Abstract

We consider the problem of deciding whether a given graph $G$ is a graph power as well as finding a root of $G$, if such a root exists. There is a wide range of applications of graph powers in practice. Besides the development of distributed algorithms, we find this concept in social media and telecommunications.

The problem of deciding whether a given graph $G$ is a square, as well as finding a square root of $G$ if existent, is already NP-hard, as was shown by Motwani and Sudan. In this thesis we concentrate on two restrictions of the general square root problem: On the one hand, we consider graphs that admit a square root that has certain properties. On the other hand, we consider input graphs that belong to certain graph classes. Both of these two approaches are already well established.

We develop a polynomial time algorithm that decides whether a given graph admits a ptolemaic square root, and if so, constructs such a root with minimum number of edges. Moreover, in linear time we can decide whether a given line graph is a square, and if so, construct a square root. For $k \in \mathbb{N}$, $k < 5$, we give polynomial time algorithms for the square root problem for the classes of $k$–split graphs and graphs with clique number less than $k$. For a given input graph $G$ with clique number at most four we can decide in polynomial time whether there exists a chordal square root of $G$ and if such a root exists, construct a
chordal square root with minimum number of edges simultaneously.