Bouncing Black Holes From Canonical Quantum Gravity

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I would still go there
if only to await
the once-in-a-lifetime
opening of truth’s flower;

if only to escape
such bought freedom, and live,
prisoner of the keyless sea,
on the mind’s bread and water.

– R.S. Thomas, "Island",
No Truce with the Furies

To represent his tangled ball of musings in a nutshell, he surmised from one side of his mouth that Time existed to make eternity pass more quickly, and from the other side that it served to make it pass more slowly. We gave him a standing ovation and then started drinking.

– Jeffrey Ford, "At Reparata"

Abstract

It is commonly believed that the ubiquitous singularities of general relativity will be cured in a theory of quantum gravity. In the absence of a complete such theory, one can still employ reduced toy models to investigate how an avoidance of singularities could be facilitated. One particular scenario for this is bouncing gravitational collapse: in it, quantum gravitational effects prevent the matter from fully collapsing to a singularity, and instead cause it to re-expand.

In the discussion of such bounces two aspects turn out to be of particular importance. First, the bounce necessitates quantum corrections not only in the high curvature region, but also at the horizon. The question is then how the behavior of the horizon is modified to accommodate the bounce. Second, since the 'black hole' is not the end result of the collapse anymore but an intermediate state, its finite lifetime is crucial as a consistency check for bouncing collapse models. In this thesis we construct and explore such models, especially with regard to these aspects.

We present a quantization of the marginally bound Lemaître-Tolman-Bondi model for inhomogeneous, spherically symmetric dust collapse, in which the model is split up into individual shells of dust and reassembled after quantization. We show that this leads to singularity avoidance via a bounce, a result that proves to be fairly robust under the quantization ambiguities. The problem is explicitly formulated from the point of view of an observer comoving with the dust, which avoids some notorious conceptual issues of quantum gravity but limits investigations of horizon behavior and lifetime.

In order to go beyond these limitations, we construct a marginally bound quantum Oppenheimer-Snyder model in which both the comoving observer and an observer exterior to the collapsing matter are included. In preparation for this, we present a phase space formulation of the classical Oppenheimer-Snyder model. The switch between the two observers is implemented by promoting the transformation between their adapted coordinates to a canonical transformation. Due to the complicated functional form of the Hamiltonian for the exterior observer an integral quantization method is used, namely affine coherent states quantization, and we focus on the investigation of quantum corrected phase space dynamics. For both observers a bounce emerges. However, for the exterior observer the minimal radius of the bounce is so large that no horizon forms.

Finally, we investigate what exterior geometries can be matched classically to a bouncing dust cloud. In particular, we show that static exteriors necessarily have a more involved causal structure, and we discuss a specific dynamic exterior in which the horizon retracts into the collapsing body at the moment of the bounce. The black hole lifetime for the latter turns out to be proportional to the mass of the cloud, and we argue that this result also applies to a larger class of dynamic exteriors.

Contents

1. Introduction

The successes of the theory of general relativity (GR) are undeniable; from explaining the perihelion motion of mercury [1], to the direct observations of its more decided departures from Newtonian gravity, such as the detection of gravitational waves [2] and the imaging of a black hole's shadow [3], it has proven to be an exceptionally effective theory of gravity at astrophysical and cosmological scales. And yet, in a sense GR is also a theory that predicts its own failure: when in a region of spacetime the curvature grows too large, up to the point where even light can't escape anymore, that spacetime is according to GR necessarily *singular*.

A singular spacetime is one in which the free fall of a test particle can abruptly come to an end; one also says that the spacetime is *geodesically incomplete*. In the cases we are interested in this occurs because in certain regions the curvature of spacetime diverges. These regions, so-called *singularities*, then have to be cut out of the spacetime, leading to sudden endpoints for the trajectories of test particles that run into them. There are also other mechanisms for the occurrence of singularities than diverging curvature, see e.g. Ref. [4], but those will not be of further importance here.

If one takes this aspect of GR at face value, the test particle would not just stop falling, it would cease to exist entirely at this endpoint (or equivalently, come into existence there). Obviously this is not physically reasonable, so one usually considers the existence of singularities as a breakdown of GR. Remarkably, as the Hawking-Penrose singularity theorems tell us [5], singularities are not just mathematical curiosities restricted to a specific niche of spacetimes, maybe those with a high degree of symmetry; they are rather a generic feature of GR, found in spacetimes used to describe important facets of gravitational physics, such as cosmology and the study of black holes.

These two at first sight antithetical statements – singularities are a ubiquitous prediction of GR, but are in themselves a bound on the predictive power of the theory – are what makes the study of singular spacetimes in GR so interesting. The prevailing wisdom is that this tension will be resolved in a theory of *quantum gravity*. Such a theory should contain classical gravity, but go beyond it and be able to make predictions where GR can not.

It is then reasonable to assume that the classical singularities will be cured in some way when quantum gravitational effects are taken into account. This poses an important question: what replaces those singularities? When a singularity would form, what happens instead? In this thesis, I will present my work on one particular scenario that strives to offer an answer, in particular for singularities arising during gravitational collapse. Before I describe this scenario, let us first cover the two ingredients that make up its construction: gravitational collapse and quantum gravity.

1.1. Gravitational collapse

Gravitational collapse is the process of an object getting compressed more and more by its own gravitational pull, eventually leading to the formation of a black hole complete with a singularity. It will be helpful for what follows to explain this phenomenon with a simple example, along with a few useful technical details. The most instructive model for gravitational collapse is the *Oppenheimer-Snyder* (OS) model [6, 7], which describes the collapse of a spherically symmetric dust cloud with homogeneous mass density. Actually it was Datt who found this solution to the Einstein equations shortly before Oppenheimer and Snyder, see Ref. [6], and accordingly the model is sometimes called Oppenheimer-Snyder-Datt. I will stick here to the former, more commonly used name.

Let me first explain the advantages of using dust as matter. In this context, dust means an ideal fluid with vanishing pressure, only interacting with itself gravitationally. Its appeal lies in this simplicity; using dust allows one to investigate purely gravitational phenomena without additional matter-related effects. This makes it particularly well suited for the use in models for gravitational collapse, since there one anyways expects gravity to eventually dominate over all other forces. Another useful feature of dust is that it naturally provides a preferred notion of time: thinking of the dust as a continuum of dust particles, one can pick out one of them, follow it along its trajectory, and measure its proper time. Repeating this for all dust particles one can then extend the *dust proper time* τ to a global time coordinate.

In the case of the OS model, there is a concise geometrical picture for this construction. Because the model is spherically symmetric, the dust cloud does not rotate; more precisely we can say that the velocity field tangential to the dust trajectories is non-rotational. One can show that for every non-rotational vector field in a four-dimensional spacetime there exists a family of three-dimensional hypersurfaces everywhere orthogonal to the vector field. This is essentially a consequence of the so-called Frobenius' theorem, see for example appendix B.3 in Ref. [8]. The global dust proper time can then be defined in such a way that it is constant on any given hypersurface from this family. In this way one can synchronize the proper times of all these individual dust particles, and merge them into τ.

The key insight of Datt, and shortly thereafter Oppenheimer and Snyder, was that dust proper time is extremely useful for solving the Einstein equations for the collapsing dust cloud. They can be reduced to an equation of motion for the radius of the surface of the cloud R_S,

$$\left(\frac{dR_S}{d\tau}\right)^2 = \frac{2GM}{R_S}\,, \tag{1.1}$$

where M is the dust cloud's total mass, and G is the gravitational constant. For simplicity we will use units in which the speed of light is unity. Note that here we are only considering the special initial condition that the dust cloud is marginally bound, meaning it is at rest for $R_S \to \infty$. We will come back to this shortly. Since we are using τ as a time coordinate, this equation describes the collapse of the cloud from the point of view of the dust, or rather an observer co-moving with it. That observer sees the dust cloud collapse with ever increasing velocity, until at $R_S \to 0$ the cloud is condensed into a single point with diverging mass density, and the singularity appears.

Eq. (1.1) and its solution can be understood in two different ways. First, since the interior of the dust cloud is homogeneous, so is its geometry. This geometry is then described by the *Friedmann-Lemaître-Robertson-Walker* (FLRW) metric. The FLRW metric appears prominently in cosmology, and the equations of motion for its remaining degree of freedom, the scale factor, are well explored for various kinds of matter. Eq. (1.1) is simply the case of these so-called Friedmann equations for that matter being dust. The collapse described above is then the time-reversal of a dust-filled universe's expansion: the formation of the singularity is a big bang in reverse.

The initial condition mentioned above can also be understood through this cosmological lens. Readers acquainted with the basics of cosmology will know that the FLRW metric comes in three different variations; spatial hypersurfaces of constant τ can be either positively or negatively curved, or they can be flat. These three cases then correspond to three different classes of initial conditions for the OS model. The dust cloud can start its collapse at rest from a finite radius, or it can start from infinity either with non-vanishing or vanishing initial velocity.

According to the initial conditions chosen in Eq. (1.1), I will only consider the flat OS model. From an astrophysical standpoint the closed (meaning positively curved) OS model, starting at rest at a finite radius, is more interesting: before a star starts collapsing because it is burned out, it is usually in a stationary state. However, at least qualitatively the behavior of the simpler flat OS model as it approaches the singularity does not differ significantly from the closed case. Further, there are some surprising technical difficulties one encounters in the closed case, which I will comment on shortly.

While the analogy to the Friedmann models is obviously useful for understanding the OS model, it cannot be exhaustively investigated via this avenue; the OS model describes the dynamics of an isolated object, hence it is not complete without an exterior to this object. Imposing vacuum and a vanishing cosmological constant in the exterior, the geometry outside of a isolated spherically symmetric object has to be the Schwarzschild geometry, and uniquely so. This is known from a uniqueness result called Birkhoff's theorem, see for example §32.2 in Ref. [9]. The full geometry of the OS model is then constructed by stitching together, or matching, the FLRW interior and the Schwarzschild exterior across a hypersurface with curvature radius $R_S(\tau)$. Details for how this matching works on a technical level can be found in Ch. 5, see also Ref. [10].

At this point a second way to understand Eq. (1.1) emerges: it also describes the trajectory of a marginally bound dust particle, radially falling towards a spherically symmetric body with mass M, see for example Eq. (25.16a) in Ref. [9]. The connection between OS collapse and these trajectories, which are geodesics in the Schwarzschild geometry, can be found in yet another interesting feature of non-rotating dust: the dynamics of every spherically symmetric thin dust shell is only determined by its initial velocity, and by the total mass of the dust contained inside it. Every shell then moves as if under the influence of a spherically symmetric body with that mass, which is exactly what Schwarzschild geodesics describe. This decoupling between different shells of dust will be very useful in Ch. 2, see there for more details.

With the Schwarzschild metric another preferred notion of time besides τ enters the picture. The Schwarzschild geometry is stationary, and with that there exists a time coordinate that makes this stationarity explicit: the so-called *Schwarzschild* or *Killing time T*. This time coordinate is of particular importance, because it is usually considered to be the time experienced by an observer far away from the object in the center of the Schwarzschild geometry; it is for example the time corresponding to us on earth observing a spherically symmetric black hole far away from us. In Schwarzschild time T, the equation of motion for OS collapse is given by

$$\left(\frac{dR_S}{dT}\right)^2 = \left(1 - \frac{2GM}{R_S}\right)^2 \frac{2GM}{R_S}\,. \tag{1.2}$$

As compared to Eq. (1.1), this equation leads to a very different behavior of R_S. However, before explaining this behavior one more concept needs to be brought up: that of a horizon.

There are quite a few different notions of horizons in GR, sometimes interchangeable but not equivalent, see for example Ref. [11]. Due to its simplicity many of them are in fact interchangeable when applied to the OS model. Here I will center the discussion on so-called *apparent horizons* in spherically symmetric spacetimes. Such an apparent horizon is the boundary between two regions in spacetime. In one region light rays behave like one would expect: outgoing light rays move outwards, ingoing light rays inwards. However, on the other side of the horizon, both out- and ingoing light rays move in the same direction. Regions where both families of light rays move inward are called *trapped*, and regions where they move outward *antitrapped*, and their associated horizons *trapping* and *antitrapping*.

What one calls the inside of a black hole is usually a trapped region (and the inside of a white hole an antitrapped region), and a trapped region indeed forms during the collapse of the OS model: since the Schwarzschild geometry is also used to describe a spherically symmetric black holes, it contains a trapped region. When the dust cloud collapses past $R_S = 2GM$ that trapped region, and with that its horizon with radius $2GM$, is not covered by the dust cloud anymore and emerges into the exterior. As one can see in Eq. (1.2), the horizon is of particular relevance to the dynamics of the dust cloud as seen by the exterior observer.

According to (1.2) the dust cloud will not collapse fully, but only asymptotically approach $R_S = 2GM$; from the point of view of the exterior observer, the dust cloud freezes right before a horizon would form. Of course the dust cloud does fully collapse to a singularity, a horizon will form at $R = 2GM$ as soon as the dust cloud crosses that radius. All of this can be confirmed by the comoving observer, but these events in the collapse are simply not visible from the perspective of the exterior observer. To fully understand the collapse, one thus needs to consider the viewpoint of both of these observers. Let me briefly emphasize the importance of this insight; it is one of the central threads running through this thesis, and has guided much of my work shown here. We will come back to it a bit further below, and discuss why it is so significant.

Once again identifying the trajectory of the dust cloud's surface with geodesics in the Schwarzschild spacetime, one can connect the two observers, or rather their corresponding time coordinates, via a coordinate transformation on that spacetime. This transformation, called Painlevé-Gullstrand transformation for the marginally bound case (see Ref. [12] for a pedagogical introduction), will play a crucial rule in Chs. 3 and 4, and to a lesser extent in Ch. 5. One can also find analogous coordinate transformations for the other initial conditions for the OS model: see again Ref. [12] for the transformation relevant for the open OS model, and Ref. [13] for the closed one. However, they are more cumbersome to work with, and in particular the latter is only defined up to a finite maximal radius. For this reason, extending the works in this thesis past the marginally bound case is a more involved task than it might initially seem.

There are also two other models for gravitational collapse I want to mention briefly. The first one is arguably even simpler than the OS model: it consists of an isolated spherical dust shell, essentially the OS model hollowed out. Of particular importance is the limit in which the dust moves at the speed of light, so-called null dust. For a concise summary of the collapse of such a null dust shell (as well as some other simple collapse models) see Ref. [14].

Further, there is also a generalization of the OS model to dust with inhomogeneous mass densities. This goes by the name *Lemaître-Tolman-Bondi* (LTB) model [15–17]. Originally intended for and extensively used in cosmology [18], it can also straightforwardly be applied to gravitational collapse. In a certain sense it is a more elegant construction than the OS model, since there is no need to strictly distinguish between interior and vacuum exterior; one can simply let the density of the dust go to zero over a finite range around the surface of the collapsing object, which leads to the geometry smoothly approaching Schwarzschild.

The dynamics of the LTB model are directly related to those of the OS model. As we have seen above, every shell in a spherically symmetric dust cloud moves according to how much mass is contained inside it, hence every individual shell in the (marginally bound) LTB model behaves according to Eq. (1.1), but instead of the total mass of the dust cloud, M denotes the total mass of the dust contained in the shell. It is then necessary to consider the mass as a field, varying over these shells, which immediately hints at a drawback of the LTB model when it comes to quantization: the OS model can essentially be treated as a mechanical theory, with its degree of freedom R_S only varying in time, whereas the LTB model is manifestly a field theory, and is as such much harder to quantize. In Ch. 2 the LTB model and its quantization are discussed in more detail.

1.2. Quantum gravity

As of yet, there is no fully consistent theory of quantum gravity, but there are quite a few contenders. There are approaches that aim to unify GR with the standard model of particle physics, such as string theory, and approaches that are content with quantizing gravity on its own as a first step towards unification. Some approaches such as causal set theory [19] start from speculative assumptions about the fundamental nature of spacetime, and attempt to construct a theory with the correct classical limit from there. Others start from classical gravity and try to quantize it. Of these, there are covariant approaches often based on a path integral, such as the asymptotic safety program [20], causal dynamical triangulation [21], and spin foam models [22], as well as canonical approaches starting from a phase space formulation, such as loop quantum gravity and quantum geometrodynamics. Further, there are also approaches that do not quite fit this categorization and make use of different windows to the problem, such as the AdS/CFT correspondence [23].

For an introduction to some of these approaches see also for example Ref. [24], and for a more complete exploration of the quantum gravity landscape see Ref. [25]. Here I want to very briefly introduce one approach, quantum geometrodynamics (QGD). In this introduction I will largely follow Ref. [24], but will focus on those aspects of the theory that are relevant for the rest of this thesis. The interested reader can find the full story in Ref. [24] and the references therein.

QGD, also referred to as the Wheeler-DeWitt approach, is one of the more conservative approaches to quantum gravity. It is the canonical quantization of a phase space formulation of GR. While this sounds straightforward at first – and compared to other approaches it is – there is an inherent tension in this statement: GR is manifestly covariant, it is not sufficient to only consider a single coordinate frame, but a phase space formulation requires a choice of an external time parameter. This conceptual obstacle will grow into the infamous *problem of time*, but for now let us solve it as follows.

In a construction similar to that of the dust proper time, one can decompose any spacetime with a benign enough causal structure into a family of spatial, non-overlapping hypersurfaces. This is called a *foliation* of the spacetime, and the individual hypersurfaces its *leaves*. There then exists a coordinate frame adapted to this foliation: spatial coordinates x^i adapted to each leaf, and a time coordinate t that labels the leaves from past to future. At this point this constructions may seem like it singles out this adapted coordinate frame. The key to keeping the theory explicitly covariant is to not specify the foliation any further, to keep it as general as possible. In this way, we have a distinguished time coordinate for the canonical formulation, but since this time is arbitrary there is still the freedom to switch between coordinate frames.

According to the foliation one can now split up tensors on the spacetime into their spatial and temporal parts. For example, the spacetime metric can be decomposed as

$$g_{\mu\nu} = \begin{pmatrix} N_i N^i - N^2 & N_j \\ N_k & h_{jk} \end{pmatrix}, \tag{1.3}$$

where greek letters denote spacetime indices, and latin letters purely spatial indices. Therein h_{ij} is the spatial metric on the leaves of the foliation, and N and N^i are called the lapse and the shift vector. The latter are closely related to the foliation, see Ref. [24] for a nice geometric illustration, and different choices for lapse and shift correspond to different foliations of the spacetime.

Accordingly one can also decompose the Einstein-Hilbert action for GR, a process called *Arnowitt-Deser-Misner* (ADM) decomposition [26], and bring it into canonical form, here with vanishing cosmological constant,

$$S_{\text{EH}} = \frac{1}{16\pi G} \int dt\, d^3x \left(p^{ij} \dot{h}_{ij} - NH - N^i H_i \right), \tag{1.4}$$

where dots denote derivatives with regard to label time t, p^{ij} is the canonically conjugate momentum to the spatial metric h_{ij}, and H and H_i are called *Hamiltonian constraint* and *momentum* or *diffeomorphism constraints*, defined as

$$H = 16\pi G\, \mathcal{G}_{ijkl} p^{ij} p^{kl} - \frac{\sqrt{h}}{16\pi G}\, {}^{(3)}\mathcal{R}, \tag{1.5}$$

$$H_i = -2\nabla_k p^k{}_i, \tag{1.6}$$

where ${}^{(3)}\mathcal{R}$ is the spatial Ricci scalar computed from h_{ij}, and $\mathcal{G}_{ijkl}$ is called the DeWitt metric. It depends only on h_{ij}, but its precise form is of no further importance here. What I want to focus on is the general structure of the total Hamiltonian density $NH + N^i H_i$.

The first thing to note is that lapse and shift are not dynamical quantities. They are arbitrary, which makes sense given their connection to the foliation; a particular evolution of N and N^i with t would break GR's general covariance. That is not to say that lapse and shift do not play a role in the canonical theory. Varying the action with regard to them gives the equations $H = 0$ and $H_i = 0$, illustrating why the constraints are called constraints: they restrict the phase space to what is called the *constraint surface* on which the dynamics of the theory take place. We can then note that H and H_i fulfill a dual purpose; they form the total Hamiltonian, and they define this constraint surface.

With N and N^i out of the picture, the only dynamical quantity left is the spatial metric h_{ij}. Classical geometrodynamics is thus a formulation of GR in terms of three-dimensional, spacelike geometries evolving with time. Making the jump to QGD, this then informs the arena it takes place in: QGD deals with wave functionals $\Psi[h_{ij}]$ defined over this space of spaces, for which Wheeler coined the term *superspace* [27]. Since we are dealing with a constrained system, not every wave functional can be a physically viable state; one needs a quantum equivalent to the constraint surface. To this end one can turn the constraints into operators $\hat{H}$ and $\hat{H}_i$, for example using Dirac's prescription of replacing the momenta by functional derivatives, and only allow states for which

$$\hat{H}\,\Psi[h_{ij}] = \left(-16\pi G\hbar^2\,\mathscr{G}_{ijkl}\frac{\delta}{\delta h_{ij}}\frac{\delta}{\delta h_{kl}} - \frac{\sqrt{h}}{16\pi G}\,{}^{(3)}\mathscr{R}\right)\Psi[h_{ij}] = 0, \tag{1.7}$$

$$\hat{H}_i\,\Psi[h_{ij}] = 2i\hbar\,\nabla_k\,h_{ij}\,\frac{\delta}{\delta h_{kj}}\,\Psi[h_{ij}] = 0. \tag{1.8}$$

Eq. (1.7) is called the *Wheeler-DeWitt equation* [27,28], while Eq. (1.8) does not have a special name, and that is for good reason: going back to the classical diffeomorphism constraints for a moment, one can show that acting on phase space functions via the Poisson bracket they generate spatial diffeomorphisms – coordinate transformations on the leaves of the foliation. (It is then tempting to think that all four constraints taken together would generate spacetime diffeomorphisms, but the full story is a bit more complicated, see Ref. [24].) When one quantizes the constraint as we did above, the corresponding operator fulfills the same role when acting on wave functionals. Hence, Eq. (1.8) imposes that Ψ should be a constant spatial scalar.

Using a more precise definition of superspace, the equation can even be fulfilled automatically; strictly speaking, superspace only refers to the space of all spatial geometries, and not all spatial metrics, the difference between the two being that a metric depends on coordinates, while the corresponding geometry is an invariant object. One should thus consider equivalence classes of metrics connected by spatial diffeomorphisms as the elements of superspace. Ψ as a functional over that space would then necessarily fulfill Eq. (1.8). However, technical difficulties make explicitly carrying out this construction infeasible; in practice one simply has to choose an ansatz for Ψ that solves (1.8).

Unfortunately, implementing the Wheeler-DeWitt equation (1.7) turns out to be a bigger hurdle. To start with, in it there is a product of two functional derivatives evaluated at the same point in space, which is not well defined and requires some form of regularization. Further, since the DeWitt metric contains h_{ij} there is a factor ordering ambiguity that has to be resolved. Yet, arguably the biggest obstacle one faces in QGD is of a more conceptual nature: time evolution in QGD would be governed by a Schrödinger equation, but since the total Hamiltonian is a sum of the constraints, any wave functional fulfilling Eqs. (1.7) and (1.8) simply vanishes when being acted upon by it. Thus, time disappears from QGD entirely. This is the problem of time.

Obviously time has to reemerge at some point, and in fact one can show that in a semiclassical approximation one can reclaim quantum field theory in a curved background from QGD. However, the question still remains whether time should be present in the fundamental theory, and how it could be reintroduced. On this depends the basic structure of the theory, and how its results should be interpreted. Should observables be self-adjoint operators? Should the constraints be? Is it even necessary to have a Hilbert space? Can one still interpret the wave functionals probabilistically? The Klein-Gordon-like form of the Wheeler-DeWitt equation (1.7) (the DeWitt metric turns out to be indefinite) certainly calls the latter into question. It is in any case rather difficult to apply the standard Copenhagen interpretation to a quantum theory that aims to describe the whole universe – there can hardly be an external measurement apparatus sharply divided from it.

These issues are by no means exclusive to QGD. Since the problem of time is rooted in the fundamentally conflicting concepts of time in quantum theories and GR, most approaches to quantum gravity have to contend with it in some shape or form, even the covariant ones. As long as this problem is not solved, along with the various other conceptual and technical difficulties of the different approaches to quantum gravity, and there is no fully formed theory, one has to resort to constructing various toy models to find indications how quantum gravitational effects could feasibly change physical scenarios of interest.

This is exactly what we will concern ourselves with in this thesis: I will present my work in which I have explored how quantum gravity could resolve the singular endstate of gravitational collapse, and how those effects imprint themselves onto the spacetime regions further away from the would-be singularity. Crucially, it is not primarily an investigation of QGD itself, I am rather using methods from QGD to build toy models to help with this investigation. QGD turns out to be very well suited to this task due to its straightforwardness. Once one circumvents the aforementioned technical problems, it's easy to see what happens during quantization if one is familiar with standard quantum mechanics, and the problem of time notwithstanding, the use of metric variables makes results comparatively simple to interpret.

Similar investigations by way of toy models, also those based on different approaches to quantum gravity, often share some aspects of their construction that help evade the issues of the full theories mentioned above. First, one often restricts the geometries considered in the model to those with a particular symmetry, for example spherically symmetry or homogeneity and isotropy; one then speaks of midi- or minisuperspaces. Especially the latter case simplifies matters quite a lot. As we have mentioned in the last section, a homogeneous and isotropic spatial geometry has only one degree of freedom, which only varies with time. The model thus loses its field theoretic nature, and along with it many of its technical problems.

Further, one often sidesteps the problem of time by declaring one of the degrees of freedom of the model, in most cases one associated with the matter content, as an internal time parameter. That makes it possible to identify from the constraints a physical Hamiltonian, which generates time evolution with regard to that parameter. In addition, particularly for gravitational collapse models where one only aims to describe an isolated object, the interpretational issues become less pressing. All of this combined makes it possible to construct and evaluate toy models for quantum gravitational collapse completely analogous to standard quantum mechanics, as we will see in Chs. 2 and 4.

Before we move on to that, let me first briefly highlight some results from similar related investigations. In quantum cosmology, a particular scenario that has garnered interest in recent years is one where the big bang singularity is replaced by a so-called *bounce*, see for example Refs. [29–35]. Therein, the expansion of the universe is not preceded by that singularity, but by the collapse of a previous iteration of that universe. Where the classical singularity would be, there is now the transition between collapse and expansion, induced by quantum gravitational effects when the universe is small. Together with for example Refs. [36, 37], where it is shown that eternal black holes might decay into white holes through quantum effects, the attentive reader can probably guess what scenario presents itself for gravitational collapse.

1.3. Bouncing collapse models

In a variety of toy models for quantum gravitational collapse, based on different approaches to quantum gravity, a similar picture has emerged: initially, the object collapses as it would classically, but when it becomes small and dense enough quantum effects start to become relevant and slow down the collapse. Eventually a turning point is reached, and the object starts expanding outwards from its minimal size. The collapse does not end in a singularity anymore, instead the object bounces. As examples, see Refs. [38–51] and for a review Ref. [52]; due to the close connection between collapse and cosmology one can also charitably add some of the aforementioned quantum cosmological bounces to this list, although this application of these models is not always made explicit.

Despite the variety of different bouncing collapse models, the underlying mechanism for the bounce can be understood along similar lines: from a quantum mechanical perspective one can say that the bounce is a superposition of the collapsing and expanding modes, while from a spacetime perspective it is facilitated by effective negative pressures and energy densities near the bounce, briefly turning gravity repulsive. Due to the simplicity of all of these models, that bounce is almost always time-reversal-symmetric, the expansion is just the collapse in reverse. However, it seems reasonable to assume that, when one moves towards more realistic models, dissipative processes would introduce an asymmetry.

Let us look at a few examples of these bouncing collapse models. Already in 1979, Frolov and Vilkovisky found that one-loop-corrections to the Einstein-Hilbert action consisting of a square of the Weyl tensor, the traceless part of the Riemann tensor, can lead spherically symmetric null shells to bounce [38]. The shells in fact reach $R = 0$, but the geometry does not become singular and they expand out again. Apart from the behavior of the null shells, Frolov and Vilkovisky also discuss how the exterior geometry is modified. Most importantly, this geometry significantly differs from the classical case not only near the would-be singularity, but also far away from it where the curvature is usually fairly small: at the horizon. Frolov and Vilkovisky speculate that accumulated quantum effects cause the horizon to eventually retract inwards. In fact, they predict the formation of an inner horizon in addition to the classical outer horizon. After the bounce both horizons would start to approach each other and eventually meet up and disappear. Unfortunately, the complexity of the equations prevented the discussion of an exact complete spacetime for the bounce, and especially post bounce many aspects of it remain speculative.

The issue of horizons is one I will keep returning to throughout this thesis. It is straightforward to see that the horizon cannot behave like it does classically: it cannot be a trapping horizon throughout the bounce, otherwise the initially collapsing matter would not be able to fully expand out again. In addition to the scenario of Frolov and Vilkovisky, a few resolutions to this issue have been proposed in the literature, along with mechanisms for how quantum gravitational effects can reach the horizon. We will comment on those in later chapters.

Similar results emerged in a model due to Hájíček and Kiefer, where a collapsing null shell (and the resulting geometry) was quantized [40–42]. The construction of that quantum theory was undertaken along geometrodynamical lines, but goes further in reducing the canonical theory at the classical level: not only spherical symmetry was directly implemented, but also further symmetries of the full classical solution, in order to make the action as simple as possible without being trivial. Helpful in this procedure is that a subgroup of those symmetries is singled out, which is connected to an exterior observer at asymptotic infinity. Hájíček and Kiefer are then able to construct a wave packet that vanishes at $R = 0$. From the expectation value of R with regard to that wave packet the bounce then emerges.

Unfortunately, since the theory is reduced down to the bare essentials before quantization, this result cannot tell us much about the exterior geometry. One can infer from the wave packet that the bounce can take place inside of the Schwarzschild radius, and hence a horizon should form. Hájíček and Kiefer then speculate that the horizon should be 'grey', in a superposition between the horizons of a black and a white hole.

Ambrus and Hájíček have further investigated the model with regard to another important aspect of bouncing collapse models [53]. Since the 'black hole' is only an intermediary state in the process, it has a finite lifetime. This lifetime is important: the black hole should exist for long enough to be consistent with astrophysical observations. However, Ambrus and Hájíček have found that the black hole's lifetime is proportional to its mass. Assuming a factor of proportionality of the order of one, that is of the order of microseconds for a solar mass black hole, and with that unfortunately much too short. One of the most disconcerting aspects of bouncing collapse scenarios is that this estimate for the lifetime appears with worrying regularity [37, 54, 55].

These two big open questions of bouncing collapse, the behavior of the horizon and relatedly the black hole lifetime, illustrate that it is crucial to understand how the exterior geometry is affected by quantum gravitational effects to arrive at a fully consistent picture; only focusing on the bouncing interior itself is not sufficient. Going back to OS collapse, this can be transferred to the two observers that we discussed previously: not only in the classical, but also in the quantum case it is important to consider the viewpoints of both a comoving and an exterior stationary observer. This is what my work in bouncing collapse centers on, informing almost all aspects of what is presented in upcoming chapters.

This idea is of course not new. It has for example recently been implemented in some of the literature in a somewhat roundabout way: since it is significantly easier to just quantize the collapsing matter and the geometry in its immediate vicinity, one can take this bouncing solution, and investigate how it can be classically extended to a full geometry. This has been done for a bouncing null shell, see Refs. [56–59], and to convert quantum cosmological bounces into bouncing collapse models [60–64].

The other option is to directly introduce quantum gravitational effects into a full collapse model with interior and exterior. Since quantum gravity is not advanced enough to simply directly quantize such models, one has to resort to effective methods. This was done recently in Ref. [49], where an improved dynamics scheme, in which loop quantum gravity inspired corrections to the Hamiltonian lead to quantum corrections, is applied to the LTB model. The resulting equations of motion can be solved exactly when restricted to the OS model, and lead to a bounce. A trapped region complete with horizon forms, but intriguingly after the bounce this region only turns antitrapped in the interior of the dust cloud, but remains trapped outside of it. The causal structure of this model was investigated in more detail in Ref. [65].

Further, the black hole lifetime was found to be proportional to the mass of the dust cloud *squared*; long enough to pass the basic consistency check, but short enough for a bounce of a small black hole formed during the early universe, a so-called *primordial black hole*, to be observable today [66]. The same estimation for the lifetime was also suggested on dimensional grounds in Ref. [56]. The price one has to pay for these nice features is that the spacetime is only approximately covariant: spacetime scalars depend on the lapse and shift through terms that vanish in the classical limit. Taking this model seriously would hence mean treating general covariance as an emergent notion, rather than a fundamental one.

Of course bounces are not the only possible scenario for singularity avoidance. Other suggestions are for example that the spacetime signature could change as the spacetime approaches the singularity, see Ref. [67], or that instead of fully collapsing the matter configuration can stabilize at some minimal size, either at the horizon or somewhere within it, see for example Refs. [68–71]. The latter is not necessarily incompatible with a bounce; it could be that the bounce is followed by another recollapse, leading to a cycle of collapses and bounces that stabilize into a new equilibrium configuration through dissipative effects [47, 48]. Further, there are also other discussions of gravitational collapse in quantum gravity with a focus not on singularity avoidance, but rather on the mass spectrum and entropy of the resulting black hole [72–74].

Related are also other quantum modifications of black holes, not necessarily connected to the collapse process. From various regularized black holes [75–77] to horizonless compact object such as gravastars [78] and fuzzballs [79], the proposals range from comparatively benign corrections to GR to rather drastic diversions from what one would usually call a black hole. On a similar note, and more relevant to our current considerations, there is also the notion of a Planck star [80, 81], essentially a slowly bouncing matter configuration that evaporates away some of its mass via Hawking radiation before expanding out of its horizon.

In the following, I will not directly consider Hawking radiation. Whether this omission is justified remains to be seen: if the black hole lifetime is significantly smaller than its evaporation time, proportional to its initial mass *cubed*, then it seems reasonable to assume that Hawking radiation will only be a small dissipative effect. If the lifetime is long enough, then it is clear that Hawking radiation could potentially alter the scenario quite significantly. For a discussion of backreaction of quantum fields in a bouncing collapse model, see for example [57].

As done in much of the existing literature in bouncing collapse, I operate under the assumption that the two processes can be considered separately, but I am convinced that there is much to be learned from the interplay between bouncing collapse and Hawking evaporation. On the one hand, it would seem like a bounce would in principle be capable of resolving the information loss paradox: if what is inside the black hole escapes it again, the information previously thought lost could be regained. This is in fact one of the main motivations to consider a scenario like Planck stars. On the other hand, as mentioned above, Hawking radiation might be capable of changing the course of the collapse. It might for example introduce a new equilibrium configuration for the collapsing matter that does not feature in the models I will present here. I will leave this for future investigations.

1.4. Structure of this thesis

This thesis is organized as follows. It is cumulative; in Chs. 2, 3, 4, and 5 I briefly introduce a published article that I am the or an author of [82–85], as well as explaining my contribution to the article. I conclude in Ch. 6. The Appendices A and B consist of two further articles [86, 87], to which my contribution is either smaller than to the articles in the main text, or for which it is harder to separate my contribution from those of my co-authors due to particularly close collaboration. Further, the results of these articles are somewhat less consequential. I still include them here for completeness.

Ch. 2 deals with a quantization of the marginally bound LTB model. The article, Ref. [82], co-authored by Claus Kiefer and me, is based on my master thesis, written under the supervision of Prof. Dr. Claus Kiefer at the University of Cologne. The material was slightly reworked and written up as the article as part of my doctoral studies. In it, we make quantizing the LTB model as simple as possible. First, we note that the dynamics of the individual dust shells in the model are effectively decoupled, as explained above, to argue that it is sufficient to only consider a single shell. Thus, we do not have to contend with the field theoretic nature of the LTB model.

Further, we explicitly take the point of view of the comoving observer, formulating the problem in terms of the dust proper time τ. This allows us to work with a physical Hamiltonian instead of a constraint, generating the dynamics of a single shell in the marginally bound LTB model with regard to τ. Quantization can then proceed in complete analogy to ordinary quantum mechanics; the Hamilton function becomes a self-adjoint operator on a Hilbert space, which generates the time evolution of states in the Hilbert space via a Schrödinger equation. Finding the domain of this self-adjoint operator is the main addition to the article not present in my master thesis, which puts the quantum model on a mathematically more rigorous footing. During this construction of the model we keep the quantization ambiguities fairly open, we consider a large class of factor orderings and all possible self-adjoint extensions of the Hamiltonian.

Investigations of the quantum model are then twofold. First, we discuss the asymptotic behavior of unitarily evolving wave packets close to where the classical singularity is in configuration space. We find that all of these wave packets, or rather the probability distributions for the radius of the shell based on them, vanish at the classical singularity. This we interpret as the quantum OS model avoiding the singularity. Further, this avoidance takes place regardless of the self-adjoint extension of the Hamiltonian, as long as the parameters controlling the factor ordering are chosen outside of a small range, and for one specific self adjoint extension it even takes place for all factor orderings; the singularity avoidance thus seems to be fairly generic.

Second, we construct a specific wave packet analytically. For this wave packet it is even possible to calculate various expectation values exactly; we then find from the expectation value of the radius of the shell that it avoids the singularity by bouncing. Based on this bouncing trajectory we finally discuss various aspects of a possible quantum corrected, bouncing LTB model: we compute the effective matter content that facilitates the bounce, which turns out to be an ideal fluid with the pressure turning negative close to the bounce. Also, we discuss possible behaviors of this horizon and determine based on the expectation value of the dust shell's velocity how fast it turns from trapping to antitrapping.

Further, we discuss the black hole lifetime. Since the model was explicitly constructed with regard to dust proper time, this is not entirely straightforward: the relevant notion of time for the lifetime is Schwarzschild Killing time, and the Painlevé-Gullstrand coordinate transformation mediating between those two times cannot be defined in the framework of our quantum model. We instead find the lifetime, via an *ad hoc* construction inspired by what was done in Refs. [54,55], to be proportional to the mass of the dust cloud cubed. Unfortunately, this result does not hold up under a more careful investigation of the lifetime in a different framework in Ch. 5.

Chs. 3 and 4 are two halves of a bigger undertaking: the construction of a quantum OS model. The idea behind this model is to still take the point of view of particular observers, as in Ch. 2, but to not restrict oneself to just the comoving observer; the exterior stationary observer will be explicitly included as well. One then has a quantum OS model comprised of two individual quantum models, one for each of the two observers. In this way, the structure of standard quantum mechanics can still be employed to full effect, while hopefully covering some of the blind spots of the approach in Ch. 2, most notably with regard to the lifetime and horizon behavior.

In the article in Ch. 3, Ref. [83], I construct a consistent phase space formulation of the classical flat OS model, Friedmann interior and Schwarzschild exterior, in preparation for quantization. To this end, I start from the canonical form of the Einstein-Hilbert action restricted to spherically symmetric spacetimes, as used for example in Kuchař's canonical quantization of the Schwarzschild black hole [88], and add to it as matter content dust in a formulation due to Brown and Kuchař [89]. In this formulation, dust proper time τ directly appears as a canonical coordinate, and hence it is suited particularly well to my purposes. I then proceed to reduce the phase space separately for interior and exterior in accordance with the OS model: in the interior I impose that the geometry be homogeneous and flat, and in the exterior I let the mass density of the dust vanish.

In the interior this leads to a Hamiltonian constraint of the form $P_\tau + H_\tau(R_S, P_S)$, where R_S is the radius of the dust cloud, P_S is its canonically conjugate momentum, and P_τ is the momentum to τ. I can thus identify H_τ as a physical Hamiltonian, generating evolution with regard to dust proper time; this identification is called *deparametrizing* the constraint. Further, H_τ is identical to the Hamiltonian for single shells in the marginally bound LTB model discussed in Ch. 2.

In the exterior, there is more work to be done. Kuchař has shown in Ref. [88] that one can bring the canonical formulation of a Schwarzschild black hole into a *fully deparametrizable* form via a series of canonical transformations; that means that one can find phase space functions which are canonically dual to the system of constraints. The dynamics of the system then simply imply that these phase space functions are superfluous and ultimately non-physical degrees of freedom. In the case of Schwarzschild, the only degree of freedom left over is then the mass of the black hole, which is of course constant. The quantized Schwarzschild black hole behaves accordingly: the quantum constraints imply that states do not depend on the superfluous degrees of freedom, but only on the mass.

The same procedure applies to the exterior in the OS model, only that not the mass of the black hole remains, but instead the degrees of freedom describing the interior. However, since in the case of the OS model one has the surface of the collapsing body as a boundary, which is not present in the case of an eternal Schwarzschild black hole, one has to make sure that this boundary does not spoil the canonical transformations. Taking this seemingly small technicality seriously proves to be worthwhile: it turns out that from the condition that the boundary term resulting from these transformations should vanish, one can find the Painlevé-Gullstrand coordinate transformation in disguise, connecting the exterior geometry to the interior dynamics.

This transformation is then also used to finally replace the dust proper time by Schwarzschild Killing time, by promoting it to a canonical transformation of the interior degrees of freedom. This leads to a second form of the Hamiltonian constraint, explicitly containing the momentum conjugate to Schwarzschild Killing time. However, since this constraint splits up into two branches, one describing the outside of the horizon and one the inside, it is not quite in a deparametrizable form. Further, the functional form of these branches is also much more complicated than what one has to deal with in H_τ, containing square roots and hyperbolic functions. These issues will be resolved in Ch. 4.

Finally I discuss quantization of the model in its two incarnations. As mentioned above, for the comoving observer the Hamiltonian matches the one discussed in Ch. 2, so the investigation done there carries over to the OS model. Although the exterior stationary observer is unfortunately less cooperative, I still present a tentative quantization of the corresponding constraint by bringing both branches into Klein-Gordon form and introducing a further *ad hoc* canonical transformation. The result of this investigation is that a bounce is in principle possible, but the haphazard nature of this quantization makes it hard to learn much more from it. What few other aspects of the bounce I speculate about unfortunately turn out to be misleading when compared to the more careful quantization undertaken in Ch. 4.

In Ch. 4, containing Ref. [84] co-authored by Włodzimierz Piechocki and me, the quantization of the OS model with regard to the two different viewpoints is discussed in much greater detail. Due to the unusual form of the constraint for the exterior observer, we do not use Dirac's prescription for quantization as in the earlier chapters, but rather a method called *affine coherent states quantization* (ACSQ) [90–92]. In this method, one identifies the phase space of the theory one wants to quantize with the affine group; this is applicable to the OS model because the radius of the dust cloud is always positive. It is then possible to construct coherent states on a Hilbert space from elements of the affine group [93]. As is well known, a family of coherent states admits a resolution of the identity; one can construct the identity on the Hilbert space by integrating over projectors on the coherent states. To quantize a phase space function according to ACSQ, one simply inserts that function into the resolution of the identity. In this way, it is at least formally possible to quantize more complicated functions.

Another advantage of ACSQ is that it lends itself well to investigations of quantum corrections to the system in question. Useful for this are *lower symbols* of phase space functions, which are the expectation values of the corresponding operators with regard to coherent states. These can be interpreted as semiclassical or quantum corrected versions of the original phase space functions. Indeed, we show in Ch. 4 that the lower symbol to a Hamiltonian approximately generates the dynamics of coherent states via the associated Hamilton equations, which can then be interpreted as dynamics of the system in a quantum corrected phase space.

This we apply to the phase space formulation of the OS model from Ch. 3, first for the Hamiltonian for the comoving observer. Remarkably, quantizing it via ACSQ yields an operator identical to what one would find via Dirac quantization, which was already discussed in Chs. 2 and 3. This is usually not the case for operators found in ACSQ, and this is not even where this analogy ends: computing the lower symbol of this Hamiltonian we find that it generates exactly the same bouncing trajectory that emerged from the expectation value of the wave packet from Ch. 2. Much of the previous analysis can then be applied to the comoving observer in our quantum OS model, as already noted in Ch. 3.

Thanks to the advantages of ACSQ, it is also possible to discuss the exterior stationary observer more carefully. First, we explain that one can turn the almost deparametrizable constraint into a proper deparametrizable one by admitting a multivalued physical Hamiltonian with two branches, one for the inside and one for the outside of the horizon. Multivalued Hamiltonians have been discussed in the literature before, see for example Refs. [94–97], but this turns out to not be relevant for our purposes: the operators to both branches of the Hamiltonian are well-defined, but still quite complicated integral operators, hence we restrict our discussion to the quantum corrected dynamics of the lower symbol of the Hamiltonian, where we can treat the two branches as largely independent of one another.

The result of these investigations is that from the viewpoint of the exterior observer the dust cloud can still bounce, but with some unfortunate restrictions. First, this bounce does not occur for every choice of the quantization ambiguities, and how one has to make that choice depends on the mass of the dust cloud; there is no choice for these ambiguities for which every cloud will bounce. Clouds which do not bounce instead behave very similarly to the classical case, they approach the horizon asymptotically. Further, the minimal radius of the bounce is much higher than for the comoving observer: it has to be outside of not only the horizon but also the *photon sphere* at $R = 3M$. In the Schwarzschild case, the photon sphere is the outer boundary of the region in which photons can be caught by the gravitational attraction of the central object. Its photon sphere, or its analogue in non-spherically symmetric spacetimes, is closely related to a black hole's shadow [98]. The fact that no photon sphere can form for our quantum OS model from the point of view of the exterior observer thus means that at no stage of the bounce the dust cloud looks like a black hole.

In hindsight it is not hard to see why this happens: in the construction in Ch. 3 we have apparently ingrained the classical Schwarzschild exterior too deeply into the model, through the use of the Painlevé-Gullstrand coordinate transformation and the classical reduction of the exterior variables. Thus, the model is forced to take the only path it can to unify a bounce and a Schwarzschild exterior, and bounces far away from the horizon. Our efforts to investigate how the exterior is affected by quantum gravitational effects to accommodate a bounce, via the detour over the exterior stationary observer, has thus unfortunately not borne fruit.

Lastly in Ch. 4, we also discuss how the two quantum theories for the two observers can be related at the quantum level. We present an approach in which a switch between those observers can be done by modifying the canonical commutation relations. The quantum theories can then be mapped onto each other purely at the level of their operator algebras; this map can not be represented as acting on the Hilbert spaces. Useful for this switch at a technical level is the fact that the identification between affine group and phase space in ACSQ is not unique. This will be the topic of Appendix B.

In Ch. 5, containing Ref. [85], I finally directly investigate corrections to the Schwarzschild exterior by determining what kind of exterior geometries can be matched classically to a bouncing Friedmann interior. This can be done via a construction similar to the LTB model: I make an ansatz that contains both the Friedmann interior and also possible exteriors, and then determine from this the boundary conditions for the exterior metric at the surface of the dust cloud. The result is a class of geometries with one free function and a particular boundary condition for this function depending on the trajectory of the cloud. This trajectory is kept open to begin with, not fixing a particular form of the bounce.

Since there is then a huge amount of freedom in choosing this free function and with that the exterior, I discuss two particular examples. First, there is a unique static exterior for every trajectory of the interior. I demonstrate that the bounce has to happen in an untrapped region, meaning that should one want a horizon to form in the exterior, there needs to be at least a second one to match it. I further show that this horizon cannot be crossed in finite Killing time. To be able to then still accommodate the bounce, the causal structure of the static exterior is necessarily somewhat complicated: the dust cloud needs to be able to expand towards a different asymptotic infinity than it started its collapse from. When one inserts the bouncing trajectory in comoving time that appeared in Chs. 2. 3 and 4, the causal structure that emerges is similar to that of a Reissner-Nordström black hole.

In order to evade these more involved causal structures one thus needs to consider dynamic exteriors. A particularly simple example that I investigate in Ch. 5 is a generalization of the Schwarzschild spacetime in Painlevé-Gullstrand form, where one allows the mass to vary with dust proper time. This time-dependent mass is fully determined by the bouncing trajectory of the dust cloud. The resulting exterior has a single dynamic horizon, trapping to start with. The horizon contracts when the dust cloud approaches its minimal size, disappears at the moment of the bounce, and emerges again afterwards as antitrapping. I discuss some aspects of this procedure, in particular the timescales involved. With one of these I quantify how long it takes the horizon to transition from trapping to antitrapping. This timescale turns out to have an upper bound proportional to the mass of the dust cloud, and is thus comparatively short.

The other timescale is the familiar black hole lifetime. Due to the non-stationarity of the exterior, we cannot use Killing time as a shorthand for a far-away observer. It is thus not straightforward to find a good definition for this lifetime. I construct it by letting null geodesics be absorbed by the horizon, and tracing them back to spatial infinity. Unfortunately, a familiar result emerges: the black hole lifetime is proportional to the dust cloud's total mass, similar to the transition timescale. I argue that this result also applies to other dynamic exteriors, all with the same classical limit as the one considered by me.

In Appendix A, containing Ref. [86] co-authored by Nick Kwidzinski, Jan J. Ostrowski, Włodzimierz Piechocki, Daniele Malafarina, and me, we construct Hamiltonians for gravitational collapse with a different procedure than in Ch. 3, both for the OS model and for single shells in the LTB model. We follow thereby Ref. [99], in which this is done for an isolated shell. The procedure is as follows: we fix the time coordinate in the interior to be dust proper time, and in the exterior Schwarzschild Killing time, and reduce the Einstein-Hilbert action accordingly. During this, the dynamical quantities are the radius of the dust cloud, and the Schwarzschild mass. Further, the hyperbolic angle between the surfaces of constant dust proper time and those of constant Schwarzschild Killing time turns out to be very useful. The resulting Hamiltonian is then the total mass of the dust cloud. Unfortunately it is of a significantly more complicated form than what I derived in Ch. 3, and thus does not seem suitable for quantization.

The largest difference between the approaches in Appendix A and Ch. 3 lies in the treatment of time. While in Ch. 3 the two notions of time where identified from the phase space coordinates fairly late in the procedure, the time coordinate in Appendix A was fixed with the foliation of the spacetime right at the beginning, similarly to Ch. 2.

Finally, in the article in Appendix B, co-authored by Andrzej Góźdź, Włodzimierz Piechocki, and me, a fairly technical aspect of ACSQ is discussed. I mentioned above that ACSQ is based on an identification of the affine group with the phase space of the to-be-quantized theory. This identification is by no means unique; one can choose to parametrize the affine group differently with the phase space coordinates. We show that different such parametrizations lead to unitarily inequivalent quantum theories.

References

[1] A. Einstein, in *The Collected Papers of Albert Einstein, Volume 6: The Berlin Years: Writings, 1914-1917*, edited by A. J. Kox, M. J. Klein, and R. Schulmann (Princeton University Press, Princeton, 1996), pp. 234–242.

[2] V. P. Abbot *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. **116**, 061102 (2016).

[3] K. Akiyama *et al.* (The Event Horizon Telescope Collaboration), Astrophys. J. **875** (1), L1 (2019).

[4] G. F. R. Ellis and B. G. Schmidt, Gen. Relativ. Gravit. **8**, 915 (1977).

[5] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, 1973).

[6] B. Datt, Z. Phys. **108** 314 (1938).

[7] J. R. Oppenheimer and H. Snyder, Phys. Rev. **56**, 455 (1939).

[8] R. M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).

[9] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Princeton University Press, Princeton, 2017).

[10] E. Poisson, *A Relativist's Toolkit: The Mathematics of Black-Hole Mechanics* (Cambridge University Press, Cambridge, 2004).

[11] V. Faraoni, *Cosmological and Black Hole Apparent Horizons*, Lecture Notes in Physics (Springer International Publishing, New York, 2015).

[12] K. Martel and E. Poisson, Am. J. Phys. **69**, 476 (2001).
[13] R. Gautreau and B. Hoffmann, Phys. Rev. D **17**, 2552 (1978).
[14] R. J. Adler, J. D. Bjorken, P. Chen, and J. S. Liu, Am. J. Phys. **73**, 1148 (2005).
[15] A. G. Lemaître, Gen. Relativ. Gravit. **29**, 641 (1997).
[16] R. C. Tolman, Proc. Natl. Acad. Sci. USA **20**, 169 (1934).
[17] H. Bondi, Mon. Not. R. Astron. Soc. **107**, 410 (1947).
[18] A. Krasiński, *Inhomogeneous Cosmological Models* (Cambridge University Press, Cambridge, 1997).
[19] S. Surya, Living Rev. Relativity **22**, 5 (2019).
[20] M. Reuter and F. Saueressig, New J. Phys. **14**, 055022 (2012).
[21] R. Loll, Classical Quantum Gravity **37**, 013002 (2020).
[22] A. Perez, Classical Quantum Gravity **20**, R43 (2003).
[23] V. E. Hubeny, Classical Quantum Gravity **32**, 124010 (2015).
[24] C. Kiefer, *Quantum Gravity*, 3rd ed. (Oxford University Press, Oxford, 2012).
[25] J. Mielczarek and T. Trześniewski, Gen. Relativ. Gravit. **50**, 68 (2018).
[26] R. Arnowitt, S. Deser, and C. W. Misner, in *Gravitation: an introduction to current research*, edited by L. Witten, (Wiley, New York, 1962), pp. 227–264.
[27] J. A. Wheeler, in *Battelle rencontres*, edited by C. M. DeWitt and J. A. Wheeler, (Benjamin, New York, 1968), pp. 242–307.
[28] B. S. DeWitt, Phys. Rev. **160**, 1113 (1967).
[29] A. Ashtekar, T. Pawlowski, and P. Singh, Phys. Rev. Lett. **96**, 141301 (2006).
[30] S. Gielen and N. Turok, Phys. Rev. Lett. **117**, 021301 (2016).
[31] N. Pinto-Neto, Universe **7**, 110 (2021).
[32] C. R. Almeida, H. Bergeron, J. P. Gazeau, and A. C. Scardua, Ann. Phys. (Berlin) **392**, 206 (2018).
[33] H. Bergeron, A. Dapor, J. P. Gazeau, and P. Małkiewicz, Phys. Rev. D **89**, 083522 (2014).
[34] H. Bergeron, E. Czuchry, J. P. Gazeau, and P. Małkiewicz, Universe **6**, 7 (2019).
[35] A. Góźdź, W. Piechocki, and G. Plewa, Eur. Phys. J. C **79**, 45 (2019); A. Góźdź and W. Piechocki, *ibid.* **80**, 142 (2020).
[36] A. Ashtekar, J. Olmedo, and P. Singh, Phys. Rev. D **98**, 126003 (2018).
[37] C. Barceló, R. Carballo-Rubio, and L. J. Garay, Classical Quantum Gravity **34**, 105007 (2017).
[38] V. P. Frolov and G. A. Vilkovisky, Phys. Lett. B **106**, 307 (1981).
[39] R. Casadio, Int. J. Mod. Phys. D **09**, 511 (2000).
[40] P. Hájíček and C. Kiefer, Nucl. Phys. B **603**, 531 (2001); P. Hájíček, *ibid.* **603**, 555 (2001).
[41] P. Hájíček and C. Kiefer, Int. J. Mod. Phys. D **10**, 775 (2001).

[42] P. Hájíček, in: *Quantum Gravity: From Theory to Experimental Search*, edited by D. Giulini, C. Kiefer, and C. Lämmerzahl (Springer, Berlin Heidelberg, 2003), pp. 255–299; C. Kiefer, arXiv:1512.08346 [gr-qc].
[43] J. Ziprick and G. Kunstatter, Phys. Rev. D **80**, 024032 (2009).
[44] C. Bambi, D. Malafarina, and L. Modesto, Phys. Rev. D **88**, 044009 (2013).
[45] Y. Liu, D. Malafarina, L. Modesto, and C. Bambi, Phys. Rev. D **90**, 044040 (2014).
[46] C. Bambi, D. Malafarina, and L. Modesto, Eur. Phys. J. C **74**, 2767 (2014).
[47] C. Barceló, L. J. Garay, and G. Jannes, Found. Phys. **41**, 1532 (2011).
[48] C. Barceló, R. Carballo-Rubio, and L. J. Garay, Int. J. Mod. Phys. D **23**, 1442022 (2014); C. Barceló, R. Carballo-Rubio, L. J. Garay, and G. Jannes, Classical Quantum Gravity **32**, 035012 (2015).
[49] J. G. Kelly, R. Santacruz, and E. Wilson-Ewing, Classical Quantum Gravity **38**, 04LT01 (2021).
[50] S. Chowdhury, K. Pal, K. Pal, and T. Sarkar, Phys. Lett. B **816**, 136269 (2021).
[51] K. Giesel, B.-F. Li, and P. Singh, arXiv:2107.05797 [gr-qc].
[52] D. Malafarina, Universe **3**, 48 (2017).
[53] M. Ambrus and P. Hájíček, Phys. Rev. D **72**, 064025 (2005).
[54] M. Christodoulou, C. Rovelli, S. Speziale, and I. Vilensky, Phys. Rev. D **94**, 084035 (2016).
[55] M. Christodoulou and F. D'Ambrosio, arXiv:1801.03027 [gr-qc].
[56] H. M. Haggard and C. Rovelli, Phys. Rev. D **92**, 104020 (2015).
[57] T. De Lorenzo and A. Perez, Phys. Rev. D **93**, 124018 (2016).
[58] C. Rovelli and P. Martin-Dussaud, Classical Quantum Gravity **35**, 147002 (2018).
[59] C. Rovelli and P. Martin-Dussaud, Classical Quantum Gravity **36**, 245002 (2019).
[60] J. Ben Achour, S. Brahma, and J.-P. Uzan, J. Cosmol. Astropart. Phys. 03 (2020) 041.
[61] J. Ben Achour and J.-P. Uzan, Phys. Rev. D **102**, 124041 (2020).
[62] J. Ben Achour, S. Brahma, S. Mukohyama, and J.-P. Uzan, J. Cosmol. Astropart. Phys. 09 (2020) 020.
[63] H. Chakrabarty, A. Abdujabbarov, D. Malafarina, and C. Bambi, Eur. Phys. J. C **80**, 373 (2020).
[64] J. Münch, arXiv:2010.13480 [gr-qc].
[65] J. Münch, arXiv:2103.17112 [gr-qc].
[66] A. Barrau, B. Bolliet, F. Vidotto, and C. Weimer, J. Cosmol. Astropart. Phys. 02 (2016) 022.
[67] M. Bojowald, Universe **6**, 125 (2020).
[68] S. Hossenfelder, L. Modesto, and I. Prémont-Schwarz, Phys. Rev. D **81**, 044036 (2010).
[69] L. Mersini-Houghton, Phys. Lett. B **738**, 61 (2014).
[70] C. Vaz, Nucl. Phys. B **891**, 558 (2015).

[71] H. Arfaei and M. Noorikuhani, J. High Energy Phys. 02 (2017) 124.

[72] F. Lund, Phys. Rev. D **8**, 3253 (1973).

[73] Y. Peleg, Phys. Lett. B **356**, 462 (1995).

[74] C. Vaz and L. Witten, Phys. Rev. D **64**, 084005 (2001).

[75] S. A. Hayward, Phys. Rev. Lett. **96**, 031103 (2006).

[76] S. Ansoldi, arXiv:0802.0330 [gr-qc].

[77] R. Carballo-Rubio, F. Di Filippo, S. Liberati, and M. Visser, Classical Quantum Gravity **37**, 145005 (2020).

[78] M. Visser and D. L. Wiltshire, Classical Quantum Gravity **21**, 1135 (2004).

[79] S. D. Mathur, Fortsch. Phys. **53**, 793 (2005).

[80] C. Rovelli and F. Vidotto, Int. J. Mod. Phys. D **23**, 1442026 (2014).

[81] T. De Lorenzo, C. Pacilio, C. Rovelli, and S. Speziale, Gen. Relativ. Gravit. **47**, 41 (2015).

[82] C. Kiefer and T. Schmitz, Phys. Rev. D **99**, 126010 (2019).

[83] T. Schmitz, Phys. Rev. D **101**, 026016 (2020).

[84] W. Piechocki and T. Schmitz, Phys. Rev. D **102**, 046004 (2020).

[85] T. Schmitz, Phys. Rev. D **103**, 064074 (2021).

[86] N. Kwidzinski, D. Malafarina, J. J. Ostrowski, W. Piechocki, and T. Schmitz, Phys. Rev. D **101**, 104017 (2020).

[87] A. Góźdź, W. Piechocki, and T. Schmitz, Eur. Phys. J. Plus **136**, 18 (2021).

[88] K. V. Kuchař, Phys. Rev. D **50**, 3961 (1994).

[89] J. D. Brown and K. V. Kuchař, Phys. Rev. D **51**, 5600 (1995).

[90] H. Bergeron and J. Gazeau, Ann. Phys. (N.Y.) **344**, 43 (2014).

[91] J. P. Gazeau and R. Murenzi, J. Math. Phys. (N.Y.) **57**, 052102 (2016).

[92] C. R. Almeida, H. Bergeron, J. P. Gazeau, and A. C. Scardua, Ann. Phys. (N.Y.) **392**, 206 (2018).

[93] A. M. Perelomov, Commun. Math. Phys. **26**, 222 (1972).

[94] M. Henneaux, C. Teitelboim, and J. Zanelli, Phys. Rev. A **36**, 4417 (1987).

[95] A. Shapere and F. Wilczek, Phys. Rev. Lett. **109**, 200402 (2012).

[96] L. Zhao, P. Yu, and W. Xu, Mod. Phys. Lett. A **28**, 1350002 (2013).

[97] S. Ruz, R. Mandal, S. Debnath, and A. K. Sanyal, Gen. Relativ. Gravit. **48**, 86 (2016).

[98] A. Grenzebach, V. Perlick, and C. Lämmerzahl, Phys. Rev. D **89**, 124004 (2014).

[99] J. Kijowski, G. Magli, and D. Malafarina, Phys. Rev. D **74**, 084017 (2006).

2. Singularity avoidance for collapsing quantum dust in the Lemaître-Tolman-Bondi model

C. Kiefer and T. Schmitz, Phys. Rev. D 99, 126010 (2019)

This article is concerned with a quantization of the marginally bound LTB model. The individual dust shells in the model are split up, quantized individually, and then reassembled into a complete, quantum corrected dust cloud. The problem is further explicitly formulated with regard to dust proper time, corresponding to the point of view of a comoving observer. Particular care is taken to leave open as many quantization ambiguities as possible, to be able to investigate the robustness of the resulting quantum model and its behavior.

It is shown that the model quite generically avoids the classical singularity, as long as the factor ordering is not of a particular class: unitarily evolving wave packets fall off towards the singularity in such a way that the shell can never actually reach it. By investigating a particular wave packet one can see that this singularity avoidance is facilitated by a bounce. Based on the dynamics of this wave packet, a quantum corrected, bouncing LTB model can be constructed, based on which the behavior of the horizon, the black hole lifetime, and the effective pressures that cause the bounce from a spacetime perspective can be discussed.

Please note that the article is based on my master thesis titled "Singularity Avoidance of the Quantum LTB Model for Gravitational Collapse" supervised by Prof. Dr. Claus Kiefer at the University of Cologne. Accordingly, most of its content can already be found in that thesis, with the exception of the construction of self-adjoint extensions for the Hamiltonian in Sec. IIIB and Appendices A and B; I have reworked them to put the model on a mathematically more rigorous footing. As a consequence, also the results concerning singularity avoidance in Sec. IIIC are slightly changed, but the general sentiment remains the same. The wave packet discussed for the rest of the paper did not need modification.

Over the course of working on my master thesis, the model discussed in the article has been developed by me under the guidance of Claus Kiefer. I wrote the first draft of the article, including the creation of all figures, which Claus Kiefer and I revised into its final form in collaboration.

3. Towards a quantum Oppenheimer-Snyder model

T. Schmitz, Phys. Rev. D 101, 026016 (2020)

In this article, a consistent phase space formulation of the classical OS model is presented, in preparation of the construction of a quantum OS model in Ch. 4. This phase space formulation contains the degrees of freedom of the homogeneous dust cloud, and initially field theoretic degrees of freedom describing the spherically symmetric exterior as well. It is however possible to reduce the latter out of the model via an elaborate series of canonical transformations.

Particular care is taken to make sure that the surface of the dust cloud as a boundary to the exterior does not spoil those canonical transformations. From the condition that the resulting boundary terms should vanish, one finds the Painlevé-Gullstrand coordinate transformation in disguise. The same coordinate transformation, promoted to a canonical transformation on phase space, is also used to switch between Schwarzschild Killing time and dust proper time in the interior degrees of freedom. This results in two versions of this formulation of the OS model, one from the point of view of the comoving observer and one from the point of view of the exterior stationary observer. Quantization of these two versions is tentatively discussed, although a detailed construction of the full quantum OS model is left for Ch. 4.

The article is single-authored; I have developed both its conceptual and technical aspects and all writing is my own, including the creation of all figures.

4. Quantum Oppenheimer-Snyder model

W. Piechocki and T. Schmitz, Phys. Rev. D 102, 046004 (2020)

In this article we present a quantum OS model, based on the classical formulation developed in Ch. 3. This model contains two separate quantum models, one using dust proper time as the notion of time, and thus taking the viewpoint of the comoving observer, and one using Schwarzschild Killing time for the exterior stationary observer. The quantization is undertaken using the integral quantization method ACSQ, necessary due to the complicated form of the Hamiltonian for the exterior observer. The focus is on quantum corrected phase space dynamics.

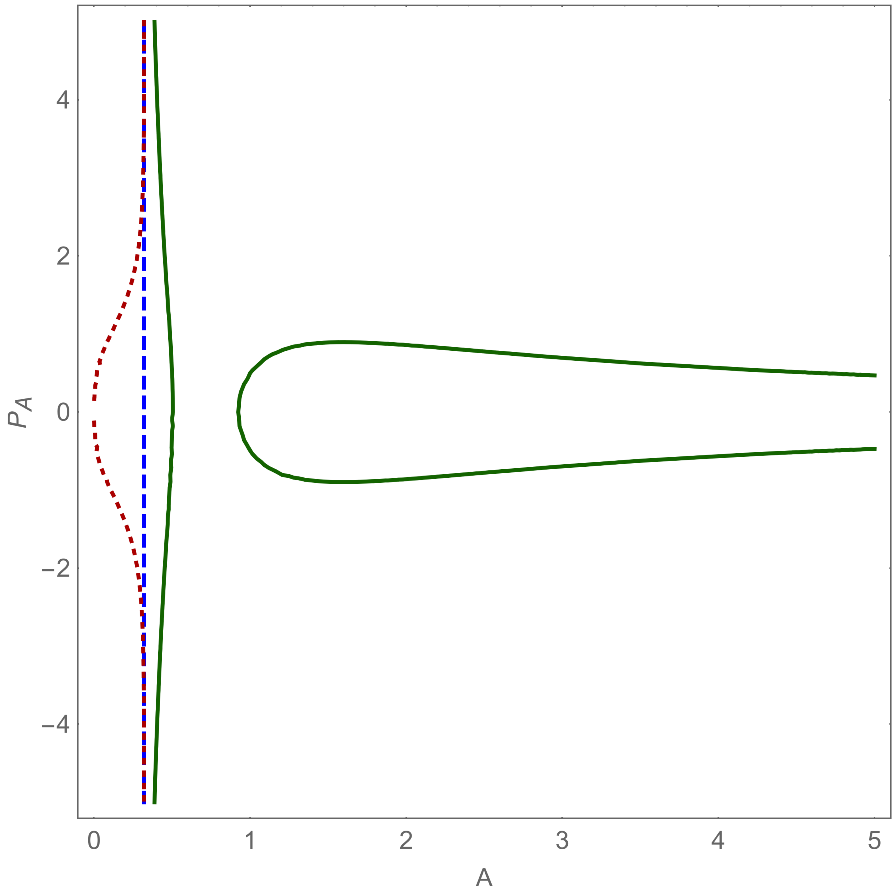
For the comoving observer, the quantum theory according to ACSQ resembles very closely that achieved by Dirac's usual prescription for quantization, which was discussed in Chs. 2 and 3. Thus many of the results of those earlier chapters carry over, in particular the familiar bouncing trajectory reappears.

For the exterior observer, the quantum corrected dynamics can also exhibit a bounce, but unfortunately this bounce is less robust under quantization ambiguities than for the comoving observer, and the minimal radius of the dust cloud is always outside of the photon sphere: nothing resembling a black hole forms during the bounce. The limitations of the model possibly leading to this result are discussed, along with potential avenues to reintroduce black holes.

Finally, a switch between the two quantum theories, and with that between the two observers, by way of modifying the commutation relations is discussed. This switch can be understood as a map between the operator algebras of the two theories, but it cannot be represented as a map between the Hilbert spaces. Integral to how this switch is implemented is a particular feature of ACSQ, discussed in more detail in Appendix B.

The article was co-authored by Włodzimierz Piechocki and me. Włodzimierz Piechocki introduced me to ACSQ and proposed to apply it to the Hamiltonian describing the LTB model in Ch. 2. I expanded the scope of the project and applied the method to my canonical formulation of the OS model discussed in Ch. 3, which includes the aforementioned Hamiltonian. All calculations were done by me, as well as writing the first draft and creating all figures. I then revised this first draft into the final article with the help of Włodzimierz Piechocki.

Unfortunately I have to add a small erratum at this point: Fig. 2c in the article below was accidentally replaced by a duplicate of Fig. 2b. In place of what is shown in the article as Fig. 2c, the following figure should appear.



(c) $M = 0.4$ and $\beta = 5$.

5. Exteriors to bouncing collapse models

T. Schmitz, Phys. Rev. D 103, 064074 (2021)

In this article, possible exterior geometries of bouncing collapse models are discussed by way of classical matching conditions to a bouncing Friedmann interior. A broad class of exteriors is presented, constructed in a procedure inspired by the classical LTB model. Since this class is quite large, two particular special cases are discussed, both for a general bouncing trajectory and for the specific one from Chs. 2, 3 and 4.

For every bouncing trajectory there is a unique static exterior. Those exteriors necessarily have a somewhat involved causal structure; the bounce takes place in an untrapped region, so if an outer horizon exists, there needs to be a complementary inner horizon. Further, the dust cloud needs to re-expand toward a different asymptotic infinity than it started its collapse from. For the specific bouncing trajectory from earlier chapters, a causal structure similar to that of a Reissner-Nordström black hole emerges.

To have a bounce with a more simple causal structure, one necessarily needs to consider dynamic exteriors. A particularly simple example of such an exterior is considered, a generalization of a Schwarzschild exterior where the mass is allowed to vary with dust proper time. There is then just a single horizon, but its position varies with time: it starts out as trapping, retracts and disappears during the bounce, and reemerges as antitrapping afterwards. Two timescales are discussed, one for the transition of the horizon from trapping to antitrapping, and the black hole lifetime. Both have an upper bound proportional to the total mass of the dust cloud. The latter of these results seems to be fairly general, it should also apply to other dynamic exteriors.

The article is single-authored; I have developed both its conceptual and technical aspects and all writing is my own, including the creation of all figures.

6. Conclusions

In this thesis we have seen that quantum gravitational effects can lead to an avoidance of the classical singularity in gravitational collapse via a bounce: the collapse is halted at a minimal radius before the singularity forms, and is followed by a re-expansion. We have investigated this scenario via the construction of a few toy models. As already discussed in the introduction, this bounce emerges in different models for gravitational collapse across various approaches to quantum gravity, and in addition in Ch. 2 we have shown that it is robust with respect to the quantization ambiguities in the model discussed there.

Unfortunately, a fully consistent model for bouncing collapse has turned out to be outside of the grasp of our investigations. In the quantum OS model in Ch. 4, from the point of view of the exterior stationary observer the bounce takes place outside of the photon sphere; no horizon ever forms, and the bouncing object never resembles a black hole. When we directly constructed exterior geometries to a bouncing interior through a classical matching procedure in Ch. 5, it emerged that one can either have stationary ones with involved causal structures, or dynamic exteriors for which the black hole lifetime is too short.

It might be the case that it is futile to search for a consistent *semiclassical* bounce, described in terms of a spacetime, and one would be better advised to focus on its quantum nature. The investigations in the spin foam formalism in [1, 2] show that considering purely quantum gravitational properties with no direct analogue in a spacetime picture can lead to a longer black hole lifetime. In Ch. 2 a similar if more makeshift construction based on discrete states for the bounce was shown to yield a lifetime proportional to M^3, where M is the total mass of the collapsing matter, a result that we also could not confirm in a quantum-corrected spacetime picture. Further examples are the idea of a grey horizon in Ref. [3] and Ch. 2, and the appealing bounce model from Refs. [4, 5], for which the authors had to forego exact covariance.

Additional support for this assertion comes from Chs. 2 and 4, where we have found that the minimal radius of the collapsing matter during the bounce is far in the sub-Planckian regime, scaling with $M^{-1/3}$. This could be taken as an indication that the region around the bounce is truly quantum gravitational, and a spacetime description of it is insufficient. The black hole as an intermediate state during the bounce would then have a small quantum core at its center. The properties of this core cannot be expected to be largely agnostic with regard to different quantum gravity approaches.

However, all that we have observed of black holes so far points toward them being described reasonably well in terms of a spacetime geometry. Those properties of bouncing collapse that truly cannot be expressed in these terms should hence have observable effects small enough to be out of reach today, or be hidden behind horizons in what one might call quantum censorship. Going beyond the spacetime picture necessarily has to be succeeded by an investigation of these observational aspects in a second step. It might thus be worthwhile to also interrogate other assumptions and simplifications we have made use of to build our models.

For example, in Ch. 5 we have assumed an exact matching between bouncing Friedmann interior and the exterior geometry, meaning the surface of the bouncing dust cloud as the matching surface does not carry any additional energy. Non-exact matching plays a role in the model from Refs. [4, 5], and was discussed in general terms in Ref. [6].

Further, the inverse scaling of the minimal radius of the bounce with the mass of the dust cloud in Chs. 2 and 4 suggests that an initially homogeneous bouncing dust cloud might necessarily turn inhomogeneous close to the bounce: generalizing the relationship between mass and minimal radius to other shells in the dust cloud than the outermost one, one finds that shells closer to the center, with less mass contained inside them, should have a larger minimal radius than shells closer to the surface. The cloud would then turn inside out, a process that certainly cannot emerge from investigations restricted to homogeneity.

Another simplifying assumption was to only consider marginally bound dust clouds, corresponding to flat interiors in the OS model. More relevant for astrophysical considerations would be dust clouds starting their collapse at rest from a finite radius, corresponding to closed OS models. Close to the singularity there is not much of a difference in the behavior of the dust cloud in those two cases. However, during the re-expansion, the different initial conditions could become relevant again, as explained for example in Refs. [7, 8]: where the flat cloud expands outwards indefinitely, the closed one would re-collapse from the radius it started its initial collapse from, leading to oscillations between collapse and expansion. With some additional dissipative mechanisms, these oscillations could stabilize into a new equilibrium configuration. We have already mentioned that the closed case is technically more demanding to treat, especially when considering the exterior stationary observer, but constructing a concrete closed bouncing model to investigate these assertions would certainly be worth the trouble.

In this scenario consisting of multiple bounces, a short lifetime of every individual bounce would even be an advantage, since it shortens the time it takes for the stable compact object to form. It has also been argued elsewhere that a short lifetime makes bouncing collapse more robust with regard to instabilities of the antitrapping horizon, namely Eardley's instability due to classical accretion [9], and instabilities due to backreaction from quantum fields [10]. In the latter reference, a different scenario was proposed as an alternative to oscillations between collapse and expansion: a time-asymmetry was introduced into the model, such that the white hole horizon can exist for short enough to not be unstable, but the black hole horizon has a long lifetime to be consistent with observations. Taking into account the possibility of a similar time-asymmetry in the exterior geometries from Ch. 5 would be interesting for future investigations.

These possible instabilities highlight that one should not be content to consider collapse models in isolation, both in the sense that astrophysical black holes do not exist in a vacuum, and also by allowing other (matter) fields in the models to begin with. Regarding the latter, it has already been shown in Ref. [11] that backreaction from Hawking radiation can halt the classical collapse close to the horizon. One could follow the same procedure to introduce Hawking evaporation into a bouncing collapse model, for example with an exterior in accordance with Ch. 5.

Allowing interactions with an environment also opens up the possibility to discuss decoherence. Decoherence has been applied before to quantum cosmology and also black holes, see for example Refs. [12–14] and for an overview Ref. [15]. In particular, it has been shown in Ref. [13] that for quantized two-dimensional dilatonic black holes, interaction with a scalar field leads on the one hand to Hawking radiation, and on the other hand to a re-emergence of classical black holes through decoherence. Thus the question presents itself whether bounces would be suppressed via this mechanism, and whether a new avenue for singularity avoidance would appear. Concretely, an investigation of this could for example involve coupling a bouncing model to an environment of one's choice, and finding the pointer basis for the bouncing model via a so-called predictability sieve [16]; the pointer basis, the basis selected by the environment through decoherence, is then the one that best preserves its purity throughout these environmental interactions.

Relatedly, a further interesting question concerns the behavior of entropy during the bounce. Through their Bekenstein-Hawking entropy, black holes are considered highly entropic systems, hence letting one disappear via a bounce or otherwise is no small feat. A similar issue is much discussed in the context of Hawking evaporation. There, small remnants as the endstate of evaporation are usually considered unphysical, since such remnants would need to carry a large amount of entanglement entropy. However, much of this discussion of black hole entropy is based on stationary, eternal black holes and their *event* horizons. Black holes from bouncing collapse only have apparent horizons, with finite lifetime and possibly non-stationary according to Ch. 5, so it should be carefully considered which arguments carry over, and which do not.

A complete discussion of this would also require including more realistic matter models. Astrophysical collapse and black holes as they exist in the universe are much more complicated systems than we have considered with our simple models, hence one could make the models almost arbitrarily more involved. A further obvious avenue for such an improvement would be to add rotation. This is a technically quite demanding ask, one only needs to compare the line elements for a Schwarzschild and a Kerr black hole to appreciate the difference in complexity, but this step will eventually need to be taken if one wants to connect the bouncing models to potential observations.

Possible observable effects of bouncing collapse have been discussed before [17–21], largely qualitatively due to the simple nature of the bouncing collapse models available. Particularly the relationship between lifetime and mass of the black hole is important, as it determines the mass of black holes that can be observed to bounce today. In light of the astrophysical observations connected to black holes in recent years, namely the detection of gravitational waves and the imaging of a black hole's shadow, naturally the question emerges whether one could find signatures of quantum gravitational corrections there. So far, all observations agree very well with the classical predictions of GR, but there are indications that with increased precision of the measurements some quantum gravity predictions could be tested [22–25]. Previous investigations in this direction wisely consider fairly arbitrary departures from classical GR, but it would be interesting to see how in particular the shadow for a dynamical exterior with a non-stationary horizon in the vein of Ch. 5 would be modified around the bounce.

In summary one can say that plenty of work remains to be done. As we have seen, there are quite a few avenues one can follow to improve on the existing bouncing collapse models, and hopefully find a consistent such scenario. While the investigations presented in this thesis were not entirely successful in this regard, I believe that they will still be useful in future endeavors. For example, how the LTB model was quantized in Ch. 2 by splitting it up into individual shells, to be reassembled afterwards, can be adapted to other approaches to quantum gravity. The construction in Chs. 3 and 4 can be generalized to closed quantum OS models to investigate possible oscillations between collapse and expansion. The same can be done for the exteriors in Ch. 5. Further, the method of finding possible exteriors in Ch. 5 via an LTB-like geometry is quite general and much more convenient than directly considering the matching conditions.

Bouncing collapse might so far resist our efforts to fully explain how it might emerge from a full theory of quantum gravity in a way that is consistent, both with present observations and our rich conceptual understanding of black holes, but it is still a promising scenario for singularity avoidance in quantum gravitational collapse; it only relies on basic features of quantum theories and GR, and accordingly indications in its favor are found across many different approaches to quantum gravity. Aside from presenting exciting possible observational windows into quantum gravity, by necessitating quantum modifications of the geometry as far away from the singularity as the near-horizon region, it is also a fascinating subject of study on purely conceptual grounds, forcing us to re-evaluate many aspects of black holes usually taken for granted.

References

[1] M. Christodoulou, C. Rovelli, S. Speziale, and I. Vilensky, Phys. Rev. D **94**, 084035 (2016).

[2] M. Christodoulou and F. D'Ambrosio, arXiv:1801.03027 [gr-qc].

[3] P. Hájíček and C. Kiefer, Nucl. Phys. B **603**, 531 (2001); P. Hájíček, *ibid.* **603**, 555 (2001).

[4] J. G. Kelly, R. Santacruz, and E. Wilson-Ewing, Classical Quantum Gravity **38**, 04LT01 (2021).

[5] J. Münch, arXiv:2103.17112 [gr-qc].

[6] J. Ben Achour, S. Brahma, and J.-P. Uzan, J. Cosmol. Astropart. Phys. 03 (2020) 041.

[7] C. Barceló, L. J. Garay, and G. Jannes, Found. Phys. **41**, 1532 (2011).

[8] C. Barceló, R. Carballo-Rubio, and L. J. Garay, Int. J. Mod. Phys. D **23**, 1442022 (2014); C. Barceló, R. Carballo-Rubio, L. J. Garay, and G. Jannes, Classical Quantum Gravity **32**, 035012 (2015).

[9] C. Barceló, R. Carballo-Rubio, and L. J. Garay, J. High Energy Phys. 01 (2016) 157.

[10] T. De Lorenzo and A. Perez, Phys. Rev. D **93**, 124018 (2016).

[11] L. Mersini-Houghton, Phys. Lett. B **738**, 61 (2014).

[12] C. Kiefer and H. D. Zeh, Phys. Rev. D **51**, 4145 (1995).

[13] J.-G. Demers and C. Kiefer, Phys. Rev. D **53**, 7050 (1996).

[14] C. Kiefer, Classical Quantum Gravity **18**, L151 (2001).

[15] C. Kiefer, *Quantum Gravity*, 3rd ed. (Oxford University Press, Oxford, 2012).

[16] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).

[17] A. Barrau and C. Rovelli, Phys. Lett. B **739**, 405 (2014).

[18] A. Barrau, C. Rovelli, and F. Vidotto, Phys. Rev. D **90**, 127503 (2014).

[19] A. Barrau, B. Bolliet, F. Vidotto, and C. Weimer, J. Cosmol. Astropart. Phys. 02 (2016) 022.

[20] F. Vidotto, Found. Phys. **48**, 1380 (2018).

[21] A. Barrau, K. Martineau, and F. Moulin, Universe **4**, 102 (2018).

[22] S. B. Giddings and D. Psaltis, Phys. Rev. D **97**, 084035 (2018).

[23] S. B. Giddings, Universe **5**, 201 (2019).

[24] A. Held, R. Gold, and A. Eichhorn, J. Cosmol. Astropart. Phys. 06 (2019) 029.

[25] F. Moulin, A. Barrau, and K. Martineau, Universe **5**, 202 (2019).

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Finally, thank you to Tim, Christoph and Nick for many much needed diversions. Thank you to my parents Gaby and Michael and my brother Jan for their unwavering support whatever I decide to pursue. And thank you, Ilona. Without you I would be someone else entirely.

A. Hamiltonian formulation of dust cloud collapse

N. Kwidzinski, D. Malafarina, J. J. Ostrowski, W. Piechocki, and T. Schmitz, Phys. Rev. D 101, 104017 (2020)

Hamiltonians for both the OS model and single shells in the LTB model are derived in an approach where the foliation of the spacetime is fixed, in contrast to what was done in Ch. 3. In the interior, surfaces of constant dust proper time are considered, and in the exterior surfaces of constant Schwarzschild Killing time. The hyperbolic angle between these surfaces turns out to be a useful quantity, which makes the derivation of the Hamiltonian much more tractable. The value of this Hamiltonian turns out to match the total mass of the dust cloud, but is unfortunately of a much more involved form than what was found in Ch. 3.

The article was co-authored by Nick Kwidzinski, Jan J. Ostrowski, Włodzimierz Piechocki, Daniele Malafarina, and me. The project originally started with Nick Kwidzinski, Włodzimierz Piechocki and Daniele Malafarina. I then joined after some time and worked closely with Nick Kwidzinski to construct a canonical theory for the OS model, see Sec. III A and B, and Sec. IV. This process was guided by Włodzimierz Piechocki and Daniele Malafarina. Jan J. Ostrowski then applied our methods to the LTB model. Writing and revisions were done by everyone collaboratively. Nick Kwidzinski and I wrote the first draft of the sections mentioned above.

B. Dependence of the affine coherent states quantization on the parametrization of the affine group

A. Góźdź, W. Piechocki, and T. Schmitz,
Eur. Phys. J. Plus 136, 18 (2021)

During the construction of a quantum theory via ACSQ one has to identify the phase space of the corresponding classical theory with the affine group: the affine group is *parametrized* with the phase space coordinates. It is shown that different such parametrizations lead to unitarily inequivalent quantum theories.

The article was co-authored by Andrzej Góźdź, Włodzimierz Piechocki, and me. The project was based on my observation that different parametrizations of the affine group in ACSQ lead to different quantization maps. Włodzimierz Piechocki and Andrzej Góźdź then worked out a proof for the unitary inequivalence of the resulting quantum theories. In collaboration we revised their manuscript. I further investigated how one can choose a suitable quantization by considering commutation relations in different quantum theories, which can be found in Appendix B in the final article.

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Hiermit versichere ich an Eides statt, dass ich die vorliegende Dissertation selbstständig und ohne die Benutzung anderer als der angegebenen Hilfsmittel und Literatur angefertigt habe. Alle Stellen, die wörtlich oder sinngemäß aus veröffentlichten und nicht veröffentlichten Werken dem Wortlaut oder dem Sinn nach entnommen wurden, sind als solche kenntlich gemacht. Ich versichere an Eides statt, dass diese Dissertation noch keiner anderen Fakultät oder Universität zur Prüfung vorgelegen hat; dass sie - abgesehen von unten angegebenen Teilpublikationen und eingebundenen Artikeln und Manuskripten - noch nicht veröffentlicht worden ist sowie, dass ich eine Veröffentlichung der Dissertation vor Abschluss der Promotion nicht ohne Genehmigung des Promotionsausschusses vornehmen werde. Die Bestimmungen dieser Ordnung sind mir bekannt. Darüber hinaus erkläre ich hiermit, dass ich die Ordnung zur Sicherung guter wissenschaftlicher Praxis und zum Umgang mit wissenschaftlichem Fehlverhalten der Universität zu Köln gelesen und sie bei der Durchführung der Dissertation zugrundeliegenden Arbeiten und der schriftlich verfassten Dissertation beachtet habe und verpflichte mich hiermit, die dort genannten Vorgaben bei allen wissenschaftlichen Tätigkeiten zu beachten und umzusetzen. Ich versichere, dass die eingereichte elektronische Fassung der eingereichten Druckfassung vollständig entspricht.

Teilpublikationen:
Alle Publikationen sind in die Dissertation eingebunden, siehe Inhaltsverzeichnis.

Tim Schmitz, der 02.11.2021

Tim Schmitz