Essays on Macroeconomics, Collateral, and Default Risk

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1 Introduction

Financial collateral, best thought of as debt securities such as government and corporate bonds, is essential to both key functions of financial markets. First, it facilitates *lending* from market participants with excess funding to participants with funding deficits via repurchase agreements (repos). Second, it enables market participants to *share risk* on derivatives markets, where collateral is placed in margin accounts. Collateral protects repo lenders and derivative traders from default by the repo borrower or by the derivative contract counterparty, since the pledged collateral can be seized in this case. It therefore addresses asymmetric information and moral hazard problems that typically plague financial markets.[1]

In addition to facilitating trade among financial market participants, collateral is also of high importance for the implementation of monetary policy. Central banks typically implement monetary policy by lending to private banks against collateral. The principle of collateralization is characteristic of central bank lending, and in the case of the European Central Bank even enshrined in its statutes (Bindseil and Papadia, 2006).[2] Ensuring that a sufficient amount of collateral is available to market participants and that it is distributed appropriately among them is a crucial task for financial regulators and central banks alike.

Similarly, credit expansions associated with excessive risk-taking have been a major issue for financial regulators and central banks, in particular since the Great Financial Crisis of 2008 (Schularick and Taylor, 2012) and the European Debt Crisis of 2011 (Lane, 2012). The implications of unsustainable credit expansions are extensively studied in the literature (see Borio (2014) for a survey). This literature usually takes a macroeconomic approach and derives a key trade-off between limiting excessive risk-taking - which reduces likelihood and costs of financial distress - and ensuring smooth functioning of financial markets - which facilitates risk-sharing and efficient lending during normal times. It is however less well established, which instruments are well-suited and how they should be designed to address this trade-off. To design effective regulatory and central bank policies, it is therefore necessary to understand in which ways instruments at the disposal of regulators and central banks affect risk-taking on financial markets.

Collateralization and credit expansions are intertwined phenomena, as outlined in an important contribution by Lorenzoni (2008) for the case of *real* collateral. In contrast, this thesis focuses on the interactions between collateralization of financial transactions with *financial* collateral (defaultable debt securities) and risk-taking on credit markets. The starting point of the analysis is the observation that financial intermediaries are unable to produce sufficient collateral

[1] For a comprehensive overview of collateral on private markets, see Corradin et al. (2017).

[2] For a detailed account of the use of collateral in central bank operations, with a focus on the European Central Bank, see Nyborg (2017).

to satisfy their own needs.[3] This argument has been put forward in the seminal contribution of Holmström and Tirole (1998), who argue that, in the presence of aggregate risk, liquidity provision on private financial markets is restricted to a point where it impairs trading on financial markets. Their model predicts that government bonds contain a liquidity premium (Krishnamurthy and Vissing-Jorgensen, 2012) and derive a role for active liquidity management by the public sector (see also Greenwood et al., 2015). A similar argument on liquidity provision by the corporate sector has been made by Greenwood et al. (2010) and corroborated by the empirical analysis of Mota (2021). In both cases, collateral supply resembles *outside liquidity provision* from financial intermediaries' point of view. This notion of outside liquidity provision is a key premise of the analytical framework used in this thesis: the models used in this thesis assume a clear distinction between the collateral demand side and the collateral supply side, which may be subject to default risk.

In this framework, interactions between collateralization and default risk stem from (i) financial intermediaries' willingness to pay premia on (financial) assets that can be used as collateral and (ii) collateral supply responses by the issuers of assets that are eligible as collateral. All else equal, supplying additional debt securities *can* increase the default risk of issuers of eligible assets, with potentially negative consequences for real sector investment, sovereign default risk, and financial stability. This thesis provides an equilibrium analysis of collateralization with defaultable debt securities in a setting with endogenous collateral supply (the amount of eligible securities issued) and endogenous credit quality (the default risk of all securities issued). This sets the present thesis apart from existing literature on financial collateral, which focuses on its allocation among market participants (see Caballero and Krishnamurthy, 2008; Heider et al., 2015; Choi et al., 2021 for example), but usually treats supply and quality of collateral as exogenously given.

1.1 Overview of the Thesis

The main contribution of this thesis is to propose an analytical framework that permits a qualitative *and* quantitative analysis of endogenous collateral supply and credit risk. Different aspects of this analytical framework are laid out in four chapters, each based on a different research paper. The first two papers focus on (potentially unintended) consequences of central bank collateral policy on the corporate bond market. The third and fourth paper explore the interactions of collateral premia and default risk on the sovereign bond market.

Eligibility Requirements and Collateral Premia on the Corporate Bond Market. The second chapter of the thesis, which is based on the research paper *Risky Financial Collateral, Firm Heterogeneity, and the Impact of Eligibility Requirements* (Kaldorf and Wicknig, 2022), evaluates how endogenous firm responses to collateral premia affect the debt and default behavior of the corporate sector. Collateral premia stem from an exogenously given liquidity demand by

[3] Financial intermediaries will be referred to as banks or investors in different chapters of this thesis. They encompass all institutions that (i) invest into defaultable debt securities and (ii) demand collateral.

banks, who are the only corporate bond investors in this model. Since bonds are eligible as collateral only if they satisfy a minimum rating requirement set by the central bank, bank's demand for corporate bonds exhibits a discontinuity. The empirical literature has shown that the corporate sector responds to this discontinuous increase in the demand for their bonds in a heterogeneous way. We propose a novel theoretical framework, which captures the heterogeneity of firm responses to collateral premia and eligibility requirements.

A key concept of our framework is the 'eligible debt capacity', which is defined as the amount of bonds a firm can issue without losing eligibility by dropping below the minimum rating requirement. Profitable firms respond to collateral premia by increasing bond issuance: a risk-taking effect. Less profitable firms find it worthwhile to reduce their debt issuance to become eligible and benefit from collateral premia: a disciplining effect. While our theoretical framework can be applied to many situations in which firms face a discontinuous demand for their bonds, we apply the framework to the ECB's collateral easing policy in 2008. Using a calibration to euro area data, we find that risk-taking dominates in the aggregate, which translates into an adverse side effect on corporate default risk. Collateral policy trades-off higher collateral supply against elevated resource losses of default. We show that adding covenants conditioning eligibility on current default risk *and* current debt issuance partially mitigates adverse risk-taking effects without dis-incentivizing corporate collateral supply altogether. Covenants reduce resource losses of default risk while ensuring a high level of collateral supply and, thereby, shift the collateral policy frontier outwards.

The Preferential (Collateral) Treatment of Green Bonds. Chapter three, which is based on the paper *The Preferential Treatment of Green Bonds* (Giovanardi et al., 2022) uses the conceptual framework laid out in Chapter two to study the effects of tilting the central bank collateral framework towards green bonds. Such a policy is actively discussed by several central banks, for example within the ECB's strategy review concluded in 2021, and has already been implemented by the People's Bank of China in 2018. We contribute to a better understanding of this policy by providing a characterization of preferential collateral policy in a DSGE model. In the model, the optimal collateral framework is determined by a trade-off between increasing collateral supply to banks, mitigating negative effects of subsidizing firm leverage, and increasing the green investment share.

We use a calibration to the euro area as a laboratory for numerical policy analysis. Our findings can be summarized as follows. First, maximal preferential treatment can substantially affect the share of conventional (non-green) production by driving a wedge between the financing costs of green and conventional firms. Second, this has adverse side effects. Preferential treatment incentivizes green firms to increase bond issuance and default risk, which has adverse effects on financial stability. Therefore, the optimal collateral framework features a smaller degree of potential treatment than possible. Third, benchmarking these results against an optimal Pigouvian tax on pollution, we find that the welfare improvement of preferential treatment is relatively small. Moreover, since taxes do not affect firms' leverage choice, preferential collateral policy is a qualitatively inferior instrument. Therefore, there is scope for (welfare-enhancing) preferential treatment if and only if the optimal Pigouvian tax on pollution is not implemented.

Collateral Premia on the Government Bond Market. While chapters two and three put emphasis of corporate bonds as collateral, chapters four and five consider the government bond market, where collateral premia are an equally - or even more - important feature as well. The European debt crisis of 2011 forcefully revealed that even many advanced economy sovereign bonds do not qualify as risk-free, but are still frequently pledged as collateral. Chapter four, based on the paper *Convenient but Risky Government Bonds* (Kaldorf and Roettger, 2022), explores the extent to which convenience yield - a conceptually similar, but slightly wider concept than collateral premia - affects the debt and default dynamics of sovereign borrowers. We augment the canonical quantitative sovereign default model (Arellano, 2008) by convenience yield as a primitive of investor preferences. Convenience yield can conceptually be decomposed into a collateral valuation component, which decreases in government bond supply, and a haircut component, which in turn are an increasing function of sovereign risk.

We then calibrate the model to Italian data and demonstrate that it is able to generate long periods of negative government bond spreads and occasional debt crisis with highly positive spreads. Using the calibration, we investigate the role of collateral valuation and haircuts for government debt and default dynamics. Both components affect the elasticity of the bond price with respect to debt issuance. In our baseline calibration, where collateral valuation depends negatively on bond supply, a reduction in the elasticity of haircut schedules to default risk translates into a sizable reduction in sovereign default risk. In contrast, if collateral valuation were independent of bond supply, reducing the elasticity of haircut schedules results in an increase of sovereign default risk. This suggests that taking into account collateral demand elasticities *and* endogenous bond supply responses has important implications for the design of haircuts, both on private market segments and for central banks.

While chapter four considers the single-borrower case, the analysis is extended to the case of heterogeneous (sovereign) borrowers in chapter five, which is based on the paper *Flight-to-Quality via the Repo Market* (Kaldorf, 2022). The cross-sectional dynamics are relevant in a financially integrated but fiscally fragmented economic union, since banks' willingness to pay collateral premia depends on the aggregate supply of safe government bonds, which collapses in periods of high aggregate default risk. Collateral scarcity induces a flight-to-quality, implying an increase in the bond price of the safest governments. This feature can jointly rationalize the cross-sectional and time-series dynamics on the government bond market during the European debt crisis, which featured a divergence of borrowing costs, even though default risk (as measured by CDS spreads) exhibited strongly positive co-movement. A full collateral backstop policy that - temporarily - accepts all government bonds as collateral reduces the large valuations of the safest government bonds and simultaneously increases the price of riskier bonds, which has a small positive effect on sovereign default risk.

1.2 Contribution to each Chapter

Chapter 2 is based on joint work with Florian Wicknig. The research question, the model, and its formal analysis were developed jointly. Florian Wicknig was providing the data work on which our quantitative analysis is based, while I was responsible for code, calibration, and quantitative analysis of the model. The current draft of the paper was written jointly.

Chapter 3 is based on joint work with Francesco Giovanardi, Lucas Radke, and Florian Wicknig. We developed the research idea and the model together. Francesco Giovanardi, Florian Wicknig, and I developed the simplified example. The coding of the model was mostly conducted by Francesco Giovanardi, Lucas Radke, and me, while I was responsible for the calibration. Florian Wicknig contributed the empirical work. The initial draft was written by Florian Wicknig and me together, which was then revised by Francesco Giovanardi and Lucas Radke.

Chapter 4 is based on a joint project with Joost Roettger. While Joost Roettger was formalizing the first version of our model, we developed it into its present form together. The code was produced jointly and I was responsible for calibrating the model and collecting the data on which the quantitative analysis is based on. Each of us contributed different sections to the first draft of the paper, which were then revised subsequently by the other.

Chapter 5 is based on a single-authored paper.

2 Risky Financial Collateral, Firm Heterogeneity, and the Impact of Eligibility Requirements

Authors: Matthias Kaldorf and Florian Wicknig

2.1 Introduction

Central banks implement monetary policy by lending to banks against collateral, which makes a sufficiently high supply of collateral essential to the functioning of the financial system. During the financial crisis of 2008, this restriction required many central banks to expand the pool of assets they accept as collateral to facilitate the conduct of expansionary monetary policy. For example, the European Central Bank (ECB) engaged in collateral easing when switching towards a full allotment regime in its Main Refinancing Operations and prior to introducing Long-Term Refinancing Operations. To expand the pool of collateral, the ECB added corporate sector assets of intermediate quality, such as BBB-rated corporate bonds and securitized bank loans, to the list of eligible assets.[1] The inclusion of corporate sector assets is quantitatively relevant: corporate bonds and credit claims make up 27% of collateral in ECB operations.[2]

While collateral easing facilitates the smooth conduct of monetary policy, a thorough assessment of this policy must also account for endogenous responses of the corporate sector. Firm responses arise, since banks increase demand for assets if they become eligible as collateral and firms cater to this demand by increasing their debt issuance and indebtedness (Mésonnier et al., 2022; Pelizzon et al., 2020; Mota, 2021). The positive response of debt issuance and indebtedness is particularly pronounced for high-rated borrowers (Grosse-Rueschkamp et al., 2019). While debt supply effects are desirable for monetary policy implementation, higher indebtedness of firms also increases their risk of default, which can also limit the efficacy of collateral easing, if higher indebtedness is associated with rating downgrades in future periods. This paper presents a novel theoretical framework to study endogenous firm responses to eligibility requirements in the presence of default risk. While this framework can be applied to many situations where

[1] See Wolff (2014), Heider et al. (2015), Nyborg (2017), and Blot and Hubert (2018) for a discussion on the collateral eligibility of risky private sector assets and the monetary policy implementation by the ECB. We show the collateral treatment of corporate sector assets by different central banks in appendix A.1.

[2] As of 2020Q4, corporate bonds are the second largest asset class accepted as collateral by the ECB with a market value of EUR 1871 billion. This is only exceeded by government bonds (see European Central Bank, 2022).

eligibility is specified in a discontinuous way through minimum rating requirements, we present an application to the ECB's collateral easing policy of 2008.

We study endogenous firm responses to eligibility requirements through the lens of a model with heterogeneous firms that issue risky debt securities (corporate bonds) to banks.[3] Firms are subject to idiosyncratic revenue shocks and have an incentive to issue bonds, because they are more impatient than their creditors. Firms default on their bonds if revenues fall short of current repayment obligations, in which case all current revenues are wasted. Thus, bond issuance is determined by a trade-off between relative impatience and expected default costs. Bonds are held and priced by banks. We assume that banks value these bonds if they can be used to collateralize borrowing from the central bank. Consistent with actual central bank practice, only sufficiently safe bonds are eligible as collateral and the central bank sets the minimum quality (rating) requirement as a policy instrument. The dual role of bonds as investment object and collateral implies that spreads on *eligible* bonds contain a fundamental component and a collateral premium that, ceteris paribus, shifts the pricing schedule outwards in a discontinuous way.

As our first contribution, we provide a characterization of firm responses to collateral eligibility in a model with discontinuous demand for corporate bonds. We obtain analytical solutions in a setting with one period bonds, i.i.d. revenue shocks, and permanent differences in firm productivity. Making corporate bonds eligible affects the firm's borrowing decision in a discontinuous way. The discussion of firm responses is organized around a key firm characteristic, the *eligible debt capacity*, defined as the maximum amount of bonds a firm can issue without losing eligibility.

Compared to the case of no collateral premia, firms' debt choices differ in sign above and below the discontinuity in bond demand induced by eligibility requirements. High-quality firms (with a large eligible debt capacity) take advantage of banks' high valuation of corporate bonds and issue more bonds to front-load dividend payouts: firms increase their *risk-taking*.[4] In contrast, medium-quality firms (which issue bonds at or near their eligible debt capacity) may find it worthwhile to reduce their debt issuance, if this makes their bonds eligible: a *disciplining* effect.[5] Both firm level effects imply that bond prices and debt choices are not solely determined by firm fundamentals. The latter case will be referred to as *market discipline*, while risk-taking and disciplining effects constitute violations thereof.[6]

[3] We refer to these securities as corporate bonds, even though they can be interpreted as securitized bank loans or other marketable corporate sector assets, which are also eligible in many central bank collateral frameworks.

[4] Front-loading dividends can be thought of as an intertemporal substitution effect. On the other hand, those firms can sustain the same dividend-payout by issuing a smaller face value of bonds: an income effect. Under a standard assumptions on firm's revenue distribution, the former effect dominates.

[5] Disciplining effects associated with rating up- and downgrades are well documented in the literature. Kisgen (2006) and Kisgen (2009) show that firms near rating thresholds reduce their debt issuance, either due to the threat of losing a high rating or due to the opportunity of being upgraded into a higher category.

[6] There are two potential confounding factors. First, firms may substitute from loan to bond financing and leave total debt constant. Pelizzon et al. (2020) provide evidence for this substitution, but still find a sizable positive effect on leverage. A second potential concern arises if firms simultaneously increase investment and keep leverage constant. However, Grosse-Rueschkamp et al., 2019 identify sizable leverage responses of firms rated A or higher after the ECB's corporate sector purchase program (CSPP). Moreover Todorov (2020) and Santis and Zaghini (2021) find that QE-eligible firms primarily issued bonds to increase dividend payouts.

While both firm level effects increase collateral supply, they have an opposing effect on default risk. This makes a heterogeneous firm model essential to study aggregate implications, because the relative strength of both effects depends on the firm distribution. To illustrate the aggregate effect, consider collateral easing, which increases the eligible debt capacity for all firms. The change of aggregate collateral supply contains a *mechanical component*, the change caused by a lower rating threshold all else equal, and *endogenous firm responses*, which depend on the relative size of risk-taking and disciplining effects and the mass of firms subject to each effect. Within this framework, we show that endogenous firm responses unambiguously amplify the positive mechanical effect of collateral easing. Risk-taking and disciplining effects positively contribute to the increase. However, they have an ambiguous impact on aggregate default cost. Note that we obtain these results under the assumptions of one-period bonds and i.i.d. revenue shocks, which we relax in the following.

As our second contribution, we illustrate the relevance of firm responses in the context of the ECB's collateral easing policy in a setting with long-term debt and persistent revenue shocks. We solve the model using global methods and calibrate the firm cross-section to euro area data by employing a merged dataset of corporate bonds from *IHS Markit* and corporate balance sheet data from *Compustat Global*. The calibrated model can replicate several features of firm debt issuance, corporate bond spreads, and collateral premia, which are crucial to evaluate the impact of eligibility requirements.

We study two different policies: our benchmark scenario are tight eligibility requirements, which only accept bonds rated A or higher, corresponding to the ECB collateral framework before the 2008 crisis. Second, we consider lenient eligibility requirements, under which all bonds rated BBB or higher are accepted, in line with ECB practice after 2008. In the setting with long-term bonds and persistent revenues, firm responses dampen the positive mechanical effect of collateral easing on collateral supply and increase aggregate cost from default. These effects are sizable: instead of mechanically expanding by 71%, collateral supply only increases by 62% if firm responses are taken into account. This is reflected by the relative importance of risk-taking and disciplining effects. While 51% of firms engage in risk-taking and 19% are subject to disciplining under a tight policy, these numbers shift to 79% and 3% under a lenient policy. Aggregate default costs increase by 8%.

The dampening effect of firm responses on collateral supply is associated with the large debt issuance of high-revenue firms. While the risk-taking response at the firm level is increasing the *current* market value of bonds outstanding, it exposes firms to rollover risk: when hit by a series of adverse revenue shocks, previously issued bonds experience a drop in their market value. If firms also lose their eligibility status, this rollover problem becomes more severe, since bonds also lose the collateral premium. As a result, indebtedness increases, default becomes more likely, and collateral supply contracts. Notably, increasing bond issuance is still optimal for firms that experience high revenue draws due to the relative impatience of firm managers: the adverse effects of rollover risk are discounted sufficiently heavily. Key to this phenomenon is the combination of persistent shocks and long-term debt. It is important to note that this mechanism is also present under tight collateral policy but becomes stronger after collateral easing. Similar effects have been described in the macro-finance (Gomes et al., 2016 or Jungherr and Schott, 2022) and sovereign default literature (Hatchondo et al., 2016).

We investigate an *eligibility covenant* as a potential instrument to tackle the dampening effect of endogenous firm responses. Our focus is on covenants depending on *current* debt outstanding, which effectively is public information for firms that are large enough to issue corporate bonds. A covenant limits the eligible debt capacity of firms with high levels of debt outstanding and provides deleveraging incentives. This reduces the rollover burden of these firms once they experience a sequence of adverse revenue draws. Conceptually, most collateral frameworks condition the eligibility of bonds only on ratings, which is a one-dimensional measure of default risk. Conditioning eligibility also on debt outstanding allows the central bank to disincentivize 'unsustainable' debt issuance of high-revenue firms, while at the same time allowing low-revenue firms with low debt outstanding to roll over their bonds.

The policy problem in choosing a covenant is to set a sufficiently tight covenant to provide deleveraging incentives for high-revenue firms without shutting down the issuance of bonds altogether. Restricting our attention to a simple parametric class for the covenant, we demonstrate the existence of a *collateral Laffer curve* for any given minimum rating requirement. Our numerical results suggest that conditioning eligibility not only on default risk but also on current debt outstanding, has a positive effect on collateral supply. For example, under a BBB minimum rating requirement, covenants can expand collateral supply by up to 22% compared to the case without covenant. Finally, we investigate the potential of adding covenants to the central bank toolkit in addition to the minimum rating requirement. Since the representation of banks and liquidity risk is too simplistic to allow for a fully-fledged welfare analysis, we build on the literature on optimal collateral policy (Koulischer and Struyven, 2014 and Choi et al., 2021 among others) and embed our analysis in a central bank trade-off between maintaining high collateral supply and limiting the additional default cost from violating market discipline. Our model predicts that using both instruments increases collateral supply, lowers cost from corporate default, and, thereby, shifts the *collateral policy frontier* outward.

While we propose a model that is suited to study discontinuous collateral eligibility, our framework can also be applied in other situations where firms respond endogenously to a discontinuous demand schedule for their debt. This includes the eligibility for asset purchase programs, where the anticipation of substantial demand increases for targeted assets induces a willingness to pay eligibility premia on them. Furthermore, pension funds are typically restricted to investment grade bonds for regulatory reasons, such that bond demand exhibits a jump from BB+ to BBB- ratings (Boot et al., 2005). Similarly, Kisgen (2006) argues that speculative grade firms (CCC+ or lower) are subject to more stringent disclosure requirements, while only firms rated at least AA- are able to issue commercial paper in the US (Hahn, 1993).

Related Literature. Our paper builds on a large strand of literature providing empirical results on the bond market impact of collateral policy and eligibility for QE programs. Ashcraft et al. (2011) find a sizable impact of haircuts on bond prices using an event study around announcement and implementation of the Term Asset-Backed Securities Loan Facility in the US.

Exploiting an unexpected policy change regarding eligibility of Chinese corporate bonds, Chen et al. (2019) and Fang et al. (2020) identify large pledgeability premia. Mésonnier et al. (2022) use an extension of eligibility criteria as part of the Additional Credit Claims program and find a premium of 8bp on bank loans relative to a non-eligible control group. Firm indebtedness are presented in Grosse-Rueschkamp et al. (2019), who identify heterogeneous responses of firms in different rating brackets.

While the previous group of papers uses surprise policy changes to identify causal effects, there are two complementary approaches leading to similar findings. First, Pelizzon et al. (2020) document collateral eligibility premia and bond supply effects using security-level data from the euro area. Their identification relies on ECB-discretion over when formally eligible bonds are put on the list of eligible assets. They identify collateral eligibility premia of 11-24bp. Second, building an identification strategy around the US treasury safety premium, Mota (2021) uses US corporate bond data and finds that non-financial corporate bonds carry a premium, which can be related to collateral service. The premium decreases in the bond default risk and depends on idiosyncratic firm characteristics as well as an aggregate component encompassing economy-wide collateral supply and demand factors. Mota (2021) finds that firm debt issuance and dividend payout responses rise in the size of the premium. We complement this literature by proposing a model that captures the empirically documented heterogeneity of firm responses regarding debt issuance and indebtedness.

The results of our paper can be related to a group of papers studying the collateral eligibility of risky assets and implications for central bank policy. Chapman et al. (2011) propose a model where the central bank faces a collateral policy trade-off between relaxing banks' liquidity constraints and incentivizing them to invest into illiquid and risky assets. Koulischer and Struyven (2014) argue that relaxing eligibility requirements can alleviate credit crunches if collateral supply or collateral quality fall below specific levels, as banks' ability to extend credit depends on both. Cassola and Koulischer (2019) quantify a collateral policy trade-off between liquidity provision and risk-taking by the central bank. In Choi et al. (2021) banks prefer to use high-quality collateral on the interbank market so that central banks negatively affect access to liquidity when accepting only high-quality assets. At the same time, lending against low-quality collateral exposes the central bank to counterparty default risk. In contrast to these papers, we make collateral supply and its riskiness endogenous but abstract from further frictions on money markets. Combining these approaches might deliver interesting interactions, which we leave to future research.

Outline. The paper is structured as follows. Section 2.2 introduces collateral premia and eligibility requirements into a corporate capital structure model and presents our main conceptual results. We extend and apply the model to the ECB's collateral easing policy in Section 2.3. In Section 2.4, we conduct policy experiments regarding eligibility covenants. Section 2.5 concludes.

2.2 A Model of Eligibility Requirements

This section introduces a model of endogenous collateral supply and firm default risk to analyze the impact of eligibility requirements on firms. Time is discrete and there are two groups of agents: a non-financial sector (*firms*) and financial intermediaries (*banks*). The *central bank* sets an eligibility requirement, which we treat as an exogenous parameter.

2.2.1 Environment

Firms are endowed with a technology that generates stochastic revenues, which can be interpreted as earnings before interest and taxes (EBIT). To maintain tractability, we do not endogenize investment. Revenue shocks realize at the beginning of each period t and are i.i.d. across firms and over time. In addition to being subject to idiosyncratic revenue shocks, firms are ex-ante heterogeneous with respect to the probability distribution over revenue shocks: some firms are permanently more productive than others and we denote this heterogeneity by the parameter s. We will use this parameter to index bonds issued by each firm as well.

Each period, firms issue debt instruments to banks. These debt instruments are referred to as corporate bonds but reflect all marketable debt instruments including securitized bank loans. Bonds are real one-period discount bonds, i.e., they promise to pay one unit of the all-purpose good in period $t+1$. In our model, firms are the natural borrowers, because they are more impatient than banks. Given their shock realization and bonds outstanding, firms either default or repay. Bonds have a dual role in the economy, since banks can pledge eligible bonds with the central bank to obtain funding. The demand for central bank funding can be motivated by liquidity deficits that require immediate settlement, such as net deposit outflows (see Bianchi and Bigio, 2022, and De Fiore et al., 2019). We follow Mota (2021) and assume a constant willingness to pay collateral premia. We present a robustness check where the size of collateral premia depends on collateral supply in Appendix A.4.

Banks. There is a unit mass of perfectly competitive banks, which price bonds risk-neutrally without discounting. They purchase bonds $b_{t+1}(s)$ issued by firm s at price $q(b_{t+1}|s)$, which reflects its value as an investment object, i.e., the repayment probability (described below), and the collateral benefit they provide. The collateral premium on an eligible bond will be denoted by L. Consistent with actual central bank policy, we assume that bonds are only eligible as collateral if their default probability $F(b_{t+1}|s)$ does not exceed an eligibility threshold $\overline{F}$ set by the central bank. The eligibility indicator Ψ is given by

$$\Psi(b_{t+1}|s) = \begin{cases} 1 & \text{if } F(b_{t+1}|s) \leq \overline{F} \ , \\ 0 & \text{else} \ . \end{cases}$$

We model collateral policy in terms of bond eligibility thresholds, i.e., bonds either receive a 100% or a 0% haircut.[7]

Firms. Firm managers/owners are risk neutral and their discount factor is denoted by $\beta < 1$. They operate a technology generating random revenues $\mu_t \in [\underline{\mu}, \overline{\mu}]$ with $\underline{\mu} < 0$ and $\overline{\mu} > 0$.[8] We assume that μ_t is independent across firms and over time, and denote its pdf and cdf by $f(\mu_t|s)$ and $F(\mu_t|s)$, respectively. Firms are *ex-ante* heterogeneous with respect to their probability distribution over revenues, which allows us to analytically disentangle how individual firms react to eligibility requirements and how firm heterogeneity affects aggregate collateral supply responses to collateral easing. The ex-ante heterogeneity is governed by a productivity parameter s, which characterizes the revenue distribution in a first order stochastic dominance sense. Firms with a high s are more productive on average. In particular, s shifts the probability mass according to $F(\mu_t|s) = F(\mu_t - s)$. We assume that s follows some continuous distribution $G(s)$ over the open interval $S \equiv [s^-, \infty]$, to which we refer as the *firm type space*. Furthermore, we assume that s^- is sufficiently low such that at least one firm is not eligible even when it chooses not to issue any bonds, i.e., $F(0|s^-) = F(s^-) > \overline{F}$.

Firm managers maximize the present value of dividends. Dividends can become negative, which we interpret as equity issuance. Firms issue bonds $b_{t+1}(s)$ to banks. These bonds are subject to default risk: if firm revenues μ_t fall short of the repayment obligation b_t, the firm is unable to raise funds by issuing additional equity and defaults. The default and repayment probabilities implied by the debt choice b_{t+1} are, therefore, given by $F(b_{t+1}|s)$ and $1 - F(b_{t+1}|s)$, respectively. In case of default, all firm revenues are lost and there is no recovery for banks.[9]

Bond Pricing. Expressing the expected payoff from investing into bonds of firm s in terms of the revenue distribution, the bond pricing condition can be written as

$$q(b_{t+1}|s) = \big(1 - F(b_{t+1}|s)\big)\left(1 + \Psi(b_{t+1}|s) \cdot L\right) \ . \tag{2.1}$$

It depends on the expected payoff, determined by the firm default decision in $t + 1$, and the collateral premium L if bond s is eligible, which, in turn, depends on firm default risk. In the absence of a collateral premium, bond prices merely reflect the expected payoffs.

2.2.2 Debt Choices at the Firm Level

In this section, we analyze how firms' debt choice is affected by the eligibility of their corporate bonds as collateral. We assume there are no delays in the restructuring of defaulted bonds and no exclusion from the corporate bond market after a default. The maximization problem of firm

[7] In practice, eligible bonds have collateral values in between due to other risk factors, like market illiquidity, which are not present in our setup. Nevertheless, collateral frameworks exhibit large discontinuities at the eligibility thresholds, as we show in Appendix A.1.

[8] Allowing for negative realizations of the revenue shock is consistent with the interpretation of μ_t as EBIT.

[9] Our approach is motivated by Lian and Ma (2021), who show that most corporate borrowing is tied to the going-concern value of the firm. Allowing for a positive recovery rate would not change our qualitative results.

s in period t can be written as

$$V(b_{t+1}|s) = q(b_{t+1}|s)b_{t+1} + \beta \int_{b_{t+1}}^{\overline{\mu}} (\mu_t - b_{t+1})\, dF(\mu_t|s) \ . \tag{2.2}$$

Maximizing (2.2) over b_{t+1} yields the first order condition

$$\beta(1 - F(b_{t+1}|s)) = q(b_{t+1}|s) + \frac{\partial q(b_{t+1}|s)}{\partial b_{t+1}} b_{t+1} \ ,$$

which we express using the bond price derivative $\frac{\partial q(b_{t+1}|s)}{\partial b_{t+1}} = -f(b_{t+1}|s)(1 + \Psi(F(b_{t+1}|s)) \cdot L)$ as

$$\beta\big(1 - F(b_{t+1}|s)\big) = \big(1 - F(b_{t+1}|s)\big) - f(b_{t+1}|s) \cdot b_{t+1} \qquad \text{if} \quad F(b_{t+1}|s) > \overline{F} \ , \tag{2.3}$$

$$\beta\big(1 - F(b_{t+1}|s)\big) = \big(1 - F(b_{t+1}|s)\big) \cdot (1 + L) - f(b_{t+1}|s) \cdot b_{t+1} \cdot (1 + L) \qquad \text{if} \quad F(b_{t+1}|s) \leq \overline{F} \ . \tag{2.4}$$

The eligibility requirement introduces a discontinuity into the first order condition. To make the implied discontinuity in the debt choice explicit, we refer to the debt levels satisfying (2.3) and (2.4) as non-eligible debt choice $b^n_{t+1}(s)$ and eligible debt choice $b^e_{t+1}(s)$, respectively. Non-eligible firms choose their bond issuance according to (2.3): the LHS of this expression reflects discounted expected repayment obligations from issuing another unit of bond, which has to equal the current revenue from issuing this bond net of debt dilution on the RHS. This case is consistent with the concept of *market discipline*, since the debt choice is determined solely by fundamentals. Collateral premia distort this trade-off by making debt issuance more attractive, since they increase the amount of funds raised per unit of bonds (first term on the RHS of (2.4)). At the same time, collateral premia increase the costs of debt dilution (second term on the RHS), which makes debt issuance less attractive.

Without further restrictions on the revenue distribution, the total effect of bond eligibility is ambiguous. However, guided by empirical evidence on increased debt issuance at the firm level in response to bond eligibility (Pelizzon et al., 2020) and consistent with the standard assumption in the literature (Bernanke et al., 1999), we assume that the distribution satisfies a monotonicity condition on the hazard rate $h(\mu_{t+1}|s) \equiv \frac{f(\mu_{t+1}|s)}{1-F(\mu_{t+1}|s)}$.

Proposition 1. If the revenue distribution satisfies $\frac{\partial h(\mu_{t+1}|s))}{\partial \mu_{t+1}} > 0$, the non-eligible debt choice is increasing in the productivity parameter $\frac{\partial b^n_{t+1}}{\partial s} > 0$, while the implied default risk decreases $\frac{\partial F(b^n_{t+1})}{\partial s} < 0$. Likewise, for eligible firms it holds that $\frac{\partial b^e_{t+1}}{\partial s} > 0$ and $\frac{\partial F(b^e_{t+1})}{\partial s} < 0$. Moreover, the optimal debt issuance of an eligible firm exceeds that of an otherwise identical non-eligible firm $b^e_{t+1}(s) > b^n_{t+1}(s)$.
Proof: See Appendix A.2.1.

Proposition 1 establishes two important results. First, more productive firms (higher s) issue more debt but their default risk falls compared to less productive firms. Because firms are risky, they increase debt issuance less than one-for-one with improving fundamentals. Second, collateral premia induce additional debt issuance of eligible firms. Those firms take advantage of their better bond valuation to front-load dividend payouts.

So far, we established differences between the non-eligible and eligible debt choice. Next, we focus on how firm choices are affected by eligibility requirements, since the eligibility status of firms is *endogenously* determined. It will be helpful to define the *eligible debt capacity* $\widetilde{b}_{t+1}(s) \equiv F^{-1}(\overline{F}|s)$ as the highest debt choice for which the default probability of firm s does not exceed the threshold $\overline{F}$. Naturally, more productive firms have a higher eligible debt capacity. As shown in Proposition 2, the ex-ante heterogeneous revenue distribution determines how firms select themselves into eligible and non-eligible regions, taking the eligibility threshold as given.

Proposition 2. There are two cut-off values s_0, implicitly defined through $V^e(\widetilde{b}_{t+1}(s_0)|s_0) = V^n(b^n_{t+1}(s_0)|s_0)$, and s_2, defined through $b^e_{t+1}(s_2) = \widetilde{b}_{t+1}(s_2)$, such that

- firms with $s < s_0$ are *non-eligible* and choose $b^n_{t+1}(s)$ according to (2.3).
- firms with $s_0 < s < s_2$ are *constrained eligible* in the sense that they borrow up to their eligible debt capacity $\widetilde{b}_{t+1}(s)$.
- firms with $s > s_2$ are *unconstrained eligible* and choose $b^e_{t+1}(s)$ according to (2.4).

Proof: See Appendix A.2.2.

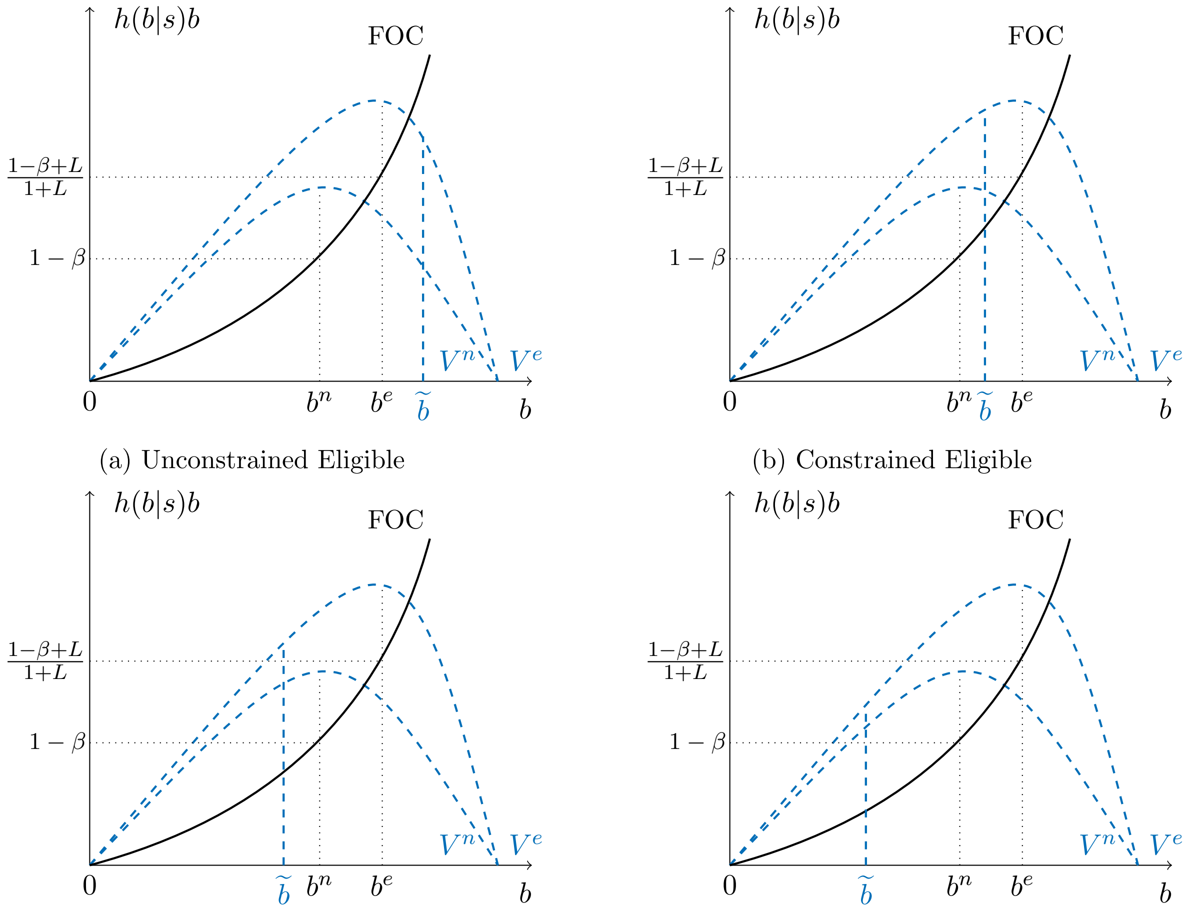
In Figure 2.1, we provide an illustration of Proposition 2. We plot the first order conditions in solid black lines, expressed in terms of the hazard rate $h(b|s)$. Objective functions for the case of non-eligibility and eligibility are denoted by V^n and V^e (blue dashed lines) and are obtained from evaluating (2.2) at the respective debt choices. There are four possible combinations of $b^n_{t+1}(s)$, $b^e_{t+1}(s)$, and $\widetilde{b}_{t+1}(s)$.

Figure 2.2a shows the case of a highly productive firm with a high draw of s so that $b^e_{t+1}(s) < \widetilde{b}_{t+1}(s)$. The eligible debt capacity of an *unconstrained eligible* firm is sufficiently high, such that it can satisfy (2.4). Figure 2.2b shows a firm with insufficient debt capacity to satisfy (2.4), whereas satisfying (2.3) would be possible, $b^n_{t+1}(s) < \widetilde{b}_{t+1}(s) < b^e_{t+1}(s)$. However, the value of the objective $V^e(\widetilde{b}_{t+1}(s)|s)$ exceeds the value $V^e(b^n_{t+1}(s)|s)$, because V^e is upward sloping for all $b < b^e_{t+1}(s)$. Thus, the firm chooses to be just eligible at debt level $\widetilde{b}_{t+1}(s)$. Such a firm is *constrained eligible*. Within the case of $\widetilde{b}_{t+1}(s) < b^n_{t+1}(s)$, there are two sub-cases: first, choosing $b^n_{t+1}(s)$ is feasible, but the firm can be better off by choosing $\widetilde{b}_{t+1}(s)$, since $V^n(b^n_{t+1}(s)|s) < V^e(\widetilde{b}_{t+1}(s)|s)$, as in Figure 2.2c. Such a firm chooses to be just eligible and is also classified as *constrained eligible*. Second, *non-eligible* firms with a sufficiently low s optimally choose $b^n_{t+1}(s)$, since the debt reduction required for eligibility is too large $V^e(\widetilde{b}_{t+1}(s)|s) < V^n(b^n_{t+1}(s)|s)$, as in Figure 2.2d.

2.2.3 Eligibility Requirements and Macroeconomic Aggregates

Having discussed how firm policies are characterized in the presence of collateral premia, we now turn to the effects of collateral easing. We consider an increase of the threshold default probability from a low value $\overline{F}^A$ to a higher value $\overline{F}^{BBB}$, akin to the ECB policy change in response to the 2008 financial crisis and also corresponding to our numerical experiments in Section 2.3. Formally, we characterize the change of aggregate collateral $\overline{B}$ in terms of the debt

Figure 2.1: Debt Choice Across Endogenous Firm Types



(a) Unconstrained Eligible

(b) Constrained Eligible

(c) Constrained Eligible

(d) Non-eligible

Notes: Endogenous partitioning of the firm space following Proposition 2. The blue lines denote the objective function (2.2) for each eligibility status. The black line represents the LHS of firms' first order conditions, rewritten as $h(b_{t+1}|s) \cdot b_{t+1}$, which maximizes V^e at b^e and V^n at b^n.

choice across the firm type space S. Let the cut-off values, which determine the partitioning of firms into constrained and unconstrained eligible, associated with $\overline{F}^A$ and $\overline{F}^{BBB}$ be denoted by (s_0^A, s_2^A) and (s_0^{BBB}, s_2^{BBB}), respectively. The threshold productivity levels partitioning firms into eligibility regions decrease in response to collateral easing, which we summarize in Lemma 1.

Lemma 1. Increasing the eligibility threshold from $\overline{F}^A$ to $\overline{F}^{BBB}$ decreases the threshold levels to $s_0^{BBB} < s_0^A$ and $s_2^{BBB} < s_2^A$.
Proof: See Appendix A.2.3.

We can use Lemma 1 to write the total effect of collateral easing on collateral supply as

$$\begin{aligned}\overline{B}^{BBB} - \overline{B}^A =&(1+L)\Bigg(\int_{s_0^{BBB}}^{s_2^{BBB}} \Big(1 - F\big(\widetilde{b}_{t+1}^{BBB}(s)\big)\Big)\widetilde{b}_{t+1}^{BBB}(s)dG(s) + \int_{s_2^{BBB}}^{\infty} \Big(1 - F\big(b_{t+1}^e(s)\big)\Big)b_{t+1}^e(s)dG(s) \\ &- \int_{s_0^A}^{s_2^A} \Big(1 - F\big(\widetilde{b}_{t+1}^A(s)\big)\Big)\widetilde{b}_{t+1}^A(s)dG(s) - \int_{s_2^A}^{\infty} \Big(1 - F\big(b_{t+1}^e(s)\big)\Big)b_{t+1}^e(s)dG(s)\Bigg). \qquad (2.5)\end{aligned}$$

Collateral supply, that is, the market value of eligible bonds, can be divided into two parts: the two integrals over $[s_0, s_2]$ contain all constrained eligible firms, respectively, while the integrals over $[s_2, \infty)$ summarize unconstrained eligible firms.

A central point of our framework is that such a policy has a mechanical effect by lowering the eligibility threshold and that it implies an endogenous firm response. To highlight this decomposition, we introduce a third threshold productivity s_1, where $s_0 < s_1 < s_2$. For the threshold firm s_1, the debt choice $b^n_{t+1}(s_1)$ equals its eligible debt capacity $\widetilde{b}_{t+1}(s_1)$. Mechanical effects are present if threshold levels satisfy $s_1^{BBB} < s_0^A$. This means that at least the firm exactly satisfying eligibility requirements after the policy change $F(b^n_{t+1}(s_1^{BBB})) = \overline{F}^{BBB}$ was not eligible before the policy change $\overline{F}^A < F(b^n_{t+1}(s_1^{BBB}))$. This firm was non-eligible under the tight policy, where it chooses $b^n_{t+1}(s)$, but becomes eligible without changing its debt issuance. To ease the exposition, we further restrict attention to 'small' changes to eligibility requirements and assume $s_0^A < s_2^{BBB}$. This implies that there is no firm directly switching from non-eligible to unconstrained eligible. We summarize the impact of collateral easing on collateral supply and default cost in Lemma 2.

Lemma 2. If $s_1^{BBB} < s_0^A$, the mechanical effect of collateral easing on collateral supply is positive and given by

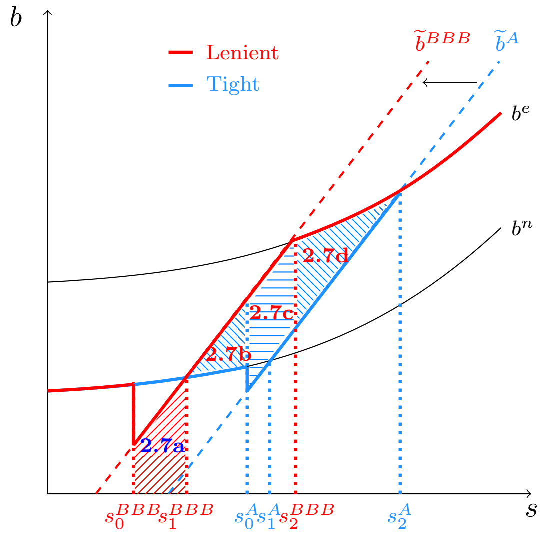
$$\overline{B}^{BBB} - \overline{B}^{A}\Big|_{mech} = (1+L)\left(\int_{s_1^{BBB}}^{s_0^A} \Big(1 - F\big(b^n_{t+1}(s)\big)\Big) b^n_{t+1}(s) dG(s) \right). \tag{2.6}$$

If $s_0^A < s_2^{BBB}$, endogenous firm responses on collateral supply can be expressed as

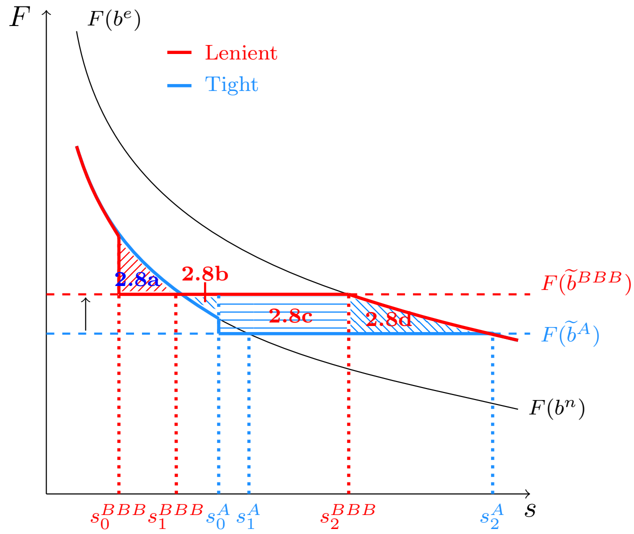
$$\begin{aligned}
\overline{B}^{BBB} - \overline{B}^{A}\Big|_{endo} =&(1+L)\Bigg(\int_{s_0^{BBB}}^{s_1^{BBB}} \Big(1 - F\big(\widetilde{b}^{BBB}_{t+1}(s)\big)\Big) \widetilde{b}^{BBB}_{t+1}(s) dG(s) && \text{(2.7a)}\\
&+ \int_{s_1^{BBB}}^{s_0^A} \Big(1 - F\big(\widetilde{b}^{BBB}_{t+1}(s)\big)\Big) \widetilde{b}^{BBB}_{t+1}(s) - \Big(1 - F\big(b^n_{t+1}(s)\big)\Big) b^n_{t+1}(s) dG(s) && \text{(2.7b)}\\
&+ \int_{s_0^A}^{s_2^{BBB}} \Big(1 - F\big(\widetilde{b}^{BBB}_{t+1}(s)\big)\Big) \widetilde{b}^{BBB}_{t+1}(s) - \Big(1 - F\big(\widetilde{b}^{A}_{t+1}(s)\big)\Big) \widetilde{b}^{A}_{t+1}(s) dG(s) && \text{(2.7c)}\\
&+ \int_{s_2^{BBB}}^{s_2^A} \Big(1 - F\big(b^e_{t+1}(s)\big)\Big) b^e_{t+1}(s) - \Big(1 - F\big(\widetilde{b}^{A}_{t+1}(s)\big)\Big) \widetilde{b}^{A}_{t+1}(s) dG(s) \Bigg). && \text{(2.7d)}
\end{aligned}$$

Denoting the resource loss of firm s from defaulting by $M(b_{t+1}(s)|s) \equiv \int_{\underline{\mu}}^{b_{t+1}(s)} \mu_{t+1} dF(\mu_{t+1}|s)$,

Figure 2.2: Firm Responses After Collateral Easing



(a) Debt Choice Across Firm Types



(b) Default Risk Across Firm Types

Notes: We compare the change in debt issuance (left) and default risk (right) across firm types after an easing from tight (blue) to lenient (orange) eligibility requirements. Colored dashed lines represent the eligible debt capacities or the associated default risk under either policy. Black solid lines denote the debt choices (or the corresponding default risk) satisfying (2.3) and (2.4). Colored bold lines denote the firm debt choices and the associated default risk. Colored dotted lines denote the threshold productivities as described in Proposition 2 and Lemma 1.

the change in aggregate default cost $\mathcal{M}_t$ can be expressed as follows

$$\mathcal{M}^{BBB} - \mathcal{M}^{A} = \int_{s_0^{BBB}}^{s_1^{BBB}} M\left(\widetilde{b}_{t+1}^{BBB}(s)\right) - M\left(b_{t+1}^{n}(s)|s\right) dG(s) \tag{2.8a}$$

$$+ \int_{s_1^{BBB}}^{s_0^{A}} M\left(\widetilde{b}_{t+1}^{BBB}(s)\right) - M\left(b_{t+1}^{n}(s)\right) dG(s) \tag{2.8b}$$

$$+ \int_{s_0^{A}}^{s_2^{BBB}} M\left(\widetilde{b}_{t+1}^{BBB}(s)\right) - M\left(\widetilde{b}_{t+1}^{A}(s)\right) dG(s) \tag{2.8c}$$

$$+ \int_{s_2^{BBB}}^{s_2^{A}} M\left(b_{t+1}^{e}(s)\right) - M\left(\widetilde{b}_{t+1}^{A}(s)\right) dG(s)\ . \tag{2.8d}$$

Proof: see Appendix A.2.4.

Equation (2.6) reflects the additionally eligible collateral under the assumption that firms do not change their debt choice. These firms were non-eligible under tight eligibility requirements and, therefore, issue bonds according to $b_{t+1}^n(s)$. The mechanical effect of collateral easing is positive.

The collateral supply effect of collateral easing associated to firm responses is given by (2.7). To aid intuition, Figure 2.3a shows the impact of collateral easing on firm debt issuance across firm types s. The lines b^n and b^e denote the debt choices that satisfy (2.3) and (2.4), respectively. As shown in Lemma 1 they increase in firm productivity. Dashed lines denote the eligible debt capacity under tight (blue) or lenient (orange) collateral policy. Under a lenient policy, the eligible debt capacity increases for every productivity level compared to a tight policy so that the line shifts to the left. Bold colored lines denote the actual firm debt choices.

For a given collateral policy, we can distinguish the three firm types from Proposition 2. Highly productive firms (above s_2) choose debt according to (2.4), i.e., they are unconstrained eligible. Firms of medium quality, between the jump at s_0 and the kink at s_2, choose their eligible debt capacity and are constrained eligible. Last, low productivity firms (below s_0) are non-eligible and choose debt b^n following (2.3). Risk-taking and disciplining effects for a given policy are related to the difference between the firm choice (bold lines) and b^n (solid black line). First, firms between s_0 and s_1 reduce debt issuance compared to non-eligibility, i.e., they *discipline* themselves to be eligible. Second, firms above s_1 issue more debt compared to b^n, which is a *risk-taking* effect.

Lowering the eligibility threshold, going from the blue to the orange lines, changes productivity cut-off values (see Lemma 2). The first integral (2.7a) relates to firms that reduce their debt issuance relative to the tight policy to benefit from being eligible, which is graphically represented by the red area in Figure 2.3a. It is associated with the disciplining effect across the firm distribution. All other parts of (2.7) relate to risk-taking effects (denoted as blue areas), i.e., firms that increase debt issuance compared to tight policy: the second integral (2.7b) corresponds to firms issuing debt at their eligible debt capacity, but above $b^n_{t+1}(s)$, which exceeds their borrowing under tight eligibility requirements. Likewise, the third integral (2.7c) captures firms that remain constrained eligible but with a higher eligible debt capacity $\widetilde{b}^{BBB} > \widetilde{b}^{A}$. Last, the fourth integral (2.7d) summarizes firms that switch from constrained to unconstrained eligible.

Disciplining and risk-taking have a positive collateral supply effect. Firms becoming newly eligible via the disciplining effect automatically increase collateral supply. For firms that take on more risk, this can be seen by noting that those firms will not issue debt beyond a point where debt dilution exceeds the funds raised by issuing an additional unit of debt.[10]

In contrast, effects on the aggregate cost from default (2.8) are ambiguous. We illustrate the change in firm default risk across types s in Figure 2.3b. Default risk associated to b^n and b^e (solid black lines) falls for more productive firms as seen from Lemma 2. The eligibility thresholds are given by the horizontal colored lines and default risk related to the firm debt choice is given by bold colored lines. As in Figure 2.3a, one can distinguish the three firm types of Proposition 2. The effect of collateral easing on aggregate default costs is closely related to the change in default risk across firm types: while disciplining effects in the first integral (2.8a) lead to a reduction in aggregate default cost (red area), the other three integrals (2.8b)-(2.8d) related to risk-taking effects (blue areas), imply an increase of aggregate default costs.

Our framework with short-term debt predicts that collateral easing has a positive mechanical impact on collateral supply, which is amplified by firm responses. At the same time, the effect of firm responses on aggregate default cost is ambiguous and depends on the relative strength of risk-taking and disciplining effects, making a heterogeneous firm model necessary to adequately

[10]Differentiating the market value of eligible bonds yields $\frac{\partial((1+L)(1-F(b^e_{t+1}))b^e_{t+1})}{\partial b^e_{t+1}} = (1+L)(1-h(b^e_{t+1})b^e_{t+1})(1-F(b^e_{t+1}))$. Rewriting in terms of the hazard rate (2.4), this simplifies to $\beta(1-F(b^e_{t+1})) > 0$.

study aggregate effects. To quantify the relevance of endogenous firm responses and determine their sign, we extend our framework with persistent revenue shocks and long-term debt in the next section. In this setting, there is a pronounced dampening effect on collateral supply, which has been described in several settings with long-term debt and default risk (see Gomes et al., 2016; Jungherr and Schott, 2022). This gives rise to a negative relationship between increasing collateral supply (which facilitates monetary policy implementation) and incentivizing risk-taking at the firm level (which increases resource losses of default). We discuss the implied central bank policy trade-off and a potential remedy in Section 2.4.

2.3 Application to the ECB Collateral Easing Policy

This section applies our framework to the ECB's collateral easing policy in response to the financial crisis of 2008. We extend the model by long-term bonds and persistent revenue shocks, solve it using global methods, and present its calibration to euro area data. We then use the calibrated model to shed light on corporate bond spreads and study the aggregate impact of endogenous firm responses induced by collateral easing. The characterization of aggregate effects forms the basis for our analysis of collateral policy in Section 2.4.

2.3.1 Extended Model

Firm heterogeneity takes the form of persistent revenue shocks rather than permanent differences in the idiosyncratic firm revenue distribution. In addition, bonds are long-term and a firm defaults if it cannot repay the maturing share of outstanding bonds out of current revenues. As in the previous section, we maintain the assumption of no delays in restructuring so that the value of non-maturing bonds is not affected by a default event. This permits us to abstract from the firm credit status as a state variable.

Firms. There is a continuum of competitive firms, indexed by j. Firms receive random revenues $e^{\mu_t^j}$ following an log-AR(1) process

$$\mu_t^j = \rho\mu_{t-1}^j + \sqrt{\sigma}\epsilon_t^j \quad \text{with} \quad \epsilon_t^j \sim N(0,1) \ .$$

The idiosyncratic revenue shock is independent across firms. We denote the (conditional) pdf of μ_{t+1}^j by $f(\mu_{t+1}^j|\mu_t^j)$ and the associated (conditional) cdf by $F(\mu_{t+1}^j|\mu_t^j)$. Firms issue long-term bonds b_{t+1}^j, which enables us to generate realistic debt ratios. Each period, a share π of outstanding bonds matures. The non-maturing share of bonds is valued like new issues at price q_t, according to the law of one price. Firms default on their current repayment obligation πb_t^j if they exceed current revenues $e^{\mu_t^j}$. We can write the default probability as

$$F(b_{t+1}^j|\mu_t^j) = \Phi\left(\frac{\log\left(\pi b_{t+1}^j\right) - \rho\mu_t^j}{\sigma}\right) \ . \tag{2.9}$$

Banks and Bond Pricing. Banks are modeled in a similar way as in Section 2.2. They are risk-neutral and discount the future at the constant rate r^{rf}. The per-unit price schedule for corporate bonds can be written

$$q(b^j_{t+1}, \mu^j_t) = \frac{1 + \Psi(F(b^j_{t+1}|\mu^j_t))L}{1 + r^{rf}} \left(\pi \left(1 - F(b^j_{t+1}|\mu^j_t) \right) + (1-\pi)\mathbb{E}_t \Big[q \left(\mathcal{B}(b^j_{t+1}, \mu^j_{t+1}), \mu^j_{t+1} \right) \Big] \right) \ . \tag{2.10}$$

Note that the rollover value of bonds is obtained from evaluating the bond price schedule at next period's debt choice $\mathcal{B}(b^j_{t+1}, \mu^j_{t+1})$, which we describe below. As in the baseline model from Section 2.2, the total payoff contains a pecuniary part and a collateral premium L. The pecuniary part depends on default in $t+1$. If the firm repays, the maturing fraction π is redeemed and the remainder $1-\pi$ is rolled over at the next period's market price. In the case of default, banks lose the maturing fraction of the bond. Due to the assumption of immediate restructuring, the payoff still contains the rollover value of the non-maturing fraction.

Characterization of Debt Choices. Firms choose issue bonds b^j_{t+1} to maximize shareholder value, taken as given the bond price schedule (2.10). The maximization problem of firm j can be represented by the Bellman equation

$$W(b^j_t, \mu^j_t) = \max_{b^j_{t+1}} \quad V(b^j_{t+1}, \mu^j_t) \quad \text{with} \tag{2.11}$$

$$V(b^j_{t+1}, \mu^j_t) = \mathbb{1}\{e^{\mu^j_t} > \pi b^j_t\} \left(e^{\mu^j_t} - \pi b^j_t \right) + q(b^j_{t+1}, \mu^j_t) \left(b^j_{t+1} - (1-\pi) b^j_t \right) + \beta \mathbb{E}_t \left[W(b^j_{t+1}, \mu^j_{t+1}) \right] \ . \tag{2.12}$$

Current dividends are given by revenues and debt service obligations $e^{\mu^j_t} - \pi b^j_t$, conditional on repayment, and net debt issuance $q(b^j_{t+1}, \mu^j_t) \left(b^j_{t+1} - (1-\pi) b^j_t \right)$. Note that the debt choice b^j_{t+1} does not depend on a potential default in period t, which again follows from the assumption of immediate restructuring. A higher debt choice increases current dividends but reduces next period's dividends due to (i) higher default risk, (ii) elevated debt service conditional on no default, and (iii) increasing the rollover burden in the next period. Plugging in the bond pricing condition (2.10), the first order condition can be written as

$$\frac{\partial q(b^j_{t+1}, \mu^j_t)}{\partial b^j_{t+1}} \left(b^j_{t+1} - (1-\pi) b^j_t \right) + q(b^j_{t+1}, \mu^j_t) = \beta \left(\pi (1 - F(b^j_{t+1}|\mu^j_t)) + (1-\pi) \mathbb{E}_t \left[q_{t+1} \right] \right) \ , \tag{2.13}$$

where the derivative of the bond price schedule (2.10) is given by

$$\frac{\partial q(b^j_{t+1}, \mu^j_t)}{\partial b^j_{t+1}} = \begin{cases} -f(b^j_{t+1}) \pi \frac{1}{1+r^{rf}} \ , & \text{if } F^j_{t+1} > \overline{F} \ , \\ -f(b^j_{t+1}) \pi \frac{1+L}{1+r^{rf}} \ , & \text{if } F^j_{t+1} \leq \overline{F} \ . \end{cases} \tag{2.14}$$

Let the solution to (2.13) in the case without eligibility be denoted by $b^{j,n}_{t+1}$ and in the case of eligibility by $b^{j,e}_{t+1}$. The debt choice depends on the feasibility of $b^{j,e}_{t+1}$ and the value of the objective function (2.12) under both candidate debt choices. The eligible debt capacity in closed form is obtained from evaluating the default probability (2.9) at $\overline{F}$ and re-arranging to

$$\widetilde{b}^j_{t+1} = \frac{\exp\{\sigma\Phi^{-1}(\overline{F}) + \rho\mu^j_t\}}{\pi} \ , \tag{2.15}$$

which we can use to obtain the debt choice $\mathcal{B}(b^j_t, \mu^j_t)$

$$\begin{aligned} \mathcal{B}(b^j_t, \mu^j_t) = \mathbb{1}\left\{V(b^{j,n}_{t+1}, \mu^j_t) \leq V(\min\{b^{j,e}_{t+1}, \widetilde{b}^j_{t+1}\}, \mu^j_t)\right\} \cdot \min\{b^{j,e}_{t+1}, \widetilde{b}^j_{t+1}\} \\ + \mathbb{1}\left\{V(b^{j,n}_{t+1}, \mu^j_t) > V(\min\{b^{j,e}_{t+1}, \widetilde{b}^j_{t+1}\}, \mu^j_t)\right\} \cdot b^{j,n}_{t+1} \ . \end{aligned} \tag{2.16}$$

If $b^{j,e}_{t+1} > \widetilde{b}^j_{t+1}$, the eligible debt choice is not feasible. Therefore, the debt choice $\mathcal{B}(b^j_t, \mu^j_t)$ depends on the value attained by exhausting the eligible debt capacity $V(\widetilde{b}^j_{t+1}, \mu^j_t)$ and the value of forgoing eligibility $V(b^{j,n}_{t+1}, \mu^j_t)$. Conversely, if $b^{j,e}_{t+1} < \widetilde{b}^j_{t+1}$, the firm can issue the optimal level of bonds without losing eligibility. Consistent with the one-period model in Section 2.2, the firm will issue $b^{j,e}_{t+1}$ in this case, since $b^{j,n}_{t+1} < b^{j,e}_{t+1}$ and $V(b^{j,n}_{t+1}, \mu^j_t) < V(b^{j,e}_{t+1}, \mu^j_t)$ by definition. Since there is no aggregate risk and banks' bond pricing condition is independent of the firm distribution, the debt choice of firms and the bond pricing condition of banks fully characterize the equilibrium of our model. The equilibrium bond price $\mathcal{Q}(b^j_t, \mu^j_t)$ obtains from evaluating the bond price schedule (2.10) at the debt choice (2.16)

$$\mathcal{Q}(b^j_t, \mu^j_t) = q(\mathcal{B}(b^j_t, \mu^j_t), \mu^j_t) \ .$$

Recursive Competitive Equilibrium. A competitive equilibrium is given by the bond price schedule $q(b^j_{t+1}, \mu^j_t)$, the firm value function $W(b^j_t, \mu^j_t)$, and the debt choice $\mathcal{B}(b^j_t, \mu^j_t)$ such that

- given the pricing schedules for bonds, the debt choice solves the firm problem (2.11).
- bonds are priced according to (2.10).
- the law of motion for the distribution of firms over bond holdings and firm-specific revenues follows

$$G_{t+1}\left(b_{t+1}, \mu_{t+1}\right) = \int\int \mathbb{1}\left\{b_{t+1} = \mathcal{B}(b_t, \mu_t)\right\} \times \mathbb{1}\left\{\mu_{t+1} = \rho\mu_t + \sigma\epsilon_{t+1}\right\} \times G_t\left(b_t, \mu_t\right) f(\epsilon_{t+1}) d\epsilon_{t+1} db_{t+1} \ .$$

Numerical Solution Method. We solve the full model computationally using policy function iteration on a discrete grid for revenues and bond issuance. The algorithm contains four steps at each iteration: first, we compute both potentially optimal debt choices by solving (2.13), given the bond price schedule (2.10). Second, we compute the eligible debt capacity (2.15) and check whether the optimal debt choice under eligibility is feasible. If this is not the case, we replace it by the eligible debt capacity $\widetilde{b}$. We randomize over the value function under both candidate debt choices using Gumbel-distributed taste shocks as proposed by Gordon (2019) to compute the debt choice (2.16). Third, given these policies, we compute the distribution of firms over individual states. The fourth step consists of updating bond price schedules. For a detailed description of the algorithm and the parameters governing our numerical approximation, we refer to Appendix A.3.2.

2.3.2 Calibration

We calibrate the model to euro area data between 2004Q1, the earliest data with reliable corporate bond data, and 2008Q3, the last quarter before the ECB relaxed its collateral framework. One period corresponds to one quarter. Our calibration is divided into two parts: the first part contains parameters determining the pricing of bond payoffs and eligibility benefits by banks, while the second set of parameters is related to firm fundamentals and the payoff profile of corporate bonds. These two blocks are connected by the central bank eligibility requirement, which is the policy variable of interest. We consider two policies: the baseline calibration is associated with tight eligibility requirements (A-rating or higher) and collateral easing refers to a scenario with lenient eligibility requirements (BBB-rating or higher). These thresholds are based on the ECB policy before and after the financial crisis of 2008.

Eligibility Requirement. We begin with discussing the eligibility thresholds $\overline{F}^A$ and $\overline{F}^{BBB}$. The ECB's collateral framework is based on ratings by external credit assessment agencies that are difficult to model parsimoniously. Therefore, we adopt an indirect approach based on macroeconomic aggregates. Specifically, we obtain data from *IHS Markit* on the total fixed income securities universe in Europe and extract the subset for non-financial corporate bonds. Using data from September 2008, the last month prior to the relaxation of eligibility requirements, 50% of all corporate bonds in our sample carried a rating of A or higher and were formally eligible as collateral. To match this share of eligible bonds, we set the baseline eligibility threshold to $\overline{F}^A = 4.15\%$, expressed in annualized terms. Similarly, we choose the eligibility threshold for a BBB-rating as $\overline{F}^{BBB} = 18.59\%$ to match the share of bonds rated BBB or higher in the *IHS Markit* sample, which was 86% in September 2008. We interpret this observed 72% increase of collateral supply as measure of the *mechanical* effect, since it is based on data prior to the policy relaxation.

Collateral Premium. We proxy the time-invariant real risk-free interest rate by a short-term interbank rate from which we subtract the consumer price inflation rate. Specifically, we use the time-series average of the 3M-EURIBOR minus the euro area inflation and obtain $r^{rf} = 0.0035$. The collateral premium L is based on the empirical findings from Pelizzon et al. (2020).[11] Their paper makes use of the ECB having discretion in including bonds that formally satisfy eligibility requirements in the list of eligible assets. This discretion generates a randomly selected control group of bonds that eventually become eligible but are not yet accepted. Depending on the econometric specification, they estimate a yield drop to surprise eligibility of 11-24bp. We pick the most conservative value of 11bp. Our structural model permits an explicit calculation of the yield effect of a surprise inclusion. We set $\Psi = 0$ when pricing the bond (holding firm behavior fixed) and compare this hypothetical price to the equilibrium bond price. The price of this

[11] Notably, L does not depend on aggregate collateral supply. We relax this assumption in Appendix A.4.4.

Table 2.1: Baseline Parameterization

Parameter	Value	Source
Bank discount rate r^{rf}	0.0035	Real risk-free rate
Borrower discount factor β	0.993	Calibrated
Maturity Parameter π	0.06	Calibrated
Collateral premium L	0.004	Calibrated
Revenue persistence ρ	0.93	Calibrated
Revenue shock std. dev. σ	0.027	Calibrated
(Annualized) A-eligibility threshold $\overline{F}^{A}$	4.15%	Calibrated
(Annualized) BBB-eligibility threshold $\overline{F}^{BBB}$	18.59%	Calibrated

hypothetical bond is given by

$$q^n(b^j_{t+1}, \mu^j_t) = \frac{1}{1+r^{rf}} \left(\left(1 - F(b^j_{t+1}|\mu^j_t)\right) \pi + (1-\pi)\mathbb{E}_t\Big[q\left(b^j_{t+1}, \mu^j_{t+1}\right) \Big] \right) , \tag{2.17}$$

and contains a collateral premium from $t+1$ onward via the bond continuation value. The yield-to-redemption $\tilde{r}^j$ is determined by the internal rate of return of a perpetuity with constant decay,

$$q^j_t = \sum_{\tau=t+1}^{\infty} \frac{\pi(1-\pi)^{\tau-1}}{(1+r^j_t)^{\tau}} = \frac{\pi}{\pi + r^j_t} .$$

It follows that $r^j_t = \pi/q^j_t - \pi$. The corporate bond spread is defined as $x^j_t \equiv r^j_t - r^{rf}$. Using an entirely analogous derivation, the yield on the hypothetical non-eligible bond is given by $r^{j,n}_t$ and the eligibility premium follows as $r^j_t - r^{j,n}_t$, which is always negative.

Firm Fundamentals. The second part of the calibration is related to firms, i.e., the parameters governing the idiosyncratic revenue process ρ and σ, the maturity parameter π characterizing the repayment profile of corporate bonds, and the discount factor of firms β affecting the relative impatience over investors. By setting β to a lower value than the time discount factor of banks $\frac{1}{1+r^{rf}}$, we ensure that even absent collateral premia, firms would have an incentive to issue bonds.

These parameters are chosen to match selected data moments characterizing the firm cross-section. We merge our corporate bond dataset from *IHS Markit* with company data available through *Compustat Global*. A description of the construction of our dataset is given in Appendix A.3. Specifically, we target the median debt/EBIT-ratio b^j_t/μ^j_t as a measure of firm indebtedness and the bond spread distribution, characterized by its quartiles. The time-series averages over the sample period 2004Q1-2008Q3 are $Q_{0.25}(x) = 24$bp, $Q_{0.50}(x) = 39$bp, and $Q_{0.75}(x) = 62$bp. We conduct a sensitivity analysis with respect to a higher level of spreads computed over an extended sample period in Appendix A.4.3. Table 2.1 summarizes all parameters for our baseline calibration and Table 2.2 shows the targeted moments in our baseline

Table 2.2: Targeted Moments

Moment	Data	Model
Collateral premium ave$(r - r^n)$	-11	-11
Debt/EBIT $Q_{0.50}(b/\mu\|\overline{F}^A)$	3.9	3.9
Bond spread $Q_{0.25}(x\|\overline{F}^A)$	24	27
Bond spread $Q_{0.50}(x\|\overline{F}^A)$	39	52
Bond spread $Q_{0.75}(x\|\overline{F}^A)$	62	72
Eligible bond share $\overline{B}/(QB)\|\overline{F}^A$	50%	50%
Eligible bond share $\overline{B}/(QB)\|\overline{F}^{BBB}$	86%	83%

Notes: The collateral premium and spreads are annualized and expressed in basis points.

calibration.

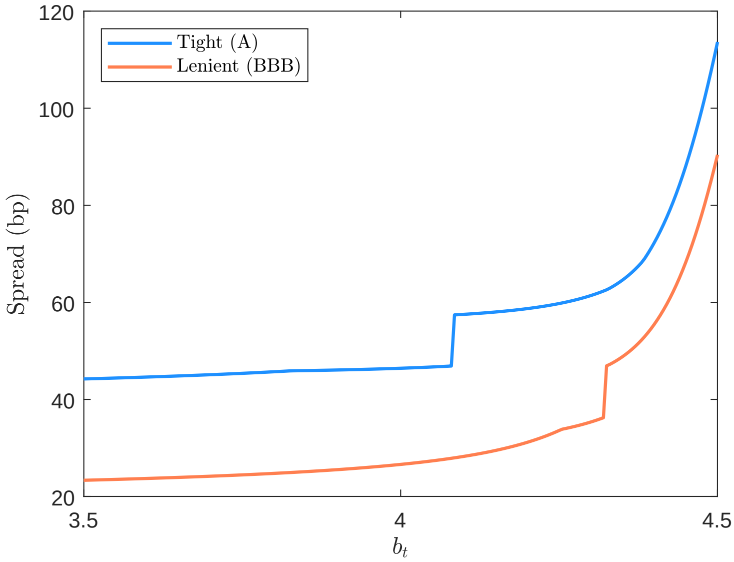
2.3.3 Corporate Bond Spreads

To illustrate how eligibility requirements affect the pricing of corporate bonds, we show the corporate bond spreads implied by banks' first order condition in Figure 2.3. Spreads are derived by evaluating the bond pricing condition at any candidate debt choice b^j_{t+1}, fixing revenues at their median. The baseline calibration (A-rating or higher) is marked in blue, while lenient eligibility requirements (BBB-rating or higher) are marked in orange. The discontinuity in each bond price schedule represents the location of the eligibility threshold. To the left of this point, bonds are currently eligible and investors are willing to pay collateral premia, which results in lower spreads. For a debt choice to the right of the discontinuity, bonds are not eligible and spreads jump upwards. The effect of relaxing eligibility requirements can be inferred from the location of the discontinuities. Intuitively, lenient eligibility requirements increase the eligible debt capacity, so that the discontinuity shifts to the right. Notably, since bonds are long-term, this also affects bond spreads away from the eligibility threshold: bonds are more likely to be eligible in future periods, which increases their price and lowers the spread already in the current period via the continuation value in (2.10). Hence, spreads under lenient eligibility requirements are uniformly lower.

2.3.4 Reconciling Cross-Sectional Evidence

Before discussing macroeconomic aggregates, we test the model's capability to replicate the (heterogeneous) impact of eligibility requirements across firms identified by several empirical papers. We run regressions on a simulated cross-section of firms, which is drawn from the equilibrium firm distribution under tight eligibility requirements. Regressions based on the firm distribution associated with lenient eligibility requirements yield similar results. We run the

Figure 2.3: Corporate Bond Spreads



Notes: The blue (orange) line represents the corporate spread under a tight (lenient) eligibility requirement, conditional on firm legacy debt b_t^j, for a firm with median revenues.

following (cross-sectional) regression

$$y_t^j = \beta_1 Eligible_t^j + \beta_2 Eligible_t^j \cdot \frac{b_t^j}{\mu_t^j} + \epsilon_t^j \ , \tag{2.18}$$

for three different specifications, that differ in the outcome variable. First, we examine the bond yield reaction to surprise eligibility $r_t^j - r_t^{j,n}$, following Pelizzon et al. (2020). Since we can control for firm indebtedness as a measure of default risk, this approach is similar to Grosse-Rueschkamp et al. (2019), Mota (2021), and Todorov (2020). Second, we evaluate the effects of a surprise inclusion on debt issuance $\mathcal{B}_{t+1}^j - b_{t+1}^{j,n}$ and, third, on dividends $\mathcal{D}_t^j - d_t^{j,n}$.[12]

The sign of the model-implied regression coefficients in all three specifications are collected in the right panel of Table 2.3. Since we sample from a parsimonious structural model, all coefficients are highly significant. We benchmark the model-implied coefficients against findings from the literature, reported in the left panel. The eligibility premium $r_t^j - r_t^{j,n}$ is a calibration target. Therefore, the coefficient on eligibility is negative by construction. The positive impact

[12]The equilibrium dividend in period t is given by

$$\mathcal{D}(b_t^j, \mu_t^j) = e^{\mu_t^j} - \pi b_t^j + q(\mathcal{B}(b_t^j, \mu_t^j), \mu_t^j) \left(\mathcal{B}(b_t^j, \mu_t^j) - (1-\pi) b_t^j \right) \ ,$$

while the dividend of an non-eligible, but otherwise identical, firm can be written as

$$d^n(b_t^j, \mu_t^j) = e^{\mu_t^j} - \pi b_t^j + q(b^n(b_t^j, \mu_t^j), \mu_t^j) \left(b^n(b_t^j, \mu_t^j) - (1-\pi) b_t^j \right) \ .$$

Table 2.3: Cross-Sectional Regression Results

	Empirical Literature			Model		
Coefficient	$r_t^j - r_t^{j,n}$	$\mathcal{B}_{t+1}^j - b_{t+1}^{j,n}$	$\mathcal{D}_t^j - d_t^{j,n}$	$r_t^j - r_t^{j,n}$	$\mathcal{B}_{t+1}^j - b_{t+1}^{j,n}$	$\mathcal{D}_t^j - d_t^{j,n}$
Eligibility	-	+	+	-	+	+
Indebtedness × Eligibility	+	-	-	+	-	-

Notes: Signs in the left panel are taken from the empirical literature. Signs in the right panel are obtained from running (2.18) on the simulated firm cross-section. Model-implied coefficient signs are independent of the tightness of eligibility requirements.

of eligibility on debt issuance is consistent with findings by Pelizzon et al. (2020), while the positive effect of eligibility on dividends has been described in Todorov (2020).

The coefficient signs of the interaction terms are informative about heterogeneous firm responses. Consistent with our theory, debt issuance and dividend payouts respond more strongly for less risky firms in the model, as the negative coefficients on the interaction term of eligibility and beginning-of-period indebtedness demonstrate. This negative relationship is consistent with the findings of Mota (2021), who documents a positive relationship of eligibility premia, debt issuance, and dividend payouts with firm safety as measured by ratings. The negative coefficient on the interaction term eligibility × indebtedness is also in line with the results of Grosse-Rueschkamp et al. (2019). They report that firms rated A or higher increased their leverage ratio by 1.8 percentage points in response to CSPP-eligibility, as opposed to eligible BBB-rated firms, which only weakly increase leverage by 0.8 percentage points. Taken together, our model can capture the impact of eligibility requirements on multiple firm outcome variables documented in the data.

2.3.5 Aggregate Effects

We now turn to the impact of collateral easing on macroeconomic aggregates. As demonstrated Appendix A.4.1, the heterogeneous risk-taking and disciplining effects from Section 2.2 carry over to the case of long-term debt and persistent revenue shocks and we will organize our discussion around these two effects as well. The changes to the cross-sectional firm distribution induced by collateral easing are relegated to Appendix A.4.2.

Similar to the one-period bond model in Section 2.2, our discussion is based on a decomposition of collateral supply into a *mechanical effect* and endogenous *firm responses*. Formally, this decomposition obtains from expanding the total effect as follows:

$$\begin{aligned}
\overline{B}^{BBB} - \overline{B}^{A} \equiv & \int \mathbb{1}\{F^{BBB} < \overline{F}^{BBB}\} q^{BBB} b^{BBB} dG^{BBB}(\mu, b) - \int \mathbb{1}\{F^{A} < \overline{F}^{A}\} q^{A} b^{A} dG^{A}(\mu, b) \\
= & \underbrace{\int \mathbb{1}\{F^{BBB} < \overline{F}^{BBB}\} q^{BBB} b^{BBB} dG^{BBB}(\mu, b) - \int \mathbb{1}\{F^{A} < \overline{F}^{BBB}\} q^{A} b^{A} dG^{A}(\mu, b)}_{\text{Firm Response}} \\
& + \underbrace{\int \mathbb{1}\{F^{A} < \overline{F}^{BBB}\} q^{A} b^{A} dG^{A}(\mu, b) - \int \mathbb{1}\{F^{A} < \overline{F}^{A}\} q^{A} b^{A} dG^{A}(\mu, b)}_{\text{Mechanical Effect}} .
\end{aligned}$$

The total effect on collateral supply is given by the difference between the market value of bonds issued by all eligible firms under either policy. The mechanical effect in the third line keeps firm behavior and the cross-sectional distribution at the baseline calibration, varying only the eligibility requirement. Firm responses are given residually and encompass changes in the market value of bonds and default risk but evaluate the eligibility status at the same minimum rating requirement $\overline{F}^{BBB}$.

In the first panel of Table 2.4, we apply this decomposition to the collateral easing experiment. It stands out that the percentage change of collateral supply from the (targeted) mechanical effect (+71%) exceeds the total effect (+62%). In contrast to Section 2.2, firm responses *dampen* the impact of eligibility requirements on collateral supply. This result is associated with the shares of firms being subject to risk-taking and disciplining effects. In particular, the share of firms disciplining themselves to be eligible falls from 19% under tight to 3% under lenient collateral policy. At the same time, the share of firms engaging in risk-taking rises from 51% to 79%. Consequently, there are also adverse effects on the corporate bond market as measured by a 8% increase of default costs.[13]

Intuitively, under lenient policy corporate bonds are eligible for worse fundamentals, which reduces disciplining incentives at the expense of risk-taking. The dampening effect of risk-taking on collateral supply is directly related to the persistence of revenue shocks and the stickiness of indebtedness: high-revenue firms find it optimal to increase their debt issuance and increase current dividends. If revenues are sufficiently persistent and firm managers sufficiently impatient, this only leads to a modest increase in default risk in the current period. Ultimately, however, firms will receive adverse revenue shocks and, due to the inherent stickiness of indebtedness, find themselves with a large amount of debt outstanding, which makes default more likely (see Jungherr and Schott, 2022, or Gomes et al., 2016). Default risk not only leads to a drop in the market value of eligible bonds but may also imply that those firms default. Both effects lower collateral supply. This feature is not present in our setting of Section 2.2 with i.i.d. shocks and one-period bonds. In such a setting, it is never optimal for firms to increase debt issuance beyond a point where it decreases the market value of bonds outstanding. The overall dampening requires the central bank to relax eligibility requirements more aggressively to achieve a specific increase in collateral supply. In addition, collateral easing induces adverse side effects on the

[13]There are no mechanical effects on aggregate default costs by construction.

Table 2.4: Macroeconomic Effects of Collateral Easing

	Total Effect	Mechanical Effect
Collateral Supply $\overline{B}$	+62%	+71%
Default Costs $\mathcal{M}$	+8%	
Firm Responses	Disciplining	Risk-Taking
Tight (A)	19%	51%
Lenient (BBB)	3%	79%

Notes: Values in the upper panel refer to collateral easing from A to BBB and are denoted as percentage difference from the A-baseline. The lower panel displays the fraction of *all* firms that is subject to disciplining or risk-taking effects.

corporate bond market in our model. In practice, higher prevalence of default risk can directly increase restructuring costs or inefficient liquidation of firms and indirectly make the financial system fragile, e.g., due to counterparty default risk.

2.4 Eligibility Covenants and the Central Bank Policy Frontier

In the previous section, we discussed the macroeconomic effects of changing the eligibility requirement. Firm responses increase resource losses from default and dampen the positive reaction on collateral supply. In this section, we extend the central bank toolkit by an eligibility covenant, which targets the large risk-taking effects associated with long-term debt and persistent revenues.

We embed our previous results in a discussion of optimal central bank collateral policy and assume that the central bank aims to minimize *violations of market discipline*, i.e., incentivizing firm risk-taking, while ensuring sufficient collateral to *facilitate monetary policy implementation.* Even though our model is too simplistic to quantitatively assess optimal policy, it is still useful to outline the key policy trade-off arising from our analysis and to discuss its potential implications for optimal collateral policy. In assuming a trade-off between violating market discipline and increasing collateral supply, we follow central banks' stated objectives (see Bindseil et al., 2017) and the literature on risky assets in the central bank collateral portfolio. In Koulischer and Struyven (2014), lenient central bank collateral policy increases credit supply and output in the private sector but implies central bank losses, because central banks are second-best user of collateral in case of a counterparty default. Similarly, Choi et al. (2021) offer a macroprudential approach to collateral policy. In their model, accepting low-quality collateral has a positive effect on bank lending, because banks can use high-quality collateral on the interbank market instead. At the same time, this exposes the central bank to potential losses.

While we do not specifically model the sources of collateral demand, we are consistent with these papers in so far that larger collateral supply is desirable but comes at a cost if this implies accepting risky collateral. Our analysis offers a complementary view, since the risk-taking decision is made at the firm level in our model. Therefore, we propose eligibility covenants to mitigate adverse collateral supply effects from firm risk-taking, while at the same time maintaining a positive quantity effect. We stress the microprudential nature of this instrument due to the absence of aggregate shocks to collateral supply and demand in our analysis.

Leverage-Based Covenants. Covenants restrict the eligible debt capacity of firms in addition to the default risk threshold $\overline{F}$ that applies uniformly to all firms. We condition covenants on firm-specific states and focus on debt-based covenants in the following. Since debt outstanding is common knowledge for firms that are sufficiently large to issue marketable debt securities, such a policy is in principle implementable. However, it still leaves us with all functions mapping from the debt state space into the binary eligibility indicator $\Psi \in \{0,1\}$. In the following, we focus on the exponential class, parametrized by $\gamma > 0$, such that the eligible debt capacity is decreasing in debt outstanding

$$\widetilde{b}^j_{t+1} = \exp\{-\gamma b^j_t\} \cdot \frac{\exp\{\sigma \Phi^{-1}(\overline{F}) + \rho \mu^j_t\}}{\pi} \,. \tag{2.19}$$

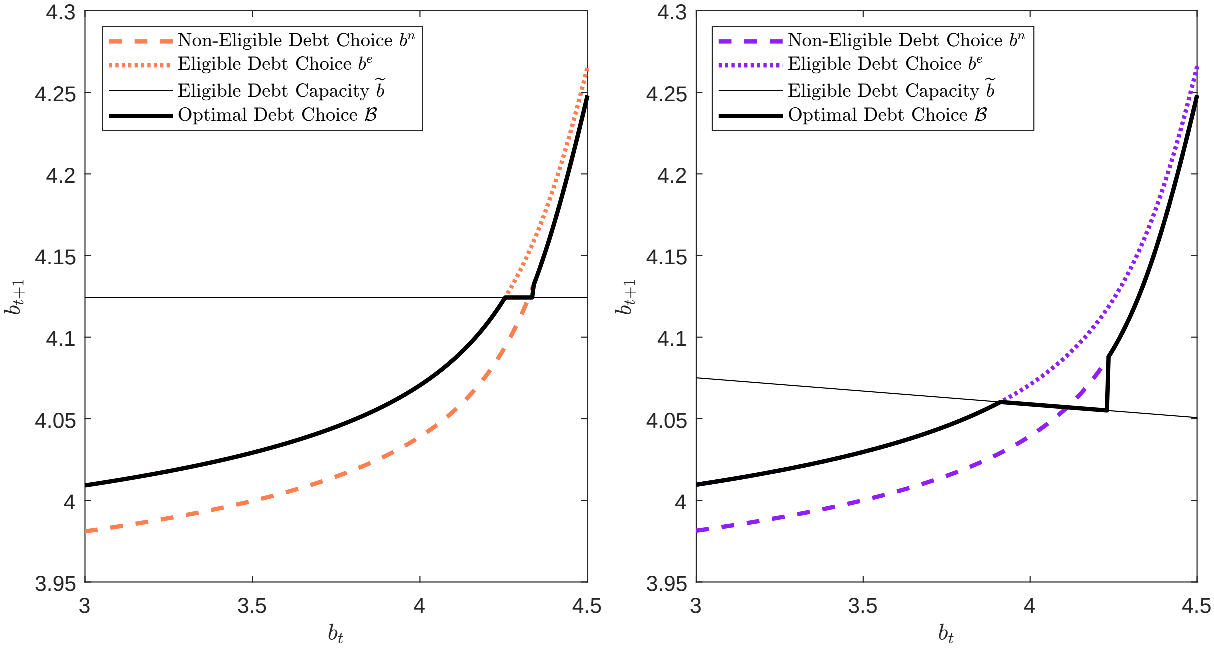
The eligibility covenant $\exp\{-\gamma b^j_t\}$ effectively lowers the eligible debt capacity of firms with high debt outstanding and, thereby, provides deleveraging incentives. We fix revenues at the median and eligibility requirements at the BBB-level to visualize the impact of an eligibility covenant on the firm debt choice in Figure 2.4. The left panel shows the case *without* an eligibility covenant. The bold black line denotes the debt choice $\mathcal{B}_{t+1}$ for a firm with median revenues under lenient eligibility requirements. This function maps bonds outstanding b_t into (gross) bond issuance b_{t+1} and exhibits a kink and a jump. These points are associated with the debt levels where firms switch from non-eligible to constrained eligible and, then, to unconstrained eligible (see Proposition 2). The orange dashed (dotted) line represents the debt choice if the firm is non-eligible (eligible) and the horizontal black lines denotes the firm's eligible debt capacity.

Firm risk-taking and disciplining effects are related to the difference between the orange dashed line b^n_{t+1} and the equilibrium debt choice $\mathcal{B}_{t+1}$ (bold black line). The disciplining effect is represented by firms reducing their debt issuance below b^n_{t+1}, which applies to firms located near the jump of the policy function. The risk-taking effect is reflected by firms issuing debt according to $\mathcal{B}_{t+1} > b^n_{t+1}$, that is, wherever the bold black line is above the dashed orange one. Compared to the mass of firms being disciplined by collateral eligibility, the risk-taking effect is sizable.

The right panel represents the case *with* an eligibility covenant. Intuitively, introducing an eligibility covenant reduces the eligible debt capacity (light black line) if firms enter the period with large legacy debt. Compared to the debt choice without covenants, the downward sloping shape of the eligible debt capacity induces a larger disciplining effect, the optimal debt choice $\mathcal{B}^{j,n}_{t+1}$ is located below $b^{j,n}_{t+1}$ for a broader range of legacy debt stocks, and, conversely, reduces the risk-taking effect. As a result, the dampening effect of firm responses on collateral supply will be limited in the presence of an eligibility covenant.

Optimal Covenants Given a Minimum Rating. We now turn to how the eligibility covenant influences collateral supply and aggregate default cost for a given eligibility requirement. The

Figure 2.4: Debt Choice with Eligibility Covenant



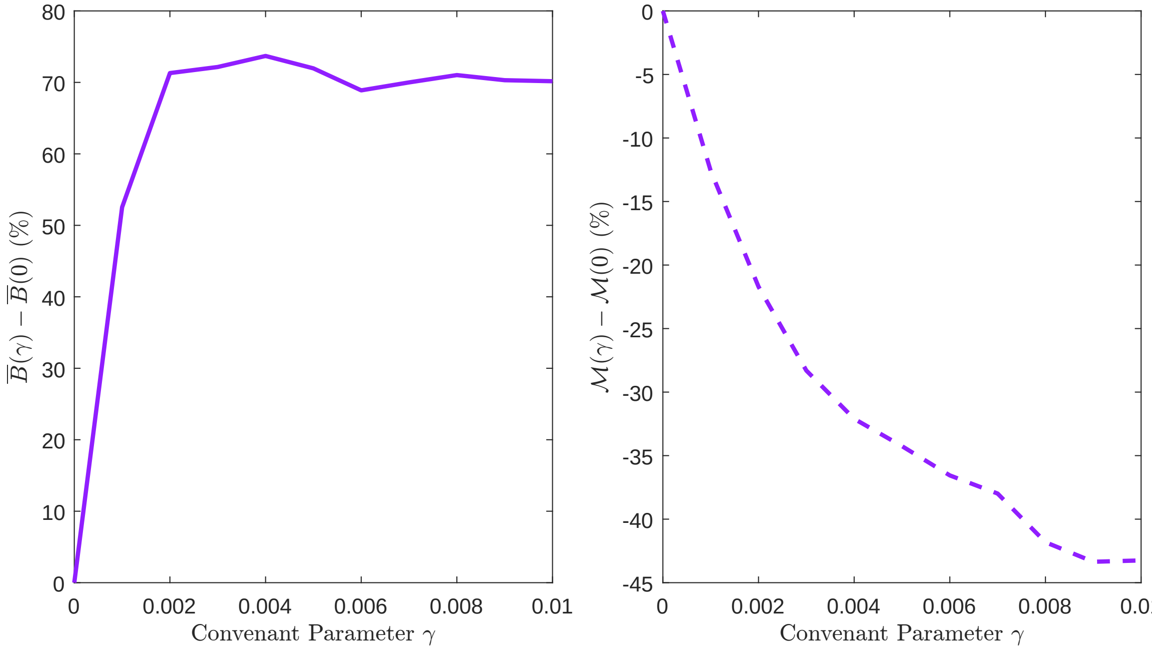
Notes: The bold black line represents the debt choice of a firm with median revenue conditional on legacy debt (see (2.16)). The purple and orange lines denote the hypothetical debt choice of an always (non-)eligible firm. The light black line is the eligible debt capacity. In the left (right) panel we depict the case without (with) covenant.

covenant has an ambiguous collateral supply impact. On the one hand, setting an overly harsh covenant (a large γ) reduces collateral supply, since it dis-incentives firms from issuing bonds. On the other hand, an overly lenient covenant (a small γ) fails to limit the risk-taking by eligible firms. We make the dependency of collateral supply $\overline{B}(\overline{F}, \gamma)$ and aggregate default cost $\mathcal{M}(\overline{F}, \gamma)$ on both policy parameters explicit in the following, compute them for different covenant parameters and show the results in Figure 2.5. The covenant gives rise to a collateral Laffer curve, that reflects this trade-off. We observe that the covenant increases collateral supply by *up to* 74% (left panel), which is very similar to the 72% increase induced by collateral easing from $\overline{F}^A$ to $\overline{F}^{BBB}$. At the same time, we observe a potential reduction in aggregate default cost of *up to* 42% (right panel). Thus, already for a fixed collateral eligibility policy, the covenant has powerful impacts.

Eligibility Covenants and the Collateral Policy Frontier. Next, we investigate how adding covenants to the central bank toolkit, in addition to the eligibility threshold, affects the collateral policy trade-off between high collateral supply and maintaining a high level of market discipline. We define the cost of violating market discipline as additional default cost that derive from making corporate bonds eligible, i.e., default cost are expressed relative to an economy without bond eligibility, where all firms would issue debt according to $b_{t+1}^{j,n}$.

Figure 2.6 shows the results in terms of the collateral policy frontier. Each dot is associated with a fixed collateral supply target on the x-axis. For a given supply target, we then choose the policy parameters to minimize the *additional default cost* relative to the case of market

Figure 2.5: Aggregate Effects of Eligibility Covenant



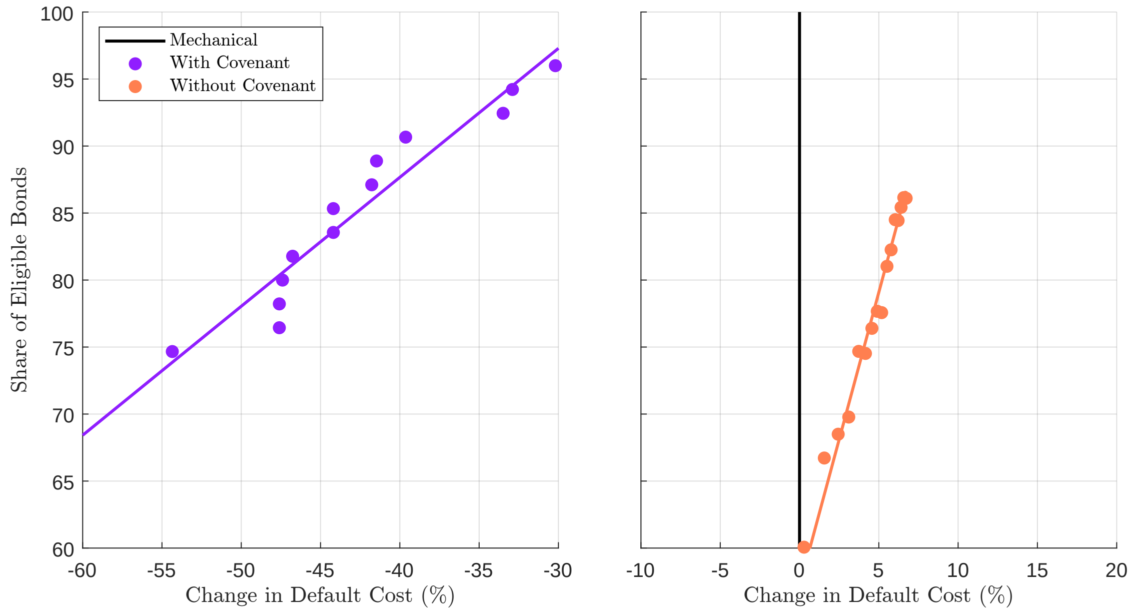
Notes: We show the effect of varying the covenant parameter γ for lenient eligibility requirements $\overline{F}^{BBB}$. The y-axis in the left (right) panel shows the percentage-increase in collateral supply (aggregate default cost) relative to the no-covenant case $\gamma = 0$.

discipline, which the central bank has to allow to satisfy its supply target, as shown on the y-axis. We distinguish between the baseline case with eligibility thresholds only (orange) and the extended central bank toolkit that also comprises covenants (purple). The vertical black line indicates the benchmark with no firm responses, where default cost are the same as in the case of market discipline. In this setting, the central bank could simply pick a collateral supply target and set $\overline{F}$ accordingly without adverse effects on the corporate bond market. Endogenizing firm responses gives rise to a trade-off between collateral supply and default cost, which is reflected by the positive slope of the policy frontier.

In the case without covenant (right panel), increasing the collateral supply necessitates increasing $\overline{F}$, which leads to additional default cost compared to the market discipline case. With covenant (left panel), the positive relation between collateral supply and default risk persists, but the overall level of default cost is significantly lower and even falls compared to the market discipline case. At the same time, the associated collateral supply levels are substantially higher, since the covenant incentivizes firms to issue more eligible *and* default less frequently. Consequently, the collateral policy frontier is shifted outward, making this instrument a potentially powerful extension of collateral frameworks.

Implementation. In our model, the eligibility covenant is expressed in terms of firm-specific eligible debt capacities, which are negatively dependent on debt outstanding $\partial\widetilde{b}^j_{t+1}/\partial b^j_t < 0$. In practice, implementing such covenants would require information on the indebtedness of firms, i.e., about current revenues and its dynamics, as well as the maturity structure of outstanding

Figure 2.6: Collateral Policy Frontier



Notes: Both panels display the additional default cost (relative to market discipline) that are necessary to satisfy a given collateral supply target. The collateral supply target is expressed as the share of eligible bonds $\overline{B}$ relative to the market value of all bonds under market discipline. In the left panel, we vary the eligibility threshold $\overline{F}$ and the covenant parameter γ. In the right panel, the covenant parameter is fixed at $\gamma = 0$.

liabilities. However, revenue dynamics and debt repayment schedules of large firms are often difficult to determine, particularly if firms have multiple subsidiaries. Therefore, several collateral frameworks (the ECB's among them) are based on credit assessments by external rating agencies.

While we abstract from modeling credit ratings and we assume that revenues and debt outstanding at the firm level is common knowledge, it is of practical importance that eligibility covenants in our model can be expressed in terms of firm-specific eligibility thresholds, which negatively dependent on debt outstanding $\partial\overline{F}_t^j/\partial b_t^j < 0$. T For sufficiently large firms, the eligibility status could be made dependent on a measure of debt outstanding *and* CDS-spreads. This would allow for a more granular classification of firms into different eligibility categories based on spreads and debt outstanding.[14] An alternative way to implement covenants is to condition eligibility on rating notches combined with debt outstanding or on the *rating outlook*, if these take into account the sustainability of firm debt in a satisfactory way. Firms rated A but with negative outlook can for example be interpreted as being on a financially unsustainable path and could, therefore, be subjected to a tighter eligibility requirement than a firm rated BB+ but with a positive outlook. This would be especially useful for firms on which no CDS are actively traded.

Last, note that collateral frameworks not only comprise eligibility thresholds but also haircuts on eligible assets. From the firm's point of view, a higher haircut reduces the collateral value of its

[14] A discussion regarding the usage of market-based credit risk assessments, such as CDS-spreads, is given by Nyborg (2017).

bonds and could in principle be made dependent on debt outstanding, which would also reduce risk-taking incentives. However, covenants provide much more salient deleveraging incentives, if a firm would observe its eligible debt capacity through the investment bank handling the underwriting process of new bond issues. In contrast, a haircut would still leave new bond issues eligible. From the central bank's point of view, haircuts are often set to account for losses in asset liquidation in the event of counterparty default. Haircuts typically address a different form of risk so that the covenant remains a potentially useful extension of collateral frameworks.

2.5 Conclusion

This paper evaluates the effects of central bank eligibility requirements on the debt and default decision of firms, i.e., the collateral supply side. Adding collateral premia and eligibility requirements to a heterogeneous firm model with default risk reveals that firms can be affected in different ways: low-risk firms increase their debt issuance and risk-taking, whereas medium-risk firms are disciplined by the prospect of benefiting from collateral premia. Both effects increase aggregate collateral supply, while they have opposing effects on cost from corporate default. Which of these two effects is the dominating force is, therefore, a numerical question. Consistent with empirical evidence at the firm level, our numerical findings suggest that risk-taking is the dominating force in the aggregate. Endogenous firm responses are quantitatively relevant and substantially dampen the impact of collateral easing on collateral supply. Eligibility covenants are suitable instruments to alleviate adverse risk-taking effects on collateral supply and aggregate default cost.

Our work can be extended along multiple dimensions. Interacting endogenous collateral supply with frictions on the collateral demand side, such as aggregate liquidity risk, can potentially generate interesting interactions with implications for the conduct of collateral policy. It should also be stressed that we take investment opportunities as exogenous. A model with endogenous investment allows to study real effects of eligibility requirements using a richer trade-off between distributing cashflows as dividends and investment. We also do not account for bank loans as alternative source of financing, which is also a margin affected by eligibility requirements. All extensions add additional layers of complexity to our present framework and we leave them to future research.

3 The Preferential Treatment of Green Bonds

Authors: Francesco Giovanardi, Matthias Kaldorf, Lucas Radke, and Florian Wicknig

3.1 Introduction

> *The ECB [...] stands ready to support innovation in the area of sustainable finance [...], exemplified by its decision to accept sustainability-linked bonds as collateral.*
> Strategy Review (European Central Bank, 2021a)

The European Central Bank (ECB) announced to take a more active role in environmental policy after concluding its strategy review. In addition to accepting sustainability-linked (*green*) bonds as collateral, several central banks contemplate to take one step further and treat them preferentially within their collateral frameworks, i.e., the conditions under which banks can pledge assets to obtain funding from the central bank.[1] The People's Bank of China (PBoC) started accepting green bonds as collateral on preferential terms already in 2018, which resulted in a substantial decline of green bond yields relative to conventional ones (Macaire and Naef, 2022). However, there is limited knowledge about the macroeconomic impact of a preferential collateral policy on green bond issuance, green investment, pollution, and potential adverse side effects on financial markets.

To study the positive and normative implications of preferential treatment, this paper extends the standard RBC-model by an environmental externality, green and conventional firms issuing corporate bonds subject to default risk, and a banking sector using these bonds as collateral. The extent to which corporate bonds can be used as collateral depends on central bank haircuts. Reducing haircuts on green bonds makes holding such bonds more attractive to banks and implies that they pay higher collateral premia on them. This in turn relaxes financing conditions for green firms, which increase bond issuance, investment and leverage in response: the equilibrium shares of green bonds and capital rise. We quantitatively assess these effects in a calibration to the euro area.

We uncover four main results. First, *maximal* preferential treatment, which applies a 100% haircut on conventional bonds and a 0% haircut on green bonds, increases the share of green bonds (capital) by almost 6% (3.7%), which reduces pollution. Second, in response to a preferential treatment green firms increase leverage, default risk, and dividend payouts. This risk-taking effect dampens the transmission of preferential treatment on green investment and increases resource losses from costly default. Because of these adverse effects, *optimal* collateral policy features a smaller degree of preferential treatment than the maximum case of only accepting green

[1] A similar policy was also proposed in Brunnermeier and Landau (2020).

bonds. Third, Pigouvian taxes on pollution as alternative instrument do not induce risk-taking and the associated welfare gains exceed the gains from optimal collateral policy considerably. Fourth, preferential treatment is an *imperfect substitute* for Pigouvian taxation. The optimal degree of preferential treatment decreases, the closer Pigouvian taxes get to their optimal level. When Pigouvian taxation is optimal, green and conventional bonds are treated *symmetrically*. In this case, however, the central bank optimally relaxes the collateral framework to address negative effects of environmental policy on collateral availability.

Our analysis is based on an extended RBC-model that connects collateral policy to financial market and environmental frictions. We assume that there are two types of intermediate good firms, green and conventional. Conventional firms generate a negative externality (pollution) during the production of intermediate goods, while green firms have access to a clean technology. Following Heutel (2012) and Golosov et al. (2014), final good firms combine green and conventional intermediate goods with labor. Pollution has a negative effect of final good firms' output, implying sub-optimally low investment into the green technology.

Collateral policy is linked to the real sector by the corporate bond market, where both intermediate good firms issue bonds to banks. Firms have an incentive to issue bonds, because their owners are assumed to be more impatient than households, who own banks. Moreover, firms are subject to idiosyncratic shocks to their productivity and default on their bonds if revenues from production fall short of current repayment obligations. Corporate bond issuance is determined by a trade-off between relative impatience and bankruptcy costs, similar to Gomes et al. (2016).[2] Banks collect deposits from households, invest into corporate bonds, and incur liquidity management costs. In the spirit of Piazzesi and Schneider (2021), these costs are decreasing in the amount of available corporate collateral reflecting that banks may use it to collateralize short-term borrowing. This introduces a willingness of banks to pay *collateral premia* on corporate bonds.[3]

The central bank sets haircuts on corporate bonds that determine the degree to which bonds can be used as collateral. While low haircuts increase collateral availability for banks, the central bank incurs costs from accepting risky bonds as collateral. The literature has associated these costs with risk management expenses and counterparty default risk that depend on the riskiness of collateral (Bindseil and Papadia, 2006; Hall and Reis, 2015). As in Choi et al. (2021), optimal collateral policy balances these two effects. Starting from this point, our paper studies the welfare gains of adding a second variable (the green haircut) to the central bank collateral framework.

The link between collateral policy and the real sector via banks' demand for bonds allows the central bank to affect the relative prices of green and conventional bonds by tilting the collateral framework in favor of green bonds. In this case, banks pay higher collateral premia on green

[2]Since our focus is on the collateral framework and thereby on firms that are sufficiently large to issue bonds and related marketable assets, we employ a financial friction that restricts *debt issuance* rather than overall *external financing* as in the canonical financial accelerator model of Bernanke et al. (1999). Moreover, our framework encompasses all marketable debt securities issued by non-financial firms, like syndicated bank loans and commercial paper.

[3]Collateral premia on corporate bonds are documented by Pelizzon et al. (2020) for the euro area, Mota (2021) for the US, and Fang et al. (2020) and Chen et al. (2019) for China.

bonds, ceteris paribus, since holding them lowers liquidity management costs more effectively. Green firms respond to higher collateral premia on their bonds by increasing bond issuance and investment, while conventional firms reduce their bond and investment positions. However, since higher collateral premia make debt financing more attractive, green firms also increase leverage and risk-taking. Notably, the effect on the green investment share is *permanent*, i.e., central bank collateral policy is *not neutral* even in the long run.[4] Higher risk-taking reduces the expected return on green investment so that the equilibrium green investment share is smaller than the green bond share under such a policy. As a result, the transmission of preferential treatment on the green investment share is substantially dampened. The endogeneity of risk-taking, which is key for the imperfect pass-through result, is consistent with the data.[5]

To quantify the optimal degree of preferential treatment, we calibrate the model to euro area data and conduct a number of policy experiments. First, we study a maximal preferential treatment policy, which makes conventional bonds non-eligible and accepts all green bonds without a haircut. This policy induces a green-conventional bond spread (also referred to as *greenium*) of 160 basis points in equilibrium, which translates into a change in the relative share of green bonds from 20% to 21.17%, while the share of green capital only increases from 20% to 20.74%. However, maximal preferential treatment reduces the collateral supply below its optimal level and distorts the risk choice of green firms. Therefore, we maximize the welfare objective over both haircut parameters. Optimal collateral policy still treats green bonds preferentially but increases the haircuts on conventional bonds to less than 100% to keep aggregate collateral supply approximately constant. In this case, the greenium amounts to 16 bp, the relative share of green bonds goes up to 20.11%, whereas the share of green capital increases to 20.07%.

While our numerical findings suggest that collateral frameworks can induce a shift towards green technologies, this shift is small and accompanied by adverse side effects. To put the effects of preferential treatment into perspective, we consider Pigouvian taxation of pollution, which is the natural policy instrument to address environmental frictions. Such a policy increases the share of green capital to 27.70% and substantially reduces the pollution externality *without* adverse effects on firm risk-taking. It should be noted that even without the adverse effects on firm risk-taking, Pigouvian taxes are an order of magnitude more powerful in addressing the environmental friction than *maximal* preferential treatment: the changes in borrowing costs are unable to induce a sufficiently strong shift towards green technologies under any plausible calibration. If firms' financing choices are taken into account, the effectiveness of an optimal preferential collateral policy decreases by another order of magnitude.

These results should not be misinterpreted as a call for central bank inaction. The level of

[4] Asset purchase programs have an anti-cyclical component by design and, therefore, seem less well suited in an environmental policy context, which is concerned with long-run problems.

[5] Risk-taking, as reflected by firms' financing decision, has been reported in the empirical literature on unconventional monetary policy. Bekkum et al. (2018) observe a decrease in repayment performance on the mortgage backed securities market following an eligibility easing. Pelizzon et al. (2020) document positive leverage responses of eligible firms. Harpedanne de Belleville (2019) finds a sizable increase in investment by issuers of newly eligible bonds following a reduction of collateral requirements. Grosse-Rueschkamp et al. (2019) and Giambona et al. (2020) document a positive investment and leverage impact of firms eligible for quantitative easing. Kaldorf and Wicknig (2022) provide a structural analysis of collateral premia and corporate default risk.

the Pigouvian tax that optimally addresses the environmental externality also reduces the collateral supply to an inefficiently low level. The central bank optimally addresses this by slightly decreasing haircuts in a symmetric way to restore the efficient level of aggregate collateral. In contrast, if public policy is restricted in its ability to set carbon taxes optimally, e.g., due to political economy frictions, the central bank can increase welfare by tilting the collateral framework towards green bonds. The extent of preferential treatment declines, the closer Pigouvian taxation gets to its optimal level: preferential treatment is a qualitatively and quantitatively imperfect substitute for taxation.

Related Literature. There is a small but fast-growing literature that adds environmental aspects to DSGE models suitable for central bank policy analysis at business cycle frequencies, building on Heutel (2012). Punzi (2019) extends this setup by adding financial intermediation of loans to a credit-constrained corporate sector. Her paper explicitly considers differentiated capital requirements to relax financial frictions of green firms. Due to our focus on the collateral framework and marketable assets instead of bank loans, our model uses a financial friction related to leverage rather than external financing. Moreover, we endogenize firm risk-taking, while the extent of financial frictions is exogenous in Punzi (2019).

In a specific assessment of green QE, Ferrari and Nispi Landi (2020) find only a modestly positive impact on aggregate environmental performance. Similarly, Abiry et al. (2021) document a small impact of QE, in particular in comparison to a carbon tax, which is similar to our results on collateral policy. Hong et al. (2021) study sustainable investment mandates, which have a similar transmission mechanism, since they affect asset demand by financial intermediaries. In their setup, sustainable investment mandates, in the form of minimum portfolio shares, increase welfare, since they widen the cost of capital wedges between green and conventional firms. Closest to our paper is the work of Papoutsi et al. (2021) who show how central banks can tilt their asset purchases towards green assets to address environmental frictions. However, they assume that central banks can buy firm equity and are silent about the pass-through via the corporate bond market, which is generating a limited policy transmission in our model. Similar to us, they show that in the presence of an optimal carbon tax, asset purchases play no role in addressing the environmental friction, consistently with the Tinbergen Principle in the public economics literature. On a more general level, all policies that change the relative demand for green and conventional bonds, such as green QE and preferential green capital requirements, will induce firm responses along several dimensions, that have not been studied extensively in the literature so far. However, in our view, a thorough analysis of these additional response margins is necessary to fully assess the effectiveness and efficiency of green policies.

It should be stressed, that we abstract from an analysis of transition risk, which arises if demand for conventional goods suddenly decreases due to ambitious environmental policy. Diluiso et al. (2021) and Carattini et al. (2021) argue that macroprudential policies can address this issue. Similar to these papers, we document an interaction between environmental policy and collateral policy and show how haircuts should be adjusted to account for these interactions.

Outline. The paper is structured as follows. We introduce our model in Section 3.2 and illustrate the pass-through of collateral policy to the real sector in Section 3.3. Section 3.4 presents our calibration, while we discuss our policy experiments in Section 3.5. Section 3.6 concludes.

3.2 Model

Time is discrete and indexed by $t = 1, 2, ...$ The model features a representative *household*, two types of *intermediate goods firms*, a perfectly competitive *wholesale firm*, aggregating both types of intermediate goods into a composite intermediate good, a competitive *final good producer*, financial intermediaries (*banks*), and a public sector consisting of a fiscal authority and the central bank. One type of intermediate goods producers (*conventional*) causes an externality when producing intermediate goods, to which we refer as *pollution*. The technology of the *green* firm does not cause the externality. Both types of intermediate goods are aggregated into a composite intermediate good by a perfectly competitive wholesale firm. A competitive final good producer uses the composite intermediate good and labor to produce the final consumption good, which it sells to the household. Banks raise deposits from the household to invest into corporate bonds and incur a liquidity management cost. Finally, the fiscal authority can levy a proportional pollution tax on the conventional firms' output, while the central bank sets the collateral framework and incurs a cost from collateral default.

3.2.1 Households and Banks

Households. The representative household derives utility from consumption c_t and disutility from supplying labor l_t at the wage w_t. To transfer resources across time, the household saves in deposits d_t. Deposits held from time $t-1$ to t earn the interest rate i_{t-1}. The household's discount factor is denoted by β, ω_l is the utility-weight on labor, and γ_c and γ_l are the inverses of the intertemporal elasticity of substitution and of the Frisch elasticity of labor supply, respectively. The maximization problem of the representative household is given by

$$\begin{aligned} V(d_t) = & \max_{c_t, l_t, d_{t+1}} \frac{c_t^{1-\gamma_c}}{1-\gamma_c} - \omega_l \cdot \frac{l_t^{1+\gamma_l}}{1+\gamma_l} + \beta \mathbb{E}_t \left[V(d_{t+1}) \right] \ , \\ & \text{s.t.} \quad c_t + d_{t+1} = w_t l_t + (1 + i_{t-1}) d_t + \Pi_t \ , \end{aligned} \tag{3.1}$$

where Π_t collects profits from banks and final goods producers and we omit the dependency of $V()$ on the aggregate state for simplicity. Solving (3.1) yields standard inter- and intratemporal optimality conditions

$$c_t^{-\gamma_c} = \beta \mathbb{E}_t \left[(1 + i_t) c_{t+1}^{-\gamma_c} \right] \ , \tag{3.2}$$

$$c_t^{-\gamma_c} w_t = \omega_l l_t^{\gamma_l} \ . \tag{3.3}$$

Banks. There is a unit mass of perfectly competitive banks $i \in (0,1)$ that supply deposits to households and invest into corporate bonds. We assume that financial intermediation is subject to liquidity management costs, which can be represented by the function $\Omega(\overline{b}^i_{t+1})$, which satisfies $\Omega_{\overline{b},t} \equiv \partial\Omega/\partial\overline{b}^i_{t+1} < 0$, i.e., liquidity management costs depend negatively on the collateral value of bank i's corporate bond portfolio,

$$\overline{b}^i_{t+1} = (1-\phi_c) q_{c,t} b^i_{c,t+1} + (1-\phi_g) q_{g,t} b^i_{g,t+1} \ .$$

The collateral value of a bank's portfolio is given by the market value its bonds $q_{\tau,t} b^i_{\tau,t+1}$ weighted by one minus the respective central bank haircut parameter ϕ_τ.[6] The higher the haircut, the lower collateral value the bond has. Banks directly benefit from a relaxation in collateral policy, since this increases available collateral $\overline{b}_{t+1}$ ceteris paribus. The literature has motivated such liquidity management costs as arising from idiosyncratic liquidity shocks associated with deposit or credit line withdrawals (De Fiore et al., 2019 and Piazzesi and Schneider, 2021). The assumption $\Omega_{\overline{b},t} < 0$ then captures in reduced form the benefits of collateral to settle idiosyncratic liquidity shocks on interbank markets or by tapping central bank facilities.[7]

We follow Cúrdia and Woodford (2011) and assume that banks maximize profits, defined as equity value net of liquidity management costs in (3.4), subject to the solvency condition (3.5). Taken the behavior of other banks, intermediate firms, and central bank policy as given, the maximization problem of bank i reads

$$\max_{d^i_{t+1}, b^i_{c,t+1}, b^i_{g,t+1}} \quad \Pi^i_t = d^i_{t+1} - q_{c,t} b^i_{c,t+1} - q_{g,t} b^i_{g,t+1} - \Omega(\overline{b}^i_{t+1}) \ , \tag{3.4}$$

$$\text{s.t.} \quad (1+i_t) d^i_{t+1} = \mathbb{E}_t \left[\mathcal{R}_{c,t+1}\right] b^i_{c,t+1} + \mathbb{E}_t \left[\mathcal{R}_{g,t+1}\right] b^i_{g,t+1} \ . \tag{3.5}$$

The bond payoff $\mathcal{R}_{\tau,t+1}$ depends on firm τ's bond issuance and capital choice via the default decision in period $t+1$ (see below). Taking first order conditions we obtain the bond price equation

$$q_{\tau,t} = \frac{\mathbb{E}_t \left[\mathcal{R}_{\tau,t+1}\right]}{(1+i_t)(1+(1-\phi_\tau)\Omega_{\overline{b},t})} \ . \tag{3.6}$$

Liquidity management costs introduce a willingness to pay a premium for eligible bonds, reflected by the term $(1-\phi_\tau)\Omega_{\overline{b},t}$, which we refer to as *collateral premium*.

[6] We restrict the analysis to time-invariant haircuts. While collateral frameworks are occasionally adjusted in practice, this usually happens in response to large shocks to the financial systems. These events are not of first order importance for our analysis of preferential treatment.

[7] Since neither the sources of liquidity demand, nor the reason why this market is collateralized are at the heart of our paper, we introduce this feature in reduced form and refer to appendix B.1.1 for details on a micro-foundation.

3.2.2 Firms

Final Good Producer. A competitive firm produces the final good y_t using a Cobb-Douglas production function that combines an intermediate good z_t and labor l_t

$$y_t = (1 - \mathcal{P}_t)\, A_t z_t^{\theta} l_t^{1-\theta} \;, \tag{3.7}$$

where θ is a technology parameter. Final good production is negatively affected by pollution $\mathcal{P}_t$ generated by the conventional firm (described below). The economy-wide TFP shock A_t evolves according to

$$\log(A_{t+1}) = \rho_A \log(A_t) + \sigma_A \epsilon_{t+1}^{A} \;, \quad \epsilon_{t+1}^{A} \sim N(0,1) \;,$$

Solving the maximization problem of the firm, we get standard first order conditions that equate the marginal product of the inputs to their market price

$$\begin{aligned} p_{z,t} &= (1 - \mathcal{P}_t)\, \theta A_t z_t^{\theta-1} l_t^{1-\theta} \;, \\ w_t &= (1 - \mathcal{P}_t)\, (1-\theta) A_t z_t^{\theta} l_t^{\theta} \;, \end{aligned}$$

where $p_{z,t}$ denotes the intermediate good price.

Wholesale Firm. The competitive wholesale firm bundles green and conventional intermediate goods into an input used by the final good firm using a Cobb-Douglas technology

$$z_t = z_{g,t}^{\nu} z_{c,t}^{1-\nu} \;, \tag{3.8}$$

where ν determines the relative share of green intermediate goods.[8] The prices of the intermediate good types τ are denoted by $p_{\tau,t}$. Solving the profit maximization problem yields

$$\nu p_{z,t} z_t = p_{g,t} z_{g,t} \;, \tag{3.9}$$

$$(1-\nu) p_{z,t} z_t = p_{c,t} z_{c,t} \;. \tag{3.10}$$

Intermediate Good Firms: Technology. There are two types of intermediate good firms producing a green or a conventional good z_τ. Within each type $\tau = \{c, g\}$, there is a unit mass of firms, indexed by j, that invest in physical capital $k_{j,\tau,t}$. The production technology of all firms is linear and subject to an idiosyncratic productivity shock $m_{j,\tau,t}$, which is i.i.d. across and within firm types

$$z_{j,\tau,t} = m_{j,\tau,t} k_{j,\tau,t} \;. \tag{3.11}$$

Following Bernanke et al. (1999), the idiosyncratic shock is log-normally distributed with $\mathbb{E}[m] = 1$. The log-normal distribution satisfies a monotone hazard rate property of the form $\partial(h(m)m)/\partial m >$

[8] In appendix B.2.1 we conduct a robustness analysis using a CES-function and find only minor differences.

0, where $h(m) \equiv f(m)/(1-F(m))$ denotes the hazard rate and $f(m)$ and $F(m)$ denote the pdf and cdf, respectively. Capital $k_{j,\tau,t}$ depreciates at rate δ, which is common to both production technologies. Sector-specific investment is denoted $i_{\tau,t}$. Since our model permits exact aggregation into representative firms, the law of motion for capital of type τ is given by

$$k_{\tau,t+1} = i_{\tau,t} + (1-\delta)k_{\tau,t} \ . \tag{3.12}$$

As common in environmental DSGE models (see Heutel, 2012), the aggregate production of conventional firms $z_{c,t}$ induces pollution $\mathcal{P}_t$, which satisfies $\partial \mathcal{P}_t / \partial z_{c,t} > 0$. Revenues are subject to a time-invariant, type-specific tax χ_τ. When χ_τ is negative, it can be interpreted as a subsidy and it will be set to zero in the baseline calibration.[9]

Intermediate Good Firms: Financial Side. As in Gomes et al. (2016), we assume that each firm j of each type τ is managed on behalf of a risk-averse and impatient representative firm owner who consumes dividends $\widetilde{c}_t = \int_j \Pi_{j,c,t} dj + \int_j \Pi_{j,g,t} dj$. The firm owner's period utility is given by $\frac{\widetilde{c}_t^{1-\gamma_c}}{1-\gamma_c}$, where the utility parameter is the same as the one of households. There is no agency friction between firm managers and owners. The representative firm owner discounts the future with a discount factor $\widetilde{\beta} < \beta$. This assumption ensures that firms borrow from banks in equilibrium. We impose the following timing structure:

- At the beginning of each period, firms enter with (type-specific) capital $k_{\tau,t}$ and bonds outstanding $b_{\tau,t}$.
- Each firm j draws an idiosyncratic productivity shock $m_{j,\tau,t}$, produces and either repays its maturing debt obligations or defaults (described below).
- Firms adjust capital $k_{j,\tau,t+1}$ and bonds outstanding $b_{j,\tau,t+1}$.
- Firms transfer their dividends $\Pi_{j,\tau,t}$ to the firm owner.

Firms finance their activities by issuing equity, modeled as negative dividends, or by issuing corporate bonds. Bonds mature stochastically each period with probability $0 < s \leq 1$ and pay one unit of the final good in $t+1$ in case of no default.[10] Firms mechanically default, if their repayment obligation exceeds revenues from production.[11] The default productivity threshold is given by $\overline{m}_{\tau,t}$ and is implicitly defined as the productivity level at which revenues $(1-\chi_\tau)p_{\tau,t}m_{\tau,t}k_{\tau,t}$ equal repayment obligations $sb_{\tau,t}$. In case of default, banks holding distressed bonds effectively replace the firm owner as shareholder: they seize the output *only in the default period*, restructure the firm, and resume to being creditors after the firm's debt has been restructured. With probability $1-s$, the bond does not mature, is unaffected by the restructuring

[9] It is not relevant in our setup, whether the intermediate or wholesale firms pay the tax. Attributing it to intermediate good producers, however, gives the cleanest comparison to collateral policy, as both instruments operate through the investment decision.

[10] Using long-term bonds allows to obtain realistic leverage ratios in the calibration, but is not required for the transmission of collateral policy. Moreover, bonds are cast in real terms. We consider nominal bonds in appendix B.2.2.

[11] We implicitly assume that there is no transfer of resources from productive to unproductive firms.

process, and is rolled over at next period's market price $q_{\tau,t+1}$. While in practice restructuring takes several periods, we follow Gomes et al. (2016) and take a shortcut by assuming that capital owners can restructure their liabilities without delays.

Firms maximize the present value of dividends, discounted using the firm owner's stochastic discount factor $\widetilde{\Lambda}_{t,t+1} \equiv \widetilde{\beta}\left(\widetilde{c}_{t+1}/\widetilde{c}_t\right)^{-\gamma_c}$. We conjecture that all firms enter any period t with the same legacy debt stock and capital to express dividends as

$$\Pi_{j,\tau,t} = \mathbb{1}\{m_{j,\tau,t} > \overline{m}_{\tau,t}\}\Big((1-\chi_\tau)p_{\tau,t}m_{j,\tau,t}k_{\tau,t} - sb_{\tau,t}\Big) - k_{j,\tau,t+1} + (1-\delta)k_{\tau,t}$$
$$+ q_{\tau,t}\left(b_{j,\tau,t+1} - (1-s)b_{\tau,t}\right) \ .$$

Under the assumption of no delays in restructuring and i.i.d. productivity shocks, next period's productivity can be integrated out in the objective function and the problem reduces to a two-period consideration

$$\max_{k_{j,\tau,t+1},b_{j,\tau,t+1}} - k_{j,\tau,t+1} + q_{\tau,t}\Big(b_{j,\tau,t+1} - (1-s)b_{\tau,t}\Big)$$
$$+ \mathbb{E}_t\Bigg[\widetilde{\Lambda}_{t,t+1}\Bigg((1-G(\overline{m}_{j,\tau,t+1}))(1-\chi_\tau)p_{\tau,t+1}k_{j,\tau,t+1} + (1-\delta)k_{j,\tau,t+1}$$
$$- s\Big(1-F(\overline{m}_{j,\tau,t+1})\Big)b_{j,\tau,t+1} + q_{\tau,t+1}\Big(b_{j,\tau,t+1} - (1-s)b_{\tau,t}\Big)\Bigg)\Bigg] \ ,$$

subject to the default threshold $\overline{m}_{j,\tau,t+1} \equiv \frac{sb_{j,\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k_{j,\tau,t+1}}$, the bond pricing condition (3.6) and taken as given the continuation value of bonds $q_{\tau,t+1}$. Since dividends of all firms are transferred to the firm owner and firms can access capital and bond markets irrespective of current default, idiosyncratic productivity risk washes out in the aggregate: current productivity is not relevant for the investment and debt issuance decisions and all type τ firms make the same choices $k_{\tau,t+1}$ and $b_{\tau,t+1}$. This allows aggregation into a representative green and conventional firm, respectively.

Let the average productivity of a defaulting firm be denoted by $G\left(\overline{m}_{\tau,t}\right) \equiv \int_0^{\overline{m}_{\tau,t}} mdF(m)$. In case of default, the bank pays restructuring costs φ and is entitled to the entire production, valued at price $p_{\tau,t}$, while the payoff in case of repayment is $b_{\tau,t}$.[12] In summary, the *per-unit* bond payoff entering the bond pricing condition of banks (3.6) is given by

$$\mathcal{R}_{\tau,t} = s\left(G(\overline{m}_{\tau,t})\frac{p_{\tau,t}(1-\chi_\tau)k_{\tau,t}}{sb_{\tau,t}} + 1 - F(\overline{m}_{\tau,t})\right) - F(\overline{m}_{\tau,t})\varphi + (1-s)q_{\tau,t} \ . \tag{3.13}$$

The first term reflects the payoff from the share s of maturing bonds: it consists of the production revenues banks seize in case of default (first term in parenthesis) and the repayment of the principal in case of no default (second term). The term $F(\overline{m}_{\tau,t})\varphi$ reflects default costs incurred by banks. The share of bonds that are rolled over is valued at the bond market price $q_{\tau,t}$.

[12]Attributing restructuring costs to green and conventional firms yields similar mechanics.

Intermediate Good Firms: Bond Issuance and Investment. As in Gomes et al. (2016), the bond price depends only on the default threshold $q_{\tau,t} = q(\overline{m}_{\tau,t+1})$. Plugging investment (3.12) and banks' bond pricing condition (3.6) into the Bellman equation, the Euler conditions for bond issuance and capital read

$$\begin{aligned} q'(\overline{m}_{\tau,t+1})\frac{\mathbb{E}_t[\overline{m}_{\tau,t+1}]}{b_{\tau,t+1}}\Big(b_{\tau,t+1} - (1-s)b_{\tau,t}\Big) + q(\overline{m}_{\tau,t+1}) \\ = \mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\Big(s(1-F(\overline{m}_{\tau,t+1})) + (1-s)q(\overline{m}_{\tau,t+1})\Big)\right], \end{aligned} \tag{3.14}$$

$$\begin{aligned} 1 = -q'(\overline{m}_{\tau,t+1})\frac{\mathbb{E}_t[\overline{m}_{\tau,t+1}]}{k_{\tau,t+1}}\Big(b_{\tau,t+1} - (1-s)b_{\tau,t}\Big) \\ + \mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\Big((1-\delta) + (1-\chi_\tau)p_{\tau,t+1}\big(1-G(\overline{m}_{\tau,t+1})\big)\Big)\right]. \end{aligned} \tag{3.15}$$

The analytical steps are relegated to appendix B.1.2. Equation (3.14) is a standard optimality condition equating the marginal benefit of issuing more bonds (LHS) to the marginal costs (RHS). Each additional unit of bonds increases funds available in period t by $q(\overline{m}_{\tau,t+1})$ units. At the same time, the bond price schedule is a decreasing function of the default threshold, which we also refer to as the *risk choice*. Since we characterize bond prices by the risk choice $\overline{m}_{\tau,t+1}$, the term $\mathbb{E}_t[\overline{m}_{\tau,t+1}]/b_{\tau,t+1}$ captures the increase of default risk arising from the issuance of an additional bond. This dilutes the value of existing bond investment $b_{\tau,t+1} - (1-s)b_{\tau,t}$. The risk choice has also implications for firm consumption in $t+1$. Each unit of bonds involves repayment of s, conditional on not defaulting. In addition, bond issuance also increases the rollover burden in $t+1$, further reducing expected consumption.

The optimality condition for capital (3.15) requires that the cost of purchasing capital (equal to one, LHS) equals its payoff, the RHS, consisting of two parts. The first line affects consumption in period t and represents an increase of the bond price that is due to a decrease of the default probability. The part in the second line increases consumption in period $t+1$ and is composed of the value of capital after depreciation and the marginal value of production net of taxes.

3.2.3 Public Policy and Resource Constraint

The central bank sets the collateral framework (ϕ_c, ϕ_g) and incurs costs from collateral default Λ_t. These costs depend positively on the default risk of pledged collateral $\Lambda_{\overline{F},t} > 0$, defined as the firms' probability of default, weighted by the repo size

$$\overline{F}_t \equiv \sum_\tau (1-\phi_\tau)b_{\tau,t}q_{\tau,t}F_{\tau,t} \ .$$

The weighting $(1-\phi_\tau)b_{\tau,t}q_{\tau,t}$ can be interpreted as the repo size collateralized by green and conventional bonds, respectively. By setting haircuts, the central bank has a direct effect on the costs. Making Λ_t dependent on default risk captures in reduced form a risk management consideration of accepting risky bonds as collateral. In appendix B.1.3 we discuss a potential micro-foundation of the cost function, based on central bank solvency concerns (Hall and Reis, 2015). This is a frequently employed argument for why central banks are only willing to lend against sufficiently safe securities. For example, Bindseil and Papadia (2006) argue that central banks are not specialized credit risk management agencies and that higher default risk of accepted collateral makes monetary policy implementation more resource-intensive.

Together with the assumptions that liquidity management costs decrease in collateral supply, $\Omega_{\bar{b},t} < 0$, the assumption $\Lambda_{\overline{F},t} > 0$ introduces a collateral policy trade-off. To close the model, we assume that the fiscal authority rebates all tax revenues to green firms to balance its budget,

$$\chi_c p_{c,t} z_{c,t} + \chi_g p_{g,t} z_{g,t} = 0 \; . \tag{3.16}$$

This fiscal rule allows us to abstract from additional fiscal instruments that would otherwise be necessary to balance the government budget. The resource constraint is given by

$$y_t = c_t + \sum_{\tau} (c_{\tau,t} + i_{\tau,t}) + \Omega(\bar{b}_{t+1}) + \Lambda(\overline{F}_{t+1}) + \sum_{\tau} \varphi F(\overline{m}_{\tau,t}) b_{\tau,t} \; , \tag{3.17}$$

where the last three terms represent the resource losses due to the liquidity management costs, collateral default costs, and corporate defaults.

3.3 The Transmission of Preferential Treatment

Before numerically evaluating optimal policy in Section 3.4, we illustrate the pass-through of collateral policy and Pigouvian taxation in a simplified setting. The discussion will be organized around intermediate good firms' first order conditions and the equilibrium green capital ratio. For the ease of exposition, we consider the case of one-period bonds and full capital depreciation ($s = \delta = 1$). Since we do not focus on macroeconomic dynamics in this section, we do not endogenize output prices and the interest rate and also set firm owner's stochastic discount factor $\Lambda_{t,t+1} = \widetilde{\beta}$.

A Benchmark Without Default Risk. To isolate the role of financial frictions in the production sector, it is informative to relate our model to a framework without default risk and with collateral premia. Specifically, we consider the case where capital has to be fully debt-financed but where there are no idiosyncratic productivity shocks. In this case, both intermediate good firms will issue exactly as many bonds as necessary to finance their capital $q_{\tau,t} b_{\tau,t+1} = k_{\tau,t+1}$, taken as given bond prices. In the absence of default risk, the bond price $q_{\tau,t} = \frac{1}{(1+i_t)(1+(1-\phi_\tau)\Omega_{\bar{b}})}$ merely reflects the discounted value of consumption in $t + 1$ and collateral benefits. The bond price is increasing in the collateral premium $\frac{\partial q_{\tau,t}}{\partial (1-\phi_\tau)\Omega_{\bar{b}}} > 0$, which in turn increases if the central bank lowers the haircut. Firms maximize the present value of dividends, which yields the following first order condition for capital

$$1 = \underbrace{(1-\chi_\tau)\mathbb{E}_t[p_{\tau,t+1}]q_{\tau,t}}_{\equiv \Gamma^{\text{no default}}_{\tau,t+1}} \ . \tag{3.18}$$

This condition states that the marginal cost of investment (LHS, equal to one) equals the marginal benefit of investment (RHS, return on capital $\Gamma^{\text{no default}}_{\tau,t+1}$). Given that the marginal cost of capital is constant, any increase of the return on capital will stimulate investment. A relaxation in collateral policy will then increase the return on capital proportionally to any increase of the bond price, which we refer to as *perfect pass-through*:

$$\frac{\partial \Gamma^{\text{no default}}_{\tau,t+1}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}} = (1-\chi_\tau)\mathbb{E}_t[p_{\tau,t+1}] \frac{\partial q_{\tau,t}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}} \ . \tag{3.19}$$

Combining the investment decision (3.18) for both firm types with the intermediate good demands (3.9) and (3.10) yields the green capital ratio

$$\frac{k_{g,t}}{k_{c,t}} = \frac{q_{g,t}}{q_{c,t}} \frac{\nu(1-\chi_g)}{(1-\nu)(1-\chi_c)} \ . \tag{3.20}$$

Equation (3.20) shows that in the no-default benchmark, a decrease in the relative borrowing costs of green firms increases the green capital ratio.

The Role of Default Risk. Now, consider the model with default risk. With one-period bonds, the default threshold is given by $\overline{m}_{\tau,t+1} = \frac{b_{\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}$ and the first order conditions for bonds and capital simplify to

$$q'(\overline{m}_{\tau,t+1})\mathbb{E}_t[\overline{m}_{\tau,t+1}] + q(\overline{m}_{\tau,t+1}) = \widetilde{\beta}\mathbb{E}_t[1-F(\overline{m}_{\tau,t+1})] \ , \tag{3.21}$$

$$1 = \underbrace{(1-\chi_\tau)\mathbb{E}_t\Big[p_{\tau,t+1}\Big(\widetilde{\beta}\big(1-G(\overline{m}_{\tau,t+1})\big) - q'(\overline{m}_{\tau,t+1})\overline{m}^2_{\tau,t+1}\Big)\Big]}_{\equiv \Gamma_{\tau,t+1}} \ . \tag{3.22}$$

The return on capital in (3.22) contains, first, the future output produced by an additional unit of capital conditional on not defaulting, $\widetilde{\beta}\big(1-G(\overline{m}_{\tau,t+1})$. Second, it contains a bond price appreciation term, $q'(\overline{m}_{\tau,t+1})\overline{m}^2_{\tau,t+1}$, reflecting the reduction in default risk from higher investment. Combining (3.21) and (3.22) and differentiating the return on capital with respect to the collateral premium, we obtain

$$\begin{aligned}\frac{\partial \Gamma_{\tau,t+1}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}} = (1-\chi_\tau)\mathbb{E}_t\Bigg[p_{\tau,t+1}\Bigg\{&\Big(\frac{\partial q_{\tau,t}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}} + q'(\overline{m}_{\tau,t+1})\frac{\partial \overline{m}_{\tau,t+1}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}}\Big)\overline{m}_{\tau,t+1} \\ &+ q_{\tau,t}\frac{\partial \overline{m}_{\tau,t+1}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}} - \widetilde{\beta}\big(1-F(\overline{m}_{\tau,t+1})\big)\frac{\partial \overline{m}_{\tau,t+1}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}}\Bigg\}\Bigg] \ .\end{aligned} \tag{3.23}$$

As in the no-default case, the effect of collateral policy on the return on capital directly depends on the change in borrowing cost $\frac{\partial q_{\tau,t}}{\partial(1-\phi_\tau)\Omega_{\bar{b}}}$. Moreover, it also depends on the risk choice, which itself is endogenously determined. To characterize the risk choice, we exploit that banks' bond pricing condition is available in closed form. To simplify the exposition, assume that banks cannot seize output of defaulting firms (their revenues are wasted) and do not incur restructuring costs ($\varphi = 0$). The bond pricing condition and its derivative with respect to risk-taking can then be written as

$$q(\overline{m}_{\tau,t+1}) = \mathbb{E}_t \frac{1 - F(\overline{m}_{\tau,t+1})}{(1+i_t)(1+(1-\phi_\tau)\Omega_{\bar{b}})} \quad \text{and} \quad q'(\overline{m}_{\tau,t+1}) = \mathbb{E}_t \frac{-f(\overline{m}_{\tau,t+1})}{(1+i_t)(1+(1-\phi_\tau)\Omega_{\bar{b}})} \, .$$

The effect of collateral policy on risk-taking can be illustrated by plugging the bond pricing condition into (3.21):

$$(1+i_t)\left(\frac{1}{1+i_t} - (1+(1-\phi_\tau)\Omega_{\bar{b}})\widetilde{\beta}\right) = \mathbb{E}_t\left[\frac{f(\overline{m}_{\tau,t+1})}{1-F(\overline{m}_{\tau,t+1})}\overline{m}_{\tau,t+1}\right] . \tag{3.24}$$

In the absence of collateral premia ($\phi_\tau = 1$), the risk choice is determined by equating relative impatience and marginal default costs. Holding the interest rate fixed, a reduction of the haircut ϕ_τ increases the LHS of (3.24). Due to the monotonicity assumption on the hazard rate, the RHS of (3.24) increases in $\overline{m}_{\tau,t+1}$. Hence, the effect of relaxing collateral policy on risk-taking is unambiguously positive. Intuitively, firms increase their risk-taking, because lower financing costs make investment *and* front-loading dividend payouts more attractive, holding expected default cost constant.

We can now re-consider the effect of haircuts on the return on capital (3.23). The term $q'(\overline{m}_{\tau,t+1})\frac{\partial \overline{m}_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\bar{b}}} < 0$ is a negative risk-taking effect, which lowers the bond price and thereby makes investment less attractive in period t. The positive term $q_{\tau,t}\frac{\partial \overline{m}_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\bar{b}}}$ captures bond price appreciation from investment. Last, $\widetilde{\beta}\big(1-F(\overline{m}_{\tau,t+1})\big)\frac{\partial \overline{m}_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\bar{b}}}$ reflects the dividend reduction in $t+1$ due to higher default rates. Using the definitions of $q(\overline{m}_{\tau,t+1})$ and $q'(\overline{m}_{\tau,t+1})$, we can simplify (3.23) to

$$\begin{aligned}
\frac{\partial \Gamma_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\bar{b}}} &= (1-\chi_\tau)\mathbb{E}_t\Bigg[p_{\tau,t+1}\Bigg\{\frac{\partial q_{\tau,t}}{\partial(1-\phi_\tau)\Omega_{\bar{b}}}\overline{m}_{\tau,t+1} \\
&\quad + \Bigg(\underbrace{\frac{1-\frac{f(\overline{m}_{\tau,t+1})}{(1-F(\overline{m}_{\tau,t+1}))}\overline{m}_{\tau,t+1}}{(1+i_t)(1+(1-\phi_\tau)\Omega_{\bar{b}})}}_{=\widetilde{\beta}}(1-F(\overline{m}_{\tau,t+1})) - \widetilde{\beta}\big(1-F(\overline{m}_{\tau,t+1})\big)\Bigg)\frac{\partial \overline{m}_{\tau,t+1}}{(1-\phi_\tau)\Omega_{\bar{b}}}\Bigg\}\Bigg] \\
&= (1-\chi_\tau)\mathbb{E}_t\Big[p_{\tau,t+1}\overline{m}_{\tau,t+1}\Big]\frac{\partial q_{\tau,t}}{\partial(1-\phi_\tau)\Omega_{\bar{b}}} < \frac{\partial \Gamma^{\text{no default}}_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\bar{b}}} \, .
\end{aligned}$$

Hence, relaxing collateral policy has a positive but unambiguously smaller effect on investment than in the no-default case. In (partial) equilibrium, the green capital ratio in the presence of financial frictions can be written

$$\frac{k_{g,t}}{k_{c,t}} = \frac{\mathbb{E}_t\left[\widetilde{\beta}\big(1-G(\overline{m}_{g,t+1})\big) - q'(\overline{m}_{g,t+1})\overline{m}^2_{g,t+1}\right]}{\mathbb{E}_t\left[\widetilde{\beta}\big(1-G(\overline{m}_{c,t+1})\big) - q'(\overline{m}_{c,t+1})\overline{m}^2_{c,t+1}\right]} \frac{\nu(1-\chi_g)}{(1-\nu)(1-\chi_c)} . \tag{3.25}$$

Absent preferential treatment, the risk choice and bond prices are identical across firm types and the terms related to financial frictions in the return on investment cancel. Then, as in the no-default case, the relative size of both sectors would be directly determined by the technology parameter ν and the environmental policy regime. Setting $\chi_c > 0$ and $\chi_g < 0$ directly increases the green capital ratio. Note that this policy also operates through the return on capital, which increases (decreases) in the subsidy (tax) from (3.18). However, in sharp contrast to haircut policies, the tax rate χ_τ does not affect risk-taking, as demonstrated in (3.24). The preferential treatment of green bonds in the collateral framework also increases the green capital ratio, but the pass-through of this policy is impaired. We quantify relevance of this impairment in the next section.

Last, note that two partial effects shape the effect of preferential treatment and the extent to which financial frictions dampen it: (i) the response of relative borrowing costs between sectors to preferential treatment and (ii) the elasticities of leverage and capital to bond price changes. In Section 3.4.2, we relate the model-implied reactions in these dimensions to the data. Separating between-sector effects on borrowing costs from sector-specific effects of borrowing conditions on real outcomes is relevant from an empirical point of view: as preferential policies are not enacted yet, this decomposition allows to assess the model predictions' plausibility.

3.4 Quantitative Analysis

In this section, we provide a calibration of the model to euro area data. All data sources are summarized in appendix B.4. We then show the model's fit regarding (untargeted) macroeconomic dynamics and demonstrate the model's ability to replicate the effect of preferential treatment on borrowing costs between sectors and the response of financial market and real sector variables to collateral policy.

3.4.1 Calibration

Each period corresponds to one quarter. We assume log-utility over consumption, fix the inverse Frisch elasticity at 1, and set the household discount factor β to 0.99. The Cobb-Douglas coefficient in the final good production technology is set to $\theta = 1/3$ to get a labor share of 2/3, and we choose the weight ω_l in the household utility function to be consistent with a steady state labor supply of 1/3. The TFP shock parameters are conventional values in the RBC literature. The depreciation rate is set to $\delta = 0.017$ to target the capital to GDP ratio.

Parameters regarding pollution and the green technology share are important drivers of environmental DSGE models. For the relative share of the green sector, we use the most recent data on the share of renewable energies in the euro area. Although this is only a subset of intermediate goods, it has the advantage that, since renewable energy is a prominent feature of the public discussion, the data quality is excellent. From this data set we find that the relative share of the green sector is 20%, which directly informs the Cobb-Douglas parameter of the wholesale goods producers ν.[13] In spirit of Heutel (2012) and Golosov et al. (2014), we assume that pollution costs are expressed as

$$\mathcal{P}_t = 1 - \exp\{-\gamma_P z_{c,t}\} ,$$

which, through final good production (3.7), generates a percentage loss in the production of the final good producer. The function captures the mapping from pollution to real economic damage and the parameter γ_P governs the pass-through from pollution to production losses. We inform the parameter γ_P using estimates of direct costs from pollution and indirect costs from adverse environmental conditions. From the model, we can directly relate this quantity $1 - \exp\{-\gamma_P z_c\}$ to observable (long-run) quantities $1 - y/z^\theta l^{1-\theta}$. We use the estimate of Muller (2020), who quantifies Damage/GDP at 10% in 2016 for the US. The value of 10% has also been reported in the fourth National Climate Assessment in the US (Reidmiller et al., 2018). Since economic activity in this dimension can be assumed to be similar in the US and the euro area, we adopt the same value.

The next group of parameters is associated with intermediate good firms. We assume that both firm types are subject to the same financial friction. This assumption is supported by the findings of Larcker and Watts (2020) and Flammer (2021), who find no effect of environmental performance on spreads in the US fixed income market. Average maturity of corporate bonds is set to five years ($s = 0.05$) and corresponds to average maturity in the *Markit iBoxx* corporate bond index between 2010 and 2019. Following Gomes et al. (2016), restructuring costs φ are set such that they are consistent with a recovery rate of 30%, defined as realized payoff in default over the promised payoff. The idiosyncratic productivity shock is log-normally distributed with variance ς_M and mean $-\varsigma_M/2$ to ensure that it satisfies $\mathbb{E}[m_{\tau,t}] = 1$. This leaves us with two free parameters, the discount factor $\widetilde{\beta}$ of firm owners and the idiosyncratic productivity variance ς_M. They are set to match time-series means of spreads and the corporate debt-GDP ratio. The model-implied bond spread is defined as

$$x_{\tau,t} \equiv (1 + s/q_{\tau,t} - s)^4 - (1 + i_t)^4 .$$

For the data moment on spreads, we use the *IHS Markit* data from 2010 until 2019. We compute the median bond spread over the entire corporate bond sample and average over time, which yields a value of around 100bp. The debt-to-GDP ratio taken from the ECB, where we restrict attention to debt issued by non-financial firms.

The final group of parameters is related to banks and collateral policy. We impose symmetric collateral treatment and set $\phi_{sym} \equiv \phi_c = \phi_g = 0.26$, which corresponds to the current haircut on BBB-rated corporate bonds with five to seven years maturity. Liquidity management costs

[13] Renewable energy statistics for the EU are accessible here. See also the guide by Eurostat (2020).

Table 3.1: Baseline Calibration

Parameter	**Value**	**Source/Target**
Households		
CRRA-coefficient γ_c	1	Log-utility
Household discount factor β	0.99	Annual riskless rate 4%
Labor disutility convexity γ_l	1	Frisch elasticity = 1
Labor disutility weight ω_l	6.68	Labor supply = 1/3
Firms		
Cobb-Douglas coefficient θ	1/3	Labor share = 2/3
Green goods share ν	0.20	Renewable share in Europe 2018
Externality Parameter γ_P	1.5e-2	Pollution damage/GDP = 0.1
Banks		
Bond maturity parameter s	0.05	*IHS Markit*
Restructuring costs φ	0.50	Recovery rate = 30%
Collateral default cost parameter η_1	0.0463	Ex-post optimality of $\phi_{sym} = 0.26$
Liquidity management intercept l_0	0.05	Ensures positive costs
Liquidity management slope l_1	0.0085	Eligibility premium = -11bp
Conventional and Green Firms		
Depreciation rate δ	0.067/4	Capital/GDP = 2.1
Discount factor $\widetilde{\beta}$	0.9835	Debt/GDP = 0.8
Standard deviation idiosyncratic risk ς_M	0.175	Bond spread = 100bp
Central Bank		
Haircut parameter ϕ_{sym}	0.26	ECB collateral framework
Shocks		
Persistence TFP shock ρ_A	0.95	Standard
Variance TFP shock σ_A	0.005	Standard

are specified as

$$\Omega\left(\overline{b}_{t+1}^{i}\right) = \max\left\{l_0 - 2l_1\left(\overline{b}_{t+1}^{i}\right)^{0.5}, 0\right\} . \tag{3.26}$$

Their concave shape captures that the marginal cost reduction of collateral is decreasing (in absolute terms), e.g., due to the re-use of existing collateral. The intercept parameter l_0 will be set sufficiently high to ensure that $\Omega(\overline{b}_{t+1}^{i})$ is positive for all considered collateral policy specifications.[14] Plugging in $\overline{b}_{t+1}^{i} = 0$ can be interpreted as the cost level of an entirely uncollateralized banking system.

The slope of the liquidity management costs l_1 governs the cost reduction per unit of collateral. We calibrate it to $l_1 = 0.0085$, matching the eligibility premium reported by the empirical literature: using the ECB list of collateral eligible for main refinancing operations, Pelizzon

[14] We verify that l_0 does not visibly affect our results.

et al. (2020) identify an eligibility premium of -11bp. The model implied eligibility premium is given by the yield differential of the traded bond and a synthetic bond that is not eligible in period t, but becomes eligible in $t+1$, corresponding to the identification strategy of Pelizzon et al. (2020). The advantage of this procedure is that the eligibility premium can be backed out from bond prices *in deterministic steady state*. The eligibility premium is available in closed form and given by

$$\widetilde{x}_{\tau,t} \equiv (1 + s/q_{\tau,t} - s)^4 - (1 + s/(q_{\tau,t}(1 + (1 - \phi_\tau)\Omega_{\bar{b},t})) - s)^4 \; .$$

In the spirit of Bindseil and Papadia (2006), the costs of accepting risky collateral follow

$$\Lambda(\overline{F}_t) = 2\eta_1 \cdot \left(\overline{F}_t\right)^{0.5} \; .$$

The concave specification reflects that there is a fixed cost component to set up a proper risk management infrastructure as well as a marginal cost component from adding additional risk to the central bank's collateral portfolio, for example through more frequent collateral default. The parameter η_1 governs the level of collateral default costs and is set so that the empirical haircut value $\phi_{sym} = 0.26$ is optimal according to an utilitarian welfare criterion. Put differently, we assume that the status-quo ECB collateral policy is optimal under the restriction of symmetric collateral policy and parametrize the cost function accordingly. Finally, we define the *greenium* as the spread of conventional over green bonds with corresponding maturity

$$\widehat{x}_t = x_{g,t} - x_{c,t} \; .$$

Note that the greenium is zero in our baseline calibration due to the assumption of symmetric financial frictions and symmetric collateral treatment. The parameterization is summarized in Table 3.1.

Macroeconomic Dynamics. In Table 3.2, we compare the model-implied second moments with the data. Notably, they are broadly consistent with each other, even though our model only uses one exogenous shock and does not feature frictions related to firm investment, labor markets, and the relationship between households and banks. The time series volatility of bond spreads is slightly smaller than the value reported by Gilchrist and Zakrajšek (2012) for US data, since bond prices in our model are priced using a log-utility pricing kernel and only contain default risk compensation and the collateral premium.

The excess volatilities of consumption and investment are broadly consistent with euro area data. The elevated investment volatility and its low autocorrelation can at least partly be attributed to the absence of investment adjustment costs. The model is also able to capture the cyclical properties of key financial market variables, debt b, leverage at market values $qb/(pk)$, and default risk F. In addition, we also match the cyclicality of emissions, which has been estimated by Doda (2014) for a large sample of countries.

Table 3.2: Model Fit – Second Moments

Moment	Model	Data	Source
Volatilities			
Bond Spread Vol. $\sigma(x)$	30 bp	50-100 bp	Gilchrist and Zakrajšek (2012)
Excess Vol. Consumption $\sigma(c)/\sigma(y)$	0.59	0.70	Euro area data
Excess Vol. Investment $\sigma(i)/\sigma(y)$	6.50	3.80	Euro area data
Persistence			
GDP $corr(y_t, y_{t-1})$	0.70	0.90	Euro area data
Consumption $corr(c_t, c_{t-1})$	0.87	0.80	Euro area data
Investment $corr(i_t, i_{t-1})$	0.60	0.80	Euro area data
Correlations with GDP			
Consumption $corr(y, c)$	0.86	0.60	Euro area data
Investment $corr(y, i)$	0.90	0.70	Euro area data
Debt $corr(y, b)$	0.70	0.65	Jungherr and Schott (2022)
Leverage $corr(y, lev)$	-0.77	-0.30	Kuehn and Schmid (2014)
Default risk $corr(y, F)$	-0.77	-0.55	Kuehn and Schmid (2014)
Pollution $corr(y, \mathcal{P})$	0.31	0.30	Doda (2014)

Notes: We calculate theoretical moments after solving the model under the productivity shock. We compare the model moments to Hodrick-Prescott-filtered data of the euro area or to moments from the literature.

3.4.2 Financial and Real Effects of Preferential Treatment

Before using the calibrated model to study optimal preferential treatment, we compare the model-implied impact of preferential treatment to results from the empirical literature, which corroborates the external validity of our quantitative analysis. Guided by the simplified setting in Section 3.3, we first discuss the effect of preferential treatment on relative borrowing costs of green and conventional firms. In a second step, we then consider the effect of changes in borrowing costs on bond issuance and investment.

Preferential Treatment and Relative Borrowing Costs. To examine the effect of preferential central bank policy on (relative) bond prices, we exploit the yield reaction of green and conventional bonds around ECB announcements regarding environmental policy.[15] We identify four relevant speeches by ECB board members between 2018 and 2020, which explicitly mention environmental concerns for the conduct of central bank policy. Using data from *IHS Markit* and *Thomson Reuters Datastream*, we generate a panel of green-conventional bond pairs, obtained by a nearest-neighbor matching. We then compute the average yield difference between green bonds and their respective conventional counterparts for a 20 trading day window around each announcement. Averaging over all announcements and the entire post-treatment window, the announcement effect is significant in statistical terms: after each ECB announcement, green bond yields drop by 4.8 bp on average over a 20 trading day window. This is economically meaningful and lies in a plausible range, compared to the empirical literature on collateral pre-

[15]See Appendix B.3 for details on the announcements and the data.

Table 3.3: Greenium Reaction – Announcement Effects

Data	**Model: Horizon**			
	2 years	3 years	4 years	5 years
-4.8 bp	-8.8 bp	-6.8 bp	-5.3 bp	-4.1 bp

Notes: The data value results from the analysis of announcement effects. Model-implied values obtain from introducing news shocks (3.27).

mia of corporate bonds. The result indicates that bond investors are willing to pay premia on green bonds already if there is the prospect of preferential treatment.

Since the ECB so far did not implement preferential treatment, these announcements can be mapped into our model by interpreting them as a news shock (see Beaudry and Portier, 2004 and Barsky and Sims, 2011). Specifically, we assume that preferential treatment will be implemented with certainty but at an unknown point in the future. We enrich the baseline calibration by a news shock to the green collateral parameter ϕ_g for various time horizons,

$$\log(\phi_{g,t}) = (1 - \rho_\phi)\log(\phi_{sym}) + \rho_\phi \log(\phi_{g,t-1}) + \sigma_\phi \epsilon^\phi_{t-h} \quad \epsilon^\phi_{t-h} \sim N(0,1)\ , \tag{3.27}$$

where ϕ_{sym} is the green collateral parameter corresponding to the baseline calibration and h denotes the announcement horizon. We choose a high value of $\rho_\phi = 0.95$ for the haircut persistence, since changes to the collateral framework only occur infrequently. The shock size σ_ϕ is set such that $\phi_g = 0.045$ in two, three, four, or five years. The haircut value of 4.5% corresponds to the treatment of AAA-rated securities in the ECB collateral framework. This haircut appears to be a reasonable value for a strong preferential policy and opens a considerable haircut gap. Moreover, the considered horizons appear plausible, given that the ECB strategy review itself took two years and that the actual implementation of preferential treatment takes some additional time. The announcement effect on the greenium is shown in Table 3.3 and lies between -8.8 bp and -4.1 bp. Naturally, the effect peters out as the announcement horizon increases. The model-implied yield response closely resembles the data value at the four-year horizon.

Relative Borrowing Costs and their Real Effects. In the second step, we consider the firm level effect of a change in borrowing cost induced by central bank policy. We build on literature studying firm responses following QE-programs and collateral framework changes. From the point of view of firms (the collateral supply side), the effects of QE and collateral eligibility are identical, since in both cases banks increase demand for their bonds for reasons unrelated to firm fundamentals. Specifically, we compare estimate from the literature to the effects of a haircut decrease from $\phi_{sym} = 1$ (no eligibility) to $\phi_{sym} = 0.26$ (our baseline value). We assume that the collateral policy relaxation is *unanticipated*, comes into effect *immediately*, and is *permanent*. We focus on the reaction of bond yields, capital, and leverage, since our discussion of the imperfect pass-through of preferential treatment in Section 3.3 is centered around these variables.

Since the eligibility premium as defined in Pelizzon et al. (2020) is a calibration target, we instead examine the yield spread between eligible and non-eligible bonds. Fang et al. (2020) study the impact of an easing of collateral eligibility requirements by the PBoC and identify a yield reaction on treated bonds of 42-62 bp (their Table 5). Using a similar approach, Chen et al. (2019) find a yield reaction of 39-85 bp (their Tables 5 and 8).

Regarding the financing of firms, Grosse-Rueschkamp et al. (2019) show that the introduction of the Corporate Sector Purchase Program (CSPP) triggered a positive response of total debt to assets for eligible firms relative to non-eligible firms prior to CSPP. The magnitude of the effect is estimated between 1.1 pp and 2.0 pp, depending on the econometric specification (see their Table 2). Pelizzon et al. (2020) report an increase of total debt/total assets between 2.5 pp and 10.8 pp (see their Table 10). Giambona et al. (2020) consider the impact of QE and find increases in total debt/total assets of around 1.8pp (see their Table 15).

On the same sample, they report an increase in investment between 4.9 pp and 6.0 pp for QE-eligible firms when controlling for firm characteristics (see their Table 3). Harpedanne de Belleville (2019), Table 4.1, finds a 5.4 pp increase in investment after the introduction of the Additional Credit Claims program using French data, which contains a large amount of small firms, which also contains firms without bond market access. Grosse-Rueschkamp et al. (2019) on the other hand only document a mild effect of 1pp on asset growth (their Table 5).

Table 3.4: Firm Reaction: Model vs. Data

	Δ Yield	**Δ Capital**	**Δ Leverage**
Model	58 bp	2.1 pp	1.4 pp
Data	39 - 85 bp	1.0 - 6.0 pp	1.1 - 10.8 pp

Notes: In the first line, we compare our baseline to an economy with 100% haircut. The second line displays the range of estimated effects taken from the empirical literature.

Table 3.4 displays our results. Consistent with the empirical literature, we observe a strong yield response to eligibility of around 58 bp. The capital response comfortably falls into the range of empirical estimates, while the model-implied leverage response is at the lower bound of the firm reaction observed in the data. The relatively modest leverage response will imply that the role of the adverse effects of preferential treatment on firm risk-taking are quantified in a conservative manner.

3.5 Policy Analysis

In this section, we conduct policy experiments regarding the collateral framework and its interactions with direct Pigouvian taxation of pollution. Throughout the analysis, we employ an utilitarian welfare criterion based on household's (unconditional) expected utility (3.1) and follow Schmitt-Grohé and Uribe (2007) by approximating it, together with the policy functions, up to second order. Given the log-utility assumption on consumption, the consumption equivalent

(CE) welfare gain follows as

$$c^{CE,policy} \equiv 100 \left(\exp\{(1-\beta)(V^{policy} - V^{base})\} - 1 \right) ,$$

where V^{base} and V^{policy} are obtained from evaluating (3.1) under the baseline and alternative policies, respectively.[16]

3.5.1 Optimal Collateral Policy With Preferential Treatment

Since intermediate good firms are at the heart of the transmission mechanism of both policies, we begin by showing the model-implied means of financial market variables for different green haircuts in Figure 3.1. The green and red line denote the green and conventional firms, respectively. The upper panels show that green collateral premia strongly increase, the smaller the green collateral haircut ϕ_g, while at the same time the green bond spread declines, relative to the baseline calibration (solid vertical line).

[16] We also explore welfare gains conditionally on being at the deterministic steady state of the baseline calibration and taking into account the transition period to the new steady state. Results are virtually unchanged.

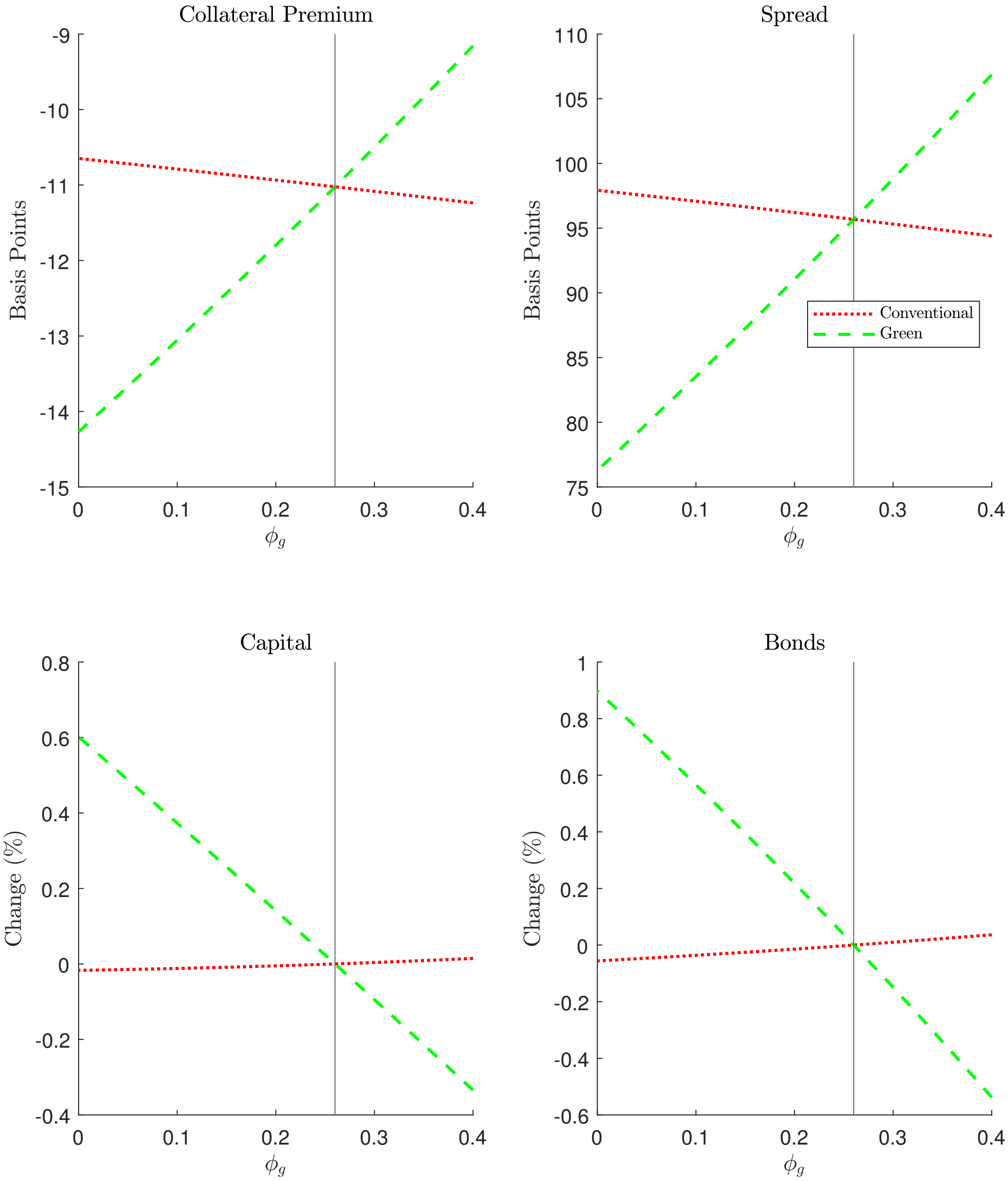


Figure 3.1: Firm Response to Preferential Treatment

Notes: We display long-run means for different green haircuts ϕ_g. The collateral premium and spreads are expressed in basis points, leverage and default rates in percentage points. Bonds outstanding and capital are shown relative to the baseline of $\phi_{sym} = 0.26$ (vertical line).

The lower financing costs of green firms increase the green capital holdings. For every haircut, the increase in investment falls short of the increase in bond issuance (lower panels), consistent with an increase in leverage: the pass-through of collateral policy is imperfect as outlined in Section 3.3. For all variables, the reaction of conventional firms mirrors the response of their green counterparts, although to a smaller extent. This is an equilibrium effect operating through the perfect substitutability of green and conventional bonds as collateral: the conventional collateral premium $(1 - \phi_c)\Omega_{\bar{b},t}$ depends on haircuts and collateral supply. If $\bar{b}_t$ increases due to preferential treatment, this has a negative effect on the conventional collateral premium.

Table 3.5: Time Series Means for Different Policies

Moment	**Baseline**	**Max Pref**	**Opt Coll**	**Only Tax**	**Glob Opt**
Tax Parameter χ_c	0	0	0	9.6%	9.6%
Haircut ϕ_g	26%	0%	11%	26%	21%
Haircut ϕ_c	26%	100%	29%	26%	21%
Welfare Change (CE)	0%	-0.6462%	0.0064%	+0.6640%	+0.6646%
Conv. Leverage	42.1%	41.4%	42.0%	42.1%	42.1%
Green Leverage	42.1%	42.5%	42.2%	42.1%	42.1%
Conv. Bond Spread	96bp	162bp	99bp	96bp	94bp
Green Bond Spread	96bp	2bp	83bp	96bp	94bp
Conv. Coll. Premium	-11bp	0bp	-11bp	-11bp	-11bp
Green Coll. Premium	-11bp	-27bp	-13bp	-11bp	-11bp
GDP	0.8494				
Change from Baseline	-	+0.05%	+0.02%	+0.58%	+0.61%
Restructuring Cost/GDP	2.30%				
Change from Baseline	-	-20.18%	+0.19%	-0.13%	+1.37%
Coll. Default Cost/GDP	1.65%				
Change from Baseline	-	-29.75%	+0.74%	-0.36%	+3.76%
Liq. Man. Cost/GDP	2.91%				
Change from Baseline	-	+46.91%	-0.47%	-0.86%	-4.37%
Pollution Cost/GDP	9.74%				
Change from Baseline	-	-1.64%	-0.06%	-8.66%	-8.59%
Green Bond Share	20%	21.17%	20.11%	27.68%	27.68%
Green Capital Share	20%	20.74%	20.07%	27.68%	27.68%

Notes: Maximal preferential treatment (*Max Pref*) only allows green bonds as collateral ($\phi_g = 0, \phi_c = 1$). The optimal collateral policy (*Opt Coll*) is derived from maximizing CE over a grid of haircuts. For the optimal tax (*Only Tax*), we hold haircuts fixed and vary the tax rate. The global optimum (*Glob Opt*) is obtained by maximizing over taxes and haircuts.

Our first finding considers the welfare and pollution impact of *maximal preferential treatment* in the second column of Table 3.5. We set $\phi_g = 0$ and $\phi_c = 1$ to provide an upper bound for the central bank's ability to induce investment into green technologies. The collateral premium on conventional bonds is zero in this case. This policy induces an increase in the green bond share to 21.17%, while green investment rises to 20.74%, translating into an 6% (3.7%) increase relative to the symmetric baseline (first column), respectively.

Around 40% of the effect on the corporate (green) bond market does not carry over to the investment decision due to the financial friction in the production sector. The converse holds for conventional firms, who reduce their bond issuance and capital holdings. This in turn lowers pollution. At the same time, setting $\phi_c = 1$ implies a strong contraction of collateral, leading to a substantial increase in liquidity management costs. Given the reduction in conventional bond issuance, we observe a decrease in the cost from debt restructuring and collateral default. Since optimal collateral policy trades off pollution with resources losses from corporate default and liquidity management costs, this combination of aggregate default rates and collateral supply is sub-optimal and substantially decreases welfare relative to the baseline collateral framework. Therefore, we maximize welfare over the collateral framework (ϕ_c, ϕ_g), to which we refer as the *optimal collateral policy*, and report results in third column of Table 3.5.

The optimal haircut levels of $\phi_g = 0.11$ and $\phi_c = 0.29$ imply a preferential treatment of green bonds. Subtracting the green bond spread of 83 bp from the conventional one of 99 bp gives a greenium of 16 bp, which is considerably smaller than under maximal preferential treatment. Consequently, the increase in the green bond (0.11 pp) and green capital (0.07 pp) shares is smaller as well. On the one hand, this policy avoids a sharp drop in available collateral. On the other hand, it reduces pollution less effectively. In Appendix B.2.1, we also show that nominal rigidities are not driving these results.

3.5.2 Interaction with Direct Taxation

While our analysis reveals that the central bank can affect the relative size of green and conventional firms and thereby reduce the pollution externality, this effect is relatively small and induces non-negligible side-effects. In this section, we benchmark these results against direct Pigouvian taxation of pollution externalities. Section 3.3 indicated that its effect on capital shares is more direct than the one of collateral policy. The exercise serves a dual purpose: first, we can put the effectiveness of preferential collateral treatment into perspective relative to Pigouvian taxation. Second, this allows us to examine a mix of direct taxation and collateral policies. By assuming a balanced budget in (3.16), we compare different policy instruments regarding their effectiveness to address environmental policy trade-offs without imposing assumptions on the financing of subsidies or the distribution of tax revenues.

The fourth column of Table 3.5 corresponds to optimal Pigouvian taxation, holding the collateral framework at its baseline value. The optimal tax on conventional production is at 9.6%, which implies a subsidy of around 40% on green intermediate goods, since taxes are rebated to conventional firms proportional to their relative sizes, as determined by the parameter ν in the wholesale good production function (3.8). The green capital share rises by 7.7 pp, which strongly tilts production towards green inputs and reduces the pollution externality. The welfare improvement of optimal Pigouvian taxation exceeds the improvement from optimal preferential treatment by two orders of magnitude. At the same time, there are no adverse effects on firm risk-taking, since the first order condition for leverage (3.14) is not affected by a tax on production. This suggests that fiscal instruments dominate preferential treatment in addressing environmental frictions.

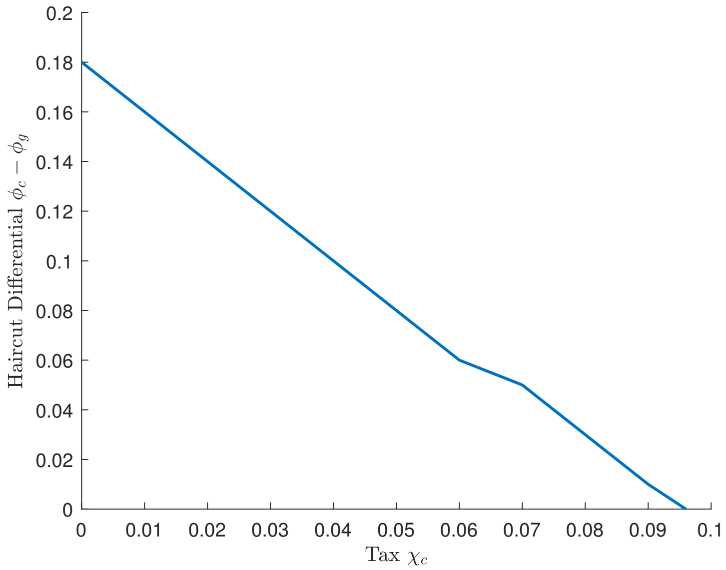


Figure 3.2: Optimal Collateral Policy Under Sub-Optimal Taxation

Notes: For different levels of the Pigouvian tax, we maximize CE over a grid of haircuts and display the result as the haircut differential.

However, this should not be misinterpreted as a call for central bank inaction, since Pigouvian taxation has also a collateral policy impact as reported in the fourth column of the third panel in Table 3.5. We observe a simultaneous decline of liquidity management expenses and costs associated to default relative to GDP, respectively. The former results from a slight increase in available collateral driven by green firm bond issuance and the latter is a result of a drop in the bond issuance of conventional firms that lowers the size of costs from default. Both changes lower resources wasted but also shift the economy away from a configuration of the collateral trade-off for which the baseline haircuts are optimal. Relative to the global optimum, when collateral policy and taxes are set jointly, reported in the last column of Table 3.5, the collateral supply is too small at the old collateral framework driven by the reduction in conventional bond issuance. As a result, the collateral framework becomes more lenient. Notably, this relaxation is symmetric. This incentivizes all firms to increase their bond issuance, implying a slight increase of default cost while liquidity management cost decline substantially.[17] The welfare gains of adjusting collateral frameworks to mitigate negative effects on collateral availability are positive, but of small size compared to the welfare gains of optimal taxation.

The symmetry result hinges on the assumption that optimal Pigouvian taxes are available, which is arguably not an empirically plausible case. In Figure 3.2, we compute the optimal degree of preferential treatment, represented by the haircut differential, for different levels of the Pigouvian tax. At $\chi_c = 0$, the haircut gap is 18%, corresponding to the third column of Table 3.5, i.e., optimal collateral policy in the absence of taxation. At the globally optimal tax of

[17] This is similar to Carattini et al. (2021), who show that macroprudential policy can alleviate adverse effects of carbon taxation. In their model, adverse effects take the form of asset stranding, while in our case adverse effects are linked to collateral availability, if conventional firms shrink their balance sheet size. Notably, optimal macroprudential policy is also symmetric in their model.

$\chi_c = 0.096$, the gap is zero as in column five. While we are not explicit about why the Pigouvian might be too low, our model implies that the central bank can improve on sub-optimal taxation. However, the optimal degree of preferential treatment decreases, the closer environmental policy gets to implementing the optimal Pigouvian tax.[18]

3.6 Conclusion

In this paper, we examine the effectiveness of the preferential collateral treatment of green bonds in an augmented RBC-model. Preferential treatment stimulates investment into green bonds. However, this only partially transmits to investment into green technologies due to an increase in green firms' leverage and default risk. In a calibration to euro area data, we find that this policy can be fairly powerful in addressing environmental policy concerns, but is still considerably less effective than Pigouvian taxes. Due to the adverse effects on firm risk-taking, the optimal collateral framework features only a small degree of preferential treatment, but still increases welfare. Preferential treatment is a qualitatively and quantitatively imperfect substitute for Pigouvian taxes and is only optimal if Pigouvian taxes cannot be set to their optimal level. If the optimal tax is implemented, the optimal collateral framework is characterized by a symmetric relaxation to solve the trade-off between the benefits of higher collateral supply and the costs of higher risk-taking.

Our results can be read as a call for (i) central bank action if tax policy is not able to adequately address pollution and climate change externalities, (ii) a careful calibration of preferential treatment that takes into account the side effects on firm risk-taking, and (iii) coordination between direct tax policy and central bank collateral policy, to mitigate adverse effects that environmental policy can inflict on the aggregate collateral supply.

[18] For a tax above the optimal value, the collateral framework optimally treats conventional bonds more preferentially to correct for an overly ambitious environmental policy.

4 Convenient But Risky Government Bonds

Authors: Matthias Kaldorf and Joost Roettger[1]

4.1 Introduction

Government bonds play a special role in the financial system of developed economies. One, if not the most important, reason for this is the exceptional degree of liquidity and safety that they provide to investors relative to alternative asset classes. These special attributes make government bonds a key ingredient for the functioning of various financial market segments, such as repo and securities lending markets, where they are posted as collateral by market participants. Investors' willingness to pay a price markup for government bonds due to non-pecuniary benefits that they provide is well documented (see Bansal and Coleman, 1996; Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood et al., 2015; Du et al., 2018). The associated premium is usually referred to as *convenience yield*.

The literature has documented the negative dependence of convenience yield on government bond supply (Krishnamurthy and Vissing-Jorgensen, 2012), its potential to reconcile the high valuations of government debt (Jiang et al., 2022b; Mian et al., 2022) and the implications for government debt management (Greenwood et al., 2015; Jiang et al., 2022a; Gorton and Ordoñez, 2022). However, the focus so far has usually been on the United States and Japan, where sovereign risk is effectively not a concern. By contrast, several European countries facing non-negligible default risk are still able to issue bonds at sizable convenience yields (Jiang et al., 2021).[2] When sovereign risk becomes a non-negligible component of government bond spreads, studying the implications of convenience yield for the conduct of fiscal policy consequently has to take into account potential interactions between convenience yield, sovereign risk, and the supply of government bonds.

This paper contributes to an understanding of this interaction, which is a non-trivial task since all of these factors are jointly and endogenously determined in equilibrium. All else equal, the presence of convenience yield improves borrowing conditions for a government by raising bond prices, which makes it more attractive for the sovereign to issue debt. However, as documented by Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood et al. (2015) for the United States, an increase in *government bond supply* lowers convenience yield by making bonds less scarce. The presence of *sovereign risk* amplifies this negative effect of bond supply on convenience

[1]The views expressed in this Chapter are those of the authors and do not necessarily represent those of the Deutsche Bundesbank or the Eurosystem.

[2]Risky government bonds are still safer and more liquid than most alternative asset classes available to investors. We will therefore focus on this type of security in this paper and abstract from privately issued debt.

yield. To cushion against price risk in general and credit risk in particular, collateral pledged in market transactions is subject to haircuts. The higher the haircut, the less collateral can effectively be pledged. Collateral services and hence convenience yield thus negatively depend on the haircut. Since haircuts are positively related to a security's credit risk, an increase in sovereign risk will adversely affect bond prices not just by raising default risk premia charged by investors, but also by lowering the convenience yield of government bonds.

The supply of public debt and the risk of default are however not exogenously given but a reflection of government behavior, which in turn will respond to changes in borrowing conditions and, therefore, convenience yield. So how do convenience yield and haircuts affect a government's incentives to borrow and default? To study the interaction between convenience yield, sovereign risk, and government bond supply, we extend a quantitative model of sovereign debt and default (see Chatterjee and Eyigungor, 2012; Bocola et al., 2019). In the model, debt issuance and default arise endogenously as optimal decisions of a risk-averse government that issues bonds to investors without commitment. In the spirit of Krishnamurthy and Vissing-Jorgensen (2012), we assume that investors derive utility from the collateral services of government bonds, which depends negatively on bond supply and haircuts. Consistent with mark-to-market practice on repo and securities lending markets, haircuts are positively linked to default risk.[3] This yields a pricing schedule for government bonds which decreases in debt issuance even absent default risk, since collateral becomes less scarce in this case and convenience yield declines. In the risky borrowing region, default risk further depresses bond prices and this increase is amplified by mark-to-market haircuts. The additional responsiveness of bond prices to debt issuance due to convenience yield is a key novelty of our analysis and will crucially determine debt and default dynamics.

In addition to default risk and convenience yield, the literature has identified market illiquidity as a potentially important determinant of bond pricing (see e.g. He and Milbradt, 2014; Pelizzon et al., 2016). We address the concern that interactions between debt, default risk and convenience yield are driven by market illiquidity as a confounding factor by explicitly adding market illiquidity to the model. To do so, we introduce trading frictions as in Lagos and Rocheteau (2009) into our model. Our interpretation of market illiquidity reflects the notion that investors face trading frictions when selling or buying a security on secondary markets, resulting in illiquidity discounts that are usually measured via bid-ask spreads. It is important to note that government bond markets are typically characterized by exceptionally high liquidity, such that bid-ask spreads are usually very low during safe times.[4] However, illiquidity discounts increase with credit risk (see Chaumont, 2018; Passadore and Xu, 2020) and therefore can be a

[3]In our model, the effective convenience yield that a bond provides depends on the haircut it is subject to on various market segments. When using the term "haircut", we refer to an average haircut over different segments. For example, Drechsler et al. (2016) document that the ECB applied a flatter haircut schedule for its standing facilities than haircuts set by a large central clearing counterparty (CCP). The average haircut encompasses this heterogeneity in a tractable way. For our comparative statics exercises with respect to the steepness of haircut schedules, we do not take a stand on whether it reflects risk management practice of CCPs or central bank collateral policies (see e.g. Nyborg, 2017).

[4]Bid-ask spreads are a common measure of market illiquidity, whose interaction with bond spreads is well established in the finance literature (see e.g. He and Milbradt, 2014).

distinct channel that might amplify the increase of government bond spreads in times of high default risk. Consistent with the data, our model of trading frictions implies that bid-ask spreads increase with credit risk.

We calibrate our model to Italian data for 2001-2012. Our main variable of interest is the government bond spread, which is the difference between the yield of a government bond and the risk-free interest rate, which we proxy by the EURIBOR. The bond spread reflects credit risk, convenience yield and market illiquidity. We use credit default swap (CDS) spreads as a measure of credit risk since derivatives *(i)* do not provide convenience yield and *(ii)* are not affected by market illiquidity the way bond markets are.[5] Our model permits direct pricing of credit default swaps and the computation of bid-ask spreads, such that we can directly compare model-implied statistics with their empirical counterparts for all variables of interest.

Free model parameters are calibrated to match the mean and volatility of bond spreads, CDS spreads, debt outstanding, and bid-ask spreads over the full sample. Although the model is quite parsimonious, it picks up several important (non-targeted) features of crisis episodes observed in the data. While government bond spreads are negative during safe times in the data, they turn positive in 2008Q4, which prompts us to interpret the time period 2008Q4-2012Q4 as the crisis sub-sample. When applying this filter to model-generated time series, we find that the model-based crisis sub-sample features higher spread volatility and a moderate increase in debt issuance, which is consistent with the data. We can therefore broadly distinguish between two endogenous regimes. In crisis times, default risk is found to dominate the convenience yield component of the bond spread, whereas the opposite is true for safe times, where bond spreads are usually negative.

To further corroborate the validity of our calibrated model, we investigate the role of convenience yield and bond supply for government bond spreads. Specifically, we use the simulated time series to regress the government bond spread on bond supply, while controlling for credit risk and bid-ask spreads. Both data and model-implied regressions reveal a highly significant positive effect of bond supply, which supports and extends the findings of Krishnamurthy and Vissing-Jorgensen (2012) to risky government bonds. Our empirical results complement Jiang et al. (2021) who identify convenience yield in a panel of risky European government bonds. At the same time, we only find a small and hardly significant effect of various measures of market illiquidity on bond spreads, suggesting that credit risk and bond supply are the most important drivers of government bond spreads.

Using our calibrated model, we shed light on how convenience yield affects debt and default dynamics by exploring various alternative model versions. To study the impact of convenience yield and market illiquidity, we first keep the haircut schedule at its baseline form, set collateral valuation and market illiquidity to zero, respectively, and recalibrate the model to match average bond and CDS spreads. The effects on debt and default dynamics are small compared to the

[5] These two features make them appropriate measures of default risk in our model, while we do not (need to) assume that CDS spreads are a pure measure of default risk. Other potential risk factors in CDS spreads include counterparty risk, i.e., uncertainty about CDS payoffs in the event of a sovereign default (see Salomao, 2017, for a discussion). In this setting, CDS spreads increase less than one-for-one in expected default risk, since they do not provide perfect insurance against default.

baseline case with positive collateral valuation. By contrast, market illiquidity has no visible impact, consistent with our empirical findings. The reason for this finding is that the shape of the bond price schedule hardly differs between the baseline model and the recalibrated model versions, such that borrowing and default incentives do not differ much as well once the same targets are matched by recalibrating the model.

In a second experiment, we therefore focus on the responsiveness of the collateral valuation and haircuts to default risk, complementing the analysis of Mian et al. (2022) for the case of (default)-risk-free government bonds. The effects on debt and default risk are much larger in this case since the bond price schedule exhibits a different curvature in the risky borrowing region, which is pivotal for debt and default dynamics. Specifically, a reduction in the elasticity of the haircut schedule translates into a reduction in average CDS spreads and their volatility. The long-run default rate declines as well. This is due to the disciplining impact of collateral scarcity effects on government borrowing that result in more debt-elastic bond pricing. However, if collateral valuation is positive but constant, which we capture by a recalibrated model specification with debt-inelastic convenience yield, the same reduction in the elasticity of haircut schedules translates into a higher and more volatile CDS spread and more frequent default on average.

Our findings suggest a U-shaped relationship between the elasticity of convenience yield and default risk, which depends on the interaction between collateral scarcity and haircuts. If the response of convenience yield with respect to debt issuance is similar to the response of the default risk compensation, there is only a level effect while debt and default dynamics are not affected. If the response of convenience yield to debt issuance exceeds the response of the default risk compensation, elevated debt rollover risk increases default rates, spreads, and reduces the cyclicality of debt issuance. If the response of convenience yield is low, this incentivizes the government to increase debt issuance, which again has adverse effects on default risk. Taking into account the endogeneity of government bond supply and default risk has potentially important implications for the design of haircuts, both on private and public market segments.

Related Literature. Our paper also relates to the quantitative sovereign default literature (see Aguiar et al., 2016), which mostly focuses on external public debt and emerging economies. Similar to Bocola et al. (2019), we re-interpret the established sovereign default framework à la Eaton and Gersovitz (1981) and Arellano (2008) as the relationship between a government and lenders regardless of their place of residence and consider an application to a developed economy.[6] In the model, the government receives an exogenous stream of tax revenues, draws utility from public spending and uses bond markets to smooth spending intertemporally.

Our paper is related to recent studies that emphasize the role of convenience yield for optimal debt management. Canzoneri et al. (2016) study optimal Ramsey monetary and fiscal policy when government bonds provide transaction services, leading to a trade-off between tax smoothing and liquidity provision. Jiang et al. (2022a) analyze convenience yield in a model the government trades off insuring bondholders and taxpayers. In order to reap the benefits of

[6] Bocola et al. (2019) show that total government debt, rather than external government debt, matters for default risk, which is captured by our model.

convenience yield, the government has to ensure that their bonds have a beta of zero, which comes at the expense of tax smoothing. By contrast, Greenwood et al. (2015) study optimal debt maturity management when short-term government bonds are valued by investors for having money-like properties. While these papers consider the interaction between public debt and non-pecuniary benefits of government bonds from an optimal policy perspective, their focus is on analytical results and – more importantly – they abstract from sovereign risk, market illiquidity and lack of commitment.[7]

Convenience yield on public debt is also discussed in Fisher (2015), who relates changes to the demand for safe and liquid assets to risk premium shocks. Perez (2018) analyzes the role of government bonds as public liquidity, which provides additional repayment incentives in a small open economy model. In contrast to his paper, we assume that investors do not enter the government objective, which in our view is a more plausible assumption in light of our application to Euro area data, which features a large degree of financial integration. Market illiquidity of government bonds has been identified as an important component of sovereign spreads during the Euro area debt crisis (see Chaumont, 2018). While micro-founded trading frictions are also considered in Chaumont (2018) and Passadore and Xu (2020), our approach differs from theirs since we link heterogeneity in bond valuation to non-pecuniary benefits.

Layout. The paper is structured as follows. Section 4.2 introduces our model. Our calibration is presented in Section 4.3, while Section 4.4 discusses the interactions between convenience yield, credit risk, and government bond supply. Section 4.5 concludes.

4.2 Model

Time is discrete and denoted as $t = 0, 1, 2, ..., \infty$. The model features a *government* of an economy that issues bonds to a mass-one continuum of ex-ante homogeneous *investors*. The government receives random tax revenues, cares about the supply of a public good and can trade long-term government bonds with investors. Following the quantitative sovereign default literature, the government cannot commit to future repayment and debt issuance (see Arellano, 2008). Investors value government debt for their pecuniary future benefits as well as their *collateral services*, such that they are willing to pay a premium on government bonds. We follow Gorton and Ordoñez (2022) and refer to this valuation of collateral services as convenience yield.

Each period is divided into two sub-periods. In the first sub-period, investors are subject to uninsurable idiosyncratic shocks to the valuation of collateral service, which introduces benefits from trading government bonds. However, we assume that the government bond market is not open at this stage and investors can only trade bonds with *dealers* on a decentralized over-the-counter market.[8] Restricting their ability to trade with each other will give rise to endogenous bid ask spreads. We assume that the consumption of investors and dealers does

[7] Bonam (2020) studies the implications of convenience yield for fiscal policy in a New Keynesian setting that models fiscal policy via exogenous policy rules and abstracts from sovereign risk.

[8] Dividing a period into two sub-periods that represent a decentralized and a centralized market is in the spirit of Lagos and Wright (2005).

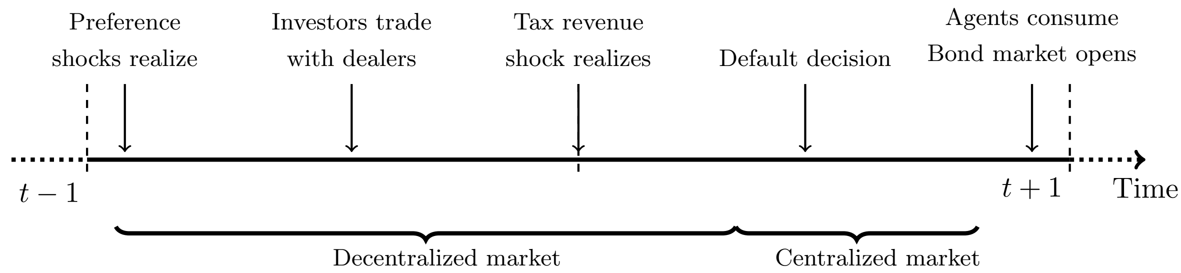


Figure 4.1: Within-Period Timing

not enter the government's objective, either because they reside outside the economy or can trade on international financial markets in a frictionless manner. Given that our model does not distinguish between domestic and external investors, the supply of government bonds in the model corresponds to total debt rather than external debt.[9] Tax revenues realize at the beginning of the second sub-period. All agents consume in the second sub-period, when the competitive and centralized government bond market is open. The timing assumption is summarized in Figure 4.1.

We begin with the investor problem, which will provide a pricing schedule for government bonds. We then describe the government's policy problem, taken as given the pricing schedule, which, together with bond market clearing, characterizes the equilibrium of the model.

Investors. Investors receive a large, constant endowment e and discount consumption at the exogenously given, time-invariant rate $r^{rf} > 0$. They maximize their expected life-time utility

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{c_t+u_i(\theta_t)}{(1+r^{rf})^t}\right]$$

where c_t denotes an investor's consumption and $u_i(\theta_t)$ is the convenience benefit of holding government bonds. The collateral value of government bonds $\theta_t \equiv (1-\kappa_t)m_t b_{t-1}$ consists of three parts: the stock of government bonds b_{t-1} purchased at the end of last period, the cum-coupon price of a bond in the next centralized market $m_t \equiv \mathbb{E}_{t-1}[k_t]$, with k_t defined below, and the haircut κ_t imposed by financial market participants on government bonds for transactions like repos.[10] The haircut will, consistent with actual practice on repo or securities lending markets, depend on the credit risk of a security (see further below). Letting the collateral value θ_t depend on the cum-coupon bond price is consistent with mark-to-market practices on repo and securities lending markets. The elasticity of $u_i(\theta_t)$ with respect to the (haircut-adjusted) market value of government bonds outstanding will have non-trivial effects on the government's debt and default behavior.

To give rise to a trading motive in the first sub-period, we assume that investors are subject to

[9]For a detailed discussion, we refer to Bocola et al. (2019).

[10]Since no information about government bond payoffs arrives during the first sub-period, we condition the expectations operator on the information set available to investors at the end of the second sub-period of $t-1$.

i.i.d. preference shocks. With probability $\frac{1}{2}$, investors are a high- or low collateral-valuation type, which we denote by $i \in \{L, H\}$. Specifically, investor types differ utility derived from a bond's collateral services, with $u_L(\cdot) < u_H(\cdot)$ and $u_i'(\cdot), -u_i''(\cdot) > 0$, for $i \in \{L, H\}$. Investors cannot directly trade bonds among each other or with the government in the first sub-period. They can however adjust their bond holdings by trading with dealers in exchange for an endogenously determined fee $\phi_{i,t}$. Dealers have access to a competitive inter-dealer market and intermediate between buyers and sellers as in Lagos and Rocheteau (2009). The terms of trade are determined bilaterally between a dealer and an investor via Nash bargaining. The bargaining power of both sides is normalized to $\frac{1}{2}$ in the following. We assume that there are no search frictions, i.e., each buyer/seller is matched to a dealer. In appendix C.1, we derive the bond pricing condition consistent with investors' optimality conditions:

$$q_t = \frac{1}{1 + r^{rf}} \left(\underbrace{m_{t+1}}_{\text{pecuniary benefits}} \times \underbrace{(1 + \Lambda_{t+1})}_{\text{convenience yield}} \right), \tag{4.1}$$

where

$$\Lambda_{t+1} \equiv \underbrace{(1 - \kappa_{t+1})}_{1-\text{haircut}} \times \underbrace{\mathbb{E}_i \left[u_i'(\theta_{t+1}) + \frac{1}{2} \times \left\{ u_i'(\widetilde{\theta}_{i,t+1}) - u_i'(\theta_{t+1}) \right\} \right]}_{\text{collateral valuation}}, \tag{4.2}$$

$\theta_{t+1} = (1 - \kappa_{t+1}) m_{t+1} B_t$ is the collateral value of government bond holdings B_t. The bond price q_t reflects expected pecuniary benefits as well as expected non-pecuniary benefits due to the collateral services provided by a bond. Non-pecuniary benefits in turn are affected by potential trading frictions when dealers have bargaining power vis-à-vis investors. Specifically, the term $u_i'(\widetilde{\theta}_{i,t+1}) - u_i'(\theta_{t+1})$ measures the net marginal gains from trading, with $\widetilde{\theta}_{i,t+1}$ denoting the collateral value of the bond position held by an investor of type i after trading on the decentralized market (see also appendix C.1). The haircut $\kappa_t = \kappa(\lambda_t, h_t)$ only depends the government's credit status h_t and the default probability $\lambda_t \equiv \mathbb{E}_{t-1}[d_t]$. Since tax revenues are not known at this stage and only the revenue state from period $t - 1$ is known, the default probability is expressed in terms of the information available to investors at the end of the previous period.

Government. Each period, the government receives exogenous revenues τy_t, where τ is a constant income tax rate and y_t random domestic income, supplies the public good g_t, issues long-term bonds on the centralized market and decides whether to repay its creditors in order to maximize

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t v(g_t) \right], \quad \beta \in (0, 1), v'(\cdot), -v''(\cdot) > 0,$$

subject to the government budget constraint

$$g_t = \tau y_t + h_t \times \left(q_t \left(B_t - (1-\delta) B_{t-1} \right) - \widetilde{\delta} B_{t-1} \right) .$$

Budget constraint and bond payoff depend on the government's credit status $h_t \in \{0,1\}$. In the autarky case ($h_t = 0$), the government finances the public good g_t with revenues τy_t only. In the repayment case ($h_t = 1$), it can issue new bonds, but has to make debt payments.

Following Chatterjee and Eyigungor (2012), government bonds are modelled as random-maturity bonds. An outstanding bond matures in the centralized market of the subsequent period with constant probability δ, with $0 < \delta \leq 1$, whereas it does not mature with probability $1-\delta$, pays a fixed coupon χ, and is valued – like newly issued bonds – at (centralized) market price q_t in this case. The per unit bond payoff k_t depends on the default decision $d_t \in \{0,1\}$ of the issuing government, where $d_t = 1$ if the government defaults and $d_t = 0$ if it repays.

When entering a period with a good credit status $h_{t-1} = 1$, a default immediately changes the government's credit status to $h_t = 0$. By contrast, if the government enters a period with $h_{t-1} = 0$, with constant probability ϑ, it is given the offer to repay the constant fraction $\omega \in [0,1]$ of its debt and immediately leave autarky in return. The indicator variable $\xi_t \in \{0,1\}$ denotes whether such an offer is received ($\xi_t = 1$) or not ($\xi_t = 0$). Following Hatchondo et al. (2016), if the government does not accept an offer ($d_t = 1$), it remains in autarky but its bond position is still reduced by $1-\omega$. If the government declines an offer, it will be excluded ($h_t = 0$) until the next period, where, with probability ϑ, it might get a new chance to settle its debt and leave autarky. The law of motion for the credit status can therefore conveniently be written as

$$h_t = \xi_t(1-d_t)(1-h_{t-1}) + (1-d_t)h_{t-1},$$

with the bond payoff in the centralized market given by

$$\begin{aligned} k_t = {} & \mathbf{1}_{\{h_{t-1}=1 \wedge d_t=0\}} \left(\widetilde{\delta} + (1-\delta)\, q_t \right) + \ \mathbf{1}_{\{(h_{t-1}=1 \wedge d_t=1) \vee (h_{t-1}=1 \wedge \xi_t=0)\}} q_t \\ & + \mathbf{1}_{\{h_{t-1}=0 \wedge \xi_t=1 \ \wedge d_t=1\}} \omega q_t + \mathbf{1}_{\{h_{t-1}=0 \wedge \xi_t=1 \wedge d_t=0\}} \left(\omega(\widetilde{\delta} + (1-\delta)\, q_t) \right), \end{aligned}$$

where $\widetilde{\delta} \equiv \delta + (1-\delta)\chi$. Bond issuance is therefore given by $I_t = B_t - (1-\delta) B_{t-1}$, while $\widetilde{\delta} B_{t-1}$ is the period t debt service. The government is assumed lack the ability to credibly commit to future debt and default policies. Following Bianchi et al. (2018), a default leads to utility costs $\phi(\tau y_t) \geq 0$, which are a function of income, and exclusion from financial markets until debt repayment is settled.

Policy Problem. As it is common in the literature, we focus on Markov-perfect equilibria, such that government policy in a given period t only depends on the payoff-relevant aggregate state variables, which consist of the aggregate public debt position B_{t-1}, tax revenues y_t, the government's credit status h_{t-1}, and the offer indicator ξ_t. In the repayment case, assuming bond market clearing $b = B$, the government faces the following bond pricing schedule for an arbitrary debt choice B', and current income y:

$$q(B',y) = \frac{1}{1+r^{rf}}\Big(m^r(B',y)\left(1+\Lambda^r(B',y)\right)\Big). \tag{4.3}$$

The bond price schedule consists of two terms. The first one, given by the function

$$m^r(B',y) = \mathbb{E}_{y'|y}\left[\ (1-\mathcal{D}(B',y'))\left(\widetilde{\delta}+(1-\delta)\mathcal{Q}^r(B',y')\right)+\mathcal{D}(B',y')\mathcal{Q}^d(B',y')\ \right],$$

captures the expected pecuniary value of the bond in the upcoming centralized market, where $\mathcal{D}(\cdot)$ denotes the default policy function. Bond prices exceed the expected pecuniary value, which is reflected in the *collateral valuation* function

$$\Lambda^r(B',y) = \left(1-\kappa\left(\lambda(B',y),0\right)\right)\times\mathbb{E}_i\Bigg[u_i'\Big(\Theta^r\left(B',y,B'\right)\Big) + \frac{1}{2}\times\Big\{u_i'\Big(\Theta^r\left(B',y,\widetilde{b}_i(B')\right)\Big)-u_i'\Big(\Theta^r\left(B',y,B'\right)\Big)\Big\}\Bigg].$$

Here, the term

$$\Theta^r\left(B',y,\widetilde{B}\right) = \left(1-\kappa\left(\lambda(B',y),0\right)\right)\times m^r(B',y)\times\widetilde{B},$$

is the effective collateral service for both realizations of the preference shock, which takes into account the adjusted bond positions $\widetilde{b}_i(B')$ held by an i-type investor when entering the centralized market (see appendix C.1). The default probability is given by the one-period ahead default policy, weighted by the transition probability of exogenous revenue shocks

$$\lambda(B',y) = \mathbb{E}_{y'|y}\left[\mathcal{D}(B',y')\right].$$

The functions $\mathcal{Q}^r(\cdot)$ and $\mathcal{Q}^d(\cdot)$ determine the equilibrium bond prices in the next period for the repayment and default case, respectively, and are determined in equilibrium. The equilibrium bond price in the repayment case is obtained from evaluating the price schedule at the debt policy function $\mathcal{B}(\cdot)$

$$\mathcal{Q}^r(B,y) = q\left(\mathcal{B}(B,y),y\right). \tag{4.4}$$

In the default (and autarky) case, debt service is suspended and the debt position is rolled over, such that the associated equilibrium bond price is given as

$$\mathcal{Q}^d(B,y) = \frac{1}{1+r^{rf}}\left(m^d(B,y)(1+\Lambda^d(B,y))\right), \tag{4.5}$$

where the pecuniary benefits now are given by

$$m^d(B',y) = \vartheta\omega m^r(\omega B',y) + \mathbb{E}_{y'|y}\left[(1-\vartheta)\mathcal{Q}^d(B',y')\right],$$

and the non-pecuniary ones by

$$\Lambda^d(B',y) = (1-\kappa(1,0)) \times \mathbb{E}_i \Bigg[u_i'(\Theta^d(B',y,B')) + \frac{1}{2} \times \left\{ u_i'(\Theta^d\left(B',y,\widetilde{b}_i(B')\right)) - u_i'(\Theta^d(B',y,B')\right\} \Bigg],$$

with collateral services

$$\Theta^d\left(B',y,\widetilde{B}\right) = (1-\kappa(1,0)) \times m^d(B',y) \times \widetilde{B}.$$

The government's problem can be written recursively. When not in autarky, the government decision problem is given by

$$\mathcal{F}(B,y) = \max_{d\in\{0,1\}} (1-d)\mathcal{F}^r(B,y) + d\mathcal{F}^d(B,y), \tag{4.6}$$

where the value of repayment $\mathcal{F}^r(B,y)$ satisfies the Bellman equation

$$\mathcal{F}^r(B,y) = \max_{B'} v\left(\tau y + q(B',y)\left(B'-(1-\delta)B\right) - \widetilde{\delta}B\right) + \beta\mathbb{E}_{y'|y}\left[\mathcal{F}(B',y')\right], \tag{4.7}$$

and the value of default (and autarky) $\mathcal{F}^d(B,y)$ satisfies

$$\mathcal{F}^d(B,y) = v(\tau y) - \phi(\tau y) + \beta\,\mathbb{E}_{y'|y}\left[\vartheta\mathcal{F}(\omega B,y') + (1-\vartheta)\mathcal{F}^d(B,y')\right]. \tag{4.8}$$

Equilibrium. A Markov-perfect equilibrium consists of value, policy and bond price functions $\{\mathcal{F}(\cdot), \mathcal{F}^r(\cdot), \mathcal{F}^d(\cdot), \mathcal{B}(\cdot), \mathcal{D}(\cdot), q(\cdot), \mathcal{Q}^r(\cdot), \mathcal{Q}^d(\cdot)\}$, such that

(i) $\mathcal{F}(B,y)$, $\mathcal{F}^r(B,y)$, and $\mathcal{F}^d(B,y)$ satisfy eqs. (4.6) to (4.8),

(ii) $\mathcal{B}(B,y)$ and $\mathcal{D}(B,y)$ solve eqs. (4.6) to (4.8),

(iii) $q(B,y)$, $\mathcal{Q}^r(B,y)$ and $\mathcal{Q}^d(B,y)$ satisfy eqs. (4.3) to (4.5).

Having defined the model equilibrium, it is now possible to price a derivative security that will serve as the model equivalent of credit default swaps and obtain expressions for model-implied bid-ask spreads. A CDS can be mapped into our model by removing convenience yield from the pricing condition (4.3):

$$q^{CDS}(B',y) = \frac{1}{1+r^{rf}}\mathbb{E}_{y'|y}\left[\begin{array}{c}(1-\mathcal{D}(B',y'))\left(\widetilde{\delta}+(1-\delta)\mathcal{Q}^{CDS,r}(B',y')\right)\\ +\mathcal{D}(B',y')\mathcal{Q}^{CDS,d}(B',y'),\end{array}\right] \tag{4.9}$$

To get the CDS-price that is consistent with equilibrium, the CDS pricing conditions are evaluated at the debt policy arising from trading on markets for bonds that provide collateral services, since these are relevant for the borrower's debt decision:

$$\mathcal{Q}^{CDS,r}(B,y) = q^{CDS}\left(\mathcal{B}(B,y),y\right), \tag{4.10}$$

where $\mathcal{B}(\cdot)$ denotes the debt policy function for the government. In the default (and autarky) case, debt service is suspended and the debt position is rolled over, such that the associated equilibrium CDS price is given as

$$\mathcal{Q}^{CDS,d}(B,y) = \vartheta\omega\mathcal{Q}^{CDS,r}(\omega B,y) + \frac{1}{1+r^{rf}}\mathbb{E}_{y'|y}\left[(1-\vartheta)\mathcal{Q}^{CDS,d}(B,y')\right]. \tag{4.11}$$

The crucial difference to the bond pricing expressions eqs. (4.4) and (4.5) is the absence of collateral service, such that the bond will trade at a lower price and a higher spread vis-a-vis the risk-free rate whenever collateral service is positive. The bid-ask spread as a measure of illiquidity is given by

$$ba^j(B,y) = \frac{1}{2}\Big[\frac{u_H(\Theta_H^j(B,y,\widetilde{b}_H(B))) - u_H(\Theta^j(B,y,B))}{\widetilde{b}_H(B)-B} - \frac{u_L(\Theta_L^j(B,y,\widetilde{b}_L(B))) - u_L(\Theta^j(B,y,B))}{\widetilde{b}_L(B)-B}\Big],$$

for $j \in \{r,d\}$.

4.3 Calibration

In this section, we present a calibration of our model to Italian data from 2001Q1-2012Q4, such that one period corresponds to one quarter. By choosing this truncation point, we exclude periods with unconventional monetary policy measures of the ECB, since these are difficult to address in our framework. The data are compiled from various sources; a complete list can be found in Table C.1. We solve the model numerically using value function iteration over a discretized state space. The computational algorithm is described in detail in appendix C.3.

4.3.1 Functional Forms and Parameter Choices

Government. For the income process, we impose a log-normal AR(1) specification:

$$\log y_t = \rho_y \log y_{t-1} + \sigma_y \epsilon_{y,t}\ , \quad \epsilon_{y,t} \sim N(0,1) \tag{4.12}$$

We estimate the income process parameters using Italian GDP data for 1991Q1-2012Q4. All data is in real terms, seasonally adjusted and de-trended using a linear-quadratic trend. Shocks are discretized as proposed by Tauchen (1986). For the period utility function of the government, we assume a CRRA specification

$$v(g_t) = \frac{(g_t - \underline{g})^{1-\gamma} - 1}{1-\gamma}.$$

Here, $\underline{g}$ denotes a subsistence level of government spending as in Bocola et al. (2019). We set the government's risk aversion to $\gamma = 2$, which is in line with similar models, such as Arellano

(2008), Chatterjee and Eyigungor (2012) and Hatchondo et al. (2016). In order to calibrate default risk dynamics, we use the same utility cost function as in Bianchi et al. (2018):

$$\phi(y) = \max\{d_0 + d_1 \log(y), 0\} \; .$$

Bond Structure. The maturity parameter δ is chosen such that - without default - debt has an average maturity of 5 years, which corresponds to our empirical specification. Although the average life of Italian bonds slightly exceeds five years during most parts of the sample, using this maturity, has several advantages: the benchmark bonds actually issued by the Italian Treasury have a maturity of 5 years at issuance, bid-ask spreads are computed using the on-the-run 5-year-bond and CDS contracted over 5 years are the most commonly traded credit derivative. Fixing the maturity parameter to on-the-run bonds allows us to easily obtain a plausible value for the coupon parameter. We collect coupon rates of all issues of 5-year-BTP bonds between 2001 and 2012 from the Italian Treasury Department. The (value-weighted) coupon was 4.62, which translates into a quarterly coupon rate of 1.15. The probability for a country in default to receive an debt restructuring offer ϑ and the corresponding recovery rate ω are in the range identified by Cruces and Trebesch (2013).

Investor Preferences. Period utility of investors is specified by the CARA functions

$$u_L(\theta_t) = -\zeta_1 \exp\{-\theta_t\},$$
$$u_H(\theta_t) = -\zeta_1 \exp\{-(\theta_t - \zeta_2)\},$$

with $\zeta_1, \zeta_2 > 0$. Bond holdings of high- and low-valuation investor types are then given by

$$\theta_{L,t} = (1-\kappa_t) m_t B_{t-1} - \frac{1}{2}\zeta_2 \; ,$$
$$\theta_{H,t} = (1-\kappa_t) m_t B_{t-1} + \frac{1}{2}\zeta_2 \; .$$

It follows that the wedge between bond and CDS prices is given by

$$\Lambda_t = (1-\kappa_t)\zeta_1 \exp\{\zeta_2 - (1-\kappa_t) m_t B_{t-1}\} \; ,$$

which increases in ζ_1 and ζ_2 and decreases in the amount of available collateral $(1-\kappa_t)m_t B_{t-1}$. The size of convenience yield is then governed by ζ_1, while ζ_2 can be used to control the trading motive between high- and low-valuation investors. The discount (real) quarterly discount rate of investors is proxied by the average 3-month-EURIBOR over quarterly inflation as measured by HCPI growth rates. Consistent with this proxy, we use the EURIBOR-swap rate with corresponding maturity as risk-free reference rate to compute government bond spreads in the data.

Haircut Function. For the haircut function $\kappa(\lambda, h)$ we use a simple exponential functional form (see Bindseil, 2014) to capture the negative relationship between the haircut value and default

Table 4.1: External Parameters

Parameter	Value	Source
Autocorrelation ρ_y	0.937	Estimate of eq. (4.12)
Variance σ_y^2	8.45e-05	Estimate of eq. (4.12)
Government risk aversion γ	2	Conventional value
Income tax rate τ	1	Normalization
Investor discount rate r^{rf}	0.0013	3-month-EURIBOR minus inflation
Coupon parameter χ	0.0115	Average coupon of 5-year treasury bonds
Maturity parameter δ	0.05	5-year average maturity
Recovery rate ω	0.63	Cruces and Trebesch (2013)
Offer probability ϑ	0.08	Cruces and Trebesch (2013)
Default risk propagation μ	0.2	Bindseil (2014)
Maximum haircut $\overline{\kappa}$	0.4	Nyborg (2017)

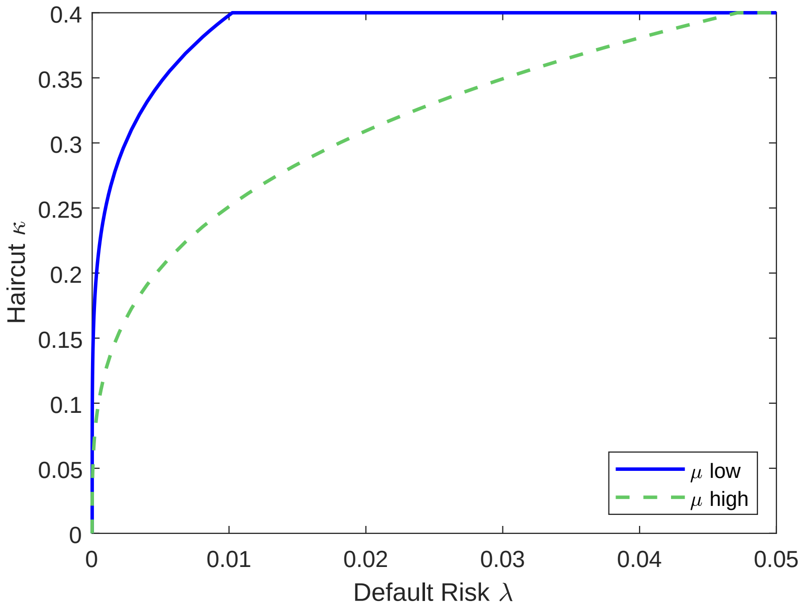
risk

$$\kappa(\lambda, h) = \begin{cases} \min\{\lambda^{\mu}, \overline{\kappa}\}, & \text{if} \quad h = 1 \ , \\ \overline{\kappa}, & \text{if} \quad h = 0 \ , \end{cases} \tag{4.13}$$

where λ is the risk-neutral default probability. The sensitivity parameter is restricted to $\mu \in (0, 1]$, i.e., haircut schedules punish default risk, which is in line with collateral frameworks on the private market and implemented by central banks. Here, a larger μ is associated with a high sensitivity of haircuts to default risk, as we show in Figure 4.2. The solid blue line corresponds to our baseline calibration, where we set $\mu = 0.2$, following Bindseil (2014). Increasing this parameter implies that haircuts are less responsive to default risk, as demonstrated by the dashed green line in Figure 4.2. We will later vary this parameter to investigate its importance for the model's predictions. We apply a ceiling to haircuts, which is set to equal to the extraordinary haircut applied to distressed Cypriot and Greek government debt (see Nyborg, 2017). In the case of default or autarky ($h_t = 0$), the default probability is zero and the haircut is given by $\overline{\kappa}$. The cap is binding around 1.4% of all periods in which the government is not in autarky. Our quantitative results do not depend critically on $\overline{\kappa}$. In Table 4.1, we summarize the parameters that are set based on external sources.

Free Parameters. We chose the remaining seven parameters $(\beta, d_0, d_1, \underline{g}, \zeta_1, \zeta_2)$ to match key moments of financial market variables observed between 2001Q1-2012Q4. The government's discount factor β, subsistence consumption $\underline{g}$, and the output loss parameters d_0 and d_1 are directly linked to borrowing and default incentives. The subsistence consumption level introduces a right-skewed distribution of default risk and countercyclical fiscal policy. We target the average and volatility of the government debt level and CDS spreads. These mechanics are well-known in quantitative sovereign default models, we refer to Chatterjee and Eyigungor (2012) and Bocola et al. (2019) for a detailed discussion and turn to the parameters that shape

Figure 4.2: Haircut Function



convenience yield, illiquidity risk and their interaction with default risk. ζ_1 is chosen to match the average government bonds spread over the total sample, which was 30 basis points lower than the CDS spread. The preference shifter ζ_2 is chosen to match the mean bid-ask spread. Our calibration is summarized in Table 4.2.

4.3.2 Model Fit

First, we show the most relevant financial market variables in this context, which are the levels and volatilities of government bonds-, CDS-, and bid-ask-spreads. We then demonstrate that the model is capable of generating countercyclical debt issuance and finally provide evidence that the central result regarding the interaction of bond supply and bond spreads of Krishnamurthy and Vissing-Jorgensen (2012) prevails even in the presence of default risk, complementing the findings of Jiang et al. (2021).

All model-implied statistics are based on simulations of 50,000 periods after a burn-in period with length of 5,000. We exclude all periods where the government is in financial autarky as well as 40 quarters after re-entering financial markets following an exclusion period.

Financial Markets. Results for targeted moments are reported in the left panel of Table 4.3. The level of debt and all three spreads are matched, whereas the volatility targets show some discrepancy with the data, which is difficult to overcome with our risk-neutral pricing setting. To examine the interaction between convenience yield and credit risk in greater detail, we also report statistics for all periods with a positive bond spread, i.e., in times where the credit risk

Table 4.2: Calibrated Parameters

Parameter	Value	Target
Government discount Factor β	0.969	Average debt/GDP
Default cost Parameter d_0	23.5596	Average CDS spread
Default cost Parameter d_1	50	Volatility of bond spread
Minimum consumption $\underline{g}$	0.86	Volatility of debt/GDP
Collateral service weight ζ_1	0.05	Average bond spread
Collateral service shifter ζ_2	0.0003	Average bid-ask spread

Table 4.3: Model Fit: Financial Market Variables

Variable	Full Sample		Crisis Episodes	
	Data	Model	Data	Model
ave(s_t)	48	50	161	175
ave(cds_t)	78	77	204	213
ave(log(Q_tB_t/y_t))	1.50	1.52	1.58	1.58
ave(ba_t)	11	12	29	12
sd(s_t)	114	160	143	249
sd(cds_t)	113	177	117	278
sd(log(Q_tB_t/y_t))	0.058	0.066	0.032	0.033
sd(ba_t)	16	5	17	8

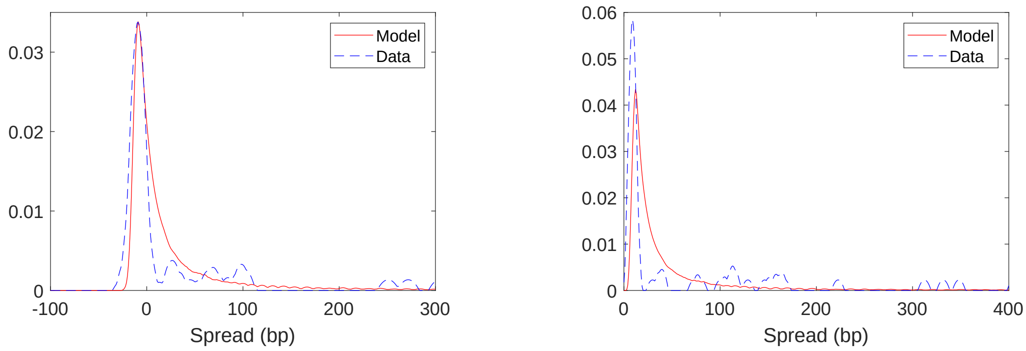
Notes: Spreads are annualized and in basis points. Crisis episodes are all periods with a positive government bond spread. Targeted moments are in color.

component of bond spreads dominates the convenience yield component. We refer to this sample as crisis times. In the our data sample, the bond spread was negative from 2001Q1 until 2008Q4 and turned positive thereafter. In particular, the rise in debt levels associated with the financial crisis is captured by our model. Since our model only features a single exogenous shock as well as risk-neutral bond pricing, the increase of all spreads, bid-ask spreads in particular, is less pronounced than observed in the data.

We also plot kernel density estimations of bond- and CDS spreads obtained from our model against their data counterparts in Figure 4.3. Both spreads show sizable positive skewness and considerable probability mass at negative bond spreads, consistent with the data.

Debt Management. Debt management in developed economies is countercyclical in the sense that the government increases borrowing in response to negative fiscal shocks. Note that such behavior is typically not present in standard models of sovereign debt and default (see Arellano, 2008). The reason for this is that increases in debt severely lower bond prices in bad times due to increased risk of default, which typically incentivizes the government to lower its debt

Figure 4.3: Model Fit: Distribution of Spreads



(a) Bond Spread over Risk-Free Rate (b) CDS Spread over Risk-Free Rate

Table 4.4: Model Fit: Debt Management

Variable	Full Sample		Crisis Episodes	
	Data	Model	Data	Model
ave($Q_{t+1}B_t/y_t$)	4.500	4.584	4.847	4.866
ave($Q_{t+1}(B_{t+1}-B_t)/y_t$)	0.029	0.010	0.041	0.029
sd($Q_{t+1}B_t/y_t$)	0.267	0.298	0.158	0.159
sd($Q_{t+1}(B_{t+1}-B_t)/y_t$)	0.049	0.050	0.051	0.042
cor($Q_{t+1}(B_{t+1}-B_t)/y_t, y_t$)	-0.216	-0.532	-0.293	-0.610

Notes: Spreads are annualized and in basis points. Crisis episodes are all periods with a positive government bond spread.

issuance. However, as in Bocola et al. (2019), the inclusion of a minimum consumption level makes the government less responsive to debt-elastic interest rate hikes in such instances, such that countercyclical debt and default risk can arise simultaneously. Mechanically, this works as follows: in bad (low revenue) states, bond prices tend to move downward. Although borrowing is countercyclical and thus collateral becomes more abundant in principle, the increase in nominal debt outstanding is dominated by default risk, especially because the haircut also increases in this case. The total effect results in a negative co-movement between borrowing and bond spreads, which is in line with the data. The model is able to generate sizable negative correlation between government revenues and debt issuance.

Decomposing Government Bond Spreads. Finally, we examine how the pricing of different risk factors is reflected in bond prices during times of crisis, corresponding to the high-risk episode of our sample. Therefore, we regress the key endogenous variable from our model, the spread of 5-year Italian government bonds, on potential drivers of the spread, namely credit risk (measured by the CDS spread), market illiquidity (measured by bid-ask spreads or turnover) and convenience yield (measured by the supply of total or long-term government bonds). Formally,

Table 4.5: Determinants of Government Bond Spreads

	Data				Model
Debt proxy	Total Debt		Long-term Debt		Total debt
Illiquidity proxy	Bid-ask	Turnover	Bid-ask	Turnover	Bid-ask
Constant	-2747	456	-1392	-1569	-70
	(254)***	(457)***	(266)***	(155)***	(1.216)***
CDS spread	0.710	0.992	0.419	0.812	0.911
	(0.132)***	(0.098)***	(0.117)**	(0.063)***	(2e-04)***
Debt proxy	1727	1328	1080	1025	0.12
	(166)***	(333)***	(214)***	(157)***	(0.002)***
Illiquidity proxy	1.36	29.6	0.38	25.7	-0.397
	(0.877)	(28.3)	(0.822)	(27)*	(0.004)***

Notes: Newey-West standard errors in parenthesis. Significance levels at 10 % (*), 1 % (**) and 0.1 % (***).

we run the regression

$$s_t = \beta_0 + \beta_1 cds_t + \beta_2 \frac{\mathcal{Q}_t B_t}{y_t} + \beta_3 ba_t + \epsilon_t \ . \tag{4.14}$$

Market illiquidity measures refer to 5-year Italian government bonds, traded on the Milan stock exchange. Results are shown in Table 4.5. Despite the small sample size, all four regression specifications draw a conclusive picture: while credit risk was the dominant driver, bond supply has a strongly positive effect on bond spreads, which is highly significant even after controlling for market illiquidity proxies, consistent with the results of Jiang et al. (2021). Market illiquidity seems to have played a minor role as the coefficients on all liquidity measures are insignificant and inconclusive in their sign. While higher bid-ask spreads translate into higher bond spreads, the coefficient on relative turnover switches signs, depending on the regression specification.

We run the same regression on the crisis periods of the simulated time series implied by our model. Since there is only one persistent exogenous shock, tax revenues, and the i.i.d. taste shock, all coefficients are highly significant, but capture a significant effect of bond supply on spreads: Higher bond supply translates into higher bond spreads after controlling for credit risk, i.e., convenience yield declines when bonds become less scarce, which confirms the findings of Krishnamurthy and Vissing-Jorgensen (2012) for a setting with risky government bonds and complements the results of Jiang et al. (2021).

4.4 Model Mechanics: A Quantitative Exploration

With our calibration at hand, this section examines the impact of convenience yield on financial market variables and the conduct of public debt management. We proceed in two steps. First, we present a recalibration of the model without convenience yield and without bid-ask spreads, and compare the dynamics of public debt and default risk to the baseline calibration. These experiments suggest that convenience yield has only small effects on debt and default dynamics, while market illiquidity has no visible effects at all. This is consistent with the determinants of bond spreads in Table 4.5.

As a second experiment, we separately examine both components of convenience yield: collateral valuation and haircuts. Specifically, we conduct a comparative statics exercise *(i)* regarding the elasticity of collateral valuation with respect to bond supply and *(ii)* regarding the haircut parameter μ, which governs the pass-through of default risk to collateral services. While the size of convenience yield and bid-ask spreads does not affect the shape of the bond pricing schedule, the shapes of collateral valuation and haircut schedules have sizable effects on debt and default incentives – precisely because they affect the shape of the bond price schedule which matters more for public debt management.

4.4.1 The Role of Convenience Yield and Market Illiquidity

We start by setting $\zeta_1 = 0$, which eliminates convenience yield and – as a by-product – bid-ask spreads. We then recalibrate the model to match the same average debt-to-GDP ratio as in the baseline calibration. For the baseline calibration, eliminating convenience yield would imply a lower average debt-to-GDP ratio. To match the target for the debt-to-GDP ratio therefore requires the government to be more impatient relative to the baseline case. To counter the positive impact of higher impatience on the incentive to default, the recalibrated default cost parameter d_0 needs to be increased to match our target. The resulting adjustment requires setting $\beta = 0.963$ and $d_0 = 26.6113$.

The results are displayed in the right column of Table 4.6. In contrast to the baseline calibration, the government bond spread mechanically increases to the level of the CDS spread. This feature stresses our baseline model's ability to reconcile sovereign risk and very low government bond spreads. Compared to the baseline calibration, the volatility of financial market variables does not change much. Similarly, the effect of convenience yield on government debt management is also very small, as shown in Table 4.7.

As the next step, we consider a recalibration of the model with $\zeta_2 = 0$ to isolate the role of bid-ask spreads. In this case, there is no trading motive for investors, such that the valuation equation (4.2) reduces to

$$\Lambda(B_t, y_t) = (1 - \kappa(\lambda_t, y_t)) \times u'(\theta_{t+1}) ,$$

which is smaller in size relative to the baseline case. Therefore, we set $\zeta_1 = 0.06$ and $d_0 = 23.5962$ to match average bond- and CDS spreads. As can be seen in the last column of Table 4.7, the moments for the model without market illiquidity do not differ much relative to the baseline case.

To understand why the level of convenience yield and market illiquidity hardly affect the model moments, it is helpful to take a look at the bond spread as a function of debt for $\zeta_1 = 0$ (dashed green line), $\zeta_2 = 0$ (dotted purple line) and the baseline case (solid blue line) in Figure 4.4.

Table 4.6: Financial Variables without Convenience Yield and Market Illiquidity

Variable	Baseline	No Convenience Yield ($\zeta_1 = 0$)	No Market Illiquidity ($\zeta_2 = 0$)
ave(s_t)	50	76	47
ave(cds_t)	77	76	78
ave($\log(Q_t B_t / y_t)$)	1.52	1.51	1.53
sd(s_t)	160	178	161
sd(cds_t)	177	178	182
sd($\log(Q_t B_t / y_t)$)	0.066	0.063	0.068

Notes: Spreads are annualized and in basis points.

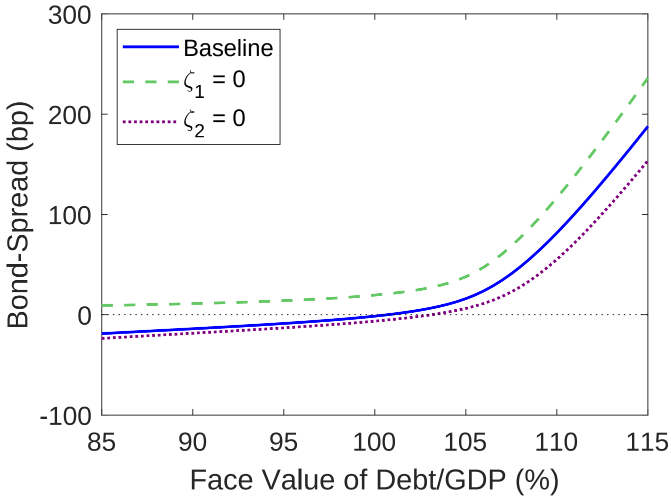
Table 4.7: Debt Management without Convenience Yield and Market Illiquidity

Variable	Baseline	No Convenience Yield ($\zeta_1 = 0$)	No Market Illiquidity ($\zeta_2 = 0$)
ave($Q_{t+1} B_t / y_t$)	4.584	4.537	4.641
ave($Q_{t+1}(B_{t+1} - B_t)/y_t$)	0.010	0.009	0.011
sd($Q_{t+1} B_t / y_t$)	0.298	0.281	0.308
sd($Q_{t+1}(B_{t+1} - B_t)/y_t$)	0.050	0.047	0.050
cor($Q_{t+1}(B_{t+1} - B_t)/y_t, y_t$)	-0.532	-0.498	-0.532

Consistent with the discussion so far, the spread is higher for $\zeta_1 = 0$ compared to the baseline case and slightly lower for $\zeta_2 = 0$. Importantly, the difference between the lines is rather invariant to the level of debt, implying that the debt-elasticity of borrowing conditions is almost the same for the three cases. Given that it is however this debt-elasticity that is important for the way public debt is managed, the respective statistics hardly differ between the three scenarios considered in this section.

An important feature highlighted by Figure 4.4 for the baseline case, which standard sovereign default models cannot address by construction, is that the bond spread is negative when collateral is scarce (and thus convenience yield is high). In regions with low debt and in the absence of default risk, the bond spread is still increasing with debt issuance however, since each additional unit of debt makes collateral less scarce. Somewhere in the relevant region of the debt state space (a debt-to-GDP ratio between 100 and 115 percent), default risk and the associated negative effect on bond prices start to dominate the pricing of sovereign bonds. The bond spread turns positive and becomes more sensitive to debt compared to the risk-free borrowing region.

Figure 4.4: Convenience Yield and Market Illiquidity: Bond Spread



Notes: The face value of debt is in terms of end-of-period debt. Government revenues are at their median level.

4.4.2 The Role of Haircuts and Collateral Scarcity

The previous section suggests that convenience yield per se hardly has an effect on default risk as measured by average CDS spread, and only a small impact on the conduct of public debt management. In this section, we show that this observation depends on how collateral valuation and haircuts react to bond issuance. There are two effects of collateral scarcity and the haircut sensitivity, with opposing influence on sovereign risk. *First*, the amount of collateral service provided by a risky bond is less sensitive to default risk, implying a flatter bond price schedule in the risky borrowing region. Therefore, the government will experience less frequent debt crises ex-post, since roll-over becomes easier if there is a negative shock to fiscal revenues. *Second*, a relatively flat bond pricing schedule incentivizes the government to issue more bonds in the risky borrowing region, since the revenues generated by issuing an additional unit of bonds only weakly respond to debt issuance. This increases default risk and spreads in equilibrium.

We evaluate the relative importance of these two effects by solving our model for different values of μ and a different specification for collateral valuation. The first panel of Tables 4.8 and 4.9 reports the results for the baseline specification. We also recalibrate the model under the assumption of a constant marginal utility of collateral service, such that collateral valuation is given by

$$\Lambda(B_t, y_t) = (1 - \kappa(\lambda_t, y_t)) \ \times \ \zeta_1 \ .$$

This specification implies that collateral valuation introduces to a constant wedge in the bond pricing condition (4.1) and that there are no collateral scarcity effects. Note that convenience yield still negatively depends on bond supply through the haircut function. We again recalibrate

Table 4.8: Financial Variables, Different Haircut Schedules

Variable	$\mu = 0.1$	$\mu = 0.2$	$\mu = 1$
With collateral scarcity			
ave(s_t)	53	50	44
ave(cds_t)	80	77	71
sd(s_t)	164	160	155
sd(cds_t)	183	177	171
Default rate (%)	1.67	1.55	1.41
No collateral scarcity			
ave(s_t)	51	50	54
ave(cds_t)	83	83	87
sd(s_t)	185	185	188
sd(cds_t)	189	190	193
Default rate (%)	1.75	1.76	1.82

Notes: Effects of varying the haircut parameter μ. A low μ corresponds to a highly elastic haircut schedule (see also Figure 4.2). Spreads are annualized and in basis points.

the model ($\zeta_1 = 0.07$ and $d_0 = 23.3$) to ensure that average CDS spread and debt-to-GDP ratio remain consistent with the data moments. We report the results in the lower panel of Tables 4.8 and 4.9.

The upper panel of Table 4.8 shows that both the level and volatility of spreads decrease with μ. The reason for this observation lies in the responsiveness of collateral valuation to the supply of government bonds. If a highly elastic collateral valuation is combined with a highly elastic haircut schedule, the disciplining effect of a more debt-elastic bond price schedule is dominated by the increased vulnerability to default when hit by a bad income shock. The impact of haircut schedules is reversed in the lower panel of Table 4.8, where less elastic haircut schedules (with a higher μ) are associated with higher default rates and spreads. In this setting, the disciplining effect of stricter haircut schedules dominates. As Table 4.9 reveals, the combination of these effects changes the government's ability to conduct a countercyclical debt policy, as measured by the correlation of net debt issuance $Q_{t+1}(B_{t+1} - B_t)/y_t$ with income y_t.

Taken together, our experiments indicate that the *composition* of effective convenience yield matters for sovereign risk and the conduct of fiscal policy. Altering the responsiveness of haircut schedules can change CDS spread level and volatility by up to 10 basis points and default rates by up to 0.25 percentage points. These effects are quantitatively relevant, given that average government bond spreads are around 50 basis points and the default rate around 1.5% in our baseline calibration. The results furthermore suggest that the design of haircut schedules (either on public or private market segments) could benefit from taking into account the nature of collateral demand.

Table 4.9: Debt Management, Different Haircut Schedules

Variable	$\mu = 0.1$	$\mu = 0.2$	$\mu = 1$
With collateral scarcity			
ave$(Q_{t+1}B_t/y_t)$	4.462	4.462	4.459
ave$(Q_{t+1}(B_{t+1} - B_t)/y_t)$	0.010	0.010	0.011
sd$(Q_{t+1}B_t/y_t)$	0.281	0.279	0.276
sd$(Q_{t+1}(B_{t+1} - B_t)/y_t)$	0.050	0.050	0.050
cor$(Q_{t+1}(B_{t+1} - B_t)/y_t, y_t)$	-0.493	-0.495	-0.508
No collateral scarcity			
ave$(Q_{t+1}B_t/y_t)$	4.586	4.584	4.591
ave$(Q_{t+1}(B_{t+1} - B_t)/y_t)$	0.010	0.010	0.009
sd$(Q_{t+1}B_t/y_t)$	0.305	0.298	0.302
sd$(Q_{t+1}(B_{t+1} - B_t)/y_t)$	0.050	0.050	0.049
cor$(Q_{t+1}(B_{t+1} - B_t)/y_t, y_t)$	-0.534	-0.510	-0.510

Notes: Effects of varying the haircut parameter μ. A low μ corresponds to a highly elastic haircut schedule (see also Figure 4.2).

4.5 Conclusion

In this paper, we have studied how convenience yield, sovereign default risk, and the supply of government bonds interact through the lenses of a quantitative macroeconomic model. Specifically, we have modified an otherwise standard quantitative sovereign default model by allowing for convenience yield as well as endogenous market illiquidity discounts, which both interact with the government's risk of default and debt management. Calibrating the model to Italy, we showed that, despite its parsimonious structure, our model can generate the basic observed properties of sovereign debt, credit risk, sovereign bond spreads, credit default swap spreads and bid-ask spreads. To understand the role of convenience yield in the presence of default risk, we provide a decomposition of convenience yield into individual components. It suggests that the elasticity of collateral valuation and haircut schedules applied to collateral with respect to government bond supply can have sizable effects on debt and default dynamics.

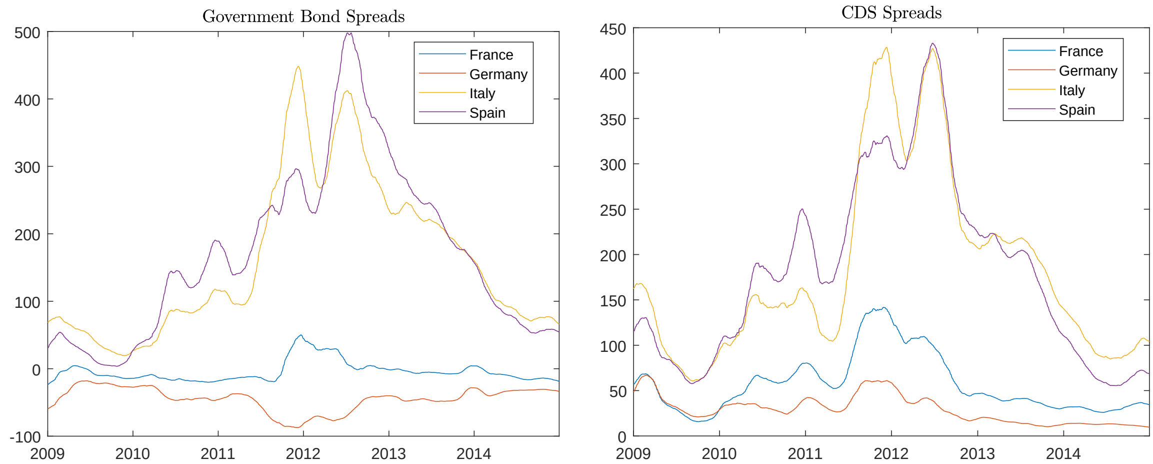
5 Flight-to-Quality via the Repo Market

Author: Matthias Kaldorf

5.1 Introduction

The European debt crisis of 2011 has been characterized by an unprecedented divergence in borrowing costs for euro area members. The left panel of Figure 5.1 depicts government bond spreads - computed over default risk-free interest rate swaps - around the European debt crisis for the four largest sovereign borrowers. While the bond spreads of France, Italy, and Spain exhibit substantial increases relative to their pre-crisis level, German bond spreads declined to around -100bp. At the same time, credit default swaps (CDS) show a positive co-movement, as shown in the right panel of Figure 5.1: similar to French, Italian, and Spanish CDS-spreads, the German CDS-spread also strongly increased in the fall of 2011, peaking at almost 80pb. This suggests that investors (1) required a higher default risk compensation on *all* euro area government bonds, including the German bund, but (2) were still willing to pay lower yields to hold the German bunds.

Figure 5.1: Government bond- and CDS-spreads, 5 year maturity, rolling means



Notes: Spreads are computed with respect to interest rate swaps of corresponding maturity. All values are 60 business day rolling means and expressed in basis points. Source: *Thomson Reuters Datastream.*

To reconcile this puzzling observation, this paper proposes a model of a monetary union with heterogeneous governments (countries) receiving random tax revenues and issuing long-term bonds to competitive investors. There are five key assumptions. *First*, governments issue bonds subject to default risk, which implies that CDS-spreads for all countries are strictly positive.[1] *Second*, there is no fiscal coordination among union members: country-specific shocks to fiscal revenues are not optimally shared among union members and default risk is heterogeneously distributed across countries. *Third*, investors use government bonds to collateralize repos, either on the interbank market or with the central bank: government bond spreads contain a default risk component and a collateral premium, such that CDS-spreads are higher than corresponding bond spreads for each country.[2] *Fourth*, the monetary union is financially integrated in the sense that investors can use all government bonds as collateral equally well. Collateral premia therefore depend negatively on *aggregate collateral supply*, which will be defined as the haircut-adjusted market value of all government bonds outstanding. *Fifth*, the collateral premium on each government bond declines in its default risk, since private repo market participants and the central bank specify haircuts that negatively depend on sovereign credit ratings, see Bindseil and Papadia (2006), Nyborg (2017), and Orphanides (2017) for a discussion of central bank collateral frameworks and Nguyen (2020), Fontana and Scheicher (2016), and Jiang et al. (2021) for empirical evidence.

The cross-sectional distribution of sovereign risk, government bond spreads, and haircuts is jointly determined with aggregate collateral supply in equilibrium. An unanticipated negative *aggregate* shock to tax revenues increases sovereign default risk and CDS-spreads for all countries, consistent with the data.[3] Elevated sovereign default risk is also associated with rating downgrades. Holding central bank collateral policy constant, this mechanically increases haircuts on all government bonds: aggregate collateral supply contracts and collateral service becomes more valuable to investors. To see how such a contraction of aggregate collateral supply affects the cross-section of government bond spreads, it is helpful to decompose the change of collateral premia into a *collateral valuation* effect, reflecting the value that an additional unit of collateral provides to investors, and a *haircut* effect, which captures how much collateral service any specific government bond provides.

The haircut effect dominates for countries most affected by default risk, who exhibit a decline of collateral premia. Their government bond spreads increase due to higher default risk compensation *and* due to lower collateral premia. At the same time, the collateral valuation component dominates for countries that are only weakly affected by default risk and, therefore, subject to smaller haircuts. Even though their haircuts increase during a fiscal crisis, the collateral premium on their bonds increases, which reduces their government bond spreads. In a calibration to euro area data, I demonstrate that the model can quantitatively replicate the dynamics of haircuts, CDS-spreads and government bond spreads observed during the European

[1] Default risk on local currency debt is relevant in a monetary union, since governments can not use currency devaluation to decrease the real value of their debt service burden if monetary policy is delegated to a supranational central bank.

[2] At the peak of the European debt crisis, government bond spreads exceeded CDS-spreads for the riskiest countries. This has been attributed to market illiquidity, which does not affect CDS-spreads, but increases government bond spreads, see Passadore and Xu (2020). The interaction of default risk and collateral premia is robust to endogenizing market illiquidity as shown in Kaldorf and Roettger (2022). Therefore, this layer of complexity is omitted in this paper.

[3] The relevance of aggregate fiscal risk is supported empirically by Monfort and Renne (2013).

debt crisis. Simulating a panel of government bonds, I furthermore show that the model also performs reasonably well in reconciling the (untargeted) cross-sectional evidence on the effect of country-specific fiscal conditions on the non-pecuniary benefits of their government bonds, which is provided for euro area data by Jiang et al. (2021).

Using the calibrated model, I evaluate to which extent the ECB's collateral framework can be used to close the gap between yields on 'peripheral' and 'core' bonds during a fiscal crisis.[4] In addition to maintaining a sufficiently high collateral supply - which ensures smooth functioning of financial markets and monetary policy implementation - containing centrifugal forces threatening the currency union has been a major challenge for policymakers. This has been motivated on several different grounds: large spread heterogeneity can be costly at the monetary union level through contagion effects (Morelli et al., 2019), joint strategic default (Arellano et al., 2017), or direct spillover costs of sovereign default to other member states (Tirole, 2015).

In a fiscal crisis, the central bank can directly affect the cross-section of government bond yields through a *temporary* relaxation of collateral policy. Decreasing haircuts on all countries has a direct positive effect on collateral premia and reduces bond yields for all countries, ceteris paribus. At the same time, this relaxation increases aggregate collateral supply, making the collateral service of the safest bonds less valuable, such that their yields increase. Specifically, a *full collateral backstop* policy that accepts all bonds as collateral without haircuts most effectively reduces the cross-sectional dispersion of government bond spreads during a fiscal crisis.[5] This reduces overall sovereign risk in the monetary union during the crisis period, since it makes debt rollover easier for high-risk borrowers. Taken together, my results lend support to the ECB's temporary suspension of minimum rating requirements in April 2020 in response to the Covid-19 shock and the associated disruptions on financial markets.[6]

The full collateral backstop result is related to canonical lender-of-last-resort (LOLR) policy. Following the survey article by Freixas et al. (2000), central banks in their role as LOLR ensure that all investors with 'good collateral' are able to borrow from the central bank whenever alternative funding sources via the private sector are not available. The design of such policies trades off the benefits of LOLR-interventions, for example avoiding bank failure, contagion effects, and contractions in real sector lending, against its costs, for example due to moral hazard in the banking system and by fiscal policy.

When applied to the single-country case, where government bonds are issued in domestic currency and therefore typically are *nominally risk-free*, the LOLR-literature usually takes a sufficient aggregate supply of 'good collateral' for granted. However, canonical LOLR-policy might be infeasible in a monetary union during a fiscal crisis, because of an insufficient supply

[4] Notably, the ECB engaged in collateral easing policies prior to announcing large-scale central bank interventions on the government bond market. However, as demonstrated in D'Amico et al. (2018) for the US and Schlepper et al. (2018), Arrata et al. (2020), and Corradin and Maddaloni (2020) for the euro area, quantitative easing policies even increase high collateral valuation effects by taking government bonds out of the market, thereby potentially amplifying the flight-to-quality effect studied in this paper.

[5] Throughout the paper, I use the term *collateral backstop* to make explicit the distinction to an *outright backstop* policy that buys a large amount of distressed government bonds on secondary markets, which is discussed in Corsetti and Dedola (2016).

[6] The ECB press release can be accessed via this link.

of 'good collateral'. LOLR-policy therefore has to ensure that fundamentally solvent, but illiquid investors are able to tap liquidity facilities, even if the collateral value of government bond declines. The central bank acts similar to an *investor*-of-last-resort (Buiter and Rahbari, 2012). However, in contrast to using outright government bond purchases to reduce heterogeneity of government borrowing costs (Canofari et al., 2018), which introduces a non-trivial risk management consideration to the central bank's policy problem (Caballero et al., 2020), a full collateral backstop does not transfer risk to the central bank balance sheet, since collateral frameworks apply to central bank *repurchase agreements*. Therefore, a full collateral backstop is partially successful at carrying out investor- and lender-of-last-resort policies simultaneously.

It should be noted that, throughout my analysis, I maintain the assumption that both the aggregate fiscal shock and the full collateral backstop are unanticipated. This effectively eliminates moral hazard and time-consistency issues in both the governments' and the central bank's policy problem. The effects of government bond haircuts on fiscal policy under sovereign are discussed in Kaldorf and Roettger (2022), but can reasonably assumed to be less relevant during a flight-to-quality episode.

Related Literature I draw on three distinct strands of literature. The collateral channel around which this paper is centered relates to a group of papers studying the interactions of non-pecuniary benefits and sovereign risk in advanced economies. The negative effect of non-pecuniary benefits on government bonds yields has been documented in Jiang et al. (2021). This divergence in borrowing costs during a fiscal crisis negatively affecting all euro area members has been associated with a 'flight-to-liquidity' effect by De Santis (2014). I contribute to this strand of literature by providing an equilibrium characterization that can be used to study counterfactual policies. My model also builds on Bolton and Jeanne (2011), who study convenience yield in a setting with high and low risk countries borrowing from a representative investor under perfect financial integration. Luque (2021) proposes a repo market model with heterogeneous banks using low-risk and high-risk sovereign borrowers as collateral and shows that flight-to-liquidity can arise if high-risk bonds lose collateral eligibility. Auray et al. (2018) propose a two-country model with an interbank market that accounts for sovereign-bank doom loops but abstracts from collateral premia. Their model also shows that unconventional central bank interventions can improve welfare during a crisis. All three papers assume that low risk borrowers issue bonds free of default risk, which is at odds with the high German CDS-spread observed in 2011.

From a methodological perspective, I relate to a group of papers studying sovereign default in monetary union, see for example Arellano et al. (2017), Costain et al. (2021), and De Ferra and Romei (2021). In contrast to these papers, rather than having two borrower types ('core' and 'periphery'), I consider a *continuum of borrower types* following Le Grand and Ragot (2021). Different to their setup, I follow Jiang et al. (2021) in assuming that governments trade with investors rather than among each other. This assumption facilitates a straightforward decomposition of government bond spreads into default risk compensation and collateral premia and also eliminates the multiplicity around which the discussion in Le Grand and Ragot (2021) is centered.

From a policy perspective, the central bank's role as investor/lender-of-last-resort in the presence of sovereign risk has also received considerable attention in recent years. Corsetti and Dedola (2016) and Reis (2017) focus on unconventional central bank policies, such as outright purchases of distressed government bonds. This literature typically operates in a multiple-equilibria setting (see also Lorenzoni and Werning, 2019), where LOLR policies help to coordinate on the low-debt and low-default equilibrium with higher welfare. Closest to my approach are Bocola and Dovis (2019), who propose a single-borrower model and show that LOLR *announcements* - exemplified by Mario Draghi's 'Whatever it takes'-speech in summer 2012 - are an effective tool to reduce sovereign risk by peripheral euro area borrowers. My paper complements these approaches from a conceptual perspective, since the equilibrium is unique in my model, and from a policy perspective, since collateral policy is typically considered a conventional monetary policy instrument.

5.2 Model

Time is discrete and denoted by $t = 1, 2, ...$ and there is no aggregate risk. Each period corresponds to one year. There are two groups of agents, investors and governments, trading with each other on the government bond market, and a central bank. All agents are risk-neutral.

Investors. Investors discount the future at constant rate r^{rf}. They buy government bonds b^j_{t+1} as investment object and in addition value the collateral service provided by government bond holdings. To introduce a willingness to pay collateral premia, I assume that investors directly draw utility from holding eligible collateral.[7] Collateral benefits are represented by a CARA-function

$$\mathcal{L}(\overline{B}) = -\frac{l_0}{l_1} \exp\left\{-l_1\overline{B}\right\} \ , \tag{5.1}$$

with CARA-coefficient l_1 and weighting parameter l_0. The economy exhibits perfect financial integration in the sense that investors can use all government bonds equally well as collateral if they are subject to the same central bank haircut. While the substitutability of collateral service is difficult to test empirically, Delatte et al. (2016) and Luque (2021) provide evidence for collateral-driven portfolio reallocations in response to rating downgrades or increased margin requirements for risky (peripheral) government bonds. Armakolla et al. (2020) empirically document that higher haircuts on government bond issued by a specific country are associated with a decrease in the usage of these bonds as collateral, suggesting that investors are at least to a certain degree able to substitute between different securities that are eligible as collateral.

[7] This is similar to the literature on convenience yield, for example Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood et al. (2015). An observationally equivalent formulation can be obtained by assuming that investors need to settle uninsurable liquidity shocks by borrowing on the interbank repo market or from the central bank against eligible collateral.

Therefore, collateral benefits (5.1) depend on aggregate collateral supply $\overline{B}$, which is defined as

$$\overline{B} \equiv \int_j \frac{(1-\kappa_t^j)k_t^j}{1+r^{rf}} b_{t+1}^j dj \; . \tag{5.2}$$

Collateral supply consists of three distinct parts: bond quantities b_{t+1}^j, discounted expected payoffs $\frac{k_t^j}{1+r^{rf}}$, and haircuts κ_t^j. The expected payoff from purchasing a bond issued by government j and the haircut applied to this bond in period t are linked to the debt and default decision in period $t+1$, which will be described below. It is natural to interpret $\frac{k_t^j}{1+r^{rf}}$ as the pledgeable value of bond j: in the (unmodeled) event of an investor default between period t and $t+1$, the central bank seizes the bond and is entitled to its payoff (Bindseil and Papadia, 2006). To cushion against adverse price movements of the pledged collateral, the central bank additionally applies haircuts κ_t^j to government bonds.[8]

The benefit of holding one additional unit of collateral is then given by the first derivative of (5.1), i.e. by $l_0 \cdot e^{-l_1\overline{B}}$. Following Bindseil (2014), default risk is mapped into haircuts using the simple functional form

$$\kappa_t^j = \min\left\{ \left(F_t^j\right)^\eta, \overline{\kappa} \right\} \; . \tag{5.3}$$

The parameter η governs the sensitivity of haircuts to default risk F_t^j, which is interpreted as a variable beyond central bank control. This is motivated by the heavy usage of government bonds as collateral on the private repo market, where haircuts are either negotiated bilaterally or set by a central clearing counterparty. During the European debt crisis, peripheral government exhibited large haircut increases (Gabor and Ban, 2015) on private markets. In contrast, $\overline{\kappa}$ is the central bank policy parameter of interest and defines a maximum haircut which applies to all bonds, irrespective of their default risk. This effective cap is consistent with the ECB's practice already prior to the financial crisis (Buiter and Sibert, 2005) and during the European debt crisis (Nyborg, 2017). Since central bank haircuts apply to standing facilities, investors are able to obtain $\frac{1-\overline{\kappa}}{1+r^{rf}}$ units of funding even for the riskiest government bonds in the monetary union. This reflects the *collateral backstop* notion of central bank collateral frameworks, with $\overline{\kappa} = 0$ corresponding to the full backstop. Solving the maximization problem of investors delivers a pricing condition for government bond j

$$q_t^j = \frac{1 + (1-\kappa_t^j) \cdot l_0 \cdot e^{-l_1\overline{B}}}{1+r^{rf}} \cdot k_t^j \; . \tag{5.4}$$

The bond pricing condition contains two parts: the discounted expected payoff $\frac{k_t^j}{1+r^{rf}}$ and a collateral premium $(1-\kappa_t^j) \cdot l_0 \cdot e^{-l_1\overline{B}}$, which in turn can be decomposed into a *collateral valuation* component $l_0 \cdot e^{-l_1\overline{B}}$ and the term $1-\kappa_t^j$, to which I will also refer as the *haircut* component.

[8] If the central bank lends funds l_t at the risk-free rate against a risky bond j, full collateralization in the absence of collateral default risk requires $l_t(1+r^{rf}) = (1-\kappa_t^j)k_t^j b_t^j$. The maximum loan size a bank can obtain by pledging this bond is therefore given by $\frac{(1-\kappa_t^j)k_t^j}{1+r^{rf}} b_{t+1}^j$.

Governments There is a mass-one continuum of governments that supply a domestic public good g_t^j. Each government receives random tax revenues y_t^j, which follow a Markov chain on a discrete grid of revenue realizations with transition matrix Π_y. The cdf associated with the distribution of y_{t+1}^j, conditional on current revenues y_t^j will be denoted by $F(y_{t+1}^j|y_t^j)$.

I assume that governments are only able to use a part of government revenues $\overline{\Theta}\theta_t^j y_t^j$ for the repayment of debt obligations. This is best interpreted as the political willingness or ability of governments to use their revenues for debt service. Its cross-country average is determined by the parameter $\overline{\Theta}$. The political willingness to repay debt obligations was a prominent issue in the case of Greece during the European debt crisis and showed that this willingness can fluctuate over time, for example after an election. Therefore, I add the stochastic component θ_t^j and assume that it has mean one and follows a two-state Markov chain with transition matrix Π_θ. Its conditional cdf will be denoted by $F(\theta_{t+1}^j|\theta_t^j)$. Specifically, countries in the low state $\theta_t^{risky} < \theta_t^{safe}$ will have a higher default risk, ceteris paribus. In the context of the euro area debt crisis, countries with θ_t^{safe} can be interpreted as the 'core', while those drawing the low realization θ_t^{risky} represent the 'periphery'.

Governments issue bonds to smooth consumption across time and states. The payoff structure of government bonds follows Chatterjee and Eyigungor (2012). Bonds pay a fixed coupon rate c, mature with probability $0 < \pi \leq 1$ each period, while the non-maturing share is rolled over at current market price q_t^j. Following the ability-to-repay approach, the government repays its bonds if transferable government revenues $\overline{\Theta}\theta_t^j y_t^j$ fall short of current repayment obligations $(\pi + c)b_t^j$. The expected payoff k_t^j is thus given by

$$k_t^j = \left[1 - F\left(\frac{(\pi + c)b_{t+1}^j}{\overline{\Theta}}\middle|\theta_t^j, y_t^j\right)\right](\pi + c) + (1 - \pi)\mathbb{E}_t\left[q_{t+1}^j\right] .$$

Here, $F\left(\frac{(\pi+c)b_{t+1}^j}{\overline{\Theta}}\middle|\theta_t^j, y_t^j\right)$ is the default probability, which can be expressed using the joint cdf of $(\theta_{t+1}^j, y_{t+1}^j)$, conditional on the current exogenous state (θ_t^j, y_t^j). Plugging this into the bond pricing condition (5.4) yields the bond price *schedule*, which depends on the exogenous state (θ_t^j, y_t^j) and government j's choice variable b_{t+1}^j:

$$\begin{aligned} q(b_{t+1}^j, \theta_t^j, y_t^j) = {} & \frac{1 + \left(1 - \kappa(b_{t+1}^j, \theta_t^j, y_t^j)\right) \cdot l_0 \cdot e^{-l_1 \overline{B}}}{1 + r^{rf}} \\ & \times \left(\left[1 - F\left(\frac{(\pi + c)b_{t+1}^j}{\overline{\Theta}}\middle|\theta_t^j, y_t^j\right)\right](\pi + c) + (1 - \pi)\mathbb{E}_t\left[q\left(\mathcal{B}(b_{t+1}^j, \theta_{t+1}^j, y_{t+1}^j), \theta_{t+1}^j, y_{t+1}^j\right)\right]\right) . \end{aligned} \tag{5.5}$$

The bond price schedule links the expected pecuniary payoff to the valuation of collateral service provided by government bond j. The expected payoff consists of the expected coupon payment c and redemption share π in period $t + 1$ as well as the expected rollover value of bonds $(1 - \pi)\mathbb{E}_t[q(\cdot)]$. Note that the rollover value depends on the debt policy $\mathcal{B}(b_{t+1}^j, \theta_{t+1}^j, y_{t+1}^j)$ in $t + 1$, which in turn depends on current debt issuance b_{t+1}^j and the shock realizations in $t+1$. Collateral service is negatively related to default risk via the haircut function $\kappa(b_{t+1}^j, \theta_t^j, y_t^j)$, while collateral valuation depends on aggregate collateral $\overline{B}$. Taken as given investors' bond price schedule, each government j maximizes

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(g_t^j)\right] \quad \text{with} \quad u(g_t^j)=\begin{cases} g_t^j-\underline{g} & \text{if } g_t^j>\underline{g}\ , \\ -\infty & \text{else}\ , \end{cases} \tag{5.6}$$

subject to the sequence of budget constraints

$$\begin{aligned} g_t^j=(1-\overline{\Theta}\theta_t^j)y_t^j+&\mathbb{1}\{\overline{\Theta}\theta_t^j y_t^j>(\pi+c)b_{t+1}^j\}\left[\overline{\Theta}\theta_t^j y_t^j-(\pi+c)b_t^j\right] \\ &+q(b_{t+1}^j,\theta_t^j,y_t^j)\left(b_{t+1}^j-(1-\pi)\,b_t^j\right)\ . \end{aligned} \tag{5.7}$$

In case of default, the government loses the transferable share $\overline{\Theta}\theta_t^j y_t^j$, but does neither redeem the maturing share π of bonds nor services the coupon payment c. Since there are no delays in restructuring, the government is always able to issue bonds b_{t+1}^j and roll-over the non-maturing share of its bonds outstanding $(1-\pi)b_t^j$. The maximization problem for each government j can be written recursively as

$$\mathcal{V}(b_t^j,\theta_t^j,y_t^j)=\max_{b_{t+1}^j}\ u(g_t^j)+\beta\mathbb{E}\left[\mathcal{V}(b_{t+1}^j,\theta_{t+1}^j,y_{t+1}^j)\right] \text{ subject to (5.7)}\ . \tag{5.8}$$

Solving the maximization problem yields the debt policy $\mathcal{B}(b_t^j,\theta_t^j,y_t^j)$, which is not available in closed form. Evaluating the bond price schedule (5.5) at the debt policy then delivers the equilibrium bond price

$$\mathcal{Q}(b_t^j,\theta_t^j,y_t^j)=q\left(\mathcal{B}(b_t^j,\theta_t^j,y_t^j),\theta_t^j,y_t^j\right)\ . \tag{5.9}$$

Equilibrium The recursive competitive equilibrium of the model is given by the bond price schedule $q(b_{t+1}^j,\theta_t^j,y_t^j)$, equilibrium bond price $\mathcal{Q}(b_t^j,\theta_t^j,y_t^j)$, debt policy $\mathcal{B}(b_t^j,\theta_t^j,y_t^j)$, value function $\mathcal{V}(b_t^j,\theta_t^j,y_t^j)$, and the cross-sectional government distribution $G(b_t^j,\theta_t^j,y_t^j)$, such that

- The debt policy solves the government problem (5.6) and the value function satisfies the Bellman equation (5.8).
- Bond price schedule and the equilibrium bond price satisfy (5.5) and (5.9).
- Aggregate collateral supply $\overline{B}$ is consistent with the government distribution over idiosyncratic states.

Credit Default Swaps. The model permits the pricing of a CDS written on government bond j. The pricing schedule for CDS is given by the recursion

$$q^{CDS}(b_{t+1}^j,\theta_t^j,y_t^j)=\frac{1}{1+r^{rf}}\left((1-F(b_{t+1}^j|\theta_t^j,y_t^j))(\pi+c)+(1-\pi)\mathbb{E}_t\left[q^{CDS}\left(\mathcal{B}(b_{t+1}^j,\theta_{t+1}^j,y_{t+1}^j),\theta_{t+1}^j,y_{t+1}^j\right)\right]\right)\ . \tag{5.10}$$

Note that the continuation value is evaluated at the equilibrium debt policy. In contrast to the bond pricing condition (5.4), a CDS only reflect fundamentals, i.e., default risk implied by the government's debt choices.

5.3 Quantitative Analysis

To evaluate the model's quantitative properties, this section presents a calibration to euro area data and shows that it can replicate a flight-to-quality episode that is consistent with the cross-sectional dynamics of haircuts, government bond spreads, and CDS-spreads during the European debt crisis. Using the calibrated model, I demonstrate how central bank collateral policy can reduce the cross-sectional dispersion of government bond spreads and the overall level of CDS-spreads during a fiscal crisis.

5.3.1 Calibration

The model is solved using value function iteration on a discrete grid for debt b_t^j and the exogenous idiosyncratic states θ_t^j and y_t^j. Details on the computational algorithm and its parameters are deferred to Appendix D.2. Using the stationary competitive equilibrium, I introduce a fiscal crisis by subjecting the economy to an unanticipated shock to the average share of transferable government revenues $\overline{\Theta}$. Specifically, I assume that $\overline{\Theta}^{crisis} < \overline{\Theta}$, which makes default more likely for any given debt choice b_{t+1}^j and any exogenous state (θ_t^j, y_t^j). The aggregate shock is assumed to last for one period, after which the economy reverts back to the stationary equilibrium with certainty. Both exogenous idiosyncratic state variables are assumed to follow an AR(1)-process in logs

$$\log(\theta_t^j) = \rho_\theta \log(\theta_{t-1}^j) + \sigma_\theta \nu_t^j \ , \quad \nu_t^j \sim N(0,1) \ , \tag{5.11}$$

$$\log(y_t^j) = \rho_y \log(y_{t-1}^j) + \sigma_y \epsilon_t^j \ , \quad \epsilon_t^j \sim N(0,1) \ , \tag{5.12}$$

which will be discretized using the method proposed by Tauchen (1986), where (ρ_y, σ_y) can be estimated directly from tax revenues. The data for all countries that were members of the euro area in 2008 are obtained from the *St Louis Fed* database. I estimate (5.12) separately for each country from 1995 to 2019 and take the median estimate $\rho_y = 0.76$ and $\sigma_y^2 = 0.0032$. The maturity parameter $\pi = 0.2$ implies an average maturity of 5 years, which is an important benchmark maturity for euro-denominated government bonds and a typical value used in the literature. Setting $c = 0.045$ reflects the average coupon rate on Italian government bonds, as in Kaldorf and Roettger (2022). The sensitivity of haircuts to default risk is normalized to $\eta = 0.5$, while the haircut cap $\overline{\kappa} = 0.4$ is set to the average extraordinary ECB haircut applied to Greek and Cypriot government bonds during the euro area debt crisis (Nyborg, 2017).

The remaining eight parameters $\{\beta, \underline{g}, \rho_\theta, \sigma_\theta, \overline{\theta}, \overline{\theta}^{crisis}, l_0, l_1, \}$ are jointly calibrated to match several characteristics of the cross-sectional distribution over debt-to-GDP ratios, government bond spreads, CDS-spreads, and haircuts. Specifically, I use the upper and lower quartile (corresponding to Germany and Italy in the data) in normal times and during a fiscal crisis.

Pre-crisis spreads and haircuts are based on the sample from 2009-01-01 to 2011-06-30. I use the sample period associated with the European debt crisis from 2011-07-01 to 2012-06-30 to represent a fiscal crisis. Details on the data cross-section are presented in Appendix D.1. The mapping into the model-implied cross-section is discussed in Appendix D.2.

Even though parameters are calibrated jointly, some of them load strongly on specific targeted moments: the discount factor $\beta = 0.95$ and average pledgeable share $\overline{\theta} = 0.5$ strongly affect the average debt level and its dispersion in normal times, while $\overline{\theta}^{crisis} = 0.3$ loads on the increase of default risk during a fiscal crisis. Volatility σ_θ and persistence ρ_θ of pledgeable government revenues primarily drive the level and dispersion of spreads. Subsistence consumption $\underline{g}$ generates high levels of default risk at the right tail of the distribution (Bocola et al., 2019), consistent with the large increase in spreads on periphery countries during the European debt crisis.

Lastly, I parametrize investors' utility function over available collateral to match the yield difference of government bonds and the corresponding CDS, which is also referred to as the *CDS-bond basis*, in normal times and during a fiscal crisis. Specifically, the parameter l_0 governs the relative importance of collateral premia and therefore primarily loads on the government bond spread level, once the default risk is matched. The CARA-parameter l_1 determines the elasticity of collateral valuation to aggregate collateral supply and primarily affects the change of bond spreads during crisis periods, in particular for low-risk countries, for which collateral valuation effects are most important.

Table 5.1: Calibration

Parameter	Value	Source
Persistence income ρ_y^j	0.76	Euro area data
Vol of income shock σ_y^j	0.0032	Euro area data
Risk-free rate r^{rf}	0.005	EURIBOR-HCPI
Maturity Parameter π	0.2	5y average maturity
Coupon Parameter c	0.045	Average coupon rate
Haircut Parameter η	0.5	Normalization
Maximum Central Bank Haircut $\overline{\kappa}$	0.4	Nyborg (2017)
Government discount factor β	0.95	Calibrated
Minimum public goods provision $\underline{g}$	0.2	Calibrated
Average transferable share $\overline{\theta}$	0.5	Calibrated
Average transferable share $\overline{\Theta}^{crisis}$	0.3	Calibrated
Persistence parameter ρ_θ	0.95	Calibrated
Volatility parameter σ_θ	0.005	Calibrated
Collateral valuation slope l_0	0.003	Calibrated
Collateral valuation curvature l_1	3	Calibrated

The model fit is shown in Table 5.2. By construction, the fiscal crisis is characterized by higher default risk across the government distribution: CDS-spreads and haircuts for all borrowers increase and this increase is most pronounced for high-risk countries. The higher level of haircuts reduces aggregate collateral supply $\overline{B}$. Therefore, the collateral service of bonds least affected by default risk become more valuable. Formally, since $\frac{\partial l_0 \cdot \exp -l_1 \overline{B}}{\partial \overline{B}} < 0$, investors' *collateral valuation increases*. For the safest bonds available (corresponding to Germany, Finland, and the Netherlands in the data), this effect dominates the effect of elevated default risk: their bond spreads decline, even though their CDS-spreads increase. For riskier countries, collateral valuation has a smaller effect, because their bonds are subject to larger haircuts. Taken together, this leads to a more dispersed distribution of government bond spreads in a fiscal crisis, consistent with the data.

Table 5.2: Model Fit

	Normal Times		Fiscal Crisis	
	Data	Model	Data	Model
Debt-GDP(%), $Q_{0.25}$	87	62	89	53
Debt-GDP(%), $Q_{0.75}$	125	142	135	118
Bond-Spread, $Q_{0.25}$	-35	-27	-73	-43
Bond-Spread, $Q_{0.75}$	69	73	340	208
CDS-Spread, $Q_{0.25}$	25	26	45	40
CDS-Spread, $Q_{0.75}$	118	123	359	290

Notes: Bond and CDS-spreads are annualized in basis points. For the construction of the cross-section in the data, see Table D.2.

5.3.2 Reconciling Cross-Country Evidence

To corroborate the model's ability to replicate the interaction between sovereign risk, collateral premia, and debt issuance, I use the model-implied cross-section of countries to replicate the cross-country evidence regarding non-pecuniary benefits of government bond holdings and sovereign risk reported by Jiang et al. (2021). Even though their paper interprets the CDS-bond basis more widely as convenience yield, comparing their empirical results to the model-implied cross-section is informative, since in my model (a) collateral premia are the only driver of the CDS-bond basis (their convenience yield measure) and (b) collateral premia are directly linked to fiscal fundamentals via the haircut function. To ease the exposition, define the collateral premium as $l_t^j \equiv (1 - \kappa_t^j) \cdot l_0 \cdot e^{-l_1 \overline{B}}$. Using this definition, the following two regressions link country-level *fiscal conditions* to collateral premia:

$$l_t^j - l_t^{DE} = \alpha_0 + \alpha_1 \frac{-(\pi + c)b_t^j + q_t(b_{t+1}^j - (1-\pi)b_t^j)}{y_t^j} + \epsilon_t^j \ , \tag{5.13}$$

$$l_t^j - l_t^{DE} = \beta_0 + \beta_1 \frac{b_t^j}{y_t^j} + \nu_t^j \ . \tag{5.14}$$

I draw a sample of 10.000 countries from the stationary equilibrium distribution of the model to estimate eqs. (5.13) and (5.14) separately. Equation (5.13) uses each government's primary surplus, defined as net borrowing minus debt service payments, as explanatory variable for the collateral premium relative to the collateral premium of the safest bond, corresponding to the German bund in the data. Equation (5.14) uses the debt-to-GDP ratio as proxy of fiscal conditions. In both cases, the *largest* collateral premium will be interpreted as the premium on German government bonds, i.e. $l_t^{DE} \equiv \max_j \{l_t^j\}$. This is replicating the empirical approach of Jiang et al. (2021), who regress time-series averages of bond convenience yields relative to Germany on the average primary deficit and debt-to-GDP ratio across euro area members.

The (untargeted) model-implied effect of fiscal conditions on collateral premia is very similar to the effect reported in Jiang et al. (2021): a one-standard deviation increase in the primary surplus increases the CDS-bond basis by $\hat{\alpha}_1 = 6.6$bp in the data, compared to $\hat{\alpha}_1 = 2.1$bp in the model. Similarly, a one-standard deviation increase in the debt-to-GDP ratio reduces the CDS-bond basis by $\hat{\beta}_1 = 14$bp in the data and by $\hat{\beta}_1 = 22$bp in the model.

5.3.3 Collateral Policy during a Flight-to-Quality

Using the calibrated model, I examine the extent to which the central bank can affect the spread distribution by adjusting its collateral framework in response to a fiscal crisis. Since the collateral framework in this model is represented by the haircut cap $\overline{\kappa}$, there are two well-defined extreme values. The case of $\overline{\kappa} = 1$ can be interpreted as *strict market discipline* in the sense that investors do not obtain a funding advantage from pledging the government bond with the central bank. When $\overline{\kappa} = 0$, investors can pledge the full market value of every government bond with the central bank.

Table 5.3 shows that this full collateral backstop is able to partially reduce the yield spread between different countries and thereby mitigates - to some extent - diverging forces in the currency union. This has also an effect on sovereign risk, which slightly decreases in particular for high-risk borrowers. In contrast, the strict market discipline policy slightly reduces collateral supply and at the same time exacerbates the dispersion of bond spreads. Compared to the baseline calibration, there are only negligible on CDS-spreads in this case.

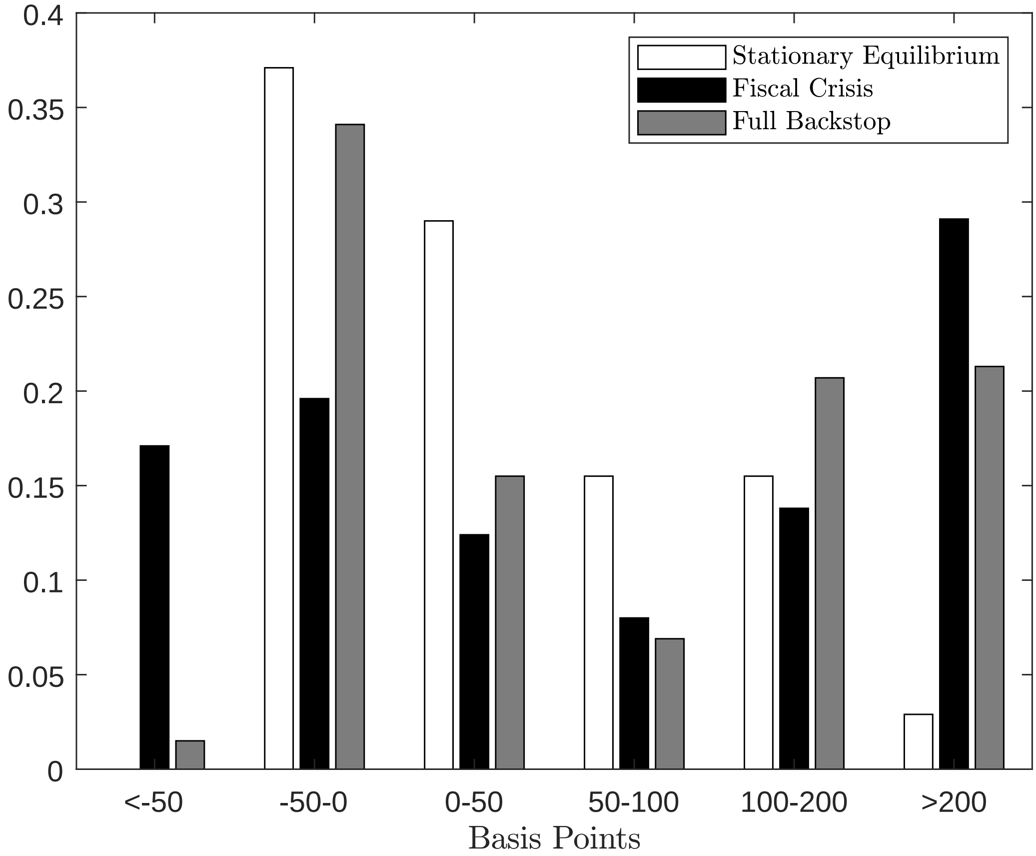
Table 5.3: Full Collateral Backstop in a Fiscal Crisis

	Baseline	Full Backstop	No Backstop
Collateral Supply (% of GDP)	55.1	65.5	54.8
Bond-Spread, $Q_{0.25}$	-43	-40	-44
Bond-Spread, $Q_{0.75}$	208	194	220
CDS-Spread, $Q_{0.25}$	39	38	39
CDS-Spread, $Q_{0.75}$	290	284	290
Haircut (%), $Q_{0.25}$	11	0	12
Haircut (%), $Q_{0.75}$	40	0	65

Notes: Bond and CDS-preads are annualized in basis points.

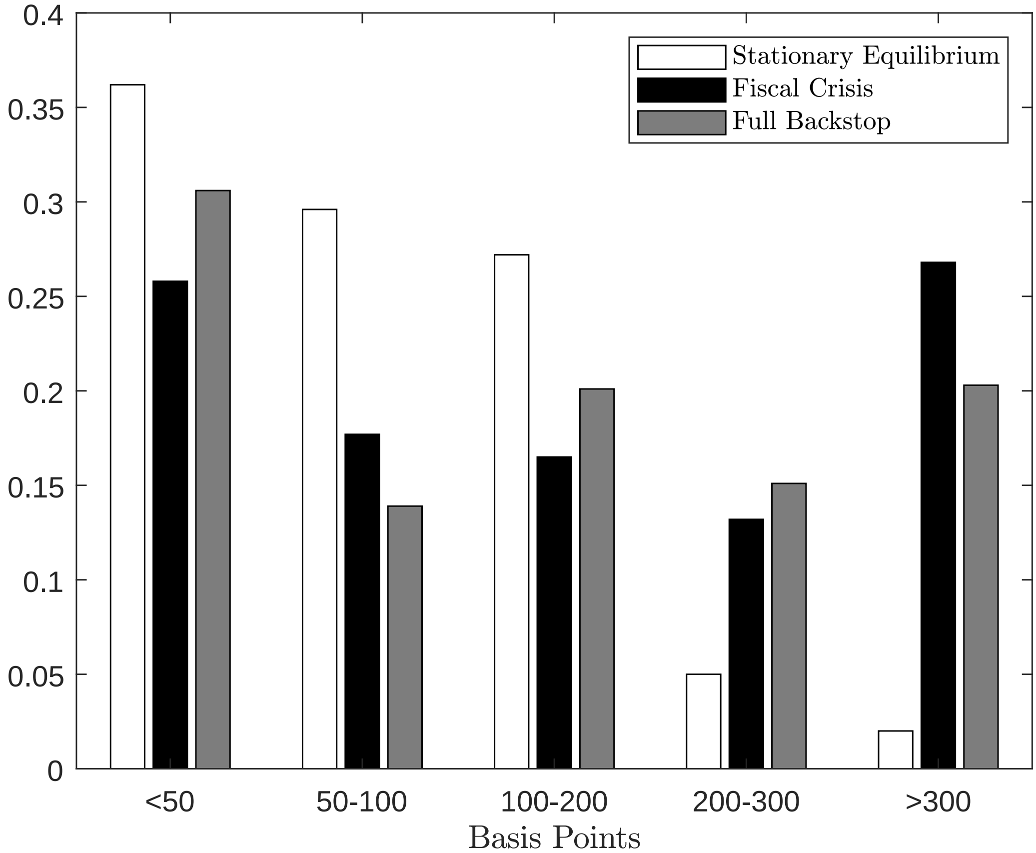
Figure 5.2 and Figure 5.3 provide an illustration of the cross-sectional distribution over bond- and CDS-spreads. The stationary equilibrium associated with the baseline calibration is indicated by white bars and shows that most government bonds trade at zero or slightly negative spreads over the risk-free rate. This corresponds to CDS-spreads below 100bp. A small share of borrowers is subject to sizable default risk, with bond- and CDS-spreads above 200bp. The distribution in the fiscal crisis period is indicated by black bars and exhibits a large mass in both the right bond spread bucket (200bp or higher) *and* the low bond spread bucket (-50bp or less). In this case, the CDS-distribution is shifted markedly to the right, with almost 25% of its mass being in the highest CDS-spread bucket.

Figure 5.2: Equilibrium Bond-Spreads



A full collateral backstop affects the cross-sectional dispersion of spreads in two ways: first, it increases collateral premia on relatively risky countries, since their bonds provide larger collateral service to investors, which has a negative effect on their spreads. The mass of government bonds trading at spreads of 200bp or more drastically reduces, as indicated by gray bars in Figure 5.2. Second, it decreases the collateral premium on relatively safe countries, since a larger collateral supply reduces investors' *collateral valuation*: the distribution over bond spreads exhibits a much smaller mass in the lowest bucket (-50bp or less). This also reduces the mass of the CDS-distribution in the high-risk bucket, because risky governments are able to roll over their legacy debt stock in the crisis period more easily.

Figure 5.3: Equilibrium Bond-Spreads



It should again be stressed that the full collateral backstop result is obtained when subjecting the model economy to an *unanticipated fiscal* crisis lasting for a known time horizon. In this setting, policy objectives are shaped by short-run considerations. In the European case, the objectives were twofold: (1) maintaining a sufficiently large collateral supply ensures smooth functioning of financial markets and facilitates monetary policy implementation through the banking sector. (2a) Reducing the government bond spread dispersion between core and peripheral borrowers contains regional imbalances and (2b) avoiding a sovereign default and its adverse consequences. Regional imbalances and sovereign defaults in turn directly threaten currency union' viability. Objective (1) in principle is a standard *lender-of-last-resort* policy, where its implementation is complicated by the elevated default risk of available collateral. Objectives (2a) and (2b) are related to the ECB's role as the *investor-of-last-resort* and are closely related in the sense that reducing the yield dispersion goes hand in hand with reducing the borrowing costs of high-risk countries, which in turn makes a debt roll-over easier for these countries. The model illustrates how a temporary relaxation of collateral policy contributes to achieving these short-term policy objectives using a single instrument.

Notably, this result obtains for an unanticipated aggregate fiscal shock. Studying the cyclical and long-run components of an optimal collateral framework in a monetary union requires a more detailed model of the adverse consequences of operating a permanently lenient collateral framework or of predictably relaxing collateral standards during a fiscal crisis. First, permanently low haircuts can increase the average default risk of individual countries since they make debt issuance permanently cheaper, as shown in Kaldorf and Roettger (2022). Criticism of treating all European government bonds as default-risk free in the lowest haircut tier at the inception of the 'single collateral list' in 2004 dates back to Buiter and Sibert (2005). Similarly, cyclical haircut relaxations can also introduce a moral hazard consideration if governments anticipate that central banks implicitly subsidize their debt rollover during a crisis, which might erode fiscal prudence. In both cases it is necessary to specify why central banks require investors to pledge collateral in the first place and how the costs of a sovereign default are distributed among member states. Likewise, designing optimal LOLR policy with defaultable government debt during a pure banking crisis requires a richer model of the collateral demand side and is potentially intertwined with the government's default behavior if a sovereign-bank nexus is at play. These extensions are left for future research.

5.4 Conclusion

This paper presented a model with heterogeneous governments issuing bonds subject to default risk. Adding collateral premia to this setting, the model generates a collateral-induced flight-to-quality during a fiscal crisis. Calibrated to the euro area debt crisis, the model can reconcile the cross-sectional distribution of haircuts, government bond spreads, and CDS-spreads. A full collateral backstop policy during crisis periods partially reduces the cross-sectional dispersion of government bond spreads. Moreover, this reduces debt rollover costs for the riskiest countries, which reduces their default risk. These results lend support to fully lenient collateral policy *on a temporary basis* in response to a fiscal crisis, as exemplified by the ECB's decision to temporarily suspend the minimum rating requirement on government bonds in 2020 as a response to the Covid-19 shock.

A Appendix to Chapter 2

A.1 Corporate Bond Eligibility in Collateral Frameworks

This section reviews the eligibility of corporate bonds in central bank collateral frameworks. As we show in Table A.1, eligibility of corporate bonds as collateral in central bank operations varies across countries and over time. The Eurosystem stands out due to its acceptance of corporate collateral before the financial crisis.

Table A.1: Non-Financial Corporate Bonds in Various Collateral Frameworks

Country	Pre 2008 (Min. Rating)	Post 2008 (Min. Rating)	Post Covid-19 (Min. Rating)
Australia	No	Yes (AAA)	Yes (BBB)
Eurosystem	Yes (A)	Yes (BBB)	Yes (BB)*
Japan	Yes (A)	Yes (BBB)†	Yes (BBB)
Switzerland	Yes (AA)	Yes (AA)	Yes (AA)
United Kingdom	No	Yes (A)	Yes (A)
United States††	Yes (AAA)	Yes (AAA)	Yes (BBB)

Notes: †: Multiple changes after Financial Crisis; *: For the duration of PEPP; ††: Only allowed in the discount window. Source: Bank for International Settlements (2013) & national CBs.

Table A.2 gives an overview of changes in the ECB collateral framework since 2007. Corporate bonds were eligible prior to the 2008 crises at a minimum rating requirement of A. In response to the financial crises, the minimum requirements were reduced from A to BBB, which substantially extended the amount of eligible assets and, thereby, broadened financial intermediaries' access to central bank liquidity. The smaller changes in 2011 and 2013 suggest that some fine-tuning was necessary after the initial relaxation. Nevertheless, the reduction of the minimum rating requirement was by far the largest adjustment, which motivates our choice of modeling collateral policy as a step function.

Table A.2: Corporate Bonds in the ECB Collateral Framework

Timespan	Regime	Haircut: A- or higher	Haircut: BBB- to BBB+
01 Jan 2007 - 24 Oct 2008	Fitch, S&P, and Moody's are accepted ECAI, minimum requirement A-.	4.5 %	100 %
25 Oct 2008 - 31 Dec 2010	DBRS accepted as ECAI, minimum requirement BBB-.	4.5 %	9.5 %
01 Jan 2011 - 30 Sep 2013	Tightening of haircuts.	5 %	25.5 %
01 Oct 2013 - 01 Dec 2019	Relaxation of haircuts.	3 %	22.5 %

Notes: Haircuts on a corporate bond with fixed coupon and maturity of 3 to 5 years; DBRS: Dominion Bond Rating Service, ECAI: external credit assessment institutions.

A.2 Proofs

This section contains the proofs of Propositions 1 and 2 and Lemma 1 and 2.

A.2.1 Proof of Proposition 1

To see why the most productive firms have the lowest default risk, we differentiate default risk with respect to s and obtain $\frac{\partial F(b^n_{t+1}|s)}{\partial s} = \frac{\partial F(b^n_{t+1}-s)}{\partial s} = f(b^n_{t+1}|s)\big(-1+\frac{\partial b^n_{t+1}}{\partial s}\big)$. We can use (A.6), which we derive below, to show that this term is unambiguously negative. Using analogous steps and (A.7), we show the same for $F(b^e_{t+1}|s)$.

We rewrite the first order conditions (2.3) and (2.4) in terms of the hazard rate as

$$h(b_{t+1}|s)\cdot b_{t+1} = 1-\beta \qquad \text{if} \quad F(b_{t+1}|s) > \overline{F}\ , \tag{A.1}$$

$$h(b_{t+1}|s)\cdot b_{t+1} = \frac{1-\beta+L}{1+L} \qquad \text{if} \quad F(b_{t+1}|s) \leq \overline{F}\ . \tag{A.2}$$

The first order condition (A.1) implies that the (debt-weighted) marginal default risk $h(b^n_{t+1}|s)\cdot b^n_{t+1}$ is identical for all s. The productivity parameter shifts the revenue distribution to the right: holding debt issuance constant, we have $\frac{\partial F(b^n_{t+1}|s)}{\partial s} < 0$ and also $\frac{\partial f(b^n_{t+1}|s)}{\partial s} < 0$ by the monotone hazard rate property so that $h(b^n_{t+1}|s)$ falls. Since the RHS of (2.3) is constant, increasing the productivity parameter implies that the debt choice has to increase $\frac{\partial b^n_{t+1}}{\partial s} > 0$. We perform analogous steps to show the same for b^e_{t+1}.

Finally, since $\frac{1-\beta+L}{1+L} > 1-\beta$ and using the monotonicity assumption on $h(\mu_{t+1})\mu_{t+1}$, we have that an eligible firm issues more debt than an otherwise non-eligible firm. □

A.2.2 Proof of Proposition 2

The partitioning of firms into different groups (unconstrained eligible, constrained eligible, and non-eligible) uses the fact that there are three *potentially* optimal debt choices for every s. The first possibility is to issue bonds $\widetilde{b}_{t+1}(s)$ to be exactly at the eligibility threshold. By the strict monotonicity of $F(b_{t+1}|s)$ in b_{t+1}, there is a unique $\widetilde{b}_{t+1}(s) \equiv F^{-1}(\overline{F}|s)$ for which the corporate bond is just eligible. Second, there is a debt level b^n_{t+1} satisfying the first order condition (A.1) for the case of non-eligibility. Third, the level b^e_{t+1} solves (A.2), the first order condition in the eligibility case. Under the monotonicity assumption on $h(b_{t+1}) \cdot b_{t+1}$, both conditions are satisfied by a unique b^n_{t+1} and b^e_{t+1}, respectively. The remainder of the proof characterizes which of these three debt levels is optimal, given the type parameter s.

Existence of Type Space Partitions. There is a positive mass of unproductive firms, such that $\widetilde{b}_{t+1}(s) = 0 < b^n_{t+1}(s) < b^e_{t+1}(s)$, which holds at least for s^- by assumption. These firms are not able to issue any bonds without exceeding the minimum quality requirement $\overline{F}$, i.e., their eligible debt capacity is zero. On the other hand, there are firms with positive eligible debt capacity. This can be shown by finding values s_1 and s_2 such that $b^n_{t+1}(s_1) = \widetilde{b}_{t+1}(s_1)$ and $b^e_{t+1}(s_2) = \widetilde{b}_{t+1}(s_2)$, i.e., firms are able to issue debt according to (A.1) and (A.2) without losing eligibility. We then show that the cut-off values satisfy $s^- < s_1 < s_2 < \infty$.

From the mass-shifting property of s, we can express the eligible debt capacity as

$$\widetilde{b}_{t+1}(s) = F^{-1}(\overline{F}) + s \, . \tag{A.3}$$

Define the hypothetical value functions for a never eligible firm $V^n(b_{t+1}|s)$ and an always eligible firm as $V^e(b_{t+1}|s)$. Plugging (A.3) into the first order conditions (2.3) and (2.4), we get

$$\left.\frac{\partial V^n(s)}{\partial b}\right|_{\widetilde{b}_{t+1}(s)} = (1-\beta)(1-\overline{F}) - \left(F^{-1}(\overline{F}) + s\right) f(F^{-1}(\overline{F})) \, , \tag{A.4}$$

$$\left.\frac{\partial V^e(s)}{\partial b}\right|_{\widetilde{b}_{t+1}(s)} = \frac{1-\beta+L}{1+L}(1-\overline{F}) - \left(F^{-1}(\overline{F}) + s\right) f(F^{-1}(\overline{F})) \, . \tag{A.5}$$

For a sufficiently productive firm with a large s, the eligible debt capacity $\widetilde{b}_{t+1}(s)$ lies on the downward sloping part of the objective function. Since the objective function is single-peaked by the monotone hazard rate assumption, $\widetilde{b}_{t+1}(s)$ is not optimal and such a firm voluntarily issues less debt than it could without losing eligibility. From $1-\beta < \frac{1-\beta+L}{1+L}$ and noting that for s_1 and s_2, (A.4) and (A.5) evaluate to zero, respectively. It follows that $s_1 < s_2$.

We can exploit the monotonicity of the first order conditions in s and monotonicity of the eligible debt capacity $\frac{\partial \widetilde{b}_{t+1}(s)}{\partial s} = 1$. Implicitly differentiating (A.1) and (A.2) with respect to s, we have

$$\frac{\partial b^n_{t+1}(s)}{\partial s} = \frac{\left(1-F(b^n_{t+1}|s)\right) f'(b^n_{t+1}|s)b^n_{t+1} + f(b^n_{t+1}|s)^2 b^n_{t+1}}{\left(1-F(b^n_{t+1}|s)\right)\left[f'(b^n_{t+1}|s)b^n_{t+1} + f(b^n_{t+1}|s)\right] + f(b^n_{t+1}|s)^2 b^n_{t+1}} < 1\ , \tag{A.6}$$

$$\frac{\partial b^e_{t+1}(s)}{\partial s} = \frac{\left(1-F(b^e_{t+1}|s)\right) f'(b^e_{t+1}|s)b^e_{t+1} + f(b^e_{t+1}|s)^2 b^e_{t+1}}{\left(1-F(b^e_{t+1}|s)\right)\left[f'(b^e_{t+1}|s)b^e_{t+1} + f(b^e_{t+1}|s)\right] + f(b^e_{t+1}|s)^2 b^e_{t+1}} < 1\ . \tag{A.7}$$

Since firms are risky by the first order conditions (2.3) and (2.4), we have $f(b^n_{t+1}|s) > 0$ and $f(b^e_{t+1}|s) > 0$ and the denominator is larger than the numerator, respectively. Therefore, the partial derivatives $\frac{\partial b^n_{t+1}(s)}{\partial s}$ and $\frac{\partial b^e_{t+1}(s)}{\partial s}$ are strictly smaller than one. Since by assumption $\widetilde{b}_{t+1}(s^-) = 0$ and $b^n_{t+1}(s^-) > 0$, $s_1 > s^-$ follows from the implicit definition $b^n_{t+1}(s_1) = \widetilde{b}_{t+1}(s_1)$. Furthermore, we can conclude that the cut-off values s_1 and s_2 are unique.

Characterizing Debt Choices. For every $s > s_2$, firms issue less debt than they could issue without losing eligibility. All firms with $s > s_2$ choose debt issuance according to their first order condition and are called *unconstrained eligible*.

Consider next firms which cannot choose their optimal borrowing without losing eligibility, i.e., firms with $s < s_2$. All firms between s_1 and s_2 choose to be just eligible and lever up until $\widetilde{b}_{t+1}(s)$, since for them $V^e(b^e_{t+1}(s)|s)$ is not feasible and $V^n(b^n_{t+1}(s)) < V^e(b^n_{t+1}(s)) < V^e(\widetilde{b}_{t+1}(s))$. The first inequality follows from $V^e(b_{t+1}|s) > V^n(b_{t+1}|s)$ for all b_{t+1}, holding s constant. The second inequality follows from the fact that V^e is increasing between $b^n_{t+1}(s)$ and $\widetilde{b}_{t+1}(s)$.

Finally, there is a threshold $s_0 < s_1$, below which firms choose $b^n_{t+1}(s)$ and are not eligible. All firms between s_0 and s_1 also choose $\widetilde{b}_{t+1}(s)$. The value s_0 is implicitly defined through the indifference condition $V^e(\widetilde{b}_{t+1}|s_0) = V^n(b^n_{t+1}|s_0)$. The assumptions on the revenue distribution will imply the existence of exactly one s_0 by the intermediate value theorem. To see this, consider their difference

$$\Delta(s) \equiv V^e(\widetilde{b}_{t+1}(s)|s) - V^n(b^n_{t+1}(s)|s)\ . \tag{A.8}$$

We have $\Delta(s_1) > 0$, since $b^n_{t+1}(s_1) = \widetilde{b}_{t+1}(s_1)$ by definition of s_1, and $V^e(\widetilde{b}_{t+1}(s_1)|s_1) > V^n(\widetilde{b}_{t+1}(s_1)|s_1)$. In addition, there exists a level s^- where $F(0|s^-) > \overline{F}$ by assumption. At this level $V^e(\widetilde{b}_{t+1}(s^-)|s^-) - V^n(b^n_{t+1}(s^-)|s^-) < 0$. Note that $\widetilde{b}_{t+1}(s^-) = 0$ and that $V^e(\widetilde{b}_{t+1}(s^-)|s^-)$ is the value of the unlevered firm. Choosing $b_{t+1} = 0$ would violate (A.1) and therefore $V^n(b^n_{t+1}(s)|s)$ exceeds the value of an unlevered firm for every s. Together with continuity of s, this already implies existence of at least one s_0 by the intermediate value theorem. To establish uniqueness, we differentiate $\Delta(s)$ with respect to s. The first part of $\Delta(s)$ can be written as

$$V^e(\widetilde{b}_{t+1}(s)|s) = (1-\overline{F})(1+L)\widetilde{b}_{t+1} + \beta \int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} \mu_{t+1} - \widetilde{b}_{t+1}(s) dF(\mu_{t+1}|s)\,,$$

and its total derivative is given by

$$\begin{aligned}
\frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial s} &= \frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial \widetilde{b}_{t+1}}\frac{\partial \widetilde{b}_{t+1}(s)}{\partial s} + \left.\frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial s}\right|_{\widetilde{b}_{t+1}} \\
&= \Big((1-\overline{F})(1+L) + \beta\int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}}(-1)dF(\mu_{t+1}|s)\Big)\frac{\partial \widetilde{b}_{t+1}}{\partial s} \\
&\quad + \beta\int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} -(\mu_{t+1}-\widetilde{b}_{t+1}(s))df(\mu_{t+1}|s) \\
&= (1-\overline{F})(1+L) - \beta(1-F(\widetilde{b}_{t+1}(s)|s)) \\
&\quad - \beta\int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} \mu_{t+1}df(\mu_{t+1}|s) + \beta\int_{\widetilde{b}_{t+1}(s)}^{\overline{\mu}} \widetilde{b}_{t+1}(s)df(\mu_{t+1}|s) \\
&= (1-\overline{F})(1+L) - \beta(1-F(\widetilde{b}_{t+1}(s)|s)) - \beta\Big(f(\overline{\mu})\overline{\mu} - f(\widetilde{b}_{t+1}(s)|s)\widetilde{b}_{t+1}(s) \\
&\quad - (1-F(\widetilde{b}_{t+1}(s)|s))\Big) + \beta\widetilde{b}_{t+1}(s)\Big(f(\overline{\mu}) - f(\widetilde{b}_{t+1}(s)|s)\Big) \\
&= (1-\overline{F})(1+L)\,. \qquad \text{(A.9)}
\end{aligned}$$

We used again that $\frac{\partial \widetilde{b}_{t+1}}{\partial s} = 1$ and $f(\overline{\mu}) = 0$. The second part of $\Delta(s)$ is given by

$$V^n(b^n_{t+1}(s)|s) = (1-F(b^n_{t+1}(s)|s)b^n_{t+1}(s) + \beta\int_{b^n_{t+1}(s)}^{\overline{\mu}} \mu_{t+1} - b^n_{t+1}(s)dF(\mu_{t+1}|s)\,.$$

The derivative of the second part of (A.8) is given by $\left.\frac{\partial V^n(b_{t+1}(s),s)}{\partial s}\right|_{b^n_{t+1}}$, since $\frac{\partial V^n(b_{t+1}(s),s)}{\partial b_{t+1}} = 0$ by the principle of optimality, when totally differentiating $V^n(b_{t+1}(s)|s)$ with respect to s. Specifically,

$$\begin{aligned}
\frac{\partial V^n(b^n_{t+1}(s)|s)}{\partial s} &= f(b^n_{t+1}(s)|s)\cdot b^n_{t+1}(s) + \beta\int_{b^n_{t+1}(s)}^{\overline{\mu}} -(\mu_{t+1}-b^n_{t+1}(s))df(\mu_{t+1}|s) \\
&= (1-\beta)\left(1-F(b^n_{t+1}(s)|s)\right) + \beta\left(1-F(b^n_{t+1}(s)|s)\right) \\
&= 1-F(b^n_{t+1}(s)|s)\,. \qquad \text{(A.10)}
\end{aligned}$$

In the second line, we directly used the first order condition (A.1). Putting both parts together

$$\frac{\partial\Delta(s)}{\partial s} = \frac{\partial V^e(\widetilde{b}_{t+1}(s)|s)}{\partial s} - \frac{\partial V^n(b^n_{t+1}(s)|s)}{\partial s} = (1-\overline{F})(1+L) - \left(1-F(b^n_{t+1}(s)|s)\right) > 0\,.$$

The sign follows from the fact that $\widetilde{b}_{t+1}(s) < b^n_{t+1}(s)$ holds in the region of interest. This implies that the default probability at $b^n_{t+1}(s)$ exceeds the eligibility threshold, i.e., $F(b^n_{t+1}(s)|s) > \overline{F}$. The inequality follows from $1 - F(b^n_{t+1}(s)|s) < 1 - \overline{F}$ and $L > 0$. Since $\Delta(s)$ is continuous and monotonically increasing, there exists an unique s_0 where the firm is indifferent between constrained eligibility and non-eligibility by the intermediate value theorem. All firms between s_0 and s_2 are called *constrained eligible*, firms below s_0 are *non-eligible*. □

A.2.3 Proof of Lemma 1

Lemma 1 can be shown by noting that collateral easing increases the eligible debt capacity across firms and that $b^n_{t+1}(s)$ and $b^e_{t+1}(s)$ are independent of the eligibility thresholds. To see that $\frac{\partial s_0}{\partial \overline{F}} < 0$, consider the indifference condition (A.8). The value of being constrained eligible $V^e(\widetilde{b}_{t+1}(s)|s)$ increases in $\overline{F}$. Differentiating the eligible debt capacity $\widetilde{b}_{t+1}(s)$ with respect to the eligibility threshold yields

$$\frac{\partial \widetilde{b}_{t+1}(s)}{\partial \overline{F}} = \frac{\partial F^{-1}\left(\overline{F}|s\right)}{\partial \overline{F}} = \frac{1}{f\left(F^{-1}(\overline{F}|s)\right)} > 0 \,, \tag{A.11}$$

where the last step follows from the inverse function theorem. A constrained eligible firm will be better off after a relaxation of eligibility requirements $V^e(\widetilde{b}^A_{t+1}(s^A_0)|s^A_0) < V^e(\widetilde{b}^{BBB}_{t+1}(s^A_0)|s^A_0)$. Note also that the value of being non-eligible $V^n(b^n_{t+1}(s)|s)$ does not depend on the eligibility threshold. Taken together, we have

$$V^e(\widetilde{b}^{BBB}_{t+1}(s^A_0)|s^A_0) > V^n(b^n_{t+1}(s^A_0)|s^A_0) = V^n(b^n_{t+1}(s^{BBB}_0)|s^A_0) \,.$$

Furthermore, for a given policy s_0 has to satisfy $V^e(\widetilde{b}_{t+1}(s)|s) = V^n(b^n_{t+1}(s)|s)$. We showed in (A.9) that the value of a constrained eligible firm is increasing in the shifting parameter. Thus, the indifference point s^{BBB}_0 shifts to the left: $s^{BBB}_0 < s^A_0$.

The effect of the eligibility threshold on s_2 can be seen from the condition $F(\widetilde{b}(s_2)) = F(b^e(s_2))$. Implicitly differentiating with respect to $\overline{F}$, we obtain

$$\frac{\partial s_2}{\partial \overline{F}} = -\frac{1}{f(\widetilde{b}(s_2)) - f(b^e(s_2))\frac{\partial b^e}{\partial s_2}} < 0$$

since $\frac{\partial b^e}{\partial s_2} < 1$. □

A.2.4 Proof of Lemma 2

Endogenous firm responses are residually given by subtracting the mechanical effect (2.6) from the total effect (2.5)

$$\begin{aligned}\overline{B}^{BBB} - \overline{B}^{A}\Big|_{endo} =& \Bigg(\underbrace{\int_{s_0^{BBB}}^{s_2^{BBB}} \left(1 - F\left(\widetilde{b}^{BBB}_{t+1}(s)\right)\right)\widetilde{b}^{BBB}_{t+1}(s)dG(s)}_{A.12.1} + \underbrace{\int_{s_2^{BBB}}^{\infty} \left(1 - F\left(b^e_{t+1}(s)\right)\right)b^e_{t+1}(s)dG(s)}_{A.12.2} \\ &- \underbrace{\int_{s_0^{A}}^{s_2^{A}} \left(1 - F\left(\widetilde{b}^{A}_{t+1}(s)\right)\right)\widetilde{b}^{A}_{t+1}(s)dG(s)}_{A.12.3} - \underbrace{\int_{s_2^{A}}^{\infty} \left(1 - F\left(b^e_{t+1}(s)\right)\right)b^e_{t+1}(s)dG(s)}_{A.12.4} \\ &- \underbrace{\int_{s_1^{BBB}}^{s_0^{A}} \left(1 - F\left(b^n_{t+1}(s)\right)\right)b^n_{t+1}(s)dG(s)}_{A.12.5} \Bigg) \cdot (1+L) \,.\end{aligned} \tag{A.12}$$

Since $s_2^{BBB} < s_2^A$ from Lemma 1, the terms A.12.2 and A.12.4 reduce to

$$\int_{s_2^{BBB}}^{s_2^A} \Big(1 - F\big(b_{t+1}^e(s)\big)\Big) b_{t+1}^e(s) dG(s) \ . \tag{A.13}$$

Due to the assumption $s_0^A < s_2^{BBB}$ we can split A.12.3 into two sub-integrals, ranging from $[s_0^A, s_2^{BBB}]$ and $[s_2^{BBB}, s_2^A]$, respectively. The second sub-integral can be combined with (A.13) and yields the last line of (2.7). The first sub-integral ranging from $[s_0^A, s_2^{BBB}]$ is used in the next step.

Note that the ordering of threshold productivity values arising from our assumptions and Lemma 1 is $s_0^{BBB} < s_1^{BBB} < s_0^A < s_2^{BBB}$. As a result, we can split A.12.1 into three sub-integrals ranging from $[s_0^{BBB}, s_1^{BBB}]$, $[s_1^{BBB}, s_0^A]$, and $[s_0^A, s_2^{BBB}]$, respectively. The third of these sub-integrals and the remaining sub-integral from A.12.3 are combined to line three in (2.7). Moreover, we combine the second sub-integral of A.12.1 with A.12.5 to obtain the second line in (2.7). Finally, the first sub-integral of A.12.1 corresponds to the first line in (2.7).

The aggregate default cost can be decomposed in a similar way. Notably, it contains *all* bonds and not only eligible ones:

$$\begin{aligned}\mathcal{M}^{BBB} - \mathcal{M}^A = &\underbrace{\int_{s^-}^{s_0^{BBB}} M\big(b_{t+1}^n(s)\big) dG(s)}_{A.14.1} + \underbrace{\int_{s_0^{BBB}}^{s_2^{BBB}} M\big(\widetilde{b}_{t+1}^{BBB}(s)\big) dG(s)}_{A.14.2} + \underbrace{\int_{s_2^{BBB}}^{\infty} M\big(b_{t+1}^e(s)\big) dG(s)}_{A.14.3} \\ &- \underbrace{\int_{s^-}^{s_0^A} M\big(b_{t+1}^n(s)\big) dG(s)}_{A.14.4} - \underbrace{\int_{s_0^A}^{s_2^A} M\big(\widetilde{b}_{t+1}^A(s)\big) dG(s)}_{A.14.5} - \underbrace{\int_{s_2^A}^{\infty} M\big(b_{t+1}^e(s)\big) dG(s)}_{A.14.6} \ .\end{aligned} \tag{A.14}$$

Again, since $s_2^{BBB} < s_2^A$ from Lemma 1, the terms A.14.3 and A.14.6 reduce to

$$\int_{s_2^{BBB}}^{s_2^A} M\big(b_{t+1}^e(s)\big) dG(s) \ . \tag{A.15}$$

Splitting A.14.5 into two sub-integrals, ranging from $[s_0^A, s_2^{BBB}]$ and $[s_2^{BBB}, s_2^A]$, we can combine the second of these with (A.15) to obtain the last line of (2.8).

Given the ordering of threshold productivity values arising from our assumptions and Lemma 1, we can split A.14.2 into three sub-integrals ranging from $[s_0^{BBB}, s_1^{BBB}]$, $[s_1^{BBB}, s_0^A]$, and $[s_0^A, s_2^{BBB}]$, respectively. Combining the last of these with the remaining sub-integral of A.14.5 yields the third line of (2.8).

Since $s_0^{BBB} < s_0^A$, we can summarize A.14.1 and A.14.4 to

$$-\int_{s_0^{BBB}}^{s_0^A} M\big(b_{t+1}^n(s)\big) dG(s) = -\int_{s_0^{BBB}}^{s_1^{BBB}} M\big(b_{t+1}^n(s)\big) dG(s) - \int_{s_1^{BBB}}^{s_0^A} M\big(b_{t+1}^n(s)\big) dG(s) \ .$$

Combining these two integrals with the remaining two sub-integrals of A.14.2 yields the first and second lines of (2.8). □

A.3 Data and Computation

A.3.1 Corporate Bond Data

We merge monthly data on the corporate bond universe in Europe from the iBoxx High Yield and Investment Grade Index families, provided by *IHS Markit*. We apply the following inclusion criteria:

1. Bond issuers are head-quartered in euro area member countries.
2. Issuers are non-financial firms.
3. The bond is denominated in euro, senior, not callable, uncollateralized, and fixed coupon.
4. The issuer is part of the constituent list for at least 48 months.

Bond issuers are provided by *Markit* and we consider only the parent company level, since it can be reasonably assumed that dedicated financial management subsidiaries are identical from an economic perspective to the respective parent company.

Company Data. We match company names to their unique *Compustat* identifier (`gvkey`) and drop all companies which are not represented in the *Compustat Global* database. For the remaining firms we query *Compustat* for long-term liabilities (`dltt`) in the firmq database and EBIT (`ebit`) in the firma database.

A.3.2 Computational Algorithm

We solve the individual firm problem using policy function iteration over a discrete set of collocation points using piecewise linear interpolation. The revenue shock is discretized using the method of Tauchen on an equispaced grid with $n_\mu = 25$ points over the interval $[-3\hat{\sigma}, +3\hat{\sigma}]$ with $\hat{\sigma} = \frac{\sigma}{1-\rho^2}$ denoting the unconditional variance of the revenue process. We denote the corresponding transition matrix Π_μ. Debt is discretized on an equispaced grid with $n_b = 21$ points over the interval $[5.5, 15.5]$.

To overcome the typical convergence issues in models with long-term debt and default, we use taste shocks when computing the debt choice (2.16), as proposed by Gordon (2019). The mass shifter for endogenous states follows immediately from the debt choice and is denoted Π_b. This matrix maps the current idiosyncratic state (μ_t^j, b_t^j), into next period's endogenous state b_{t+1}^j, i.e., has dimension $n_\mu \cdot n_b \times n_\mu \cdot n_b$.

Together with the transition matrix of idiosyncratic revenues, the combined mass shifter is given by $\Pi_g = \Pi_b \otimes \Pi_\mu$. The mass shifter implicitly defines the firm distribution G via $G^T = G^T \Pi_g$, where G denotes the firm distribution. Extracting the distribution, thus, boils down to computing the right eigenvector to Π_g.

Starting with a guess for firm policies and bond prices, each iteration ι consists of four different steps:

(i) Solve the firm problem taken as given the bond price schedule and value function from the previous iteration.

(ii) Compute the eligible debt capacity (2.15), the associated values of the objective function, and determine the debt choice according to (2.16).

(iii) Obtain the ensuing mass shifter Π_g from the policy functions and the transition matrix for revenue shock Π_μ and update the distribution G by iterating on $G^T = G^T \Pi_g$.

(iv) Update bond price schedules and value functions.

We then iterate on the policy functions until convergence, i.e., $||\mathcal{B}^\iota(b_t^j, \mu_t^j) - \mathcal{B}^{\iota-1}(b_t^j, \mu_t^j)||_\infty < 10^{-5}$. The standard deviation of the taste shock is set to 0.01 to ensure convergence. This is typically achieved within 200 iterations.

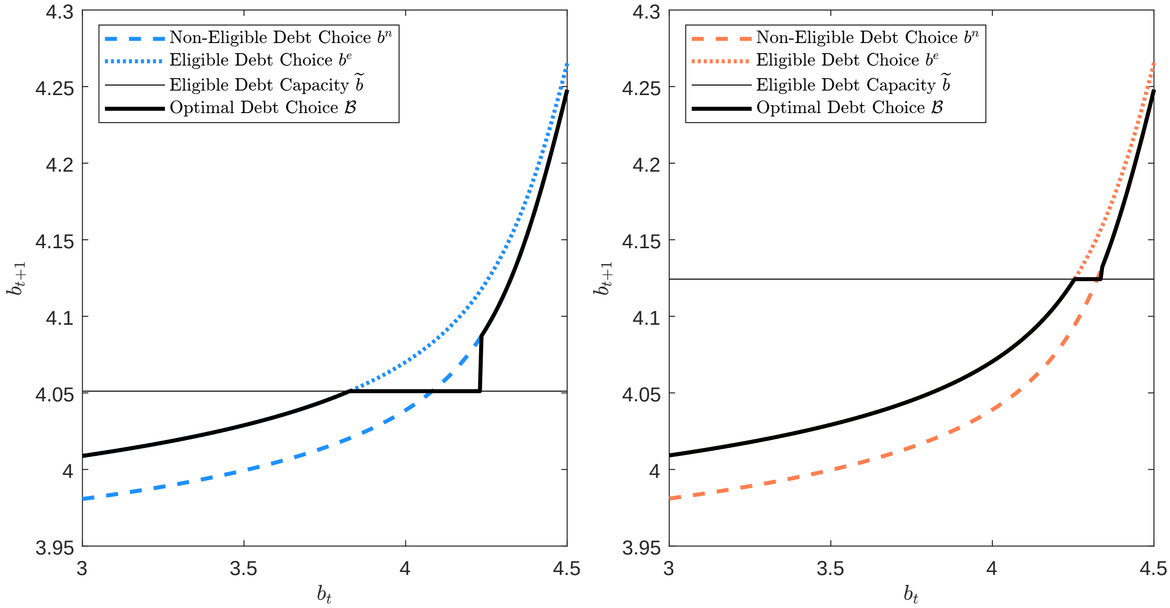
A.4 Additional Numerical Results

This section contains supplementary numerical results to our quantitative policy analysis. In Appendix A.4.1 we compare optimal debt choices under tight and lenient collateral policy. Appendix A.4.2 provides details on the distribution of bond spreads and default risk across firms. Appendix A.4.3 presents a robustness check that also includes data from the financial crisis of 2008 and consequently has a higher level of default risk. Appendix A.4.4 endogenizes the size of collateral premia.

A.4.1 Firm Debt Choices

We now illustrate how the characterization of firm debt choices carries over to the case of long-term debt. The black solid line in each panel of Figure A.1 denotes the debt choice given current debt b_t for a firm with median revenues under tight (left) or lenient (right) eligibility requirements. The colored dashed and dotted lines in either panel denote the debt choice if a firm is non-eligible or eligible, respectively. The firms' eligible debt capacity (2.15), which is independent of legacy debt b_t, is given by the horizontal black line. The debt choice exhibits a kink and a jump that represent the debt levels where firms change type (from non-eligible to constrained eligible and, eventually, to unconstrained eligible). The optimal debt choice (bold black line) is equal to b_{t+1}^e until it reaches its eligible debt capacity (first kink). For legacy debt levels between the kink and the jump, the firm exhausts its eligible debt capacity and is constrained eligible. Last, for debt outstanding above those at the jump, firms choose b_{t+1}^n. Similar to the one-period bond model, the effects of bond eligibility correspond to the difference between the non-eligible debt choice b_{t+1}^n and the equilibrium debt choice $\mathcal{B}_{t+1}$ (bold black line). Firms subject to risk-taking choose debt above b_{t+1}^n. Disciplined firms choose debt lower than b_{t+1}^n instead. Comparing the left to the right panel, we observe that under lenient eligibility requirements, where the eligible debt capacity shifts upwards, the risk-taking effect becomes more prominent, while the relative size of the disciplining effect falls.

Figure A.1: Debt Choice with Collateral Easing



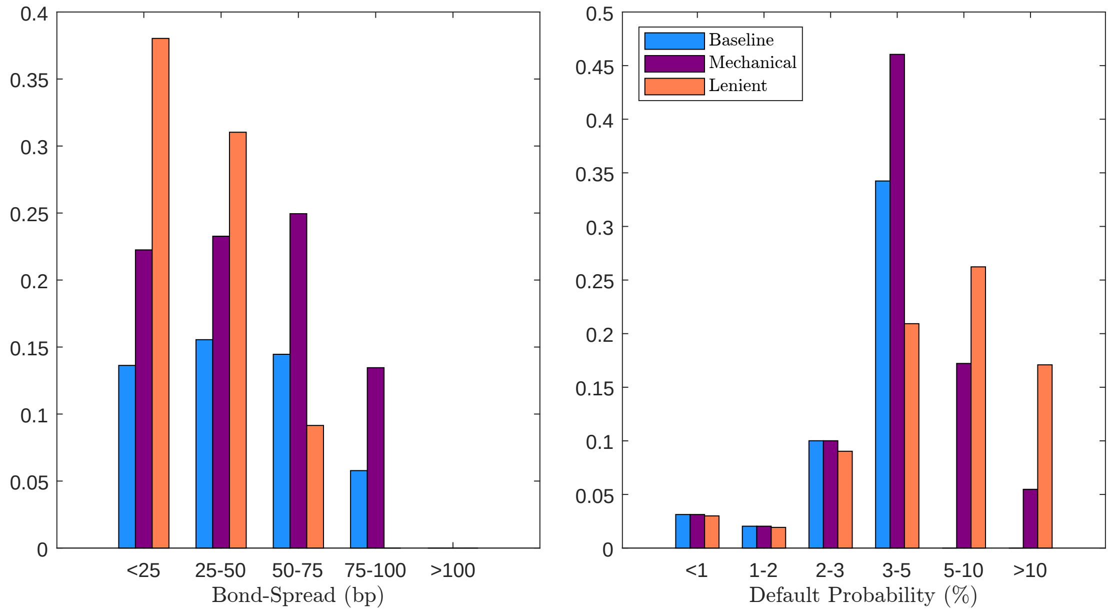
Notes: The bold black line represents the debt choice of a firm with median revenue conditional on legacy debt (see (2.16)). The colored lines denote the hypothetical debt choice of an always (non-)eligible firm. The light black line is the eligible debt capacity. In the left (right) panel we depict the case of tight (lenient) collateral policy.

A.4.2 Firm Distribution

While Table 2.4 condenses firm responses into the shares of risk-taking and disciplined firms, this section provides supplementary information on the firm distribution. Specifically, we compare the bond spread and default risk distributions of eligible firms for the baseline calibration (blue) to those under lenient eligibility requirements (orange), and to those under lenient eligibility requirements with constant firm behavior (purple).

The left panel of Figure A.2 divides eligible firms into different spread buckets. For the mechanical effect, we observe a rightward shift of bond spreads compared to the baseline of tight eligibility requirements, corresponding to newly eligible risky firms. Accordingly, in the right panel, we observe a similar rightward shift of the distribution of eligible firms' default probabilities. Taking firms responses into account markedly increases the share of firms in the left tail of the spread distribution. This follows from the high likelihood of satisfying the minimum rating requirement in future periods, which is associated with low bond spreads. However, the distribution over default risk in the right panel reveals that firm responses raise the mass of eligible firms in the higher risk buckets, reflecting the overall dominance of risk-taking effects.

Figure A.2: Eligible Firm Distribution over Bond Spreads



Notes: We show bond spread (left) and default risk (right) distributions across firms. Blue bars denote the economy with tight collateral policy, purple bars an economy with lenient collateral policy but fixed firm responses, and the orange bars an economy with lenient collateral policy.

A.4.3 Extended Sample Period

As a robustness check, we recalibrate the model and target the higher spread level over a sample encompassing the financial crisis of 2008. To match the elevated level of spreads, we set $\pi = 0.058$ and $\rho = 0.94$ to match the higher debt/EBIT-ratio as well as the increased level and cross-sectional dispersion of spreads. We calibrate $\overline{F}^{A} = 1.7\%$ and $\overline{F}^{BBB} = 18.5\%$ to recover the share of eligible bonds before and after relaxing eligibility requirements.

In Table A.4, we observe that firm responses dampen the impact of eligibility requirements to a similar extent as in the baseline calibration, but the mechanical and total effect are of smaller magnitude: since the firm distribution over default risk exhibits a larger dispersion, collateral easing increases $\overline{B}$ in a less effective way. However, the shares of risk-taking and disciplining firms under either policy are similar to the baseline calibration, suggesting that our characterization of endogenous firm responses does not crucially depend on the aggregate level of default risk.

Table A.3: Targeted Moments, Extended Sample

Moment	Data	Model
Collateral premium $r - r^n$	-11	-11
Debt/EBIT $Q_{0.50}(b/\mu\|\overline{F}^A)$	4.1	3.06
Bond spread $Q_{0.25}(x\|\overline{F}^A)$	45	58
Bond spread $Q_{0.50}(x\|\overline{F}^A)$	72	92
Bond spread $Q_{0.75}(x\|\overline{F}^A)$	115	118
Eligible bond share $\overline{B}/(QB)\|\overline{F}^A$	50%	50%
Eligible bond share $\overline{B}/(QB)\|\overline{F}^{BBB}$	86%	77%

Notes: Collateral premium and spreads are annualized and expressed in basis points.

Table A.4: Macroeconomic Effects of Collateral Easing, Extended Sample

	Total Effect	Mechanical Effect
Collateral Supply $\overline{B}$	+58%	+67%
Default Costs $\mathcal{M}$	+7%	
Firm Responses	Disciplining	Risk-Taking
Tight (A)	16%	52%
Lenient (BBB)	0%	77%

Notes: Values in the upper panel refer to collateral easing from A to BBB and are denoted as percentage difference from the A-baseline. The lower panel displays the fraction of *all* firms that is subject to disciplining or risk-taking effects.

A.4.4 Endogenous Size of Collateral Premia

This section presents a robustness check of our results by endogenizing the *size* of collateral premia. While these have been fixed to a constant L in the baseline, we make them dependent on aggregate collateral supply. In this case, collateral premia decline after collateral easing, which reduces both risk-taking incentives for eligible firms and disciplining effects for firms slightly below the eligibility requirement. Whether and how this affects the macroeconomic effects of collateral easing can, therefore, only be assessed numerically. Assume that banks directly draw utility from holding collateral. For numerical and analytical tractability, we impose a CARA-functional form

$$\mathcal{L}(\overline{B}) = -\frac{l_0}{l_1} \exp\left\{-l_1\overline{B}\right\} \ . \tag{A.16}$$

The collateral premium in this case is given by $L = l_0 \exp\{-l_1\overline{B}\}$. While we calibrate l_0 to match the eligibility premium of -11bp, the CARA-parameter l_1 governs the curvature of (A.16) and will be normalized to $l_1 = 1$. In Table A.5 we show the model fit corresponding to a parameter choice of $\beta = 0.994$, $\rho = 0.93$, $\sigma = 0.03$, and and $l_0 = 8.25$, while the (annualized) threshold default risk levels are given by $\overline{F}^A = 1.4\%$ and $\overline{F}^{BBB} = 18.5\%$.

Table A.5: Targeted Moments, Endogenous L

Moment	Data	Model
Collateral premium $r - r^n$	-11	-11
Debt/EBIT $Q_{0.50}(b/\mu \vert \overline{F}^A)$	3.9	3.9
Bond spread $Q_{0.25}(x \vert \overline{F}^A)$	24	25
Bond spread $Q_{0.50}(x \vert \overline{F}^A)$	39	49
Bond spread $Q_{0.75}(x \vert \overline{F}^A)$	62	72
Eligible bond share $\overline{B}/(QB) \vert \overline{F}^A$	50%	50%
Eligible bond share $\overline{B}/(QB) \vert \overline{F}^{BBB}$	86%	85%

Notes: Collateral premium and spreads are annualized and expressed in basis points.

Table A.6: Macroeconomic Effects of Collateral Easing, Endogenous L

	Total Effect	Mechanical Effect
Collateral Supply $\overline{B}$	+53%	+66%
Default Costs $\mathcal{M}$	-2%	
Firm Responses	Disciplining	Risk-Taking
Tight (A)	17%	51%
Lenient (BBB)	0%	82%

Notes: Values in the upper panel refer to collateral easing from A to BBB and are denoted as percentage difference from the A-baseline. The lower panel displays the fraction of *all* firms that is subject to disciplining or risk-taking effects.

Different to the baseline model with constant collateral premia, the large increase in collateral supply induces a drastic decline of the collateral premium to $L \approx 1$ bp in response to collateral easing, which decreases the *extent* of risk-taking in our model. Even though risk-taking effects still have a dampening effect on collateral supply, this is smaller than in the baseline calibration (see Table A.6). Furthermore, default costs experience a slight *decline*: firms take on more risk and are less likely to be eligible, but they default less often.

B Appendix to Chapter 3

B.1 Model Appendix

B.1.1 Bank Liquidity Management Costs

In the quantitative analysis, we assume that banks incur liquidity management costs $\Omega(\overline{b}^i_{t+1})$, which gives rise to collateral premia. In this section, we demonstrate that the resulting first order conditions for corporate bonds are observationally equivalent to the most common microfoundation used in this context, which are stochastic bank deposit withdrawals, see Corradin et al. (2017), De Fiore et al. (2019), Piazzesi and Schneider (2021), or Bianchi and Bigio (2022). The standard modeling device in this literature is a two sub-period structure, where banks participate in asset markets sequentially: in the first sub-period, banks trade with households on the deposit market and with intermediate good firms on the corporate bond market. In the second sub-period, bank i faces a liquidity deficit $\omega^i_t > 0$, which it settles on a collateralized short-term funding market, e.g., with the central bank.

If bank i is unable to collateralize its entire funding need, it must borrow on the (more expensive) unsecured segment. More specifically, since all banks hold the same amount of collateral $\overline{b}_{t+1}$ before the deposits are withdrawn, there is a cut-off withdrawal $\overline{\omega}_t = \overline{b}_{t+1}$ above which a bank needs to tap the unsecured segment. The amount borrowed on the unsecured segment for all banks follows as

$$\widetilde{b}_{t+1} \equiv \int_{\overline{b}_{t+1}}^{\infty} \left(\omega^i_{t+1} - \overline{b}_{t+1}\right) dW(\omega) ,$$

where W denotes the cdf of the withdrawal shock distribution. Due to its analytical tractability, it is convenient to assume that withdrawals follow a Lomax distribution. This distribution is supported on the right half-line and characterized by a shape $\widetilde{\alpha}$ and a scale $\widetilde{\lambda}$ parameter. This allows us to write the expected amount of borrowing on the unsecured segment in closed form:

$$\begin{aligned}
\widetilde{b}_{t+1} &= \int_{\overline{b}_{t+1}}^{\infty} \omega^i_{t+1} \frac{\widetilde{\alpha}}{\widetilde{\lambda}} \left(1 + \frac{\omega^i_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}-1} d\omega - \overline{b}_{t+1} \int_{\overline{b}_{t+1}}^{\infty} \frac{\widetilde{\alpha}}{\widetilde{\lambda}} \left(1 + \frac{\omega^i_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}-1} d\omega \\
&= \overline{b}_{t+1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}} + \frac{\widetilde{\lambda}}{\widetilde{\alpha}-1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}+1} - \overline{b}_{t+1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}} \\
&= \frac{\widetilde{\lambda}}{\widetilde{\alpha}-1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}+1} .
\end{aligned}$$

When $\widetilde{\alpha} > 1$, the aggregate amount of unsecured borrowing falls, the more collateral is held.

The benefit of holding collateral corresponds to the secured-unsecured spread ξ that is paid on borrowing $\widetilde{b}_{t+1}$, which we assume to be an exogenous parameter. These expected cost $\xi\widetilde{b}_{t+1}$ enter bank profits in the first sub-period

$$\Pi^i_t = d^i_{t+1} - q_{c,t+1}b^i_{c,t+1} - q_{g,t+1}b^i_{g,t+1} - \xi\widetilde{b}_{t+1} \ .$$

The cost depend negatively on $\overline{b}_{t+1}$, but the marginal cost reduction is falling in $\overline{b}_{t+1}$. Since very large withdrawal shocks are unlikely, the additional benefit of holding another unit of collateral is positive but decreasing. The properties of our concave liquidity cost function $\Omega(\overline{b}^i_{t+1})$ are closely related to the common micro-foundation using bank liquidity risk.

B.1.2 Intermediate Good Firms

We start with observing that the default threshold of a type-τ intermediate good firm in period $t+1$ is given by $\overline{m}_{\tau,t+1} \equiv \frac{sb_{\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}$. The threshold satisfies the following properties:

$$\frac{\partial \overline{m}_{\tau,t+1}}{\partial b_{\tau,t+1}} = \frac{s}{(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}} = \frac{b_{\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}\frac{s}{b_{\tau,t+1}} = \frac{\overline{m}_{\tau,t+1}}{b_{\tau,t+1}} \ , \tag{B.1}$$

$$\frac{\partial \overline{m}_{\tau,t+1}}{\partial k_{\tau,t+1}} = -\frac{sb_{\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k^2_{\tau,t+1}} = -\frac{b_{\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}\frac{s}{k_{\tau,t+1}} = -\frac{\overline{m}_{\tau,t+1}}{k_{\tau,t+1}} \ . \tag{B.2}$$

We assume that $\log(m_{\tau,t})$ is normally distributed with mean μ_M and standard deviation ς_M. In the calibration, we ensure that $\mathbb{E}[m_{\tau,t}] = 1$ by setting $\mu_M = -\varsigma^2_M/2$. The CDF of $m_{\tau,t}$ is given by $F(m_{\tau,t}) = \Phi\left(\frac{\log m_{\tau,t} - \mu_M}{\varsigma_M}\right)$, where $\Phi(\cdot)$ is the cdf of the standard normal distribution. The conditional mean of m at the threshold value $\overline{m}_{\tau,t+1}$ can be expressed as

$$G(\overline{m}_{\tau,t+1}) = \int_0^{\overline{m}_{\tau,t+1}} mf(m)dm = e^{\mu_M + \frac{\varsigma^2_M}{2}}\Phi\left(\frac{\log \overline{m}_{\tau,t+1} - \mu_M - \varsigma^2_M}{\varsigma_M}\right) \ ,$$

$$1 - G(\overline{m}_{\tau,t+1}) = \int_{\overline{m}_{\tau,t+1}}^{\infty} mf(m)dm = e^{\mu_M + \frac{\varsigma^2_M}{2}}\Phi\left(\frac{-\log \overline{m}_{\tau,t+1} + \mu_M + \varsigma^2_M}{\varsigma_M}\right) \ .$$

Note that the derivative of the conditional mean $g(\overline{m}_{\tau,t+1})$ satisfies

$$g(\overline{m}_{\tau,t+1}) = \overline{m}_{\tau,t+1}f(\overline{m}_{\tau,t+1}) \ . \tag{B.3}$$

For notational convenience, we write the bond price schedule as function of the default threshold $\overline{m}_{\tau,t}$ throughout this section. The bond payoff is given by

$$\mathcal{R}_{\tau,t} = s\left(G(\overline{m}_{\tau,t})\frac{(1-\chi_\tau)p_{\tau,t}k_{\tau,t}}{sb_{\tau,t}} + 1 - F(\overline{m}_{\tau,t})\right) - F(\overline{m}_{\tau,t})\varphi + (1-s)q_{\tau,t} \ ,$$

such that we can write the bond price only in terms of the default threshold $\overline{m}_{\tau,t+1}$

$$q(\overline{m}_{\tau,t+1}) = \mathbb{E}_t \frac{s\left(\frac{G(\overline{m}_{\tau,t+1})}{\overline{m}_{\tau,t+1}} + 1 - F(\overline{m}_{\tau,t+1})\right) - F(\overline{m}_{\tau,t+1})\varphi + (1-s)q_{\tau,t+1}}{(1+(1-\phi_\tau)\Omega_{b,t})(1+i_t)} \,. \tag{B.4}$$

The derivative with respect to the default threshold is given by

$$q'(\overline{m}_{\tau,t+1}) = \mathbb{E}_t \frac{-\frac{sG(\overline{m}_{\tau,t+1})}{\overline{m}^2_{\tau,t+1}} - \varphi f(\overline{m}_{\tau,t+1})}{(1+(1-\phi_\tau)\Omega_{b,t})(1+i_t)} \,. \tag{B.5}$$

FOC w.r.t $b_{\tau,t+1}$**.** The first order condition for bonds is given by

$$\begin{aligned}
0 =& q'(\overline{m}_{\tau,t+1})\frac{\partial \overline{m}_{t+1}}{\partial b_{\tau,t+1}}\left(b_{\tau,t+1} - (1-s)b_{\tau,t}\right) + q(\overline{m}_{\tau,t+1}) \\
&+ \mathbb{E}_t\Bigg[\widetilde{\Lambda}_{t,t+1}\Bigg(-(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}G'(\overline{m}_{\tau,t+1})\frac{\partial \overline{m}_{\tau,t+1}}{\partial b_{\tau,t+1}} \\
&- s\Big(-f(\overline{m}_{\tau,t+1})\frac{\partial \overline{m}_{\tau,t+1}}{\partial b_{\tau,t+1}}b_{\tau,t+1} + 1 - F(\overline{m}_{\tau,t+1})\Big) - q_{\tau,t+1}(1-s)\Bigg)\Bigg] \,,
\end{aligned}$$

which can be expressed as

$$\begin{aligned}
0 =& q'(\overline{m}_{\tau,t+1})\frac{\mathbb{E}_t[\overline{m}_{\tau,t+1}]}{b_{\tau,t+1}}\left(b_{\tau,t+1} - (1-s)b_{\tau,t}\right) + q(\overline{m}_{\tau,t+1}) \\
&+ \mathbb{E}_t\Bigg[\widetilde{\Lambda}_{t,t+1}\Bigg(-sG'(\overline{m}_{\tau,t+1})\frac{\overline{m}_{\tau,t+1}(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}{sb_{\tau,t+1}} \\
&- s\Big(-f(\overline{m}_{\tau,t+1})\overline{m}_{\tau,t+1} + 1 - F(\overline{m}_{\tau,t+1})\Big) - q_{\tau,t+1}(1-s)\Bigg)\Bigg] \,,
\end{aligned}$$

and then yields (3.14).

FOC w.r.t $k_{\tau,t+1}$**.** The first order condition for capital is given by

$$\begin{aligned}
1 =& q'(\overline{m}_{\tau,t+1})\frac{\partial \overline{m}_{\tau,t+1}}{\partial k_{\tau,t+1}}\left(b_{\tau,t+1} - (1-s)b_{\tau,t}\right) \\
&+ \mathbb{E}_t\Bigg[\widetilde{\Lambda}_{t,t+1}\Bigg(-G'(\overline{m}_{\tau,t+1})\frac{\partial \overline{m}_{\tau,t+1}}{\partial k_{\tau,t+1}}(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1} + (1-G(\overline{m}_{\tau,t+1}))(1-\chi_\tau)p_{\tau,t+1} \\
&+ sb_{\tau,t+1}f(\overline{m}_{\tau,t+1})\frac{\partial \overline{m}_{\tau,t+1}}{\partial k_{\tau,t+1}} + 1 - \delta\Bigg)\Bigg] \,,
\end{aligned}$$

which can be rearranged to

$$1 = -q'(\overline{m}_{\tau,t+1})\frac{\mathbb{E}_t[\overline{m}_{\tau,t+1}]}{k_{\tau,t+1}}\bigg(b_{\tau,t+1} - (1-s)b_{\tau,t}\bigg)$$
$$+ \mathbb{E}_t\Bigg[\widetilde{\Lambda}_{t,t+1}\bigg(G'(\overline{m}_{\tau,t+1})\overline{m}_{\tau,t+1}(1-\chi_\tau)p_{\tau,t+1} + \big(1 - G(\overline{m}_{\tau,t+1})\big)(1-\chi_\tau)p_{\tau,t+1}$$
$$- sb_{\tau,t+1}f(\overline{m}_{\tau,t+1})\frac{\overline{m}_{\tau,t+1}}{k_{\tau,t+1}}\frac{(1-\chi_\tau)p_{\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}} + 1 - \delta\bigg)\Bigg] ,$$

and further to (3.15).

B.1.3 Collateral Default Costs

In the main text we assume an exogenous cost function from collateral default $\Lambda(\overline{F}_t)$. In this section, we provide a micro-foundation based on central bank solvency concerns (see Hall and Reis, 2015). We show that this yields a loss function $\Lambda(\overline{F}_t)$, which is increasing in $\overline{F}_t$, consistent with our assumption in the main text. Similar to appendix B.1.1, assume that banks incur a fixed liquidity shock in every period ω, which they settle by borrowing from the central bank. Since the collateral banks pledge is subject to default risk, the central bank will subject itself to these risks when entering repurchase agreements. The central bank haircut ϕ directly affects exposure to this risk. The timing is as follows: in the beginning of period t, banks invest into risky bonds. In the end of period t, they incur the exogenous liquidity need and tap the central bank facility. Repos mature in the beginning of period $t+1$ and banks repay the central bank. Each bank holds corporate bonds b_{t+1} at price q_t and borrows

$$\omega = (1-\phi)q_t b_{t+1} ,$$

from the central bank. Because every bank i incurs the liquidity shock, i indexes both banks and repo contracts. We assume that bank default can be represented by the i.i.d. random variable ζ^i with cdf Z and pdf z, and with support $[0,1]$. The bond-specific default risk is denoted F_t. In case of a bank default, the central bank seizes the posted collateral to cover its losses. However, since the collateral itself defaults at rate F_t, the central bank will not recover the full amount of the defaulted repo. The expected loss on repo i follows as

$$\mathcal{F}_t^i = \zeta^i \cdot \omega \cdot F_t = \zeta^i \cdot (1-\phi)q_t b_{t+1} \cdot F_t .$$

To make the results more easily interpretable, it is helpful to assume central bank also generates seigniorage revenues from lending through its facilities. As customary in the literature, we assume that seigniorage revenues are bounded from above by the (time-invariant) constant $\mathcal{M}$. Consequently, the central bank incurs a loss from bank default if the default shock exceeds $\bar{\zeta}_t = \mathcal{M}/(\omega \cdot F_t)$. We can then denote the expected central bank loss as

$$\mathcal{L}_t = \int_{\bar{\zeta}_t}^1 \zeta^i \cdot (1-\phi)q_t b_{t+1} \cdot F_t \cdot z(\zeta)d\zeta = (1-\phi)q_t b_{t+1}F_t \cdot \int_{\bar{\zeta}_t}^1 \zeta^i z(\zeta)d\zeta . \quad \text{(B.6)}$$

Table B.1: Time Series Means with $\epsilon_\nu = 1.6$

Moment	**Baseline**	**Max Pref**	**Opt Coll**	**Only Tax**	**Glob Opt**
Tax Parameter χ_c	0	0	0	12.5%	12.5%
Haircut ϕ_g	26%	0%	5%	26%	18%
Haircut ϕ_c	26%	100%	31%	26%	18%
Welfare Change (CE)	0%	-0.5450%	+0.0135%	+1.1560%	+1.1577%
Conv. Leverage	39.7%	38.7%	39.7%	39.7%	39.7%
Green Leverage	39.7%	40.4%	39.9%	39.7%	39.7%
Conv. Bond Spread	97bp	157bp	101bp	97bp	94bp
Green Bond Spread	97bp	16bp	81bp	97bp	94bp
Conv. Coll. Premium	-11bp	0bp	-10bp	-11bp	-11bp
Green Coll. Premium	-11bp	-26bp	-14bp	-11bp	-11bp
GDP	0.8253				
Change from Baseline	-	+0.14%	+0.03%	+1.01%	+1.05%
Restructuring Cost/GDP	2.17%				
Change from Baseline	-	-20.38%	+0.13%	-0.23%	+2.42%
Coll. Default Cost/GDP	1.52%				
Change from Baseline	-	-28.56%	+0.81%	-0.62%	+6.46%
Liq. Man. Cost/GDP	3.29%				
Change from Baseline	-	+37.57%	-0.28%	-1.40%	-6.35%
Pollution Cost/GDP	9.81%				
Change from Baseline	-	-1.79%	-0.14%	-16.17%	-16.06%
Green Bond Share	20.49%	22.24%	20.73%	34.57%	34.56%
Green Capital Share	20.49%	21.52%	20.64%	34.57%	34.56%

Notes: Maximal preferential treatment (*Max Pref*) only allows green bonds as collateral ($\phi_g = 0, \phi_c = 1$). The optimal collateral policy (*Opt Coll*) is derived from maximizing CE over a grid of haircuts. For the optimal tax (*Only Tax*), we hold haircuts fixed and vary the tax rate. The global optimum (*Glob Opt*) is obtained by maximizing over taxes and haircuts.

The central bank haircut and bond default risk affect the expected loss in two ways. The first part of (B.6) show that irrespective of the distributional assumption on ζ^i, the expected loss rises in bond default risk F_t and that a higher haircut ϕ, by lowering the repo size, reduces cost. Second, note that $\overline{\zeta}_t = \mathcal{M}/((1-\phi)q_t b_{t+1} \cdot F_t)$. A higher haircut increases the default risk threshold beyond which central bank income is negative. Thus, it lowers the expected loss. Conversely, higher bond default risk increases expected cost. Defining collateral default risk as the repo size-weighted default risk $\overline{F}_t = (1-\phi)q_t b_{t+1} F_t$, this behavior is directly reflected in $\Lambda(\overline{F}_t)$ in the main text.

B.2 Additional Numerical Results

B.2.1 The Role of the Green-Conventional Substitution Elasticity

In Table B.1, we provide robustness checks regarding the production technology of wholesale goods producers. By assuming a Cobb-Douglas production function in (3.8), we implicitly assume an elasticity of substitution of one between green and conventional intermediate goods. When strictly interpreting green and conventional firms as energy producers, this elasticity is usually estimated to be larger than one. Therefore, we repeat our policy analysis when replacing the wholesale producers' technology by a general CES-function

$$z_t = \left(\nu z_{g,t}^{\frac{\epsilon_\nu - 1}{\epsilon_\nu}} + (1-\nu) z_{c,t}^{\frac{\epsilon_\nu - 1}{\epsilon_\nu}} \right)^{\frac{\epsilon_\nu}{\epsilon_\nu - 1}} ,$$

and set the elasticity of substitution $\epsilon_\nu = 1.6$, following the point estimate in Papageorgiou et al. (2017). The parameter ν is set to keep the green production share at 20%, consistent with the baseline. Results are shown in Table B.1. To ensure an apples-to-apples comparison with the baseline model, we re-calibrate the idiosyncratic productivity variance to $\varsigma_M = 0.195$, the firm owners' discount factor $\widetilde{\beta} = 0.984$, the externality parameter $\gamma_P = 0.015$, the slope parameter $\eta_1 = 0.0432$ in the collateral default cost function, and the slope parameter $l_1 = 0.008$ in the liquidity management cost function. While the main results from the Cobb-Douglas baseline carry over to the CES case, the optimal tax is much higher and optimal collateral policy implies a much larger degree of preferential treatment. Intuitively, when conventional and green intermediate goods are easier to substitute, any policy-induced reduction in the size of conventional firms is less harmful to production.

B.2.2 The Role of Nominal Rigidities

In this section, we add nominal rigidities to the model following the standard New Keynesian model. In particular, bonds are assumed to be denominated in nominal terms, i.e., inflation has a direct effect on corporate bonds and the supply side. Households consume a final goods basket c_t given by

$$c_t = \left(\int_0^1 c_{i,t}^{\frac{\varepsilon - 1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon - 1}} ,$$

where $\varepsilon > 1$ is the elasticity of substitution among the differentiated final goods. The demand schedule for final good i is given by

$$c_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} c_t \ , \tag{B.7}$$

where P_t denotes the CES price index for the final consumption bundle. Final good firms sell their differentiated good with a markup over their marginal costs. However, the price of firm j, $P_{j,t}$, can only be varied by paying a quadratic adjustment cost à la Rotemberg (1982) that is proportional to the nominal value of aggregate production, $P_t y_t$. Firm j's marginal costs are denoted by $\mathrm{mc}_{j,t} \equiv \partial \mathcal{C}_t^W / \partial y_{j,t}$, where the wholesale firm's cost minimization problem is given by

$$\mathcal{C}_t^W(y_{j,t}) = \min_{z_{j,t}, l_{j,t}} \quad P_{z,t} z_{j,t} + W_t l_{j,t} \quad \text{s.t.} \quad y_{j,t} = (1 - \mathcal{P}_t) \, A_t z_{j,t}^{\theta} l_{j,t}^{1-\theta} \ ,$$

and $P_{z,t}$ is the price of the wholesale good. From the minimization problem we obtain *real* marginal costs

$$\mathrm{mc}_t = \frac{1}{(1-\mathcal{P}_t)\,A_t}\left(\frac{p_{z,t}}{\theta}\right)^{\theta}\left(\frac{w_t}{1-\theta}\right)^{1-\theta} ,$$

where $p_{z,t} = P_{z,t}/P_t$ is the relative price of the wholesale good and w_t is the real wage. Hence, total nominal profits of firm j in period t are given by

$$\widehat{\Pi}_{j,t} = (P_{j,t} - \mathrm{mc}_t P_t)\,y_{j,t} - \frac{\psi}{2}\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)^2 P_t y_t ,$$

where ψ measures the degree of the nominal rigidity. Each wholesale good firm j maximizes the expected sum of discounted profits

$$\max_{P_{j,t+s},y_{j,t+s}} \mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^s\frac{c_{t+s}^{-\gamma_c}/P_{t+s}}{c_t^{-\gamma_c}/P_t}\,\widehat{\Pi}_{j,t+s}\right] ,$$

subject to the demand schedule (B.7). Plugging in the demand function yields the first order condition

$$\left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} Y_t - \varepsilon\,(P_{j,t} - \mathrm{mc}_t P_t)\left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon}\frac{y_t}{P_t} - \psi\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)\frac{P_t}{P_{j,t-1}}y_t$$
$$+\,\mathbb{E}_t\left[\frac{c_{t+1}^{-\gamma_c}/P_{t+1}}{c_t^{-\gamma_c}/P_t}\psi\left(\frac{P_{j,t+1}}{P_{j,t}} - 1\right)\frac{P_{j,t+1}}{P_{j,t}^2}P_{t+1}y_{t+1}\right] = 0 .$$

In a symmetric price equilibrium, $P_{j,t} = P_t$ for all j. Using this, we rearrange and get

$$(1-\varepsilon(1-\mathrm{mc}_t))\,y_t + \mathbb{E}_t\left[\beta\frac{c_{t+1}^{-\gamma_c}/P_{t+1}}{c_t^{-\gamma_c}/P_t}y_{t+1}\pi_{t+1}\psi\,(\pi_{t+1}-1)\,\pi_{t+1}\right] = \psi\,(\pi_t - 1)\,\pi_t y_t ,$$

where $\pi_t = \frac{P_t}{P_{t-1}}$. Dividing both sides by y_t and Ψ we arrive at the New Keynesian Phillips Curve

$$\mathbb{E}_t\left[\beta\frac{c_{t+1}^{-\gamma_c}/P_{t+1}}{c_t^{-\gamma_c}/P_t}\frac{y_{t+1}\pi_{t+1}}{y_t}\,(\pi_{t+1}-1)\,\pi_{t+1}\right] + \frac{\varepsilon}{\psi}\left(\mathrm{mc}_t - \frac{\varepsilon-1}{\varepsilon}\right) = (\pi_t - 1)\,\pi_t .$$

In addition, nominal rigidities also affect intermediate good firms, since inflation affects the default threshold $\overline{m}_{\tau,t+1} \equiv \frac{sb_{\tau,t+1}}{\pi_{t+1}(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}$ and the *real per-unit* bond payoff is

$$\mathcal{R}_{\tau,t} = s\bigg(G(\overline{m}_{\tau,t})\frac{\pi_t p_{\tau,t}(1-\chi_\tau)k_{\tau,t}}{sb_{\tau,t}} + 1 - F(\overline{m}_{\tau,t})\bigg) - F(\overline{m}_{\tau,t})\varphi + (1-s)q_{\tau,t} .$$

Their first order conditions are now given by

$$q'(\overline{m}_{\tau,t+1})\frac{\mathbb{E}_t[\overline{m}_{\tau,t+1}]}{b_{\tau,t+1}}\left(b_{\tau,t+1}-(1-s)\frac{b_{\tau,t}}{\pi_t}\right)+q(\overline{m}_{\tau,t+1})$$
$$=\widetilde{\beta}\mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\frac{s(1-F(\overline{m}_{\tau,t+1}))+(1-s)q_{\tau,t+1}}{\pi_{t+1}}\right]$$

and

$$1=-\,q'(\overline{m}_{\tau,t+1})\frac{\mathbb{E}_t[\overline{m}_{\tau,t+1}]}{k_{\tau,t+1}}\left(b_{\tau,t+1}-(1-s)\frac{b_{\tau,t}}{\pi_t}\right)$$
$$+\,\mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\left(\widetilde{\beta}(1-\delta)+\widetilde{\beta}(1-\chi_\tau)p_{\tau,t+1}\big(1-G(\overline{m}_{\tau,t+1})\big)\right)\right].$$

The resource constraint now also includes Rotemberg costs

$$y_t=c_t+\sum_\tau\left(c_{\tau,t}+i_{\tau,t}\right)+\Lambda(\overline{F}_{t+1})+\Omega(\overline{b}_{t+1})+\frac{\psi}{2}\left(\pi_t-1\right)^2y_t+\sum_\tau\varphi F(\overline{m}_{\tau,t})\frac{b_{\tau,t}}{\pi_t}\,.$$

To close the model, we assume that the central bank sets i_t according to a Taylor rule

$$i_t=i\pi_t^{\phi_\pi}\,. \tag{B.8}$$

We choose standard parameters for the final goods elasticity $\epsilon=6$, implying a markup of 20% in the deterministic steady state, and a Rotemberg parameter $\psi=57.8$, consistent with a Calvo parameter of 0.75. The parameter on inflation stabilization in the monetary policy rule is set to $\phi_\pi=5$, which ensures determinacy for all policy experiments. We slightly re-calibrate the slope parameter $\eta_1=0.0407$ in the collateral default cost function, and the slope parameter $l_1=0.007$ in the liquidity management cost function. Results are reported in Table B.2 and show very similar implications for optimal collateral policy and its interaction with Pigouvian taxation.[1] In particular, the inflation volatility under optimal preferential treatment is almost unchanged with respect to the baseline in column one, alleviating concerns that preferential treatment jeopardizes price stability, the central bank's primary policy objective.

B.3 Yield Reaction to Central Bank Policy Announcements

B.3.1 Construction of the Dataset

The first step of our analysis is to identify a list of relevant pieces of ECB communication with significant space or time devoted to environmental policy. To identify relevant speeches for our empirical analysis, we rely on a dataset published by the ECB that contains date, title (including sub-titles), speaker, content, and footnotes of nearly all speeches by presidents and

[1] We only observe small differences in the reactions of the cost terms for optimal collateral policy. The haircut on conventional bonds increases by more compared to the main text. As a result, liquidity management costs rise and costs from debt restructuring fall.

Table B.2: Time Series Means with Nominal Rigidities

Moment	**Baseline**	**Max Pref**	**Opt Coll**	**Only Tax**	**Glob Opt**
Tax Parameter χ_c	0	0	0	10%	10%
Haircut ϕ_g	26%	0%	7%	26%	19%
Haircut ϕ_c	26%	100%	31%	26%	19%
Welfare Change (CE)	0%	-0.4608%	0.0088%	0.6979%	0.6987%
Conv. Leverage	42.1%	41.4%	42.1%	42.1%	42.1%
Green Leverage	42.1%	42.5%	42.2%	42.1%	42.1%
Conv. Bond Spread	96bp	162bp	100bp	96bp	94bp
Green Bond Spread	96bp	2bp	80bp	96bp	94bp
Conv. Coll. Premium	-11bp	0bp	-10bp	-11bp	-11bp
Green Coll. Premium	-11bp	-27bp	-14bp	-11bp	-11bp
GDP	0.6869				
Change from Baseline	-	+0.00%	+0.02%	+0.60%	+0.64%
Restructuring Cost/GDP	1.92%				
Change from Baseline	-	-20.22%	-0.04%	-0.14%	+1.96%
Coll. Default Cost/GDP	1.48%				
Change from Baseline	-	-29.68%	+0.26%	-0.37%	+5.36%
Liq. Man. Cost/GDP	4.79%				
Change from Baseline	-	+23.88%	+0.03%	-0.75%	-3.25%
Pollution Cost/GDP	9.95%				
Change from Baseline	-	-1.69%	-0.10%	-9.02%	-8.92%
Inflation Volatility	0.10%				
Change from Baseline	-	-7.73%	-0.05%	+0.49%	+0.51%
Green Bond Share	20.00%	21.17%	20.15%	28%	28%
Green Capital Share	20.00%	20.74%	20.09%	28%	28%

Notes: Maximal preferential treatment (*Max Pref*) only allows green bonds as collateral ($\phi_g = 0, \phi_c = 1$). The optimal collateral policy (*Opt Coll*) is derived from maximizing welfare over haircuts. For the optimal tax (*Only Tax*), we hold haircuts fixed and vary the tax rate. The global optimum (*Glob Opt*) is obtained by maximizing over taxes and haircuts.

board members since 1999 (see European Central Bank, 2021b). We perform the following steps:

- We string-match titles and content separately for the following keywords: climate, green, sustainable, greenhouse, environment, warming, climatic, carbon, coal.
- We designate a speech for manual inspection as soon as we have one match for a title or three matches for content (variations did not change results).
- We exclude a speech if insufficient space is devoted to the topic, there is no monetary policy relation, or for a wrong positive (e.g., *environment* refers to low interest rates).
- We exclude speeches that address climate risk or transition risk.
- Speeches within 20 trading days of the previous speech are excluded to avoid overlapping treatment periods.

We exclude communication that refer to *climate risk* and *transition risk*, since these refer to improving disclosure standards, the extent to which climate risk should be considered in credit risk assessment, and asset stranding. These issues are important for the conduct of central bank policy in general, but do not specifically address bond markets. This leaves us with four speeches. Table B.3 contains details regarding the key content that motivates our classification.

Table B.3: Relevant ECB Policy Announcements

Date	**Person**	**Link**	**Relevant Quotes**
08-11-2018	Benoît Cœuré	ECB	• (...) the ECB, acting within its mandate, can – and should – actively support the transition to a low carbon economy (...) second, by acting accordingly, without prejudice to price stability. • Purchasing green bonds (...) could be an option, as long as the markets are deep and liquid enough.
27-02-2020	Christine Lagarde	ECB	• (...) reviewing the extent to which climate-related risks are understood and priced by the market (...) • (...) evaluate the implications for our own management of risk, in particular through our collateral framework.
17-07-2020	Isabel Schnabel	ECB	• (...) way in which we can contribute is by taking climate considerations into account when designing and implementing our monetary policy operations. • (...) Of course, central banks would need to be mindful of their effects on market functioning. • (...) severe risks to price stability, central banks are required, within their traditional mandates, to strengthen their efforts (...)
21-09-2020	Christine Lagarde	ECB	• We cannot miss this opportunity to reduce and prevent climate risks and finance the necessary green transition. • The ECB's ongoing strategy review will ensure that its monetary policy strategy is fit for purpose (...) • (...) Jean Monnet's words, (...) opportunity for Europe to take a step towards the forms of organisation of the world of to-morrow.

Notes: Speeches are taken from European Central Bank (2021b).

The classification of securities into "green" and "conventional" is based on bonds listed in the "ESG" segments of *Euronext*, the *Frankfurt Stock Exchange* and the *Vienna Stock Exchange*, all of which offer publicly available lists. We limit the analysis to bonds classified as "green" or "sustainable". Since many green bonds do not show up in the *IHS Markit* database, we additionally obtain data from *Thomson Reuters Datastream.* We match green and conventional bonds *one trading-day before* each announcement date using a nearest-neighbors procedure involving coupon, bid-ask spread, maturity, notional amount, and yield spreads. Specifically, we identify an appropriate untreated bond as control group, which is the conventional bond with the smallest distance to the green bond. We drop a green bond if the distance to the closest conventional bond is too high. Table B.4 contains summary statistics regarding the matching. Coupon and bid-ask spreads are very similar for both types of bonds. Spreads of green bonds are higher by between 5 and 8bp, while their maturity is higher by 1.5 years on average.

Table B.4: Matching Green to Conventional Bonds: Summary Statistics

Date		BA-Spread		Coupon		Spread		Maturity		Amount	
	#	Green	Conv.	Green	Conv.	Green	Conv.	Green	Conv.	Green	Conv.
08-11-2018	80	0.34	0.33	1.08	1.05	47.50	42.20	7.6	6.0	716	719
27-02-2020	83	0.36	0.32	1.18	1.15	51.66	44.82	6.7	5.2	695	690
17-07-2020	77	0.45	0.38	1.22	1.22	77.49	72.00	6.6	4.9	693	689
21-09-2020	79	0.38	0.36	1.18	1.14	64.94	56.68	6.3	4.6	701	709

Notes: We denote the number of matches by #. Conv. denotes a *conventional* bond. Bond yield spreads over the Euribor/Swap are expressed in basis points. Bid-ask spread and coupon are relative to a face value of 100, maturity is in years. Amount outstanding is in million EUR.

B.3.2 Yield Reactions

In Figure B.1, we display the average response across treatment dates. The greenium becomes significant two trading days after each announcement and widens to around 16 bp after 20 trading days.

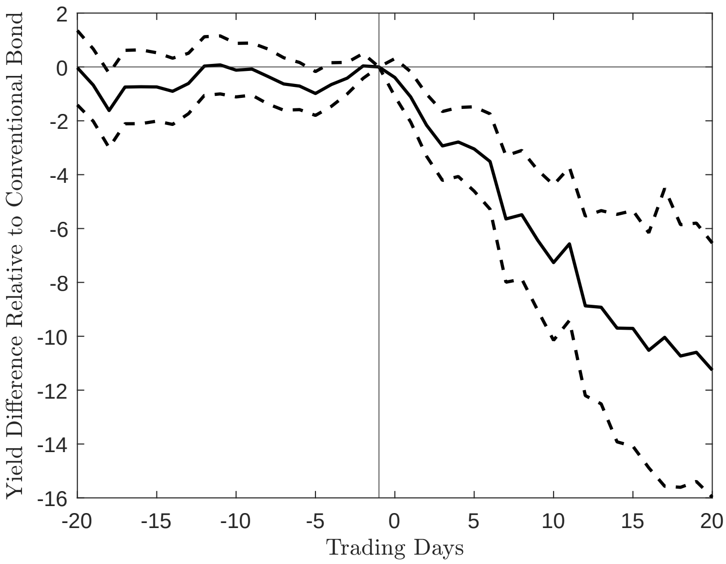


Figure B.1: Average Yield Reaction around Treatment Window

Notes: Results are averaged over all policy announcements. Dashed lines represent 95% confidence intervals. All values in basis points.

Table B.5 gives details on single events. We observe significantly negative premia for green bonds up to one month after the treatment events. The strongest effect is visible for ECB president Christine Lagarde's speech on February 27^{th} 2020, which included the first explicit reference to the ECB's collateral framework. Moreover, the speech delivered by Isabel Schnabel on July 17^{th} 2020 stands out, since yields on green bonds significantly increased compared to their conventional counterparts following the event. However, the tone regarding future ECB environmental policy is much more modest than in other speeches. There is also no explicit prospect of preferential treatment in this speech.[2]

Table B.5: Yield Reaction Around ECB Policy Announcements

Date	**Type**	**Yield Reaction**	**Standard Error**
08-11-2018	Board Member Speech	-7.9***	1.78
27-02-2020	President Speech	-19.4***	3.89
17-07-2020	Board Member Speech	6.8***	1.67
21-09-2020	President Speech	1.3	1.23

Notes: We display the *average* yield over 20 days after minus *average* yield over 20 trading day before the policy announcement , relative to the matched control group (in basis points). Significance levels correspond to 10 % (*), 1 % (**) and 0.1 % (***) of Welch's t-test.

We also perform our analysis for five speeches that are unrelated to environmental policy in Table B.6. We do not find any significantly negative effects and conclude that the overall impact of ECB environmental policy announcement is unlikely to be explained by a general negative trend in the greenium.

Table B.6: Yield Reaction Around Non-Related ECB Policy Announcements

Date	**Type**	**Yield Reaction**	**Standard Error**
01-10-2019	President Speech (ECB)	1.61**	0.82
06-11-2019	Board Member Speech (ECB)	0.77	0.69
16-12-2019	Board Member Speech (ECB)	5.06***	0.80
10-06-2020	Board Member Speech (ECB)	3.39*	2.54
27-08-2020	Board Member Speech (ECB)	1.64**	0.81

Notes: We display the *average* yield over 20 days after minus *average* yield over 20 trading day before the policy announcement , relative to the matched control group (in basis points). Significance levels correspond to 10 % (*), 1 % (**) and 0.1 % (***) of Welch's t-test.

B.4 Data Sources

Table B.7 summarizes the data sources on which our empirical analysis and calibration are based. The classification of bonds as "green" is based on publicly available lists of securities traded via various stock exchanges. Based on the list of ISINs, we retrieve bond-specific info from Datastream. Data on conventional bonds in the control group is taken from Markit. EURIBOR data are also obtained through Datastream. We use the ECB to obtain data on non-financial firm debt, GDP, employment, gross fixed capital formation, private consumption, and the GDP deflator.

[2] For example, central banks "*need to be mindful of their effects on market functioning*" and are required to exert effort towards environmental concerns only "*within their traditional mandates*".

Table B.7: Data Sources and Ticker

Series	**Source**	**Mnemonic**
Green Bond List I	Euronext	List retrieved Nov-30-2020
Green Bond List II	Frankfurt SE	List retrieved Nov-30-2020
Green Bond List III	Vienna SE	List retrieved Nov-30-2020
Constant Maturity Ask Price	Datastream	CMPA
Constant Maturity Bid Price	Datastream	CMPB
Coupon	Datastream	C
Issue Date	Datastream	ID
Amount Outstanding	Datastream	AOS
Currency	Datastream	PCUR
Life At Issue	Datastream	LFIS
Redemption Date	Datastream	RD
EURIBOR rates (... = maturity)	Datastream	TRE6S...Y
Debt-to-GDP	ECB	QSA.Q.N.I8.W0.S11.S1.C.L.LE.F3T4.T._Z.XDC_R_B1GQ_CY._T.S.V.N._T
Markit iBoxx Components	IHS Markit	-
GDP	ECB	MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.EUR.V.N
Gross fixed capital formation	ECB	MNA.Q.Y.I8.W0.S1.S1.D.P51G.N11G._T._Z.EUR.V.N
Consumption	ECB	MNA.Q.Y.I8.W0.S1M.S1.D.P31._Z._Z._T.EUR.V.N
GDP Deflator	ECB	MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.IX.D.N
Employment	ECB	ENA.Q.Y.I8.W2.S1.S1._Z.EMP._Z._T._Z.PS._Z.N

C Appendix to Chapter 4

C.1 Investor Problem

This section presents details on the investor problem and the derivation of the bond pricing formula.

Maximization Problem. The investor decision problem is formulated recursively. An investor's value function in the second sub-period (centralized market) is given as

$$\mathcal{W}_t(\widetilde{b}_t, a_t) = \max_{b_t, c_t} \left\{ c_t + \frac{1}{1+r^{rf}} \mathbb{E}_t \left[\mathcal{V}_{t+1}(b_t) \right] \right\}$$

subject to the budget constraint

$$c_t = e - a_t + k_t \widetilde{b}_t - q_t b_t,$$

where a_t are payments owed to a dealer from trading in the decentralized market in the first sub-period of period t (described below). The probability of becoming an L-type investor is normalized to $\frac{1}{2}$. An investor's value function at the beginning of a period, before idiosyncratic preference shocks are realized, is given by

$$\mathcal{V}_t(b_{t-1}) = \frac{1}{2}\mathcal{V}_{L,t}(b_{t-1}) + \frac{1}{2}\mathcal{V}_{H,t}(b_{t-1}),$$

with the value of being an i-type investor given by

$$\begin{aligned}\mathcal{V}_{i,t}(b_{t-1}) =& u_i\big((1-\kappa_t)m_t\widetilde{b}_{i,t}\big) + \mathbb{E}_{t-1}\left[\mathcal{W}_t(\widetilde{b}_{i,t}, -\widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t})\right] \\ =& u_i\big((1-\kappa_t)m_t\widetilde{b}_{i,t}\big) + m_t(\widetilde{b}_{i,t} - b_t) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t} + \mathbb{E}_{t-1}\left[\mathcal{W}_t(b_t, 0)\right] \ .\end{aligned}$$

The second equality is due to linearity of $\mathcal{W}_t(\cdot)$ with respect to $\widetilde{b}_t$ and a_t.

Decentralized Market. To adjust their bond holdings in response to the i-shock, investors contact dealers. The terms of trade between dealers and investors are determined bilaterally via Nash bargaining. For an i-type investor, the bargaining threat point is

$$\overline{\mathcal{V}}_{i,t}(b_{t-1}) = u_i\big((1-\kappa_t)m_t b_{t-1}\big) + \mathbb{E}_{t-1}\left[\mathcal{W}_t(b_{t-1}, 0)\right],$$

such that the surplus from trading is given as

$$S_{i,t}(b_{t-1}) = u_i(\widetilde{\theta}_{i,t}) - u_i\big((1-\kappa_t)m_t b_{t-1}\big) + m_t(\widetilde{b}_{i,t} - b_{t-1}) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t},$$

where $\widetilde{\theta}_{i,t} \equiv (1-\kappa_t)m_t\widetilde{b}_{i,t}$, $\widetilde{q}_t$ is the competitive bond price on the inter-dealer market and $(\widetilde{b}_{i,t}, \phi_{i,t})$ are the terms of trade, which consist of the investor's bond holdings after the meeting $\widetilde{b}_{i,t}$ and the fee charged by the dealer $\phi_{i,t}$. Payments owed to dealers consist of the fee and the desired adjustment of the bond positions,

$$a_{i,t} = \phi_{i,t} + \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) \ ,$$

and are settled in the subsequent centralized market. A dealer's surplus in an i-type meeting simply equals $\phi_{i,t}$, which is consumed by the dealer in the centralized market. Dealers do not acquire bonds in the centralized market. The investors' bargaining power is α, the terms of trade solve the generalized Nash bargaining problem

$$\max_{\widetilde{b}_{i,t},\phi_{i,t}} \ \Big[u_i\big((1-\kappa_t)m_t\widetilde{b}_{i,t}\big) - u_i\big((1-\kappa_t)m_t b_{t-1}\big) + m_t(\widetilde{b}_{i,t} - b_{t-1}) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t}\Big]^{\alpha} \phi_{i,t}^{1-\alpha},$$

which leads to the two first-order conditions

$$\widetilde{q}_t = (1-\kappa_t)m_t u_i'(\widetilde{\theta}_{i,t}) + m_t, \tag{C.1}$$

$$\phi_{i,t} = (1-\alpha)\Big(u_i(\widetilde{\theta}_{i,t}) - u_i(\theta_t) + m_t(\widetilde{b}_{i,t} - b_{t-1}) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1})\Big). \tag{C.2}$$

Note that the dealer fee simply equals the dealer's bargaining power $1-\alpha$ times the total surplus $S_{i,t}(b_{t-1}) + \phi_{i,t}$. As in Lagos and Rocheteau (2009), the dealer fee can be expressed in terms of a meeting-specific bond price $\widetilde{q}_{i,t}$,

$$\phi_{i,t} = (\widetilde{q}_{i,t} - \widetilde{q}_t)(\widetilde{b}_{i,t} - b_{t-1}).$$

Using this relationship, one can derive the expression

$$\widetilde{q}_{i,t} = \alpha\widetilde{q}_t + (1-\alpha)\frac{u_i(\widetilde{\theta}_{i,t}) - u_i(\theta_t) + m_t(\widetilde{b}_{i,t} - b_{t-1})}{\widetilde{b}_{i,t} - b_{t-1}}.$$

The meeting-specific price $\widetilde{q}_{i,t}$ equals a weighted average of the inter-dealer market price $\widetilde{q}_t$ and the total surplus net of payments $\widetilde{q}_{i,t}(\widetilde{b}_{i,t} - b_{t-1})$ divided by the net trading position. If the investor holds all bargaining power ($\alpha = 1$), the dealer does not charge a mark-up/mark-down, i.e. $\widetilde{q}_{i,t} = \widetilde{q}_t$. If the investor is a net-buyer ($\widetilde{b}_{i,t} > b_{t-1}$), then the ask price of the dealer is $\widetilde{q}_{i,t}$, which exceeds the inter-dealer price $\widetilde{q}_t$ whenever $\alpha < 1$. Similarly, if the investor is a net-seller, $\widetilde{q}_{i,t}$ is the bid price which is below $\widetilde{q}_t$ for $\alpha < 1$. Consistent with the quantitative analysis we normalize the bargaining power to $\alpha = \frac{1}{2}$ in the following. A commonly used measure of the extent to which a market is affected by trading frictions is the bid-ask spread, which is given as

$$\widetilde{q}_{H,t} - \widetilde{q}_{L,t} = \frac{\phi_{H,t}}{\widetilde{b}_{H,t} - b_{t-1}} - \frac{\phi_{L,t}}{\widetilde{b}_{L,t} - b_{t-1}},$$

in our model. Market clearing in the competitive inter-dealer market requires

$$\frac{1}{2}\widetilde{b}_{L,t} + \frac{1}{2}\widetilde{b}_{H,t} = B_{t-1},$$

with government bond supply B_{t-1}, such that

$$u'_L\big((1-\kappa_t)m_t\widetilde{b}_{L,t}\big) = u'_H\Big((1-\kappa_t)m_t \times 2 \times \Big(B_{t-1} - \frac{1}{2}\widetilde{b}_{L,t}\Big)\Big),$$

determines $\widetilde{b}_{L,t}$ and $\widetilde{b}_{H,t}$ via market clearing.

Centralized Market. For the centralized market, the investor first-order condition is

$$-q_t + \frac{1}{1+r^{rf}}\frac{\partial \mathbb{E}_t\left[\mathcal{V}_{t+1}(b_t)\right]}{\partial b_t} = 0. \tag{C.3}$$

The value $\mathcal{V}_{t+1}(\cdot)$ can be written as

$$\mathcal{V}_{t+1}(b_t) = \mathbb{E}_i\left[u_i(\theta_t) + S_{i,t+1}(b_t)\right] + \mathbb{E}_t\left[\mathcal{W}_{t+1}(b_t, 0)\right],$$

such that

$$\begin{aligned}\frac{\partial \mathcal{V}_{t+1}(b_t)}{\partial b_t} &= \mathbb{E}_i\left[(1-\kappa_{t+1})m_{t+1}u'_i(\theta_{t+1}) + \frac{\partial S_{i,t+1}(b_t)}{\partial b_t}\right] + \frac{\partial \mathbb{E}_t\left[\mathcal{W}_{t+1}(b_t,0)\right]}{\partial b_t} \\ &= \mathbb{E}_i\left[(1-\kappa_{t+1})m_{t+1}u'_i(\theta_{t+1}) + \frac{\partial S_{i,t+1}(b_t)}{\partial b_t}\right] + m_{t+1}\ .\end{aligned}$$

The effect of individual bond holdings on trading frictions is given by

$$\begin{aligned}\frac{\partial S_{i,t+1}(b_t)}{\partial b_t} =&(1-\kappa_{t+1})m_{t+1}\frac{\widetilde{b}_i(b_t)}{\partial b_t}u'_i(\widetilde{\theta}_{i,t+1}) - (1-\kappa_{t+1})m_{t+1}u'_i(\theta_{t+1}) \\ &+ m_{t+1}\left(\frac{\widetilde{b}_i(b_t)}{\partial b_t} - 1\right) - \widetilde{q}_{t+1}\left(\frac{\widetilde{b}_i(b_t)}{\partial b_t} - 1\right) - \frac{\partial \phi_i(b_t)}{\partial b_t}.\end{aligned}$$

Using

$$\frac{\partial \phi_i(b_t)}{\partial b_t} = \frac{1}{2}\left[\begin{array}{c}(1-\kappa_{t+1})m_{t+1}\frac{\widetilde{b}_i(b_t)}{\partial b_t}u'_i(\widetilde{\theta}_{i,t+1}) - (1-\kappa_{t+1})m_{t+1}u'_i(\theta_{t+1}) \\ +m_{t+1}\left(\frac{\widetilde{b}_i(b_t)}{\partial b_t} - 1\right) - \widetilde{q}_{t+1}\left(\frac{\widetilde{b}_i(b_t)}{\partial b_t} - 1\right)\end{array}\right],$$

together with (C.1), this derivative can be written as

$$\frac{\partial S_{i,t+1}(b_t)}{\partial b_t} = \frac{1}{2}\left[u'_i(\widetilde{\theta}_{i,t+1}) - u'_i(\theta_{t+1})\right](1-\kappa_{t+1})m_{t+1},$$

such that

$$\frac{\partial \mathcal{V}_{t+1}(b_t)}{\partial b_t} = \mathbb{E}_i \left[u_i'(\theta_{t+1}) + \frac{1}{2} \times \left\{ u_i'(\widetilde{\theta}_{i,t+1}) - u_i'(\theta_{t+1}) \right\} \right] (1 - \kappa_{t+1}) m_{t+1} + m_{t+1}.$$

Combining this condition with the investor first-order condition (C.3) then yields the bond pricing equation (4.1). As in Lagos and Wright (2005), linearity of investor preferences with respect to consumption and idiosyncratic preference shocks being i.i.d. implies that investors choose the same bond holdings b_t in the centralized market regardless of their current holdings $\widetilde{b}_{i,t-1}$.

C.2 Data

This section presents details on the data sources used in this paper.

Debt. Debt service is taken from monthly reports of the Italian Department of Treasury. To focus on debt with appropriate maturity, we focus on redemption and coupon payments of bonds with a maturity of at least one year. Including short-term liabilities (treasury bills, called BOT) would introduce a relatively large amount of debt outstanding, that is rolled over potentially multiple times each period. Coupon data are obtained from annual reports on debt issuance published also by the Treasury.

Income Processes. GDP data for Italy is obtained from the St Louis Fed database, going from 1961Q1 to 2012Q4. Data is in real terms and seasonally adjusted. Data is logged and de-trended using deviations from a linear-quadratic trend. We subsequently impose an AR(1)-structure and get $(\rho_y, \sigma_y^2) = (0.937, 8.45e - 05)$.

Investors. To proxy the discount rate of investors, we use 3-month-EURIBOR data (1991Q1-2012Q4) from the Bundesbank and (quarterly) Euro Area inflation rates. The real rate is simply obtained by subtracting quarterly inflation from 3-month-EURIBOR. Data on German bonds used to construct the maximum convenience yield is taken from Datastream.

C.3 Numerical Solution Method

This section presents details on the numerical solution algorithm used to solve the government problem.

Taste Shocks. We solve the model numerically using value function iteration on a discretize state space without interpolation. Let $\mathfrak{B}$ denote the debt grid. Define $F(B', y) \equiv \mathbb{E}_{y'|y} \mathcal{F}(B', y')$. As already pointed out by Chatterjee and Eyigungor (2012), convergence problems typically arise in this class of models. We follow Gordon (2019) by introducing Gumbel-distributed taste

Table C.1: Data Sources and Ticker

Series	Source	Mnemonic	Frequency
Redemption yield, 5-year benchmark BTP	Bank of Italy	MFN_BMK.M.020.922.0.EUR.205	Monthly average
Turnover, 5-year BTP	Bank of Italy	MFN_QMTS.D.020.926.MKV.EUR.9	Monthly average
Total debt	Bank of Italy	FPI_FP.M-IT.S13.MGD.SBI3.101.112.FAV.EUR.EDP	Monthly
Net issuance, medium and long-term	Bank of Italy	FPI_FP.M.IT.S13.F32.SBI3.103.115.COV.EUR.FPBI	Monthly
Debt service, medium and long-term	Treasury Dep.	Monthly reports	Monthly
Turnover, total debt	Bank of Italy	MFN_QMTS.D.100010.926.MKV.EUR.9	Monthly average
Bid-ask spreads, 5-year BTP	Treasury Dep.	Quarterly Bulletins	Monthly
EURIBOR-SWAP, 5-Year	Datastream	ICEIB5Y	Daily
CDS spread Italy, 5-Year	Bocola (2016)		Daily
3-month-EURIBOR	Bundesbank	BBK01.SU0316	Daily
Euro Area CPI	St Louis Fed	EA19CPALTT01IXOBQ	Quarterly
Redemption yield Germany, 5-Year	Datastream	TRBD5YT	Daily
GDP, Italy	St Louis Fed	LORSGPORITQ661S	Quarterly

shocks z to randomize over debt and default choices:

$$\mathcal{F}^r(B, y', z) = \max_{B'} \left\{ v\left(\tau y + q\left(B', y\right)\left(B' - (1-\delta) B\right) - \widetilde{\delta} B\right) + \widetilde{\beta} F(B', y) + \sigma_z z^{B'}. \right\}$$

Throughout the algorithm we standardize the value function prior to computing choice probabilities and expected choices. The standardization immediately cancels out when choice probabilities are computed. For the expected value of the maximum choice, consider the well-known expression

$$\mathbb{E}\left(\max_j V(j) + \sigma\epsilon_j\right) = \sigma \log\left(\sum_j \exp\left\{\frac{V(j)}{\sigma}\right\}\right)$$

where j denotes next period's choice. Dependence on current persistent states is omitted. Expanding both sides by an arbitrary constant $\overline{V}$ yields

$$\overline{V} + \mathbb{E}\left(\max_j V(j) - \overline{V} + \sigma\epsilon_j\right) = \sigma \log\left(\sum_j \exp\left\{\frac{V(j) - \overline{V}}{\sigma}\right\}\right) \tag{C.4}$$

the right-hand side of (C.4) can be rearranged:

$$\begin{aligned}
\overline{V} + \sigma \log\left(\sum_j \exp\left\{\frac{V(j) - \overline{V}}{\sigma}\right\}\right) &= \overline{V} + \sigma \log\left(\exp\left\{-\frac{-\overline{V}}{\sigma}\right\} \sum_j \exp\left\{\frac{V(j)}{\sigma}\right\}\right) \\
&= \overline{V} + \sigma \log(\exp\left\{-\frac{\overline{V}}{\sigma}\right\}) + \sigma \log\left(\sum_j \exp\left\{\frac{V(j)}{\sigma}\right\}\right) \\
&= \sigma \log\left(\sum_j \exp\left\{\frac{V(j)}{\sigma}\right\}\right)
\end{aligned}$$

Plugging back into (C.4) gives

$$\mathbb{E}\left(\max_j V(j) + \sigma\epsilon_j\right) = \overline{V} + \sigma \log\left(\sum_j \exp\left\{\frac{V(j) - \overline{V}}{\sigma}\right\}\right). \tag{C.5}$$

When $\overline{V} = \max_j V(j)$ is chosen, each term in the exponent on the left hand side is negative, so the problem remains numerically tractable. To speed up computation, we use a version of the divide-and-conquer algorithm proposed by Gordon and Qiu (2018) and Gordon (2019), that truncates numerically irrelevant choices based on policy function monotonicity. The improved convergence properties compared to a brute-force algorithm are particularly relevant for the large state space required by our model. Since there are some (numerically small) non-monotonicities at some points of the state-space, we use a guess-and-verify approach. We compute the equilibrium using the divide-and-conquer approach until convergence. After the final step, we use that solution as to compute another iteration. We consider the solution obtained using divide-and-conquer appropriate if the equilibrium objects stay within a close neighborhood of the solution from the previous iteration.

The Algorithm. To start off the algorithm, we use the final period of a finite-horizon model to determine default states for iteration 0. Given the results of iteration ι (or initialization), each iteration $\iota+1$ proceeds as follows:

1. For all states (B, y'), evaluate debt choices B' according to

$$\left\{\overline{B}, 0, \frac{1}{2}\overline{B}, \frac{3}{4}\overline{B}, \frac{1}{4}\overline{B}, \frac{7}{8}\overline{B}, \frac{5}{8}\overline{B}, \dots\right\}$$

   a) Determine the option value of default F

$$\mathcal{F}^d_{\iota+1}(B, y') = v\left(\tau y' - \underline{g} - \phi(\tau y')\right) + F^d_\iota(B, y')$$

   b) Evaluate every numerically relevant debt choice in $\mathfrak{B}$, given current debt over the interval $[B'_-, B'_+]$ (elements of the grid $\mathfrak{B}$) with

$$B'_- = \min\Big(B' | P\left(\mathcal{B}(B_-)\right) > \epsilon\Big), \quad \text{and} \quad B'_+ \max\Big(B' | P\left(\mathcal{B}(B_+)\right) > \epsilon\Big),$$

   where B_+ and B_- denote the next larger/smaller current debt stock, that has already been evaluated. At the upper bound, the choices are not bounded, while for $\underline{B} = 0$, they are only bounded from above. The value of choosing $B^j \in \mathfrak{B} | B_- < B^j < B_+$ is given by

$$\mathcal{F}^r_{\iota+1}(B^j | B, y') = v\left(\tau y' - \underline{g} + \mathcal{Q}^r_\iota(B^j, y')\left(B^j - (1-\delta)B\right) - \widetilde{\delta} B\right) + \widetilde{\beta} F_\iota(B^j, y')$$

   c) Compute debt choice and default probabilities using the Type I-extreme value distribution:

$$\Pr(d = 1 | B, y') = \frac{\exp\left(\mathcal{F}^d_{\iota+1}(B, y')/\sigma_z\right)}{\exp\left(\mathcal{F}^d_{\iota+1}(B, y')/\sigma_z\right) + \sum_j \exp\left(\mathcal{F}^r_{\iota+1}(B^j, y')/\sigma_z\right)}$$
$$\Pr(B' = B^j | B, y') = \frac{\exp\left(\mathcal{F}^r_{\iota+1}(B^j, y')/\sigma_z\right)}{\exp\left(\mathcal{F}^d_{\iota+1}(B, y')/\sigma_z\right) + \sum_j \exp\left(\mathcal{F}^r_{\iota+1}(B^j, y')/\sigma_z\right)}$$

   d) Determine expected pecuniary payoffs (k^r and k^d) w.r.t. z using the probabilities computed in (c):

$$\begin{aligned}
\mathcal{D}_{\iota+1}(B, y') &= \Pr(d = 1 | B, y') \\
k^r_{\iota+1}(B, y') &= \sum_j \Pr(B^j | B, y') \left[\widetilde{\delta} + (1-\delta)\mathcal{Q}^r_\iota(B^j, y')\right] \\
k^d_{\iota+1}(B, y') &= \vartheta\omega \left\{\begin{array}{c} (1 - \mathcal{D}_\iota(\omega B, P(\omega B, y'), y')) \\ \times\left(\widetilde{\delta} + (1-\delta)\mathcal{Q}^r_\iota(\omega B, y')\right) \\ + \mathcal{D}_\iota(\omega B, y')\mathcal{Q}^d_\iota(\omega B, y') \end{array}\right\} + (1-\vartheta)\mathcal{Q}^d_\iota(B, y')
\end{aligned}$$

where the value under re-entry is interpolated linearly, since $(1-\omega)B$ is not necessarily part of $\mathfrak{B}$.

e) Update expected value functions $\mathcal{F}$, default probability λ and pecuniary payoffs m w.r.t. the persistent exogenous state:

$$
\begin{aligned}
&F_{\iota+1}(B,y) = \Pi\mathcal{F}_{\iota+1}(B,y')\ , \quad F^d_{\iota+1}(B,y) = \Pi\mathcal{F}^d_{\iota+1}(B,y'),\\
&m^r_{\iota+1}(B,y) = \Pi k^r_{\iota+1}(B,y')\ , \quad m^d_{\iota+1}(B,y) = \Pi k^d_{\iota+1}(B,y'),\\
&\lambda_{\iota+1}(B,y) = \Pi\mathcal{D}_{\iota+1}(B,y').
\end{aligned}
$$

f) Compute haircuts κ, haircut weighted collateral Θ, and adjusted bond holdings for both investor types in the repayment case, and combine these elements to get the non-pecuniary part of the payoff, Λ:

$$
\begin{aligned}
\kappa^r_{\iota+1}(B,y) =&\kappa(\lambda_{\iota+1})\ ,\\
\Theta^r_{\iota+1}(B,y) =&(1-\kappa^r_{\iota+1}(B,y)) \times m^r_{\iota+1}(B,y') \times B\ ,\\
\Theta^d_{\iota+1}(B,y) =&(1-\overline{\kappa}) \times m^d_{\iota+1}(B,y') \times B\ ,\\
\Theta^r_{H,\iota+1}(B,y) =&\Theta^r_{\iota+1}(B,y) - \frac{1}{2}\zeta_2\ ,\\
\Theta^d_{H,\iota+1}(B,y) =&\Theta^d_{\iota+1}(B,y) - \frac{1}{2}\zeta_2\ ,\\
\Theta^r_{L,\iota+1}(B,y) =&\Theta^r_{\iota+1}(B,y) + \frac{1}{2}\zeta_2\ ,\\
\Theta^d_{L,\iota+1}(B,y) =&\Theta^d_{\iota+1}(B,y) + \frac{1}{2}\zeta_2\ ,\\
\Lambda^r_{\iota+1}(B,y) =&(1-\kappa^r_{\iota+1}(B,y))\Big(\frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^r_{L,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^r_{\iota+1}(B,y')\big)\big)\\
&+\frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^r_{H,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^r_{\iota+1}(B,y')\big)\big)\Big)\\
\Lambda^d_{\iota+1}(B,y) =&(1-\overline{\kappa})\Big(\frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^d_{L,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^d_{\iota+1}(B,y')\big)\big)\ ,\\
&+\frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^d_{H,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^d_{\iota+1}(B,y')\big)\big)\Big)\ .
\end{aligned}
$$

g) Update bond prices in repayment and default

$$
\begin{aligned}
&\mathcal{Q}^r_{\iota+1}(B,y) = m^r_{\iota+1}(B,y)\big(1+\Lambda^r_{\iota+1}(B,y)\big)/(1+r^{rf}),\\
&\mathcal{Q}^d_{\iota+1}(B,y) = m^d_{\iota+1}(B,y)\big(1+\Lambda^d_{\iota+1}(B,y)\big)/(1+r^{rf})\ .
\end{aligned}
$$

h) Apply the cap on bond prices.

Table C.2: Parameters of Computational Algorithm

Number of grid points for income	$n_y = 201$
Size of income grid	$y \in [-3\sigma_y, +3\sigma_y]$
Number of grid points for debt	$n_B = 301$
Size of debt grid	$B \in [0, 6]$
Taste shock parameter	$\sigma_z = 1$
Maximum default probability	$\overline{\lambda} = 0.75$
Minimum bond spread	$\underline{s} = -105\text{bp}$

2. Compute convergence criteria:

$$\Delta \mathcal{F}_{\iota+1} = \max \left\{ \left|\left| \mathcal{F}_{\iota+1} - \mathcal{F}_{\iota} \right|\right|_{\infty}, \left|\left| \mathcal{F}^d_{\iota+1} - \mathcal{F}^d_{\iota} \right|\right|_{\infty} \right\}$$

$$\Delta \mathcal{Q}_{\iota+1} = \max \left\{ \left|\left| \mathcal{Q}^r_{\iota+1} - \mathcal{Q}^r_{\iota} \right|\right|_{\infty}, \left|\left| \mathcal{Q}^d_{\iota+1} - \mathcal{Q}^d_{\iota} \right|\right|_{\infty} \right\}$$

3. If $\Delta \mathcal{Q}_{\iota+1} < \epsilon \ \wedge \ \Delta \mathcal{W}_{\iota+1} < \epsilon$ BREAK, else $\iota = \iota + 1$ and go to step 1.

Table C.2 summarizes grid width and granularity for income and debt, respectively. Results do not visibly change when grid sizes are increased further. The standard deviation of the taste shock is chosen to ensure stable convergence for all parameter combinations, which is typically achieved in less then 1000 iterations.

Following Chatterjee and Eyigungor (2015), we impose a maximum default probability to rule out extreme debt dilution when the economy is close to default. The maximum default probability is set to 0.75. To rule out implausibly high valuation of collateral services for small amounts of debt outstanding, we also impose a minimum bond spread (or, alternatively, maximum convenience yield). The minimum spread of a 5-year German government bond over the 5-year-interest-rate-swap rate observed in our sample, which was –105bp, serves as lower bound on bond spreads in our calibration. Like the maximum default probability, this constraint does not bind during model simulation and all our numerical results are robust to changes to either constraint.

D Appendix to Chapter 5

D.1 Data

This section provide an overview of the cross-section of euro area members, before and during the European sovereign debt crisis. The sample is excluding Austria, Cyprus and Luxembourg due to their small size and limited data availability. Government bond spreads are computed from the yield-to-maturity of 5-year benchmark bonds, obtained from *Thomson Reuters Datastream*. The risk-free rate is proxied by 5y-EURIBOR-swaps.

The cross-sectional distribution in the pre-crisis period is shown in Table D.1, where the reported spreads are averages from daily data between 2009-01-01 until 2011-06-30. For government bond spreads For debt outstanding and ratings, I use data from 2010Q4 as a cut-off. For all countries except Greece, CDS-spreads are substantially larger than government bond spreads. This so-called 'CDS-bond basis' is largest for the German bund (69bp), still large for Italian (49bp) and Spanish (54bp) bonds and becomes small for the riskiest countries (9bp for Portuguese and 7bp for Irish bonds). The empirical cumulative distribution function of bond- and CDS-spreads is given in the last column, where the weighting corresponds to the market value of debt outstanding. The upper and lower quartile correspond to the German (-35bp) and Italian spread (69bp), respectively.

Table D.1: Cross-Section of Euro Area Members, Pre-crisis

Country	Bond-Spread	CDS-Spread	Rating	Debt/GDP (%)	Debt (billions)	Cumulative (%)
Germany	-35	34	AAA	87	2271	26
Finland	-15	35	AAA	55	105	28
Netherlands	-14	33	AAA	70	449	33
France	-12	49	AA	101	2032	56
Belgium	30	89	AA	109	398	61
Italy	69	118	A	124	2018	84
Spain	86	140	A	67	723	93
Portugal	220	229	A	106	139	95
Ireland	291	298	A	83	190	97
Greece	627	570	HY	131	283	100
Total					8609	

Notes: Bond and CDS-preads are annualized in basis points. Countries are sorted by bond spreads in the pre-crisis period. The rating refers to the highest rating by the four recognized credit assessment agencies Moody's, S&P, Fitch, and DRBS, which is in accordance with the ECB collateral framework. Market value of debt outstanding in EUR.

Table D.2 shows the data for the crisis period. I use average spreads from 2011-07-01 until 2012-06-30, which is typically considered to be the most severe phase of the European sovereign debt crisis. Debt outstanding refers to 2011Q4. All rating downgrades relevant for collateral valuation occurred in the crisis period. Compared to the pre-crisis period, CDS-spreads increased for all countries, where the increase is most pronounced in the 'periphery' countries (Italy, Spain, Portugal, Ireland and Greece). The rating downgrades in the case of Italy, Spain and Portugal imply larger haircuts on their bonds according to the ECB's collateral framework (Nyborg, 2017). For example, government bonds with a maturity of 5 to 7 years were subject to an 8% haircut after the downgrade, compared to a 3% haircut before.

In contrast, the spreads on German, Finish, and Dutch government bonds show a decline compared to the pre-crisis subsample, which is most pronounced for the German bund. The CDS-bond basis increases to 118bp for the bund, while it reduces to 19bp (11bp) in the case of Italy (Spain). The CDS-bond basis for Portugal, Ireland, and Greece is less reliable in the crisis period, due to exceptionally large volatilities and market illiquidity.

Table D.2: Cross-Section of Euro Area Members, Crisis

Country	Bond-Spread	CDS-Spread	Rating	Debt/GDP (%)	Debt (billions)	Cumulative (%)
Germany	-73	45	AAA	89	2375	25
Finland	-29	60	AAA	63	121	26
Netherlands	-27	66	AAA	79	507	31
France	15	110	AA	112	2290	55
Belgium	109	180	AA	121	450	60
Italy	340	359	BBB	135	2109	82
Spain	326	337	BBB	93	955	92
Portugal	1153	984	BBB	137	228	94
Ireland	558	600	A	129	220	97
Greece	3493	10120	HY	168	310	100
Total				9565		

Notes: Bond and CDS-spreads are annualized in basis points. Countries are sorted by bond spreads in the pre-crisis period. The rating refers to the highest rating by the four recognized credit assessment agencies Moody's, S&P, Fitch, and DRBS, which is in accordance with the ECB collateral framework. Market value of debt outstanding in EUR.

D.2 Numerical Details

The model is solved using value function iteration on a discrete grid for government bond holdings with $n_b = 301$ points equally distributed over the grid $[0, 3]$. The idiosyncratic income shock is discretized using the method of Tauchen (1986) on an equispaced grid with $n_y = 101$ points over the interval $[-3\frac{\sigma_\mu}{1-\rho_\mu^2}, 3\frac{\sigma_\mu}{1-\rho_\mu^2}]$. As in Gordon (2019), I use taste shocks over potential debt choices b_{t+1}^j to address the typical convergence issues in this class of model.

To compute the cross-sectional distribution, note that the debt policy function $\mathcal{B}(b_t^j, \theta_t^j, y_t^j)$ defines an endogenous mass-shifter Π_b mapping idiosyncratic states into debt choices. Similar to the computational algorithm in Kaldorf and Wicknig (2022), I use the mode of the distribution over debt choices $\mathcal{B}(b_t^j, \theta_t^j, y_t^j)$ when setting up the mass-shifter. Together with the transition matrices of the transferable revenue share Π_θ and government revenues Π_y, the combined mass shifter is given by $\Pi_g = \Pi_b \otimes \Pi_\theta \otimes \Pi_y$. Π_z is a sparse matrix that implicitly defines the firm distribution G via $G' = G'\Pi_g$. Extracting the distribution thus boils down to computing the right Eigenvalue to Π_g'. This is numerically feasible since Π_g is sparse. Once G is known, collateral supply $\overline{B}$ is obtained by aggregating over the collateral supply $(1-\kappa_t^j)b_{t+1}^j k_t^j$ by individual governments.

Computational Algorithm.

1. At iteration step $\iota = 0$, guess government debt policy as $\mathcal{B}(b_t^j, \theta_t^j, y_t^j) = b_t^j$ and compute implied aggregate collateral supply and the bond price schedule.

2. Given the bond price schedule and value function from the previous iteration

   (i) solve the government problem and compute the corresponding endogenous mass shifter Π_b,

   (ii) obtain the mass shifter Π_g and update the cross-sectional distribution G,

   (iii) compute $\overline{B}$ and the associated collateral valuation $l_0 \cdot \exp\left\{-l_1\overline{B}\right\}$,

   (iv) update haircuts and bond prices using the updated debt policy,

   (v) if $\mathcal{V}(b_t^j, \theta_t^j, y_t^j)$ and $q(b_{t+1}^j, \theta_t^j, y_t^j)$ converge, STOP, else go back to (i).

Mapping the Cross-Sectional Distribution to the Data. Due to the small number of euro area members making up the cross-section of borrowers to which the model is calibrated, some remarks on the model implied cross-sectional distribution are in order. While in principle, this is an infinite-dimensional object, it becomes finite-dimensional in the process of discretization. To illustrate the mapping from the model into the sample of euro area governments presented in appendix D.1, consider the left tail of the bond spread distribution, represented in the calibration by the 25%-quantile. As shown in the last column of Table D.1, this corresponds to Germany in the data. Since Germany accounted for 26% of the euro area government bond market, I numerically integrate over the interval $[0, 0.26]$ of the model-implied distribution over bond spreads, CDS-spreads, haircuts, and debt outstanding. Likewise, the 75%-quantile is given by Italy in the data and I integrate the model-implied cross-section over the interval $[0.61, 0.84]$.

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