Essays in Monetary Economics

Inauguraldissertation
zur
Erlangung des Doktorgrades
der
Wirtschafts- und Sozialwissenschaftlichen Fakultät
der
Universität zu Köln

2023

vorgelegt
von

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Tag der Promotion:

Acknowledgements

Having written down the last lines of the present thesis, I would like to express my gratitude to those people who supported me during my doctoral studies.

First and foremost, I would like to sincerely thank my advisors for their support at all stages of my dissertation. Johannes Pfeifer's office door was always open when I asked for guidance on technical or intuitive questions and when I needed help to structure my thoughts. My papers greatly benefited from his technical rigour and his sense for conciseness. Michael Krause significantly supported me in straightening the exposition and analysis of my papers. I would like to thank him for always reminding me to never lose sight of the intuition of certain assumptions and results as well as the policy implications of my research.

Further, I would like to thank Martin Barbie, Peter Funk, Emanuel Hansen, Erik Hornung, Andreas Schabert and Paul Schempp for helpful suggestions and comments on my papers and during internal seminars. I thank Sebastian Hauptmeier and Christophe Kamps for numerous discussions as part of the joint research project at the European Central Bank. These discussions were helpful for developing the research idea for Chapter 2. Moreover, I would like to thank Benjamin Born who motivated me to dig deeper into New Keynesian models and played a decisive role for my way as a doctoral student at the University of Cologne.

Working on my dissertation was considerably facilitated by the productive and at the same time pleasant and joyful environment at the Center for Macroeconomic Research. I would like to thank my (former) colleagues Raphael Flore, Tobias Föll, Philipp Giesa, Anna Hartmann, Jonas Loebbing, Carola Stapper and Marius Vogel. In particular, I would like to thank Eduardo Hildago and Stefan Hasenclever for various funny coffee breaks, hilarious stories, and long evenings in Köln Süd. In addition, I thank Ina Dinstühler, Diana Frangenberg, Yvonne Havertz, Sylvia Hoffmeyer and Kristin Winnefeld for their support on administrative matters.

I am indebted to my co-authors and friends Francesco Giovanardi, Matthias Kaldorf and Florian Wicknig. Without them, my dissertation would not have been possible. We spent quite a lot time in deriving model solutions at the white board, in coding, and in jointly promoting our own research. These efforts resulted in Chapter 1 and 3 of this dissertation. Besides our joint research projects, we also spent a lot of time off-campus, which yielded various unforgettable memories.

Moreover, I would like to thank my parents for their invaluable and never-ceasing support – *come rain, come shine.* In particular, I would like to thank my father for his time and diligence in reading my work since the beginning of my studies. My parents-in-law were a huge support in taking care of my son at the end of my dissertation and always accommodated my son, my wife, and me when I needed time to work on my thesis. Finally, I would like to thank my wife for her absolute support and for clearing up any doubts along the long way. During this journey, she was always by my side, which I will never forget.

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Introduction

In response to various economic changes and challenges that emerged during the preceding two decades, the European Central Bank (ECB) recently conducted a review of its monetary policy strategy which was concluded in July 2021 (European Central Bank, 2021a). Three important topics considered within its strategy review were (i) the role of inflation expectations for monetary policy's transmission on macroeconomic variables, (ii) the supporting role of fiscal policy for stabilising the economy in face of the zero lower bound (ZLB) on the nominal interest rate, and (iii) an evaluation of monetary policy instruments to actively curb the adverse effects of climate change on the economy.

A major challenge for central banks was posed by the prolonged period in which monetary policy's primary policy tool, the short-term nominal interest rate, was effectively zero, while inflation and inflation expectations fell persistently below the central bank's target value. Policymakers were concerned that the persistent undershooting of inflation below its target value could un-anchor inflation expectations (Powell, 2020; Schnabel, 2021). In consequence, the ECB's strategy review launched an analysis of the determinants of inflation expectations to assess their impact on the transmission of monetary policy as well as central bank's effectiveness in anchoring expectations (European Central Bank, 2021a, pp. 3-4).

At the same time, the ECB emphasised that unconventional monetary policy instruments, such as forward guidance and quantitative easing, mitigated the disinflationary pressures during the ZLB episode. However, the academic literature cautions that there are circumstances in which these alternative monetary policy instruments alone are insufficient to stabilise the economy (Corsetti et al., 2019). For that reason, the ECB underscored the importance of fiscal policy for supporting monetary policy in stabilising the economy in face of the ZLB (European Central Bank, 2021a, p. 10).

In addition to the challenges resulting from the low interest rate environment, the ECB acknowledged that the economic implications of climate change affect the conduct of monetary policy (European Central Bank, 2021a, p. 13). As a result, the ECB analysed how to adapt its monetary policy framework to the economic and financial risks associated with climate change and, further, asserted potential monetary policy instruments to actively curb climate change.

The present thesis contains three chapters that contribute to the discussion on the aforementioned topics that were launched by the ECB's strategy review. Chapter 1 extends the understanding of the effect of private sector expectations on monetary policy's transmission mechanism on macroeconomic variables. Chapter 2 broadens the discussion on the role of fiscal policy for macroeconomic stabilisation when monetary policy stabilisation via the nominal interest rate is occasionally constrained by the ZLB. Further, Chapter 3 examines the effectiveness of monetary policy in curbing the adverse effects of climate change for the economy. The common theme of the three chapters is the analysis of topics that are of high relevance for central bank's monetary policy strategy, both in the recent past and ongoing present.

Chapter 1, which is based on Radke and Wicknig (2023), analyses the implications of heterogeneity in expectations for the transmission of monetary policy on macroeconomic variables. The workhorse model used for a quantitative analysis of monetary policy (Galí, 2015, Ch. 3) typically assumes rational expectations, which implies that expectations are homogenous among private agents. Empirically, however, expectations display strong cross-sectional differences. Malmendier and Nagel (2016) document a heterogeneity in households' inflation expectations across age groups. They provide empirical evidence that a significant share of this heterogeneity can be explained by differences in agents' inflation experiences over their lifetime. The research in Chapter 1 builds on their empirical evidence and analyses the implications of experience-based heterogeneity in expectations for the transmission of monetary policy.

We rely on a New Keynesian model with overlapping generations and assume that private agents form expectations as specified in Malmendier and Nagel (2016). Their experience-based learning (EBL) approach builds on the literature on adaptive learning, which is based on Evans and Honkapohja (2001). The adaptive learning approach assumes that agents behave like econometricians who constantly revise their beliefs as new information becomes available. In contrast to the standard learning approach, however, EBL assumes that agents only use lifetime observations to revise their beliefs and that young agents attach a higher weight to more recent observations than old individuals. To illustrate the implications of EBL for the monetary policy transmission, we compare it to the one under constant-gain learning (CGL), which is commonly used in the adaptive learning literature (Branch and Evans, 2006). Under CGL, agents put the same weight on new observations such that expectations are homogenous across age groups.

The key finding of Chapter 1 is that EBL endogenously reduces agents' perceived persistence of inflation and the output gap and, thereby, impairs the transmission of monetary policy on inflation via expectations. As a result of the impaired transmission of monetary policy on inflation, there are three policy implications for central banks. First, the impact of unanticipated monetary policy shocks on inflation under EBL is less pronounced than under CGL. Second, monetary policy's trade-off between stabilising inflation and the output-gap under supply shocks aggravates in that any reduction in inflation volatility is associated with a larger increase in the output gap volatility. Third, since expectations are heterogeneous across cohorts, variations in the age distribution affect aggregate expectations through a composition effect. We show that an increase in the share of old individuals strengthens monetary policy's transmission on inflation, which is reinforced under EBL through the composition effect. Thereby, the monetary policy stabilisation trade-off under supply shocks attenuates in old economies.

Chapter 2, which is based on Radke (2023), contributes to the debate on the role of fiscal stabilisation policy when the central bank's primary policy tool, i.e. the short-term nominal interest rate, is occasionally constrained by the ZLB. The previous literature has analysed both normative and positive implications of fiscal stabilisation policy during ZLB episodes. However, even in periods where the ZLB is not binding, the mere possibility of encountering the ZLB in the future, which is referred to as ZLB risk, exacerbates monetary policy's stabilisation policy. Bianchi et al. (2021) and Hills et al. (2019) show that, even in the absence of fundamental shocks, inflation falls below the central bank's desired target value, which they refer to as deflationary bias. While the literature has analysed adjustments of monetary policy rules to address the deflationary bias, the same does not hold for fiscal policy rules.

For this reason, Chapter 2 analyses how a systematic response of government spending to output affects the deflationary bias that is caused by the ZLB risk. The analysis is based on a New Keynesian model in which the representative household's utility positively depends on the level of government spending. Fiscal policy finances government spending via lump-sum taxes and follows a simple rule that sets government spending as a function of output. Monetary policy sets the nominal interest rate according to a simple rule. Due to exogenous demand-side shocks, the ZLB on the nominal interest rate is occasionally binding.

The results in Chapter 2 bear four policy implications for central banks. First, counter-cyclical government spending can substantially reduce the deflationary bias, thereby mitigating the adverse effects of the ZLB risk on monetary policy's stabilisation policy. In turn, pro-cyclical fiscal policy can substantially increase the deflationary bias and may even prevent the existence of an equilibrium. Second, the fiscal output feedback that minimises the welfare cost relative to the optimal policy under commitment renders government spending strongly counter-cyclical. The optimised degree of counter-cyclicality balances the gains from an improved stabilisation of inflation and output against the cost resulting from a higher volatility in consumption and government spending. Third, an increase in the deflationary bias that results from heightened ZLB risk or a lower degree of price stickiness is mitigated by counter-cyclical fiscal policy. Fourth, a more aggressive response of monetary policy to inflation already reduces the deflationary bias and generates a sizeable reduction in the welfare cost relative to the optimal commitment policy. In this case, fiscal stabilisation policy becomes relatively less important in supporting the central bank to stabilise inflation at the desired target value. Further, the additional reduction in the welfare cost from counter-cyclical fiscal policy attenuates when the monetary policy inflation response increases.

Chapter 3, which is based on Giovanardi et al. (2023), scrutinises the preferential treatment of green bonds in central bank's collateral framework as an instrument to mitigate the adverse implications of climate change for the economy. Such tilting of the collateral framework towards green bonds was proposed within the ECB's strategy review (Drudi et al., 2021, p. 18). The People's Bank of China already implemented such a policy tool in 2018, which empirically led to a decline in the yields of green relative to conventional bonds (Macaire and Naef, 2022). However, the implications of such a policy tool for macroeconomic dynamics, the emission of greenhouse gas (GHG), green investment and bond issuance as well as for financial stability have not been considered so far by the literature. The research in Chapter 3 adds upon the literature by jointly analysing the effect of a preferential treatment of green bonds on macroeconomic, financial, and environmental variables.

Our quantitative analysis relies on a real-business cycle model, which we extend along three dimensions. We incorporate an intermediate goods sector that includes green and conventional firms. The production of the conventional intermediate good generates pollution and, thereby, decreases the output of the final good. Further, intermediate good firms finance their capital investment by issuing bonds that are subject to default risk. The default of bonds generates an economic resource loss. Moreover, we incorporate banks who collect deposits from households, purchase bonds of firms and incur liquidity management costs. The latter are decreasing in the amount of bonds that banks hold in their balance sheet, which reflects that banks can use these bonds in collateralised borrowing with the central bank. The aggregate supply of collateral depends on central bank's (potentially different) haircuts on green and conventional bonds, which control the degree by which banks can use these bonds as collateral.

We calibrate the model to the euro area and unveil four important policy implications for central banks. First, the preferential treatment of green bonds reduces the financing costs of green firms relative to those of conventional firms. The resulting relative increase in investments of green firms reduces the emission of GHG. Second, the preferential treatment increases green firms' bond issuance and, importantly, their leverage which increases the resource loss generated by heightened bond default. Hence, the optimal collateral framework balances the gains from a higher collateral supply to banks and a reduction in the pollution externality against the costs stemming from a heightened default of bonds. Third, we show that a preferential treatment of green bonds is an imperfect substitute for a Pigouvian emission tax. The tax is substantially more effective in reducing emissions and generates sizeable welfare gains without generating adverse side effects on firms' risk-taking behaviour. Fourth, we demonstrate that the central bank's collateral framework should only treat green bonds preferentially if and only if the Pigouvian tax is below its optimal level.

Contribution to each Chapter. Chapter 1 is joint work with Florian Wicknig. We jointly developed the research idea and the model framework. While Florian Wicknig provided most of the paper's draft, I mostly implemented the computer code for the quantitative analysis. The refinement of the final draft was conducted jointly.

Chapter 2 is single-authored. The research idea in Chapter 2 was developed as part of the joint research project with Sebastian Hauptmeier and Christophe Kamps, which resulted in the paper Hauptmeier et al. (2022). In the latter paper, Sebastian Hauptmeier and Christophe Kamps formulated the research idea, while I developed the model as well as the computer code for the quantitative analysis. Further, I drafted the initial version of the paper, which was refined by Sebastian Hauptmeier and Christophe Kamps.

The model presented in the second section of Chapter 2 relates to the one in Hauptmeier et al. (2022) but (i) excludes government debt as an additional state variable and (ii) considers a fiscal rule for government spending that only includes a response to deviations of output from its deterministic steady state. Further, the research in Chapter 2 differs from the one in Hauptmeier et al. (2022) in three important ways.

1. The focus in Chapter 2 is distinct from the one in Hauptmeier et al. (2022). In the latter work, we focus on the determination of the optimal rule-based interaction between monetary and fiscal policy. In particular, we evaluate the welfare effects of a systematic response of government spending to inflation relative to one to output. In contrast, Chapter 2 focuses on the effect of cyclical government spending on the deflationary bias that is caused by the ZLB risk.

2. Hauptmeier et al. (2022) evaluate the monetary and fiscal policy rules in comparison to the first best allocation, that maximises household utility taking into account only the technological and resource constraints.
   In contrast, Chapter 2 evaluates the simple government spending rule in comparison to the optimal monetary and fiscal policy under commitment, which maximises households' welfare taking into account the constraints present in the decentralised economy.

3. Chapter 2 extends the analysis in Hauptmeier et al. (2022) and emphasises the adverse effects of pro-cyclical government spending on the welfare and the deflationary bias.

Chapter 3 is joint work with Francesco Giovanardi, Matthias Kaldorf and Florian Wicknig. The research idea, the simple model as well as the quantitative model framework were developed in collaboration. Florian Wicknig mostly contributed to the empirical analysis. In joint work with Francesco Giovanardi and Matthias Kaldorf, I contributed to the computer code that generated the results for the quantitative policy analysis. The initial draft was mostly written by Matthias Kaldorf and Florian Wicknig, while Francesco Gionvanardi and I refined the draft later on.

Chapter 1

Experience-Based Heterogeneity in Expectations and Monetary Policy

This chapter is based on Radke and Wicknig (2023).

1.1 Introduction

Private sector expectations are a key determinant for the implementation of monetary policy. Most New Keynesian models employ rational, that is, homogeneous expectations. Empirically, however, expectations exhibit substantial cross-sectional heterogeneity (see Mankiw et al., 2003) that has been shown to alter the propagation of shocks and the transmission of monetary policy (see Branch and McGough, 2018).

Earlier studies introduce expectational heterogeneity exogenously.[1] In this paper, we instead relax the assumption of homogeneous expectations using endogenous expectation heterogeneity in an overlapping generations New Keynesian model featuring *experience-based learning* (EBL) (Malmendier and Nagel, 2016). Expectations are a function of the different economic experiences that individuals made over their lifetime. In consequence, agents of different age have heterogeneous expectations. Since aggregate expectations are a size-weighted average over cohort-specific expectations, a variation in the demographic structure affects aggregate expectations through a composition effect that we call the *Experience Channel*. Aside from investigating the impact of an empirically-relevant form of expectation heterogeneity, EBL allows us to explore how demographic factors affect the monetary policy transmission, which is becoming a relevant topic for central banks (e.g. Eggertsson et al., 2019).

We show that experience-based heterogeneity in expectations across age groups weakens the transmission of monetary policy on inflation and the output gap. The weaker

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[1] Those studies often exogenously divide the population into individuals with different forecasting models or endow agents with a discrete choice from a finite set of predictors.

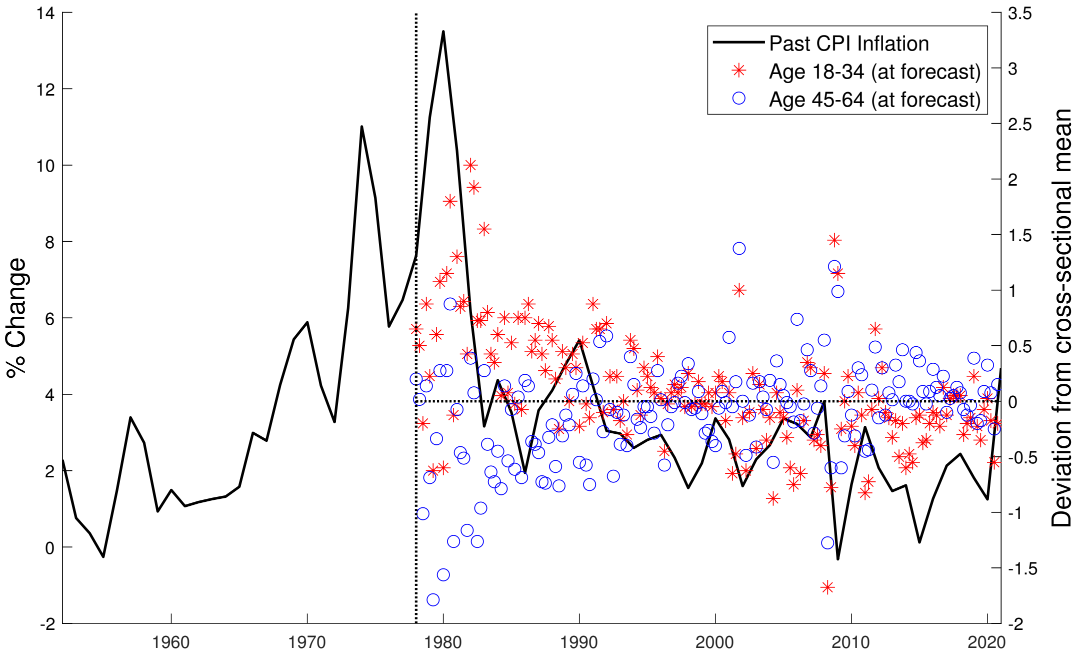

Figure 1.1: Realised Inflation vs. One-Year Ahead Inflation Expectations Across Cohorts

Notes: left y-axis: annual percentage change of the seasonally-adjusted U.S.-CPI (solid black line). Right y-axis: quarterly 4-quarter moving average of cohort inflation expectations, expressed as percentage deviations from the cross-sectional mean (markers). We take a moving-average to concentrate on lower frequency variation. Inflation expectations are based on the Michigan Survey of Consumers (question: *Expected Change in Prices During the Next Year*) from 1978Q1 to 2020Q4. Cohorts define persons of a certain age group at a specific point in time, i.e., no age group is tracked over time.

transmission stems from the fact that monetary policy's influence on expectations is lower if only lifetime experiences are used to form expectations. As a result of monetary policy's reduced impact on expectations, the stabilisation trade-off under supply shocks aggravates, as any reduction in inflation volatility causes a stronger increase in the output gap volatility. Moreover, due to the Experience Channel, the effect of a demographic variation on the transmission of monetary policy is substantially more pronounced under EBL. The transmission of monetary policy on inflation increases in older societies, while the stabilisation trade-off attenuates.

Malmendier and Nagel (2016) provide empirical evidence that differences in inflation expectations across age groups are largely driven by differences in their experienced inflation history. Figure 1.1 illustrates that young cohorts' expectations are more sensitive to recent observations than those of old individuals.[2] The markers denote one-year ahead inflation expectations of a "young" (red) and an "old" (blue) cohort as deviation from the cross-sectional mean (percentage points, right y-axis). The black solid line depicts the year-on-year realised CPI-inflation from the previous year (left y-axis). The heterogeneity in expectations across different age groups is particularly pronounced in the early 1980s, because young individuals whose entire experienced inflation history consists of

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[2] We extend Figure I from Malmendier and Nagel (2016) by plotting the annual percentage change of the seasonally-adjusted U.S.-CPI to facilitate the comparison of *expected* inflation to *experienced* inflation.

the high inflation rates during the 1970s tend to have higher inflation expectations than those individuals who also observed low inflation during the 1950s and 1960s. Until the mid-2000s inflation stabilized on a lower level so that those new experiences led to more homogeneous expectations. Importantly, the high inflation expectations of young agents at the start of the sample are not a function of their age but of their lifetime experience as can be inferred from the reversal of the ordering at the end of the sample where recent inflation experiences were low.

These experience effects describe the data well. Malmendier et al. (2021) show that inflation experiences of Federal Open Market Committee (FOMC) members affect their inflation expectations. Cavallo et al. (2017) document age-related experience effects in the formation of inflation expectations in the U.S. and in Argentina. Beyond inflation, experience effects are also documented for other dimensions, like investment or consumption decisions (Kaustia and Knüpfer, 2008; Malmendier and Shen, 2018). Malmendier (2021) provides a brief overview of neuroscientific facts on brain functioning that give rise to experience effects: repeated stimuli (experiences) build long-lasting connections to synapses, synapse formation continuously responds to new experiences, potentially, undoing older changes (new experiences are overweighted), and, importantly, such experience-driven behavior is the result of biology, i.e., even highly-informed FOMC members' expectation formation exhibits experience effects (Malmendier et al., 2021).

The present paper embeds EBL into a New Keynesian model with overlapping generations à la Blanchard (1985) and Yaari (1965). We assume that agents behave like econometricians who form expectations about future economic variables based on forecasting models whose parameters they constantly revise as new data becomes available. In particular, agents forecast future variables with a covariance stationary auto-regressive process of order one with time-varying AR parameter, which we denote as an agent's *perceived persistence*. Following the empirical analysis of Malmendier and Nagel (2016), agents put more weight on recently observed data points rather than those observed early in life, while ignoring any data *prior* to their birth. The weight attached to new observations when updating beliefs decreases in age, rendering young individuals more sensitive to new data points than older ones. It is the specification of the weight by which EBL differs from standard approaches in the learning literature like constant-gain learning (CGL) where all cohorts attach the same constant weight to new information so that expectations are homogenous across cohorts.

Our calibrated model generates quantitatively substantial heterogeneity in expectations across age groups, which stems from differences in the perceived persistence that different cohorts attach to economic variables. On average, the perceived persistence is more dispersed for young cohorts, because the parameter estimates of their forecasting rules are based on fewer observations and because more recent observations are overweighted. Both features also imply that recurrent reversals in inflation and the output gap make young agents perceive both variables to be less persistent, on average.

We show that EBL endogenously reduces the aggregate perceived persistence in the economy relative to CGL due to the heterogeneity of expectations at the cohort-level. The decrease in the aggregate perceived persistence reduces the impact of monetary policy via expectations on current macroeconomic variables. Under adaptive expectations, current monetary policy has a delayed impact on expectations, because it influences current macroeconomic variables which are used to form beliefs only in the next period. Since household under EBL attach a smaller persistence to macroeconomic variables, monetary policy affects current variables via expectations by less. Intuitively, the aggregate perceived persistence can be interpreted as the weight agents attach to past monetary policy. Because of the lower impact of past monetary policy actions on current variables, the transmission of monetary policy on inflation and the output gap is impaired under EBL. Hence, when neglecting experience effects on expectations the impact of monetary policy is *overstated*. The lower influence of monetary policy on expectations under EBL also affects its stabilisation trade-off under supply shocks in two ways. First, due to the lower perceived persistence, a *given* level of inflation volatility is associated with a lower output gap volatility. Second, a given reduction in inflation volatility is related to a stronger increase in output gap volatility. To see why, note that due to the backward-looking nature of expectations, monetary policy affects inflation also via past changes of the output gap. Those changes are reflected in past inflation and thereby in today's inflation expectations. Since the perceived persistence attached to inflation is reduced, the pass-through via (past) changes in demand on current inflation is lower so that inflation is stabilised less effectively. As a result, the policy trade-off is understated in models that abstract from experience effects.

Our framework introduces a role for the age distribution that is absent from models with rational expectations and from standard learning models. Since under EBL the perceived persistence is heterogeneous across age groups, a variation of the age distribution directly affects the aggregate perceived persistence through the Experience Channel. In response to an increase in the share of old individuals, the aggregate perceived persistence considerably increases under EBL, while it is hardly affected under CGL. Consequently, the aggregate weight attached to past monetary policy actions when forming expectations rises. Thereby, the transmission of monetary policy via expectations also increases, while its stabilisation trade-off under supply shocks attenuates as inflation can be stabilised at the expense of less additional output gap volatility.[3] Due to the increased reaction of inflation expectations to monetary policy, past changes in demand have a stronger impact

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[3] The empirical literature on the relation of demography and monetary policy is still developing. Several studies point into the direction of our theoretical results (see below).

on current inflation.

Our analysis suggests that monetary policy could increase its transmission on current variables by recommending young to medium aged households to base their forecasts on a broader set of observations.[4] The resulting increase in the aggregate perceived persistence would mitigate monetary policy's trade-off between stabilising the output-gap and inflation under supply shocks. In consequence, if monetary policy aims to narrow the inflation target range, recommending young to medium aged households to base their forecast on a broader information set could mitigate the resulting increase in the output gap volatility.[5]

Related Literature. Our work is related to several strands of literature. First, we relate to the adaptive learning literature that studies monetary policy within a New Keynesian model as surveyed in Eusepi and Preston (2018). In contrast to this literature, we assume that individuals do not know the true state-space representation of the economy. Instead, they form expectations based on simple auto-regressive forecasting models as indicated by the empirical evidence of Malmendier and Nagel (2016). We further assume that the parameters of the forecasting model are recursively updated and differ across age groups so that expectations are heterogeneous across generations.

Second, we contribute to the literature that analyses monetary policy within a New Keynesian model with heterogeneous expectations. This literature is mostly concerned with determinacy properties in the presence of heterogeneous expectations (e.g. Branch and McGough, 2009; Gasteiger, 2014; Massaro, 2013). Expectation formation in these studies is time-invariant in contrast to our approach that allows for real-time updates. Further, these studies take expectation heterogeneity as given, whereas in our model it arises endogenously from cohorts that made different lifetime experiences. This gives rise to a novel channel by which the demographic structure affects the pass-through of monetary policy.

Third, there is a literature analysing experience-based expectation heterogeneity in theoretical models. However, most focus on asset pricing in partial equilibrium, as Collin-Dufresne et al. (2016), Ehling et al. (2018), Malmendier et al. (2020), Nagel and Xu (2022), and Schraeder (2015). The only model using EBL in a general equilibrium framework that we are aware of is Acedański (2017), who explores its implication on the wealth distribution. The present paper is the first one to investigate the impact of experience effects on monetary policy.

Lastly, we contribute to the growing literature that studies the relation between demo-

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[4] Coibion et al. (2022) show that communicating the most recent inflation observations to households significantly affects their inflation expectations.

[5] Erceg (2002) argues that the choice of a target range for the inflation rate should be based on the stabilisation trade-off between inflation and output gap volatility.

graphic changes and monetary policy pass-through. Leahy and Thapar (2019) and Berg et al. (2021) consider how the transmission of monetary policy differs across age groups. Both studies document a positive empirical relationship between the impact of monetary policy shocks on employment, income, or consumption and the share of agents that are at the end of their working life. These studies lend support to our theoretical result of a stronger monetary policy transmission in old economies. Likewise, Juselius and Takáts (2018) and Baksa and Munkácsi (2020) show that inflation volatility and the share of old agents are positively related similar to our finding of a policy trade-off that involves more inflation volatility in the old economy. Our work points to a novel channel of how shifts in demography lead to these results through experience effects.

Outline. The remainder of the paper is organised as follows. Section 1.2 presents our New Keynesian model with an overlapping generations structure. In Section 1.3 we explain the details of EBL. Section 1.4 gives an overview of our parameter choices and simulation algorithm. In Section 1.5, we inspect the key difference between EBL and CGL. Section 1.6 discusses the implications of EBL for monetary policy. Finally, Section 1.7 concludes.

1.2 Model

In the present section, we consider a New Keynesian model with overlapping generations of the "perpetual youth" type á la Blanchard (1985) and Yaari (1965).[6] We deviate from the standard model by assuming that households' expectations are not rational and heterogeneous across cohorts. Conceptually, we follow the statistical learning literature advocated by Evans and Honkapohja (2001). Agents behave like econometricians who constantly revise their beliefs as new observations become available. Heterogeneity in expectations across age groups results from the assumption that the revision of beliefs depends on an agent's age.

1.2.1 Households

Households consume the final consumption good, supply labour, form expectations according to EBL, and own intermediate good firms. At each point in time, the mass of households is constant and normalised to one. They face an age-independent probability $\omega \in [0, 1]$ of surviving into the following period. In turn, at the beginning of each period a share of $1-\omega$ households deceases and is replaced by new-born households of equal mass. Consequently, the mass of a cohort born in period k at time $t \geq k$ is given by $(1-\omega)\omega^{t-k}$.

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[6] A similar assumption on demography is made by Ehling et al. (2018) and Galí (2021).

A household born in period k maximises the discounted sum of lifetime utility

$$\tilde{E}_t^k \sum_{j=0}^{\infty} (\beta\omega)^j \, u(c_{t+j|k}, l_{t+j|k}) \ , \tag{1.1}$$

subject to the flow nominal budget constraint in period t

$$p_t c_{t|k} + b_{t|k} = r_{t-1}(b_{t-1|k} + z_{t|k}) + p_t w_t l_{t|k} + \mathcal{D}_{t|k} \ , \tag{1.2}$$

where β denotes the discount factor, p_t the price level, and $\tilde{E}_t^k$ denotes the *subjective* expectations operator, which potentially differs across cohorts k. Importantly, in contrast to its counterpart under rational expectations (i.e. the mathematical expectations operator, $\mathbb{E}$), the information set entailed in $\tilde{E}_t^k$ does *not* include the knowledge of the model. We will be more specific about the specification of $\tilde{E}_t^k$ in section 1.3. Further, $c_{t|k}$ denotes the consumption of households from cohort k, $l_{t|k}$ cohort k's working hours and w_t the real wage per hour worked. They invest in private one-period nominal bonds $b_{t|k}$, which pay the nominal interest rate r_t tomorrow. We assume each household owns equal shares in the intermediate good firms so that nominal dividends are equal across cohorts, i.e., $\mathcal{D}_{t|k} = \mathcal{D}_t$.

The time of death is uncertain and households may die with wealth. To avoid the inefficiency of accidental bequests we follow Blanchard (1985) and introduce insurance companies that make annuity payments $z_{t|k}$ and that receive all assets at the time of death. Profits for a particular company contracting with cohort k are

$$\pi_t^I = (1-\omega)\, b_{t-1|k} - \omega z_{t|k} \ .$$

Due to free entry, insurers make zero-profits so that the annuity payment equals a fraction of cohort bond holdings $z_{t|k} = \frac{1-\omega}{\omega} b_{t-1|k}$. The above sequence of period budget constraints is supplemented with a solvency condition of the form

$$\lim_{T\to\infty} \tilde{E}_t^k \left\{ \mathcal{R}_{t,T} b_{T|k} \right\} = 0 \ , \tag{1.3}$$

where $\mathcal{R}_{t,T} = (\prod_{s=t+1}^{T} r_s)^{-1}$. We assume the following form of the felicity function

$$u(c_{t|k}, l_{t|k}) = \ln\left(c_{t|k}\right) + \psi_n \ln\left(1 - l_{t|k}\right) \ ,$$

where ψ_n is a utility weight. Maximising (1.1) subject to (1.2) yields the optimal consumption/saving decision:

$$1 = \tilde{E}_t^k \left\{ \beta \frac{p_t c_{t|k}}{p_{t+1} c_{t+1|k}} r_t \right\} . \tag{1.4}$$

Equation (1.4) denotes the Euler equation of households in cohort k. While households of all ages face the same nominal interest rate, they have different expectations of the real rate. Hence, a household expecting a high future return, saves more than a household whose past experiences make her believe in dismal real future returns. Notwithstanding that age-related heterogeneity in expectations implies differences in cohort wealth, we aggregate the economy without considering the wealth distribution as an additional state variable as outlined in Section 1.2.4.[7] We also derive the labour supply of a household from cohort k that, via different consumption choices among cohorts, is cohort specific

$$\psi_n \frac{c_{t|k}}{(1 - l_{t|k})} = w_t . \tag{1.5}$$

1.2.2 Firms

There are two types of firms. Final good firms use intermediate inputs to provide an aggregate consumption good. Intermediate good firms are owned by households and operate on a monopolistically competitive market. The choice to set up firms as in the usual New Keynesian model makes our departure from the standard case minimal.

Final Good Firm. The aggregate consumption good in the economy y_t is produced by a perfectly competitive firm, which is aggregating intermediate goods $i \in [0, 1]$ produced by intermediate good firms according to the technology

$$y_t = \left[\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} , \tag{1.6}$$

where $\varepsilon > 0$ is the elasticity of substitution among the intermediate goods $y_{i,t}$. The final good firm chooses the quantities of intermediate goods to maximise its profits. The demand for intermediate good i is given by

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t} \right)^{-\varepsilon} y_t , \tag{1.7}$$

where $p_{i,t}$ is the price at which the intermediate good firm i sells to final good producers.

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[7] Note that it is common practice in the literature on heterogeneous expectations to make certain assumptions in order to abstract from the wealth distribution as an additional state (e.g. Adam et al., 2016; Ehling et al., 2018; Mankiw and Reis, 2007). These assumptions facilitate the analysis of the implications of EBL via the expectation operator *only*.

Intermediate Good Firms. All households alive in period t own an equal share in each intermediate good firm $i \in [0,1]$ that produces a differentiated good on a monopolistically competitive market. Since households are involved in firms to an equal degree, the latter employ average expectations as detailed below. We assume that the share of a deceasing household is transmitted to a new-born one instantaneously. Production of intermediate good i uses the technology

$$y_{i,t} = l_{i,t}^{\alpha} \ , \tag{1.8}$$

where $l_{i,t}$ denotes labour demand of firm i and $0 < \alpha \leq 1$. Intermediate good firm i sells its good at price $p_{i,t}$ and, when changing its price, pays quadratic nominal price adjustment costs à la Rotemberg (1982) proportional to the nominal value of aggregate production

$$\frac{\phi}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 p_t y_t \ ,$$

where ϕ measures the degree of nominal rigidity. Hence, the firm faces an inter-temporal problem that stems from the effect of $p_{i,t}$ on future price adjustment costs. Current real period profits of firm i are denoted by $d_{i,t} = \frac{\mathcal{D}_{i,t}}{p_t}$. Taking aggregate prices as given, firm i chooses $p_{i,t+j}$ and $y_{i,t+j}$ to maximise discounted profits

$$\max_{p_{i,t+j}, y_{i,t+j}} \bar{E}_t \sum_{j=0}^{\infty} \omega^j Q_{t,t+j} \left(\frac{p_{i,t+j}}{p_{t+j}} y_{i,t+j} - w_{t+j} l_{i,t+j} - \frac{\phi}{2}\left(\frac{p_{i,t+j}}{p_{i,t-1+j}} - 1\right)^2 y_{t+j} \right) \ ,$$

subject to the demand schedule of final good firms (1.7). The expectation operator $\bar{E}_t \mathbf{z}_{t+1} \equiv (1-\omega)\sum_{k=-\infty}^{t} \omega^{t-k} \tilde{E}_t^k \mathbf{z}_{t+1}$ denotes the aggregated expectations across all cohorts alive in period t for a generic variable $\mathbf{z}$ and is a size-weighted sum of cohort expectations. Note that the generational structure matters for aggregating the decisions of the households of different age and especially when aggregating the expectations of differently aged households. Since households hold equal shares in every firm, firms use a weighted average of household expectations. Further, $Q_{t,t+j} \equiv \beta^j \frac{c_t}{c_{t+j}}$ denotes the aggregate real stochastic discount factor of households, where $c_t = (1-\omega)\sum_{k=-\infty}^{t} \omega^{t-k} c_{t|k}$.

1.2.3 Government

The nominal interest rate on bonds is determined by a monetary policy authority that sets it according to a feedback rule:

$$r_t = \bar{r}\left(\frac{\pi_t}{\pi}\right)^{\varphi_\pi} \left(\frac{y_t}{y_t^n}\right)^{\varphi_y} \exp\left(m_t\right) \ , \tag{1.9}$$

$$m_t = \rho_m m_{t-1} + \nu_t^m \quad \text{with} \quad \nu_t^m \overset{iid}{\sim} \left(0, \sigma_m^2\right) \ , \tag{1.10}$$

where $\bar{r}$ and π denote the steady state values of the nominal interest rate and aggregate inflation, respectively. Further, y_t^n denotes the natural level of output. For the sake of convenience, we assume that $\pi = 1$, i.e., we consider a zero-inflation steady state. The parameters φ_π and φ_y denote the feedback coefficients that determine the sensitivity to deviations of inflation from its steady state value and of output from its natural level, respectively. Last, m_t serves as monetary policy shock and evolves according to an AR(1) process with zero mean. We specify the monetary policy authority to use *current* inflation (opposed to its expectation), to avoid taking a stance on which type of expectations the monetary policymaker has.

1.2.4 Equilibrium

Labour Market Equilibrium. As all intermediate firms produce with the same technology, equilibrium labour demand is symmetric. Aggregate working hours follow as

$$l_t^d \equiv \int_0^1 l_{i,t} di = \int_0^1 (y_{i,t})^{\frac{1}{\alpha}} \, di = (y_t)^{\frac{1}{\alpha}} \, \Delta_t^p = l_t^s \equiv (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} l_{t|k} \ , \tag{1.11}$$

where $\Delta_t^p \equiv \int_0^1 \left(\frac{p_{i,t}}{p_t}\right)^{-\frac{\varepsilon}{\alpha}} di$ is an index of relative price distortions. Since all firms face a symmetric maximisation problem, we focus on a symmetric price equilibrium, i.e., $\Delta_t^p = 1$.

Goods Market Equilibrium. An equilibrium on the aggregate goods market requires that the total number of goods produced y_t equals the total amount of goods demanded, taking into account the dead-weight loss due to repricing cost

$$y_t = c_t + \frac{\phi}{2}(\pi_t - 1)^2 y_t \ , \tag{1.12}$$

where $\pi_t = \frac{p_t}{p_{t-1}}$ denotes (gross) inflation.

Bond Market Equilibrium. Private bonds are in zero net supply, that is

$$(1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} b_{t|k} = 0 \ . \tag{1.13}$$

New Keynesian Phillips Curve. Using the FOC on prices of intermediate good firms and symmetry, one derives

$$(\pi_t - 1)\,\pi_t = \omega \bar{E}_t \left[Q_{t,t+1} \frac{y_{t+1}}{y_t} (\pi_{t+1} - 1)\,\pi_{t+1} \right] + \frac{\varepsilon(1+\eta_l)}{\phi} \left(\mathrm{mc}_t^r - \mu\right) \ , \tag{1.14}$$

where mc_t^r are the real marginal cost, $\mu \equiv \frac{\varepsilon - 1}{\varepsilon}$ denotes the steady state markup, and $\eta_l = \frac{l}{1-l}$ denotes the stationary labour-leisure share. Note that *aggregate* inflation expectations, π_{t+1}, affect π_t. According to (1.14), optimal price setting requires inflation to be a function of current real marginal cost and expected future inflation.

Linearised Equilibrium Conditions. Expectation heterogeneity matters for households' Euler equations and the New Keynesian Phillips Curve (NKPC). To arrive at an aggregated dynamic IS-curve, we follow the literature and rely on Branch and McGough (2009). First, we adopt their assumption on higher-order beliefs: household i's expectation about what another household k expects, is its own expectation: $\tilde{E}_t^i \tilde{E}_t^k \mathbf{z}_{t+1} = \tilde{E}_t^i \mathbf{z}_{t+1}$, $i \neq k$ for some generic variable $\mathbf{z}$, which reduces the complexity imposed on the model considerably.[8] Second, we assume agents expect to hold the same wealth in the limit $t \to \infty$. For each agent i, consumption then equals the long-run consumption: $\tilde{E}_t^i \left(\hat{c}_\infty - \hat{c}_\infty^i\right) = 0$. This assumption prevents the wealth distribution from appearing in the aggregated IS-curve.[9] After linearisation around the deterministic steady state and aggregation, we rewrite the model in terms of the output gap $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$.[10] Here, $\hat{y}^n$ denotes the deviation of the natural level of output from its steady state value. We arrive at a system of five equations and five variables $\{\tilde{y}_t, \hat{\pi}_t, \hat{r}_t, m_t, u_t\}_{t=0}^{\infty}$:

$$\tilde{y}_t = \bar{E}_t \tilde{y}_{t+1} - \left(\hat{r}_t - \bar{E}_t \hat{\pi}_{t+1}\right) \ , \tag{1.15a}$$

$$\hat{\pi}_t = \beta\omega \bar{E}_t \hat{\pi}_{t+1} + \kappa \tilde{y}_t + u_t \ , \tag{1.15b}$$

$$\hat{r}_t = \varphi_\pi \hat{\pi}_t + \varphi_y \tilde{y}_t + m_t \ , \tag{1.15c}$$

$$m_t = \rho_m m_{t-1} + \nu_t^m \ , \tag{1.15d}$$

$$u_t = \rho_u u_{t-1} + \nu_t^u \ , \tag{1.15e}$$

where $\kappa \equiv \frac{(\varepsilon - 1)}{\phi} \frac{(1+\eta_l)}{\alpha}$ denotes the slope of the New Keynesian Phillips curve.[11] We introduce a cost-push shock u_t that could stem from a firm-specific shock to marginal cost to have a source of exogenous variation apart from the monetary policy innovation (see Ireland, 2004). To solve the model, we next need to specify how agents form expectations.

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[8] For approaches considering higher order beliefs, see Angeletos et al. (2018) or Farhi and Werning (2019).

[9] See Appendix 1.A for further details.

[10] In the following, a variable with a hat denotes the percentage deviation of that variable from its deterministic steady state value.

[11] The updating equations of household beliefs (1.18a) and (1.18b) are also part of the model.

1.3 Expectation Formation

In this section, we discuss how different cohorts form expectations on inflation and the output gap based on Malmendier and Nagel (2016). First, we explain how a single cohort forms expectations and discuss why experience effects play an important role. Then, we highlight how EBL differs from CGL, which is among the most popular learning approaches.

1.3.1 Learning

We follow Evans and Honkapohja (2001) and Slobodyan and Wouters (2012a) who require near-rational agents to forecast variables only one period ahead, e.g., of variables in their Euler equation so that the approach is called *Euler equation learning*.[12] A large part of the literature assumes that agents know the true state-space representation, i.e., they know the relevant variables for the economy's evolution, but have to learn about the coefficients of this representation (e.g. Milani, 2007). Instead, we assume that agents employ a misspecified forecasting rule. In comparison to having all state variables as regressors, the agents' perceived law of motion (PLM) is based on only a subset of them or no states at all. We adapt the set-up in Malmendier and Nagel (2016) and specify agents' PLM as an AR(1) process.[13] The PLM of a generic variable $\mathbf{z} \in Y^f \equiv \{\tilde{y}, \hat{\pi}\}$ for a household in cohort k at time t is given by

$$\mathbf{z}_{t|k} = \delta^{\mathbf{z}}_{t-1|k} \mathbf{z}_{t-1} + \varepsilon^{\mathbf{z}}_{t|k} \ , \tag{1.16}$$

where $\varepsilon^{\mathbf{z}}_{t|k}$ is a disturbance term which is serially-uncorrelated with zero mean and constant variance and $\delta^{\mathbf{z}}_{t-1|k}$ denotes the AR(1) parameter estimate of household k at time $t-1$. Intuitively, $\delta^{\mathbf{z}}_{t-1|k}$ can be interpreted as household k's *perceived* persistence of $\mathbf{z}$, respectively.[14]

We assume that individuals form expectations at time t using only information available at time $t-1$. By doing so, we avoid a simultaneity problem that arises when agents use time t endogenous variables to forecast future realisations, which in turn affects the time t endogenous variables (see Evans and Honkapohja, 2001). We summarise the timing assumption in Figure 1.2.

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[12] An alternative approach is the Infinite Horizon Forecast as developed in Preston (2005). For a comparison of these approaches see Evans et al. (2013).

[13] Slobodyan and Wouters (2012a) use similar specifications of agents' forecasting rule. The choice of order one is further consistent with the model under RE where variables are also Markov-processes of order one.

[14] The PLM for both the output gap and inflation excludes a constant (see also Milani, 2007). Hence, we assume that agents in each cohort know the true mean of both variables. We validated that including a constant in agents' PLM does not qualitatively affect our main results.

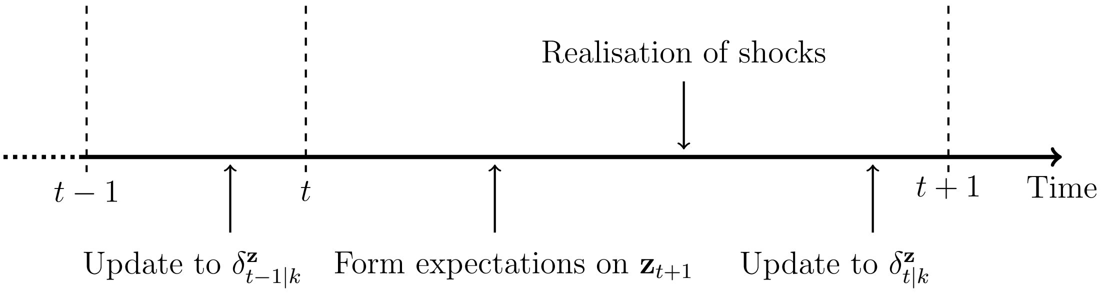

Figure 1.2: Timing Assumption

The last element of the PLM is how its coefficients develop over time. Let $\mathbf{I}_t$ be the information set on which households base their forecast at time t. The information set $\mathbf{I}_t$ includes all model variables up to $t-1$. Consequently, the formation of expectations occurs *before* the realisation of the endogenous variables included in Y^f such that $\tilde{E}^k_t(\mathbf{z}_t) = \tilde{E}^k\left(\mathbf{z}_t|\mathbf{I}_t\right) \neq \mathbf{z}_t$. Instead, using (1.16) and presuming that the law of iterated expectations holds for the subjective expectations, households in cohort k forecast variable $\mathbf{z}_{t+1}$ as follows:

$$\begin{aligned}\tilde{E}^k_t\left(\mathbf{z}_{t+1}\right) &= \tilde{E}^k_t\left(\delta^{\mathbf{z}}_{t|k}\mathbf{z}_t\right) = \tilde{E}^k_t\left(\delta^{\mathbf{z}}_{t-1|k}\mathbf{z}_t\right)\\ &= \tilde{E}^k_t\left(\delta^{\mathbf{z}}_{t-1|k}\left(\delta^{\mathbf{z}}_{t-1,k}\mathbf{z}_{t-1} + \varepsilon^{\mathbf{z}}_{t|k}\right)\right)\\ &= \left(\delta^{\mathbf{z}}_{t-1|k}\right)^2\mathbf{z}_{t-1}\ , \end{aligned} \tag{1.17}$$

where for the first equality we use the PLM (dated $t+1$) and for the second that point estimates of the PLM parameters only include information up to $t-1$. The third equality uses that agents form expectations before the current realisation of $\mathbf{z}$ such that also today's realisation is forecasted using the PLM. Finally, the last equality uses that the PLM parameter estimated with information up to time $t-1$ is uncorrelated with the error term at time t, i.e., $\tilde{E}^k_t\left(\delta^{\mathbf{z}}_{t-1|k}\varepsilon^{\mathbf{z}}_{t|k}\right) = 0$.

After the realisation of time t shocks, agents update their PLM parameters from $\delta^{\mathbf{z}}_{t-1|k}$ to $\delta^{\mathbf{z}}_{t|k}$ using the following recursive least-squares algorithm

$$\delta^{\mathbf{z}}_{t|k} = \delta^{\mathbf{z}}_{t-1|k} + \gamma_{t|k}\left(R^{\mathbf{z}}_{t|k}\right)^{-1}\mathbf{z}_{t-1}\hat{\varepsilon}^{\mathbf{z}}_{t|k} \tag{1.18a}$$

$$R^{\mathbf{z}}_{t|k} = R^{\mathbf{z}}_{t-1|k} + \gamma_{t|k}(\mathbf{z}^2_{t-1} - R^{\mathbf{z}}_{t-1|k})\ , \tag{1.18b}$$

for each $\mathbf{z} \in Y^f$. Here, $\hat{\varepsilon}^{\mathbf{z}}_{t|k} \equiv \mathbf{z}_t - \delta^{\mathbf{z}}_{t-1|k}\mathbf{z}_{t-1}$ denotes the forecast error of cohort k and $\gamma_{t|k}$ gives the (potentially) age-dependent Kalman gain of cohort k that governs how sensitive estimate revisions are to forecast errors $\hat{\varepsilon}^{\mathbf{z}}_{t|k}$ based on the old parameter estimate. The variance of the regressor, $R^{\mathbf{z}}_{t-1|k}$, also influences the revision of the estimates to forecast

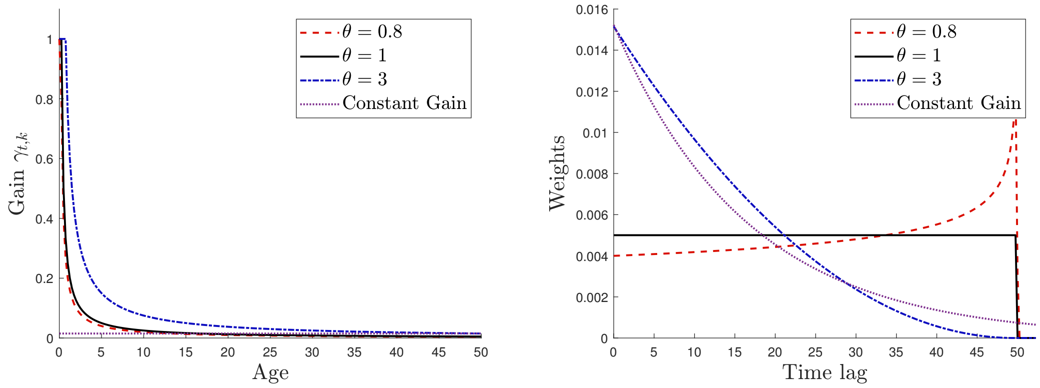

Figure 1.3: Gain and Weights on Past Data

Notes: The left panel depicts the evolution of the gain parameter over age (in years) for three different values of θ (the estimate of Malmendier and Nagel (2016) is around 3). The right panel shows how a 50 year old agent weights past information when estimating the parameters of the PLM (again for different θ). The purple line denotes the case of CGL.

errors.[15] Similar to the PLM parameters $\delta^{\mathbf{z}}_{t|k}$, the variance of the regressor is updated recursively. We will discuss the simulation algorithm in section 1.4.

1.3.2 Experience-Based Learning (EBL)

The novelty of EBL lies in the age-dependent form of parameter updating. Malmendier and Nagel (2016) provide evidence that the gain parameter $\gamma_{t|k}$ depends on the amount of lifetime data (or equivalently age), $t-k$, of individuals in cohort k

$$\gamma_{t|k} = \begin{cases} \frac{\theta}{t-k} & \text{if } t-k \geq \theta \\ 1 & \text{if } t-k < \theta \ , \end{cases} \tag{1.19}$$

where $\theta > 0$ determines the degree to which individuals react to recent observations. Above specification implies, firstly, that expectations are heterogeneous between cohorts. Secondly, it implies that young agents have higher gains than older ones so that they update their PLM's parameters more strongly.

Both aspects are captured in Figure 1.3. The left panel plots the gain parameter over age for different values of θ.[16] Young agents have high gains, consistent with the idea that they have less lifetime observations and, therefore, rely more strongly on current data. The size of gains also decreases in age; the more so, the higher θ. The right panel of Figure 1.3 shows the implied weights a 50 year (200 quarter) old individual puts on data observed over her lifetime for different values of θ.[17] For $\theta > 1$, data observed early in life

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[15] The lower the variance in the explanatory variable, the stronger the update.

[16] The graph is based on Malmendier and Nagel (2016). Their appendix shows how to derive it.

[17] Note that recursive least-squares is the recursive formulation of weighted least squares. The weights

receives negligible weights as an individual ages so that recent data is more important to update the PLM (data before birth has weight zero, as only lifetime information is used). Note also that agents of different age use a different amount of information.[18] Although in our perpetual youth structure there may be individuals who use information from the far past, the mass of such a cohort declines as time passes by. Further, the weight such an individual would put on this information is small, such that this informations' influence on the current aggregate expectation is negligible.

1.3.3 Constant-Gain Learning (CGL)

Under EBL agents have different gain parameters depending on their age. As mentioned above, studies of DSGE models with dynamic non-rational expectations often employ learning algorithms with a constant gain. Under CGL, all agents react equally to new observations. In practice, this amounts to replacing $\gamma_{t|k}$ in (1.18) with a the constant scalar g so that the left panel of Figure 1.3 shows a constant (purple line) across ages. Under this assumption, agents of different cohorts are *homogeneous* with respect to their expectation formation, i.e.,

$$\tilde{E}_t^k \mathbf{z}_{t+1} = \tilde{E}_t \mathbf{z}_{t+1} = (b_{t-1}^{\mathbf{z}})^2 \mathbf{z}_{t-1} \ ,$$

for all cohorts k. This setup still retains the feature that new observations (and, hence, forecast errors) are weighted higher than old observations (right panel of Figure 1.3). However, in contrast to EBL, individuals put a non-zero weight to *all* data points. In the following, we will interpret the CGL approach as the counterpart of EBL where we *shut off* experience effects on individuals' expectations. We simulate our model for this specification to study the *additional* endogenous source of variation that stems from experience effects alone.

1.4 Calibration and Simulation Algorithm

This section briefly discusses our parameter choices for the structural parameters and gives an overview of the simulation algorithm.

1.4.1 Calibration

One period in the model corresponds to one quarter. We calibrate the model's deep parameters to U.S. data (Table 1.1 provides a summary). Our choice of the survival

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inside the weighting matrix contain the gain parameter $\gamma_{t|k}$ and, thus, depend on θ.

[18] Strictly speaking agents have access to all observations. However, as seen in the right panel of Figure 1.3, individuals do not put any weight on observations before birth.

Table 1.1: Parameter Choices (quarterly)

Variable		**Value**	
β	Discount factor	0.995	Ann. riskless rate 4%
ψ_n	Utility weight on leisure	1.17	Steady state labour supply of 1/3
ε	Elasticity of substitution	9	Mark-up of 12.5%
α	DRS parameter	0.66	U.S. labor share
ξ	(Inverse) Frisch elasticity	2	Standard choice
ϕ	Rotemberg parameter	93.2	Share non-adjusters 75%
φ_π	Taylor parameter π	1.5	Galí (2015)
φ_y	Taylor parameter y	0.125	Galí (2015)
π	Inflation Target	1	Zero-inflation steady state
ω	Survival probability	0.995	50 year working-life
ρ_u	Persistence u	0.96	Ireland (2004)
ρ_m	Persistence m	0.60	Standard choice
σ_u	Standard deviation ν^u (in %)	0.15	Ireland (2004)
θ	EBL parameter	3.044	Malmendier and Nagel (2016)
g	Gain under CGL	0.015	Standard choice

probability $\omega = 0.995$ is guided to meet an average life span of 200 quarters, which represents the working-life of an agent. Most of the other parameters are taken from Galí (2015). The households' discount factor β is calibrated to get a steady state real *annualised* return on riskless bonds of about 4% given our choice for ω. Furthermore, we set the steady state elasticity of substitution to $\varepsilon = 9$, which implies a steady state mark-up of 12.5%. The parameter of the production function α is chosen to be 0.66 in line with the labor share in U.S. data. The choice of the Rotemberg adjustment cost parameter matches a fraction of non-adjusters of 0.75 in a model with Calvo price setting $\phi = 93.2$. We set $\varphi_\pi = 1.5$ and $\varphi_y = 0.125$ to standard choices. Moreover, the values for the serial correlation coefficient of the cost-push shock ρ_u and the standard deviation of the innovation σ_u are set 0.96 and 0.0015, respectively, which correspond to the values estimated in Ireland (2004). Further, we set the serial correlation coefficient the monetary policy shock ρ_m to 0.6. We choose the learning parameter that governs the age-dependent gain under EBL as $\theta = 3.044$ (Malmendier and Nagel, 2016) and the CGL parameter g as 0.015 according to Milani (2007) and much of the learning literature. Finally, steady state gross inflation is targeted to be one, while the steady state labour supply is 1/3.

1.4.2 Simulation Algorithm

We simulate the model under adaptive learning (EBL and CGL) and RE for the same random sequence of supply shocks, while setting the monetary policy innovation to zero. To initialise the PLM parameters for the learning models, we simulate the economy under RE for $T_{\text{init}} = 10{,}000$ quarters and estimate an AR(1) model for inflation and the output gap. The estimated AR(1) coefficients for both variables serve as the respective initial PLM parameters for the models with EBL and CGL. To start model simulations, we endow *each* cohort with the same initial PLM parameters $\delta^{\mathbf{z}}_{-1|k} = \delta^{\mathbf{z}}_{-1}$ and the same estimate for $R^{\mathbf{z}}_{-1|k} = R^{\mathbf{z}}_{-1}$ for $\mathbf{z} \in Y^f$ both of which are updated subsequently. Under EBL, however, in *each* period t a new cohort is born which needs to be endowed with initial values for the PLM parameters, $\delta^{\mathbf{z}}_{t|1}$, and the regressor variance, $R^{\mathbf{z}}_{t|1}$. We assume that newly-born agents are endowed with the aggregate persistence parameter of the previous period.[19] In turn, we assume that $R^{\mathbf{z}}_{t|1} = R^{\mathbf{z}}_{-1|k}$ for all t. We then simulate the economy for $T_{\text{sim}} = T_b + 300{,}000$ quarters, where $T_b = 1000$ is the burn-in discarded to wash out the impact of the initial values from the simulation of the RE economy. Each period members of cohort k update their parameter estimate and the covariance matrix according to equation (1.18). Similar to Slobodyan and Wouters (2012a) we restrict agents to rely on covariance stationary forecasting models: we invoke a projection facility and restrict the new estimate to induce a stationary AR(1) process, which requires $|\delta^{\mathbf{z}}_{t|k}| < 1$ for all cohorts k and $\mathbf{z} \in Y^f$. In case the new estimate exceeds the bounds of ±1, the old estimate is kept and no updating takes place.[20] Intuitively, Evans and Honkapohja (2001) argue that agents avoid explosive paths of the economy such that the agent chooses its parameter estimate accordingly. Further, there is an infinite number of cohorts so that we need to restrict the number of cohorts for our simulations. A high number of cohorts reduces the approximation error but comes at the cost of greater computational time. Since the baseline model calibrates the survival probability so that the expected lifetime is 200 quarters, we restrict the number of cohorts in the aggregation to be 200 and normalise cohort weights to sum to one. Appendix 1.B provides a detailed description of the algorithm.

1.5 Inspecting the Mechanism

In this section, we discuss the key implications of EBL that matter for the analysis of monetary policy. In particular, young individuals' perceived persistence is, on average, lower and more volatile relative to old individuals under EBL. Consequently, the *aggregate* perceived persistence of inflation and the output gap turns out to be, on average, lower compared to an economy where we shut-off experience effects, i.e., where we assume

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[19] Under this assumption, a newly-born agent follows the "conventional wisdom". While several papers consider learning-from-experiences, the treatment of initial beliefs varies: Schraeder (2015) uses initial beliefs that correspond to RE, Ehling et al. (2018) endow young agents with a small initial information set to deduct an initial belief, and Collin-Dufresne et al. (2016) assume young agents inherit beliefs from their parents.

[20] The restriction is invoked in only 6% of updates for PLM parameters under EBL. This falls to 2% when we marginally increase the bound on parameter estimates. Its usage is similar to the one in Slobodyan and Wouters (2012b) for high gains.

Table 1.2: Moments of PLM Parameters for Inflation and Output Gap

<table>
<tr><th>Parameter</th><th>Mean</th><th>Std. Dev.</th><th>2.5 Percentile</th><th>97.5 Percentile</th></tr>
<tr><td>Panel A. Cohort-specific PLM Parameters under EBL</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(y | o)</td><td>(y | o)</td><td>(y | o)</td><td>(y | o)</td></tr>
<tr><td>Perceived Persistence - Output Gap</td><td>0.645 | 0.961</td><td>0.421 | 0.065</td><td>-0.660 | 0.919</td><td>0.992 | 0.999</td></tr>
<tr><td>Perceived Persistence - Inflation</td><td>0.652 | 0.938</td><td>0.335 | 0.026</td><td>-0.315 | 0.884</td><td>0.999 | 0.992</td></tr>
<tr><td>Panel B. Aggregate PLM Parameters</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(EBL | CGL)</td><td>(EBL | CGL)</td><td>(EBL | CGL)</td><td>(EBL | CGL)</td></tr>
<tr><td>Perceived Persistence - Output gap</td><td>0.893 | 0.976</td><td>0.044 | 0.012</td><td>0.809 | 0.952</td><td>0.975 | 0.999</td></tr>
<tr><td>Perceived Persistence - Inflation</td><td>0.880 | 0.956</td><td>0.047 | 0.016</td><td>0.781 | 0.925</td><td>0.969 | 0.991</td></tr>
</table>

Notes: We simulate the model for 300,000 periods and calculate the summary statistics based on the obtained sample. Panel A compares young (y) and old (o) agents whereas in Panel B we compare aggregate values under EBL and CGL.

CGL.[21]

PLM Parameters. In our model, heterogeneity in expectations across cohorts stems from the heterogeneity in individuals' estimated parameters of the PLM for inflation and the output gap. In panel A of Table 1.2, we show moments of the ergodic distribution of the PLM parameters for a young ($k = 10$; labeled "y") and an old ($k = 158$; labeled "o") individual.

The perceived persistence of the output gap and of inflation for young agents is more dispersed and, *on average*, lower relative to the ones for old agents, respectively. Looking at the left tails of the distributions confirms that young agents' estimates can strikingly deviate from the one of old agents. There are two driving forces behind these findings. First, young individuals rely on a lower amount of information when updating their PLM parameters. Second, they are more sensitive towards new observations. As a result, the dispersion of estimates is higher, since the variance in parameter estimates decreases in the number of observations and since young agents overweight new observations in updating. In particular, this implies a lower perceived persistence, on average.[22]

Intuition Behind the Learning Process. The heterogeneity in inflation expectations is linked to the sensitivity by which agents of different age revise their PLM parameters as new information becomes available. The left panel of Figure 1.4 displays a snapshot of one simulation path for inflation. The right panel of Figure 1.4 shows the corresponding path for the PLM parameters of inflation for the young cohort (black solid) and the old cohort (blue dashed). Figure 1.4 clearly shows that the young household updates its

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[21] Under EBL, the *aggregate* perceived persistence corresponds to the size weighted average over the cohort-specific perceived persistences. See Appendix 1.B for a formal definition.

[22] Recall that agents use covariance stationary forecasting models. The combination of a higher dispersion and the truncation of the PLM parameter distribution may further decrease the mean of young agents' perceived persistence. However, we find this to not be a key driving force.

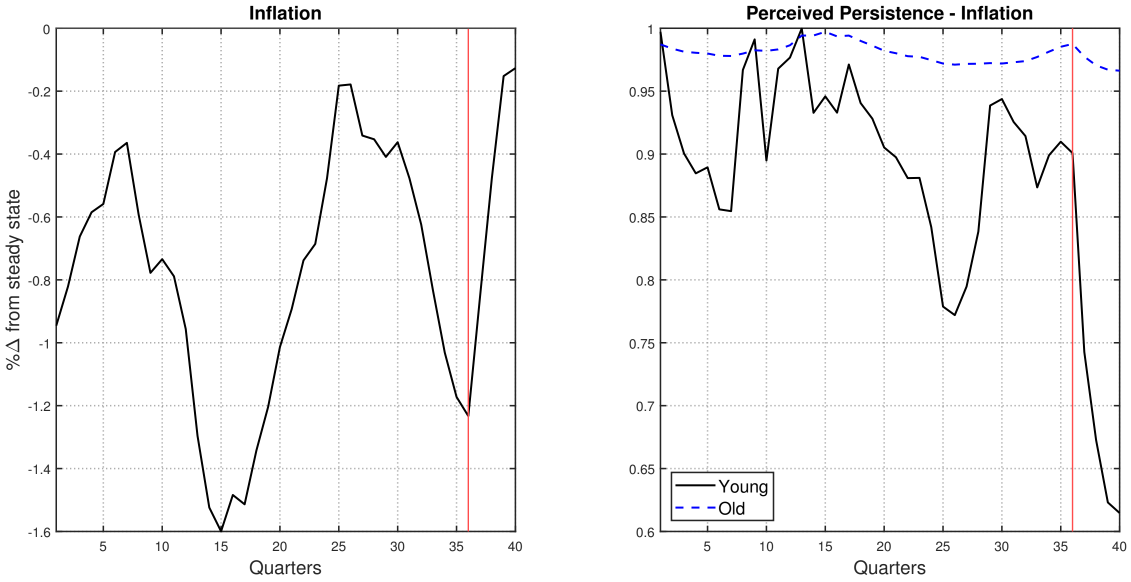

Figure 1.4: Simulation Path for Inflation

Notes: Left panel: Snapshot of a simulation path for inflation. Inflation is shown as percentage deviation from its steady state. Right panel: Snapshot of a simulation path for the perceived persistence parameter of inflation for a young ($k = 10$) and old ($k = 158$) household.

PLM parameters much more strongly than the old one. The higher updating results from the larger weight that the young household attaches to forecast errors made with its previous PLM parameters. Consider the inflation path between period 30 and 37. During this time span, actual inflation is persistently falling and reaches its trough at period 37 (indicated by the red vertical line). Afterwards, inflation increases which generates a positive forecast error for both the young and the old household.[23] Since inflation is still negative, the positive forecast error induces a downward revision in households' perceived persistence.[24] Because the young household puts a higher weight on the forecast error when updating its PLM parameter, its downward revision is more pronounced. Intuitively, under EBL, households' updating of their PLM parameters only makes use of observations that they actually "experienced" during their lifetime. The young household in period 37 mainly observed downward trending inflation during his lifetime. Hence, a reversal in inflation has a more pronounced impact on its perceived persistence than for the old household that has seen both upswings and downswings in inflation. Finally, Figure 1.4 demonstrates the perceived persistence for the young household is, on average, smaller than the one of the old household which reflects the intuition of panel A in Table 1.2.

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[23] Intuitively, using their linear PLM, households' inflation expectations follow the recent downward trend. Hence, at $t = 37$, they *under*-predict inflation which generates a *positive* forecast error.

[24] To see this, consider equation (1.18a). The positive forecast error, $\hat{\epsilon}^{\pi}_{t|k}$, gets multiplied by negative inflation which generates a downward revision in $\delta^{\pi}_{t|k}$.

Aggregate PLM Parameters. The heterogeneity in the PLM parameters for inflation and the output gap across cohorts has important implications for the *aggregate* PLM parameters. In panel B of Table 1.2 we compare the distributions of the aggregate perceived persistence for the output gap and inflation under EBL to those obtained under CGL. Under EBL, the aggregate perceived persistence for inflation and the output gap is lower, on average, and more dispersed, respectively. To understand this finding, note that the commonly used gain of the representative agent under CGL $g = 0.015$ maps into the gain of an individual with a 50-year working life under EBL. For any individual in the model with EBL that is younger than the representative agent under CGL, the PLM parameter of both inflation and the output gap are more dispersed and lower, on average. If the mass of such individuals is sufficiently high, also the aggregate PLM parameters are affected in the same way. Hence, the age distribution affects the first two moments of the distribution of the PLM parameters. In particular, EBL *endogenously* pushes down the aggregate perceived persistence of inflation and the output gap, on average.

Experience Channel. Since the aggregate PLM parameters are a size-weighted average over the cohort-specific PLM parameter, a variation in the age distribution directly affects the aggregate values through a composition effect, which we call the *Experience Channel*. In our model, a variation in the age distribution corresponds to a variation in the survival probability ω. As ω falls, the share of young individuals increases, while the share of old individuals decreases. As shown above, young individuals' perceived persistence is, on average, lower than the one of old ones so that the aggregate perceived persistence for inflation and the output gap decrease in the share of young individuals. This effect does not exist under CGL.

1.6 The Effect of Monetary Policy Under EBL

We use the results from the previous section and analyse their implications for monetary policy. First, we investigate the transmission of monetary policy shocks under EBL, which we contrast with those from models with RE and CGL.[25] We also explore how different relative sizes of young to old cohorts (called "demography") affect results. Finally, we compare the monetary policy trade-off between output gap and inflation stabilisation in a model with EBL to the model with CGL and, again, consider the role of demography.

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[25] The transmission of monetary policy shocks under RE is extensively analysed (e.g. Galí, 2015). We primarily RE as a reference point to clarify the mechanism by which monetary policy affects expectations under adaptive learning.

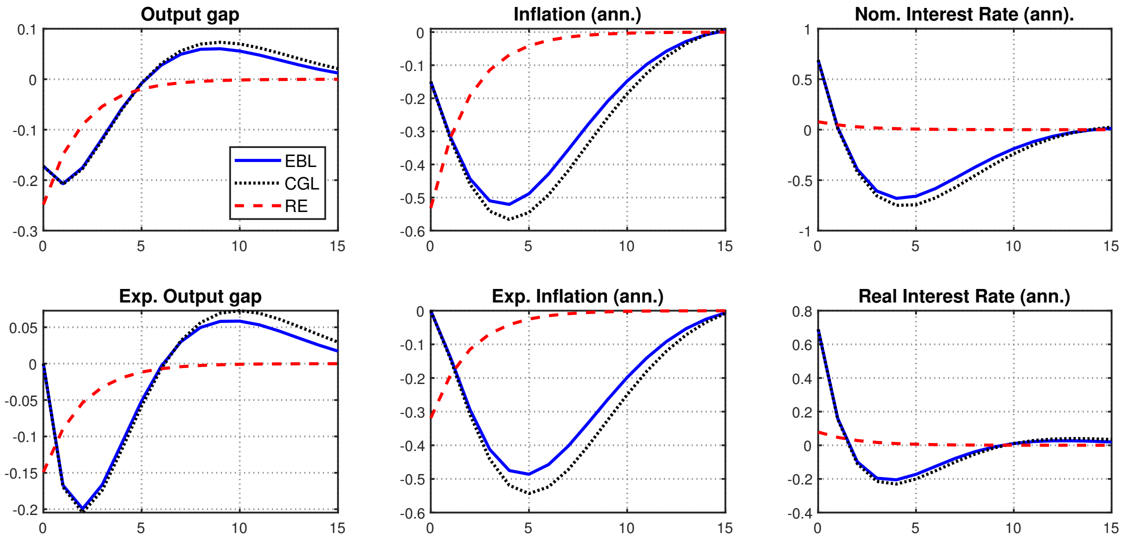

Figure 1.5: Generalised Impulse Response Functions to a Monetary Policy Shock

Notes: We show the generalised impulse response functions for key variables in the economies under EBL (blue solid), CGL (black dotted), and RE (red dashed). We average responses over 8,000 iterations. The output gap and output gap expectations are measured as percentage deviations from their respective steady state, while the other variables are measured as (annualised) deviation from their respective steady state.

1.6.1 Transmission of a Monetary Policy Shock

First, we analyse the effect of EBL on the transmission of monetary policy shocks and how this effect depends on the age distribution. Since under adaptive learning, the model responses depend on the initial values of the PLM parameters and the moment matrix, we compute *generalised* impulse response functions based on Koop et al. (1996). The monetary policy shock corresponds to an innovation of 25 basis points to m_t.[26] The algorithm used to compute the generalised impulse response functions is described in Appendix 1.B.

Impact. We show our results in Figure 1.5. On impact, a contractionary monetary policy shock lowers the output gap and inflation. In response, the central bank pushes the nominal interest rate down. This decrease is, however, insufficient to offset the exogenous shift such that the nominal interest rate increases, which increases the real interest rate. The differences in the initial responses of the output gap and inflation under EBL and CGL are negligible but become stronger when expectations are revised. In contrast, the initial response under adaptive learning (EBL and CGL) is less pronounced than under RE which is primarily driven by the different response of expectations.

Revision of Expectations. In the lower left and middle panels of Figure 1.5, we show the responses of expectations on the output gap and inflation, respectively. Under adaptive

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[26] In Appendix 1.C, we consider responses to a supply shock. The basic result remains the same.

learning, expectations are backward-looking so that they are revised only in the period *after* the shock. In consequence of the initial drop in the output gap and inflation, individuals revise expectations on both variables downwards, which further lowers the *current* output gap and inflation. Note that the downward revision under EBL is less pronounced than under CGL. In response to the contraction in the output gap and inflation, agents lower their perceived persistence on those variables. Since under EBL, young agents update their PLM parameters comparatively strong as new observation become available, the downward revision in the *aggregate* perceived persistence is more pronounced than under CGL. This downward revision *mitigates* the drop in expectations so that the downturn in inflation under EBL is less pronounced than under CGL.[27]

Under adaptive learning, current monetary policy only affects expectations in the next period, i.e., the perceived persistence reflects past shocks and monetary policy. Since the aggregate perceived persistence for both the output gap and inflation under EBL is lower, monetary policy is less effective in influencing expectations and, thereby, contemporaneous variables. As a result, the pass-through of monetary policy on inflation is impaired under EBL. However, the difference in the response of the output gap under EBL and CGL is smaller because the simultaneous decrease in the real rate, which is slightly stronger under CGL, partly counteracts the impact of more negative CGL-expectations.[28] Forward-looking RE stand in contrast to adaptive learning. Agents under RE take into account the Taylor rule's impact on the future path of real interest rates and the shock's persistence when forming expectations. Under RE, expectations of the output gap and inflation recover quickly and, thereby, have a different trajectory than expectations under EBL or CGL that respond to the previous realisations of the output gap and inflation with a lag.

Dynamics. The dynamic response of macroeconomic variables under adaptive learning (EBL and CGL) is quite different compared to the one under RE. While under RE the economy reverts back to the steady state roughly after ten quarters, deviations in the economy under adaptive learning are more persistent. Under adaptive learning, the effect of the shock is more slowly transmitted into agents' expectations than under RE, where individuals perfectly incorporate the shock persistence as well as the impact of the Taylor rule into their expectations. In addition, inflation and the output gap display a hump-shaped response, which results from the backward-looking expectations of agents and their lagged response to the monetary policy shock.

To bring inflation back to its steady state, monetary policy decreases the nominal

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[27] A similar result is obtained by Slobodyan and Wouters (2012a). The aggregate perceived persistence in the economy affects the response of the endogenous variables in response to exogenous disturbances.

[28] The small difference is at first sight surprising given that, ceteris paribus, the response of expected inflation under EBL is less pronounced, i.e., the real rate should move by more. However, the smaller response of inflation expectations is counteracted by a weaker response of the nominal rate due to a lower response of *current* inflation under EBL. Both effects approximately neutralise.

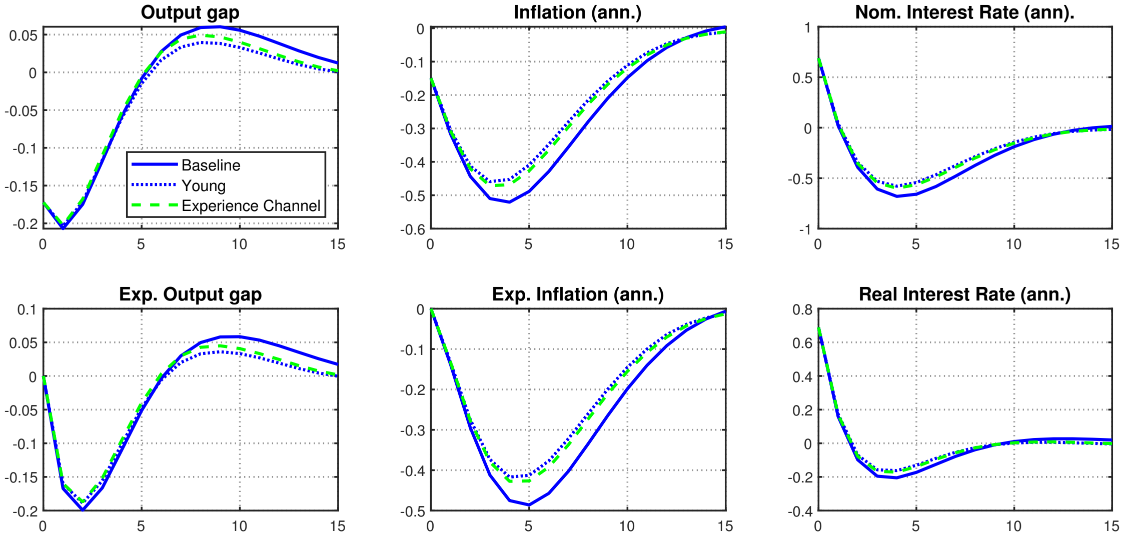

Figure 1.6: Generalised Impulse Response Functions to a Monetary Policy Shock: Decomposition Under EBL

Notes: We show the generalised impulse response functions for key variables in economies under EBL. We distinguish the baseline case (blue solid), the young economy (blue dotted), and the Experience Channel (green). We average responses over 8,000 iterations. The output gap and output gap expectations are measured as percentage deviations from their respective steady state, while the other variables are measured as (annualised) deviation from their respective steady state.

interest rate by that much that the real interest rate undershoots two quarters after the shock hits. This, in turn, shifts the output gap upwards that eventually overshoots, which boosts inflation upwards towards its steady state. Compared to the model with CGL, the overshooting is slightly less pronounced under EBL which results from the smaller aggregate perceived persistence.

Demography. Next, we focus on the impact of a demographic shift in the model with EBL and analyse the effect on the transmission of the monetary policy shock. In the perpetual youth model, a change in the demographic structure corresponds to a change in the survival probability ω. To illustrate the effect of the demographic structure we reduce the survival probability to 0.96. A change in the survival probability affects

1. the effective discount factor, $\tilde{\beta} \equiv \beta\omega$ and
2. aggregate expectations $\bar{E}_t$ *under EBL* (Experience Channel; see Section 1.5).

In Figure 1.6, the solid blue line denotes responses in the baseline ($\omega = 0.995$) economy and the dotted blue line denotes responses for a lower survival probability ($\omega = 0.96$). The green line shows the impulse response function under EBL when the effective discount factor $\tilde{\beta}$ is fixed at the baseline value by varying β accordingly so that only the Experience Channel (channel 2.) operates.[29]

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[29] In Appendix 1.C, we also vary ω under RE and CGL. Given no Experience Channel exists, differences between a young and an old economy are small.

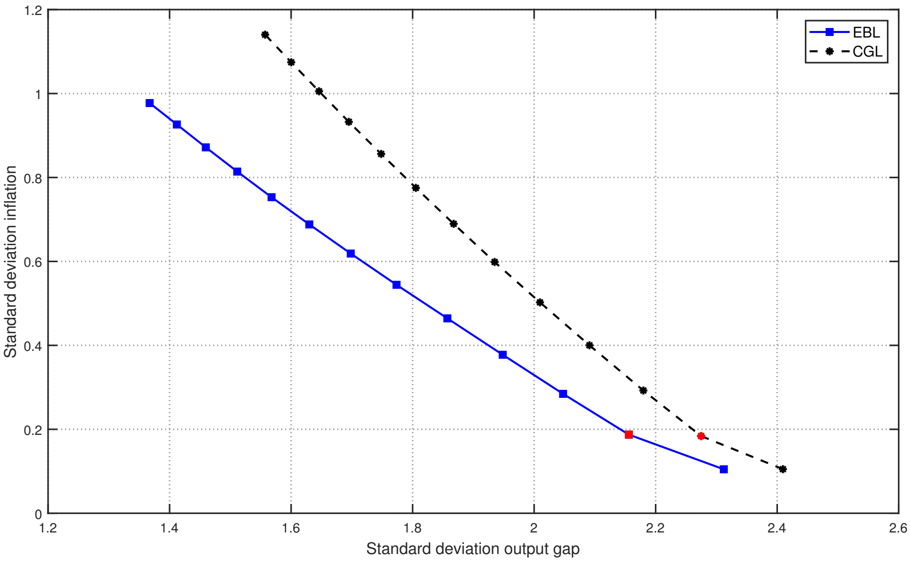

Figure 1.7: Monetary Policy Frontier Under a Supply Shock

Notes: We plot the standard deviation of the output gap (σ_y) against the one for inflation (σ_π) for the models under EBL (blue) and CGL (black). Units are in percent deviation from the steady state. The markers denote the values of λ on the grid. The red markers on both policy frontiers show the (σ_y, σ_π) combination under the baseline parametrisation of the monetary policy rule, respectively.

As discussed in Section 1.5, an increase in the share of young agents reduces the aggregate perceived persistence of inflation and the output gap via the Experience Channel. Thus, in the young economy the pass-through of monetary policy via expectations is reduced compared to the baseline economy (blue solid). Indeed, the Experience Channel alone (green line) generates a quantitatively considerable decrease in the size and persistence of the responses of the output gap and inflation. The impact response is slightly less pronounced when considering the full effect, i.e., when channel 1. is active as well (blue dotted line). This stems from the lower effective discount factor that makes current inflation less sensitive to inflation expectations. For the same reason, the persistence of the response is lower for the full effect. Intuitively, channel 1. has a similar impact as the Experience Channel with the difference that it only affects expectations in the NKPC.[30]

1.6.2 Trade–Off Under Supply Shocks

As a next step, we compare different Taylor rule calibrations with respect to their ability to close the output gap and to stabilise inflation under supply shocks. In the context of the New Keynesian model, it is a well-known result that the Taylor rule is incapable to simultaneously close the output gap and stabilise inflation when supply shocks perturb

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[30] Since the Rotemberg parameter ϕ is a function of $\tilde{\beta}$, one would need to change ϕ in order to maintain a calibration that targets a share of non-adjusters of 0.75. We ascertained that the variation in ϕ neither qualitatively nor quantitatively changes the results. As the interpretation of results is more intuitive for a fixed Rotemberg parameter, we refrain from this change.

the economy (see Galí, 2015). Since EBL impairs the monetary policy transmission via expectations, this trade-off is also affected. Consider the following reformulation of the linearised Taylor rule (assuming that $m_t = 0$ for all t):

$$\hat{r}_t = \varphi_\pi \left(\hat{\pi}_t + \lambda \tilde{y}_t \right) \ ,$$

where $\lambda \equiv \varphi_y / \varphi_\pi$. We interpret λ as monetary policy's output gap stabilisation motive. We consider values of $\lambda \in [0, 1]$, where monetary policy puts no weight on output gap stabilisation when $\lambda = 0$ and weights output gap and inflation stabilisation equally important when $\lambda = 1$.[31] The policy frontiers are shown in Figure 1.7, where we plot the standard deviation of the output gap σ_y against the one of inflation σ_π for each value of λ. The blue solid line with squares displays the results under EBL while the black dashed line with asterisks shows results under CGL.[32]

The monetary policy trade-off between closing the output gap and stabilising inflation under supply shocks also arises under adaptive learning. As we increase λ (move up the policy frontier), output gap volatility decreases, while inflation volatility increases. However, the policy frontier under EBL is flatter than the one under CGL, which indicates that any reduction in inflation volatility is more costly in terms of output gap volatility. To understand the flattening of the policy frontier under EBL, recall that, under adaptive learning, the nominal interest rate affects expectations only with a delay of one period. Intuitively, monetary policy affects the *current* output gap via the dynamic IS curve, which, in turn, affects *current* inflation via the NKPC. The current realisation of the output gap and inflation, however, affects agents' revision of their PLM parameters in the *subsequent* period. Consider an increase in monetary policy's output gap stabilisation motive, λ. In response to an inflationary cost push shock that decreases the output gap, a higher value for λ dampens the downturn in the output gap but reinforces the upturn in inflation, ceteris paribus. Under backward looking expectations, the reinforced increase in inflation in the current period feeds back on next period's inflation via expectations. Under EBL, the latter feedback mechanism is weaker because, as discussed above, the aggregate perceived persistence is lower than under CGL, on average. Consequently, an increase in λ increases the inflation volatility by less under EBL.

Note that the policy frontier under EBL also shifts inwards relative to the one under CGL. Hence, the output gap volatility under EBL is lower than under CGL. The lower output gap volatility again results from the lower aggregate perceived persistence of the

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[31] Specifically, we define a grid of points for λ in the interval $[0, 1]$ and simulate the economy as described above for 300,000 periods for each grid point. Note that we keep φ_π fixed so that increase in λ corresponds to an increase in φ_y. Under the baseline calibration of the monetary policy rule, $\lambda = 1/12$. If we increase φ_π to 3.0, the results are qualitatively unaffected.

[32] We refrain from showing the policy frontier obtained under RE because the misspecification of households forecasting model under adaptive learning impedes a proper comparison.

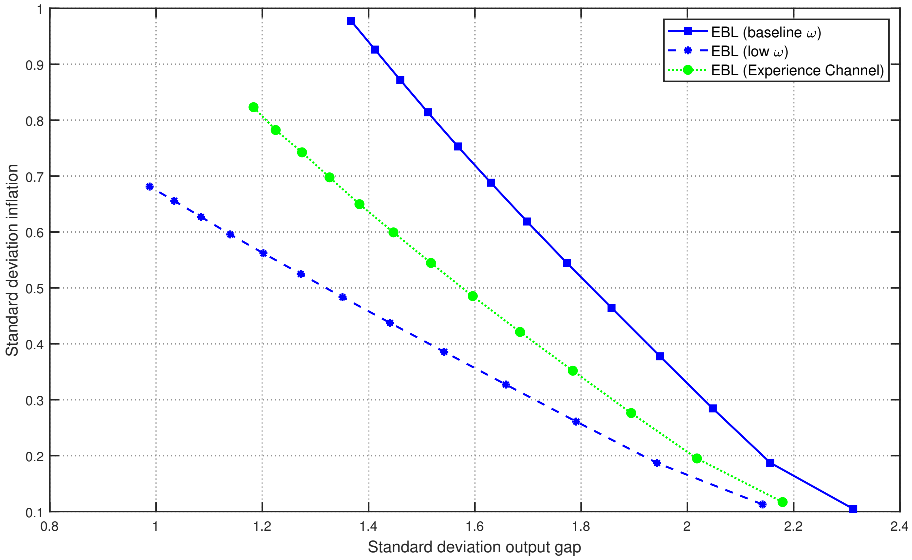

Figure 1.8: Monetary Policy Frontier Under a Supply Shock: Experience Channel

Notes: Policy frontier for baseline value of survival probability ω (blue solid), for low value of ω (blue dashed) and for the case where we hold the effective discount rate $\tilde{\beta} = \beta\omega$ constant (green dotted).

output gap and inflation. In consequence, expectations and, thereby, current variables, react by less to past monetary policy actions or past shocks so that the output gap is less volatile and the policy frontier shifts inwards compared to CGL.[33]

Demography. Next, we consider how the policy frontier is affected by a change in the demographic structure of the economy by reducing ω to 0.96 (corresponding to a high share of young agents). In Figure 1.8, the full impact of a variation in ω on the policy frontier, when channels 1.–2. operate, is shown by the blue dashed line with circles. The policy frontier when only the Experience Channel is considered is given by the green line. In total, we observe a downward shift and flattening of the policy frontier. Hence, in an economy with a high share of young individuals, a given level of inflation volatility is associated with a lower output gap volatility. Moreover, the lower slope of the policy frontier indicates that in a young society a given reduction in inflation volatility is more costly in terms of additional output gap volatility compared to an old society (blue rectangles).

Much of this change is driven by the Experience Channel, as it already generates a substantial downward shift and flattening of the policy frontier (green line). Recall that an increase in the share of young agents reduces the aggregate perceived persistence under EBL through the Experience Channel. As we have seen when comparing EBL and CGL, a reduction in the perceived persistence flattens the policy frontier. The same

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[33] Appendix 1.D deepens the comparison between EBL and CGL and shows that only when setting the constant gain parameter, g, to an empirically implausible value, one can obtain a similar policy frontier as under EBL.

mechanism applies when comparing young to old economies under EBL.[34] The total effect of a demographic shift is more pronounced, as the effective discount factor is decreasing (channel 1.), which further attenuates the effect of expectations on current outcomes of inflation and the output gap. As a result, the curve further flattens.

1.6.3 Implications of EBL for Monetary Policy

The last sub-sections revealed that considering experience effects changes the transmission of monetary policy and the stabilisation trade-off under supply shocks. We can draw three types of conclusions for policymakers.

The first type is related to the expectation formation under EBL. We have shown that, under EBL, the aggregate perceived persistence of inflation and the output gap is lower, on average. In consequence, the limited experience of young to medium aged households leads them to expect a less persistent impact of monetary policy. The resulting impaired monetary policy transmission on inflation could be counteracted by a more aggressive response of the nominal interest rate to inflation. A further possibility to increase the transmission on inflation would be to stabilise fluctuations in the perceived persistence of young to medium aged households. The resulting increase in the aggregate perceived persistence would strengthen the monetary policy transmission on inflation via expectations. For this purpose, monetary policy could aim to broaden the set of observations of young to medium old households and recommend to attach a weight on these observations when updating their PLM parameters. Hence, monetary policy communication could also include the provision of information of past observations of inflation and the output gap. Coibion et al. (2022) show that the communication of the most recent inflation rates significantly affects households' inflation expectations. Our analysis emphasises the particular importance for monetary policy to broaden the information set of young to medium aged households. The provision of information to these age groups would increase the monetary policy transmission on inflation via expectations.

The second type is related to the flattening of the policy frontier under EBL. Erceg (2002) contends that many central banks have established a symmetric "inflation band" around the inflation target. He argues that these inflation bands should be based on the monetary policy frontier of the economy. Under EBL, a certain inflation band is associated with a lower output gap volatility than under CGL (inward shift of policy frontier). If, however, monetary policy aims to tighten the inflation band, the corresponding increase in the output gap volatility is more pronounced under EBL than under CGL (smaller slope of policy frontier). The flattening of the policy frontier under EBL results from the lower aggregate perceived persistence of the output gap and inflation. To mitigate

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[34] In Appendix 1.D, we show that under CGL, the demography-driven shift in the policy frontier is substantially less pronounced.

the stabilisation trade-off, monetary policy could strengthen its transmission on inflation by communicating the history of both the output gap and inflation. As discussed above, recommending young and medium aged households to base their forecasts on a broader set of observations would increase their average PLM parameters, thereby mitigating monetary policy's stabilisation trade-off.

The third type relates to the role of demography. Our analysis provides a novel channel by which demographic variations affect monetary policy: the Experience Channel. Aggregate expectations are a function of the age distribution so that demographic variations affect aggregate expectations through a composition effect. We have shown that the Experience Channel matters for the monetary policy transmission of inflation and, thereby, for its stabilisation trade-off. In consequence, our analysis suggests that central banks need to account for the Experience Channel when evaluating the consequences of demographic shifts for their monetary policy.

1.7 Conclusion

This paper discusses how experience-based heterogeneity in expectations affects monetary policy. To address this issue, we introduce a New Keynesian model with overlapping generations in which individuals' expectations of inflation and the output gap depend on their lifetime experiences, which creates heterogeneity in expectations. Expectations heterogeneity in our model is based on the heterogeneity in the perceived persistence across cohorts. We show that under experience-based learning (EBL), the *aggregate* perceived persistence in the economy is pushed down relative to a model with constant-gain learning (CGL) in which agents attach the same constant weight to new information.

Under EBL, the pass-through of monetary policy via expectations on inflation and the output gap weakens relative to models with CGL or rational expectations. Hence, abstracting from experience effects overstate the impact of monetary policy on inflation. Further, due to the lower (delayed) influence of monetary policy on expectations, the stabilisation trade-off under supply shocks aggravates, as any reduction in inflation volatility is more costly in terms of output gap volatility. The demographic structure directly affects aggregate expectations, which are a size-weighted average across cohort, through a composition effect. Consistent with the empirical literature we show, first, that the response of inflation to a monetary policy shock is more pronounced and more persistent in an economy with a higher share of old individuals. Second, the monetary policy trade-off between output gap and inflation stabilisation under supply shocks occurs for higher variable volatilities in older economies. At the same time, stabilising inflation involves less additional output gap volatility so that the trade-off attenuates. Thus, the age structure, through EBL, is a relevant factor to determine the transmission of monetary policy.

Appendix

1.A Equilibrium under EBL

We define equilibrium conditions for the EBL economy. Importantly, we perform aggregation of the cohorts' Euler equations into an aggregate IS-curve.

Aggregation. We follow the literature on heterogeneous expectations that relies on the axiomatic approach of Branch and McGough (2009) to aggregate the decisions of agents with heterogeneous expectations without including the wealth distribution as an additional state variable.[35] To do so, we rely on two key assumptions

1. The structure of higher order beliefs: $\tilde{E}_t^i \tilde{E}_t^k x_{t+1} = \tilde{E}_t^i x_{t+1},\ i \neq k$.
2. Agents expect to return to the same wealth in the long-run: $\tilde{E}_t^i \left(\hat{c}_\infty - \hat{c}_\infty^i\right) = 0$.

Consider the Euler equation given in (1.4)

$$c_{t|k} = \beta \tilde{E}_t^k \left(c_{t+1|k} \frac{r_t}{\pi_{t+1}} \right) .$$

The linearised Euler equation of a household in cohort i is given by

$$\hat{c}_{t|i} = \tilde{E}_t^i \hat{c}_{t+1|i} - \left(\hat{r}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right) \ \forall i .$$

Forward iteration of the Euler equation yields

$$\hat{c}_{t|i} = \underbrace{\lim_{j\to\infty} \tilde{E}_t^i \hat{c}_{\infty|i}}_{\equiv \tilde{E}_t^i \hat{c}_\infty^i} - \tilde{E}_t^i \sum_{j=0}^{\infty} \left(\hat{r}_{t+j} - \hat{\pi}_{t+1+j} \right) \ \forall i , \tag{1.A.1}$$

where we used Assumption A5. of Branch and McGough (2009), which states that the Law of Iterated Expectations is satisfied, i.e., $\tilde{E}_t^k \left(\tilde{E}_{t+1}^k \left(c_{t+2} \right) \right) = \tilde{E}_t^k (c_{t+2})$.

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[35] Examples in the literature that rely on this approach are Gasteiger (2014), Di Bartolomeo et al. (2016), Hagenhoff (2018).

The aggregated linearised resource constraint in t and in $t+1$ is

$$\begin{aligned}
\hat{c}_t &= (1-\omega)\sum_{k=-\infty}^{t} \omega^{t-k}\hat{c}_{t|k} = \hat{y}_t \ , \\
\hat{c}_{t+1} &= (1-\omega)\sum_{k=-\infty}^{t+1} \omega^{t+1-k}\hat{c}_{t+1|k} = \hat{y}_{t+1} \ . \qquad (1.A.2)
\end{aligned}$$

Next, insert the forward iterated Euler equation (1.A.1) into the $t+1$-resource constraint (1.A.2) for $\hat{c}_{t+1|k}$ (for each cohort in $t+1$, respectively) and take expectations of cohort i

$$\tilde{E}_t^i\left[(1-\omega)\sum_{k=-\infty}^{t+1}\omega^{t+1-k}\left(\tilde{E}_t^k\hat{c}_\infty^k - \tilde{E}_t^k\sum_{j=1}^{\infty}\left(\hat{r}_{t+j}-\hat{\pi}_{t+1+j}\right)\right)\right] = \tilde{E}_t^i\left(\hat{y}_{t+1}\right) \ . \qquad (1.A.3)$$

Following Branch and McGough (2009), the treatment of higher-order beliefs matters for the further steps to aggregation. They impose that agents' expectations about what other agents expect are equal to their own expectation, which corresponds to assumption 1. Having departed from RE and, therefore, having assumed that agents do not know the underlying structure of the economy, imposing that they do not foresee how others form expectations can be seen as consequential. We rewrite (1.A.3) as

$$\begin{aligned}
\tilde{E}_t^i\left(\hat{y}_{t+1}\right) &= (1-\omega)\sum_{k=-\infty}^{t+1}\omega^{t+1-k}\left(\tilde{E}_t^i c_\infty^k - \tilde{E}_t^i\sum_{j=1}^{\infty}\left(\hat{r}_{t+j}-\hat{\pi}_{t+1+j}\right)\right) \\
&= \tilde{E}_t^i\left((1-\omega)\sum_{k=-\infty}^{t+1}\omega^{t+1-k}c_\infty^k\right) \\
&\quad - \tilde{E}_t^i\left((1-\omega)\sum_{k=-\infty}^{t+1}\omega^{t+1-k}\sum_{j=1}^{\infty}\left(\hat{r}_{t+j}-\hat{\pi}_{t+1+j}\right)\right) \\
&= \tilde{E}_t^i c_\infty - \tilde{E}_t^i\sum_{j=1}^{\infty}\left(\hat{r}_{t+j}-\hat{\pi}_{t+1+j}\right) \ ,
\end{aligned}$$

where the last equality uses assumption 2, which we discuss below, and that weights sum to one. We use this to substitute the infinite sum of real interest rates in (1.A.1)

$$\begin{aligned}
\hat{c}_{t|i} &= \tilde{E}_t^i c_\infty^i - \tilde{E}_t^i\sum_{j=1}^{\infty}\left(\hat{r}_{t+j}-\hat{\pi}_{t+1+j}\right) - \left(\hat{r}_t - \tilde{E}_t^i\hat{\pi}_{t+1}\right) \\
&= \tilde{E}_t^i\left(\hat{y}_{t+1}\right) - \tilde{E}_t^i\left(c_\infty - c_\infty^i\right) - \left(\hat{r}_t - \tilde{E}_t^i\hat{\pi}_{t+1}\right) \ .
\end{aligned}$$

Of particular interest is the term $\tilde{E}_t^i\left(c_\infty - c_\infty^i\right)$, which denotes expected differences of own consumption and aggregate household consumption in the limit. Branch and McGough

(2009) deal with such a term by assuming that agents agree on expected differences in limiting consumption so that in aggregation it vanishes. Equivalently, Hagenhoff (2018) assumes that agents expect to be back at the steady state in the long-run, which also eliminates the term. We use the same assumption but adopt it for our usage in the following sense: take cohorts i and k that both expect to have steady state consumption in the long-run

$$\tilde{E}_t^j \hat{c}_\infty^j = \tilde{E}_t^j \hat{c}_\infty \qquad \text{for } j = i, k \ .$$

Now, let cohort i take expectations of the limiting expectations for cohort k and invoke assumption 1: $\tilde{E}_t^i \left(\tilde{E}_t^k \hat{c}_\infty^k \right) \overset{2}{=} \tilde{E}_t^i \left(\tilde{E}_t^k \hat{c}_\infty \right) \overset{1}{=} \tilde{E}_t^i \hat{c}_\infty$. Cohorts not only expect to be back at the steady state but also expect this for others. Assuming agents expect to be back at the steady state in the long run, in which all consume equally, and having a unit mass of agents implies $c = c^k \ \forall \ k$ for non-explosive PLM-parameters. Under assumption 2 we get

$$\hat{c}_{t|i} = \tilde{E}_t^i \left(\hat{y}_{t+1} \right) - \left(\hat{r}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right) \ . \tag{1.A.4}$$

Note that (1.A.4) holds for all cohorts. Insert into the aggregate resource constraint in t

$$\begin{aligned} \hat{y}_t &= (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \hat{c}_{t|k} = (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \left(\tilde{E}_t^k \left(\hat{y}_{t+1} \right) - \left(\hat{r}_t - \tilde{E}_t^k \hat{\pi}_{t+1} \right) \right) \\ &= (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \tilde{E}_t^k \left(\hat{y}_{t+1} \right) - (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \left(\hat{r}_t - \tilde{E}_t^k \hat{\pi}_{t+1} \right) \ . \end{aligned}$$

Finally, we use the definition of aggregate expectations, $\bar{E}_t x_{t+1} = (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \tilde{E}_t^k x_{t+1}$ to receive the aggregate dynamic IS-curve

$$\hat{y}_t = \bar{E}_t \left(\hat{y}_{t+1} \right) - \left(\hat{r}_t - \bar{E}_t \hat{\pi}_{t+1} \right) \ . \tag{1.A.5}$$

It remains to rewrite the IS curve in terms of the output gap. We get

$$\tilde{y}_t = \bar{E}_t \left(\tilde{y}_{t+1} \right) - \left(\hat{r}_t - \bar{E}_t \hat{\pi}_{t+1} - r_t^n \right) \ ,$$

where $r_t^n = \bar{E}_t \left(\Delta \hat{x}_{t+1} \right) = 0$. The derivation of the New Keynesian Phillips Curve stays unchanged.

Summary. We receive the following system of equations:

$$\tilde{y}_t = \bar{E}_t \tilde{y}_{t+1} - \left(\hat{r}_t - \bar{E}_t \hat{\pi}_{t+1} \right) \ , \tag{1.A.6}$$

$$\hat{\pi}_t = \beta \omega \bar{E}_t \hat{\pi}_{t+1} + \kappa \tilde{y}_t + u_t \ , \tag{1.A.7}$$

$$\hat{r}_t = \varphi_\pi \hat{\pi}_t + \varphi_y \tilde{y}_t + m_t \ , \tag{1.A.8}$$

$$m_t = \rho m_{t-1} + \nu_t^m \ , \tag{1.A.9}$$

$$u_t = \rho_u u_{t-1} + \nu_t^u \ , \tag{1.A.10}$$

where the expectations in the New Keynesian Phillips Curve and in the dynamic IS-curve follow EBL.

1.B Simulation Algorithm

Let $X_t = (\tilde{y}_t, \hat{\pi}_t)'$ be the vector of endogenous variables, $\hat{\epsilon}_t = (m_t, u_t)'$ be the vector exogenous shocks and $\nu_t = (\nu_t^m, \nu_t^u)'$ be the vector of innovations. The state space representation of the model economy (1.15) under learning at time t is given by

$$\begin{aligned} AX_t &= B_{t-1}X_{t-1} + D\hat{\epsilon}_t \qquad (1.B.1)\\ \hat{\epsilon}_t &= \mathcal{R}\hat{\epsilon}_{t-1} + \Omega\nu_t \ , \qquad (1.B.2)\end{aligned}$$

where A, B_{t-1}, D, $\mathcal{R}$ and Ω are 2×2 matrices given by

$$A = \begin{bmatrix} 1+\varphi_y & \varphi_\pi \\ -\kappa & 1 \end{bmatrix} \ , \quad B_{t-1} = \begin{bmatrix} (b_{t-1}^y)^2 & (b_{t-1}^\pi)^2 \\ 0 & \beta\omega(b_{t-1}^\pi)^2 \end{bmatrix} \ , \quad D = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \ ,$$
$$\mathcal{R} = \begin{bmatrix} \rho_m & 0 \\ 0 & \rho_u \end{bmatrix} \ , \quad \Omega = \begin{bmatrix} \sigma_m & 0 \\ 0 & \sigma_u \end{bmatrix} \ .$$

Note, that the matrix B_{t-1} is time-varying because it involves the recursively updated parameter estimate of individuals' forecasting model for the output gap (b_{t-1}^y) and inflation (b_{t-1}^π). The updating process is given by equations (1.18a) – (1.18b).

While the PLM parameters under CGL are homogenous across age groups (i.e., $\delta_{t-1|k}^{\mathbf{z}} = \delta_{t-1}^{\mathbf{z}}\ \forall k$ and $\mathbf{z} \in \{\tilde{y}, \hat{\pi}\}$), the *aggregate* PLM parameters under EBL are given by

$$b_{t-1}^{\mathbf{z}} = (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} b_{t-1|k}^{\mathbf{z}} \ . \qquad (1.B.3)$$

Theoretically, agents can live forever (and the number of cohorts is infinite) so that we need to choose a finite number of cohorts when simulating the model. A higher number approximates the "true" economy more closely but comes at the cost of a higher computational time. We, therefore, choose the number of cohorts as 200 (equivalent to a 50-year working life) and normalise cohort weights to sum to one.

Remember, that due to our timing assumption depicted in Figure 1.2, it follows that (see equation (1.17))

$$\tilde{E}_t^k\left(\mathbf{z}_{t+1}\right) = (b_{t-1|k}^{\mathbf{z}})^2 \mathbf{z}_{t-1} \ .$$

Under CGL, it holds that $(b_{t-1|k}^{\mathbf{z}})^2 = (b_{t-1}^{\mathbf{z}})^2$ for each cohort k, whereas under EBL

$$(b_{t-1}^{\mathbf{z}})^2 = (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} (b_{t-1|k}^{\mathbf{z}})^2 \ . \qquad (1.B.4)$$

This has to be taken into account when simulating the model using equation (1.B.1).

ALGORITHM TO SIMULATE THE MODEL[36]

The simulation algorithm works as follows. To start the recursion, we need the initial parameter estimates, (δ^{y}_{-1}) and (δ^{π}_{-1}), and the initial moment matrices, R^{y}_{-1} and R^{π}_{-1}. To obtain those objects, we first simulate the model under RE for $T_{\text{init}} = 10{,}000$ periods. The PLM parameters are obtained by estimating a simple AR(1) for both the output gap and inflation (depending on whether the PLM includes a constant or not, the AR(1) process either includes or excludes a constant as well). In the initial period, we endow all cohorts with the same PLM parameters and moment matrices.

1. In period t, we simulate X_t based on equation (1.B.1) given the PLM parameters $\delta^{\mathbf{z}}_{t-1}$ and the exogenous shocks, $\hat{\epsilon}_t$.
2. Based on the new observation of $\mathbf{z} \in \{\hat{\pi}, \tilde{y}\}$, we update $\delta^{\mathbf{z}}$ and $R^{\mathbf{z}}$ for each cohort using (1.18a) and (1.18b).[37]
3. We repeat steps 1 – 2 for T_{sim} periods.

The actual simulation displayed in the figures discards the first $T_b = 1000$ iterations to allow the impact of initial values from the RE economy to wash out.

In step 2, we use a so-called projection facility to ensure the model under learning can be solved (see Orphanides and Williams, 2007; Slobodyan and Wouters, 2012a). Conceptually, it reinitializes the updating step as soon as new simulated data makes agents update their PLM parameters that renders their forecasting model non-stationary. We proceed as follows:

PROJECTION FACILITY

1. We take the updated PLM parameters and check whether they make the forecasting model explosive, i.e., $|b^{\mathbf{z}}_{t|k}| > 1$. If the forecasting model generates non-explosive behaviour, we allow the updating step.
2. If $|b^{\mathbf{z}}_{t|k}| > 1$, the new PLM parameter and the new moment matrix are set to the values from the previous period.

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[36] For the simulation, we rely on Matlab 2020a.

[37] Note, that, under EBL, we endow the newly born cohort with PLM parameters equal to the aggregate PLM parameter (1.B.3) of the last period. A variation of the initial belief did not greatly alter results.

The initial parameter estimates are obtained by the same procedure as state above. Then, we proceed as follows.

i. Draw a sequence of random innovations to the cost-push shock, $\{\nu_t^u\}_{t=0}^{T_s}$, from its normal distribution. The length of the sequence is set to $T_s = T_b + H$, where T_b denotes the number of burn-in periods and H denotes the horizon of the impulse response function considered.

ii. Generate two sequences for the monetary policy shock:

   a. $\{\nu_{1,t}^m\}_{t=0}^{T_s}$ where $\nu_{1,t}^m = 0$ for all $t \in \{0, \ldots, T_s\}$
   b. $\{\nu_{2,t}^m\}_{t=0}^{T_s}$, where at $t = T_{\text{imp}} \equiv T_b + 1$, $\nu_{2,t}^m = 0.25$, and $\nu_{2,t}^m = 0$ otherwise.

iii. Simulate the model with adaptive learning (either EBL or CGL) under the sequence of innovations $\nu_{1,t} = (\nu_{1,t}^m, \nu_t^u)$ following the steps in 1. – 2.

iv. Simulate the model with adaptive learning (either EBL or CGL) under the sequence of innovations $\nu_{2,t} = (\nu_{2,t}^m, \nu_t^u)$ following the steps in 1. – 2.

iv. Repeat step i. to iv. $N = 8{,}000$ times. For $n = 1, \ldots, N$, let the sequence for a generic endogenous variable of interest x obtained under step iii. and step iv. be given by $\{x_{1,t}^n\}_{t=0}^{T_s}$ and $\{x_{2,t}^n\}_{t=0}^{T_s}$, respectively.

v. The impulse response of the generic variable x to a monetary policy shock at horizon $h \in \{1, \ldots, H\}$ is defined as:

$$\text{irf}_{x,h} = \frac{1}{N}\sum_{n=1}^{N} x_{T_b+h,2}^n - \frac{1}{N}\sum_{n=1}^{N} x_{T_b+h,1}^n$$

1.C Generalised Impulse Responses

Supply Shock. Figure 1.C.1 denotes generalised impulse responses after a negative supply shock. The difference in responses between EBL and CGL that are driven by expectations remain unchanged.

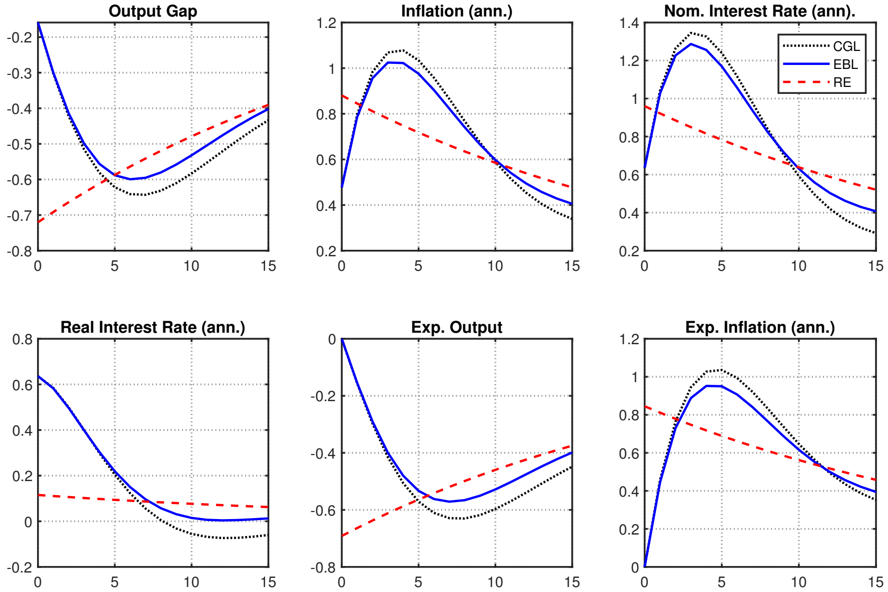

Figure 1.C.1: Generalised Impulse Response Functions to a Supply Shock

Notes: We show the generalised impulse response functions for key variables in the economies under CGL (black), EBL (blue), and RE (red). We average responses over 8,000 iterations. The output gap and output gap expectations are measured as percentage deviations from their respective steady state, while the other variables are measured as (annualised) deviation from their respective steady state.

Types of Expectation Formation. In the main text we discuss how a change in the mass of young agents affects the response to monetary policy shocks under EBL. Figure 1.C.2 depicts results for all types of expectation formation.

The effect of a change in ω on the GIRFs under RE (third row) is negligible. We also compare the effect of the age-distribution on the GIRF in a model with CGL (second row) in which all cohorts have the same expectations of the output gap and inflation to the model with EBL (first row). The aggregate perceived persistence is lower under the latter assumption on expectation formation. When the share of young agents increases under EBL, the aggregate perceived persistence falls further compared to the baseline economy due to the Experience Channel. In contrast, under CGL without experience effects, such changes do not occur. In consequence, the difference in the GIRFs in the baseline and the young economy is more pronounced when considering the model with EBL in the first

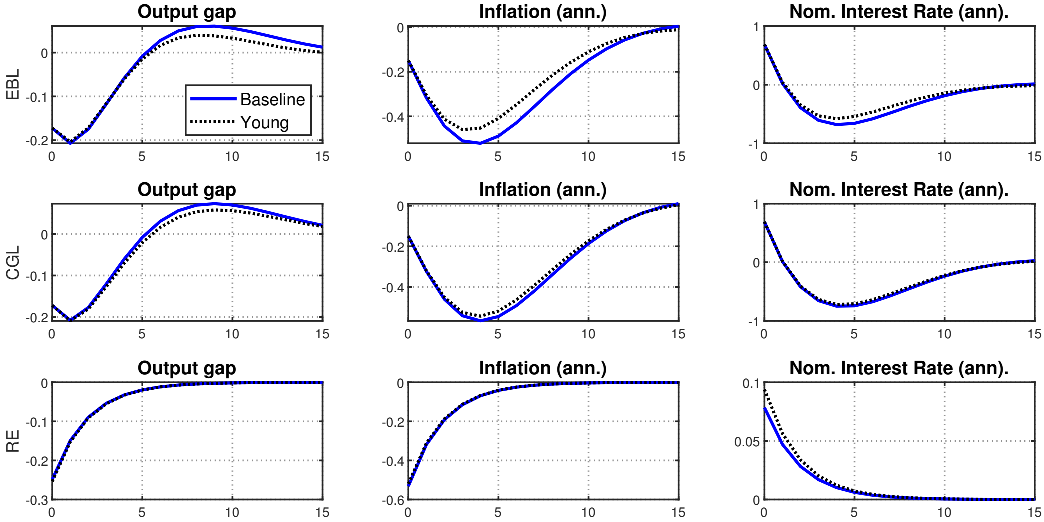

Figure 1.C.2: Generalised Impulse Response Functions to a Monetary Policy Shock Shock for Different Demographic Structures

Notes: We show the generalised impulse response functions for key variables in the economies under EBL (first row), CGL (second), and RE (third). The black solid lines denote the IRF under the baseline calibration of $\omega = 0.995$ while the black dashed lines denote the IRF in the young economy when $\omega = 0.96$. We average responses over 8,000 iterations. The output gap and output gap expectations are measured as percentage deviations from their respective steady state, while the other variables are measured as (annualised) deviation from their respective steady state.

row of Figure 1.C.2. Under CGL, differences to the baseline economy solely stem from a change in discounting (see channel 1.).

1.D Monetary Policy Trade-Off

In this Appendix, we further discuss the differences between policy frontiers under EBL and CGL.

1.D.1 The Role of the Gain Parameter

The key difference between EBL and CGL is the weighting of the most recent observation in the updating process. Under EBL, the gain of agents $\gamma_{t|k}$, depending on age, ranges from 0 to 1.[38] If in contrast, we choose a model based on CGL the literature suggests a gain of $g = 0.015$. Technically, it is possible to replicate our finding regarding the position of the policy frontier also with CGL for very high (constant) gains. Figure 1.D.1 depicts policy frontiers for models with EBL (blue) and CGL (black). Only when setting the CGL parameter to $g^* = 0.12$ we approximately replicate the EBL frontier with CGL expectation formation (light blue). Yet, such a value for the gain parameter is empirically implausible.[39]

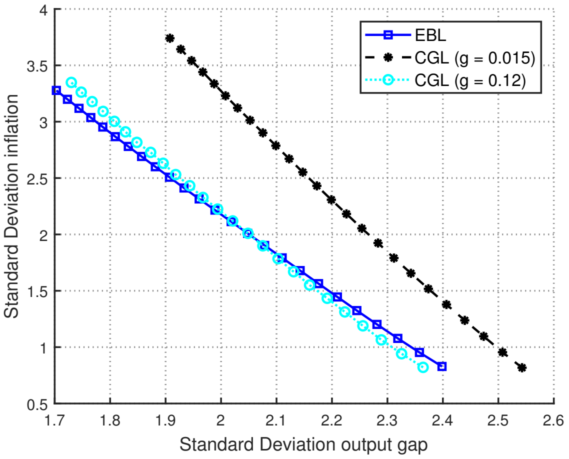

Figure 1.D.1: Monetary Policy Frontier Under a Supply Shock: High Constant Gain

Notes: The figure shows the policy frontier in the model under the baseline calibration under (i) EBL, (ii) CGL when the gain parameter is set to $g = 0.015$, and (iii) CGL when the gain parameter is set such that the policy frontier under CGL comes close to the one obtained under EBL ($g^* = 0.12$).

1.D.2 Demographic Shift: EBL vs. CGL

Recall that the shift of the policy frontier under EBL for an increase in the mass of young agents was mainly driven by the Experience Channel, that is the impact of lower aggregate

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[38] Due to the "perpetual youth" assumption, agents can potentially live forever. Hence, as age goes to infinity, the gain parameter goes to zero.

[39] Milani (2007) finds a 95% highest posterior density interval for the gain parameter of $[0.0133, 0.0231]$. Across different specifications of the PLM and initial beliefs Slobodyan and Wouters (2012b) estimate a maximum gain of 0.036.

perceived persistence on the output gap and inflation in a young economy.

Figure 1.D.2 shows the shift in the policy frontier under EBL (blue) and CGL (black) when we decrease the survival probability from its baseline value (baseline ω) to 0.96 (low ω). Importantly, under the CGL framework all agents update equally and, hence, no effect from experience effects arises. Comparing both panels, we see that without experience effects, i.e. under CGL, the policy frontier shift is less pronounced than under EBL.

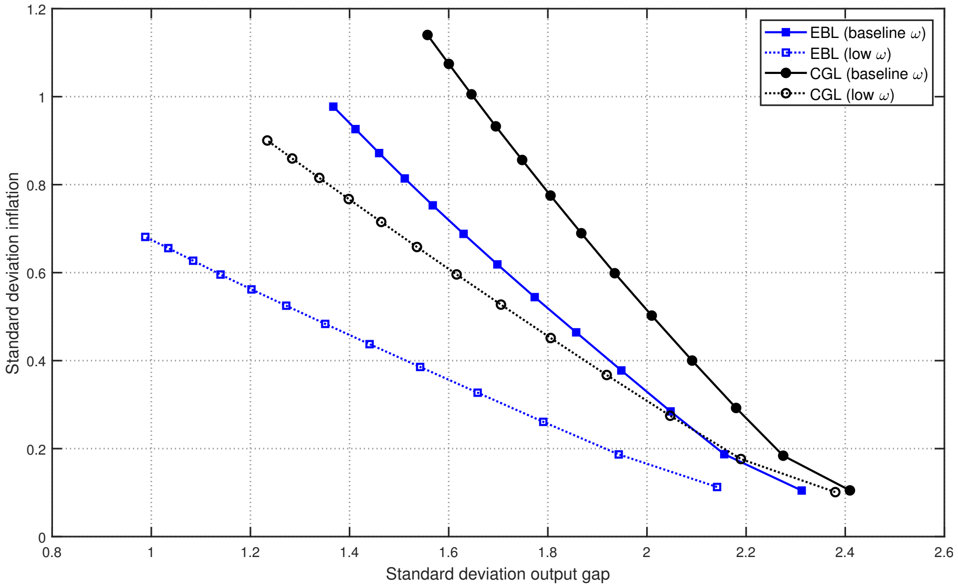

Figure 1.D.2: Monetary Policy Trade-off Under a Supply Shock: Demography

Notes: Comparison of policy frontier under EBL (blue lines) to the one under CGL (black lines) for baseline demographic structure ($\omega = 0.995$; solid line with filled marker) and young economy ($\omega = 0.96$; dashed line with empty marker). We plot the standard deviation of the output gap against the one for inflation. Units are in percent deviation from the steady state. The markers denote the grid points for λ.

Chapter 2

Cyclical Government Spending and Zero Lower Bound Risk

This chapter is based on Radke (2023).

2.1 Introduction

A binding zero lower bound (ZLB) on the nominal interest rate impairs the monetary policy stabilisation of inflation and real activity. However, even if the ZLB is *currently* not binding, the possibility to encounter the ZLB in the future – which Hills et al. (2019) call ZLB risk – exacerbates monetary policy's stabilisation policy. If agents' expectations are forward-looking, the ZLB risk affects their economic decisions already today, which generates a wedge between the economy's risky steady state and its deterministic steady state.[1] In particular, the ZLB risk causes a *deflationary bias*, i.e. an undershooting of the inflation rate in the risky steady state below its deterministic steady state, which corresponds to the central bank's desired target value.

The literature has analysed various potential adjustments of monetary policy rules to address the deflationary bias (e.g. Bianchi et al., 2021; Hills et al., 2019; Mertens and Williams, 2019). However, the role of *fiscal* rules in supporting the central bank to counteract the adverse effects of the ZLB risk has not been considered so far. Instead, the literature mainly focused on the effectiveness of *exogenous* increases in government spending during ZLB episodes (e.g. Christiano et al., 2011; Woodford, 2011). In turn, Corsetti et al. (2019) argue that government spending should be *systematically* more accommodative to support monetary policy in face of the ZLB.[2]

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[1] The risky steady state corresponds to the economy's ergodic mean in the absence of shocks (see Born and Pfeifer, 2014). In contrast to the deterministic steady state, private agents in the risky steady state take into account the macroeconomic risk caused by economic shocks in the future and adjust their economic decisions accordingly. Intuitively, Schmidt (2022) argues that the risky steady state can be considered as a formalisation of the central bank's medium-term orientation.

[2] In its recent strategy review, the European Central Bank reaffirmed the role of fiscal policy for supporting

This paper analyses the effect of *cyclical* government spending on the deflationary bias that is caused by the ZLB risk. The main finding of the paper is that fiscal policy can substantially reduce the deflationary bias and improve welfare by relying on a strong counter-cyclical rule for government spending. In turn, *pro*-cyclical government spending exacerbates the adverse effects of the ZLB risk on the risky steady state, deteriorates welfare, and might even prevent the existence of an equilibrium. Further, I show that counter-cyclical government spending effectively mitigates an increase in the deflationary bias caused by heightened ZLB risk or by a lower degree of price stickiness. Finally, a more aggressive monetary policy response to inflation already reduces the deflationary bias and substantially improves welfare, so that the *additional* stabilisation gain provided by counter-cyclical fiscal policy turns weaker.

I analyse the effect of cyclical government spending on the deflationary bias within a standard New Keynesian model (e.g. Galí, 2015). The representative household dislikes labour and values consumption as well as government spending. Fiscal policy uses lump-sum taxes to finance government spending and follows a simple rule that sets the latter as a function of output deviations from its deterministic steady state. Monetary policy sets the nominal interest rate according to a Taylor (1993)-type rule. The ZLB on the nominal interest rate is *occasionally* binding due to exogenous variations in household's discount factor, which the literature commonly uses as a proxy for demand shocks (e.g. Fernández-Villaverde et al., 2015). The model is solved with global methods in its non-linear specification to avoid potentially large approximation errors that arise in a model version that linearises all equilibrium conditions except for the ZLB constraint (e.g. Braun and Körber, 2011).

Relative to the baseline case where government spending is a-cyclical, i.e. irresponsive to fluctuations in output, a counter-cyclical government spending rule reduces the deflationary bias via two channels. First, forward-looking firms anticipate that government spending increases during ZLB episodes, which mitigates the corresponding downturn in inflation. As a result, the deflationary pressure on firms' inflation expectations caused by the ZLB risk reduces. Second, the fiscal stimulus in response to large contractionary demand shocks endogenously lowers the ZLB risk by reducing the range of demand shocks for which the ZLB is binding. Hence, counter-cyclical fiscal policy not only mitigates the downturn of inflation *during* ZLB episodes, but also the one at the economy's risky steady state which is caused by the ZLB *risk*.

In contrast, *pro*-cyclical government spending aggravates the deflationary pressure on firms' inflation expectations and endogenously increases the ZLB risk, which increases the deflationary bias. If government spending gets sufficiently pro-cyclical, the deflationary

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monetary policy in face of the ZLB (see European Central Bank, 2021a, p. 10).

pressure on firms' expectations might even be that strong so that no equilibrium exists.[3]

Further, the *marginal* reduction in the deflationary bias decreases as government spending becomes more counter-cyclical. On the one hand, a stronger *negative* response of government spending to output mitigates deviations of output from its target value. The lower fluctuations in fiscal policy's response variable endogenously diminish the additional fiscal stimulus provided by a stronger negative output coefficient. On the other hand, a higher degree of counter-cyclicality endogenously reduces the length and the frequency of ZLB episodes where fiscal policy is particularly effective in stabilising inflation.

Moreover, I determine the fiscal output coefficient that minimises the welfare costs relative to the optimal policy under commitment. The optimised fiscal output coefficient is strongly negative, i.e. government spending is strongly counter-cyclical. In comparison to the baseline case where government is a-cyclical, the optimised simple government spending rule generates a sizeable reduction in the deflationary bias and the welfare costs. However, the optimised degree of counter-cyclicality requires to balance the gains from the stabilisation of labour and inflation, on the one hand, against the increased fluctuations in private consumption and government spending, on the other hand.

Furthermore, counter-cyclical government spending mitigates the increase in the deflationary bias that is caused by (i) an increase in the standard deviation of the demand shock that increases the ZLB risk and (ii) a lower degree of price stickiness. In both cases, the optimised degree of counter-cyclicality increases.

Finally, I evaluate the interaction between monetary and fiscal policy in addressing the ZLB risk. An increase in the monetary policy inflation coefficient decreases the deflationary bias and generates a sizeable reduction in the welfare cost relative to the optimal commitment policy. Given the improved monetary policy stabilisation, counter-cyclical government spending becomes comparatively less important in supporting the central bank in stabilising inflation at the desired target value and the welfare gains from fiscal stabilisation policy attenuate.

This paper builds upon the work of Adam and Billi (2007) and Nakov (2008) who show that the possibility of encountering the ZLB generates a deflationary bias. The main contribution of the present paper is the analysis of the effectiveness of a simple fiscal rule for government spending in supporting monetary policy to mitigate the deflationary bias.

Thereby, I relate to the literature that analyses a rule-based interaction between monetary and fiscal policy in face of the ZLB. Typically, government spending is assumed to follow a simple rule and responds to variations in public debt. Bianchi and Melosi (2019) show that the ZLB can be avoided if monetary and fiscal policy coordinate to inflate away government debt that was accumulated during large recessions. Billi and Walsh

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[3] Bianchi et al. (2021) refer to such a situation as *deflationary spiral*, in which prices keep falling unboundedly.

(2022) show that a switch to a regime with passive monetary and active fiscal policy can substantially decrease the ZLB frequency and generate welfare gains relative to a regime with active monetary and passive fiscal policy.[4] However, none of these papers analyses the effect of systematic variations in government spending on the deflationary bias. Importantly, I show that passive fiscal policy can effectively reduce the ZLB frequency as well as the deflationary bias relying on counter-cyclical government spending to output. Closely related to this chapter is the paper by Hauptmeier et al. (2022). The authors show that a government spending rule that responds negatively to both government debt and inflation improves welfare and reduces the ZLB frequency. However, they do not consider the implications of cyclical government spending on the deflationary bias caused by the ZLB risk. Schmidt (2016) shows that simple "Ricardian" government spending rules can avoid expectation-driven liquidity traps, but does not analyse the effect of these rules on the deflationary bias caused by the ZLB risk. Erceg and Lindé (2014) show that a systematic negative response of government spending to output reduces the fiscal multiplier during ZLB episodes.[5]

My paper, further, contributes to the literature that analyses the normative implications of fiscal policy in face of the ZLB. Nakata (2016) and Schmidt (2013) show that fiscal policy under the optimal commitment solution should increase in response to contractionary demand shocks. Schmidt (2017) shows that a policymaker under discretion can improve welfare by putting less weight on stabilising government spending relative to inflation than society. Instead, government spending in this paper follows a simple rule and its welfare effects are evaluated based on a comparison to the optimal interaction of monetary and fiscal policy under commitment.

The remainder of this paper is structured as follows. Section 2.2 presents the New Keynesian model with an occasionally binding zero lower bound on the nominal interest rate. In Section 2.3, I describe the calibration of the model and give an overview of the solution method. Section 2.4 analyses the effect of cyclical government spending on the deflationary bias caused by the ZLB risk and welfare. In Section 2.5, I analyse how variations in structural parameters of the model affect the deflationary bias as well as the effect of cyclical government spending. Section 2.6 concludes.

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[4] Leeper (1991) refers to a regime with active monetary policy and passive fiscal policy as a regime where monetary policy controls inflation while fiscal policy adjusts the primary surplus to stabilise government debt.

[5] Similarly, Leeper et al. (2010) and Roulleau-Pasdeloup (2021) show that cyclical government spending substantially affects the fiscal multiplier, both in the short- and the long-run.

2.2 Model

The effect of cyclical government spending on the deflationary bias is considered within a standard New Keynesian framework (e.g. Galí, 2015). The economy is populated by a representative household, a perfectly competitive final good firm, a continuum of monopolistically competitive intermediate good firms that face quadratic price adjustment costs à la Rotemberg (1982) as well as a monetary and a fiscal authority. Economic fluctuations are driven by shocks to household's discount factor, that render the ZLB on the nominal interest rate occasionally binding.

2.2.1 Households

The representative household's preferences are given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \delta_t \Big\{ \ln(c_t) + \omega_g \ln(g_t) - \omega_h \frac{h_t^{1+\eta}}{1+\eta} \Big\} \, , \tag{2.1}$$

where $\mathbb{E}_0$ is the conditional expectations operator, c_t denotes private consumption, g_t government spending, and h_t hours worked. Further, $\beta \in (0,1)$ denotes the subjective discount factor, η the inverse Frisch elasticity of labour supply, and $\omega_g, \omega_h > 0$ are utility weights on government spending and hours worked, respectively. Moreover, δ_t is a preference shock that follows the process:

$$\delta_t = \delta_{t-1}^{\rho_\delta} \exp\left(\epsilon_t\right) \, , \tag{2.2}$$

where $\rho_\delta \in [0,1)$, $\epsilon_t \sim \mathbb{N}(0, \sigma_\epsilon^2)$, and $\delta_{-1} = 1$.[6] In period t, the household's budget constraint (in nominal terms) is given by:

$$P_t c_t + q_t B_t^d = B_{t-1}^d + P_t w_t h_t + P_t d_t - P_t \tau_t \tag{2.3}$$

where P_t denotes the price of the consumption good, w_t the real wage earned on each unit of labour effort, d_t denotes the total amount of real lump-sum profits from the ownership of firms, and τ_t denotes a real lump-sum tax. Moreover, B_t^d denotes nominal one-period non-state-contingent government bonds that the household purchases at price $q_t = R_t^{-1}$, where R_t is the gross one-period, riskless, nominal interest rate. The household chooses the sequence $\{c_t, h_t, B_t^d\}_{t=0}^{\infty}$ to maximise (2.1) subject to (2.3). The first order necessary

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[6] The steady state value of the discount factor shock is one.

conditions for household optimality are given by:

$$1 = \beta R_t \mathbb{E}_t \left(\frac{\delta_{t+1}}{\delta_t} \frac{c_t}{c_{t+1}} \frac{1}{\Pi_{t+1}} \right) \tag{2.4}$$

$$w_t = \omega_h h_t^{\eta} c_t \ , \tag{2.5}$$

as well as a suitable transversality condition for government bonds. Equation (2.4) denotes the standard Euler equation, equation (2.5) the inverse labour supply, and $\Pi_{t+1} = P_{t+1}/P_t$ denotes the gross inflation rate at time $t+1$.

2.2.2 Firms

Final Good Firms. The aggregate consumption good in the economy, y_t, is produced by perfectly competitive firms, which are aggregating intermediate goods $i \in [0,1]$ produced by intermediate good firms according to the technology:

$$y_t = \left[\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \ , \tag{2.6}$$

where $\theta > 0$ is the intratemporal elasticity of substitution among the intermediate goods, $y_{i,t}$. Final good producerss choose the quantities of intermediate goods to maximise their profits. The demand for intermediate good i is given by:

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} y_t \ , \tag{2.7}$$

where $P_{i,t}$ denotes the price at which the intermediate good producer i sells the input to final good producers.

Intermediate Good Firms. The representative household owns an equal share in each intermediate good firm $i \in [0,1]$ that produces a differentiated good on a monopolistically competitive market. Production of intermediate good i follows the technology:

$$y_{i,t} = l_{i,t} \ , \tag{2.8}$$

where $l_{i,t}$ denotes the labour input of intermediate good firm i. Intermediate firm i sells its good at price $P_{i,t}$ but, when changing its price, pays quadratic nominal price adjustment costs à la Rotemberg (1982). The costs of changing prices are proportional to the nominal value of aggregate production:

$$\frac{\psi}{2} \left(\frac{P_{i,t}}{\Pi P_{i,t-1}} - 1 \right)^2 P_t y_t, \tag{2.9}$$

where ψ measures the degree of nominal rigidity and Π denotes the inflation target of the central bank.[7] Current *real* period profits $d_{i,t}$ of firm i are given by:

$$d_{i,t} = \frac{P_{i,t}}{P_t} y_{i,t} - w_t l_{i,t} - \frac{\psi}{2}\left(\frac{P_{i,t}}{\Pi P_{i,t-1}} - 1\right)^2 y_t \ .$$

Taking aggregate prices P_t as given, firm i chooses $P_{i,t}$ to maximise the expected discounted present value of real profits:

$$\mathbb{E}_t \sum_{j=0}^{\infty} Q_{t,t+j}\ d_{i,t+j} \tag{2.10}$$

subject to the demand schedule of final good firms (2.7) and the production technology (2.8). Here, $Q_{t,t+j} = \beta^j(\delta_{t+j}/\delta_t)(c_t/c_{t+j})$ denotes household's stochastic discount factor for real payoffs in period $t+j$ with $j \geq 0$. I focus on a symmetric price equilibrium where each intermediate good firm i chooses the same price. Then, the optimality condition of firm i's profit-maximisation problem gives the New Keynesian Phillips Curve (NKPC):

$$\left(\frac{\Pi_t}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} = \mathbb{E}_t\left[Q_{t,t+1}\frac{y_{t+1}}{y_t}\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\frac{\Pi_{t+1}}{\Pi}\right] + \kappa\left(w_t - \mu\right)\ , \tag{2.11}$$

where $\kappa \equiv \theta/\psi$ denotes the slope of the NKPC and $\mu \equiv (\theta - 1)/\theta$ denotes the inverse of the desired gross mark-up over real marginal costs.

2.2.3 Government

Monetary Policy. The monetary authority sets the gross nominal interest rate according to:

$$R_t = \max\left[\tilde{R}_t, 1\right]\ , \tag{2.12}$$

where

$$\tilde{R}_t = R\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi}\ . \tag{2.13}$$

Here, $\tilde{R}_t$ denotes the *notional* nominal interest rate which corresponds to the nominal interest rate that the monetary authority would like to set in the absence of the ZLB, which in turn is captured by equation (2.12). Further, R and Π denote the steady state values of the interest rate and the inflation target, respectively. The policy parameter ϕ_Π

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[7] It is a common assumption in the literature that intermediate good firms perfectly index their price to the central bank's inflation target (e.g. Braun and Körber, 2011; Nakata, 2017). Hence, monetary policy has credibly communicated its inflation target to private agents.

denotes the feedback coefficient that determines the sensitivity of the nominal interest rate to deviations of inflation from its target value.[8]

Fiscal Policy. The fiscal authority finances government spending and interest payments on the outstanding government debt via lump-sum taxes and by issuing new government bonds. The government budget constraint is given by:

$$b_t = R_t \left(b_{t-1} \Pi_t^{-1} + g_t - \tau_t \right) \ , \tag{2.14}$$

where $b_t \equiv B_t / P_t$ denotes real one-period government debt supply. Due to Ricardian equivalence, the mix between taxes and government debt is indeterminate and irrelevant, so that I set $b_t = 0$. In consequence, the government budget constraint reduces to

$$g_t = \tau_t \ , \tag{2.15}$$

so that the government budget is balanced in each period. Further, I assume that government spending is set according to the rule:

$$g_t = g - \varphi_y \left(y_t - y \right) \ , \tag{2.16}$$

where g denotes the deterministic steady state value of government spending. The fiscal output coefficient, φ_y determines the strength by which government spending responds to deviations of output from its value in the deterministic steady state. Government spending is said to be counter-cyclical if $\varphi_y > 0$, pro-cyclical if $\varphi_y < 0$, and a-cyclical if $\varphi_y = 0$.

2.2.4 Market Clearing and Equilibrium

Market Clearing. Labour market clearing requires that aggregate working hours supplied by the household, h_t equal intermediate good producers' labour demand:

$$h_t = l_t \ , \tag{2.17}$$

where $l_t \equiv \int_0^1 l_{i,t} di$.

Market clearing for government bonds requires that household's demand for government bonds equals government's supply of bonds:

$$B_t^d = 0 \ . \tag{2.18}$$

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[8] In line with the simplified models considered in Bianchi et al. (2021) and Hills et al. (2019), I exclude a feedback to output in the monetary policy rule. This assumption facilitates the analysis of the effects of fiscal policy on the deflationary bias and welfare.

Clearing of the goods market requires that the total number of goods produced, y_t, equals the sum of private and public goods demand, taking into account the dead-weight loss due to repricing cost:

$$(1 - \varrho_t) y_t = c_t + g_t \ , \tag{2.19}$$

where $\varrho_t \equiv \frac{\psi}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2$ denotes the efficiency wedge arising from the Rotemberg adjustment cost. Due do Walras' law, the market for goods clears whenever labour and bond markets clear.

Equilibrium. Given δ_{-1}, a rational expectations equilibrium consists of sequences for quantities $\{c_t, g_t, h_t, y_t, l_t\}_{t=0}^{\infty}$, prices $\{\Pi_t, w_t, R_t\}_{t=0}^{\infty}$, and exogenous states $\{\delta_t\}_{t=0}^{\infty}$ such that (i) household's and firms' optimality conditions (2.4), (2.5), the suitable transversality condition for government bonds, and (2.11) are satisfied, (ii) the government budget constraint (2.15) is satisfied, (iii) government spending follows the rule (2.16) and the gross nominal interest rate is set according to the rule (2.13), taking into account the ZLB (2.12), (iv) markets for labour, bonds, and goods clear, and (v) the law of motion for δ_t is given by (2.2).

As shown by Benhabib et al. (2001), models that account for the ZLB on the nominal interest rate have two deterministic steady state equilibria. In the first steady state equilibrium, the ZLB on the nominal interest rate is binding and the net inflation rate is negative. The second steady state is characterised by a positive inflation rate that corresponds to the central bank's target value and a gross nominal interest rate strictly above one. In this paper, I follow the common approach in the literature and focus on a rational expectations equilibrium that fluctuates around the second steady state with a positive net inflation rate and the gross nominal interest rate above one (Fernández-Villaverde et al., 2015; Nakata, 2017).[9]

2.3 Calibration and Solution Method

2.3.1 Calibration

I calibrate the model to the U.S. economy where one period in the model corresponds to one quarter. The parameter values chosen are standard in the literature and are

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[9] Nakata (2017) argues that, from an empirical perspective, an equilibrium that fluctuates around the steady state with a positive inflation rate seems to be more reasonable than one that fluctuates around a steady state with deflation. For a discussion, see Gavin et al. (2015) and their section 3.2.

summarised in Table 2.1.[10] The households' discount factor β is calibrated to get a steady state real annualised return on risk-free bonds of 1.75%. This value lies within the range of estimates provided by the literature (e.g. Del Negro et al., 2019). The elasticity of substitution among intermediate goods is set to 6.0, which implies a steady state mark-up of 20%. The Rotemberg adjustment cost parameter is chosen to match a fraction of roughly 86% non-adjusters in a linearised version of the model with Calvo (1983) price setting, which implies an average price duration of six quarters.[11] Under the baseline calibration, the Taylor coefficient on inflation is set to 1.5, which is the standard value (e.g. Galí, 2015). In turn, government spending under the baseline calibration is assumed to be a-cyclical, i.e. $\varphi_y = 0$. The persistence of the preference shock, ρ_δ is set to 0.8, which as well is the standard value chosen in the literature (e.g. Nakata, 2017). The standard deviation of the innovation to the preference shock, σ_ϵ, is set such that the ZLB frequency under the baseline parametrisation of the monetary and fiscal policy rules is 10%. The ZLB frequency is defined as the unconditional probability that the economy encounters the ZLB (in percent):

Table 2.1: Baseline Calibration

Parameter	Description	Value
β	Discount factor	0.9956
ω_g	Utility weight (government spending)	0.227
ω_h	Utility weight (labour effort)	9.57
η	Inverse Frisch elasticity of labour supply	1
θ	Elasticity of substitution among intermediate goods	6
ψ	Rotemberg parameter	200
Π	Inflation Target	1.005
ϕ_Π	Inflation coefficient in monetary policy rule	1.5
φ_y	Output coefficient in fiscal policy rule	0
ρ_δ	Persistence preference shock	0.8
σ_ϵ	Std. deviation demand shock innovation ϵ	0.0189

$$p_{\text{zlb}} \equiv \Pr\left(R_t = 1\right) \times 100 \, . \tag{2.20}$$

Following Fernández-Villaverde et al. (2015), I approximate the ZLB frequency as the share of periods in the simulations in which the ZLB is binding:

$$\Pr\left(R_t = 1\right) \simeq \widehat{\Pr}\left(R_t = 1\right) = \frac{\sum_{i=1}^{T} \mathcal{I}_{\{R_i=1\}}}{T}$$

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[10] I closely follow the parameter choices of the simplified model version in Hills et al. (2019).

[11] The resulting slope of the New Keynesian Phillips Curve in the linearised version of the model amounts to $(\epsilon - 1)/\psi = 0.025$, which is in line with estimates of Boneva et al. (2016).

where T is the simulation length and $\mathcal{I}_{\{R_i=1\}}$ is an indicator function that is defined as follows:

$$\mathcal{I}_{\{R_i=1\}} = \begin{cases} 1 & \text{if ZLB is binding in period } i \\ 0 & \text{else .} \end{cases}$$

The implied ZLB frequency is lower than the one chosen by Hills et al. (2019) but is in line with the ZLB frequency observed in the U.S. post-war period (see Dordal i Carreras et al., 2016). The utility weight on labour ω_h is set such that labour hours in the deterministic steady state are 0.33. In turn, the utility weight on government spending is set so that the steady state government spending share g/y is 20% under the optimal commitment solution (described below). Moreover, monetary policy's target value for gross inflation is set such that the annualised net inflation rate (defined in section 2.4) in the deterministic steady state is 2%.

2.3.2 Solution Method

I solve the model relying on the policy function iteration algorithm as described in Richter et al. (2014). First, I approximate the lognormal $AR(1)$ process

$$\log \delta_t = \rho_\delta \log \delta_{t-1} + \epsilon_t$$

by a finite state Markov chain based on the method developed by Tauchen (1986). The finite realisations of the logarithm of δ_t obtained are then exponentiated to obtain the finite number of grid points of the state variable δ_t. The policy function iteration algorithm starts from a guess of the values that the policy functions take on the finite number of grid points of the state variable δ_t. Under the assumption that the guessed policy functions are in use *tomorrow*, the equilibrium system of the model is solved to determine the policy functions *today*. These policy functions are then assumed to be in use for the next period in the next iteration step. The iteration procedure continues until the policy functions tomorrow are sufficiently close to the policy functions today. For a detailed description of the algorithm, see Appendix 2.A.

2.4 Deflationary Bias and Fiscal Policy

2.4.1 Definition of Deflationary Bias

This section formally defines the deflationary bias. To do so, I first define the risky steady state of the annualised net inflation rate. The risky steady state corresponds to the ergodic mean of the economy in the absence of innovations to the exogenous variables. In the model, the annualised net inflation rate at the risky steady state (in percent) is defined as follows:

$$\pi^{a}_{\mathrm{rss}} = 400 \times \mathbb{E}\left(\pi_t \mid \delta_t = 1\right) \ , \tag{2.21}$$

where $\pi_t = \Pi_t - 1$ denotes the *net* inflation rate. In contrast to the deterministic steady state, agents in the risky steady state are aware of the macroeconomic risk stemming from the shock variances. Hence, π^{a}_{rss} measures the annualised net inflation rate in the state of the economy where, in the absence of shocks in the past and in the current period, private agents choose to stay, taking into account the possibility of future shocks. In a fully linear model, the stochastic steady state coincides with the deterministic steady state due to certainty equivalence. However, the model in this paper is non-linear due to the specification of household's preferences, the non-linear price adjustment costs and, notably, the presence of the ZLB. Let χ denote the wedge between the risky steady state and the deterministic steady state of the annualised net inflation rate (in basis points):

$$\chi = 100 \times \left(\pi^{a}_{\mathrm{rss}} - \pi^{a}\right) \tag{2.22}$$

where $\pi^{a} = 400 \times (\Pi - 1)$ denotes the deterministic steady state of the annualised net inflation rate, which corresponds to the central bank's target value. The economy features a deflationary bias if $\chi < 0$.

Intuitively, Schmidt (2022) argues that the economy's risky steady state can be considered as a formalisation of central bank's medium-term orientation. Many central banks, including the ECB, define their price stability objective to achieve a desired inflation rate over the medium term (Hammond, 2012). Then, a deflationary bias can be interpreted as a failure of monetary policy's price stability objective.[12]

2.4.2 Zero Lower Bound Risk and the Deflationary Bias

To understand the causes of the deflationary bias, it is useful to consider how the ZLB affects the dynamics of the model. Figure 2.1 displays the policy functions of endogenous variables of interest, i.e. the equilibrium responses of these variables to the discount factor shock at time t, δ_t. The black solid lines show the policy functions in the model with ZLB, while the blue solid lines show the policy functions in a hypothetical economy which ignores the ZLB in equation (2.12).

A smaller value of δ_t makes consumption today relatively less valuable so that house-

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[12] In Appendix 2.B, I consider the wedge between the average inflation rate and the model's deterministic steady state and discuss the differences of this wedge to the one defined in equation (2.22).

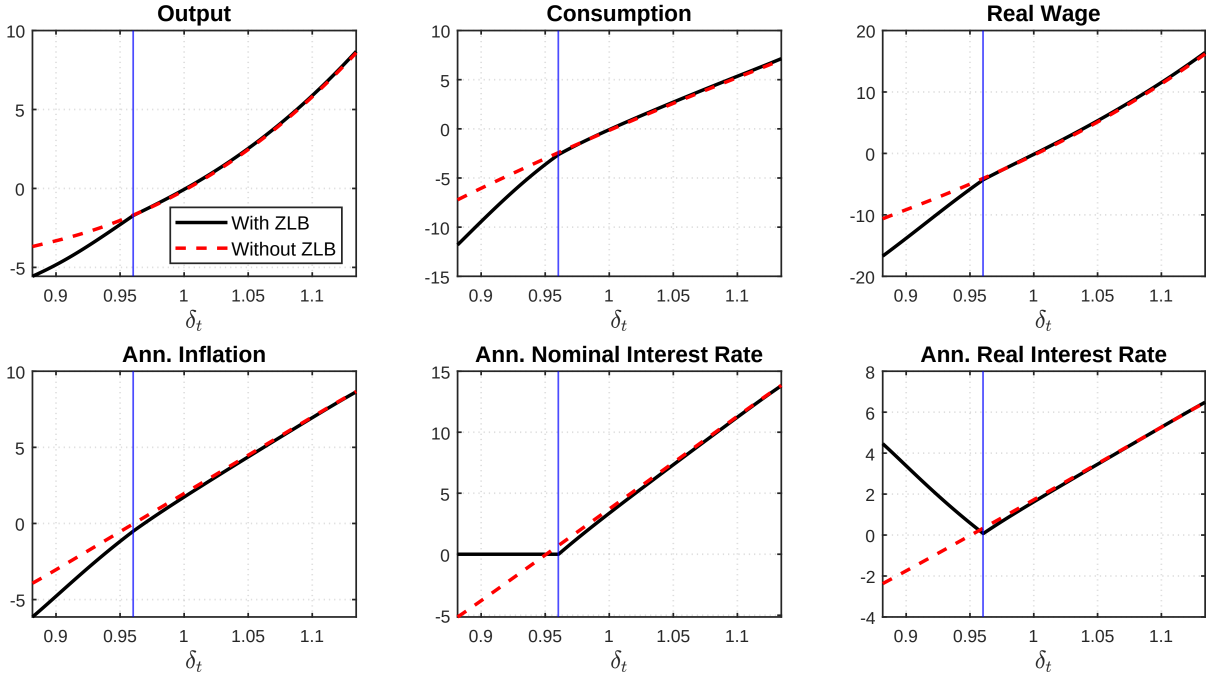

Figure 2.1: Policy Functions: The Role of the Zero Lower Bound

Notes. Equilibrium responses of endogenous variables to the discount factor shock, δ_t. Black solid line: Baseline calibration. Red dashed line: Model without ZLB. For values of δ_t to the left of the blue vertical line, the ZLB is binding in the model with ZLB. Output, consumption and the real wage are presented as percentage deviation from their respective steady state. All other variables are presented as annualised rates.

hold's demand for the final good decreases. In equilibrium, the reduction in consumption lowers output and firms' demand for labour, which reduces the real wage and inflation. To counteract the reduction in inflation, monetary policy lowers the nominal interest rate, which, due to the Taylor principle, reduces the real interest rate. However, for values of δ_t to the left of the vertical blue line, the ZLB starts to bind and prevents the nominal interest rate to fall as stipulated by the monetary policy rule (2.13). Then, the reduction in inflation expectations caused by a further decrease in δ_t *increases* the real interest rate, which further depresses consumption and, thereby, output, the real wage and inflation. In contrast, the nominal interest rate in the hypothetical economy without ZLB is allowed to fall below zero, so that the real interest rate continues to decrease. As a result, the downturn in output, inflation, consumption and the real wage is mitigated relative to the economy with ZLB.

Crucially, the ZLB not only affects the equilibrium responses of the endogenous variables when the ZLB is binding, but also in states of the economy where it its *not* binding. Since firms are forward-looking, their inflation expectations depend on *all* possible future realisations of the discount factor shock – including those where the ZLB is binding. Assume that the current nominal interest rate is away from the ZLB. In response to a large positive discount factor shock in the next period, monetary policy increases the nominal interest rate as stipulated by the monetary policy rule (2.13) to counteract the inflationary pressure. Consider now a large *negative* discount factor shock in the next period of the same absolute size. If the negative discount factor shock is sufficiently large, the ZLB might become binding. The resulting increase in the real interest rate exacerbates the drop in output, consumption and the real wage. In contrast, the resulting downturn in output, consumption and the real wage is less pronounced in the hypothetical economy without ZLB. In consequence, the presence of the ZLB reduces firms expectations on the real wage, which lowers their inflation expectations and, thereby, inflation already today.

Note that the downward pressure on firms' inflation expectations caused by the ZLB risk is particularly pronounced at or in the proximity of the ZLB, which is consistent with Nakata (2017). For low states of δ_t where the ZLB is binding in the model economy with ZLB, there is a pronounced gap between the policy functions in the model with ZLB and those in the hypothetical economy without ZLB. However, for sufficiently large values of δ_t, the policy functions in the model with ZLB hardly differ from those in the hypothetical economy without ZLB. Intuitively, the downward pressure on firms' inflation expectations is more pronounced when the discount factor shock today is already low. Assume δ_t is sufficiently low so that the current nominal interest rate is at the ZLB. A further decrease in the discount factor shock in the next period keeps the nominal interest rate at the ZLB and further depresses output, consumption, and the real wage. In consequence, the downward pressure on firms' inflation expectations caused by the ZLB risk is even more pronounced for low values of δ_t. In contrast, for sufficiently large values of δ_t, the probability of a negative shock to the discount factor that renders the ZLB binding in the next period is negligible. Hence, the downward pressure on firms' inflation expectations is substantially lower for large values of δ_t. This explains the lower difference to the policy functions in the hypothetical economy without ZLB.

Table 2.2: Deflationary Bias and the Role of the Zero Lower Bound

	Model	
	with ZLB	w/o ZLB
Annualised Net Inflation Rate		
Deterministic Steady State	2	2
Risky Steady State	1.73	1.98
Deflationary Bias (in basis points)	−27	−2

While the ZLB risk affects equilibrium responses to a greater extent when the ZLB is binding, it also affects the risky steady state of the economy, which corresponds to the value of the endogenous variables evaluated at $\delta_t = 1$.[13] The second row of Table 2.2 shows the deterministic steady state value of the annualised net inflation rate in the model with ZLB (first column) and the hypothetical economy without ZLB (second column). In both cases, the deterministic steady state value corresponds to the central bank's target value of two percent. The third row shows the risky steady state value of the annualised net inflation rate for both model versions, while the fourth row shows the deflationary bias. At the risky steady state, the annualised net inflation rate falls below the target value by 27 basis points.[14] In turn, the deflationary bias in the hypothetical model without ZLB amounts only to two basis points. In consequence, roughly 93% of the overall deflationary bias is driven by the ZLB. As shown by Hills et al. (2019), the fact that the model features a deflationary bias even without ZLB results from the presence of the model's other non-linearities, i.e. household's preferences and the quadratic price adjustment cost.

2.4.3 The Effect of Cyclical Government Spending

In the preceding section, I discussed the causes of the deflationary bias. I now analyse the mechanism by which cyclical government spending affects the deflationary bias. In Figure 2.2, I plot the deflationary bias as a function of the fiscal coefficient on output, φ_y. Under the baseline calibration where $\varphi_y = 0$ (indicated by the black dot), the deflationary bias amounts to roughly 27 basis points. The deflationary bias is strictly decreasing in φ_y. Hence, strong *counter*-cyclical fiscal policy is effective in supporting the central bank to stabilise inflation at its desired target value. For example, the deflationary bias reduces to 6 basis points when $\varphi_y = 2.0$. In turn, *pro*-cyclical fiscal policy aggravates the adverse effects of the ZLB risk on the economy's risky steady state. The deflationary bias increases to 50 basis points when $\varphi_y = -0.2$. Notably, the policy function iteration algorithm fails to converge for sufficiently negative values of φ_y, which is indicated by the grey shaded area.

Equilibrium responses. To gain an intuition about the mechanism by which cyclical government spending affects the deflationary bias, consider Figure 2.3 that shows the policy functions of various endogenous variables. The black solid lines correspond to the black lines in Figure 2.1 and show the policy functions under the baseline calibration where government spending is a-cyclical, i.e. held constant at its deterministic steady

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[13] Note that the discount factor shock corresponds to the only *exogenous* state variable in the model economy. In the absence of any *endogenous* state variable, the risky steady state corresponds to the vector of policy functions evaluated at $\delta_t = 1$ (see Hills et al., 2019).

[14] Note that the ZLB is not binding in the risky steady state and the nominal interest rate is set according to the rule (2.13). Since the rule obeys the Taylor principle ($\phi_\Pi > 1$), the deflationary bias causes the nominal interest and the real interest rate in the risky steady state to fall below their respective deterministic steady state value. In consequence, consumption and output in the risky steady state overshoot their respective deterministic steady state value.

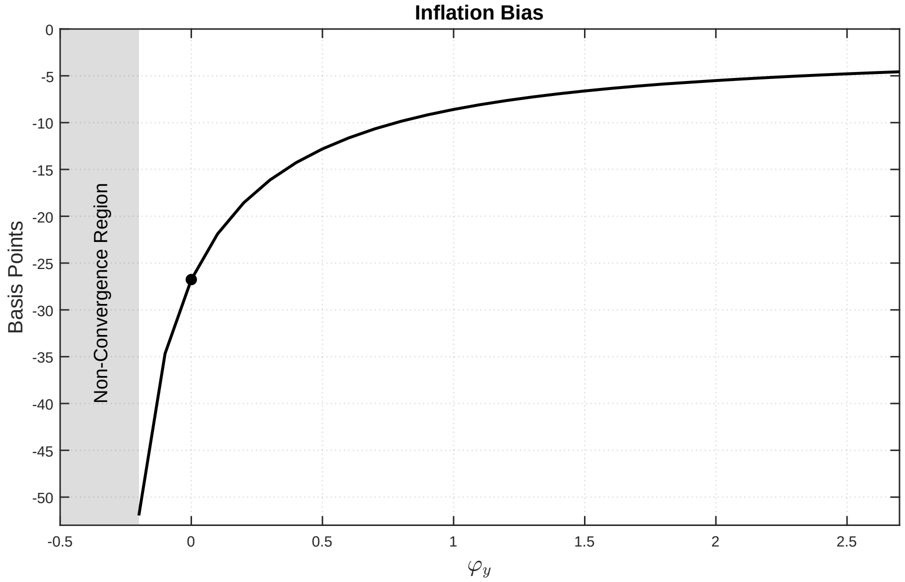

Figure 2.2: Deflationary Bias and Counter-Cyclical Fiscal Policy

Notes. Deflationary bias as function of the fiscal coefficient on output, φ_y. The grey shaded area indicates the range of values of φ_y where the policy function iteration algorithm does not converge. Black dot: Inflation bias under the baseline calibration ($\varphi_y = 0$).

state value. In turn, the red dashed lines show the corresponding responses in case government spending is strongly counter-cyclical ($\varphi_y = 2.0$), while the green dash-dotted lines show those when government spending is pro-cyclical ($\varphi_y = -0.2$). For values of δ_t to the left of the vertical lines, the ZLB is binding in the baseline economy (blue solid), in the economy with counter-cyclical fiscal policy (blue dashed), and in the economy with pro-cyclical fiscal policy (blue dash-dotted).

If being counter-cyclical, the policy function of government spending increases in δ_t. Notably, counter-cyclical government spending renders output, inflation, and the real wage more stable for all states of δ_t, compared to the case where government spending is a-cyclical. For low values of δ_t where output is below its deterministic steady state value, the counter-cyclical fiscal rule stipulates an increase in government spending to mitigate the reduction in aggregate demand. Since government spending is financed via lump-sum taxes, the increase in government spending constitutes a negative wealth effect for the household.[15] Via the negative wealth effect, the increase in government spending raises household's labour supply, which mitigates the downturn in equilibrium labour hours, output, and the real wage (Monacelli and Perotti, 2008). In turn, the effect of the increase in government spending on consumption depends on the monetary policy response. If the ZLB is not binding, the increase in government spending crowds-out consumption via the standard negative wealth effect (Baxter and King, 1993). However, if the ZLB is

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[15] To be more precise, the increase in government spending causes an equivalent increase in lump-sum taxes *in present value terms*.

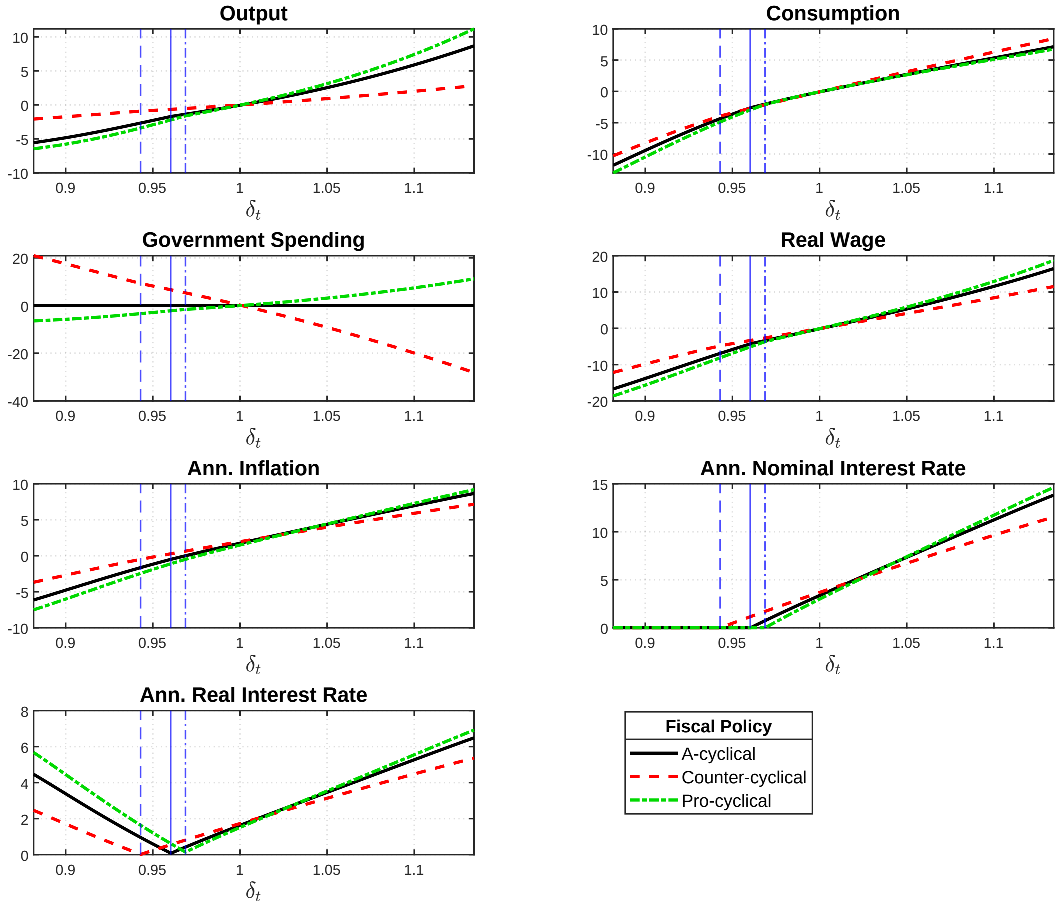

Figure 2.3: Policy Functions: The Role of Fiscal Policy

Notes. Equilibrium responses of endogenous variables to the discount factor shock, δ_t. Black solid line: Baseline calibration. Red dashed line: Negative fiscal response to output ($\varphi_y = 2.0$). Green dash-dotted line: Positive fiscal response to output ($\varphi_y = -0.2$). For values of δ_t left of the vertical lines, the ZLB is binding in the economy with a-cyclical (blue solid), counter-cyclical (blue dashed), and pro-cyclical (blue dash-dotted) government spending. Output, consumption, government spending and the real wage are presented as percentage deviation from their respective steady state. All other variables are presented as annualised rates.

binding, the increase in government spending crowds-*in* consumption. Intuitively, since the government spending hike increases the real wage, intermediate good firms raise their current prices. In the presence of nominal rigidities resulting from the price adjustment costs, the increase in firms' current prices raises their inflation expectations. Since the nominal interest rate is stuck at the ZLB, the increase in inflation expectations lowers the *real* interest rate, which increases consumption. The increase in consumption generates a beneficial feedback loop by further increasing output, marginal costs, inflation, and firms' inflation expectations.[16]

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[16] For values of δ_t where output is above its deterministic steady state value, fiscal policy lowers government spending, which reduces the household's labour supply but raises private consumption via a positive wealth effect. In equilibrium, labour hours reduce, which lowers output, the real wage, and inflation, despite the increase in private consumption.

The fiscal stabilisation of inflation for low states of δ_t reduces the deflationary bias via two channels. First, forward-looking firms anticipate that fiscal policy increases government spending during ZLB episodes, which mitigates the corresponding downturn in inflation. In consequence, the fiscal stimulus provided during ZLB episodes reduces the deflationary pressure on firms' inflation expectations at the risky steady state where the ZLB is *not* binding. Second, since the fiscal stimulus attenuates the downturn of inflation for low values of δ_t, the threshold value of δ_t for which the ZLB starts binding reduces (the vertical blue line shifts to the left relative to the baseline case where government spending is a-cyclical). Hence, counter-cyclical fiscal policy *endogenously* reduces the range of demand shocks for which the ZLB is binding and, thereby, reduces the *risk* of encountering the ZLB.

Under pro-cyclical government spending, the aforementioned intuition is reversed. Output, inflation and the real wage are less stable for all states of the discount factor shock. Moreover, the cut in public spending at the ZLB crowds-*out* private consumption because the reduction in government spending increases the real interest rate. In particular, the drop in inflation during ZLB episodes is more pronounced under pro-cyclical fiscal policy. In consequence, pro-cyclical fiscal policy increases the range of demand shocks for which the ZLB is binding and, thereby, the ZLB risk. Both firms' anticipation of more pronounced downturn in inflation for low states of δ_t and the increase in the ZLB risk cause an increase in the deflationary bias under pro-cyclical fiscal policy.

Diminishing Effectiveness of Fiscal Policy. Note that Figure 2.2 reveals that the *marginal* reduction in the deflationary bias in response to a marginal increase in φ_y diminishes. To gain an intuition about the diminishing effect of φ_y on the deflationary bias, consider Figure 2.4. The top row shows generalised impulse response functions (GIRFs) to a large reduction in the disturbance term ϵ_t that pushes δ_t three unconditional standard deviations below its steady state.[17] Black solid lines denote the GIRFs when government spending is constant, red dashed lines when $\varphi_y = 0.5$ and green dash-dotted lines when $\varphi_y = 1.0$. The bottom row shows the difference between the GIRFs when government spending is constant to the one when $\varphi_y = 0.5$ (red dashed) and to the one when $\varphi_y = 1.0$ (green dash-dotted).

In response to the negative discount factor shock, output, inflation, and the real wage decrease. Since the shock is sufficiently negative, the ZLB becomes binding so that the real interest rate increases, on impact. Consistent with the results above, counter-cyclical fiscal policy attenuates the reduction in output, inflation and the real wage and lowers the real interest rate. Relative to the baseline economy, the downturn in output and the annualised inflation rate reduces by 1.5 and 1 percentage point, respectively, if $\varphi_y = 0.5$.

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[17] The algorithm to generate the GIRFs is described in Appendix 2.A.

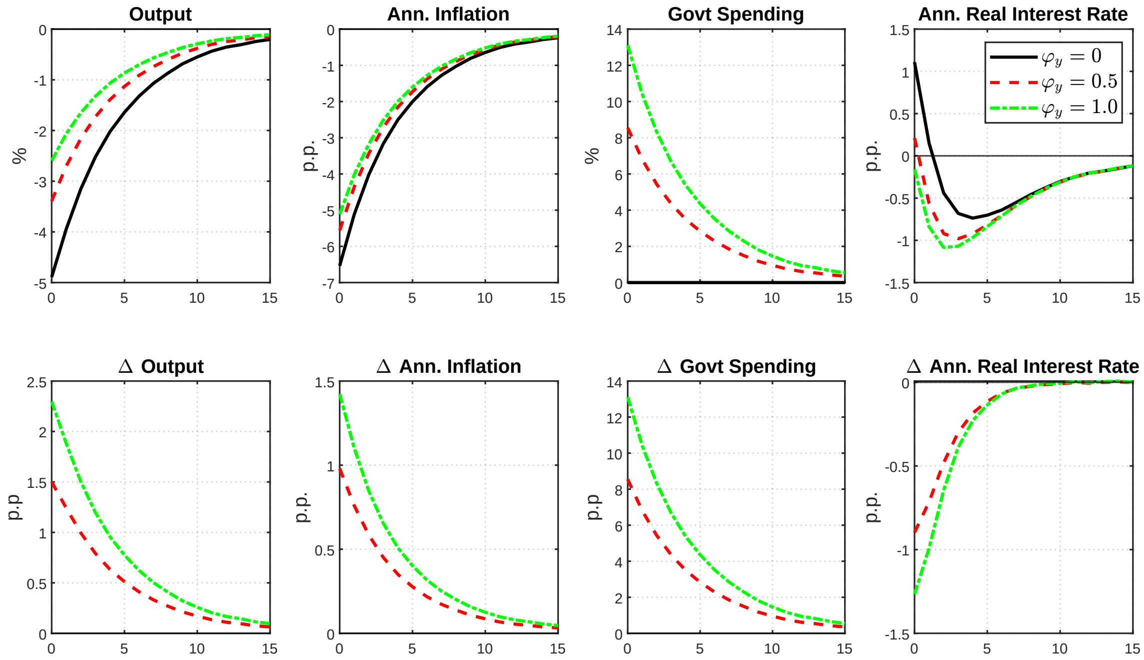

Figure 2.4: Generalised Impulse Response Functions to Discount Factor Shock and the Diminishing Effect of Fiscal Policy

Notes. Top row: Generalised Impulse Response Functions (GIRF) to a negative three standard deviation innovation to δ_t. The fiscal output coefficient is set to $\varphi_y = 0$ (black solid), $\varphi_y = 0.5$ (red dashed) and $\varphi_y = 1.0$ (green dash-dotted). Bottom row: Difference in the GIRFs under constant government spending and the one when $\varphi_y = 0.5$ (red dashed) and the one when $\varphi_y = 1.0$ (green dash-dotted).

In turn, increasing φ_y by another 0.5 units causes a further reduction in output and the annualised inflation rate by only 0.79 and 0.44 percentage points, respectively. Hence, the stabilising effect on output and inflation that results from an increase in φ_y diminishes.

Intuitively, an increase in φ_y attenuates the downturn in output so that the increase in government spending caused by the higher value of φ_y diminishes as well. Further, the fiscal stimulus counteracts the deflationary pressure of the negative discount factor shock so that the nominal interest rate leaves the ZLB earlier, which can be inferred from the fact that the real interest rate falls below zero earlier. Hence, counter-cyclical fiscal policy *endogenously* reduces the duration of the ZLB episode.[18] In consequence, an increase in φ_y not only endogenously reduces the size of the additional fiscal stimulus during ZLB episodes but also endogenously reduces the duration of episodes where it is particularly effective in stabilising inflation. As a result, the reduction in the deflationary bias caused by an increase in φ_y diminishes.[19]

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[18] The lower frequency and duration of ZLB episodes is consistent with the result that the threshold value of δ_t for which the ZLB is binding decreases (see Figure 2.3).

[19] The finding is reminiscent of the decreasing average government spending multiplier shown by Erceg and Lindé (2014). They show that the multiplier declines in the level of government spending if the duration of the ZLB episode endogenously reduces with higher government spending.

Non-Convergence under Pro-cyclical Fiscal Policy. The grey shaded area in Figure 2.2 indicates the range of values for φ_y where the policy function iteration algorithm described in section 2.3 fails to converge. As discussed in Richter and Throckmorton (2015), the algorithm fails to converge if the ZLB frequency is sufficiently high. Intuitively, the ZLB partitions the state space into two distinct regions: one where monetary policy obeys the Taylor principle, and one where it pegs the nominal interest rate at the ZLB. In the terminology of Leeper (1991), monetary policy in the former region of the state space is active, while it is passive in the latter region. As long as the private sector has sufficient expectations of returning to the region with active monetary policy, Richter and Throckmorton (2015) show that a stable equilibrium exists and the algorithm converges.

Figure 2.2 shows that fiscal policy matters for the convergence of the algorithm as well. Under pro-cyclical fiscal policy, government spending decreases during downturns, thereby crowding-out private consumption and aggravating the downturn in output and inflation. At the same time, relative to a- and counter-cyclical fiscal policy, the region of the state for which the ZLB is binding increases (see the vertical lines in Figure 2.3). Hence, as fiscal policy becomes more pro-cyclical, it endogenously increases the ZLB frequency and the ZLB risk. As the pro-cyclical fiscal response gets sufficiently strong, the resulting increase in the ZLB frequency renders the private sector expectations of returning to the Taylor rule too low for the policy functions to converge. Basu and Bundick (2015) offer an heuristic analysis on the consequences of too high ZLB risk that causes non-existence of the equilibrium. If the private sector expects the ZLB to bind in a sufficiently large number of the state space, the expected increase in the real interest rate causes the household to reduce consumption and firms to lower prices. As a result, the number of states where the ZLB is binding further increases. This process continues without converging.[20] The results above indicate that pro-cyclical fiscal policy can severely deteriorate the adverse effects of the ZLB risk on private sector's expectations and might even prevent the existence of a stable equilibrium.[21]

2.4.4 Welfare Analysis

The previous section has shown that a systematic counter-cyclical government spending rule reduces the deflationary bias. In this section, I scrutinise whether and to what extent it is desirable to use counter-cyclical government spending to reduce the deflationary bias.[22]

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[20] Bianchi et al. (2021) refer to such a situation as deflationary spiral.

[21] Notably, the empirical literature provides evidence for pro-cyclical fiscal policy at the national level. Roulleau-Pasdeloup (2021) shows that median value of the fiscal output coefficient is -0.23, which lies within the non-convergence region in Figure 2.2.

[22] Note that the purpose of the analysis is *not* to determine the fully optimal simple monetary and fiscal policy rules. Instead, the analysis scrutinises the welfare implications of cyclical government spending,

Welfare Measure. I follow Schmitt-Grohé and Uribe (2007) and search for the fiscal output coefficient, φ_y, that maximises $\mathbb{E}\left(V_0\right)$, where

$$V_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \delta_t \Big\{ \ln(c_t) + \omega_g \ln(g_t) - \omega_h \frac{h_t^{1+\eta}}{1+\eta} \Big\} \ ,$$

and $\mathbb{E}$ denotes the unconditional expectations operator. Let the policy regime under a specific parametrisation for φ_y be denoted by a. To rank alternative policy regimes, I compute the welfare costs of a specific policy regime relative to the stochastic equilibrium allocations under the optimal commitment policy.[23] Let the optimal commitment solution be denoted by c. The *unconditional* lifetime utility under the optimal commitment policy is given by:

$$\mathbb{E}V_0^c = \sum_{t=0}^{\infty} \beta^t \mathbb{E} \left(\delta_t \Big\{ \ln\left(c_t^c\right) + \omega_g \ln\left(g_t^c\right) - \omega_h \frac{\left(h_t^c\right)^{1+\eta}}{1+\eta} \Big\} \right) \ ,$$

where the superscript c indicates the value of the endogenous variable at time t under the commitment solution. In turn, the unconditional lifetime utility obtained under policy regime a is defined by:

$$\mathbb{E}V_0^a = \sum_{t=0}^{\infty} \beta^t \mathbb{E} \left(\delta_t \Big\{ \ln\left(c_t^a\right) + \omega_g \ln\left(g_t^a\right) - \omega_h \frac{\left(h_t^a\right)^{1+\eta}}{1+\eta} \Big\} \right) \ .$$

Let λ_u be the *unconditional* welfare cost obtained under policy regime a relative to the optimal commitment policy.[24] Formally, λ_u is defined as:

$$\mathbb{E}V_0^a = \frac{1}{1-\beta} \mathbb{E} \left(\delta_t \Big\{ \ln\left((1-\lambda_u) c_t^c\right) + \omega_g \ln\left(g_t^c\right) - \omega_h \frac{\left(h_t^c\right)^{1+\eta}}{1+\eta} \Big\} \right) \ .$$

Since household's preferences over consumption are logarithmic, λ_u can be determined analytically:

$$\lambda_u = 1 - \exp\Big\{ (1-\beta) \left(\mathbb{E}V_0^a - \mathbb{E}V_0^c\right) \Big\} \ , \tag{2.23}$$

where I used the fact that $\mathbb{E}(\delta_t) = 1$. Intuitively, λ_u measures the fraction of consumption that the household has to forgo in economy c to be indifferent between staying in economy c and joining economy a.

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taken as given the simple monetary policy rule.

[23] In Appendix 2.C.1, I present the maximisation problem of the policymaker under commitment. For a detailed analysis of the optimal commitment policy in face of the ZLB in a in a fully non-linear model version similar to the one in this paper, see Nakata (2016).

[24] In the following, I refer to the unconditional welfare cost simply as welfare cost, for the sake of brevity.

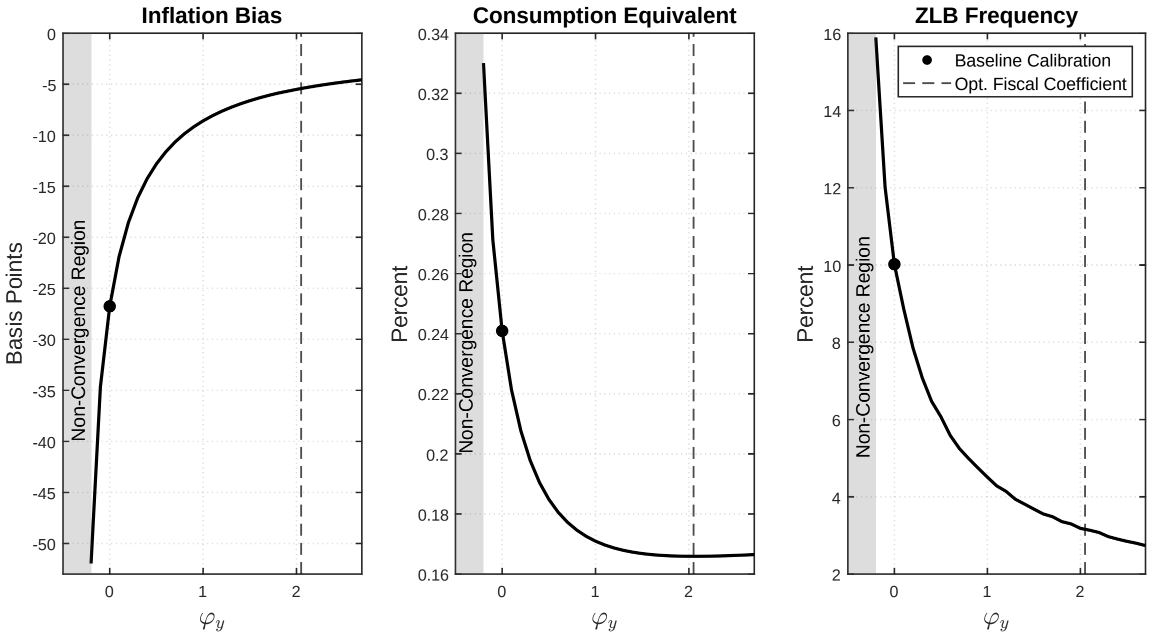

Figure 2.5: Evaluation of Counter-Cyclical Fiscal Policy

Notes. Deflationary bias (left panel), consumption equivalent (middle panel) and ZLB frequency (right panel) as a function of the fiscal output coefficient, φ_y, respectively. Vertical black dashed line: Optimised fiscal output coefficient. Black dot: Value of respective variable under the baseline calibration.

Optimised Fiscal Output Coefficient. In Figure 2.5, I plot the deflationary bias (left panel), the unconditional consumption equivalent (middle panel), and the ZLB frequency (right panel) as a function of the fiscal output coefficient, φ_y, respectively. The vertical black dashed line shows the value of φ_y that minimises the unconditional consumption equivalent.[25]

Consider, first, the benchmark case where government spending is constant ($\varphi_y = 0$). As intended, the ZLB frequency under the baseline calibration amounts to ten percent. The annualised net inflation rate in the risky steady steady state falls below the central bank's target value by 27 basis points. Moreover, the baseline calibration generates a sizeable welfare cost relative to the optimal commitment policy. Under the optimised simple fiscal rule, government spending is not held constant but is strongly counter-cyclical. The optimised fiscal coefficient on output amounts to 2.05. In line with the results from the preceding section, counter-cyclical government spending decreases the deflationary bias (to six basis points) and the ZLB frequency (to three percent).[26] Further, the optimised simple fiscal rule considerably reduces the welfare cost by 0.07 percentage

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[25] For each value of φ_y, the standard deviations are computed based on a simulation of 300,000 periods under the same sequence of shocks.

[26] In face of the ZLB, it is well known that the mean of the annualised inflation rate under the optimal commitment policy is positive (Billi, 2011). Appendix 2.C.2 shows that also the risky steady state value of the annualised inflation rate lies above the central bank's target value. Hence, the deflationary bias is sub-optimal from a welfare perspective.

points relative to the baseline case where government spending is a-cyclical. Notably, the welfare cost sharply increase as fiscal policy becomes more pro-cyclical (i.e., φ_y becomes more negative). For the threshold value of φ_y where the policy function iteration algorithm converges ($\varphi_y = -0.2$), the welfare cost increases by roughly 0.09 percentage points relative to the baseline calibration where fiscal policy is a-cyclical.

Fiscal Stabilisation Trade-off. While a counter-cyclical fiscal policy improves welfare, the associated fluctuations in government spending itself generate welfare costs for the household. To see this, it is helpful to rewrite household's unconditional lifetime utility as follows:

$$\sum_{t=0}^{\infty} \beta^t \, \mathbb{E} \left(\delta_t \Big\{ \ln \left(h_t (1 - \varrho_t) - g_t \right) + \omega_g \ln g_t - \omega_h \frac{h_t^{1+\eta}}{1+\eta} \Big\} \right) \ ,$$

where $\varrho_t \equiv 0.5\psi \left(\Pi_t / \Pi - 1 \right)^2$. Here, I used the production function (2.8) and the resource constraint (2.19) to rewrite consumption as $c_t = h_t(1 - \varrho_t) - g_t$. Figure 2.6 shows the unconditional standard deviation of the variables entering the rewritten unconditional lifetime utility, i.e. labour, government spending, inflation, and, in addition, consumption as a function of the fiscal coefficient to output, φ_y.

Both the volatility of labour and inflation are decreasing functions of φ_y, which is consistent with the finding in Section 2.4.3 that counter-cyclical government spending renders labour and inflation more stable for *all* states of the discount factor shock. Since the disutility of labour is decreasing and convex, a lower labour volatility improves welfare. The lower labour volatility reduces fluctuations in output, which, via the resource constraint, reduces fluctuations in consumption. In addition, the lower inflation volatility reduces fluctuations in the resource loss associated with price adjustment costs, ϱ_t, which further reduces fluctuations in consumption. At the same time, however, if government spending is cyclical, fluctuations in output increase the volatility in government spending, which generates a welfare cost stemming from two sources. First, the household's preferences are concave in government spending, so that increasing its volatility reduces household's welfare. Second, the increased volatility of government spending also affects the volatility of private consumption via the resource constraint. As can be seen in Figure 2.6, for sufficiently low positive values of φ_y, consumption volatility decreases since the stabilising effect on labour and inflation dominates the destabilising effect resulting from the increased government spending volatility. However, if φ_y gets sufficiently large, the latter effect dominates the former so that consumption volatility increases. Hence, the counter-cyclical fiscal policy requires to balance the gains resulting from the lower volatility of labour and inflation against the costs stemming from the increased volatility in consumption and government spending.

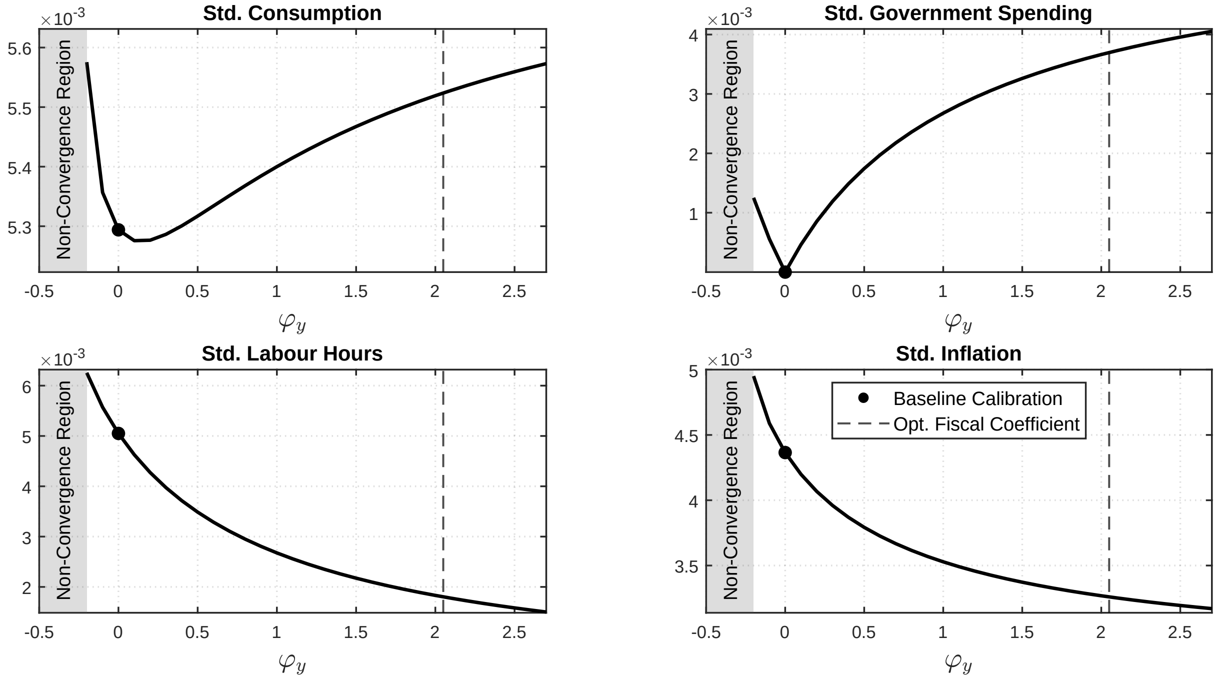

Figure 2.6: Stabilisation Trade-off of Fiscal Policy

Notes. Unconditional standard deviations of consumption, inflation, labour, and government spending as a function of the fiscal output coefficient, φ_y. Grey shaded area: Range of values for φ_y where policy function iteration algorithm is not converging. Vertical black dashed line: Optimised value of φ_y. Black dot: Value of respective variable under baseline calibration.

Notably, the volatility of all four variables depicted in Figure 2.6 strictly increases as fiscal policy becomes more pro-cyclical. Consistent with the equilibrium responses shown in Figure 2.3, pro-cyclical fiscal policy renders labour and inflation less stable for all states of the discount factor shock. Moreover, the pro-cyclical fiscal response during ZLB episodes further depresses private consumption due to the resulting increase in the real interest rate. As a result, the welfare cost as a function of φ_y sharply increases as φ_y becomes more negative and becomes particularly large close to the non-convergence region.

2.5 Sensitivity Analysis

This section analyses how the effect of cyclical government spending on the deflationary bias depends on selected structural parameters of the model. As outlined above, the wedge between the model's risky and deterministic steady state arises due to forward-looking firms' anticipation of ZLB episodes. In consequence, I consider a variation in the risk of encountering the ZLB and the strength by which firms' price setting decision depends on expected marginal costs. Further, I analyse the interaction of the simple monetary and fiscal policy rules in addressing the deflationary bias caused by the ZLB risk.

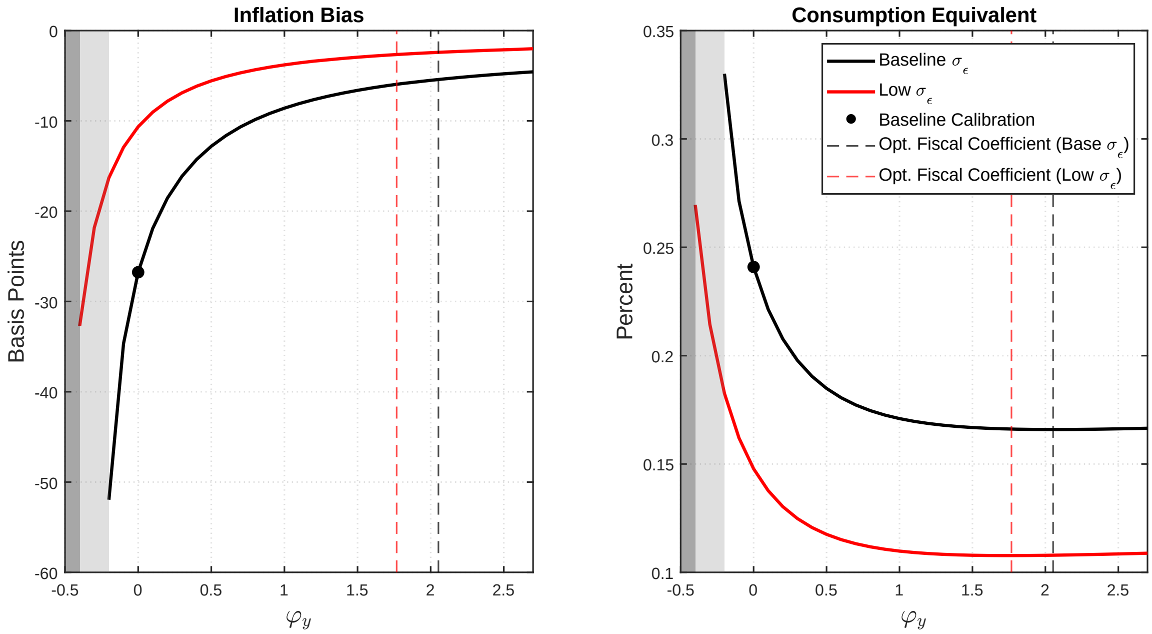

Figure 2.7: The Role of the Zero Lower Bound Risk

Notes. Deflationary bias (left panel) and consumption equivalent (right panel) as a function of the fiscal output coefficient, φ_y, under the baseline ZLB risk (black solid; $\sigma_\epsilon = 0.0189$) and low ZLB risk (red solid; $\sigma_\epsilon = 0.0166$). Vertical lines: Optimised output coefficient under the baseline (black dashed) and low (red dashed) ZLB risk. Light and dark grey shaded areas denote the non-convergence region under the baseline and the low ZLB risk, respectively. Black dot: Value of respective variable under the baseline calibration.

2.5.1 The Zero Lower Bound Risk

In this section, I analyse the effect of a variation in the ZLB risk, which corresponds to a variation in the standard deviation of the innovation to the discount factor shock, σ_ϵ. I focus on a reduction in σ_ϵ and set it to a value of 0.0166, which generates a ZLB frequency of five percent, assuming government spending is a-cyclical. Figure 2.7 shows the deflationary bias (left panel) and the consumption equivalent (right panel) as a function of the fiscal output coefficient for the low ZLB risk (red solid) and the baseline ZLB risk (black solid; $\sigma_\epsilon = 0.0189$). The vertical dashed black and red lines denote the optimised fiscal output coefficient under the baseline and low ZLB risk, respectively. The black dots in the left and right panel denote the deflationary bias and the consumption equivalent under the baseline calibration, respectively.

For each value of φ_y, the gap between the risky and the deterministic steady state of the annualised net inflation rate increases in σ_ϵ, which is consistent with the results of Bianchi et al. (2021) and Hills et al. (2019). Intuitively, a higher risk of encountering the ZLB in the future increases the downward pressure on firms' inflation expectations and, thereby, the deflationary bias.

Further, strong counter-cyclical government spending mitigates the adverse effects of heightened ZLB risk on the economy's risky steady state. In particular, the increase in the deflationary bias that is caused by a higher ZLB risk attenuates when government spending becomes more counter-cyclical. Consider the gap between the black and the red solid line in the left panel of Figure 2.7. In case government spending is a-cyclical ($\varphi_y = 0$), the deflationary bias under the baseline ZLB risk increases by 16 basis points relative to the one under the low ZLB risk. In turn, the corresponding increase under a strong counter-cyclical response ($\varphi_y = 2.0$) amounts only to three basis points.

Moreover, the optimised fiscal output coefficient increases in σ_ϵ. In consequence, fiscal stabilisation becomes more important when the ZLB risk is high. This finding resembles the result of Nakata (2016), who shows that the optimal government spending response increases during ZLB episodes when the ZLB risk increases. Notably, even under a low ZLB risk where the ZLB frequency is zero, the optimised fiscal output coefficient is large.[27] The desirability of fiscal stabilisation even in the absence of ZLB episodes stems from the fact that monetary policy itself is not conducted optimally but follows a simple Taylor (1993)-type rule. Eser (2009) shows in a linearised version of the standard New Keynesian model without ZLB that, if both monetary and fiscal policy are determined optimally, government spending should not be used as a stabilisation instrument. If, however, monetary policy follows a simple rule like the one in equation (2.13), government spending should adjust to stabilise economic fluctuations.

The strength of the ZLB risk also affects the range of values for φ_y for which no equilibrium exists. Under the low ZLB risk, the algorithm does not converge for values of φ_y below -0.4 (dark grey shaded area). In turn, under the baseline ZLB risk, the algorithm does not converge for values of φ_y below -0.2 (light grey shaded area). Hence, pro-cyclical fiscal policy is particularly inadvisable when the ZLB risk is high, because the resulting amplifications of the adverse effects of the ZLB risk might be sufficiently strong to prevent the existence of an equilibrium.

2.5.2 The Price Adjustment Cost Parameter

In this subsection, I consider the effect of a reduction in the Rotemberg price adjustment cost parameter, ψ, from its baseline value of 200 to 181.81. The lower value of ψ causes an increase of the slope if the NKPC.[28] Hence, firms' price setting behaviour gets more sensitive to their expectations on future marginal costs. Figure 2.8 shows the deflationary bias (left panel) and the consumption equivalent (right panel) as a function of the fiscal output coefficient, φ_y, for the baseline value of ψ (black solid) and the low value of ψ

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[27] I computed the threshold value for σ_ϵ where the ZLB frequency turns zero when all other parameters are set to their values under the baseline calibration. Even for this small value of σ_ϵ, the optimised fiscal output coefficient is 1.52.

[28] In the linearised version of the model, the reduction in ψ increases the slope of the NKPC from 0.025 to 0.0275.

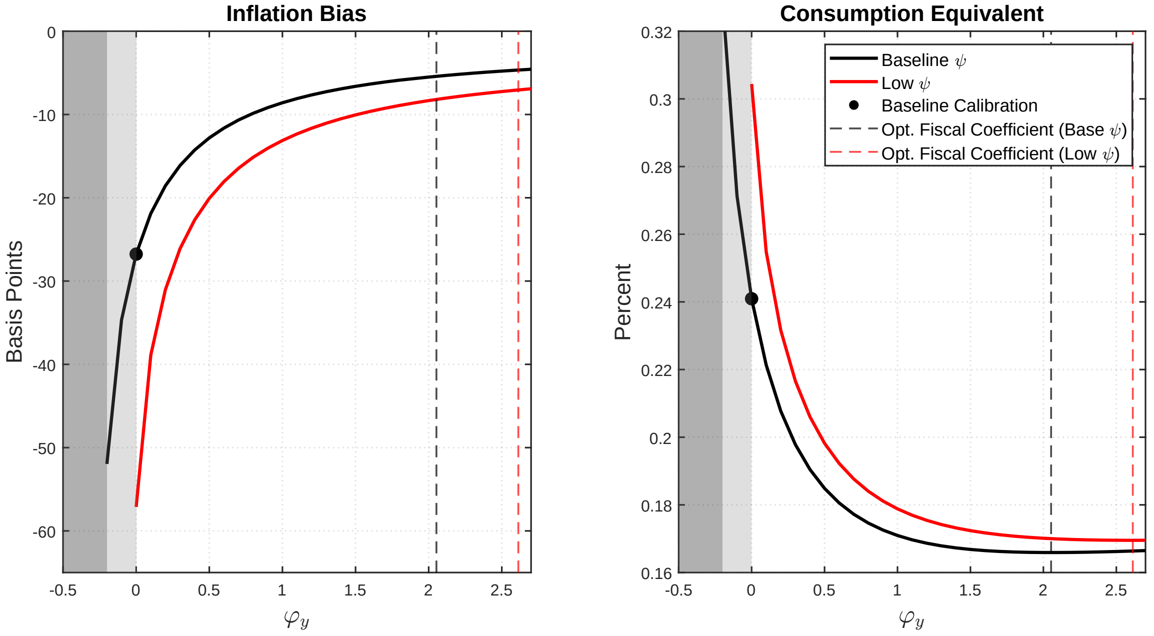

Figure 2.8: The Role of the Price Adjustment Cost Parameter

Notes. Deflationary bias (left panel) and consumption equivalent (right panel) as a function of the fiscal output coefficient, φ_y for different levels of the Rotemberg parameter, ψ: baseline (black solid; $\psi = 200$) and low (red solid; $\psi = 181.81$). Vertical lines: Optimised fiscal output coefficient under baseline value for ψ (black dashed) and low value for ψ (red dashed). Dark and light grey shaded areas denote the non-convergence region under the baseline and the low value of ψ, respectively. Black dot: Value of respective variable under the baseline calibration.

(red solid). The vertical dashed black and red lines denote the optimised fiscal output coefficient under the baseline and low value for ψ, respectively. The black dots in the left and right panel denote the deflationary bias and the consumption equivalent under the baseline calibration, respectively.

Despite the comparatively small increase in the slope of the NKPC, the deflationary bias as well as the welfare cost substantially increase when government spending is a-cyclical ($\varphi_y = 0$). Intuitively, with a higher slope of the NKPC, firms' inflation expectations become more sensitive to fluctuations in real activity. In consequence, the risk of encountering the ZLB in future periods generates a more pronounced downward pressure on firms' inflation expectations and, thereby, on current inflation. As a result, a larger slope of the NKPC increases the deflationary bias, ceteris paribus. Again, the effect of a variation in the slope of the NKPC on the deflationary bias attenuates as government spending becomes more counter-cyclical. Further, since fluctuations in real activity have a larger impact on inflation and inflation expectations, the volatility of inflation and firms' labour demand increases, which deteriorates welfare. At the same time, the increased sensitivity of inflation to variations in real activity improves the pass-through of fiscal demand stabilisation on inflation. In consequence, fiscal policy becomes more effective in addressing the deflationary bias and in stabilising fluctuations in inflation. As a result,

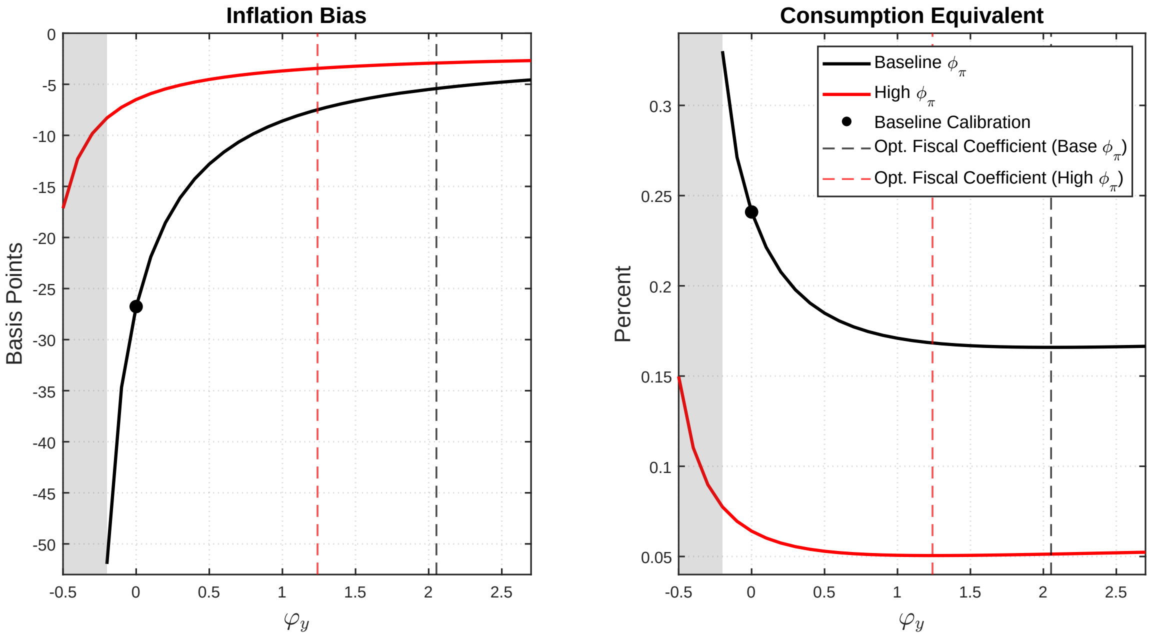

Figure 2.9: The Role of Monetary Policy

Notes. Deflationary bias (left panel) and consumption equivalent (right panel) as a function of the fiscal output coefficient, φ_y for two different values of the monetary policy coefficient on inflation, ϕ_Π: baseline value (black solid; $\phi_\Pi = 1.5$) and high value (red solid; $\phi_\Pi = 3.0$). Vertical lines: Optimised fiscal output coefficient under baseline value for ϕ_Π (black dashed) and high value for ϕ_Π (red dashed). Grey shaded area: non-convergence region under the baseline value of ϕ_Π. Black dot: Value of respective variable under the baseline calibration.

the optimised fiscal output coefficient decreases in ψ.

Also the Rotemberg cost parameter affects the non-convergence region. A reduction in ψ shifts the range of admissible values of φ_y to the right. Notably, the small increase in the slope of the linearised NKPC prevents convergence of the algorithm for *any* pro-cyclical response of government spending. In addition, the welfare deteriorating effects of pro-cyclical fiscal policy increase as the price adjustment cost parameter decreases, which can be inferred from the fact that the distance between black and the red solid line in the right panel of Figure 2.8 increases as fiscal policy becomes more pro-cyclical. Hence, reverting to pro-cyclical fiscal policy gets even less desirable as inflation becomes more responsive to real activity.

2.5.3 The Monetary Policy Response to Inflation

This section discusses the interaction between monetary and fiscal policy in face of the ZLB risk. In particular, I discuss how the response of monetary policy to inflation affects the deflationary bias, the welfare cost and, thereby, the stabilisation role of fiscal policy. I consider an increase in the inflation coefficient of monetary policy, ϕ_Π, to a value of

3.0.[29] Hence, monetary policy responds more "aggressively" to deviations of inflation from its target value. In Figure 2.9, I plot the deflationary bias (left panel) and the consumption equivalent as a function of the fiscal output coefficient for the baseline value of ϕ_Π (black solid) and the aggressive monetary policy response to inflation (red solid). The vertical dashed black and red lines denote the optimised fiscal output coefficient under the baseline and high value for ϕ_Π, respectively. The black dots in the left and right panel denote the deflationary bias and the consumption equivalent under the baseline calibration, respectively.

Increasing ϕ_Π while keeping government spending fixed substantially reduces the welfare cost relative to the baseline calibration from 0.24 to 0.06 percent and reduces the deflationary bias by roughly 18 basis points.[30] The optimised fiscal output coefficient under the aggressive monetary policy response to inflation still features a strong negative response to output. However, the optimised fiscal output coefficient decreases relative to the one obtained under the baseline value of ϕ_Π to 1.24. Further, the additional welfare gain generated by the fiscal stabilisation amounts to 0.015 percentage points while the deflationary bias reduces by another six basis points.

Hence, the effectiveness of the simple fiscal rule in reducing the deflationary bias as well as its welfare effects crucially depend on the *monetary* policy response to inflation. In terms of welfare, counter-cyclical government spending can be regarded as an imperfect substitute for a more aggressive monetary policy response to inflation.[31] Nonetheless, fiscal policy should contribute to the macroeconomic stabilisation for two reasons. First, monetary policy's stabilisation tool is occasionally constrained by the ZLB, which makes fiscal stabilisation desirable. Second, as already discussed above, monetary policy is not conducted optimally but follows a simple rule.

Note that the range of admissible values of φ_y ensuring convergence of the algorithm increases with ϕ_Π. The flip-side of this finding is that the peril of pro-cyclical fiscal policy for the existence of an equilibrium increases as the monetary policy response to inflation becomes more lenient.

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[29] Schmitt-Grohé and Uribe (2007) argue that values for ϕ_Π above 3.0 are difficult to communicate to both policymakers and the public. Moreover, Erceg and Lindé (2014) show that a value of 3.0 is consistent with estimates based on a simple regression analysis using instrumental variables.

[30] The fact that a more aggressive monetary policy response to inflation reduces the deflationary bias and improves welfare resembles the finding of Nakata and Schmidt (2019). They show that the appointment of a "conservative" central banker, who puts more weight on inflation stabilisation as society, enhances welfare and reduces the deflationary bias.

[31] The result that the nominal interest rate is the preferred instrument to stabilise the economy is consistent with the findings of Schmidt (2013). He shows in a linearised model that the welfare gain of optimal government spending is negligible if monetary policy is conducted optimally.

2.6 Conclusion

This paper analyses the effect of cyclical government spending on the deflationary bias that is caused by the risk of encountering the ZLB. Fiscal policy can substantially reduce the deflationary bias and improve welfare by relying on counter-cyclical government spending. In turn, pro-cyclical fiscal policy increases the deflationary bias, deteriorates welfare and might even prevent the existence of an equilibrium. Further, counter-cyclical fiscal policy mitigates the increase in the deflationary bias that is caused by heightened ZLB risk or by a lower degree of price stickiness. In contrast, a stronger monetary policy response to inflation already reduces the deflationary bias and improves welfare so that the additional stabilisation gain provided by counter-cyclical government spending diminishes.

Appendix

2.A Solution Method and Simulation Algorithm

2.A.1 Time Iteration Approach

This section of the Appendix outlines the algorithm used to solve the non-linear version of the model described in the main text. The description of the algorithm closely follows the one of Gavin et al. (2015). Formally, the dynamics of the economy is defined by:

$$\mathbb{E}\left[f\left(\mathbf{z}_{t+1}, \mathbf{v}_{t+1}, \mathbf{z}_t, \mathbf{v}_t\right) \mid \mathbf{I}_t\right] \equiv \mathbb{E}_t\left[f\left(\mathbf{z}_{t+1}, \mathbf{v}_{t+1}, \mathbf{z}_t, \mathbf{v}_t\right)\right] = 0 \ ,$$

where $f(\cdot)$ is a vector valued function that contains the equilibrium system of the model, $\mathbf{z}$ and $\mathbf{v}$ are the vectors of endogenous and exogenous variables, and $\mathbf{I}_t \equiv \{M, P, \mathbf{w}_t\}$ denotes the private sector's information set in period t that contains the structural model, M, the structural parameters, P, and the state vector, $\mathbf{w}_t$. Specifically, for the model in this paper, the equilibrium system contained in $f(\cdot)$ is given by:

$$1 = \beta R_t \mathbb{E}_t\left(\frac{\delta_{t+1}}{\delta_t}\frac{c_t}{c_{t+1}}\frac{1}{\Pi_{t+1}}\right) \tag{2.A.1}$$

$$\left(\frac{\Pi_t}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} = \beta\mathbb{E}_t\left[\frac{\delta_{t+1}}{\delta_t}\frac{c_t}{c_{t+1}}\frac{y_{t+1}}{y_t}\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\frac{\Pi_{t+1}}{\Pi}\right] + \kappa\left(w_t - \mu\right) \tag{2.A.2}$$

$$w_t = \omega_h h_t^{\eta} c_t \tag{2.A.3}$$

$$y_t = h_t \tag{2.A.4}$$

$$c_t = y_t\left(1 - \frac{\psi}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2\right) - g_t \tag{2.A.5}$$

$$R_t = \max\left\{R\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi}, 1\right\} \tag{2.A.6}$$

$$g_t = g - \varphi_y\left(y_t - y\right) \tag{2.A.7}$$

$$\delta_t = \delta_{t-1}^{\rho_d}\,\exp\left(\epsilon_t\right) \ . \tag{2.A.8}$$

In the model of this paper, $\mathbf{z}_t = (h_t, \Pi_t, c_t, w_t, y_t, g_t, R_t)'$ and $\mathbf{v}_t = \mathbf{w}_t = \delta_t$. Hence, the only exogenous variable corresponds to the discount factor shock, which is also the only state variable of the economy.

The model is solved with global methods using the *time-iteration approach* as described in Richter et al. (2014). For each endogenous variable x contained in the vector $\mathbf{z}$, the algorithm approximates the policy function, $Z_x(\cdot)$, as a function of δ with the piecewise linear function $\widehat{Z}_x(\delta)$. In particular, I solve the equilibrium system by iterating on the policy function for labour, $\widehat{Z}_h(\delta)$, and inflation, $\widehat{Z}_\Pi(\delta)$.

To do so, the discount factor shock is discretised into a finite number of grid points. The $AR(1)$ process of the discount factor shock, δ, is approximated using a first order Markov process based on the method developed by Tauchen (1986).[32] Let $\mathcal{D} \equiv \{\delta^1, \dots, \delta^N\}$ denote the discrete grid for discount factor shock where N denotes the number of grid points used to discretise δ. For the computation, I choose $N = 1001$. The bounds of the grid for δ is set to encompass 99.999% of the probability mass of its distribution.

The discretised state space is represented by a set of unique N-dimensional nodes. The approximated policy function on the node $n \in \{1, \dots, N\}$ for the generic variable x is given by $\widehat{Z}_x(\delta^n)$ with $\delta^n \in \mathcal{D}$ denotes the state of the discount factor shock on node n. The time-iteration algorithm can be summarised as follows.

OUTLINE OF ALGORITHM[33]

Let $k \in \{0, \dots, K\}$ denote the k-th iteration of the algorithm and $n \in \{1, \dots, N\}$ denote the n-th node of the discretised state space. Further, let $\widehat{Z}_{h,k}$ and $\widehat{Z}_{\Pi,k}$ denote the policy function for labour and inflation at the k-th iteration of the algorithm, respectively.

1. For $k = 0$, obtain an initial guess of the policy functions for labour and inflation on each node of the discretised state space.[34]

2. For $k \in \{1, \dots, K\}$, proceed on each node n as follows.

   i. Let any endogenous or exogenous variable with a superscript n denote its value on node n of the discretised state space. Use the equilibrium system (B.26) – (B.33) to solve for all time–t variables – given the policy functions for labour and inflation determined in the last iteration step, $k-1$:

$$h_t^n = \widehat{Z}_{h,k-1}(\delta^n)$$

---

[32] To be more precise, I approximate the logarithm of δ using a first order Markov process and then exponentiate the resulting grid points obtained from the Tauchen (1986) method.

[33] For the computation, I rely on Julia 1.6.2.

[34] Following Nakata (2017), I use for each variable flat functions at the respective deterministic steady state.

$$
\begin{aligned}
\Pi_t^n &= \widehat{Z}_{\Pi,k-1}(\delta^n) \\
y_t^n &= h_t^n \\
g_t^n &= g - \varphi_y\left(y_t^n - y\right) \\
c_t^n &= y_t^n\left(1 - \frac{\psi}{2}\left(\frac{\Pi_t^n}{\Pi} - 1\right)^2\right) - g_t^n \\
w_t^n &= \omega_h (h_t^n)^\eta c_t^n \\
R_t^n &= \max\left\{R\left(\frac{\Pi_t^n}{\Pi}\right)^{\phi_\Pi}, 1\right\} .
\end{aligned}
$$

ii. Calculate the errors for the Euler equation (2.A.1) and the New-Keynesian Phillips-Curve (2.A.2):

$$
\begin{aligned}
\mathrm{err}_1 &= (c_t^n)^{-1} - \beta\frac{R_t^n}{\delta^n}\sum_{j=1}^{N} p_j^n\, L_{t+1}^j \\
\mathrm{err}_2 &= \left(\frac{\Pi_t^n}{\Pi} - 1\right)\frac{\Pi_t^n}{\Pi} - \beta\frac{1}{\delta^n}\sum_{j=1}^{n_\delta} p_j^n\, M_{t+1}^j - \kappa\left(w_t^n - \mu\right) ,
\end{aligned}
$$

where

$$
p_j^n = \Pr(\delta_{t+1} = \delta^j \mid \delta_t = \delta^n) .
$$

Here, p_j^n denotes the probability for the discount factor shock to arrive at $\delta^j \in \mathcal{D}$ tomorrow when its current value is on the n-th node of the discretised state space, δ^n.[35] Further, L_{t+1}^j and M_{t+1}^j are defined as follows:

$$
\begin{aligned}
L_{t+1}^j &\equiv \frac{\delta^j}{c_{t+1}^j \Pi_{t+1}^j} \\
M_{t+1}^j &\equiv \delta^j \frac{c_t^n}{c_{t+1}^j}\frac{y_{t+1}^j}{y_t^n}\left(\frac{\Pi_{t+1}^j}{\Pi} - 1\right)\frac{\Pi_{t+1}^j}{\Pi} .
\end{aligned}
$$

where

$$
\begin{aligned}
\Pi_{t+1}^j &= \widehat{Z}_{\Pi,k-1}(\delta^j) \\
y_{t+1}^j &= \widehat{Z}_{h,k-1}(\delta^j) \\
c_{t+1}^j &= y_{t+1}^j\left(1 - \frac{\psi}{2}\left(\frac{\Pi_{t+1}^j}{\Pi} - 1\right)^2\right) - g_{t+1}^j
\end{aligned}
$$

---

[35] The transition probabilities are obtained as part of the approximation of the underlying $AR(1)$ process based on the method developed by Tauchen (1986).

$$g^j_{t+1} = g - \varphi_y \left(y^j_{t+1} - y \right) \ .$$

   iii. Use a numerical root finder to search for $\widehat{Z}_{h,k}(\delta^n)$ and $\widehat{Z}_{\Pi,k}(\delta^n)$ so that $\text{err}_1 \approx 0$ and $\text{err}_2 \approx 0$.[36]
   iv. Define $\text{dist}^n_k = \max\Big\{ \mid \widehat{Z}_{h,k}(\delta^n) - \widehat{Z}_{h,k-1}(\delta^n) \mid, \mid \widehat{Z}_{\Pi,k}(\delta^n) - \widehat{Z}_{\Pi,k-1}(\delta^n) \mid \Big\}$.

3. If at the k-th iteration, $\text{dist}^n_k < 10^{-8}$ for all $n \in \{1, \dots, N\}$, then the algorithm converged to a Minimum State Variable (MSV) solution.

4. In turn, I define the algorithm to be not converging to a MSV solution if at least one of the following conditions holds:
   i. If $k > K$ (Algorithm times out).
   ii. For all n, if $\Pi^n_t < 0.5$, $c^n_t < 0$, $g^n_t < 0$, or $h^n_t < 0$.
   iii. If $\text{dir}_k = \text{dist}_k - \text{dist}_{k-1} > 0$ for 15 consecutive iterations (Algorithm diverges), where $\text{dist}_k = \max\{\text{dist}^1_k, \dots, \text{dist}^N_k\}$.

2.A.2 Generalised Impulse Response Functions

Since the model is solved in its original non-linear form, the impulse response functions depend on both private sector's expectations about future shocks and the current point of the state space of the economy. Koop et al. (1996) propose to consider *generalised* impulse response functions (GIRFs), which allow to consider impulse response functions that, given a certain point in the state space, average out the future realisations of shocks.

The algorithm used to compute the GIRFs again closely follows Gavin et al. (2015) and is outlined in the following.

1. Compute the initial state of the economy which corresponds to the stochastic steady state. In the model of this paper, the stochastic steady state of a variable x corresponds to the value of the policy function when the discount factor shock is at its steady state, i.e. $Z_x(\delta_t = 1)$.

2. Simulate a Markov chain for the discount factor shock, $\{\delta_t\}^N_{t=0}$.

3. Given the simulated Markov chain for the exogenous state variable, simulate the economy beginning with the initial state of the discount factor being equal to its steady state. Obtain equilibrium paths for the variables of interest.

4. Repeat point 2, but replace the discount factor shock in period 1 with the first value of δ that is three unconditional standard deviations below its deterministic steady state. Let this value be denoted by δ_{shock}.

---

[36] I rely on the function `nlsolve` that is provided in the Julia package NLsolve.jl.

5. Repeat points 2 - 4 for R times, beginning with the same respective initial state for δ.

6. Obtain the average path for a generic variable x at time t across the R simulations:

$$\bar{x}_{1,t}(w_0) = \frac{1}{R}\sum_{j=1}^{R} x_t^j(w_0) \tag{2.A.9}$$

$$\bar{x}_{2,t}(w_0) = \frac{1}{R}\sum_{j=1}^{R} x_t^j(w_0 \mid \delta_1 = \delta_{\text{shock}}) \tag{2.A.10}$$

   where x_t^j denotes the value of x at time t under the j-th Markov chain. Further, $\bar{x}_{1,t}(w_0^s)$ denotes the average path for x at time t when the discount factor shock in the first period is generated as part of the Markov chain. In turn, $\bar{x}_{1,t}(w_0)$ denotes the average path for x at time t when the discount factor shock in the first period is set to δ_{shock}.

7. The GIRFs for the quantity variables at time t are presented in percentage terms and are calculated as $100\,(\bar{x}_{2,t}(w_0)/\bar{x}_{1,t}(w_0) - 1)$. In turn, the GIRFs for prices are presented in annualised percentage points and are calculated as $400\,(\bar{x}_{2,t}(w_0) - \bar{x}_{1,t}(w_0))$.

2.B Mean Bias

This section of the Appendix considers an alternative specification of the deflationary bias, which is defined as the undershooting of the ergodic mean below the central bank's target value. The ergodic mean of the annualised net inflation rate (in percent) is defined by:

$$\bar{\pi}^a = 400 \times \mathbb{E}\left(\pi_t\right) \ . \tag{2.B.1}$$

where $\pi_t = \Pi_t - 1$ denotes the net inflation rate. Then, the *mean bias* of inflation is defined (in basis points) as:

$$\bar{\chi} = 100 \times \left(\bar{\pi}^a - \pi\right) \ ,$$

where $\pi = 400 \times (\Pi - 1)$ denotes the central bank's target value for the annualised inflation rate. Following the alternative specification, the economy features a *deflationary* bias if the ergodic mean of the annualised net inflation rate undershoots the central bank's target value, i.e. $\bar{\chi} < 0$.

It is instructive to differentiate between the ergodic mean of the annualised inflation rate, $\bar{\pi}^a$, and its risky steady state, π^a_{rss}. Recall that the model presented in section 2.2 does not include any endogenous state variable and that the sole exogenous state variable corresponds to the discount factor shock, δ_t. Then, $\bar{\pi}^a$ corresponds to the average of inflation *in all* states of the economy. Hence, it includes the actually realised inflation rates for all states for δ_t – including those where the ZLB *is* binding. In turn, π^a_{rss} denotes the ergodic mean of inflation in the absence of shocks. Hence, π^a_{rss} corresponds to the annualised inflation rate in the state of the economy where, in the absence of shocks in that period, agents choose to stay, taking into account the risk of future shocks – including those where the ZLB *could* be binding. Since the focus of this paper is to evaluate the effect of cyclical government spending on the distortions caused by the ZLB *risk*, the variable π^a_{rss} is better suited for this purpose.

However, π^a_{rss} is not directly observable in the data but needs to be inferred from a structural model. The average inflation is more directly observable in the data. Consider Table 2.B.1, where I show various moments of the inflation distribution. The distribution of inflation is negatively skewed, so that the average inflation falls below its median. The fact that the ZLB renders the distribution of inflation negatively skewed has already been noted by Reifschneider and Williams (2000). Intuitively, the ZLB prevents monetary policy to further lower the nominal interest rate to mitigate the downturn in demand and inflation caused by large negative innovations to the discount factor shock. In consequence, the left tail of the distribution of inflation is longer, which moves the average of the distribution to the left. As shown by Hills et al. (2019), in the absence of endogenous state variables, the median of the distribution corresponds to the model's risky steady state. As discussed in section 2.4, the fact that the median of the inflation distribution falls below the central bank's target value follows from the forward-looking behaviour of firms and their anticipation of the risk of encountering the ZLB in the future.

Table 2.B.1: Moments of Distribution of Annualised Net Inflation Rate

	Moment				
	Mean	Median	Std.	Skewness	RSS
Baseline	1.71	1.73	1.75	−0.12	1.73
Counter-cyclical Fiscal Policy	1.94	1.95	1.30	−0.04	1.95

Notes. Calculations of the moments are based on a simulation of the economy for 301,000 periods, where the first 1,000 observations are discarded. Baseline refers to the economy under the baseline calibration. Counter-cyclical Fiscal Policy refers to the economy under the optimised fiscal output coefficient, given the other structural parameters are fixed at their baseline value in Table 2.1. RSS denotes the abbreviation for risky steady state.

Further, as shown in section 2.4, counter-cyclical fiscal policy endogenously reduces the ZLB frequency, which reduces the skewness of the inflation distribution, thereby shifting its average to the right. Hence, counter-cyclical government spending not only reduces the adverse effects of ZLB risk, but also gives monetary policy more room to manoeuvre in response to large negative innovations to the discount factor shock, which also reduces the wedge between the average inflation rate and the desired target value.

2.C Optimal Policy under Commitment

2.C.1 Model with Commitment

The problem of the policymaker under commitment is to choose a state-contingent sequence of the model's variables at time zero in order to maximise the welfare of the household:

$$\max_{\{c_t,g_t,h_t,\Pi_t,R_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \delta_t \Big\{ \ln(c_t) + \omega_g \ln(g_t) - \omega_h \frac{h_t^{1+\eta}}{1+\eta} \Big\} ,$$

subject to

$$1 = \beta R_t \mathbb{E}_t \left(\frac{\delta_{t+1}}{\delta_t} \frac{c_t}{c_{t+1}} \frac{1}{\Pi_{t+1}} \right)$$

$$\left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} = \beta \mathbb{E}_t \left[\frac{\delta_{t+1}}{\delta_t} \frac{c_t}{c_{t+1}} \frac{h_{t+1}}{h_t} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \right] + \kappa \left(\omega_h h_t^{\eta} c_t - \mu \right)$$

$$c_t = h_t \left[1 - 0.5\psi \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 \right] - g_t$$

$$R_t \geq 1$$

$$\delta_t = \delta_{t-1}^{\rho_\delta} \exp(\epsilon_t)$$

with $\delta_{-1} = 1$ and $\epsilon_t \sim \mathbb{N}(0, \sigma_\epsilon^2)$. Note that, by assumption, monetary policy in the decentralised economy has credibly communicated its inflation target so that intermediate good firms index their prices to the inflation target. Hence, the optimal commitment solution takes into account firms' indexing of their prices to the communicated target value. The Lagrangian of the optimal commitment problem is given by:

$$\begin{aligned} \mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \delta_t \Bigg\{ & \ln(c_t) + \omega_g \ln(g_t) - \omega_h \frac{h_t^{1+\eta}}{1+\eta} \\ & + \phi_{1,t} \left[1 - \beta R_t \mathbb{E}_t \left(\frac{\delta_{t+1}}{\delta_t} \frac{c_t}{c_{t+1}} \frac{1}{\Pi_{t+1}} \right) \right] \\ & + \phi_{1,t} \left[\left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} - \beta \mathbb{E}_t \left[\frac{\delta_{t+1}}{\delta_t} \frac{c_t}{c_{t+1}} \frac{h_{t+1}}{h_t} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \right] - \kappa \left(\omega_h h_t^{\eta} c_t - \mu \right) \right] \\ & + \phi_{3,t} \left[c_t - h_t \left[1 - 0.5\psi \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 \right] + g_t \right] \\ & + \phi_{4,t} \left[R_t - 1 \right] \Bigg\} , \end{aligned}$$

given the process of the discount factor shock, δ_t. The first order conditions at time t are given by:

$$
\begin{aligned}
0 &= c_t^{-1} - \phi_{1,t}\frac{c_t^{-2}}{R_t} + \phi_{1,t-1}\frac{c_t^{-2}}{\Pi_t} - \phi_{2,t}c_t^{-2}y_t\left[\left(\frac{\Pi_t}{\Pi}-1\right)\frac{\Pi_t}{\Pi}+\kappa\mu\right] \\
&\quad + \phi_{2,t-1}c_t^{-2}y_t\left(\frac{\Pi_t}{\Pi}-1\right)\frac{\Pi_t}{\Pi} - \phi_{3,t} && (2.C.1)\\
0 &= \omega_g g_t^{-1} - \phi_{3,t} && (2.C.2)\\
0 &= -\omega_h h_t^{\eta} + \phi_{2,t}c_t^{-1}\left[\left(\frac{\Pi_t}{\Pi}-1\right)\frac{\Pi_t}{\Pi}+\kappa\mu\right] - \phi_{2,t}(1+\eta)h_t^{\eta}\kappa\omega_h \\
&\quad + \phi_{3,t}\left[1-0.5\psi\left(\frac{\Pi_t}{\Pi}-1\right)^2\right] - \phi_{2,t-1}c_t^{-1}\left(\frac{\Pi_t}{\Pi}-1\right)\frac{\Pi_t}{\Pi} && (2.C.3)\\
0 &= \phi_{2,t}h_t c_t^{-1}\left(2\frac{\Pi_t}{\Pi}-1\right)\frac{1}{\Pi} + \phi_{1,t-1}\frac{1}{c_t\Pi_t^2} - \phi_{2,t-1}c_t^{-1}h_t\left(2\frac{\Pi_t}{\Pi}-1\right)\frac{1}{\Pi} \\
&\quad - \phi_{3,t}\psi\left(\frac{\Pi_t}{\Pi}-1\right)h_t && (2.C.4)\\
0 &= -\phi_{1,t}\frac{1}{c_t R_t^2} + \phi_{4,t} && (2.C.5)\\
R_t &\geq 1 && (2.C.6)\\
\phi_{4,t} &\geq 0 && (2.C.7)\\
0 &= (R_t-1)\phi_{4,t}\ , && (2.C.8)
\end{aligned}
$$

given $\phi_{2,-1}$ and $\phi_{1,-1} = 0$, i.e. the ZLB is not binding at time $t = -1$.[37]

2.C.2 Inflation Distribution under Optimal Commitment Policy

Table 2.C.1 gives an overview over selected moments of the inflation distribution under the optimal commitment policy. The mean, median and risky steady state value of the annualised inflation rate slightly exceed the central bank's target value by 1 basis points. Further, the standard deviation of the annualised net inflation rate corresponds to 3 basis points so that it is substantially less volatile than under the simple rules (see Table 2.B.1). Moreover, the distribution of the annualised net inflation rate under optimal commitment is strongly positively skewed.

Consider Figure 2.C.1 that shows generalised impulse response functions under the optimal commitment policy for selected endogenous variables. On impact, there is a negative innovation to the discount factor shock that pushes δ_t three unconditional standard deviations below its steady state value. In response, the ZLB on the nominal interest

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[37] The algorithm used to solve for the policy functions under optimal commitment follows the one described in the Online Appendix of Nakata (2016).

Table 2.C.1: Moments of Distribution of Annualised Net Inflation Rate under Optimal Commitment Policy

	Moment				
	Mean	Median	Std.	Skewness	RSS
Optimal Commitment	2.01	2.01	0.03	6.42	2.01

Notes. Calculations of the moments are based on a simulation of the economy for 301,000 periods, where the first 1,000 observations are discarded. RSS denotes the abbreviation for risky steady state.

rate becomes binding. As shown by Adam and Billi (2006) and Nakov (2008), a binding ZLB generates an incentive for the policymaker under commitment to create inflationary expectations. Higher inflation expectations at the ZLB reduce the real interest rate and, thereby, stimulate consumption. Hence, the policymaker promises to keep the nominal interest 'lower-for-longer', which generates an overshooting in inflation and output above their respective steady state values. Government spending, in turn, inversely follows output and consumption by increasing initially, then undershooting and eventually returning to its ergodic mean. The path of government spending follows from the policymaker's incentive to balance the marginal utility of government spending with the marginal utility of labour, i.e. the negative of the marginal disutility of labour hours (see Werning, 2011).

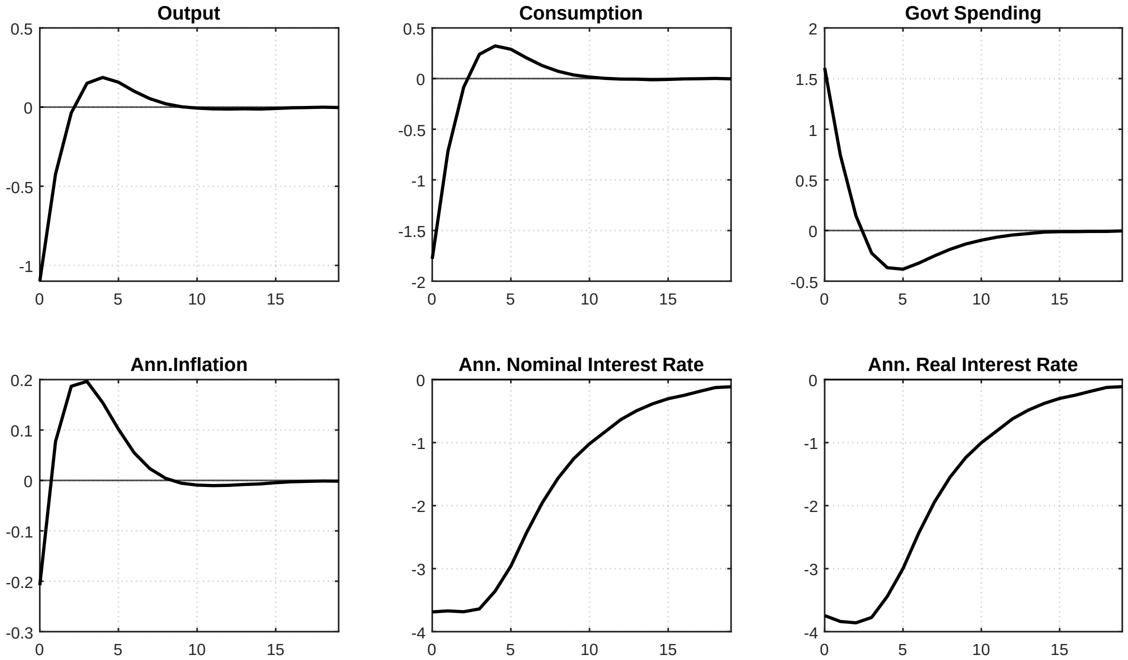

Figure 2.C.1: Generalised Impulse Response Functions to Discount Factor Shock: Optimal Commitment Policy

Notes. Generalised Impulse Response Functions (GIRF) under optimal commitment to a large decrease in the disturbance term ϵ_t that pushes the discount factor shock δ_t three unconditional standard deviations below its steady state value.

As shown by Billi (2011), the incentive of the policymaker under commitment to generate inflationary expectations at the ZLB generates an overshooting of the average annualised inflation rate above its deterministic steady state value. Table 2.C.1 demonstrates that also the risky steady state value of the annualised inflation rate exceeds its deterministic counterpart. Firms anticipate the improved stabilisation of inflation during ZLB episodes, which lowers the deflationary pressure on their inflation expectations. Likewise, the incentive to generate inflationary expectations at the ZLB also causes the median of the inflation distribution to exceed the deterministic steady state value and generates a positive skewness of the distribution of the annualised inflation rate. Despite the strong positive skewness, the wedge between the mean and the median is small because the standard deviation of the net annualised inflation rate is small.

Chapter 3

The Preferential Treatment of Green Bonds

This chapter is based on Giovanardi et al. (2023).

3.1 Introduction

The European Central Bank (ECB) announced, after concluding its strategy review in 2021, that it will take a more active role in climate policy. In addition to accepting sustainability-linked (*green*) bonds as collateral, several central banks contemplate to take one step further and treat them preferentially within their collateral frameworks, i.e., the conditions under which banks can pledge assets to obtain funding from the central bank.[1] The People's Bank of China (PBoC) started accepting green bonds as collateral on preferential terms already in 2018, which resulted in a substantial decline of green bond yields relative to conventional ones (Macaire and Naef, 2022). However, there is limited knowledge about the macroeconomic impact of a preferential collateral policy on green bond issuance, green investment, carbon emissions, and potential adverse side effects on financial markets.

To study the positive and normative implications of preferential treatment, this paper extends the standard RBC-model by (i) a climate externality (emissions), (ii) green and conventional firms issuing corporate bonds subject to default risk, and (iii) banks using these bonds as collateral. The extent to which corporate bonds can be used as collateral depends on central bank haircuts. Reducing haircuts on green bonds makes holding such bonds more attractive to banks and implies that they pay higher collateral premia on them. This in turn improves financing conditions to green firms, which increases bond issuance, investment, and leverage in response. Consequently, the equilibrium green capital share and corporate default risk rise.

We quantitatively assess the strength of these effects in a calibration to the euro area and uncover four main results. First, treating green bonds preferentially can have

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[1] A similar policy was also proposed in Brunnermeier and Landau (2020).

quantitatively relevant effects on the investment by green firms. Reducing the haircut on green bonds from 26% to 4.5% (corresponding to the ECB haircut on BBB- and AAA-rated corporate bonds, respectively), green firms increase investment by 0.5%. Second, green firms increase leverage by 1.1%, which increases resource losses from costly default. Because of these adverse effects, optimal collateral policy features a green haircut of 10%, such that green capital increases by 0.4%. Third, due to the small elasticity of investment to borrowing conditions in the presence of default risk, the real effects of optimal collateral policy are sizable, but considerably smaller than those of an optimal Pigouvian tax on emissions. Consequently, the welfare gain of an optimal tax exceeds the welfare gain of optimal collateral policy by two orders of magnitude. Fourth, the emission tax does not induce risk-taking, i.e., preferential treatment is an *imperfect substitute* for carbon taxation. The optimal degree of preferential treatment decreases, the closer the carbon tax gets to its optimal level. When emissions are taxed optimally, green and conventional bonds are treated symmetrically.

Our analysis is based on an extended RBC-model that connects collateral policy to financial and climate frictions. There are two types of intermediate good firms, green and conventional. Conventional firms generate emissions during the production of intermediate goods, while green firms have access to a clean technology. Following Heutel (2012) and Golosov et al. (2014), final good firms combine green and conventional intermediate goods with labor. Accumulated emissions are a real externality, because they have a negative effect on final good firms' productivity. This implies a sub-optimally low investment into the green technology in the competitive equilibrium.

Collateral policy is linked to the real sector by the corporate bond market, where both intermediate good firms issue bonds to banks. Firms have an incentive to issue bonds, because their owners are assumed to be more impatient than households, who own banks. Moreover, firms are subject to idiosyncratic shocks to their productivity and default on their bonds if revenues from production fall short of current repayment obligations. Absent collateral premia, corporate bond issuance is solely determined by a trade-off between relative impatience and bankruptcy costs, similar to Gomes et al. (2016).[2] Banks collect deposits from households, invest into corporate bonds, and incur liquidity management costs. In the spirit of Piazzesi and Schneider (2018), these costs are decreasing in the amount of available corporate collateral reflecting that banks may use it to collateralize short-term borrowing. This introduces a willingness of banks to pay collateral premia on corporate bonds.[3]

---

[2] Since our focus is on the collateral framework and thereby on firms that are sufficiently large to issue bonds and related marketable assets, we employ a financial friction that restricts debt issuance rather than overall external financing as in the canonical financial accelerator model of Bernanke et al. (1999).

[3] Collateral premia on corporate bonds are documented by Pelizzon et al. (2020) for the euro area, Mota (2020) for the US, and Chen et al. (forthcoming) and Fang et al. (2020) for China.

The central bank sets haircuts on corporate bonds that determine the degree to which bonds can be used as collateral. While low haircuts increase collateral availability for banks, the central bank incurs costs from accepting risky bonds as collateral. The literature has associated these costs with risk management expenses and counterparty default risk that depend on the riskiness of collateral (Bindseil and Papadia, 2006; Hall and Reis, 2015). As in Choi et al. (2021), optimal collateral policy balances the adverse effects of accepting risky collateral with the benefits of liquidity provision to banks. Starting from this point, our paper studies the welfare gains of adding a novel instrument (the green haircut) to the central bank toolkit.

The link between collateral policy and the real sector via banks' demand for bonds allows the central bank to affect the relative prices of green vis-à-vis conventional bonds by tilting the collateral framework in favor of them. In this case, banks pay higher collateral premia on green bonds, ceteris paribus, since holding them lowers liquidity management costs more effectively. Higher collateral premia make debt financing more attractive to green firms, such that their trade-off between relative impatience and bankruptcy costs is distorted: green firms increase their bond issuance, leverage, and investment. In contrast, conventional firms reduce their bond issuance, leverage, and investment: the green investment share rises. However, we can show analytically that higher risk-taking reduces the expected payoff from green investment compared to a benchmark without endogenous risk-taking. As a result, the transmission of preferential treatment to the green investment share is dampened: it increases by less than the green bond share. The endogeneity of risk-taking, which is key for this imperfect pass-through, is consistent with empirical observations.[4]

To quantify the optimal degree of preferential treatment, we calibrate the model to euro area data and show that it can replicate the joint dynamics of macroeconomic variables, financial markets, and emissions. We also provide evidence that the model can reconcile the effects of collateral premia on corporate bond spreads, investment, and leverage observed in the data. We then conduct a number of policy experiments. Starting from the baseline haircut of 26%, we first study a strong preferential policy, which treats all green bonds as if they where AAA-rated, implying a 4.5% haircut. This policy features a haircut gap of 21.5 percentage points and induces a green-conventional bond spread (also referred to as *greenium*) of 19 basis points, which increases the share of green capital from the calibration target of 20% to 20.09%. However, such a policy increases the collateral

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[4] Risk-taking, as reflected by firms' financing decision, has been reported in the empirical literature on unconventional monetary policy. Bekkum et al. (2018) observe a decrease in repayment performance on the mortgage backed securities market following an eligibility easing. Pelizzon et al. (2020) document positive leverage responses of eligible firms. Harpedanne de Belleville (2019) finds a sizable increase in investment by issuers of newly eligible bonds following a reduction of collateral requirements. Grosse-Rueschkamp et al. (2019), Giambona et al. (2020), and Cahn et al. (2022) document a positive investment and leverage impact of firms eligible for QE and LTRO operations.

supply beyond its optimal level and distorts the risk choice of green firms. Therefore, while still treating green bonds preferentially, optimal collateral policy is characterized by a substantial decrease of green haircuts to 10% and a modest increase of the haircut on conventional bonds to 30%, which keeps aggregate collateral supply approximately constant. The relative share of green capital increases to 20.08% in this case.

To put this modest effect of preferential treatment into perspective, we consider Pigouvian carbon taxation, which is the benchmark policy instrument to address climate externalities. The optimal tax increases the share of green capital to 28% and substantially reduces the pollution externality without adverse effects on firm risk-taking. These results should not be misinterpreted as a call for central bank inaction. If public policy is restricted in its ability to set carbon emission taxes optimally, the central bank can increase welfare by tilting the collateral framework towards green bonds. The optimal extent of preferential treatment declines, the closer carbon taxation gets to its optimal level: preferential treatment is a qualitatively and quantitatively an imperfect substitute for carbon taxes.

Related Literature. There is a small but fast-growing literature that adds climate aspects to DSGE models suitable for central bank policy analysis at business cycle frequencies, building on Heutel (2012). Punzi (2019) extends this setup by a credit-constrained corporate sector to study differentiated capital requirements on green and conventional firms. Due to our focus on the collateral framework and marketable assets instead of bank loans, our model uses a financial friction related to leverage rather than external financing.

In a specific assessment of green QE, Ferrari and Nispi Landi (forthcoming) find only a modestly positive impact on climate policy objectives. Similarly, Abiry et al. (2022) document a small impact of green QE, in particular in comparison to a carbon tax, which is similar to our results on collateral policy. Hong et al. (2021) study sustainable investment mandates which, similarly to our paper, affect the cost of capital wedges between green and conventional firms. We also relate to the work of Papoutsi et al. (2021) who study the carbon bias of central bank bond portfolios and the principle of market neutrality. Their paper stresses heterogeneous financial frictions between green and conventional sectors but abstracts from endogenous risk-taking. Similar to our results green-tilted asset purchases play no role in addressing the climate externality if optimal carbon taxes are available.

It should be stressed that all these papers at least implicitly add a second policy instrument (preferential treatment) together with the climate dimension to a setup in which financial policy (size of QE or capital requirements) solves a trade-off related to financial frictions. Our analysis is the first to provide a quantitative evaluation of optimal policy jointly addressing climate and financial frictions. Moreover, green financial policies will induce firm responses along several dimensions, that have not been studied in the literature so far. Our model is the first that endogenizes risk-taking on financial markets in this context, which enables us to explicitly consider downsides of preferential treatment in the optimal policy problem. In our view, a thorough analysis of these response margins is necessary to fully assess the effectiveness and efficiency of green financial policies.

Throughout the paper, we abstract from an analysis of transition risk, which arises if demand for conventional goods suddenly decreases due to ambitious climate policy. Diluiso et al. (2021) and Carattini et al. (2021) argue that macroprudential policies can address this issue. Similar to these papers, we document an interaction between climate policy and financial frictions and show how policy instruments should be adjusted to account for these interactions.

Outline. The paper is structured as follows. We introduce our model in Section 3.2. Section 3.3 presents our calibration, which we validate externally in Section 3.4. Our policy experiments are shown in Section 3.5. We provide an analytical characterization of the pass-through of preferential treatment to the real sector in Section 3.6. Section 3.7 concludes.

3.2 Model

Time is discrete and indexed by $t = 1, 2, ...$ The model is in real terms and features a representative *household*, two types of *intermediate goods firms*, a perfectly competitive *final good producer*, financial intermediaries (*banks*), and a public sector consisting of a fiscal authority and the central bank. One type of intermediate goods producers (*conventional*) emits greenhouse gases, which accumulate over time into socially costly *pollution*. The technology of the *green* firm does not contribute to pollution. The final good producer uses both types of intermediate goods and labor to produce the final consumption good. Banks raise deposits from the household to invest into corporate bonds issued by both intermediate good producers. Finally, the fiscal authority can levy a proportional carbon tax on the conventional firms' output, while the central bank sets the collateral framework. The full set of equilibrium conditions is provided in Section 3.C.

3.2.1 Households

The representative household derives utility from consumption c_t and disutility from supplying labor l_t at the wage w_t and saves in deposits d_t. Deposits held from time $t-1$ to t earn the interest rate i_{t-1}. The household's discount factor is denoted by β, ω_l is the utility-weight on labor and γ_l is the inverse Frisch elasticity of labor supply. The maximization problem of the representative household is given by

$$V_t = \max_{c_t, l_t, d_{t+1}} \log(c_t) - \omega_l \cdot \frac{l_t^{1+\gamma_l}}{1+\gamma_l} + \beta \mathbb{E}_t \left[V_{t+1} \right] \tag{3.1}$$

$$\text{s.t.} \quad c_t + d_{t+1} = w_t l_t + (1 + i_{t-1}) d_t + \Pi_t \ ,$$

where Π_t collects profits from banks and final goods producers.

3.2.2 Banks

There is a representative bank that supplies deposits to households and invests into corporate bonds of green (g) and conventional (c) firms. We assume that financial intermediation is subject to liquidity management costs, represented by the function $\Omega(\bar{b}_{t+1})$. They depend negatively on the collateral value of banks' corporate bond portfolio and are specified as

$$\Omega\left(\bar{b}_{t+1}\right) = \max\left\{ l_0 - \frac{l_1}{0.5}\left(\bar{b}_{t+1}\right)^{0.5}, 0 \right\} \tag{3.2}$$

where $\bar{b}_{t+1} = \sum_{\tau \in \{g,c\}} (1-\phi_\tau) q_{\tau,t} b_{\tau,t+1}$ denotes the collateral value of banks' bond portfolio. The collateral value is given by the market value of its bonds, weighted by one minus the respective central bank haircut ϕ_τ for firms of type $\tau \in \{g, c\}$.[5] Note that the collateral value of bonds is decreasing in haircuts. Banks directly benefit from smaller haircuts, since this, ceteris paribus, increases available collateral $\bar{b}_{t+1}$.

Making liquidity management costs dependent on available collateral $\bar{b}_{t+1}$ captures in reduced form the benefits of collateral to settle liquidity shocks on interbank markets or by tapping central bank facilities.[6] Plugging $\bar{b}_{t+1} = 0$ into equation (3.2) can be interpreted as the cost level of an entirely un-collateralized banking system. The concave shape of $\Omega(\cdot)$ captures that the marginal cost reduction of an additional unit of collateral decreases in the collateral level, such that banks will have a very high willingness to pay collateral premia if collateral is scarce. The marginal benefit of collateral is then given by $\Omega_{\bar{b},t} = -l_1(\bar{b}_{t+1})^{-0.5}$, which increases in the parameter l_1 and declines in available collateral $\bar{b}_{t+1}$.

We follow Cúrdia and Woodford (2011) and assume that banks maximize profits,

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[5] We restrict the analysis to time-invariant haircuts. In practice, collateral frameworks are occasionally adjusted in response to large shocks to the financial system. These events are not of first order importance to the analysis of preferential treatment.

[6] The literature has motivated liquidity management costs as arising from idiosyncratic liquidity shocks, such as deposit or credit line withdrawals (De Fiore et al., 2019 and Piazzesi and Schneider, 2018 among others). Since neither the sources of liquidity demand, nor the reason why the interbank market or central bank facilities are collateralized is at the heart of our paper, we introduce this feature in reduced form and refer to Section 3.B.1 for a micro-foundation.

defined as the equity value net of liquidity management costs in equation (3.3), subject to the solvency condition (3.4). The maximization problem of the representative bank reads

$$\max_{d_{t+1},b_{c,t+1},b_{g,t+1}} \Pi_t = d_{t+1} - q_{c,t}b_{c,t+1} - q_{g,t}b_{g,t+1} - \Omega(\bar{b}_{t+1}) \ , \tag{3.3}$$

$$\text{s.t.} \quad (1+i_t)d_{t+1} = \mathbb{E}_t\left[\mathcal{R}_{c,t+1}\right]b_{c,t+1} + \mathbb{E}_t\left[\mathcal{R}_{g,t+1}\right]b_{g,t+1} \ . \tag{3.4}$$

The expected bond payoff $\mathbb{E}_t\left[\mathcal{R}_{\tau,t+1}\right]$ depends on the bond issuance and capital choice of firm type τ via the possibility of default in future periods (see equation (3.11) below). Taking first order conditions, we obtain the bond price schedule

$$q_{\tau,t} = \frac{\mathbb{E}_t\left[\mathcal{R}_{\tau,t+1}\right]}{(1+i_t)(1+(1-\phi_\tau)\Omega_{\bar{b},t})} \ . \tag{3.5}$$

Equation (3.5) contains the marginal reduction of liquidity management cost from holding an additional unit of collateral $\Omega_{\bar{b},t}$, weighted by one minus the type-specific haircut. We refer to the term $(1-\phi_\tau)\Omega_{\bar{b},t}$ as the collateral premium.

3.2.3 Firms

Final Good Firms. A representative firm produces the final good y_t using a Cobb-Douglas production function that combines conventional and green intermediate goods $z_{c,t}$ and $z_{g,t}$ with labor l_t

$$y_t = (1-\mathcal{P}_t)\,A_t\left(z_{g,t}^{\nu}z_{c,t}^{1-\nu}\right)^{\theta} l_t^{1-\theta} \ , \tag{3.6}$$

where θ governs the labor intensity of production and ν determines the relative share of green intermediate goods.[7] Final good production is negatively affected by pollution damages $\mathcal{P}_t$ (described below). Aggregate TFP A_t is normalized to one in the long run and follows an AR(1) process in logs with persistence ρ_A. The TFP shock standard deviation is denoted by σ_A. Solving the maximization problem of the firm, we get standard first order conditions that equate the marginal product of the inputs to their market price

$$p_{c,t} = (1-\mathcal{P}_t)\,A_t(1-\nu)\theta z_{g,t}^{\nu\theta}z_{c,t}^{(1-\nu)\theta-1}l_t^{1-\theta} \ , \tag{3.7}$$

$$p_{g,t} = (1-\mathcal{P}_t)\,A_t\nu\theta z_{g,t}^{\nu\theta-1}z_{c,t}^{(1-\nu)\theta}l_t^{1-\theta} \ , \tag{3.8}$$

$$w_t = (1-\mathcal{P}_t)\,A_t(1-\theta)\left(z_{g,t}^{\nu}z_{c,t}^{1-\nu}\right)^{\theta} l_t^{\theta} \ , \tag{3.9}$$

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[7] In Section 3.E.1, we repeat our policy experiments using a CES-function and find only minor differences in terms of macroeconomic aggregates and welfare.

where $p_{c,t}$ and $p_{g,t}$ denote the conventional and green intermediate goods prices, respectively.

Intermediate Good Firms: Technology. There is a mass-one continuum of green and conventional firms that invest in capital $k_{\tau,t}$ to produce intermediate goods $z_{\tau,t}$ with a linear production technology.[8] Firm-specific output is subject to an idiosyncratic productivity shock $m_{\tau,t}$, which is i.i.d. across and within firm types ($z_{\tau,t} = m_{\tau,t} k_{\tau,t}$). Following Bernanke et al. (1999), the idiosyncratic shock is log-normally distributed with variance ς_M^2 and mean $-\varsigma_M^2/2$ to ensure that it satisfies $\mathbb{E}[m_{\tau,t}] = 1$. The log-normal distribution satisfies a monotone hazard rate property of the form $\partial(h(m)m)/\partial m > 0$, where $h(m) \equiv f(m)/(1 - F(m))$ denotes the hazard rate and $f(m)$ and $F(m)$ denote the pdf and cdf, respectively. Capital depreciates at rate δ_k, which is common to both technologies.

In the spirit of Heutel (2012), we assume that only conventional firms emit greenhouse gases which accumulate over time and only depreciate slowly. We refer to cumulated emissions as pollution $\mathcal{Z}_t$, which evolves according to $\mathcal{Z}_t = \delta_z \mathcal{Z}_{t-1} + z_{c,t}$ with $\delta_z < 1$.[9] The cost of pollution incurred by final goods producers satisfies $\partial \mathcal{P}/\partial \mathcal{Z} > 0$. Revenues $p_{\tau,t} z_{\tau,t}$ are subject to a time-invariant, type-specific tax χ_τ. A positive tax on conventional production χ_c is the Pigouvian instrument at disposal of the fiscal authority to address the climate externality.

Intermediate Good Firms: Financial Side. We assume that each firm is managed on behalf of a risk-averse and impatient representative firm owner who consumes only firm dividends $\widetilde{c}_t = \Pi_{c,t} + \Pi_{g,t}$. The firm owner's period utility is given by $\log(\widetilde{c}_t)$, while the discount factor is denoted by $\widetilde{\beta}$. Since there is a continuum of firms, individual firm behavior has no marginal effect on firm owner consumption: when maximizing the present value of their dividends, firms take the firm owner's stochastic discount factor $\widetilde{\Lambda}_{t,t+1} \equiv \widetilde{\beta}\widetilde{c}_t/\widetilde{c}_{t+1}$ as given. As in Gomes et al. (2016), firms finance their activities by issuing equity (negative dividends) or by issuing corporate bonds.[10]

Making the firm owner relatively impatient ($\widetilde{\beta} < \beta$) ensures that firms borrow from banks in equilibrium. Bonds mature stochastically each period with probability $0 < s \leq 1$

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[8] We do not explicitly index individual firms since our setup facilitates aggregation into two representative firm types. See also Section 3.A.

[9] We do not explicitly model emissions by the rest-of-the-world, since our main goal is an analysis of the role of financial frictions when climate policy operates through firm financing. In unreported policy experiments with positive rest-of-the-world emissions, welfare gains of optimal preferential treatment and optimal tax are smaller than in the closed economy case. However, the relative welfare gain of preferential treatment relative to the gain of carbon taxes is almost identical.

[10] We verify that aggregate dividends are always positive. Imposing the same degree of risk aversion over consumption rules out that firm owners insure households against aggregate shocks or vice versa.

and pay one unit of the final good in the repayment case.[11] With probability $1-s$, a bond does not mature and is rolled over at this period's market price. With probability s, the bond matures and firms default if their repayment obligation exceeds revenues from production.[12] In case of default, banks effectively replace the firm owner as shareholder: they seize the output *only in the default period*, pay a restructuring cost φ per unit of defaulted bonds and restructure the firm. After the firm's debt has been restructured, banks resume to being creditors. While in practice restructuring takes several periods, we follow Gomes et al. (2016) and take a shortcut by assuming that firm debt is restructured without delays, which implies that the debt choice is unaffected by the default decision in the current period and the default decision does not become a firm-specific state variable. Indeed, as we demonstrate in Section 3.A, all firms of type τ make the same debt and investment decision. The type-specific default threshold $\overline{m}_{\tau,t}$ is then implicitly defined as the productivity level at which revenues $(1-\chi_\tau)p_{\tau,t}m_{\tau,t}k_{\tau,t}$ equal repayment obligations $sb_{\tau,t}$.

Each firm of type τ maximizes the present value of dividends $\mathbb{E}_0 \sum_{t=0}^{\infty} \widetilde{\Lambda}_{t,t+1}\Pi_{\tau,t+1}$ subject to the default threshold and banks' bond pricing condition (3.5). Period t profits of each firm can be written by

$$\Pi_{\tau,t} = \mathbb{1}\{m_{\tau,t} > \overline{m}_{\tau,t}\}\Big((1-\chi_\tau)p_{\tau,t}m_{\tau,t}k_{\tau,t} - sb_{\tau,t}\Big) - k_{\tau,t+1} + (1-\delta_k)k_{\tau,t} \\ + q_{\tau,t}\left(b_{\tau,t+1} - (1-s)b_{\tau,t}\right) . \tag{3.10}$$

Integrating out the idiosyncratic shock $m_{\tau,t}$ in (3.10) yields aggregate profits in each sector τ. Let the expected productivity of type τ defaulting firms be denoted by $G\left(\overline{m}_{\tau,t}\right) \equiv \int_0^{\overline{m}_{\tau,t}} m dF(m)$. The default probability is given by $F(\overline{m}_{\tau,t}) \equiv \int_0^{\overline{m}_{\tau,t}} dF(m)$. In the default case, the entire output is distributed among holders of the defaulted bond, i.e., the revenues of the defaulting firm are divided by $sb_{\tau,t}$. The payoff in case of repayment is simply given by s. The realized per-unit bond payoff entering the bond pricing condition of banks (3.5) is given by

$$\mathcal{R}_{\tau,t} = s\Bigg(G(\overline{m}_{\tau,t})\underbrace{\frac{p_{\tau,t}(1-\chi_\tau)k_{\tau,t}}{sb_{\tau,t}}}_{=1/\overline{m}_{\tau,t}} + 1 - F(\overline{m}_{\tau,t})\Bigg) - F(\overline{m}_{\tau,t})\varphi + (1-s)q_{\tau,t} . \tag{3.11}$$

The first term reflects the payoff from the share s of maturing bonds: it consists of the revenues banks seize in case of default and the repayment of the principal in case of

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[11] Using long-term bonds allows to obtain realistic leverage ratios in the calibration, but is not required for the transmission of collateral policy to leverage and investment.

[12] We implicitly assume that there is no transfer of resources from productive to unproductive firms. This is consistent with the notion of uninsurable idiosyncratic productivity shocks.

repayment. The term $F(\overline{m}_{\tau,t})\varphi$ represents restructuring costs, while the share $(1-s)$ of bonds that are rolled over is valued at market price $q_{\tau,t}$. Note that the bond payoff (3.11) only depends on the firms' choices of debt and capital through the risk choice $\overline{m}_{\tau,t+1}$. We will make the dependency of the bond price on the default threshold explicit by writing $q(\overline{m}_{\tau,t+1})$ in the following.

Intermediate Good Firms: Bond Issuance and Investment. We show in Section 3.A that the first order conditions for bond issuance and capital read

$$\begin{aligned} &\frac{\partial q(\overline{m}_{\tau,t+1})}{\partial b_{\tau,t+1}}\Big(b_{\tau,t+1}-(1-s)b_{\tau,t}\Big)+q(\overline{m}_{\tau,t+1}) \\ &\qquad\qquad =\mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\Big(s(1-F(\overline{m}_{\tau,t+1}))+(1-s)q(\overline{m}_{\tau,t+2})\Big)\right] , \end{aligned} \tag{3.12}$$

$$\begin{aligned} 1=&\frac{\partial q(\overline{m}_{\tau,t+1})}{\partial k_{\tau,t+1}}\Big(b_{\tau,t+1}-(1-s)b_{\tau,t}\Big) \\ &\qquad\qquad +\mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\Big((1-\delta_k)+(1-\chi_\tau)p_{\tau,t+1}\big(1-G(\overline{m}_{\tau,t+1})\big)\Big)\right] . \end{aligned} \tag{3.13}$$

Equation (3.12) requires that the marginal benefit of issuing more bonds (LHS) equals marginal costs (RHS). Each unit of bonds increases funds available in period t by $q(\overline{m}_{\tau,t+1})$ units. By increasing the likelihood of future default, reflected by the term $\frac{\partial q(\overline{m}_{\tau,t+1})}{\partial b_{\tau,t+1}}<0$, issuing more bonds also dilutes the resources that can be raised by net debt issuance $(b_{\tau,t+1}-(1-s)b_{\tau,t})$. Issuing bonds has also implications for firm dividends in $t+1$. Each unit of bonds involves repayment of s, conditional on not defaulting. In addition, bond issuance also increases the rollover burden in the next period by $(1-s)q(\overline{m}_{\tau,t+2})$.

The optimality condition for capital (3.13) requires that the cost of purchasing capital (normalized to one) equals its payoff, which consists of the undepreciated value of capital next period and two parts related to firm risk-taking. The first part reflects the bond price increase due to a decrease of the default probability $\frac{\partial q(\overline{m}_{\tau,t+1})}{\partial k_{\tau,t+1}}>0$, which increases dividends in period t. Investment also increases dividends in $t+1$ directly by raising the marginal value of production net of taxes and conditional on not defaulting.

3.2.4 Government

The central bank sets the collateral framework (ϕ_c,ϕ_g) and incurs costs from collateral default $\Lambda(\overline{F}_t)$ which depend positively on the default risk of collateral pledged in the previous period:

$$\Lambda(\overline{F}_t)=\frac{\eta_1}{0.5}\left(\overline{F}_t\right)^{0.5}\quad\text{with}\quad\overline{F}_t\equiv\sum_{\tau\in\{g,c\}}(1-\phi_\tau)b_{\tau,t}q_{\tau,t}F(\overline{m}_{\tau,t}) . \tag{3.14}$$

This functional form implies that the marginal cost of accepting risky collateral is positive but decreasing. Following Bindseil and Papadia (2006), the concave specification reflects that there is a fixed cost component to set up an appropriate risk management infrastructure as well as a marginal cost component from adding additional risk to the central bank's collateral portfolio, for example through more frequent collateral default. The weighting $(1-\phi_\tau)b_{\tau,t}q_{\tau,t}$ can be interpreted as the repo size collateralized by green and conventional bonds, respectively. By setting haircuts, the central bank has a direct effect on these costs. In Section 3.B.2 we present a potential micro-foundation of the cost function, based on central bank solvency concerns (Hall and Reis, 2015).

Taken together, lowering haircuts reduces banks' liquidity management cost, but increases the central bank's cost due to collateral default. To close the model, we assume that the fiscal authority rebates all tax revenues raised from the conventional sector to green firms to balance its budget,

$$\chi_c p_{c,t} z_{c,t} + \chi_g p_{g,t} z_{g,t} = 0 \ . \tag{3.15}$$

This fiscal rule allows us to abstract from additional fiscal instruments that would otherwise be necessary to balance the government budget.

3.2.5 Resource Constraint

The resource constraint is given by

$$y_t = c_t + \sum_{\tau\in\{g,c\}} (\widetilde{c}_{\tau,t} + i_{\tau,t}) + \Omega(\bar{b}_{t+1}) + \sum_{\tau\in\{g,c\}} \varphi F(\overline{m}_{\tau,t}) b_{\tau,t} + \Lambda(\overline{F}_t) \ , \tag{3.16}$$

where the last three terms represent the resource losses due to the liquidity management costs and corporate default costs, suffered by banks, and collateral default costs, suffered by the central bank.

3.3 Calibration

We calibrate the model to euro area data. Each period corresponds to one quarter. The parameters governing macroeconomic dynamics at business cycle frequencies are set to standard values. We fix the inverse Frisch elasticity at 1 and set the household discount factor β to 0.995. The Cobb-Douglas coefficient in the final good production technology is set to $\theta = 1/3$ to imply a labor share of 2/3. We set the weight ω_l in the household utility function to be consistent with a steady state labor supply of 0.3. The TFP shock parameters are conventional values in the RBC literature. The capital depreciation rate

δ_k is set to match the capital/GDP ratio of 2.1. Having fixed these parameters, the calibration can then be divided into parameters related to climate-, firm-, and bank-specific frictions.

Emissions and Pollution Damage. For the relative size of the green sector, we use the most recent data on the share of renewable energies in the euro area. Although this is only a subset of intermediate goods, it has the advantage that the data quality is excellent. In 2018, renewable energy sector's size was 20%, which directly informs the Cobb-Douglas parameter ν of the wholesale goods producers ν.[13] The persistence of pollution is set to $\delta_z = 0.997$, following Heutel (2012). We assume that pollution costs can be expressed as

$$\mathcal{P}_t = 1 - \exp\{-\gamma_P \mathcal{Z}_t\} \ , \tag{3.17}$$

which corresponds to a percentage loss in the final good producer's production (3.6). We can directly relate pollution costs $\mathcal{P}$ to observable long run quantities $1 - y/z^{\theta} l^{1-\theta}$. We inform γ_P in equation (3.17) using estimates of direct costs from pollution and indirect costs from adverse climate conditions, which Muller (2020) and Reidmiller et al. (2017) quantify at 10% for the US, which can reasonably assumed to be similar in the euro area.[14]

Intermediate Good Firms and Corporate Bonds. The next group of parameters is associated with intermediate good firms. We assume that financial frictions are symmetric across firm types.[15] Average maturity of corporate bonds is set to five years ($s = 0.05$) and corresponds to average maturity in the *Markit iBoxx* corporate bond index between 2010 and 2019. Firm owners' discount factor $\widetilde{\beta}$ and the variance of idiosyncratic productivity shocks ς_M^2 are set to match long run means of corporate bond spreads and the corporate debt/GDP ratio. The model-implied bond spread is defined as

$$x_{\tau,t} \equiv \left(1 + \frac{s}{q(\overline{m}_{\tau,t+1})} - s\right)^4 - (1 + i_t)^4 \ .$$

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[13] Renewable energy statistics for the EU are accessible here. See also the guide by Eurostat (2020).

[14] The climate block of our model is intentionally simplistic, since we consider time-invariant Pigouvian carbon taxes and haircuts. In this case, optimal policy is primarily governed by long run default risk and climate damage, both of which are calibration targets. Adding "climate tipping points" which might interact with financial risk is a promising extension of our analysis, but beyond the scope of this paper.

[15] Our paper abstracts from risk factors affecting both sectors in a heterogeneous way, such as transition risk. In this context, differentiated haircuts can be motivated on risk management considerations for monetary policy operations, but are not used as climate policy instrument. We also abstract from investor preferences for holding green bonds. These premia are typically very small, as shown in Larcker and Watts (2020) and Flammer (2021), who find at most a modest effect of environmental performance on spreads in the US fixed income market.

For the data moment on spreads, we use *IHS Markit* data from 2010 until 2019. We compute the median bond spread over the entire corporate bond sample and average over time, which yields a value of 100bp. The data moment on corporate debt is the ratio of non-financial firm debt to GDP, taken from the ECB.

Table 3.1: Baseline Calibration

Parameter	**Value**	**Source/Target**
Households		
Household discount factor β	0.995	Standard
Labor disutility convexity γ_l	1	Inverse Frisch
Labor disutility weight ω_l	12	Labor supply
Firms		
Cobb-Douglas coefficient θ	1/3	Labor share
Green goods share ν	0.20	Renewable energy share
Externality parameter γ_P	5.5e-5	Pollution damage/GDP
Pollution decay parameter δ_z	0.997	Heutel (2012)
Capital depreciation rate δ_k	0.0288	Capital/GDP
Discount factor $\widetilde{\beta}$	0.988	Debt/GDP
Standard deviation idiosyncratic risk ς_M	0.21	Bond spread
Bond maturity parameter s	0.05	*IHS Markit*
Financial Markets		
Restructuring costs φ	1.2	Leverage
Collateral default cost parameter η_1	0.0555	Optimality of ϕ_{sym}
Liquidity management intercept l_0	0.05	Ensures positive cost
Liquidity management slope l_1	0.0075	Eligibility premium
Haircut parameter ϕ_{sym}	0.26	ECB collateral framework
Shocks		
Persistence TFP ρ_A	0.95	Standard
Variance TFP shock σ_A	0.005	Standard

Banks and Collateral Premia. The final group of parameters is related to banks and collateral policy. Restructuring costs φ are set to obtain a leverage ratio qb/k of 40% as in Gomes et al. (2016). We impose symmetric collateral treatment and set $\phi_{sym} \equiv \phi_c = \phi_g = 0.26$, which corresponds to the haircut on BBB-rated coupon-bearing corporate bonds with five to seven years maturity in the ECB collateral framework in 2015.[16]

The intercept parameter l_0 of the liquidity management cost function (3.2) is set sufficiently high to ensure that $\Omega(\bar{b}_{t+1})$ is positive for all considered collateral policy spec-

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[16] The BBB-haircut can be reasonably assumed to be representative for the firm cross-section. We use 2015 values since they are consistent with the sample period over which we compute the target for corporate bond spreads. Furthermore, these values were in effect during the first policy announcements we use in Section 3.4 to demonstrate that the model delivers reasonable effects of preferential treatment on corporate bond yields. The full set of haircut values in force in 2015 is accessible under this link. For a comprehensive treatment of the ECB collateral framework over time, see also Nyborg (2017).

ifications, but does not visibly affect our results. We calibrate l_1 to match the eligibility premium reported by the empirical literature: using the ECB list of collateral eligible for main refinancing operations, Pelizzon et al. (2020) identify an eligibility premium of -11bp. The model implied eligibility premium is given by the yield differential of the traded bond and a synthetic bond that is not eligible in period t, but becomes eligible in $t+1$, corresponding to the identification strategy of Pelizzon et al. (2020). The advantage of this procedure is that the eligibility premium can be backed out from bond prices in closed form and is given by

$$\widetilde{x}_{\tau,t} \equiv \left(1 + \frac{s}{q(\overline{m}_{\tau,t+1})} - s\right)^4 - \left(1 + \frac{s}{q(\overline{m}_{\tau,t+1})(1+(1-\phi_\tau)\Omega_{\overline{b},t})} - s\right)^4 .$$

The level parameter η_1 in the collateral default cost function (3.14) is set so that the empirical haircut value $\phi_{sym} = 0.26$ is optimal according to an utilitarian welfare criterion. Put differently, we assume that the status-quo ECB collateral policy is optimal under the restriction of symmetric collateral policy and parameterize (3.14) accordingly. Finally, we define the *greenium* as the spread of conventional over green bonds with corresponding maturity $\widehat{x}_t = x_{g,t} - x_{c,t}$. Note that the greenium is zero in our baseline calibration due to the assumption of symmetric financial frictions and haircuts. The parameterization is summarized in Table 3.1, while targeted moments are presented in Table 3.C.1. All data sources are summarized in Section 3.D.

Table 3.2: Model Fit - Untargeted Moments

Moment	Model	Data	Source
Annualized default rate	0.02	0.01	Gomes et al. (2016)
Volatilities			
Bond spread $\sigma(x)$	31bp	50-100bp	Gilchrist and Zakrajšek (2012)
Relative vol. consumption $\sigma(c)/\sigma(y)$	0.64	0.73	Euro area data
Relative vol. investment $\sigma(i)/\sigma(y)$	4.38	3.58	Euro area data
Persistence			
GDP $corr(y_t, y_{t-1})$	0.71	0.83	Euro area data
Consumption $corr(c_t, c_{t-1})$	0.87	0.78	Euro area data
Investment $corr(i_t, i_{t-1})$	0.61	0.65	Euro area data
Correlations with GDP			
Consumption $corr(y, c)$	0.89	0.73	Euro area data
Investment $corr(y, i)$	0.90	0.69	Euro area data
Debt $corr(y, b)$	0.71	0.65	Jungherr and Schott (2022)
Default risk $corr(y, F)$	-0.76	-0.55	Kuehn and Schmid (2014)
Emissions $corr(y, z_{c,t})$	0.34	0.30	Doda (2014)

Notes: The data moments are based on quarterly euro area data from 1995-2019. All second moments are based on HP-filtered data.

In Table 3.2, we compare untargeted model-implied first and second moments with euro area data. The relative volatilities of consumption and investment to output are consistent with euro area data, even though our model only uses one exogenous shock and does not feature frictions directly related to firm investment or to the relationship between households and banks. The slightly elevated investment volatility and its low autocorrelation can at least partly be attributed to the absence of investment adjustment costs. The time series volatility of bond spreads is slightly smaller than the value reported by Gilchrist and Zakrajšek (2012) for US data, since bonds are priced using a log-utility pricing kernel and only contain default risk compensation and the collateral premium. The model also captures the cyclical properties of debt and default risk, which are key financial market variables in the context of risk-taking effects induced by collateral policy. In addition, we also match closely the cyclicality of emissions, which has been estimated by Doda (2014) for a large sample of countries.

3.4 The Financial and Real Effects of Preferential Treatment

To additionally corroborate the external validity of our quantitative analysis, we compare the model-implied effect of preferential haircuts to effects estimated in the empirical literature. There are two partial effects that shape the financial and real effects of preferential treatment: (i) the response of relative borrowing costs for green and conventional firms (the greenium) to preferential haircuts and (ii) the elasticities of leverage and capital to bond yield changes. We proceed in two steps and separate the between-sector effects of preferential treatment on borrowing costs from sector-specific effects of borrowing conditions on real outcomes. This is relevant from an empirical point of view as preferential policies are not implemented yet: this decomposition allows us to assess the plausibility of our model predictions.

Preferential Treatment and Relative Borrowing Costs. To examine the effect of preferential central bank policy on (relative) bond prices, we exploit the yield reaction of green and conventional bonds around ECB announcements regarding climate policy.[17] We identify four relevant speeches by ECB board members between 2018 and 2020, which explicitly mention climate policy concerns for the conduct of central bank policy. Using data from *IHS Markit* and *Thomson Reuters Datastream*, we generate a panel of green-conventional bond pairs, obtained by a nearest-neighbor matching. We then compute the average yield difference between green bonds and their respective conventional counterparts for a 20 trading day window around each announcement.

The treatment window is longer than the typical intra-day windows used in the literature on monetary policy shock identification, which are based on reactions of highly

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[17] See Section 3.F for details on the announcements and the data.

liquid interbank market rates or government bond yields. Corporate bonds in general and green bonds in particular are traded on considerably less liquid markets. Especially the latter are furthermore often owned by buy-and-hold investors (Flammer, 2021), such that it is reasonable to expand the event window. Section 3.F suggests that the announcement effect only peters out after about 15 trading days, which is consistent with results in Eliet-Doillet and Maino (2022). Averaging over all announcements and the entire post-treatment window, the announcement effect is statistically significant and economically meaningful: green bond yields drop by 4.8bp, which is again similar to the effect reported in Eliet-Doillet and Maino (2022). The result indicates that bond investors are willing to pay premia on green bonds if there is the prospect of preferential treatment by the central bank.

Since the ECB so far did not implement preferential treatment, these announcements can be mapped into our model by interpreting them as a news shock (see Beaudry and Portier, 2004 and Barsky and Sims, 2011). Specifically, we assume that preferential treatment will be implemented with certainty at some point in the future. We enrich the baseline calibration by a news shock to the green collateral parameter ϕ_g,

$$\log(\phi_{g,t}) = (1-\rho_\phi)\log(\phi_{sym}) + \rho_\phi \log(\phi_{g,t-1}) + \sigma_\phi \epsilon^{\phi}_{t-h} \quad \epsilon^{\phi}_{t-h} \sim N(0,1)\ , \tag{3.18}$$

where ϕ_{sym} is the green collateral parameter corresponding to the baseline calibration and h denotes the announcement horizon. We choose a high value of $\rho_\phi = 0.95$ for the haircut persistence, since changes to the collateral framework only occur infrequently. The shock size σ_ϕ is set such that $\phi_g = 0.045$, corresponding to the AAA-haircut we will use in the next section to study a strong preferential policy. We use announcement horizons of two, three, four, or five years which appear plausible, given that the ECB strategy review itself took two years and that the actual implementation of preferential treatment takes some additional time. The announcement effect is shown in Table 3.3 and lies between -7.1bp and -3.3bp. Naturally, the effect size declines as the announcement horizon increases. The model-implied yield response closely resembles the value estimated in our event-study at the four-year announcement horizon.

Table 3.3: Greenium Reaction - Announcement Effects

Data	**Model: News Shock Horizon**			
	2 years	3 years	4 years	5 years
-4.8bp	-7.1bp	-5.5bp	-4.2bp	-3.3bp

Relative Borrowing Costs and their Real Effects. In the second step, we consider the firm level effects of a change in borrowing cost induced by central bank policy. We build on the literature studying firm responses following QE-programs and collateral framework changes. From the point of view of firms (the collateral supply side), the effects of QE and collateral eligibility are identical, since in both cases banks increase demand for their bonds for reasons unrelated to firm fundamentals. Specifically, we compare empirical estimates from the literature to the model-implied effect of a haircut reduction from $\phi_{sym} = 1$ (no eligibility) to $\phi_{sym} = 0.26$ (our baseline value). We assume that the collateral policy relaxation is unanticipated, comes into effect immediately, and is permanent. To ensure consistency with the event-study approaches of the empirical literature, we do not take into account general equilibrium effects on collateral supply and fix $\Omega_{\bar{b},t}$ at the baseline value. We focus on the reaction of bond yields, capital, and leverage, which are crucial for the pass-through of preferential collateral policy.

Since the eligibility premium as defined in Pelizzon et al. (2020) is a calibration target, we instead examine the yield spread between eligible and non-eligible bonds. Fang et al. (2020) study the impact of an easing of collateral eligibility requirements by the PBoC and identify a yield reaction on treated bonds of 42 to 62bp (their Table 5). Using a similar approach, Chen et al. (forthcoming) find a yield reaction of 39 to 85bp (their Tables 5 and 8).

Cahn et al. (2022) document that newly eligible firms increase their leverage by 1.2 to 2.4pp in response to a relaxation of collateral eligibility requirements for the ECB's very long term refinancing operations. Pelizzon et al. (2020) report an increase of total debt/-total assets between 2.5pp and 10.8pp (their Table 10) after a firm's first inclusion into the list of eligible assets. Regarding the firm financing structure, Grosse-Rueschkamp et al. (2019) show that the introduction of the Corporate Sector Purchase Program (CSPP) triggered a positive response of total debt/total assets for eligible firms relative to non-eligible firms prior to CSPP. The magnitude of the effect is estimated between 1.1pp and 2.0pp, depending on the empirical specification (see their Table 2). Giambona et al. (2020) consider the impact of QE and find increases in total debt/total assets of around 1.8pp (their Table 15).

On the same sample, Giambona et al. (2020) report an increase in investment between 4.9pp and 15.1pp for QE-eligible firms when controlling for firm characteristics (their Tables 3-13). Harpedanne de Belleville (2019), Table 5.1, finds a 5.4pp increase in investment after the introduction of the Additional Credit Claims program using French data, which contains a large amount of small firms without bond market access. Grosse-Rueschkamp et al. (2019) and Cahn et al. (2022) on the other hand only document a mild effect of 1pp and 2.6pp on asset growth (their Table 5 and Table 9, respectively). Table 3.4 shows that the elasticities of bond yields, capital, and leverage comfortably fall into the range of empirical estimates.

Table 3.4: Firm Reaction: Model vs. Data

	Δ **Yield**	Δ **Capital**	Δ **Leverage**
Model	70bp	4.9pp	5.1pp (market value) 2.1pp (book value)
Data	39 - 85bp	1 - 15pp	1 - 11pp

Notes: Difference between a 26% to 100% haircut in the first row. Range of estimated effects taken from the literature in the second row.

3.5 Policy Analysis

Using the calibrated model, we now conduct policy experiments regarding the collateral framework and its interactions with direct Pigouvian carbon taxation.

The Effects of Preferential Treatment. Since intermediate good firms are at the heart of the transmission mechanism of both policies, we begin by showing the model-implied means of bond spreads, capital, and leverage for different green haircuts in Figure 3.1. The left panel shows that lowering the green haircut induces a sizable decline of the green bond spread relative to the baseline calibration (solid vertical line), which is accompanied by an increase in capital and leverage. The reaction of conventional firms mirrors the response of their green counterparts, although to a smaller extent. This is an equilibrium effect operating through the perfect substitutability of green and conventional bonds as collateral: the conventional collateral premium $(1-\phi_c)\Omega_{\bar{b},t}$ negatively depends on haircuts and aggregate collateral supply $\bar{b}_{t+1}$. If $\bar{b}_{t+1}$ increases due to preferential treatment, the conventional collateral premium declines, since the conventional haircut is kept constant in this experiment.

The case of strong preferential treatment ($\phi_g = 0.045$) is shown in the first column of Table 3.5. We use this haircut value since it corresponds to the treatment of AAA-rated corporate bonds in the ECB collateral framework and opens a considerable haircut gap.[18] The eligibility premium on green bonds widens to -14bp, translating into a green bond spread reduction of 16bp, which is sizable when comparing it to the (targeted) baseline bond spread of 100bp. The implied greenium is around 20bp. This increases green investment by 0.5%, while leverage of green firms (at market prices) increases by 1.1%. Since conventional capital falls by less than 0.1%, pollution costs fall only marginally. Due to the increase in leverage, restructuring and collateral default costs are higher than under

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[18] Our model is not necessarily well suited to study the general equilibrium effects of more drastic haircut values, for example a 100% haircut on conventional bonds, sometimes referred to as complete decarbonization of monetary policy operations. Since corporate bonds are the only financial assets that can be used as collateral in our model, such a policy would imply a severe reduction in available collateral and might therefore predict unreasonably large effects on green collateral premia.

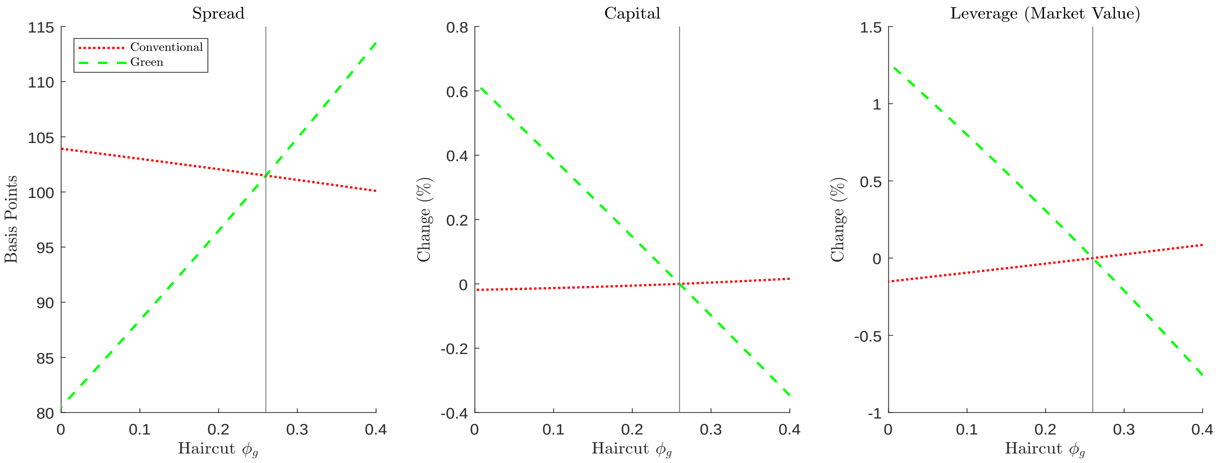

Figure 3.1: Firm Response to Preferential Treatment

Notes: We display long run means for different green haircuts ϕ_g. Spreads are expressed in basis points, leverage is relative to the baseline calibration of $\phi_{sym} = 0.26$ (vertical line).

the baseline scenario. At the same time, liquidity management cost decrease due to the higher corporate bond issuance. Conversely, a strongly punishing haircut on conventional bonds would imply a substantial increase of liquidity management costs and a decrease in aggregate cost of corporate default.

Having demonstrated that tilts to green collateral policy reduce pollution costs but have non-negligible side effects on financial markets, we next compute optimal haircuts. We employ an utilitarian welfare criterion based on household's (unconditional) expected utility and follow Schmitt-Grohé and Uribe (2007) by approximating it, together with the policy functions, up to second order. Given the log-utility assumption on consumption, the consumption equivalent (CE) welfare gain follows as $c^{CE,policy} \equiv 100(\exp\{(1-\beta)(V^{policy} - V^{base})\} - 1)$, where V^{base} and V^{policy} are obtained from evaluating the household's value function (3.1) under the baseline and alternative policies, respectively.[19]

The second column of Table 3.5 shows the effects of optimal collateral policy, conditional on an emission tax of zero. While the haircut gap of 20% is very similar to the case of strong preferential treatment, the average haircut ($\nu\phi_g + (1-\nu)\phi_c = 0.26$) equals the value in the symmetric baseline. This implies that liquidity management and default cost hardly change relative to the baseline. The optimal average haircut is determined by the trade-off between default and liquidity management cost, while the optimal haircut gap trades off the climate impact and risk-taking effects of a green-tilted collateral policy. Under optimal collateral policy, the green capital share is slightly smaller than in the strong preferential treatment case and the greenium attains a slightly smaller value of 18bp. At the same time however, the pollution cost/GDP fall by 0.08%, compared to a

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[19] We also explore welfare gains conditionally on being at the deterministic steady state of the baseline calibration and taking into account the transition period to the new steady state. Results are virtually unchanged. In Section 3.E.2, we also show that nominal rigidities do not affect our results.

Table 3.5: Time Series Means for Different Policies

Moment	**Strong Pref**	**Opt Coll**	**Opt Tax**
Tax χ_c	0	0	10.2%
Conventional Haircut ϕ_c	26%	30%	26%
Green Haircut ϕ_g	4.5%	10%	26%
Welfare (CE, Change)	+0.014%	+0.016%	+1.490%
Conv. Elig. Premium	-11bp	-10bp	-11bp
Green Elig. Premium	-14bp	-13bp	-11bp
Conv. Bond Spread	104bp	105bp	102bp
Green Bond Spread	84bp	87bp	102bp
Conv. Leverage (Change)	-0.1%	-0.2%	0%
Green Leverage (Change)	+1.1%	+0.9%	0%
Conv. Capital (Change)	0%	-0.1%	-9.5%
Green Capital (Change)	+0.5%	+0.4%	+40.8%
Green Capital Share	0.209	0.208	0.280
GDP	+0.04%	+0.02%	+0.61%
Default Cost/GDP (Change)	+2.37%	+0.13%	-0.24%
Liq. Man. Cost/GDP (Change)	-1.74%	-0.04%	-0.77%
Pollution Cost/GDP (Change)	-0.02%	-0.08%	-9.01%

Notes: Strong preferential treatment (*Strong Pref*) is based on a collateral framework set to $\phi_g = 0.045$ and $\phi_c = 0.26$. The optimal collateral policy (*Opt Coll*) is computed holding $\chi_g = 0$ constant. For the optimal tax (*Opt Tax*), we hold haircuts fixed at their baseline values and vary the tax rate. *Change* refers to percentage differences from the baseline calibration. Default costs are the sum of collateral default costs (3.14) and aggregate restructuring cost $\varphi(\nu F_{g,t} + (1-\nu)F_{c,t})$. The baseline green capital share of 0.2 is a calibration target.

fall of 0.02% for strong preferential treatment. This additional decline is due to the higher haircut on conventional bonds, which reduces investment in the conventional technology. Since pollution damages are smaller under this policy, GDP and welfare increase.

Interaction with Direct Taxation. So far, our analysis showed that the central bank can affect the relative size of green and conventional capital, reduce carbon emissions, and increase welfare. In this section, we benchmark the effects of preferential treatment against direct Pigouvian carbon taxation. The third column of Table 3.5 corresponds to the optimal tax (10%), holding the collateral framework at its baseline value. Compared to the baseline scenario, the green capital share rises from the (targeted) baseline value of 0.2 to 0.28, which reduces pollution cost/GDP by around 9%. At the same time, there are no adverse effects on firm risk-taking: the leverage ratio of firms and their default rate are unchanged. Quantitatively, both the emission reduction and the welfare improvement of optimal carbon taxation exceed the improvement from optimal preferential treatment by two orders of magnitude.

In a last step, we explore the welfare effects of optimal collateral policy as carbon taxes gradually approach their optimal level. Therefore, we vary carbon taxes and the

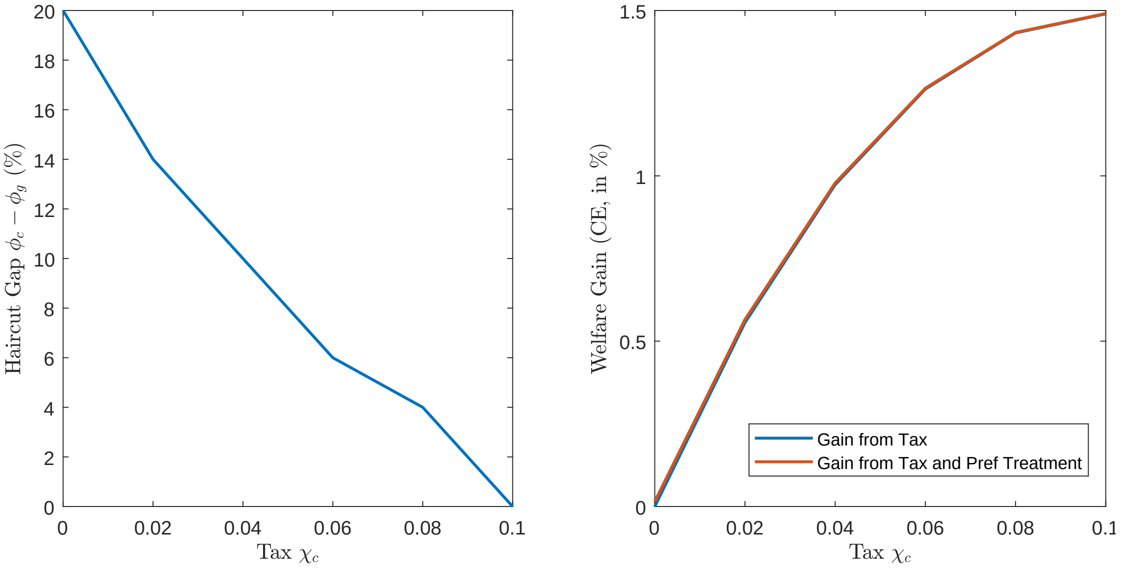

Figure 3.2: Optimal Collateral Policy Under Sub-Optimal Taxation

Notes: For different levels of the Pigouvian carbon tax, we show welfare and display the result as the difference between optimal conventional and green haircuts (left panel) and in terms of welfare gains relative to the baseline calibration (in CE, right panel).

collateral framework simultaneously. The left panel of Figure 3.2 shows the optimal haircut gap for different levels of the tax. Absent taxes ($\chi_c = 0$), the optimal haircut gap is 20%. As the carbon tax increases, the optimal haircut gap declines until, at the optimal tax of $\chi_c = 0.1$, the optimal haircut gap reduces to zero. Once optimal taxes are available, the Tinbergen principle of targeting applies: collateral policy is determined by the trade-off between default and liquidity management cost in symmetric fashion. Our model implies that preferential treatment is welfare-enhancing if and only if climate policy is unable to implement the optimal tax. This qualitative result holds irrespective of the parameterization as long as firm risk-taking effects are present (see also Section 3.6). Quantitatively, the additional welfare gain of preferential treatment turns out to be very small even for a tax of zero (see the right panel of Figure 3.2). It becomes negligible for larger carbon taxes and eventually zero at the optimal tax.

3.6 On the Transmission of Preferential Treatment

Our quantitative analysis demonstrates that a green-tilted collateral policy is optimal if and only if carbon taxes are sub-optimally low, implying that green-tilted haircuts are an imperfect substitute for carbon taxes. In this section, we show that this is due to the endogeneity in risk-taking, which generates an imperfect pass-through of a collateral policy relaxation via improved borrowing conditions to real investment. We illustrate such imperfect pass-through analytically in a simplified setting. The discussion will be organized around intermediate good firms' first order condition for capital, which relates the cost of investment to its payoff. We show how haircuts affect the investment payoff and the green capital ratio and discuss the role of endogenous firms risk-taking.

For the ease of exposition, we consider the case of one-period bonds and full capital depreciation ($s = \delta_k = 1$). Furthermore, banks cannot seize output of defaulting firms and do not incur restructuring costs ($\varphi = 0$), such that the only real cost of default is a production loss. Since we do not focus on macroeconomic dynamics in this section, we also set firm owner's stochastic discount factor to $\widetilde{\Lambda}_{t,t+1} = \widetilde{\beta}$.

A Benchmark Without Default Risk. To isolate the role of financial frictions in the production sector, it is informative to relate our model to a framework without default risk, but with collateral premia. In this case, firms choose bonds $b_{\tau,t+1}$ and capital $k_{\tau,t+1}$ to maximize the present value of dividends $\Pi_{\tau,t} + \widetilde{\beta}\mathbb{E}_t[\Pi_{\tau,t+1}]$ subject to non-negativity constraints on dividends $\Pi_{\tau,t}, \Pi_{\tau,t+1} \geq 0$. Define with $\widetilde{q}_{\tau,t}$ the price of the default-free bond. Without default risk, only expected productivity $\mathbb{E}_t[m_{\tau,t+1}]$ is relevant for the firm problem, which equals one by assumption. Therefore, we can re-write the maximization problem as

$$\max_{b_{\tau,t+1},k_{\tau,t+1}} - k_{\tau,t+1} + \widetilde{q}_{\tau,t} b_{\tau,t+1} + \widetilde{\beta}\mathbb{E}_t\left[(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1} - b_{\tau,t+1}\right]$$

$$\text{s.t.} \quad k_{\tau,t+1} \leq \widetilde{q}_{\tau,t} b_{\tau,t+1} \quad \text{and} \quad (1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1} \geq b_{\tau,t+1} \; .$$

Note that the bond price $\widetilde{q}_{\tau,t}$ does not depend on firm decisions in the default-free benchmark. Since firms effectively have linear preferences with a unitary weight over dividends in t and weight $\widetilde{\beta}$ over dividends in $t+1$, it is optimal to issue bonds up to $b_{\tau,t+1} = (1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}$, holding the capital stock constant. Intuitively, while debt issuance allows firms to front-load dividends, there are no (default) costs associated with high debt issuance and the firm optimally chooses a leverage of 100%.[20] Using this, the maximization problem reduces further to

$$\max_{k_{\tau,t+1}} \quad -k_{\tau,t+1} + \widetilde{q}_{\tau,t}(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1} \; ,$$

which yields the following first order condition for capital

$$1 = \underbrace{(1-\chi_\tau)\mathbb{E}_t[p_{\tau,t+1}]\widetilde{q}_{\tau,t}}_{\text{Investment payoff } \Xi^{\text{no default}}_{\tau,t+1}} \; . \tag{3.19}$$

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[20] This definition of leverage relates repayment obligations $b_{\tau,t+1}$ to the production value of assets $(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}$, rather than the re-sale value of capital, which is not well-defined in this model.

This condition requires that the marginal cost of investment (normalized to one) equals the investment payoff $\Xi^{\text{no default}}_{\tau,t+1}$, which also depends on firms' financing conditions, i.e., the bond price $\widetilde{q}_{\tau,t}$. In partial equilibrium, an increase of the expected payoff $\Xi^{\text{no default}}_{\tau,t+1}$ stimulates investment.

The presence of the bond price in equation (3.19) links central bank policy to the real sector through banks' demand for corporate bonds. The bond price $\widetilde{q}_{\tau,t} = \frac{1}{(1+i_t)(1+(1-\phi_\tau)\Omega_{\overline{b}})}$ reflects the discounted bond payoff in $t+1$ and the collateral premium. Since $\widetilde{q}_{\tau,t}$ is increasing in the collateral premium ($\frac{\partial \widetilde{q}_{\tau,t}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}} > 0$), a haircut reduction will increase the investment payoff. Since the investment payoff is proportional to the bond price, we refer to the default-free case as *perfect pass-through*:

$$\frac{\partial \Xi^{\text{no default}}_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}} = (1-\chi_\tau)\mathbb{E}_t[p_{\tau,t+1}] \frac{\partial \widetilde{q}_{\tau,t}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}} \; . \tag{3.20}$$

Combining the investment decision (3.19) for both firm types with the respective intermediate good demand (3.7) and (3.8) yields the equilibrium green capital ratio

$$\frac{k_{g,t}}{k_{c,t}} = \frac{\widetilde{q}_{g,t}}{\widetilde{q}_{c,t}} \frac{\nu(1-\chi_g)}{(1-\nu)(1-\chi_c)} \; . \tag{3.21}$$

Equation (3.21) shows that, in the default-free benchmark, any policy affecting the relative price of green bonds, such as a preferential collateral treatment, will proportionally affect the equilibrium green capital ratio.

The Role of Default Risk. Next, consider the model with default risk. With one-period bonds, the default threshold is given by $\overline{m}_{\tau,t+1} = \frac{b_{\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}$ and the first order conditions for bonds and capital simplify to

$$q'(\overline{m}_{\tau,t+1})\mathbb{E}_t[\overline{m}_{\tau,t+1}] + q(\overline{m}_{\tau,t+1}) = \widetilde{\beta}\mathbb{E}_t[1 - F(\overline{m}_{\tau,t+1})] \; , \tag{3.22}$$

$$1 = (1-\chi_\tau)\mathbb{E}_t\Big[p_{\tau,t+1} \underbrace{\Big(\widetilde{\beta}\big(1 - G(\overline{m}_{\tau,t+1})\big) - q'(\overline{m}_{\tau,t+1})\overline{m}^2_{\tau,t+1}\Big)}_{\text{Financial wedge } \Gamma_{\tau,t+1}}\Big] \; . \tag{3.23}$$

As in the default-free case, condition (3.23) requires that the investment payoff $\Xi^{\text{default}}_{\tau,t+1} \equiv (1-\chi_\tau)\mathbb{E}_t[p_{\tau,t+1}\Gamma_{\tau,t+1}]$ equals the price of capital, still normalized to one. In contrast to the default-free case however, the investment payoff now also depends on the firm's financing decision. The financial wedge entering equation (3.23) contains, first, the discounted future output produced by an additional unit of capital conditional on not defaulting, $\widetilde{\beta}\big(1 - G(\overline{m}_{\tau,t+1})$. Second, it contains a bond price appreciation term, $q'(\overline{m}_{\tau,t+1})\overline{m}^2_{\tau,t+1}$, reflecting the reduction in default risk from higher investment. The transmission of collateral policy on the investment payoff is given by

$$\frac{\partial \Xi^{\text{default}}_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}} = (1-\chi_\tau)\mathbb{E}_t\left[p_{\tau,t+1}\frac{\partial \Gamma_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}}\right] , \tag{3.24}$$

and depends on the financial wedge $\Gamma_{\tau,t+1}$, which itself is endogenously determined. To characterize the effect of collateral policy on the financial wedge ($\frac{\partial\Gamma_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}}$), we exploit that banks' bond pricing condition is available in closed form. The bond pricing condition and its derivative with respect to the risk choice $\overline{m}_{\tau,t+1}$ can then be written as

$$\begin{aligned}
q(\overline{m}_{\tau,t+1}) &= \mathbb{E}_t\left[\frac{1-F(\overline{m}_{\tau,t+1})}{(1+i_t)(1+(1-\phi_\tau)\Omega_{\overline{b}})}\right] \\
q'(\overline{m}_{\tau,t+1}) &= \mathbb{E}_t\left[\frac{-f(\overline{m}_{\tau,t+1})}{(1+i_t)(1+(1-\phi_\tau)\Omega_{\overline{b}})}\right] .
\end{aligned}$$

Plugging these into equation (3.22), we can express the risk-choice as

$$(1+i_t)\left(\frac{1}{1+i_t}-(1+(1-\phi_\tau)\Omega_{\overline{b}})\widetilde{\beta}\right) = \mathbb{E}_t\Big[\frac{f(\overline{m}_{\tau,t+1})}{1-F(\overline{m}_{\tau,t+1})}\overline{m}_{\tau,t+1}\Big] . \tag{3.25}$$

In the absence of collateral premia ($\phi_\tau = 1$), the risk choice is determined by equating relative impatience (LHS) and marginal default costs (RHS). Holding the interest rate i_t and the marginal collateral benefit $\Omega_{\overline{b}}$ fixed, a reduction of the haircut ϕ_τ increases the LHS of (3.25). Due to the monotonicity assumption on the hazard rate, the RHS of (3.25) is increasing in the default threshold $\overline{m}_{\tau,t+1}$. Hence, relaxing collateral policy requires firms to choose a higher $\overline{m}_{\tau,t+1}$ to satisfy equation (3.25). Intuitively, firms increase their risk-taking, because lower financing costs make investment and front-loading dividend payouts more attractive. Lemma 1 demonstrates that the effect of collateral policy on the financial wedge can be simplified into an expression that is directly comparable to the default-free benchmark.

Lemma 1. Imperfect Pass-Through. The effect of collateral policy on the investment payoff can be expressed as

$$\frac{\partial \Xi^{\text{default}}_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}} = (1-\chi_\tau)\mathbb{E}_t\Big[p_{\tau,t+1}\overline{m}_{\tau,t+1}\big(1-F(\overline{m}_{\tau,t+1})\big)\Big]\frac{\partial\widetilde{q}_{\tau,t}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}} , \tag{3.26}$$

where $\frac{\partial\widetilde{q}_{\tau,t}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}}$ is the response of a default-free bond to collateral premia. Proof: Section 3.A.2.

This expression closely resembles the default-free case (3.20). When $\frac{\partial \Xi^{\text{default}}_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}} < \frac{\partial \Xi^{\text{no default}}_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}}$, the financial wedge dampens the transmission of collateral policy to investment payoffs, which obtains if $\big(1 - F(\overline{m}_{\tau,t+1})\big)\overline{m}_{\tau,t+1} < 1$. This condition is trivially satisfied for any $\overline{m}_{\tau,t+1} < 1$, since we normalize the distribution to have a mean of one. Furthermore, $\overline{m}_{\tau,t+1} = 1$ already corresponds to a default rate of more than 50% per quarter, while the default rate in the calibrated model and the data is around 1% in annualized terms. For any economically reasonable parameterization, the inequality will hold and relaxing collateral policy has a positive but unambiguously smaller effect on investment compared to the default-free case. In equilibrium, the green capital ratio in the presence of endogenous risk-taking can be written

$$\frac{k_{g,t}}{k_{c,t}} = \frac{\mathbb{E}_t\left[\Gamma_{g,t+1}\right]}{\mathbb{E}_t\left[\Gamma_{c,t+1}\right]} \frac{\nu(1-\chi_g)}{(1-\nu)(1-\chi_c)} \,. \tag{3.27}$$

Absent preferential treatment, risk choice and bond prices are identical across firm types such that the financial wedges $\Gamma_{\tau,t+1}$ in the payoffs from investment $\Xi_{\tau,t+1}$ cancel. Then, as in the default-free case, the relative size of both sectors would be solely determined by the technology parameter ν and the climate policy regime. Setting $\chi_c > 0$ and $\chi_g < 0$ directly increases the green capital ratio. Note that this policy also operates through the investment payoff, which increases (decreases) in the subsidy (tax), irrespective of whether there is default risk or not. However, in sharp contrast to collateral policy, the tax rate χ_τ does not affect firm risk-taking, since it does not directly enter the risk-choice (3.25). The preferential treatment of green bonds in the collateral framework also increases the green capital ratio, but endogenous default risk impairs the effectiveness of this policy.

3.7 Conclusion

In this paper, we examine the effectiveness of the preferential collateral treatment of green bonds in an augmented RBC-model. Preferential treatment stimulates investment into green capital, but simultaneously induces an increase in green firms' leverage and default risk. In a calibration to euro area data, we show that optimal collateral policy takes into account these adverse effects and is considerably less effective than Pigouvian carbon taxes, but still increases welfare. Preferential treatment is a qualitatively and quantitatively imperfect substitute for carbon taxes and is desirable if and only if carbon taxes are set below their optimal level.

Appendix

3.A Model Appendix

3.A.1 Intermediate Good Firms' Debt and Investment Choice

Our model allows for exact aggregation into a representative green and conventional firm, which greatly simplifies exposition of the channels at play and allows for a tractable welfare analysis. The necessary assumptions are similar to Gomes et al. (2016): productivity shocks are i.i.d., defaulting firms are restructured immediately, and there is a continuum of small firms all owned by the same representative firm owner. Formally, for each type $\tau \in \{c, g\}$ there is a unit mass of perfectly competitive firms and we index individual firms by j. Firm owner consumption is given by $\widetilde{c}_t = \int_j \Pi_{j,c,t} dj + \int_j \Pi_{j,g,t} dj$. We impose the following within-period timing:

- Each firm j of type τ draws an idiosyncratic productivity shock $m_{j,\tau,t}$, produces and either repays its maturing debt obligations or defaults.
- In the default case, revenues are transferred to banks and the firm immediately re-enters the bond market.
- Firms adjust capital $k_{j,\tau,t+1}$ and bonds outstanding $b_{j,\tau,t+1}$.
- Firms transfer their dividends $\Pi_{j,\tau,t}$ to the firm owner.

The present value of dividends paid by firm j of type τ is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \widetilde{\Lambda}_{0,t} \Big\{ \mathbf{1}\{m_{j,\tau,t} > \overline{m}_{j,\tau,t}\} \Big((1-\chi_\tau) p_{\tau,t} m_{j,\tau,t} k_{j,\tau,t} - s b_{j,\tau,t} \Big) - k_{j,\tau,t+1} \\ + (1-\delta_k) k_{j,\tau,t} + q_{j,\tau,t} \left(b_{j,\tau,t+1} - (1-s) b_{\tau,t} \right) \Big\} .$$

Under the assumption of no delays in restructuring and i.i.d. productivity shocks, next period's productivity can be integrated out in the objective function and the problem reduces to a two-period consideration. The relevant part of the maximization problem then becomes

$$\begin{aligned}\max_{k_{j,\tau,t+1},b_{j,\tau,t+1},\overline{m}_{j,\tau,t+1}} \quad & -k_{j,\tau,t+1} + q_{j,\tau,t}\Big(b_{j,\tau,t+1} - (1-s)b_{j,\tau,t}\Big) \\ & + \mathbb{E}_t\Bigg[\widetilde{\Lambda}_{t,t+1}\Bigg((1-\chi_\tau)p_{\tau,t+1}\int_{\overline{m}_{j,\tau,t+1}}^{\infty} m\cdot k_{j,\tau,t+1}dF(m) \\ & + (1-\delta_k)k_{j,\tau,t+1} - \int_{\overline{m}_{j,\tau,t+1}}^{\infty} sb_{j,\tau,t+1}dF(m) \\ & + q_{j,\tau,t+1}\Big(b_{j,\tau,t+2} - (1-s)b_{j,\tau,t+1}\Big)\Bigg)\Bigg] ,\end{aligned}$$

subject to the default threshold $\overline{m}_{j,\tau,t+1} \equiv \frac{sb_{j,\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k_{j,\tau,t+1}}$ and the bond pricing condition (3.5), taking as given next period's bond price $q_{j,\tau,t+1}$. Since dividends of all firms are transferred to the firm owner each period and firms can access capital and bond markets irrespective of a potential default event in the current period, all type τ firms make the same choices $k_{\tau,t+1}$ and $b_{\tau,t+1}$. This allows aggregation into a representative green and conventional firm, respectively, and we will omit the firm index j in the following.

To derive the first-order condition for bonds and capital, we start with observing that the default threshold of a type-τ intermediate good firm in period $t+1$ is given by $\overline{m}_{\tau,t+1} \equiv \frac{sb_{\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}$. The threshold satisfies the following properties:

$$\frac{\partial \overline{m}_{\tau,t+1}}{\partial b_{\tau,t+1}} = \frac{s}{(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}} = \frac{b_{\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}\frac{s}{b_{\tau,t+1}} = \frac{\overline{m}_{\tau,t+1}}{b_{\tau,t+1}} , \tag{3.A.1}$$

$$\frac{\partial \overline{m}_{\tau,t+1}}{\partial k_{\tau,t+1}} = -\frac{sb_{\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}^2} = -\frac{b_{\tau,t+1}}{(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}\frac{s}{k_{\tau,t+1}} = -\frac{\overline{m}_{\tau,t+1}}{k_{\tau,t+1}} . \tag{3.A.2}$$

We assume that $\log(m_{\tau,t})$ is normally distributed with mean μ_M and standard deviation ς_M. In the calibration, we ensure that $\mathbb{E}[m_{\tau,t}] = 1$ by setting $\mu_M = -\varsigma_M^2/2$. The CDF of $m_{\tau,t}$ is given by $F(m_{\tau,t}) = \Phi\left(\frac{\log m_{\tau,t} - \mu_M}{\varsigma_M}\right)$, where $\Phi(\cdot)$ is the cdf of the standard normal distribution. The conditional mean of m at the threshold value $\overline{m}_{\tau,t+1}$ can be expressed as

$$\begin{aligned} G(\overline{m}_{\tau,t+1}) &= \int_0^{\overline{m}_{\tau,t+1}} mf(m)dm = e^{\mu_M + \frac{\varsigma_M^2}{2}}\Phi\left(\frac{\log \overline{m}_{\tau,t+1} - \mu_M - \varsigma_M^2}{\varsigma_M}\right) , \\ 1 - G(\overline{m}_{\tau,t+1}) &= \int_{\overline{m}_{\tau,t+1}}^{\infty} mf(m)dm = e^{\mu_M + \frac{\varsigma_M^2}{2}}\Phi\left(\frac{-\log \overline{m}_{\tau,t+1} + \mu_M + \varsigma_M^2}{\varsigma_M}\right) . \end{aligned}$$

Note that the derivative $g(\overline{m}_{\tau,t+1})$ of this expression satisfies

$$g(\overline{m}_{\tau,t+1}) = \overline{m}_{\tau,t+1}f(\overline{m}_{\tau,t+1}) . \tag{3.A.3}$$

For notational convenience, we write the bond price schedule as function of the default threshold $\overline{m}_{\tau,t}$ throughout this section. The bond payoff is given by

$$\mathcal{R}_{\tau,t} = s\left(G(\overline{m}_{\tau,t}) \frac{(1-\chi_\tau)p_{\tau,t}k_{\tau,t}}{sb_{\tau,t}} + 1 - F(\overline{m}_{\tau,t}) \right) - F(\overline{m}_{\tau,t})\varphi + (1-s)q(\overline{m}_{\tau,t}) \, ,$$

such that we can write the bond price as

$$q(\overline{m}_{\tau,t+1}) = \mathbb{E}_t \left[\frac{s\left(\frac{G(\overline{m}_{\tau,t+1})}{\overline{m}_{\tau,t+1}} + 1 - F(\overline{m}_{\tau,t+1}) \right) - F(\overline{m}_{\tau,t+1})\varphi + (1-s)q(\overline{m}_{\tau,t+2})}{(1+(1-\phi_\tau)\Omega_{b,t})(1+i_t)} \right] . \tag{3.A.4}$$

The partial derivatives with respect to bonds and capital is given by

$$q'(\overline{m}_{\tau,t+1}) = -\mathbb{E}_t \left[\frac{sG(\overline{m}_{\tau,t+1})/\overline{m}^2_{\tau,t+1} + \varphi f(\overline{m}_{\tau,t+1})}{(1+(1-\phi_\tau)\Omega_{b,t})(1+i_t)} \right] , \tag{3.A.5}$$

Taken as given the bond pricing condition, firms choose $k_{\tau,t+1}$ and $b_{\tau,t+1}$ to maximize

$$\begin{aligned} & - k_{\tau,t+1} + q(\overline{m}_{\tau,t+1})\Big(b_{\tau,t+1} - (1-s)b_{\tau,t} \Big) \\ & + \mathbb{E}_t \Bigg[\widetilde{\Lambda}_{t,t+1} \Bigg((1 - G(\overline{m}_{\tau,t+1}))(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1} + (1-\delta_k)k_{\tau,t+1} \\ & \quad - s\Big(1 - F(\overline{m}_{\tau,t+1})\Big) b_{\tau,t+1} + q(\overline{m}_{\tau,t+2})\Big(b_{\tau,t+2} - (1-s)b_{\tau,t+1} \Big) \Bigg) \Bigg] . \end{aligned}$$

FOC w.r.t $b_{\tau,t+1}$. The first order condition for bonds is given by

$$\begin{aligned} 0 = & \frac{\partial q(\overline{m}_{\tau,t+1})}{\partial b_{\tau,t+1}} \Big(b_{\tau,t+1} - (1-s)b_{\tau,t} \Big) + q(\overline{m}_{\tau,t+1}) \\ & + \mathbb{E}_t \Bigg[\widetilde{\Lambda}_{t,t+1} \Bigg(-(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1} g(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial b_{\tau,t+1}} \\ & \qquad - s\Big(- f(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial b_{\tau,t+1}} b_{\tau,t+1} + 1 - F(\overline{m}_{\tau,t+1}) \Big) - (1-s)q(\overline{m}_{\tau,t+2}) \Bigg) \Bigg] , \end{aligned}$$

which can be expressed as

$$\begin{aligned} 0 = & \frac{\partial q(\overline{m}_{\tau,t+1})}{\partial b_{\tau,t+1}} \Big(b_{\tau,t+1} - (1-s)b_{\tau,t} \Big) + q(\overline{m}_{\tau,t+1}) \\ & + \mathbb{E}_t \Bigg[\widetilde{\Lambda}_{t,t+1} \Bigg(-sg(\overline{m}_{\tau,t+1}) \frac{\overline{m}_{\tau,t+1}(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}{sb_{\tau,t+1}} \\ & - s\Big(-f(\overline{m}_{\tau,t+1})\overline{m}_{\tau,t+1} + 1 - F(\overline{m}_{\tau,t+1}) \Big) - (1-s)q(\overline{m}_{\tau,t+2}) \Bigg) \Bigg] , \end{aligned}$$

and then yields (3.12).

FOC w.r.t $k_{\tau,t+1}$. The first order condition for capital is given by

$$
\begin{aligned}
1 = {} & \frac{\partial q(\overline{m}_{\tau,t+1})}{\partial k_{\tau,t+1}} \Big(b_{\tau,t+1} - (1-s) b_{\tau,t} \Big) \\
& + \mathbb{E}_t \Bigg[\widetilde{\Lambda}_{t,t+1} \Bigg(1 - \delta_k - g(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial k_{\tau,t+1}} (1-\chi_\tau) p_{\tau,t+1} k_{\tau,t+1} \\
& \qquad + (1 - G(\overline{m}_{\tau,t+1}))(1-\chi_\tau) p_{\tau,t+1} + s b_{\tau,t+1} f(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial k_{\tau,t+1}} \Bigg] ,
\end{aligned}
$$

which can be rearranged to

$$
\begin{aligned}
1 = {} & \frac{\partial q(\overline{m}_{\tau,t+1})}{\partial k_{\tau,t+1}} \Big(b_{\tau,t+1} - (1-s) b_{\tau,t} \Big) \\
& + \mathbb{E}_t \Bigg[\widetilde{\Lambda}_{t,t+1} \Bigg(1 - \delta_k + g(\overline{m}_{\tau,t+1}) \overline{m}_{\tau,t+1} (1-\chi_\tau) p_{\tau,t+1} + \big(1 - G(\overline{m}_{\tau,t+1})\big)(1-\chi_\tau) p_{\tau,t+1} \\
& \qquad - f(\overline{m}_{\tau,t+1}) \overline{m}^2_{\tau,t+1} (1-\chi_\tau) p_{\tau,t+1} \Bigg) \Bigg] ,
\end{aligned}
$$

and further to equation (3.13). Gomes et al. (2016) consider explicitly the impact of today's debt choice on tomorrows debt choice, which further reduces tomorrow's bond price. Since tomorrow's bond price is part of today's bond price by the rollover value in the bond pricing condition, this stickiness of leverage has a dynamic feedback effect into today's debt choice. We verify that this effect does not materially change the cyclical properties of our model or the optimal policy results.

3.A.2 Proof of Lemma 1

The proof uses the closed-form expressions for bond prices together with the first order conditions for capital and bonds to express the effect of collateral policy on the investment payoff in an easily interpretable way.

Combining the first order conditions (3.22) with (3.23), and differentiating the financial wedge $\Gamma_{\tau,t+1}$ with respect to the collateral premium, we can decompose the total effect into the increase of bond prices $\frac{\partial q(m_{\tau,t+1})}{\partial (1-\phi_\tau)\Omega_{\overline{b}}}$ that is also present in the default-free case and three terms associated with risk-taking:

$$
\begin{aligned}
\frac{\partial \Gamma_{\tau,t+1}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}} = \mathbb{E}_t \Bigg[& \Big(\frac{\partial q(m_{\tau,t+1})}{\partial (1-\phi_\tau)\Omega_{\overline{b}}} + q'(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}} \Big) \overline{m}_{\tau,t+1} \\
& + q(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}} - \widetilde{\beta} \big(1 - F(\overline{m}_{\tau,t+1})\big) \frac{\partial \overline{m}_{\tau,t+1}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}} \Bigg] .
\end{aligned}
\tag{3.A.6}
$$

The first term $\frac{\partial q(m_{\tau,t+1})}{\partial (1-\phi_\tau)\Omega_{\overline{b}}}$ reflects the reduction in financing costs, holding firm behavior constant, and is closely related to the default-free benchmark. The term $q'(\overline{m}_{\tau,t+1}) \frac{\partial \overline{m}_{\tau,t+1}}{\partial (1-\phi_\tau)\Omega_{\overline{b}}} <$

0 is a negative risk-taking effect, which lowers the bond price and thereby makes investment less attractive in period t. The positive term $q_{\tau,t}\frac{\partial \overline{m}_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}}$ captures a bond price appreciation from investment, since higher investment lowers default risk, ceteris paribus. Last, $\widetilde{\beta}\big(1-F(\overline{m}_{\tau,t+1})\big)\frac{\partial \overline{m}_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}}$ reflects the dividend reduction in $t+1$ due to higher default rates.

Using the definitions of $q(\overline{m}_{\tau,t+1})$ and $q'(\overline{m}_{\tau,t+1})$, we can express (3.A.6) as

$$\begin{aligned}\frac{\partial \Gamma_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}} &= \left(\underbrace{\frac{1-\frac{f(\overline{m}_{\tau,t+1})}{(1-F(\overline{m}_{\tau,t+1}))}\overline{m}_{\tau,t+1}}{(1+i_t)(1+(1-\phi_\tau)\Omega_{\overline{b}})}}_{=\widetilde{\beta}\text{ from (3.25)}}(1-F(\overline{m}_{\tau,t+1}))-\widetilde{\beta}\big(1-F(\overline{m}_{\tau,t+1})\big)\right)\frac{\partial \overline{m}_{\tau,t+1}}{(1-\phi_\tau)\Omega_{\overline{b}}}\\ &+\frac{\partial q(\overline{m}_{\tau,t+1})}{\partial(1-\phi_\tau)\Omega_{\overline{b}}}\overline{m}_{\tau,t+1}=\frac{\partial q(\overline{m}_{\tau,t+1})}{\partial(1-\phi_\tau)\Omega_{\overline{b}}}\overline{m}_{\tau,t+1}\ .\end{aligned}$$

Using the derivative of the bond pricing condition with respect to the collateral premium, this expression further simplifies to

$$\frac{\partial \Gamma_{\tau,t+1}}{\partial(1-\phi_\tau)\Omega_{\overline{b}}}=\overline{m}_{\tau,t+1}(1-F(\overline{m}_{\tau,t+1}))\frac{\partial \widetilde{q}(\overline{m}_{\tau,t+1})}{\partial(1-\phi_\tau)\Omega_{\overline{b}}}\ ,$$

where $\frac{\partial \widetilde{q}(\overline{m}_{\tau,t+1})}{\partial(1-\phi_\tau)\Omega_{\overline{b}}}$ is the response of the bond in the case without default risk. Plugging this condition into equation (3.24) we get to equation (3.26). $\square$

3.B Microfoundations for Cost Functions

In this section, we provide microfoundations for both cost functions used in the quantitative analysis.

3.B.1 Bank Liquidity Management Costs

In the quantitative analysis, we assume that banks incur liquidity management costs $\Omega(\overline{b}_{t+1})$, which gives rise to collateral premia. In this section, we demonstrate that the resulting first order conditions for corporate bonds are observationally equivalent to the most common micro-foundation used in this context, which are stochastic bank deposit withdrawals, see Corradin et al. (2017), De Fiore et al. (2019), Piazzesi and Schneider (2018), or Bianchi and Bigio (2022). The standard modeling device in this literature is a two sub-period structure, where banks participate in asset markets sequentially: in the first sub-period, banks trade with households on the deposit market and with intermediate good firms on the corporate bond market. In the second sub-period, the representative bank faces an uninsurable liquidity deficit $\omega_t > 0$, which it settles using short-term funding from the central bank against collateral.

If a bank is unable to collateralize its entire funding need, it must borrow on the (more expensive) unsecured segment or obtain funds elsewhere, for example by attracting new deposits or issuing equity. More specifically, since all banks hold the same amount of collateral $\overline{b}_{t+1}$ before the deposits are withdrawn, there is a cut-off withdrawal $\overline{\omega}_t = \overline{b}_{t+1}$ above which a bank needs to tap more expensive funding sources. The expected amount of funds raised from alternative funding sources is given by

$$\widetilde{b}_{t+1} \equiv \int_{\overline{b}_{t+1}}^{\infty} \left(\omega_{t+1} - \overline{b}_{t+1}\right) dW(\omega_{t+1}) \ ,$$

where $W(\omega_{t+1})$ denotes the cdf of the withdrawal shock distribution. Due to its analytical tractability, it is convenient to assume that withdrawals follow a Lomax distribution. This distribution is supported on the right half-line and characterized by a shape parameter $\widetilde{\alpha} > 1$ and a scale parameter $\widetilde{\lambda} > 0$. This allows us to write the expected funding shortfall in closed form:

$$\begin{aligned}
\widetilde{b}_{t+1} &= \int_{\overline{b}_{t+1}}^{\infty} \omega_{t+1} \frac{\widetilde{\alpha}}{\widetilde{\lambda}} \left(1 + \frac{\omega_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}-1} d\omega_{t+1} - \overline{b}_{t+1} \int_{\overline{b}_{t+1}}^{\infty} \frac{\widetilde{\alpha}}{\widetilde{\lambda}} \left(1 + \frac{\omega_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}-1} d\omega_{t+1} \\
&= \overline{b}_{t+1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}} + \frac{\widetilde{\lambda}}{\widetilde{\alpha} - 1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}+1} - \overline{b}_{t+1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}} \\
&= \frac{\widetilde{\lambda}}{\widetilde{\alpha} - 1} \left(1 + \frac{\overline{b}_{t+1}}{\widetilde{\lambda}}\right)^{-\widetilde{\alpha}+1} .
\end{aligned}$$

It follows that the expected amount of expensive borrowing falls, the more collateral is held. The benefit of holding collateral corresponds to spread ξ that is paid on uncollateralized, more expensive funding $\widetilde{b}_{t+1}$. We assume that ξ is an exogenous parameter, such that expected costs $\xi\widetilde{b}_{t+1}$ enter bank profits in the first sub-period

$$\Pi_t = d_{t+1} - q_{c,t+1}b_{c,t+1} - q_{g,t+1}b_{g,t+1} - \xi\widetilde{b}_{t+1} \ .$$

The cost term depends negatively on $\overline{b}_{t+1}$, but the marginal cost reduction is falling in $\overline{b}_{t+1}$. Since very large withdrawal shocks are unlikely, the additional benefit of holding another unit of collateral is positive but decreasing. The properties of our concave liquidity cost function $\Omega(\overline{b}_{t+1})$ are closely related to the common micro-foundation using bank liquidity risk.

3.B.2 Collateral Default Costs

In the main text, we assume an exogenous cost function from collateral default $\Lambda(\overline{F}_t)$. In this section, we provide a micro-foundation based on central bank solvency concerns (see Hall and Reis, 2015). We show that this yields a loss function $\Lambda(\overline{F}_t)$ which is increasing in $\overline{F}_t$, consistent with our assumption in the main text. In addition to the liquidity shocks introduced in Section 3.B.1, we assume that some banks will default on their loans obtained from the central bank. This introduces an economically meaningful role for collateral. Since the collateral that banks pledge is subject to default risk, the central bank will subject itself to corporate default risk when entering repurchase agreements. The central bank haircut ϕ directly affects exposure to this risk.

The timing is as follows: in the beginning of period t, before the corporate bond market opens, banks incur the exogenous liquidity need and tap central bank facilities. Repos mature at the end of period t, after corporate and bank default materializes, but before the corporate bond market opens again. Therefore, only corporate bonds b_t held from the previous period can be used as collateral. Since the bond payoff is still uncertain when banks enter repos, they are valued at price q_t. Banks can borrow up to the (haircut-corrected) market value of bonds $(1-\phi)q_t b_t$. To ensure positive collateral premia, we assume that the liquidity shock exceeds available collateral with positive probability, see for example the distributional assumption in Section 3.B.1. Lowering the haircut will then increase the aggregate amount of funds borrowed from the central bank $\widetilde{\omega}_t$, also referred to as the repo size, which is smaller than the aggregate liquidity shock, since some banks will be constrained by their collateral holdings.

We assume for simplicity that the bank default rate ζ_t is an i.i.d. random variable with cdf $Z(\cdot)$ and support $[0,1]$. The aggregate default rate of corporate bonds is denoted F_t. In case of a bank default, the central bank seizes the posted collateral to cover its losses. However, since the collateral itself defaults at rate F_t, the central bank will not recover the full amount of the defaulted repo. The aggregate loss from lending to banks follows as $\zeta_t\widetilde{\omega}_t F_t$, which increases in repo size $\widetilde{\omega}_t$ and the default rates of banks and corporate bonds. At the same time, the central bank also generates seignorage revenues from lending through its facilities. As customary in the literature, we assume that seignorage revenues are bounded from above by the (time-invariant) constant $\mathcal{M}$ (Hall and Reis, 2015). Consequently, the central bank incurs a loss if $\zeta_t\widetilde{\omega}_t\varpi F_t > \mathcal{M}$. We can then denote expected central bank revenues

$$\mathcal{L}_t = \mathcal{M} - \int_0^1 \zeta_t\widetilde{\omega}_t\overline{F}_t dZ(\zeta_t) \ . \tag{3.B.1}$$

Equation (3.B.1) shows that, irrespective of the distributional assumption on ζ_t, the expected revenues fall in the bond default rate F_t. At the same time, a higher haircut ϕ, by lowering the repo size $\widetilde{\omega}_t$ increases revenues. Define the collateral default rate $\overline{F}_t$ as the repo size times bond default rate ($\overline{F}_t \equiv (1-\phi)q_t b_t F_t$). Under the assumptions of Section 3.B.1, the repo size is convex in the collateral value of corporate bonds $(1-\phi)$, since high liquidity shocks are less likely. Therefore, the convex relationship between haircuts and central bank revenues is reflected in the cost function $\Lambda(\overline{F}_t)$ that we use in the quantitative analysis.

3.C Full System of Equilibrium Conditions

For any given exogenous process A_t, taxes on conventional/green production χ_c, χ_g, and central bank haircuts ϕ_c, ϕ_g, an equilibrium is a set of 30 endogenous variables: $\{c_t, y_t, l_t, z_t, z_{c,t}, z_{g,t}, k_{c,t}, k_{g,t}, i_{c,t}, i_{g,t}, b_{c,t}, b_{g,t}, \bar{b}_t, d_t, \mathcal{Z}_t, \mathcal{P}_t, w_t, p_{z,t}, p_{c,t}, p_{g,t},$ $i_t, \mathcal{R}_{c,t}, \mathcal{R}_{g,t}, \overline{m}_{c,t}, \overline{m}_{g,t}, q_{c,t}, q_{g,t}, \Pi_{c,t}, \Pi_{g,t}, \widetilde{\Lambda}_{t,t+1}\}$ solving the following set of conditions:

1. Household Euler equation:

$$c_t^{-1} = \beta(1 + i_t)\mathbb{E}_t[c_{t+1}^{-1}]$$

2. Household labor supply:

$$w_t c_t^{-1} = \omega_L l_t^{\gamma_L}$$

3. Demand for composite intermediate good:

$$\theta y_t = p_{z,t} z_t$$

4. Demand for labor:

$$(1 - \theta)y_t = w_t l_t$$

5. Final good production:

$$y_t = (1 - \mathcal{P}_t)A_t z_t^{\theta} l_t^{1-\theta}$$

6. Demand for green intermediate good:

$$\nu p_{z,t} z_t = z_{g,t} p_{g,t}$$

7. Demand for conventional intermediate good:

$$(1 - \nu)p_{z,t} z_t = z_{c,t} p_{c,t}$$

8. Composite intermediate good:

$$z_t = z_{g,t}^{\nu} z_{c,t}^{1-\nu}$$

9. Default threshold - conventional firm:

$$\overline{m}_{c,t} = \frac{s b_{c,t}}{(1 - \chi_c)p_{c,t} z_{c,t}}$$

10. Default threshold - green firm:

$$\overline{m}_{g,t} = \frac{s b_{g,t}}{(1-\chi_g) p_{g,t} z_{g,t}}$$

11. Conventional bond payoff:

$$\mathcal{R}_{c,t} = s\left[\frac{G(\overline{m}_{c,t})}{\overline{m}_{c,t}} + 1 - F(\overline{m}_{c,t})\right] + (1-s) q_{c,t} - F(\overline{m}_{c,t})\varphi$$

12. Green bond payoff:

$$\mathcal{R}_{g,t} = s\left[\frac{G(\overline{m}_{g,t})}{\overline{m}_{g,t}} + 1 - F(\overline{m}_{g,t})\right] + (1-s) q_{g,t} - F(\overline{m}_{g,t})\varphi$$

13. Conventional firm FOC on bonds:

$$\frac{\partial q(\overline{m}_{c,t+1})}{\partial b_{c,t+1}}\Big(b_{c,t+1} - (1-s) b_{c,t}\Big) + q(\overline{m}_{c,t+1}) = \mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\Big(s(1-F(\overline{m}_{c,t+1})) + (1-s) q(\overline{m}_{c,t+2})\Big)\right]$$

14. Green firm FOC on bonds:

$$\frac{\partial q(\overline{m}_{g,t+1})}{\partial b_{g,t+1}}\Big(b_{g,t+1} - (1-s) b_{g,t}\Big) + q(\overline{m}_{g,t+1}) = \mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\Big(s(1-F(\overline{m}_{g,t+1})) + (1-s) q(\overline{m}_{g,t+2})\Big)\right]$$

15. Conventional firm FOC on capital:

$$1 = \frac{\partial q(\overline{m}_{c,t+1})}{\partial k_{c,t+1}}\Big(b_{c,t+1} - (1-s) b_{c,t}\Big) + \mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\Big((1-\delta_k) + (1-\chi_c) p_{c,t+1}\big(1 - G(\overline{m}_{c,t+1})\big)\Big)\right]$$

16. Green firm FOC on capital:

$$1 = \frac{\partial q(\overline{m}_{g,t+1})}{\partial k_{g,t+1}}\Big(b_{g,t+1} - (1-s) b_{g,t}\Big) + \mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\Big((1-\delta_k) + (1-\chi_g) p_{g,t+1}\big(1 - G(\overline{m}_{g,t+1})\big)\Big)\right]$$

17. Conventional production:

$$z_{c,t} = k_{c,t}$$

18. Green production:

$$z_{g,t} = k_{g,t}$$

19. Law of motion of conventional capital:

$$k_{c,t+1} = (1-\delta_k) k_{c,t} + i_{c,t}$$

20. Law of motion of green capital:

$$k_{g,t+1} = (1-\delta_k)k_{g,t} + i_{g,t}$$

21. Conventional firm dividends:

$$\Pi_{c,t} = (1-\chi_c)(1-G(\overline{m}_{c,t}))p_{c,t}z_{c,t} - sb_{c,t}[1-F(\overline{m}_{c,t})] - i_{c,t} + q_{c,t}(b_{c,t+1} - (1-s)b_{c,t})$$

22. Green firm dividends:

$$\Pi_{g,t} = (1-\chi_g)(1-G(\overline{m}_{g,t}))p_{g,t}z_{g,t} - sb_{g,t}[1-F(\overline{m}_{g,t})] - i_{g,t} + q_{g,t}(b_{g,t+1} - (1-s)b_{g,t})$$

23. Firm owner's SDF:

$$\widetilde{\Lambda}_{t,t+1} = \widetilde{\beta}\mathbb{E}_t\left[\frac{(\Pi_{c,t+1} + \Pi_{g,t+1})^{-1}}{(\Pi_{c,t} + \Pi_{g,t})^{-1}}\right]$$

24. Pollution:

$$\mathcal{Z}_t = \delta_z \mathcal{Z}_{t-1} + z_{c,t}$$

25. Pollution damage:

$$\mathcal{P}_t = 1 - \exp\{-\gamma_P \mathcal{Z}_t\}$$

26. Collateral value of bank portfolio:

$$\bar{b}_{t+1} = (1-\phi_c)q_{c,t}b_{c,t+1} + (1-\phi_g)q_{g,t}b_{g,t+1}$$

27. Price of conventional bond:

$$q_{c,t} = \frac{\mathbb{E}_t[\mathcal{R}_{c,t+1}]}{(1+i_t)\left[1 + (1-\phi_c)\Omega_{\bar{b},t}\right]}$$

28. Price of green bond:

$$q_{g,t} = \frac{\mathbb{E}_t[\mathcal{R}_{g,t+1}]}{(1+i_t)\left[1 + (1-\phi_g)\Omega_{\bar{b},t}\right]}$$

29. Bank's balance sheet constraint:

$$(1+i_t)d_{t+1} = \mathbb{E}_t[\mathcal{R}_{c,t+1}]b_{c,t+1} + \mathbb{E}_t[\mathcal{R}_{g,t+1}]b_{g,t+1}$$

30. Market clearing:

$$y_t = c_t + \Pi_{c,t+1} + \Pi_{g,t+1} + \varphi[b_{c,t}F(\overline{m}_{c,t}) + b_{g,t}F(\overline{m}_{g,t})] + \Lambda(\overline{F}_t) + \Omega(\bar{b}_{t+1})$$

Targeted Moments. All codes used to solve and simulate the baseline model and its extensions are available online.[21] Table 3.C.1 summarizes the targeted moments in our calibration (Table 3.1).

Table 3.C.1: Targeted Moments

Moment	**Data**	**Model**
Labor supply	0.3	0.305
Damage/GDP	0.1	0.105
Capital/GDP	2.1	2.13
Debt/GDP	0.8	0.83
Bond spread	100bp	99bp
Leverage	0.4	0.39
Eligibility premium	-11bp	-11bp

---

[21] For the quantitative analysis, we rely on Matlab 2021a and Dynare 4.6.4 (see Adjemian et al., 2021).

3.D Data Sources

Table 3.D.1 summarizes the data sources on which our empirical analysis and calibration are based. The classification of bonds as "green" is based on publicly available lists of securities traded via various stock exchanges. Based on the list of ISINs, we retrieve bond-specific info from *Datastream*. Data on conventional bonds in the control group is taken from *IHS Markit*. EURIBOR data are also obtained through *Datastream*. We use the ECB to obtain data on non-financial firm debt, GDP, employment, gross fixed capital formation, private consumption, and the GDP-deflator.

Table 3.D.1: Data Sources and Ticker

Series	**Source**	**Mnemonic**
Green Bond List I	Euronext	List retrieved Nov-30-2020
Green Bond List II	Frankfurt SE	List retrieved Nov-30-2020
Green Bond List III	Vienna SE	List retrieved Nov-30-2020
Constant Maturity Ask Price	Datastream	CMPA
Constant Maturity Bid Price	Datastream	CMPB
Coupon	Datastream	C
Issue Date	Datastream	ID
Amount Outstanding	Datastream	AOS
Currency	Datastream	PCUR
Life At Issue	Datastream	LFIS
Redemption Date	Datastream	RD
EURIBOR rates (... = maturity)	Datastream	TRE6S...Y
Debt-to-GDP	ECB	QSA.Q.N.I8.W0.S11.S1.C.L.LE.F3T4.T._Z.XDC_R_B1GQ_CY._T.S.V.N._T
Markit iBoxx Components	IHS Markit	-
GDP	ECB	MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.EUR.V.N
Gross fixed capital formation	ECB	MNA.Q.Y.I8.W0.S1.S1.D.P51G.N11G._T._Z.EUR.V.N
Consumption	ECB	MNA.Q.Y.I8.W0.S1M.S1.D.P31._Z._Z._T.EUR.V.N
GDP Deflator	ECB	MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.IX.D.N
Employment	ECB	ENA.Q.Y.I8.W2.S1.S1._Z.EMP._Z._T._Z.PS._Z.N

3.E Additional Numerical Results

3.E.1 The Role of the Green-Conventional Substitution Elasticity

In this section, we provide a robustness check regarding the production technology of wholesale goods producers. By assuming a Cobb-Douglas production function in (3.6), we implicitly assume an elasticity of substitution of one between green and conventional intermediate goods. When strictly interpreting green and conventional firms as energy producers, this elasticity is usually estimated to be larger than one. Therefore, we repeat our policy analysis when replacing the final good producers' technology by a nested CES-function

$$y_t = (1 - \mathcal{P}_t)\, A_t \left(\nu z_{g,t}^{\frac{\epsilon_\nu - 1}{\epsilon_\nu}} + (1-\nu) z_{c,t}^{\frac{\epsilon_\nu - 1}{\epsilon_\nu}} \right)^{\frac{\theta \epsilon_\nu}{\epsilon_\nu - 1}} l_t^{1-\theta} z_t = , \tag{3.E.1}$$

and set the elasticity of substitution $\epsilon_\nu = 1.6$, which has been reported in Popp (2004) and Papageorgiou et al. (2017). The parameter ν is set to keep the green production share at 20%, consistent with the baseline. To ensure an apples-to-apples comparison with the baseline model, we re-calibrate the idiosyncratic productivity variance to $\varsigma_M = 0.195$, the externality parameter to $\gamma_P = 6 \times 10^{-5}$, the slope parameter in the collateral default cost function to $\eta_1 = 0.0352$, and the slope parameter in the liquidity management cost function to $l_1 = 0.0065$.

Table 3.E.1: Greenium Reaction - Announcement Effects with CES-Production

Data	**Model: News Shock Horizon**			
	2 years	3 years	4 years	5 years
-4.8bp	-7.6bp	-5.4bp	-4.1bp	-3.2bp

Results are shown in table 3.E.3. The optimal tax is much higher and optimal collateral policy implies a much larger degree of preferential treatment. Intuitively, when conventional and green intermediate goods are easier to substitute, any reduction in the size of conventional firms is less costly in terms of final good production, irrespective of whether the reduction is induced by carbon taxes or by preferential treatment. In contrast, the effect of collateral policy on bond prices, leverage, and investment is similar to the baseline calibration (see Table 3.E.1 and Table 3.E.2). Put differently, the side effects of preferential treatment are hardly affected by the green-conventional elasticity of substitution. Therefore, the relative welfare gains of optimal taxation still exceed the gains of optimal collateral policy by a very similar factor as in the baseline case of a Cobb-Douglas production function.

Table 3.E.2: Firm Reaction: Model vs. Data with CES-Production

	Δ Yield	**Δ Capital**	**Δ Leverage**
Model	81bp	5.2pp	6.3pp (market value) 3.0pp (book value)
Data	39 - 85bp	1 - 15pp	1 - 11pp

Notes: Difference between baseline of 26% to 100% haircut in the first row. Range of estimated effects in the literature in the second row.

3.E.2 The Role of Nominal Rigidities

In this section, we add nominal rigidities to the model following the standard New Keynesian model. In particular, bonds are assumed to be denominated in nominal terms, i.e., inflation has a direct effect on corporate bonds and the supply side. Households consume a final goods basket c_t given by

$$c_t = \left(\int_0^1 c_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} ,$$

where $\varepsilon > 1$ is the elasticity of substitution among the differentiated final goods. The demand schedule for final good i is given by

$$c_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} c_t \, , \tag{3.E.2}$$

where P_t denotes the CES price index for the final consumption bundle. Final good firms sell their differentiated good with a markup over their marginal costs. However, the price of firm j, $P_{j,t}$, can only be varied by paying a quadratic adjustment cost à la Rotemberg (1982) that is proportional to the nominal value of aggregate production, $P_t y_t$. Firm j's marginal costs are denoted by $\mathrm{mc}_{j,t} \equiv \partial \mathcal{C}_t^W / \partial y_{j,t}$, where the wholesale firm's cost minimization problem is given by

$$\mathcal{C}_t^W(y_{j,t}) = \min_{z_{j,t}, l_{j,t}} \quad P_{z,t} z_{j,t} + W_t l_{j,t} \quad \text{s.t.} \quad y_{j,t} = (1 - \mathcal{P}_t) \, A_t z_{j,t}^{\theta} l_{j,t}^{1-\theta} \, ,$$

and $P_{z,t}$ is the price of the wholesale good. From the minimization problem we obtain *real* marginal costs

$$\mathrm{mc}_t = \frac{1}{(1 - \mathcal{P}_t) \, A_t} \left(\frac{p_{z,t}}{\theta} \right)^{\theta} \left(\frac{w_t}{1-\theta} \right)^{1-\theta} ,$$

Table 3.E.3: Time Series Means for Different Policies with CES-Production

Moment	**Strong Pref**	**Opt Coll**	**Opt Tax**
Tax χ_c	0	0	12%
Conventional Haircut ϕ_c	26%	32%	26%
Green Haircut ϕ_g	4.5%	4%	26%
Welfare (CE, Change)	+0.012%	+0.015%	+1.306%
Conv. Elig. Premium	-10bp	-10bp	-11bp
Green Elig. Premium	-13bp	-14bp	-11bp
Conv. Bond Spread	99bp	102bp	98bp
Green Bond Spread	84bp	81bp	98bp
Conv. Leverage (Change)	-0.1%	-0.3%	0%
Green Leverage (Change)	+0.9%	+1.1%	0%
Conv. Capital (Change)	-0.1%	-0.2%	-16.2%
Green Capital (Change)	+0.6%	+0.8%	+67.9%
Green Capital Share	0.2060	0.2065	0.3406
GDP (Change)	+0.04%	+0.03%	+1.06%
Default Cost/GDP (Change)	+2.45%	+0.07%	-0.41%
Liq. Man. Cost/GDP (Change)	-1.39%	+0.05%	-1.28%
Pollution Cost/GDP (Change)	-0.06%	-0.16%	-15.47%

Notes: Strong preferential treatment (*Strong Pref*) is based on a collateral framework set to $\phi_g = 0.045$ and $\phi_c = 0.26$. The optimal collateral policy (*Opt Coll*) is computed holding $\chi_g = 0$ constant. For the optimal tax (*Opt Tax*), we hold haircuts fixed at their baseline values and vary the tax rate. *Change* refers to percentage differences from the baseline calibration. Default cost are the sum of collateral default cost (3.14) and aggregate restructuring cost $\varphi(\nu F_{g,t} + (1-\nu)F_{c,t})$. The baseline green capital share of 0.2 is a calibration target.

where $p_{z,t} = P_{z,t}/P_t$ is the relative price of the wholesale good and w_t is the real wage. Hence, total nominal profits of firm j in period t are given by

$$\widehat{\Pi}_{j,t} = (P_{j,t} - \mathrm{mc}_t P_t)\, y_{j,t} - \frac{\psi}{2}\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)^2 P_t y_t \ ,$$

where ψ measures the degree of the nominal rigidity. Each wholesale good firm j maximizes the expected sum of discounted profits

$$\max_{P_{j,t+s}, y_{j,t+s}} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \frac{c_{t+s}^{-1}/P_{t+s}}{c_t^{-1}/P_t}\, \widehat{\Pi}_{j,t+s}\right] \ ,$$

subject to the demand schedule (3.E.2). Plugging in the demand function yields the first order condition

$$\left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} Y_t - \varepsilon\left(P_{j,t} - \mathrm{mc}_t P_t\right)\left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} \frac{y_t}{P_t} - \psi\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)\frac{P_t}{P_{j,t-1}} y_t$$

$$+\mathbb{E}_t\left[\frac{c_{t+1}^{-1}/P_{t+1}}{c_t^{-1}/P_t}\psi\left(\frac{P_{j,t+1}}{P_{j,t}}-1\right)\frac{P_{j,t+1}}{P_{j,t}^2}P_{t+1}y_{t+1}\right]=0\ .$$

In a symmetric price equilibrium, $P_{j,t}=P_t$ for all j. Using this, we rearrange and get

$$(1-\varepsilon(1-\mathrm{mc}_t))\,y_t+\mathbb{E}_t\left[\beta\frac{c_{t+1}^{-1}/P_{t+1}}{c_t^{-1}/P_t}y_{t+1}\pi_{t+1}\psi\left(\pi_{t+1}-1\right)\pi_{t+1}\right]=\psi\left(\pi_t-1\right)\pi_t y_t\ ,$$

where $\pi_t=\frac{P_t}{P_{t-1}}$. Dividing both sides by y_t and Ψ we arrive at the New Keynesian Phillips Curve

$$\mathbb{E}_t\left[\beta\frac{c_{t+1}^{-1}/P_{t+1}}{c_t^{-1}/P_t}\frac{y_{t+1}\pi_{t+1}}{y_t}\left(\pi_{t+1}-1\right)\pi_{t+1}\right]+\frac{\varepsilon}{\psi}\left(\mathrm{mc}_t-\frac{\varepsilon-1}{\varepsilon}\right)=\left(\pi_t-1\right)\pi_t\ .$$

In addition, nominal rigidities also affect intermediate good firms, since inflation affects the default threshold $\overline{m}_{\tau,t+1}\equiv\frac{sb_{\tau,t+1}}{\pi_{t+1}(1-\chi_\tau)p_{\tau,t+1}k_{\tau,t+1}}$ and the real per-unit bond payoff is

$$\mathcal{R}_{\tau,t}=s\Bigg(G(\overline{m}_{\tau,t})\frac{\pi_t p_{\tau,t}(1-\chi_\tau)k_{\tau,t}}{sb_{\tau,t}}+1-F(\overline{m}_{\tau,t})\Bigg)-F(\overline{m}_{\tau,t})\varphi+(1-s)q_{\tau,t}\ .$$

Their first order conditions are now given by

$$\begin{aligned}&q'(\overline{m}_{\tau,t+1})\frac{\mathbb{E}_t[\overline{m}_{\tau,t+1}]}{b_{\tau,t+1}}\left(b_{\tau,t+1}-(1-s)\frac{b_{\tau,t}}{\pi_t}\right)+q(\overline{m}_{\tau,t+1})\\&\quad=\widetilde{\beta}\mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\frac{s(1-F(\overline{m}_{\tau,t+1}))+(1-s)q_{\tau,t+1}}{\pi_{t+1}}\right]\end{aligned}$$

and

$$\begin{aligned}1=&-q'(\overline{m}_{\tau,t+1})\frac{\mathbb{E}_t[\overline{m}_{\tau,t+1}]}{k_{\tau,t+1}}\left(b_{\tau,t+1}-(1-s)\frac{b_{\tau,t}}{\pi_t}\right)\\&+\mathbb{E}_t\left[\widetilde{\Lambda}_{t,t+1}\Big((1-\delta_k)+(1-\chi_\tau)p_{\tau,t+1}\big(1-G(\overline{m}_{\tau,t+1})\big)\Big)\right]\ .\end{aligned}$$

The resource constraint now also includes Rotemberg cost

$$y_t=c_t+\sum_\tau\left(c_{\tau,t}+i_{\tau,t}\right)+\Lambda(\overline{F}_{t+1})+\Omega(\overline{b}_{t+1})+\frac{\psi}{2}\left(\pi_t-1\right)^2y_t+\sum_\tau\varphi F(\overline{m}_{\tau,t})\frac{b_{\tau,t}}{\pi_t}\ .$$

To close the model, we assume that the central bank sets i_t according to a Taylor rule

$$i_t=i\pi_t^{\phi_\pi}\ . \tag{3.E.3}$$

We choose standard parameters for the final goods elasticity $\epsilon=6$, implying a markup of 20% in the deterministic steady state, and a Rotemberg parameter $\psi=57.8$, consistent with a Calvo parameter of 0.75. The parameter on inflation stabilization in the monetary policy rule is set to $\phi_\pi = 5$, which ensures determinacy for all policy experiments. We slightly re-calibrate the slope parameter $\eta_1 = 0.0371$ in the collateral default cost function, the slope parameter $l_1 = 0.0065$ in the liquidity management cost function, the capital depreciation rate $\delta_k = 0.0175$, and the idiosyncratic shock volatility $\varsigma_M = 0.16$. The relationship between haircuts, bond prices, and firms' financing and investment decision is largely unaffected by the presence of nominal rigidities (see Table 3.E.4 and Table 3.E.5).

Table 3.E.4: Greenium Reaction - Announcement Effects with Nominal Rigidities

Data	**Model: News Shock Horizon**			
	2 years	3 years	4 years	5 years
-4.8bp	-9.1bp	-6.6bp	-5.1bp	-4.1bp

Table 3.E.5: Firm Reaction: Model vs. Data with Nominal Rigidities

	Δ Yield	**Δ Capital**	**Δ Leverage**
Model	72bp	4.1pp	5.2pp (market value) 2.1pp (book value)
Data	39 - 85bp	1 - 15pp	1 - 11pp

Notes: Difference between baseline of 26% to 100% haircut in the first row. Range of estimated effects in the literature in the second row.

Results are reported in table 3.E.6 and show very similar implications for optimal collateral policy and its interaction with carbon taxation. In particular, the inflation volatility (measured by Rotemberg cost) under optimal preferential treatment is almost unchanged with respect to the baseline in column one, alleviating concerns that preferential treatment jeopardizes price stability, the central bank's primary policy objective. As before, the welfare gain of optimal taxation exceeds the welfare gain by a factor of almost 100.

Table 3.E.6: Time Series Means for Different Policies with Nominal Rigidities

Moment	**Strong Pref**	**Opt Coll**	**Opt Tax**
Tax χ_c	0	0	12%
Conventional Haircut ϕ_c	26%	30%	26%
Green Haircut ϕ_g	4.5%	10%	26%
Welfare (CE, Change)	+0.019%	+0.021%	+1.483%
Conv. Elig. Premium	-11bp	-11bp	-11bp
Green Elig. Premium	-14bp	-14bp	-11bp
Conv. Bond Spread	102bp	104bp	100bp
Green Bond Spread	82bp	80bp	100bp
Conv. Leverage (Change)	-0.1%	-0.3%	0%
Green Leverage (Change)	+1.1%	+1.2%	0%
Conv. Capital (Change)	0.0%	-0.1%	-9.6%
Green Capital (Change)	+0.5%	+0.6%	+41.3%
Green Capital Share	0.2009	0.2011	0.281
GDP (Change)	+0.04%	+0.03%	+0.62%
Default Cost/GDP (Change)	+2.45%	+0.82%	-0.25%
Liq. Man. Cost/GDP (Change)	-1.31%	-0.38%	-0.74%
Rotemberg Cost/GDP (Change)	+0.11%	-0.14%	+0.53%
Pollution Cost/GDP (Change)	-0.02%	-0.09%	-9.10%

Notes: Strong preferential treatment (*Strong Pref*) is based on a collateral framework set to $\phi_g = 0.045$ and $\phi_c = 0.26$. The optimal collateral policy (*Opt Coll*) is computed holding $\chi_g = 0$ constant. For the optimal tax (*Opt Tax*), we hold haircuts fixed at their baseline values and vary the tax rate. *Change* refers to percentage differences from the baseline calibration. Default cost are the sum of collateral default cost (3.14) and aggregate restructuring cost $\varphi(\nu F_{g,t} + (1-\nu)F_{c,t})$. The baseline green capital share of 0.2 is a calibration target.

3.F Yield Reaction to Central Bank Policy Announcements

Construction of the Dataset. The first step of our analysis is to identify a list of relevant pieces of ECB communication with significant space devoted to climate policy. To identify relevant speeches for our empirical analysis, we rely on a dataset published by the ECB that contains date, title (including sub-titles), speaker, content, and footnotes of nearly all speeches by presidents and board members since 1999 (see European Central Bank, 2021b). We perform the following steps:

- We string-match titles and content separately for the following keywords: climate, green, sustainable, greenhouse, environment, warming, climatic, carbon, coal.
- We designate a speech for manual inspection as soon as we have one match for a title or three matches for content (variations did not change results).
- We exclude a speech if insufficient space is devoted to the topic, there is no monetary policy relation, or for a wrong positive (e.g., *environment* refers to low interest rates).
- We exclude speeches that address climate risk or transition risk.
- Speeches within 20 trading days of the previous speech are excluded to avoid overlapping treatment periods.

We exclude communication that refers to *climate risk* and *transition risk*, since these refer to improving disclosure standards, the extent to which climate risk should be considered in credit risk assessment, and asset stranding. These issues are potentially important for the conduct of central bank policy in general, but neither specifically address bond markets nor refer to an active ECB climate policy. This leaves us with four speeches. Table 3.F.1 contains details regarding the key content that motivates our classification.

The classification of securities into ”green” and ”conventional” is based on bonds listed in the ”ESG” segments of *Euronext*, the *Frankfurt Stock Exchange* and the *Vienna Stock Exchange*, all of which offer publicly available lists. We limit the analysis to bonds classified as ”green” or ”sustainable”. Since many green bonds are not part of the *IHS Markit* database, we additionally obtain data from *Thomson Reuters Datastream.* We match green and conventional bonds one trading-day before each announcement date using a nearest-neighbors procedure involving coupon, bid-ask spread, maturity, notional amount, and yield spreads. Specifically, we identify an appropriate untreated bond as control group, which is the conventional bond with the smallest distance to the green bond. We drop a green bond if the distance to the closest conventional bond is too high. Table 3.F.2 contains summary statistics regarding the matching. Coupon and bid-ask spreads are very similar for both types of bonds. Spreads of green bonds are higher by between 5 and 8bp, while their maturity is higher by 1.5 years on average.

Table 3.F.1: Relevant European Central Bank Policy Announcements

Date	Person	Link	Relevant Quotes
08-11-2018	Benoît Cœuré	ECB	• (. . .) the ECB, acting within its mandate, can – and should – actively support the transition to a low carbon economy (. . .) second, by acting accordingly, without prejudice to price stability. • Purchasing green bonds (. . .) could be an option, as long as the markets are deep and liquid enough.
27-02-2020	Christine Lagarde	ECB	• (. . .) reviewing the extent to which climate-related risks are understood and priced by the market (. . .) • (. . .) evaluate the implications for our own management of risk, in particular through our collateral framework.
17-07-2020	Isabel Schnabel	ECB	• (. . .) way in which we can contribute is by taking climate considerations into account when designing and implementing our monetary policy operations. • (. . .) Of course, central banks would need to be mindful of their effects on market functioning. • (. . .) severe risks to price stability, central banks are required, within their traditional mandates, to strengthen their efforts (. . .)
21-09-2020	Christine Lagarde	ECB	• We cannot miss this opportunity to reduce and prevent climate risks and finance the necessary green transition. • The ECB's ongoing strategy review will ensure that its monetary policy strategy is fit for purpose (. . .) • (. . .) Jean Monnet's words, (. . .) opportunity for Europe to take a step towards the forms of organisation of the world of tomorrow.

Notes: Speeches are taken from European Central Bank (2021b).

Yield Reactions. Table 3.F.3 gives details on the greenium reaction after each speech. The strongest effect is visible for ECB president Christine Lagarde's speech on February 27*th* 2020, which included the first explicit reference to the ECB's collateral framework. Moreover, the speech delivered by Isabel Schnabel on July 17*th* 2020 stands out, since yields on green bonds significantly increased compared to their conventional counterparts following the event. However, the tone regarding ECB climate policy is much more modest than in other speeches.[22] There is also no explicit prospect of preferential treatment given in this speech.

In figure 3.F.1, we display the average response across treatment dates. After two trading days, we observe a significantly negative greenium reaction, which flattens out only after 15 trading days and widens to around 11bp after 20 trading days.

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[22] For example, central banks "*need to be mindful of their effects on market functioning*" and are required to exert effort towards climate concerns only "*within their traditional mandates*". See also table 3.F.1.

Table 3.F.2: Matching Green to Conventional Bonds: Summary Statistics

Date		BA-Spread		Coupon		Spread		Maturity		Amount	
	#	Green	Conv.	Green	Conv.	Green	Conv.	Green	Conv.	Green	Conv.
08-11-2018	80	0.34	0.33	1.08	1.05	47.50	42.20	7.6	6.0	716	719
27-02-2020	83	0.36	0.32	1.18	1.15	51.66	44.82	6.7	5.2	695	690
17-07-2020	77	0.45	0.38	1.22	1.22	77.49	72.00	6.6	4.9	693	689
21-09-2020	79	0.38	0.36	1.18	1.14	64.94	56.68	6.3	4.6	701	709

Notes: We denote the number of matches by #. Bond yield spreads over the Euribor/Swap are expressed in basis points. Bid-ask spread and coupon are relative to a face value of 100, maturity is in years. Amount outstanding is in million EUR.

Table 3.F.3: Yield Reaction Around European Central Bank Policy Announcements

Date	Type	Yield Reaction	Standard Error
08-11-2018	Board Member Speech	-7.9***	1.78
27-02-2020	President Speech	-19.4***	3.89
17-07-2020	Board Member Speech	6.8***	1.67
21-09-2020	President Speech	1.3	1.23

Notes: We display the average yield over 20 days after minus average yield over 20 trading day before the policy announcement , relative to the matched control group (in basis points). Significance levels correspond to 10 % (*), 1 % (**) and 0.1 % (***) of Welch's t-test.

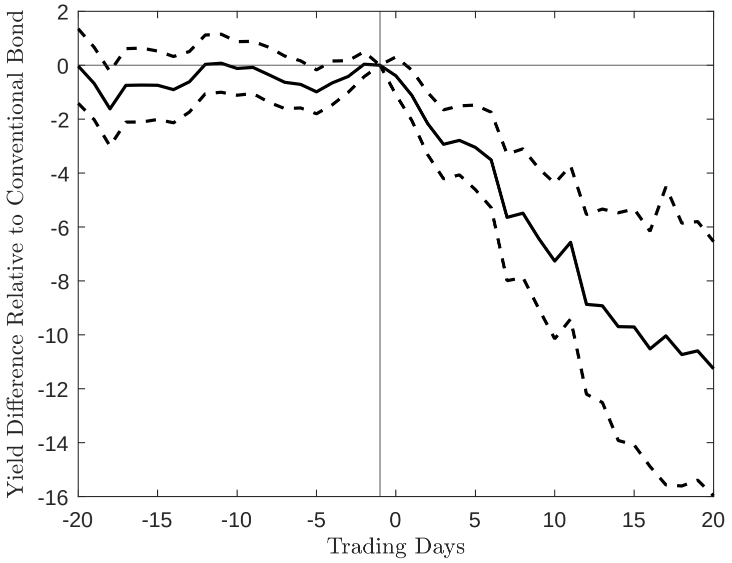

Figure 3.F.1: Average Yield Reaction around Treatment Window

Notes: Results are averaged over all policy announcements. Dashed lines represent 95% confidence intervals. All values in basis points.

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List of Applied Software

- Matlab (Version 2021a): Used to generate the quantitative and empirical analysis in Chapter 1 and Chapter 3.
- Julia (Version 1.6.2): Used to generate the quantitative results in Chapter 2.
- Dynare (Version 4.6.4): Used to simulate the quantitative model in Chapter 3.
- R: Used to perform the empirical analysis in Chapter 3.