Two-sided projective resolutions, periodicity and local algebras

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Abstract

We consider minimal projective bimodule-resolutions of finite dimensional algebras. They are useful to compute Hochschild cohomology.

There is still no general form of such a minimal resolution. Only in the monomial case there is a comprehensive description. To understand the resolution in the general case we need some new methods.

We classify the monomial algebras by means of their resolutions. They divide into algebras with finite, (eventually) periodic and infinite, but non-periodic resolution. Whatever the case may be can be read off the quiver and the relations of the algebra. It turns out that a monomial algebra is periodic if and only if it is a product of self-injective Nakayama algebras.

The remaining part of the thesis describes a new technique to modify the algebra. By gluing the vertices of the quiver we get a new local algebra which contains still many properties of the original algebra. If two algebras have the same associated local algebra, we call them locally equivalent. Every algebra in such a local equivalence class can be constructed by splitting certain relations. Then local properties of the algebra hold for the whole equivalence class and depend on the locally minimal algebras in this class.

The projective resolution of the simple module of the local algebra is completely determined by the resolutions of the simple modules of the original algebra. Hence we get an explicit description of the resolution of the local algebra. In particular, the Anick/Green-resolution is minimal for the local algebra if and only if it is minimal for the original algebra.

In the general case we associate to every non-monomial algebra a monomial algebra. For this algebra the Anick/Green-resolution is minimal and is an upper bound for the original algebra. With the aid of the local algebra we get inequalities of the form: If the resolution of the associated monomial algebra is locally quasi finite, then the resolution of the non-monomial algebra is locally finite. If the resolution of the monomial algebra is locally quasi periodic, then the resolution of the original algebra is either locally almost periodic or locally finite.