Essays in Monetary and Financial Economics

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Referent: Univ.-Prof. Dr. Andreas Schabert Korreferent: Univ.-Prof. Michael Krause, Ph.D. Tag der Promotion: 17.06.2022 To my parents

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Chapter 1

Introduction

The last decade entailed different changes and challenges for the conduct of monetary policy. Central banks introduced a range of new tools in the wake of the financial crisis of 2008 that became tools of primary policy relevance. Moreover, the conditions under which central banks operate changed: the last years were not only marked by persistently low inflation but also by a rising importance of the non-bank financial sector. Lately, the debate on central banks' role in fighting climate change gained traction. This changing environment and the discussions that it induced are a common theme across the chapters of this thesis.

More specifically, the second chapter considers the impact of heterogeneous expectation formation on the transmission of monetary policy, which is one facet of the discussion on the causes of low inflation (see Ciccarelli et al., 2017). The third and fourth chapter focus on the central bank collateral framework that became increasingly relevant to support newly introduced central bank policies. More recently, it came into public focus following the ECB's decision to consider environmental disclosure requirements as eligibility criterion. The last chapter considers the macroprudential regulation of investment funds and, therefore, captures an increasingly relevant change to credit markets that itself are key to the monetary policy transmission (see Holm-Hadulla et al., 2021).

Chapter 2, which is joint work with Lucas Radke, contributes to the debate on the interaction between expectations and monetary policy (see Coibion et al., 2018; Hazell et al., 2022). One recurrent topic in this debate is the assumption of homogeneous expectations. However, empirical evidence points to non-negligible heterogeneity in inflation expectations (Mankiw et al., 2003). One seminal contribution is the work of Malmendier and Nagel (2016) that documents heterogeneity in inflation expectations across age groups and proposes a channel related to agents' lifetime experiences that endogenizes expectation heterogeneity. Their empirical study uncovers, first, that agents rely on lifetime experiences when forming expectations on inflation and, second, that young individuals

over-weight recent experiences relatively more than their older peers. As a result, inflation expectations depend on age via lifetime experiences. The latter aspect reflects a broader debate on the impact of an ageing society on the macroeconomy that is also of interest for central bankers (Eggertsson et al., 2019). This raises two questions with respect to monetary policy. First, how is the transmission of monetary policy affected when introducing this type of expectation heterogeneity? Most studies document important impacts of expectation heterogeneity on monetary policy but assume this feature exogenously. Second, since expectation formation is age-dependent, how does a demographic shift affect the monetary policy transmission?

Chapter 2 builds on the New Keynesian framework and extends it along two dimensions. First, we introduce an overlapping generation framework that gives rise to households of different age with different lifetime experiences. Second, those lifetime experiences become the centerpiece of expectation formation: households form expectations based on a forecasting model of the economy, whose parameters they re-estimate in every period. Following Malmendier and Nagel (2016) agents use only lifetime observations and react more strongly to new observations, the younger they are. Hence, expectations differ across age groups. Conceptually, this approach is closely related to constant-gain learning (Evans and Honkapohja, 1998) that assumes that all agents possess the full amount of information and react equally to new observations. As a result, expectations are homogeneous. To highlight the relevance of experience effects introduced by our framework of experience-based learning, we compare against a model with constant-gain learning.

The key mechanism arising from experience effects is straightforward. Even the oldest agent has limited lifetime experience. We show that this limited amount of information paired with the high sensitivity to new observations leads agents to attach macroeconomic variables a low persistence compared to a model with constant-gain learning. Given the amount of data rises and the sensitivity to new data points falls in age, this effect is stronger, the younger agents are.

The heterogeneity at the household level matters for aggregate expectations. We show that a monetary policy shock transmits less to inflation under experience-based learning compared to an economy with constant-gain learning. While monetary policy still directly affects current variables via the nominal interest rate, its impact via expectations is weaker. Today's monetary policy feeds back into expectations only in the next period when expectations are revised based on current macroeconomic variables. Since agents attach macroeconomic variables a smaller persistence in expectation formation, monetary policy's impact through expectations attenuates. We also investigate how the transmission of monetary policy changes in an economy with a higher share of young agents and find weaker inflation and output reactions for the same monetary policy shock. The shift of the demographic structure towards more young and less old agents drives this result, because it reduces the aggregate persistence attached to macroeconomic variables.

Furthermore, we show that the introduction of experience-based learning changes the monetary policy trade-off between stabilizing inflation and closing the output gap under supply shocks. Changes in comparison to constant-gain learning happen along two dimensions: first, the smaller pass-through via expectations leads to an overall smaller volatility of the economy so that the trade-off occurs on a lower level. Second, by reducing the response of expected inflation the trade-off aggravates, i.e., stabilizing inflation is more costly in terms of additional output gap variation. When comparing an old to a young economy these results magnify, because the reaction of expectations to monetary policy is weaker in young economies. In comparison to constant-gain learning, where young and old agents form equal expectations, we find the trade-off to be attenuated in old societies.

Chapter 2 has important implications for the debate on the interaction between monetary policy and expectations. First, the type of expectation formation assumed matters: the impact of monetary policy on inflation is overstated whereas the degree of the monetary policy trade-off is understated, when abstracting from experience effects. Second, the result of a weaker trade-off in old economies implies that a given level of inflation volatility can be reduced less costly. This can have implications for the choice of an inflation target range by the central bank.

Chapter 3, which is joint work with Matthias Kaldorf, contributes to the understanding of the extended central bank toolkit in the aftermath of the financial crisis of 2008. A key feature of this crisis was a dry-up of liquidity in the banking sector that led central banks to increase the scope of their market operations. To obtain liquidity from the central bank, banks need to post collateral that satisfies certain criteria like a minimum rating. Accordingly, the ECB lowered the minimum rating threshold in October 2008 to support collateral availability. Changes to these criteria, aside from a direct effect on collateral supply, may also work indirectly through the response of the issuers of eligible assets. Specifically, the policy change greatly increased the amount of eligible corporate assets. The change in the eligibility status of their debt implied lower cost of finance for these firms, which led to an increase of their debt issuance (Pelizzon et al., 2020). If firms increase debt issuance to take advantage of cheaper financing, this raises eligible debt but increases its default risk, which may limit the positive impact on collateral supply. This raises two important questions. First, how does eligibility affect firm debt and default policies? Such endogenous firm responses are so far unconsidered in the discussion of collateral frameworks. Second, taking firm responses into account, what is the impact of collateral easing, like in 2008, on collateral supply and aggregate default cost?

Chapter 3 approaches these questions in two steps. First, we develop a framework to highlight the different channels at work. We analyze endogenous firm responses to eligibility requirements in the presence of default risk and discuss the effect of collateral easing. Second, we illustrate the mechanisms in an extended model that can replicate different empirical features of firm debt issuance and bond spreads.

In our model, banks pay a premium on corporate bonds if they do not exceed a default risk threshold. Firms issue those one-period bonds, because they are more impatient than banks. Moreover, firms are heterogeneous as they draw revenue from a firm-specific distribution implying that some firms are permanently more productive than others. In case of low revenues, a firm defaults on its debt and all revenues are lost. We establish two key firm level effects when introducing collateral eligibility. First, the eligibility of bonds prompts firm risk-taking. Mostly firms of high quality, i.e., high productivity, take advantage of lower financing cost and become riskier by issuing more debt. Second, we demonstrate a disciplining effect as firms of medium quality reduce their debt issuance to become sufficiently safe to be classified as eligible. Evidence for such heterogeneous firm responses is documented by Grosse-Rueschkamp et al. (2019).

Risk-taking and disciplining effects matter for the response of collateral supply and aggregate default cost, defined as lost revenues, after collateral easing. In our model, collateral easing is given by an increase in the default risk threshold below which bonds are eligible. We show that collateral easing mechanically increases the supply of collateral by making more bonds eligible all else equal. In our baseline model, firm responses further increase collateral supply. However, they have an ambiguous impact on aggregate default cost, since the risk-taking and disciplining effects affect default risk in different directions. Notably, these results are obtained under the assumptions of short-term debt and permanent firm revenue distributions.

Therefore, the second set of results is based on a model featuring long-term debt and persistent revenue shocks. High-revenue firms' risk-taking still increases their current debt issuance in response to broader bond eligibility. However, if debt is long-term and revenue non-permanent, some firms will experience a series of adverse revenue shocks that increases their rollover burden. This makes default more likely and not only lowers the market value of eligible bonds but also reduces the amount of eligible bonds if some of those firms become non-eligible. When calibrating our model to euro area data, we find that in response to collateral easing endogenous firm responses dampen the mechanical effect on collateral supply by 9 percentage points. At the same time, cost from default rise by 8%. These results are driven by an increasingly relevant risk-taking effect: after a collateral easing 79% of firms are subject to risk-taking compared to 51% before the policy change.

Finally, we discuss an instrument to address the dampening effect of firm risk-taking. We propose an eligibility covenant that makes the amount of debt that a firm can issue without losing eligibility negatively dependent on current debt outstanding. An eligibility covenant lowers firm risk-taking (and incentives deleveraging, i.e., disciplining) in response to bond eligibility and, thereby, alleviates the negative collateral supply effect from higher rollover risk. Our results suggest that a covenant can successfully increase collateral supply, while limiting additional default cost.

Chapter 3 adds a novel perspective to the debate on the collateral framework as a policy tool. We show that collateral easing has adverse effects in terms of firm risk-taking that dampen the increase of available collateral. To achieve the same expansion of collateral supply, collateral easing would need to be more aggressive. We propose an eligibility covenant to address the dampening effect of firm responses.

Chapter 4 is joint work with Francesco Giovanardi, Matthias Kaldorf, and Lucas Radke. Building on the firm risk-taking mechanism presented in Chapter 3, it investigates the role of the central bank collateral framework in addressing climate change. During the last decade the debate on climate change intensified and central banks began to explore their policy options to take a more active role in environmental policy. A key aspect in these considerations is the sizable financing need for the reduction in carbon-intensive production. For example, the European Commission (2020) estimates that in addition to current energy-related investment, 90 billion EUR more per year need to be raised for reaching the EU climate targets. Since central banks are in a unique position to influence prices on financial markets, they can play a crucial role to close this funding gap. One tool that has been debated in this respect is a preferential treatment of green bonds in the central bank collateral framework, e.g., by applying more favorable haircuts (Brunnermeier and Landau, 2020). The proceeds collected by issuing these bonds are dedicated to projects with environmental focus so that a preferential treatment should ease financing conditions for sustainable investment. However, little is known about preferential treatment's pass-through on green investment, pollution, its side-effects, or its interaction with carbon taxation.

Chapter 4 looks at these aspects in detail by extending the standard real-business cycle model along three dimensions. First, we introduce an environmental externality. We distinguish between two types of firms, green and conventional. Conventional firms' technology creates pollution, which decreases final good production. Second, production of both firm types is risky as it involves uncertain productivity. In case firms cannot repay the bonds issued to finance investment, they default. Third, banks demand bonds not only as investment but also to reduce cost from liquidity management capturing that a bank can use bonds in collateralized borrowing with the central bank. The central bank accepts risky bonds only with a haircut and incurs costs, which depend positively on the default risk of pledged collateral, from accepting these bonds. Thus, the collateral policy is associated with a trade-off between collateral availability for banks and costs from accepting risky bonds as collateral. Addressing pollution is then an additional goal.

In our model, a preferential treatment of green bonds works through a relatively more favorable haircut for green bonds. Ceteris paribus, banks can reduce liquidity management cost by more for any unit of green bond they purchase. As a result, they are willing to pay a higher collateral premium on those bonds so that green firms finance more cheaply. This shifts the usage of capital towards sustainable production that does not generate pollution so that output and consumption rise.

We show that such a preferential treatment of bonds, indeed, lowers pollution by increasing the green capital share. However, the additional bond issuance is not fully invested into capital. Key to this result is that the increase in firm debt lowers the expected return on capital due to higher firm risk-taking. Capital investment is less attractive in this case. Despite the imperfect pass-through, we find optimal haircuts to fall by half for green bonds and to markedly increase for conventional bonds compared to our baseline of symmetric treatment. This policy change also affects the original collateral policy trade-off. The relatively worse treatment of conventional bonds lowers the supply of collateral, thereby increasing cost from liquidity management and lowering resource losses from default risk.

To put our results into perspective, we introduce a carbon tax on conventional firms, a widely discussed and, in theory, the most powerful tool to address the pollution externality. Indeed, we find that a carbon tax at its optimal level is highly effective in decreasing pollution. However, by changing the relative size of conventional to green firms, it also changes the availability of collateral for banks. Thus, in the global optimum the haircuts of green and conventional bonds are also adjusted: haircuts are reduced to increase the availability of collateral. Since these adjustments are symmetric, there is no preferential treatment of green bonds. However, as setting the carbon tax to its optimal level is arguably not an empirically plausible case, we also investigate the scope for preferential treatment given sub-optimally low carbon taxes. We find that preferential treatment falls, the closer the tax is to its optimal level.

Chapter 4 bears relevant implications for the debate on the role of the central bank collateral framework to address climate change. First, the pass-through of a preferential treatment of green bonds is impaired in the presence of endogenous default risk. Second, in the absence of optimal carbon taxation, the central bank should introduce a preferential treatment of green bonds despite its weak pass-through. Finally, the socially optimal outcome arises from a coordination between the fiscal and monetary sides, where the latter supports rather than leads the effort to reduce pollution.

Chapter 5, which is joint work with Giovanni di Iasio and Christoph Kaufmann, dis-

cusses the liquidity regulation of investment funds. While Chapters 2 to 4 consider the tools and transmission of monetary policy, this chapter focuses on financial stability, i.e., a dimension that gained relevance in central bank considerations over the last decade. In response to the financial crisis of 2008, policymakers refined macroprudential policy to reduce the risks from imbalances on financial markets. Tighter bank regulation was one factor that contributed to the growth of the non-bank financial sector of which investment funds represent a major share. While their growth made the financing of euro area firms more diverse, their business model also allows for short-term redemptions. Since investment funds reduced holdings of liquid assets, this creates scope for a liquidity mismatch that might lead to a rapid liquidation of bonds to settle redemptions (see Chernenko and Sunderam, 2020; Falato et al., 2021). The debate on the regulatory options for addressing this liquidity risk also involve the introduction of liquidity buffers for investment funds. Since little is known about the macroeconomic and welfare effects of liquidity buffers, Chapter 5 contributes to this debate by developing and analyzing a model with investment fund liquidity risk that allows for an examination of such a regulation.

Our model features two types of financial intermediaries. Banks finance themselves with deposits and grant loans to firms. Importantly, bank deposits provide utility to households. Investment funds issue shares to households and use their proceeds to invest into corporate bonds and deposits. While firms have access to both, loans and bonds, these are not perfect substitutes. In contrast to banks, each investment fund faces stochastic periodic redemptions from households, which it settles through deposits and selling corporate bonds on a secondary market to households. The latters' willingness to pay for bonds falls in the amount of sales due to management cost that lead to resource losses (see Gertler and Kiyotaki, 2015). Investment funds first use deposits to settle redemptions as sales on the secondary market are decreasing their dividends. Similar to Chernenko and Sunderam (2020), this is captured by resource cost that can be thought of as the cost of being active on an illiquid market.

The presence of periodic redemptions creates an inefficiency: investment funds only consider the cost from bond sales via their liquidity cost and do not internalize the positive impact of their deposit choice on prices. Since deteriorating secondary bond market prices make more sales necessary that, in turn, increase resource losses, this implies a loss in consumption. Moreover, this also implies investment fund shares to be a less profitable investment, which decreases their market value and, thereby, their ability to purchase corporate bonds. Due to the imperfect substitution of loans and bonds, total credit to firms falls.

We show that the introduction of a regulatory liquidity buffer (a fraction of assets invested in deposits) above the voluntary level raises welfare. The optimal level of the buffer is 7.6% in contrast to the calibrated voluntary buffer of 2%. The higher share

of deposits directly lowers aggregate bond sales of investment funds. The associated reduction of resource losses drives the positive welfare impact. However, regulation also has negative welfare impacts. First, investment funds invest into deposits at low interest rates. This reduces the return of deposits relative to fund shares so that households demand less deposits, which, due to deposits' utility benefit, depresses welfare. Second, while increasing regulatory liquidity buffers initially leads to more bond intermediation by increasing investment fund shares' return, already for intermediate values of the buffer bond intermediation falls. The reason is that a rising fraction of shares needs to be invested into deposits instead of bonds. This pushes financial intermediation away from investment funds towards banks and decreases output. We show that the former effect, a reduction in deposits held by households, is the main welfare cost associated with regulation.

We also investigate the optimal liquidity buffer after a shock to household saving preferences. The shock reduces savings in investment fund shares akin to the Covid-19 related dynamics in March 2020. As a result of their shrinking liability side, investment funds reduce assets in the unregulated economy. Funds scale back on deposits relatively more than on bonds due to deposits' smaller return. In an economy without regulatory liquidity buffer the impact of the shock is amplified by periodic fund redemptions: since investment funds reduce deposits, they are more exposed to redemptions so that bond sales and resource losses increase. This reduces consumption and the return from investment fund shares. Hence, households reduce their savings in fund shares even more, which amplifies the initial aggregate shock. Instead, when imposing the regulatory liquidity buffer, investment funds cannot reduce deposits to the same degree. We show that the optimal buffer eliminates the amplification from periodic redemptions and increases welfare compared to an unregulated economy.

Chapter 5 offers relevant insights for the debate on investment fund regulation. It is the first contribution in investigating welfare effects of a liquidity buffer and demonstrates potential trade-offs. It further shows that a liquidity buffer is suited to contain amplification after an aggregate shock to investment fund funding.

Contribution

Chapter 2 is based on joint work with Lucas Radke. The research idea and the framework were developed in collaboration. Lucas Radke provided most of the code and I wrote most of the draft. We then jointly refined the code and the text.

Chapter 3 is based on joint work with Matthias Kaldorf. The research question, the model, and its formal analysis were developed jointly. I provided the data work on which our quantitative analysis is based, while Matthias Kaldorf was responsible for code and calibration. The first draft as well as all revisions of the paper were written jointly.

Chapter 4 is based on joint work with Matthias Kaldorf, Francesco Giovanardi, and Lucas Radke. We developed the research idea and the model together. Moreover. we analyzed the simplified example in collaboration. The coding of the model was mostly conducted by Francesco Giovanardi, Lucas Radke, and Matthias Kaldorf. I contributed the empirical sections. The initial draft was written by Matthias Kaldorf and me, which was then revised by Francesco Giovanardi and Lucas Radke.

Chapter 5 is based on joint work with Christoph Kaufmann and Giovanni di Iasio. We developed the research idea together. Christoph Kaufmann provided the empirical analysis in Section 2. I developed, calibrated, and coded the model. Moreover, I conducted the model analysis and wrote the first draft of the paper. Later revisions were joint work by all of us.

Chapter 2

Experience-Based Heterogeneity in Expectations and Monetary Policy

This chapter is based on Radke and Wicknig (2022).¹

2.1 Introduction

Private sector expectations are a key determinant for the implementation of monetary policy. Most New Keynesian models employ rational, that is, homogeneous expectations. However, empirically expectations exhibit substantial cross-sectional heterogeneity (see Mankiw et al., 2003) that has been shown to alter the propagation of shocks and the transmission of monetary policy (see Branch and McGough, 2018).

Earlier studies introduce expectational heterogeneity exogenously.² In this paper, we relax the assumption of homogeneous expectations using an endogenous form of expectation heterogeneity based on *Experience-Based Learning* (EBL) (Malmendier and Nagel, 2016). In our model, expectations are a function of the different economic experiences that individuals made over their lifetime. In consequence, agents of different age have heterogeneous expectations. Since aggregate expectations are a size-weighted average over cohort-specific expectations, a variation in the demographic structure affects aggregate expectations through a composition effect that we call the *Experience Channel*. Using this empirically relevant form of endogenous expectations for monetary policy but also

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²Those studies often exogenously divide the population into individuals with different forecasting models or endow agents with a choice from a set of predictors.



FIGURE 2.1: Realized Inflation vs. One-Year Ahead Inflation Expectations Across Cohorts

to explore how demographic factors affect the monetary policy transmission, which is becoming a topic of interest for central banks (see Eggertsson et al., 2019).

We show that experience-based heterogeneity in expectations across age groups weakens the transmission of monetary policy on inflation and the output gap. The weaker transmission stems from the fact that monetary policy's influence on expectations is lower if only lifetime experiences are used to form expectations. As a result of monetary policy's reduced impact on expectations, also the stabilization trade-off under supply shocks is affected. The trade-off aggravates, as any reduction in inflation volatility causes a stronger increase in output gap volatility. Moreover, due to the Experience Channel, the effect of a demographic variation on the transmission of monetary policy is substantially more pronounced under EBL. The transmission of monetary policy on inflation increases in older societies, while the stabilization trade-off attenuates.

Malmendier and Nagel (2016) provide empirical evidence that differences in inflation expectations across age groups are largely driven by differences in their experienced inflation history.³ Figure 2.1 illustrates that young cohorts' expectations are more sensitive

Notes: left y-axis: annual percentage change of the seasonally-adjusted US-CPI (solid black line). Right y-axis: quarterly 4-quarter moving average of cohort inflation expectations, expressed as percentage deviations from the cross-sectional mean (markers). We take a moving-average to concentrate on lower frequency variation. Inflation expectations are based on the Michigan Survey of Consumers (question: *Expected Change in Prices During the Next Year*) from 1978Q1 to 2017Q4. Cohorts define persons of a certain age group at a specific point in time, i.e., no age group is tracked over time.

 $^{{}^{3}}$ Experience effects are also documented for other dimensions, like consumption or investment decisions, and different regions. See Malmendier (2021) for an overview of the literature.

to recent observations than those of old individuals.⁴ The markers denote one-year ahead inflation expectations of a "young" (red) and an "old" (blue) cohort as deviation from the cross-sectional mean (percentage points, right *y*-axis). The black solid line depicts the year-on-year realized CPI-inflation from the previous year (left *y*-axis). The heterogeneity in expectations across different age groups is particularly pronounced in the early 1980s, because young individuals, whose whole inflation history consists of the high inflation rates during the 1970s, tend to have higher inflation expectations than those individuals who also observed low inflation during the 1950s and 1960s. Importantly, the high inflation expectations of young agents are not a function of their age but of their lifetime experience as can be inferred from the reversal of the ordering at the end of the sample where recent inflation experiences were extremely low.

The present paper embeds EBL into a New Keynesian model with overlapping generations à la Blanchard (1985) and Yaari (1965). We assume that agents behave like econometricians who form expectations about future economic variables based on forecasting models whose parameters they constantly revise as new data becomes available. In particular, agents forecast future variables with a covariance stationary auto-regressive process of order one with time-varying auto-regressive parameter, which we denote as an agent's *perceived persistence*. Following the empirical analysis of Malmendier and Nagel (2016), agents put more weight on recently observed data points rather than those observed early in life, while ignoring any data *prior* to their birth. The weight attached to new observations when updating beliefs decreases in age, rendering young individuals more sensitive to new data points than older ones. It is the specification of the weight by which EBL differs from standard approaches in the learning literature, like constant-gain learning (CGL), where all cohorts attach the same constant weight to new information so that expectations are homogeneous across cohorts.

Our calibrated model generates substantial heterogeneity in expectations across age groups, which stems from differences in the perceived persistence that different cohorts attach to economic variables. On average, the perceived persistence is more dispersed for young cohorts, because the parameter estimates of their forecasting rules are based on fewer observations and because more recent observations are over-weighted. Both features also imply that recurrent reversals in inflation and the output gap make young agents perceive both variables to be less persistent, on average.

We show that EBL endogenously reduces the aggregate perceived persistence in the economy relative to CGL due to the heterogeneity of expectations at the cohort-level.

⁴We extend Figure I from Malmendier and Nagel (2016) by plotting the annual percentage change of the seasonallyadjusted US-CPI to facilitate the comparison of *expected* inflation to *experienced* inflation.

The decrease in the aggregate perceived persistence reduces the impact of monetary policy via expectations on current macroeconomic variables. Under adaptive expectations, current monetary policy has a delayed impact on expectations, because it influences current macroeconomic variables, which are used to form beliefs only in the next period. Since household under EBL attach a smaller persistence to macroeconomic variables, monetary policy affects current variables via expectations by less. The aggregate perceived persistence can then be interpreted as the weight agents attach to past monetary policy. Because of the lower impact of past monetary policy actions on current variables, the transmission of monetary policy on inflation and the output gap is impaired under EBL. Hence, when neglecting experience effects on expectations, the impact of monetary policy is overstated. The lower influence of monetary policy on expectations under EBL also affects its stabilization trade-off under supply shocks. First, due to the lower perceived persistence, a *given* level of inflation volatility is associated with a lower output gap volatility. Second, a given reduction in inflation volatility is related to a stronger increase in output gap volatility. To see why, note that due to the backward-looking nature of expectations, monetary policy affects inflation also via past changes of the output gap. Those changes are reflected in past inflation and, thereby, in today's inflation expectations. Since the perceived persistence attached to inflation is reduced, the pass-through via (past) changes in demand on current inflation is lower so that inflation is stabilized less effectively. As a result, the policy trade-off is understated in models that abstract from experience effects.

Our framework introduces a role for the age distribution that is absent from models with rational expectations and from standard learning models. Since under EBL the perceived persistence is heterogeneous across age groups, a variation of the age distribution directly affects the aggregate perceived persistence through the Experience Channel. In response to an increase in the share of old individuals, the aggregate perceived persistence considerably increases under EBL, while it is hardly affected under CGL. Consequently, the aggregate weight attached to past monetary policy actions when forming expectations rises. Thereby, the transmission of monetary policy via expectations also increases, while its stabilization trade-off under supply shocks attenuates as inflation can be stabilized at the expense of less additional output gap volatility.⁵ Due to the increased reaction of inflation expectations to monetary policy, past changes in demand have a stronger impact on current inflation.

Our model shows that experience-based heterogeneity in expectations affects monetary policy's transmission on current variables. Under EBL, the effect of demographic shifts on the transmission of monetary policy intensifies via the Experience Channel. Since this

⁵The empirical literature on the relation of demography and monetary policy is still developing. Several studies point into the direction of our theoretical results (see below).

alters the degree of the output gap-inflation trade-off between young and old societies, our result has also relevant implications for the choice of a certain inflation target range.⁶

Related Literature. Our work is related to several strands of literature. First, we relate to the adaptive learning literature that studies monetary policy within a New Keynesian model as surveyed in Eusepi and Preston (2018). In contrast to this literature, we assume that individuals do not know the true state-space representation of the economy. Instead, they form expectations based on simple auto-regressive forecasting models as indicated by the empirical evidence of Malmendier and Nagel (2016). We further assume that the parameters of the forecasting model are recursively updated and differ across age groups so that expectations are heterogeneous across generations.

Second, we contribute to the literature that analyses monetary policy within a New Keynesian model with heterogeneous expectations. This literature is mostly concerned with determinacy properties in the presence of heterogeneous expectations (see, e.g., Branch and McGough, 2009; Gasteiger, 2014; Massaro, 2013). Expectation formation in these studies is time-invariant in contrast to our approach that allows for real-time updates. Further, these studies take expectation heterogeneity as given, whereas in our model it arises endogenously from cohorts that made different lifetime experiences. This gives rise to a novel channel by which the demographic structure affects the pass-through of monetary policy.

Third, there is a literature analyzing experience-based expectation heterogeneity in theoretical models. However, most focus on asset pricing in partial equilibrium, as Collin-Dufresne et al. (2016), Ehling et al. (2018), Malmendier et al. (2020), Nagel and Xu (2021), and Schraeder (2015). The only model using EBL in a general equilibrium framework that we are aware of is Acedański (2017), who explores its implication on the wealth distribution. The present paper is the first one to investigate the impact of experience effects on monetary policy.

Last, we contribute to the growing literature that studies the relation between demographic changes and monetary policy pass-through. Leahy and Thapar (2019) and Berg et al. (2021) consider how the transmission of monetary policy differs across age groups. Both studies document a positive empirical relationship between the impact of monetary policy shocks on employment, income, or consumption and the share of agents that are at the end of their working life. These studies lend support to our theoretical result of a stronger monetary policy transmission in old economies. Likewise, Juselius and Takáts (2018) and Baksa and Munkácsi (2020) show that inflation volatility and the share of old agents are positively related similar to our finding of a policy trade-off that involves more

⁶See, for example, Erceg (2002) who argues that the choice of a target range for the inflation rate should be based on the stabilization trade-off between inflation and output gap volatility.

inflation volatility in the old economy. Our work points to a novel channel of how shifts in demography lead to these results through experience effects.

Outline. The remainder of the paper is organized as follows. Section 2.2 presents our New Keynesian model with overlapping generations. In Section 2.3 we explain the details of EBL. In Section 2.4, we inspect the key difference between EBL and CGL. Section 2.5 discusses the implications of EBL for monetary policy. Finally, Section 2.6 concludes.

2.2 The Model

In the present section, we consider a New Keynesian framework. However, we deviate from the standard model by assuming that households form expectations based on their lifetime experiences. Conceptually, we follow the statistical learning literature that assumes agents to forecast future outcomes by using a simplified model of the economy instead of knowing its actual law of motion. Agents constantly revise their beliefs as new observations become available. Since we allow this revision of beliefs to depend on an agent's age, we assume households to face a constant probability of death á la Blanchard (1985) and Yaari (1965) to introduce an age distribution.⁷ Their saving decision involves the formation of inflation and output gap expectations, which depend on their age and lifetime experiences. This is the only source of heterogeneity across agents. Taken by itself, heterogeneous expectations lead to different saving decisions across agents. However, similar to other literature using boundedly rational agents (e.g., Mankiw and Reis, 2007, Adam et al., 2016, and Ehling et al., 2018), we abstract from the wealth distribution as an additional state to focus on the effects stemming from EBL via the expectation operator.

2.2.1 Households

Households provide labor, form expectations according to EBL, and own intermediate good firms. At each point in time, the mass of households is constant and normalized to one. They face an age-independent probability $\omega \in [0, 1]$ of surviving into the following period. In turn, at the beginning of each period a share of $1 - \omega$ households deceases and is replaced by new-born households of equal mass. Consequently, the mass of a cohort born in period k at time $t \geq k$ is given by $(1 - \omega)\omega^{t-k}$.

A household born in period k maximizes the discounted sum of lifetime utility

$$\tilde{E}_{t}^{k} \sum_{j=0}^{\infty} (\beta \omega)^{j} u(c_{t+j|k}, l_{t+j|k}) , \qquad (2.1)$$

subject to the flow nominal budget constraint in period t

$$p_t c_{t|k} + b_{t|k} = r_{t-1}(b_{t-1|k} + z_{t|k}) + p_t w_t l_{t|k} + \mathcal{D}_{t|k} , \qquad (2.2)$$

 $^{^{7}}$ A similar assumption on demography is made by Ehling et al. (2018) and Galí (2021).

where β denotes the discount factor, p_t the price level, and \tilde{E}_t^k denotes the subjective expectations operator, which potentially differs across cohorts k and is specified below. Households from cohort k receive labor income, which is the product of nominal hourly wages $p_t w_t$ and working hours in cohort k, $l_{t|k}$. They invest in private one-period nominal bonds $b_{t|k}$, which pay the nominal interest rate r_t tomorrow. We assume each household owns equal shares in the intermediate good firms so that nominal dividends are equal across cohorts, i.e., $\mathcal{D}_{t|k} = \mathcal{D}_t$.

The time of death is uncertain and households may die with wealth. To avoid the inefficiency of accidental bequests, we follow Blanchard (1985) and introduce insurance companies that make annuity payments $z_{t|k}$ and that receive all assets at the time of death. Profits for a particular company contracting with cohort k are

$$\pi_t^I = (1 - \omega) b_{t-1|k} - \omega z_{t|k}$$
.

Due to free entry insurers make zero-profits so that the annuity payment equals a fraction of cohort bond holdings $z_{t|k} = \frac{1-\omega}{\omega}b_{t-1|k}$. The above sequence of period budget constraints is supplemented with a solvency condition of the form

$$\lim_{T \to \infty} \tilde{E}_t^k \left\{ \mathcal{R}_{t,T} b_{T|k} \right\} = 0 , \qquad (2.3)$$

where $\mathcal{R}_{t,T} = (\prod_{s=t+1}^{T} r_s)^{-1}$. We assume the following form of the felicity function

$$u(c_{t|k}, l_{t|k}) = \ln\left(c_{t|k}\right) + \psi_n \ln\left(1 - l_{t|k}\right) ,$$

where ψ_n is a utility weight. Maximizing (2.1) subject to (2.2) yields the consumption/saving decision,

$$1 = \tilde{E}_t^k \left\{ \beta \frac{p_t c_{t|k}}{p_{t+1} c_{t+1|k}} r_t \right\} .$$
(2.4)

Equation (2.4) denotes the household's Euler equation. While households of all ages face the same nominal interest rate, they have different expectations of the real rate. Hence, a household expecting a high future return, saves more than a household whose past experiences make her believe in dismal real future returns. Notwithstanding that agerelated heterogeneity in expectations implies differences in cohort wealth, we aggregate the economy without considering the wealth distribution as an additional state variable as outlined in Section 2.2.4. We also derive the labor supply of a household from cohort k that, via different consumption choices among cohorts, is cohort specific

$$\psi_n \frac{c_{t|k}}{(1 - l_{t|k})} = w_t .$$
(2.5)

2.2.2 Firms

There are two types of firms. Final good firms use intermediate inputs to provide an aggregate consumption good. Intermediate good firms are owned by households and operate on a monopolistically competitive market. The choice to set up firms as in the usual New Keynesian model makes our departure from the standard case minimal.

2.2.2.1 Final Good Firm

The aggregate consumption good in the economy y_t is produced by a perfectly competitive firm, which is aggregating intermediate goods $i \in [0, 1]$ produced by intermediate firms according to the technology

$$y_t = \left[\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} , \qquad (2.6)$$

where $\varepsilon > 0$ is the elasticity of substitution among the intermediate goods $y_{i,t}$. The final good firm chooses the quantities of intermediate goods to maximize its profits. The demand for intermediate good *i* is given by

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t}\right)^{-\varepsilon} y_t , \qquad (2.7)$$

where $p_{i,t}$ is the price at which the intermediate good firm *i* sells to final good producers.

2.2.2.2 Intermediate Good Firms

All households alive in period t own an equal share in each intermediate good firm $i \in [0, 1]$ that produces a differentiated good on a monopolistically competitive market. Since households are involved in firms to an equal degree, the latter employ average expectations as detailed below. We assume that the share of a deceasing household is transmitted to a new-born one instantaneously. Production of intermediate good i uses the technology

$$y_{i,t} = l_{i,t}^{\alpha} , \qquad (2.8)$$

where $l_{i,t}$ denotes labor demand of firm *i* and $0 < \alpha \leq 1$. Intermediate firm *i* sells its good at price $p_{i,t}$ and, when changing its price, pays quadratic nominal price adjustment costs à la Rotemberg (1982) proportional to the nominal value of aggregate production

$$\frac{\phi}{2} \left(\frac{p_{i,t}}{p_{i,t-1}} - 1 \right)^2 p_t y_t ,$$

where ϕ measures the degree of nominal rigidity. Hence, the firm faces an inter-temporal problem that stems from the effect of $p_{i,t}$ on future price adjustment costs. Current real

period profits of firm *i* are denoted by $d_{i,t} = \mathcal{D}_{i,t}/p_t$. Taking aggregate prices as given, firm *i* chooses $p_{i,t+j}$ and $y_{i,t+j}$ to maximize discounted profits

$$\max_{p_{i,t+j},y_{i,t+j}} \bar{E}_t \sum_{j=0}^{\infty} \omega^j Q_{t,t+j} \left(\frac{p_{i,t+j}}{p_{t+j}} y_{i,t+j} - w_{t+j} l_{i,t+j} - \frac{\phi}{2} \left(\frac{p_{i,t+j}}{p_{i,t-1+j}} - 1 \right)^2 y_{t+j} \right) ,$$

subject to the demand schedule of final good firms (2.7). The expectation operator $\bar{E}_t \mathbf{z}_{t+1} \equiv (1-\omega) \sum_{k=-\infty}^t \omega^{t-k} \tilde{E}_t^k \mathbf{z}_{t+1}$ denotes the aggregated expectations across all cohorts alive in period t for a generic variable \mathbf{z} and is a size-weighted sum of cohort expectations. Note that the generational structure matters for aggregating the decisions of the households of different age and especially when aggregating the expectations of differently aged households. Since households hold equal shares in every firm, firms use a weighted average of household expectations. Further, $Q_{t,t+j} \equiv \beta^j \frac{c_t}{c_{t+j}}$ denotes the aggregate real stochastic discount factor of households, where $c_t = (1-\omega) \sum_{k=-\infty}^t \omega^{t-k} c_{t|k}$.

2.2.3 Monetary Policy

The nominal interest rate on bonds is determined by a monetary policy authority that sets it according to a feedback rule:

$$r_t = \bar{r} \left(\frac{\pi_t}{\pi}\right)^{\varphi_\pi} \left(\frac{y_t}{y_t^n}\right)^{\varphi_y} \exp\left(\epsilon_t^m\right) , \qquad (2.9)$$

$$\epsilon_t^m = \rho_m \epsilon_{t-1}^m + \nu_t^m \quad \text{with} \quad \nu_t^m \stackrel{iid}{\sim} \left(0, \sigma_m^2 \right) , \qquad (2.10)$$

where \bar{r} , π , and y_t^n are the steady state interest rate, aggregate inflation, and the natural level of output, respectively. For the sake of convenience, we assume that $\pi = 1$, i.e., we consider a zero-inflation steady state. The parameters φ_{π} and φ_y denote the feedback coefficients that determine the sensitivity to deviations of inflation from its steady state and of output from its natural level, respectively. Last, ϵ_t^m serves as monetary policy shock and evolves according to an AR(1)-process. We specify the monetary policy authority to use *current* inflation (opposed to its expectation), to avoid taking a stance on which type of expectations the monetary policymaker has.

2.2.4 Equilibrium

Labor Market Equilibrium. As all intermediate firms produce with the same technology, equilibrium labor demand is symmetric. Aggregate working hours follow as

$$l_t^d \equiv \int_0^1 l_{i,t} di = \int_0^1 (y_{i,t})^{\frac{1}{\alpha}} di = (y_t)^{\frac{1}{\alpha}} \Delta_t^p = l_t^s \equiv (1-\omega) \sum_{k=-\infty}^t \omega^{t-k} l_{t|k} , \qquad (2.11)$$

where $\Delta_t^p \equiv \int_0^1 \left(\frac{p_{i,t}}{p_t}\right)^{-\frac{\varepsilon}{\alpha}} di$ is an index of relative price distortions. Since all firms face a symmetric maximization problem, we focus on a symmetric price equilibrium, i.e., $\Delta_t^p = 1$.

Goods Market Equilibrium. An equilibrium on the aggregate goods market requires that the total number of goods produced y_t equals the total amount of goods demanded, taking into account the dead-weight loss due to repricing cost

$$y_t = c_t + \frac{\phi}{2} (\pi_t - 1)^2 y_t , \qquad (2.12)$$

where $\pi_t = \frac{p_t}{p_{t-1}}$ denotes (gross) inflation.

Bond Market Equilibrium. Private bonds are in zero net supply, that is

$$(1-\omega)\sum_{k=-\infty}^{t}\omega^{t-k}b_{t|k} = 0.$$
(2.13)

New Keynesian Phillips Curve. Using the FOC on prices of intermediate good firms and symmetry, one derives

$$(\pi_t - 1) \pi_t = \omega \bar{E}_t \left[Q_{t,t+1} \frac{y_{t+1}}{y_t} (\pi_{t+1} - 1) \pi_{t+1} \right] + \frac{\varepsilon (1 + \eta_l)}{\phi} (\mathrm{mc}_t^r - \mu) , \qquad (2.14)$$

where $\operatorname{mc}_{t}^{r}$ are the real marginal cost, $\mu \equiv \frac{\varepsilon-1}{\varepsilon}$ denotes the steady state markup, and $\eta_{l} = \frac{l}{1-l}$ denotes the stationary labor-leisure share. Note that *aggregate* inflation expectations, π_{t+1} , affect π_{t} . According to (2.14), optimal price setting requires inflation to be a function of current real marginal cost and expected future inflation.

Linearized Equilibrium Conditions. Expectation heterogeneity matters for households' Euler equations and the New Keynesian Phillips Curve (NKPC). To arrive at an aggregated dynamic IS-curve, we follow the literature and rely on Branch and McGough (2009). First, we adopt their assumption on higher-order beliefs: household *i*'s expectation about what another household *k* expects, is its own expectation: $\tilde{E}_t^i \tilde{E}_t^k \mathbf{z}_{t+1} = \tilde{E}_t^i \mathbf{z}_{t+1}, \ i \neq k$ for some generic variable \mathbf{z} , which reduces the complexity imposed on the model considerably.⁸ Second, we assume agents expect to hold the same wealth in the limit $t \to \infty$. For each agent *i*, consumption then equals the long-run consumption: $\tilde{E}_t^i (\hat{c}_{\infty} - \hat{c}_{\infty}^i) = 0$. This assumption prevents the wealth distribution from appearing in the aggregated IS-curve.⁹ After linearization (a linearized variable *x* is denoted \hat{x}) and aggregation, we rewrite the model in terms of the output gap $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$, where \hat{y}^n is the deviation of the natural level of output from its steady state. We arrive at a system of five equations and five variables $\{\tilde{y}_t, \hat{\pi}_t, \hat{r}_t, \hat{e}_t^m, u_t\}_{t=0}^{\infty}$:

$$\tilde{y}_t = \bar{E}_t \tilde{y}_{t+1} - \left(\hat{r}_t - \bar{E}_t \hat{\pi}_{t+1}\right) , \qquad (2.15a)$$

⁸For approaches considering higher order beliefs, see Angeletos et al. (2018) or Farhi and Werning (2019).

⁹This is seen when aggregating the linearized Euler equations (see Appendix A.1). The assumption cancels the differences in expected consumption at $t \to \infty$ that occur when aggregating expected consumption for households that have heterogeneous expectations and solve a dynamic problem.

$$\hat{\pi}_t = \beta \omega \bar{E}_t \hat{\pi}_{t+1} + \kappa \tilde{y}_t + u_t , \qquad (2.15b)$$

$$\hat{r}_t = \varphi_\pi \hat{\pi}_t + \varphi_y \tilde{y}_t + \hat{\epsilon}_t^m , \qquad (2.15c)$$

$$\hat{\epsilon}_t^m = \rho_m \hat{\epsilon}_{t-1}^m + \nu_t^m , \qquad (2.15d)$$

$$u_t = \rho_u u_{t-1} + \nu_t^u \,, \tag{2.15e}$$

where $\kappa \equiv \frac{(\varepsilon-1)}{\phi} \frac{(1+\eta_l)}{\alpha}$ denotes the slope of the New Keynesian Phillips curve.¹⁰ We introduce a cost-push shock u_t that could stem from a firm-specific shock to marginal cost to have a source of exogenous variation apart from the monetary policy innovation (see Ireland, 2004). To solve the model, we next need to specify how agents form expectations.

2.3 Expectation Formation

In this section, we discuss how different cohorts form expectations on inflation and the output gap based on Malmendier and Nagel (2016). First, we explain how a single cohort forms expectations and discuss why experience effects play an important role. Then, we highlight how EBL differs from CGL, which is among the most popular learning approaches.

2.3.1 Learning

We follow Evans and Honkapohja (2012) and Slobodyan and Wouters (2012a) who require near-rational agents to forecast variables only one period ahead, e.g., of variables in their Euler equation so that the approach is called *Euler equation learning*.¹¹ A large part of the literature assumes that agents know the true state-space representation, i.e., they know the relevant variables for the economy's evolution, but have to learn about the coefficients of this representation (see, e.g., Milani, 2007). Instead, we assume that agents employ a mis-specified forecasting rule. In comparison to having all state variables as regressors, the agents' perceived law of motion (PLM) is based on only a subset of them or no states at all. We adapt the set-up in Malmendier and Nagel (2016) and specify agents' PLM as AR(1).¹² However, we assume that it does not include a constant so that agents know the true mean of the model. Consequently, the PLM of a generic variable \mathbf{z}_t for a household in cohort k is given by

$$\mathbf{z}_{t|k} = b_{t-1,k}^{\mathbf{z}} \mathbf{z}_{t-1} + \varepsilon_{t|k}^{\mathbf{z}} , \qquad (2.16)$$

 $^{^{10}}$ The updating equations of household beliefs (2.18a) and (2.18b) are also part of the model.

¹¹An alternative approach is the Infinite Horizon Forecast as developed in Preston (2005). For a comparison of these approaches see Evans et al. (2013).

¹²Orphanides and Williams (2005) and Slobodyan and Wouters (2012a) use similar specifications of agents' forecasting rule. The choice of order one is further consistent with the model under RE where variables are also Markov-processes of order one.



FIGURE 2.2: Timing Assumption

where $\varepsilon_{t|k}^{\mathbf{z}}$ is a disturbance term which is serially-uncorrelated with zero mean and constant variance and $b_{t-1|k}^{\mathbf{z}}$ is the estimated parameter of household k at time t-1. In our model, agents form expectations on the output gap and inflation. Hence, the set of variables on which agents form expectations is given by $Y^f \equiv \{\tilde{y}, \hat{\pi}\}$ with $\mathbf{z} \in Y^f$.

We assume that individuals form expectations at time t using only information available at time t - 1. By doing so, we avoid a simultaneity problem that arises when agents use time t endogenous variables to forecast future realizations, which, in turn, affects the time t endogenous variables (see Evans and Honkapohja, 2012). We summarize the timing assumption in Figure 2.2.

The last element of the PLM is how its coefficients develop over time. Let \mathbf{I}_t be the information set on which households base their forecast at time t. The information set \mathbf{I}_t includes all model variables up to t - 1. Consequently, the formation of expectations occurs before the realization of the endogenous variables included in Y^f such that $\tilde{E}_t^k(\mathbf{z}_t) = \tilde{E}^k(\mathbf{z}_t | \mathbf{I}_t) \neq \mathbf{z}_t$. Instead, using (2.16) and presuming that the law of iterated expectations holds for the subjective expectations, households in cohort k forecast

$$\tilde{E}_{t}^{k}\left(\mathbf{z}_{t+1}\right) = \tilde{E}_{t}^{k}\left(b_{t|k}^{\mathbf{z}}\mathbf{z}_{t}\right) = \tilde{E}_{t}^{k}\left(b_{t-1|k}^{\mathbf{z}}\mathbf{z}_{t}\right) \\
= \tilde{E}_{t}^{k}\left(b_{t-1|k}^{\mathbf{z}}\left(b_{t-1,k}^{\mathbf{z}}\mathbf{z}_{t-1} + \varepsilon_{t|k}^{\mathbf{z}}\right)\right) = (b_{t-1|k}^{\mathbf{z}})^{2}\mathbf{z}_{t-1},$$
(2.17)

where for the first equality we use the PLM (dated t + 1) and for the second that point estimates of the PLM parameters only include information up to t - 1. The third equality uses that agents form expectations before the current realization of \mathbf{z} such that also today's realization is forecasted using the PLM. Finally, the last equality uses that the PLM parameter estimated with information up to time t - 1 is uncorrelated with the error term at time t, i.e., $\tilde{E}_t^k \left(b_{t-1|k}^{\mathbf{z}} \varepsilon_{t|k}^{\mathbf{z}} \right) = 0$.

After the realization of time t shocks, agents update their PLM parameters from $b_{t-1|k}^{\mathbf{z}}$ to $b_{t|k}^{\mathbf{z}}$ using the following recursive least-squares algorithm

$$b_{t|k}^{\mathbf{z}} = b_{t-1|k}^{\mathbf{z}} + \gamma_{t|k} \left(R_{t|k}^{\mathbf{z}} \right)^{-1} \mathbf{z}_{t-1} \hat{\varepsilon}_{t|k}^{\mathbf{z}}$$
(2.18a)

$$R_{t|k}^{\mathbf{z}} = R_{t-1|k}^{\mathbf{z}} + \gamma_{t|k} (\mathbf{z}_{t-1} \mathbf{z}_{t-1}' - R_{t-1|k}^{\mathbf{z}}) , \qquad (2.18b)$$

for each $\mathbf{z} \in Y^f$. Here, $\hat{\varepsilon}_{t|k}^{\mathbf{z}} \equiv \mathbf{z}_t - b_{t-1|k}^{\mathbf{z}} \mathbf{z}_{t-1}$ denotes the forecast error of cohort k and $\gamma_{t|k}$ gives the (potentially) age-dependent Kalman gain of cohort k that governs how sensitive estimate revisions are to forecast errors $\hat{\varepsilon}_{t|k}^{\mathbf{z}}$ based on the old parameter estimate. The covariance matrix of the regressor, $R_{t-1|k}^{\mathbf{z}}$, also influences the revision of the estimates to forecast errors.¹³ Similar to the PLM parameter $b_{t|k}^{\mathbf{z}}$, the covariance matrix is updated recursively. Last, we assume newly-born agents are endowed with a PLM parameter that is equal to the aggregate persistence parameter of the previous period.¹⁴

2.3.2 Experience-Based Learning

The novelty of EBL lies in the age-dependent form of parameter updating. Malmendier and Nagel (2016) provide evidence that the gain parameter $\gamma_{t|k}$ depends on the amount of lifetime data (or equivalently age), t - k, of individuals in cohort k

$$\gamma_{t|k} = \begin{cases} \frac{\theta}{t-k} & \text{if } t-k \ge \theta\\ 1 & \text{if } t-k < \theta \end{cases},$$
(2.19)

where $\theta > 0$ determines the degree to which individuals react to recent observations. Above specification implies, firstly, that expectations are heterogeneous between cohorts. Secondly, it implies that young agents have higher gains than older ones so that they update their PLM's parameters more strongly.

Both aspects are captured in Figure 2.3. The left panel plots the gain parameter over age for different values of θ .¹⁵ Young agents have high gains, consistent with the idea that they have less lifetime observations and, therefore, rely more on current data. The size of gains also decreases in age; the more so, the higher θ . The right panel of Figure 2.3 shows the implied weights a 50 year (200 quarter) old individual puts on data observed over her lifetime for different values of θ .¹⁶ For $\theta > 1$, data observed early in life receives negligible weights as an individual ages so that recent data is more important to update the PLM (data before birth has weight zero, as only lifetime information is used). Note also that agents of different age use a different amount of information. Although in our perpetual youth structure there are individuals who use information from the far past, the mass of such a cohort declines as time passes by. Further, the weight such an individual would

¹³The lower the variance in the explanatory variables, the stronger the update.

¹⁴A newly-born agent follows the "conventional wisdom". While several papers consider learning-from-experiences, the treatment of initial beliefs varies: Schraeder (2015) uses initial beliefs that correspond to RE, Ehling et al. (2018) endow young agents with a small initial information set to deduct an initial belief, and Collin-Dufresne et al. (2016) assume young agents inherit beliefs from their parents. In Appendix A.3 we assume that new-born cohorts draw the initial belief from a normal distribution around the RE estimate, where the normal distribution is truncated at ± 1 . Key results remain unchanged.

¹⁵The graph is based on Malmendier and Nagel (2016). Their appendix shows how to derive it.

¹⁶Note that recursive least-squares is the recursive formulation of weighted least squares. The weights inside the weighting matrix contain the gain parameter $\gamma_{t|k}$ and, thus, depend on θ .



FIGURE 2.3: Gain and Weights on Past Data

Notes: The left panel depicts the evolution of the gain parameter over age (in years) for three different values of θ (the estimate of Malmendier and Nagel (2016) is around 3). The right panel shows how a 50 year old agent weights past information when estimating the parameters of the PLM (again for different θ). The purple line denotes the case of CGL.

put on this information is small, such that this information's influence on the current aggregate expectation is negligible.

2.3.3 Constant-Gain Learning

Under EBL, agents have different gain parameters depending on their age. As mentioned above, studies of DSGE models with dynamic non-rational expectations often employ learning algorithms with CGL so that all agents react equally to new observations. In practice this amounts to replacing $\gamma_{t|k}$ in (2.18) with a constant g so that the left panel of Figure 2.3 shows a constant (purple line) across ages. Under this assumption, agents of different cohorts are homogeneous with respect to their expectation formation, i.e.,

$$\tilde{E}_t^k \mathbf{z}_{t+1} = \tilde{E}_t \mathbf{z}_{t+1} = (b_{t-1}^{\mathbf{z}})^2 \mathbf{z}_{t-1} ,$$

for all cohorts k. This setup still retains the feature that new observations (and, hence, forecast errors) are weighted higher than old observations (right panel of Figure 2.3). However, in contrast to EBL, individuals put a non-zero weight to *all* data points. In the following, we will interpret the CGL approach as the counterpart of EBL where we *shut off* experience effects on individuals' expectations. We simulate our model for this specification to study the *additional* endogenous source of variation that stems from experience effects alone.

2.4 Inspecting the Mechanism

This section discusses the mechanism by which EBL affects the dynamics of the economy relative to a model with CGL. We start with a brief description of our parameter choices.

Variable		Value	
β	Discount factor	0.995	Ann. riskless rate 4%
ψ_n	Utility weight on leisure	1.17	Steady state labor supply of $1/3$
ε	Elasticity of substitution	9	Mark-up of 12.5%
α	DRS parameter	0.66	US labor share
ξ	(Inverse) Frisch elasticity	2	Standard choice
ϕ	Rotemberg parameter	93.2	Share non-adjusters 75%
φ_{π}	Taylor parameter π	1.5	Galí (2015)
φ_y	Taylor parameter y	0.125	Galí (2015)
π	Inflation Target	1	Zero-inflation steady state
ω	Survival probability	0.995	50 year working-life
$ ho_u$	Persistence $\hat{\epsilon}^u$	0.96	Ireland (2004)
$ ho_m$	Persistence $\hat{\epsilon}^m$	0.60	Standard choice
σ_u	Standard deviation ν^u (in %)	0.15	Ireland (2004)
θ	EBL parameter	3.044	Malmendier and Nagel (2016)
g	Gain under CGL	0.015	Standard choice

TABLE 2.1: Parameter Choices (quarterly)

To discuss the key difference between EBL and CGL, we simulate both models under supply shocks.

2.4.1 Parameterization and Simulation Algorithm

Parameterization. One period in the model corresponds to one quarter. We calibrate the model's deep parameters to US data (Table 2.1 provides a summary). Our choice of the survival probability $\omega = 0.995$ is guided to meet an average life span of 200 quarters, which represents the working-life of an agent. Most of the other parameters are taken from Galí (2015). The households' discount factor β is calibrated to get a steady state real annualized return on riskless bonds of 4% given our choice for ω . Furthermore, we set the steady state elasticity of substitution to $\varepsilon = 9$, which implies a steady state mark-up of 12.5%. The parameter of the production function α is chosen to be 0.66 in line with the labor share in US data. The choice of the Rotemberg adjustment cost parameter matches a fraction of non-adjusters of 0.75 in a model with Calvo price setting $\phi = 93.2$. We set $\varphi_{\pi} = 1.5$ and $\varphi_{\eta} = 0.125$ to standard choices. Moreover, the values for the serial correlation coefficient of the cost-push shock ρ_u and the standard deviation of the innovation σ_u are set 0.96 and 0.0015, respectively, which correspond to the values estimated in Ireland (2004). Further, we set the serial correlation coefficient the monetary policy shock ρ_m to 0.6. We choose the learning parameter that governs the age-dependent gain under EBL as $\theta = 3.044$ (Malmendier and Nagel, 2016) and the CGL parameter g as 0.015 according to Milani (2007) and much of the learning literature. Finally, steady state gross inflation is targeted to be one, while the steady state labor supply is 1/3.
Simulation. We simulate the model under (i) EBL, (ii), RE, and (iii) CGL for the same random sequence of supply shocks, while setting the monetary policy innovation to zero. To initialize the PLM parameters for the learning models, we simulate the economy under RE for $T_{\text{init}} = 1000$ quarters and estimate an AR(1) model for inflation and the output gap. The estimated AR(1) coefficients for both variables serve as the respective initial PLM parameters for the models with EBL and CGL. To start model simulations, we endow each cohort with the same initial persistence parameter $b_{-1|k}^{\mathbf{z}} = b_{-1}^{\mathbf{z}}$ and the same covariance matrix of estimates $R_{-1|k}^{\mathbf{z}} = R_{-1}^{\mathbf{z}}$ for $\mathbf{z} \in Y^{f}$ both of which are updated subsequently. We then simulate the economy for $T_{\rm sim} = T_b + 300,000$ quarters, where $T_b = 1000$ is the burn-in discarded to wash out the impact of the initial values from the simulation of the RE economy. Each period members of cohort k update their parameter estimate and the covariance matrix according to equation (2.18). Similar to Slobodyan and Wouters (2012a), we restrict agents to rely on covariance stationary forecasting models: we invoke a projection facility and restrict the new estimate to induce a stationary AR(1) process, which requires $|b_{t|k}^{\mathbf{z}}| < 1$ for all cohorts k and $\mathbf{z} \in Y^{f}$. In case the new estimate exceeds the bounds of ± 1 , the old estimate is kept and no updating takes place.¹⁷ Intuitively, Evans and Honkapohja (2012) argue that agents avoid explosive paths of the economy such that the agent chooses its parameter estimate accordingly. Each newly born cohort's initial parameter equals the aggregate persistence parameter of the previous period. Further, there is an infinite number of cohorts so that we need to restrict the number of cohorts for our simulations. A high number of cohorts reduces the approximation error but comes at the cost of greater computational time. Since the baseline model calibrates the survival probability so that the expected lifetime is 200 quarters, we restrict the number of cohorts in the aggregation to be 200 and normalize cohort weights to sum to one. A detailed description of the algorithm is given in Appendix A.2.

2.4.2 The Effect of Experiences on the Perceived Persistence

In this section we discuss the key implications of EBL that matter for the analysis of monetary policy. In particular, young individuals' perceived persistence is on average lower and more volatile relative to old individuals under EBL. Consequently, the *aggregate* perceived persistence of inflation and the output gap turns out to be on average lower compared to an economy where we shut-off experience effects, i.e., where we assume CGL. Put differently, under EBL the micro-level heterogeneity affects the macro-level.

Perceived Persistence. In our model, heterogeneity in expectations across cohorts stems from the heterogeneity in individuals' perceived persistence of inflation and the output

¹⁷The restriction is invoked in only 6% of updates for PLM parameters under EBL. This falls to 2% when we marginally increase the bound on parameter estimates. Its usage is similar to the one in Slobodyan and Wouters (2012b) for high gains.



FIGURE 2.4: Distribution of Cohort PLM Parameters in the Model with EBL

Notes: The blue histogram denotes the distribution of the PLM parameters for young (k = 10) agents. The red histogram shows the distribution for old agents (k = 158). The left part shows the PLM parameters for the output gap and the right part shows the PLM parameters for inflation. Vertical lines denote means. We simulate the economy for 300,000 periods.

gap. In Figure 2.4 we plot the ergodic distribution of PLM parameters for individuals of cohort k = 10 (young) and k = 158 (old) for 300,000 simulated periods. The left part shows the distribution of the PLM parameters for the output gap and the right part shows the distribution of the PLM parameters for inflation. The vertical lines denote the average perceived persistence across simulations for young (solid) and old (dashed) agents.

Figure 2.4 demonstrates that the perceived persistence of the output gap and of inflation for young agents (blue) is more dispersed and *on average* lower relative to the ones for old agents (red), respectively. There are two driving forces behind this finding. First, young individuals rely on a lower amount of information when updating their PLM parameters. Second, they are more sensitive towards new observations. As a result, the dispersion of estimates is higher, since the variance in parameter estimates decreases in the number of observations and since young agents overweight new observations in updating. Moreover, this implies a lower perceived persistence, on average. Unless inflation or the output gap stay almost constant for multiple periods, recurrent reversals make young agents perceive both variables to be less persistent compared to old individuals.¹⁸

¹⁸Recall that agents use covariance stationary forecasting models. The combination of a higher dispersion and the truncation of the PLM parameter distribution may further decrease the mean of young agents' perceived persistence. However, we find this to not be a key driving force.



FIGURE 2.5: Actual Inflation and PLM Parameters of Young and Old

Notes: The blue line depicts the annualized realized inflation deviation from steady state under EBL. The red lines show the estimated slope of the PLM for young (dashed) and old (dotted) agents. Throughout the simulation we compare agents of ages 10 and 158 quarters. Hence, we do not follow two single age groups through time.

Intuition Behind the Learning Process. The dynamics of the difference in inflation expectations is linked to the experienced inflation of each cohort. Figure 2.5 displays a snapshot of one simulation path. The blue bold line depicts simulated *realized* inflation (left y-axis) and the red lines denote the slope of the PLM for a young (dashed) and an old (dotted) cohort (right y-axis). PLM parameters of young and old households differ, which determines differences in expected inflation. A key reason for the difference in the perceived persistence is the fact that young individuals only use the most recent data to estimate the parameter of their forecasting model. Consider the periods before t = 50. During the preceding ten quarters, inflation stayed roughly at a 3% deviation from the steady state so that young individuals perceived inflation to be highly persistent. In fact, their perceived persistence is higher than the one of old individuals, who already experienced sharp reversals in inflation. In contrast, in the ten quarters before t = 30, inflation went through a reversal so that young individuals perceive inflation to be less persistent than old individuals, who also observed more stable times of inflation. Figure 2.5 demonstrates that young individuals on average perceive inflation to be less persistent which reflects the results in Figure 2.4.

Aggregate Perceived Persistence. The heterogeneity in the perceived persistence across cohorts has important implications for the *aggregate* perceived persistence, which, under EBL, is a size-weighted average over the cohort-specific perceived persistence. In Figure 2.6 we compare the distribution of the aggregate perceived persistence under EBL (red)



FIGURE 2.6: Distribution of Aggregate PLM Parameters in the Models with EBL and CGL

Notes: The blue histogram denotes the distribution of the PLM parameters in the model under EBL. The red histogram shows the distribution under CGL. The left part shows the PLM parameters for the output gap and the right part shows the PLM parameters for inflation. We simulate the economy with EBL and CGL for 300,000 periods.

to the one obtained under CGL (blue). Vertical lines denote the average values for CGL (solid) and EBL (dashed). Under EBL, the aggregate perceived persistence for inflation (right panel) and the output gap (left panel) is lower on average and more dispersed, respectively. To understand this finding, note that the gain of the representative agent under CGL g = 0.015 maps into the gain of an individual with a 50-year working life under EBL. For any individual in the model with EBL that is younger than the representative agent under CGL, the perceived persistence of both inflation and the output gap is lower on average and more dispersed. If the mass of such individuals is sufficiently high, also the aggregate perceived persistence is lower and more dispersed. Hence, the age distribution affects the first two moments of the distribution of the aggregate perceived persistence. EBL endogenously pushes down the aggregate perceived persistence of inflation and the output gap on average.¹⁹ In the next section, we show that this is of relevance for the impact of monetary policy on expectations and its transmission on inflation.

Experience Channel. Since the aggregate perceived persistence is a size-weighted average over the cohort-specific perceived persistence, a variation in the age distribution directly affects the aggregate perceived persistence through a composition effect, which we call the *Experience Channel*. In our model, a variation in the age distribution corresponds to a variation in the survival probability ω . As ω falls, the share of young indi-

¹⁹In Appendix A.3, we perform robustness checks by varying key parameters of the model and the type of the shock. The core result regarding the aggregate perceived persistence remains unchanged.

viduals increases, while the share of old individuals decreases. As shown above, young individuals' perceived persistence is on average lower than the one of old ones so that the aggregate perceived persistence for inflation and the output gap decrease in the share of young individuals. This effect does not exist under CGL. As we will see next, EBL opens a new channel by which a variation in the age distribution affects the transmission of monetary policy.

2.5 The Effect of Monetary Policy Under EBL

We use the results from the previous section and analyze their implications for monetary policy. First, we investigate impulse responses to monetary policy shocks under EBL, which we contrast with those from models with RE and CGL. We also explore how different relative sizes of young to old cohorts (called "demography") affect results. Finally, we compare the monetary policy trade-off between output gap and inflation stabilization in a model with EBL to models with RE and CGL and, again, consider the role of demography.

2.5.1 Transmission of a Monetary Policy Shock

First, we analyze the effect of EBL on the transmission of monetary policy shocks and how this effect depends on the age distribution.²⁰ To explore this, we compute *generalized* impulse response functions in the model with (i) EBL, (ii) CGL, and (iii) RE. The monetary policy shock corresponds to an innovation of 25 basis points to $\hat{\epsilon}_t^m$. The algorithm used to compute the generalized impulse response functions is described in Appendix A.2.

Impact. Consider Figure 2.7. On impact, a contractionary monetary policy shock affects the output gap and inflation negatively. In response, the central bank pushes the nominal interest rate down. This decrease is, however, insufficient to offset the exogenous shift such that the nominal interest rate increases, which increases the real interest rate. Comparing the initial responses, both the output gap and inflation react less under EBL compared to the RE model. If we shut off experience effects on expectations (CGL), we observe similar impact responses of the output gap and of inflation that become stronger the period after the shock hits the economy when expectations are revised.

Revision of Expectations. Under both EBL and CGL, individuals revise their expectations with a delay of one period. This backward-looking expectation formation is visualized in the lower middle and right panels of Figure 2.7, which illustrate responses of expectations. Only in the period *after* the shock, individuals revise expectations on

 $^{^{20}}$ In Appendix A.4 we consider responses to a supply shock. The basic result remains the same.



FIGURE 2.7: IRFs to a Monetary Policy Shock

Notes: We show the generalized impulse response functions for key variables in the economies under CGL (black), EBL (blue), and RE (red). We average responses over 8,000 iterations. The output gap and output gap expectations are measured as percentage deviations from their respective steady state, while the other variables are measured as (annualized) deviation from their respective steady state.

future inflation and the output gap downwards, which feeds back into the current output gap and inflation. The lower impact response of inflation and, to a smaller extent, the output gap under EBL compared to CGL results from the smaller reaction of expectations that is driven by a smaller aggregate perceived persistence attached to these two variables (see Figure 2.6).²¹ Under adaptive expectations, current monetary policy only affects expectations in the next period, i.e., the perceived persistence reflects past shocks and monetary policy. Since agents under EBL attach on average a lower perceived persistence to variables, monetary policy is less effective in influencing expectations and, thereby, contemporaneous variables. As a result, the pass-through of monetary policy on inflation is impaired under EBL. However, the difference in the response of the output gap under EBL and CGL is smaller because the simultaneous decrease in the real rate, which is slightly stronger under CGL, partly counteracts the impact of more negative CGL-expectations. Note that the small difference is at first sight surprising given that, ceteris paribus, the response of expected inflation under EBL is less pronounced, i.e., the real rate should move by more. However, the smaller response of inflation expectations is counteracted by a weaker response of the nominal rate due to a lower response of current

²¹A similar result is obtained by Slobodyan and Wouters (2012a). The aggregate perceived persistence in the economy affects the response of the endogenous variables in response to exogenous disturbances.

inflation under EBL. Both effects approximately neutralize.

Forward-looking RE stand in contrast to adaptive expectations. Agents take into account the Taylor rule's impact on the future path of real interest rates and the shock's persistence when forming expectations. Rational expectations of the output gap and inflation recover quickly and, thereby, have a different trajectory than expectations under EBL or CGL that respond to the previous realization of the output gap and inflation with a lag.

Dynamics. The dynamic response of macroeconomic variables under EBL (or CGL) is quite different compared to the one under RE. While under RE the economy reverts back to the steady state roughly after ten quarters, deviations in the economy under EBL are more persistent. Under EBL, agents revise their beliefs downwards and the shock is more slowly transmitted into their expectations compared to RE, where monetary policy affects expectations more effectively. Under RE, individuals perfectly incorporate the impact of the Taylor rule into their expectations. In addition, inflation and the output gap display a hump-shaped response under EBL, which results from the backward-looking expectations of agents and their lagged response to the monetary policy shock.

To bring inflation back to its steady state, monetary policy decreases the nominal interest rate by that much that the real interest rate undershoots two quarters after the shock hits. This, in turn, shifts the output gap upwards that eventually overshoots, which boosts inflation upwards towards its steady state. Compared to the model with EBL, this overshooting is more pronounced under CGL so that the gap between the inflation responses under EBL and CGL closes.

Demography. Next, we focus on the impact of a demographic shift in the model with EBL and analyze the effect on the transmission of the monetary policy shock. In the perpetual youth model, a change in the demographic structure corresponds to a change in the survival probability ω . To illustrate the effect of the demographic structure, we reduce the survival probability to 0.96. A change in the survival probability affects

- 1. the effective discount factor, $\tilde{\beta} \equiv \beta \omega$ and
- 2. aggregate expectations \overline{E}_t under EBL (Experience Channel).

In Figure 2.8, the solid blue line denotes responses in the baseline ($\omega = 0.995$) economy and the dotted blue line denotes responses for a lower survival probability ($\omega = 0.96$). The green line shows the impulse response function under EBL when the effective discount factor $\tilde{\beta}$ is fixed at the baseline value by varying β accordingly so that only the Experience Channel (channel 2.) operates.²²

 $^{^{22}}$ In Appendix A.4, we also vary ω under RE and CGL. Given no Experience Channel exists, differences between a young and an old economy are small.



FIGURE 2.8: IRFs to a Monetary Policy Shock – Decomposition Under EBL

Notes: We show the generalized impulse response functions for key variables in economies under EBL. We distinguish the baseline case (blue solid), the young economy (blue dotted), and the Experience Channel (green). We average responses over 8,000 iterations. The output gap and output gap expectations are measured as percentage deviations from their respective steady state, while the other variables are measured as (annualized) deviation from their respective steady state.

As discussed in Section 2.4.2, an increase in the share of young agents reduces the aggregate perceived persistence of inflation and the output gap via the Experience Channel. Thus, in the young economy the pass-through of monetary policy via expectations is reduced compared to the baseline economy (blue solid). Indeed, the Experience Channel alone (green line) generates a quantitatively considerable decrease in the size and persistence of the responses of the output gap and inflation. The impact response is slightly less pronounced when considering the full effect, i.e., when channel 1. is active as well (blue dotted line). This stems from the higher effective discount factor that makes current inflation less sensitive to inflation expectations. For the same reason, the persistence of the response is lower for the full effect. Intuitively, channel 1. has a similar impact as the Experience Channel with the difference that it only affects expectations in the NKPC.²³

2.5.2 Trade–Off Under Supply Shocks

Next, we compare different Taylor rule calibrations with respect to their ability to close the output gap and to stabilize inflation under supply shocks. In the context of the

²³To maintain a calibration that targets a share of non-adjusters of 0.75, one would need to also change the Rotemberg parameter. However, qualitatively this does not change results and quantitatively the impact is marginal. As the interpretation of results is more intuitive for a fixed Rotemberg parameter, we refrain from implementing this change.



FIGURE 2.9: Monetary Policy Frontier Under a Supply Shock

Notes: We plot the standard deviation of the output gap against the one for inflation for the models under EBL (blue), CGL (black), and with RE (red). Units are in percent deviation from the steady state. The markers denote the combinations of $(\varphi_{\pi}, \varphi_{y})$ on the grid.

New Keynesian model, it is a well-known result that the Taylor rule is incapable to simultaneously close the output gap and stabilize inflation when supply shocks perturb the economy (see Galí, 2015). Since EBL impairs the monetary policy transmission via expectations, this trade-off is also affected. Starting from the baseline parameterization of the Taylor parameters ($\varphi_{\pi}, \varphi_{y}$) we successively increase monetary policy's output gap stabilization motive (increase φ_{y} up to 1), while holding the Taylor parameter on inflation φ_{π} constant.²⁴ The policy frontiers are shown in Figure 2.9, where we plot the standard deviation of the output gap σ_{y} against the one of inflation σ_{π} for each combination of $(\varphi_{\pi}, \varphi_{y})$. The blue line displays the results under EBL, the black line shows results when we consider CGL, and the red line displays the simulation results under RE.

On the one hand, the monetary policy trade-off between closing the output gap and stabilizing inflation under supply shocks also arises under EBL. As we increase the Taylor coefficient on the output gap (move up the policy frontier), output gap volatility decreases, while inflation volatility increases. On the other hand, the policy frontier shifts inwards under EBL so that a given level of inflation volatility is associated with a lower output gap volatility.²⁵ Moreover, the policy frontier under EBL is flatter than the one under CGL, which indicates that any reduction in inflation volatility is more costly in terms of output gap volatility.

The *inward shift* of the EBL policy frontier is explained by two effects. First, the passthrough of monetary policy is weaker under adaptive expectations, in general. Under RE,

²⁴Specifically, we define a grid of points for φ_y and simulate the economy as described above for 300,000 periods for each grid point, while setting $\varphi_{\pi} = 1.5$.

 $^{^{25}}$ Similarly, inflation volatility falls for most parameter combinations so that the trade-off occurs on an overall lower level of volatility.

private sector expectations are forward-looking and consider the Taylor rule's impact on the future path of real interest rates, which directly affects the current output gap via the Dynamic IS Curve. Under EBL (and CGL), in contrast, agents' expectations are backward-looking. They fail to project the effect of monetary policy on the future path of the real interest rate. Therefore, the effect of monetary policy on aggregate demand is subdued, which lowers its transmission on inflation via the NKPC. Second, the aggregate perceived persistence of inflation and the output gap is lower on average under EBL compared to CGL. In consequence, expectations and current variables react by less to past monetary policy actions or past supply-side shocks so that the output gap is less volatile and the policy frontier shifts inwards compared to CGL.²⁶

The *flattening* of the policy frontier in comparison to CGL is a result of the lower aggregate perceived persistence. Assume monetary policy increases the relative weight it puts on stabilizing inflation. In response to a supply shock that increases inflation but decreases the output gap, monetary policy increases the interest rate more forcefully, which strengthens the downward shift in the output gap. Due to the lower perceived persistence, inflation expectations are less responsive to past inflation. Since past inflation is influenced by past output gap realizations, the downward shift in current inflation, that usually arises from the changes in the output gap, is dampened, on average. As a result, inflation is stabilized less effectively so that a given reduction in inflation volatility requires a higher increase in output gap volatility under EBL.

Demography. Next, we consider how the policy frontier is affected by a change in the demographic structure of the economy by reducing ω to 0.96 (corresponding to a high share of young agents). In Figure 2.10, the full impact of a variation in ω on the policy frontier, when channels 1.-2. operate, is shown by the line with blue circles. The policy frontier when only the Experience Channel is considered is given by the green line. In total, we observe a downward shift and flattening of the policy frontier. Hence, in an economy with a high share of young individuals, a given level of inflation volatility is associated with a lower output gap volatility. Moreover, the lower slope of the policy frontier indicates that in a young society a given reduction in inflation volatility is more costly in terms of additional output gap volatility compared to an old society (blue rectangles).

Much of this change is driven by the Experience Channel, as it already generates a substantial downward shift and flattening of the policy frontier (green line). Recall that an increase in the share of young agents reduces the aggregate perceived persistence under EBL through the Experience Channel. As we have seen when comparing EBL and CGL, a reduction in the perceived persistence flattens the policy frontier and shifts it downwards.

²⁶In Appendix A.5 we perform the analysis for a higher inflation focus. Results remain unchanged. Further, the appendix deepens the comparison between EBL and CGL and shows that only when setting empirically implausible CGL parameters, one can obtain a similar policy frontier as under EBL.



FIGURE 2.10: Monetary Policy Frontier Under a Supply Shock – Experience Channel

The same mechanism applies when comparing young to old economies under EBL.²⁷ The total effect of a demographic shift is more pronounced, as the effective discount factor is decreasing (channel 1.), which further attenuates the effect of inflation expectations on current inflation. As a result, the curve shifts downwards and flattens.

2.6 Conclusion

This paper discusses how experience-based heterogeneity in expectations affects monetary policy. To address this issue, we introduce a New Keynesian model with overlapping generations in which individuals' expectations of inflation and the output gap depend on their lifetime experiences. This creates heterogeneity in expectations. Expectations heterogeneity in our model is based on the heterogeneity in the perceived persistence across cohorts. We show that under experience-based learning (EBL), the *aggregate* perceived persistence in the economy is pushed down relative to a model with constant-gain learning (CGL) in which agents attach the same constant weight to new information.

Under EBL, the pass-through of monetary policy via expectations on inflation and the output gap weakens relative to models with CGL or rational expectations. Hence, models that abstract from experience effects are found to overstate the impact of monetary policy on inflation. Further, due to the lower (delayed) influence of monetary policy on expectations, the stabilization trade-off under supply shocks aggravates, as any reduction

Notes: We increase the share of young agents, while holding the effective discount rate $\tilde{\beta} = \beta \omega$ constant. We also show the frontiers for the models under EBL in an old economy (blue rectangles) and in a young economy. The latter is divided into the full effect (blue circles) and into the effect from different experiences alone (green).

²⁷In Appendix A.5, we show that under CGL, the demography-driven shift in the policy frontier is substantially less pronounced and stays close to the policy frontier under RE.

in inflation volatility is more costly in terms of output gap volatility. The demographic structure directly affects aggregate expectations, which are a size-weighted average across cohort, through a composition effect. Consistent with the empirical literature we show, first, that the response of inflation to a monetary policy shock is more pronounced and more persistent in an economy with a higher share of old individuals. Second, the monetary policy trade-off between output gap and inflation stabilization under supply shocks occurs for higher variable volatilities in older economies. At the same time, stabilizing inflation involves less additional output gap volatility so that the trade-off attenuates. Thus, the age structure, through EBL, is a relevant factor to determine the transmission of monetary policy.

A worthwhile extension of our analysis is to consider the effect of experience-based heterogeneity on the conduct of *optimal* monetary policy. Aside from its theoretical contribution, such an extension adds a practical benefit. As shown by Di Bartolomeo et al. (2016), under heterogeneous expectations the welfare loss function entails consumption dispersion across households with different expectations. However, such consumption heterogeneity is not observable in the data. In contrast, the distribution of consumption across age groups is perfectly observable for policymakers. Hence, EBL might be beneficial for practical reasons when analyzing optimal monetary policy under heterogeneous expectations.

Another interesting extension of our model is to incorporate long-term expectations and a central bank inflation target. Given a spell of persistently low inflation, a counterfactual policy analysis could then investigate monetary policy's ability to stimulate inflation by changing the inflation target. As a result of its strategy review, the ECB recently adjusted its target towards a symmetric inflation goal, which highlights the policy relevance of such an extension.

Appendix

A.1 Equilibrium under EBL

We define equilibrium conditions for the EBL economy. Importantly, we perform aggregation of the cohorts' Euler equations into an aggregate IS-curve.

Aggregation. We follow the literature on heterogeneous expectations that relies on the axiomatic approach of Branch and McGough (2009) to aggregate the decisions of agents with heterogeneous expectations without including the wealth distribution as an additional state variable.²⁸ To do so, we rely on two key assumptions

- 1. The structure of higher order beliefs: $\tilde{E}^i_t \tilde{E}^k_t x_{t+1} = \tilde{E}^i_t x_{t+1}, \ i \neq k$.
- 2. Agents expect to return to the same wealth in the long-run: $\tilde{E}_t^i(\hat{c}_{\infty} \hat{c}_{\infty}^i) = 0.$

Consider the Euler equation given in (2.4)

$$c_{t|k} = \beta \tilde{E}_t^k \left(c_{t+1|k} \frac{r_t}{\pi_{t+1}} \right)$$

The linearized Euler equation of a household in cohort i is given by

$$\hat{c}_{t|i} = \tilde{E}_t^i \hat{c}_{t+1|i} - \left(\hat{r}_t - \tilde{E}_t^i \hat{\pi}_{t+1}\right) \ \forall i \ .$$

Forward iteration of the Euler equation yields

$$\hat{c}_{t|i} = \underbrace{\lim_{j \to \infty} \tilde{E}_t^i \hat{c}_{\infty|i}}_{\equiv \tilde{E}_t^i \hat{c}_{\infty}^i} - \tilde{E}_t^i \sum_{j=0}^{\infty} \left(\hat{r}_{t+j} - \hat{\pi}_{t+1+j} \right) \,\forall i , \qquad (A.1.1)$$

where we used Assumption A5. of Branch and McGough (2009), which states that the Law of Iterated Expectations is satisfied, i.e., $\tilde{E}_t^k\left(\tilde{E}_{t+1}^k\left(c_{t+2}\right)\right) = \tilde{E}_t^k(c_{t+2})$. The aggregated linearized resource constraint in t and in t + 1 is

$$\hat{c}_t = (1 - \omega) \sum_{k = -\infty}^t \omega^{t-k} \hat{c}_{t|k} = \hat{y}_t ,$$

²⁸Examples in the literature that rely on this approach are Gasteiger (2014), Di Bartolomeo et al. (2016), Hagenhoff (2018).

$$\hat{c}_{t+1} = (1-\omega) \sum_{k=-\infty}^{t+1} \omega^{t+1-k} \hat{c}_{t+1|k} = \hat{y}_{t+1} .$$
(A.1.2)

Next, insert the forward iterated Euler equation (A.1.1) into the t + 1-resource constraint (A.1.2) for $\hat{c}_{t+1|k}$ (for each cohort in t+1, respectively) and take expectations of cohort i

$$\tilde{E}_{t}^{i}\left[(1-\omega)\sum_{k=-\infty}^{t+1}\omega^{t+1-k}\left(\tilde{E}_{t}^{k}\hat{c}_{\infty}^{k}-\tilde{E}_{t}^{k}\sum_{j=1}^{\infty}\left(\hat{r}_{t+j}-\hat{\pi}_{t+1+j}\right)\right)\right]=\tilde{E}_{t}^{i}\left(\hat{y}_{t+1}\right) .$$
(A.1.3)

Following Branch and McGough (2009), the treatment of higher-order beliefs matters for the further steps to aggregation. They impose that agents' expectations about what other agents expect are equal to their own expectation, which corresponds to assumption 1. Having departed from RE and, therefore, having assumed that agents do not know the underlying structure of the economy, imposing that they do not foresee how others form expectations can be seen as consequential. We rewrite (A.1.3) as

$$\begin{split} \tilde{E}_{t}^{i}\left(\hat{y}_{t+1}\right) &= (1-\omega) \sum_{k=-\infty}^{t+1} \omega^{t+1-k} \left(\tilde{E}_{t}^{i} c_{\infty}^{k} - \tilde{E}_{t}^{i} \sum_{j=1}^{\infty} \left(\hat{r}_{t+j} - \hat{\pi}_{t+1+j} \right) \right) \\ &= \tilde{E}_{t}^{i} \left((1-\omega) \sum_{k=-\infty}^{t+1} \omega^{t+1-k} c_{\infty}^{k} \right) \\ &- \tilde{E}_{t}^{i} \left((1-\omega) \sum_{k=-\infty}^{t+1} \omega^{t+1-k} \sum_{j=1}^{\infty} \left(\hat{r}_{t+j} - \hat{\pi}_{t+1+j} \right) \right) \\ &= \tilde{E}_{t}^{i} c_{\infty} - \tilde{E}_{t}^{i} \sum_{j=1}^{\infty} \left(\hat{r}_{t+j} - \hat{\pi}_{t+1+j} \right) , \end{split}$$

where the last equality uses assumption 2, which we discuss below, and that weights sum to one. We use this to substitute the infinite sum of real interest rates in (A.1.1)

$$\hat{c}_{t|i} = \tilde{E}_t^i c_{\infty}^i - \tilde{E}_t^i \sum_{j=1}^{\infty} \left(\hat{r}_{t+j} - \hat{\pi}_{t+1+j} \right) - \left(\hat{r}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right) \\ = \tilde{E}_t^i \left(\hat{y}_{t+1} \right) - \tilde{E}_t^i \left(c_{\infty} - c_{\infty}^i \right) - \left(\hat{r}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right) .$$

Of particular interest is the term $\tilde{E}_t^i (c_{\infty} - c_{\infty}^i)$, which denotes expected differences of own consumption and aggregate household consumption in the limit. Branch and McGough (2009) deal with such a term by assuming that agents agree on expected differences in limiting consumption so that in aggregation it vanishes. Equivalently, Hagenhoff (2018) assumes that agents expect to be back at the steady state in the long-run, which also eliminates the term. We use the same assumption, but adapt it for our usage in the following sense: take cohorts *i* and *k* that both expect to have steady state consumption in the long-run

$$\tilde{E}_t^j \hat{c}_\infty^j = \tilde{E}_t^j \hat{c}_\infty \quad \text{for } j = i, k .$$

Now, let cohort *i* take expectations of the limiting expectations for cohort *k* and invoke assumption 1: $\tilde{E}_t^i \left(\tilde{E}_t^k \hat{c}_\infty^k\right) \stackrel{?}{=} \tilde{E}_t^i \left(\tilde{E}_t^k \hat{c}_\infty\right) \stackrel{1}{=} \tilde{E}_t^i \hat{c}_\infty$. Cohorts not only expect to be back at the steady state, but also expect this for others. Assuming agents expect to be back at the steady state in the long run, in which all consume equally, and having a unit mass of agents implies $c = c^k \forall k$ for non-explosive PLM-parameters. Under assumption 2 we get

$$\hat{c}_{t|i} = \tilde{E}_t^i \left(\hat{y}_{t+1} \right) - \left(\hat{r}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right) .$$
(A.1.4)

Note that (A.1.4) holds for all cohorts. Insert into the aggregate resource constraint in t

$$\hat{y}_{t} = (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \hat{c}_{t|k} = (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \left(\tilde{E}_{t}^{k} \left(\hat{y}_{t+1} \right) - \left(\hat{r}_{t} - \tilde{E}_{t}^{k} \hat{\pi}_{t+1} \right) \right) \\ = (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \tilde{E}_{t}^{k} \left(\hat{y}_{t+1} \right) - (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \left(\hat{r}_{t} - \tilde{E}_{t}^{k} \hat{\pi}_{t+1} \right) .$$

Finally, we use the definition of aggregate expectations, $\bar{E}_t x_{t+1} = (1-\omega) \sum_{k=-\infty}^t \omega^{t-k} \tilde{E}_t^k x_{t+1}$ to receive the aggregate dynamic IS-curve

$$\hat{y}_t = \bar{E}_t \left(\hat{y}_{t+1} \right) - \left(\hat{r}_t - \bar{E}_t \hat{\pi}_{t+1} \right) .$$
(A.1.5)

It remains to rewrite the IS curve in terms of the output gap. We get

$$\tilde{y}_t = \bar{E}_t \left(\tilde{y}_{t+1} \right) - \left(\hat{r}_t - \bar{E}_t \hat{\pi}_{t+1} - r_t^n \right) ,$$

where $r_t^n = \bar{E}_t(\Delta \hat{x}_{t+1}) = 0$. The derivation of the New Keynesian Phillips Curve stays unchanged.

Summary. We receive the following system of equations:

$$\tilde{y}_t = \bar{E}_t \tilde{y}_{t+1} - \left(\hat{r}_t - \bar{E}_t \hat{\pi}_{t+1}\right) , \qquad (A.1.6)$$

$$\hat{\pi}_t = \beta \omega \bar{E}_t \hat{\pi}_{t+1} + \kappa \tilde{y}_t + u_t , \qquad (A.1.7)$$

$$\hat{r}_t = \varphi_\pi \hat{\pi}_t + \varphi_y \tilde{y}_t + \epsilon_t^m , \qquad (A.1.8)$$

$$\hat{\epsilon}_t^m = \rho \hat{\epsilon}_{t-1}^m + \nu_t^m , \qquad (A.1.9)$$

$$u_t = \rho_u u_{t-1} + \nu_t^u \,, \tag{A.1.10}$$

where the expectations in the New Keynesian Phillips Curve and in the dynamic IS-curve follow EBL.

A.2 Simulation Algorithm

Let $X_t = (\tilde{y}_t, \hat{\pi}_t)'$ be the vector of endogenous variables and $\hat{\epsilon}_t = (\hat{\epsilon}_t^m, u_t)'$ be the vector exogenous shocks. The state space representation of the model economy (2.15) under learning at time t is given by

$$AX_t = B_t X_{t-1} + \Omega \hat{\epsilon}_t , \qquad (A.2.1)$$

where A, B_t , and Ω are 2×2 matrices given by

$$A = \begin{bmatrix} 1 + \varphi_y & \varphi_\pi \\ -\kappa & 1 \end{bmatrix}, \quad B_t = \begin{bmatrix} (b_{t-1}^y)^2 & -(b_{t-1}^\pi)^2 \\ 0 & \beta\omega(b_{t-1}^\pi)^2 \end{bmatrix}, \quad \Omega = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Note, that the matrix B_t is time-varying because it involves the recursively updated parameter estimates of individuals' forecasting model for the output gap (b_{t-1}^y) and inflation (b_{t-1}^{π}) . The updating process is given by equations (2.18a) - (2.18b).

While the PLM parameters under CGL are homogeneous across age groups (i.e., $b_{t-1|k}^{\mathbf{z}} = b_{t-1}^{\mathbf{z}} \forall k$ and $\mathbf{z} \in \{\tilde{y}, \hat{\pi}\}$), the *aggregate* PLM parameter under EBL is given by

$$b_{t-1}^{\mathbf{z}} = (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} b_{t-1|k}^{\mathbf{z}} .$$
(A.2.2)

Theoretically, agents can live forever (and the number of cohorts is infinite) so that we need to choose a finite number of cohorts when simulating the model. A higher number approximates the "true" economy more closely but comes at the cost of a higher computational time. We, therefore, choose the number of cohorts as 200 (equivalent to a 50-year working life) and normalize cohort weights to sum to one.

Remember, that due to our timing assumption depicted in Figure 2.2, it follows that (see equation (2.17))

$$\tilde{E}_t^k\left(\mathbf{z}_{t+1}\right) = (b_{t-1|k}^{\mathbf{z}})^2 \mathbf{z}_{t-1}$$

Under CGL, it holds that $(b_{t-1|k}^{\mathbf{z}})^2 = (b_{t-1}^{\mathbf{z}})^2$ for each cohort k, whereas under EBL

$$(b_{t-1}^{\mathbf{z}})^2 = (1-\omega) \sum_{k=-\infty}^t \omega^{t-k} (b_{t-1|k}^{\mathbf{z}})^2 .$$
(A.2.3)

This has to be taken into account when simulating the model using equation (A.2.1).

Algorithm to simulate the model²⁹

The simulation algorithm works as follows. To start the recursion, we need the initial parameter estimates, (b_{-1}^y) and (b_{-1}^π) , and the initial moment matrices, R_{-1}^y and R_{-1}^π . To obtain those objects, we first simulate the model under RE for $T_{\text{init}} = 1000$ periods. The PLM parameters are obtained by estimating a simple AR(1) for both the output gap and inflation. In the initial period, we endow all cohorts with the same PLM parameters and moment matrices.

1. In period t, we simulate X_t based on equation (A.2.1) given the PLM parameters $b_{t-1}^{\mathbf{z}}$ and the exogenous shocks, $\hat{\epsilon}_t$.

²⁹For the simulation, we rely on Matlab 2020a.

- 2. Based on the new observation of $\mathbf{z} \in \{\hat{\pi}, \tilde{y}\}$, we update $b^{\mathbf{z}}$ and $R^{\mathbf{z}}$ for each cohort using (2.18a) and (2.18b).³⁰
- 3. We repeat steps 1 2 for $T_{\rm sim}$ periods.

The actual simulation displayed in the figures discards the first $T_b = 1000$ iterations to allow the impact of initial values from the RE economy to wash out.

In step 2, we use a so-called projection facility to ensure the model under learning can be solved (see Orphanides and Williams, 2007; Slobodyan and Wouters, 2012a). Conceptually, it re-initializes the updating step as soon as new simulated data implies unstable PLM parameters. We proceed in 2 steps:

PROJECTION FACILITY

- 1. We take the updated PLM parameters and check whether they make the forecasting model explosive, i.e., $|b_{t|k}^{\mathbf{z}}| > 1$. If the forecasting model generates non-explosive behavior, we allow the updating step.
- 2. If behavior is explosive we proceed as follows: we invoke the projection facility and ignore the updating step. In this case, the new PLM parameter and the new moment matrix are set to the values from the previous period.

GENERALIZED IMPULSE RESPONSE FUNCTIONS

To compute the initial parameters for agents' PLM, we simulate the model with RE for $T_{\text{init}} = 1000$ periods and obtain $b_{-1}^{\mathbf{z}}$ by estimating an AR(1) process for $\mathbf{z} \in \{\tilde{y}, \hat{\pi}\}$. In the next step, we simulate both the model with EBL under a supply shock for $T_{\text{sim}} = T_b + T_{\text{irf}}$ periods as described in 1.–2., while setting the monetary policy shock to zero. We then simulate the model under the *same* path of supply shocks and add an innovation of 25 basis points to the monetary policy shock at time $T_{\text{imp}} = T_b + 1$. We then take the difference between these two series as the impulse response function to a monetary policy shock. In an analogous way, we compute the impulse response function to a monetary policy shock in the model with CGL.

 $^{^{30}}$ Note, that, under EBL, we endow the newly born cohort with PLM parameters equal to the aggregate PLM parameter (A.2.2) of the last period. A variation of the initial belief did not greatly alter results (see Appendix A.3).

A.3 Sensitivity Analysis of the Perceived Persistence

Robustness w.r.t. θ . The gain to new information is age-dependent under EBL. To parameterize its behavior across age groups, we use the estimated shape parameter θ of Malmendier and Nagel (2016). While we set $\theta = 3.044$ to the estimate of their preferred specification, the authors also provide values for different regression specifications. In fact our choice of θ is the lowest value provided so that we check the robustness of our results against the highest estimate in Malmendier and Nagel (2016), $\theta = 4.144$.

Recall that the shape parameter governs how quickly gains reduce across age groups (see Figure 2.3). A higher value, hence, will intensify the difference between young and old agents, which drives expectation differences and our results. Figure A.1 compares the distribution of aggregate PLM parameters under both specifications of θ . As expected, the dispersion of parameters increases. In that sense, our results constitute a lower bound. In particular, the difference to a model with CGL will increase, because results under CGL are unaffected by the choice of θ .



FIGURE A.1: Distribution of PLM Parameters – Variation of θ

Notes: The blue histogram denotes the distribution of the PLM parameters in the model under the baseline calibration ($\theta = 3.044$). The red histogram shows the distribution of the PLM parameters for $\theta = 4.144$. The left part shows the PLM parameters for the output gap and the right part shows the PLM parameters for inflation.

Robustness w.r.t. κ . In the next step, we explore the sensitivity of the results with respect to the slope of the NKPC κ . We increase the Rotemberg parameter from its baseline value in Table 2.1 to 140. This reduces the slope of the NKPC. In this case, inflation becomes less sensitive to variations in output. As seen in Figure A.2, the mean of the distribution of the PLM parameters for inflation and output under EBL is lower



FIGURE A.2: Distribution of PLM Parameters – Variation of κ

Notes: The blue histogram denotes the distribution of the PLM parameters in the model under EBL if $\phi = 140$. The red one shows the distribution of the PLM parameters in the model with CGL for $\phi = 140$. The left part shows the PLM parameters for the output gap and the right part shows the PLM parameters for inflation.

relative to the corresponding value under CGL. Hence, the key mechanism that the perceived persistence of inflation and the output gap in the model with EBL is lower on average relative to the model with CGL is unaffected. We verified that, under EBL, the stabilization trade-off of monetary policy remains less severe relative to the model with CGL as κ decreases. Moreover, also the transmission of monetary policy shocks under EBL remains less pronounced and less persistent relative to the one under CGL.³¹

Robustness w.r.t. the Initial Belief. Each new cohort needs to be given an initial set of PLM parameters to start the recursive least-squares updating process. In the main text we assume that each new cohort uses previous period's aggregate PLM parameter. Yet, initial beliefs may be of relevance for the general result with respect to the aggregate persistence, as they change the young agents' starting point in updating parameters.

Now, we instead assume that each newly born cohort draws its initial parameter from a normal distribution around the RE estimate, where the normal distribution is truncated at ± 1 . Figure A.3 shows the histogram of aggregate PLM parameters of inflation and the output gap for the models with EBL and CGL. The key result that the aggregate persistence is smaller and more dispersed under EBL remains unchanged.

Robustness w.r.t. the Shock. We also check the validity of our result for another type of shock. Figure A.4 displays the histograms of PLM parameters for economies that are

³¹The corresponding simulation results are available upon request.



FIGURE A.3: Distribution of PLM Parameters – Variation of Initial Beliefs

Notes: The blue histogram denotes the distribution of the PLM parameters in the model under EBL. The red one shows the distribution of the PLM parameters in the model with CGL. The left part shows the PLM parameter for the output gap and the right part shows the PLM parameter for inflation.

simulated under monetary policy shocks. The shock persistence is smaller than the one of a supply shock so that also the aggregate perceived persistence for inflation and the output gap are smaller. However, the key result of a smaller and more dispersed perceived persistence under EBL relative to CGL still holds.



FIGURE A.4: Distribution of PLM Parameters – Monetary Policy Shock

Notes: The blue histogram denotes the distribution of PLM parameters in the model under EBL. The red histogram shows the distribution of the PLM parameters in the model with CGL. The left part shows the PLM parameters for the output gap and the right part shows the PLM parameters for inflation.

A.4 Impulse Responses

Supply Shock. Figure A.5 denotes impulse responses after a negative supply shock. The difference in responses between EBL and CGL that are driven by expectations remain unchanged.



FIGURE A.5: IRFs to a Supply Shock

Notes: We show the generalized impulse response functions for key variables in the economies under CGL (black), EBL (blue), and RE (red). We average responses over 8,000 iterations. The output gap and output gap expectations are measured as percentage deviations from their respective steady state, while the other variables are measured as (annualized) deviation from their respective steady state.

Types of Expectation Formation. In the main text we discuss how a change in the mass of young agents affects the response to monetary policy shocks under EBL. Figure A.6 depicts results for all types of expectation formation.

The effect of a change in ω on the IRFs under RE (third row) is negligible. We also compare the effect of the age-distribution on the IRF in a model with CGL (second row) in which all cohorts have the same expectations of the output gap and inflation to the model with EBL (first row). The aggregate perceived persistence is lower under the latter assumption on expectation formation. When the share of young agents increases under EBL, the aggregate perceived persistence falls further compared to the baseline economy due to the Experience Channel. In contrast, under CGL without experience effects, such changes do not occur. In consequence, the difference in the IRFs in the baseline and the young economy is more pronounced when considering the model with EBL in the first



FIGURE A.6: IRF to a MP Shock for Different Demographic Structures

row of Figure A.6. Under CGL, differences to the baseline economy solely stem from a change in discounting (see channel 1.).

A.5 Monetary Policy Trade-off

We discuss further the differences between policy frontiers under EBL and CGL depending on the gain parameter. We also vary the strength of the inflation fighting motive. Last, we consider a shift of the demographic structure also for models with CGL and RE.

Gain Parameter. The key difference between EBL and CGL is the weighting of the most recent observation in the updating process. Under EBL, the gain of agents $\gamma_{t|k}$, depending on age, ranges from 0.015 to 1. If in contrast, we choose a model based on CGL the literature suggests a gain of g = 0.015. Technically, it is possible to replicate our finding regarding the position of the policy frontier also with CGL for very high (constant) gains. Figure A.7 depicts policy frontiers for models with EBL (blue) and CGL (black). Only when setting the CGL parameter to $g^* = 0.12$ we approximately replicate the EBL frontier with CGL expectation formation (light blue). Yet, such a value for the gain parameter is empirically implausible.³²

Notes: We show the generalized impulse response functions for key variables in the economies under EBL (first row), CGL (second), and RE (third). The black solid lines denote the IRF under the baseline calibration of $\omega = 0.995$ while the black dashed lines denote the IRF in the young economy when $\omega = 0.96$. We average responses over 8,000 iterations. The output gap and output gap expectations are measured as percentage deviations from their respective steady state, while the other variables are measured as (annualized) deviation from their respective steady state.

³²Milani (2007) finds a 95% highest posterior density interval for the gain parameter of [0.0133, 0.0231]. Across different specifications of the PLM and initial beliefs Slobodyan and Wouters (2012b) estimate a maximum gain of 0.036.



FIGURE A.7: Monetary Policy Frontier Under a Supply Shock – High Constant Gain

Stabilization Motive for Inflation. Figures A.8 and A.9 repeat the analysis in the main text but for a higher focus in inflation stabilization: φ_{π} rises from 1.5 to 2. Our core results are not affected as can be seen by the relative positions of the policy frontiers. For a higher focus on inflation stabilization the intersection of the policy frontiers for RE and CGL occurs later, since the interest rate is still strongly affected by inflation even for comparatively high values on φ_y . Given RE factor in the impact of current and future nominal rate changes on the output gap, the volatility of the output gap is higher for more values of φ_y .



FIGURE A.8: Monetary Policy Frontier – High Inflation Focus

Notes: We plot the standard deviation of the output gap against the one for inflation for the models under EBL (blue), CGL (black), and with RE (red). Units are in percent deviation from the steady state. The markers denote the combinations of $(\varphi_{\pi}, \varphi_{y})$ on the grid.

Notes: The figure shows the policy frontier in the model under the baseline calibration under (i) EBL, (ii) CGL when the gain parameter is set to g = 0.015, and (iii) CGL when the gain parameter is set such that the policy frontier under CGL comes close to the one obtained under EBL ($g^* = 0.12$).



FIGURE A.9: Monetary Policy Frontier – Experience Channel

Notes: We increase the share of young agents, while holding the effective discount rate $\tilde{\beta} = \beta \omega$ constant. We also show the frontiers for the models under EBL in an old economy (blue rectangles) and in a young economy. The latter is divided into the full effect (blue circles) and into the effect from different experiences alone (green).

Stabilization Trade-off of Monetary Policy. The shift of the policy frontier under EBL for an increase in the mass of young agents was mainly driven by the Experience Channel, i.e., the impact of lower aggregate perceived persistence on the output gap and inflation.

The left panel of Figure A.10 shows the full shift in the policy frontier under EBL and RE for the baseline and young economies. The right panel includes a model with CGL. Importantly, under the CGL framework all agents update equally and, hence, no effect from experience effects arises. Comparing both panels, we see that without experience effects, i.e., under CGL, the policy frontier shift is comparable to the one occurring under RE. Under EBL, however, experience effects lead to a stronger downward shift.



FIGURE A.10: Monetary Policy Trade-off Under a Supply Shock

Notes: The left panel compares the monetary policy trade-off under EBL (blue lines) relative to the one under RE (red lines). The right panel compares the monetary policy trade-off in the model with CGL (black lines) relative to the one under RE (red lines). Filled marker denote the baseline economy ($\omega = 0.995$) while the empty marker represent the young economy ($\omega = 0.96$). We plot the standard deviation of the output gap against the one for inflation. Units are in percent deviation from the steady state. The markers denote the combinations of ($\varphi_{\pi}, \varphi_{y}$) on the grid.

Chapter 3

Risky Financial Collateral, Firm Heterogeneity, and the Impact of Eligibility Requirements

This chapter is based on Kaldorf and Wicknig (2022).¹

3.1 Introduction

Central banks implement monetary policy by lending to banks against collateral, which makes a sufficiently high supply of collateral essential to the functioning of the financial system. During the financial crisis of 2008, this restriction required many central banks to expand the pool of assets they accept as collateral to facilitate the conduct of expansionary monetary policy. For example, the European Central Bank (ECB) engaged in collateral easing when switching towards a full allotment regime in its Main Refinancing Operations and prior to introducing Long-Term Refinancing Operations. To expand the pool of collateral, the ECB added corporate sector assets of intermediate quality, such as BBB-rated corporate bonds and securitized bank loans, to the list of eligible assets.²

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²See Wolff (2014), Heider et al. (2015), Nyborg (2017), and Blot and Hubert (2018) for a discussion on the collateral eligibility of risky private sector assets and the monetary policy implementation by the ECB. We show the collateral treatment of corporate sector assets by different central banks in Appendix B.1.

inclusion of corporate sector assets is quantitatively relevant: corporate bonds and credit claims make up 27% of collateral in ECB operations.³

While collateral easing facilitates the smooth conduct of monetary policy, a thorough assessment of this policy must also account for endogenous responses of the corporate sector. Firm responses arise, since banks increase demand for assets if they become eligible as collateral and firms cater to this demand by increasing their debt issuance (see Mésonnier et al., 2022; Mota, 2021; Pelizzon et al., 2020). Crucially, firm responses have also been shown to be heterogeneous across firms with different rating (Grosse-Rueschkamp et al., 2019). While debt supply effects are desirable for monetary policy implementation, higher indebtedness of firms also increases their risk of default, which, in turn, may also limit the efficacy of collateral easing. This paper presents a novel theoretical framework to study endogenous firm responses to eligibility requirements in the presence of default risk. While this framework can be applied to many situations where eligibility is specified in a discontinuous way through minimum rating requirements, we present an application to the ECB's collateral easing policy of 2008.

We study endogenous firm responses to eligibility requirements through the lens of a model with heterogeneous firms that issue risky debt securities (corporate bonds) to banks.⁴ Firms are subject to idiosyncratic revenue shocks and have an incentive to issue bonds, because they are more impatient than their creditors. Firms default on their bonds if revenues fall short of current repayment obligations, in which case all current revenues are wasted. Thus, bond issuance is determined by a trade-off between relative impatience and expected default costs. Bonds are held and priced by banks. We assume that banks value these bonds if they can be used to collateralize borrowing from the central bank. Consistent with actual central bank practice, only sufficiently safe bonds are eligible as collateral and the central bank sets the minimum quality requirement as a policy instrument. The dual role of bonds as investment object and collateral implies that spreads on *eligible* bonds contain a fundamental component and a collateral premium that, ceteris paribus, shifts the pricing schedule outwards in a discontinuous way.

As our first contribution, we provide a characterization of firm responses to the introduction of a collateral framework in a model with discontinuous demand for corporate bonds. We obtain analytical solutions in a setting with one period bonds, i.i.d. revenue shocks, and permanent differences in firm productivity. Making corporate bonds eligible affects the firm's borrowing decision in a discontinuous way. The discussion of firm responses is organized around a key firm characteristic, the *eligible debt capacity*, defined as the maximum amount of bonds a firm can issue without losing eligibility.

 $^{^{3}}$ As of 2020Q4, corporate bonds are the second largest asset class accepted as collateral by the ECB with a market value of EUR 1871 billion. This is only exceeded by government bonds (see European Central Bank, 2022).

⁴We refer to these securities as corporate bonds, even though they can be interpreted as securitized bank loans or other marketable corporate sector assets, which are also eligible in many central bank collateral frameworks.

Compared to the case of no collateral premia, firms' debt choices differ in sign above and below the discontinuity in bond demand induced by eligibility requirements. Highquality firms (with a large eligible debt capacity) take advantage of banks' high valuation of corporate bonds and issue more bonds to front-load dividend payouts: firms increase their *risk-taking*.⁵ In contrast, medium-quality firms (which issue bonds at or near their eligible debt capacity) may find it worthwhile to reduce their debt issuance, if this makes their bonds eligible: a *disciplining* effect.⁶ Both firm level effects imply that bond prices and debt choices are not solely determined by firm fundamentals. The latter case will be referred to as *market discipline*, where risk-taking and disciplining effects constitute violations thereof.

While both firm level effects, risk-taking and disciplining, increase collateral supply, they have an opposing effect on default risk. This makes a heterogeneous firm model essential to study aggregate implications, because the relative strength of both effects depends on the firm distribution. To illustrate the aggregate effect, consider collateral easing, which increases the eligible debt capacity for all firms. The change of aggregate collateral supply contains a *mechanical component*, the change caused by a lower rating threshold all else equal, and *endogenous firm responses*, which depend on the relative size of risk-taking and disciplining effects and the mass of firms affected by each effect. Within this framework, we show that endogenous firm responses unambiguously amplify the positive mechanical effect of collateral easing. Risk-taking and disciplining effects positively contribute to the increase. However, they have an ambiguous impact on aggregate default cost. Note that we obtain these results under the assumptions of one-period bonds and i.i.d. revenue shocks, which we relax in the following.

As our second contribution, we illustrate firm responses to the ECB's collateral easing policy in a setting with long-term debt and persistent revenue shocks. We solve the model using global methods and calibrate the firm cross-section to euro area data by employing a merged dataset of corporate bonds from *IHS Markit* and corporate balance sheet data from *Compustat Global*. The calibrated model can replicate several features of firm debt issuance, corporate bond spreads, and collateral premia, which are crucial to evaluate the impact of eligibility requirements.

We study two different policies: our benchmark scenario are tight eligibility requirements, which only accept bonds rated A or higher, corresponding to the ECB collateral framework before the 2008 crisis. Second, we consider lenient eligibility requirements, under which all bonds rated BBB or higher are accepted, in line with ECB practice after

⁵This can be thought of as an intertemporal substitution effect. On the other hand, those firms can sustain the same dividend-payout by issuing a smaller face value of bonds: an income effect. We show that, under a standard monotone hazard rate assumption on firm revenues, the former effect dominates.

⁶The heterogeneous response is consistent with empirical evidence in Grosse-Rueschkamp et al. (2019). They show that firms rated A or higher increased their leverage in response to the ECB's corporate sector purchase program (CSPP). BBB-rated firms did not materially increase their leverage.

2008. Since revenue shocks are persistent, introducing a lenient policy increases the probability of a firm to be eligible in the future, thereby lowering spreads for all states through the continuation value. Hence, collateral policy is not only relevant for firms that are near their eligible debt capacity in the current period but affects all firms via the rollover value of bonds.

In the framework with long-term debt and persistent revenues, firm responses dampen the positive mechanical effect of collateral easing on collateral supply and, furthermore, increase aggregate cost from default. These effects are sizable: instead of mechanically expanding by 71%, collateral supply only increases by 62% if firm responses are taken into account. This is reflected by the relative importance of risk-taking and disciplining effects. While 51% of firms engage in risk-taking and 19% are subject to disciplining under a tight policy, these numbers shift to 79% and 3% under a lenient policy. Consequently, aggregate default costs increase by 8%.

The dampening effect of firm responses on collateral supply is associated with the large debt issuance of high-revenue firms. While the risk-taking response at the firm level is increasing the *current* market value of bonds outstanding, it exposes firms to rollover risk: when hit by a series of adverse revenue shocks, previously issued bonds experience a drop in their market value. If firms also lose their eligibility status, this rollover problem becomes more severe, since bonds also lose the collateral premium. As a result, indebtedness increases, default becomes more likely, and collateral supply contracts. Notably, increasing bond issuance is still optimal for firms that experience high revenue draws due to the relative impatience of firm managers: the adverse effects of rollover risk are discounted sufficiently heavily. Key to this phenomenon is the combination of persistent shocks and long-term debt. It is important to note that this mechanism is also present under tight collateral policy but becomes stronger after collateral easing. Similar effects have been described in the macro-finance (Gomes et al., 2016 or Jungherr and Schott, 2022) and sovereign default literature (Hatchondo et al., 2016).

We investigate an *eligibility covenant* as a potential instrument to tackle the dampening effect of endogenous firm responses. Our focus is on covenants depending on *current* debt outstanding, which effectively is public information for firms that are large enough to issue corporate bonds. A covenant limits the eligible debt capacity of firms with high levels of debt outstanding and provides deleveraging incentives. This reduces the rollover burden of these firms once they experience a sequence of adverse revenue draws. Conceptually, most collateral frameworks condition the eligibility of bonds only on ratings, which is a onedimensional measure of default risk. Conditioning eligibility also on debt outstanding allows the central bank to dis-incentivize 'unsustainable' debt issuance of high-revenue firms, while at the same time allowing low-revenue firms with low debt outstanding to roll over their bonds.

The policy problem in choosing a covenant is to set a sufficiently tight covenant to provide deleveraging incentives for high-revenue firms without shutting down the issuance of bonds altogether. Restricting our attention to a simple parametric class for the covenant, we demonstrate the existence of a *collateral Laffer curve* for any given minimum rating requirement. Our numerical results suggest that conditioning eligibility not only on default risk but also on current debt outstanding, has a positive effect on collateral supply. For example, under a BBB minimum rating requirement, covenants can expand collateral supply by up to 22% compared to the case without covenant. Finally, we investigate the potential of adding covenants to the central bank toolkit in addition to the minimum rating requirement. Since the representation of banks and liquidity risk is too simplistic to allow for a fully-fledged welfare analysis, we build on the literature on optimal collateral policy (Koulischer and Struyven, 2014 and Choi et al., 2021 among others) and embed our analysis in a central bank trade-off between maintaining high collateral supply and limiting the additional default cost from violating market discipline. Our model predicts that using both instruments increases collateral supply, lowers cost from corporate default, and, thereby, shifts the *collateral policy frontier* outward.

While we propose a model that is well suited to study discontinuous collateral eligibility, our analysis remains valid in many cases where firms respond endogenously to a discontinuous demand schedule for their debt. Our model can also be applied to eligibility for asset purchase programs, where the anticipation of substantial demand increases for targeted assets may induce a willingness to pay eligibility premia.

Related Literature. Our paper builds on a large strand of literature providing empirical results on the bond market impact of collateral policy and eligibility for QE programs. Ashcraft et al. (2011) find a sizable impact of haircuts on bond prices using an event study around announcement and implementation of the Term Asset-Backed Securities Loan Facility in the US. Exploiting an unexpected policy change regarding eligibility of Chinese corporate bonds, Chen et al. (2021) identify a pledgeability premium of around 50 basis points (bp) for AA-bonds. Mésonnier et al. (2022) use an extension of eligibility criteria as part of the Additional Credit Claims program and find a premium of 8 bp on bank loans relative to a non-eligible control group. Santis and Zaghini (2021) find that firms eligible in the corporate sector purchase program increase their debt issuance and use some of the funds to repurchase their own stocks. Todorov (2020) finds that issuers of QE-eligible bonds increase their dividend payouts fourfold, relative to pre-treatment levels but do not increase investment. Adverse effects on firm risk-taking are presented in Grosse-Rueschkamp et al. (2019), who furthermore identify heterogeneous responses of firms in different rating brackets. This highlights potentially unintended behavior on the firm side and the role of firm heterogeneity, which are central ingredients of our model.

While the previous group of papers uses surprise policy changes to identify causal effects, there are two complementary approaches leading to similar findings. First, Pelizzon et al. (2020) document collateral eligibility premia and bond supply effects using security-level data from the euro area. Their identification relies on ECB-discretion over when formally eligible bonds are put on the list of eligible assets. They identify collateral eligibility premia of 11-24 bp. Second, building an identification strategy around the US treasury safety premium, Mota (2021) uses US corporate bond data and finds that non-financial corporate bonds carry a premium, which can be related to collateral service. The premium decreases in the bond default risk and depends on idiosyncratic firm characteristics as well as an aggregate component encompassing economy-wide collateral supply and demand factors. Mota (2021) finds that firm debt issuance and dividend payout responses rise in the size of the premium.

The results of our paper can be related to a group of papers studying the collateral eligibility of risky assets and implications for central bank policy. Chapman et al. (2011) propose a model where the central bank faces a collateral policy trade-off between relaxing banks' liquidity constraints and incentivizing them to invest into illiquid and risky assets. Koulischer and Struyven (2014) argue that relaxing eligibility requirements can alleviate credit crunches if collateral supply or collateral quality fall below specific levels, as banks' ability to extend credit depends on both. Cassola and Koulischer (2019) quantify a collateral policy trade-off between liquidity provision and risk-taking by the central bank. In Choi et al. (2021) banks prefer to use high-quality collateral on the interbank market so that central banks negatively affect access to liquidity when accepting only high-quality assets. At the same time, lending against low-quality collateral exposes the central bank to counterparty default risk. In contrast to these papers, we make collateral supply and its riskiness endogenous but abstract from further frictions on money markets. Combining these approaches might deliver interesting interactions, which we leave to future research.

Outline. The paper is structured as follows. Section 3.2 introduces collateral premia and eligibility requirements into a corporate capital structure model and presents our main conceptual results. We extend and apply the model to the ECB's collateral easing policy in Section 3.3. In Section 3.4, we conduct policy experiments regarding eligibility covenants. Section 3.5 concludes.

3.2 A Model of Eligibility Requirements

This section introduces a model of endogenous collateral supply and firm default risk to analyze the impact of eligibility requirements on firms. Time is discrete and there are two groups of agents: a non-financial sector (*firms*) and financial intermediaries (*banks*). The *central bank* sets an eligibility requirement, which we treat as an exogenous parameter.

3.2.1 Environment

Firms are endowed with a technology that generates stochastic revenues, which can be interpreted as earnings before interest and taxes (EBIT). To maintain tractability, we do not endogenize investment. Revenue shocks realize at the beginning of each period t and are i.i.d. across firms and over time. In addition to being subject to idiosyncratic revenue shocks, firms are ex-ante heterogeneous with respect to the probability distribution over revenue shocks: some firms are permanently more productive than others and we denote this heterogeneity by the parameter s. We will use this parameter to index bonds issued by each firm as well.

Each period, firms issue debt instruments to banks. These debt instruments are referred to as corporate bonds but reflect all marketable debt instruments including securitized bank loans. Bonds are real one-period discount bonds, i.e., they promise to pay one unit of the all-purpose good in period t+1. In our model, firms are the natural borrowers, because they are more impatient than banks. Given their shock realization and bonds outstanding, firms either default or repay. Bonds have a dual role in the economy, since banks can pledge eligible bonds with the central bank to obtain funding. The demand for central bank funding can be motivated by liquidity deficits that require immediate settlement, such as net deposit outflows (see Bianchi and Bigio, 2022, and De Fiore et al., 2019). We follow Mota (2021) and assume a constant willingness to pay collateral premia. We present a robustness check where the size of collateral premia depends on collateral supply in Appendix B.4.

Banks. There is a unit mass of perfectly competitive banks, which price bonds riskneutrally without discounting. They purchase bonds $b_{t+1}(s)$ issued by firm s at price $q(b_{t+1}|s)$, which reflects its value as an investment object, i.e., the repayment probability (described below), and the collateral benefit they provide. The collateral premium on an eligible bond will be denoted by L. Consistent with actual central bank policy, we assume that bonds are only eligible as collateral if their default probability $F(b_{t+1}|s)$ does not exceed an eligibility threshold \overline{F} set by the central bank. The eligibility indicator Ψ is given by

$$\Psi(b_{t+1}|s) = \begin{cases} 1 & \text{if } F(b_{t+1}|s) \le \overline{F} \\ 0 & \text{else} \end{cases}$$

We model collateral policy in terms of bond eligibility thresholds, i.e., bonds either receive a 100% or a 0% haircut.⁷

⁷In practice, eligible bonds have collateral values in between due to other risk factors, like market illiquidity, which are not present in our setup. Nevertheless, collateral frameworks exhibit large discontinuities at the eligibility thresholds, as we show in Appendix B.1.

Firms. Firm managers/owners are risk neutral and their discount factor is denoted by $\beta < 1$. They operate a technology generating random revenues $\mu_t \in [\underline{\mu}, \overline{\mu}]$ with $\underline{\mu} < 0$ and $\overline{\mu} > 0$.⁸ We assume that μ_t is independent across firms and over time, and denote its pdf and cdf by $f(\mu_t|s)$ and $F(\mu_t|s)$, respectively. Firms are *ex-ante* heterogeneous with respect to their probability distribution over revenues, which allows us to analytically disentangle how individual firms react to eligibility requirements and how firm heterogeneity affects aggregate collateral supply responses to collateral easing. The ex-ante heterogeneity is governed by a productivity parameter s, which characterizes the revenue distribution in a first order stochastic dominance sense. Firms with a high s are more productive on average. In particular, s shifts the probability mass according to $F(\mu_t|s) = F(\mu_t - s)$. We assume that s follows some continuous distribution G(s) over the open interval $S \equiv [s^-, \infty]$, to which we refer as the firm type space. Furthermore, we assume that s^- is sufficiently low such that at least one firm is not eligible even when it chooses not to issue any bonds, i.e., $F(0|s^-) = F(s^-) > \overline{F}$.

Firm managers maximize the present value of dividends. Dividends can become negative, which we interpret as equity issuance. Firms issue bonds $b_{t+1}(s)$ to banks. These bonds are subject to default risk: if firm revenues μ_t fall short of the repayment obligation b_t , the firm is unable to raise funds by issuing additional equity and defaults. The default and repayment probabilities implied by the debt choice b_{t+1} are, therefore, given by $F(b_{t+1}|s)$ and $1 - F(b_{t+1}|s)$, respectively. In case of default, all firm revenues are lost and there is no recovery for banks.⁹

Bond Pricing. Expressing the expected payoff from investing into bonds of firm s in terms of the revenue distribution, the bond pricing condition can be written as

$$q(b_{t+1}|s) = \left(1 - F(b_{t+1}|s)\right) \left(1 + \Psi(b_{t+1}|s) \cdot L\right) .$$
(3.1)

It depends on the expected payoff, determined by the firm default decision in t + 1, and the collateral premium L if bond s is eligible, which, in turn, depends on firm default risk. In the absence of a collateral premium, bond prices merely reflect the expected payoffs.

3.2.2 Debt Choices at the Firm Level

In this section, we analyze how firms' debt choice is affected by the eligibility of their corporate bonds as collateral. We assume there are no delays in the restructuring of defaulted bonds and no exclusion from the corporate bond market after a default. The

⁸Allowing for negative realizations of the revenue shock is consistent with the interpretation of μ_t as EBIT.

⁹Our approach is motivated by Lian and Ma (2021), who show that most corporate borrowing is tied to the going-concern value of the firm. Allowing for a positive recovery rate would not change our qualitative results.

maximization problem of firm s in period t can be written as

$$V(b_{t+1}|s) = q(b_{t+1}|s)b_{t+1} + \beta \int_{b_{t+1}}^{\overline{\mu}} (\mu_t - b_{t+1}) \, dF(\mu_t|s) \,. \tag{3.2}$$

Maximizing (3.2) over b_{t+1} yields the first order condition

$$\beta(1 - F(b_{t+1}|s)) = q(b_{t+1}|s) + \frac{\partial q(b_{t+1}|s)}{\partial b_{t+1}}b_{t+1} ,$$

which we express using the bond price derivative $\frac{\partial q(b_{t+1}|s)}{\partial b_{t+1}} = -f(b_{t+1}|s)(1+\Psi(F(b_{t+1}|s))L)$ as

$$\beta \left(1 - F(b_{t+1}|s) \right) = \left(1 - F(b_{t+1}|s) \right) - f(b_{t+1}|s) \cdot b_{t+1} \quad \text{if } F(b_{t+1}|s) > \overline{F} , \qquad (3.3)$$

$$\beta \left(1 - F(b_{t+1}|s) \right) = \left(1 - F(b_{t+1}|s) \right) \cdot (1+L)$$

$$-f(b_{t+1}|s) \cdot b_{t+1} \cdot (1+L) \qquad \text{if } F(b_{t+1}|s) \le \overline{F} . \tag{3.4}$$

The eligibility requirement introduces a discontinuity into the first order condition. To make the implied discontinuity in the debt choice $b_{t+1}^n(s)$ and eligible debt choice $b_{t+1}^e(s)$, respectively. Non-eligible firms choose their bond issuance according to (3.3): the left hand side (LHS) of this expression reflects discounted expected repayment obligations from issuing another unit of bond, which has to equal the current revenue from issuing this bond net of debt dilution on the right hand side (RHS). This case is consistent with the concept of *market discipline*, since the debt choice is determined solely by fundamentals. Collateral premia distort this trade-off by making debt issuance more attractive, since they increase the amount of funds raised per unit of bonds (first term on the RHS of (3.4)). At the same time, collateral premia increase the costs of debt dilution (second term on the RHS), which makes debt issuance less attractive.

Without further restrictions on the revenue distribution, the total effect of bond eligibility is ambiguous. However, guided by empirical evidence on increased debt issuance at the firm level in response to bond eligibility (Pelizzon et al., 2020) and consistent with the standard assumption in the literature (Bernanke et al., 1999), we assume that the distribution satisfies a monotonicity condition on the hazard rate $h(\mu_{t+1}|s) \equiv \frac{f(\mu_{t+1}|s)}{1-F(\mu_{t+1}|s)}$.

Proposition 1. If the revenue distribution satisfies $\frac{\partial(\mu_{t+1}h(\mu_{t+1}|s))}{\partial\mu_{t+1}} > 0$, the non-eligible debt choice is increasing in the productivity parameter $\frac{\partial b_{t+1}^n}{\partial s} > 0$, while the implied default risk decreases $\frac{\partial F(b_{t+1}^n)}{\partial s} < 0$. Likewise, for eligible firms it holds that $\frac{\partial b_{t+1}^e}{\partial s} > 0$ and $\frac{\partial F(b_{t+1}^e)}{\partial s} < 0$. Moreover, the optimal debt issuance of an eligible firm exceeds that of an otherwise identical non-eligible firm $b_{t+1}^e(s) > b_{t+1}^n(s)$. *Proof:* See Appendix B.2.1. Proposition 1 establishes two important results. First, more productive firms (higher s) issue more debt but their default risk falls compared to less productive firms. Because firms are risky, they increase debt issuance less than one-for-one with improving fundamentals. Second, collateral premia induce additional debt issuance of eligible firms. Those firms take advantage of their better bond valuation to front-load dividend payouts.

So far, we established differences between the non-eligible and eligible debt choice. Next, we focus on how firm choices are affected by eligibility requirements, since the eligibility status of firms is *endogenously* determined. It will be helpful to define the *eligible debt capacity* $\tilde{b}_{t+1}(s) \equiv F^{-1}(\overline{F}|s)$ as the highest debt choice for which the default probability of firm s does not exceed the threshold \overline{F} . Naturally, more productive firms have a higher eligible debt capacity. As shown in Proposition 2, the ex-ante heterogeneous revenue distribution determines how firms select themselves into eligible and non-eligible regions, taking the eligibility threshold as given.

Proposition 2. There are two cut-off values s_0 , implicitly defined through $V^e(\tilde{b}_{t+1}(s_0)|s_0) = V^n(b_{t+1}^n(s_0)|s_0)$, and s_2 , defined through $b_{t+1}^e(s_2) = \tilde{b}_{t+1}(s_2)$, such that

- firms with $s < s_0$ are *non-eligible* and choose $b_{t+1}^n(s)$ according to (3.3).
- firms with $s_0 < s < s_2$ are constrained eligible in the sense that they borrow up to their eligible debt capacity $\tilde{b}_{t+1}(s)$.
- firms with $s > s_2$ are unconstrained eligible and choose $b_{t+1}^e(s)$ according to (3.4).

Proof: See Appendix B.2.2.

In Section 3.2.2, we provide an illustration of Proposition 2. We plot the first order conditions in solid black lines, expressed in terms of the hazard rate h(b|s). Objective functions for the case of non-eligibility and eligibility are denoted by V^n and V^e (blue dashed lines) and are obtained from evaluating (3.2) at the respective debt choices. There are four possible combinations of $b_{t+1}^n(s)$, $b_{t+1}^e(s)$, and $\tilde{b}_{t+1}(s)$.

Figure 3.1a shows the case of a highly productive firm with a high draw of s so that $b_{t+1}^e(s) < \tilde{b}_{t+1}(s)$. The eligible debt capacity of an unconstrained eligible firm is sufficiently high, such that it can satisfy (3.4). Figure 3.1b shows a firm with insufficient debt capacity to satisfy (3.4), whereas satisfying (3.3) would be possible, $b_{t+1}^n(s) < \tilde{b}_{t+1}(s) < b_{t+1}^e(s)$. However, the value of the objective $V^e(\tilde{b}_{t+1}(s)|s)$ exceeds the value $V^e(b_{t+1}^n(s)|s)$, because V^e is upward sloping for all $b < b_{t+1}^e(s)$. Thus, the firm chooses to be just eligible at debt level $\tilde{b}_{t+1}(s)$. Such a firm is constrained eligible. Within the case of $\tilde{b}_{t+1}(s) < b_{t+1}^n(s)$, there are two sub-cases: first, choosing $b_{t+1}^n(s)$ is feasible, but the firm can be better off by choosing $\tilde{b}_{t+1}(s)$, since $V^n(b_{t+1}^n(s)|s) < V^e(\tilde{b}_{t+1}(s)|s)$, as in Figure 3.1c. Such a firm chooses to be just eligible and is also classified as constrained eligible. Second, non-eligible



FIGURE 3.1: Debt Choice Across Endogenous Firm Types

Notes: Endogenous partitioning of the firm space following Proposition 2. The blue lines denote the objective function (3.2) for each eligibility status. The black line represents the LHS of firms' first order conditions, rewritten as $h(b_{t+1}|s) \cdot b_{t+1}$, which maximizes V^e at b^e and V^n at b^n .

firms with a sufficiently low s optimally choose $b_{t+1}^n(s)$, since the debt reduction required for eligibility is too large $V^e(\tilde{b}_{t+1}(s)|s) < V^n(b_{t+1}^n(s)|s)$, as in Figure 3.1d.

3.2.3 Eligibility Requirements and Macroeconomic Aggregates

Having discussed how firm policies are characterized in the presence of collateral premia, we now turn to the effects of collateral easing. We consider an increase of the threshold default probability from a low value \overline{F}^A to a higher value \overline{F}^{BBB} , akin to the ECB policy change in response to the 2008 financial crisis and also corresponding to our numerical experiments in Section 3.3. Formally, we characterize the change of aggregate collateral \overline{B} in terms of the debt choice across the firm type space S. Let the cut-off values, which determine the partitioning of firms into constrained and unconstrained eligible, associated with \overline{F}^A and \overline{F}^{BBB} be denoted by (s_0^A, s_2^A) and (s_0^{BBB}, s_2^{BBB}) , respectively. The threshold productivity levels partitioning firms into eligibility regions decrease in response to collateral easing, which we summarize in Lemma 1.

Lemma 1. Increasing the eligibility threshold from \overline{F}^A to \overline{F}^{BBB} decreases the threshold levels to $s_0^{BBB} < s_0^A$ and $s_2^{BBB} < s_2^A$. *Proof*: See Appendix B.2.3.

We can use Lemma 1 to write the total effect of collateral easing on collateral supply as

$$\overline{B}^{BBB} - \overline{B}^{A} = (1+L) \left(\int_{s_{0}^{BBB}}^{s_{2}^{BBB}} \left(1 - F\left(\tilde{b}_{t+1}^{BBB}(s)\right) \right) \tilde{b}_{t+1}^{BBB}(s) dG(s) + \int_{s_{2}^{BBB}}^{\infty} \left(1 - F\left(b_{t+1}^{e}(s)\right) \right) b_{t+1}^{e}(s) dG(s) - \int_{s_{0}^{A}}^{s_{2}^{A}} \left(1 - F\left(\tilde{b}_{t+1}^{A}(s)\right) \right) \tilde{b}_{t+1}^{A}(s) dG(s) - \int_{s_{2}^{A}}^{\infty} \left(1 - F\left(b_{t+1}^{e}(s)\right) \right) b_{t+1}^{e}(s) dG(s) \right).$$
(3.5)

Collateral supply, that is, the market value of eligible bonds, can be divided into two parts: the two integrals over $[s_0, s_2]$ contain all constrained eligible firms, respectively, while the integrals over $[s_2, \infty)$ summarize unconstrained eligible firms.

A central point of our framework is that such a policy has a mechanical effect by lowering the eligibility threshold and that it implies an endogenous firm response. To highlight this decomposition, we introduce a third threshold productivity s_1 , where $s_0 < s_1 < s_2$. For the threshold firm s_1 , the debt choice $b_{t+1}^n(s_1)$ equals its eligible debt capacity $\tilde{b}_{t+1}(s_1)$. Mechanical effects are present if threshold levels satisfy $s_1^{BBB} < s_0^A$. This means that at least the firm exactly satisfying eligibility requirements after the policy change $F(b_{t+1}^n(s_1^{BBB})) = \overline{F}^{BBB}$ was not eligible before the policy change $\overline{F}^A < F(b_{t+1}^n(s_1^{BBB}))$. This firm was non-eligible under the tight policy, where it chooses $b_{t+1}^n(s)$, but becomes eligible without changing its debt issuance. To ease the exposition, we further restrict attention to 'small' changes to eligibility requirements and assume $s_0^A < s_2^{BBB}$. This implies that there is no firm directly switching from non-eligible to unconstrained eligible. We summarize the impact of collateral easing on collateral supply and default cost in Lemma 2.

Lemma 2. If $s_1^{BBB} < s_0^A$, the mechanical effect of collateral easing on collateral supply is positive and given by

$$\overline{B}^{BBB} - \overline{B}^{A}\Big|_{mech} = (1+L) \left(\int_{s_{1}^{BBB}}^{s_{0}^{A}} \left(1 - F(b_{t+1}^{n}(s)) \right) b_{t+1}^{n}(s) dG(s) \right).$$
(3.6)
If $s_0^A < s_2^{BBB}$, endogenous firm responses on collateral supply can be expressed as

$$\overline{B}^{BBB} - \overline{B}^{A}\Big|_{endo} = (1+L) \left(\int_{s_{0}^{BBB}}^{s_{1}^{BBB}} \left(1 - F\left(\widetilde{b}_{t+1}^{BBB}(s)\right) \right) \widetilde{b}_{t+1}^{BBB}(s) dG(s)$$
(3.7a)

$$+ \int_{s_{1}^{BBB}}^{s_{0}^{A}} \left(1 - F\left(\tilde{b}_{t+1}^{BBB}(s)\right) \right) \tilde{b}_{t+1}^{BBB}(s) - \left(1 - F\left(b_{t+1}^{n}(s)\right) \right) b_{t+1}^{n}(s) dG(s)$$
(3.7b)

$$+ \int_{s_0^A}^{s_2^{BBB}} \left(1 - F(\tilde{b}_{t+1}^{BBB}(s)) \right) \tilde{b}_{t+1}^{BBB}(s) - \left(1 - F(\tilde{b}_{t+1}^A(s)) \right) \tilde{b}_{t+1}^A(s) dG(s)$$
(3.7c)

$$+ \int_{s_2^{BBB}}^{s_2^A} \left(1 - F\left(b_{t+1}^e(s)\right) \right) b_{t+1}^e(s) - \left(1 - F\left(\tilde{b}_{t+1}^A(s)\right) \right) \tilde{b}_{t+1}^A(s) dG(s) \right) .$$
(3.7d)

Denoting the resource loss of firm s from default by $M(b_{t+1}(s)|s) \equiv \int_{\underline{\mu}}^{b_{t+1}(s)} \mu_{t+1} dF(\mu_{t+1}|s)$, the change in aggregate default cost \mathcal{M}_t can be expressed as follows

$$\mathcal{M}^{BBB} - \mathcal{M}^{A} = \int_{s_{0}^{BBB}}^{s_{1}^{BBB}} M\left(\tilde{b}_{t+1}^{BBB}(s)\right) - M\left(b_{t+1}^{n}(s)|s\right) dG(s)$$
(3.8a)

$$+ \int_{s_1^{BBB}}^{s_0^A} M(\tilde{b}_{t+1}^{BBB}(s)) - M(b_{t+1}^n(s)) dG(s)$$
(3.8b)

$$+ \int_{s_0^A}^{s_2^{BBB}} M(\tilde{b}_{t+1}^{BBB}(s)) - M(\tilde{b}_{t+1}^A(s)) dG(s)$$
(3.8c)

$$+ \int_{s_2^{BBB}}^{s_2^A} M\left(b_{t+1}^e(s)\right) - M\left(\tilde{b}_{t+1}^A(s)\right) dG(s) .$$
(3.8d)

Proof: see Appendix B.2.4.

Equation (3.6) reflects the additionally eligible collateral under the assumption that firms do not change their debt choice. These firms were non-eligible under tight eligibility requirements and, therefore, issue bonds according to $b_{t+1}^n(s)$. The mechanical effect of collateral easing is positive.

The collateral supply effect of collateral easing associated to firm responses is given by (3.7). To aid intuition, Figure 3.2a shows the impact of collateral easing on firm debt issuance across firm types s. The lines b^n and b^e denote the debt choices that satisfy (3.3) and (3.4), respectively. As shown in Lemma 1 they increase in firm productivity. Dashed lines denote the eligible debt capacity under tight (blue) or lenient (orange) collateral policy. Under a lenient policy, the eligible debt capacity increases for every productivity level compared to a tight policy so that the line shifts to the left. Bold colored lines denote the actual firm debt choices.

For a given collateral policy, we can distinguish the three firm types from Proposition 2. Highly productive firms (above s_2) choose debt according to (3.4), i.e., they are unconstrained eligible. Firms of medium quality, between the jump at s_0 and the kink at s_2 , choose their eligible debt capacity and are constrained eligible. Last, low productivity firms (below s_0) are non-eligible and choose debt b^n following (3.3). Risk-taking and



FIGURE 3.2: Firm Responses After Collateral Easing

disciplining effects for a given policy are related to the difference between the firm choice (bold lines) and b^n (solid black line). First, firms between s_0 and s_1 reduce debt issuance compared to non-eligibility, i.e., they *discipline* themselves to be eligible. Second, firms above s_1 issue more debt compared to b^n , which is a *risk-taking* effect.

Lowering the eligibility threshold, going from the blue to the orange lines, changes productivity cut-off values (see Lemma 2). The first integral (3.7a) relates to firms that reduce their debt issuance relative to the tight policy to benefit from being eligible, which is graphically represented by the red area in Figure 3.2a. It is associated with the disciplining effect across the firm distribution. All other parts of (3.7) relate to risk-taking effects (denoted as blue areas), i.e., firms that increase debt issuance compared to tight policy: the second integral (3.7b) corresponds to firms issuing debt at their eligible debt capacity, but above $b_{t+1}^n(s)$, which exceeds their borrowing under tight eligibility requirements. Likewise, the third integral (3.7c) captures firms that remain constrained eligible but with a higher eligible debt capacity $\tilde{b}^{BBB} > \tilde{b}^A$. Last, the fourth integral (3.7d) summarizes firms that switch from constrained to unconstrained eligible.

Disciplining and risk-taking have a positive collateral supply effect. Firms becoming newly eligible via the disciplining effect automatically increase collateral supply. For firms that take on more risk, this can be seen by noting that those firms will not issue debt beyond a point where debt dilution exceeds the funds raised by issuing an additional unit

Notes: We compare the change in debt issuance (left) and default risk (right) across firm types s after an easing from tight (blue) to lenient (orange) collateral requirements. Colored dashed lines represent the eligible debt capacities or the associated default risk under either policy. Black solid lines denote the debt choices (or the corresponding default risk) satisfying (3.3) and (3.4). Colored bold lines denote the firm debt choices and the related default risk. Colored dotted lines denote the threshold productivities as described in Proposition 2 and Lemma 1.

of debt.¹⁰

In contrast, effects on the aggregate cost from default (3.8) are ambiguous. We illustrate the change in firm default risk across types s in Figure 3.2b. Default risk associated to b^n and b^e (solid black lines) falls for more productive firms as seen from Lemma 2. The eligibility thresholds are given by the horizontal colored lines and default risk related to the firm debt choice is given by bold colored lines. As in Figure 3.2a, one can distinguish the three firm types of Proposition 2. The effect of collateral easing on aggregate default costs is closely related to the change in default risk across firm types: while disciplining effects in the first integral (3.8a) lead to a reduction in aggregate default cost (red area), the other three integrals (3.8b)-(3.8d) related to risk-taking effects (blue areas), imply an increase of aggregate default costs.

Our framework with short-term debt predicts that collateral easing has a positive mechanical impact on collateral supply, which is amplified by firm responses. At the same time, the effect of firm responses on aggregate default cost is ambiguous and depends on the relative strength of risk-taking and disciplining effects, making a heterogeneous firm model necessary to adequately study aggregate effects. To quantify the relevance of endogenous firm responses and determine their sign, we extend our framework with persistent revenue shocks and long-term debt in the next section. In this setting, there is a pronounced dampening effect on collateral supply, which has been described in several settings with long-term debt and default risk (see Gomes et al., 2016; Jungherr and Schott, 2022). This gives rise to a negative relationship between increasing collateral supply (which facilitates monetary policy implementation) and incentivizing risk-taking at the firm level (which increases resource losses of default). We discuss the implied central bank policy trade-off and a potential remedy in Section 3.4.

3.3 Application to the ECB Collateral Easing Policy

In this section, we evaluate the ECB's collateral easing policy in response to the financial crisis of 2008. We first extend the one-period bond model from the previous section and show its calibration to euro area data. We then use the calibrated model to shed light on corporate bond spreads and study the aggregate impact of endogenous firm responses induced by collateral easing. The characterization of aggregate effects forms the basis for our analysis of collateral policy in Section 3.4.

¹⁰Differentiating the market value of bonds outstanding for any eligible debt choice $\frac{\partial((1+L)(1-F(b_{t+1}^e))b_{t+1}^e)}{\partial b_{t+1}^e} = (1+L)(1-h(b_{t+1}^e)b_{t+1}^e)(1-F(b_{t+1}^e))$. Using (3.4), this simplifies to $\beta(1-F(b_{t+1}^e)) > 0$.

3.3.1 Extended Model

Firm heterogeneity takes the form of persistent revenue shocks rather than permanent differences in the idiosyncratic firm revenue distribution. In addition, bonds are longterm and a firm defaults if it cannot repay the maturing share of outstanding bonds out of current revenues. As in the previous section, we maintain the assumption of no delays in restructuring so that the value of non-maturing bonds is not affected by a default event. This permits us to abstract from the firm credit status as a state variable.

Firms. There is a continuum of competitive firms, indexed by j. Firms receive random revenues $e^{\mu_t^j}$ following an log-AR(1) process

$$\mu_{t}^{j} = \rho \mu_{t-1}^{j} + \sqrt{\sigma} \epsilon_{t}^{j} \quad \text{with } \epsilon_{t}^{j} \sim N\left(0,1\right) \; .$$

The idiosyncratic revenue shock is independent across firms. We denote the (conditional) pdf of μ_{t+1}^j by $f(\mu_{t+1}^j|\mu_t^j)$ and the associated (conditional) cdf by $F(\mu_{t+1}^j|\mu_t^j)$. Firms issue long-term bonds b_{t+1}^j , which enables us to generate realistic debt ratios. Each period, a share π of outstanding bonds matures. The non-maturing share of bonds is valued like new issues at price q_t , according to the law of one price. Firms default on their current repayment obligation πb_t^j if they exceed current revenues $e^{\mu_t^j}$. We can write the default probability as

$$F(b_{t+1}^j|\mu_t^j) = \Phi\left(\frac{\log\left(\pi b_{t+1}^j\right) - \rho \mu_t^j}{\sigma}\right) .$$
(3.9)

Banks and Bond Pricing. Banks are modeled in a similar way as in Section 3.2. They are risk-neutral and discount the future at the constant rate r^{rf} . The per-unit price schedule for corporate bonds can be written

$$q(b_{t+1}^{j}, \mu_{t}^{j}) = \frac{1 + \Psi(F(b_{t+1}^{j} | \mu_{t}^{j}))L}{1 + r^{rf}} \left(\pi \left(1 - F(b_{t+1}^{j} | \mu_{t}^{j}) \right) + (1 - \pi)\mathbb{E}_{t} \left[q \left(\mathcal{B} \left(b_{t+1}^{j}, \mu_{t+1}^{j} \right), \mu_{t+1}^{j} \right) \right] \right)$$

$$(3.10)$$

Note that the rollover value of bonds is obtained from evaluating the bond price schedule at next period's debt choice $\mathcal{B}(b_{t+1}^j, \mu_{t+1}^j)$, which we describe below. As in the baseline model from Section 3.2, the total payoff contains a pecuniary part and a collateral premium L. The pecuniary part depends on default in t + 1. If the firm repays, the maturing fraction π is redeemed and the remainder $1 - \pi$ is rolled over at the next period's market price. In the case of default, banks lose the maturing fraction of the bond. Due to the assumption of immediate restructuring, the payoff still contains the rollover value of the non-maturing fraction.

Characterization of Debt Choices. Firms choose issue bonds b_{t+1}^{j} to maximize shareholder value, taken as given the bond price schedule (3.10). The maximization problem of firm j can be represented by the Bellman equation

$$W(b_{t}^{j}, \mu_{t}^{j}) = \max_{b_{t+1}^{j}} V(b_{t+1}^{j}, \mu_{t}^{j}) \text{ with}$$

$$V(b_{t+1}^{j}, \mu_{t}^{j}) = \mathbb{1}\left\{e^{\mu_{t}^{j}} > \pi b_{t}^{j}\right\} \left(e^{\mu_{t}^{j}} - \pi b_{t}^{j}\right) + q(b_{t+1}^{j}, \mu_{t}^{j}) \left(b_{t+1}^{j} - (1 - \pi)b_{t}^{j}\right)$$

$$+ \beta \mathbb{E}_{t} \left[W(b_{t+1}^{j}, \mu_{t+1}^{j})\right].$$

$$(3.11)$$

Current dividends are given by revenues and debt service obligations $e^{\mu_t^j} - \pi b_t^j$, conditional on repayment, and net debt issuance $q(b_{t+1}^j, \mu_t^j) \left(b_{t+1}^j - (1-\pi)b_t^j \right)$. Note that the debt choice b_{t+1}^j does not depend on a potential default in period t, which again follows from the assumption of immediate restructuring. A higher debt choice increases current dividends but reduces next period's dividends due to (i) higher default risk, (ii) elevated debt service conditional on no default, and (iii) increasing the rollover burden in the next period. Plugging in the bond pricing condition (3.10), the first order condition can be written as

$$\frac{\partial q(b_{t+1}^{j}, \mu_{t}^{j})}{\partial b_{t+1}^{j}} \left(b_{t+1}^{j} - (1-\pi)b_{t}^{j} \right) + q(b_{t+1}^{j}, \mu_{t}^{j}) \\
= \beta \left(\pi (1 - F(b_{t+1}^{j} | \mu_{t}^{j})) + (1-\pi)\mathbb{E}_{t} \left[q_{t+1} \right] \right) ,$$
(3.13)

where the derivative of the bond price schedule (3.10) is given by

$$\frac{\partial q(b_{t+1}^j, \mu_t^j)}{\partial b_{t+1}^j} = \begin{cases} -f(b_{t+1}^j)\pi \frac{1}{1+r^{rf}}, & \text{if } F_{t+1}^j > \overline{F}, \\ -f(b_{t+1}^j)\pi \frac{1+L}{1+r^{rf}}, & \text{if } F_{t+1}^j \le \overline{F}. \end{cases}$$
(3.14)

Let the solution to (3.13) in the case without eligibility be denoted by $b_{t+1}^{j,n}$ and in the case of eligibility by $b_{t+1}^{j,e}$. The debt choice depends on the feasibility of $b_{t+1}^{j,e}$ and the value of the objective function (3.12) under both candidate debt choices. The eligible debt capacity in closed form is obtained from evaluating the default probability (3.9) at \overline{F} and re-arranging to

$$\tilde{b}_{t+1}^{j} = \frac{\exp\{\sigma\Phi^{-1}(\overline{F}) + \rho\mu_{t}^{j}\}}{\pi} , \qquad (3.15)$$

which we can use to obtain the debt choice $\mathcal{B}(b_t^j, \mu_t^j)$

$$\mathcal{B}(b_t^j, \mu_t^j) = \mathbb{1}\left\{ V(b_{t+1}^{j,n}, \mu_t^j) \le V(\min\{b_{t+1}^{j,e}, \widetilde{b}_{t+1}^j\}, \mu_t^j) \right\} \cdot \min\{b_{t+1}^{j,e}, \widetilde{b}_{t+1}^j\} \\ + \mathbb{1}\left\{ V(b_{t+1}^{j,n}, \mu_t^j) > V(\min\{b_{t+1}^{j,e}, \widetilde{b}_{t+1}^j\}, \mu_t^j) \right\} \cdot b_{t+1}^{j,n} .$$

$$(3.16)$$

If $b_{t+1}^{j,e} > \tilde{b}_{t+1}^{j}$, the eligible debt choice is not feasible. Therefore, the debt choice $\mathcal{B}(b_{t}^{j}, \mu_{t}^{j})$ depends on the value attained by exhausting the eligible debt capacity $V(\tilde{b}_{t+1}^{j}, \mu_{t}^{j})$ and the

value of forgoing eligibility $V(b_{t+1}^{j,n}, \mu_t^j)$. Conversely, if $b_{t+1}^{j,e} < \tilde{b}_{t+1}^j$, the firm can issue the optimal level of bonds without losing eligibility. Consistent with the one-period model in Section 3.2, the firm will issue $b_{t+1}^{j,e}$ in this case, since $b_{t+1}^{j,n} < b_{t+1}^{j,e}$ and $V(b_{t+1}^{j,n}, \mu_t^j) < V(b_{t+1}^{j,e}, \mu_t^j)$ by definition. Since there is no aggregate risk and banks' bond pricing condition is independent of the firm distribution, the debt choice of firms and the bond pricing condition of banks fully characterize the equilibrium of our model. The equilibrium bond price $\mathcal{Q}(b_t^j, \mu_t^j)$ obtains from evaluating the bond price schedule (3.10) at the debt choice (3.16)

$$\mathcal{Q}(b_t^j, \mu_t^j) = q(\mathcal{B}(b_t^j, \mu_t^j), \mu_t^j)$$

Recursive Competitive Equilibrium. A competitive equilibrium is given by the bond price schedule $q(b_{t+1}^j, \mu_t^j)$, the firm value function $W(b_t^j, \mu_t^j)$, and the debt choice $\mathcal{B}(b_t^j, \mu_t^j)$ such that

- given the pricing schedules for bonds, the debt choice solves the firm problem (3.11).
- bonds are priced according to (3.10).
- the law of motion for the distribution of firms over bond holdings and firm-specific revenues follows

$$G_{t+1}(b_{t+1},\mu_{t+1}) = \int \int \left[\mathbb{1} \left\{ b_{t+1} = \mathcal{B}(b_t,\mu_t) \right\} \right] \times \mathbb{1} \left\{ \mu_{t+1} = \rho \mu_t + \sigma \epsilon_{t+1} \right\}$$
$$\times G_t(b_t,\mu_t) f(\epsilon_{t+1}) d\epsilon_{t+1} db_{t+1}$$

Numerical Solution Method. We solve the full model computationally using policy function iteration on a discrete grid for revenues and bond issuance. The algorithm contains four steps at each iteration: first, we compute both potentially optimal debt choices by solving (3.13), given the bond price schedule (3.10). Second, we compute the eligible debt capacity (3.15) and check whether the optimal debt choice under eligibility is feasible. If this is not the case, we replace it by the eligible debt capacity \tilde{b} . We randomize over the value function under both candidate debt choices using Gumbel-distributed taste shocks as proposed by Gordon (2019) to compute the debt choice (3.16). Third, given these policies, we compute the distribution of firms over individual states. The fourth step consists of updating bond price schedules. For a detailed description of the algorithm and the parameters governing our numerical approximation, we refer to Appendix B.3.2.

3.3.2 Calibration

We calibrate the model to euro area data between 2004Q1, the earliest data with reliable corporate bond data, and 2008Q3, the last quarter before the ECB relaxed its collateral framework. One period corresponds to one quarter. Our calibration is divided into two

Parameter	Value	Source
Bank discount rate r^{rf}	0.0035	Real risk-free rate
Borrower discount factor β	0.993	Calibrated
Maturity Parameter π	0.06	Calibrated
Collateral premium L	0.004	Calibrated
Revenue persistence ρ	0.93	Calibrated
Revenue shock std. dev. σ	0.027	Calibrated
(Annualized) A-eligibility threshold \overline{F}^A	4.15%	Calibrated
(Annualized) BBB-eligibility threshold \overline{F}^{BBB}	18.59%	Calibrated

TABLE 3.1: Baseline Parameterization

parts: the first part contains parameters determining the pricing of bond payoffs and eligibility benefits by banks, while the second set of parameters is related to firm fundamentals and the payoff profile of corporate bonds. These two blocks are connected by the central bank eligibility requirement, which is the policy variable of interest. We consider two policies: the baseline calibration is associated with tight eligibility requirements (A-rating or higher) and collateral easing refers to a scenario with lenient eligibility requirements (BBB-rating or higher). These thresholds are based on the ECB policy before and after the financial crisis of 2008.

Eligibility Requirement. We begin with discussing the eligibility thresholds \overline{F}^A and \overline{F}^{BBB} . The ECB's collateral framework is based on ratings by external credit assessment agencies that are difficult to model parsimoniously. Therefore, we adopt an indirect approach based on macroeconomic aggregates. Specifically, we obtain data from *IHS Markit* on the total fixed income securities universe in Europe and extract the subset for non-financial corporate bonds. Using data from September 2008, the last month prior to the relaxation of eligibility requirements, 50% of all corporate bonds in our sample carried a rating of A or higher and were formally eligible as collateral. To match this share of eligible bonds, we set the baseline eligibility threshold to $\overline{F}^A = 4.15\%$, expressed in annualized terms. Similarly, we choose the eligibility threshold for a BBB-rating as $\overline{F}^{BBB} = 18.59\%$ to match the share of bonds rated BBB or higher in the *IHS Markit* sample, which was 86% in September 2008. We interpret this observed 72% increase of collateral supply as measure of the *mechanical* effect, since it is based on data prior to the policy relaxation.

Collateral Premium. We proxy the time-invariant real risk-free interest rate by a shortterm interbank rate from which we subtract the consumer price inflation rate. Specifically, we use the time-series average of the 3M-EURIBOR minus the euro area inflation and obtain $r^{rf} = 0.0035$. The collateral premium L is based on the empirical findings from

Moment	Data	Model
Collateral premium $\operatorname{ave}(r-r^n)$	-11	-11
Debt/EBIT $Q_{0.50}(b/\mu \overline{F}^A)$	3.9	3.9
Bond spread $Q_{0.25}(x \overline{F}^A)$	24	27
Bond spread $Q_{0.50}(x \overline{F}^A)$	39	52
Bond spread $Q_{0.75}(x \overline{F}^A)$	62	72
Eligible bond share $\overline{B}/(QB) \overline{F}^A$	50%	50%
Eligible bond share $\overline{B}/(QB) \overline{F}^{BBB}$	86%	83%

TABLE 3.2: Targeted Moments

Notes: The collateral premium and spreads are annualized and expressed in basis points.

Pelizzon et al. (2020).¹¹ Their paper makes use of the ECB having discretion in including bonds that formally satisfy eligibility requirements in the list of eligible assets. This discretion generates a randomly selected control group of bonds that eventually become eligible but are not yet accepted. Depending on the econometric specification, they estimate a yield reaction to surprise eligibility of 11-24 bp. We pick the most conservative value of 11 bp. Our structural model permits an explicit calculation of the yield effect of a surprise inclusion. We set $\Psi = 0$ when pricing the bond (holding firm behavior fixed) and compare this hypothetical price to the equilibrium bond price. The price of this hypothetical bond is given by

$$q^{n}(b_{t+1}^{j},\mu_{t}^{j}) = \frac{1}{1+r^{rf}} \left(\left(1-F(b_{t+1}^{j}|\mu_{t}^{j})\right)\pi + (1-\pi)\mathbb{E}_{t} \left[q\left(b_{t+1}^{j},\mu_{t+1}^{j}\right)\right] \right) , \qquad (3.17)$$

and contains a collateral premium from t + 1 onward via the bond continuation value. The yield-to-redemption \tilde{r}^{j} is determined by the internal rate of return of a perpetuity with constant decay,

$$q_t^j = \sum_{\tau=t+1}^{\infty} \frac{\pi (1-\pi)^{\tau-1}}{(1+r_t^j)^{\tau}} = \frac{\pi}{\pi + r_t^j}$$

It follows that $r_t^j = \pi/q_t^j - \pi$. The corporate bond spread is defined as $x_t^j \equiv r_t^j - r^{rf}$. Using an entirely analogous derivation, the yield on the hypothetical non-eligible bond is given by $r_t^{j,n}$ and the eligibility premium follows as $r_t^j - r_t^{j,n}$, which is always negative.

Firm Fundamentals. The second part of the calibration is related to firms, i.e., the parameters governing the idiosyncratic revenue process ρ and σ , the maturity parameter π characterizing the repayment profile of corporate bonds, and the discount factor of firms β affecting the relative impatience over investors. By setting β to a lower value than the time discount factor of banks $\frac{1}{1+r^{rf}}$, we ensure that even absent collateral premia, firms would have an incentive to issue bonds.

¹¹Notably, L does not depend on aggregate collateral supply. We relax this assumption in Appendix B.4.4.



FIGURE 3.3: Corporate Bond Spreads

Notes: The blue (orange) line represents the corporate spread under a tight (lenient) eligibility requirement, conditional on firm legacy debt b_t^j , for a firm with median revenues.

These parameters are chosen to match selected data moments characterizing the firm cross-section. We merge our corporate bond dataset from *IHS Markit* with company data available through *Compustat Global*. A description of the construction of our dataset is given in Appendix B.3. Specifically, we target the median debt/EBIT-ratio b_t^j/μ_t^j as a measure of firm indebtedness and the bond spread distribution, characterized by its quartiles. The time-series averages over the sample period 2004Q1-2008Q3 are $Q_{0.25}(x) =$ 24 bp, $Q_{0.50}(x) = 39$ bp, and $Q_{0.75}(x) = 62$ bp. We conduct a sensitivity analysis with respect to a higher level of spreads computed over an extended sample period in Appendix B.4.3. Table 3.1 summarizes all parameters for our baseline calibration and Table 3.2 shows the targeted moments in our baseline calibration.

3.3.3 Corporate Bond Spreads

To illustrate how eligibility requirements affect the pricing of corporate bonds, we show the corporate bond spreads implied by banks' first order condition in Figure 3.3. Spreads are derived by evaluating the bond pricing condition at any candidate debt choice b_{t+1}^{j} , fixing revenues at their median. The baseline calibration (A-rating or higher) is marked in blue, while lenient eligibility requirements (BBB-rating or higher) are marked in orange. The discontinuity in each bond price schedule represents the location of the eligibility threshold. To the left of this point, bonds are currently eligible and investors are willing to pay collateral premia, which results in lower spreads. For a debt choice to the right of the discontinuity, bonds are not eligible and spreads jump upwards. The effect of relaxing eligibility requirements can be inferred from the location of the discontinuities. Intuitively, lenient eligibility requirements increase the eligible debt capacity, so that the discontinuity shifts to the right. Notably, since bonds are long-term, this also affects bond spreads away from the eligibility threshold: bonds are more likely to be eligible in future periods, which increases their price and lowers the spread already in the current period via the continuation value in (3.10). Hence, spreads under lenient eligibility requirements are uniformly lower.

3.3.4 Reconciling Cross-Sectional Evidence

Before discussing macroeconomic aggregates, we test the model's capability to replicate the (heterogeneous) impact of eligibility requirements across firms identified by several empirical papers. We run regressions on a simulated cross-section of firms, which is drawn from the equilibrium firm distribution under tight eligibility requirements. Regressions based on the firm distribution associated with lenient eligibility requirements yield similar results. We run the following (cross-sectional) regression

$$y_t^j = \beta_1 Eligible_t^j + \beta_2 Eligible_t^j \cdot \frac{b_t^j}{\mu_t^j} + \epsilon_t^j , \qquad (3.18)$$

for three different specifications, that differ in the outcome variable. First, we examine the bond yield reaction to surprise eligibility $r_t^j - r_t^{j,n}$, following Pelizzon et al. (2020). Since we can control for firm indebtedness as a measure of default risk, this approach is similar to Grosse-Rueschkamp et al. (2019), Mota (2021), and Todorov (2020). Second, we evaluate the effects of a surprise inclusion on debt issuance $\mathcal{B}_{t+1}^j - b_{t+1}^{j,n}$ and, third, on dividends $\mathcal{D}_t^j - d_t^{j,n}$.¹²

The sign of the model-implied regression coefficients in all three specifications are collected in the right panel of Table 3.3. Since we sample from a parsimonious structural model, all coefficients are highly significant. We benchmark the model-implied coefficients against findings from the literature, reported in the left panel. The eligibility premium $r_t^j - r_t^{j,n}$ is a calibration target. Therefore, the coefficient on eligibility is negative by construction. The positive impact of eligibility on debt issuance is consistent with findings

 $\mathcal{D}(b_t^j, \mu_t^j) = e^{\mu_t^j} - \pi b_t^j + q(\mathcal{B}(b_t^j, \mu_t^j), \mu_t^j) \left(\mathcal{B}(b_t^j, \mu_t^j) - (1 - \pi) b_t^j \right) \;,$

while the dividend of an non-eligible, but otherwise identical, firm can be written as

 $d^{n}(b^{j}_{t},\mu^{j}_{t}) = e^{\mu^{j}_{t}} - \pi b^{j}_{t} + q(b^{n}(b^{j}_{t},\mu^{j}_{t}),\mu^{j}_{t}) \left(b^{n}(b^{j}_{t},\mu^{j}_{t}) - (1-\pi)b^{j}_{t}\right) \ .$

¹²The equilibrium dividend in period t is given by

	Empirical Literature			Empirical Literature Model				
Coefficient	$r_t^j - r_t^{j,n}$	$\mathcal{B}_{t+1}^j - b_{t+1}^{j,n}$	$\mathcal{D}_t^j - d_t^{j,n}$	$r_t^j - r_t^{j,n}$	$\mathcal{B}_{t+1}^j - b_{t+1}^{j,n}$	$\mathcal{D}_t^j - d_t^{j,n}$		
Eligibility	-	+	+	-	+	+		
Indebtedness \times Eligibility	+	-	-	+	-	-		

TABLE 3.3: Cross-Sectional Regression Results

Notes: Signs in the left panel are taken from the empirical literature. Signs in the right panel are obtained from running (3.18) on the simulated firm cross-section. Model-implied coefficient signs are independent of the tightness of eligibility requirements.

by Pelizzon et al. (2020), while the positive effect of eligibility on dividends has been described in Todorov (2020).

The coefficient signs of the interaction terms are informative about heterogeneous firm responses. Consistent with our theory, debt issuance and dividend payouts respond more strongly for less risky firms in the model, as the negative coefficients on the interaction term of eligibility and beginning-of-period indebtedness demonstrate. This negative relationship is consistent with the findings of Mota (2021), who documents a positive relationship of eligibility premia, debt issuance, and dividend payouts with firm safety as measured by ratings. The negative coefficient on the interaction term eligibility \times indebtedness is also in line with the results of Grosse-Rueschkamp et al. (2019). They report that firms rated A or higher increased their leverage ratio by 1.8 percentage points in response to CSPP-eligibility, as opposed to eligible BBB-rated firms, which only increase leverage by 0.8 percentage points. Taken together, our model can capture the impact of eligibility requirements on multiple firm outcome variables documented in the data.

3.3.5 Aggregate Effects

We now turn to the impact of collateral easing on macroeconomic aggregates. As demonstrated Appendix B.4.1, the heterogeneous risk-taking and disciplining effects from Section 3.2 carry over to the case of long-term debt and persistent revenue shocks and we will organize our discussion around these two effects as well. The changes to the cross-sectional firm distribution induced by collateral easing are relegated to Appendix B.4.2.

Similar to the one-period bond model in Section 3.2, our discussion is based on a decomposition of collateral supply into a *mechanical effect* and endogenous *firm responses*. Formally, this decomposition obtains from expanding the total effect as follows:

$$\begin{split} \overline{B}^{BBB} - \overline{B}^A &\equiv \int \mathbbm{1}\{F^{BBB} < \overline{F}^{BBB}\}q^{BBB}b^{BBB}dG^{BBB}(\mu, b) - \int \mathbbm{1}\{F^A < \overline{F}^A\}q^Ab^AdG^A(\mu, b) \\ &= \underbrace{\int \mathbbm{1}\{F^{BBB} < \overline{F}^{BBB}\}q^{BBB}b^{BBB}dG^{BBB}(\mu, b) - \int \mathbbm{1}\{F^A < \overline{F}^{BBB}\}q^Ab^AdG^A(\mu, b)}_{\text{Firm Response}} \end{split}$$

	Total Effect	Mechanical Effect
Collateral Supply \overline{B} Default Costs \mathcal{M}	+62% +8%	+71%
Firm Responses	Disciplining	Risk-Taking

TABLE 3.4: Macroeconomic Effects of Collateral Easing

Notes: Values in the upper panel refer to collateral easing from A to BBB and are denoted as percentage difference from the A-baseline. The lower panel displays the fraction of *all* firms that is subject to disciplining or risk-taking effects.

$$+\underbrace{\int \mathbbm{1}\{F^A < \overline{F}^{BBB}\}q^A b^A dG^A(\mu, b) - \int \mathbbm{1}\{F^A < \overline{F}^A\}q^A b^A dG^A(\mu, b)}_{\text{Mechanical Effect}} \ .$$

The total effect on collateral supply is given by the difference between the market value of bonds issued by all eligible firms under either policy. The mechanical effect in the third line keeps firm behavior and the cross-sectional distribution at the baseline calibration, varying only the eligibility requirement. Firm responses are given residually and encompass changes in the market value of bonds and default risk but evaluate the eligibility status at the same minimum rating requirement \overline{F}^{BBB} .

In the first panel of Table 3.4, we apply this decomposition to the collateral easing experiment. It stands out that the percentage change of collateral supply from the (targeted) mechanical effect (+71%) exceeds the total effect (+62%). In contrast to Section 3.2, firm responses *dampen* the impact of eligibility requirements on collateral supply. This result is associated with the shares of firms being subject to risk-taking and disciplining effects. In particular, the share of firms disciplining themselves to be eligible falls from 19% under tight to 3% under lenient collateral policy. At the same time, the share of firms engaging in risk-taking rises from 51% to 79%. Consequently, there are also adverse effects on the corporate bond market as measured by a 8% increase of default costs.¹³

Intuitively, under lenient policy corporate bonds are eligible for worse fundamentals, which reduces disciplining incentives at the expense of risk-taking. The dampening effect of risk-taking on collateral supply is directly related to the persistence of revenue shocks and the stickiness of indebtedness: high-revenue firms find it optimal to increase their debt issuance and increase current dividends. If revenues are sufficiently persistent and firm managers sufficiently impatient, this only leads to a modest increase in default risk in the current period. Ultimately, however, firms will receive adverse revenue shocks and, due to the inherent stickiness of indebtedness, find themselves with a large amount of debt outstanding, which makes default more likely (see Jungherr and Schott, 2022, or Gomes

¹³There are no mechanical effects on aggregate default costs by construction.

et al., 2016). Default risk not only leads to a drop in the market value of eligible bonds but may also imply that those firms default. Both effects lower collateral supply. This feature is not present in our setting of Section 3.2 with i.i.d. shocks and one-period bonds. In such a setting, it is never optimal for firms to increase debt issuance beyond a point where it decreases the market value of bonds outstanding. The overall dampening might require the central bank to relax eligibility requirements more aggressively to achieve a specific increase in collateral supply. In addition, collateral easing induces adverse side effects on the corporate bond market in our model. In practice, higher prevalence of default risk can directly increase restructuring costs or inefficient liquidation of firms and indirectly make the financial system fragile, e.g., due to counterparty default risk.

3.4 Eligibility Covenants and the Central Bank Policy Frontier

In the previous section, we discussed the macroeconomic effects of changing the eligibility requirement. Firm responses increase resource losses from default and dampen the positive reaction on collateral supply. In this section, we extend the central bank toolkit by an eligibility covenant, which targets the large risk-taking effects associated with long-term debt and persistent revenues.

We embed our previous results in a discussion of optimal central bank collateral policy and assume that the central bank aims to minimize *violations of market discipline*, i.e., incentivizing firm risk-taking, while ensuring sufficient collateral to *facilitate monetary policy implementation*. Even though our model is too simplistic to quantitatively assess optimal policy, it is still useful to outline the key policy trade-off arising from our analysis and to discuss its potential implications for optimal collateral policy. In assuming a tradeoff between violating market discipline and increasing collateral supply, we follow central banks' stated objectives (see Bindseil et al., 2017) and the literature on risky assets in the central bank collateral portfolio. In Koulischer and Struyven (2014), lenient central bank collateral policy increases credit supply and output in the private sector but implies central bank losses, because central banks are second-best user of collateral in case of a counterparty default. Similarly, Choi et al. (2021) offer a macroprudential approach to collateral policy. In their model, accepting low-quality collateral has a positive effect on bank lending, because banks can use high-quality collateral on the interbank market instead. At the same time, this exposes the central bank to potential losses.

While we do not specifically model the sources of collateral demand, we are consistent with these papers in so far that larger collateral supply is desirable but comes at a cost if this implies accepting risky collateral. Our analysis offers a complementary view, since the risk-taking decision is made at the firm level in our model. Therefore, we propose eligibility covenants that mitigate adverse collateral supply effects from firm risk-taking. We stress the microprudential nature of this instrument due to the absence of aggregate shocks to collateral supply and demand in our analysis.

Leverage-Based Covenants. Covenants restrict the eligible debt capacity of firms in addition to the default risk threshold \overline{F} that applies uniformly to all firms. We condition covenants on firm-specific states and focus on debt-based covenants in the following. Since debt outstanding is common knowledge for firms that are sufficiently large to issue marketable debt securities, such a policy is in principle implementable. However, it still leaves us with all functions mapping from the debt state space into the binary eligibility indicator $\Psi \in \{0, 1\}$. In the following, we focus on the exponential class, parameterized by $\gamma > 0$, such that the eligible debt capacity is decreasing in debt outstanding

$$\tilde{b}_{t+1}^{j} = \exp\{-\gamma b_{t}^{j}\} \cdot \frac{\exp\{\sigma \Phi^{-1}(\overline{F}) + \rho \mu_{t}^{j}\}}{\pi} .$$
(3.19)

The eligibility covenant $\exp\{-\gamma b_t^j\}$ effectively lowers the eligible debt capacity of firms with high debt outstanding and, thereby, provides deleveraging incentives. We fix revenues at the median and eligibility requirements at the BBB-level to visualize the impact of an eligibility covenant on the firm debt choice in Figure 3.4. The left panel shows the case without an eligibility covenant. The bold black line denotes the debt choice \mathcal{B}_{t+1} for a firm with median revenues under lenient eligibility requirements. This function maps bonds outstanding b_t into (gross) bond issuance b_{t+1} and exhibits a kink and a jump. These points are associated with the debt levels where firms switch from non-eligible to constrained eligible and, then, to unconstrained eligible (see Proposition 2). The orange dashed (dotted) line represents the debt choice if the firm is non-eligible (eligible) and the horizontal black lines denotes the firm's eligible debt capacity.

Firm risk-taking and disciplining effects are related to the difference between the orange dashed line b_{t+1}^n and the equilibrium debt choice \mathcal{B}_{t+1} (bold black line). The disciplining effect is represented by firms reducing their debt issuance below b_{t+1}^n , which applies to firms located near the jump of the policy function. The risk-taking effect is reflected by firms issuing debt according to $\mathcal{B}_{t+1} > b_{t+1}^n$, that is, wherever the bold black line is above the dashed orange one. Compared to the mass of firms being disciplined by collateral eligibility, the risk-taking effect is sizable.

The right panel represents the case with an eligibility covenant. Intuitively, introducing an eligibility covenant reduces the eligible debt capacity (light black line) if firms enter the period with large debt outstanding. Compared to the debt choice without covenants, the downward sloping shape of the eligible debt capacity induces a larger disciplining effect, the optimal debt choice $\mathcal{B}_{t+1}^{j,n}$ is located below $b_{t+1}^{j,n}$ for a broader range of legacy debt, and, conversely, reduces the risk-taking effect. As a result, the dampening effect of firm responses on collateral supply will be limited in the presence of an eligibility covenant.



FIGURE 3.4: Debt Choice with Eligibility Covenant

Notes: The bold black line represents the debt choice of a firm with median revenue conditional on legacy debt (see (3.16)). The purple and orange lines denote the hypothetical debt choice of an always (non-)eligible firm. The light black line is the eligible debt capacity. In the left (right) panel we depict the case without (with) covenant.

Optimal Covenants Given a Minimum Rating. We now turn to how the eligibility covenant influences collateral supply and aggregate default cost for a given eligibility requirement. The covenant has an ambiguous collateral supply impact. On the one hand, setting an overly harsh covenant (a large γ) reduces collateral supply, since it dis-incentives firms from issuing bonds. On the other hand, an overly lenient covenant (a small γ) fails to limit the risk-taking by eligible firms. We make the dependency of collateral supply $\overline{B}(\overline{F}, \gamma)$ and aggregate default cost $\mathcal{M}(\overline{F}, \gamma)$ on both policy parameters explicit in the following, compute them for different covenant parameters, and show the results in Figure 3.5. The covenant gives rise to a collateral supply by up to 74% (left panel), which is very similar to the 72% increase induced by collateral easing from \overline{F}^A to \overline{F}^{BBB} . At the same time, we observe a potential reduction in aggregate default cost of up to 42% (right panel). Thus, already for a fixed collateral eligibility policy, the covenant has powerful impacts.

Eligibility Covenants and the Collateral Policy Frontier. Next, we investigate how adding covenants to the central bank toolkit, in addition to the eligibility threshold, affects the collateral policy trade-off between high collateral supply and maintaining a high level of market discipline. We define the cost of violating market discipline as additional default cost that derive from making corporate bonds eligible, i.e., default cost are expressed



Notes: We show the effect of varying the covenant parameter γ for lenient eligibility requirements \overline{F}^{BBB} . The y-axis in the left (right) panel shows the percentage-increase in collateral supply (aggregate default cost) relative to the no-covenant case $\gamma = 0$.

relative to an economy without bond eligibility, where all firms would issue debt according to $b_{t+1}^{j,n}$.

Figure 3.6 shows the results in terms of the collateral policy frontier. Each dot is associated with a fixed collateral supply target on the x-axis. Supply targets are denoted as the fraction of eligible bonds \overline{B} relative to the market value of all bonds under market discipline. For a given supply target, we then choose the policy parameters to minimize the *additional default cost* relative to the case of market discipline, which the central bank has to allow to satisfy its supply target, as shown on the y-axis. We distinguish between the baseline case with eligibility thresholds only (orange) and the extended central bank toolkit that also comprises covenants (purple). The vertical black line indicates the benchmark with no firm responses, where default cost are the same as in the case of market discipline. In this setting, the central bank could simply pick a collateral supply target and set \overline{F} accordingly without adverse effects on the corporate bond market. Endogenizing firm responses gives rise to a trade-off between collateral supply and default cost, which is reflected by the positive slope of the policy frontier.

In the case without covenant (right panel), increasing the collateral supply necessitates increasing \overline{F} , which leads to additional default cost compared to the market discipline case. With covenant (left panel), the positive relation between collateral supply and default risk persists, but the overall level of default cost is significantly lower and even falls compared to the market discipline case. At the same time, the associated collateral supply levels



FIGURE 3.6: Collateral Policy Frontier

Notes: Both panels display the additional default cost (relative to market discipline) that are necessary to satisfy a given collateral supply target. The collateral supply target is expressed as the share of eligible bonds \overline{B} relative to the market value of all bonds under market discipline. In the left panel, we vary the eligibility threshold \overline{F} and the covenant parameter γ . In the right panel, the covenant parameter is fixed at $\gamma = 0$.

are substantially higher, since the covenant incentivizes firms to issue more eligible *and* default less frequently. Consequently, the collateral policy frontier is shifted outward, making this instrument a potentially powerful extension of collateral frameworks.

Implementation. In our model, the eligibility covenant is expressed in terms of firmspecific eligible debt capacities, which are negatively dependent on debt outstanding $\partial \tilde{b}_{t+1}^j / \partial b_t^j < 0$. In practice, implementing such covenants would require information on the indebtedness of firms, i.e., about current revenues and its dynamics, as well as the maturity structure of outstanding liabilities. However, revenue dynamics and debt repayment schedules of large firms are often difficult to determine, particularly if firms have multiple subsidiaries. Therefore, several collateral frameworks (the ECB's among them) are based on credit assessments by external rating agencies.

While we abstract from modeling credit ratings and assume that revenues and debt outstanding at the firm level is common knowledge, it is of practical importance that eligibility covenants in our model can be expressed in terms of firm-specific eligibility thresholds, which negatively dependent on debt outstanding $\partial \overline{F}_t^j / \partial b_t^j < 0$. For sufficiently large firms, the eligibility status could be made dependent on a measure of debt outstanding and CDS-spreads. This would allow for a more granular classification of firms into different eligibility categories based on spreads and debt outstanding.¹⁴ An alterna-

 $^{^{14}}$ A discussion regarding the usage of market-based credit risk assessments, such as CDS-spreads, is given by

tive way to implement covenants is to condition eligibility on rating notches combined with debt outstanding or on the *rating outlook*, if these take into account the sustainability of firm debt in a satisfactory way. Firms rated A, but with negative outlook, can, for example, be interpreted as being on a financially unsustainable path and could, therefore, be subjected to a tighter eligibility requirement than a firm rated BB+ but with a positive outlook. This would be especially useful for firms on which no CDS are actively traded.

Last, note that collateral frameworks not only comprise eligibility thresholds but also haircuts on eligible assets. From the firm's point of view, a higher haircut reduces the collateral value of its bonds and could, in principle, be made dependent on debt outstanding, which would also reduce risk-taking incentives. However, covenants provide more salient deleveraging incentives, if a firm would observe its eligible debt capacity through the investment bank handling the underwriting process of new bond issues. In contrast, a haircut would still leave the bond eligible. From the central bank's point of view, haircuts are often set to account for losses in asset liquidation in the event of counterparty default. Haircuts typically address a different form of risk so that the covenant remains a potentially useful extension of collateral frameworks.

3.5 Conclusion

This paper evaluates the effects of central bank eligibility requirements on the debt and default decision of firms, i.e., the collateral supply side. Adding collateral premia and eligibility requirements to a heterogeneous firm model with default risk reveals that firms can be affected in different ways: low-risk firms increase their debt issuance and risk-taking, whereas medium-risk firms are disciplined by the prospect of benefiting from collateral premia. Both effects increase aggregate collateral supply, while they have opposing effects on cost from corporate default. Which of these two effects is the dominating force is, therefore, a numerical question. Consistent with empirical evidence at the firm level, our numerical findings suggest that risk-taking is the dominating force in the aggregate. Endogenous firm responses are quantitatively relevant and substantially dampen the impact of collateral easing on collateral supply. Eligibility covenants are suitable instruments to alleviate adverse risk-taking effects on collateral supply and aggregate default cost.

Our work can be extended along multiple dimensions. Interacting endogenous collateral supply with frictions on the collateral demand side, such as aggregate liquidity risk, can potentially generate interesting interactions with implications for the conduct of collateral policy. It should also be stressed that we take investment opportunities as exogenous. A model with endogenous investment allows to study real effects of eligibility

Nyborg (2017).

requirements using a richer trade-off between distributing cashflows as dividends and investment. We also do not account for bank loans as alternative source of financing, which is also a margin affected by eligibility requirements. All extensions add additional layers of complexity to our present framework and we leave them to future research.

Appendix

B.1 Corporate Bond Eligibility in Collateral Frameworks

This section reviews the eligibility of corporate bonds in central bank operations. As we show in Table B.1, eligibility of corporate bonds as collateral in central bank operations varies across countries and over time. The Eurosystem stands out due to its acceptance of corporate collateral before the financial crisis.

Country	Pre 2008 (Min. Rating)	Post 2008 (Min. Rating)	Post Covid-19 (Min. Rating)
Australia	No	Yes (AAA)	Yes (BBB)
Eurosystem	Yes (A)	Yes (BBB)	Yes $(BB)^*$
Japan	Yes (A)	Yes $(BBB)^{\dagger}$	Yes (BBB)
Switzerland	Yes (AA)	Yes (AA)	Yes (AA)
United Kingdom	No	Yes (A)	Yes (A)
United States ^{††}	Yes (AAA)	Yes (AAA)	Yes (BBB)

TABLE B.1: Non-Financial Corporate Bonds in Various Collateral Frameworks

Notes: [†]: Multiple changes after Financial Crisis; ^{*}: For the duration of PEPP; ^{††}: Only allowed in the discount window. Source: BIS (2013) & national CBs.

Table B.2 gives an overview of changes in the ECB collateral framework since 2007. Corporate bonds were eligible prior to the 2008 crises at a minimum rating requirement of A. In response to the financial crises, the minimum requirements were reduced from A to BBB, which substantially extended the amount of eligible assets and, thereby, broadened financial intermediaries' access to central bank liquidity. The smaller changes in 2011 and 2013 suggest that some fine-tuning was necessary after the initial relaxation. Nevertheless, the reduction of the minimum rating requirement was by far the largest adjustment, which motivates our choice of modeling collateral policy as a step function.

Timespan	Regime	A- or higher	BBB- to BBB+
01 Jan 2007 - 24 Oct 2008	Fitch, S&P, and Moody's are accepted ECAI, minimum		100 07
25 Oct 2008 - 31 Dec 2010	requirement A DBRS accepted as ECAI,	4.5 %	100 %
	minimum requirement BBB	$4.5 \ \%$	9.5~%
01 Jan 2011 - 30 Sep 2013	Tightening of haircuts.	5~%	25.5~%
01 Oct 2013 - 01 Dec 2019	Relaxation of haircuts.	3~%	22.5~%

TABLE B.2: Corporate Bonds in the ECB Collateral Framework

Notes: Haircuts on a corporate bond with fixed coupon and maturity of 3 to 5 years; DBRS: Dominion Bond Rating Service, ECAI: external credit assessment institutions.

B.2 Proofs

This section contains the proofs of Propositions 1 and 2 and Lemma 1 and 2.

B.2.1 Proof of Proposition 1

To see why the most productive firms have the lowest default risk, we differentiate default risk with respect to s and obtain $\frac{\partial F(b_{t+1}^n|s)}{\partial s} = \frac{\partial F(b_{t+1}^n-s)}{\partial s} = f(b_{t+1}^n|s)[-1+\frac{\partial b_{t+1}^n}{\partial s}]$. We can use (B.6), which we derive below, to show that this term is unambiguously negative. Using analogous steps and (B.7), we show the same for $F(b_{t+1}^e|s)$.

We rewrite the first order conditions (3.3) and (3.4) in terms of the hazard rate as

$$h(b_{t+1}|s) \cdot b_{t+1} = 1 - \beta$$
 if $F(b_{t+1}|s) > \overline{F}$, (B.1)

$$h(b_{t+1}|s) \cdot b_{t+1} = \frac{1 - \beta + L}{1 + L}$$
 if $F(b_{t+1}|s) \le \overline{F}$. (B.2)

The first order condition (B.1) implies that the (debt-weighted) marginal default risk $h(b_{t+1}^n|s) \cdot b_{t+1}^n$ is identical for all s. The productivity parameter shifts the revenue distribution to the right: holding debt issuance constant, we have $\frac{\partial F(b_{t+1}^n|s)}{\partial s} < 0$ and also $\frac{\partial f(b_{t+1}^n|s)}{\partial s} < 0$ by the monotone hazard rate property so that $h(b_{t+1}^n|s)$ falls. Since the RHS of (3.3) is constant, increasing the productivity parameter implies that the debt choice has to increase $\frac{\partial b_{t+1}^n}{\partial s} > 0$. We perform analogous steps to show the same for b_{t+1}^e . Finally, since $\frac{1-\beta+L}{1+L} > 1-\beta$ and using the monotonicity assumption on $h(\mu_{t+1})\mu_{t+1}$,

Finally, since $\frac{1-\beta+L}{1+L} > 1-\beta$ and using the monotonicity assumption on $h(\mu_{t+1})\mu_{t+1}$, we have that an eligible firm issues more debt than an otherwise identical non-eligible firm.

B.2.2 Proof of Proposition 2

The partitioning of firms into different groups (unconstrained eligible, constrained eligible, and non-eligible) uses the fact that there are three *potentially* optimal debt choices for

every s. The first possibility is to issue bonds $\tilde{b}_{t+1}(s)$ to be exactly at the eligibility threshold. By the strict monotonicity of $F(b_{t+1}|s)$ in b_{t+1} , there is a unique $\tilde{b}_{t+1}(s) \equiv F^{-1}(\overline{F}|s)$ for which the corporate bond is just eligible. Second, there is a debt level b_{t+1}^n satisfying the first order condition (B.1) for the case of non-eligibility. Third, the level b_{t+1}^e solves (B.2), the first order condition in the eligibility case. Under the monotonicity assumption on $h(b_{t+1}) \cdot b_{t+1}$, both conditions are satisfied by a unique b_{t+1}^n and b_{t+1}^e , respectively. The remainder of the proof characterizes which of these three debt levels is optimal, given the type parameter s.

Existence of Type Space Partitions. There is a positive mass of unproductive firms, such that $\tilde{b}_{t+1}(s) = 0 < b_{t+1}^n(s) < b_{t+1}^e(s)$, which holds at least for $s = s^-$ by assumption. These firms are not able to issue any bonds without exceeding the minimum quality requirement \overline{F} , i.e., their eligible debt capacity is zero. On the other hand, there are firms with positive eligible debt capacity. This can be shown by finding values s_1 and s_2 such that $b_{t+1}^n(s_1) = \tilde{b}_{t+1}(s_1)$ and $b_{t+1}^e(s_2) = \tilde{b}_{t+1}(s_2)$, i.e., firms are able to issue debt according to (B.1) and (B.2) without losing eligibility. We then show that the cut-off values satisfy $s^- < s_1 < s_2 < \infty$.

From the mass-shifting property of s, we can express the eligible debt capacity as

$$\widetilde{b}_{t+1}(s) = F^{-1}(\overline{F}) + s . \tag{B.3}$$

Define the hypothetical value functions for a never eligible firm $V^n(b_{t+1}|s)$ and an always eligible firm as $V^e(b_{t+1}|s)$. Plugging (B.3) into the first order conditions (3.3) and (3.4), we get

$$\frac{\partial V^n(s)}{\partial b}\Big|_{\widetilde{b}_{t+1}(s)} = (1-\beta)(1-\overline{F}) - \left(F^{-1}(\overline{F}) + s\right)f(F^{-1}(\overline{F})) , \qquad (B.4)$$

$$\frac{\partial V^e(s)}{\partial b}\Big|_{\widetilde{b}_{t+1}(s)} = \frac{1-\beta+L}{1+L}(1-\overline{F}) - \left(F^{-1}(\overline{F})+s\right)f(F^{-1}(\overline{F})) . \tag{B.5}$$

For a sufficiently productive firm with a large s, the eligible debt capacity $\tilde{b}_{t+1}(s)$ lies on the downward sloping part of the objective function. Since the objective is concave by the monotone hazard rate assumption, $\tilde{b}_{t+1}(s)$ is not optimal and such a firm voluntarily issues less debt than it could without losing eligibility, i.e., s_2 exists. From $1 - \beta < \frac{1-\beta+L}{1+L}$ and noting that for s_1 and s_2 (B.4) and (B.5) evaluate to zero, it follows that $s_1 < s_2$.

We can exploit the monotonicity of the first order conditions in s and monotonicity of the eligible debt capacity $\frac{\partial \tilde{b}_{t+1}(s)}{\partial s} = 1$. Implicitly differentiating (B.1) and (B.2) with respect to s, we have

$$\frac{\partial b_{t+1}^n(s)}{\partial s} = \frac{\left(1 - F(b_{t+1}^n|s)\right) f'(b_{t+1}^n|s) b_{t+1}^n + f(b_{t+1}^n|s)^2 b_{t+1}^n}{\left(1 - F(b_{t+1}^n|s)\right) \left[f'(b_{t+1}^n|s) b_{t+1}^n + f(b_{t+1}^n|s)\right] + f(b_{t+1}^n|s)^2 b_{t+1}^n} < 1 , \qquad (B.6)$$

$$\frac{\partial b_{t+1}^e(s)}{\partial s} = \frac{\left(1 - F(b_{t+1}^e|s)\right) f'(b_{t+1}^e|s)b_{t+1}^e + f(b_{t+1}^e|s)^2 b_{t+1}^e}{\left(1 - F(b_{t+1}^e|s)\right) \left[f'(b_{t+1}^e|s)b_{t+1}^e + f(b_{t+1}^e|s)\right] + f(b_{t+1}^e|s)^2 b_{t+1}^e} < 1.$$
(B.7)

Since firms are risky by the first order conditions (3.3) and (3.4), we have $f(b_{t+1}^n|s) > 0$ and $f(b_{t+1}^e|s) > 0$ and the denominator is larger than the numerator, respectively. Therefore, the partial derivatives $\frac{\partial b_{t+1}^n(s)}{\partial s}$ and $\frac{\partial b_{t+1}^e(s)}{\partial s}$ are strictly smaller than one. Since by assumption $\tilde{b}_{t+1}(s^-) = 0$ and $b_{t+1}^n(s^-) > 0$, $s_1 > s^-$ follows from the implicit definition $b_{t+1}^n(s_1) = \tilde{b}_{t+1}(s_1)$. Furthermore, we can conclude that the cut-off values s_1 and s_2 are unique.

Characterizing Debt Choices. For every $s > s_2$, firms issue less debt than they could issue without losing eligibility. All firms with $s > s_2$ choose debt issuance according to their first order condition and are called *unconstrained eligible*.

Consider next firms which cannot choose their optimal borrowing without losing eligibility, i.e., firms with $s < s_2$. All firms between s_1 and s_2 choose to be just eligible and lever up until $\tilde{b}_{t+1}(s)$, since for them $V^e(b^e_{t+1}(s)|s)$ is not feasible and $V^n(b^n_{t+1}(s)) <$ $V^e(b^n_{t+1}(s)) < V^e(\tilde{b}_{t+1}(s))$. The first inequality follows from $V^e(b_{t+1}|s) > V^n(b_{t+1}|s)$ for all b_{t+1} , holding s constant. The second inequality follows from the fact that V^e is increasing between $b^n_{t+1}(s)$ and $\tilde{b}_{t+1}(s)$.

Finally, there is a threshold $s_0 < s_1$, below which firms choose $b_{t+1}^n(s)$ and are not eligible. All firms between s_0 and s_1 also choose $\tilde{b}_{t+1}(s)$. The value s_0 is implicitly defined through the indifference condition $V^e(\tilde{b}_{t+1}|s_0) = V^n(b_{t+1}^n|s_0)$. The assumptions on the revenue distribution will imply the existence of exactly one s_0 by the intermediate value theorem. To see this, consider their difference

$$\Delta(s) \equiv V^{e}(\tilde{b}_{t+1}(s)|s) - V^{n}(b_{t+1}^{n}(s)|s) .$$
(B.8)

Obviously $\Delta(s_1) > 0$, because $b_{t+1}^n(s_1) = \tilde{b}_{t+1}(s_1)$ and $V^e(\tilde{b}_{t+1}(s_1)|s_1) > V^n(\tilde{b}_{t+1}(s_1)|s_1)$. In addition, there exists a level s^- where $F(0|s^-) > \overline{F}$ by assumption. At this level $V^e(\tilde{b}_{t+1}(s^-)|s^-) - V^n(b_{t+1}^n(s^-)|s^-) < 0$. Note that $\tilde{b}_{t+1}(s^-) = 0$ and that $V^e(\tilde{b}_{t+1}(s^-)|s^-)$ is the value of the unlevered firm. Choosing $b_{t+1} = 0$ would violate (B.1) and therefore $V^n(b_{t+1}^n(s)|s)$ exceeds the value of an unlevered firm for every s. Together with continuity of s, this already implies existence of at least one s_0 by the intermediate value theorem. To establish uniqueness, we differentiate $\Delta(s)$ with respect to s. The first part of $\Delta(s)$ can be written as

$$V^{e}(\tilde{b}_{t+1}(s)|s) = (1 - \overline{F})(1 + L)\tilde{b}_{t+1} + \beta \int_{\tilde{b}_{t+1}(s)}^{\overline{\mu}} \mu_{t+1} - \tilde{b}_{t+1}(s)dF(\mu_{t+1}|s),$$

and its total derivative is given by

$$\frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial s} = \frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial \widetilde{b}_{t+1}} \frac{\partial \widetilde{b}_{t+1}(s)}{\partial s} + \frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial s} \bigg|_{\widetilde{b}_{t+1}}$$

$$= \left((1 - \overline{F})(1 + L) + \beta \int_{\tilde{b}_{t+1}(s)}^{\overline{\mu}} (-1)dF(\mu_{t+1}|s) \right) \frac{\partial \tilde{b}_{t+1}}{\partial s} + \beta \int_{\tilde{b}_{t+1}(s)}^{\overline{\mu}} -(\mu_{t+1} - \tilde{b}_{t+1}(s))df(\mu_{t+1}|s) = (1 - \overline{F})(1 + L) - \beta(1 - F(\tilde{b}_{t+1}(s)|s)) - \beta \int_{\tilde{b}_{t+1}(s)}^{\overline{\mu}} \mu_{t+1}df(\mu_{t+1}|s) + \beta \int_{\tilde{b}_{t+1}(s)}^{\overline{\mu}} \tilde{b}_{t+1}(s)df(\mu_{t+1}|s) = (1 - \overline{F})(1 + L) - \beta(1 - F(\tilde{b}_{t+1}(s)|s)) - \beta \left(f(\overline{\mu})\overline{\mu} - f(\tilde{b}_{t+1}(s)|s)\tilde{b}_{t+1}(s) - (1 - F(\tilde{b}_{t+1}(s)|s))\right) + \beta \tilde{b}_{t+1}(s) \left(f(\overline{\mu}) - f(\tilde{b}_{t+1}(s)|s)\right) = (1 - \overline{F})(1 + L) .$$
(B.9)

We used again that $\frac{\partial \tilde{b}_{t+1}}{\partial s} = 1$ and $f(\overline{\mu}) = 0$.

The second part of $\Delta(s)$ is given by

$$V^{n}(b_{t+1}^{n}(s)|s) = (1 - F(b_{t+1}^{n}(s)|s)b_{t+1}^{n}(s) + \beta \int_{b_{t+1}^{n}(s)}^{\overline{\mu}} \mu_{t+1} - b_{t+1}^{n}(s)dF(\mu_{t+1}|s).$$

The derivative of the second part of (B.8) is given by $\frac{\partial V^n(b_{t+1}(s),s)}{\partial s}\Big|_{b_{t+1}^n}$, since $\frac{\partial V^n(b_{t+1}(s),s)}{\partial b_{t+1}} = 0$ by the principle of optimality, when totally differentiating $V^n(b_{t+1}(s)|s)$ with respect to s. Specifically,

$$\frac{\partial V^n(b_{t+1}^n(s)|s)}{\partial s} = f(b_{t+1}^n(s)|s) \cdot b_{t+1}^n(s) + \beta \int_{b_{t+1}^n(s)}^{\overline{\mu}} -(\mu_{t+1} - b_{t+1}^n(s))df(\mu_{t+1}|s)$$
$$= (1 - \beta) \left(1 - F(b_{t+1}^n(s)|s)\right) + \beta \left(1 - F(b_{t+1}^n(s)|s)\right)$$
$$= 1 - F(b_{t+1}^n(s)|s) . \tag{B.10}$$

In the second line, we directly used the first order condition (B.1). Putting both parts together

$$\frac{\partial\Delta(s)}{\partial s} = \frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial s} - \frac{\partial V^n(b_{t+1}^n(s)|s)}{\partial s} = (1 - \overline{F})(1 + L) - \left(1 - F(b_{t+1}^n(s)|s)\right) > 0$$

The sign follows from the fact that $\tilde{b}_{t+1}(s) < b_{t+1}^n(s)$ holds in the region of interest. This implies that the default probability at $b_{t+1}^n(s)$ exceeds the eligibility threshold, i.e., $F(b_{t+1}^n(s)|s) > \overline{F}$. The inequality follows from $1 - F(b_{t+1}^n(s)|s) < 1 - \overline{F}$ and L > 0. Since $\Delta(s)$ is continuous and monotonically increasing, there exists an unique s_0 where the firm is indifferent between constrained eligibility and non-eligibility by the intermediate value theorem. All firms between s_0 and s_2 are called *constrained eligible*, firms below s_0 are *non-eligible*.

B.2.3 Proof of Lemma 1

Lemma 1 can be shown by noting that collateral easing increases the eligible debt capacity across firms and that $b_{t+1}^n(s)$ and $b_{t+1}^e(s)$ are independent of the eligibility thresholds. To see that $\frac{\partial s_0}{\partial \overline{F}} < 0$, consider the indifference condition (B.8). The value of being constrained eligible $V^e(\tilde{b}_{t+1}(s)|s)$ increases in \overline{F} . Differentiating the eligible debt capacity $\tilde{b}_{t+1}(s)$ with respect to the eligibility threshold yields

$$\frac{\partial \tilde{b}_{t+1}(s)}{\partial \overline{F}} = \frac{\partial F^{-1}\left(\overline{F}|s\right)}{\partial \overline{F}} = \frac{1}{f\left(F^{-1}(\overline{F}|s)\right)} > 0 , \qquad (B.11)$$

where the last step follows from the inverse function theorem. A constrained eligible firm will be better off after a relaxation of eligibility requirements $V^e(\tilde{b}_{t+1}^A(s_0^A)|s_0^A) < V^e(\tilde{b}_{t+1}^{BBB}(s_0^{BBB})|s_0^A)$. Note also that the value of being non-eligible $V^n(b_{t+1}^n(s)|s)$ does not depend on the eligibility threshold. Taken together, we have

$$V^{e}(\tilde{b}_{t+1}^{BBB}(s_{0}^{BBB})|s_{0}^{A}) > V^{n}(b_{t+1}^{n}(s_{0}^{A})|s_{0}^{A}) = V^{n}(b_{t+1}^{n}(s_{0}^{BBB})|s_{0}^{A}) .$$

Furthermore, for a given policy s_0 has to satisfy $V^e(\tilde{b}_{t+1}(s)|s) = V^n(b_{t+1}^n(s)|s)$. We showed in (B.9) that the value of a constrained eligible firm is increasing in the shifting parameter. Thus, the indifference point s_0^{BBB} shifts to the left: $s_0^{BBB} < s_0^A$.

To see the effect of eligibility thresholds on s_2 , it suffices to note that $V^e(b^e_{t+1}|s)$ is independent of \overline{F} and restrict attention to the condition pinning down the eligible debt capacity $F(\tilde{b}_{t+1} - s) = \overline{F}$. Rearranging for s and differentiating w.r.t. \overline{F} yields $\frac{\partial s_2}{\partial \overline{F}} = -\frac{1}{f(F^{-1}(\overline{F}))} < 0.$

B.2.4 Proof of Lemma 2

Endogenous firm responses are residually given by subtracting the mechanical effect (3.6) from the total effect (3.5)

$$\begin{split} \overline{B}^{BBB} - \overline{B}^{A} \Big|_{endo} &= (1+L) \left(\underbrace{\int_{s_{0}^{BBB}}^{s_{0}^{BBB}} \left(1 - F\left(\tilde{b}_{t+1}^{BBB}(s)\right) \right) \tilde{b}_{t+1}^{BBB}(s) dG(s)}_{B.12.1} \\ &+ \underbrace{\int_{s_{0}^{BBB}}^{\infty} \left(1 - F\left(b_{t+1}^{e}(s)\right) \right) b_{t+1}^{e}(s) dG(s)}_{B.12.2} \\ &- \underbrace{\int_{s_{0}^{A}}^{s_{0}^{A}} \left(1 - F\left(\tilde{b}_{t+1}^{A}(s)\right) \right) \tilde{b}_{t+1}^{A}(s) dG(s)}_{B.12.3} \\ &- \underbrace{\int_{s_{2}^{A}}^{\infty} \left(1 - F\left(b_{t+1}^{e}(s)\right) \right) b_{t+1}^{e}(s) dG(s)}_{B.12.4} \end{split}$$

$$-\underbrace{\int_{s_{1}^{BBB}}^{s_{0}^{A}} \left(1 - F\left(b_{t+1}^{n}(s)\right)\right) b_{t+1}^{n}(s) dG(s)}_{B.12.5}\right).$$
 (B.12)

Since $s_2^{BBB} < s_2^A$ from Lemma 1, the terms B.12.2 and B.12.4 reduce to

$$\int_{s_2^{BBB}}^{s_2^A} \left(1 - F\left(b_{t+1}^e(s)\right)\right) b_{t+1}^e(s) dG(s) .$$
(B.13)

Due to the assumption $s_0^A < s_2^{BBB}$ we can split B.12.3 into two sub-integrals, ranging from $[s_0^A, s_2^{BBB}]$ and $[s_2^{BBB}, s_2^A]$, respectively. The second sub-integral can be combined with (B.13) and yields the last line of (3.7). The first sub-integral ranging from $[s_0^A, s_2^{BBB}]$ is used in the next step.

Note that the ordering of threshold productivity values arising from our assumptions and Lemma 1 is $s_0^{BBB} < s_1^{BBB} < s_0^A < s_2^{BBB}$. As a result, we can split B.12.1 into three sub-integrals ranging from $[s_0^{BBB}, s_1^{BBB}]$, $[s_1^{BBB}, s_0^A]$, and $[s_0^A, s_2^{BBB}]$, respectively. The third of these sub-integrals and the remaining sub-integral from B.12.3 are combined to line three in (3.7). Moreover, we combine the second sub-integral of B.12.1 with B.12.5 to obtain the second line in (3.7). Finally, the first sub-integral of B.12.1 corresponds to the first line in (3.7).

The aggregate default cost can be decomposed in a similar way. Notably, it contains *all* bonds and not only eligible ones:

$$\mathcal{M}^{BBB} - \mathcal{M}^{A} = \underbrace{\int_{s^{-}}^{s_{0}^{BBB}} M(b_{t+1}^{n}(s)) dG(s)}_{B.14.1} + \underbrace{\int_{s_{0}^{BBB}}^{s_{0}^{BBB}} M(\tilde{b}_{t+1}^{BBB}(s)) dG(s)}_{B.14.2} + \underbrace{\int_{s_{0}^{2}^{BBB}}^{\infty} M(b_{t+1}^{e}(s)) dG(s)}_{B.14.3} - \underbrace{\int_{s^{-}}^{s_{0}^{A}} M(b_{t+1}^{n}(s)) dG(s)}_{B.14.4} - \underbrace{\int_{s_{0}^{A}}^{s_{0}^{A}} M(\tilde{b}_{t+1}^{A}(s)) dG(s)}_{B.14.5} - \underbrace{\int_{s_{0}^{A}}^{\infty} M(b_{t+1}^{e}(s)) dG(s)}_{B.14.6}$$
(B.14)

Again, since $s_2^{BBB} < s_2^A$ from Lemma 1, the terms B.14.3 and B.14.6 reduce to

$$\int_{s_2^{BBB}}^{s_2^A} M(b_{t+1}^e(s)) dG(s) .$$
(B.15)

Splitting B.14.5 into two sub-integrals, ranging from $[s_0^A, s_2^{BBB}]$ and $[s_2^{BBB}, s_2^A]$, we can combine the second of these with (B.15) to obtain the last line of (3.8).

Given the ordering of threshold productivity values arising from our assumptions and Lemma 1, we can split B.14.2 into three sub-integrals ranging from $[s_0^{BBB}, s_1^{BBB}]$, $[s_1^{BBB}, s_0^A]$, and $[s_0^A, s_2^{BBB}]$, respectively. Combining the last of these with the remaining sub-integral of B.14.5 yields the third line of (3.8).

Since $s_0^{BBB} < s_0^A$, we can summarize B.14.1 and B.14.4 to

$$-\int_{s_0^{BBB}}^{s_0^A} M\Big(b_{t+1}^n(s)\Big) dG(s) = -\int_{s_0^{BBB}}^{s_1^{BBB}} M\Big(b_{t+1}^n(s)\Big) dG(s) - \int_{s_1^{BBB}}^{s_0^A} M\Big(b_{t+1}^n(s)\Big) dG(s) - \int_{s_1^{BBB}}^{s_0^A} M\Big(b_{t+1}^n(s)\Big) dG(s) - \int_{s_0^{BBB}}^{s_0^A} M\Big(b_{t+1}^n(s)\Big) dG(s) - \int_{s_0^A}^{s_0^A} M\Big(b_{t+1}^n(s)\Big) dG(s) - \int_{s_0^A}^{s_0^A} M\Big(b_{t+$$

Combining these two integrals with the remaining two sub-integrals of B.14.2 yields the first and second lines of (3.8).

B.3 Data and Computation

B.3.1 Corporate Bond Data

We merge monthly data on the corporate bond universe in Europe from the iBoxx High Yield and Investment Grade Index families, provided by *IHS Markit*. We apply the following inclusion criteria:

- 1. Bond issuers are head-quartered in euro area member countries.
- 2. Issuers are non-financial firms.
- 3. The bond is denominated in euro, senior, not callable, uncollateralized, and fixed coupon.
- 4. The issuer is part of the constituent list for at least 48 months.

Bond issuers are provided by *Markit* and we consider only the parent company level, since it can be reasonably assumed that dedicated financial management subsidiaries are identical from an economic perspective to the respective parent company.

Company Data. We match company names to their unique *Compustat* identifier (gvkey) and drop all companies which are not represented in the *Compustat Global* database. For the remaining firms we query *Compustat* for long-term liabilities (dltt) in the firmq database and EBIT (ebit) in the firma database.

B.3.2 Computational Algorithm

We solve the individual firm problem using policy function iteration over a discrete set of collocation points using piecewise linear interpolation. The revenue shock is discretized using the method of Tauchen on an equi-spaced grid with $n_{\mu} = 25$ points over the interval $[-3\hat{\sigma}, +3\hat{\sigma}]$ with $\hat{\sigma} = \frac{\sigma}{1-\rho^2}$ denoting the unconditional variance of the revenue process. We denote the corresponding transition matrix Π_{μ} . Debt is discretized on an equispaced grid with $n_b = 21$ points over the interval [5.5, 15.5].

To overcome the typical convergence issues in models with long-term debt and default, we use taste shocks when computing the debt choice (3.16), as proposed by Gordon (2019). The mass shifter for endogenous states follows immediately from the debt choice and is denoted Π_b . This matrix maps the current idiosyncratic state (μ_t^j, b_t^j) , into next period's endogenous state b_{t+1}^j , i.e., has dimension $n_{\mu} \cdot n_b \times n_{\mu} \cdot n_b$.

Together with the transition matrix of idiosyncratic revenues, the combined mass shifter is given by $\Pi_g = \Pi_b \otimes \Pi_\mu$. The mass shifter implicitly defines the firm distribution G via $G^T = G^T \Pi_g$, where G denotes the firm distribution. Extracting the distribution, thus, boils down to computing the right eigenvector to Π_g .

Starting with a guess for firm policies and bond prices, each iteration ι consists of four different steps:

- 1. Solve the firm problem taken as given the bond price schedule and value function from the previous iteration.
- 2. Compute the eligible debt capacity (3.15), the associated values of the objective function, and determine the debt choice according to (3.16).
- 3. Obtain the ensuing mass shifter Π_g from the policy functions and the transition matrix for revenue shock Π_{μ} and update the distribution G by iterating on $G^T = G^T \Pi_q$.
- 4. Update bond price schedules and value functions.

We then iterate on the policy functions until convergence, $||\mathcal{B}^{\iota}(b_t^j, \mu_t^j) - \mathcal{B}^{\iota-1}(b_t^j, \mu_t^j)||_{\infty} < 10^{-5}$. The standard deviation of the taste shock is set to 0.01 to ensure convergence. This is typically achieved within 200 iterations.

B.4 Additional Numerical Results

This section contains supplementary numerical results to our quantitative policy analysis. In Appendix B.4.1, we compare optimal debt choices under tight and lenient collateral policy. Appendix B.4.2 provides details on the distribution of bond spreads and default risk across firms. Appendix B.4.3 presents a robustness check that also includes data from the financial crisis of 2008 and consequently has a higher level of default risk. Appendix B.4.4 endogenizes the size of collateral premia.

B.4.1 Firm Debt Choices

We now illustrate how the characterization of firm debt choices carries over to the case of long-term debt. The black solid line in each panel of Figure B.1 denotes the debt choice given current debt b_t for a firm with median revenues under tight (left) or lenient



FIGURE B.1: Debt Choice with Collateral Easing

Notes: The bold black line represents the debt choice of a firm with median revenue conditional on legacy debt (see (3.16)). The colored lines denote the hypothetical debt choice of an always (non-)eligible firm. The light black line is the eligible debt capacity. In the left (right) panel we depict the case of tight (lenient) collateral policy.

(right) eligibility requirements. The colored dashed and dotted lines in either panel denote the debt choice if a firm is non-eligible or eligible, respectively. The firms' eligible debt capacity (3.15), which is independent of legacy debt b_t , is given by the horizontal black line. The debt choice exhibits a kink and a jump that represent the debt levels where firms change type (from non-eligible to constrained eligible and, eventually, to unconstrained eligible). The optimal debt choice (bold black line) is equal to b_{t+1}^e until it reaches its eligible debt capacity (first kink). For legacy debt levels between the kink and the jump, the firm exhausts its eligible debt capacity and is constrained eligible. Last, for debt outstanding above those at the jump, firms choose b_{t+1}^n . Similar to the one-period bond model, the effects of bond eligibility correspond to the difference between the non-eligible debt choice b_{t+1}^n and the equilibrium debt choice \mathcal{B}_{t+1} (bold black line). Firms subject to risk-taking choose debt above b_{t+1}^n . Disciplined firms choose debt lower than b_{t+1}^n instead.

Comparing the left to the right panel, we observe that under lenient eligibility requirements, where the eligible debt capacity shifts upwards, the risk-taking effect becomes more prominent, while the relative size of the disciplining effect falls.

B.4.2 Firm Distribution

While Section 3.3.5 condenses firm responses into the shares of risk-taking and disciplined firms, this section provides supplementary information on the firm distribution. Specifically, we compare the bond spread and default risk distributions of eligible firms for the



Notes: We show bond spread (left) and default risk (right) distributions across firms. Blue bars denote the economy with tight collateral policy, purple bars an economy with lenient collateral policy but fixed firm responses, and the orange bars an economy with lenient collateral policy.

baseline calibration (blue) to those under lenient eligibility requirements (orange), and to those under lenient eligibility requirements with constant firm behavior (purple). Differentiating between the full equilibrium response and the share of eligible bonds with constant firm behavior allows us to decompose the total collateral supply response into *mechanical* and firm effects.

The left panel of Figure B.2 divides eligible firms into different spread buckets. For the mechanical effect, we observe a rightward shift of bond spreads compared to the baseline of tight eligibility requirements, corresponding to newly eligible risky firms. Accordingly, in the right panel we observe a similar rightward shift of the distribution of eligible firms' default probabilities. Taking firms responses into account markedly increases the share of firms in the left tail of the spread distribution. This follows from the high likelihood of satisfying the minimum rating requirement in future periods, which is associated with low bond spreads. However, the default probability distribution in the right panel reveals that firm responses raise the mass of eligible firms in the higher risk buckets, reflecting risk-taking effects.

B.4.3 Extended Sample Period

As a robustness check, we re-calibrate the model and target the higher spread level over a sample encompassing the financial crisis of 2008. To match the elevated level of spreads, we set $\pi = 0.058$ and $\rho = 0.94$ to match the higher debt/EBIT-ratio as well as the

Moment	Data	Model
Collateral premium $r - r^n$	-11	-11
Debt/EBIT $Q_{0.50}(b/\mu \overline{F}^A)$	4.1	3.06
Bond spread $Q_{0.25}(x \overline{F}^A)$	45	58
Bond spread $Q_{0.50}(x \overline{F}^A)$	72	92
Bond spread $Q_{0.75}(x \overline{F}^A)$	115	118
Eligible bond share $\overline{B}/(QB) \overline{F}^A$	50%	50%
Eligible bond share $\overline{B}/(QB) \overline{F}^{BBB}$	86%	77%

TABLE B.3: Targeted Moments – Extended Sample

Notes: Collateral premium and spreads are annualized and expressed in basis points.

increased level and cross-sectional dispersion of spreads. We calibrate $\overline{F}^A = 1.7\%$ and $\overline{F}^{BBB} = 18.5\%$ to recover the share of eligible bonds before and after relaxing eligibility requirements.

In Table B.4, we observe that firm responses dampen the impact of eligibility requirements to a similar extent as in the baseline calibration, but the mechanical and total effect are of smaller magnitude: since the firm distribution over default risk exhibits a larger dispersion, collateral easing increases \overline{B} in a less effective way. However, the shares of risk-taking and disciplining firms under either policy are similar to the baseline calibration, suggesting that our characterization of endogenous firm responses does not crucially depend on the aggregate level of default risk.

B.4.4 Endogenous Size of Collateral Premia

This section presents a robustness check of our results by endogenizing the *size* of collateral premia. While these have been fixed to a constant L in the baseline, we make them dependent on aggregate collateral supply. In this case, collateral premia decline after collateral easing, which reduces both risk-taking incentives for eligible firms and disciplining

	Total Effect	Mechanical Effect
Collateral Supply \overline{B} Default Costs \mathcal{M}	+58% +7%	+67%
Firm Responses	Disciplining	Risk-Taking
Tight (A) Lenient (BBB)	16%	52% 77%

TABLE B.4: Macroeconomic Effects of Collateral Easing – Extended Sample

Notes: Values in the upper panel refer to collateral easing from A to BBB and are denoted as percentage difference from the A-baseline. The lower panel displays the fraction of *all* firms that is subject to disciplining or risk-taking effects.

Moment	Data	Model
Collateral premium $r - r^n$	-11	-11
Debt/EBIT $Q_{0.50}(b/\mu \overline{F}^A)$	3.9	3.9
Bond spread $Q_{0.25}(x \overline{F}^A)$	24	25
Bond spread $Q_{0.50}(x \overline{F}^A)$	39	49
Bond spread $Q_{0.75}(x \overline{F}^A)$	62	72
Eligible bond share $\overline{B}/(QB) \overline{F}^A$	50%	50%
Eligible bond share $\overline{B}/(QB) \overline{F}^{BBB}$	86%	85%

TABLE B.5: Targeted Moments – Endogenous L

Notes: Collateral premium and spreads are annualized and expressed in basis points.

effects for firms slightly below the eligibility requirement. Whether and how this affects the macroeconomic effects of collateral easing can, therefore, only be assessed numerically. Assume that banks directly draw utility from holding collateral. For numerical and analytical tractability, we impose a CARA-functional form

$$\mathcal{L}(\overline{B}) = -\frac{l_0}{l_1} \exp\left\{-l_1\overline{B}\right\} . \tag{B.16}$$

The collateral premium in this case is given by $L = l_0 \exp\{-l_1\overline{B}\}$. While we calibrate l_0 to match the eligibility premium of -11 bp, the CARA-parameter l_1 governs the curvature of (B.16) and will be normalized to $l_1 = 1$. In Table B.5 we show the model fit corresponding to a parameter choice of $\beta = 0.994$, $\rho = 0.93$, $\sigma = 0.03$, and and $l_0 = 8.25$, while the (annualized) threshold default risk levels are given by $\overline{F}^A = 1.4\%$ and $\overline{F}^{BBB} = 18.5\%$.

Different to the baseline model with constant collateral premia, the large increase in collateral supply induces a drastic decline of the collateral premium to $L \approx 1$ bp in response to collateral easing, which decreases the *extent* of risk-taking in our model. Even though risk-taking effects still have a dampening effect on collateral supply (see Table B.6), this is smaller than in the baseline calibration (see Table 3.4). Furthermore, default costs experience a slight *decline*: firms take on more risk and are less likely to be eligible, but they default less often.

	Total Effect	Mechanical Effect
Collateral Supply \overline{B} Default Costs \mathcal{M}	+53% -2%	+66%
Firm Responses	Disciplining	Risk-Taking

TABLE B.6: Macroeconomic Effects of Collateral Easing – Endogenous I	TABLE B.6:	Macroeconomic	Effects of	Collateral	Easing –	Endogenous L
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Notes: Values in the upper panel refer to collateral easing from A to BBB and are denoted as percentage difference from the A-baseline. The lower panel displays the fraction of *all* firms that is subject to disciplining or risk-taking effects.

Chapter 4

The Preferential Treatment of Green Bonds

This chapter is based on Giovanardi et al. (2022).¹

4.1 Introduction

The ECB (...) stands ready to support innovation in the area of sustainable finance (...), exemplified by its decision to accept sustainability-linked bonds as collateral.

Strategy Review (European Central Bank, 2021a)

The European Central Bank (ECB) announced to take a more active role in environmental policy after concluding its strategy review. In addition to accepting sustainabilitylinked (green) bonds as collateral, several central banks contemplate to go one step further and treat them preferentially within their collateral frameworks, i.e., the conditions under which banks can pledge assets to obtain funding from the central bank.² The People's Bank of China (PBoC) started accepting green bonds as collateral on preferential terms already in 2018, which resulted in a substantial decline of green bond yields relative to conventional ones (Macaire and Naef, 2022). However, there is limited knowledge about the macroeconomic impact of preferential collateral treatment on green bond issuance, green investment, pollution, and potential adverse side effects on financial markets.

To study the positive and normative implications of preferential treatment, this paper extends the standard real business-cycle (RBC) model by an environmental externality,

¹Elena Carletti, Christian Engels (discussant), Givi Melkadze, Alain Naef, Andreas Schabert, Eline ten Bosch (discussant), Stylianos Tsiaras (discussant), Christiaan v.d. Kwaak, and seminar participants at Bonn Macro Brown Bag, E-axes Environmental Economics Forum, German Council of Economic Experts, Bundesbank Research Centre, University of Konstanz as well as the 2021 International Conference on European Studies, SMYE, AEFIN, EEA, CFS, GRAFSI, CEFin Symposium, and the 4th Conference on Contemporary Issues in Banking (St Andrews) provided useful comments and suggestions. An earlier version of this paper was circulated under the title 'Directing Investment to Green Finance: How Much Can Central Banks Do?'. Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy - EXC 2126/1- 390838866 is gratefully acknowledged.

²A similar policy was also proposed in Brunnermeier and Landau (2020).

green and conventional firms issuing corporate bonds subject to default risk, and a banking sector using these bonds as collateral. The extent to which corporate bonds can be used as collateral depends on central bank haircuts. Reducing haircuts on green bonds makes holding such bonds more attractive to banks and implies that they pay higher collateral premia on them. This, in turn, relaxes financing conditions for green firms, which increase bond issuance, investment, and leverage in response: the equilibrium shares of green bonds and capital rise. We quantitatively assess these effects in a calibration to the euro area.

We uncover four main results. First, maximal preferential treatment, which applies a 100% haircut on conventional bonds and a 0% haircut on green bonds, increases the share of green bonds (capital) by almost 6% (3.7%), which reduces pollution. Second, in response to a preferential treatment green firms increase leverage, default risk, and dividend payouts. This risk-taking effect dampens the transmission of preferential treatment on green investment and increases resource losses from costly default. Because of these adverse effects, optimal collateral policy features a smaller degree of preferential treatment than the maximum case of only accepting green bonds. Third, Pigouvian taxes on pollution as alternative instrument do not induce risk-taking and the associated welfare gains exceed the gains from optimal collateral policy considerably. Fourth, preferential treatment is an *imperfect substitute* for Pigouvian taxes get to their optimal degree of preferential treatment decreases, the closer Pigouvian taxes get to their optimal level. When Pigouvian taxation is optimal, green and conventional bonds are treated symmetrically. In this case, however, the central bank optimally relaxes the collateral framework to address negative effects of environmental policy on collateral availability.

Our analysis is based on an extended RBC model that connects collateral policy to financial market and environmental frictions. We assume that there are two types of intermediate good firms, green and conventional. Conventional firms generate a negative externality (pollution) during the production of intermediate goods, while green firms have access to a clean technology. Following Heutel (2012) and Golosov et al. (2014), final good firms combine green and conventional intermediate goods with labor. Pollution has a negative effect of final good firms' output, implying sub-optimally low investment into the green technology.

Collateral policy is linked to the real sector by the corporate bond market, where both types of intermediate good firms issue bonds to banks. Firms have an incentive to issue bonds, because their owners are assumed to be more impatient than households, who own banks. Moreover, firms are subject to idiosyncratic shocks to their productivity and default on their bonds if revenues from production fall short of current repayment obligations. Corporate bond issuance is determined by a trade-off between relative impatience and bankruptcy costs, similar to Gomes et al. (2016).³ Banks collect deposits

³Since our focus is on the collateral framework and, thereby, on firms that are sufficiently large to issue bonds

from households, invest into corporate bonds, and incur liquidity management costs. In the spirit of Piazzesi and Schneider (2021), these costs are decreasing in the amount of available corporate collateral reflecting that banks may use it to collateralize short-term borrowing. This introduces a willingness of banks to pay *collateral premia* on corporate bonds.⁴

The central bank sets haircuts on corporate bonds that determine the degree to which bonds can be used as collateral. While low haircuts increase collateral availability for banks, the central bank incurs costs from accepting risky bonds as collateral. The literature has associated this cost with risk management expenses and counterparty default risk that depend on the riskiness of collateral (Bindseil and Papadia, 2006; Hall and Reis, 2015). As in Choi et al. (2021), optimal collateral policy balances these two effects. Starting from this point, our paper studies the welfare gains of adding a second variable (the green haircut) to the central bank collateral framework.

The link between collateral policy and the real sector via banks' demand for bonds allows the central bank to affect the relative prices of green and conventional bonds by tilting the collateral framework in favor of green bonds. In this case, banks pay higher collateral premia on green bonds, ceteris paribus, since holding them lowers liquidity management costs more effectively. Green firms respond to higher collateral premia on their bonds by increasing bond issuance and investment, while conventional firms reduce their bond and investment positions. Notably, the effect on the green investment share is *permanent*, i.e., central bank collateral policy is *not neutral* even in the long run.⁵ However, since higher collateral premia make debt financing more attractive, green firms also increase leverage and risk-taking. Higher risk-taking reduces the expected return on green investment so that the equilibrium green investment share is smaller than the green bond share under such a policy. As a result, the transmission of preferential treatment on the green investment share is substantially dampened. The endogeneity of risk-taking, which is key for the imperfect pass-through result, is consistent with the data.⁶

To quantify the optimal degree of preferential treatment, we calibrate the model to

and related marketable assets, we employ a financial friction that restricts *debt issuance* rather than overall *external financing* as in the canonical financial accelerator model of Bernanke et al. (1999). Moreover, our framework encompasses all marketable debt securities issued by non-financial firms, like syndicated bank loans and commercial paper.

⁴Collateral premia on corporate bonds are documented by Pelizzon et al. (2020) for the euro area, Mota (2021) for the US, and Chen et al. (2021) and Fang et al. (2020) for China.

⁵Asset purchase programs have an anti-cyclical component by design and, therefore, seem less well suited in an environmental policy context, which is concerned with long-run problems.

⁶Risk-taking, as reflected by firms' financing decision, has been reported in the empirical literature on unconventional monetary policy. Bekkum et al. (2018) observe a decrease in repayment performance on the mortgage backed securities market following an eligibility easing. Pelizzon et al. (2020) document positive leverage responses of eligible firms. Harpedanne de Belleville (2019) finds a sizable increase in investment by issuers of newly eligible bonds following a reduction of collateral requirements. Grosse-Rueschkamp et al. (2019) and Giambona et al. (2020) document a positive investment and leverage impact of firms eligible for quantitative easing (QE). Kaldorf and Wicknig (2022) provide a structural analysis of collateral premia and corporate default risk.
euro area data and conduct a number of policy experiments. First, we study a maximal preferential treatment policy, which makes conventional bonds non-eligible and accepts all green bonds without a haircut. This policy induces a green-conventional bond spread (also referred to as *greenium*) of 160 basis points (bp) in equilibrium, which translates into a change in the relative share of green bonds from 20% to 21.17%, while the share of green capital only increases from 20% to 20.74%. However, maximal preferential treatment reduces the collateral supply below its optimal level and distorts the risk choice of green firms. Therefore, we maximize the welfare objective over both haircut parameters. Optimal collateral policy still treats green bonds preferentially but increases the haircuts on conventional bonds to less than 100% to keep aggregate collateral supply approximately constant. In this case, the greenium amounts to 16 bp, the relative share of green bonds goes up to 20.11%, whereas the share of green capital increases to 20.07%.

While our numerical findings suggest that collateral frameworks can induce a shift towards green technologies, this shift is small and accompanied by adverse side effects. To put the effects of preferential treatment into perspective, we consider Pigouvian taxation of pollution, which is the natural policy instrument to address environmental frictions. Such a policy increases the share of green capital to 27.70% and substantially reduces the pollution externality *without* adverse effects on firm risk-taking. It should be noted that even without the adverse effects on firm risk-taking, Pigouvian taxes are an order of magnitude more powerful in addressing the environmental friction than maximal preferential treatment: the changes maximal preferential treatment induces in borrowing costs are unable to generate a sufficiently strong shift towards green technologies under any plausible calibration. If firms' financing choices are taken into account, the effectiveness of an optimal preferential collateral policy decreases by another order of magnitude.

These results should not be misinterpreted as a call for central bank inaction. The level of the Pigouvian tax that optimally addresses the environmental externality also reduces the collateral supply to an inefficiently low level. The central bank optimally addresses this by slightly decreasing haircuts in a symmetric way to restore the efficient level of aggregate collateral. In contrast, if public policy is restricted in its ability to set carbon taxes optimally, e.g., due to political economy frictions, the central bank can increase welfare by tilting the collateral framework towards green bonds. The extent of preferential treatment declines, the closer Pigouvian taxation gets to its optimal level.

Related Literature. There is a small but fast-growing literature that adds environmental aspects to DSGE models suitable for central bank policy analysis at business cycle frequencies, building on Heutel (2012). Punzi (2019) extends this setup by adding financial intermediation of loans to a credit-constrained corporate sector. Her paper explicitly considers differentiated capital requirements to relax financial frictions of green firms. Due to our focus on the collateral framework and marketable assets instead of bank loans, our model uses a financial friction related to leverage rather than external financing. Moreover, we endogenize firm risk-taking, while the extent of financial frictions is exogenous in Punzi (2019).

In a specific assessment of green QE, Ferrari and Nispi Landi (2020) find only a modestly positive impact on aggregate environmental performance. Similarly, Abiry et al. (2021) document a small impact of QE, in particular in comparison to a carbon tax, which is similar to our results on collateral policy. Hong et al. (2021) study sustainable investment mandates, which have a similar transmission mechanism, since they affect asset demand by financial intermediaries. In their setup, sustainable investment mandates, in the form of minimum portfolio shares, increase welfare, since they widen the cost of capital wedges between green and conventional firms. Closest to our paper is the work of Papoutsi et al. (2021) who show how central banks can tilt their asset purchases towards green assets to address environmental frictions. However, they assume that central banks can to buy firm equity and are silent about the pass-through via the corporate bond market, which is generating a limited policy transmission in our model. Similar to us, they show that in the presence of an optimal carbon tax, asset purchases play no role in addressing the environmental friction, consistently with the Tinbergen Principle in the public economics literature. On a more general level, all policies that change the relative demand for green and conventional bonds, such as green QE and preferential green capital requirements, will induce firm responses along several dimensions, that have not been studied extensively in the literature so far. However, in our view, a thorough analysis of these additional response margins is necessary to fully assess the effectiveness and efficiency of green policies.

It should be stressed, that we abstract from an analysis of transition risk, which arises if demand for conventional goods suddenly decreases due to ambitious environmental policy. Diluiso et al. (2021) and Carattini et al. (2021) argue that macroprudential policies can address this issue. Similar to these papers, we document an interaction between environmental policy and collateral policy and show how haircuts should be adjusted to account for these interactions.

Outline. The paper is structured as follows. We introduce our model in Section 4.2 and illustrate the pass-through of collateral policy to the real sector in Section 4.3. Section 4.4 presents our calibration, while we discuss our policy experiments in Section 4.5. Section 4.6 concludes.

4.2 Model

Time is discrete and indexed by t = 1, 2, ... The model features a representative household, two types of intermediate goods firms, a perfectly competitive wholesale firm, aggregating both types of intermediate goods into a composite intermediate good, a competitive final good producer, financial intermediaries (banks), and a public sector consisting of a fiscal authority and the central bank. One type of intermediate goods producers (conventional) causes an externality, to which we refer as pollution, when producing intermediate goods. The technology of the green firm does not cause the externality. Both types of intermediate goods are aggregated into a composite intermediate good by a perfectly competitive wholesale firm. A competitive final good producer uses the composite intermediate good and labor to produce the final consumption good, which it sells to the household. Banks raise deposits from the household to invest into corporate bonds and incur liquidity management cost. Finally, the fiscal authority can levy a proportional pollution tax on the conventional firms' output, while the central bank sets the collateral framework and incurs a cost from collateral default.

4.2.1 Households and Banks

Households. The representative household derives utility from consumption c_t and disutility from supplying labor l_t at the wage w_t . To transfer resources across time, the household saves in deposits d_t . Deposits held from time t - 1 to t earn the interest rate i_{t-1} . The household's discount factor is denoted by β , ω_l is the utility-weight on labor, and γ_c and γ_l are the inverses of the intertemporal elasticity of substitution and of the Frisch elasticity of labor supply, respectively. The maximization problem of the representative household is given by

$$V(d_t, S_t) = \max_{c_t, l_t, d_{t+1}} \frac{c_t^{1-\gamma_c}}{1-\gamma_c} - \omega_l \cdot \frac{l_t^{1+\gamma_l}}{1+\gamma_l} + \beta \mathbb{E}_t \left[V(d_{t+1}) \right] ,$$

s.t. $c_t + d_{t+1} = w_t l_t + (1+i_{t-1})d_t + \Pi_t ,$ (4.1)

where Π_t collects profits from banks and final goods producers and where we omit the dependency of V on the aggregate state for simplicity. Solving (4.1) yields standard interand intratemporal optimality conditions

$$c_t^{-\gamma_c} = \beta \mathbb{E}_t \left[(1+i_t) c_{t+1}^{-\gamma_c} \right] , \qquad (4.2)$$

$$c_t^{-\gamma_c} w_t = \omega_l l_t^{\gamma_l} \ . \tag{4.3}$$

Banks. There is a unit mass of perfectly competitive banks $i \in (0, 1)$ that supply deposits to households and invest into corporate bonds. We assume that financial intermediation is subject to liquidity management costs, which can be represented by the function $\Omega(\overline{b}_{t+1}^i)$.

Costs satisfy $\Omega_{\bar{b},t} \equiv \partial \Omega / \partial \bar{b}_{t+1}^i < 0$, i.e., liquidity management costs depend negatively on the collateral value of bank *i*'s corporate bond portfolio,

$$\bar{b}_{t+1}^{i} = (1 - \phi_c)q_{c,t}b_{c,t+1}^{i} + (1 - \phi_g)q_{g,t}b_{g,t+1}^{i} .$$

The collateral value of a bank's portfolio is given by the market value its bonds $q_{\tau,t}b_{\tau,t+1}^i$ weighted by one minus the respective central bank haircut parameter ϕ_{τ} .⁷ The higher the haircut, the lower collateral value the bond has. Banks directly benefit from a relaxation in collateral policy, since this increases available collateral \bar{b}_{t+1} ceteris paribus. The literature has motivated such liquidity management costs as arising from idiosyncratic liquidity shocks associated with deposit or credit line withdrawals (De Fiore et al., 2019 and Piazzesi and Schneider, 2021). The assumption $\Omega_{\bar{b},t} < 0$ then captures in reduced form the benefits of collateral to settle idiosyncratic liquidity shocks on interbank markets or by tapping central bank facilities.⁸

We follow Cúrdia and Woodford (2011) and assume that banks maximize profits, defined as equity value net of liquidity management costs in (4.4), subject to the solvency condition (4.5). Taken the behavior of other banks, intermediate firms, and central bank policy as given, the maximization problem of bank i reads

$$\max_{d_{t+1}^i, b_{c,t+1}^i, b_{g,t+1}^i} \prod_{t=1}^i d_{t+1}^i - q_{c,t} b_{c,t+1}^i - q_{g,t} b_{g,t+1}^i - \Omega(\overline{b}_{t+1}^i) , \qquad (4.4)$$

s.t.
$$(1+i_t)d_{t+1}^i = \mathbb{E}_t \left[\mathcal{R}_{c,t+1} \right] b_{c,t+1}^i + \mathbb{E}_t \left[\mathcal{R}_{g,t+1} \right] b_{g,t+1}^i$$
 (4.5)

The bond payoff $\mathcal{R}_{\tau,t+1}$ depends on firm τ 's bond issuance and capital choice via the default decision in period t + 1 (see below). Taking first order conditions we obtain the bond price equation

$$q_{\tau,t} = \frac{\mathbb{E}_t \left[\mathcal{R}_{\tau,t+1} \right]}{(1+i_t)(1+(1-\phi_\tau)\Omega_{\bar{b},t})} \,. \tag{4.6}$$

Liquidity management costs introduce a willingness to pay a premium for eligible bonds, reflected by the term $(1 - \phi_{\tau})\Omega_{\bar{h},t}$, which we refer to as *collateral premium*.

4.2.2 Firms

Final Good Producer. A competitive firm produces the final good y_t using a Cobb-Douglas production function that combines an intermediate good z_t and labor l_t

$$y_t = (1 - \mathcal{P}_t) A_t z_t^{\theta} l_t^{1-\theta} , \qquad (4.7)$$

⁷We restrict the analysis to time-invariant haircuts. While collateral frameworks are occasionally adjusted in practice, this usually happens in response to large shocks to the financial systems. These events are not of first order importance for our analysis of preferential treatment.

⁸Since neither the sources of liquidity demand, nor the reason why this market is collateralized are at the heart of our paper, we introduce this feature in reduced form and refer to Appendix C.1.1 for details on a micro-foundation.

where θ is a technology parameter. Final good production is negatively affected by pollution \mathcal{P}_t generated by the conventional firm (described below). The economy-wide TFP shock A_t evolves according to

$$\log (A_{t+1}) = (1 - \rho_A) \log(A_{ss}) + \rho_A \log (A_t) + \sigma_A \epsilon_{t+1}^A, \quad \epsilon_{t+1}^A \sim N(0, 1)$$

where A_{ss} , is normalized to one. Solving the maximization problem of the firm, we get standard first order conditions that equate the marginal product of the inputs to their market price

$$p_{z,t} = (1 - \mathcal{P}_t) \,\theta A_t z_t^{\theta - 1} l_t^{1 - \theta} ,$$
$$w_t = (1 - \mathcal{P}_t) \,(1 - \theta) A_t z_t^{\theta} l_t^{\theta} ,$$

where $p_{z,t}$ denotes the intermediate good price.

Wholesale Firm. The competitive wholesale firm bundles green and conventional intermediate goods into an input used by the final good firm using a Cobb-Douglas technology

$$z_t = z_{g,t}^{\nu} z_{c,t}^{1-\nu} , \qquad (4.8)$$

where ν determines the relative share of green intermediate goods.⁹ The prices of the intermediate good types τ are denoted by $p_{\tau,t}$. Solving the profit maximization problem yields

$$\nu p_{z,t} z_t = p_{g,t} z_{g,t} , \qquad (4.9)$$

$$(1-\nu)p_{z,t}z_t = p_{c,t}z_{c,t} . (4.10)$$

Intermediate Good Firms: Technology. There are two types of intermediate good firms producing a green or a conventional good z_{τ} . Within each type $\tau = \{c, g\}$, there is a unit mass of firms, indexed by j, that invest in physical capital $k_{j,\tau,t}$. The production technology of all firms is linear and subject to an idiosyncratic productivity shock $m_{j,\tau,t}$, which is i.i.d. across and within firm types

$$z_{j,\tau,t} = m_{j,\tau,t} k_{j,\tau,t}$$
 (4.11)

Following Bernanke et al. (1999), the idiosyncratic shock is log-normally distributed with $\mathbb{E}[m_{j,\tau,t}] = 1$. The log-normal distribution satisfies a monotone hazard rate property of the form $\partial(h(m_{j,\tau,t})m_{j,\tau,t})/\partial m_{j,\tau,t} > 0$, where $h(m_{j,\tau,t}) \equiv f(m_{j,\tau,t})/(1 - F(m_{j,\tau,t}))$ denotes the hazard rate and $f(m_{j,\tau,t})$ and $F(m_{j,\tau,t})$ denote the pdf and cdf, respectively. Capital $k_{j,\tau,t}$ depreciates at rate δ , which is common to both production technologies. Sector-specific

⁹In Appendix C.2.1 we conduct a robustness analysis using a CES-function and find only minor differences.

investment is denoted $i_{\tau,t}$. Since our model permits exact aggregation into representative firms, the law of motion for capital of type τ is given by

$$k_{\tau,t+1} = i_{\tau,t} + (1-\delta)k_{\tau,t} .$$
(4.12)

As common in environmental DSGE models (see Heutel, 2012), the aggregate production of conventional firms $z_{c,t}$ induces pollution \mathcal{P}_t , which satisfies $\partial \mathcal{P}_t / \partial z_{c,t} > 0$. Revenues are subject to a time-invariant, type-specific tax χ_{τ} . When χ_{τ} is negative, it can be interpreted as a subsidy and it will be set to zero in the baseline calibration.¹⁰

Intermediate Good Firms: Financial Side. As in Gomes et al. (2016), we assume that each firm j of each type τ is managed on behalf of a risk-averse and impatient representative firm owner who consumes dividends $\tilde{c}_t = \int_j \prod_{j,c,t} dj + \int_j \prod_{j,g,t} dj$. The firm owner's period utility is given by $\frac{\tilde{c}_t^{1-\gamma_c}}{1-\gamma_c}$, where the utility parameter is the same as the one of households. There is no agency friction between firm managers and owners. The representative firm owner discounts the future with a discount factor $\tilde{\beta} < \beta$. This assumption ensures that firms borrow from banks in equilibrium. We impose the following timing structure:

- At the beginning of each period, firms enter with (type-specific) capital $k_{\tau,t}$ and bonds outstanding $b_{\tau,t}$.
- Each firm j draws an idiosyncratic productivity shock $m_{j,\tau,t}$, produces and either repays its maturing debt obligations or defaults (described below).
- Firms adjust capital $k_{j,\tau,t+1}$ and bonds outstanding $b_{j,\tau,t+1}$.
- Firms transfer their dividends $\Pi_{j,\tau,t}$ to the firm owner.

Firms finance their activities by issuing equity, modeled as negative dividends, or by issuing corporate bonds. Bonds mature stochastically each period with probability $0 < s \leq 1$ and pay one unit of the final good in t+1 in case of no default.¹¹ Firms mechanically default, if their repayment obligation exceeds revenues from production.¹² The default productivity threshold is given by $\overline{m}_{\tau,t}$ and is implicitly defined as the productivity level at which revenues $(1 - \chi_{\tau})p_{\tau,t}m_{\tau,t}k_{\tau,t}$ equal repayment obligations $sb_{\tau,t}$. In case of default, banks holding distressed bonds effectively replace the firm owner as shareholder: they seize the output only in the default period, restructure the firm, and resume to being

¹⁰It is not relevant in our setup, whether the intermediate or wholesale firms pay the tax. Attributing it to intermediate good producers, however, gives the cleanest comparison to collateral policy, as both instruments operate through the investment decision.

¹¹Using long-term bonds allows to obtain realistic leverage ratios in the calibration but is not required for the transmission of collateral policy. Moreover, bonds are cast in real terms. We consider nominal bonds in Appendix C.2.2.

 $^{^{12}}$ We implicitly assume that there is no transfer of resources from productive to unproductive firms.

creditors after the firm's debt has been restructured. With probability 1 - s, the bond does not mature, is unaffected by the restructuring process, and is rolled over at next period's market price $q_{\tau,t+1}$. While in practice restructuring takes several periods, we follow Gomes et al. (2016) and take a shortcut by assuming that capital owners can restructure their liabilities without delays.

Firms maximize the present value of dividends, discounted using the firm owner's stochastic discount factor $\tilde{\Lambda}_{t,t+1} \equiv \tilde{\beta} (\tilde{c}_{t+1}/\tilde{c}_t)^{-\gamma_c}$. We conjecture that all firms enter any period t with the same legacy debt stock and capital to express dividends as

$$\Pi_{j,\tau,t} = \mathbb{1}\{m_{j,\tau,t} > \overline{m}_{\tau,t}\} \left((1 - \chi_{\tau}) p_{\tau,t} m_{j,\tau,t} k_{\tau,t} - s b_{\tau,t} \right) - k_{j,\tau,t+1} + (1 - \delta) k_{\tau,t} + q_{\tau,t} \left(b_{j,\tau,t+1} - (1 - s) b_{\tau,t} \right) \ .$$

Under the assumption of no delays in restructuring and i.i.d. productivity shocks, next period's productivity can be integrated out in the objective function and the problem reduces to a two-period consideration

$$\max_{k_{j,\tau,t+1},b_{j,\tau,t+1}} - k_{j,\tau,t+1} + q_{\tau,t} \Big(b_{j,\tau,t+1} - (1-s)b_{\tau,t} \Big) \\ + \mathbb{E}_t \Big[\widetilde{\Lambda}_{t,t+1} \Big((1 - G(\overline{m}_{j,\tau,t+1}))(1 - \chi_{\tau}) p_{\tau,t+1} k_{j,\tau,t+1} + (1-\delta) k_{j,\tau,t+1} \\ - s \Big(1 - F(\overline{m}_{j,\tau,t+1}) \Big) b_{j,\tau,t+1} + q_{\tau,t+1} \Big(b_{j,\tau,t+1} - (1-s)b_{\tau,t} \Big) \Big) \Big] ,$$

subject to the default threshold $\overline{m}_{j,\tau,t+1} \equiv \frac{sb_{j,\tau,t+1}}{(1-\chi_{\tau})p_{\tau,t+1}k_{j,\tau,t+1}}$, the bond pricing condition (4.6) and taken as given the continuation value of bonds $q_{\tau,t+1}$. Since dividends of all firms are transferred to the firm owner and firms can access capital and bond markets irrespective of current default, idiosyncratic productivity risk washes out in the aggregate: current productivity is not relevant for the investment and debt issuance decisions and all type τ firms make the same choices $k_{\tau,t+1}$ and $b_{\tau,t+1}$. This allows aggregation into a representative green and conventional firm, respectively.

Let the average productivity of a defaulting firm be denoted by $G(\overline{m}_{\tau,t}) \equiv \int_0^{\overline{m}_{\tau,t}} m dF(m)$. In case of default, the bank pays restructuring costs φ and is entitled to the entire production, valued at price $p_{\tau,t}$, while the payoff in case of repayment is $b_{\tau,t}$.¹³ In summary, the *per-unit* bond payoff entering the bond pricing condition of banks (4.6) is given by

$$\mathcal{R}_{\tau,t} = s \left(G(\overline{m}_{\tau,t}) \frac{p_{\tau,t}(1-\chi_{\tau})k_{\tau,t}}{sb_{\tau,t}} + 1 - F(\overline{m}_{\tau,t}) \right) - F(\overline{m}_{\tau,t})\varphi + (1-s)q_{\tau,t} .$$
(4.13)

The first term reflects the payoff from the share s of maturing bonds: it consists of the production revenues banks seize in case of default (first term in parenthesis) and the repayment of the principal in case of no default (second term). The term $F(\overline{m}_{\tau,t})\varphi$ reflects

¹³Attributing restructuring costs to green and conventional firms yields similar mechanics.

default costs incurred by banks. The share of bonds that are rolled over is valued at the bond market price $q_{\tau,t}$.

Intermediate Good Firms: Bond Issuance and Investment. As in Gomes et al. (2016), the bond price depends only on the default threshold $q_{\tau,t} = q(\overline{m}_{\tau,t+1})$. Plugging investment (4.12) and banks' bond pricing condition (4.6) into the Bellman equation, the Euler conditions for bond issuance and capital read

$$q'(\overline{m}_{\tau,t+1}) \frac{\mathbb{E}_{t}[\overline{m}_{\tau,t+1}]}{b_{\tau,t+1}} \left(b_{\tau,t+1} - (1-s)b_{\tau,t} \right) + q(\overline{m}_{\tau,t+1})$$

$$= \mathbb{E}_{t} \left[\widetilde{\Lambda}_{t,t+1} \left(s(1 - F(\overline{m}_{\tau,t+1})) + (1-s)q(\overline{m}_{\tau,t+1}) \right) \right],$$

$$1 = -q'(\overline{m}_{\tau,t+1}) \frac{\mathbb{E}_{t}[\overline{m}_{\tau,t+1}]}{k_{\tau,t+1}} \left(b_{\tau,t+1} - (1-s)b_{\tau,t} \right)$$

$$+ \mathbb{E}_{t} \left[\widetilde{\Lambda}_{t,t+1} \left((1-\delta) + (1-\chi_{\tau})p_{\tau,t+1} \left(1 - G(\overline{m}_{\tau,t+1}) \right) \right) \right].$$
(4.14)
$$(4.14)$$

$$(4.15)$$

The analytical steps are relegated to Appendix C.1.2. Equation (4.14) is a standard optimality condition equating the marginal benefit of issuing more bonds (left-hand side, LHS) to the marginal costs (right-hand side, RHS). Each additional unit of bonds increases funds available in period t by $q(\overline{m}_{\tau,t+1})$ units. At the same time, the bond price schedule is a decreasing function of the default threshold, which we also refer to as the risk choice. Since we characterize bond prices by the risk choice $\overline{m}_{\tau,t+1}$, the term $\mathbb{E}_t[\overline{m}_{\tau,t+1}]/b_{\tau,t+1}$ captures the increase of default risk arising from the issuance of an additional bond. This dilutes the value of existing bond investment $b_{\tau,t+1} - (1-s)b_{\tau,t}$. The risk choice has also implications for firm consumption in t + 1. Each unit of bonds involves repayment of s, conditional on not defaulting. In addition, bond issuance also increases the rollover burden in t + 1, further reducing expected consumption.

The optimality condition for capital (4.15) requires that the cost of purchasing capital (equal to one, LHS) equals its payoff, the RHS, consisting of two parts. The one in the first line affects consumption in period t and represents an increase of the bond price that is due to a decrease of the default probability. The part in the second line increases consumption in period t + 1 and is composed of the value of capital after depreciation and the marginal value of production net of taxes.

4.2.3 Public Policy and Resource Constraint

The central bank sets the collateral framework (ϕ_c, ϕ_g) and incurs costs from collateral default Λ_t . These costs depend positively on the default risk of pledged collateral $\Lambda_{\overline{F},t} > 0$,

defined as the firms' probability of default, weighted by the repo size

$$\overline{F}_t \equiv \sum_{\tau} (1 - \phi_{\tau}) b_{\tau,t} q_{\tau,t} F_{\tau,t} \; .$$

The weighting $(1 - \phi_{\tau})b_{\tau,t}q_{\tau,t}$ can be interpreted as the repo size collateralized by green and conventional bonds, respectively. By setting haircuts, the central bank has a direct effect on the costs. Making Λ_t dependent on default risk captures in reduced form a risk management consideration of accepting risky bonds as collateral. In Appendix C.1.3 we discuss a potential micro-foundation of the cost function, based on central bank solvency concerns (Hall and Reis, 2015). This is a frequently employed argument for why central banks are only willing to lend against sufficiently safe securities. For example, Bindseil and Papadia (2006) argue that central banks are not specialized credit risk management agencies and that higher default risk of accepted collateral makes monetary policy implementation more resource-intensive.

Together with the assumptions that liquidity management costs decrease in collateral supply, $\Omega_{\bar{b},t} < 0$, the assumption $\Lambda_{\bar{F},t} > 0$ introduces a collateral policy trade-off. To close the model, we assume that the fiscal authority rebates all tax revenues to green firms to balance its budget,

$$\chi_c p_{c,t} z_{c,t} + \chi_g p_{g,t} z_{g,t} = 0.$$
(4.16)

This fiscal rule allows us to abstract from additional fiscal instruments that would otherwise be necessary to balance the government budget. The resource constraint is given by

$$y_t = c_t + \sum_{\tau} \left(c_{\tau,t} + i_{\tau,t} \right) + \Omega(\overline{b}_{t+1}) + \Lambda(\overline{F}_{t+1}) + \sum_{\tau} \varphi F(\overline{m}_{\tau,t}) b_{\tau,t} , \qquad (4.17)$$

where the last three terms represent the resource losses due to the liquidity management costs, collateral default costs, and corporate defaults.

4.3 The Transmission of Preferential Treatment

Before numerically evaluating optimal policy in Section 4.4, we illustrate the pass-through of collateral policy and Pigouvian taxation in a simplified setting. The discussion will be organized around intermediate good firms' first order conditions and the equilibrium green capital ratio. For the ease of exposition, we consider the case of one-period bonds and full capital depreciation ($s = \delta = 1$). Since we do not focus on macroeconomic dynamics in this section, we do not endogenize output prices and the interest rate and also set firm owner's stochastic discount factor $\Lambda_{t,t+1} = \tilde{\beta}$. A Benchmark Without Default Risk. To isolate the role of financial frictions in the production sector, it is informative to relate our model to a framework without default risk and with collateral premia. Specifically, we consider the case where capital has to be fully debt-financed but where there are no idiosyncratic productivity shocks. In this case, both intermediate good firms will issue exactly as many bonds as necessary to finance their capital $q_{\tau,t}b_{\tau,t+1} = k_{\tau,t+1}$, taken as given bond prices. In the absence of default risk, the bond price $q_{\tau,t} = \frac{1}{(1+i_t)(1+(1-\phi_{\tau})\Omega_{\tilde{b}})}$ merely reflects the discounted value of consumption in t + 1 and collateral benefits. The bond price is increasing in the collateral premium $\frac{\partial q_{\tau,t}}{\partial (1-\phi_{\tau})\Omega_{\tilde{b}}} > 0$, which, in turn, increases if the central bank lowers the haircut. Firms maximize the present value of dividends, which yields the following first order condition for capital

$$1 = \underbrace{(1 - \chi_{\tau}) \mathbb{E}_t[p_{\tau,t+1}]q_{\tau,t}}_{\equiv \Gamma_{\tau,t+1}^{\text{no default}}} .$$

$$(4.18)$$

This condition states that the marginal cost of investment (LHS, equal to one) equals the marginal benefit of investment (RHS, return on capital $\Gamma_{\tau,t+1}^{\text{no default}}$). Given that the marginal cost of capital is constant, any increase of the return on capital will stimulate investment. A relaxation in collateral policy will then increase the return on capital proportionally to any increase of the bond price, which we refer to as *perfect pass-through*:

$$\frac{\partial \Gamma_{\tau,t+1}^{\text{no default}}}{\partial (1-\phi_{\tau})\Omega_{\overline{b}}} = (1-\chi_{\tau})\mathbb{E}_t[p_{\tau,t+1}]\frac{\partial q_{\tau,t}}{\partial (1-\phi_{\tau})\Omega_{\overline{b}}} .$$
(4.19)

Combining the investment decision (4.18) for both firm types with the intermediate good demands (4.9) and (4.10) yields the green capital ratio

$$\frac{k_{g,t}}{k_{c,t}} = \frac{q_{g,t}}{q_{c,t}} \frac{\nu(1-\chi_g)}{(1-\nu)(1-\chi_c)} \,. \tag{4.20}$$

Equation (4.20) shows that in the no-default benchmark, a decrease in the relative borrowing costs of green firms increases the green capital ratio.

The Role of Default Risk. Now, consider the model with default risk. With one-period bonds, the default threshold is given by $\overline{m}_{\tau,t+1} = \frac{b_{\tau,t+1}}{(1-\chi_{\tau})p_{\tau,t+1}k_{\tau,t+1}}$ and the first order conditions for bonds and capital simplify to

$$q'(\overline{m}_{\tau,t+1})\mathbb{E}_t[\overline{m}_{\tau,t+1}] + q(\overline{m}_{\tau,t+1}) = \widetilde{\beta}\mathbb{E}_t[1 - F(\overline{m}_{\tau,t+1})], \qquad (4.21)$$

$$1 = \underbrace{(1 - \chi_{\tau})\mathbb{E}_t \left[p_{\tau,t+1} \left(\widetilde{\beta} \left(1 - G(\overline{m}_{\tau,t+1}) \right) - q'(\overline{m}_{\tau,t+1}) \overline{m}_{\tau,t+1}^2 \right) \right]}_{\equiv \Gamma_{\tau,t+1}} .$$
(4.22)

The return on capital in (4.22) contains, first, the future output produced by an additional unit of capital conditional on not defaulting, $\tilde{\beta} \left(1 - G(\overline{m}_{\tau,t+1})\right)$. Second, it contains a bond price appreciation term, $q'(\overline{m}_{\tau,t+1})\overline{m}_{\tau,t+1}^2$, reflecting the reduction in default risk from higher investment. Combining (4.21) and (4.22) and differentiating the return on capital with respect to the collateral premium we obtain

$$\frac{\partial\Gamma_{\tau,t+1}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}} = (1-\chi_{\tau})\mathbb{E}_{t}\left[p_{\tau,t+1}\left\{\left(\frac{\partial q_{\tau,t}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}} + q'(\overline{m}_{\tau,t+1})\frac{\partial\overline{m}_{\tau,t+1}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}}\right)\overline{m}_{\tau,t+1} + q_{\tau,t}\frac{\partial\overline{m}_{\tau,t+1}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}} - \widetilde{\beta}\left(1-F(\overline{m}_{\tau,t+1})\right)\frac{\partial\overline{m}_{\tau,t+1}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}}\right\}\right].$$
(4.23)

As in the no-default case, the effect of collateral policy on the return on capital directly depends on the change in borrowing cost $\frac{\partial q_{\tau,t}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}}$. Moreover, it also depends on the risk choice, which itself is endogenously determined. To characterize the risk choice, we exploit that banks' bond pricing condition is available in closed form. To simplify the exposition, assume that banks cannot seize output of defaulting firms (their revenues are wasted) and do not incur restructuring costs ($\varphi = 0$). The bond pricing condition and its derivative with respect to risk-taking can then be written as

$$q(\overline{m}_{\tau,t+1}) = \mathbb{E}_t \frac{1 - F(\overline{m}_{\tau,t+1})}{(1 + i_t)(1 + (1 - \phi_\tau)\Omega_{\overline{b}})} \text{ and } q'(\overline{m}_{\tau,t+1}) = \mathbb{E}_t \frac{-f(\overline{m}_{\tau,t+1})}{(1 + i_t)(1 + (1 - \phi_\tau)\Omega_{\overline{b}})}.$$

The effect of collateral policy on risk-taking can be illustrated by plugging the bond pricing condition into (4.21):

$$(1+i_t)\left(\frac{1}{1+i_t} - (1+(1-\phi_\tau)\Omega_{\overline{b}})\widetilde{\beta}\right) = \mathbb{E}_t\left[\frac{f(\overline{m}_{\tau,t+1})}{1-F(\overline{m}_{\tau,t+1})}\overline{m}_{\tau,t+1}\right].$$
(4.24)

In the absence of collateral premia ($\phi_{\tau} = 1$), the risk choice is determined by equating relative impatience and marginal default costs. Holding the interest rate fixed, a reduction of the haircut ϕ_{τ} increases the LHS of (4.24). Due to the monotonicity assumption on the hazard rate, the RHS of (4.24) increases in $\overline{m}_{\tau,t+1}$. Hence, the effect of relaxing collateral policy on risk-taking is unambiguously positive. Intuitively, firms increase their risk-taking, because lower financing costs make investment *and* front-loading dividend payouts more attractive, holding expected default cost constant.

We can now re-consider the effect of haircuts on the return on capital (4.23). The term $q'(\overline{m}_{\tau,t+1})\frac{\partial \overline{m}_{\tau,t+1}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}} < 0$ is a negative risk-taking effect, which lowers the bond price and, thereby, makes investment less attractive in period t. The positive term $q_{\tau,t}\frac{\partial \overline{m}_{\tau,t+1}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}}$ captures bond price appreciation from investment. Last, $\tilde{\beta}\left(1-F(\overline{m}_{\tau,t+1})\right)\frac{\partial \overline{m}_{\tau,t+1}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}}$ reflects the dividend reduction in t+1 due to higher default rates. Using the definitions of $q(\overline{m}_{\tau,t+1})$ and $q'(\overline{m}_{\tau,t+1})$, we can simplify (4.23) to

$$\begin{split} \frac{\partial\Gamma_{\tau,t+1}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}} &= (1-\chi_{\tau})\mathbb{E}_{t} \left[p_{\tau,t+1} \Biggl\{ \frac{\partial q_{\tau,t}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}} \overline{m}_{\tau,t+1} + \frac{\partial\overline{m}_{\tau,t+1}}{(1-\phi_{\tau})\Omega_{\overline{b}}} \\ & \cdot \left(\underbrace{\frac{1-\frac{f(\overline{m}_{\tau,t+1})}{(1-F(\overline{m}_{\tau,t+1}))} \overline{m}_{\tau,t+1}}_{=\widetilde{\beta}}}_{=\widetilde{\beta}} (1-F(\overline{m}_{\tau,t+1})) - \widetilde{\beta} \left(1-F(\overline{m}_{\tau,t+1})\right) \right) \Biggr\} \Biggr] \\ &= (1-\chi_{\tau})\mathbb{E}_{t} \Biggl[p_{\tau,t+1} \overline{m}_{\tau,t+1} \Biggr] \frac{\partial q_{\tau,t}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}} < \frac{\partial\Gamma_{\tau,t+1}^{\text{no default}}}{\partial(1-\phi_{\tau})\Omega_{\overline{b}}} \,. \end{split}$$

Hence, relaxing collateral policy has a positive but unambiguously smaller effect on investment than in the no-default case. In (partial) equilibrium, the green capital ratio in the presence of financial frictions can be written

$$\frac{k_{g,t}}{k_{c,t}} = \frac{\mathbb{E}_t \left[\widetilde{\beta} \left(1 - G(\overline{m}_{g,t+1}) \right) - q'(\overline{m}_{g,t+1}) \overline{m}_{g,t+1}^2 \right]}{\mathbb{E}_t \left[\widetilde{\beta} \left(1 - G(\overline{m}_{c,t+1}) \right) - q'(\overline{m}_{c,t+1}) \overline{m}_{c,t+1}^2 \right]} \frac{\nu(1-\chi_g)}{(1-\nu)(1-\chi_c)} \,. \tag{4.25}$$

Absent preferential treatment, the risk choice and bond prices are identical across firm types and the terms related to financial frictions in the return on investment cancel. Then, as in the no-default case, the relative size of both sectors would be directly determined by the technology parameter ν and the environmental policy regime. Setting $\chi_c > 0$ and $\chi_g < 0$ directly increases the green capital ratio. Note that this policy also operates through the return on capital, which increases (decreases) in the subsidy (tax) from (4.18). However, in sharp contrast to haircut policies, the tax rate χ_{τ} does not affect risk-taking, as demonstrated in (4.24). The preferential treatment of green bonds in the collateral framework also increases the green capital ratio, but the pass-through of this policy is impaired. We quantify relevance of this impairment in the next section.

Last, note that two partial effects shape the effect of preferential treatment and the extent to which financial frictions dampen it: (i) the response of relative borrowing costs between sectors to preferential treatment and (ii) the elasticities of leverage and capital to bond price changes. In Section 4.4.2, we relate the model-implied reactions in these dimensions to the data. Separating between-sector effects on borrowing costs from sector-specific effects of borrowing conditions on real outcomes is relevant from an empirical point of view: as preferential policies are not enacted yet, this decomposition allows to assess the model predictions' plausibility.

4.4 Quantitative Analysis

In this section, we provide a calibration of the model to euro area data. All data sources are summarized in Appendix C.4. We then show the model's fit regarding (untargeted) macroeconomic dynamics and demonstrate the model's ability to replicate the effect of

preferential treatment on borrowing costs between sectors and the response of financial market and real sector variables to collateral policy.

4.4.1 Calibration

Each period corresponds to one quarter. We assume log-utility over consumption, fix the inverse Frisch elasticity at 1, and set the household discount factor β to 0.99. he Cobb-Douglas coefficient in the final good production technology is set to $\theta = 1/3$ to obtain a labor share of 2/3, and we choose the weight ω_l in the household utility function to be consistent with a steady state labor supply of 1/3. The TFP shock parameters are conventional values in the RBC literature. The depreciation rate is set to $\delta = 0.017$ to target the capital to GDP ratio.

Parameters regarding pollution and the green technology share are important drivers of environmental DSGE models. For the relative share of the green sector, we use the most recent data on the share of renewable energies in the euro area. Although this is only a subset of intermediate goods, it has the advantage that, since renewable energy is a prominent feature of the public discussion, the data quality is excellent. From this data set we find that the relative share of the green sector is 20%, which directly informs the Cobb-Douglas parameter of the wholesale goods producers ν .¹⁴ In spirit of Golosov et al. (2014) and Heutel (2012), we assume that pollution costs are expressed as

$$\mathcal{P}_t = 1 - \exp\{-\gamma_P z_{c,t}\} ,$$

which, through final good production (4.7), generates a percentage loss in the production of the final good producer. The function captures the mapping from pollution to real economic damage and the parameter γ_P governs the pass-through from pollution to production losses. We inform the parameter γ_P using estimates of direct costs from pollution and indirect costs from adverse environmental conditions. From the model, we can directly relate this quantity $1 - \exp\{-\gamma_P z_c\}$ to observable (long-run) quantities $1 - y/z^{\theta}l^{1-\theta}$. We use the estimate of Muller (2020), who quantifies Damage/GDP at 10% in 2016 for the US. The value of 10% has also been reported in the fourth National Climate Assessment in the US (Reidmiller et al., 2018). Since economic activity in this dimension can be assumed to be similar in the US and the euro area, we adopt the same value.

The next group of parameters is associated with intermediate good firms. We assume that both firm types are subject to the same financial friction. This assumption is supported by the findings of Larcker and Watts (2020) and Flammer (2021), who find no effect of environmental performance on spreads in the US fixed income market. The average maturity of corporate bonds is set to five years (s = 0.05) and corresponds to the average maturity in the *Markit iBoxx* corporate bond index between 2010 and 2019. Following

¹⁴Renewable energy statistics for the EU are accessible here. See also the guide by Eurostat (2020).

Gomes et al. (2016), restructuring costs φ are set such that they are consistent with a recovery rate of 70%, defined as realized payoff in default over the promised payoff. The idiosyncratic productivity shock is log-normally distributed with variance ς_M and mean $-\varsigma_M/2$ to ensure that it satisfies $\mathbb{E}[m_{\tau,t}] = 1$. This leaves us with two free parameters, the discount factor $\tilde{\beta}$ of firm owners and the idiosyncratic productivity variance ς_M . They are set to match time-series means of spreads and the corporate debt to GDP ratio. The model-implied bond spread is defined as

$$x_{\tau,t} \equiv (1 + s/q_{\tau,t} - s)^4 - (1 + i_t)^4$$
.

For the data moment on spreads, we use the *IHS Markit* data from 2010 until 2019. We compute the median bond spread over the entire corporate bond sample and average over time, which yields a value of around 100 bp. The data moment on corporate debt is the non-financial firm debt to GDP ratio taken from the ECB. Our parameterization implies an (annualized) default rate of 1% and a leverage ratio of 42%, which is closely aligned with the data moments reported by Gomes et al. (2016) for the US.

The final group of parameters is related to banks and collateral policy. We impose symmetric collateral treatment and set $\phi_{sym} \equiv \phi_c = \phi_g = 0.26$, which corresponds to the current haircut on BBB-rated corporate bonds with five to seven years maturity. Liquidity management costs are specified as

$$\Omega\left(\bar{b}_{t+1}^{i}\right) = \max\left\{l_{0} - 2l_{1}\left(\bar{b}_{t+1}^{i}\right)^{0.5}, 0\right\}$$
(4.26)

Their concave shape captures that the marginal cost reduction from another unit of collateral is decreasing, e.g., due to the re-use of existing collateral. The intercept parameter l_0 will be set sufficiently high to ensure that $\Omega(\overline{b}_{t+1}^i)$ is positive for all considered collateral policy specifications.¹⁵ Plugging in $\overline{b}_{t+1}^i = 0$ can be interpreted as the cost level of an entirely un-collateralized banking system.

The slope of the liquidity management costs l_1 governs the cost reduction per unit of collateral. We calibrate it to $l_1 = 0.0085$, matching the eligibility premium reported by the empirical literature: using the ECB list of collateral eligible for main refinancing operations, Pelizzon et al. (2020) identify an eligibility premium of -11 bp. The model implied eligibility premium is given by the yield differential of the traded bond and a synthetic bond that is not eligible in period t but becomes eligible in t + 1, corresponding to the identification strategy of Pelizzon et al. (2020). The advantage of this procedure is that the eligibility premium can be backed out from bond prices in the deterministic steady state. The eligibility premium is available in closed form and given by

$$\widetilde{x}_{\tau,t} \equiv (1 + s/q_{\tau,t} - s)^4 - (1 + s/(q_{\tau,t}(1 + (1 - \phi_\tau)\Omega_{\overline{b},t})) - s)^4 .$$

¹⁵We verify that l_0 does not affect our results markedly.

Parameter	Value	Source/Target
Households		
CRRA-coefficient γ_c Household discount factor β Labor disutility convexity γ_l	$1 \\ 0.99 \\ 1 \\ 0.00$	Log-utility Annual riskless rate 4% Frisch elasticity= 1
Labor disutility weight ω_l	6.68	Labor supply= $1/3$
$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	1/3 0.20 1.5e-2	Labor share $= 2/3$ Renewable share in Europe 2018 Pollution damage/GDP $= 0.1$
Banks		
Bond maturity parameter s Restructuring costs φ Collateral default cost parameter η_1 Liquidity management intercept l_0 Liquidity management slope l_1	$\begin{array}{c} 0.05\\ 0.50\\ 0.0463\\ 0.05\\ 0.0085\end{array}$	$\begin{array}{l} IHS \ Markit\\ Recovery \ rate = 70\%\\ Ex-post \ optimality \ of \ \phi_{sym} = 0.26\\ Ensures \ positive \ costs\\ Eligibility \ premium = -11bp \end{array}$
Conventional and Green Firms		
$\begin{array}{c} \mbox{Depreciation rate } \delta \\ \mbox{Discount factor } \widetilde{\beta} \\ \mbox{Standard deviation idiosyncratic risk } \varsigma_M \end{array}$	$\begin{array}{c} 0.067/4 \\ 0.9835 \\ 0.175 \end{array}$	$\begin{array}{l} {\rm Capital/GDP}=2.1\\ {\rm Debt/GDP}=0.8\\ {\rm Bond\ spread}=100 {\rm bp} \end{array}$
Central Bank		
Haircut parameter ϕ_{sym}	0.26	ECB collateral framework
Shocks	_	
Persistence TFP shock ρ_A Variance TFP shock σ_A	$\begin{array}{c} 0.95 \\ 0.005 \end{array}$	Standard Standard

 TABLE 4.1: Baseline Calibration

In the spirit of Bindseil and Papadia (2006), the costs of accepting risky collateral follow

$$\Lambda(\overline{F}_t) = 2\eta_1 \cdot \left(\overline{F}_t\right)^{0.5}$$

The concave specification reflects that there is a fixed cost component to set up a proper risk management infrastructure as well as a marginal cost component from adding additional risk to the central bank's collateral portfolio, for example through more frequent collateral default. The parameter η_1 governs the level of collateral default costs and is set so that the empirical haircut value $\phi_{sym} = 0.26$ is optimal according to an utilitarian welfare criterion. Put differently, we assume that the status-quo ECB collateral policy is optimal under the restriction of symmetric collateral policy and parameterize the cost function accordingly. Finally, we define the greenium as the spread of conventional over green bonds with corresponding maturity

$$\widehat{x}_t = x_{g,t} - x_{c,t} \; .$$

Moment	Model	Data	Source
Volatilities			
Bond Spread Vol. $\sigma(x)$	30 bp	50-100 bp	Gilchrist and Zakrajšek (2012)
Excess Vol. Consumption $\sigma(c)/\sigma(y)$	0.59	0.70	Euro area data
Excess Vol. Investment $\sigma(i)/\sigma(y)$ Persistence	6.50	3.80	Euro area data
GDP $corr(y_t, y_{t-1})$	0.70	0.90	Euro area data
Consumption $corr(c_t, c_{t-1})$	0.87	0.80	Euro area data
Investment $corr(i_t, i_{t-1})$ Correlations with GDP	0.60	0.80	Euro area data
Consumption $corr(y, c)$	0.86	0.60	Euro area data
Investment $corr(y, i)$	0.90	0.70	Euro area data
Debt $corr(y, b)$	0.70	0.65	Jungherr and Schott (2022)
Leverage $corr(y, lev)$	-0.77	-0.30	Kuehn and Schmid (2014)
Default risk $corr(y, F)$	-0.77	-0.55	Kuehn and Schmid (2014)
Pollution $corr(y, \mathcal{P})$	0.31	0.30	Doda (2014)

TABLE 4.2: Model Fit – Second	d Moments
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Notes: We calculate theoretical moments after solving the model under the productivity shock. We compare the model moments to Hodrick-Prescott-filtered data of the euro area or to moments from the literature.

Note that the greenium is zero in our baseline calibration due to the assumption of symmetric financial frictions and symmetric collateral treatment. The parameterization is summarized in Table 4.1.

Macroeconomic Dynamics. In Table 4.2, we compare the model-implied second moments with the data. Notably, they are broadly consistent with each other, even though our model only uses one exogenous shock and does not feature frictions related to firm investment, labor markets, and the relationship between households and banks. The time series volatility of bond spreads is slightly smaller than the value reported by Gilchrist and Zakrajšek (2012) for US data, since bond prices in our model are priced using a log-utility pricing kernel and only contain default risk compensation and the collateral premium.

The excess volatilities of consumption and investment are broadly consistent with euro area data. The elevated investment volatility and its low autocorrelation can at least partly be attributed to the absence of investment adjustment costs. The model is also able to capture the cyclical properties of key financial market variables, debt b, leverage at market values qb/(pk), and default risk F. In addition, we also match the cyclicality of emissions, which has been estimated by Doda (2014) for a large sample of countries.

4.4.2 Real Effects of Preferential Treatment

Before using the calibrated model to study optimal preferential treatment, we compare the model-implied impact of preferential treatment to results from the empirical literature, which corroborates the external validity of our quantitative analysis. Guided by the simplified analysis in Section 4.3, we first discuss the effect of preferential treatment on relative borrowing costs of green and conventional firms. In a second step, we consider the effect of changes in borrowing costs on bond issuance and investment.

Preferential Treatment and Relative Borrowing Costs. To examine the effect of preferential central bank policy on (relative) bond prices, we exploit the yield reaction of green and conventional bonds around ECB announcements regarding environmental policy.¹⁶ We identify four relevant speeches by ECB board members between 2018 and 2020, which explicitly mention environmental concerns for the conduct of central bank policy. Using data from *IHS Markit* and *Thomson Reuters Datastream*, we generate a panel of green-conventional bond pairs, obtained by a nearest-neighbor matching. We then compute the average yield difference between green bonds and their respective conventional counterparts for a 20 trading day window around each announcement. Averaging over all announcements and the entire post-treatment window, the announcement effect is significant in statistical terms: after each ECB announcement, green bond yields drop by 4.8 bp on average over a 20 trading day window. This is economically meaningful and lies in a plausible range, compared to the empirical literature on collateral premia of corporate bonds. The result indicates that bond investors are willing to pay premia on green bonds already if there is the prospect of preferential treatment.

Since the ECB so far did not implement preferential treatment, these announcements can be mapped into our model by interpreting them as a news shock (see Beaudry and Portier, 2004 and Barsky and Sims, 2011). Specifically, we assume that preferential treatment will be implemented with certainty but at an unknown point in the future. We enrich the baseline calibration by a news shock to the green collateral parameter ϕ_g for various time horizons,

$$\log(\phi_{g,t}) = (1 - \rho_{\phi})\log(\phi_{sym}) + \rho_{\phi}\log(\phi_{g,t-1}) + \sigma_{\phi}\epsilon^{\phi}_{t-h} \quad \epsilon^{\phi}_{t-h} \sim N(0,1) , \qquad (4.27)$$

where ϕ_{sym} is the green collateral parameter corresponding to the baseline calibration and h denotes the announcement horizon. We choose a high value of $\rho_{\phi} = 0.95$ for the haircut persistence, since changes to the collateral framework only occur infrequently. The shock size σ_{ϕ} is set such that $\phi_g = 0.045$ in two, three, four, or five years. The haircut value of 4.5% corresponds to the treatment of AAA-rated securities in the ECB collateral framework. This haircut appears to be a reasonable value for a strong preferential policy and

¹⁶See Appendix C.3 for details on the announcements and the data.

Data		Model:	Horizon	
	2 years	3 years	4 years	5 years
-4.8 bp	-8.8 bp	-6.8 bp	-5.3 bp	-4.1 bp

 TABLE 4.3:
 Greenium Reaction – Announcement Effects

Notes: The data value results from the analysis of announcement effects. Model-implied values obtain from introducing news shocks (4.27).

opens a considerable haircut gap. Moreover, the considered horizons appear plausible, given that the ECB strategy review itself took two years and that the actual implementation of preferential treatment takes some additional time. The announcement effect on the greenium is shown in Table 4.3 and lies between -8.8 bp and -4.1 bp. Naturally, the effect peters out as the announcement horizon increases. The model-implied yield response closely resembles the data value at the four-year horizon.

Relative Borrowing Costs and their Real Effects. In the second step, we consider the firm level effect of a change in borrowing cost induced by central bank policy. We build on literature studying firm responses following QE-programs and collateral framework changes. From the point of view of firms (the collateral supply side), the effects of QE and collateral eligibility are identical, since in both cases banks increase demand for their bonds for reasons unrelated to firm fundamentals. Specifically, we compare estimates from the literature to the effects of a haircut decrease from $\phi_{sym} = 1$ (no eligibility) to $\phi_{sym} = 0.26$ (our baseline value). We assume that the collateral policy relaxation is unanticipated, comes into effect immediately, and is permanent. We focus on the reaction of bond yields, capital, and leverage, since our discussion of the imperfect pass-through of preferential treatment in Section 4.3 is centered around these variables .

Since the eligibility premium as defined in Pelizzon et al. (2020) is a calibration target, we instead examine the yield spread between eligible and non-eligible bonds. Fang et al. (2020) study the impact of an easing of collateral eligibility requirements by the PBoC and identify a yield reaction on treated bonds of 42-62 bp (their Table 5). Using a similar approach, Chen et al. (2021) find a yield reaction of 39-85 bp (their Tables 5 and 8).

Regarding the financing of firms, Grosse-Rueschkamp et al. (2019) show that the introduction of the Corporate Sector Purchase Program (CSPP) triggered a positive response of total debt to assets for eligible firms relative to non-eligible firms prior to CSPP. The magnitude of the effect is estimated between 1.1 pp and 2.0 pp, depending on the econometric specification (see their Table 2). Pelizzon et al. (2020) report an increase of total debt/total assets between 2.5 pp and 10.8 pp (see their Table 10). Giambona et al. (2020) consider the impact of QE and find increases in total debt/total assets of around 1.8pp (see their Table 15). On the same sample, they report an increase in investment between 4.9 pp and 6.0 pp for QE-eligible firms when controlling for firm characteristics (see their Table 3). Harpedanne de Belleville (2019), Table 4.1, finds a 5.4 pp increase in investment after the introduction of the Additional Credit Claims program using French data. Grosse-Rueschkamp et al. (2019), on the other hand, only document a mild effect of 1pp on asset growth (their Table 5).

	Δ Yield	Δ Capital	Δ Leverage
Model	$58 \mathrm{\ bp}$	2.1 pp	1.4 pp
Data	39 - $85~{\rm bp}$	1.0 - $6.0~{\rm pp}$	1.1 - 10.8 pp

TABLE 4.4: Firm Reaction: Model vs. Data

Notes: In the first line, we compare our baseline to an economy with 100% haircut. The second line displays the range of estimated effects taken from the empirical literature.

Table 4.4 displays our results. Consistent with the empirical literature, we observe a strong yield response to eligibility of around 58 bp. The capital response comfortably falls into the range of empirical estimates, while the model-implied leverage response is at the lower bound of the firm reaction observed in the data. The relatively modest leverage response will imply that the role of the adverse effects of preferential treatment on firm risk-taking are quantified in a conservative manner.

4.5 Policy Analysis

In this section, we conduct policy experiments regarding the collateral framework and its interactions with direct Pigouvian taxation of pollution. Throughout the analysis, we employ an utilitarian welfare criterion based on household's (unconditional) expected utility (4.1) and follow Schmitt-Grohé and Uribe (2007) by approximating it, together with the policy functions, up to second order. Given the log-utility assumption on consumption, the consumption equivalent (CE) welfare gain follows as

$$c^{CE,policy} \equiv 100 \left(\exp\{(1-\beta)(V^{policy} - V^{base})\} - 1 \right) ,$$

where V^{base} and V^{policy} are obtained from evaluating (4.1) under the baseline and alternative policies, respectively. The CE is defined as the fraction of the baseline consumption path that the household would need to receive to be indifferent between both policies.¹⁷

4.5.1 Optimal Collateral Policy With Preferential Treatment

Since intermediate good firms are at the heart of the transmission mechanism, we begin by showing the model-implied means of financial market variables for different green bond

¹⁷We also explore welfare gains conditionally on being at the deterministic steady state of the baseline calibration and taking into account the transition period to the new steady state. Results are virtually unchanged.



Notes: We display long-run means conditional on the green bond haircut. The collateral premium and spreads are expressed in basis points, leverage and default rates in percentage points. Bonds outstanding and capital are relative to the baseline of $\phi_{sym} = 0.26$ (vertical line).

haircuts in Figure 4.1. The green and red line denote the green and conventional firms, respectively. The upper panels show that green collateral premia strongly increase, the smaller the green collateral haircut ϕ_g , while at the same time the green bond spread declines, relative to the baseline calibration (solid vertical line). Leverage increases by around 0.15 pp, which translates into a 15% increase in default rates (middle panels). Notably, the increase in collateral premia dominates the effect on corporate bond spreads, which are lower despite elevated default risk.

The lower financing costs of green firms increase the green capital holdings. For every haircut, the increase in investment falls short of the increase in bond issuance (lower panels) consistent with higher leverage: the pass-through of collateral policy is imperfect as outlined in Section 4.3. For all variables, the reaction of conventional firms mirrors the response of their green counterparts, although to a smaller extent. This is an

Moment	Baseline	Max Pref	Opt Coll	Only Tax	Glob Opt
Tax Parameter χ_c	0	0	0	9.6%	9.6%
Haircut ϕ_q	26%	0%	11%	26%	21%
Haircut ϕ_c	26%	100%	29%	26%	21%
Welfare Change (CE)	0%	-0.6462%	0.0064%	+0.6640%	+0.6646%
Conv. Leverage	42.1%	41.4%	42.0%	42.1%	42.1%
Green Leverage	42.1%	42.5%	42.2%	42.1%	42.1%
Conv. Bond Spread	$96\mathrm{bp}$	$162 \mathrm{bp}$	$99\mathrm{bp}$	$96\mathrm{bp}$	94bp
Green Bond Spread	$96\mathrm{bp}$	$2\mathrm{bp}$	$83\mathrm{bp}$	$96\mathrm{bp}$	$94\mathrm{bp}$
Conv. Coll. Premium	-11bp	$0\mathrm{bp}$	-11bp	-11bp	-11bp
Green Coll. Premium	-11bp	-27bp	-13bp	-11bp	-11bp
GDP	0.8494				
Change from Baseline	-	+0.05%	+0.02%	+0.58%	+0.61%
Restructuring Cost/GDP	2.30%				
Change from Baseline	-	-20.18%	+0.19%	-0.13%	+1.37%
Coll. Default Cost/GDP	1.65%				
Change from Baseline	-	-29.75%	+0.74%	-0.36%	+3.76%
Liq. Man. Cost/GDP	2.91%				
Change from Baseline	-	+46.91%	-0.47%	-0.86%	-4.37%
Pollution Cost/GDP	9.74%				
Change from Baseline	-	-1.64%	-0.06%	-8.66%	-8.59%
Green Bond Share	20%	21.17%	20.11%	27.68%	27.68%
Green Capital Share	20%	20.74%	20.07%	27.68%	27.68%

 TABLE 4.5: Time Series Means for Different Policies

Notes: Maximal preferential treatment (Max Pref) only allows green bonds as collateral ($\phi_g = 0, \phi_c = 1$). The optimal collateral policy (*Opt Coll*) is derived from maximizing CE over a grid of haircuts. For the optimal tax (*Only Tax*), we hold haircuts fixed and vary the tax rate. The global optimum (*Glob Opt*) is obtained by maximizing over taxes and haircuts.

equilibrium effect operating through the perfect substitutability of green and conventional bonds as collateral: the conventional collateral premium $(1 - \phi_c)\Omega_{\bar{b},t}$ depends on haircuts and collateral supply. If green firms increase bond issuance due to preferential treatment, this increases the collateral supply so that $\Omega_{\bar{b},t}$ declines.

Our first finding considers the welfare and pollution impact of maximal preferential treatment in the second column of Table 4.5. We set $\phi_g = 0$ and $\phi_c = 1$ to provide an upper bound for the central bank's ability to induce investment into green technologies. The collateral premium on conventional bonds is zero in this case. This policy induces an increase in the green bond share to 21.17%, while green investment rises to 20.74%, translating into a 6% (3.7%) increase relative to the symmetric baseline (first column), respectively.

Around 40% of the effect on the corporate (green) bond market does not carry over to the investment decision due to the financial friction in the production sector. The converse holds for conventional firms, who reduce their bond issuance and capital holdings. This, in turn, lowers pollution. At the same time, setting $\phi_c = 1$ implies a strong contraction of collateral, leading to a substantial increase in liquidity management costs. Given the reduction in conventional bond issuance, we observe a decrease in the cost from debt restructuring and collateral default. Since optimal collateral policy trades off pollution with resources losses from corporate default and liquidity management costs, this combination of aggregate default rates and collateral supply is sub-optimal and substantially decreases welfare relative to the baseline collateral framework. Therefore, we maximize welfare over the collateral framework (ϕ_c, ϕ_g), to which we refer as the *optimal collateral policy*, and report results in third column of Table 4.5.

The optimal haircut levels of $\phi_g = 0.11$ and $\phi_c = 0.29$ imply a preferential treatment of green bonds. Subtracting the green bond spread of 83 bp from the conventional one of 99 bp gives a greenium of 16 bp, which is considerably smaller than under maximal preferential treatment. Consequently, the increase in the green bond (0.11 pp) and green capital (0.07 pp) shares is smaller as well. On the one hand, this policy avoids a sharp drop in available collateral, so that liquidity management costs even fall marginally. On the other hand, this policy comes at the cost of increasing default risk of green firms and reducing pollution less effectively. In Appendix C.2.1, we also show that nominal rigidities are not driving these results

4.5.2 Interaction with Direct Taxation

While our analysis reveals that the central bank can affect the relative size of green and conventional firms and, thereby, reduce the pollution externality, this effect is relatively small and induces non-negligible side-effects. In this section, we benchmark these results against direct Pigouvian taxation of pollution externalities. Section 4.3 indicated that its effect on capital shares is more direct than the one of collateral policy. The exercise serves a dual purpose: first, we can put the effectiveness of preferential collateral treatment into perspective relative to Pigouvian taxation. Second, this allows us to examine a mix of direct taxation and collateral policies. By assuming a balanced budget in (4.16), we compare different policy instruments regarding their effectiveness to address environmental policy trade-offs without imposing assumptions on the financing of subsidies or the distribution of tax revenues.

The fourth column of Table 4.5 corresponds to optimal Pigouvian taxation, holding the collateral framework at its baseline value. The optimal tax on conventional production is at 9.6%, which implies a subsidy of around 40% on green intermediate goods, since taxes are rebated to conventional firms proportional to their relative sizes, as determined by the parameter ν in the wholesale good production function (4.8). The green capital share rises by 7.7 pp, which strongly tilts production towards green inputs and reduces the pollution externality. The welfare improvement of optimal Pigouvian taxation exceeds the improvement from optimal preferential treatment by two orders of magnitude. At the same time, there are no adverse effects on firm risk-taking, since the first order condition



FIGURE 4.2: Optimal Collateral Policy Under Sub-Optimal Taxation

Notes: For different levels of the Pigouvian tax, we maximize CE over a grid of haircuts and display the result as the haircut differential.

for leverage (4.14) is not affected by a tax on production. This suggests that fiscal instruments dominate preferential treatment in addressing environmental frictions.

However, this should not be misinterpreted as a call for central bank inaction, since Pigouvian taxation has also a collateral policy impact as reported in the fourth column of the third panel in Table 4.5. We observe a simultaneous decline of liquidity management expenses and costs associated to default relative to GDP, respectively. The former results from a slight increase in available collateral driven by green firm bond issuance and the latter is a result of a drop in the bond issuance of conventional firms that lowers the size of costs from default. Both changes lower resources wasted but also shift the economy away from a configuration of the collateral trade-off for which the baseline haircuts are optimal. Relative to the global optimum, when collateral policy and taxes are set jointly, reported in the last column of Table 4.5, the collateral supply is too small at the old collateral framework driven by the reduction in conventional bond issuance. As a result, the collateral framework becomes more lenient. Notably, this relaxation is symmetric $(\phi_c = \phi_g)$ and there is no preferential treatment. This incentivizes all firms to extend their bond issuance, implying a slight increase of default costs, while liquidity management cost decline substantially.¹⁸ The welfare gains of adjusting collateral frameworks to mitigate negative effects on collateral availability are positive but of small size compared to the welfare gains of optimal taxation.

¹⁸This is similar to Carattini et al. (2021), who show that macroprudential policy can alleviate adverse effects of carbon taxation. In their model, adverse effects take the form of asset stranding, while in our case adverse effects are linked to collateral availability, if conventional firms shrink their balance sheet size. Notably, optimal macroprudential policy is also symmetric in their model.

The symmetry result hinges on the assumption that optimal Pigouvian taxes are available, which is arguably not an empirically plausible case. In Figure 4.2, we compute the optimal degree of preferential treatment, represented by the haircut differential, for different levels of the Pigouvian tax. At $\chi_c = 0$, the haircut differential is 18%, corresponding to the third column of Table 4.5, i.e., optimal collateral policy in the absence of taxation. At the globally optimal tax of $\chi_c = 0.096$, the differential is zero as in column five. While we are not explicit about why the Pigouvian might be too low, our model implies that the central bank can improve on sub-optimal taxation.¹⁹ However, the optimal degree of preferential treatment decreases, the closer environmental policy gets to implementing the optimal Pigouvian tax.²⁰

4.6 Conclusion

In this paper, we examine the effectiveness of the preferential collateral treatment of green bonds in an augmented RBC model. Preferential treatment stimulates investment into green bonds. However, this only partially transmits to investment into green technologies due to an increase in green firms' leverage and default risk. In a calibration to euro area data, we find that this policy can be fairly powerful in addressing environmental policy concerns but is still considerably less effective than Pigouvian taxes. Due to the adverse effects on firm risk-taking, the optimal collateral framework features only a small degree of preferential treatment but still increases welfare. Preferential treatment is a qualitatively and quantitatively imperfect substitute for Pigouvian taxes and is only optimal if Pigouvian taxes cannot be set to their optimal level. If the optimal tax is implemented, the optimal collateral framework is characterized by a symmetric relaxation to solve the tradeoff between the benefits of higher collateral supply and the costs of higher firm risk-taking.

Our results can be read as a call for (i) central bank action if tax policy is not able to adequately address pollution and climate change externalities, (ii) a careful calibration of preferential treatment that takes into account the side effects on firm risk-taking, and (iii) coordination between direct tax policy and central bank collateral policy, to mitigate adverse effects that environmental policy can inflict on the aggregate collateral supply.

 $^{^{19}\}mathrm{Welfare}$ gains of introducing preferential treatment for a given tax are always positive.

²⁰For a tax above the optimal value, the collateral framework optimally treats conventional bonds more preferentially to correct for ambitious environmental policy.

Appendix

C.1 Model Appendix

C.1.1 Bank Liquidity Management Costs

In the quantitative analysis, we assume that banks incur liquidity management costs $\Omega(\overline{b}_{t+1}^i)$, which gives rise to collateral premia. In this section, we demonstrate that the resulting first order conditions for corporate bonds are observationally equivalent to the most common micro-foundation used in this context, which are stochastic bank deposit withdrawals, see Corradin et al. (2017), De Fiore et al. (2019), Piazzesi and Schneider (2021), or Bianchi and Bigio (2022). The standard modeling device in this literature is a two sub-period structure, where banks participate in asset markets sequentially: in the first sub-period, banks trade with households on the deposit market and with intermediate good firms on the corporate bond market. In the second sub-period, bank *i* faces a liquidity deficit $\omega_t^i > 0$, which it settles on a collateralized short-term funding market, e.g., with the central bank.

If bank *i* is unable to collateralize its entire funding need, it must borrow on the (more expensive) unsecured segment. More specifically, since all banks hold the same amount of collateral \overline{b}_{t+1} before the deposits are withdrawn, there is a cut-off withdrawal $\overline{\omega}_t = \overline{b}_{t+1}$ above which a bank needs to tap the unsecured segment. The amount borrowed on the unsecured segment for all banks follows as

$$\widetilde{b}_{t+1} \equiv \int_{\overline{b}_{t+1}}^{\infty} \left(\omega_{t+1}^{i} - \overline{b}_{t+1} \right) dW(\omega)$$

where W denotes the cdf of the withdrawal shock distribution. Due to its analytical tractability, it is convenient to assume that withdrawals follow a Lomax distribution. This distribution is supported on the right half-line and characterized by a shape $\tilde{\alpha}$ and a scale $\tilde{\lambda}$ parameter. This allows us to write the expected amount of borrowing on the unsecured segment in closed form:

$$\widetilde{b}_{t+1} = \int_{\overline{b}_{t+1}}^{\infty} \omega_{t+1}^{i} \frac{\widetilde{\alpha}}{\widetilde{\lambda}} \left(1 + \frac{\omega_{t+1}^{i}}{\widetilde{\lambda}} \right)^{-\widetilde{\alpha}-1} d\omega - \overline{b}_{t+1} \int_{\overline{b}_{t+1}}^{\infty} \frac{\widetilde{\alpha}}{\widetilde{\lambda}} \left(1 + \frac{\omega_{t+1}^{i}}{\widetilde{\lambda}} \right)^{-\widetilde{\alpha}-1} d\omega$$
$$= \overline{b}_{t+1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}} \right)^{-\widetilde{\alpha}} + \frac{\widetilde{\lambda}}{\widetilde{\alpha}-1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}} \right)^{-\widetilde{\alpha}+1} - \overline{b}_{t+1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}} \right)^{-\widetilde{\alpha}}$$

$$= \frac{\widetilde{\lambda}}{\widetilde{\alpha} - 1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}} \right)^{-\widetilde{\alpha} + 1}$$

When $\tilde{\alpha} > 1$, the aggregate amount of unsecured borrowing falls, the more collateral is held. The benefit of holding collateral corresponds to the secured-unsecured spread ξ that is paid on borrowing \tilde{b}_{t+1} , which we assume to be an exogenous parameter. These expected cost $\xi \tilde{b}_{t+1}$ enter bank profits in the first sub-period

$$\Pi_t^i = d_{t+1}^i - q_{c,t+1} b_{c,t+1}^i - q_{g,t+1} b_{g,t+1}^i - \xi \widetilde{b}_{t+1}$$

The cost depend negatively on \bar{b}_{t+1} , but the marginal cost reduction is falling in \bar{b}_{t+1} . Since very large withdrawal shocks are unlikely, the additional benefit of holding another unit of collateral is positive but decreasing. The properties of our concave liquidity cost function $\Omega(\bar{b}_{t+1}^i)$ are closely related to the common micro-foundation using bank liquidity risk.

C.1.2 Intermediate Good Firms

We start with observing that the default threshold of a type- τ intermediate good firm in period t + 1 is given by $\overline{m}_{\tau,t+1} \equiv \frac{sb_{\tau,t+1}}{(1-\chi_{\tau})p_{\tau,t+1}k_{\tau,t+1}}$. The threshold satisfies the following properties:

$$\frac{\partial \overline{m}_{\tau,t+1}}{\partial b_{\tau,t+1}} = \frac{s}{(1-\chi_{\tau})p_{\tau,t+1}k_{\tau,t+1}} = \frac{b_{\tau,t+1}}{(1-\chi_{\tau})p_{\tau,t+1}k_{\tau,t+1}} \frac{s}{b_{\tau,t+1}} = \frac{\overline{m}_{\tau,t+1}}{b_{\tau,t+1}} , \qquad (C.1.1)$$

$$\frac{\partial \overline{m}_{\tau,t+1}}{\partial k_{\tau,t+1}} = -\frac{sb_{\tau,t+1}}{(1-\chi_{\tau})p_{\tau,t+1}k_{\tau,t+1}^2} = -\frac{b_{\tau,t+1}}{(1-\chi_{\tau})p_{\tau,t+1}k_{\tau,t+1}}\frac{s}{k_{\tau,t+1}} = -\frac{\overline{m}_{\tau,t+1}}{k_{\tau,t+1}} . \quad (C.1.2)$$

We assume that $\log(m_{\tau,t})$ is normally distributed with mean μ_M and standard deviation ς_M . In the calibration, we ensure that $\mathbb{E}[m_{\tau,t}] = 1$ by setting $\mu_M = -\varsigma_M^2/2$. The CDF of $m_{\tau,t}$ is given by $F(m_{\tau,t}) = \Phi\left(\frac{\log m_{\tau,t}-\mu_M}{\varsigma_M}\right)$, where $\Phi(\cdot)$ is the cdf of the standard normal distribution. The conditional mean of m at the threshold value $\overline{m}_{\tau,t+1}$ can be expressed as

$$G(\overline{m}_{\tau,t+1}) = \int_0^{\overline{m}_{\tau,t+1}} mf(m)dm = e^{\mu_M + \frac{\varsigma_M^2}{2}} \Phi\left(\frac{\log \overline{m}_{\tau,t+1} - \mu_M - \varsigma_M^2}{\varsigma_M}\right),$$

$$1 - G(\overline{m}_{\tau,t+1}) = \int_{\overline{m}_{\tau,t+1}}^\infty mf(m)dm = e^{\mu_M + \frac{\varsigma_M^2}{2}} \Phi\left(\frac{-\log \overline{m}_{\tau,t+1} + \mu_M + \varsigma_M^2}{\varsigma_M}\right).$$

Note that the derivative of the conditional mean $g(\overline{m}_{\tau,t+1})$ satisfies

$$g(\overline{m}_{\tau,t+1}) = \overline{m}_{\tau,t+1} f(\overline{m}_{\tau,t+1}) .$$
(C.1.3)

For notational convenience, we write the bond price schedule as function of the default threshold $\overline{m}_{\tau,t}$ throughout this section. The bond payoff is given by

$$\mathcal{R}_{\tau,t} = s \left(G(\overline{m}_{\tau,t}) \frac{(1-\chi_{\tau}) p_{\tau,t} k_{\tau,t}}{s b_{\tau,t}} + 1 - F(\overline{m}_{\tau,t}) \right) - F(\overline{m}_{\tau,t}) \varphi + (1-s) q_{\tau,t} ,$$

such that we can write the bond price only in terms of the default threshold $\overline{m}_{\tau,t+1}$

$$q(\overline{m}_{\tau,t+1}) = \mathbb{E}_t \frac{s\left(\frac{G(\overline{m}_{\tau,t+1})}{\overline{m}_{\tau,t+1}} + 1 - F(\overline{m}_{\tau,t+1})\right) - F(\overline{m}_{\tau,t+1})\varphi + (1-s)q_{\tau,t+1}}{(1 + (1-\phi_{\tau})\Omega_{b,t})(1+i_t)} .$$
(C.1.4)

The derivative with respect to the default threshold is given by

$$q'(\overline{m}_{\tau,t+1}) = \mathbb{E}_t \frac{-\frac{sG(m_{\tau,t+1})}{\overline{m}_{\tau,t+1}^2} - \varphi f(\overline{m}_{\tau,t+1})}{(1 + (1 - \phi_\tau)\Omega_{b,t})(1 + i_t)} .$$
(C.1.5)

FOC w.r.t $b_{\tau,t+1}$. The first order condition for bonds is given by

$$0 = q'(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{t+1}}{\partial b_{\tau,t+1}} \left(b_{\tau,t+1} - (1-s)b_{\tau,t} \right) + q(\overline{m}_{\tau,t+1}) + \mathbb{E}_t \left[\widetilde{\Lambda}_{t,t+1} \left(-(1-\chi_{\tau})p_{\tau,t+1}k_{\tau,t+1}G'(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial b_{\tau,t+1}} - s \left(-f(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial b_{\tau,t+1}} b_{\tau,t+1} + 1 - F(\overline{m}_{\tau,t+1}) \right) - q_{\tau,t+1}(1-s) \right) \right],$$

which can be expressed as

$$0 = q'(\overline{m}_{\tau,t+1}) \frac{\mathbb{E}_{t}[\overline{m}_{\tau,t+1}]}{b_{\tau,t+1}} \left(b_{\tau,t+1} - (1-s)b_{\tau,t} \right) + q(\overline{m}_{\tau,t+1}) \\ + \mathbb{E}_{t} \left[\widetilde{\Lambda}_{t,t+1} \left(-sG'(\overline{m}_{\tau,t+1}) \frac{\overline{m}_{\tau,t+1}(1-\chi_{\tau})p_{\tau,t+1}k_{\tau,t+1}}{sb_{\tau,t+1}} - s\left(-f(\overline{m}_{\tau,t+1})\overline{m}_{\tau,t+1} + 1 - F(\overline{m}_{\tau,t+1}) \right) - q_{\tau,t+1}(1-s) \right) \right],$$

and then yields (4.14).

FOC w.r.t $k_{\tau,t+1}$. The first order condition for capital is given by

$$\begin{split} 1 = & q'(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial k_{\tau,t+1}} \bigg(b_{\tau,t+1} - (1-s) b_{\tau,t} \bigg) \\ & + \mathbb{E}_t \bigg[\widetilde{\Lambda}_{t,t+1} \bigg(-G'(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial k_{\tau,t+1}} (1-\chi_{\tau}) p_{\tau,t+1} k_{\tau,t+1} + (1-G(\overline{m}_{\tau,t+1})) (1-\chi_{\tau}) \\ & p_{\tau,t+1} + s b_{\tau,t+1} f(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial k_{\tau,t+1}} + 1 - \delta \bigg) \bigg] , \end{split}$$

which can be rearranged to

$$1 = -q'(\overline{m}_{\tau,t+1}) \frac{\mathbb{E}_t[\overline{m}_{\tau,t+1}]}{k_{\tau,t+1}} \left(b_{\tau,t+1} - (1-s)b_{\tau,t} \right) \\ + \mathbb{E}_t \left[\widetilde{\Lambda}_{t,t+1} \left(G'(\overline{m}_{\tau,t+1})\overline{m}_{\tau,t+1}(1-\chi_{\tau})p_{\tau,t+1} + \left(1 - G(\overline{m}_{\tau,t+1})\right)(1-\chi_{\tau})p_{\tau,t+1} - sb_{\tau,t+1}f(\overline{m}_{\tau,t+1})\frac{\overline{m}_{\tau,t+1}}{k_{\tau,t+1}} \frac{(1-\chi_{\tau})p_{\tau,t+1}}{(1-\chi_{\tau})p_{\tau,t+1}} + 1 - \delta \right) \right],$$

and further to (4.15).

C.1.3 Collateral Default Costs

In the main text we assume an exogenous cost function from collateral default $\Lambda(\overline{F}_t)$. In this section, we provide a micro-foundation based on central bank solvency concerns (see Hall and Reis, 2015). We show that this yields a loss function $\Lambda(\overline{F}_t)$, which is increasing in \overline{F}_t , consistent with our assumption in the main text. Similar to Appendix C.1.1, assume that banks incur a fixed liquidity shock in every period ω , which they settle by borrowing from the central bank. Since the collateral banks pledge is subject to default risk, the central bank will subject itself to these risks when entering repurchase agreements. The central bank haircut ϕ directly affects exposure to this risk. The timing is as follows: in the beginning of period t, banks invest into risky bonds. In the end of period t, they incur the exogenous liquidity need and tap the central bank facility. Repos mature in the beginning of period t + 1 and banks repay the central bank.

Each bank holds corporate bonds b_{t+1} at price q_t and borrows

 $\omega = (1 - \phi)q_t b_{t+1} ,$

from the central bank. Because every bank *i* incurs the liquidity shock, *i* indexes both banks and repo contracts. We assume that bank default can be represented by the i.i.d. random variable ζ^i with cdf Z and pdf z, and with support [0, 1]. The bond-specific default risk is denoted F_t . In case of a bank default, the central bank seizes the posted collateral to cover its losses. However, since the collateral itself defaults at rate F_t , the central bank will not recover the full amount of the defaulted repo. The expected loss on repo *i* follows as

$$\mathcal{F}_t^i = \zeta^i \cdot \omega \cdot F_t = \zeta^i \cdot (1 - \phi) q_t b_{t+1} \cdot F_t .$$

To make the results more easily interpretable, it is helpful to assume central bank also generates seigniorage revenues from lending through its facilities. As customary in the literature, we assume that seigniorage revenues are bounded from above by the (timeinvariant) constant \mathcal{M} . Consequently, the central bank incurs a loss from bank default if the default shock exceeds $\overline{\zeta}_t = \mathcal{M}/(\omega \cdot F_t)$. We can then denote the expected central bank loss as

$$\mathcal{L}_t = \int_{\overline{\zeta}_t}^1 \zeta^i \cdot (1-\phi) q_t b_{t+1} \cdot F_t \cdot z(\zeta) d\zeta = (1-\phi) q_t b_{t+1} F_t \cdot \int_{\overline{\zeta}_t}^1 \zeta^i z(\zeta) d\zeta .$$
(C.1.6)

The central bank haircut and bond default risk affect the expected loss in two ways. The first part of (C.1.6) show that irrespective of the distributional assumption on ζ^i , the expected loss rises in bond default risk F_t and that a higher haircut ϕ , by lowering the repo size, reduces cost. Second, note that $\overline{\zeta}_t = \mathcal{M}/((1-\phi)q_tb_{t+1}\cdot F_t)$. A higher haircut increases the default risk threshold beyond which central bank income is negative. Thus,

Moment	Baseline	Max Pref	Opt Coll	Only Tax	Glob Opt
Tax Parameter χ_c	0	0	0	12.5%	12.5%
Haircut ϕ_q	26%	0%	5%	26%	18%
Haircut ϕ_c	26%	100%	31%	26%	18%
Welfare Change (CE)	0%	-0.5450%	+0.0135%	+1.1560%	+1.1577%
Conv. Leverage	39.7%	38.7%	39.7%	39.7%	39.7%
Green Leverage	39.7%	40.4%	39.9%	39.7%	39.7%
Conv. Bond Spread	$97\mathrm{bp}$	$157 \mathrm{bp}$	101bp	$97\mathrm{bp}$	$94\mathrm{bp}$
Green Bond Spread	$97\mathrm{bp}$	16bp	81bp	97bp	94bp
Conv. Coll. Premium	-11bp	$0\mathrm{bp}$	-10bp	-11bp	-11bp
Green Coll. Premium	-11bp	-26bp	-14bp	-11bp	-11bp
GDP	0.8253				
Change from Baseline	-	+0.14%	+0.03%	+1.01%	+1.05%
Restructuring Cost/GDP	2.17%				
Change from Baseline	-	-20.38%	+0.13%	-0.23%	+2.42%
Coll. Default Cost/GDP	1.52%				
Change from Baseline	-	-28.56%	+0.81%	-0.62%	+6.46%
Liq. Man. Cost/GDP	3.29%				
Change from Baseline	-	+37.57%	-0.28%	-1.40%	-6.35%
Pollution Cost/GDP	9.81%				
Change from Baseline	-	-1.79%	-0.14%	-16.17%	-16.06%
Green Bond Share	20.49%	22.24%	20.73%	34.57%	34.56%
Green Capital Share	20.49%	21.52%	20.64%	34.57%	34.56%

TABLE C.1: Time Series Means with $\epsilon_{\nu} = 1.6$

Notes: Maximal preferential treatment (Max Pref) only allows green bonds as collateral ($\phi_g = 0, \phi_c = 1$). The optimal collateral policy (*Opt Coll*) is derived from maximizing CE over a grid of haircuts. For the optimal tax (*Only Tax*), we hold haircuts fixed and vary the tax rate. The global optimum (*Glob Opt*) is obtained by maximizing over taxes and haircuts.

it lowers the expected loss. Conversely, higher bond default risk increases expected cost. Defining collateral default risk as the repo size-weighted default risk $\overline{F}_t = (1-\phi)q_t b_{t+1}F_t$, this behavior is directly reflected in $\Lambda(\overline{F}_t)$ in the main text.

C.2 Additional Numerical Results

C.2.1 The Role of the Green-Conventional Substitution Elasticity

In Table C.1, we provide robustness checks regarding the production technology of wholesale goods producers. By assuming a Cobb-Douglas production function in (4.8), we implicitly assume an elasticity of substitution of one between green and conventional intermediate goods. When strictly interpreting green and conventional firms as energy producers, this elasticity is usually estimated to be larger than one. Therefore, we repeat our policy analysis when replacing the wholesale producers' technology by a general CES-function

$$z_t = \left(\nu z_{g,t}^{\frac{\epsilon_\nu - 1}{\epsilon_\nu}} + (1 - \nu) z_{c,t}^{\frac{\epsilon_\nu - 1}{\epsilon_\nu}}\right)^{\frac{\epsilon_\nu}{\epsilon_\nu - 1}},$$
(C.2.1)

and set the elasticity of substitution $\epsilon_{\nu} = 1.6$, following the point estimate in Papageorgiou et al. (2017). The parameter ν is set to keep the green production share at 20%, consistent with the baseline. Results are shown in Table C.1. To ensure an apples-toapples comparison with the baseline model, we re-calibrate the idiosyncratic productivity variance to $\varsigma_M = 0.195$, the firm owners' discount factor $\tilde{\beta} = 0.984$, the externality parameter $\gamma_P = 0.015$, the slope parameter $\eta_1 = 0.0432$ in the collateral default cost function, and the slope parameter $l_1 = 0.008$ in the liquidity management cost function. While the main results from the Cobb-Douglas baseline carry over to the CES case, the optimal tax is much higher and optimal collateral policy implies a much larger degree of preferential treatment. Intuitively, when conventional and green intermediate goods are easier to substitute, any policy-induced reduction in the size of conventional firms is less harmful to production.

C.2.2 The Role of Nominal Rigidities

In this section, we add nominal rigidities to the model following the standard New Keynesian model. In particular, bonds are assumed to be denominated in nominal terms, i.e., inflation has a direct effect on corporate bonds and the supply side. Households consume a final goods basket c_t given by

$$c_t = \left(\int_0^1 c_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon > 1$ is the elasticity of substitution among the differentiated final goods. The demand schedule for final good *i* is given by

$$c_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} c_t , \qquad (C.2.2)$$

where P_t denotes the CES price index for the final consumption bundle. Final good firms sell their differentiated good with a markup over their marginal costs. However, the price of firm j, $P_{j,t}$, can only be varied by paying a quadratic adjustment cost à la Rotemberg (1982) that is proportional to the nominal value of aggregate production, $P_t y_t$. Firm j's marginal costs are denoted by $mc_{j,t} \equiv \partial C_t^W / \partial y_{j,t}$, where the wholesale firm's cost minimization problem is given by

$$\mathcal{C}_{t}^{W}(y_{j,t}) = \min_{z_{j,t}, l_{j,t}} \quad P_{z,t} z_{j,t} + W_{t} l_{j,t} \quad \text{s.t.} \quad y_{j,t} = (1 - \mathcal{P}_{t}) A_{t} z_{j,t}^{\theta} l_{j,t}^{1-\theta} ,$$

and $P_{z,t}$ is the price of the wholesale good. From the minimization problem we obtain *real* marginal costs

$$\operatorname{mc}_{t} = \frac{1}{(1 - \mathcal{P}_{t}) A_{t}} \left(\frac{p_{z,t}}{\theta}\right)^{\theta} \left(\frac{w_{t}}{1 - \theta}\right)^{1 - \theta} ,$$

where $p_{z,t} = P_{z,t}/P_t$ is the relative price of the wholesale good and w_t is the real wage. Hence, total nominal profits of firm j in period t are given by

$$\widehat{\Pi}_{j,t} = (P_{j,t} - \mathrm{mc}_t P_t) y_{j,t} - \frac{\psi}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 P_t y_t ,$$

where ψ measures the degree of the nominal rigidity. Each wholesale good firm j maximizes the expected sum of discounted profits

$$\max_{P_{j,t+s},y_{j,t+s}} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \frac{c_{t+s}^{-\gamma_c}/P_{t+s}}{c_t^{-\gamma_c}/P_t} \,\widehat{\Pi}_{j,t+s} \right] ,$$

subject to the demand schedule (C.2.2). Plugging in the demand function yields the first order condition

$$\left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} Y_t - \varepsilon \left(P_{j,t} - \mathrm{mc}_t P_t\right) \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} \frac{y_t}{P_t} - \psi \left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right) \frac{P_t}{P_{j,t-1}} y_t \\ + \mathbb{E}_t \left[\frac{c_{t+1}^{-\gamma_c}/P_{t+1}}{c_t^{-\gamma_c}/P_t} \psi \left(\frac{P_{j,t+1}}{P_{j,t}} - 1\right) \frac{P_{j,t+1}}{P_{j,t}^2} P_{t+1} y_{t+1}\right] = 0 .$$

In a symmetric price equilibrium, $P_{j,t} = P_t$ for all j. Using this, we rearrange and get

$$(1 - \varepsilon(1 - \mathrm{mc}_t)) y_t + \mathbb{E}_t \left[\beta \frac{c_{t+1}^{-\gamma_c} / P_{t+1}}{c_t^{-\gamma_c} / P_t} y_{t+1} \pi_{t+1} \psi \left(\pi_{t+1} - 1 \right) \pi_{t+1} \right] = \psi \left(\pi_t - 1 \right) \pi_t y_t ,$$

where $\pi_t = \frac{P_t}{P_{t-1}}$. Dividing both sides by y_t and Ψ we arrive at the New Keynesian Phillips Curve

$$\mathbb{E}_t \left[\beta \frac{c_{t+1}^{-\gamma_c}/P_{t+1}}{c_t^{-\gamma_c}/P_t} \frac{y_{t+1}\pi_{t+1}}{y_t} \left(\pi_{t+1}-1\right)\pi_{t+1} \right] + \frac{\varepsilon}{\psi} \left(\mathrm{mc}_t - \frac{\varepsilon-1}{\varepsilon} \right) = (\pi_t - 1)\pi_t \,.$$

In addition, nominal rigidities also affect intermediate good firms, since inflation affects the default threshold $\overline{m}_{\tau,t+1} \equiv \frac{sb_{\tau,t+1}}{\pi_{t+1}(1-\chi_{\tau})p_{\tau,t+1}k_{\tau,t+1}}$ and the *real per-unit* bond payoff is

$$\mathcal{R}_{\tau,t} = s \left(G(\overline{m}_{\tau,t}) \frac{\pi_t p_{\tau,t} (1-\chi_\tau) k_{\tau,t}}{s b_{\tau,t}} + 1 - F(\overline{m}_{\tau,t}) \right) - F(\overline{m}_{\tau,t}) \varphi + (1-s) q_{\tau,t} \; .$$

Their first order conditions are now given by

$$q'(\overline{m}_{\tau,t+1})\frac{\mathbb{E}_t[\overline{m}_{\tau,t+1}]}{b_{\tau,t+1}}\left(b_{\tau,t+1}-(1-s)\frac{b_{\tau,t}}{\pi_t}\right)+q(\overline{m}_{\tau,t+1})$$
$$=\widetilde{\beta}\mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\frac{s(1-F(\overline{m}_{\tau,t+1}))+(1-s)q_{\tau,t+1}}{\pi_{t+1}}\right]$$

and

$$1 = -q'(\overline{m}_{\tau,t+1})\frac{\mathbb{E}_t[\overline{m}_{\tau,t+1}]}{k_{\tau,t+1}} \left(b_{\tau,t+1} - (1-s)\frac{b_{\tau,t}}{\pi_t}\right)$$

Moment	Baseline	Max Pref	Opt Coll	Only Tax	Glob Opt
Tax Parameter χ_c	0	0	0	10%	10%
Haircut ϕ_g	26%	0%	7%	26%	19%
Haircut ϕ_c	26%	100%	31%	26%	19%
Welfare Change (CE)	0%	-0.4608%	0.0088%	0.6979%	0.6987%
Conv. Leverage	42.1%	41.4%	42.1%	42.1%	42.1%
Green Leverage	42.1%	42.5%	42.2%	42.1%	42.1%
Conv. Bond Spread	$96\mathrm{bp}$	$162 \mathrm{bp}$	$100 \mathrm{bp}$	$96\mathrm{bp}$	$94\mathrm{bp}$
Green Bond Spread	$96\mathrm{bp}$	$2\mathrm{bp}$	$80\mathrm{bp}$	$96\mathrm{bp}$	94bp
Conv. Coll. Premium	-11bp	$0\mathrm{bp}$	-10bp	-11bp	-11bp
Green Coll. Premium	-11bp	-27bp	-14bp	-11bp	-11bp
GDP	0.6869				
Change from Baseline	-	+0.00%	+0.02%	+0.60%	+0.64%
Restructuring Cost/GDP	1.92%				
Change from Baseline	-	-20.22%	-0.04%	-0.14%	+1.96%
Coll. Default Cost/GDP	1.48%				
Change from Baseline	-	-29.68%	+0.26%	-0.37%	+5.36%
Liq. Man. Cost/GDP	4.79%				
Change from Baseline	-	+23.88%	+0.03%	-0.75%	-3.25%
Pollution Cost/GDP	9.95%				
Change from Baseline	-	-1.69%	-0.10%	-9.02%	-8.92%
Inflation Volatility	0.10%				
Change from Baseline	-	-7.73%	-0.05%	+0.49%	+0.51%
Green Bond Share	20.00%	21.17%	20.15%	28%	28%
Green Capital Share	20.00%	20.74%	20.09%	28%	28%

TABLE C.2: Time Series Means with Nominal Rigidities

Notes: Maximal preferential treatment (Max Pref) only allows green bonds as collateral ($\phi_g = 0, \phi_c = 1$). The optimal collateral policy (*Opt Coll*) is derived from maximizing CE over a grid of haircuts. For the optimal tax (*Only Tax*), we hold haircuts fixed and vary the tax rate. The global optimum (*Glob Opt*) is obtained by maximizing over taxes and haircuts.

+
$$\mathbb{E}_t \left[\widetilde{\Lambda}_{t,t+1} \left(\widetilde{\beta}(1-\delta) + \widetilde{\beta}(1-\chi_{\tau}) p_{\tau,t+1} \left(1 - G(\overline{m}_{\tau,t+1}) \right) \right) \right].$$

The resource constraint now also includes Rotemberg costs

$$y_{t} = c_{t} + \sum_{\tau} \left(c_{\tau,t} + i_{\tau,t} \right) + \Lambda(\overline{F}_{t+1}) + \Omega(\overline{b}_{t+1}) + \frac{\psi}{2} \left(\pi_{t} - 1 \right)^{2} y_{t} + \sum_{\tau} \varphi F(\overline{m}_{\tau,t}) \frac{b_{\tau,t}}{\pi_{t}}$$

To close the model, we assume that the central bank sets i_t according to a Taylor rule

$$i_t = i\pi_t^{\phi_\pi} . \tag{C.2.3}$$

We choose standard parameters for the final goods elasticity $\epsilon = 6$, implying a markup of 20% in the deterministic steady state, and a Rotemberg parameter $\psi = 57.8$, consistent with a Calvo parameter of 0.75. The parameter on inflation stabilization in the monetary policy rule is set to $\phi_{\pi} = 5$, which ensures determinacy for all policy experiments. We slightly re-calibrate the slope parameter $\eta_1 = 0.0407$ in the collateral default cost function, and the slope parameter $l_1 = 0.007$ in the liquidity management cost function. Results

are reported in Table C.2 and show very similar implications for optimal collateral policy and its interaction with Pigouvian taxation.²¹ In particular, the inflation volatility under optimal preferential treatment is almost unchanged with respect to the baseline in column one, alleviating concerns that preferential treatment jeopardizes price stability, the central bank's primary policy objective.

C.3 Yield Reaction to Central Bank Policy Announcements

C.3.1 Construction of the Dataset

The first step of our analysis is to identify a list of relevant pieces of ECB communication with significant space or time devoted to environmental policy. To identify relevant speeches for our empirical analysis, we rely on a dataset published by the ECB that contains date, title (including sub-titles), speaker, content, and footnotes of nearly all speeches by presidents and board members since 1999 (see European Central Bank, 2021b). We perform the following steps:

- We string-match titles and content separately for the following keywords: climate, green, sustainable, greenhouse, environment, warming, climatic, carbon, coal.
- We designate a speech for manual inspection as soon as we have one match for a title or three matches for content (variations did not change results).
- We exclude a speech if insufficient space is devoted to the topic, there is no monetary policy relation, or for a wrong positive (e.g., *environment* refers to low interest rates).
- We exclude speeches that address climate risk or transition risk.
- Speeches within 20 trading days of the previous speech are excluded to avoid overlapping treatment periods.

We exclude communication that refer to *climate risk* and *transition risk*, since these refer to improving disclosure standards, the extent to which climate risk should be considered in credit risk assessment, and asset stranding. These issues are important for the conduct of central bank policy in general but do not specifically address bond markets. This leaves us with four speeches. Table C.3 contains details regarding the key content that motivates our classification.

²¹We only observe small differences in the reactions of the cost terms for optimal collateral policy. The haircut on conventional bonds increases by more compared to the main text. As a result, liquidity management costs rise and costs from debt restructuring fall.

Date	Person	Link	Relevant Quotes
08-11-2018	Benoît Cœuré	ECB	 () the ECB, acting within its mandate, can – and should actively support the transition to a low carbon economy () second, by acting accordingly, without prejudice to price stability. Purchasing green bonds () could be an option, as long as the markets are deep and liquid enough.
27-02-2020	Christine Lagarde	ECB	 () reviewing the extent to which climate-related risks are understood and priced by the market () () evaluate the implications for our own management of risk, in particular through our collateral framework.
17-07-2020	Isabel Schnabel	ECB	 () way in which we can contribute is by taking climate considerations into account when designing and implementing our monetary policy operations. () Of course, central banks would need to be mindful of their effects on market functioning. () severe risks to price stability, central banks are required, within their traditional mandates, to strengthen their efforts ()
21-09-2020	Christine Lagarde	ECB	 We cannot miss this opportunity to reduce and prevent climate risks and finance the necessary green transition. The ECB's ongoing strategy review will ensure that its monetary policy strategy is fit for purpose () () Jean Monnet's words, () opportunity for Europe to take a step towards the forms of organisation of the world of tomorrow.

TABLE C.3: Relevant ECB Policy Announcements

Notes: Speeches are taken from European Central Bank (2021b).

The classification of securities into "green" and "conventional" is based on bonds listed in the "ESG" segments of *Euronext*, the *Frankfurt Stock Exchange* and the *Vienna Stock Exchange*, all of which offer publicly available lists. We limit the analysis to bonds classified as "green" or "sustainable". Since many green bonds do not show up in the *IHS Markit* database, we additionally obtain data from *Thomson Reuters Datastream*. We match green and conventional bonds *one trading-day before* each announcement date using a nearest-neighbors procedure involving coupon, bid-ask spread, maturity, notional amount, and yield spreads. Specifically, we identify an appropriate untreated bond as control group, which is the conventional bond with the smallest distance to the green bond.

TABLE C.4: Matching Green to Conventional Bonds – Summary Statistics

Date		BA-S	pread	Cou	pon	\mathbf{Spr}	ead	Mat	urity	Amo	ount
	#	Green	Conv.	Green	Conv.	Green	Conv.	Green	Conv.	Green	Conv.
08-11-2018	80	0.34	0.33	1.08	1.05	47.50	42.20	7.6	6.0	716	719
27-02-2020	83	0.36	0.32	1.18	1.15	51.66	44.82	6.7	5.2	695	690
17-07-2020	77	0.45	0.38	1.22	1.22	77.49	72.00	6.6	4.9	693	689
21-09-2020	79	0.38	0.36	1.18	1.14	64.94	56.68	6.3	4.6	701	709

Notes: We denote the number of matches by #. Conv. denotes a *conventional* bond. Bond yield spreads over the Euribor/Swap are expressed in basis points. Bid-ask spread and coupon are relative to a face value of 100, maturity is in years. Amount outstanding is in million EUR.



FIGURE C.1: Average Yield Reaction Around Treatment Window

Notes: Results are averaged over all policy announcements. Dashed lines represent 95% confidence intervals. All values in basis points.

We drop a green bond if the distance to the closest conventional bond is too high. Table C.4 contains summary statistics regarding the matching. Coupon and bid-ask spreads are very similar for both types of bonds. Spreads of green bonds are higher by between 5 and 8bp, while their maturity is higher by 1.5 years on average.

C.3.2 Yield Reactions

In Figure C.1, we display the average response across treatment dates. The greenium becomes significant two trading days after each announcement and widens to around 16 bp after 20 trading days.

Table C.5 gives details on single events. We observe significantly negative premia for green bonds up to one month after the treatment events. The strongest effect is visible for ECB president Christine Lagarde's speech on February 27^{th} 2020, which included the first explicit reference to the ECB's collateral framework. Moreover, the speech delivered by Isabel Schnabel on July 17^{th} 2020 stands out, since yields on green bonds significantly increased compared to their conventional counterparts following the event. However, the tone regarding future ECB environmental policy is much more modest than in other speeches. There is also no explicit prospect of preferential treatment in this speech.²²

²²For example, central banks "need to be mindful of their effects on market functioning" and are required to exert effort towards environmental concerns only "within their traditional mandates". Indeed, as our analysis in Section 4.4 and in Appendix C.2.2 predict, considering environmental concerns is not clearly motivated by the

Date	Person	Yield Reaction	Standard Error
08-11-2018	Benoît Cœuré	-7.9***	1.78
27-02-2020	Christine Lagarde	-19.4***	3.89
17-07-2020	Isabel Schnabel	6.8^{***}	1.67
21-09-2020	Christine Lagarde	1.3	1.23

TABLE C.5: Yield Reaction Around ECB Policy Announcements

Notes: We display the *average* yield over 20 days after minus *average* yield over 20 trading day before the policy announcement, relative to the matched control group (in basis points). Significance levels correspond to 10 % (*), 1 % (**) and 0.1 % (***) of Welch's t-test. Speeches are taken from European Central Bank (2021b).

We also perform our analysis for five speeches that are unrelated to environmental policy in Table C.6. We do not find any significantly negative effects and conclude that the overall impact of ECB environmental policy announcement is unlikely to be explained by a general negative trend in the greenium.

TABLE C.6: Yield Reaction Around Non-Related ECB Policy Announcements

Date	Person	Yield Reaction	Standard Error
01-10-2019	Mario Draghi (ECB)	1.61**	0.82
06-11-2019	Luis de Guindos (ECB)	0.77	0.69
16 - 12 - 2019	Luis de Guindos (ECB)	5.06^{***}	0.80
10-06-2020	Isabel Schnabel (ECB)	3.39^{*}	2.54
27-08-2020	Philip R. Lane (ECB)	1.64^{**}	0.81

Notes: We display the *average* yield over 20 days after minus *average* yield over 20 trading day before the policy announcement, relative to the matched control group (in basis points). Significance levels correspond to 10 % (*), 1 % (**) and 0.1 % (***) of Welch's t-test. Speeches are taken from European Central Bank (2021b).

C.4 Data Sources

Table C.7 summarizes the data sources on which our empirical analysis and calibration are based. The classification of bonds as "green" is based on publicly available lists of securities traded via various stock exchanges. Based on the list of ISINs, we retrieve bond-specific info from Datastream. Data on conventional bonds in the control group is taken from Markit. EURIBOR data are also obtained through Datastream. We use the ECB to obtain data on non-financial firm debt, GDP, employment, gross fixed capital formation, private consumption, and the GDP deflator.

mandates of price and financial stability.
Series	Source	Mnemonic
Green Bond List I	Euronext	List retrieved Nov-30-2020
Green Bond List II	Frankfurt SE	List retrieved Nov-30-2020
Green Bond List III	Vienna SE	List retrieved Nov-30-2020
Constant Maturity Ask Price	Datastream	CMPA
Constant Maturity Bid Price	Datastream	CMPB
Coupon	Datastream	С
Issue Date	Datastream	ID
Amount Outstanding	Datastream	AOS
Currency	Datastream	PCUR
Life At Issue	Datastream	LFIS
Redemption Date	Datastream	RD
EURIBOR rates $(\dots = \text{maturity})$	Datastream	TRE6SY
Debt-to-GDP	ECB	QSA.Q.N.I8.W0.S11.S1.C.L.LE.F3T4.T.
		_Z.XDC_R_B1GQ_CYT.S.V.NT
Markit iBoxx Components	IHS Markit	-
GDP	ECB	MNA.Q.Y.I8.W2.S1.S1.B.B1GQZZ.EUR.V.N
Gross fixed capital formation	ECB	MNA.Q.Y.I8.W0.S1.S1.D.P51G.N11GTZ.EUR.V.N
Consumption	ECB	MNA.Q.Y.I8.W0.S1M.S1.D.P31ZZT.EUR.V.N
GDP Deflator	ECB	MNA.Q.Y.I8.W2.S1.S1.B.B1GQZZ.IX.D.N
Employment	ECB	$ENA.Q.Y.I8.W2.S1.S1._Z.EMP._Z._T._Z.PS._Z.N$

TABLE C.7: Data Sources and Ticker

Chapter 5

Macroprudential Regulation of Investment Funds

This chapter is based on Di Iasio et al. (2022).¹

5.1 Introduction

The investment fund sector grew significantly over the last years. In the euro area, assets held by investment funds increased almost fourfold, from around 3.6 EUR trillion in 2002 to more than 14 EUR trillion in 2020 (Figure 5.1, top left panel). Investment funds' assets now amount to 35% of those of the banking sector.² About a third of euro area non-financial corporate bonds is held by the fund sector (Figure 5.1, top right panel).

These developments make the euro area financial system, traditionally bank-based, more diverse. But vulnerabilities associated to investment funds' activities are on the rise as well. Although fund shares are often redeemable at a very short notice, funds became more active in less liquid market segments. At the same time, funds' liquidity buffers, i.e., the share of cash and cash-like instruments in total assets, markedly declined (Figure 5.1, bottom left panel). Small liquidity buffers in combination with large redemptions, such as those registered in March 2020 during the beginning of the Covid-19 pandemic (Figure 5.1, bottom right panel), can force funds to sell relatively illiquid assets.³ Sales can amplify

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The views expressed in this paper are those of the authors only and do not necessarily reflect the views of the Banca d'Italia, the European Central Bank, or the Eurosystem.

²Similar trends are visible at the global level (see FSB, 2020a). Investment funds, also abbreviated as "funds" when ambiguity can be ruled out, are the largest component of the non-bank financial intermediation sector, formerly known as "shadow banking system".

³Outflows reversed only when central banks intervened in financial markets, for example, by means of the ECB's Pandemic Emergency Purchase Program (PEPP) (see Breckenfelder et al., 2021).



FIGURE 5.1: Stylized Facts on Euro Area Investment Funds

asset price deterioration, leading to broader adverse effects on the financing of the economy (see Falato et al., 2021, Morris et al., 2017 and Chernenko and Sunderam, 2016). These events gave additional momentum to the policy debate on macroprudential regulatory options that address vulnerabilities in funds, including minimum liquidity buffers.⁴ Such a regulation could contribute to contain adverse spillovers from the investment fund sector to wider financial markets and the real economy.

In this paper, we develop a dynamic stochastic general equilibrium model (DSGE) to study the macroeconomic effects of liquidity risk in the investment fund sector. Moreover, we analyze the macroeconomic and welfare effects of a macroprudential liquidity buffer of funds. We also discuss the different mechanisms through which the regulation affects our economy. To the best of our knowledge, this is the first paper that studies these issues in a macroeconomic model setting.

Notes: Top left (A): areas show the balance sheet size of banks and investment funds. Black line shows size of the investment fund relative to the banking sector in %. Top right (B): holdings of euro area non-financial corporate bonds in 2019 (all maturities) by holding sectors. Bottom left (C): boxplots show the distribution of cash holdings in % of total assets across funds for two periods, 2009 - 2014 and 2015 - 2019. Cash includes bank deposits, call accounts, call money, and repos. Bottom right (D): fund flows shown as cumulative changes in assets under management of different investment fund types in the euro area. Changes are expressed as relative deviation from assets as of 19 February 2020. Sources: ECB Investment Fund Statistics, ECB Quarterly Sectoral Accounts, Refinitiv Lipper, EPFR Global.

⁴See, for example, IMF (2021), FSB (2020b), and Cominetta et al. (2018).

In our model, non-financial firms issue bonds and receive bank loans, which are imperfectly substitutable, to finance investment. Households invest in fund shares and bank deposits. We assume that the latter also provide liquidity benefits to households in terms of utility, as in Begenau and Landvoigt (2021). Banks issue deposits and use the proceeds to invest into loans directly. Investment funds issue shares, purchase corporate bonds, and hold liquidity in the form of bank deposits. We capture liquidity risk in investment funds by assuming that funds periodically face stochastic redemptions. When outflows exceed the liquidity buffer in terms of deposit holdings, the fund must sell bonds to households who, as second-best users, incur management cost and purchase the assets at a discount, leading to resource losses as in Gertler and Kiyotaki (2015).

Our model includes a pecuniary externality: individual fund managers do not internalize the aggregate price impact of their bond sales. Instead, they only consider how their sales reduce their own profits via a liquidity cost. As a result, funds hold inefficiently low liquidity buffers, which generate bond liquidation.⁵ Bond liquidation implies resource losses and depresses investment fund intermediation. We calibrate the model to the euro area in the late 2010s.

Our main results can be summarized as follows. In the unregulated economy, investment funds hold inefficiently low liquidity buffers because of the pecuniary externality. We calibrate the model such that investment funds voluntarily hold a liquidity buffer of 2% of their assets under management, in line with euro area data (see Figure 5.1, bottom left panel). A higher regulatory liquidity buffer helps to meet periodic redemptions and improves upon the unregulated economy. The optimal regulatory liquidity buffer is about four times higher than in the unregulated economy, amounting to 7.6% of funds' assets under management.

This regulation has benefits and costs. On the one hand, the liquidity buffer improves welfare by reducing periodic bond liquidation and the associated resource losses, which depress consumption. On the other hand, by forcing investment funds to hold a larger fraction of their assets in bank deposits, the regulation is associated with lower bond intermediation. Already for intermediate values of the liquidity buffer, this implies a drop in output due to a change in the firms' financing mix. In addition, if funds hold more deposits, less deposits are held by households, who derive utility benefits from them. Tighter regulation induces investment funds to demand deposits at lower interest rates, thereby increasing households' opportunity cost to hold them. Altogether, fund liquidity regulation trades off the resource gains from lower bond sales against a reduction in (i) bond intermediation and (ii) households' utility from holding deposits. The welfaredecreasing effect of lower bond intermediation is found to be of second-order importance,

⁵For empirical evidence on the relevance of this externality, see Chernenko and Sunderam (2016, 2020) and Falato et al. (2021).

whereas the reduction of household deposits is the main welfare cost associated with the liquidity regulation.⁶ We show that, in absence of this latter mechanism, e.g., in an economy where an alternative storage of liquidity is available to investment funds, the optimal liquidity buffer is much higher, at around 12% of assets under management.

We also consider the response of our economy to a sudden change in household saving preferences for liquid assets (similarly to Fisher, 2015). This shock leads to a shift of households' asset allocation towards bank deposits and away from investment fund shares. A similar dynamic was observed during March 2020. Investment funds respond to the resulting loss of funding by reducing both, deposit and bond holdings. In the absence of the regulation, funds reduce deposits relatively more than bonds given deposits' lower return. This exposes funds to the periodic redemptions by more, ultimately magnifying resource losses from bond sales and reducing fund share dividends. Consequently, investment funds attract even less savings from households and must scale down their bond portfolios as well. At the same time, households' higher preference for bank deposits implies more funding for banks and a lower loan rate. This beneficial effect on production is dampened by the imperfect substitutability of loans and bonds.

The overall effect is an amplification of the initial shock, which leads to a drop in output and consumption. The optimal regulatory liquidity buffer substantially limits this amplification, as investment funds cannot reduce deposits to the same degree as in the absence of the regulation. Redemptions then lead to smaller bond sales and resource losses. As a result, the regulation stabilizes output and consumption, which increases welfare compared to the unregulated economy.

The negative effect of the loss in investment fund financing on output in our simulations is also consistent with empirical evidence that we provide to showcase the macroeconomic relevance of the investment fund sector. Based on a vector-autoregression (VAR) model, we show that fund outflows, which reduce the amount of financial intermediation investment funds can conduct on corporate bond markets, lead to persistent adverse effects on real economic activity in a sample of euro area data starting in 2007.

Related Literature. Closest to our paper is the work by Begenau and Landvoigt (2021). In their model, non-banks face runs that force them to sell capital to households, who are less productive users of capital such that output contracts. Non-banks can default, in which case additional resource losses in terms of capital depreciation and default cost occur. Non-bank leverage is an important aspect to generate default and model dynamics. To reduce the magnitude of sales of capital, the authors propose a tax on non-bank borrowing.

⁶This result is similar to findings by Begenau (2020) in the context of optimal bank regulation.

Our analysis, instead, focuses on the liquidity risk of investment funds, the largest and fastest-growing sector in the non-bank universe. For these entities, and in line with the recent policy discussion, liquidity mismatch rather than leverage is the key vulnerability. In fact, most types of investment funds are legally prohibited from using leverage at a significant scale. Our paper is the first contribution that explicitly analyses investment fund liquidity risks and the effects of macroprudential liquidity regulation of investment funds in a macroeconomic model.

Leverage or insufficient risk-controls have been used extensively in other studies to model non-banks in the context of the 2007-2009 crises, where both features played a prominent role. Verona et al. (2013) show that the presence of shadow banks that differ from banks by the markets they serve and their propensity to underestimate risk leads to a boom-bust cycle in financial markets. Gertler and Kiyotaki (2015) and Gertler et al. (2016) highlight that non-banks are more efficient in financial intermediation at the cost of higher risk, since they are prone to funding shocks. They show that caps on non-bank leverage can reduce such roll-over risk. Based on Gertler et al. (2016), several papers extend the analysis of shadow banks and potential regulatory responses. Rottner (2021) proposes a leverage tax on shadow banks to limit their risk-taking, similar to Begenau and Landvoigt (2021). Poeschl (2020) considers an intervention of central banks on the wholesale funding market after shadow bank runs that are induced by excessive leverage. However, neither paper discusses optimal responses.

Fève et al. (2019) and Meeks et al. (2017) study the role of non-banks in asset securitization, which relaxes funding constraints of banks.⁷ Meeks et al. (2017) introduce central bank asset purchases to address asset price deterioration and the propagation of losses. Ferrante (2018) assumes that non-banks have a superior ability in risk diversification; yet, since they are highly leveraged and subject to runs, their presence adds fragility to financial markets. Ferrante (2018) finds that central bank asset purchases can prevent negative price spirals during runs. As a policy tool, our paper considers instead the macroprudential regulation of non-banks, rather than central bank support interventions. While a fully-fledged comparison between these different policy options is beyond the scope of our paper, regulation can have several advantages, e.g., in terms of moral hazard that is usually associated with ex-post central bank support.

In Gebauer (2021) and Gebauer and Mazelis (2020) tighter capital regulation of banks leads to leakages of financial intermediation to the non-bank sector. Consistent with this result, we find that fund intermediation falls and bank intermediation rises, when a macroprudential liquidity regulation for funds is introduced.

There is also considerable work on non-banks in microeconomic models. For example,

⁷While non-bank securitization vehicles played a large role in the global financial crisis, especially in the United States, their importance has receded afterwards. In European markets, securitization vehicles only make up a small share of the non-bank financial sector, so that we abstract from this feature.

based on the work by Stein (2012), Hanson et al. (2015) assume that traditional banking and shadow banking are different ways to create safe claims. In their setting, shadow bank liabilities are subject to fire sales that give rise to a pecuniary externality. In Di Iasio and Kryczka (2021) investment funds hold inefficiently low amounts of liquidity. Asset sales increase the cost of meeting redemptions and depress fund returns. In line with our results, the liquidity regulation of funds improves upon competitive equilibrium allocations.

Outline. The rest of the paper is structured as follows. In Section 5.2, we provide empirical evidence on the macroeconomic relevance of the investment fund sector. Section 5.3 describes our model. The calibration of the model, all results, and robustness checks are discussed in Section 5.4. Finally, Section 5.5 concludes.

5.2 The Macroeconomic Effect of Fund Outflows

Before presenting our DSGE model analysis, we empirically assess the impact of outflows from investment funds on macroeconomic outcomes in euro area data. As outflows reduce the amount of financial intermediation investment funds can conduct on corporate bond markets, we think of this measure as a proxy for a non-bank credit supply shock. This exercise showcases the macroeconomic relevance of the investment fund sector and represents a useful empirical benchmark for the subsequent model analysis.

We use a VAR to estimate the effects of fund outflows on macro variables in monthly data between April 2007 and June 2019. We consider a VAR with seven variables in the following ordering: the annual inflation in the harmonized index of consumer prices, the log of industrial production, the annual growth in lending of euro area banks, cumulative flows to European corporate bond funds, the spread between BBB-rated euro non-financial corporate bond yields and the 5-year German government bond yield, the yield of the 5-year German Bund itself, and the VSTOXX volatility measure.

Our analysis focuses on funds domiciled in the euro area that have an investment focus on European corporate bond markets. Cumulative flows are measured in percent of lagged assets under management. The corporate bond spread serves as a measure for the severity of financial frictions that has been shown to be a relevant ingredient for deriving sensible macro responses in VAR analyses (see, e.g., Gertler and Karadi, 2015 and Jarociński and Karadi, 2020). The 5-year German Bund is used to capture monetary policy in the model.⁸ Corporate bond spreads and German Bunds are measured in percent. The VSTOXX – the 30-day implied volatility of the EURO STOXX 50 – captures investor risk sentiment, widely acknowledged as a major determinant of fund flows.

⁸By including a yield with a long maturity, we also capture the effects of unconventional monetary policy when short-term interest rates are close to their effective lower bound.



FIGURE 5.2: Impulse Responses to a Fund Outflow Shock

Notes: Impulse response functions to a 1 percentage point shock to bond fund flows obtained from a structural VAR model identified via Cholesky ordering. The blue (grey) areas show 68% (90%) confidence intervals. The y-axes are given in percent for the first four variables, in percentage points for the bond spread and the 5-year yield, and in index points for the VSTOXX. The x-axis shows months after the shock. Data is taken from EPFR Global (Investment fund flows), Markit (bond spread), Datastream (VSTOXX) and various ECB datasets (industrial production, inflation, bank lending, Bund yield).

We choose a lag length of four based on comparing Akaike and Bayesian information criteria. The VAR is conventionally estimated with ordinary least squares. We use the estimated VAR to compute impulse responses for a shock to cumulative fund flows. The shock is identified via Cholesky ordering. The ordering of variables reflects the assumption that industrial production, inflation, and bank lending can respond to changes in the fund flows only with a lag, while financial variables can react immediately.⁹

 $^{^{9}}$ All findings are highly robust to a change in the ordering of the variables, e.g., with fund flows ordered first, and to the inclusion of less and more lags.

Figure 5.2 shows the impulse responses to a 1%-shock on cumulative bond fund flows. The shock implies higher financing costs for firms on corporate bond markets, as visible from the increase in the bond spread. The positive response of the VSTOXX indicates increased uncertainty and a reduction of risk appetite in financial markets. Bank lending does not respond significantly to the shock in the first 11 months, after which it starts falling. Banks may, accordingly, not be able to fully compensate for the reduction in non-bank financial intermediation. The macroeconomic variables react significantly to the shock, from both an economic and statistical perspective. The decrease in fund flows reduces real economic activity, as measured by industrial production, by about 0.4 percentage points after six months before reaching a through of -0.6 after one year. Inflation also falls by up to 0.15 percentage points one year after the shock.

In sum, we find that a decrease in fund financing leads to persistent adverse macroeconomic effects. The related literature, while being still relatively small, arrives at similar conclusions. For example, Ben-Rephael et al. (2021) find that flows towards high-yield bond mutual funds are a highly informative lead indicator for the business cycle. Kaufmann (2020) shows that investment fund flows, triggered by changes in US monetary policy, affect global financial conditions as well as real economic activity in both the United States and the euro area. Barauskaite et al. (2021) estimate the effects of bank and market-based (non-bank) credit supply shocks on euro area GDP in a VAR model. They find that both types of shocks are important drivers of the business cycle and have a similar explanatory power for output.

5.3 The Model

The model consists of households, a financial sector with banks and investment funds, a firm sector that is made up of entrepreneurs, intermediate, capital and final goods producers, and a macroprudential regulator (see Figure 5.3 for an overview). All derivations are provided in Appendix D.1. Unless stated differently, all variables are formulated in real terms.

5.3.1 Households

The representative household derives utility from consumption c_t and from holding bank deposits d_t^{hh} , which provide liquidity services for transactions. The household has disutility from labor n_t , which is supplied to the final good producer. Period-t utility is given by

$$U(c_t, d_t^{hh}, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \delta_t^d \frac{\left(d_t^{hh}\right)^{1-\sigma_d}}{1-\sigma_d} - \psi_n \frac{n_t^{1+\sigma_n}}{1+\sigma_n} ,$$



FIGURE 5.3: Model Overview

where $\sigma, \sigma_n, \sigma_d \geq 0$ denote the relative risk aversion, the inverse Frisch elasticity, and a liquidity preference parameter, respectively. Utility weights for labor and liquidity are given by ψ_n and δ_t^d . We assume that the latter can be stochastic to capture a shock to household's preferences for liquid assets in the spirit of Fisher (2015) and Smets and Wouters (2007). Besides holding bank deposits, households can save in investment fund shares s_t , which pay dividends div_t^{if} but carry no liquidity benefit.¹⁰ The price of shares is q_t^s .

The period-t real budget constraint is

$$c_t + d_t^{hh} + q_t^s s_t + f\left(\tilde{b}_t\right) = w_t n_t + (1 + i_{t-1}^d) d_{t-1}^{hh} + (q_t^s + div_t^{if}) s_{t-1} + \Pi_t , \qquad (5.1)$$

where w_t is the real wage, Π_t are total profits of the financial and non-financial sectors, and i_{t-1}^d is the deposit rate, which is agreed in period t-1 and paid in period t. The term $f\left(\tilde{b}_t\right) = \kappa_{hh}/2 \cdot \tilde{b}_t^2$ captures costs that are associated with intra-period bond sales \tilde{b}_t . We assume that households are second-best users of bonds and face convex management costs when holding corporate bonds sold by investment funds. These costs represent a resource loss as in Gertler and Kiyotaki (2015) and capture households' relative disadvantage in screening bonds. Intuitively, bond investment requires detailed knowledge of firms or the market, which is costly to collect for non-experts. These costs increase in the amount traded and become higher, at the margin, due to the complexity of managing a large portfolio. This not only reflects direct cost from information acquisition, e.g., gaining access to information and processing it, but also that agents might have specific risk-

¹⁰This assumption can be relaxed without changing our results, as long as deposits grant a sufficiently higher liquidity benefit than fund shares.

preferences and need to choose bonds accordingly (see Gârleanu and Pedersen, 2007).¹¹ In comparison, investment into bonds via investment funds is costless, since the latter are assumed to be experts. A related approach that assumes households are less productive users of capital is used in Brunnermeier and Sannikov (2014) and Begenau and Landvoigt (2021). Section 5.3.2.2 describes the mechanism behind bond sales.

Households maximize the discounted sum of life-time utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, d_t^{hh}, n_t) , \qquad (5.2)$$

subject to the sequence of period budget constraints (5.1), where β is the discount factor. The first order conditions (FOCs) of the household for deposits, fund shares, and labor are given by

$$c_t^{-\sigma} = \delta_t^d (d_t^{hh})^{-\sigma_d} + \mathbb{E}_t \left[\beta c_{t+1}^{-\sigma} (1+i_t^d) \right] , \qquad (5.3)$$

$$c_t^{-\sigma} = \mathbb{E}_t \left[\beta c_{t+1}^{-\sigma} \frac{q_{t+1}^s + div_{t+1}^{ij}}{q_t^s} \right] , \qquad (5.4)$$

$$\psi_n(c_t)^\sigma n_t^{\sigma_n} = w_t . ag{5.5}$$

Equation (5.3) is the Euler equation related to deposits. The left-hand side represents the opportunity cost of investing in deposits in terms of forgone marginal utility. The right-hand side denotes the marginal utility benefit from holding deposits plus the expected marginal utility of repayment. Equation (5.4) is the corresponding asset-pricing equation for investment fund shares. Equation (5.5) describes the labor supply decision. We define $\Lambda_{t,t+s} \equiv \beta^s (c_{t+s}/c_t)^{-\sigma}$ as the stochastic discount factor of households.

5.3.2 Financial Sector

There are two types of financial intermediaries, banks and investment funds. Besides their specialization on different types of financial intermediation (loans and bonds, respectively), they differ across important dimensions.

First, by issuing deposits, banks engage in liquidity creation, which provides a utility benefit to households and, as a result, gives banks access to a cheap form of funding.

Second, we assume that households never redeem bank deposits before maturity. This can be motivated by an implicit assumption that bank liabilities are backed by some form of government guarantee, such as a deposit insurance. Investment funds, in turn, are subject to liquidity risk in the form of early redemptions, which can only be settled with liquid assets in the form of deposits or by selling bonds on secondary markets.

¹¹An alternative interpretation is suggested by Pagano and Röell (1996) who show that uninformed investors negotiate disadvantageous terms of trade in the form of higher spreads. Goldstein et al. (2007) estimate substantial gains in trading cost from reducing such an information asymmetry.

5.3.2.1 Banks

Since the focus of the paper is on investment funds, we consider a stylized banking sector. Appendix D.5 proposes a version of the model where banks have more structure, but we show that this affects our results only marginally.

The banking sector finances loans with deposits d_t . Households' non-pecuniary benefit from deposits drives down the deposit rate i_t^d and, thus, banks' cost of funding. Banks grant loans l_t to entrepreneurs at the loan rate i_t^l . Table 5.1 depicts the bank balance sheet.

TABLE 5.1: Bank Balance SheetAssetsLiabilitiesLoans l_t Deposits d_t

Banks are owned by households to whom they transfer their profits as dividends. They maximize the discounted sum of cash-flows div_t^b ,

$$\max_{d_{t},l_{t}} \quad \mathbb{E}_{0} \sum_{t=0}^{\infty} \Lambda_{0,t} \left[d_{t+1} - \left(1 + i_{t}^{d} \right) d_{t} + (1 + i_{t}^{l}) l_{t} - l_{t+1} \right] ,$$

subject to a balance sheet constraint $d_t = l_t$. After repeated substitutions, this leads to the static bank problem

$$\max_{d_t, l_t} \quad i_t^l l_t - i_t^d d_t \;,$$

where $i_t^l l_t$ denotes revenues from lending and $i_t^d d_t$ are the interest payments to depositors. FOCs imply that the deposit rate equals the loan rate,

$$i_t^l = i_t^d . (5.6)$$

5.3.2.2 Investment Funds

Investment fund j issues shares to households and invests in bonds $b_{j,t}$ and bank deposits $d_{j,t}^{if}$. Fund shares are subject to redemption risk that we capture with a two sub-period setup, similarly to Kara and Ozsoy (2020).

Sub-Period I. In the first sub-period, the only market that opens is the secondary market for bonds, where investment funds can sell bonds to households. Fund j enters the sub-period I of period t with its end-of-t - 1 period positions. In the spirit of Bianchi and Bigio (2022), a stochastic fraction $\phi_{j,t}$ of the fund's shares is redeemed by households.¹²

¹²As pointed out by De Fiore et al. (2019), one may think of (not modeled) random idiosyncratic consumption needs of households. Intuitively, households would not use deposits to satisfy the consumption needs as they would lose the utility benefit.

When faced with redemptions, the fund either uses its deposit holdings or sells a fraction $1 - \vartheta_{j,t}$ of its bonds,

$$\phi_{j,t}q_{j,t-1}^{s}s_{j,t-1} \le d_{j,t-1}^{if} + (1 - \vartheta_{j,t})\tilde{q}_{t}^{b}b_{j,t-1} .$$
(5.7)

Investment funds sell bonds to households that value the bonds at the secondary market price \tilde{q}_t^b . As selling bonds is costly, investment funds only do so when deposits are insufficient to cover the liquidity need. Investment funds with insufficient deposits choose to retain the maximum share of bonds $\vartheta_{j,t}$. For these investment funds the redemption constraint (5.7) holds with equality,

$$1 - \vartheta_{j,t} = \frac{\phi_{j,t}q_{j,t-1}^s s_{j,t-1} - d_{j,t-1}^{ij}}{\tilde{q}_t^b b_{j,t-1}}$$

The fraction of bonds sold to households rises in the size of the draw and in the value of fund shares issued. Larger deposits or a higher secondary market bond price imply that a smaller fraction of bonds needs to be sold. Bond sales by fund j are given by

$$\tilde{b}_{j,t} \equiv (1 - \vartheta_{j,t}) b_{j,t-1} = \frac{\phi_{j,t} q_{j,t-1}^s s_{j,t-1} - d_{j,t-1}^{ij}}{\tilde{q}_t^b}$$

Let $\tilde{\phi}_t \equiv d_{t-1}^{if}/(q_{t-1}^s s_{t-1})$ denote the redemption threshold above which investment funds must sell bonds. Since all investment funds hold equal positions at the start of a period, the threshold is not fund-specific. The aggregate bond sales are given by the sum of sales by individual funds with a redemption draw above $\tilde{\phi}_t$

$$\tilde{b}_t = \int_{\tilde{\phi}_t}^1 \tilde{b}_{j,t} g(\phi) d\phi \; ,$$

where $g(\phi)$ denotes the probability density function of the stochastic redemptions ϕ_i .

When purchasing bonds, households face convex costs $f(\tilde{b}_t)$. These can be seen as management costs that reflect households' lack of expertise that increase with bond holdings. As the value of a bond just before maturity is one, we derive the bond price schedule on the secondary market as

$$\tilde{q}_t^b = 1 - f'\left(\tilde{b}_t\right)$$

Since households have convex costs from accepting bonds, the secondary market price is decreasing in sales. We assume households sell their bond holdings back to investment funds at the end of sub-period I. Hence, the liquidity need of investment funds is only temporary and positions are equal again across investment funds at the end of the subperiod.¹³

¹³We follow De Fiore et al. (2019), who assume that a reverse redemption shock hits financial intermediaries at the end of the sub-period, such that households re-invest the redemptions. We also built a more structural version of the model that did not rely on the assumption of full redemption reversion. While it allows to track financial flows more rigorously, it yields little additional insights.

TABLE 5.2: Investment Fund Balance Sheet

Assets	Liabilities
Bonds $q_t^b b_t$ Deposits d_t^{if}	Shares $q_t^s s_t$

-

Sub-Period II. In the second sub-period, all markets open and investment funds make their portfolio choice. Investment funds maximize the discounted sum of dividends

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} di v_t^{if} , \qquad (5.8)$$

where dividends are

$$div_t^{if} = b_{t-1} - q_t^b b_t - d_t^{if} + (1 + i_{t-1}^d) d_{t-1}^{if} - L\left(\tilde{b}_t\right) .$$
(5.9)

In each period, investment funds invest in bank deposits and purchase bonds in the primary market at the price q_t^b . The last term $L(\tilde{b}_t) = \kappa_{if}/2 \cdot \tilde{b}_t^2$ represents convex costs from trading bonds with households on an illiquid secondary market in sub-period I. These costs can be motivated along different dimensions. First, the literature models over-thecounter trading using matching frictions or transaction costs in search and bargaining frameworks. The former represent search expenses or delays in selling the asset. The latter capture broker fees or bid-ask spreads. These models imply rising cost induced by the counterparty's bargaining power or market tightness (see Duffie et al., 2005, 2007, or Rocheteau and Weill, 2011). Second, an alternative interpretation of convex cost arising from asset sales is suggested by the results of Alexander et al. (2007) and Edelen (1999). Sales after redemptions increase cost due to price changes, deviations from the own investment strategy, or commissions, thereby weakening returns and dividends. Chen et al. (2010) show that outflows related to bad performance increase, the higher investment fund illiquidity. We capture such frictions parsimoniously by assuming the convex costs L that increase in the amount of trading similar to Chernenko and Sunderam (2020). Ultimately, costs L create a motive for investment funds to voluntarily hold deposits. Investment funds maximize (5.8) subject to the balance sheet constraint

$$q_t^s s_t = q_t^b b_t + d_t^{if} , (5.10)$$

which is depicted in Table 5.2. This leads to the following FOCs for deposits and bonds:

$$1 + \lambda_t^{if} = \mathbb{E}_t \Lambda_{t,t+1} \left[(1 + i_t^d) - \frac{dL}{d\tilde{b}_{t+1}} \frac{d\tilde{b}_{t+1}}{dd_t^{if}} \right] , \qquad (5.11)$$

$$1 + \lambda_t^{if} = \mathbb{E}_t \Lambda_{t,t+1} \frac{1}{q_t^b} .$$

$$(5.12)$$

Equation (5.11) captures investment funds' deposit investment trade-off. Investing today reduces available resources and tightens the balance sheet constraint (5.10), whose Lagrange multiplier is given by λ_t^{if} . Next period, the deposits yield interest income i_t^d . The second term of the right-hand side is the reduction in liquidity costs. These costs fall because bond sales in sub-period I are reduced for any additional unit of deposits. Equation (5.12) is the FOC related to bond investment.

5.3.3 Non-Financial Sector

Entrepreneurs produce inputs for the final good producer. The latter combines labor with the entrepreneur inputs into the final good that is sold to households and capital producers. The capital producers provide capital and face investment adjustment cost.

5.3.3.1 Entrepreneurs

In each period, there is a unit mass of entrepreneurs who raise funding from banks or investment funds to purchase capital from capital producers at real price $q_t^{k,\tau}$ with $\tau = l, b$. We assume that financing is obtained from one type of financial intermediary only. Accordingly, we distinguish between bond and loan financed entrepreneurs. To retain the notion of an endogenous financing choice while limiting model complexity, we assume both entrepreneur types sell their good to a firm that aggregates their output into an intermediate good sold to the final good producer.

Bond User. Bond using entrepreneurs buy new capital K^b from and sell old depreciated capital to specialized capital producers at the end of a period. They finance the acquisition of new capital by issuing one-period bonds. Entrepreneurs sell their product at price p_t^b to the intermediate good producer. Their profits are

$$div_t^b = p_t^b \left(K_{t-1}^b \right)^{\gamma} - b_{t-1} + q_t^b b_t + (1-\delta) q_t^{k,b} K_{t-1}^b - q_t^{k,b} K_t^b$$

with $q_t^b b_t = q_t^{k,b} K_t^b$,

where δ denotes the rate of capital deprecation. The FOCs yield

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \frac{1}{q_t^b} \right] = \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\gamma p_t^b (K_t^b)^{\gamma-1} + (1-\delta) q_{t+1}^{k,b}}{q_t^{k,b}} \right] .$$
(5.13)

The marginal value from investing into capital, including its marginal product and the re-sale value, is equated with the capital financing cost on bond markets.

Loan User. These entrepreneurs operate the same technology but finance their capital with loans. Profits are given by

$$\begin{aligned} div_t^l &= p_t^l \left(K_{t-1}^l \right)^{\gamma} - (1+i_{t-1}^l) l_{t-1} + l_t + (1-\delta) q_t^{k,l} K_{t-1}^l - q_t^{k,l} K_t^l \\ \text{with } l_t &= q_t^{k,l} K_t^l , \end{aligned}$$

where K_t^l and p_t^l denote loan-user capital and the price of their output, respectively. The FOCs imply

$$\mathbb{E}_t \left[\Lambda_{t,t+1}(1+i_t^l) \right] = \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\gamma p_t^l (K_t^l)^{\gamma-1} + (1-\delta) q_{t+1}^{k,l}}{q_t^{k,l}} \right] .$$

Intermediate Good Producer. Both types of entrepreneurs sell to an intermediate good producer that aggregates these inputs using a CES-technology

$$z_t = \left(\nu(z_t^l)^{\tilde{\epsilon}} + (1-\nu)(z_t^b)^{\tilde{\epsilon}}\right)^{\frac{1}{\tilde{\epsilon}}} , \qquad (5.14)$$

where ν denotes a production weight and $\tilde{\epsilon}$ guides the elasticity of substitution. Input demand follows as

$$\begin{split} z_t^l &= (\nu)^{\frac{1}{1-\bar{\epsilon}}} \left(\frac{p_t^z}{p_t^l}\right)^{\frac{1}{1-\bar{\epsilon}}} z_t \ ,\\ z_t^b &= (1-\nu)^{\frac{1}{1-\bar{\epsilon}}} \left(\frac{p_t^z}{p_t^b}\right)^{\frac{1}{1-\bar{\epsilon}}} z_t \end{split}$$

Since both entrepreneur types operate the same technology but use different sources of funding, one can think of the optimal input mix chosen by the intermediate firm as an endogenous financing choice. The technology parameters ν and $\tilde{\epsilon}$ play an important role to determine the relative sizes of bond and loan finance as well as the ability to switch between financing choices. The two financing choices are not perfect substitutes but, according to the calibrated parameter $\tilde{\epsilon}$, still good substitutes. The notion underpinning this imperfect substitutability is that entrepreneurs face obstacles to switch across financing options, including the time and costs needed to arrange a bond issuance or effort and time to build a bank-relationship.

5.3.3.2 Capital Good Producers

At the end of each period, capital producers purchase depreciated capital from bond and loan financed entrepreneurs and refurbish it into new capital. They purchase the final good to invest into new capital. Their technology only allows them to do so subject to quadratic adjustment cost $\Phi(\frac{I_t^{\tau}}{I_{t-1}^{\tau}}) = \frac{\kappa^I}{2} \left(\frac{I_t^{\tau}}{I_{t-1}^{\tau}} - 1\right)^2$ with $\tau = l, b$. Capital evolves as follows:

$$K_t^{\tau} = (1 - \delta) K_{t-1}^{\tau} + I_t^{\tau} \left(1 - \Phi \left(\frac{I_t^{\tau}}{I_{t-1}^{\tau}} \right) \right)$$

New capital is sold to entrepreneurs at real price $q^{k,\tau}$. Given that the marginal rate of transformation between depreciated and new capital is one, old capital is also valued at this price. The FOC for investment is

$$q_t^{k,\tau} \left[1 - \Phi\left(\frac{I_t^{\tau}}{I_{t-1}^{\tau}}\right) - \frac{I_t^{\tau}}{I_{t-1}^{\tau}} \Phi'\left(\frac{I_t^{\tau}}{I_{t-1}^{\tau}}\right) \right] - \mathbb{E}_t \Lambda_{t,t+1} q_{t+1}^{k,\tau} \left(\frac{I_{t+1}^{\tau}}{I_t^{\tau}}\right)^2 \Phi'\left(\frac{I_{t+1}^{\tau}}{I_t^{\tau}}\right) = 1 \; .$$

5.3.3.3 Final Good Producer

There is a final good producer owned by households that produces the good Y_t . It is produced using the intermediate good z_t and labor of households n_t . The production technology reads

$$Y_t = A_t \left(n_t \right)^{\alpha} \left(z_t \right)^{1-\alpha} , \qquad (5.15)$$

where $\alpha \in (0, 1)$ is the labor share and A_t is the total factor productivity that evolves according to an AR(1) process. The final good producer pays p_t^z for intermediate inputs z_t and the real wage w_t per unit of labor. Profits in period t are

$$\Gamma_t = Y_t - w_t n_t - p_t^z z_t \, .$$

FOCs equalize marginal products with marginal cost,

$$\alpha \frac{Y_t}{n_t} = w_t , \qquad (5.16)$$

$$(1-\alpha)\frac{Y_t}{z_t} = p_t^z \,. \tag{5.17}$$

5.3.4 Resource Constraint and Market Clearing

The aggregate resource constraint is given by

$$Y_t = c_t + \sum_{\tau = l, b} I_t^{\tau} + f(\tilde{b}_t) + L(\tilde{b}_t) .$$
(5.18)

We define net output, i.e., the usage of production aside from cost terms, as

$$Y_t^{net} = c_t + \sum_{\tau = l, b} I_t^{\tau} .$$
 (5.19)

Market clearing for deposits implies

$$d_t = d_t^{hh} + d_t^{if} . ag{5.20}$$

The equilibrium conditions of the model as well as the derivation of the steady state are given in Appendix D.2.

5.3.5 Macroprudential Liquidity Regulation

In this section, we discuss the rationale behind a macroprudential liquidity regulation of investment funds and describe how we integrate it to the model.

Investment funds are subject to a pecuniary externality because of which they operate with an inefficiently low liquidity buffer. In each sub-period I, funds face stochastic redemptions. When the latter are sufficiently high, funds must liquidate bonds, thereby depressing bond prices. Individual funds do not internalize the aggregate price impact of their sales but only consider their own expected liquidity cost L that they face in sub-period II.

In other terms, there is a 'wedge' between the private and social valuation of holding deposits. This is because investment funds take the secondary market bond price as given and do not internalize how their individual choice of deposit holdings affects prices via the aggregate amount of sales (see Chernenko and Sunderam (2016, 2020) and Falato et al. (2021) for empirical evidence). From a social perspective, this has two adverse effects. First, bond sales bring about resource losses that depress consumption via the cost terms f and L in the resource constraint (5.18). Second, bond sales decrease investment fund dividends and the value of fund shares. Eventually, this reduces total bond intermediation via the balance sheet constraint of investment funds (5.10).

The pecuniary externality can be addressed by a regulation that imposes a liquidity buffer for investment funds. This intervention is macroprudential as it considers the general equilibrium effects of the individual investment fund choices on financial markets and the economy. The regulation reduces bond sales and the associated drop in bond prices. This implies lower resource losses and higher dividends paid by investment funds, which increase the market value of shares, $q_t^s s_t$. This can lead to higher bond intermediation if $q_t^s s_t$ increases relatively more than the deposit holdings.

For our model economy, consider a macroprudential regulator who requires investment funds to hold a fraction ρ of its fund shares, a liquidity buffer, in the form of bank deposits,

$$d_t^{if} = \varrho q_t^s s_t . agenum{5.21}{}$$

Note that while the regulatory requirement (5.21) must be met at the end of each period, within periods the liquidity buffer is usable, in the sense that funds can deplete deposits to meet the periodic redemptions. Under the regulation, investment funds attach extra value to deposit holdings, which is also reflected in first order conditions. The equilibrium conditions for the model with regulation are given in Appendix D.3. In the next section, we discuss the effects of regulation in a calibrated version of the model.

5.4 Quantitative Analysis

Section 5.4.1 presents the calibration of the model. We conduct a welfare analysis and solve for the optimal regulatory liquidity buffer in Section 5.4.2. Section 5.4.3 analyses the effects of the regulation in stabilizing the economy after a shock to households' liquidity preferences. This aims to capture some of the dynamics experienced in financial markets during the Covid-19 event in March 2020. Finally, Section 5.4.4 deepens the analysis regarding the asset used for the liquidity buffer.

5.4.1 Calibration

We calibrate the model to euro area data and our period length is one quarter. Some parameters are set in line with the relevant literature, while others are set to target data moments.¹⁴

		Value	Source	
Househo	lds			
σ	Risk aversion	1	Broader literature	
σ_n	(Inverse) Frisch elasticity	3	Broader literature	
ψ_n	Utility weight labor	19.79	Steady state labor $1/3$	
σ_d	Liquidity parameter	1	Broader literature	
δ^d	Steady state utility weight on liquidity	0.02	Broader literature	
Firms				
δ	Depreciation rate of physical capital	0.025	Gerali et al. (2010)	
γ	DRS parameter of entrepreneurs	0.627	Hennessy and Whited (2007)	
α	Labor share	0.67	Labor income share 67%	
Financia	al Sector			
$\tilde{\lambda}$	Scale Lomax Distribution	2.23	Bond fund flow data	
$\tilde{\alpha}$	Shape Lomax Distribution	57.02	Bond fund flow data	
		Calibrated	Target	
β	Household discount factor	0.994	Annual fund return 2.5%	
κ_{if}	IF cost parameter	0.198	Deposit share in IFs' AuM 1.96 $\%$	
κ_{hh}	HH cost parameter	2.84	Bond share HH $2.5~\%$	
ν	Production weight	0.678	Bond to loan finance: 29%	
$\tilde{\epsilon}$	Entrepreneur Aggregator	0.499	Loan-to-GDP 1.5	
ρ_{δ}	Persistence preference shock	0.60	Auto-correlation deposits (HH) 0.8	
σ_{δ}	Std. dev. preference shock	0.001	$\sigma_c/\sigma_Y = 0.59$	
$ ho_a$	Persistence TFP shock	0.96	Auto-correlation GDP 0.85	
σ_a	Std. dev. TFP shock	0.0054	$\sigma_Y = 0.72$	
κ^{I}	Investment Adjustment Cost	0.33	$\sigma_I / \sigma_Y = 3.35$	

TABLE 5.3: Parameter Choices and Calibrated Parameters

We assume log-utility from consumption by setting the risk aversion parameter $\sigma = 1$ and set the Frisch elasticity of labor supply $\sigma_n = 3$, both within the range of common choices. Likewise, we set n = 1/3 in the steady state and choose the utility weight of labor ψ_n accordingly. Utility from deposits is also logarithmic, $\sigma_m = 1$, and the steady state utility weight δ^d is set to the standard value of 0.02 (see, e.g., Begenau, 2020). The labor share is set to $\alpha_n = 0.67$, in accordance with European data. Productivity A is normalized to a value of 1 in the steady state. The entrepreneur return to scale parameter is $\gamma = 0.627$, based on the estimates of Hennessy and Whited (2007).

Notes: We use the following abbreviations. DRS: decreasing returns to scale; AuM: assets under management; IF: investment fund; HH: household; TFP: total factor productivity; Std. dev.: standard deviation; $\sigma_Y, \sigma_c, \sigma_I$: standard deviations of output, consumption, investment.

 $^{^{14}\}mathrm{See}$ Appendix D.4 for additional information on data sources and definitions.

Redemptions of investment fund shares are drawn from a distribution, which is calibrated to data on outflows from euro area corporate bond funds between 2007 and 2019. The data source for this is EPFR Global. We fit a Lomax distribution to the aggregate quarterly outflows using a methods of moments approach. We set the shape parameter of the distribution $\tilde{\alpha} = 57.02$ and the scale parameter $\tilde{\lambda} = 2.23$. This allows us to target the quarterly median redemptions of 2.48% and a standard deviation of 4.05% in the data.¹⁵

The discount factor is set to $\beta = 0.994$ to match an annualized investment fund share return of 2.5%, based on data for representative corporate bond indices from Markit that cover both the investment grade and high yield segments for the period 2010 to 2019.

The next set of parameters is derived jointly by minimizing the distance between data and model moments over a discrete grid. Investment funds and households are subject to quadratic costs. The parameter κ_{hh} in the household bond management cost f directly affects the willingness of households to pay for bonds and, thereby, the amount of bonds sold in the first sub-period. We calibrate κ_{hh} to match the household share in non-financial corporate bond holdings, which is equal to 2.5% in the euro area.¹⁶ The investment fund cost parameter κ_{if} affects the willingness of investment funds to hold deposits. We calibrate the parameter by targeting the median liquidity share in the portfolio of euro area corporate bond funds between 2015 and 2019 of 1.96% (see Figure 5.1). The parameters κ_{if} and κ_{hh} both affect investment funds' deposits and households' bonds. Therefore, we perform sensitivity checks and verify that no other combination of parameters offers a better fit.

The production function of the intermediate good producer (5.14) features two parameters, ν and $\tilde{\epsilon}$. We set ν by targeting the relative size of firm financing via investment funds relative to banks, which is 29%. The ability to substitute bond and loan finance is captured by the parameter $\tilde{\epsilon}$, which we set to target a loan-to-GDP share of 1.5.

The model features two types of shocks. On the supply side, there is a shock to total factor productivity. On the demand side, we use a shock to the household preferences for liquid assets in the form of bank deposits:

$$\log(A_t) = (1 - \rho_a)\log(A) + \rho_a\log(A_{t-1}) + \sigma_a\epsilon_a , \qquad (5.22)$$

$$\log(\delta_t^d) = (1 - \rho_\delta) \log(\delta^d) + \rho_\delta \log(\delta_{t-1}^d) + \sigma_\delta \epsilon_\delta .$$
(5.23)

We calibrate the standard deviations and persistence of the shocks by setting the four parameters σ_a , ρ_a , σ_δ , ρ_δ to target the auto-correlations of output and household deposits, the standard deviation of output, and the relative standard deviation of consumption to

¹⁵Chernenko and Sunderam (2020) use an exponential distribution to model redemption draws. Its shape is comparable to the one of the Lomax distribution.

¹⁶In Begenau and Landvoigt (2021) households are four times less productive than financial intermediaries. Similar to our formulation, a resource loss occurs as soon as households hold capital.

Targeted Moments	Data	Model
IF return	2.50~%	2.50~%
Bond to loan finance	29~%	29.03~%
Deposits in IF assets	1.96~%	1.96~%
Loan-to-GDP	150~%	138~%
Bond share HH	2.50~%	2.65~%
σ_I/σ_Y	3.35	3.14
σ_c/σ_Y	0.59	0.59
σ_Y	0.72	0.73
Auto-correlation Y	0.85	0.75
Auto-correlation d^{hh}	0.86	0.57
Non-Targeted Moments	Data	Model
IF shares in HH saving	17.2~%	23.0~%
Bonds-to-GDP	0.44	0.40
Investment-to-GDP	0.21	0.18
Auto-correlation c	0.82	0.54
Auto-correlation b	0.71	0.60
Auto-correlation s	0.82	0.60

TABLE 5.4: Empirical and Model-Implied Moments

output. Finally, we set the investment adjustment cost parameter κ^{I} by targeting the relative standard deviation of investment to GDP.

Table 5.4 provides a comparison between targeted and non-targeted moments in the model and the data. With respect to targeted parameters, first and second moments are mostly in accordance with the data.

We linearize the model around its deterministic steady state and solve it using Dynare (see Adjemian et al., 2021).

5.4.2 Optimal Liquidity Regulation

In this section, we show that the macroprudential liquidity regulation expressed in (5.21) can improve welfare in the economy and discuss the welfare-relevant trade-offs of the regulation. To this end, we solve a second-order approximation of the model and simulate the economy under the productivity and the deposit preference shocks for different levels of the liquidity buffer.

We compute a utilitarian measure of welfare based on conditional expected utility (5.2) and compare welfare in the economy without the liquidity regulation V^{woreg} to welfare in the economy with the regulation $V^{reg,\varrho}$. We then derive consumption equivalents (CE)

Notes: We calculate theoretical moments after solving and simulating the model under the productivity and the liquidity preference shock. We compare the model moments to Hodrick-Prescott-filtered data of the euro area. $\sigma_Y, \sigma_c, \sigma_I$: standard deviations of output, consumption, investment; IF: investment fund; HH: household.

for different levels of the liquidity buffer ρ ,

$$CE_{\rho} = 100 \cdot \left(\exp\left(1-\beta\right)\left(V^{reg,\varrho}-V^{woreg}\right)-1\right),\,$$

where CE_{ρ} represents the fraction of 'no policy' consumption that the household would be willing to forego to live in an economy with liquidity regulation ρ .

Figure 5.4 plots the welfare measure and long-run means of key variables for different levels of the liquidity buffer. Welfare follows a hump-shaped curve (top left panel). A liquidity buffer of 7.57% is associated with the highest welfare. This optimal buffer is about four times higher than the median value of 1.96% observed in the data (see Figure 5.1).¹⁷

The regulation increases welfare by reducing resource losses, which, in turn, allows higher consumption (top middle panel). Without regulation, funds hold too little deposits because of the pecuniary externality and need to sell bonds. The bond sales lead to resource losses via the liquidity and management costs of investment funds and households. The sales also make fund shares less attractive by decreasing dividends (5.9). Instead, the liquidity regulation ρ forces investment funds to hold a higher liquidity buffer, thereby reducing bond sales and increasing secondary market bond prices (top right panel). The regulatory buffer raises the redemption threshold $\tilde{\phi}$ above which sales occur, i.e., less funds must liquidate bonds. At the optimal buffer, only 15% of investment funds draw a shock in excess of their deposits and must sell bonds, compared to 60% in the equilibrium without regulation. This mechanism is responsible for most of the reduction in bond sales.

The regulation can have two negative effects on welfare. The first one is related to household savings. By imposing mandatory deposit holdings to investment funds, the regulation lowers the return on deposits, thereby inducing households to hold less of them (bottom left panel, blue line). Households' utility from holding deposits, therefore, falls with higher regulatory liquidity buffers.

A second downside of the regulation is due to changes in the financing mix of the economy that can lead to lower net output. Although the regulatory buffer boosts fund dividends and makes fund shares more attractive (bottom left panel, red line), it eventually *reduces* bond intermediation (bottom middle panel, red line). This is because funds' deposit holdings increase relatively more than the market value of fund shares when the regulatory buffer rises. This implies a drop in bond holdings according to the balance sheet constraint (5.10). Overall, the regulation prompts a shift in credit intermediation from funds towards banks.¹⁸ A higher demand for deposits from investment funds lowers the deposit rate, reducing funding costs for banks and making loans and, thus, the goods

¹⁷When simulating the model without regulation for a large number of periods, we find that the buffer held by funds voluntarily fluctuates between 1.84% and 2.08%, i.e., one can interpret our optimal 7.57%-buffer as a minimum regulatory buffer consistent with the policy discussion.

¹⁸Vice versa, studies that discuss *bank regulation* in the presence of non-bank financial intermediaries document leakages of activity from banks to non-banks (see Begenau and Landvoigt, 2021; Gebauer and Mazelis, 2020).



FIGURE 5.4: Optimal Liquidity Regulation Trade-Offs

Notes: We solve a second-order approximation of the model and simulate the economy under a productivity and a preference shock for different levels of the macroprudential liquidity buffer. We calculate conditional welfare to derive consumption equivalents (CE) in terms of the final good. The solid lines depict theoretical long-run means for each simulation. The x-axes start at 1.96%, which is the liquidity buffer voluntarily held by investment funds in absence of regulation. The vertical dashed lines denote the welfare-maximizing liquidity buffer of 7.57%.

produced by loan financed entrepreneurs cheaper. The relative increase of loan financed goods boosts intermediate good production (bottom right panel) for lower values of the buffer, which supports the increase in consumption. But for liquidity buffers above 6.1%, and, hence, before reaching the optimal buffer, intermediate output starts falling due to the imperfect substitutability of loan and bond financed inputs. The increase in loan finance is not sufficient to maintain higher production. The drop in output then weighs down on consumption and welfare.

Figure 5.5 sheds light on the relevance of the mechanisms through which the buffer affects welfare. The left panel focuses on the welfare effects of the reduction of households' deposits. The blue line shows welfare under the regulation, as in Figure 5.4. Using the same model specification, the red line depicts an alternative welfare measure in which household deposits are kept constant. The vertical distance between both curves captures the change in welfare associated with the reduction of household deposits. Ignoring the drop in household deposits, the optimal buffer is 11.15% (red dashed line), well above the 7.57% that is optimal in the baseline case. Thus, this effect is an important driver of the welfare trade-off related to regulation and we explore it further in Section 5.4.4.

The right panel of Figure 5.5 shows how the resource losses evolve when liquidity buffers rise. The reduction of resource losses leads to higher welfare, as it allows for



FIGURE 5.5: Adverse Effects of Liquidity Regulation

Notes: We solve a second-order approximation of the model and simulate the economy under a productivity and a preference shock for different levels of the macroprudential liquidity buffer. We calculate conditional welfare to derive consumption equivalents (CE) in terms of the final good. The solid lines depict theoretical long-run means for each simulation. The x-axes start at 1.96%, which is the liquidity buffer voluntarily held by investment funds in absence of regulation. The left panel compares baseline welfare (blue line) to an alternative welfare measure that keeps household deposits fixed (red line). The dashed lines depict the respective optimal buffers. The right panel shows a decomposition of the resource cost components in (5.18) related to household bond management cost f (red) and fund liquidity cost L (blue).

higher consumption. The colored areas provide a decomposition of the resource cost components in (5.18). These are the household bond management cost f (red area) and the fund liquidity cost L (blue area). Most of the drop in resource costs is achieved via lower household bond management cost, while the liquidity costs of funds play a limited role.

To highlight the contribution of different channels to the welfare trade-off and as a robustness check, Table 5.5 shows optimal buffers, welfare values, and long-run means of relevant variables for different assumptions on a selected set of parameters. To ensure comparability, we express values in terms of the percentage deviations from the respective long-run means of the economies without regulation. Column I of the table shows results for our baseline calibration with regulation. In Column II, utility benefits δ^d that households derive from bank deposits are doubled. As banks become more important as creators of liquidity for households, the welfare trade-off changes: the drop in household deposits induced by the regulation implies a more sizable loss in welfare. Consequently, the optimal liquidity buffer falls to 6.42% and the shift of household saving towards shares is smaller. In Column III, we increase the parameter of household bond management cost κ_{hh} by a factor of four. As a result, the optimal buffer increases to 8.50%, since any reduction in the amount sold to households has a bigger impact on the secondary market price. Finally, in Column IV, we consider larger periodic redemptions. The parameter $\tilde{\lambda}$ governing the random draws from the Lomax distribution is changed. In this calibration,

	Baseline (I)	Deposit utility (II)	Household cost (III)	Redemptions (IV)
Optimal buffer (%)	7.57	6.42	8.50	11.50
CE (%)	0.18	0.31	0.18	0.19
HH Deposits (d^{hh})	-1.44	-1.01	-1.38	-1.31
Fund shares $(q^s s)$	5.54	4.93	5.48	5.14
Capital (b)	-0.51	-0.93	-0.52	-0.10
Capital (l)	0.36	0.51	0.42	0.33
Bond price (\tilde{q}^b)	5.54	7.43	36.63	10.81
Intermediate output (z)	0.08	0.10	0.11	0.14

TABLE 5.5: Sensitivity of Optimal Liquidity Buffers

Notes: Values denote percentage deviations from the respective long-run means of the economies without regulation. Column I "Baseline": calibration as in Section 5.4.1; Column II "Deposit utility": $2 \cdot \delta^d$; Column III "Household Cost": $4 \cdot \kappa_{hh}$; Column IV "Redemptions": $2 \cdot \tilde{\lambda}$.

redemptions are on average twice as high as in the baseline. The amount of bonds sold by investment funds rises, implying higher resource losses. Reducing the price impact from bond sales, thus, becomes even more important. The optimal liquidity buffer rises to 11.50%.

5.4.3 Liquidity Regulation and Aggregate Outflow Shocks

Regulation alleviates the adverse welfare effects of periodic idiosyncratic redemptions by limiting the fraction of funds that must liquidate bonds in every period. To gain further intuition for the findings of the welfare analysis in the last section, we now turn to the analysis of an aggregate shock that triggers a shift in household savings from fund shares into bank deposits. This would test the ability of the optimal liquidity regulation to reduce adverse macroeconomic outcomes related to the investment fund sector. The analysis is motivated by the large-scale outflows from investment funds in March 2020 that can be interpreted as an abrupt change in savers' risk preferences (Figure 5.1, bottom right panel).

Figure 5.6 shows the impulse response functions of a positive shock to the liquidity weight δ_t^d in (5.23) that generates an outflow from the investment fund sector of 1% on impact. Blue solid lines represent the effects of the shock in the economy without regulation.

The shock leads to an overall increase of household savings but to a reduction of those allocated to investment funds. This implies lower financial intermediation through funds, as shown by a decline in bond financed capital investment, as well as a drop in deposits held by funds. Funds reduce their deposit holdings disproportionately more than their bond investment, given the return differential between the two assets. This increases the amount of bonds that the investment fund must sell to cover the periodic redemptions. As



FIGURE 5.6: Impulse Response Functions to a Deposit Preference Shock

Notes: Impulse response functions are shown for a positive shock to δ_t^d (5.23) inducing an outflow from the investment fund sector of 1% on impact. Blue lines denote the economy without regulation, blue dashed lines an economy without periodic redemptions, and red lines an economy with liquidity buffer regulation. Responses of the variables are given in percentage deviations from steady state. The x-axis denotes quarters after the shock. Output is defined as in (5.19). IF: investment fund; HH: household.

a result, the secondary market price drops sharply, while bond sales and resource losses increase. This lowers consumption beyond the decline from the overall shift into savings. Dividends paid by funds fall, leading to a further reduction in the market value of fund shares and, hence, fund assets.

The increase in deposits held by households more than compensates for the drop in fund deposits. Thus, loan financed investment increases. Although bond and loan finance are imperfectly substitutable, the rise in loan financed investment, driven by a falling loan rate, more than compensates the drop in bond financed investment such that aggregate investment increases slightly. In sum, output still falls because of the drop in consumption.

Before analyzing the effects of regulation, we display the amplification effect of our model's key inefficiency, i.e., the liquidity risk in the investment fund sector that generates bond liquidation. The blue dashed lines refer to an economy without periodic redemptions. In this case, investment funds do not need to sell bonds on secondary markets and they have no reason to hold deposits. The amplification effect via funds does not exist. As a result, the decline in fund shares is significantly smaller than for the corresponding solid blue line. Bond financed investment, accordingly, falls by less. The increase in loan financed investment after impact is in fact *higher*, since the dampening effect on loan provision from a decrease in fund deposits (implying a loss of funding for banks) in the

model with redemptions is now absent.

The initial declines of output and consumption are of similar size as in the economy with redemptions (solid blue lines). But while in the case without redemptions both start recovering immediately after the shock, output and consumption decline significantly more in presence of the redemption risk. The reason is that the amplification through bond sales becomes relevant in the period after impact, when investment funds have reduced their deposits.¹⁹

Now, consider the case of regulation. Compared to the economy without regulation (solid blue lines), the optimal liquidity buffer of 7.57% alleviates the shock's impact on fund shares considerably (red lines). The fall in the deposits of the investment funds is almost absent, as deposits are now a fixed fraction of the value of fund shares by regulation. As a result, less bonds need to be sold and the secondary market price hardly reacts. Accordingly, resource losses decline as well. The drop in bond financed investment is reduced, yet bond financed investment is lower when compared to the economy without redemptions (dashed blue lines). This is because under the regulation investment funds always need to maintain the mandatory amount of deposits so that they cannot fully invest in bonds. Again, loan financed investment increases since banks' total deposits, and, thus, their balance sheet size, grow.

Overall, the introduction of the liquidity buffer dampens the negative effects of the shock not only in the investment fund sector but also on macroeconomic aggregates, like consumption and output. Under the optimal regulation, the response of the economy closely follows the one where redemptions do not take place at all.

We follow Gertler and Karadi (2011) and compute the welfare gains from the optimal liquidity buffer compared to the unregulated economy. Based on the second-order approximation of the model, we assume the economy is hit with the single aggregate outflow shock and calculate the CE in every period afterwards, for the regulated and unregulated economies. As we are considering a one-off event – as opposed to a sequence of shocks as in the previous section – we discount and add up the single CE in every period following the shock. We are interested in any welfare gains beyond those taking place in the long-run (see Section 5.4.2) and, therefore, deduct the long-run CE. We find that, when the economy faces the aggregate outflow shock, the optimal buffer increases welfare beyond the long-run gain. A household in the unregulated economy is willing to forego an additional 0.02% of long-run consumption to switch to an economy with the optimal buffer. Hence, the macroprudential liquidity regulation is an effective instrument to address the wider economic ramifications of the liquidity risks in the investment fund sector.

¹⁹Empirically, Ma et al. (2022) find that mutual fund outflows in March 2020 were amplified due to the liquidity mismatch between their assets and liabilities. Our findings are, thus, consistent with their results.



FIGURE 5.7: Optimal Liquidity Regulation Trade-Offs – Government Debt

Notes: We solve a second-order approximation of the model and simulate the economy under a productivity and a preference shock for different levels of the macroprudential liquidity buffer. We calculate conditional welfare to derive consumption equivalents (CE) in terms of the final good. The solid lines depict theoretical long-run means for each simulation. The x-axes start at 1.96%, which is the liquidity buffer voluntarily held by investment funds in absence of regulation. The simulations are based on a model version where government debt is used to fulfill the regulatory liquidity buffer. The vertical dashed lines denote the welfare-maximizing liquidity buffer of 12.08%.

5.4.4 Alternative Storage of Liquidity

The previous sections suggest that an important effect of liquidity regulation on welfare is the reduction of deposits held by households. In this section, we discuss a case where funds use an alternative asset to store liquidity.²⁰ This allows us to consider a situation in which there is no competition among agents for the liquid asset.

To minimize changes to the model, we consider that the alternative asset m_t investment funds can hold is one-period government debt. Since households are now the only investor in bank deposits, aggregate deposits and household deposits coincide, $d_t = d_t^{hh}$, replacing the previous deposit market clearing condition (5.20). We add a public budget constraint

$$m_t + t_t = \left(1 + i_{t-1}^d\right) m_{t-1} , \qquad (5.24)$$

where t_t is a lump-sum tax paid by households that clears the government budget. We assume the interest rate on government debt is equal to the deposit rate, as in Gertler and Karadi (2011), while we keep parameters unchanged.

Figure 5.7 shows results for the welfare analysis with government debt. The optimal liquidity buffer is at 12.08%, which is significantly higher than the level of 7.57% in the

²⁰In practice, asset managers use a variety of instruments to hold liquidity, including bank deposits, reverse repos with banks, as well as short-term government securities.

baseline case of Section 5.4.2. Importantly, bank deposits held by households no longer decrease but increase instead (bottom left panel). This finances the increase in bank loans (bottom middle panel), whereas in our baseline case, the increased loan origination is financed by additional deposits held by investment funds.

This difference affects the intermediate production (bottom right panel), which already declines for small buffers. In contrast to the economy of Figure 5.5, loans do not get cheaper when fund regulation tightens. The increase in loans, triggered by falling bond intermediation from tighter regulation, is mirrored by rising household deposits. Due to a falling marginal utility from deposits, there is an increase of the deposit and, thus, the loan rate, leading to a reduction in production in equilibrium. The optimal liquidity buffer is reached as soon as the resource gain from reducing bond sales is more than offset by the reduction in output. In the baseline case, the utility loss from declining household deposit drives the hump-shape of the welfare curve instead (see Figure 5.5, left panel).

In an economy with an alternative storage of investment fund liquidity, there remains to be a welfare trade-off, yet adverse welfare effects of regulation are smaller. Regulation no longer lowers deposit returns and, effectively, pushes households out of deposits. On a more general level, this shows that there are relevant interactions between the availability of liquid assets and the liquidity regulation of non-banks. The result can inform the debate on the possibility to grant certain non-banks access to central bank liabilities (see Stein, 2012). Indeed, when liquid assets are in high demand by multiple sectors, central banks could expand the supply of such assets to certain non-banks by establishing dedicated deposit facilities. Examples include the Reverse Repo Facility introduced by the Federal Reserve in 2013 (see Anderson and Kandrac, 2018).

5.5 Conclusion

The last two decades witnessed an extraordinary growth of the investment fund sector and of its importance in the financing of the real economy. The financial market disruptions in March 2020 showed that investment funds can contribute to amplify macroeconomic and financial shocks. Large-scale outflows put extreme pressure on funds that were forced to sell assets in increasingly illiquid markets. These developments catalyzed the debate on regulatory options to mitigate vulnerabilities in the investment fund sector.

In this paper, we analyze the role of the investment fund sector in the macroeconomy as well as its regulation from two angles. First, in a motivational empirical analysis, we document that outflows from investment funds, by reducing the sector's financial intermediation capacity, have significant and persistent adverse macroeconomic effects. Second, as the main contribution of the paper, we develop a DSGE model with two types of financial intermediaries, banks and investment funds. The latter are subject to stochastic periodic redemptions that can lead to costly bond sales. Individual funds fail to internalize the full impact of sales on the bond price and hold inefficiently low liquidity buffers. This pecuniary externality eventually results in lower bond intermediation and resource losses.

We show that a macroprudential liquidity buffer improves upon the unregulated economy by limiting bond sales. The optimal liquidity buffer is 7.57%, which is about fourtimes the median liquidity holdings of investment funds in the euro area. Our model allows us to identify different channels through which the regulation affects welfare and we disentangle benefits and costs of the regulation. Aside from reducing welfare losses stemming from the periodic redemptions, the regulation successfully contains the amplification of financial shocks and limits their adverse macroeconomic effects in a scenario reminiscent of the March 2020 episode.

Our paper constitutes the first analysis of macroprudential policies that addresses the liquidity risk of investment funds in a macroeconomic model. In future research, our model could be enriched to explore other policy tools, including central bank asset purchases or granting certain non-banks access to central bank facilities. This avenue would also allow us to study interactions between monetary policy and macroprudential policies for the non-bank financial sector.

Appendix

D.1 Model Derivations

D.1.1 Household

Households maximize the discounted value of life-time utility subject to the real period budget constraints. The Lagrangian reads

$$\mathcal{L} = \mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} \left[\frac{c_{t}^{1-\sigma}}{1-\sigma} + \delta_{t}^{d} \frac{d_{t}^{1-\sigma_{d}}}{1-\sigma_{d}} - \psi_{n} \frac{n_{t}^{1+\sigma_{n}}}{1+\sigma_{n}} + \lambda_{t} \left(w_{t}n_{t} + \left(1 + i_{t-1}^{d}\right) d_{t-1}^{hh} + \left(q_{t}^{s} + div_{t}^{if}\right) s_{t-1} + \Pi_{t} - c_{t} - d_{t}^{hh} - q_{t}^{s} s_{t} - f(\tilde{b}_{t}) \right) \right].$$

The FOCs are

$$\begin{split} &\frac{\partial \mathcal{L}}{\partial c_t} = c_t^{-\sigma} - \lambda_t = 0 ,\\ &\frac{\partial \mathcal{L}}{\partial d_t^{hh}} = \delta_t^d d_t^{-\sigma_d} - \lambda_t + \beta \mathbb{E}_t \left[\lambda_{t+1} (1+i_t^d) \right] = 0 ,\\ &\frac{\partial \mathcal{L}}{\partial s_t} = -\lambda_t + \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{q_{t+1}^s + div_{t+1}^{if}}{q_t^s} \right] = 0 ,\\ &\frac{\partial \mathcal{L}}{\partial n_t} = -\psi_n n_t^{\sigma_n} + \lambda_t w_t = 0 . \end{split}$$

D.1.2 Banks

The representative bank faces the balance sheet constraint $d_t = l_t$ in every period and maximizes the discounted sum of profits. Repeated substitution of the balance sheet renders its problem static. It maximizes the cash-flow from its portfolio in period t

$$\max_{d_t, l_t} \quad i_t^l l_t - i_t^d d_t \; .$$

subject to the balance sheet constraint. The FOCs are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial d_t} &= -i_t^d + \lambda_t = 0 ,\\ \frac{\partial \mathcal{L}}{\partial l_t} &= i_t^l - \lambda_t = 0 , \end{aligned}$$

where λ_t denotes the multiplier on the period t balance sheet constraint.

D.1.3 Investment Funds

Investment funds maximize the discounted value of their dividends. They issue fund shares and invest into bonds and bank deposits. In the first part of a period, they are subject to redemption risk in the sense that a fraction of their fund shares is redeemed early by households which requires an immediate settlement. Settlement of redemptions is done through deposits or selling bonds to households.

Redemptions occur for all investment funds but differ in size. The i.i.d. draws follow a Lomax distribution with parameters $\tilde{\alpha}$ and $\tilde{\lambda}$. Note that given $g(\phi) = \frac{\tilde{\alpha}}{\tilde{\lambda}} (1 + \frac{\phi}{\tilde{\lambda}})^{-(\tilde{\alpha}+1)}$,

$$\begin{split} \int_{\tilde{\phi}_t}^{\infty} \phi_{j,t} g(\phi) d\phi &= \left[-\phi_{j,t} \left(1 + \frac{\phi_{j,t}}{\tilde{\lambda}} \right)^{-\tilde{\alpha}} - \frac{\tilde{\lambda}}{\tilde{\alpha} - 1} \left(1 + \frac{\phi_{j,t}}{\tilde{\lambda}} \right)^{-\tilde{\alpha} + 1} \right]_{\tilde{\phi}_t}^{\infty} ,\\ \int_{\tilde{\phi}_t}^{\infty} g(\phi) d\phi &= \left[1 - \left(1 + \frac{\phi_{j,t}}{\tilde{\lambda}} \right)^{-\tilde{\alpha}} \right]_{\tilde{\phi}_t}^{\infty} . \end{split}$$

Aggregating across all draws gives the aggregate redemption,

$$\int_0^\infty \phi_{j,t} g(\phi) d\phi = \frac{\tilde{\lambda}}{\tilde{\alpha} - 1} ,$$

which is just the mean redemption. Note that we use an unbounded Lomax distribution so that draws above one are possible. However, given the fitted parameters of our distribution the probability to have draws above one is 7e - 8% so that we stick with the general distribution instead of a bounded Pareto distribution. Investment fund j sells the fraction,

$$1 - \vartheta_{j,t} = \frac{\phi_{j,t} q_{j,t-1}^s s_{j,t-1} - d_{j,t-1}^{if}}{\tilde{q}_t^b b_{j,t-1}} ,$$

of beginning-of-period bonds. The amount sold by investment fund j is then,

$$\tilde{b}_{j,t} \equiv (1 - \vartheta_{j,t}) b_{j,t-1} = \frac{\phi_{j,t} q_{j,t-1}^s s_{j,t-1} - d_{j,t-1}^{if}}{\tilde{q}_t^b}$$

Aggregating across i.i.d draws gives (using that all investment funds hold the same initial positions, e.g., $s_{j,t-1} = s_{t-1}$ and $q_{j,t-1}^s = q_{t-1}^s$),

$$\tilde{b}_t = \frac{1}{\tilde{q}_t^b} \left(1 + \frac{\tilde{\phi}_t}{\tilde{\lambda}} \right)^{-\tilde{\alpha}} \left(\frac{\tilde{\lambda} + \tilde{\alpha}\tilde{\phi}_t}{\tilde{\alpha} - 1} q_{t-1}^s s_{t-1} - d_{t-1}^{if} \right)$$

Since the probability of a draw above the threshold value is given by $1 - G = \left(1 + \frac{\tilde{\phi}_t}{\tilde{\lambda}}\right)^{-\tilde{\alpha}}$, the amount of bonds sold is weighted by the probability of a (sufficiently) high draw. G denotes the cumulative density function of the redemption distribution. Intuitively, cost fall in the amount of deposits and increase in the amount of fund shares issued. The latter is multiplied by a factor that corresponds to the mean redemption plus a term

that increases in the threshold draw above which investment funds start to sell bonds. Theoretically, a high threshold has an ambiguous effect. While it lowers the probability that sales occur, it also implies that, if sales occur, redemptions are higher. However, given $\tilde{\alpha} > 1$ the net effect can be shown to be negative.

The secondary market price can be expressed as

$$\tilde{q}_t^b = 1 - \kappa_{hh} \tilde{b}_t \; .$$

Investment funds further obey a balance sheet constraint $q_t^b b_t + d_t^{if} = q_t^s s_t$. At the end of a period, they transfer all income as dividends to households,

$$div_t^{if} = b_{t-1} - q_t^b b_t - d_t^{if} + (1 + i_{t-1}^d) d_{t-1}^{if} - L(\tilde{b}_t) ,$$

where we assume $L(\tilde{b}_t) = \frac{\kappa_{if}}{2} (\tilde{b}_t)^2$. The problem of investment funds can then be written as follows:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[b_{t-1} - q_t^b b_t - d_t^{if} + (1 + i_{t-1}^d) d_{t-1}^{if} - \frac{\kappa_{if}}{2} \left(\tilde{b}_t \right)^2 + \lambda_t^{if} \left(q_t^s s_t - q_t^b b_t - d_t^{if} \right) \right]$$

FOCs for bonds and bank deposits follow,

$$\frac{\partial \mathcal{L}}{\partial b_t} = -q_t^b + \mathbb{E}_t \Lambda_{t,t+1} - \lambda_t^{if} q_t^b = 0 ,$$

$$\frac{\partial \mathcal{L}}{\partial d_t^{if}} = \mathbb{E}_t \Lambda_{t,t+1} \left(\left(1 + i_t^d \right) + \frac{1}{\tilde{q}_{t+1}^b} \left(\kappa_{if} \tilde{b}_{t+1} \left(1 + \frac{\tilde{\phi}_{t+1}}{\tilde{\lambda}} \right)^{-\tilde{\alpha}} \right) \right) - 1 - \lambda_t^{if} = 0 ,$$

with λ_t^{if} denoting the multiplier on the balance sheet constraint.

D.1.4 Entrepreneurs

The problem of loan-financed entrepreneurs reads

$$\max_{l_t, K_t^l} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left(p_t^l \left(K_{t-1}^l \right)^{\gamma} - (1+i_{t-1}^l) l_{t-1} + l_t + (1-\delta) q_t^{k,l} K_{t-1}^l - q_t^{k,l} K_t^l \right)$$

subject to $l_t = q_t^{k,l} K_t^l$.

The FOCs for capital and loans from maximizing the discounted value of dividends are

$$1 - \lambda_t^l = \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\gamma p_{t+1}^l (K_t^l)^{\gamma - 1} + (1 - \delta) q_{t+1}^{k,l}}{q_t^{k,l}} \right] ,$$

$$1 - \lambda_t^l = \mathbb{E}_t \left[\Lambda_{t,t+1} (1 + i_t^l) \right] ,$$

where λ_t^l is the multiplier on the financing constraint. Similarly, for bond-financed entrepreneurs we have,

$$\max_{b_t, K_t^b} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left(p_t^b \left(K_{t-1}^b \right)^{\gamma} - b_{t-1} + q_t^b b_t + (1-\delta) q_t^{k,b} K_{t-1}^b - q_t^{k,b} K_t^b \right)$$

subject to
$$q_t^b b_t = q_t^{k,b} K_t^b$$
.

The FOCs for capital and bonds are

$$1 - \lambda_t^b = \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\gamma p_{t+1}^b (K_t^b)^{\gamma - 1} + (1 - \delta) q_{t+1}^{k,b}}{q_t^{k,b}} \right] ,$$

$$1 - \lambda_t^b = \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{1}{q_t^b} \right] ,$$

where λ_t^b is the multiplier on the financing constraint. The intermediate good producer buys input from both types and uses a CES-technology. It maximizes

$$div_t^{entr} = p_t^z \left(\nu(z_t^l)^{\tilde{\epsilon}} + (1-\nu)(z_t^b)^{\tilde{\epsilon}} \right)^{\frac{1}{\tilde{\epsilon}}} - p_t^l z_t^l - p_t^b z_t^b .$$

The FOCs yield the demand equations

$$\begin{split} z_t^l &= (\nu)^{\frac{1}{1-\tilde{\epsilon}}} \left(\frac{p_t^z}{p_t^l}\right)^{\frac{1}{1-\tilde{\epsilon}}} z_t \ ,\\ z_t^b &= (1-\nu)^{\frac{1}{1-\tilde{\epsilon}}} \left(\frac{p_t^l}{p_t^b}\right)^{\frac{1}{1-\tilde{\epsilon}}} z_t \end{split}$$

D.1.5 Capital Good Producer

There are two types of capital good producers that purchase depreciated capital from loan- and bond-financed firms, invest into new capital subject to adjustment cost, and resell the new capital to entrepreneurs. Derivations hold for $\tau = l, b$. Capital evolves as

$$K_t^{\tau} = (1 - \delta) K_{t-1}^{\tau} + I_t^{\tau} \left(1 - \frac{\kappa^I}{2} \left(\frac{I_t^{\tau}}{I_{t-1}^{\tau}} - 1 \right)^2 \right) .$$

New capital is sold at price $q_t^{k,\tau}$ to entrepreneur of type τ . The problem of the capital good producer is given by maximizing real profits

$$\max_{I_t^{\tau}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[q_t^{k,\tau} \left((1-\delta) K_{t-1}^{\tau} + I_t^{\tau} \left(1 - \frac{\kappa^I}{2} \left(\frac{I_t^{\tau}}{I_{t-1}^{\tau}} - 1 \right)^2 \right) \right) - I_t^{\tau} - q_t^{k,\tau} (1-\delta) K_{t-1}^{\tau} \right] .$$

The FOC with respect to investment reads

$$q_{t}^{k,\tau} \left[1 - \frac{\kappa^{I}}{2} \left(\frac{I_{t}^{\tau}}{I_{t-1}^{\tau}} - 1 \right) - \frac{I_{t}^{\tau}}{I_{t-1}^{\tau}} \kappa^{I} \left(\frac{I_{t}^{\tau}}{I_{t-1}^{\tau}} - 1 \right) \right] + \mathbb{E}_{t} \Lambda_{t,t+1} q_{t+1}^{k,\tau} \left(\frac{I_{t+1}^{\tau}}{I_{t}^{\tau}} \right)^{2} \left(\frac{I_{t+1}^{\tau}}{I_{t}^{\tau}} - 1 \right) = 1 \; .$$

D.1.6 Final Good Firms

The final good producer uses labor and the intermediate good to produce the final good. Their profits read

$$\Gamma_t = A_t \left(n_t \right)^{\alpha} \left(z_t \right)_t^{1-\alpha} - w_t n_t - p_t^z z_t .$$

FOCs for labor and intermediate goods equate the marginal products with input prices:

$$\alpha A_t (n_t)^{\alpha - 1} (z_t)_t^{1 - \alpha} = w_t , \qquad (D.1.1)$$

$$(1 - \alpha)A_t (n_t)^{\alpha} (z_t)_t^{-\alpha} = p_t^z.$$
 (D.1.2)

D.1.7 Derivation of Resource Constraint

When deriving the resource constraint, we take the household budget and insert profits of entrepreneurs, the intermediate good producer, the final good producer, capital producers, and banks,

$$\begin{aligned} c_t + d_t^{hh} + q_t^s s_t + f(\tilde{b}_t) &= w_t n_t + (1 + i_{t-1}^d) d_{t-1}^{hh} + \left(q_t^s + div_t^{if}\right) s_{t-1} + p_t^z z_t - p_t^b z_t^b - p_t^l z_t^l \\ &+ p_t^b z_t^b - b_{t-1} + q_t^b b_t + (1 - \delta) q_t^{k,b} K_{t-1}^b - q_t^{k,b} K_t^b + p_t^l z_t^l - (1 + i_{t-1}^l) l_{t-1} + (1 - \delta) q_t^{k,l} K_{t-1}^l \\ &+ l_t - q_t^{k,l} K_t^l + Y_t - p_t^z z_t - w_t n_t + div_t^b + \sum_{\tau = l, b} \left(q_t^{k,\tau} K_t^\tau - I_t^\tau - q_t^{k,\tau} (1 - \delta) K_{t-1}^\tau \right) \,. \end{aligned}$$

Many terms cancel directly. We normalize $s_t = 1$.

$$c_t + d_t^{hh} + f(\tilde{b}_t) = (1 + i_{t-1}^d) d_{t-1}^{hh} + div_t^{if} - b_{t-1} + q_t^b b_t - (1 + i_{t-1}^l) l_{t-1} + l_t + Y_t - \sum_{\tau = l, b} I_t^{\tau} + div_t^b .$$

Next, we eliminate bank-related terms. Recall $d_t = d_t^{hh} + d_t^{if}$ and $d_t = l_t$ so $d_t^{hh} = l_t - d_t^{if}$,

$$c_t + l_t - d_t^{if} + f(\tilde{b}_t) = (1 + i_{t-1}^d) d_{t-1}^{hh} + div_t^{if} - b_{t-1} + q_t^b b_t - (1 + i_{t-1}^l) l_{t-1} + l_t + Y_t - \sum_{\tau = l, b} I_t^{\tau} + div_t^b .$$

Using the bank balance sheet for t-1, $d_{t-1}^{hh} = l_{t-1} - d_{t-1}^{if}$ and that $i_{t-1}^l = i_{t-1}^d$ yields

$$c_t - d_t^{if} + f(\tilde{b}_t) = (1 + i_{t-1}^d)(l_{t-1} - d_{t-1}^{if}) + div_t^{if} - b_{t-1} + q_t^b b_t - (1 + i_{t-1}^d)l_{t-1} + Y_t - \sum_{\tau = l, b} I_t^{\tau} .$$

Next, we insert investment fund dividends

$$div_t^{if} = b_{t-1} - q_t^b b_t - d_t^{if} + (1 + i_{t-1}^d) d_{t-1}^{if} - L(\tilde{b}_t) .$$

Inserting yields:

$$Y_t = c_t + \sum_{\tau=l,b} I_t^{\tau} + f(\tilde{b}_t) + L(\tilde{b}_t) .$$

D.2 Equilibrium Without Regulation

Households

$$c_t^{-\sigma} = \delta^d (d_t^{hh})^{-\sigma_d} + \mathbb{E}_t \left[\beta c_{t+1}^{-\sigma} (1+i_t^d) \right] , \qquad (D.2.1)$$

$$c_t^{-\sigma} = \mathbb{E}_t \left[\beta c_{t+1}^{-\sigma} \frac{q_{t+1}^s + div_{t+1}^s}{q_t^s} \right] , \qquad (D.2.2)$$

$$\psi_n(c_t)^\sigma n_t^{\sigma_n} = w_t , \qquad (D.2.3)$$

$$\Lambda_{t,t+1} = \beta \left(\frac{c_t}{c_{t+1}}\right)^{\sigma}. \tag{D.2.4}$$

Banks

$$i_t^l = i_t^d , \qquad (D.2.5)$$

$$d_t = l_t , \qquad (D.2.6)$$

$$d_t = d_t^{hh} + d_t^{if} . ag{D.2.7}$$

Investment Funds

$$div_t^{if} = b_{t-1} - q_t^b b_t + (1 + i_{t-1}^d) d_{t-1}^{if} - d_t^{if} - L(\tilde{b}_t) , \qquad (D.2.8)$$

$$1 + \lambda_t^{if} = \mathbb{E}_t \Lambda_{t,t+1} \left(\left(1 + i_t^d \right) + \frac{\kappa_{if} b_{t+1}}{\tilde{q}_{t+1}^b} \left(1 + \frac{\phi_{t+1}}{\tilde{\lambda}} \right)^{-\alpha} \right) , \qquad (D.2.9)$$

$$q_t^b = -q_t^b \lambda_t^{if} + \mathbb{E}_t \Lambda_{t,t+1} , \qquad (D.2.10)$$

$$q_t^s s_t = q_t^o b_t + d_t^{ij} , \qquad (D.2.11)$$

$$\tilde{\phi}_t = \frac{d_{t-1}^{if}}{q_{t-1}^s s_{t-1}} \,. \tag{D.2.12}$$

Bond Sales

$$\tilde{b}_{t} = \frac{1}{\tilde{q}_{t}^{b}} \left(1 + \frac{\tilde{\phi}_{t}}{\tilde{\lambda}} \right)^{-\tilde{\alpha}} \left(\frac{\tilde{\lambda} + \tilde{\alpha}\tilde{\phi}_{t}}{\tilde{\alpha} - 1} q_{t-1}^{s} s_{t-1} - d_{t-1}^{if} \right) , \qquad (D.2.13)$$

$$\tilde{q}_t^b = 1 - \kappa_{hh} \tilde{b}_t . \tag{D.2.14}$$

Loan-Using Entrepreneur

$$l_t = q_t^{k,l} K_t^l , \qquad (D.2.15)$$

$$z_t^l = \left(K_{t-1}^l\right)^\gamma , \qquad (D.2.16)$$

$$\mathbb{E}_t \left[\Lambda_{t,t+1} (1+i_t^l) \right] = \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\gamma p_t^l (K_t^l)^{\gamma-1} + (1-\delta) q_{t+1}^{k,l}}{q_t^{k,l}} \right] .$$
(D.2.17)

Bond-Using Entrepreneur

$$q_t^b b_t = q_t^{k,b} K_t^b ,$$
 (D.2.18)
$$z_t^b = \left(K_{t-1}^b\right)^\gamma , \qquad (D.2.19)$$

$$\mathbb{E}_{t}\left[\Lambda_{t,t+1}\frac{1}{q_{t}^{b}}\right] = \mathbb{E}_{t}\left[\Lambda_{t,t+1}\frac{\gamma p_{t}^{b}(K_{t}^{b})^{\gamma-1} + (1-\delta)q_{t+1}^{k,b}}{q_{t}^{k,b}}\right]$$
(D.2.20)

Intermediate Good Producer

$$z_t = \left(\nu(z_t^l)^{\tilde{\epsilon}} + (1-\nu)(z_t^b)^{\tilde{\epsilon}}\right)^{\frac{1}{\tilde{\epsilon}}} , \qquad (D.2.21)$$

$$z_t^l = \left(\nu \frac{p_t^z}{p_t^l}\right)^{1-\epsilon} z_t , \qquad (D.2.22)$$

$$z_t^b = \left((1-\nu) \frac{p_t^z}{p_t^b} \right)^{\frac{1}{1-\epsilon}} z_t . \tag{D.2.23}$$

Capital Producer

$$K_t^l = (1 - \delta) K_{t-1}^l + I_t^l \left(1 - \Phi\left(\frac{I_t^l}{I_{t-1}^l}\right) \right) , \qquad (D.2.24)$$

$$1 = q_t^{k,l} \left[1 - \frac{\kappa^I}{2} \left(\frac{I_t^l}{I_{t-1}^l} - 1 \right) - \frac{I_t^l}{I_{t-1}^l} \kappa^I \left(\frac{I_t^l}{I_{t-1}^l} - 1 \right) \right] \\ + \mathbb{E}_t \Lambda_{t,t+1} q_{t+1}^{k,l} \left(\frac{I_{t+1}^l}{I_t^l} \right)^2 \left(\frac{I_{t+1}^l}{I_t^l} - 1 \right) , \qquad (D.2.25)$$

$$K_t^b = (1 - \delta) K_{t-1}^b + I_t^b \left(1 - \Phi\left(\frac{I_t^b}{I_{t-1}^b}\right) \right) , \qquad (D.2.26)$$

$$1 = q_t^{k,b} \left[1 - \frac{\kappa^I}{2} \left(\frac{I_t^b}{I_{t-1}^b} - 1 \right) - \frac{I_t^b}{I_{t-1}^b} \kappa^I \left(\frac{I_t^b}{I_{t-1}^b} - 1 \right) \right] \\ + \mathbb{E}_t \Lambda_{t,t+1} q_{t+1}^{k,b} \left(\frac{I_{t+1}^b}{I_t^b} \right)^2 \left(\frac{I_{t+1}^b}{I_t^b} - 1 \right) .$$
(D.2.27)

Final Good Producer

$$\alpha \frac{Y_t}{n_t} = w_t , \qquad (D.2.28)$$

$$(1-\alpha)\frac{Y_t}{z_t} = p_t^z$$
, (D.2.29)

$$Y_t = A_t (n_t)^{\alpha_n} (z_t)^{1-\alpha_n}$$
 (D.2.30)

Resource Constraint

$$Y_{t} = c_{t} + \sum_{\tau=l,b} \frac{\kappa^{I}}{2} \left(\frac{I_{t}^{\tau}}{I_{t-1}^{\tau}} - 1 \right)^{2} I_{t}^{\tau} + \sum_{\tau=l,b} q_{t}^{K,\tau} \left(K_{t}^{\tau} - (1-\delta) K_{t-1}^{\tau} \right) + f(\tilde{b}_{t}) + L(\tilde{b}_{t}) .$$
(D.2.31)

Shocks

$$\log A_{t+1} = (1 - \rho^a) \log A^* + \rho^a \log A_t + \epsilon_t , \qquad (D.2.32)$$

$$\log \delta_{t+1}^d = \left(1 - \rho^\delta\right) \log \delta^{d,*} + \rho^\lambda \log \delta_t^d + \epsilon_\delta . \tag{D.2.33}$$

We set s = 1. This gives 33 equations and 33 unknowns:

- Quantities: $c, n, z, z^b, z^l, K^b, K^l, l, b, Y, q^s, d, d^{if}, d^{hh}, I^l, I^b, \tilde{b}_t, div^{if}$
- Prices & Interest Rates: $w, i^d, i^l, p^z, p^b, p^l, q^b, \tilde{q}^b, q^{k,l}, q^{k,b}$
- Shocks: δ^d, A
- Auxiliary: $\Lambda, \tilde{\phi}, \lambda^{if}$

Steady State

The capital price is $q^{k,\tau} = 1$ in the steady state. Further, $\Lambda = \beta$. Consider the final good producer. Inserting equation (D.2.29),

$$z = (1 - \alpha) \frac{Y}{p^z} ,$$

into the production function (D.2.30) gives

$$Y = n \cdot A^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{p^z}\right)^{\frac{1-\alpha}{\alpha}} ,$$

where Y and p^z are unknown. Equating with the resource constraint (D.2.31) yields

$$n \cdot A^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{p^z}\right)^{\frac{1-\alpha}{\alpha}} = c + \delta(l+q^bb) + f(\tilde{b}) + L(\tilde{b}) , \qquad (D.2.34)$$

where we use that $K^l = l$ and $K^b = q^b b$. We will use this equation at a later stage. Next, consider the equilibrium conditions of banks. Note (D.2.5)

$$i^l = i^d \; .$$

Using $K^l = l$ and $K^b = q^b b$ as well as the entrepreneur production functions, the entrepreneur and intermediate good producer equilibrium conditions yield

$$\left(\frac{i^d + \delta}{\gamma p^l}\right)^{\frac{1}{\gamma - 1}} = l \tag{D.2.35}$$

$$\left(\frac{1/q^b - 1 + \delta}{\gamma p^b}\right)^{\frac{1}{\gamma - 1}} = q^b b \tag{D.2.36}$$

$$(l)^{\gamma} = \left(\nu \frac{p^z}{p^l}\right)^{\frac{1}{1-\tilde{\epsilon}}} \left((1-\alpha)\frac{A}{p^z}\right)^{\frac{1}{\alpha}} n , \qquad (D.2.37)$$

$$\left(q^{b}b\right)^{\gamma} = \left((1-\nu)\frac{p^{z}}{p^{b}}\right)^{\frac{1}{1-\tilde{\epsilon}}} \left((1-\alpha)\frac{A}{p^{z}}\right)^{\frac{1}{\alpha}} n , \qquad (D.2.38)$$

$$\left((1-\alpha)\frac{A}{p^z}\right)^{\frac{1}{\alpha}}n = \left(\nu(l^\gamma)^{\tilde{\epsilon}} + (1-\nu)(\left(q^bb\right)^\gamma)^{\tilde{\epsilon}}\right)^{\frac{1}{\tilde{\epsilon}}},\qquad(D.2.39)$$

where we use $z = \left((1-\alpha)\frac{A}{p^z}\right)^{\frac{1}{\alpha}} n$ and $i^l = i^d$.

The deposit and investment fund share demand of households read

$$1 = \delta^d \frac{(d_t^{hh})^{-\sigma_d}}{c^{-\sigma}} + \beta(1+i^d) , \qquad (D.2.40)$$

$$1 = \beta \frac{q^s + (1 - q^b)b + i^d d^{if} - L(\tilde{b})}{q^s} .$$
 (D.2.41)

Finally, consider the equilibrium conditions of investment funds:

$$\frac{1}{\beta} = (1+i^d) + \frac{1}{\tilde{q}^b} \left(1 + \frac{\tilde{\phi}}{\tilde{\lambda}}\right)^{-\alpha} \kappa_{if} \tilde{b},\tag{D.2.42}$$

$$q^b = q^b \lambda^{if} + \beta , \qquad (D.2.43)$$

$$q^s = q^b b + d^{if} , \qquad (D.2.44)$$

$$\tilde{\phi} = \frac{d^{ij}}{q^s} , \qquad (D.2.45)$$

$$\tilde{b} = \frac{1}{\tilde{q}^b} \left(1 + \frac{\tilde{\phi}}{\tilde{\lambda}} \right)^{-\alpha} \left(\frac{\tilde{\lambda} + \tilde{\alpha}\tilde{\phi}}{\tilde{\alpha} - 1} q^s - d^{if} \right) , \qquad (D.2.46)$$

$$\tilde{q}^b = 1 - \kappa_{hh} \tilde{b} . \tag{D.2.47}$$

We use (D.2.34)-(D.2.47) and deposit market clearing

$$d = l = d^{hh} + d^{if}$$

to solve for the unknowns $c, p^l, l, \tilde{\phi}, p^z, b, \tilde{q}^b, i^d, p^b, q^s, d^{if}, d^{hh}, \tilde{b}, \lambda^{if}, q^b$. We use the marginal rate of substitution between consumption and labor (D.2.3) to choose ψ_n to satisfy $n = \frac{1}{3}$.

D.3 Equilibrium With Regulation of Investment Funds

The equilibrium is the same as without investment fund regulation except for the following equations:

Investment Funds – The FOC for deposits is changed and a regulatory constraint is added:

$$1 + \lambda_t^{if} = \mathbb{E}_t \Lambda_{t,t+1} \left((1 + i_t^d) + \frac{1}{\tilde{q}_{t+1}^b} \left(1 + \frac{\tilde{\phi}_{t+1}}{\tilde{\lambda}} \right)^{-\tilde{\alpha}} \kappa_{if} \tilde{b}_{t+1} \right) + \mu_t , \qquad (D.3.1)$$

$$\varrho q_t^s s_t = d_t^{if} , \qquad (D.3.2)$$

where μ_t is the multiplier on the regulatory constraint. This adds one equation and one unknown μ to the calculation of the steady state.

Var.	Description	ID	Source
l_t	Short term loans MFIs	QSA.Q.N.I8.W2.S124.S11.N.A.LE.F4.SZ.XDCT.S.V.NT	ECB
	Long term loans MFI	QSA.Q.N.I8.W2.S12K.S11.N.A.LE.F4.LZ.XDCT.S.V.NT	ECB
	Listed shares MFI	QSA.Q.N.I8.W2.S12K.S11.N.A.LE.F511ZZ.XDCT.S.V.NT	ECB
b_t	Debt Sec Short Mat. IF	QSA.Q.N.I8.W2.S124.S11.N.A.LE.F3.SZ.XDCT.S.V.NT	ECB
	Debt Sec. Long Mat. IF	QSA.Q.N.I8.W2.S124.S11.N.A.LE.F3.LZ.XDCT.S.V.NT	ECB
	Listed shares IF	QSA.Q.N.I8.W2.S124.S11.N.A.LE.F511ZZ.XDCT.S.V.NT	ECB
d_t^{hh}	Overnight deposits, Total	BSI.M.U2.N.A.L21.A.1.U2.2250.EUR.E	ECB
	Deposits with agreed maturity, $<2Y$	BSI.M.U2.N.A.L22.L.1.U2.2250.EUR.E	ECB
	Deposits redeemable at notice, $<3M$	BSI.M.U2.N.A.L23.D.1.U2.2250.EUR.E	ECB
Y_t	GDP at market prices	MNA.Q.Y.I8.W2.S1.S1.B.B1GQZZ.EUR.V.N	ECB
c_t	Consumption Expenditure	MNA.Q.Y.I8.W0.S1M.S1.D.P31ZZT.EUR.V.N	ECB
I_t	GFCF	MNA.Q.Y.I8.W0.S1.S1.D.P51G.N11GTZ.EUR.V.N	ECB
π	GDP Deflator	MNA.Q.Y.I8.W2.S1.S1.B.B1GQZZZ.IX.D.N	ECB
	Total Employment	ENA.Q.Y.I8.W2.S1.S1Z.EMPZTZ.PSZ.N	ECB
	Corp. bond fund (IG): Ann. Yield	iBox x ${\ensuremath{ \in }}$ Non-Financials	Markit
	Corp. bond fund (HY): Ann. Yield	iBoxx EUR High Yield core Non-Financials ex crossover LC	Markit

TABLE D.1: Data Sources

Notes: IF: investment fund; MFI: monetary financial institutions; GFCF: gross fixed capital formation; IG: investment grade; HY: high yield.

D.4 Data and Calibration

Data Sources. We take most data from the Statistical Data Warehouse of the ECB. We employ a broad definition of bank-based and investment fund-based finance. We aggregate loans and listed shares vis-a-vis non-financial corporations held by monetary financial institutions (MFIs) excluding euro area central banks to obtain the measure for bank-financing. To obtain a measure of investment fund-finance, we aggregate debt securities and listed shares vis-a-vis non-financial corporations held by investment funds. Our measures encompass debt securities, loans, and equity, since in the context of our model, we do not discriminate between debt and equity funding sources for the firm sector. Our loan and bond measures are used to calculate the size of bond-to-loan finance, loans-and bonds-to-GDP as well as the autocorrelation of loans and bonds.

To obtain household deposits, we follow Gerali et al. (2010) who obtain deposits as the sum of different series of short-term deposits (all with maturity below three months). To obtain the size of the investment fund sector in our model, we proceed as follows. Using our measure of bond finance and the data on the liquidity share of corporate bond investment funds (see Figure 5.1), we back out the consistent amount of deposits by applying the balance sheet constraint of investment funds ($d = \text{Liq.Share} \cdot (d + b)$). This eventually yields fund shares s = d + b. We use the measure of household deposits and investment fund shares to calculate the fraction household save in investment fund shares. We also use fund shares to obtain their autocorrelation.

We use the GDP deflator and total employment to calculate real per capita series for output, consumption, investment, shares, loans, bonds, and deposits. Second moments



FIGURE D.1: Empirical and Fitted Distribution of Corporate Bond Fund Outflows

Notes: Blue bars show empirical distribution of quarterly outflows from euro area corporate bond funds between 2007 and 2019. The red line shows the fitted Lomax (Generalized Pareto) distribution.

are calculated based on the log of the respective series using a Hodrick-Prescott filter with smoothing parameter 1600 and discarding the first and last 1.5 years of data.

Finally, we target the return on investment fund shares using the annual yield on Markit indices that reflect the non-financial corporate bond universe in the euro area.

Redemptions. To parameterize the distribution of fund redemptions, we use weekly data on corporate bond investment fund flows from EPFR Global from 2007 to 2019. We obtain quarterly flows by aggregating across the weekly flows and dividing by assets under management of all funds at the start of a quarter. We focus on the outflow episodes to obtain a measure for the redemptions in the model. We match the histogram of the empirical outflow distribution to a Generalized Pareto distribution in Figure D.1. As we obtain a location parameter of zero, we obtain a Lomax distribution, which is a special case of Generalized Pareto distributions.

Robustness of the Calibration to Parameter Changes. Table D.2 shows the sensitivity of our calibration targets to changes in the calibrated parameters. All parameters are increased by one percent except for the liquidity and management cost parameters, which are increased by two percent, and for the parameters governing the preference shock. Due to their relatively small size, we increase the persistence by fifty percent and increase the shock standard deviation by one-half. Values denote the percentage change of the target. We highlight in bold the respective parameter targets. The responses of targets to their main parameter have the expected sign and are not unusually large.

Target	ν	$\tilde{\epsilon}$	κ_{if}	κ_{hh}	$ ho^a$	σ^{a}	$ ho^{\delta}$	σ^{δ}	κ^{I}
Bond-to-Loan Finance	-4.286	-0.544	-0.008	-0.001	0	0	0	0	0
Deposits in IF Assets	-2.576	-0.332	1.851	0.302	0	0	0	0	0
Loan-to-GDP	0.976	0.123	0.002	0	0	0	0	0	0
Bond Share HH	1.053	0.135	-0.931	0.007	0	0	0	0	0
σ_I/σ_Y	0.002	0	0.001	-0.001	-4.412	0	0	0	-0.100
σ_c/σ_Y	0.002	0	0.001	-0.001	4.317	0	0.001	0	0.175
σ_Y	-0.004	-0.001	-0.002	0.002	-0.315	1	0	0	-0.039
Auto-correlation Y	0.001	0	0	0	0.040	0	0	0	0.005
Auto-correlation d^{hh}	0.054	0.003	0.003	-0.004	1.390	0	0.002	0.002	-0.274

TABLE D.2: Robustness of Calibration

Notes: The table shows the percentage change in calibration targets from their baseline in Table 5.4 after small changes in parameters. IF: investment fund; HH: household; $\sigma_Y, \sigma_c, \sigma_I$: standard deviations of output, consumption, investment.

D.5 Model With Bank Frictions

As a robustness check we add more structure to the banking sector. Specifically, we assume that banks are subject to capital regulation that constrains their ability to issue loans: banks must have equity e_t to extend loans.

The banking sector finances loans with deposits d_t and equity e_t accumulated out of retained earnings. Loans l_t are granted to entrepreneurs at the loan rate i_t^l . Following Gerali et al. (2010), banks incur quadratic cost when deviating from the target capital ratio χ :²¹

$$\frac{\kappa}{2} \left(\frac{e_t}{l_t} - \chi \right)^2 e_t \; ,$$

where κ is a cost parameter. These costs are proportional to bank's equity and impose a limit to the size and the speed of adjustment of the balance sheet. Banks are owned by households to whom they pay a fraction ψ of their profits as dividends. They maximize the discounted sum of cash-flows div_t^b ,

$$\max_{d_{t},l_{t}} \quad \psi \mathbb{E}_{0} \sum_{t=0}^{\infty} \Lambda_{0,t} \left[d_{t+1} - \left(1 + i_{t}^{d} \right) d_{t} + (e_{t+1} - e_{t}) + (1 + i_{t}^{l}) l_{t} - l_{t+1} - \frac{\kappa}{2} \left(\frac{e_{t}}{l_{t}} - \chi \right)^{2} e_{t} \right],$$

subject to a balance sheet constraint $d_t + e_t = l_t$. After repeated substitutions, the static bank problem is

$$\max_{d_t, l_t} \quad \psi \cdot \left(i_t^l l_t - i_t^d d_t - \frac{\kappa}{2} \left(\frac{e_t}{l_t} - \chi \right)^2 e_t \right) \,,$$

²¹This ratio can be seen as the result of limited commitment. Intuitively, this could be the result of moral hazard on the bank's side. For example, in a model in which the borrower (bankers) can misbehave, lenders (depositors) are willing to lend only if the borrower has sufficient pledgeable income, which increases with its equity capital (see Gertler and Karadi, 2011).



FIGURE D.2: Welfare Effect of Liquidity Regulation – Relevance of Bank Frictions

Notes: We solve a second-order approximation of the model and simulate the economy under a productivity and a preference shock for different levels of the macroprudential liquidity buffer. We calculate conditional welfare to derive consumption equivalents (CE) in terms of the final good. The x-axis starts at 1.96%, which is the liquidity buffer voluntarily held by investment funds in absence of regulation. We compare the welfare in our full model (blue line) to the one in an economy with bank frictions (red line). The dashed lines depict the optimal buffers in the two economies.

where $i_t^l l_t$ denotes revenues from lending and $i_t^d d_t$ are the interest payments to depositors. The last term captures the costs of deviations from the target capital ratio. The fraction $1 - \psi$ of dividends that is retained is used to build equity capital, which evolves as:

$$e_t = (1 - \delta^b)e_{t-1} + (1 - \psi)div_{t-1}^b$$

The parameter $0 < \delta^b < 1$ captures exogenous factors that erode bank capital in every period, such as resources used to manage the bank or equity losses due to defaulting loans. It is chosen to ensure that bank equity equals the target χ in the steady state.

From the FOCs we can derive

$$i_t^l = i_t^d - \kappa \left(\frac{e_t}{l_t} - \chi\right) \left(\frac{e_t}{l_t}\right)^2 \,. \tag{D.5.1}$$

Equation (D.5.1) is the loan supply schedule and defines the spread between the loan and the deposit rate. Whenever the bank increases lending, this implies a costly deviation from the capital ratio target, as equity builds up only sluggishly out of the retained earnings. This leads to a higher loan rate that, in turn, contributes to increasing dividends and lowering loan demand. These two factors support the capital ratio, which can converge back to the target. Finally, the resource constraint changes to

$$Y_t = c_t + \frac{\kappa}{2} \left(\frac{e_{t-1}}{l_{t-1}} - \chi \right)^2 e_{t-1} + \sum_{\tau = l, b} I_t^{\tau} + f(\tilde{b}_t) + L(\tilde{b}_t) + \delta^b e_{t-1} .$$

Figure D.2 depicts our welfare measure for the baseline economy (blue) and an economy without bank frictions (red).²² With the bank frictions, the optimal buffer is 7.95%, only slightly higher than our optimal buffer in the full model. These findings reveal that bank frictions are not an important driver of our results. Indeed, in the long-run mean, banks fulfill their target capital ratio because otherwise they pay cost for deviating permanently.

²²We follow Gerali et al. (2010) and set $\kappa = 11$. We set the dividend payout ratio of banks to 0.6 to match the euro area data for 2010-2019 in Muñoz (2021). According to ECB supervisory banking statistics, the Core Equity Tier 1 ratio of euro area banks is around 15%, hence the ratio of bank equity to loans is $\chi = 0.15$. Other parameters are re-calibrated to retain a good fit of the model.

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List of Applied Software

- Matlab R2021a: Applied to solve the theoretical models and to perform the empirical analysis.
- Excel 2016: Applied to calculate preparatory empirical analysis and basic data cleaning.
- TeXstudio 4.0.3: Used to compile the drafts of the single papers and the final thesis.

Eidesstattliche Erklärung nach § 8 Abs. 3 der Promotionsordnung vom 17.02.2015

Hiermit versichere ich an Eides Statt, dass ich die vorgelegte Arbeit selbstständig und ohne die Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe. Die aus anderen Quellen direkt oder indirekt übernommenen Aussagen, Daten und Konzepte sind unter Angabe der Quelle gekennzeichnet. Bei der Auswahl und Auswertung folgenden Materials haben mir dienachstehend aufgeführten Personen in der jeweils beschriebenen Weise entgeltlich/unentgeltlich geholfen:

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