

# Calculation of street traffic emissions with a queuing model

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## Abstract

The aim of this article is to show the applicability of a simple traffic simulator for the calculation of emissions. Traffic is simulated by using a queuing model (Q-model) originally introduced to solve the dynamic traffic assignment problem (DTA). Although vehicle dynamics is modeled on a quite coarse level it can be shown that, given certain conditions, emissions computed with the Q-model agree fairly good with those computed with more detailed models. For this purpose, we have done extensive comparisons of emissions computed with a car-following model which explicitly mimics single vehicle dynamics.

**Keywords:** Traffic simulation, Queuing-model, Emissions.

## 1 Introduction

Traffic represents one of the largest sources of primary air pollutants in urban areas. Air-quality management becomes more and more important [1, 2]. For an air pollution abatement strategy it is first of all necessary to identify pollution sources and to quantify their emissions. An urban emission inventory should at

least include industrial sources, road transport and private heating systems. It should provide the emission-input data for air pollution models with a sufficient resolution in time and space. This emission inventory can either be generated by measurements, by simulation models or by a combination of both.

In particular, simulation models determining the effects of street traffic on air pollution are particularly valuable, since usually there are no emission measurements covering a whole urban area. Knowing the travel demand however, either from traffic counts or from statistical methods, simulation models allow to calculate the flows through the individual links of the network. For this, most of the models use static methods to solve the assignment problem, i.e. the distribution of the travel demand over the network. The emissions are then calculated by means of mean traffic loads and speeds [3].

However, the amount of emitted pollutants by traffic is not only dependent on mean traffic loads but also on dynamic effects as jamming and single vehicular dynamics [4]. A class of models that can provide dynamic information on traffic are microsimulation models that work on the level of individual vehicles. In addition, due to the microscopic approach they are suitable for scenario calculations to evaluate reduction

strategies since changes in infrastructure, traffic composition or route choice behavior are easy to build in. If they are also computationally efficient even the traffic of large cities can be computed in acceptable time. Therefore, dynamic microsimulation models can be a component of air-quality management systems [5, 6].

## 2 Model description

This article deals with the calculation of emissions by street traffic using microscopic simulation models. The focus will be put on the dynamical aspects which play an important role in this respect. In section 2.1, the Q-model is introduced. Then in section 2.2, a car-following model (SK-model) is presented. This will be used as a reference model to compute the emissions. Finally we will give a short description of the database used for the calculation of the emission.

### 2.1 Queuing-model

The Q-model is a vehicle oriented traffic flow model which was originally introduced to solve the dynamic traffic assignment problem (DTA) by simulation [7, 8]. One of its major features is its computational efficiency which allows the computation of traffic flows in large networks still being microscopic and dynamic. Microscopic here means that its input is a set of drivers with specified trips. This set is the solution of the assignment step in which the route choice for a given travel demand is calculated [7, 9, 10]. In the Q-model each link of a road network is represented by a *priority queue* and an *outgoing queue*. Its properties are characterized by the length  $l$ , a maximum flow  $j_{max}$  (*capacity*), a maximum speed  $v_{max}$  and the maximum number

of cars  $n_{max}$  that fits into the link (according to the number of lanes). When a car enters a link, the travel time is calculated by  $l/v_{max}$ . The vehicle is then put into the priority queue with the corresponding time of arrival at the end of the link. The model is updated in discrete time steps  $q$  ( $t \rightarrow t+q$ ). At each time step, a certain number of cars whose waiting time in the queue exceed the calculated travel time is passed to the outgoing queue. If there are several outgoing links, the outgoing queue is chosen according to the associated route plan. The number of cars that can leave a link is constrained by its capacity and by the maximum number of cars which fit into the next link. If more cars than the link capacity arrive, a queue starts to build at this link. Doing so the model allows to take into account dynamic effects like spill-back.

### 2.2 Car-following model

In contrast to the Q-model, microscopic *car-following models* are based on the modelling of single vehicle dynamics. In principle, for each time step the speed of each car is calculated according to its state and the state of the neighborhood [11]. In the last decade car-following models based on cellular automaton received a lot of attention, e.g. the well known Nagel-Schreckenberg model (NaSch-model) [12]. The NaSch-model has been extended in [13, 14, 15] to continuous space and bounded values for deceleration and acceleration (SK-model). This leads to an improved description of single vehicle dynamics. The SK-model is a minimal model based on the following assumptions: (1) Each driver tries to reach a maximal velocity  $v_{max}$ , (2) acceleration and deceleration are bounded, (3) there is a stochastic component and (4) the movement is collision free. The latter leads to a

safety condition for the velocity of a car following some other. In each time step a safe velocity is calculated for each car according to this condition. Doing so, the gap between the two cars, the current velocities of following and leading car and the braking ability are taken into account [14].

The SK-model results in a realistic description of traffic flow properties as the relation between density and flow or density and lane occupancy. It was shown that the state of networks can be reproduced by using real traffic counts as input. Indeed, the dynamic properties of jam formation are reproduced with good agreement to experimental facts.

Since the model allows to model speed changes of cars describing the driving state of individual vehicles in a realistic way, it is a good candidate to calculate the emissions.

For implementation details of both models, Q-model and SK-model, the reader is referred to [16].

### 2.3 Emission-factors

Both models described in the previous subsections provide information on the individual speed of cars in the system. Nevertheless, this is quantified in different ways in the two models. In the Q-model the travel time of each car is taken after passing an edge. The current mean speed on each edge of the network is then computed. In the SK-model instead, the vehicle speeds are observed for each car at each time step. In order to transform these velocities into the amount of emitted pollutants, tables relating the two quantities are needed (*emission factors*). A comprehensive widely-used data-base in this field is given in [17]. It provides information for different classes of cars generated by

dynamometric tests on a representative sample of more than 300 vehicles. In addition, forecasts for the development of emission factors is given based on legal norms for exhaust gases. Figure 8 plots tables of emission-factors extracted from [17] that we used in this work. They show quite plainly the strong dependence of the emissions with the velocity.

Due to this dependency, it is crucial for the applicability of a model in calculating emissions to be able to reproduce dynamic effects. In the following section a particular dynamic situation, i.e. the temporal evolution of a queue due to a bottleneck, will be investigated for the Q-model. Its dynamics is compared to those of the SK-model. A final remark on how the discreteness of the emission factors affects the calculated amount of pollutants in the Q-model will be given in the appendix.

## 3 Dynamic effects – Queuing

The crucial question discussed in this section is whether the simple Q-model is able to map dynamic effects like spill-back properly.

In order to study the influence of time-dependent queues of cars we set up a system with a merging of two lanes into one. The system consists of two road sections (one with two lanes, the other with one) of  $37.5 \text{ km}$  length each and maximum allowed speed  $v_{max} = 159 \text{ km/h}$ . They are connected with a merging region of  $450 \text{ m}$  length. In the SK-model overtaking of cars and lane changes in the merging region are explicitly modeled by a set of rules. In the Q-model instead we can represent the bottleneck by an additional road-section with a maximum throughput and a maximum number of cars. This value is taken equal to  $n_{max}$  in the SK-model. If the

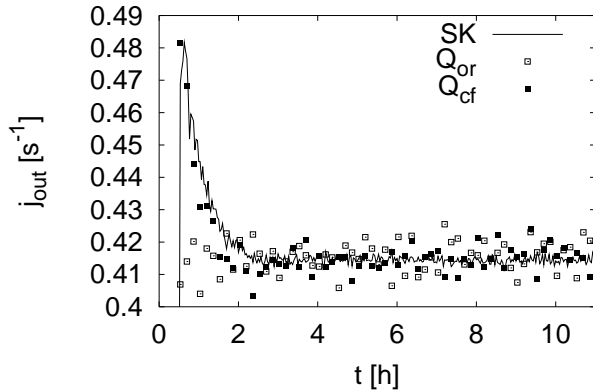


Figure 1: Comparison of the flow through the bottleneck in the SK–model and Q–model for constant inflow  $j = 0.49 \text{ s}^{-1}$ .  $Q_{or}$  represents the flow without an explicit modelling of the dependency of the flow on the number of queued cars. In  $Q_{cf}$  this dependency is modeled as explained in section 3.1.

inflow of the bottleneck exceeds the maximum throughput, i.e. the capacity, of the merging region, it acts as a bottleneck and a queue of cars starts to build up.

In favour to compare the two models with more statistical confidence, averages over 100 runs have been taken for all the simulations presented here. For each run, quantities as the flow  $j$ , the velocity  $v$  or travel times  $t_{tr}$  are sampled on intervals of 120 s. In the following, the time step was fixed to  $q = 1$  in the Q–model in order to minimize the effects of the discreteness of the emission–factors (see appendix).

### 3.1 Bottleneck with constant inflow

To investigate the relation between the throughput at the bottleneck and the number of queued cars, the system has been fed with constant

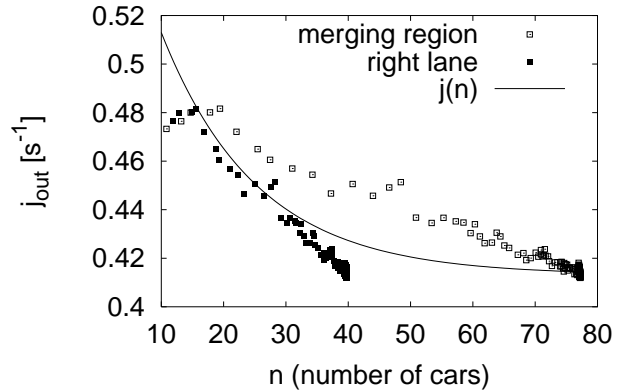


Figure 2: Flow through the bottleneck versus the number of cars in the merging region at the SK–model (averages over 100 runs). The solid line represents an exponential fit  $A \exp^{Bn} + C$  with  $A = 0.191$ ,  $B = -0.065$  and  $C = 0.413$ . For  $n < 15$  the throughput is not affected by the number of queued cars.

flow  $j_{in} = 0.49 \pm 0.02 \text{ s}^{-1}$  which exceeds the maximum possible capacity of the bottleneck. That leads to a constantly increasing number of cars that are queued in the merging region. The asymptotic outflow of the bottleneck in the SK–model is  $j_{out}^{t \rightarrow \infty} = 0.414 \pm 0.009 \text{ s}^{-1}$  with  $n^{t \rightarrow \infty} = 77 \pm 1.5$  being the maximum number of cars queued in the merging region.

### Outflow of the bottleneck

In figure 1 the outflow  $j_{out}$  of the bottleneck during the building of the queue is shown. The reason for the smooth decay of the flow observed in the SK–model is that the “performance” of the bottleneck depends on the number of cars that are queued in the merging region. In figure 2 the relation between the outflow and the number of queued cars is plotted. There, the mean value

is computed averaging 100 simulations. In contrast, looking at a single run this dependency is not visible. Nonetheless, what is important for us is the overall behaviour which shows clearly the expected decay of flow with increasing queue length.

It is obvious that fixing the maximum capacity in the Q-model to the asymptotic value measured in the SK-model, the decay over time in the flow can not be reproduced. In that case the capacity just acts as a cut-off parameter (figure 1). The resulting error has a noticeable influence on the travel time and therefore on the amount of emitted pollutants. The setup with a fixed maximum capacity at the bottleneck will further be referred to as  $Q_{or}$ .

In order to model the relation between the number of cars and  $j_{out}$  in the Q-model, a parameterization for the observed correlation of the two quantities has to be found. Since at the beginning of the queuing process the behaviour is dominated by the filling of the right lane (the lane to which the cars have to change to, in order to pass the bottleneck), data points of this lane have been used to determine the beginning part of the decay. The filling of the complete merging region, e.g. both lanes, was used to determine the asymptotic behaviour. To combine the two behaviours an exponential function has been used. The resulting fitted function is also shown in figure 2. Since the behaviour of the system is mainly determined by the state of the right lane when the queue starts to build up, one might also think to use a linear fit. However, simulations show that the decay of the flow becomes too rapid in this case. Instead, using the exponential fit one finds a good agreement for the flows between the two models (see figure 1). The exponential fit has been used for all the following simulations, referred to as  $Q_{cf}$ .

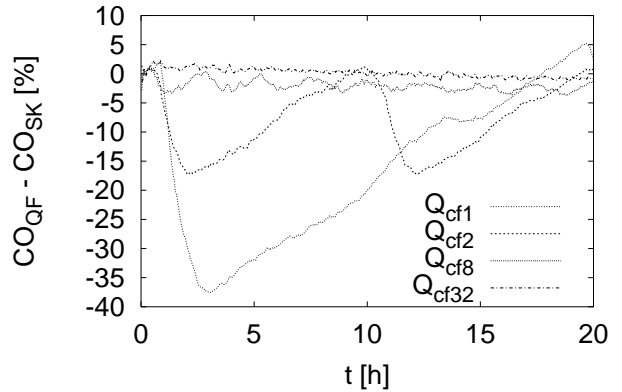


Figure 3: Relative error of the total amount of CO in the Q-model using the exponential fit for the capacity compared to the SK-model. The edge leading to the bottleneck has been split into several parts. The number of splits is indicated. With increasing number of splits the accuracy of the Q-model compared to the SK-model grows rapidly.

### Emission-calculation

Figure 3 shows the emitted carbon monoxide (CO) over time in the Q-model in comparison to the SK-model. CO has been chosen here since dynamic effects can be observed quite precisely due to the strong increase of its emission at high velocities (figure 8).

Focusing first at the setup  $Q_{cf1}$  (equal to  $Q_{cf}$ ) one can see, that the temporal development of the emitted CO follows quite well the curve of the SK-model around the building of the queue, i.e. during the first hour. However, later on a rapid drop occurs, leading to a dramatic underestimation of the total emission of CO (up to 30 % error). This effect is even more pronounced in situations in which the inflow of the system varies over time (see section 3.2). It should be

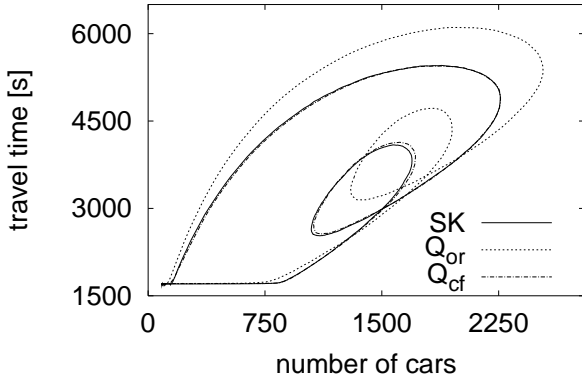


Figure 4: Comparison of travel time versus number of cars for the Q-model and the SK-model. Using the exponential fit for the capacity at the bottleneck (see figure 2) even the dynamic changes in the length of the queue are well reproduced ( $Q_{cf}$ ). In the case of a constant maximum capacity ( $Q_{or}$ ) the travel time is increased.

noted that this significant discrepancy is found, even though the Q-model reproduces well the behaviour of the mean velocity of the SK-model over the full simulation time. That demonstrates that to model emissions by traffic it is not sufficient to take just the mean speed of the cars into account.

The reason for the observed discrepancy in the emission of CO is that the queuing starts to grow at the far end of the edge while in the remaining part of the edge the cars flow freely. The free-flow part is then reduced until the complete filling of the edge. This effect is well reproduced in the SK-model while, for the Q-model, some tricks are needed. In order to model periods of free flows interlaced with the formation of queues also in the Q-model, the edge leading to the bottleneck was subdivided into several shorter

pieces. The maximum number of cars that fit into each piece has been reallocated respectively. Indeed a clear improvement of the results can be seen in figure 3. Using 32 pieces ( $Q_{cf32}$ ) an excellent agreement compared to the SK-model is already achieved, reducing the maximum error below 3% for the CO emissions. The same holds for the other pollutants. Note, that the quite high number of short edges necessary to obtain such good agreement depends in part on the inflow conditions used here. The system was constantly fed with a flow that exceeds the capacity of the bottleneck until the complete link was filled. As can be seen in figure 6 for a situation with more reasonable flows, it is sufficient to split the edges in a way that free-flow and congested parts can coexist.

### 3.2 Bottleneck with periodic inflow

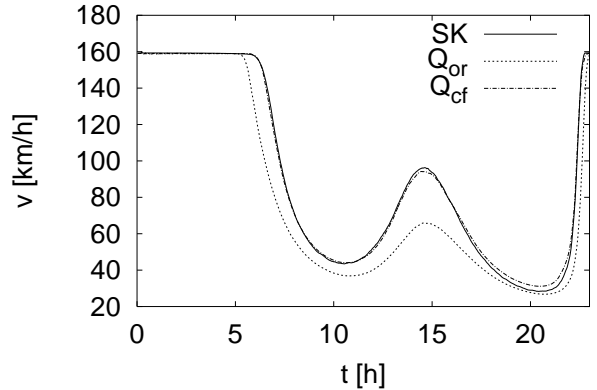


Figure 5: Mean velocity over time in front of the bottleneck. Due to the shift towards higher travel times in the case of a constant capacity ( $Q_{or}$ ), that speed is lower than in the car-following model (SK). With the exponential fit ( $Q_{cf}$ ) an excellent agreement is achieved.

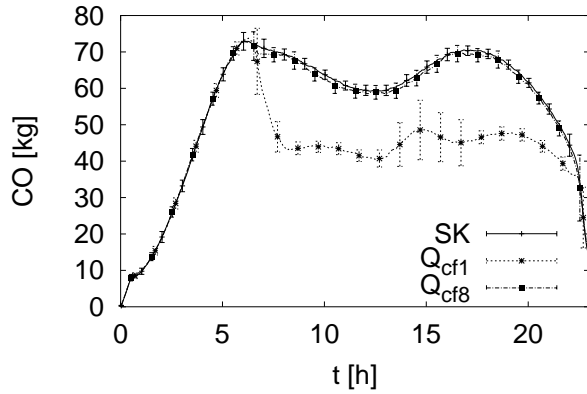


Figure 6: Emission of CO over time. Representing the edge leading to the bottleneck by one link ( $Q_{cf1}$ ) big discrepancies compared to the SK-model occur. By splitting the edge to several links ( $Q_{cf8}$ ) the time dependence of the emission of CO is quantitatively well reproduced.

Having determined the relation between the number of queued cars and the throughput at the bottleneck, the same system is used to investigate the dynamics of the Q-model with a more “natural” travel demand. To do so, a periodically changing input flow to the bottleneck is used which shows the typical structure of two “rush-hours” per day, namely

$$j_{in}(t) = 0.35 - 0.15 \left( \cos \frac{2\pi t}{\text{day}} + \cos \frac{4\pi t}{\text{day}} \right).$$

The parameters were chosen in a way that the queue does not fully dissolve between the rush-hours.

In figure 4 the relation between travel time and the number of cars in the system is plotted for both models. The mean values are very well reproduced by the Q-model using the exponential fit of the capacity ( $Q_{cf}$ ) derived in section 3.1. The same holds for the standard deviation of the

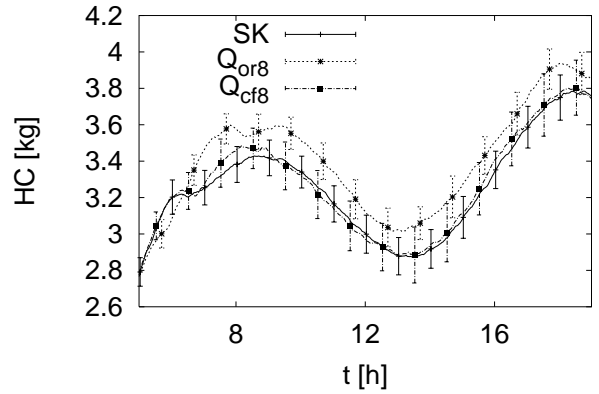


Figure 7: Emission of HC over time. Using a fixed capacity in the Q-model ( $Q_{or8}$ ) the emission of HC is overestimated, while a good agreement is achieved by using the exponential fit for the capacity ( $Q_{cf8}$ ). In both cases the edge was splitted into eight pieces.

quantities. It can be seen that the Q-model is even able to reproduce the dynamic decrease of the queue length at noon, which leads to the “loop” structure in figure 4. In the case of a fixed maximum capacity ( $Q_{or}$ ) the travel times are shifted to higher values since higher flows are just chopped. The mean error of the travel time in comparison to the SK-model amounts to 15% for  $Q_{or}$ , while for  $Q_{cf}$  it is below 1%.

Before having a look at the amount of emitted pollutants it is worth to focus on the mean velocity in front of the bottleneck. The shape over time is similar to the SK-model in both cases  $Q_{or}$  and  $Q_{cf}$  (shown in figure 5), but  $Q_{or}$  always underestimates the mean speed. Moreover, the Q-model also reproduces the standard deviation of the travel time. Consider now only  $Q_{cf}$ . Although the mean velocities are virtually equal, one finds quite big discrepancies in the amount

of emissions (figure 6). Recalling section 3.1, the reason is that the queuing takes place only at the end of the edge. We then use several subsequent links to represent the edge leading to the bottleneck to locate the queuing there. With eight links one already obtains an excellent agreement compared to the SK-model. The result for this case ( $Q_{cf8}$ ) is also shown in figure 6.

In summary, to be able to use the Q-model to compute emissions it is crucial to separate regions where queuing occurs from free-flow parts. It should be noted, however, that a correct mapping of the relation between the number of queued cars and the throughput of the bottleneck not only influences travel time in the system but also the amount of emissions. Hydrocarbons (HC) for example show a quite high emission level at low velocities (figure 8). Modelling the bottleneck with constant maximum flow, the mean velocity in the system is lowered compared to the SK-model (figure 5) which leads to an overestimation of emitted HC as shown in figure 7.

## 4 Summary

We have shown that a simple queuing model (Q-model) can be used to compute emissions in street traffic. Comparison with a more detailed microsimulation car-following model have been performed. It seems that the crucial point to achieve high accuracy is the ability of the Q-model to reproduce the dynamic effects like the building of queues and the throughput of bottlenecks. The latter can be reached by using a relation for the flow according to the number of cars queued at the bottleneck using an exponential decay. Another important point is that if we take into account only the mean ve-

locity on an edge without differentiating between free-flow and jammed regions, we get an over- or underestimation of emissions (depending on the pollutant). It could be shown, that by subdividing an edge in front of a bottleneck to several links, one can override this problem.

In further work we will investigate the introduction of a dynamic automated splitting of the edges to obtain the required separation of free-flow and jammed regions. One should note, however, that for the calculation of emissions in city networks, the splitting of the edges is already given due to their typical shortness. Moreover, future work will focus on the properties of the Q-model in larger street networks to be used to calculate the environmental impacts of traffic by means of scenario simulations.

## Appendix

The accuracy one is able to reach with the Q-model is limited by the discreteness of the given emission factors (figure 8) and the time step  $q$ . To demonstrate this limitations a setup of a single lane is used. The length of the link is set to  $37.5 \text{ km}$ . Cars enter the system with a constant flow  $j_{in} = 0.37 \pm 0.05 \text{ s}^{-1}$ , which is lower than the maximum capacity of the link. Therefore no queues can build up in the system. The maximum velocity is chosen as  $v_{max} = 102.5 \text{ km/h}$ . As before, averages over 100 simulation runs have been taken for each situation.

The temporal discretization  $q$  leads to a systematic error in the travel time compared to the SK-model. Due to the constant inflow of cars the error in  $\langle t_{tr} \rangle$  increases by approximately  $q/2$ . The average travel time is shifted towards higher values which corresponds to a reduction of the mean speed  $\langle v_m \rangle$ .



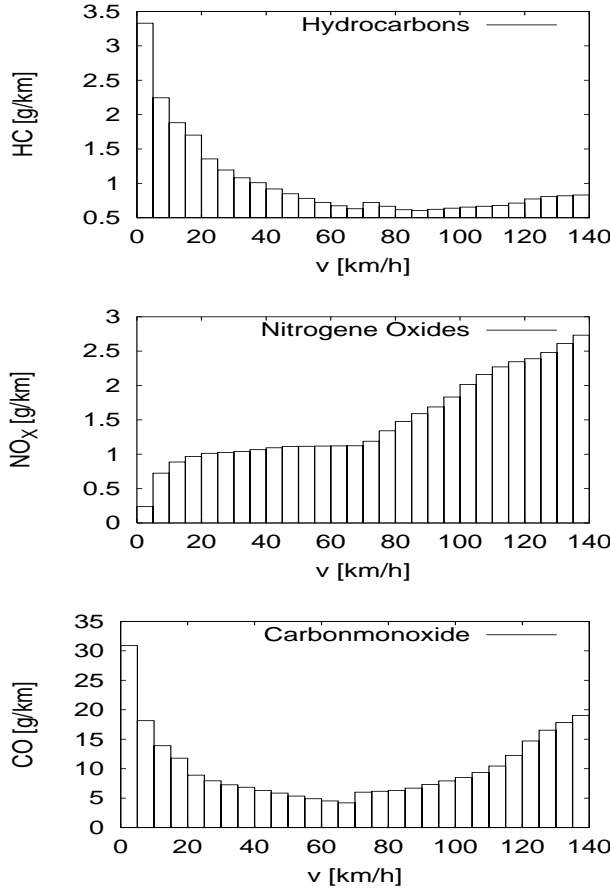


Figure 8: Emission-factors for different kind of pollutants as discrete function of velocity. The relations were constructed by using data from [17]. As one can see, the amount of emission is strongly dependent on the vehicle’s velocity.

The resulting error of the amount of emitted pollutants is plotted in figure 9. As expected, a slight increase of the total emission can be observed in the regime of small  $q$ . Since the cars “stay” longer on the edge, the produced emission is increased. At  $q \approx 65$  a steep decrease in the total emission of the system occurs. The reason for

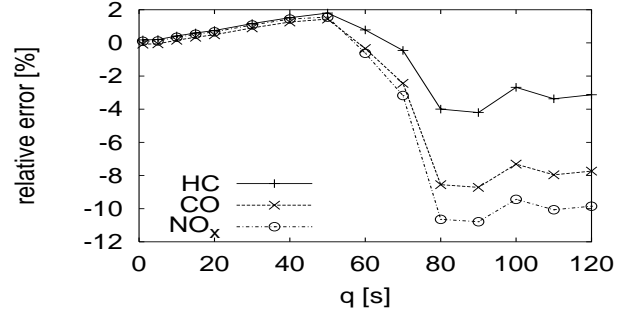


Figure 9: Relative error of the total amount of emissions in the Q-model compared to the SK-model. Due to the shift of the average travel time with increasing  $q$  the error grows rapidly at a certain value of the time discretization ( $q \approx 65$ ).

the rapid growth in the error is the discreteness of the emission-factors. At a certain  $q$ ,  $\langle v_m \rangle$  is shifted to values corresponding to the precedent bin of the emission-factors. Assuming  $q/2$  to be the approximative error in travel time, the value for  $q$  at which  $\langle v_m \rangle$  is shifted into the precedent bin can be estimated by

$$q^* \approx \frac{l/v_{max}}{v_{max}/\Delta v - 1/2}$$

with  $l$ ,  $v_{max}$  and  $\Delta v$  being the length of the link, the maximum velocity and the bin-size in terms of velocity, respectively. The system parameters chosen here ( $\Delta v = 5 \text{ km/h}$ ) give  $q^* \approx 65$ , which is in good agreement with the simulation. The validity of the relation has been stated for systems of different link lengths. Real street networks usually consist of shorter edges with lower speed limits. Therefore, the maximum time step that one is able to chose in favour of speeding up the simulation is bounded for example to  $q^* \approx 10$  for edges of  $1 \text{ km}$  length and a maximum speed of  $v_{max} = 50 \text{ km/h}$ .

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