the durables price and therefore the importance of the indirect effects has been at the maximum. Empirical studies, however, show that the price sensitivity of housing, which is the most important collateral asset of the household sector (see e.g. Hintermaier and Koeniger (2016)), is not homogeneous across areas in the US and that this heterogeneity is driven to a large extent by heterogeneous supply elasticities (see e.g. Glaeser et al. (2008), Mian and Sufi (2009), Saiz (2010) or Mian et al. (2013)). These results suggest that model versions with a fixed supply of collateral - as in section 3.3.2, Guerrieri and Iacoviello (2017) or Bianchi and Mendoza (2018) - generate an upper bound for the importance of indirect price effects on household saving and consumption.[34] This section, therefore, re-analyzes the relative magnitude of indirect effects in a model version where the supply of durables is more elastic and the price sensitivity thereby reduced

[34] Studies with a completely elastic supply (e.g. Diaz and Luengo-Prado (2010)) generate a lower bound for the importance of indirect price effects on household saving and consumption.

compared to the benchmark scenario. In this case, an increase in demand for durables increases both the price and the supply of durables while the change in prices is reduced compared to the fixed supply case.

As explained in section 3.3.1, the price of durables is given by $q_t = T'(\Delta d_t)$ in the model with an elastic durables supply where T is the transformation technology. This relationship implies that the price sensitivity under a preference shock depends on the transformation function T. As noted above, when the assumption of a completely inelastic supply is dropped, the price sensitivity reduces such that the endogenous increase in the price path under the same preference shock considered in 3.3.2 is lower. In the following, *only* those transformation technologies are considered that endogenously generate price paths that are *parallel* to the path under a fixed supply of durables shown in figure 3.3.[35] Thus, this analysis does *not* calibrate the transformation technology to get an empirically relevant supply elasticity or price sensitivity. Instead, this section simply analyzes the relative magnitude of direct and indirect effects under ad-hoc increases in the elasticity of durables supply while the empirical relevance of the chosen elasticity is disregarded.

Total effects under a different supply elasticity The first part of this section analyzes household decisions under the given preference shock in the following three versions of the model. In the first version, the aggregate supply of durables is fixed such that the price sensitivity is at the maximum. The total effects in this version have been analyzed in section 3.3.2. In the second version, aggregate supply is completely elastic such that the price of durables is time-invariant. The total effects in this version are identical to the direct effects discussed in section 3.3.2. In the third version, the supply elasticity is in between the two extreme versions such that the increase in the price of durables in the impact period is half as much as under fixed supply.[36] To facilitate the analysis, the stationary equilibrium in period $t-1$ is assumed to be the same in all three versions and given by the stationary equilibrium under a fixed supply of durables. In period t then, the preference parameter unexpectedly and permanently increases. In the model version with a fixed supply, the price path of durables increases and aggregate durables supply is constant. In the version with a completely elastic supply, the durables price is constant and aggregate durables supply increases.[37] In the version with an incompletely elastic supply, the price of durables as well as aggregate supply rise.[38]

Figure 3.8 depicts the endogenous paths of the durables price (left plot) and of aggregate durables supply (right plot) under the same preference shock considered in section 3.3.2. Figure 3.9 shows the

[35]If aggregate durables supply is *not* fixed, the price path of durables as well as aggregate supply increase under the same preference shock considered in 3.3.2. In the following, however, only those elasticties are considered under which the increase in the price path has the same curvature as under fixed supply.

[36]This is an ad-hoc chosen elasticity.

[37]Details about the numerical solution procedure for the version with a fixed supply can be found in the Appendix B.3.2. The version with a completely elastic supply is identical to the direct effects analyzed in 3.3.2 (see Appendix B.3.3).

[38]See Appendix B.3.5

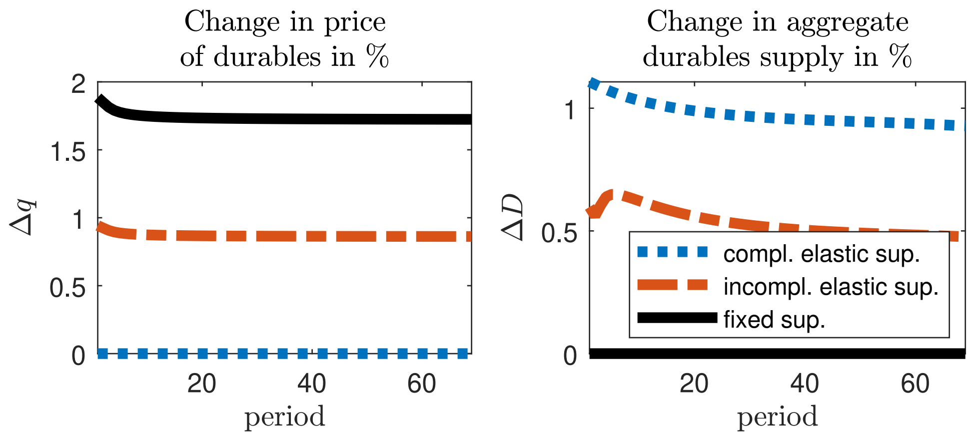


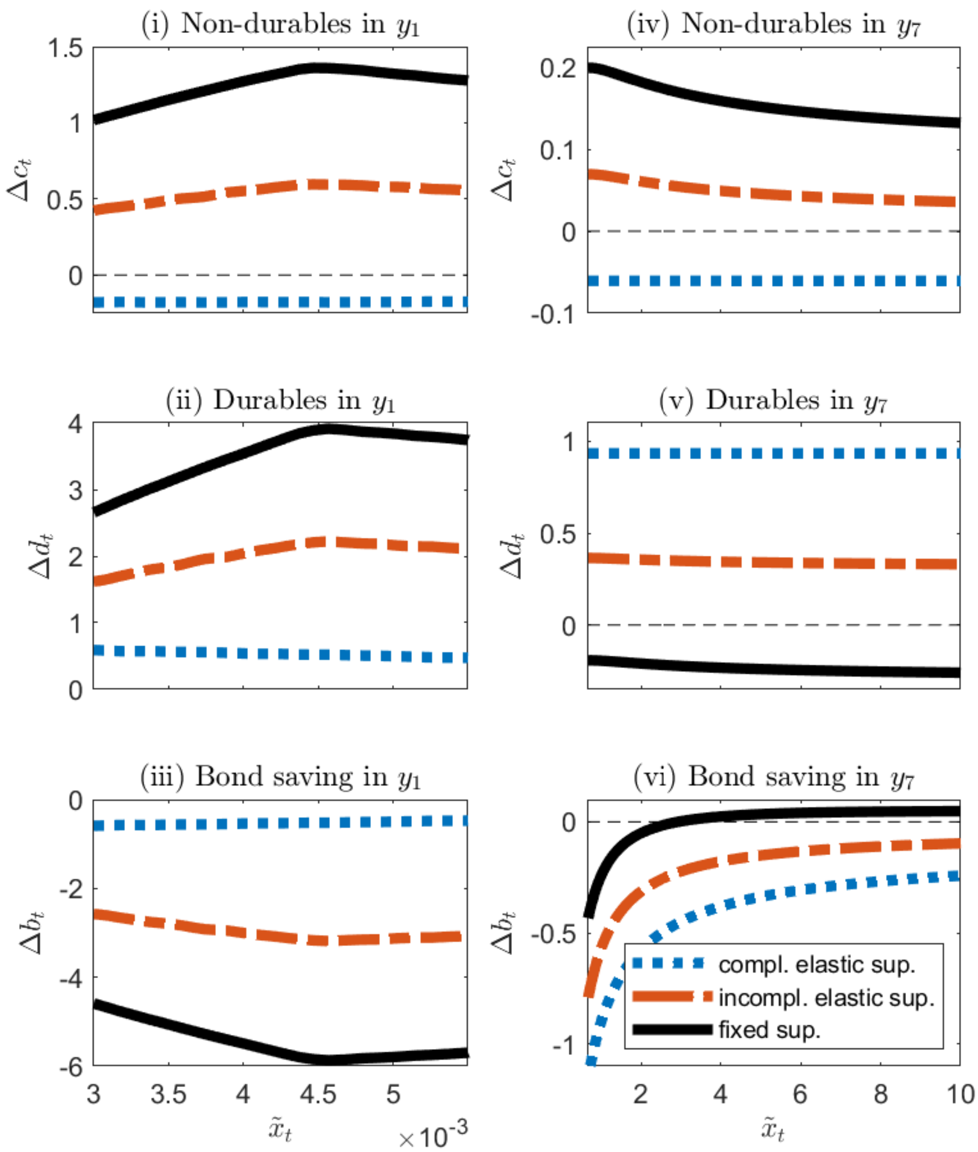
Figure 3.8: Transition paths for durables price and for aggregate supply of durables after a positive durables preference shock under different supply elasticities - first part of the analysis

associated change in saving and consumption of constrained borrowers (left column) and savers (right column). If aggregate supply of durables is completely elastic (dotted lines), the price of durables does not change over time under higher preferences for these goods while the growth rate of aggregate supply is ca. 1.1% in the impact period and then converges over time to ca. 0.9% compared to the old stationary equilibrium. As explained in section 3.3.2, under a fixed supply (solid lines) the price of durables increases on impact by ca. 1.9% and then converges to a slightly lower value over time while aggregate supply does not change. If the elasticity of aggregate supply is such that the price of durables only increases by half as much as under fixed supply (dashed dotted lines), the growth rate of aggregate supply is approximately half as high as under a completely elastic supply.

What is the total effect of the preference shock under the different elasticities of aggregate durables supply? If aggregate supply of durables is *completely elastic*, the total effects of the preference shock on household decisions are identical to the direct effects discussed in section 3.3.2. If preferences for durables increase, both groups of households decrease non-durable consumption and saving in bonds to increase demand for durable goods (see dotted lines in figure 3.9). Constrained borrowers decrease non-durable consumption by ca. 0.18%, increase borrowing by ca. $0.47\% - 0.59\%$ and raise demand for durables by ca. $0.47\% - 0.59\%$.

If aggregate supply of durables is, however, *fixed* such that the price of durables relatively strongly rises, constrained households increase borrowing by even $4.62\% - 5.87\%$ and durable consumption by ca. $2.68 - 3.89\%$ while demand for non-durable goods does not decrease but rises by ca. $1.03\% - 1.36\%$ (see solid lines). Thus, compared to the version with a completely elastic supply the increase in saving and durable consumption rises by a multiple under a fixed supply while the adjustment in non-durable consumption is even qualitatively different. For savers, the relatively strong change in the durables

Figure 3.9: Percentage changes in individual decisions after a positive durables preference shock under different supply elasticities - first part of the analysis



Note: This figure shows the decisions of constrained borrowers in the lowest income state (lhs) and savers in the highest income state (rhs).

price under a fixed supply even changes the direction of adjustment in durable consumption. Instead of increasing durable goods under higher preferences for durables, these households decrease durable consumption.

How do household decisions change when the price sensitivity is halved compared to the version with a fixed supply? In this case (see dashed dotted lines), the change in saving/borrowing and consumption of both groups - constrained borrowers and savers - is also (approximately) in the middle between the scenario with a fixed supply and a completely elastic one. Changes in non-durable consumption of constrained borrowers and savers are still driven by the price effects such that these adjustments go in the same direction as the changes under a fixed supply. However, the change in savers' durable consumption as well as the adjustment in bond saving of wealthy savers are qualitatively different under the halved price sensitivity compared to the changes under a fixed supply. Instead of decreasing durable consumption and increasing bond saving, these households raise consumption of durables and reduce saving in bonds if the price sensitivity is halved.

Direct and indirect effects under a higher supply elasticity The second part of this section analyzes the impact of an even higher supply elasticity on the relative magnitude of direct and indirect effects under the given preference shock on durables. As explained above, the higher the elasticity of aggregate durables supply is, the lower is the relative importance of indirect price effects. In this part of the analysis, we examine by how much the supply elasticity at least has to be increased compared to the fixed supply case such that the indirect price effects do *not* dominate the direct ones on household decisions anymore. Indirect effects dominate direct ones if the total effect of the preference shock on household decisions is qualitatively different from the direct one. Section 3.3.2 has shown that this is indeed the case for non-durable consumption of both groups of households under a maximum price sensitivity induced by a fixed supply of durables (see figure 3.4). In this case, the direct effect of the increase in preferences for durables is a reduction in non-durable consumption while the total effect is an increase in demand for these goods. In the following, we calculate how low the price sensitivity at least has to be such that the total effect is a *reduction* in non-durable consumption under the increase in preferences for durables.[39]

As explained above, this analysis only considers those elasticities of aggregate durables supply that create price paths of the durable good that are parallel to path under a fixed supply. In the following, it is examined by how much the price path has to be shifted downwards compared to the fixed supply case - by increasing the supply elasticity - such that the direct and indirect effects on non-durable consumption

[39]Section 3.3.2 has furthermore shown that indirect effects dominate direct ones with respect to *savers' consumption of durable goods.* While the direct effect is an increase in demand for these goods, the total effect is a reduction in durable consumption under fixed supply.

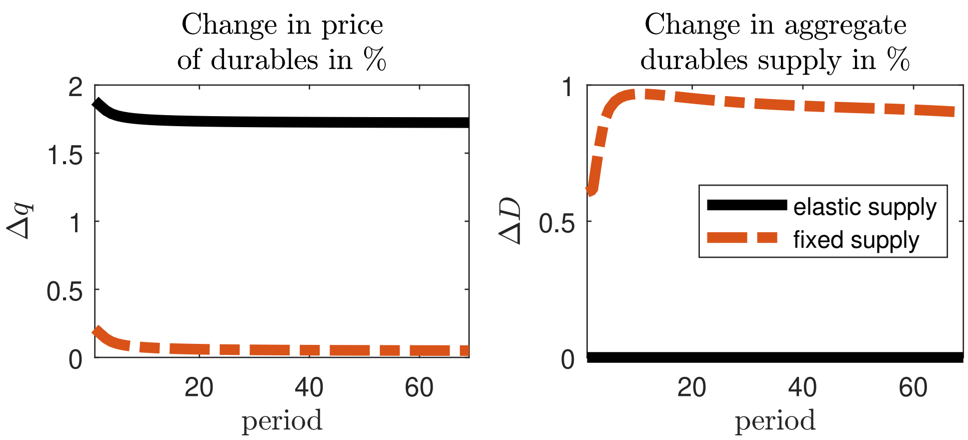


Figure 3.10: Transition paths for durables price and for aggregate supply of durables after a positive durables preference shock under different supply elasticities - second part of the analysis

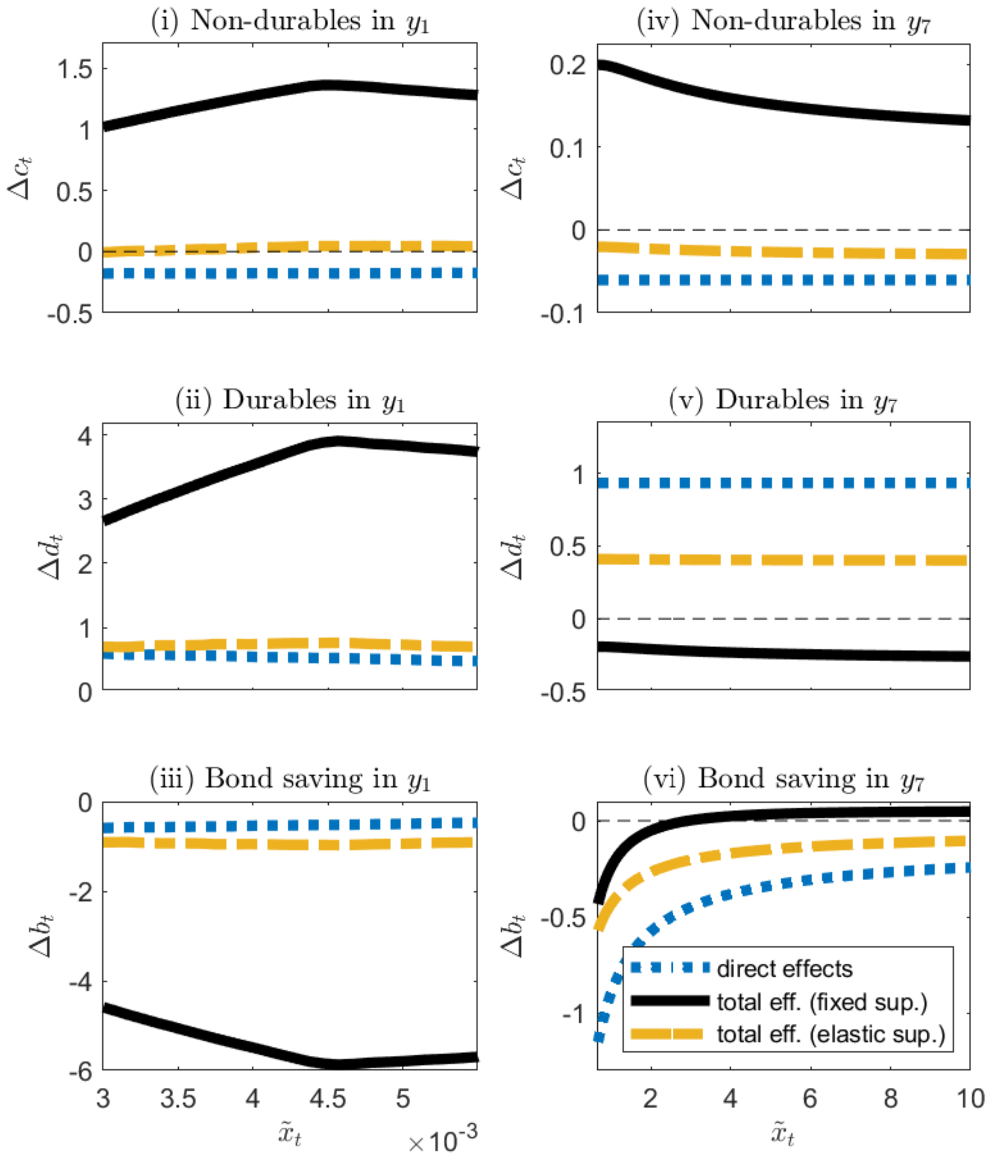
of constrained borrowers cancel out under the given preference shock, i.e. non-durable consumption does not change at all. Denote the resulting price path by q^*. This price path is endogenously generated by a specific transformation technology. In this section, however, we do not explicitly model this technology that endogenously generates the price path q^* but implicitly assume that there exists one that is able to do so. Note that by considering only price paths that have the same curvature as in the experiment with a fixed supply, changes in household decisions between an experiment with an elastic supply and the one with a fixed supply can be compared more easily.

Figure 3.10 depicts the endogenous paths of the durables price (left plot) and of aggregate durables supply (right plot) in this experiment. Figure 3.11 shows the associated change in saving and consumption of constrained borrowers (left column) and savers (right column). The dotted lines show the direct effects of the preference shock while the solid lines are the total effects under a fixed supply of durables already discussed in section 3.3.2. The dashed-dotted lines show the results for the one elasticity at which indirect and direct effects on *constrained borrowers* balance out such that these households do not change non-durable consumption under the given increase in preferences for durable goods.[40]

As already explained in section 3.3.2, the direct effects of higher preferences for durables on constrained borrowers are an *increase* in demand for durable goods and a *reduction* in non-durable consumption as well as in bond saving (see dotted lines). If the supply of durables is fixed (see solid lines), these direct effects induce an increase in the durables price on impact by ca. 1.9% while the aggregate supply of durables does not change. In contrast to the direct effects, the total impact on constrained borrowers under fixed supply is an *increase* in non-durable consumption. Furthermore, changes in demand for

[40]As in the first part of the analysis, the stationary equilibrium in $t-1$ of all versions is identical to the one under fixed supply.

Figure 3.11: Percentage changes in individual decisions after a positive durables preference shock under different supply elasticities - second part of the analysis



Note: This figure shows the decisions of constrained borrowers in the lowest income state (lhs) and savers in the highest income state (rhs).

durables and borrowing relatively strongly increase compared to the direct effects.

How do the total effects change when the price sensitivity relatively strongly decreases? Assume that the elasticity of aggregate durables supply increases such that the resulting endogenous price path of durables under the given preference shock is given by the dashed-dotted line in the left plot of figure 3.10. If the elasticity rises, the increase in the price path of durables is mitigated while aggregate supply rises. The price increases on impact by ca. 0.2% and then converges down to a value that is still ca. 0.05% higher. Aggregate supply of durables rises on impact by ca. 0.6% and then converges to a value that is ca. 0.9% higher compared to the case with a fixed supply.

Figure 3.11 shows that even under such a relatively high supply elasticity which generates an increase in the durables price in the impact period by only 0.2% the direct effects of the preference shock do not dominate saving and consumption of constrained borrowers. Although relative preferences for non-durable consumption decrease, these households do not change demand for these goods (see dashed-dotted lines in (i)).[41] Furthermore, while the magnitude of the price effects on durable consumption and borrowing is reduced compared to the case with the high price sensitivity under fixed supply (see dashed-dotted lines in (ii) and (iii)), indirect price effects still explain ca. $15\% - 32\%$ of the total rise in durable consumption and ca. $35\% - 48\%$ of the total increase in borrowing of these households compared to the old stationary equilibrium - the rest is due to the direct effects. Note that if the elasticity of durables supply is lower such that the durables price rises by more than ca. 0.2% on impact, constrained borrowers even *increase* non-durable consumption under higher preferences for durable goods. These results imply that even under relatively small price sensitivities the decisions of constrained borrowers in empirically relevant heterogeneous household economies are driven to a relatively large extent by indirect price effects.

What is the impact of the lower price sensitivity on savers' decisions? While the price sensitivity that induces an increase in the durables price by ca. 0.2% on impact does not qualitatively change saving and consumption of these households compared to the directs effects (see dashed-dotted lines in (iv)-(vi)), the indirect price effects reduce the decrease in non-durable consumption by ca. $30\% - 50\%$ and the increase in demand for durables is approximately halved compared to the direct effects. Note that if the elasticity of durables supply is such that the durables price rises by more than ca. 0.5% on impact, *indirect effects* even dominate for *savers* such that these households start to *increase* non-durable consumption under higher preferences for durable goods.[42] The threshold for savers (ca. 0.5%) is higher than the one for constrained borrowers (ca. 0.2%) because the former group is not directly affected by the price-dependent borrowing constraint.[43]

[41]The change in non-durable consumption is not exactly 0% for all households but relatively close to it.
[42]The corresponding figure is not shown here.
[43]If the elasticity of durables supply is such that the durables price rises by more than ca. 1.3% on impact, savers even *decrease durable consumption* although preferences for these goods increase. The corresponding figure is not shown here.

Summarizing, this section has shown that the total effects on household decisions are mainly driven by indirect price effects even under relatively low price sensitivities induced by relatively high elasticities of aggregate durables supply. This result implies that - even if the supply of the collateral asset is not fixed but more elastic - price effects are important for positive analyses of household saving and consumption decisions as well as for normative analyses.

3.4 Conclusion

This paper analyzes the importance of general equilibrium effects via changes in market prices in a calibrated heterogeneous household economy under collateralized loans. While the existing quantitative literature on these price effects is typically focussed on changes in collateral values in models with a relatively stylized household heterogeneity, this study analyzes the importance of price fluctuations in an economy with an empirically relevant specification of household heterogeneity. The analysis shows that the indirect effects in form of changes in the collateral price that arise from the direct effects under preference shocks are the key driver of adjustments in household saving and consumption in this type of framework. If preferences for durable consumption increase, the endogenous rise in the durables price induces savers to reduce demand for this good to increase non-durable consumption while constrained borrowers are able to increase consumption of both goods.

The indirect price effects are then decomposed into collateral and distributive effects of changes in the durables price. The former effects are induced by changes in the collateral value while the latter ones are induced by changes in the market price at which household trade the durable good. The analysis shows that the distributive effects of the change in the collateral price are more important for household decisions than the collateral effects. This result is in sharp contrast to the quantitative studies that focus only on collateral effects in frameworks with a simplified household heterogeneity (e.g. Guerrieri and Iacoviello (2017) or Bianchi and Mendoza (2018)).

In the benchmark economy, the aggregate supply of collateral is fixed. In this case, the sensitivity of the collateral price and therefore the importance of indirect price effects for households are at the maximum. The study, however, finds that even under relatively low price sensitivities of collateral - induced by a relatively high elasticity of collateral supply - the indirect price effects are still the key driver of household saving and consumption. This result shows that general equilibrium effects via price changes are relatively important for household decisions even in empirically relevant heterogeneous household economies.

Appendix

A.3 Definition of equilibrium

A.3.1 Equilibrium under a fixed supply of durables

In the following, an equilibrium under a fixed supply of durables is defined. In the quantitative analysis, the real interest rate and the wage rate are time-invariant and given by $r_t = r$ and $w_t = w$. The exogenous individual states are then, w.l.o.g., given by individual labor income $y_{i,t}$ with $y_{i,t} := e_{i,t}w$. Let $\Phi_t(x, y)$ denote the joint distribution of wealth and income across households in period t. An equilibrium of Section 3.2 under a fixed supply of durables is then defined as:

Definition 3.1 *Given an initial distribution Φ_0 and given $r_t = r$ and $w_t = w$, an equilibrium consists of a sequence of prices $\{q_t\}$, a sequence of household policy functions $\{b_t(x, y), c_t(x, y), d_t(x, y)\}$ and a sequence of joint distributions of wealth and income $\{\Phi_t\}$, such that*

(i) the policy functions $b_t(x, y)$, $c_t(x, y)$ and $d_t(x, y)$ solve the household problem given $\{q_t\}$,

(ii) the distribution Φ_t is consistent with the household policy functions,

(iii) the market for durables clears,

$$\sum d_t(x, y) d\Phi_t(x, y) \quad = \quad \bar{d}.$$

Note that under the calibrated real interest rate, the net-demand of the household sector in the bond market is positive, i.e. $\int_0^1 b_{i,t} di > 0 \; \forall t \geq 0$, such that aggregate capital K_{t+1} given by $K_{t+1} = \int_0^1 b_{i,t} di$ is also positive. Given that the policy functions ensure that all household budget constraints are satisfied and given that total labor supply L is time-invariant with $L = \int_0^1 e_{i,t} di$, the aggregate resource constraint under a fixed supply of durables in period t is $\int_0^1 c_{i,t} di + K_{t+1} = F(K_t, L)$.[44] For a stationary equilibrium, we additionally require the distribution of wealth and income as well as prices to be constant over time, i.e. $\Phi_{t+1} = \Phi$ and $q_t = q$ for all t.

A.3.2 Equilibrium under an elastic supply of durables

An equilibrium of Section 3.2 under an elastic supply of durables is defined as:

[44] Consider a model version with zero net-supply of bonds in the household sector as in Loenser and Schabert (2019). In such a version, the net-supply of bonds in the household sector is given to be zero, i.e. $\int_0^1 b_{i,t} di = 0$, which implies a specific value for the real interest rate. In the version analyzed in this paper, in contrast, the value of the real interest rate is given which then implies a specific net-supply of bonds in the household sector.

Definition 3.2 *Given an initial distribution Φ_0 and given $r_t = r$, $w_t = w$, an equilibrium consists of a sequence of prices $\{q_t\}$, a sequence of household policy functions $\{b_t(x, y), c_t(x, y), d_t(x, y)\}$ and a sequence of joint distributions of wealth and income $\{\Phi_t\}$, such that*

(i) the policy functions $b_t(x, y)$, $c_t(x, y)$ and $d_t(x, y)$ solve the household problem given $\{q_t\}$,

(ii) the distribution Φ_t is consistent with the household policy functions,

(iii) $q_t = T'(\Delta d_t)$ with $\Delta d_t = \int_0^1 d_{i,t} - d_{i,t-1} di$.

Note that under the calibrated real interest rate, the net-demand of the household sector in the bond market is positive, i.e. $\int_0^1 b_{i,t} di > 0 \; \forall t \geq 0$, such that aggregate capital K_{t+1} given by $K_{t+1} = \int_0^1 b_{i,t} di$ is also positive. Given that the policy functions ensure that all household budget constraints are satisfied and that total labor supply L is time-invariant with $L = \int_0^1 e_{i,t} di$, the aggregate resource constraint under an elastic supply of durables in period t is $\int_0^1 c_{i,t} di + K_{t+1} + T'(\Delta d_t)\Delta d_t = F(K_t, L)$. For a stationary equilibrium, we additionally require the distribution of wealth and income to be constant over time, i.e. $\Phi_{t+1} = \Phi$ for all t.

B.3 Computational algorithm

This section presents how the quantitative model from Section 3.3 is solved. It extends the algorithm in Loenser and Schabert (2019) to an economy with a collateral constraint, a durable consumption good and a production sector for the non-durable good while the real interest rate is constant. Section B.3.1 shows the income states and the corresponding transition matrix. In section B.3.2, the calculation of the total effects under the preference shock is explained for the version with a fixed supply of durables. First, it is discussed how to solve for the stationary equilibrium of the model economy. It is then shown how to solve for the transition path between two different stationary equilibria. B.3.3 and B.3.4 explain how to calculate the direct effects and the indirect effects, respectively, of the preference shock. B.3.5 shows how to solve for the transition path under a higher elasticity of durables supply (see 3.3.4). Note that for notational reasons the exogenous states for the efficiency of individual labor supply, $e_{i,t}$, are replaced by states for individual labor income denoted by $y_{i,t}$ with $y_{i,t} = we_{i,t}$ in the following description of the algorithm where w is the time-invariant wage rate.

B.3.1 Transition probabilities and income values

The individual income transition probabilities are obtained as discussed in Section 3.3. The transition matrix is given as

$$P = \begin{pmatrix} 0.9129 & 0.0867 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0005 \\ 0.0867 & 0.7867 & 0.1260 & 0.0001 & 0.0000 & 0.0000 & 0.0005 \\ 0.0000 & 0.1260 & 0.7352 & 0.1381 & 0.0001 & 0.0000 & 0.0005 \\ 0.0000 & 0.0001 & 0.1381 & 0.7352 & 0.1260 & 0.0000 & 0.0005 \\ 0.0000 & 0.0000 & 0.0001 & 0.1260 & 0.7867 & 0.0867 & 0.0005 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0867 & 0.9129 & 0.0005 \\ 0.0075 & 0.0075 & 0.0075 & 0.0075 & 0.0075 & 0.0075 & 0.9550 \end{pmatrix},$$

and the income grid values $(y_1, y_2, ..., y_7)$ are

$$(0.0123, 0.0250, 0.0376, 0.0544, 0.0819, 0.1667, 1)\,.$$

Let i denote the row index and j the column index of matrix P. The entry $P(i,j) \equiv \pi\,(y_j|y_i)$ is the probability that next period's income y_{t+1} equals y_j, conditional on current income $y_t = y_i$.

B.3.2 Calculation of the total effects under fixed supply

Calculation of the stationary equilibrium

Solving for the stationary equilibrium involves finding a time-invariant values for the price of durables as well as a time-invariant distribution of wealth implied by the household policy functions such that the market for durables clears. To do so, it will be convenient to focus on the sufficient endogenous individual state variable net-wealth: $x = b(1+r) + qd$. The numerical procedure involves the following steps:

I. Choose initial values for q.

II. Given q, compute the policy functions for non-durable consumption $c(x,y)$, end-of-period bonds $b'(x,y)$, end-of-period durables $d'(x,y)$ and end-of-period wealth $x'(x,y) = b'(x,y)(1+r)+q^i d'(x,y)$, using the endogenous grid point method (see Hintermaier and Koeniger (2010)) as outlined below.

III. Given the wealth policy function $x'(x,y)$, compute the implied stationary distribution $\lambda(x,y)$ (see below).

IV. Check whether the market for durables clears: $|\sum_{x,y} \lambda(x,y)d'(x,y) - \bar{d}|_\infty < \epsilon^d$, with $\epsilon^d > 0$. If

$||\sum_{x,y}\lambda(x,y)d'(x,y)-\bar{d}||_\infty < \epsilon^d$, stop: q is the equilibrium price. If $||\sum_{x,y}\lambda(x,y)d'(x,y)-\bar{d}|| \geq \epsilon^d$ update the price q and go to Step II.

Solving the household problem via the endogenous grid method The endogenous grid point method used to solve the household problem for q involvs the following steps:

1. Discretize next period wealth space $x' = \{x'_1, x'_2, ..., x'_m\}$, $x'_i < x'_{i+1}$. The discretized individual discretized state space then is given by $\{x'_1, x'_2, ..., x'_m\} \times \{y'_1, y'_2, ..., y'_n\}$, where y'_k, $k = 1, ..., n$, are the income states that are possible next period. Select a stopping rule parameter $\epsilon^{egm} > 0$.

2. Initialize the policy functions for non-durable and durable consumption $c_0(x'_i, y'_k)$ and $d'_0(x'_i, y'_k)$, $k \in \{1, ..., n\}$. Our guess is given by $c_0(x'_i, y'_k) = 0.5y'_k$ and $d'_0(x'_i, y'_k) = 0.5\bar{d}$ for all grid point combinations.

3. Update the consumption policy functions (using three auxiliary functions $\hat{c}_0(x'_i, y_k)$, $\hat{x}_0(x'_i, y_k)$ and $\hat{d}'_0(x'_i, y_k)$):

   - First, assume that the borrowing constraint does not bind in any state.
   - Use consumption policy functions $c_0(x'_i, y'_k)$ and $d'_0(x'_i, y'_k)$ to compute a guess for current period non-durable and durable consumption at future wealth x'_i and today's income state y_k, i.e. $\hat{c}_0(x'_i, y_k)$ and $\hat{d}'_0(x'_i, y_k)$, by applying the Euler equations for bonds and durables:

$$\begin{aligned} u_c(\hat{c}_0(x'_i, y_k), \hat{d}'_0(x'_i, y_k)) =& \beta(1+r)\sum_{j=1}^{n} p_{kj} u_c(c_0(x'_i, y'_j), d'_0(x'_i, y'_j)) \\ u_c(\hat{c}_0(x'_i, y_k), \hat{d}'_0(x'_i, y_k))q =& u_d(\hat{c}_0(x'_i, y_k), \hat{d}'_0(x'_i, y_k)) \\ &+ \beta q \sum_{j=1}^{n} p_{kj} u_c(c_0(x'_i, y'_j), d'_0(x'_i, y'_j)) \end{aligned}$$

     which are two equations in the two unknowns $\hat{c}_0(x'_i, y_k)$ and $\hat{d}'_0(x'_i, y_k)$ at given values of x'_i and y_k.
   - Now, find the states for which the borrowing constraint is violated. If the borrowing constraint is violated at given grid points x'_i and y_k, i.e. $\hat{d}'_0(x'_i, y_k) > x'_i/(q(1-\gamma))$, we set $\hat{d}'_0(x'_i, y_k) = x'_i/(q(1-\gamma))$. The corresponding value for non-durable consumption $\hat{c}_0(x'_i, y_k)$ is then given by the two Euler equations if we include the positive multiplier on the borrowing constraint. If the constraint is not binding, i.e. $\hat{d}'_0(x'_i, y_k) \leq x'_i/(q(1-\gamma))$ holds, we keep the values of $\hat{d}'_0(x'_i, y_k)$ and $\hat{c}_0(x'_i, y_k)$ calculated in the step before for this state.

- Use the budget constraint and the auxiliary functions $\hat{c}_0(x'_i, y_k)$ and $\hat{d}'_0(x'_i, y_k)$ to compute current period wealth $\hat{x}$ for x'_i and y_k:

$$\hat{x}_0(x'_i, y_k) = \hat{c}_0(x'_i, y_k) + q\hat{d}'_0(x'_i, y_k) + \left(x'_i - q\hat{d}'_0(x'_i, y_k)\right)/(1+r) - y_k.$$

  This implies $\hat{c}_0(x'_i, y_k) = \hat{c}_0(\hat{x}_0(x'_i, y_k), y_k)$ and $\hat{d'}_0(x'_i, y_k) = \hat{d'}_0(\hat{x}_0(x'_i, y_k), y_k)$.

- Calculate updates for the policy functions at $(x'_i, y'_k) \in \{x'_1, x'_2, ..., x'_m\} \times \{y'_1, y'_2, ..., y'_n\}$ by linearly interpolating $\hat{c}_0(\hat{x}_0, y_k)$ and $\hat{d'}_0(\hat{x}_0, y_k)$ at (x'_i, y'_k). This calculation gives the updated consumption policy functions $c_1(x'_i, y'_k)$ and $d'_1(x'_i, y'_k)$.

4. If $||c_1(x'_i, y'_k) - c_0(x'_i, y'_k)||_\infty < \epsilon^{egm}(1 + |c_0(x'_i, y'_k)||_\infty)$ and $||d'_1(x'_i, y'_k) - d'_0(x'_i, y'_k)||_\infty < \epsilon^{egm}(1 + |d'_0(x'_i, y'_k)||_\infty)$, stop and set $c(\cdot) = c_1(\cdot)$ and $d'(\cdot) = d'_1(\cdot)$.
   Else, set $c_0(\cdot) = c_1(\cdot)$ and $d'_0(\cdot) = d'_1(\cdot)$ and go to Step 3.

Computing the stationary distribution For given policy functions, we compute the stationary distribution by calculating the normalized eigenvalue of the Markov transition matrix implied by the policy function for wealth and the income transition probabilities:

1. We add additional grid points for wealth relative the one grid used for the calculation of the policy functions (we go from 10 to 50 thousand grid points for x) and calculate the wealth policy function values for these new states.

2. We calculate the transition probability of being in the state (x_j, y_l) in the next period conditional on currently being in state (x_i, y_k). We denote it as $\Pr((x_i, y_k)|(x_j, y_l))$. This probability is computed as $\Pr((x_i, y_k)|(x_j, y_l)) = \pi(y_l|y_k) \times I(x'(x_i, y_k) = x_j)$, where $I(x'(x_i, y_k) = x_j) = 1$ if $x'(x_i, y_k) = x_j$ and zero otherwise. The Markov transition matrix then consists of the individual transition probabilities $\Pr((x_i, y_k)|(x_j, y_l))$ for all grid point combinations.

3. Compute the eigenvector of this transition matrix associated with the largest eigenvalue (which is one). The stationary distribution of the model economy then is given by the normalization of this eigenvector.

Updating the price of durables The price is updated by using a bisection algorithm.

Calculation of the transition path to the new stationary equilibrium

In period $t-1$, the economy is in the stationary equilibrium under the lower value of the preference parameter, i.e. $\phi^d = 0.100$. Denote the parameter in the old stationary equilibrium by ϕ^d_{old}, i.e. $\phi^d_{old} = 0.100$. In period t, the parameter unexpectedly and permanently changed to a new value denoted as ϕ^d_{new} with $\phi^d_{new} = 0.102$. The economy then leaves the old stationary equilibrium in period t and gradually converges to the new stationary equilibrium under the preference parameter ϕ^d_{new}. The transition path to the new long-run equilibrium is computed as follows (see e.g. Rios-Rull (1999)):

- Calculate the stationary equilibria for an economy with ϕ^d_{old} and with ϕ^d_{new} as described in the previous section and denote the respective stationary distributions by Φ_{old} and Φ_{new}.

- The beginning-of-period distribution in period $t-1$ is denoted Φ_{t-1} and given by $\Phi_{t-1} = \Phi_{old}$. The distribution of the economy once it has converged to the new stationary equilibrium is denoted as Φ_∞. It is given as $\Phi_\infty = \Phi_{new}$. Note that the beginning-of-period distribution in period t is *not* the same as in period $t-1$, i.e. $\Phi_t \neq \Phi_{t-1}$, because the change in preferences changes the price of durables q_t and thereby also the beginning-of-period wealth in period t. Since the beginning-of-period distributions of bonds and durables in period t are however not affected and the same as in period $t-1$, it is possible to calculate Φ_t based on these distributions and the price of durables q_t. This price is however not known ex-ante and has to be calculated (see below).

- Calculate the transition path:

  1. Assume that the transition into the new stationary equilibrium takes $T > 0$ periods, i.e. the economy converges into the new stationary equilibrium in period $t+T-1$. This implies that $\Phi_{t+T-1} = \Phi_\infty$.

  2. Guess a sequence of prices of durables $\{\hat{q}_{t+j}\}_{j=0}^{T-2}$. Choose a stopping rule parameter $\epsilon^d > 0$ and $\epsilon^\Phi > 0$.

  3. With a guess $\{\hat{q}_{t+j}\}_{j=0}^{T-2}$ and with $q_{t+T-1}, \hat{c}_{t+T-1}, \hat{x}_{t+T-1}, \hat{d}_{t+T-1}$ and $\hat{b}_{t+T-1}$ given by the new stationary equilibrium, we can solve for $\left\{\hat{c}_{t+j}, \hat{x}_{t+j}, \hat{d}_{t+j}, \hat{b}_{t+j}\right\}_{j=0}^{T-2}$ via backward induction.

  4. By using the beginning-of-period distributions of bonds and durables of the old stationary equilibrium for period t together with the guess for the durables price in period t, i.e. $\hat{q}_t$, we calculate a guess for the beginning-of-period wealth distribution in period t denoted by $\hat{\Phi}_t$.

  5. Use the policy functions $\{\hat{x}_{t+j}\}$ and $\hat{\Phi}_t$ to iterate the distribution forward to get $\hat{\Phi}_{t+j}$ for $j = 1, ..., T-1$.

6. Use $\left\{\hat{\Phi}_{t+j}\right\}_{j=0}^{T-1}$ to compute excess supply $\hat{A^d}_{t+j} = \int \hat{d}_{t+j} d\hat{\Phi}_{t+j} - \bar{d}$ for $j = 0, ..., T-1$. Check for market clearance: If

$$\max_{0 \leq j < T-1} \left| \hat{A^d}_{t+j} \right| < \epsilon^d$$

holds, go to Step 7. If not, adjust the guess $\{\hat{q}_t\}_{j=0}^{T-2}$ and go to Step 3.

7. Check for $\left\| \hat{\Phi}_{t+T-1} - \Phi_{t+T-1} \right\|_{\infty} < \epsilon^{\Phi}$. If yes, the transition smoothly converges to the new stationary equilibrium. If not, go back to Step 1 and start again with a higher T.

B.3.3 Calculation of the direct effects

Consider the same preference shock as in B.3.2. The direct effects under this preference shock are given by the transition path at a constant price path of durables, i.e. $q_{t+j} = q_{t-1}$ for $j \geq 0$. To solve for these effects consider the following experiment. Assume that in period $t-1$ the economy is in the same stationary equilibrium induced by ϕ^d_{old} as in B.3.2 under fixed supply and denote the durables price in this equilibrium by $q^{old,bench}$. In period t then, the preference parameter unexpectedly and permanently increases from ϕ^d_{old} to ϕ^d_{new}. The economy then leaves the old stationary equilibrium and converges over time to a new one induced by the new preference parameter ϕ^d_{new}. In contrast to B.3.2 under fixed supply, however, the price path of durables does not react, i.e. $q_{t+j} = q^{old,bench}$ for $j \geq 0$, such that aggregate supply of durables increases over time until it converges to a value that is higher compared to the total effects. The transition path is then solved by an adequately adapted version of the algorithm explained in B.3.2.

B.3.4 Calculation of the indirect effects

Consider the price path of durables that is endogenously generated by the preference shock in B.3.2. The indirect effects of this preference shock are given by the following experiment. Assume that in period $t-1$ the economy is in the same stationary equilibrium induced by ϕ^d_{old} as in section B.3.2 under fixed supply and denote the durables price in this equilibrium by $q^{old,bench}$. In period t then, the price path of durables unexpectedly and permanently increase from $q_{t+j} = q^{old,bench}$ for $j \geq 0$ to the one that is endogenously generated in B.3.2 while the preference parameter does not change and is given by ϕ^d_{old} in all periods. The economy then leaves the old stationary equilibrium in period t and converges over time to a new induced by a new price of durables while the preference parameter is the same as in the old stationary equilibrium. Note that aggregate supply of durables decreases over time in this experiment until it converges to a value that is lower compared to the total effects. The transition path is then solved by an adequately adapted version of the algorithm explained in B.3.2.

B.3.5 Calculation of the total effects under a higher supply elasticity

In section 3.3.4, the total effects under a fixed supply are compared to the effects under a higher supply elasticity. For the latter version, we assume that the endogenous price path of durables is given while the transformation technology that induces this price path is not explicitly determined. In the following, it is explained how to solve for the transition path in this case. Consider the same preference shock as in B.3.2. Assume that in period $t-1$ the economy is in the same stationary equilibrium induced by ϕ^d_{old} as in B.3.2 under fixed supply and denote the durables price in this equilibrium by $q^{old,bench}$. In period t then, the preference parameter unexpectedly and permanently increases from ϕ^d_{old} to ϕ^d_{new}. The economy then leaves the old stationary equilibrium and converges over time to a new one induced by the new preference parameter ϕ^d_{new} and a durables price that is lower than in the new stationary equilibrium under fixed supply. As explained above, the corresponding durables price path is given and just a downward shift of the price path under fixed supply. The difference to the algorithm in B.3.2 is that here it is not necessary to solve for the durables price path that clears the durables market. It is only necessary to find a number of periods after which the economy converges into the new stationary equilibrium.

Chapter 4

Financial Regulation, Interest Rate Responses, and Distributive Effects

This chapter is based on Loenser, Röttger and Schabert (2020).

4.1 Introduction

The recent financial crisis has steered attention toward the interaction between de-leveraging and asset prices. Given that the scope to borrow against collateral crucially depends on the price of (pledgeable) assets, borrowers tend to de-leverage in states where asset prices fall, giving rise to a financial amplification mechanism (see e.g. Kiyotaki and Moore (1997)). This can even be more pronounced when adverse effects of de-leveraging induce a further decline in the price of collateral.[1] Given that (borrowing) agents do not internalize the impact of their behavior on prices, they might tend to overborrow. This pecuniary externality with regard to the collateral price provides a straightforward rationale for macroprudential financial regulation, as for example shown by Lorenzoni (2008), Bianchi (2011), Jeanne and Korinek (2019) or Bianchi and Mendoza (2018).[2] At the heart of this mechanism are borrowing limits that positively depend on the current price of collateral. As a novel contribution, we examine financial regulation and corrective taxes in a prototype heterogeneous agents framework with collateralized loans,[3] providing an empirically relevant specification of household debt.[4] While previous studies focused on asset prices and

[1]This mechanism can give rise to "fire sales", when assets are sold at dislocated prices (see Davila and Korinek (2018)).

[2]Based this mechanism, policy interventions with several types of instruments can be justified (Fornaro (2015), Benigno et al. (2016), Schmitt-Grohé and Uribe (2017), Korinek (2018)).

[3]Gottardi and Kubler (2015) examine a complete markets model with a collateral constraint and find that tighter restrictions on borrowing can enhance (constrained) efficiency.

[4]Since the 1980s, household debt secured by durable consumption goods (like vehicles or especially residential real estate) has accounted for more than 90% of US household debt in the United States (see Hintermaier and Koeniger (2016)), which we will calibrate our model to. Similar to Diaz and Luengo-Prado (2010), our model can replicate several distributional

collateral effects,[5] our analysis reveals that the welfare consequences of policy interventions in credit and asset markets mainly depend on induced interest rate responses and distributive effects.

For the analysis of financial regulation based on the above mentioned mechanism, existing studies consider the case where borrowing constraints bind only in extreme states (e.g. financial crises). When agents tend to overborrow, policy interventions can be beneficial if they induce agents to borrow less before borrowing constraints become binding. Ideally, policy should not only intervene ex ante, but also ex post to mitigate adverse effects of the financial amplification mechanism (see Benigno et al. (2016), or Jeanne and Korinek (2019)). So far, these analyses have been conducted in a framework with a single endogenous price. They usually employ infinite-horizon small open economy models (based on Mendoza (2010)), where a representative domestic agent borrows from abroad, or three-period closed economy models (e.g. Lorenzoni (2008) or Jeanne and Korinek (2019)) with distinct types of agents, who either borrow or lend. In these studies, agents can only borrow against the current market value of collateral. Given that the interest rate is exogenously fixed, changes in the terms of borrowing are mainly induced by variations of the price of collateral. The above cited studies then typically find that the pecuniary externality regarding the collateral price can be corrected by an ex-ante policy that constrains or disincentivizes borrowing, such as a reduction in the loan-to-value ratio or a Pigouvian tax on borrowing.[6]

In this paper, we assess corrective policies when the fundamental element of this amplification mechanism, namely, a borrowing limit that depends on the current value of collateral, is integrated into an incomplete markets framework (see Huggett (1993)). Individual agents cannot fully ensure against idiosyncratic income risk and might face a binding collateral constraint depending on their stochastic income and their endogenously determined wealth. In this framework, the interest rate is not invariant and the borrowing constraint occasionally binds for individual agents, while it is regularly binding for a non-zero fraction of the population. The model is calibrated to match several aggregate and distributional targets based on US data. In contrast to the above-mentioned studies on financial regulation, we abstract from aggregate risk and aim at addressing the following questions:

1. Is financial regulation in form of a loan-to-value ratio reduction recommendable under an empirically plausible distribution of secured household debt?

2. What are the distributional consequences of financial regulation and corrective taxes in the markets for debt and assets?

statistics observed in the data (see Section 4.3.1). This framework has been shown by Aaronson et al. (2012) to be consistent with individual household consumption behavior (see also Parker et al. (2013)). A related model is used by Guerrieri and Lorenzoni (2017) to quantitatively analyze a debt-deleveraging crisis.

[5]An exception is Davila and Korinek (2018) who restrict their study to an analytically tractable (rather than quantitative) framework. We relate our analysis to theirs in Section 4.2.1.

[6]An exception is Schmitt-Grohé and Uribe (2019), who demonstrate the existence of underborrowing in a small-open economy model with equilibrium multiplicity.

3. How important are changes in the interest rate compared to asset prices when borrowing is constrained by the value of assets?

To address these questions, we start with a simplified model version with two types of agents and a time horizon of three periods. This choice is made for comparability with the study of Davila and Korinek (2018), who provide an analysis of pecuniary externalities under financial frictions in a closely related framework. The main differences to their model are that agents face a borrowing constraint that depends on the current value of collateral not only in the second but also in the first period, and that there is no superior borrowers' use for assets (besides serving as collateral). As in Davila and Korinek (2018), the constrained-efficient allocation can be implemented by type-specific Pigouvian taxes on debt and on assets, which are both compensated by a set of type-dependent lump-sum transfers. Davila and Korinek (2018) show that the effect of pecuniary externalities on the collateral constraint "generally entails over-borrowing" under "natural conditions", i.e. when asset prices increase with agents' net worth. In contrast, we find that "collateral effects", i.e. effects of pecuniary externalities on the collateral constraint (see Davila and Korinek (2018)), are not unambiguous, since debt market interventions affect borrowers not only ex ante (before they are constrained) but also ex post. The analysis further shows that the design of corrective policies in markets for debt and assets depends on how changes in the asset price as well as in the interest rate exert "distributive effects" via agents' intertemporal choices.

We then examine corrective policies in a calibrated heterogeneous agents model, which is essentially a Huggett (1993) model with (utility-providing) durable goods and a borrowing limit, which is based on limited commitment and depends on the current market value of end-of-period durable goods. To isolate the main effects, the model is kept deliberately simple, while it nevertheless features elements that allow for an empirically relevant specification of household (secured) debt (see Diaz and Luengo-Prado (2010), Aaronson et al. (2012) or Guerrieri and Lorenzoni (2017)). We calibrate the model to match several aggregate and distributional targets following Diaz and Luengo-Prado (2010). In contrast to their analysis, we endogenize the durables price, which serves as a main object of our analysis. We then use the model to assess price effects and the ability of different types of corrective policies to enhance social welfare.[7] Given that our calibrated (discrete-time) model features both, an endogenous wealth distribution and a borrowing constraint that depends on the market price of collateral, numerical computation of constrained-efficient policies is – to our knowledge – not feasible.[8] For the purpose of our

[7]Following several normative studies in the incomplete markets literature (see Conesa et al. (2009), Krueger et al. (2016b) or Nuño and Moll (2018)), we measure social welfare as ex-ante expected lifetime utility, which is identical to utilitarian welfare.

[8]Davila et al. (2012) derive constrained-efficient policies for a heterogeneous-agent economy à la Aiyagari (1994), which has an endogenous distribution of wealth but price-inelastic borrowing constraints. Nuño and Moll (2018) propose a numerical strategy for computing constrained-efficient allocations in continuous-time heterogeneous-agent models. They also do not consider environments where market prices enter borrowing constraints.

analysis (see questions 1-3 above), we focus on a loan-to-value ratio reduction as a typical measure of financial regulation and anonymous (rather than type-specific) corrective taxes, i.e. Pigouvian-type tax interventions in the markets for debt and durables that equally apply to all agents, and that are intended to manipulate market prices in beneficial ways. Our main findings can be summarized as follows.

First, an unforeseen permanent reduction in the loan-to-value ratio (LTV) has direct and indirect effects. It directly limits the borrowing capacity of constrained agents and thereby leads to a decline in aggregate credit volume, which further causes indirect price effects. The reduction in credit demand leads to a lower equilibrium interest rate. Savers respond by raising their demand for durables as a store of wealth, such that the price of durables increases. The LTV reduction tends to reduce welfare of agents in the lowest income groups as well as in the highest ones. For only few borrowers with relatively high income the indirect price effects dominate the direct effect such that they experience a welfare improvement. These unconstrained agents reduce their borrowing, which contributes to the lower interest rate. Hence, these agents would have been better off under laissez faire if they were able to internalize the interest rate effects of borrowing decision. While the LTV reduction can in principle address this pecuniary externality, social welfare falls.[9] We further find that the effects of the LTV reduction on prices and welfare are slightly more pronounced in an artifical case of a price-inelastic borrowing constraint case where the borrowing limit depends on the value of collateral at a fixed price (at the laissez faire level). Thus, we do not find support for a substantial role of collateral effects in this experiment.

Second, we examine corrective taxes in the debt market. Specifically, we (unexpectedly and permanently) introduce an anonymous tax on debt, implying a subsidy on savings, which is aimed at manipulating market prices by affecting – in contrast to the LTV reduction – both sides of the credit market. Effects on agents' available resources are neutralized by type-specific lump-sum transfers/taxes (as in Davila and Korinek (2018)). Due to this Pigouvian-type tax, lenders tend to save more and borrowers tend to dis-save less, such that the interest rate declines and the price of durables increases. Compared to a LTV reduction, interest rate responses are relatively more pronounced than collateral price responses. In contrast to the LTV reduction, the debt-tax/saving-subsidy raises rather than lowers the aggregate credit volume, and also induces constrained borrowers to issue more debt. Overall, we find that aggregate welfare in all (except the highest) income states increases after this intervention and that it enhances social welfare. Specifically, borrowers tend to gain and lenders tend to lose from the decline in the interest rate. Thus, this policy intervention induces price changes that serve as partial insurance for borrowers from an ex-ante perspective, which is not internalized by individual agents in the laissez-faire economy. The analysis further shows that interest rate responses are more relevant than collateral price responses

[9] Gottardi and Kubler (2015) find that tighter restrictions on borrowing can enhance (constrained) efficiency in a model with state-contingent debt and an endogenous collateral constraint.

for the overall welfare results under the debt-tax/saving-subsidy.[10]

Third, we (unexpectedly and permanently) introduce a Pigouvian-type tax/subsidy on end-of-period holdings of durables. As a direct effect, a tax on durables dis-incentivizes purchases of durables and lowers their price. Agents substitute investment in durables in favor of investment in bonds, such that the interest rate decreases. Thus, low-wealth borrowers tend to benefit from the intervention, whereas high-wealth savers tend to lose. Like the tax on debt, the durables tax induces price changes that partially insure borrowers from an ex-ante perspective. Overall, the durables tax induces an increase in social welfare over the transition phase as well as in the long run. Notably, the welfare gain is higher under a price-inelastic borrowing constraint (since the borrowing constraint is not tightened by the lower durables price), indicating non-negligible collateral effects. Yet, the associated increase in borrowing reflects the decisive role of interest rate responses. By contrast to the durables tax, the social welfare effects of a Pigouvian-type subsidy on durables differ between the short run and the long run. Due to the increase in the durables price and in the interest rate low-wealth agents tend to lose and high-wealth agents tend to gain in the long run.[11] In the long run, the adverse effects on the former group dominate, such that the subsidy leads to a social welfare loss. Immediately after the introduction of the subsidy, when the distribution of bonds and durables is not yet adjusted, all income groups tend to gain from the wealth increase induced by the durables price appreciation. Given that a higher durables price further benefits low income agents who sell durables, social welfare including the transition phase increases under a durables subsidy. These results indicate that the distributive effects of corrective policy interventions are decisive and can qualitatively differ between the short run and the long run.

The Pigouvian-type tax interventions in the markets for debt and durables were scaled to induce equally-sized effects on the long-run price of collateral. We then observe that the simultaneous change in the interest rate is much more pronounced under the tax/subsidy on debt. More specifically, whereas a tax on debt of 5% lowers the long-run real interest rate by 5.5 percentage points, a subsidy on durables of 0.6%, which results in the same long-run price of durables, yields an interest rate increase of 0.4 percentage points. We further find that the overall welfare effects of the former tax is about 20-times larger than under the tax/subsidy on durables. Hence, the impact of corrective policies on the price of collateral is much less relevant than the impact on the interest rate in an empirically relevant model of household (collateralized) debt. This finding suggests that the role of collateral price effects is overestimated in studies on financial regulation where credit supply and interest rates changes are disregarded.

The remainder is structured as follows. Section 4.2 develops the simplified model, examines the

[10] Consistently, we find that price and welfare effects are almost identical under a price-inelastic borrowing constraint, indicating a negligible role of collateral effects.

[11] The latter do not internalize that raising their holdings of durables contributes to a beneficial increase in the durables price, which can be addressed by the durables subsidy.

constrained-efficient allocation, and describes issues of its implementation. Section 4.3 describes the Huggett (1993)-type model and its calibration, and presents results for our policy experiments. Section 4.4 concludes.

4.2 A model with limited commitment and incomplete markets

In this section, we develop a basic framework with financial frictions and assess effects of policy intervention with corrective instruments in an analytical way. As main ingredients for our analysis, the model features heterogeneous agents and a financial constraint that can induce inefficiencies due to pecuniary externalities. We examine government interventions in markets for debt and assets (here, durable goods). We start with a two-agent and three-period framework, which facilitates the derivation of analytical results and direct comparisons with Davila and Korinek (2018). In Section 4.3, we extend the analysis to an infinite-horizon model, based on Huggett (1993), where the status of each agents is endogenous, and examine the effects of policy interventions numerically.

We assume that financial markets are incomplete and that the only financial asset is a non-state-contingent one-period bond. A bond issued in period t trades at price $1/r_t$ and promises the payment of one unit of a non-durable good, which serves as the numeraire in the model, in period $t+1$. We further assume that there exists a financial friction, which gives rise to a borrowing constraint that can induce pecuniary externalities. Specifically, we assume that borrowers cannot commit to repay debt and that debt can be renegotiated after issuance in the same period. We allow borrowers to make a take-it-or-leave-it offer to reduce the value of debt. If the lender rejects the offer, he can seize a fraction γ of the borrower's assets (durable goods), which he can sell at the competitive market price q_t. Offers are therefore accepted when the repayment value of debt at least equals the current value of seizable assets. Without loss of generality, we assume that default and renegotiation never happen in equilibrium. Hence, when debt is issued, an individual borrower i has to take into account that the amount of debt $-b_{i,t}$ is constrained by

$$- b_{i,t} \leq \gamma q_t d_{i,t}, \tag{4.1}$$

where $d_{i,t}$ denotes the amount of the asset (durable good) held during the debt contract.

Given that renegotiation of debt issued in period t takes place in period t rather than in the subsequent period $t+1$, the borrowing constraint (4.1) features the price of the asset for the period of issuance q_t. This type of borrowing constraint is shared by many recent studies on macroprudential regulation (see, e.g. Stein (2012), Jeanne and Korinek (2017) or Bianchi and Mendoza (2018)), it is also common in quantitative studies with collateralized debt (see e.g. Favilukis et al. (2017), Lorenzoni and Guerrieri

(2017) or Berger et al. (2017)), and it is consistent with empirical evidence (see Cloyne et al. (2019)).

The borrowing constraint (4.1) can generate a feedback from sales of durables and price declines to a reduction of the debt limit, which induces borrowers to de-leverage. Given that the effects of individual behavior on the price of durables are not internalized, pecuniary externalities can be relevant for the allocation of resources. If, for example, the price of durables increases with borrowers' net wealth, agents tend to *overborrow* (see Davila and Korinek (2018)). In this case, borrowers do not internalize price effects of a higher debt burden, such that efficiency can principally be enhanced by corrective ex-ante policies that limit the build-up of debt.

4.2.1 A finite-horizon model with two types of agents

We develop a model that is structured to facilitate comparisons with the analysis of Davila and Korinek (2018). Our model essentially differs from theirs by *i.*) considering price dependent borrowing limits not only in the second, but also in the first period and *ii.*) by neglecting borrowers' prior use of assets (beyond their ability to serve as collateral). We consider three periods $t = 1, 2, 3$ and two mass-one groups $\{b, l\}$ with infinitely many households each. In each period t, a household $i \in \{b, l\}$ derives utility from consumption of a non-durable good, $c_{i,t}$, and a durable good, $d_{i,t}$, as given by the function

$$u_{i,t} = u(c_{i,t}, d_{i,t}), \tag{4.2}$$

which is increasing and concave with respect to both arguments. The budget constraint of a household i for period t is given by

$$c_{i,t} + q_t(d_{i,t} - d_{i,t-1}) + b_{i,t}/r_t = b_{i,t-1} + y_{i,t}, \tag{4.3}$$

where $y_{i,t}$ denotes the household's exogenous endowment of non-durable goods. Households of group b have initial assets $b_{b,0}$ and $d_{b,0}$ and an initial endowment $y_{b,1}$, and households of group l initial assets $b_{l,0}$ and $d_{l,0}$ and an initial endowment $y_{l,1}$. Households of the former (latter) group will be called borrowers (lenders). In period 2, the state of nature is random $\omega \in \{u, e\}$. With probability p, state u realizes and households face an unequal distribution of endowment (u), where borrowers receive $y_{b,2}$ and lenders $y_{l,2}$, such that they do not change roles. In state e, which realizes with probability $1 - p$, both types of households receive an equal endowment y. In period 3, endowment is identical for all households and given by y.

As discussed above, limited commitment implies that borrowing (in periods 1 and 2) is restricted by the collateral constraint (4.1). A household i aims at maximizing expected lifetime utility, given by $E_1[\sum_{t=1}^{T} \beta^{t-1} u(c_{i,t}, d_{i,t})]$, where $T = 3$ and E_1 denotes an expectation operator, subject to (4.1) and (4.3) for a given initial endowment $b_{i,0} = 0\ \forall i$ and $d_{i,0} > 0\ \forall i$. The first-order conditions for consumption, durables, and debt for $i \in \{b, l\}$ can be summarized as

$$u_c'(c_{i,1}, d_{i,1})q_1 = u_d'(c_{i,1}, d_{i,1}) + \beta\left(pq_2^u u_c'(c_{i,2}^u, d_{i,1}^u) + (1-p)q_2^e u_c'(c_{i,2}^e, d_{i,1}^e)\right) + \mu_{i,1}\gamma q_1, \tag{4.4}$$

$$u_c'(c_{i,1}, d_{i,1})/r_1 = \beta\left(pu_c'(c_{i,2}^u, d_{i,2}^u) + (1-p)\,u_c'(c_{i,2}^e, d_{i,2}^e)\right) + \mu_{i,1}, \tag{4.5}$$

$$u_c'(c_{i,2}^\omega, d_{i,2}^\omega)q_2^\omega = u_c'(c_{i,2}^\omega, d_{i,2}^\omega) + \beta q_3^\omega u_c'(c_{i,3}^\omega, d_{i,3}^\omega) + \mu_{i,2}^\omega \gamma q_2^\omega, \tag{4.6}$$

$$u_c'(c_{i,2}^\omega, d_{i,2}^\omega)/r_2^\omega = \beta u_c'(c_{i,3}^\omega, d_{i,3}^\omega) + \mu_{i,2}^\omega, \tag{4.7}$$

$$u_c'(c_{i,3}^\omega, d_{i,3}^\omega)q_3^\omega = u_d'(c_{i,3}^\omega, d_{i,3}^\omega), \tag{4.8}$$

where $\mu_{i,t}^\omega \geq 0$ denotes the multiplier on the collateral constraint (4.1), which satisfies $\mu_{l,1} = \mu_{l,2}^\omega = \mu_{b,2}^e = 0$ as well as $\mu_{b,1}(b_{b,1} + \gamma q_1^\varepsilon d_{b,1}) = 0$ and $\mu_{b,2}^u(b_{b,2}^u + \gamma q_2^u d_{b,2}) = 0$. For lenders, the multipliers $\mu_{l,1}$ and $\mu_{l,2}$ equal zero as well as the multiplier for the borrower in state $\omega = e$, $\mu_{b,2}^e = 0$.

To close the model, we assume that the supply of durables in each period is fixed and equal to $\bar{d}$. A competitive equilibrium can then be defined as follows: A competitive equilibrium is given by an allocation of durables, non-durables, and debt $\{c_{i,1}, d_{i,1}, b_{i,1}, c_{i,2}^\omega, d_{i,2}^\omega, b_{i,2}^\omega, c_{i,3}^\omega, d_{i,3}^\omega\}\ \forall i$, a set of prices $\{r_1, r_2^\omega, q_1, q_2^\omega, q_3^\omega\}$ and multipliers $\{\mu_{b,1}, \mu_{b,2}^u\}$, satisfying the budget constraints (4.3) $\forall t$, (4.4)-(4.8), the market clearing conditions $d_{b,t} + d_{l,t} = \bar{d}\ \forall t$, and $b_{b,t} + b_{l,t} = 0$ for $t \in \{1, 2\}$, and the collateral constraints (4.1) for $i = b$ and $t \in \{1, 2\}$ with $\mu_{b,1}(b_{b,1} + \gamma q_1 d_{b,1}) = 0$ and $\mu_{b,2}^\omega(b_{b,2}^\omega + \gamma q_2^\omega d_{b,2}) = 0$, given an initial distribution of assets and sequences of endowments.[12]

4.2.2 Pecuniary externalities and corrective policies

To assess the potential for welfare improvements, we consider the problem of a social planner who faces the households' constraints and optimal price taking behavior in a competitive equilibrium. We restrict our attention to the case, where the social planner is not able to implement first best, but a constrained-efficient allocation. Thus, we aim at identifying how changes in household behavior with regard to borrowing/lending and purchases of durables, which are induced by a specific set of instruments, can improve upon the equilibrium allocation under laissez faire. To facilitate comparisons, we thereby closely follow the set-up of the policy problem of Davila and Korinek (2018).

We consider two types of instruments. The social planner has access to a tax (subsidy) on borrowing $\tau_b^b > 0$ ($\tau_b^b < 0$) and a tax (subsidy) on borrowers' purchases of durables $\tau_b^d > 0$ ($\tau_b^d < 0$). Both instruments are only applied in period 1. We disregard further instruments, e.g. taxes on lenders, which

[12] Appendix A.4 describes the equilibrium solution.

would render implementation of first best possible.[13] We assume that these taxes/subsides are introduced at constant rates and are fully compensated by type-specific transfers/taxes, such that the instruments are purely corrective and exclusively affect the perceived after-tax prices for borrowers. The tax on borrowing represents regulatory instruments on borrowing, which has been the focus of studies on macroprudential regulation under pecuniary externalities (see e.g. Bianchi (2011), Benigno et al. (2016) or Bianchi and Mendoza (2018)). While the latter studies feature just one relevant endogenous price (namely, the price of collateral), we introduce a second instrument to be able to address pecuniary externalities related to the two endogenous prices in our model, q and r.[14]

Given this set of instruments, the social planner can choose the period 1 allocation $\bar{c}_{b,1}, \bar{c}_{l,1}, \bar{d}_{b,1}, \bar{d}_{l,1}, \bar{b}_{b,1}$, and $\bar{b}_{l,1}$ subject to the budget constraints of all agents, market clearing conditions, and the lenders' first-order conditions for the period 1, while accounting for the optimizing behavior of private agents for periods 2 and 3, summarized by $V_2^{i,\omega}(\cdot)$ (see Appendix A.4). The social planner problem then reads

$$\max_{\left\{\bar{c}_{i,1},\bar{d}_{i,1},\bar{b}_{i,1}\right\}_{i\in\{b,l\}}} \sum_i \theta_i \left\{u_1(\bar{c}_{i,1},\bar{d}_{i,1}) + \beta E_1\left[V_2^{i,\omega}(\bar{d}_{i,1},\bar{b}_{i,1};\bar{d}_1,\bar{b}_1)\right]\right\} \tag{4.9}$$

$$\begin{aligned}
\text{s.t. } & y_{b,1} = \bar{c}_{b,1} + q_1(\bar{d}_{b,1} - \bar{d}_{b,0}) + \bar{b}_{b,1} r_1^{-1}, \\
& y_{l,1} = \bar{c}_{l,1} + q_1(\bar{d}_{l,1} - \bar{d}_{l,0}) + \bar{b}_{l,1} r_1^{-1}, \\
& -\bar{b}_{b,1} \leq \gamma q_1 \bar{d}_{b,1}, \\
& \bar{d}_{b,1} + \bar{d}_{l,1} = \bar{d}, \\
& \bar{b}_{b,1} + \bar{b}_{l,1} = 0, \\
& u_c'(\bar{c}_{l,1},\bar{d}_{l,1})q_1 = u_d'(\bar{c}_{l,1},\bar{d}_{l,1}) + \beta E_1[q_2^\omega u_c'(\bar{c}_{l,2}^\omega,\bar{d}_{l,2}^\omega)], && \text{(L1)} \\
& u_c'(\bar{c}_{l,1},\bar{d}_{l,1})r_1^{-1} = \beta E_1\left[u_c'(\bar{c}_{l,2}^\omega,\bar{d}_{l,2}^\omega)\right], && \text{(L2)}
\end{aligned}$$

where θ_i denotes the Pareto weight assigned to household type i by the social planner. For the quantitative analysis in Section 4.3, we consider ex-ante expected utility, i.e. Pareto weights that reflect a household's ex-ante probability of being a specific type. Since the planner knows that all agents of type $i \in \{b,l\}$ act identically, the individual states, $d_{i,1}$ and $b_{i,1}$, can as arguments of the continuation value $V_2^{i,\omega}(\cdot)$ be replaced by the type-specific aggregate states, $\bar{d}_{i,1}$ and $\bar{b}_{i,1}$ in the policy problem (4.9). The distinction between individual and aggregate states is relevant to understand why the social planner solution might differ from the competitive one.

[13] A subsidy on durables can for example stimulate the collateral value such that collateral constraint never binds. Davila and Korinek (2018) allow for taxes on borrowers and lenders. The policy problem is nonetheless non-trivial, since the borrowing constraint in period 1 does not depend on prices.

[14] Notably, instrument *ii.)* can alternatively be specified as a single tax/subsidy on purchases of durables of both agents (borrowers and lender), where the borrowing constraint depends on the after tax price of collateral.

The problem for the social planner can be written as

$$\begin{aligned}\mathcal{L} \equiv \sum_i \theta_i \left\{ u_1(\bar{c}_{i,1}, \bar{d}_{i,1}) + \beta E_1 \left[V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1) \right] \right\} \\ + \theta_b \lambda_{b,1}^{bud} \left[y_{b,1} - \bar{c}_{b,1} - q_1(\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} r_1^{-1} \right] \\ + \theta_l \lambda_{l,1}^{bud} \left[y_{l,1} - \bar{c}_{l,1} - q_1(\bar{d}_{l,1} - \bar{d}_{l,0}) - \bar{b}_{l,1} r_1^{-1} \right] \\ + \theta_b \mu_{b,1} \left[\bar{b}_{b,1} + \gamma q_1 \bar{d}_{b,1} \right],\end{aligned} \tag{4.10}$$

where we use the market clearing conditions for debt, $\bar{b}_{l,1} = -\bar{b}_{b,1}$, and durables, $\bar{d} = \bar{d}_{l,1} + \bar{d}_{b,1}$, implying for the aggregate state variables $\bar{d}_1 = \left(\bar{d}_{b,1}, \bar{d} - \bar{d}_{b,1}\right)$ and $\bar{b}_1 = \left(\bar{b}_{b,1}, -\bar{b}_{b,1}\right)$. Furthermore, the planner takes into account that prices are functions of the allocation and implicitly given by (L1) and (L2). In contrast to the first period, the allocation in the second period is determined by the laissez-faire equilibrium solution, given the beginning-of-period-2 states, that the planner can control in the first period. This dependence is captured by $V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1)$. Notably, period-1-prices r_1 and q_1 not only depend on lenders' non-durables $\bar{c}_{l,1}$ and durables $\bar{d}_{l,1}$, but also on the aggregate states $\bar{d}_1$ and $\bar{b}_1$ via their impact on next period's equilibrium objects. Moreover, the end-of-period 1 aggregate states alter next period prices q_2^ω and r_2^ω via their impact through the continuation values $V_2^{l,\omega}(\cdot)$ and $V_2^{b,\omega}(\cdot)$. These price effects, i.e. $\frac{\partial q_1}{\partial \bar{x}_{j,1}}$, $\frac{\partial r_1^{-1}}{\partial \bar{x}_{j,1}}$, $\frac{\partial q_2^\omega}{\partial \bar{x}_{j,1}}$, and $\frac{\partial (r_2^\omega)^{-1}}{\partial \bar{x}_{j,1}}$ for $\bar{x} \in \{\bar{d}, \bar{b}\}$ and $j \in \{b, l\}$, are not internalized by any individual agent, but by the social planer, who accounts for their impact via the agents' budget constraints and the borrowing constraint. These effects need to be distinguished from those effects that are internalized by the households, which correspond to the derivatives of $V_2^{i,\omega}(\cdot)$ with respect to the individual state $x_{i,1}$, which however coincides with $\bar{x}_{i,1}$ in equilibrium.

The social planner's first-order conditions for $\bar{d}_{b,1}$ and $\bar{b}_{b,1}$ can be written as (see Appendix B.4)

$$u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) q_1 = u_d'(\bar{c}_{b,1}, \bar{d}_{b,1}) + \beta E_1 \left[u_c'(\bar{c}_{b,2}^\omega, \bar{d}_{b,2}^\omega) q_2^\omega \right] + \mu_{b,1} \gamma q_1 + \Delta_1^d + \beta E_1 \left[\Delta_2^{d,\omega} \right], \tag{4.11}$$

$$u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) r_1^{-1} = \beta E_1 \left[u_c'(\bar{c}_{b,2}^\omega, \bar{d}_{b,2}^\omega) \right] + \mu_{b,1} + \Delta_1^b + \beta E_1 \left[\Delta_2^{b,\omega} \right]. \tag{4.12}$$

These first-order conditions solely deviate from those of the competitive equilibrium by the wedges Δ_1^x and $\Delta_2^{x,\omega}$. They capture the social value that changes of $\bar{x}_{b,1}$ and – due to market clearing – of $\bar{x}_{l,1}$ have because of their impact on first and second period prices $(q_1, 1/r_1)$ and $(q_2^\omega, 1/r_2^\omega)$. The wedges $\Delta_2^{x,\omega}$ account for the marginal effect that an increase in $\bar{x}_1$ has on prices in the second period, conditional on the realization of ω. Since these prices only respond to aggregate quantities, the wedges depend on the derivatives of $V_2^{b,\omega}(\cdot)$ and $V_2^{l,\omega}(\cdot)$ with respect to the aggregate states, $\Delta_2^{x,\omega} = \frac{\partial V^{b,\omega}}{\partial \bar{x}_1} \frac{\partial \bar{x}_1}{\partial \bar{x}_{b,1}} - \frac{\partial V^{b,\omega}}{\partial \bar{x}_1} \frac{\partial \bar{x}_1}{\partial \bar{x}_{l,1}} + \frac{\theta_l}{\theta_b} \frac{\partial V^{l,\omega}}{\partial \bar{x}_1} \frac{\partial \bar{x}_1}{\partial \bar{x}_{b,1}} - \frac{\theta_l}{\theta_b} \frac{\partial V^{l,\omega}}{\partial \bar{x}_1} \frac{\partial \bar{x}_1}{\partial \bar{x}_{l,1}}$. The wedges for the first period Δ_1^x account for the marginal effects of changes in $\bar{x}_1$ on first period prices via agents' choices in the first period (L1) and (L2). Specifically, the wedges Δ_1^x and $\Delta_2^{x,\omega}$ are given by

$$\Delta_1^x = \left(u_c'(\bar{c}_{b,1},\bar{d}_{b,1}) - \frac{\theta_l}{\theta_b}u_c'(\bar{c}_{l,1},\bar{d}_{l,1}) - \mu_{b,1}C^b_{c_{l,1}}\right)\frac{D^b_{x_{b,1}} - D^b_{x_{l,1}}}{1+D^b_{c_{l,1}}} + \mu_{b,1}\left(C^b_{x_{b,1}} - C^b_{x_{l,1}}\right), \quad (4.13)$$

$$\Delta_2^{x,\omega} = \left(u_c'(\bar{c}^\omega_{b,2},\bar{d}^\omega_{b,2}) - \frac{\theta_l}{\theta_b}u_c'(\bar{c}^\omega_{l,2},\bar{d}^\omega_{l,2})\right)\left(D^{b,\omega}_{x_{b,2}} - D^{b,\omega}_{x_{l,2}}\right) + \mu^\omega_{b,2}\left(C^{b,\omega}_{x_{b,1}} - C^{b,\omega}_{x_{l,1}}\right), \quad (4.14)$$

for $x \in \{b,d\}$. Following Davila and Korinek (2018), effects of un-internalized price changes are separated in *distributive effects* (D), which affect the budget sets of agents, and *collateral effects* (C), which affect the borrowing constraint:

$$\begin{aligned}
D^b_{c_{l,1}} &= -\frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{b,1} - \bar{d}_{b,0}) - \frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}}\bar{b}_{b,1},\\
D^b_{x_{j,1}} &= -\frac{\partial q_1}{\partial \bar{x}_{j,1}}(\bar{d}_{b,1} - \bar{d}_{b,0}) - \frac{\partial r_1^{-1}}{\partial \bar{x}_{j,1}}\bar{b}_{b,1},\\
D^{b,\omega}_{x_{j,2}} &= -\frac{\partial q_2^\omega}{\partial \bar{x}_{j,1}}(\bar{d}^\omega_{b,2} - \bar{d}_{b,1}) - \frac{\partial \left(r_2^\omega\right)^{-1}}{\partial \bar{x}_{j,1}}\bar{b}^\omega_{b,2},\\
C^b_{c_{l,1}} &= \gamma\frac{\partial q_1}{\partial \bar{c}_{l,1}}\bar{d}_{b,1},\\
C^b_{x_{j,1}} &= \gamma\frac{\partial q_1}{\partial \bar{x}_{j,1}}\bar{d}_{b,1},\\
C^{b,\omega}_{x_{j,1}} &= \gamma\frac{\partial q_2^\omega}{\partial \bar{x}_{j,1}}\bar{d}^\omega_{b,2}.
\end{aligned}$$

The effects of pecuniary externalities in this model differ from those in Davila and Korinek (2018), since prices here are also relevant in period 1 for the social planer, whereas only second period prices matter in their analysis (which corresponds to $\Delta_1^x = 0$) As a consequence, we have to account additionally for the un-internalized effects of changes in $\bar{c}_{l,1}$, $\bar{d}_{l,1}$, $\bar{d}_{b,1}$, $\bar{b}_{b,1}$ and $\bar{b}_{l,1}$ on period 1 prices. This for example implies that the shadow price of the lender's budget constraint in the first period in (4.10), $\lambda^{bud}_{l,1}$, deviates from $u_c'(\bar{c}_{l,1})$. Given that we only consider taxes/subsidies on borrowers, the relative effects $D^{b,\omega}_{x_{b,1}} - D^{b,\omega}_{x_{l,1}}$ and $C^b_{x_{b,1}} - C^b_{x_{l,1}}$ show up in the externality terms of the social planner's first-order conditions for $x_{b,1}$, since raising $x_{b,1}$, the planner also directly lowers $x_{l,1}$ because of fixed aggregate supply and market clearing.[15] Apparently, the collateral effects in periods 1 and 2 only apply to household types $i = b$ since $\mu_{l,1} = \mu^\omega_{l,2} = 0$ holds by construction. Further note that the distributive effects on the budget sets of borrowers and lenders are – as in Davila and Korinek (2018) – symmetric, such that $\sum_{i=b,l} D^i_{x_{j,1}} = \sum_{i=b,l} D^{i,\omega}_{x_{j,2}} = 0$ where $D^i_{x_{j,1}} = -\frac{\partial q_1}{\partial \bar{x}_{j,1}}(\bar{d}_{i,1} - \bar{d}_{i,0}) - \frac{\partial r_1^{-1}}{\partial \bar{x}_{j,1}}\bar{b}_{i,1}$ and $D^{i,\omega}_{x_{j,2}} = -\frac{\partial q_2^\omega}{\partial \bar{x}_{j,1}}(\bar{d}^\omega_{i,2} - \bar{d}_{i,1}) - \frac{\partial (r_2^\omega)^{-1}}{\partial \bar{x}_{j,1}}\bar{b}^\omega_{i,2}$ for $i,j \in \{b,l\}$. Given that the social planer has access to borrower-specific Pigouvian tax instruments,

[15] If, as in Davila and Korinek (2018), taxes/subsidies were also imposed on lenders, the solution to his problem would involve separate first order conditions for $x_{b,1}$ and $x_{l,1}$.

the borrowers' first-order conditions are given by

$$(1+\tau_b^d)u_c'(\bar{c}_{b,1},\bar{d}_{b,1})q_1 = u_d'(\bar{c}_{b,1},\bar{d}_{b,1}) + \beta E_1\left[u_c'(\bar{c}_{b,2}^{\omega},\bar{d}_{b,2}^{\omega})q_2^{\omega}\right] + \mu_{b,1}\gamma q_1,$$

$$(1-\tau_b^b)u_c'(\bar{c}_{b,1},\bar{d}_{b,1})r_1^{-1} = \beta E_1\left[u_c'(\bar{c}_{b,2}^{\omega},\bar{d}_{b,2}^{\omega})\right] + \mu_{b,1}.$$

Comparing these two conditions with the first-order conditions of the social planer (4.11) and (4.12) shows that the constrained-efficient allocation can be implemented by tax rates satisfying

$$\tau_b^b = \frac{\Delta_1^b + \beta E_1\left[\Delta_2^{b,\omega}\right]}{u_c'(\bar{c}_{b,1})r_1^{-1}} \quad \text{and} \quad \tau_b^d = -\frac{\Delta_1^d + \beta E_1\left[\Delta_2^{d,\omega}\right]}{q_1 u_c'(\bar{c}_{b,1})}, \tag{4.15}$$

and by associated lump-sum taxes/transfers, which neutralize the effects of τ_b^d and τ_b^b on the borrowers' budget set.

Collateral effects The signs of the wedges Δ_1^x and $\Delta_2^{x,\omega}$ depend on a variety of potentially opposed price effects as well as on differences in the valuation of funds between agents and their asset positions (see 4.13 and 4.14). For example, an increase in durables held by lenders at the end of period 1 tends to alter the current price q_1 and the future price q_2^{ω} in opposite ways (see L2). Apparently, the collateral effects in period 1 are only relevant if the borrowing constraint is binding in period 1, $\mu_{b,1} > 0$. These externalities are evidently absent when the borrowing constraint does not depend on the price of collateral in period 1 (as in Davila and Korinek, 2018), such that $C^b_{c_{l,1}} = C^b_{x_{j,1}} = C^{b,\omega}_{x_{j,1}} = 0$. Here, the sign of the collateral effects depends on sign of the derivatives $\partial q_1/\partial \bar{c}_{l,1}$, $\partial q_1/\partial \bar{x}_{b,1}$, $\partial q_1/\partial \bar{x}_{l,1}$, $\partial q_2^{\omega}/\partial \bar{x}_{b,1}$, and $\partial q_2^{\omega}/\partial \bar{x}_{l,1}$. To assess the impact of the collateral effects on the tax/subsidy on borrowing, suppose that the following inequality holds (which relates to Condition 1 in Davila and Korinek (2018))

$$\gamma\mu_{b,2}^{\omega}\left(\frac{\partial q_2^{\omega}}{\partial \bar{b}_{b,1}} - \frac{\partial q_2^{\omega}}{\partial \bar{b}_{l,1}}\right)\bar{d}_{b,2}^{\omega} > 0. \tag{4.16}$$

Then, the collateral externality included in $\Delta_2^{b,\omega}$ (see 4.14) calls for a tax on borrowing $\tau_b^b > 0$ (see 4.15) as implied by studies on macroprudential regulation where agents tend to overborrow (see e.g. Bianchi and Mendoza (2018)). The inequality (4.16) requires that the increasing impact of a lower debt position at the beginning of period 2 $\bar{b}_{b,1}$ (or higher bond holdings) on the collateral price q_2^{ω} in period 2 is larger than the simultaneous decreasing impact of the reduction in lenders' bond holdings $\bar{b}_{l,1}$. Then, the borrowers' increased willingness to spend on durables dominates the response of lenders. Now consider the effects of a change in the borrowers' (lenders') bond position $\bar{b}_{b,1}$ ($\bar{b}_{l,1}$) at the end of period 1 on prices in period 1. Focusing on the impact on available resources in a particular period, a change in the end-of-period bond holdings tends to lead to price effects that are opposed to the price effects of equal changes in the beginning-of-period bond holdings. Thus, the term representing the collateral effects in the wedge Δ_1^b (see 4.13), i.e.

$$\gamma\mu_{b,1}\left(\frac{\partial q_1}{\partial \bar{b}_{b,1}}-\frac{\partial q_1}{\partial \bar{b}_{l,1}}\right)\bar{d}_{b,1}, \tag{4.17}$$

tends to be negative under (4.16). This tendency is further strengthened by the fact that an increase in $\bar{b}_{b,1}$ (decline in end-of-period debt) tends to reduce borrowers' marginal valuation of durables as collateral. Hence, even if agents tend to overborrow, as they do not internalize the adverse impact of de-leveraging on the collateral price in period 2, this externality is not sufficient to imply a tax on borrowing, if borrowers are already constrained in period 1, $\mu_{b,1}>0$. Apparently, the size of the effects given in (4.16) and (4.17) also depend on the tightness of the borrowing constraints and the stock of durables held by borrowers in both periods, such that the total impact of the collateral effects on the tax rate τ_b^b (see 4.15) is ambiguous. In our numerical analysis (see Section 4.3.1), we actually find that distributive effects rather than collateral effects are decisive for the welfare effects of interventions in the credit market.

Distributive effects Now suppose that the borrowing limit were price-inelastic, i.e. $-\gamma\overline{q}d_{i,t}$, with a fixed price $\overline{q}>0$ rather than an endogenous price q_t given by the RHS of (4.1). This assumption will also be used as a reference case in the subsequent quantitative analysis. Based on the quantitative results, which will be presented in Section 4.3, we will conclude that the effects of policy interventions under the price-inelastic borrowing constraint are very similar to those under the price-elastic collateral constraint (4.1). Under a price-inelastic borrowing constraint, the wedges Δ_1^b and $\Delta_2^{b,\omega}$, which are relevant for the borrowing tax τ^b (see 4.15), would solely depend on the distributive effects, i.e.

$$\begin{aligned}
\Delta_1^b &= \frac{u_c'(\bar{c}_{b,1},\bar{d}_{b,1})-\frac{\theta_l}{\theta_b}u_c'(\bar{c}_{l,1},\bar{d}_{l,1})}{1-\frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{b,1}-\bar{d}_{b,0})-\frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}}\bar{b}_{b,1}} && (4.18)\\
&\quad\times\left[\left(\frac{\partial q_1}{\partial \bar{b}_{l,1}}-\frac{\partial q_1}{\partial \bar{b}_{b,1}}\right)\{\bar{d}_{b,1}-\bar{d}_{b,0}\}+\left(\frac{\partial r_1^{-1}}{\partial \bar{b}_{l,1}}-\frac{\partial r_1^{-1}}{\partial \bar{b}_{b,1}}\right)\bar{b}_{b,1}\right], \\
\Delta_2^{b,\omega} &= \left(u_c'(\bar{c}_{b,2}^{\omega},\bar{d}_{b,2}^{\omega})-\frac{\theta_l}{\theta_b}u_c'(\bar{c}_{l,2}^{\omega},\bar{d}_{l,2}^{\omega})\right) && (4.19)\\
&\quad\times\left[\left(\frac{\partial q_2^{\omega}}{\partial \bar{b}_{l,1}}-\frac{\partial q_2^{\omega}}{\partial \bar{b}_{b,1}}\right)\{\bar{d}_{b,2}^{\omega}-\bar{d}_{b,1})\}+\left(\frac{\partial\left(r_2^{\omega}\right)^{-1}}{\partial \bar{b}_{l,1}}-\frac{\partial\left(r_2^{\omega}\right)^{-1}}{\partial \bar{b}_{b,1}}\right)\bar{b}_{b,2}^{\omega}\right].
\end{aligned}$$

The wedges Δ_1^b and $\Delta_2^{b,\omega}$, which are decisive for the size and sign of the borrowing tax, depend not only on un-internalized changes in the interest rates $\{r_1,\ r_2^{\omega}\}$ that are induced by changes in debt/savings $\{\bar{b}_{b,1},\ \bar{b}_{l,1}\}$, but also on their un-internalized impact on the durables prices $\{q_1,q_2^{\omega}\}$ (see the terms in the square brackets in 4.18 and 4.19). The subsequent quantitative analysis will reveal that interventions in the credit market alter the interest rate to a larger extent than the durables price, and that social welfare effects will be dominated by the former price effects. Consider, for example, the case where the first factor on the RHS of (4.18) is positive.[16] Given that the price of debt r_1^{-1} tends to increase with savings and to decrease with borrowing, such that $(\partial r_1^{-1}/\partial \bar{b}_{l,1}) - (\partial r_1^{-1}/\partial \bar{b}_{b,1}) > 0$ holds, and that borrowing implies $\bar{b}_{b,1} < 0$, the wedge Δ_1^b would then call for a subsidy, $\tau^b < 0$, rather than a tax on debt.

Likewise, the wedges relevant for the sign of the tax on durables τ^d, also depend on un-internalized changes of the price of durables and the interest rate as well as the allocation of bonds and durables,

$$\begin{aligned}
\Delta_1^d &= \frac{u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) - \frac{\theta_l}{\theta_b} u_c'(\bar{c}_{l,1}, \bar{d}_{l,1})}{1 - \frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{b,1} - \bar{d}_{b,0}) - \frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}} \bar{b}_{b,1}} && (4.20) \\
&\times \left[\left(\frac{\partial q_1}{\partial \bar{d}_{l,1}} - \frac{\partial q_1}{\partial \bar{d}_{b,1}} \right) \left\{ \bar{d}_{b,1} - \bar{d}_{b,0} \right\} + \left(\frac{\partial r_1^{-1}}{\partial \bar{d}_{l,1}} - \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right) \bar{b}_{b,1} \right], \\
\Delta_2^{d,\omega} &= \left(u_c'(\bar{c}_{b,2}^{\omega}, \bar{d}_{b,2}^{\omega}) - \frac{\theta_l}{\theta_b} u_c'(\bar{c}_{l,2}^{\omega}, \bar{d}_{l,2}^{\omega}) \right) && (4.21) \\
&\times \left[\left(\frac{\partial q_2^{\omega}}{\partial \bar{d}_{l,1}} - \frac{\partial q_2^{\omega}}{\partial \bar{d}_{b,1}} \right) \left\{ \bar{d}_{b,2}^{\omega} - \bar{d}_{b,1}) \right\} + \left(\frac{\partial \left(r_2^{\omega} \right)^{-1}}{\partial \bar{d}_{l,1}} - \frac{\partial \left(r_2^{\omega} \right)^{-1}}{\partial \bar{d}_{b,1}} \right) \bar{b}_{b,2}^{\omega} \right],
\end{aligned}$$

where collateral effects are again not present. In contrast to the uninternalized effects on the interest rate, the impact of un-internalized durables price changes that are induced by interventions in the durable market are not scaled with a stock variable, i.e. the stock of debt, but with a flow variable, i.e. the change in holdings of durables (see the terms in the curly brackets in 4.20 and 4.21). Hence, the impact of durables price changes increases with the adjustments of durables. In general, these adjustments are likely to be larger in the short run, i.e. on impact or during the transition after the policy intervention, than in the long run, i.e. in a new stationary equilibrium when adjustments in quantities are completed. In Section 4.3.3, we will consistently observe that durable price effects are quantitatively more relevant in the short run, whereas interest rate effects (that are scaled with the debt level) tend to dominate welfare results in the long run.

The expressions in (4.18)-(4.21) apparently indicate that without further information on preferences and the distributions of bonds and durables, the sign of these wedges are in general unclear, so that the implementation of the constrained-efficient allocation might either require taxes or subsides on debt and durables.

[16] This holds if the weighted marginal utility of non-durable consumption of the borrower exceeds the weighted marginal utility of non-durable consumption of the lender, $u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) - \frac{\theta_l}{\theta_b} u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}) > 0$, and if $D^b_{c_{l,1}} \geq -1$. We find that the former holds in our quantitative model evaluation, i.e. lenders tend to have a lower marginal utility of non-durable consumption than borrowers.

4.3 Quantitative analysis

The analysis in the previous section has shown that the identification of welfare-enhancing corrective policies requires further information on agents' preferences and the allocation of bonds and durables. To restrict our attention to an empirically relevant specification of (secured) household debt (see Diaz and Luengo-Prado (2010), Aaronson et al. (2012), Guerrieri and Lorenzoni (2017)), we use a model version with an infinite horizon and potentially varying income and wealth state of agents, which can be calibrated to reasonably match the data. We start by examining the consequences of changing the loan-to-value ratio γ, which directly affects households via the collateral constraint but also indirectly via general equilibrium price effects. Then, we look at taxes on debt and end-of-period durables, similar to the ones considered in Section 4.2.1. Given that the distribution of wealth and the market price of collateral both are endogenous in the model, the implementation of a constrained-efficient allocation would require a set of individual tax rates that depends on income and wealth states, which cannot be computed in a straightforward way.[17] For the purpose of the paper, it however suffices to locally examine anonymous Pigouvian-type taxes on debt and durables.[18] For all policy experiments, we start with the laissez-faire economy. We then unexpectedly (and permanently) change the policy tool of interest, look at the equilibrium effects along the transition path to the new long-run equilibrium, and analyze the welfare implications of these policy experiments.

4.3.1 An infinite-horizon model with an endogenous wealth distribution

In this section, we set up a version of the model that can be calibrated reasonably well and be used to quantitatively assess the effects of corrective policies. The model is an incomplete-markets economy à la Huggett (1993) that is extended to allow for durable goods and a collateral constraint (see 4.1). In contrast to the 3-period model studied so far, households are infinitely-lived ($T = \infty$), i.e. $E_1 \sum_{t=1}^{\infty} \beta^{t-1} u(c_{i,t}, d_{i,t})$, and face a random, idiosyncratic income $y_{i,t} \in Y \equiv \{y_1, y_2, ..., y_N\}$, with $y_1 < y_2 < ... < y_N$, which follows a first-order Markov process with conditional transition probabilities $\pi\left(y_{i,t+1}|y_{i,t}\right)$. The presence of uninsurable idiosyncratic risk results in an endogenous and non-degenerate wealth distribution. Importantly, whether a household is a borrower or a saver is not fixed over time (as in the model of Section 4.2.1), but an endogenous outcome that depends on an individual household's history of shocks.[19] We consider

[17]Davila et al. (2012) calculate optimal corrective taxes for an Aiyagari (1994)-type economy by first directly solving for the constrained-efficient allocation. Doing so is not feasible in our model due to the presence of a borrowing constraint that depends on the endogenous collateral price. See Nuño and Moll (2018) for an approach similar to Davila et al. (2012) in continuous time.

[18]While it would be interesting to impose such a tax on borrowers only, i.e. asymmetrically, doing so makes the household problem non-convex, such that first-order conditions are no longer sufficient to find the optimal decision rules. A solution approach like this is however necessary to solve a model with Pigouvian-type taxes.

[19]The equilibrium for the infinite-horizon model is defined in Section C.4.

Parameter	Value	Target
α	0.9480	$q\bar{d}/c = 1.4$
β	0.8811	Real interest rate 4%
γ	0.8000	Empirical LTV ratio
δ	0.4500	Literature
θ	2.0000	Standard value
ρ	0.9895	Diaz and Luengo-Prado (2010)
σ	0.1257	Diaz and Luengo-Prado (2010)
$\pi_{R,S} \times 100$	0.0125	Gini coefficient income
$\pi_{S,R} \times 100$	0.2063	Gini coefficient wealth
$\bar{d}$	0.0724	Relative durable distribution

Table 4.1: Model parameters in chapter 4

unanticipated permanent policy changes and study transition dynamics. For the quantitative analysis, we check (ex post) that, for the considered policy experiments, net wealth is always positive (and default never occurs).

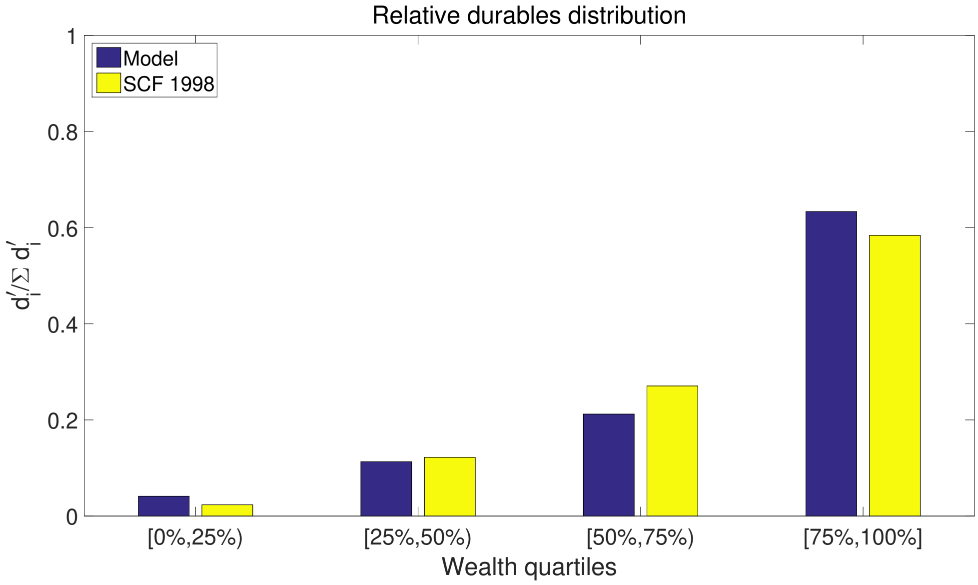
Functional forms and parameters Since the model is solved numerically, functional forms and parameters have to be specified.[20] We calibrate the model by choosing suited parameter values from related studies and by targeting selected statistics of the income, wealth, and durables distribution observed for the United States, similar to Diaz and Luengo-Prado (2010), based on data from the Survey of Consumer Finances (SCF) 2016 for the year 1998. The parameter values are summarized in Table 4.1. In contrast to the latter study we define the empirical counterpart of durable consumption not only as residential housing but add vehicles as well, given that these two categories account for the majority of collateral used for household credit. For the household utility function, we use the specification

$$u(c,d) = \frac{\left[\alpha c^{\delta} + (1-\alpha) d^{\delta}\right]^{\frac{1-\theta}{\delta}}}{1-\theta},$$

where $0 < \theta \neq 1$ is the inverse of the intertemporal elasticity of substitution with respect to a constant-elasticity-of-substitution (CES) consumption aggregate that consists of non-durable and durable consumption, c and d, with $\delta > 0$ controlling the degree of substitution between the two types of goods. For θ, we choose a standard value of two, whereas δ is set to 0.45, which is the average of values used by Benhabib et al. (1991), McGrattan et al. (1997), and Piazzesi et al. (2007), who use the same functional form for the utility function. The values for the utility function parameter α (0.948) and the discount factor β (0.8811) are set to match two empirical targets, namely, the ratio of aggregate durable-to-non-durable-consumption of 1.4 and a real interest rate of 4%. The fraction of seizable collateral γ is set at 0.8, implying an empirically plausible loan-to-value ratio of 80% (see Diaz and Luengo-Prado (2010)).

[20]Details about the numerical solution procedure can be found in Appendix D.4.

Figure 4.1: Relative durable holdings for different wealth quartiles (data vs model)

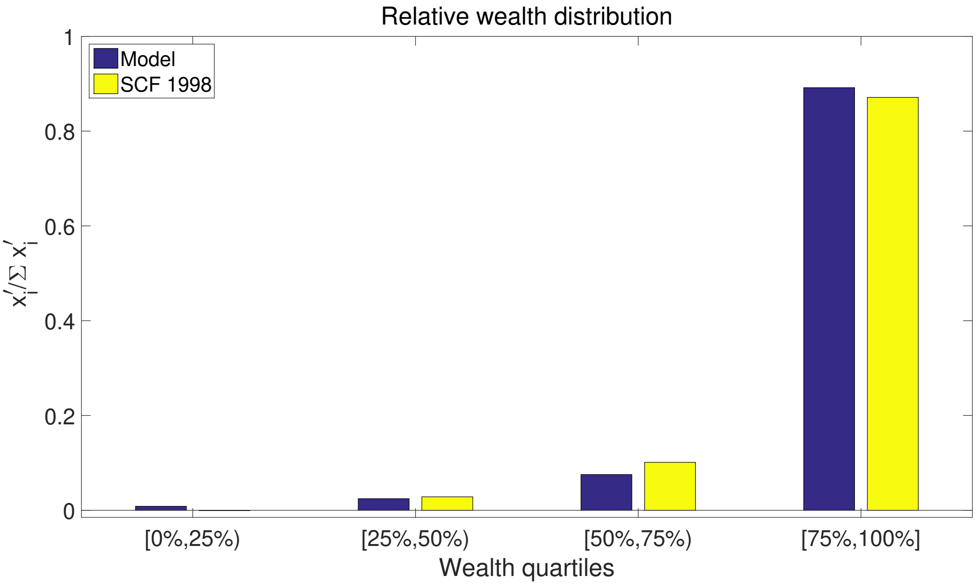


The income support Y and the associated income transition probabilities π are chosen to match the Gini coefficients for income $y_{i,t}$ (0.43) and (net-)wealth $x_{i,t} \equiv b_{i,t} + q_t d_{i,t}$ (0.8). A household's wealth position serves as the single endogenous individual state variable, which can take on finitely many values (see Appendix D.4 for details on the numerical computation). As is well known in the literature (see e.g. Di Nardi et al. (2015)), without additional assumptions, a standard Bewley-Aiyagari-Huggett-type incomplete-markets model fails to match important features of the wealth distribution, the concentration of wealth at the top in particular. To address this shortcoming, we follow Diaz and Luengo-Prado (2010) and assume that individual income follows a log-normal AR(1) process,

$$\ln y_{i,t} = \rho \ln y_{i,t-1} + \sigma \varepsilon_{i,t},$$

with autocorrelation $\rho = 0.9895$ and standard deviation $\sigma = 0.1257$, and that, additionally, there is also a small probability $\pi_{R,S}$ of transitioning to a "superstar" income state, which is left with probability $\pi_{S,R}$. While the AR(1) process provides a good fit for most of the population, it cannot suitably account for the top 1% of the income distribution. The "regular" income states y_1 to y_6 are obtained by discretizing the AR(1) process via the method proposed by Tauchen and Hussey (1991), while the superstar income value y_7 is set to match the empirical ratio $y_7/y_6 = 6$ and the transition probabilities are $\pi_{R,S} = 0.000125$ and $\pi_{S,R} = 0.002063$. Combining these values with the transition probabilities for the regular income states, obtained by discretizing the AR(1) process, yields the transition probabilities $\pi(y_{i,t+1}|y_{i,t})$, which are given in Appendix D.4.1. Lastly, the aggregate supply of the durable good $\bar{d} = 0.0724$ is chosen to provide a reasonable fit for the durable distribution, as given by Figure 4.1. Figure 4.2 shows the distribution of net-wealth for the model and the data.

Figure 4.2: Relative net-wealth for different wealth quartiles (data vs model)



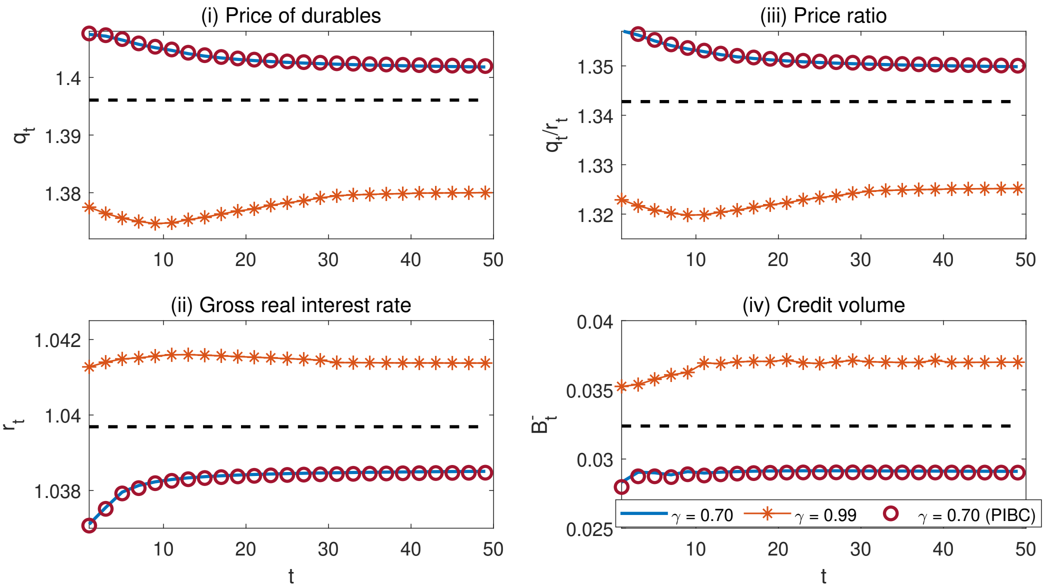
4.3.2 Loan-to-value ratio

In this section, we discuss the effects of permanently and unexpectedly changing the loan-to-value ratio in the economy. For convenience, we abstain from introducing an additional policy instrument that enters the collateral constraint and assume that the policy maker directly reduces γ.[21] Although, as illustrated in Section 4.2.1, type-dependent Pigouvian taxes on borrowing and durables can induce (welfare-improving) corrections of prices, they are unlikely going to be implemented in practice. The loan-to-value ratio by contrast is a policy instrument that is typically considered as an useful instrument to regulate borrowing. In the model, changes in the loan-to-value ratio have two types of effects. First, they directly affect households' decisions to borrow and – as a result – to buy durable goods. Second, the resulting reactions affect equilibrium prices of debt and durables, which in turn lead households to (re-)adjust their behavior. In the remainder, we will refer to effects of the first kind as “direct effects" and those of the second kind as “indirect effects". The direct effects work mechanically and as expected, such that it will be particularly important to understand the indirect effects. To do so, we will look at the responses of equilibrium prices as well as of the aggregate credit volume in the short run and in the long run following an unexpected change of γ. Figure 4.3 visualizes the results by plotting the transition path for prices, their ratio and the credit volume, which we denote as B_t^-.[22]

[21] Equivalently, we could introduce a policy instrument Γ, such that the borrowing constraint (4.1) changes to $-b_{i,t} \leq \Gamma\gamma q_t d_{i,t}$.

[22] Credit volume is calculated by aggregating all negative end-of-period bond positions across agents.

Figure 4.3: Transition paths for prices and credit after an unexpected change of loan-to-value ratio γ



Notes: The panels also show the transition paths for a price-inelastic constraint (PIBC). The dashed black lines denote the respective laissez-faire steady-state values.

Consider a reduction of the loan-to-value ratio (from $\gamma = 0.8$) to $\gamma = 0.7$, which tightens the collateral constraint for all households. Ceteris paribus, this change directly reduces the borrowing limit of constrained households and low-income/low-wealth borrowers who have previously been unconstrained become constrained as well. As a direct effect, these types of households respond by reducing their debt as well as their holdings of durables since debt-financing becomes more restricted. These direct effects suggest that the credit volume, the interest rate, and the price of durables fall. Yet, we find that the durables price q_t instead increases, as shown by the solid blue line in Panel (i) of Figure 4.3 (which almost coincides with the magenta-colored circles displaying a reference case, see below). The effect on the debt market is in line with the intuition suggested by the direct effects. Credit volume declines and the interest rate falls and settles at a higher value that is nevertheless lower compared to the old steady state. Furthermore, the price ratio q_t/r_t increases when γ declines (see Panel (iii) of Figure 4.3).

Why does the price of durables increase? The overall response of q_t particularly depends on how savers and richer unconstrained borrowers respond. These types of households are not directly affected by the tightening of the collateral constraint and also unlikely going to be in the near future. Their response will therefore mainly reflect how the prices of durables and bonds change when borrowers de-leverage. The lower real interest rate makes investing in bonds less attractive for these households who increase their holdings of durables. These responses create upward pressure on both prices, q_t and $1/r_t$, with the price of durables ultimately experiencing an increase. Although a higher value for q_t counteracts the lower loan-to-value ratio, it is not sufficient for γq_t to go up and to relax household borrowing constraints. For demonstrative purposes, we also consider a higher loan-to-value ratio $\gamma = 0.99$. Reverting the direction of the change in γ (marked orange line) leaves the propagation mechanism unchanged and switches the sign of the effects.

We further examine an artificial reference case, which allows abstracting from the collateral effects of pecuniary externalities. Specifically, we consider a price-inelastic borrowing constraint (PIBC), for which we hold the durables price in the collateral constraint fixed at the laissez-faire equilibrium level. The magenta-colored circles in Figure 4.3 show that interest rate effects as well as the impact on the durables price during the transition phase are almost identical under the price-inelastic borrowing constraint. Panel (iv) in Figure 4.3 further shows that there is only a tiny difference between both cases with respect to the credit volume, indicating that the results are hardly affected by price-induced changes in the collateral constraint.

Welfare implications What do the effects of the policy experiments imply for welfare? To assess the welfare effects, taking into account the transition to the new steady state, we consider three measures. The first one is the welfare-equivalent consumption bundle variation $CEV_i \equiv CEV(x_i, y_i)$ with

$$CEV_i = \left[\frac{\tilde{V}_1(x_{i,t}, y_{i,t})}{V(x_{i,t}, y_{i,t})}\right]^{\frac{1}{1-\theta}} - 1,$$

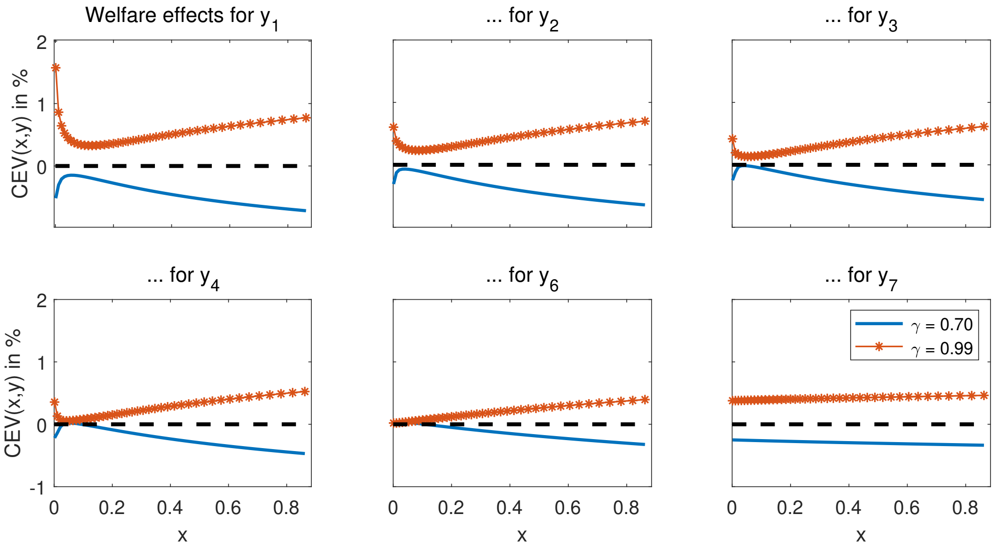
where $V(\cdot)$ denotes the value of a household with beginning-of-period wealth $x_{i,t} = b_{i,t-1} + q_t d_{i,t-1}$ and income $y_{i,t}$ in the laissez-faire economy, which satisfies

$$V(x_{i,t}, y_{i,t}) = \max_{b_{i,t}, c_{i,t}, d_{i,t}} \left\{ u(c_{i,t}, d_{i,t}) + \beta \sum_{y_{i,t+1} \in Y} \pi\left(y_{i,t+1} | y_{i,t}\right) V(b_{i,t} + q d_{i,t}, y_{i,t+1}) \right\} \quad \text{s.t. (4.1) and (4.3)},$$

and $\tilde{V}_1(\cdot)$ denotes the corresponding value of a household in the impact period $t = 1$ of an economy that is experiencing a policy change. The value $\tilde{V}_t(\cdot)$ carries a time index because the prices q_t and r_t change during the transition period and are therefore a function of time in this case. By contrast, prices in the stationary laissez-faire economy are constant in all periods and the associated individual household values only changes over time via the individual states. The welfare measure CEV_i allows assessing how welfare of individual types of households changes after the policy intervention, where a positive (negative) value for CEV_i means that a household is better (worse) off.

We further want to examine policy effects on social welfare. Pareto improvements are hardly possible under dispersed agents' endowments of wealth and income (see Davila et al. (2012)). Thus, we consider ex-ante expected lifetime utility (see e.g. Conesa et al. (2009), Krueger et al. (2016b), or Nuño and Moll

Figure 4.4: Welfare effects conditional on income and wealth type (change in LTV)



(2018)) and compute variations of this measure expressed in equivalent consumption bundle units

$$CEV = \left[\frac{\sum_{x_{i,t},y_{i,t}} \tilde{\lambda}(x_{i,t}, y_{i,t}) \tilde{V}_1(x_{i,t}, y_{i,t})}{\sum_{x_{i,t},y_{i,t}} \lambda(x_{i,t}, y_{i,t}) V(x_{i,t}, y_{i,t})} \right]^{\frac{1}{1-\theta}} - 1,$$

where $\lambda(x_{i,t}, y_{i,t})$ denotes the (unconditional) probability of individual state $(x_{i,t}, y_{i,t})$ in the laissez-faire economy and $\tilde{\lambda}(x_{i,t}, y_{i,t})$ the corresponding probability in the period of the policy change.[23] This welfare criterion, which is identical to a utilitarian welfare measure, can be interpreted as measuring whether an unborn household, who is randomly assigned to an idiosyncratic state, would prefer to be born into the laissez-faire economy or into an economy that experiences a sudden policy change. In contrast to the state-dependent and household-specific measure CEV_i, CEV can be used to assess the overall welfare implications for the economy.

The final welfare measure that we use is the income-specific measure CEV_{y_i} which is defined as

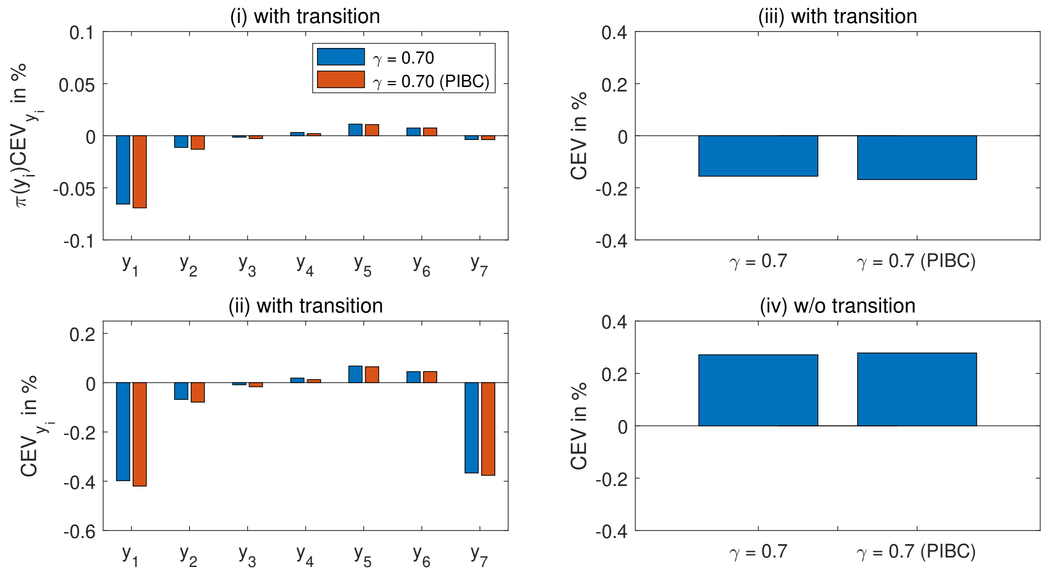
$$CEV_{y_i} = \left[\frac{\sum_{x_{i,t}} \tilde{\lambda}_{y_i}(x_{i,t}) \tilde{V}_1(x_{i,t}, y_i)}{\sum_{x_{i,t}} \lambda_{y_i}(x_{i,t}) V(x_{i,t}, y_i)} \right]^{\frac{1}{1-\theta}} - 1,$$

where $\lambda_{y_i}(x_{i,t})$ and $\tilde{\lambda}_{y_i}(x_{i,t})$ denote the probability of state $x_{i,t}$ for the laissez-faire economy and the economy subject to a policy change, respectively, given income y_i. This welfare measure is a refinement of CEV that takes into account the distribution of wealth conditional on income y_i, which will help to shed light on the source of aggregate social welfare changes (see Krueger et al. (2016b)).

[23]While individual holdings of bonds and durables are predetermined, wealth $x_{i,t} = b_{i,t} + q_t d_{i,t}$ also depends on the price q_t, which can shift the wealth distribution on impact.

Figure 4.4, which displays CEV_i for different income and wealth levels, shows that almost all types of households are worse off after a decrease in γ (see solid blue lines). Constrained borrowers can borrow less, whereas savers suffer from lower interest rates. The only types of households who gain under a lower γ-value are income-rich unconstrained borrowers who are not directly affected by the tighter collateral constraint but benefit from the lower interest rate. These unconstrained households reduce their borrowing (see Figure 4.14 in Appendix E.4), which contributes to the lower interest rate. Hence, these agents would have benefited under laissez faire if they internalized the price effects of their behavior. These types of households, however, constitute a tiny fraction of households in the economy.

Figure 4.5: Transition paths for prices and credit after an unexpected change of loan-to-value ratio γ



Notes: Panels (i) and (ii) display welfare conditional on the income state. In Panel (i) the welfare effects are weighted with the probability mass of the respective income states. Panels (iii) and (iv) display aggregate welfare with and without taking into account the transition periods. The case of a price-inelastic collateral constraint is denoted as "PIBC".

Panel (i) of Figure 4.5 depicts welfare conditional on specific income states and weighted with the respective probability, i.e. $\pi(y_i)CEV_{y_i}$. The unweighted welfare measure CEV_{y_i} is displayed in Panel (ii). These figures reveal that the welfare losses clearly dominate. Consistently, social welfare effects as measured by CEV are negative for $\gamma = 0.7$ (see Panel (iii)). When the transition is however not taken into account, these results are reversed, since the increase (decrease) in the price of durables due to the LTV reduction (increase) leads to a general upward shift in wealth (see Panel (iv)).[24] Overall, Figure 4.5 shows that the welfare losses of a LTV reduction under a price-inelastic borrowing constraint (PIBC) are slightly larger for low income groups and that social welfare effects hardly differ under a price-inelastic

[24] In this case, the computed welfare measure is $CEV = \left[\sum_{s_{i,t}} \bar{\lambda}(s_{i,t})\bar{V}(s_{i,t}) / \sum_{s_{i,t}} \lambda(s_{i,t})V(s_{i,t})\right]^{\frac{1}{1-\theta}}$, with $s_{i,t} \equiv (x_{i,t}, y_{i,t})$. The bars denote that the probability measure and the value function are associated with the new long-run equilibrium.

borrowing constraint. According to this experiment, we do not find evidence for substantial collateral effects.

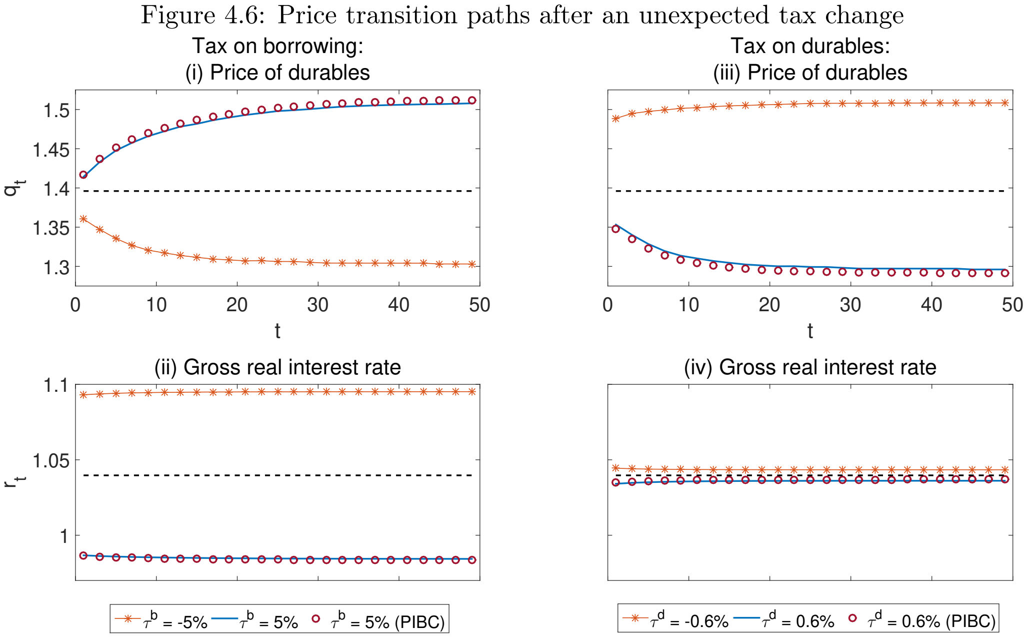
4.3.3 Corrective taxes

This section examines the effects of corrective tax policies. These anonymous taxes affect prices by altering the marginal valuation of goods and assets, while payments or receipts of funds are individually compensated in a lump-sum way. Thereby, these Pigouvian-type tax policies do not directly redistribute resources across households and – like the loan-to-value ratio – affect the economy via price effects that can address pecuniary externalities. We first consider a symmetric tax on debt $-b$ at the rate τ_b, which implies a tax on borrowing and a subsidy on savings. We focus on local effects in the neighborhood of the laissez faire equilibrium induced by $\tau^b > 0$ and $\tau^b < 0$; the latter implying a subsidy on borrowing and a savings tax. As a second Pigouvian-type tax policy, we introduce an anonymous tax on end-of-period holdings of durables τ^d. To facilitate comparisons between both Pigouvian-type tax policies, the values for τ^d are chosen to yield equally-sized changes in the long-run durable price q_t.

Debt-tax/Saving-subsidy

First, suppose that the tax on durables is kept at zero ($\tau^d = 0$) and a tax on debt, which implies a subsidy on savings, is unexpectedly and permanently imposed ($\tau^b = 0.05$).

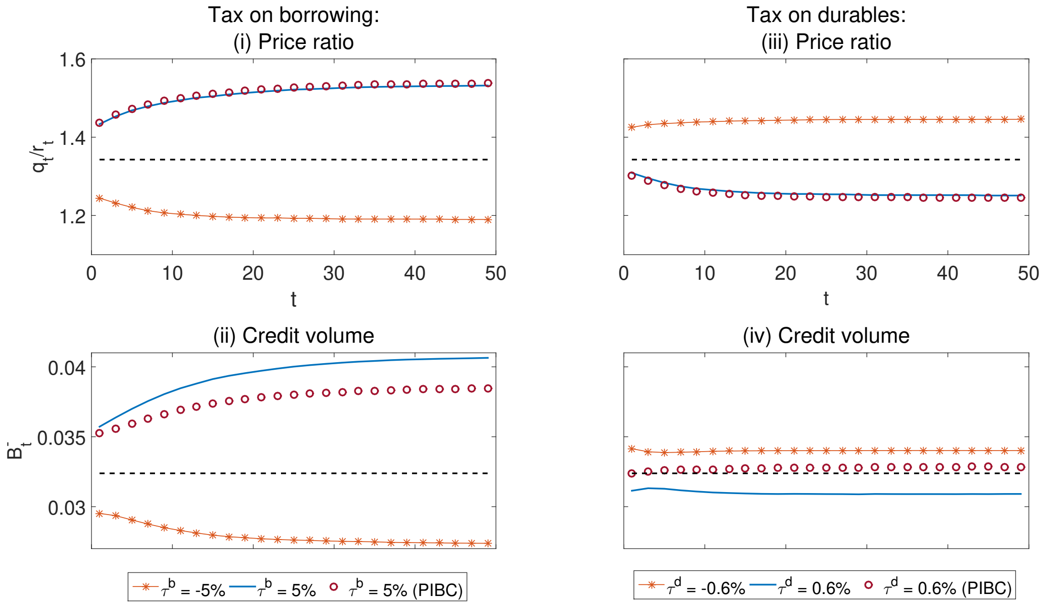
Figure 4.6: Price transition paths after an unexpected tax change



Notes: The panels also show the transition paths for a price-inelastic constraint (PIBC). The dashed black lines denote the respective laissez-faire steady-state values.

To illustrate the policy effects, we examine time paths of prices and of the credit volume in the left hand columns of Figures 4.6 and 4.7. Relative to the laissez-faire steady-state values (dashed black lines), the durables price q_t and the interest rate r_t, given by the solid blue lines in the left hand column of Figure 4.6, move into opposite directions. The price of durables q_t jumps up and then gradually moves up to a higher new steady-state value, while the interest rate r_t immediately drops and further declines until it arrives at the new (lower) long-run value.

Figure 4.7: Transition path for price ratio and credit volume after an unexpected tax change



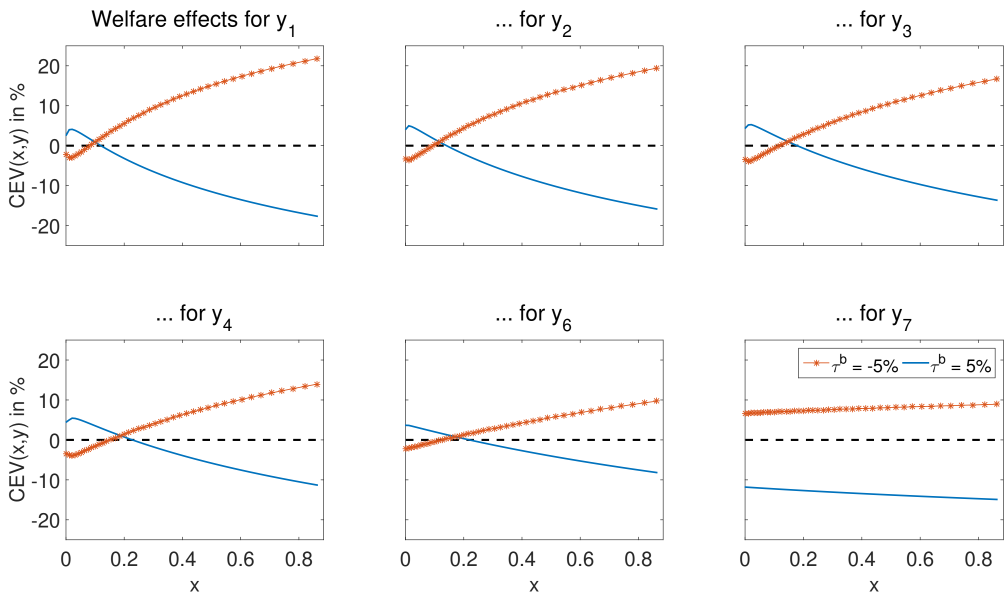
Notes: The panels also show the transition paths for a price-inelastic constraint (PIBC). The dashed black lines denote the respective laissez-faire steady-state values.

What drives these responses? Ceteris paribus, since it is imposed symmetrically, the tax τ^b induces all types households, i.e. borrowers and savers, to save more and to dis-save less, i.e. to choose higher values of $b_{i,t}$. These direct effects in turn create downward pressure on the interest rate r_t to ensure market clearing for debt and simultaneously also upward pressure on the price q_t (see solid blue line in Panels (i) and (ii) of Figure 4.6). The reason for the latter is that durables are untaxed and now hence provide a relatively higher return, which stimulates the demand for durables. These price responses tend to ease borrowing conditions, while the increase in the price ratio q/r raises the amount of funds borrowed per unit of collateral. Consistently, we observe a gradual increase in the aggregate credit volume (see solid blue line in Panel (ii) of Figure 4.7), reflecting that income- and wealth-poor households tend to increase their borrowing. It should be noted that even constrained borrowers issue more debt, even though the policy (tax on debt) tends to make it less attractive for households to borrow. At the same time, savers are however incentivized to save more, which induces the interest rate to decline and the credit volume to increase. Under a subsidy on borrowing ($\tau^b = -0.05$), given by the marked orange lines, prices and the credit volume move into the opposite direction, showing that the mechanism just discussed operates in a symmetric way.

As in Section 4.3.2, we further consider a price-inelastic borrowing constraint (PIBC) where we fix the durables price in the collateral constraint at the steady-state laissez-faire value. In this case, we hardly observe any difference with regard to the responses of q_t and r_t to the introduction of the borrowing tax compared to the benchmark case of a price-elastic borrowing constraint (see magenta circles in Figure 4.6).[25] In contrast to the LTV-reduction, the credit volume response is however substantially affected by keeping the durables price fixed in the collateral constraint: The increase in the collateral price raises the borrowing limit only in the benchmark case, such that the increase in the credit volume is apparently less pronounced under a price-inelastic borrowing constraint (see Panel (ii) of Figure 4.7).

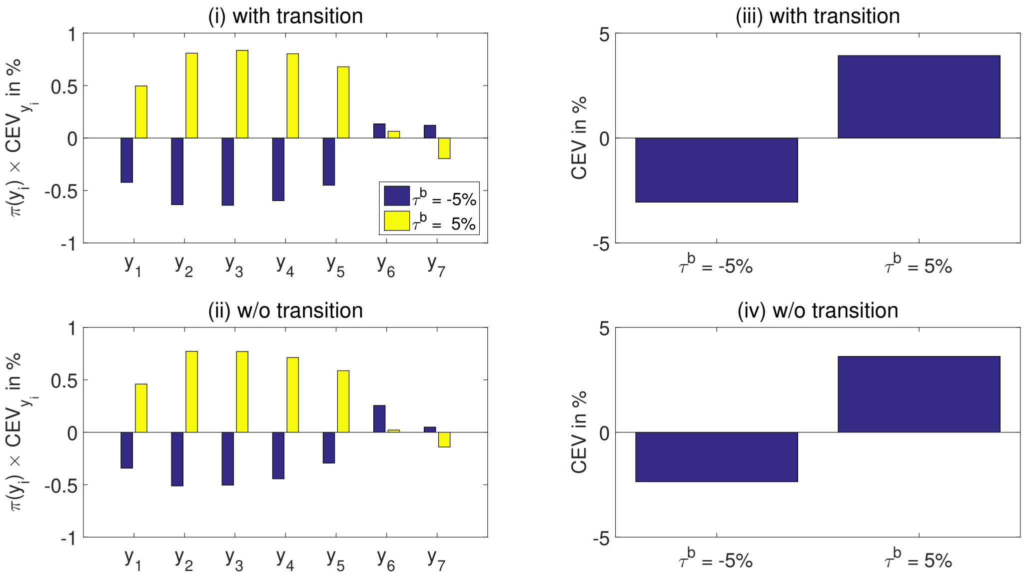
Welfare implications The welfare effects of a tax on debt are visualized in Figures 4.8 and 4.9. As shown by the solid blue lines in Figure 4.8, wealth-poor households in all income groups (except the highest) gain from a debt-tax/saving-subsidy ($\tau^b > 0$). Both price effects of the debt-tax/saving-subsidy, i.e. a higher price of durables and lower interest rates, are beneficial for borrowers. The tax on debt thus induces price responses that serve as partial insurance for borrowers from an ex-ante perspective. While the price effects are qualitatively identical to the effects of a LTV reduction, they here are associated with a higher credit volume induced by agents' increased willingness to lend.

Figure 4.8: Welfare effects conditional on income and wealth type of tax/subsidy on borrowing



[25] The debt-subsidy/saving-tax has the same qualitative implications with the responses having the opposite sign.

Figure 4.9: Welfare effects of a tax/subsidy on borrowing



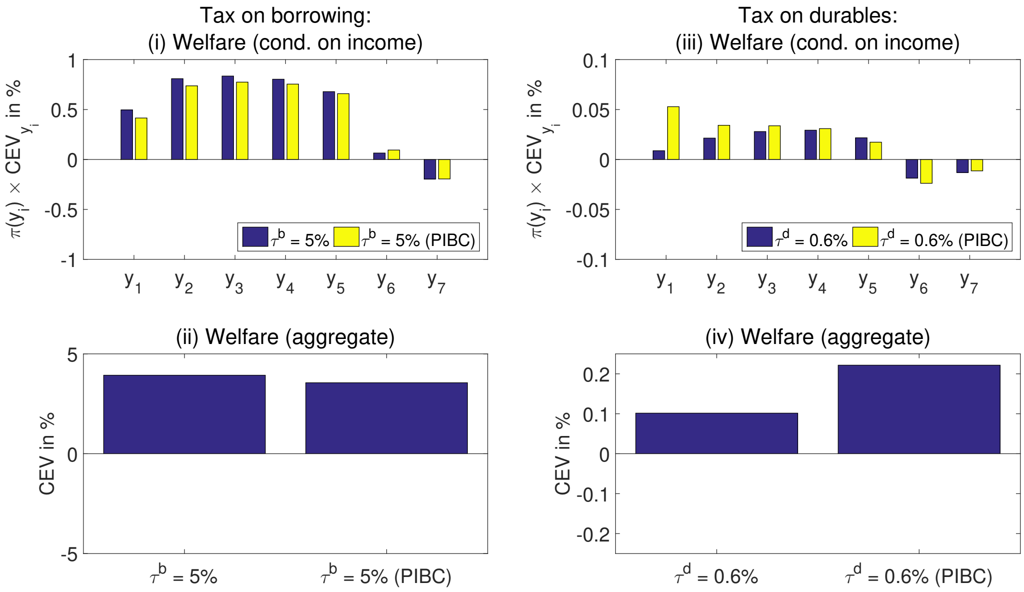
Notes: Panels (i) and (ii) display welfare conditional on the income state, with and without taking into account the transition periods Panels (iii) and (iv) display aggregate welfare with and without transition periods.

Notably, wealth-rich agents tend to lose, as they earn a lower interest rate on their savings. To understand the welfare effects displayed by the yellow bars in Panels (i) and (ii) of Figure 4.9 (compared to Figure 4.8), one has to further take into account that endogenous shifts in the durables price q_t lead to a change in agents wealth $x_{i,t} = b_{i,t-1} + q_t d_{i,t-1}$ and does so the more a household owns durables $d_{i,t-1}$. When aggregating within income groups, which takes changes of the wealth distribution into account, one can see that all income groups except the highest benefit from a tax on debt, which reflects that this group mostly consists of lenders. Panels (iii) and (iv) of Figure 4.9, which display the social welfare gains/losses with and without transition, reveal that the welfare gains from the debt-tax/saving-subsidy ($\tau^b = 0.05$) are due to positive short-run and long-run effects. Furthermore, the welfare effects hardly change under a price-inelastic borrowing constraint, indicating a negligible role of collateral effects for welfare, both within income groups as well as the aggregate (see Panels (i) and (ii) of Figure 4.10).

Durables tax

Now consider an unexpected permanent increase in the tax (subsidy) on end-of-period holdings of durables by $\tau^d = 0.006$, which is compensated by lump-sum transfers (taxes). The tax on debt is set at zero ($\tau^b = 0$). The size of the Pigouvian-type policy intervention in the market for durables is chosen to yield a change in the long-run price of durables that is of the same (long-run) magnitude as under debt market interventions, $-\tau^b \neq 0$. The associated price and credit responses are given by the solid blue lines in the Panels (iii) and (iv) of Figures 4.6 and 4.7. Ceteris paribus, the taxation of durables causes households to

Figure 4.10: Welfare effects of a Pigouvian-type tax with and without price-inelastic borrowing constraint

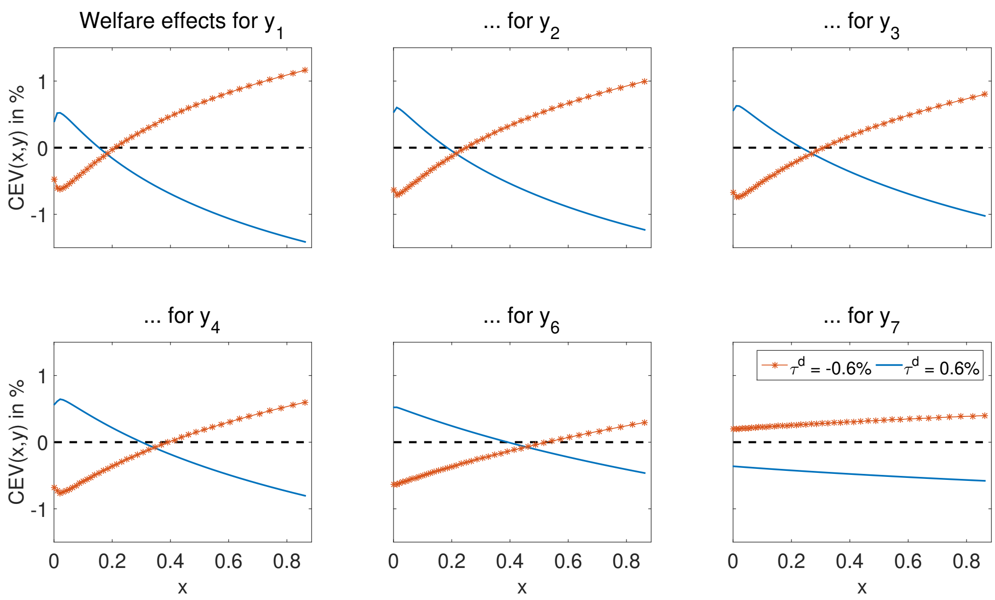


substitute durable goods in favor of non-durable goods. Furthermore, agents who are willing to transfer wealth intertemporally tend to substitute durables in favor of bonds, such that credit supply increases. These direct effects imply that q_t and r_t fall to clear markets, which is shown in the Panels (iii) and (iv) of Figure 4.6. While the taxes on durables and debt both lower the real interest rate, the responses of the aggregate credit volume substantially differ. A main difference is that the price ratio q/r decreases under the durables tax, which reduces the maximum amount of funds that can borrowed against collateral, whereas the price ratio increases under the debt tax (see solid blue lines in Panels (i) and (iii) of Figure 4.7). Correspondingly, the credit volume increases under the debt tax and decreases under the durables tax, as shown in the Panels (ii) and (iv) of Figure 4.7.

Like under the debt-tax/savings-subsidy, the responses of the durables price q_t and the interest rate r_t to a durables tax introduction hardly change when we consider a price-inelastic borrowing constraint (see magenta circles in Panels (i) and (ii) of Figure 4.6). In contrast, the credit volume responses show remarkable differences: The durables tax leads to a decline in the interest rate, which tends to stimulate borrowing, and to a decline in the durables price, which tends to reduce the borrowing limit under the benchmark (price-elastic) collateral constraint. Under a price-inelastic borrowing constraint, however, a decline in the durables price does not have a direct impact on the borrowing constraint. As a result, the credit volume increases relative to the laissez-faire case due to the decline in the interest rate, whereas it decreases under the price-elastic borrowing constraint (see Panel (iv) of Figure 4.7).

Since we considered values for the durables tax τ^d that induce long-run changes in q_t that are of the

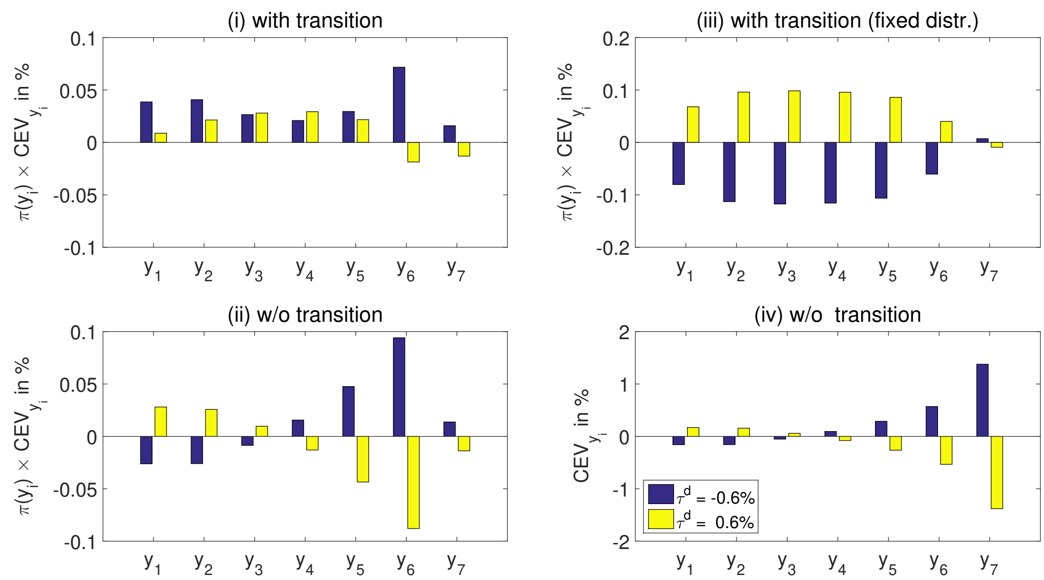
Figure 4.11: Welfare effects conditional on income and wealth type of tax/subsidy on durables



same magnitude as those associated with a debt tax $-\tau^b$, one can compare the response of the real interest rate under the two tax instruments. Figure 4.6 shows that the interest rate adjustment is much more pronounced under the borrowing tax τ^b, which directly affects the market for debt. More specifically, a tax on debt of 5% lowers the long-run real interest rate by 5.5 percentage points, while a subsidy on durables of 0.6%, which results in the same long-run price of durables, yields an interest rate increase of 0.4 percentage points.

Welfare implications Who benefits from a tax on durables? As shown by the solid blue lines in Figure 4.11, low-wealth households, which are typically borrowers, benefit from the lower interest rate in almost all income groups. Even constrained households gain despite the drop in the price of durables, which tends to tighten their borrowing constraints. Like tax on debt, the durables tax leads to responses of the interest rate that partially insure borrowers from a social perspective. Welfare declines for the highest income groups (see yellow bars in the Panels (i) and (ii) of Figure 4.12), while social welfare increases for $\tau^d > 0$ (see Panels (i) and (ii) of Figure 4.13). It should further be noted that the decline in the durables price reduces wealth of agents in terms of non-durables, which shifts the wealth distribution such that the mass of agents who gain from lower interest rates increases. Under a price-inelastic borrowing constraint, there are visibly larger social welfare gains compared to the benchmark case (see Panels (iii) and (iv) of Figure 4.10), which correspond to the qualitative difference in the credit volume response (see Figure 4.7). While this indicates the quantitative relevance of collateral effects, it should be noted that it is the decline in the interest rate which is ultimately responsible for the increase in the credit volume and in

Figure 4.12: Welfare effects of a tax/subsidy on durables conditional on income state



Notes: Panels (i) to (ii) display welfare conditional on the income state. Panel (iii) shows the measure when the transition period is taken into account but the wealth distribution in the impact period is kept fixed. Panel (iv) displays the measure in Panel (i), weighted by the probability mass of the income states.

social welfare.

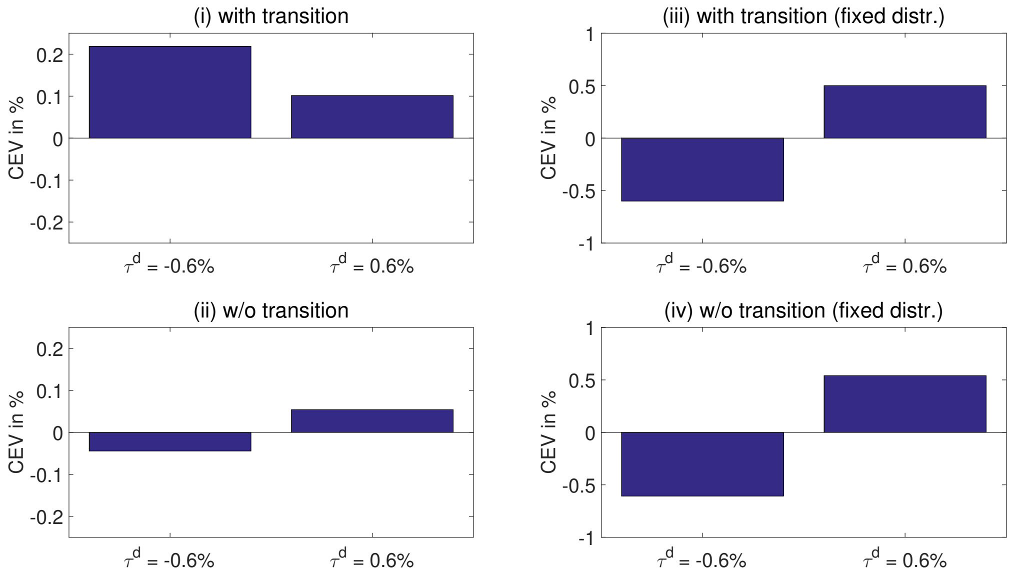
A subsidy on durables, $\tau^d < 0$, reverses the qualitative responses of prices and the credit volume (see marked orange lines in the Panels (iii) and (iv) Figures 4.6 and 4.7). The long-run welfare effects, i.e. those not accounting for the transition dynamics, of a durables subsidy ($\tau^d = -0.006$) are negative and mirror those of the tax on durables with the opposite sign, as shown in the Panel (ii) of Figures 4.12 and 4.13. Specifically, savers tend to gain from the increase in the interest rate and the price of durables compared to the laissez-faire case (see marked orange line in Figure 4.11), where they do not internalize that raising their holdings of durables contributes to the increase in the durables price.[26] However, the overall aggregate welfare effect, which includes the transition phase, is positive (see Panels (i) of Figures 4.12 and 4.13).[27] The durable subsidy leads to an increase in the durables price q_t, leading to a higher wealth level of all agents. This effect due to a higher durables price is particularly more pronounced in the short run, where adjustments of the quantities are not yet made. Computing the welfare measures by fixing the wealth distribution prior to the policy change (see Panel (iii) of Figures 4.12 and 4.13), shows that welfare then declines in (almost) all income groups, such that aggregate welfare CEV declines as well.

It should be noted that the welfare effects of the durables price increase are non-trivial, since it is beneficial for agents who are net-sellers of durables ($d_{i,t} < d_{i,t-1}$), whereas net-buyers ($d_{i,t} > d_{i,t-1}$) suffer

[26] This can be seen from the policy functions shown in Figure 4.15 in Appendix E.4.
[27] Note that CEV does not equal $\sum_{y_i} \pi(y_i) CEV_{y_i}$ by construction.

from the price increase. In the experiment, a durables subsidy thereby tends to benefit poor households more than it hurts rich ones. These effects are particularly larger in the transition phase where agents adjust their holdings of durables more than in a stationary equilibrium.

Figure 4.13: Aggregate welfare effects of a tax/subsidy on durables



Note: Panels (iii) and (iv) use the wealth distribution prior to the policy shock for the welfare calculation.

Note that these effects relate to the distributive effects included in the wedges Δ_1^d and $\Delta_2^{d,\omega}$ that are relevant for the corrective durables tax identified in Section 4.2.2 (see 4.18-4.21). The beneficial effects of higher durables prices even compensate agents that would otherwise lose under the durables subsidy (see marked orange lines in Figure 4.11). As a result, all income groups are better off and social welfare increases for the durables subsidy $\tau^d < 0$ (see Panel (i) of Figures 4.12 and 4.13).[28]

Finally, recall that the subsidy on durables and the tax on debt have been scaled to lead to equally-sized long-run changes in the durable prices. Figure 4.13 however shows that the implied aggregate welfare effects are more than 20-times larger under the debt market intervention. As revealed in Figure 4.6, this difference can mainly be attributed to the change in the interest rate, which is more pronounced under the debt tax. This observation thus implies that interest rate responses are even more relevant for the overall welfare effects of corrective policies than changes in the collateral price.

4.4 Conclusion

Pecuniary externalities with regard to the price of collateral can justify financial regulation.

[28]Within income groups, there might however be some losers, such that there is no strict Pareto improvement.

This paper examines financial regulation and corrective taxes in a prototype incomplete markets economy with limited commitment featuring collateral constraints. We find that a loan-to-value reduction positively affects welfare of only income-rich unconstrained borrowers, whereas social welfare decreases. By contrast, a Pigouvian-type debt-tax/savings-subsidy generates welfare gains for (almost) all income groups due to a higher collateral price and a lower interest rate. Borrowers tend to gain and income-rich lenders tend to lose from the decline in the interest rate, providing a partial insurance for borrowers from an ex-ante perspective. Interventions in the market for durables (collateral) exert ambiguous welfare effects due to price-induced shifts in the wealth distribution. The resulting short-run welfare effects can qualitatively overturn the long-run welfare implications, which tend to be positive (negative) for households with a low (high) income in the case of a tax (subsidy) on durables. Overall, the analysis reveals that interest rate responses and distributive effects rather than changes in the durables price and collateral effects are decisive for the total welfare effect of corrective policies. While the previous literature has found that financial regulation can be beneficial under pecuniary externalities due to financial frictions, our analysis indicates that this principle cannot be generalized to a macroeconomic structure with an endogenous interest rate and a non-trivial distribution of income and wealth.

Appendix

A.4 Competitive equilibrium of the three-period model

The competitive equilibrium can be characterized as follows. Consider the terminal period 3. Conditional upon previous choices for debt $b^{\omega}_{i,2}$ and durables $d^{\omega}_{i,2}$ as well as on the exogenous period-2-state ω and the fixed endowment y, agents $i \in \{b, l\}$ allocate their available resources between durables $d^{\omega}_{i,3}$ and non-durables $c^{\omega}_{i,3}$ according to the first-order conditions (4.8) and the budget constraint in period 3, while the relative price of durables q^{ω}_3 ensures that $d^{\omega}_{b,3} + d^{\omega}_{l,3} = \bar{d}$ holds.

In period 2, agents choose to borrow/lend $b^{\omega}_{i,2}$, to buy/sell durables $d^{\omega}_{i,2} - d_{i,1}$, and to buy non-durables $c^{\omega}_{i,2}$ contingent upon the predetermined states $\{d_{i,1}, b_{i,1}\}$ and the realization of the random variable $\omega \in \{u, e\}$. Like in Davila and Korinek (2018), we define $\bar{d}_1 \equiv \left(\bar{d}_{l,1}, \bar{d}_{b,1}\right)$ and $\bar{b}_1 \equiv \left(\bar{b}_{l,1}, \bar{b}_{b,1}\right)$ to distinguish the states variables of an individual household of type i from the state variables aggregated across all i-type households. The combined problem for periods 2 and 3 of household $i \in \{b, l\}$ can be summarized as

$$\begin{aligned} V_2^{i,\omega}(d_{i,1}, b_{i,1}; \bar{d}_1, \bar{b}_1) &= \max_{c^{\omega}_{i,2}, d^{\omega}_{i,2}, b^{\omega}_{i,2}, c^{\omega}_{i,3}, d^{\omega}_{i,3}} u(c^{\omega}_{i,2}, d^{\omega}_{i,2}) + \beta u(c^{\omega}_{i,3}, d^{\omega}_{i,3}) && (4.22)\\ \text{s.t. } b^{\omega}_{i,1} + y^{\omega}_{i,2} &= c^{\omega}_{i,2} + q^{\omega}_2(d^{\omega}_{i,2} - d_{i,1}) + b^{\omega}_{i,2}/r^{\omega}_2, && (4.23)\\ b^{\omega}_{i,2} + y &= c^{\omega}_{i,3} + q^{\omega}_3(d^{\omega}_{i,3} - d^{\omega}_{i,2}), && (4.24)\\ -b^{\omega}_{i,2} &\leq \gamma q^{\omega}_2 d^{\omega}_{i,2}, && (4.25) \end{aligned}$$

where the prices are functions of $\bar{d}_1$ and $\bar{b}_1$ which leads to the first-order conditions given by (4.6)-(4.8). The latter as well as (4.23) and (4.24) all holding for $i \in \{b, l\}$ and $\omega \in \{e, u\}$, $d^{\omega}_{b,t} + d^{\omega}_{l,t} = d$ for $\omega \in \{e, u\}$ and $t \in \{2, 3\}$, $b^{\omega}_{b,2} + b^{\omega}_{l,2} = 0$ for $\omega \in \{e, u\}$, $\mu^{\omega}_{l,2} = 0$ for $\omega \in \{e, u\}$, $\mu^{e}_{b,2} = 0$ and $-b^{u}_{b,2} = \gamma q^{u}_2 d^{u}_{b,2}$ determine the equilibrium solution for the household choices $\{c^{\omega}_{i,2}, d^{\omega}_{i,2}, b^{\omega}_{i,2}, c^{\omega}_{i,3}, d^{\omega}_{i,3}\}$ for $i \in \{b, l\}$ and $\omega \in \{e, u\}$, prices $\{q^{\omega}_2, r^{\omega}_2, q^{\omega}_3\}$ for $\omega \in \{e, u\}$, and the multipliers $\{\mu^{\omega}_{b,2},\ \mu^{\omega}_{l,2}\}$ for $\omega \in \{e, u\}$, given $\{d_{i,1}, b_{i,1}\}$ for $i \in \{b, l\}$.

Then, a household i's problem in period 1 can be summarized as

$$\begin{aligned} &\max{}_{c_{i,1}, d_{i,1}, b_{i,1}} u(c_{i,1}, d_{i,1}) + \beta E_1[V_2^{i,\omega}(d_{i,1}, b_{i,1}; \bar{d}_1, \bar{b}_1)] \\ &\text{s.t. } c_{i,1} + q_1(d_{i,1} - d_{i,0}) + b_{i,1}/r_1 = b_{i,0} + y_{i,1}, \\ &\quad - b_{i,1} \leq \gamma q_1 d_{i,1}, \end{aligned}$$

leading to the first-order conditions (4.4) and (4.5). The latter for $i \in \{b, l\}$, the budget constraint (4.3) for $t = 1$, for $i \in \{b, l\}$, $-b_{b,1} \leq \gamma q_1 d_{b,1}$ and $\mu_{b,1}(b_{b,1} + \gamma q_1^{\varepsilon} d_{b,1}) = 0$, $\mu_{l,1} = 0$, $d_{b,t} + d_{l,t} = \bar{d}$ and $b_{b,1} + b_{l,1} = 0$, then determine the equilibrium solution for the allocation $\{c_{i,1}, d_{i,1}, b_{i,1}\}$ for $i \in \{b, l\}$, prices $\{q_1, r_1\}$ and multipliers $\{\mu_{b,1}, \mu_{l,1}\}$, given initial values $b_{i,0} = 0$ and $d_{i,0} > 0$.

B.4 Social planner problem in the three-period model

In this part of the appendix, we set up the social planner problem and calculate the first-order conditions. The social planner problem is given as

$$\begin{aligned}
\mathcal{L} \equiv & \sum_i \theta_i \left\{ u_1(\bar{c}_{i,1}, \bar{d}_{i,1}) + \beta E_1 \left[V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1) \right] \right\} \\
& + \theta_b \lambda_{b,1}^{bud} \left[y_{b,1} - \bar{c}_{b,1} - q_1(\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} r_1^{-1} \right] \\
& + \theta_l \lambda_{l,1}^{bud} \left[y_{l,1} - \bar{c}_{l,1} - q_1(\bar{d}_{l,1} - \bar{d}_{l,0}) - \bar{b}_{l,1} r_1^{-1} \right] \\
& + \theta_b \mu_{b,1} \left[\bar{b}_{b,1} + \gamma q_1 \bar{d}_{b,1} \right],
\end{aligned}$$

where we use the market clearing conditions for debt, $\bar{b}_{l,1} = -\bar{b}_{b,1}$, and durables, $\bar{d} = \bar{d}_{l,1} + \bar{d}_{b,1}$. The social planner takes into account that prices r_1 and q_1 are functions of the allocation and implicitly given by

$$\begin{aligned}
r_1^{-1}(\bar{d}_1, \bar{b}_1, \bar{c}_{l,1}) &= \beta E_1 \left[u_c'(\bar{c}_{l,2}^{\omega}, \bar{d}_{l,2}^{\omega}) \right] / u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}), \\
q_1(\bar{d}_1, \bar{b}_1, \bar{c}_{l,1}) &= \left(u_d'(\bar{c}_{l,1}, \bar{d}_{l,1}) + \beta E_1 \left[u_c'(\bar{c}_{l,2}^{\omega}, \bar{d}_{l,2}^{\omega}) q_2^{\omega} \right] \right) / u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}).
\end{aligned}$$

The first-order conditions of the social planner with respect to $\bar{c}_{b,1}$, $\bar{c}_{l,1}$, $\bar{d}_{b,1}$ and $\bar{b}_{b,1}$ are given by the equations

$$0 = \theta_b u_c'(\bar{c}_{b,1}, \bar{d}_{b,1}) - \theta_b \lambda_{b,1}^{bud}, \tag{4.26}$$

$$0 = \theta_l u_c'(\bar{c}_{l,1}, \bar{d}_{l,1}) + \theta_b \lambda_{b,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{c}_{l,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}} \right) \tag{4.27}$$

$$\begin{aligned}
& + \theta_l \lambda_{l,1}^{bud} \left(-1 - \frac{\partial q_1}{\partial \bar{c}_{l,1}} (\bar{d}_{l,1} - \bar{d}_{l,0}) - \bar{b}_{l,1} \frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}} \right) \\
& + \theta_b \mu_{b,1} \gamma \frac{\partial q_1}{\partial \bar{c}_{l,1}} \bar{d}_{b,1},
\end{aligned} \tag{4.28}$$

as well as

$$
\begin{aligned}
0 &= \theta_b u_d'(\bar{c}_{b,1}, \bar{d}_{b,1}) - \theta_l u_d'(\bar{c}_{l,1}, \bar{d}_{l,1}) + \beta E_1 \left[\theta_b \frac{\partial V_2^{b,\omega}}{\partial d_{b,1}} \right] - \beta E_1 \left[\theta_l \frac{\partial V_2^{l,\omega}}{\partial d_{l,1}} \right] \\
&+ \beta \left(\theta_b E_1 \left[\frac{\partial V^{b,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{b,1}} \right] + \theta_l E_1 \left[\frac{\partial V^{l,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{b,1}} \right] \right) - \beta \left(\theta_b E_1 \left[\frac{\partial V^{b,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{l,1}} \right] + \theta_l E \left[\frac{\partial V^{l,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{l,1}} \right] \right) \\
&+ \theta_b \lambda_{b,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{d}_{b,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - q_1 - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right) + \theta_l \lambda_{l,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{d}_{b,1}} (\bar{d}_{l,1} - \bar{d}_{l,0}) - \bar{b}_{l,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right) \\
&- \theta_b \lambda_{b,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{d}_{l,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{l,1}} \right) - \theta_l \lambda_{l,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{d}_{l,1}} (\bar{d}_{l,1} - \bar{d}_{l,0}) - q_1 - \bar{b}_{l,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right) \\
&+ \theta_b \mu_{b,1} \gamma \left(\frac{\partial q_1}{\partial \bar{d}_{b,1}} \bar{d}_{b,1} + q_1 \right) + \theta_b \mu_{b,1} \gamma \left(-\frac{\partial q_1}{\partial \bar{d}_{l,1}} \bar{d}_{b,1} \right),
\end{aligned}
\tag{4.29}
$$

and

$$
\begin{aligned}
0 &= \beta E_1 \left[\theta_b \frac{\partial V_2^{b,\omega}}{\partial b_{b,1}} \right] - \beta E_1 \left[\theta_l \frac{\partial V_2^{l,\omega}}{\partial b_{l,1}} \right] + \beta \left(\theta_b E_1 \left[\frac{\partial V^{b,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{b,1}} \right] + \theta_l E_1 \left[\frac{\partial V^{l,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{b,1}} \right] \right) \\
&- \beta \left(\theta_b E \left[\frac{\partial V^{b,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{l,1}} \right] + \theta_l E_1 \left[\frac{\partial V^{l,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{l,1}} \right] \right) + \theta_b \lambda_{b,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{b}_{b,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - r_1^{-1} - b_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{b}_{b,1}} \right) \\
&+ \theta_l \lambda_{l,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{b}_{b,1}} (\bar{d}_{l,1} - \bar{d}_{l,0}) - b_{l,1} \frac{\partial r_1^{-1}}{\partial \bar{b}_{b,1}} \right) - \theta_b \lambda_{b,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{b}_{l,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - b_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{b}_{l,1}} \right) \\
&- \theta_l \lambda_{l,1}^{bud} \left(-\frac{\partial q_1}{\partial \bar{b}_{l,1}} (\bar{d}_{l,1} - \bar{d}_{l,0}) - r_1^{-1} - b_{l,1} \frac{\partial r_1^{-1}}{\partial \bar{b}_{l,1}} \right) + \theta_b \mu_{b,1} \left(1 + \gamma \frac{\partial q_1}{\partial \bar{b}_{b,1}} \bar{d}_{b,1} \right) + \theta_b \mu_{b,1} \left(-\gamma \frac{\partial q_1}{\partial \bar{b}_{l,1}} \bar{d}_{b,1} \right).
\end{aligned}
\tag{4.30}
$$

First, consider the first-order condition for durables (4.29). Note that $\partial V^{i,\omega}/\partial d_{i,1} = \lambda_{i,2}^{bud,\omega} q_2^{\omega}$. Since $-\theta_l u_d'(\bar{c}_{l,1}, \bar{d}_{l,1}) - \beta E_1 \left[\theta_l \lambda_{l,2}^{bud,\omega} q_2^{\omega} \right] - \theta_l \lambda_{l,1}^{bud} q_1 = 0$ holds because of the lender's first-order condition (see L1), the social planner's first-order condition for durables (4.29) can be written as

$$
\begin{aligned}
0 &= \theta_b u_d'(\bar{c}_{b,1}, \bar{d}_{b,1}) + \beta E_1 \left[\theta_b \lambda_{b,2}^{bud,\omega} q_2^{\omega} \right] - q_1 \theta_b \lambda_{b,1}^{bud} + \theta_b \mu_{b,1} \gamma q_1 \\
&+ \beta \left(\theta_b E_1 \left[\frac{\partial V^{b,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{b,1}} \right] + \theta_l E_1 \left[\frac{\partial V^{l,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{b,1}} \right] \right) - \beta \left(\theta_b E \left[\frac{\partial V^{b,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{l,1}} \right] + \theta_l E_1 \left[\frac{\partial V^{l,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{l,1}} \right] \right) \\
&+ \left(\theta_b \lambda_{b,1}^{bud} - \theta_l \lambda_{l,1}^{bud} \right) \left(-\frac{\partial q_1}{\partial \bar{d}_{b,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right) \\
&+ \theta_b \mu_{b,1} \gamma \left(\frac{\partial q_1}{\partial \bar{d}_{b,1}} - \frac{\partial q_1}{\partial \bar{d}_{l,1}} \right) \bar{d}_{b,1} - \left(\theta_b \lambda_{b,1}^{bud} - \theta_l \lambda_{l,1}^{bud} \right) \left(-\frac{\partial q_1}{\partial \bar{d}_{l,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{l,1}} \right),
\end{aligned}
\tag{4.31}
$$

where we used the market clearing conditions $\bar{d}_{l,t} = \bar{d} - \bar{d}_{b,t}$ and $\bar{b}_{l,t} = -\bar{b}_{b,t}$ for $t = 1$. Define

$$
\begin{aligned}
\theta_b \Delta_1^d &= \left(\theta_b \lambda_{b,1}^{bud} - \theta_l \lambda_{l,1}^{bud} \right) \left(-\frac{\partial q_1}{\partial \bar{d}_{b,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{b,1}} \right) + \theta_b \mu_{b,1} \gamma \frac{\partial q_1}{\partial \bar{d}_{b,1}} \bar{d}_{b,1} \\
&- \left(\theta_b \lambda_{b,1}^{bud} - \theta_l \lambda_{l,1}^{bud} \right) \left(-\frac{\partial q_1}{\partial \bar{d}_{l,1}} (\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1} \frac{\partial r_1^{-1}}{\partial \bar{d}_{l,1}} \right) - \theta_b \mu_{b,1} \gamma \frac{\partial q_1}{\partial \bar{d}_{l,1}} \bar{d}_{b,1},
\end{aligned}
\tag{4.32}
$$

and

$$D^i_{d_{j,1}} = -\frac{\partial q_1}{\partial \bar{d}_{j,1}}(\bar{d}_{i,1} - \bar{d}_{i,0}) - \frac{\partial r_1^{-1}}{\partial \bar{d}_{j,1}}\bar{b}_{i,1}, \quad \text{and} \quad C^i_{d_{j,1}} = \gamma \bar{d}^1_{i,1}\frac{\partial q_1}{\partial \bar{d}_{j,1}},$$

for $i, j \in \{b, l\}$, to get

$$\Delta^d_1 = \left(\lambda^{bud}_{b,1} - \frac{\theta_l}{\theta_b}\lambda^{bud}_{l,1}\right)\left(D^b_{d_{b,1}} - D^b_{d_{l,1}}\right) + \mu_{b,1}\left(C^b_{d_{b,1}} - C^b_{d_{l,1}}\right). \tag{4.33}$$

Since the planner respects the lenders' first-order conditions in the first period (see L1), the shadow prices in the first period are asymmetric and given by (see 4.27 and 4.26)

$$\begin{aligned}
\lambda^{bud}_{b,1} &= u'_c(\bar{c}_{b,1}, \bar{d}_{b,1}) \qquad (4.34)\\
\lambda^{bud}_{l,1} &= u'_c(\bar{c}_{l,1}, \bar{d}_{l,1}) + \frac{\theta_b}{\theta_l}\lambda^{bud}_{b,1}\left(-\frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{b,1} - \bar{d}_{b,0}) - \bar{b}_{b,1}\frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}}\right)\\
&\quad + \lambda^{bud}_{l,1}\left(-\frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{l,1} - \bar{d}_{l,0}) - \bar{b}_{l,1}\frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}}\right) + \frac{\theta_b}{\theta_l}\mu_{b,1}\gamma\frac{\partial q_1}{\partial \bar{c}_{l,1}}\bar{d}_{b,1}\\
&= u'_c(\bar{c}_{l,1}, \bar{d}_{l,1}) + \left(\frac{\theta_b}{\theta_l}\lambda^{bud}_{b,1} - \lambda^{bud}_{l,1}\right)D^b_{c_{l,1}} + \theta_b\mu_{b,1}C^b_{c_{l,1}},
\end{aligned}$$

where we used $\bar{d}_{l,t} = \bar{d} - \bar{d}_{b,t}$ and $\bar{b}_{l,t} = -\bar{b}_{b,t}$ and defined

$$D^b_{c_{l,1}} = -\frac{\partial q_1}{\partial \bar{c}_{l,1}}(\bar{d}_{b,1} - \bar{d}_{b,0}) - \frac{\partial r_1^{-1}}{\partial \bar{c}_{l,1}}\bar{b}_{b,1}, \quad \text{and} \quad C^b_{c_{l,1}} = \gamma \bar{d}^1_{i,1}\frac{\partial q_1}{\partial \bar{c}_{l,1}}.$$

The shadow price for the lender can be written as

$$\lambda^{bud}_{l,1} = \frac{u'_c(\bar{c}_{l,1}, \bar{d}_{l,1}) + \frac{\theta_b}{\theta_l}u'_c(\bar{c}_{b,1}, \bar{d}_{b,1})D^b_{c_{l,1}} + \frac{\theta_b}{\theta_l}\mu_{b,1}C^b_{c_{l,1}}}{1 + D^b_{c_{l,1}}}, \tag{4.35}$$

which reduces to $\lambda^{bud}_{l,1} = u'_c(\bar{c}_{l,1}, \bar{d}_{l,1})$ for $D^b_{c_{l,1}} = C^b_{c_{l,1}} = 0$. As in Davila and Korinek (2018), we use that

$$\frac{\partial V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1)}{\partial \bar{x}_{i,1}} = \frac{\partial V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1)}{\partial x_{i,1}} + \frac{\partial V_2^{i,\omega}(\bar{d}_{i,1}, \bar{b}_{i,1}; \bar{d}_1, \bar{b}_1)}{\partial \bar{x}_1}\frac{\partial \bar{x}_1}{\partial \bar{x}_{i,1}} \tag{4.36}$$

where $\partial V_2^{i,\omega}(\cdot)/\partial d_{i,1} = \lambda^{bud,\omega}_{b,2}q_2^\omega$ and $\partial V_2^{i,\omega}(\cdot)/\partial b_{i,1} = \lambda^{bud,\omega}_{b,2}$ are the derivatives of $V_2^{i,\omega}(\cdot)$ with respect to to individual states, evaluated at the equilibrium values, i.e. $x_{i,1} = \bar{x}_{i,1}$, $x \in \{b, d\}$, and the derivatives of $V_2^{i,\omega}(\cdot)$ with respect to type-specific aggregate variables are

$$\begin{aligned}
\frac{\partial V^{i,\omega}}{\partial \bar{b}_1}\frac{\partial \bar{b}_1}{\partial \bar{b}_{j,1}} &= \lambda^{bud,\omega}_{i,2}D^{i,\omega}_{b_{j,1}} + \mu^\omega_{i,2}C^{i,\omega}_{b_{j,1}}, \qquad (4.37)\\
\frac{\partial V^{i,\omega}}{\partial \bar{d}_1}\frac{\partial \bar{d}_1}{\partial \bar{d}_{j,1}} &= \lambda^{bud,\omega}_{i,2}D^{i,\omega}_{d_{j,1}} + \mu^\omega_{i,2}C^{i,\omega}_{d_{j,1}},
\end{aligned}$$

and the un-internatlized price effects are

$$D^{i,\omega}_{x_{b,2}} = -\frac{\partial q^{\omega}_2}{\partial \bar{x}_{j,1}}(\bar{d}^{\omega}_{i,2} - \bar{d}_{i,1}) - \frac{\partial \left(r^{\omega}_2\right)^{-1}}{\partial \bar{x}_{j,1}} b^{\omega}_{i,2}, \quad \text{and} \quad C^{i,\omega}_{x_{j,2}} = \gamma \bar{d}^{\omega}_{i,2} \frac{\partial q^{\omega}_2}{\partial \bar{x}_{j,1}},$$

for $i, j \in \{b, l\}$ and for $x \in \{b, d\}$. Note that $D^{l,\omega}_{x_{j,2}} = -D^{b,\omega}_{x_{j,2}}$ holds as in Davila and Korinek (2018),

$$\begin{aligned} D^{l,\omega}_{x_{j,2}} &= -\frac{\partial q^{\omega}_2}{\partial \bar{x}_{j,1}}(\bar{d}^{\omega}_{l,2} - \bar{d}_{l,1}) - \frac{\partial \left(r^{\omega}_2\right)^{-1}}{\partial \bar{x}_{j,1}} b^{\omega}_{l,2} = -\frac{\partial q^{\omega}_2}{\partial \bar{x}_{j,1}}(\bar{d} - \bar{d}^{\omega}_{b,2} - \bar{d} + \bar{d}_{b,1}) + b^{\omega}_{b,2} \frac{\partial \left(r^{\omega}_2\right)^{-1}}{\partial \bar{x}_{j,1}} \\ &= -D^{b,\omega}_{x_{j,2}}, \end{aligned}$$

which was used for $t = 1$ to get (4.31) by exploiting the market clearing conditions. Now define

$$\begin{aligned} \theta_b \Delta^{d,\omega}_2 &= \theta_b \frac{\partial V^{i,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{b,1}} - \theta_b \frac{\partial V^{i,\omega}}{\partial \bar{d}_1} \frac{\partial \bar{d}_1}{\partial \bar{d}_{l,1}} + \theta_l \frac{\partial V^{i,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{b,1}} - \theta_l \frac{\partial V^{i,\omega}}{\partial \bar{b}_1} \frac{\partial \bar{b}_1}{\partial \bar{b}_{l,1}} \\ &= \left(\theta_b \lambda^{bud,\omega}_{b,2} - \theta_l \lambda^{bud,\omega}_{l,2}\right) \left(-\frac{q^{\omega}_2}{\partial \bar{d}_{b,1}}(\bar{d}^{\omega}_{b,2} - \bar{d}_{b,1}) - b^{\omega}_{b,2} \frac{\partial \left(r^{\omega}_2\right)^{-1}}{\partial \bar{d}_{b,1}}\right) \\ &- \left(\theta_b \lambda^{bud,\omega}_{b,2} - \theta_l \lambda^{bud,\omega}_{l,2}\right) \left(-\frac{\partial q^{\omega}_2}{\partial \bar{d}_{l,1}}(\bar{d}^{\omega}_{b,2} - \bar{d}_{b,1}) - b^{\omega}_{b,2} \frac{\partial \left(r^{\omega}_2\right)^{-1}}{\partial \bar{d}_{l,1}}\right) \\ &+ \theta_b \mu^{\omega}_{b,2} \gamma \frac{\partial q^{\omega}_2}{\partial \bar{d}_{b,1}} \bar{d}^{\omega}_{b,2} - \theta_b \mu^{\omega}_{b,2} \gamma \frac{\partial q^{\omega}_2}{\partial \bar{d}_{l,1}} \bar{d}^{\omega}_{b,2} \\ &= \left(\theta_b \lambda^{bud,\omega}_{b,2} - \theta_l \lambda^{bud,\omega}_{l,2}\right) \left(D^{b,\omega}_{d_{b,2}} - D^{b,\omega}_{d_{l,2}}\right) + \theta_b \mu^{\omega}_{b,2} \left(C^{b,\omega}_{d_{b,2}} - C^{b,\omega}_{d_{l,2}}\right), \end{aligned} \tag{4.38}$$

where the second equality uses (4.37). Using (4.33) and (4.38), condition (4.31) can now be written as $q_1 \lambda^{bud}_{b,1} = u'_d(\bar{c}_{b,1}, \bar{d}_{b,1}) + \beta E_1[\lambda^{bud,\omega}_{b,2} q^{\omega}_2] + \mu_{b,1} \gamma q_1 + \Delta^d_1 + \beta E_1 \Delta^{d,\omega}_2$ or

$$q_1 u'_c(\bar{c}_{b,1}, \bar{d}_{b,1}) = u'_d(\bar{c}_{b,1}, \bar{d}_{b,1}) + \beta E_1 \left[u'_c(\bar{c}^{\omega}_{b,2}, \bar{d}^{\omega}_{b,2}) q^{\omega}_2\right] + \mu_{b,1} \gamma q_1 + \Delta^d_1 + \beta E_1 \left[\Delta^{d,\omega}_2\right], \tag{4.39}$$

with the first-period wedge

$$\begin{aligned} \Delta^d_1 &= \left(\lambda^{bud}_{b,1} - \frac{\theta_l}{\theta_b} \lambda^{bud}_{l,1}\right) \left(D^b_{d_{b,1}} - D^b_{d_{l,1}}\right) + \mu_{b,1} \left(C^b_{x_{b,1}} - C^b_{x_{l,1}}\right) \\ &= \left(\frac{u'_c(\bar{c}_{b,1}, \bar{d}_{b,1}) - \frac{\theta_l}{\theta_b} u'_c(\bar{c}_{l,1}, \bar{d}_{l,1}) - \mu_{b,1} C^b_{c_{l,1}}}{1 + D^b_{c_{l,1}}}\right) \left(D^b_{d_{b,1}} - D^b_{d_{l,1}}\right) + \mu_{b,1} \left(C^b_{x_{b,1}} - C^b_{x_{l,1}}\right), \end{aligned}$$

and with the second-period wedge

$$\begin{aligned} \Delta^{d,\omega}_2 &= \left(\lambda^{bud,\omega}_{b,2} - \frac{\theta_l}{\theta_b} \lambda^{bud,\omega}_{l,2}\right) \left(D^{b,\omega}_{d_{b,2}} - D^{b,\omega}_{d_{l,2}}\right) + \mu^{\omega}_{b,2} \left(C^{b,\omega}_{d_{b,2}} - C^{b,\omega}_{d_{l,2}}\right), \\ &= \left(u'_c(\bar{c}^{\omega}_{b,2}, \bar{d}^{\omega}_{b,2}) - \frac{\theta_l}{\theta_b} u'_c(\bar{c}^{\omega}_{l,2}, \bar{d}^{\omega}_{l,2})\right) \left(D^{b,\omega}_{d_{b,2}} - D^{b,\omega}_{d_{l,2}}\right) + \mu^{\omega}_{b,2} \left(C^{b,\omega}_{d_{b,2}} - C^{b,\omega}_{d_{l,2}}\right), \end{aligned}$$

where we used $\lambda_{b,2}^{bud,\omega} = u_c'(\bar{c}_{b,2}^{\omega}, \bar{d}_{b,2}^{\omega})$ and $\lambda_{l,2}^{bud,\omega} = u_c'(\bar{c}_{l,2}^{\omega}, \bar{d}_{l,2}^{\omega})$. To rewrite the first-order conditions for bonds (4.30), we use $\partial V^{i,\omega}/\partial b_{i,1} = \lambda_{i,2}^{bud,\omega}$ and $\theta_l \lambda_{l,1}^{bud} r_1^{-1} = \beta E_1[\theta_l \lambda_{l,2}^{bud,\omega}]$.

Proceeding as above, then gives

$$\theta_b \lambda_{b,1}^{bud} r_1^{-1} = \beta E_1 \left[\theta_b \lambda_{b,2}^{bud,\omega}\right] + \theta_b \mu_{b,1} + \Delta_1^b + \beta E_1 \left[\Delta_2^{b,\omega}\right],$$

which completes the derivation of the results in Section 4.2.2.

C.4 Definition of equilibrium for infinite-horizon model

Let $\Phi_t(x, y)$ denote the joint distribution of wealth and income across households in period t. An equilibrium for the infinite-horizon model of Section 4.3.1 is defined as:

Definition 4.1 *Given an initial distribution Φ_0, an equilibrium consists of a sequence of prices $\{r_t, q_t\}$, a sequence of household policy functions $\{b_t(x,y), c_t(x,y), d_t(x,y)\}$, a sequence of taxes $\left\{\tau_t^b, \tau_t^d\right\}$, and a sequence of joint distributions of wealth and income $\{\Phi_t\}$, such that*

(i) the policy functions $b_t(x,y)$, $c_t(x,y)$ and $d_t(x,y)$ solve the household problem given $\{r_t, q_t\}$ and $\left\{\tau_t^b, \tau_t^d\right\}$,

(ii) the distribution Φ_t is consistent with the household policy functions,

(iii) the markets for bonds and durables clear,

$$\begin{aligned} \sum b_t(x,y) d\Phi_t(x,y) &= 0, \\ \sum d_t(x,y) d\Phi_t(x,y) &= \bar{d}. \end{aligned}$$

The market for non-durable goods clears via Walras' Law, given that the policy functions ensure that all household budget constraints are satisfied. For a stationary equilibrium, we additionally require the distribution of wealth and income as well as prices to be constant over time, i.e. $\Phi_{t+1} = \Phi$, $r_t = r$ and $q_t = q$ for all t. Note that such an equilibrium requires that the tax rates are constant over time as well.

D.4 Computational algorithm

This section presents how we solve the quantitative model from Section 4.3.1. First, we discuss how to solve for the stationary equilibrium of the model economy. Then, we show how to solve for the transition path between two different stationary equilibria.

D.4.1 Transition probabilities and income values

The individual income transition probabilities are obtained as discussed in Section 4.3.1. The transition matrix is given as

$$\Pi = \begin{pmatrix} 0.9132 & 0.0867 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \\ 0.0867 & 0.7870 & 0.1200 & 0.0001 & 0.0000 & 0.0000 & 0.0001 \\ 0.0000 & 0.1260 & 0.7355 & 0.1382 & 0.0001 & 0.0000 & 0.0001 \\ 0.0000 & 0.0001 & 0.1382 & 0.7355 & 0.1260 & 0.0000 & 0.0001 \\ 0.0000 & 0.0000 & 0.0001 & 0.1260 & 0.7870 & 0.0867 & 0.0001 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0867 & 0.9132 & 0.0001 \\ 0.0021 & 0.0021 & 0.0021 & 0.0021 & 0.0021 & 0.0021 & 0.9876 \end{pmatrix},$$

and the income grid values $(y_1, y_2, ..., y_7)$ are

$$(0.0123, 0.0250, 0.0376, 0.0544, 0.0819, 0.1667, 1)\,.$$

Let i denote the row index and j the column index of matrix Π. The entry $\Pi(i, j) \equiv \pi\,(y_j|y_i)$ is the probability that next period's income y_{t+1} equals y_j, conditional on current income $y_t = y_i$.

D.4.2 Calculation of the stationary equilibrium

Solving for the stationary equilibrium involves finding time-invariant values for the real interest rate r and the price of durables q as well as a time-invariant joint distribution of wealth and income implied by the household policy functions such that the markets for durables and bonds clear (see previous section). The numerical procedure involves the following steps:

I. Choose initial values for r and q.

II. Given r and q, compute the policy functions for non-durable consumption $c(x, y)$, end-of-period bonds $b'(x, y)$, end-of-period durables $d'(x, y)$ and end-of-period wealth $x'(x, y) = b'(x, y)+qd'(x, y)$, using the endogenous grid point method (see Hintermaier and Koeniger (2010)) as outlined below.

III. Given the wealth policy function $x'(x, y)$, compute the implied stationary distribution $\lambda(x, y)$ (see below).

IV. Check whether markets for debt and durables clear. If $|\sum_{x,y} \lambda(x, y)b'(x, y)| < \epsilon^b$ and $|\sum_{x,y} \lambda(x, y)d'(x, y) - \bar{d}| < \epsilon^d$, with $\epsilon^b > 0$ and $\epsilon^d > 0$, stop: r and q are the equilibrium prices. Else, update prices (r, q) and go to Step II.

Solving the household problem via the endogenous grid method The endogenous grid point method used to solve the household problem for r and q involves the following steps:

1. Discretize next period's wealth space $\{x'_1, x'_2, ..., x'_m\}$, $x'_i < x'_{i+1}$. The discretized individual state space then is given by $\{x'_1, x'_2, ..., x'_m\} \times \{y'_1, y'_2, ..., y'_n\}$, where y'_k, $k = 1, ..., n$, are the income states that are possible next period.[29] Select a stopping rule parameter $\epsilon^{egm} > 0$.

2. Initialize the policy functions for non-durable and durable consumption $c_0(x'_i, y'_k)$ and $d'_0(x'_i, y'_k)$, $k \in \{1, ..., n\}$. Our guess is given by $c_0(x'_i, y'_k) = 0.5y'_k$ and $d'_0(x'_i, y'_k) = 0.5\bar{d}$ for all grid point combinations.

3. Update the consumption policy functions (using three auxiliary functions $\hat{c}_0(x'_i, y_k)$, $\hat{x}_0(x'_i, y_k)$ and $\hat{d}'_0(x'_i, y_k)$):

   - First, assume that the borrowing constraint does not bind in any state.
   - Use consumption policy functions $c_0(x'_i, y'_k)$ and $d'_0(x'_i, y'_k)$ to compute a guess for current-period non-durable and durable consumption at future wealth x'_i and today's income state y_k, i.e. $\hat{c}_0(x'_i, y_k)$ and $\hat{d}'_0(x'_i, y_k)$, by applying the Euler equations for bonds and durables:

$$\begin{aligned} u_c(\hat{c}_0(x'_i, y_k), \hat{d}'_0(x'_i, y_k)) =& \beta r \sum_{j=1}^{n} \pi\,(y_j|y_k)\, u_c(c_0(x'_i, y'_j), d'_0(x'_i, y'_j)), \\ u_c(\hat{c}_0(x'_i, y_k), \hat{d}'_0(x'_i, y_k))q =& u_d(\hat{c}_0(x'_i, y_k), \hat{d}'_0(x'_i, y_k)) \\ &+ \beta q \sum_{j=1}^{n} \pi\,(y_j|y_k)\, u_c(c_0(x'_i, y'_j), d'_0(x'_i, y'_j)), \end{aligned}$$

     which are two equations in the two unknowns $\hat{c}_0(x'_i, y_k)$ and $\hat{d}'_0(x'_i, y_k)$ at given values of x'_i and y_k.
   - Now, find the states for which the borrowing constraint is violated. If the borrowing constraint is violated at given grid points x'_i and y_k, i.e. $\hat{d}'_0(x'_i, y_k) > x'_i/(q(1-\gamma))$, we set $\hat{d}'_0(x'_i, y_k) = x'_i/(q(1-\gamma))$. The corresponding value for non-durable consumption $\hat{c}_0(x'_i, y_k)$ can then be calculated via the two Euler equations after having combined them by eliminating the multiplier on the borrowing constraint, which now enters both Euler equations. If the constraint is not binding, i.e. $\hat{d}'_0(x'_i, y_k) \leq x'_i/(q(1-\gamma))$ holds, we keep the values of $\hat{d}'_0(x'_i, y_k)$ and $\hat{c}_0(x'_i, y_k)$ calculated in the step before for this state.

[29] The values for the income grid and the associated transition probabilities are listed in Section D.4.1.

- Use the budget constraint and the auxiliary functions $\hat{c}_0(x_i', y_k)$ and $\hat{d}_0'(x_i', y_k)$ to compute current period wealth $\hat{x}$ for x_i' and y_k:

$$\hat{x}_0(x_i', y_k) = \hat{c}_0(x_i', y_k) + q\hat{d}_0'(x_i', y_k) + \left(x_i' - q\hat{d}_0'(x_i', y_k)\right)/r - y_k.$$

This implies $\hat{c}_0(x_i', y_k) = \hat{c}_0(\hat{x}_0(x_i', y_k), y_k)$ and $\hat{d'}_0(x_i', y_k) = \hat{d'}_0(\hat{x}_0(x_i', y_k), y_k)$.

- Calculate updates for the policy functions at $(x_i', y_k') \in \{x_1', x_2', ..., x_m'\} \times \{y_1', y_2', ..., y_n'\}$ by linearly interpolating $\hat{c}_0(\hat{x}_0, y_k)$ and $\hat{d'}_0(\hat{x}_0, y_k)$ at (x_i', y_k'). This calculation yields the updated consumption policy functions $c_1(x_i', y_k')$ and $d_1'(x_i', y_k')$.

4. If $||c_1(x_i', y_k') - c_0(x_i', y_k')||_\infty < \epsilon^{egm}(1 + |c_0(x_i', y_k')||_\infty)$ and $||d_1'(x_i', y_k') - d_0'(x_i', y_k')||_\infty < \epsilon^{egm}(1 + |d_0'(x_i', y_k')||_\infty)$, stop and set $c(\cdot) = c_1(\cdot)$ and $d'(\cdot) = d_1'(\cdot)$.
   Else, set $c_0(\cdot) = c_1(\cdot)$ and $d_0'(\cdot) = d_1'(\cdot)$ and go to Step 3.

Computing the stationary distribution For given policy functions, we compute the stationary distribution by calculating the normalized eigenvalue of the Markov transition matrix implied by the policy function for wealth and the income transition probabilities:

1. We add additional grid points for wealth relative the grid used for the calculation of the policy functions (we go from 10 to 50 thousand grid points for x) and calculate the wealth policy function values for these new states.

2. We calculate the transition probability of being in the state (x_j, y_l) in the next period conditional on currently being in state (x_i, y_k). We denote it as $\Pr((x_j, y_l)|(x_i, y_k))$. This probability is computed as $\Pr((x_j, y_l)|(x_i, y_k)) = \pi(y_l|y_k) \times I(x'(x_i, y_k) = x_j)$, where $I(x'(x_i, y_k) = x_j) = 1$ if $x'(x_i, y_k) = x_j$ and zero otherwise. The Markov transition matrix then consists of the individual transition probabilities $\Pr((x_j, y_l)|(x_i, y_k))$ for all grid point combinations.

3. Compute the eigenvector of this transition matrix that has the largest eigenvalue (which is equal to one). The stationary distribution of the model economy then is obtained by the normalizing this eigenvector.

Updating prices of debt and durables The prices are updated by using two nested bisection algorithms as follows: For a given price of durables q, we calculate the real interest rate r that clears the loan market, i.e. $|\sum_{x,y}\lambda(x,y)b'(x,y)| < \epsilon^b$, using bisection. If the market for durables is also cleared at this combination of q and r, i.e. $|\sum_{x,y}\lambda(x,y)d'(x,y) - \bar{d}| < \epsilon^d$, we can stop. If not, we update the price of durables q and then again calculate the real interest rate r that clears the loan market. The price q is updated by using bisection, too, to get the price that clears the durables market while the corresponding real interest rate at a given q is the value of r that clears the loan market.

D.4.3 Calculation of the transition path to the new stationary equilibrium

In period 0, the economy is in the laissez-faire stationary equilibrium without taxation. In the subsequent period 1, one or both tax rates are unexpectedly and permanently changed to the values τ^b_{new} and τ^d_{new}. Due to this change, the economy departs from the old stationary equilibrium in period 1 and gradually moves to the new stationary equilibrium under the tax rates τ^b_{new} and τ^d_{new}. The transition path to the new long-run equilibrium is computed by using the following steps (see e.g. Rios-Rull (1999)):

- Calculate the stationary equilibria for the laissez-faire economy and the economy with τ^b_{new} and τ^d_{new} as described above and denote the associated stationary distributions as Φ_{old} and Φ_{new}, respectively.

- The beginning-of-period distribution in period 0 is denoted Φ_0 and given by $\Phi_0 = \Phi_{old}$. The distribution of the economy once it has converged to the new stationary equilibrium is denoted as Φ_∞. It is given as $\Phi_\infty = \Phi_{new}$. Note that the beginning-of-period distribution of wealth in period 1 is not the same as in period 0, i.e. $\Phi_1 \neq \Phi_0$, because the policy change will alter household wealth via durables price q. Since the beginning-of-period distributions of bonds and durables in period 1 are however not affected and the same as in period 0, it is possible to calculate Φ_1 based on these distributions and the price of durables q_1. This price is however not known ex-ante and has to be calculated (see below).

- Compute the value function $V_0(x,y)$ in period 0, giving the expected lifetime utility of a household who is in the state (x,y) in period $t=0$, and the value function in the new stationary equilibrium $\mathcal{V}_\infty(\cdot)$.

- Computation of the transition path:

  1. Guess that the transition to the new stationary equilibrium takes $T > 0$ periods. This implies that $\Phi_T = \Phi_\infty$ and $V_T = V_\infty$.

  2. Guess a sequence of interest rates $\{\hat{r}_t\}_{t=1}^{T-1}$ as well as of durables prices $\{\hat{q}_t\}_{t=1}^{T-1}$. Choose stopping rule parameters $\epsilon^b > 0$, $\epsilon^d > 0$ and $\epsilon^\Phi > 0$.

3. With the known value $V_T(x,y)$ and guesses $\{\hat{r}_t, \hat{q}_t\}_{t=1}^{T-1}$, we can solve for $\left\{\hat{V}_t, \hat{c}_t, \hat{x}_{t+1}, \hat{d}_{t+1}, \hat{b}_{t+1}\right\}_{t=1}^{T-1}$ via backward induction.

4. By using the beginning-of-period distributions of bonds and durables of the old stationary equilibrium for period 1 together with the guess for the durables price in period 1, i.e. $\hat{q}_1$, we calculate a guess for the beginning-of-period wealth distribution in period 1 denoted by $\hat{\Phi}_1$.

5. Use the policy functions $\{\hat{x}_{t+1}\}$ and $\hat{\Phi}_1$ to iterate the distribution forward to get $\hat{\Phi}_t$ for $t = 2, ..., T$.

6. Use the sequence $\left\{\hat{\Phi}_t\right\}_{t=1}^{T}$ to compute excess supply $\hat{A}_t^b = \sum \hat{b}_{t+1} \mathrm{d}\hat{\Phi}_t$ and $\hat{A}_t^d = \sum \hat{d}_{t+1} \mathrm{d}\hat{\Phi}_t - \bar{d}$ for periods $t = 1, ..., T$. If

$$\max_{1 \le t < T} \left| \hat{A}_t^b \right| < \epsilon^b$$
$$\max_{1 \le t < T} \left| \hat{A}_t^d \right| < \epsilon^d,$$

holds, go to Step 7. Else, adjust the guesses for $\{\hat{r}_t, \hat{q}_t\}_{t=1}^{T-1}$ and go to Step 3.

7. Check whether $\left\| \hat{\Phi}_T - \Phi_T \right\|_\infty < \epsilon^\Phi$. If it does, the model economy smoothly converges to the new stationary equilibrium and the algorithm ends. If not, go back to Step 1 and start again with a higher value for T.

- The obtained $V_1(\cdot)$ is the value function at time $t = 1$ after taxation has changed, such that $V_1(x, y)$ is the expected lifetime utility of a household with income y and beginning-of-period wealth x who has just been hit by the change in taxation. This value hence accounts for the transition of the economy to the new long-run equilibrium.

E.4 Additional figures

Figure 4.14: Bond holdings and welfare for y_6 (change in loan-to-value ratio γ)

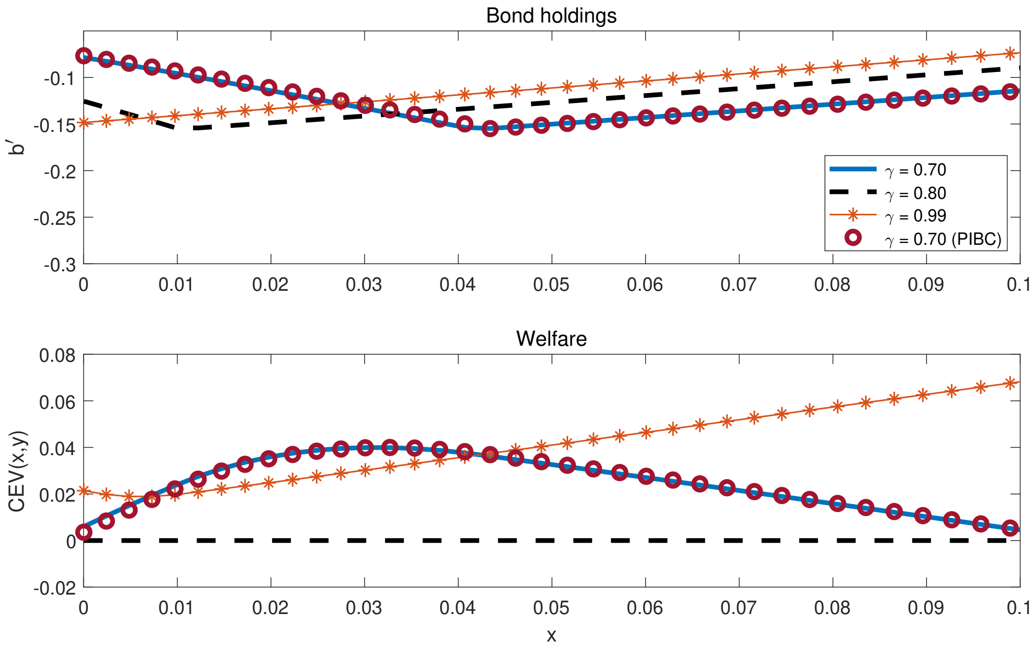
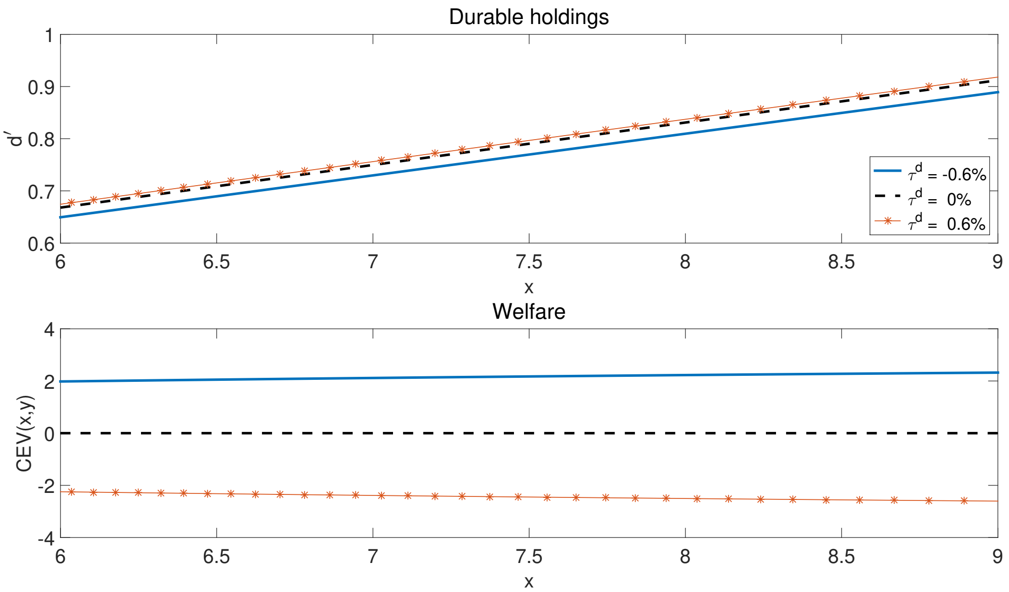


Figure 4.15: Durable holdings and welfare for y_6 (tax on durables))



Chapter 5

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List of applied software

- **Matlab R2019b:** Applied to solve the theoretical models
- **Excel 2016:** Applied to calculate empirical distributions

Eidesstattliche Erklärung

nach § 6 der Promotionsordnung vom 16. Januar 2008

Hiermit erkläre ich an Eidesstatt, dass ich die vorgelegte Arbeit ohne Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe. Die aus anderen Quellen direkt oder indirekt übernommenen Aussagen, Daten und Konzepte sind unter Angabe der Quelle gekennzeichnet. Bei der Auswahl und Auswertung folgenden Materials haben mir die nachstehend aufgeführten Personen in der jeweils beschriebenen Weise entgeltlich/unentgeltlich geholfen:

Weitere Personen – neben den in der Einleitung der Arbeit aufgeführten Koautorinnen und Koautoren - waren an der inhaltlich - materiellen Erstellung der vorliegenden Arbeit nicht beteiligt. Insbesondere habe ich hierfür nicht die entgeltliche Hilfe von Vermittlungs- bzw. Beratungsdiensten in Anspruch genommen. Niemand hat von mir unmittelbar oder mittelbar geldwerte Leistungen für Arbeiten erhalten, die im Zusammenhang mit dem Inhalt der vorgelegten Dissertation stehen. Die Arbeit wurde bisher weder im In- noch im Ausland in gleicher oder ähnlicher Form einer anderen Prüfungsbehörde vorgelegt. Ich versichere, dass ich nach bestem Wissen die reine Wahrheit gesagt und nichts verschwiegen habe.

Ort und Datum: Castrop-Rauxel, 16.02.2020

Unterschrift:

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Research Interests

Macroeconomics, Monetary Economics, Financial Economics, Computational Economics

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Research Experience & Education

03/2019 – 04/2019	Research visitor in the Financial Policy Division at the Central Bank of Chile
Since 10/2016	Ph.D. fellow and research assistance at Center for Macroeconomic Research (CMR), University of Colgne, Germany (Supervisors: Prof. Dr. Andreas Schabert & Prof. Michael Krause, Ph.D.)
10/2014 – 09/2016	Ph.D. fellow at Cologne Graduate School in Management, Economics and Social Sciences: Doctoral course programme in Economics
10/2012 – 09/2014	M.Sc. in Mathematical Economics, TU Dortmund in Germany. Thesis: "The optimal inflation target in New Keynesian macroeconomic models"
10/2009 – 09/2012	B.Sc. in Mathematical Economics, TU Dortmund in Germany. Thesis: "Financial frictions and unconventional monetary policy in a DSGE model"

Professional Experience

Since 10/2016	Research and teaching assistant, Center for Macroeconomic Research, University of Cologne
05/2014 – 07/2014	Internship at the Macroeconomic Policy Institute (IMK) in Dusseldorf
02/2014 – 04/2014	Internship at the Ifo Center "Business Cycle Analysis and Surveys" in Munich
10/2012 – 02/2014	Student assistant at the chairs Macroeconomics and Applied Economics at TU Dortmund

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Teaching Experience

Teaching assistant at CMR, University of Cologne:

- Supervision of bachelor theses
- Monetary Theory and Policy (bachelor-tutorial)
- Topics in Macroeconomics, Money and Financial Markets (bachelor-tutorial)
- Monetary Theory and Policy (master-tutorial)
- Advanced Macroeconomics II (master-tutorial)
- Monetary Policy, Financial Regulation and Theory of Incomplete Markets (master-seminar)

Teaching assistant at TU Dortmund:

- Econometrics (master-tutorial)
- Macroeconomics (bachelor-tutorial)

Research Papers

A quantitative analysis of general equilibrium effects in a heterogeneous household economy (Version: February 2020)

Monetary Policy, Financial Constraints, and Redistribution, with Andreas Schabert (Version: October 2019, Revised & Resubmitted to International Economic Review)

Financial Regulation, Interest Rate Responses, and Distributive Effects, with Joost Röttger and Andreas Schabert (Version: July 2019)

Presentations

2019	Rhineland Workshop, Cologne Financial Policy Division, Central Bank of Chile
2018	CMR Lunch Seminar, Cologne
2017	Annual Congress of the European Economic Association, Lisbon Rhineland Workshop, Bonn
2016	Doctoral Workshop on Quantitative Dynamic Economics, Marseille CMR Lunch Seminar, Cologne
2015	CMR Lunch Seminar, Cologne

Scholarships

10/2014 – 09/2016	Full scholarship for doctoral studies granted by Cologne Graduate School of Management, Economics and Social Sciences

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Skills

Languages	German (native), English (fluent), French (basic), Latin (Latinum)
Computer Skills	Matlab, Dynare, R, SQL, Python, LaTeX, MS Office

Cologne, 16th February 2020

Christian Loenser