Essays on Financial and Real Estate Markets

Dissertation

submitted in fulfillment of the requirements for the degree of
Philosophiae Doctor (PhD)

of the Faculty of Management, Economics, and Social Sciences
of the University of Cologne

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October 2024

First reviewer: Prof. Dr. Andreas Schabert

Second reviewer: Prof. Dr. Tom Zimmermann

Date of oral defense:

Acknowledgement

Writing a dissertation gives one the opportunity to satisfy their curiosity and, in doing so, contribute to academic research. Through this goal, the work on the dissertation allows, but also requires, one to engage with topics in great detail and for extended periods of time. I greatly enjoyed this opportunity, even though it was very challenging, and it helped me develop significantly on a personal level. I would like to thank the people who accompanied me along this journey.

First and foremost, I would like to thank my two supervisors, Andreas Schabert and Tom Zimmermann. I consider myself very fortunate to have had you as supervisors. Your courses were stimulating and encouraged me to think about research questions. Furthermore, you were always accessible and gave me the opportunity to seek your advice at any time. You always found a good balance between support and challenge.

I would also like to thank my co-authors and friends Florian Schuster and Marco Wysietzki, with whom I wrote the first chapter. I believe that working so intensively together on a topic that interested all of us equally made the start of our PhD journey easier. I admire your mathematical skills, from which I have learned a great deal.

Additionally, I would especially like to thank my co-author and friend Francisco Amaral. I am very grateful to you for including me in your research project, which is now known as the German Real Estate Index (Greix). I am truly proud of what we have achieved so far with this project and look forward to more exciting research projects with you. In this context, I would also like to thank Moritz Schularick, who entrusted me with significant responsibility within this research project and, in doing so, had a substantial positive influence on my dissertation.

Furthermore, I would like to thank my other co-authors Martin Kornejew, Mark Toth, and Steffen Zetzmann. It is great to work with Martin and learn from his quick grasp of concepts. It was a pleasure working with Mark and learning about spatial models. It is a joy to conduct research with Steffen, now also a part of Greix, due to his reliability and detailed understanding.

I would also like to thank everyone from whom I received feedback during personal conversations or seminars at the Center for Macroeconomic Research – thanks to Martin

Barbie, Peter Funk, Erik Hornung, Michael Krause, Max Löffler, and Sebastian Siegloch, among others. Additionally, I would like to thank Helge Braun and Julia Fath for their valuable advice during my teaching. I would also like to thank ECONtribute, the cluster of excellence, my PhD colleagues from the Young ECONtribute Program and CMR, as well as the entire staff of CMR, especially Ina Dinstühler, Eva Karnagel-Meinhardt, Yvonne Havertz, Sylvia Hoffmeyer, and Kristin Winnefeld, for helping me navigate all the administrative hurdles.

A big and very important thank you goes to my family and friends. In particular, I would like to thank my parents and my brother Lukas. My parents supported me unconditionally at all times, and for that, I am deeply grateful. Lukas, I am grateful to you for always listening to me when I talked about my research and for providing me with practical insights.

Last, but definitely not least, I would like to thank Henrike Kannen. Without exaggeration, I think I can say that I would not be writing these sentences without you, as you encouraged me to apply for the PhD program at the University of Cologne. But more importantly, you have always supported me during this time, whether things were going well or poorly. I am infinitely grateful to you for your understanding and love during this stressful period.

I wish to acknowledge support from the Deutsche Forschungsgemeinschaft (DFG) under Germany's Excellence Strategy (EXC 2126/1 – 390838866).

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Introduction

This dissertation generally addresses financial markets, with a particular focus on real estate markets. Financial markets are central to all economic activity, and in the real estate market, practically everyone participates, whether as a renter or owner. Real estate is typically the largest investment most people make in their lifetime, and it is often directly connected to financial markets since the property serves as collateral for the mortgage. In 2007, with the onset of the Great Financial Crisis, attention was especially focused on this link between real estate and financial markets, as the housing crisis in the United States triggered a global financial crisis. This event, including the preceding extreme housing boom and subsequent bust, initiated extensive research on both sides of the issue. On one hand, research focused on how collateralized borrowing, particularly related to housing and leverage, can trigger a crisis but also explain its mechanisms (Bianchi, 2011; Schularick and Taylor, 2012; Dávila and Korinek, 2018). While on the other hand, the literature has increasingly examined housing as a financial asset, whose value fluctuations play a role in both business cycles and monetary policy transmission (Del Negro and Otrok, 2007; Mian and Sufi, 2011; Kaplan et al., 2018; Cloyne et al., 2020). The models developed in these research papers are very successful in offering results for mostly aggregate or national questions, but they sometimes dismiss the heterogeneity of agents or assets, particularly the housing asset. Therefore, I tried in my research to integrate heterogeneity in financial and real estate markets.

Hence, each chapter deals with a specific aspect of heterogeneity of financial or real estate markets. The first chapter addresses collateralized lending—not necessarily related to housing—and differing expectations among agents regarding the future payouts of assets, and how this can change crises and their mechanisms. Chapter 2 focuses on real estate as a financial asset and investigates how liquidity depends on the heterogeneity of real estate, measured by its location. Chapter 3 explores the interaction between monetary policy and its transmission through real estate markets. The focus is on the heterogeneous impact of monetary policy on house prices across geographic locations.

How Heterogeneous Beliefs Trigger Financial Crises. Chapter 1 which is joint work with Florian Schuster and Marco Wysietzki combines multiple strands of literature that have examined the Great Financial Crisis. We develop a theoretical model that characterizes how the heterogeneity of economic agents' beliefs about future payoffs interacts with financial crises.

On one hand, it has been identified that heterogeneous beliefs were a key factor in the emergence of the Great Financial Crisis (Cheng et al., 2014; Gennaioli and Shleifer, 2018; Mian and Sufi, 2022). On the other hand, there is an extensive literature addressing the links between borrowing, investment, and financial crises. A well-established result in this context is that macroprudential policy has an efficiency-enhancing effect (Geanakoplos and Polemarchakis, 1986; Bianchi, 2011; Dávila and Korinek, 2018; Jeanne and Korinek, 2020).

These models typically assume a representative agent, whereas optimal macroprudential regulation in the presence of heterogeneous agents would need to address their individual contributions. Therefore, a model is required to measure these individual contributions.

We fill this gap by extending established models of collateralized, price-dependent borrowing to incorporate heterogeneous beliefs, allowing us to analyze the mechanism of financial distress. Due to belief heterogeneity, agents vary in the risks associated with their financial decisions. Since these individual financial decisions are observable, we can derive type-specific macroprudential regulation.

Our main result is that heterogeneous beliefs increase both the likelihood and extent of a financial crisis. Compared to homogeneous beliefs, less severe macroeconomic shocks are required for a crisis to occur under heterogeneous beliefs. Optimistic agents contribute more to financial instability than pessimistic ones, leading to an asymmetrical impact on financial distress.

From a policy perspective, we demonstrate that corrective interventions are generally welfare-enhancing, but the inclusion of type-specific policies further improves efficiency. We propose a system of non-linear, type-specific macroprudential taxes that outperforms the linear taxes typically suggested in the literature.

Urban Spatial Distribution of Housing Liquidity. Chapter 2 of my dissertation, which is joint work with Francisco Amaral and Mark Toth, examines how location, liquidity, and prices interact in housing markets.

Housing markets are typically modeled using spatial equilibrium frameworks, as national factors alone cannot explain regional variations (Alonso, 1964; Roback, 1982). Additionally, housing markets are characterized by information asymmetries and search frictions due to the nature of bilateral transactions (Han and Strange, 2015). Therefore, we analyze how local housing market characteristics interact with the typical constraints of real estate markets.

At the core of our analysis is a newly developed dataset that provides micro-level housing transaction data, including prices and liquidity, for major German cities over the past decade. Empirically, we show not only that the well-established result holds—that prices decrease with distance from the city center—but also that liquidity decreases with distance. Even after controlling for a large set of transaction and property characteristics, apartments closer to the city center spend significantly less time on the market and sell for prices closer to the original asking price. We call this new result the "urban liquidity gradient," which complements the well-established "urban price gradient." The urban liquidity premium can also be linked to liquidity premiums of other financial assets, such as bonds and stocks (Amihud and Mendelson, 1986).

To explain our empirical findings, we construct a spatial search model of housing within a monocentric city. We show qualitatively and quantitatively that increasing travel costs to the city center can simultaneously explain why both prices and liquidity decline with distance from the city center. We can show that spatial liquidity differences due to search frictions account for a quarter of the spatial price gradient. Finally, we conduct a counterfactual analysis, showing that prices in the city center are 5.5% higher than in the periphery due to higher liquidity. Thus, this chapter provides the first estimate of the spatial housing liquidity premium.

Monetary Policy and the Spatially Heterogeneous Response of House Prices. The final chapter of my dissertation, which is joint work with Francisco Amaral, Martin Kornejew, and Steffen Zetzmann, investigates the heterogeneous impact of monetary policy on house prices across geographical locations.

Real estate markets vary significantly across regions in terms of many characteristics (Glaeser et al., 2014; Piazzesi et al., 2020), and local housing markets can respond differently to aggregate macroeconomic shocks, thereby reinforcing regional heterogeneity. Additionally, the valuation of real estate is relevant for both borrowing capacity and consumption-saving decisions (Piazzesi and Schneider, 2016). For this reason, the transmission of monetary policy through real estate markets has been widely studied (Jordà et al., 2015; Garriga et al., 2017; Agarwal et al., 2022), though spatial differences in monetary policy transmission have been largely overlooked.

We document a striking spatial pattern in the response of house prices to monetary policy. We show that housing prices in areas with low rental yields respond more strongly to monetary policy shocks. By embedding high-frequency monetary policy shocks in a state-dependent local projection framework, we demonstrate that the effects are quantitatively significant. Specifically, our results indicate that five years after a contractionary monetary policy shock equivalent to one standard deviation, house prices in the bottom 5% of the rental yield distribution decline more than twice as much as in all other MSAs.

Based on empirical literature that has shown that local housing risk—and therefore local discount rates—are lower in cities with lower rental yields (Demers and Eisfeldt, 2022; Amaral et al., 2023b), we introduce a new mechanism. We propose that a change in national interest rates leads to larger (smaller) *relative* changes in local discount rates where they were initially small (large). The change in the *relative* discount rate, in turn, dictates *relative* price adjustments according to the standard pricing equation for (real estate) assets. Moreover, we are able to show that our results are not confounded by other potential mechanisms.

Personal contribution. All chapters of this dissertation are joint work with co-authors. Individual contributions have been made as follows.

Chapter 1 is joint work with Florian Schuster amd Marco Wysietzki. The research idea, as well as the basic setup of the theoretical framework and formal analysis, was developed together. Florian Schuster provided the first draft and developed most parts of the mathematical derivations underlying section 1.3.3. Marco Wysietzki provided the numerical application of the model. I individually prepared the section *Related literature*. We jointly refined everything.

Chapter 2 is joint work with Francisco Amaral and Mark Toth. Francisco Amaral and I developed the research idea and the empirical framework. Furthermore, we are responsible for gathering the underlying newly developed dataset on German housing transaction data. The algorithm for merging the two underlying datasets was also constructed together by the two of us. The general empirical specification was built by Francisco Amaral and myself, while each of us applied it to different cities. Mark Toth individually developed the theoretical model. Francisco Amaral drafted most of the empirical part, while I concentrated on the section *Robustness Analysis*. Mark Toth drafted the theoretical part. The introduction was written together. We jointly refined everything.

Chapter 3 is joint work with Francisco Amaral, Martin Kornejew, and Steffen Zetzmann. The research idea and the empirical setup were developed together. Steffen Zetzmann drafted the empirical analysis, which was refined collaboratively. I wrote most of the first draft, which was refined together through very close communication.

Chapter 1

How Heterogeneous Beliefs Trigger Financial Crises

by Florian Schuster, Marco Wysietzki, and Jonas Zdrzalek

Abstract

We present a theoretical framework to characterize how belief heterogeneity in financial markets interacts with financial crises. To that end, we embed belief heterogeneity in a financial market model featuring a collateral constraint, which introduces a pecuniary externality. This setup allows us to identify individual contributions to financial distress. The main result is that belief disagreements increase the likelihood and the extent of financial distress. Specifically, crises occur under less severe macroeconomic shocks, and the associated loss of efficiency is larger than in an economy populated by homogeneous individuals. This finding rests upon the fact that economic agents contribute to financial distress asymmetrically, with optimistic agents making larger contributions than pessimistic agents. In terms of policy implications, we show that, while corrective policy interventions are generally efficiency-enhancing, type-specific policies generate additional efficiency gains. Against this background, we propose a system of non-linear, i. e. type-specific, macroprudential taxes. This policy outperforms linear taxes, which are typically proposed in the literature, in reducing indebtedness and stabilizing collateral prices.

Key words: Financial amplification, Pecuniary externalities, Collateral constraint, Financial crisis, Belief heterogeneity, Macroprudential policy

JEL codes: D84, E44, G28, H23

1.1 Introduction

Belief heterogeneity was a contributing factor in prompting the 2007–2009 global financial crisis, underlying the build-up of leverage in financial balance sheets (Cheng et al., 2014; Gennaioli and Shleifer, 2018; Mian and Sufi, 2022). However, from a theory perspective, a question still to be answered is how the occurrence of a crisis and optimal macroprudential policies are impacted by the belief channel.

There is a vast literature on the link between borrowing, investment, and financial crises. An established result is the efficiency-enhancing effect of macroprudential policy, encompassing both corrective taxes and quantitative restrictions on financial decisions (Geanakoplos and Polemarchakis, 1986; Bianchi, 2011; Dávila and Korinek, 2018; Jeanne and Korinek, 2020). However, in the presence of belief heterogeneity, policy instruments would optimally address the individual behavior of market participants, and the extent to which it contributes to financial distress. That requires a characterization of how differentiated beliefs give rise to such individual contributions. Albeit the literature has provided valuable insights into measuring systemic risk contributions, it lacks an explicit consideration of the role of beliefs (Acharya et al., 2012; Adrian and Brunnermeier, 2016; Acharya et al., 2017).

We fill this gap and analyze the belief channel of financial distress. We build upon an established model that features a pecuniary externality, originating from a price-dependent collateral constraint in the credit market. It is augmented by heterogeneity of beliefs across the population. This setup gives rise to differentiated risk taking in financial decisions. The latter are observable, so we may characterize individual contributions to financial distress explicitly. That allows us to derive optimal macroprudential policies which are calibrated to each individual's type.

The main result is that belief disagreements increase the likelihood and the extent of financial distress. Specifically, crises occur under less severe macroeconomic shocks, and the associated loss of efficiency is larger than in an economy populated by homogeneous individuals. This finding rests upon the fact that economic agents contribute to financial distress asymmetrically, with optimistic agents making larger contributions than pessimistic agents. That has notable implications for macroprudential policy. We show that, while corrective policy interventions are generally efficiency-enhancing, type-specific policies generate additional efficiency gains. Against this background, we propose a system of non-linear, i. e. type-specific, macroprudential taxes. This policy outperforms linear taxes, which are typically proposed in the literature, in reducing indebtedness and stabilizing collateral prices.

To the best of our knowledge, we are the first to study the efficiency properties of an economy that features both frictional financial markets and belief heterogeneity. Specifically, our model incorporates a collateral constraint on borrowing as a function of the market-determined collateral price. This type of friction introduces a pecuniary externalitiy, as economic agents do not internalize that their decisions mutually affect borrowing capacities, which, in turn, establishes a financial amplification mechanism. Agents may hold heterogeneous beliefs in the sense of perceiving differentiated probability distributions over the future state of the world. This setup allows us to distinguish relatively more optimistic from pessimistic individuals.

We use this model to analyze the interaction of the collateral constraint and belief heterogeneity. First, we characterize how the latter impacts the probability of distress in the competitive equilibrium, as well as the equilibrium allocation, collateral prices, and externalities. We then perform an efficiency analysis. It allows us to detect inefficiences in the competitive allocation and to show how a social planner achieves a welfare improvement by internalizing the effects of individual investment and borrowing decisions on collateral prices. We characterize her optimal corrective policies numerically, and evaluate how they alter the levels of borrowing, investment, the probability of financial distress, and efficiency.

Theorem 1.1 entails a key results of our analysis, stating that, compared to an economy where agents hold a homogeneous and rational belief, belief heterogeneity raises the likelihood of financial distress. For collateral constraints to be binding, the economy does need to be hit by severe shocks to aggregate investment or net worth, as is typically the case in the literature on pecuniary externalities. Instead, it suffices that some agents' beliefs deviate from the ex post state of the world. This is likely to be the case in the presence of belief heterogeneity, which therefore raises the probability that a financial crisis is triggered.

This finding is brought about by the fact that optimistic and pessimistic agents contribute asymmetrically to systemic distress. Our analysis reveals that optimists overborrow and overinvest, while pessimists underborrow and underinvest. By this behavior, optimists and pessimists exert downward and upward pressure on collateral prices, respectively. However, pessimists prove to make a smaller impact, as their level of investment is bounded from below by the motive to hold collateral. Under the plausible assumption that beliefs are distributed normally across the population, that implies that collateral prices turn out to be lower under heterogeneous beliefs, suggesting that belief heterogeneity precipitates financial distress.

We further study the efficiency implications of the former findings. The competitive allocation is associated with inefficiencies emanating from the fact that economic agents do not internalize how their investment and borrowing decisions impact collateral prices.

A social planner, however, albeit herself constrained by the borrowing limit, internalizes these effects. Theorem 1.2 draws upon this insight, formulating policy implications on how to implement an efficient allocation. We show that, even though beliefs are agents' private information and a priori unobservable to the social planner, it can be decentralized by a set of non-linear macroprudential taxes, suitable to address type-specific contributions to financial distress. Our policy proposal goes beyond the existing literature, which takes no account of heterogeneity among agents and, hence, focuses on linear macroprudential taxes. By evaluating efficiency implications numerically, we find that, although either sort of policy intervention enhances efficiency and reduces the probability of financial distress, the non-linear policy we propose produces considerable welfare gains.

This chapter makes several important contributions. We develop a framework that helps to explicitly characterize how different market participants contribute to financial crises. That is decisive to show that the mere presence of belief disagreements poses a source of financial distress. The optimal design of prudential policies thus accounts for belief divergences and is calibrated to individual contributions. This is particularly relevant during different phases of the business cycle, as investors' beliefs prove to fluctuate and diverge largely between booms and busts (Minsky, 1977, 1986; Aliber and Kindleberger, 2015; Adam et al., 2017; Kaplan et al., 2020; Mian and Sufi, 2022). On the conceptual level, this chapter provides a formal framework which can be used for further analyses of financial amplification mechanisms in environments where economic agents do not have rational expectations, but potentially feature heterogeneous beliefs.

The remainder of this chapter is organized as follows. We review the related literature in section 1.2. Section 1.3 develops the baseline model, and analyzes the competitive equilibrium. In section 1.4, we describe the externalities present in our model, derive optimal corrective policies, and perform normative analyses numerically. We provide some final remarks in section 1.5.

1.2 Related literature

Financial amplification, pecuniary externalities and systemic risk. The chapter relates to the literature on financial amplification, including studies of pecuniary externalities in particular. This literature originates from Fisher (1933) and was extended by analyses of borrowing constraints and their effects on asset prices by Bernanke and Gertler (1990), Shleifer and Vishny (1992), Kiyotaki and Moore (1997), Brunnermeier and Pedersen (2009), and Acharya et al. (2011). Hart (1975) and Stiglitz (1982) moreover prove the presence of pecuniary externalities in incomplete markets.[1] By modeling a borrowing

[1]For survey articles, see Shleifer and Vishny (2011) and Brunnermeier and Oehmke (2013).

constraint in an incomplete credit market, our framework builds on the structures of this literature.

Welfare implications of pecuniary externalities are examined in Gromb and Vayanos (2002), Caballero and Krishnamurthy (2003), Lorenzoni (2008), and Caballero and Lorenzoni (2014). While these papers focus on externalities affecting borrowers' net worth, Jeanne and Korinek (2010), Bianchi (2011), Bianchi and Mendoza (2018), Dávila and Korinek (2018), and Jeanne and Korinek (2019) explore the collateral channel of financial amplification that can lead to financial crises. Since we are modeling externalities equivalently, we adopt their terminology and basic model structure.

Furthermore, we derive optimal corrective policies implemented by a a constrained social planner, referring to the early contributions of Stiglitz (1982) and Geanakoplos and Polemarchakis (1986). The policy maker in our model applies an ex ante macroprudential tax along the lines of Jeanne and Korinek (2010), Dávila and Korinek (2018), Jeanne and Korinek (2019), and Jeanne and Korinek (2020).[2]

In specifying individual constributions to financial distress, this chapter links to articles that focus on defining measures of systemic risk. Notably, Adrian and Brunnermeier (2016) propose $\Delta CoVar$, a measure capturing the interdependences between specific financial institutions and the entire financial system. Furthermore, Acharya et al. (2012) and Acharya et al. (2017) model individual institutions' exposure to financial crises. For an overview of quantitative measures of systemic risk, see Bisias et al. (2012). As opposed to our analysis, these studies do not account for belief disagreements as potential drivers of systemic risk contributions.

Macroeconomic perspectives on belief heterogeneity. Our work is also part of the literature on macroeconomic perspectives on belief heterogeneity. The idea of belief heterogeneity shaping market outcomes was pioneered by Keynes (1936), Minsky (1977), and Aliber and Kindleberger (2015). Since then, the literature has provided evidence that belief heterogeneity is relevant for asset prices and market volatility, in particular during the recent financial crisis (Harrison and Kreps, 1978; Scheinkman and Xiong, 2003; Reinhart and Rogoff, 2008; Simsek, 2013; Cheng et al., 2014; Gennaioli and Shleifer, 2018; Adam and Nagel, 2022).

Prior research has already combined belief heterogeneity with frictional financial markets, particularly in the context of leveraged speculation.[3] Geanakoplos (1996) was the first to model a general equilibrium with endogenous collateral constraints and heteroge-

[2] The social planner in our model has an instrument at hand which could be interpreted as a financial transaction tax. So the interested reader is referred to the literature on financial transaction taxes initiated by Tobin (1978) and extended by Summers and Summers (1989) and Stiglitz (1989).

[3] Xiong (2013) and Simsek (2021) review the literature on asset trading driven by heterogeneous beliefs in more detail.

neous beliefs, further developed in subsequent studies (Geanakoplos, 2003, 2010), showing that heterogeneity of beliefs fosters credit and leverage cycles. Simsek (2013) generalizes the framework by using a continuum of states, and focuses on various degrees of heterogeneity. The contribution of this chapter is that we add an analysis of efficiency implications to this literature, in particular by deriving optimal type-specific macroprudential policies.

As in all normative studies involving heterogeneity of beliefs, we face the challenge of how to aggregate welfare properly. Several approaches have been suggested, such as the welfare criteria put forth by Gilboa et al. (2014), Gayer et al. (2014), Brunnermeier et al. (2014), Blume et al. (2018), and Kim and Kim (2021). We overcome this challenge by an agnostic approach, equipping the social planner with no superior information, but letting her accepting individual beliefs.

Methodological approach. Lastly, our investigation of comparative statics with respect to the economy's belief structure closely relates to Dávila and Walther (2023), who study optimal leverage policies in response to changing beliefs. We follow their approach of applying methods of the calculus of variation to equilibrium variables under belief heterogeneity.

1.3 Model

The aim of this chapter is to explore a financial amplification mechanism in an environment where agents hold heterogeneous beliefs about the future. To that end, we set up a model featuring frictional financial markets and enrich it by belief heterogeneity across agents. We derive the competitive equilibrium of this economy and study how it is impacted by variations in beliefs. The framework allows us to distinguish the respective contributions of optimistic and pessimistic agents to financial amplification, and to evaluate the probability and the extent of distress in economies with different belief structures. Our results lay the ground for the study of optimal corrective policies in the next section.

1.3.1 Setup

We model a small open economy with three periods $t = 0, 1, 2$ and two classes of agents, referred to as lenders and investors. Lenders trade debt securities with investors or save in a zero return storage technology. The interest rate is exogenous and normalized to zero for simplicity, and lenders are assumed to be risk-neutral. Investors are divided into J groups indexed by $j \in \{1, ...J\}$, each of which consists of a continuum of individuals. Each group has a population share s^j that is common knowledge and derives utility from a single consumption good c_t^j according to a concave and strictly increasing utility function $u\left(c_t^j\right)$. Population shares are collected in the vector $s = \{s^j\}_{j\in\{1,...J\}}$.

In $t = 0$, investors receive an endowment $e > 0$, as well as an initial amount of assets $\bar{a} > 0$. They can borrow or save d_0^j to finance consumption and to further invest into a_0^j units of the asset.[4] The asset is traded at a price q_0 and exists in fixed supply. In $t = 1$, financial investment pays off an a priori uncertain dividend $R \in \left[\underline{R}, \overline{R}\right]$, which different groups of investors hold specific beliefs about. After all uncertainty has been resolved at the beginning of the period, investors repay former debt d_0^j, issue new debt d_1^j, and trade again, purchasing or liquidating l_1^j claims on the asset at price q_1. Debt issuance in $t = 1$ is restricted by a borrowing constraint

$$d_1^j \leq \phi q_1 \left(a_0^j - l_1^j\right).$$

The constraint implies that investors borrow against their asset position at the end of the period.[5] In $t = 2$, net of claims $a_0^j - l_1^j$ materializes and debt d_1^j must be repaid, determining final consumption c_2^j.

Our model features two important components. First, financial markets exhibit a friction, captured by the borrowing constraint. It incorporates a financial amplification mechanism within our framework, and results in a pecuniary externality. Second, we allow investors to hold different beliefs about the asset pay-off R.

Definition 1.1. *Let $F(R)$ be the true cumulative distribution function (cdf) of R, and $F^j(R)$ be the cdf perceived by type-j investors. We refer to heterogeneous beliefs if each type of investors j perceives an idiosyncratic distribution of R, i. e. $F^i(R) \neq F^j(R)$ for all $i \neq j$. We refer to homogeneous beliefs if all types of investors have rational expectations, i. e. $F^j(R) = F(R)$ for all j.*

The vector $\mathcal{F} = \{F^j(R)\}_{j\in\{1,...J\}}$ characterizes the complete set of beliefs existing in the economy, which we assume to be publicly known. Beliefs are distributed discretely

[4] Lenders' endowment is assumed to make the supply of debt securities perfectly elastic to demand. That is, all investors' borrowing preferences can be satisfied by assumption. This includes the possibility of savings $d_0^j < 0$.

[5] To rationalize this constraint, we adopt the mechanism suggested by Jeanne and Korinek (2019). The constraint bases on the presumption that investors lack commitment to repay. When investors renegotiate debt obligations, they make a take-it-or-leave-it offer in order to lower the amount of outstanding debt. If lenders reject the offer, they may seize a fraction ϕ of investors' assets and sell it at the prevailing market price. Lenders will hence accept the offer provided the repayment exceeds the current market value of seizable positions. This being said, we may assume without loss of generality that default and renegotiations never occur in equilibrium. One could further consider a similar restriction of debt issuance in period $t = 0$, which we neglect on the grounds that there is no role for macroprudential interventions in that period. Binding borrowing constraints in $t = 0$ would limit the set of cases when the period-1 constraint is binding, however without altering the results of our analysis, which focuses on states within this set.

across types, so each cdf $F^j(R)$ appears with frequency s^j.

1.3.2 Competitive equilibrium

To derive the competitive equilibrium, we first solve individual optimization problems backwards from $t = 2$ to $t = 0$. We distinguish between state variables of type-j individuals, i. e. $\{a_0^j, d_0^j\}$, and aggregate state variables of group j, denoted by $\left\{\tilde{a}_0^j, \tilde{d}_0^j\right\}$.

Optimization in $\mathbf{t = 1, 2}$. The optimization problem of type-j investors in $t = 1$ reads

$$V_1^j\left(a_0^j, d_0^j \mid \tilde{a}_0, \tilde{d}_0\right) = \max_{c_1^j, c_2^j, d_1^j, l_1^j \le a_0^j} u\left(c_1^j\right) + u\left(c_2^j\right) \quad \text{s.t.}$$

$$\left(\lambda_1^j\right) \quad c_1^j = Ra_0^j + q_1 l_1^j + d_1^j - d_0^j \tag{1.1}$$

$$\left(\lambda_2^j\right) \quad c_2^j = a_0^j - l_1^j - d_1^j \tag{1.2}$$

$$\left(\eta_1^j\right) \quad d_1^j \le \phi q_1\left(a_0^j - l_1^j\right), \tag{1.3}$$

where investors take group-wide aggregate states $\tilde{a}_0 = \left\{\tilde{a}_0^j\right\}_{j\in\{1,...J\}}$ and $\tilde{d}_0 = \left\{\tilde{d}_0^j\right\}_{j\in\{1,...J\}}$ as given because they affect the equilibrium asset price q_1. Let λ_1^j and λ_2^j be the Lagrange multipliers for the budget constraints (1.1) and (1.2), respectively, and η_1^j for the borrowing constraint (1.3).

This problem gives rise to the following pair of Euler equations for each j:

$$u'\left(c_1^j\right) - \eta_1^j = u'\left(c_2^j\right) \tag{1.4}$$

$$q_1 u'\left(c_1^j\right) - \eta_1^j \phi q_1 = u'\left(c_2^j\right), \tag{1.5}$$

jointly yielding equilibrium price equations

$$q_1 = \frac{u'\left(c_2^j\right)}{(1-\phi)u'\left(c_1^j\right) + \phi u'\left(c_2^j\right)} \tag{1.6}$$

for each j.

Optimization in $\mathbf{t = 0}$. In $t = 0$, the optimization of a type-j investor is

$$\max_{c_0^j, a_0^j \ge 0, d_0^j} u\left(c_0^j\right) + E^j\left[V_1^j\left(a_0^j, d_0^j \mid \tilde{a}_0, \tilde{d}_0\right)\right] \quad \text{s.t.}$$

$$\left(\lambda_0^j\right) \quad c_0^j = e + d_0^j + q_0\left(\bar{a} - a_0^j\right), \tag{1.7}$$

where the expectation operator is indexed by j, capturing potentially differing beliefs, and λ_0^j denotes the Lagrange multiplier for the period-0 budget constraint. Eliminating

Lagrange multipliers, we obtain the following optimality conditions for each j:

$$q_0 u'\left(c_0^j\right) = E^j\left[Ru'\left(c_1^j\right) + u'\left(c_2^j\right) + \eta_1^j \phi q_1\right] \tag{1.8}$$

$$u'\left(c_0^j\right) = E^j\left[u'\left(c_1^j\right)\right]. \tag{1.9}$$

Equilibrium. In equilibrium, the asset market is cleared in both periods $t = 0$ and $t = 1$, formalized by the conditions

$$\sum_{j=1}^{J} s^j a_0^j = \bar{a} \tag{1.10}$$

and

$$\sum_{j=1}^{J} s^j l_1^j = 0, \tag{1.11}$$

completing the set of equilibrium conditions. In a symmetric equilibrium, investors are identical within each group j, i. e. $x_t^j = \tilde{x}_t^j$ for all j with $x \in \{c, a, d, l, \lambda, \eta\}$. We may thus define the symmetric competitive equilibrium as follows.

Definition 1.2. *A competitive equilibrium consists of an allocation* $\left\{\tilde{c}_0^j, \tilde{c}_1^j, \tilde{c}_2^j, \tilde{a}_0^j, \tilde{d}_0^j, \tilde{d}_1^j, \tilde{l}_1^j\right\}_{j\in\{1,\ldots J\}}$*, a sequence of multipliers* $\tilde{\eta}_1 = \left\{\tilde{\eta}_1^j\right\}_{j\in\{1,\ldots J\}}$*, and prices* $\{q_0, q_1\}$*, satisfying equations* (1.1)*,* (1.2)*,* (1.4)*,* (1.5)*,* (1.7)*,* (1.8)*,* (1.9)*, and a complementary slackness condition for all* j*, as well as the market clearing conditions* (1.10) *and* (1.11)*, given population shares* s *and beliefs* $\mathcal{F}$*.*

The competitive equilibrium reflects the two main components of our model: the financial friction and potential belief disagreements. The financial friction introduces a wedge between market prices of the asset as well as debt and investors' marginal rates of substitution across periods. The wedge is formally represented by the multiplier $\tilde{\eta}_1^j$ that appears in equations (1.4), (1.5), and (1.8). In the latter two equations, the term $\tilde{\eta}_1^j \phi q_1$ captures the collateral premium of the asset, as each additional unit of $\tilde{a}_0^j$ and $\tilde{l}_1^j$ relaxes the constraint.

To highlight the impact of belief heterogeneity, we compare the competitive equilibrium under heterogeneous and homogeneous beliefs. If investors have heterogeneous expectations about the return R, they evaluate expected marginal benefits of investment and borrowing differently. Formally, group-specific expectation operators E^j apply in the Euler equations (1.8) and (1.9), resulting in group-specific values of $\tilde{a}_0^j$, $\tilde{d}_0^j$, and of the shadow price of borrowing $\tilde{\eta}_1^j$.

If, in contrast, investors hold a homogeneous belief, their marginal rates of substitution are identical, as is the shadow value of borrowing. Importantly, intertemporal substitution in this case is only possible through debt or savings $\tilde{d}_t^j$. The reason is that investors do not trade in excess of the initial asset endowment neither in $t = 0$ nor in $t = 1$, i. e. $\tilde{a}_0^j = \bar{a}$ and $\tilde{l}_1^j = 0$ for all j.

In the following, we restrict the set of equilibria taken into account in the analysis. Since we are only interested in situations when financial distress occurs, the model parameters, comprising risk aversion A, beliefs $\mathcal{F}$, the realized return $\hat{R}$, as well as the margin requirement ϕ, must satisfy that, in equilibrium, the asset is traded and constraints are binding ($\tilde{\eta}_1^j > 0$).[6]

Period-1 equilibrium price. Given its impact on the borrowing constraint, the equilibrium collateral price q_1 is a key variable in our model. We show its existence and uniqueness, and how it interacts with the multiplier of the borrowing constraint.

Proposition 1.1.

(i) The equilibrium price q_1 exists.

(ii) If at least one type of investors j receives a return as expected or higher, i. e. $E^j[R] \leq \hat{R}$ for at least one j and any realization $\hat{R}$ of R, the equilibrium price is unique, satisfying $q_1 \leq 1$, and the following two equivalences hold:

(1) $q_1 = 1$ if and only if $\tilde{\eta}_1^j = 0$ for all j,

(2) $q_1 < 1$ if and only if $\tilde{\eta}_1^j > 0$ for at least one j.

Proposition 1.1 first states that the equilibrium exists. Second, assuming that there is positive demand because at least one type makes a profit from investment, it asserts that the equilibrium price is unique and characterizes its relation with the borrowing constraint.[7] The constraint is binding at a price smaller than 1, but slack if $q_1 = 1$. At this price, investors are indifferent between purchasing or selling claims.

The two equivalences in part *(ii)* of Proposition 1.1 formalize this indifference property. They imply that either all or none of the investors are constrained by the borrowing limit. It is sufficient that only one group of investors is forced to liquidate claims on the market, i. e. $\tilde{l}_1^j > 0$, to reduce the price q_1 to a level below one. This deflation either constrains other investors via a tighter borrowing limit, or it gives them a pecuniary incentive to issue as much debt as possible. They do so to purchase additional claims, i. e. $\tilde{l}_1^j < 0$. To see this, recall the budget constraints (1.1) and (1.2), and note that, provided $q_1 < 1$,

[6] We make parameter restrictions explicit in the derivations of our results, provided in the appendix.

[7] However, the equilibrium price exists even if demand is zero, as this scenario corresponds to all investors being bankrupt, and infinitely many prices satisfy the Walrasian equilibrium definition. Abstracting from this case, we focus on equilibria with positive demand, which turn out to be uniquely determined.

every purchased unit of claims offers a positive return $1 - q_1 > 0$ in the final period. Hence, in order to transfer funds to $t = 2$, solvent investors prefer additional investment $\tilde{l}_1^j < 0$ over savings $\tilde{d}_1^j < 0$. For a price $q_1 = 1$, however, they are indifferent between both ways of intertemporal substitution.[8]

1.3.3 Equilibrium effects of variations in beliefs

In this section, we analyze how variations in beliefs affect the allocation and prices in the competitive equilibrium. We show how the two main ingredients of our model, the financial friction and heterogeneity of beliefs, interact. The results of this comparative statics exercise allow us to specify how different types contribute to financial amplification. We use these insights to evaluate how belief heterogeneity affects the overall probability and extent of financial distress.

To keep the model tractable, we henceforth impose the following assumption without further mention. It is useful to simplify the comparative statics analysis below.

Assumption 1.1. *Investors have exponential preferences of the form* $u\left(c_t^j\right) = -\exp\left(-Ac_t^j\right)$, *where absolute risk aversion* $A = -\frac{u''\left(c_t^j\right)}{u'\left(c_t^j\right)}$ *is constant (CARA).*[9]

We start out by examining the effect of changes in period-0 variables on the equilibrium price in $t = 1$, before analyzing how belief variations impact the equilibrium values of investment and borrowing in period $t = 0$. Note that the period-1 equilibrium price q_1 is no direct function of beliefs $\mathcal{F}$, but only through period-0 choices $\tilde{a}_0(\mathcal{F})$ and $\tilde{d}_0(\mathcal{F})$, i. e. $q_1 = q_1\left(\tilde{a}_0(\mathcal{F}), \tilde{d}_0(\mathcal{F})\right)$. Thus, this two-step procedure allows us to elaborate the relationship between the set of beliefs in the economy and the equilibrium price q_1, which defines the tightness of the borrowing constraint and measures the extent of financial distress.

Period-0 allocation and the equilibrium price. Proposition 1.2 states how the equilibrium price q_1 is linked to period-0 levels of investment and debt.

Proposition 1.2.

(i) If investors hold heterogeneous beliefs $\mathcal{F}$, *the period-1 equilibrium price* q_1 *is decreas-*

[8] Formally, one of the Euler equations (1.4) and (1.5) is redundant in the unconstrained case, i. e. if $q_1 = 1$ and $\tilde{\eta}_1^j = 0$ for all j. Intuitively, investors are indifferent between the instruments $\tilde{l}_1^j$ and $\tilde{d}_1^j$, given that both promise a zero net return. We assume without loss of generality that there is no trade in the unconstrained economy, i. e. $\tilde{l}_1^j = 0$ for all j.

[9] For expositional reasons, we continue using the general notation $u\left(c_t^j\right)$.

ing with period-0 investment and borrowing, i. e., for all j,

$$\frac{\partial q_1}{\partial \tilde{a}_0^j} < 0 \text{ and } \frac{\partial q_1}{\partial \tilde{d}_0^j} < 0.$$

(ii) If investors hold the homogeneous belief $F(R)$, the period-1 equilibrium price q_1 is decreasing with period-0 borrowing, i. e.

$$\frac{\partial q_1}{\partial \tilde{d}_0} < 0.$$

Proposition 1.2 states that more investment and borrowing in period $t = 0$ have a diminishing effect on the future equilibrium asset price. In the homogeneous case, while the former is irrelevant, as trade does not occur, the negative effect of borrowing holds true as well.

The two effects work through different channels, illustrated by the budget constraints (1.1) and (1.2). First, investment in $\tilde{a}_0^j$ increases period-2 consumption $\tilde{c}_2^j$ one-to-one, while $\tilde{c}_1^j$ rises with factor $\hat{R}$. Thus, in a sufficiently adverse state, satisfying $\hat{R} < 1$, consumption in the last period $\tilde{c}_2^j$ increases by more in response to investment than $\tilde{c}_1^j$. To smooth consumption, investors redistribute resources from $t = 2$ to $t = 1$ by liquidating $\tilde{l}_1^j$ units of their asset position (or purchasing less additional units). Second, higher indebtedness $\tilde{d}_0^j$ reduces the initial period-1 wealth $\hat{R}\tilde{a}_0^j - \tilde{d}_0^j$, raising the risk of being constrained and forced to liquidate a fraction of the portfolio. Both channels result in a higher supply (and a lower demand) of claims, which, in turn, reduce the equilibrium price q_1.

Beliefs and the period-0 allocation. We now turn to the relationship between investment $\tilde{a}_0(\mathcal{F})$ and borrowing $\tilde{d}_0(\mathcal{F})$ and investors' beliefs $\mathcal{F}$. To that end, we employ methods from the calculus of variation. We adopt the following procedure, that was first applied to heterogeneous belief environments by Dávila and Walther (2023). Recall that type-j investors' beliefs are characterized by the perceived distribution of R with cdf $F^j(R)$. Consider a perturbation to beliefs of the form $F^j(R) + \epsilon G^j(R)$, where $\epsilon > 0$ is an arbitrary number, and $G^j(R)$ captures the direction of the perturbation. $F^j(R) + \epsilon G^j(R)$ is required to be a valid cdf for small enough ϵ, so we assume it is continuous and differentiable, satisfies $G(\underline{R}) = G(\overline{R}) = 0$, and $\partial (F^j(R) + \epsilon G^j(R)) / \partial R \geq 0$ for sufficiently small ϵ.

This setup allows us to specify the concepts of optimism and pessimism. These terms are defined relative to each other in the sense of first-order stochastic dominance. A perturbation $G^j(R)$ makes type-j investors more optimistic if and only if it satisfies $F^j(R) + \epsilon G^j(R) \leq F^j(R)$ for all R. It is easy to see that a more optimistic belief requires the perturbation to have a non-positive direction, i. e. $G^j(R) \leq 0$ for all R. Analogously, investors of type j are made more pessimistic through a perturbation with direction $G^j(R) \geq 0$ for all R. Intuitively, investors are more optimistic if they assign lower probabilities than pessimists to low returns, so their cdf is shifted downwards.[10]

Using this technique, we show how a variation of a type's belief alters its individual choices of investment and debt issuance. The corresponding functional derivatives are

$$\frac{\delta \tilde{a}_0^j}{\delta F^j} \cdot G^j \text{ and } \frac{\delta \tilde{d}_0^j}{\delta F^j} \cdot G^j,$$

where δ denotes the operator for functional derivatives. Proposition 1.3 summarizes the results.

Proposition 1.3.

(i) Let investors hold heterogeneous beliefs $\mathcal{F}$ and let $G^j(R)$ be the direction of a perturbation of type-j investors' belief $F^j(R)$. More optimistic (pessimistic) investors invest and borrow more (less), i. e.

$$\frac{\delta \tilde{a}_0^j}{\delta F^j} \cdot G^j \begin{cases} \geq 0, & G^j(R) \leq 0 \\ < 0, & G^j(R) \geq 0 \end{cases} \text{ and } \frac{\delta \tilde{d}_0^j}{\delta F^j} \cdot G^j \begin{cases} \geq 0, & G^j(R) \leq 0 \\ < 0, & G^j(R) \geq 0 \end{cases}.$$

(ii) Let investors hold the homogeneous belief $F(R)$ and let $G(R)$ be the direction of a perturbation. The more optimistic (pessimistic) the homogeneous belief is, the more (less) investors borrow, i. e.

$$\frac{\delta \tilde{d}_0}{\delta F} \cdot G \begin{cases} \geq 0, & G(R) \leq 0 \\ < 0, & G(R) \geq 0 \end{cases}.$$

The essential insight from Proposition 1.3 is that investment and borrowing are monotonic functions of beliefs. The more optimistic a group of investors is, the more it invests into the asset and the more debt it issues. The opposite holds true for more pessimistic groups. If investors are homogeneous, only borrowing responds to variations in beliefs, while the asset is not traded.

Beliefs and the equilibrium price. Combining the results from Propositions 1.2 and 1.3, we describe how behavioral responses of investors to changes in beliefs $\mathcal{F}$ impact the period-1 equilibrium price $q_1\left(\tilde{a}_0(\mathcal{F}), \tilde{d}_0(\mathcal{F})\right)$ in Proposition 1.4.

[10]In the case of investors holding a homogeneous belief, a perturbation implies a variation of the true distribution $F(R)$.

Proposition 1.4.

(i) Let investors hold heterogeneous beliefs $\mathcal{F}$.

(1) Let further $G^j(R)$ be the direction of a perturbation of type-j investors' belief $F^j(R)$ and beliefs $F^i(R)$ be constant for all $i \neq j$. If the perturbation makes investors of type j more optimistic (pessimistic), the period-1 equilibrium price q_1 is lower (higher), i. e.

$$\frac{\delta q_1}{\delta F^j} \cdot G^j \begin{cases} \leq 0, & G^j(R) \leq 0 \\ > 0, & G^j(R) \geq 0 \end{cases}.$$

(2) Let further $G^j(R) < 0 < G^i(R)$ with $|G^j(R)| = |G^i(R)|$ for all R be the directions of two perturbations that make investors of type j more optimistic and investors of type i more pessimistic by the same magnitude. The behavioral responses to the perturbation with direction $G^j(R)$ have a stronger impact on the period-1 equilibrium price q_1 than those of the perturbation with direction $G^i(R)$, i. e.

$$\left| \frac{\delta q_1}{\delta F^j} \cdot G^j \right| \geq \left| \frac{\delta q_1}{\delta F^i} \cdot G^i \right|.$$

(ii) Let investors hold the homogeneous belief $F(R)$ and $G(R)$ be the direction of a perturbation. If the perturbation makes investors more optimistic (pessimistic), the period-1 equilibrium price q_1 is lower (higher), i. e.

$$\frac{\delta q_1}{\delta F} \cdot G \begin{cases} \leq 0, & G(R) \leq 0 \\ > 0, & G(R) \geq 0 \end{cases}.$$

Proposition 1.4 comprises a fundamental finding that proves pivotal in the derivation of the results below. Part (*i*) characterizes the relationship of q_1 and heterogeneous beliefs. The more optimistic investors are, the lower the collateral price is in equilibrium. Conversely, if investors hold more pessimistic beliefs, the equilibrium price is higher. This result originates from the two monotonicities we have established in Propositions 1.2 and 1.3: q_1 responds monotonically to period-0 investment and borrowing, which, in turn, are monotonically driven by beliefs.

However, according to statement (2), the equilibrium price responds asymmetrically to symmetric variations of beliefs. Consider the thought experiment of two distinct perturbations, one making investors of type j more optimistic, the other making investors of type i more pessimistic, both to the very same extent. Formally, this is equivalent to decreasing type j's and increasing type i's probability mass for each realization $\hat{R}$ by the same amount. The statement argues that the perturbation to j dominates the perturbation to i. Thus, the equilibrium price turns out to be lower. More precisely, the perturbation to the optimistic type j exerts a downward effect that outweighs the upward effect from the perturbation to the pessimistic type i, resulting in a lower equilibrium price. The asymmetry between optimistic and pessimistic investors' influence on q_1 is the main result of Proposition 1.4.

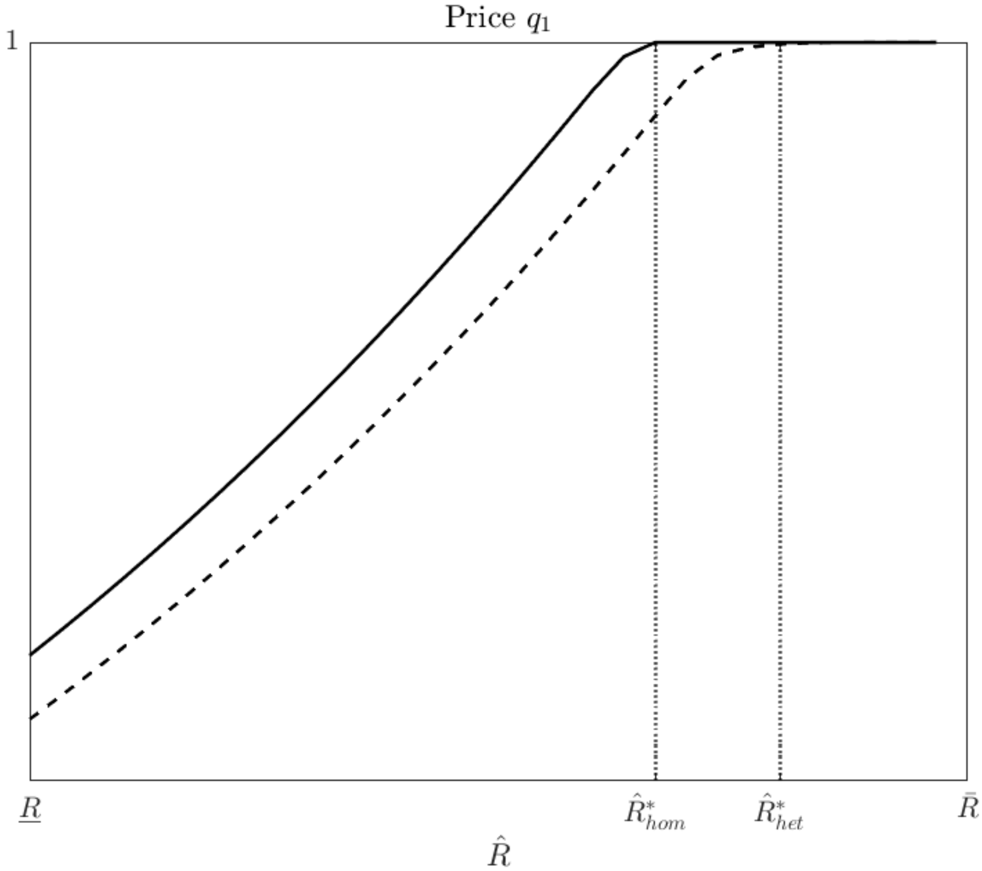
Key to understand the asymmetry is the collateral constraint. By the two perturbations, type-j investors become more optimistic, willing to invest and borrow more, while type-i investors become more pessimistic, willing to invest less and save more. Importantly, both types have the incentive to invest into the asset as collateral in $t = 1$. In $t = 0$, this incentive amplifies type j's willingness to extend investment, but it counteracts type i's willingness to reduce investment. Accordingly, it induces type j to increase period-0 borrowing by more than type i increases period-0 savings. Therefore, when the constraint is binding in the following period $t = 1$, type-j investors' supply of liquidated claims will initially exceed type-i investors' demand, which can only be equated for a lower equilibrium price q_1.

Part (ii) of Proposition 1.4 states that the former result holds true in the case of a homogeneous belief as well. A lower equilibrium price will arise if the uniform belief is more optimistic, and q_1 will be higher if it is more pessimistic.

Probability of financial distress. While Proposition 1.4 specifies how different types of investors contribute to financial amplification, we now evaluate how heterogeneity affects the overall probability of financial distress. We apply the method proposed by Dávila and Walther (2023) to prove that financial distress is more likely under heterogeneous beliefs. The probability of financial distress is determined by the lowest possible realization of R such that the constraints are slack.

Definition 1.3. *Let* $\hat{R}^*_{het} \equiv \min\left\{\hat{R} \mid \tilde{\eta}^j_1 = 0 \text{ for all } j\right\}$ *and* $\hat{R}^*_{hom} \equiv \min\left\{\hat{R} \mid \tilde{\eta}_1 = 0\right\}$ *be the lowest possible realizations of* R *such that the borrowing constraints are slack in the competitive equilibrium if investors hold heterogeneous beliefs* $\mathcal{F}$ *or the homogeneous belief* F*, respectively.*

Figure 1.1: Mapping from $\hat{R}$ to q_1 in the competitive equilibrium



Notes: This figure shows the mapping from $\hat{R}$ to q_1 for the two cases when investors hold the homogeneous belief $F(R)$ or heterogeneous beliefs $\mathcal{F}$, respectively. The solid line refers to the homogeneous case, and the dashed line refers to the heterogeneous case. $\hat{R}^*_{hom}$ and $\hat{R}^*_{het}$ are thresholds as defined in Definition 1.3. The assumptions underlying this simulation are given in section 1.4.4.

Definition 1.3 translates into the mappings $\hat{R} \mapsto q_1\left(\hat{R}\right)$, where q_1 serves as a measure of financial distress, formally written as

$$q_1 \begin{cases} = 1 & \hat{R} \geq \hat{R}^*_{het} \\ < 1 & \hat{R} < \hat{R}^*_{het} \end{cases} \text{ or } q_1 \begin{cases} = 1 & \hat{R} \geq \hat{R}^*_{hom} \\ < 1 & \hat{R} < \hat{R}^*_{hom} \end{cases}.$$

Figure 1.1 portrays an illustration of the two mappings.[11] We show that the the threshold is lower if investors hold a homogeneous belief, compared to a setting of heterogeneous beliefs varying around it.

Theorem 1.1. *Consider two distinct populations with investors holding heterogeneous beliefs $\mathcal{F}$ in one, and the homogeneous belief $F(R)$ in the other. If the homogeneous belief*

[11]Figure 1.1 is based on the numerical application provided in section 1.4.4.

is not more optimistic than any other belief in the heterogeneous case, i. e. $F^j(R) < F(R)$ *for all* R *and at least one* j*, the probability of financial distress in the competitive equilibrium is higher under heterogeneity than under homogeneity, which is equivalent to*

$$\hat{R}^*_{het} > \hat{R}^*_{hom}.$$

Theorem 1.1 constitutes the first key result of our analysis. In an environment of heterogeneous beliefs, it is more likely that financial distress occurs. In general, it occurs whenever the realized return $\hat{R}$ is insufficient so that each investor could comply with her repayment obligations. If investors share a homogeneous belief, each $\hat{R} < \hat{R}^*_{hom}$ will constrain *all* investors. However, if beliefs are heterogeneous, it is enough that $\hat{R}$ is too low for *one* group to make everyone's borrowing constraint binding. In fact, under heterogeneity, the threshold $\hat{R}^*_{het}$ corresponds to the most optimistic type reaching the constraint, as it has built up the highest exposure to low returns.

We find that the most optimistic type is financially distressed even for higher returns than if investors held a homogeneous belief. Consequently, under heterogeneity, financial distress occurs in even more favorable states of the world (as depicted in Figure 1.1) and is hence more likely. It rests on the presumption that the most optimistic belief is sufficiently off the ex post realization. Hence, Theorem 1.1 highlights the role of belief divergences as an additional source of financial distress. As is well known from the literature, a spiral of financial amplification can be initiated by adverse shocks sufficiently strong to drive excessively borrowing agents towards the constraint. Beyond that, we document that the dispersion of beliefs lays the ground for another trigger, namely that some agents' beliefs deviate sufficiently from the true shock distribution.

1.3.4 Discussion

In the previous section, we have shown that belief heterogeneity increases the probability of financial distress and how it affects the equilibrium collateral price. This price, in turn, is the main determinant of the financial friction, as it governs the tightness or slackness of the borrowing limit. Our results jointly allow us to characterize the interaction of the collateral constraint and belief divergence and to specify how different types of agents contribute to financial amplification.

The mechanism emerging from this interaction has two features. The first property is that heterogeneity of beliefs raises the *likelihood* of financial distress relative to the homogeneous benchmark, as stated by Theorem 1.1. The second property refers to the *extent* of financial distress, building upon the differences in individual contributions shown in Proposition 1.4. Principally, during financial distress, optimistic and pessimistic investors drive collateral prices in opposing directions, as the former tend to sell and the latter tend to purchase. However, we find an asymmetry of their contributions, attributing a larger impact to optimistic behavior. Hence, to distinguish the behavior of borrowing constraints in the presence of heterogeneous beliefs from the homogeneous benchmark, we must take into account how beliefs are distributed over the population.

It turns out that the financial friction tends to be more severe under heterogeneity rather than homogeneity. That holds true under the condition that the mean belief coincides with or is more optimistic than the homogeneous belief. Put differently, so long as the belief distribution is symmetric around the homogeneous belief, or skewed towards more optimistic beliefs, heterogeneity exacerbates financial amplification. The reason is that optimistic investors' (negative) contribution more than outweighs pessimistic investors' (positive) contribution.[12]

These results add new insights to the existing literature on financial amplification. It typically presumes rational expectations and establishes mechanisms where financial constraints bind in response to exogenous reductions of aggregate investment or net worth (Bianchi, 2011; Dávila and Korinek, 2018; Jeanne and Korinek, 2020). We extend this approach, and show that belief differences are sufficient to make such constraints binding. We may further quantify how market participants contribute to their tightness on the micro level. In the following section, we turn to the welfare implications of these interactions of heterogeneous beliefs and financial frictions.

1.4 Efficiency analysis

We proceed by exploring the efficiency properties of our baseline economy. Given that the borrowing constraint is price-dependent, investors are subject to a pecuniary externality, as they do not internalize how their decisions affect other agents' individual welfare. We characterize these uninternalized welfare effects and their interplay with belief heterogeneity in the following section. Subsequently, we derive a constrained-efficient allocation as a welfare benchmark to contrast the competitive equilibrium and develop optimal corrective policies. Lastly, we quantify the welfare impact of such policy interventions numerically.

[12]Belief heterogeneity may mitigate financial amplification compared to the homogeneous benchmark, on the contrary, provided that the distribution is sufficiently skewed towards more pessimistic beliefs. The skewness would have to be large enough to reverse the relation of optimistic and pessimistic investors' influence on the collateral price. However, we argue that the presumption of a symmetric distribution is likely to prevail in financial markets. A range of studies provides both empirical and theoretical evidence that financial market participants' beliefs are distributed symmetrically, if not (close to) normally (Söderlind, 2009; Cvitanic and Malamud, 2011; Atmaz, 2014; Atmaz and Basak, 2016). Under this premise, extreme beliefs are either sufficiently improbable or counteracted by an equiprobable set of contrasting beliefs.

1.4.1 Uninternalized welfare effects

The collateral price q_1 links individual choices and utilities across investors in two ways. First, it changes the value of investors' budgets in $t = 1$. Second, it determines the tightness of the borrowing constraints. Investors do not internalize these price effects. We use the terminology of Dávila and Korinek (2018) of *distributive* and *collateral* externalities.

Definition 1.4. *The uninternalized effects of changes in any type j's aggregate state variables* $\left\{\tilde{a}_0^j, \tilde{d}_0^j\right\}$ *on any i's individual welfare in periods* $t = 1, 2$ *can be written as*

$$\frac{\partial V_1^i}{\partial \tilde{a}_0^j} = \tilde{\lambda}_1^i D_{\tilde{a}_0^j}^i + \tilde{\eta}_1^i C_{\tilde{a}_0^j}^i$$
$$\frac{\partial V_1^i}{\partial \tilde{d}_0^j} = \tilde{\lambda}_1^i D_{\tilde{d}_0^j}^i + \eta_1^i C_{\tilde{d}_0^j}^i,$$

where $D_{\tilde{a}_0^j}^i$ *and* $D_{\tilde{d}_0^j}^i$ *are referred to as distributive externalities, and* $C_{\tilde{a}_0^j}^i$ *and* $C_{\tilde{d}_0^j}^i$ *are referred to as collateral externalities.*

(i) If investors hold heterogeneous beliefs $\mathcal{F}$*, distributive externalities are given by*

$$D_{\tilde{a}_0^j}^i = \frac{\partial q_1}{\partial \tilde{a}_0^j} \tilde{l}_1^i$$
$$D_{\tilde{d}_0^j}^i = \frac{\partial q_1}{\partial \tilde{d}_0^j} \tilde{l}_1^i,$$

and collateral externalities are given by

$$C_{\tilde{a}_0^j}^i = \phi \frac{\partial q_1}{\partial \tilde{a}_0^j} \left(\tilde{a}_0^i - \tilde{l}_1^i\right)$$
$$C_{\tilde{d}_0^j}^i = \phi \frac{\partial q_1}{\partial \tilde{d}_0^j} \left(\tilde{a}_0^i - \tilde{l}_1^i\right).$$

(ii) If investors hold the homogeneous belief $F(R)$*, distributive externalities are zero, and collateral externalities are given by*

$$C_{\tilde{a}_0} = \phi \frac{\partial q_1}{\partial \tilde{a}_0} \bar{a}$$
$$C_{\tilde{d}_0} = \phi \frac{\partial q_1}{\partial \tilde{d}_0} \bar{a}.$$

Distributive effects describe the price-induced redistribution between trading agents, altering their marginal rates of substitution. Collateral effects measure the price-induced change in an agent's capacity to borrow. In an environment of heterogeneous beliefs, it turns out that, the more optimistic investors are, the more likely it is that they will sell claims on the asset in $t = 1$ ($\tilde{l}_1^j \geq 0$). Accordingly, more pessimistic investors will more probably enter the market as buyers ($\tilde{l}_1^j < 0$). The reason is that a group's exposure to adverse states, reflected by its position $\tilde{a}_0^j$, is a monotonic function of beliefs (see Proposition 1.3). We use this fact, as well as Proposition 1.2, to characterize the direction of distributive and collateral externalities.

Proposition 1.5.

(i) If investors hold heterogeneous beliefs $\mathcal{F}$, distributive externalities are non-positive for period-1 sellers, i. e. $D^i_{\tilde{a}_0^j} \leq 0$ and $D^i_{\tilde{d}_0^j} \leq 0$ if $\tilde{l}_1^i \geq 0$, and non-negative for period-1 buyers, i. e. $D^i_{\tilde{a}_0^j} \geq 0$ and $D^i_{\tilde{d}_0^j} \geq 0$ if $\tilde{l}_1^i \leq 0$. If investors hold the homogeneous belief $F(R)$, distributive externalities are zero.

(ii) Collateral externalities have are non-positive for any type i and irrespective of beliefs, i. e. $C^i_{\tilde{a}_0^j} \leq 0$ and $C^i_{\tilde{d}_0^j} \leq 0$ for each i.

Distributive externalities are signed reflective of the fact that a decline of the equilibrium price q_1 benefits buyers and harms sellers in $t = 1$. Collateral externalities, in turn, are unambiguously adverse to each type of agent, as more investment and borrowing reduce the collateral value, cutting any investor's borrowing capacity. Combining Proposition 1.5 with our results from section 1.3 allows us to evaluate the welfare implications of the interaction mechanism between beliefs and the equilibrium price q_1.

Proposition 1.6.

(i) Let investors hold heterogeneous beliefs $\mathcal{F}$.

(1) Let further $G^j(R)$ be the direction of a perturbation of type-j investors' belief $F^j(R)$ and beliefs $F^i(R)$ be constant for all $i \neq j$. If the perturbation makes investors of type j more optimistic (pessimistic), uninternalized welfare effects of any type-i investor are larger (smaller) in absolute value, i. e., for each $i \neq j$ and $x \in \{a, d\}$,

$$\left| \frac{\delta D^i_{\tilde{x}_0^j}}{\delta F^j} \cdot G^j \right| \begin{cases} \geq 0, & G^j(R) \leq 0 \\ \leq 0, & G^j(R) \geq 0 \end{cases} \quad \text{and} \quad \frac{\delta C^i_{\tilde{x}_0^j}}{\delta F^j} \cdot G^j \begin{cases} \leq 0, & G^j(R) \leq 0 \\ \geq 0, & G^j(R) \geq 0 \end{cases}.$$

(2) Let further $G^j(R) < 0 < G^k(R)$ with $|G^j(R)| = \left|G^k(R)\right|$ for all R be the directions of two perturbations that make investors of type j more optimistic, and investors of type k more pessimistic by the same magnitude. Uninternalized welfare effects under the perturbation with direction $G^j(R)$ are stronger than

those under the perturbation with direction $G^k(R)$*, i. e., for each* $i \neq j, k$ *and* $x \in \{a, d\}$*,*

$$\left|\frac{\delta D^i_{\tilde{x}^j_0}}{\delta F^j} \cdot G^j\right| \geq \left|\frac{\delta D^i_{\tilde{x}^k_0}}{\delta F^k} \cdot G^k\right| \quad \textit{and} \quad \left|\frac{\delta C^i_{\tilde{x}^j_0}}{\delta F^j} \cdot G^j\right| \geq \left|\frac{\delta C^i_{\tilde{x}^k_0}}{\delta F^k} \cdot G^k\right|.$$

(ii) *Let investors hold the homogeneous belief* $F(R)$ *and* $G(R)$ *be the direction of a perturbation. If the perturbation makes investors more optimistic (pessimistic), collateral externalities are larger (smaller) in absolute value, i. e., for* $x \in \{a, d\}$

$$\frac{\delta C_{\tilde{x}_0}}{\delta F} \cdot G \begin{cases} \leq 0, & G(R) \leq 0 \\ \geq 0, & G(R) \geq 0 \end{cases}.$$

Proposition 1.6 describes the welfare effects associated with the interaction of beliefs and the equilibrium price q_1. It states that more optimistic types, exerting downward pressure on the collateral price due to large investment and borrowing, impose more intense negative distributive externalities on sellers ($\tilde{l}^i_1 > 0$) and more intense positive ones on buyers ($\tilde{l}^i_1 < 0$). In contrast, more pessimistic types' choices have an increasing impact on the collateral price, by this causing the reverse response of distributive externalities.

By the same logic, collateral externalities, being non-positive in general, turn out to be more or less pronounced in the case of more optimistic or pessimistic groups, respectively. This result holds true analogously in the homogeneous case.

Importantly, the asymmetry between optimistic and pessimistic investors' influence on q_1 translates into asymmetric welfare effects, as we formalize in statement (2) of part (i). Since the price responds more markedly to optimistic than to pessimistic behavior, the former further dominates in welfare terms. If the two groups j's and k's beliefs are made more optimistic and pessimistic to the same extent, respectively, any further type i's group-wide welfare losses from j's high investment and borrowing exceed the gains from k's precaution.

1.4.2 Constrained efficiency

Investors do not internalize the distributive or collateral side effects of their behavior which materialize through the collateral price q_1. These externalities render the competitive equilibrium allocation inefficient. To evaluate its welfare properties, we employ the concept of constrained efficiency.

The constrained-efficient allocation solves the problem of a constrained social planner who chooses investment and borrowing in period $t = 0$ while leaving all later choices to private agents. Specifically, she maximizes social welfare subject to all resource constraints, technological constraints, market clearing conditions, and financial frictions, respecting the competitive equilibrium price formation (see equation (1.6)).

Social welfare is evaluated by aggregating investors' expected lifetime utilities, and applying arbitrary Pareto weights $\omega = \{\omega^j\}_{j\in\{1,\dots,J\}}$. A relevant question in this setting is the planner's belief (Blume et al., 2018; Dávila, 2023; Kim and Kim, 2021). If we assigned a specific belief to the planner, she would naturally disagree with investors upon their beliefs. Abstracting from this trivial motive of correction, we aim at isolating ex ante corrective policies related to the financial friction, and, thus, make the following assumption.

Assumption 1.2. *The constrained social planner has no superior information and respects individual beliefs for each type j.*

We solve the following social planner problem.

$$\begin{aligned} \max_{\left\{\tilde{c}_0^j,\tilde{a}_0^j,\tilde{d}_0^j\right\}_{j\in\{1,\dots J\}}} \quad & \sum_{j=1}^{J} \omega^j s^j \left[u\left(\tilde{c}_0^j\right) + E^j\left[V_1^j\left(\tilde{a}_0^j, \tilde{d}_0^j | \tilde{a}_0, \tilde{d}_0\right)\right]\right] \quad \text{s.t.} \\ (\tilde{\lambda}_0) \quad & \sum_{j=1}^{J} s^j \tilde{c}_0^j = \sum_{j=1}^{J} s^j \left[e + \tilde{d}_0^j\right] \\ (\tilde{\psi}) \quad & \sum_{j=1}^{J} s^j \tilde{a}_0^j = \bar{a}. \end{aligned} \tag{1.12}$$

With the first order conditions for consumption, $\tilde{\lambda}_0 = \omega^j u'\left(\tilde{c}_0^j\right)$, the planner's optimality conditions for each j are

$$0 = E^j\left[Ru'\left(\tilde{c}_1^j\right) + u'\left(\tilde{c}_2^j\right) + \tilde{\eta}_1^j \phi q_1\right] - \frac{\tilde{\psi}}{\omega^j} + \sum_{i=1}^{J} \frac{\omega^i}{\omega^j} \frac{s^i}{s^j} E^i\left[D^i_{\tilde{a}_0^j} u'\left(\tilde{c}_1^i\right) + \tilde{\eta}_1^i C^i_{\tilde{a}_0^j}\right] \tag{1.13}$$

$$0 = u'\left(\tilde{c}_0^j\right) - E^j\left[u'\left(\tilde{c}_1^j\right)\right] + \sum_{i=1}^{J} \frac{\omega^i}{\omega^j} \frac{s^i}{s^j} E^i\left[D^i_{\tilde{d}_0^j} u'\left(\tilde{c}_1^i\right) + \tilde{\eta}_1^i C^i_{\tilde{d}_0^j}\right]. \tag{1.14}$$

We can now define the constrained-efficient allocation.

Definition 1.5. *The period-0 allocation $\left\{\tilde{c}_0^j, \tilde{a}_0^j, \tilde{d}_0^j\right\}_{j\in\{1,\dots J\}}$ is constrained-efficient if and only if there are shadow prices $\tilde{\lambda}_0$, $\tilde{\psi}$, $\left\{\tilde{\eta}_1^j\right\}_{j\in\{1,\dots J\}}$ and a set of Pareto weights $\{\omega^j\}_{j\in\{1,\dots J\}}$ such that it satisfies the price relation* (1.6) *for each j, the market clearing condition* (1.10)*, and the resource constraint* (1.12)*, as well as equations* (1.13)*,* (1.14)*, and $\tilde{\lambda}_0 = \omega^j u'\left(\tilde{c}_0^j\right)$ for each j, given population shares s and beliefs $\mathcal{F}$.*

Equations (1.13) and (1.14) differ from the competitive equilibrium conditions (1.8) and (1.9) by the aggregate terms of externalities on the right-hand side. They indicate formally that the competitive allocation is not constrained-efficient, whereas the social planner takes distributive and collateral externalities into account. Furthermore, she accounts for market clearing in $t = 0$, represented by the multiplier $\tilde{\psi}$.

1.4.3 Optimal corrective policies

The constrained-efficient allocation can be achieved in a decentralized market using a set of adequate policy instruments. We start out by characterizing optimal macroprudential taxes under both heterogeneous and homogeneous beliefs. We contrast a system of non-linear taxes under heterogeneity with a linear tax. The latter allows us to quantify differences in the efficiency-enhancing effects of our approach and existing policy proposals in the following section.

Decentralization. To decentralize the constrained-efficient allocation, we provide the social planner with access to macroprudential taxes, available to manipulate agents' investment and borrowing decisions, and lump-sup transfers. These instruments satisfy the conditions stated in the following proposition.

Proposition 1.7.

(i) If investors hold heterogeneous beliefs $\mathcal{F}$, the social planner can implement the constrained-efficient allocation by taxing investment and borrowing, satisfying

$$\tau_a^j = \operatorname{sgn}\left(\bar{a} - \tilde{a}_0^j\right)\left(s^j q_0 \tilde{\lambda}_0\right)^{-1} \sum_{i=1}^{J} \omega^i s^i E^i \left[D^i_{\tilde{a}_0^j} u'\left(\tilde{c}_1^i\right) + \tilde{\eta}_1^i C^i_{\tilde{a}_0^j}\right] \tag{1.15}$$

$$\tau_d^j = -\operatorname{sgn}\left(\tilde{d}_0^j\right)\left(s^j \tilde{\lambda}_0\right)^{-1} \sum_{i=1}^{J} \omega^i s^i E^i \left[D^i_{\tilde{d}_0^j} u'\left(\tilde{c}_1^i\right) + \tilde{\eta}_1^i C^i_{\tilde{d}_0^j}\right] \tag{1.16}$$

for each j and rebating revenues through type-specific lump-sum transfers $T^j = \tau_a^j \operatorname{sgn}\left(\bar{a} - \tilde{a}_0^j\right) q_0 \left(\bar{a} - \tilde{a}_0^j\right) + \tau_d^j \operatorname{sgn}\left(\tilde{d}_0^j\right) \tilde{d}_0^j$.[13]

(ii) If investors hold the homogeneous belief $F(R)$, the social planner can implement the constrained-efficient allocation by taxing borrowing, satisfying

$$\tau_d = -\tilde{\lambda}_0^{-1} E\left[\tilde{\eta}_1 C_{\tilde{d}_0}\right] \tag{1.17}$$

[13] We use a sign operator for an easier interpretation of taxes and subsidies, given the fact that investors can take short and long positions in the asset as well as borrow and save.

and rebating revenues through lump-sum transfers $T = \tau_d \tilde{d}_0$, while the tax on investment is arbitrary.

In the heterogeneous case, optimal macroprudential taxes are characterized by a range of sufficient statistics related to distributive and collateral externalities, aggregated in the squared brackets in equations (1.15) and (1.16).[14]

Three components determine distributive effects. First, when price movements induce a redistribution of funds between period-1 buyers and sellers, this affects their marginal rates of substitution. Second, price movements themselves measure the intensity of redistribution. Third, the direction of redistribution depends on whether an investor is a seller ($\tilde{l}_1^j > 0$) or a buyer ($\tilde{l}_1^j < 0$) in $t = 1$. The latter two components are captured by the distributive externalities $D^i_{\tilde{a}_0^j}$ and $D^i_{\tilde{d}_0^j}$, given in Definition 1.4.

Collateral effects are driven by another three components. First, the multiplier $\tilde{\eta}_1^j$ measures the welfare gain (loss) when the constraint is relaxed (tightened) by one unit. Second, price movements describe the change in an investor's borrowing capacity per unit of collateral, whose total magnitude available matters third. The last two elements are incorporated in the collateral externalities $C^i_{\tilde{a}_0^j}$ and $C^i_{\tilde{d}_0^j}$ from Definition 1.4.

If, however, investors hold the homogeneous and rational belief, these sufficient statistics turn out to be simpler. Since investors do not trade the asset under homogeneity, the social planner cannot manipulate investment decisions. The resulting tax on investment is arbitrary. Moreover, for the very same reason, distributive externalities are zero, rendering the tax on borrowing responsive solely to collateral externalities (see equation (1.17)).

Notably, the instruments derived in Proposition 1.7 may well be subsidies instead of taxes, depending on the extent of externalities induced by type j and its specific choices of investment and borrowing. Taxes/subsidies turn out to be zero only provided that *all* investors expect their collateral constraints to be slack. To put it another way, it suffices that one group of investors expects to be constrained to let taxes/subsidies take on either sign for the entire population. We will return to the signing of policy instruments in the next section.

Incentive compatibility. In an environment of heterogeneous agents, whose type is their private information, corrective policies may not be incentive-compatible. The instruments we have derived in Proposition 1.7 are type-specific, raising the question of knowledge required by the social planner to impose taxes in an incentive-compatible way.

Importantly, the optimal non-linear taxes in equations (1.15) and (1.16) incorporate no more than publicly known objects. To be precise, to set group-specific taxes, the social planner must be informed about the set of beliefs $\mathcal{F}$ in the economy, each type's respective

[14] For a more detailed description of sufficient statistics, see Dávila and Korinek (2018).

population share s^j, as well as investment and borrowing choices $\tilde{a}_0$ and $\tilde{d}_0$, which are publicly observable in the market. Since the latter are monotonic functions of beliefs, as we have shown in Proposition 1.3, they perfectly reveal any investor's belief.

Therefore, the constrained-efficient allocation can be implemented by means of the following system of non-linear macroprudential taxes.

Theorem 1.2. *If investors hold heterogeneous beliefs $\mathcal{F}$, the social planner can implement the constrained-efficient allocation by taxing investment and borrowing according to the tax system $(\tilde{\tau}_a, \tilde{\tau}_d)$, satisfying*

$$\tilde{\tau}_a : \quad \tilde{a}_0^k \mapsto \tilde{\tau}_a(\tilde{a}_0^k) \ s.t. \ \tilde{\tau}_a(\tilde{a}_0^k) = \begin{cases} RHS \ of \ (1.15) & if \ \tilde{a}_0^k = \tilde{a}_0^j \ for \ any \ j \ with \ \tilde{a}_0^j \in \tilde{a}_0 \\ \infty & if \ \tilde{a}_0^k \notin \tilde{a}_0 \end{cases} \tag{1.18}$$

$$\tilde{\tau}_d : \quad \tilde{d}_0^k \mapsto \tilde{\tau}_d(\tilde{d}_0^k) \ s.t. \ \tilde{\tau}_d(\tilde{d}_0^k) = \begin{cases} RHS \ of \ (1.16) & if \ \tilde{d}_0^k = \tilde{d}_0^j \ for \ any \ j \ with \ \tilde{d}_0^j \in \tilde{d}_0 \\ \infty & if \ \tilde{d}_0^k \notin \tilde{d}_0, \end{cases} \tag{1.19}$$

and corresponding lump-sum transfers.

The essential point of Theorem 1.2 is that the social planner does not rely on knowledge of individual beliefs. The peculiar nature of our optimal macroprudential taxes ensures that the constrained-efficient allocation is indeed decentralizable, even in a setting of heterogeneous beliefs.

Our results on optimal corrective policies give rise to several issues linked to the welfare implications of the interplay between belief heterogeneity and the financial friction. First, analyzing the responses of group-specific taxes/subsidies to variations of beliefs is informative on different types' contributions to changes in social welfare. Second, we seek to compare the efficiency properties of our economy under homogeneity and heterogeneity of beliefs. Third, it is enlightening to evaluate how the probability of financial distress is altered through a planner intervention of the kind sketched above. Moreover, we aim at quantifying the welfare impact of the non-linear tax instruments we propose in contrast to a linear macroprudential tax on borrowing. The latter is an instrument which has gained much attention in the literature (Bianchi, 2011; Dávila and Korinek, 2018; Jeanne and Korinek, 2019, 2020). In our model, it corresponds to equation (1.17), being a tax on borrowing calibrated to the case of homogeneous and rational expectations.

Examining these questions is analytically intractable. The clear signing of tax instruments depends on the specific belief distribution, which we have kept general thus far. To gain insights into the welfare implications of our policy proposals, we provide a numerical application of our model in the following.

1.4.4 Numerical application

The numerical analysis requires a simplified version of our model. In this section, we first describe the simplifications applied to make the baseline model numerically tractable, and briefly characterize the resulting equilibrium allocations, prices, and, importantly, optimal corrective policies for different levels of belief heterogeneity. Subsequently, we quantify the welfare implications of such policies. The final exercise of this section is an assessment of how these interventions impact the probability of financial distress.

Simplifications. Suppose the economy is populated by two groups of investors, called optimists and pessimists, indexed by o and p. We let both groups be of equal mass, i. e. $s^o = s^p = 1$, and differ in terms of their return expectations, i. e. $E^o[R] > E^p[R]$. Furthermore, there are only two states of the world. To be precise, R may take on either a *good* or a *bad* value, denoted by $R^g > R^b$.

We choose parameters in line with the assumptions underlying our theoretical analysis, simulating equilibria with significant trade volumes and binding financial constraints. Table 1.1 summarizes the parameter values chosen in the application.

Table 1.1: Parametrization

Parameter		Value
Margin requirement	ϕ	0.35
Good state	R^g	2
Bad state	R^b	0
Initial endowment of consumption goods	e	1
Initial asset endowment	$\bar{a}$	2
Risk aversion	A	0.5
Heterogeneity step	μ	0.025
Initial belief	π^g	0.5

Note: This table provides a summary of model parameter values chosen.

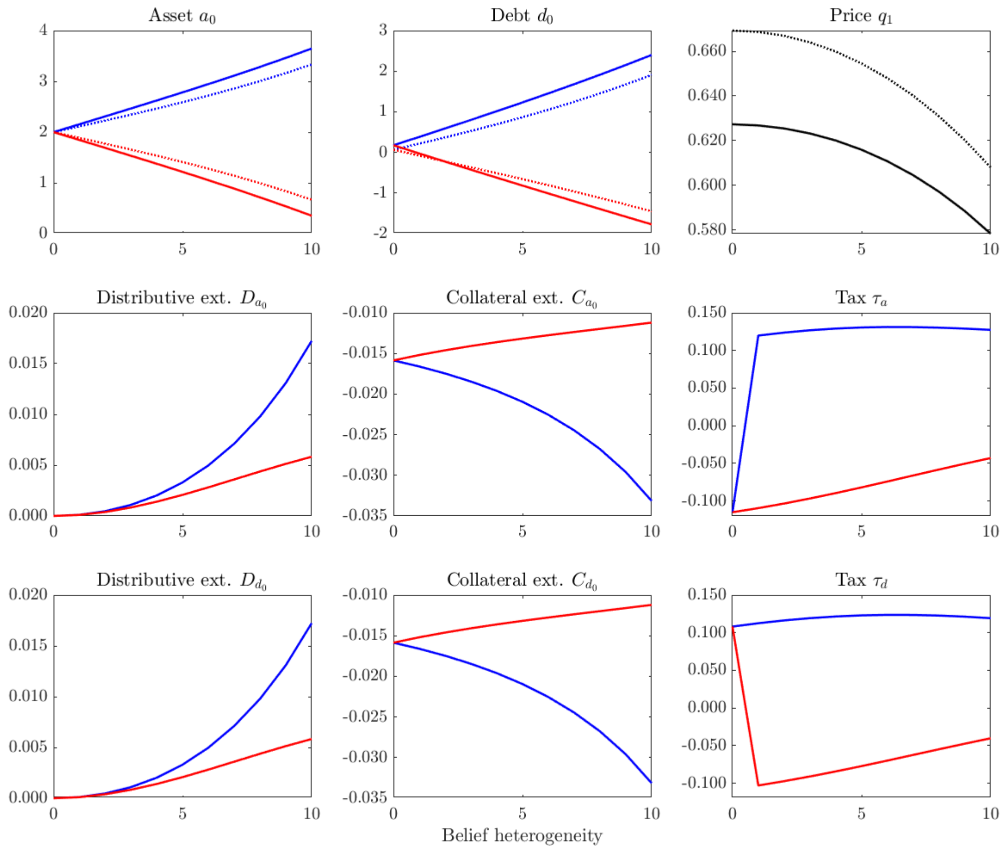
The parameter ϕ, capturing the margin requirement for borrowing, is selected following Mendoza (2002) and Bianchi (2011), who suggest that debt is required to not exceed a fraction of 30 to 40 percent of tradable assets. Averaging these values, we set $\phi = 0.35$. The two states R^g and R^b are chosen with the aim to make trading incentives strong enough, which, in turn, ensures a significant trade volume. This condition is met for $R^g = 2$ and $R^b = 0$. For the same argument, we set initial endowments of consumption goods e and assets $\bar{a}$ to $e = 1$ and $\bar{a} = 2$ and choose a moderate degree of risk aversion $A = 0.5$.

Heterogeneity itself is defined as the linear distance between the probabilities that the two types assign to the good state, i. e. $\pi^{j,g} = 1 - \pi^{j,b}$. We increase this distance symmetrically by N steps of size $\mu = 0.025$ (see Simsek (2013) for comparison). The multiples N thus serve as a measure of belief heterogeneity. The benchmark case is a population with homogeneous beliefs, where $\pi^{o,g} = \pi^{p,g} \equiv \pi^g$, which we set to $\pi^g = 0.5$. Finally, the two types' beliefs at any given level of heterogeneity N are given by

$$E^o[R] = (\pi^g + N\mu)R^g + (\pi^b - N\mu)R^b$$
$$E^p[R] = (\pi^g - N\mu)R^g + (\pi^b + N\mu)R^b.$$

Notably, we let the social planner apply Pareto weights ω such that the constrained-efficient allocation replicates the unconstrained competitive allocation, i. e. when the collateral constraints are slack. This choice ensures that the simulated corrective interventions by the planner are solely related to inefficiencies from the financial friction, but not to differences in the aggregation of social welfare.

Figure 1.2: Equilibrium allocations, prices, and optimal corrective policies



Notes: The three upper panels show period-0 choices of investment and borrowing as well as the period-1 asset price. The three middle panels show optimal taxes on investment and aggregate distributive and collateral externalities therein. The three middle panels show optimal taxes on borrowing and aggregate distributive and collateral externalities therein. The blue and red lines refer to the optimistic and the pessimistic type, respectively. Solid lines refer to variables from the competitive equilibrium, while dotted lines refer to the constrained-efficient equilibrium. Each number on the x-axis relates to the N-th heterogeneity step, where $N = 0$ stands for the benchmark case of homogeneous beliefs.

Allocations, prices, and corrective policies. Figure 1.2 displays the responses of key variables to different levels of heterogeneity. Specifically, it shows the equilibrium values of period-0 investment and borrowing, the period-1 price q_1 – the main determinant of the collateral constraint – as well as taxes and the externalities therein. The two beliefs diverge increasingly the further one follows the x-axis. The blue and red lines refer to the optimists and pessimists, respectively. Solid lines refer to variables from the competitive equilibrium, while dotted lines refer to the constrained-efficient equilibrium.

The top-left and top-central panels illustrate the monotonicity of period-0 investment and borrowing in beliefs. Starting from a no-trade equilibrium under homogeneous beliefs, where investors keep their initial asset position constant, investment and borrowing increase (decrease) the more optimistic (pessimistic) they become. Contrasting the competitive allocation, the social planner induces agents to trade, borrow, and save less. Importantly, the planner reduces optimists' borrowing by more than pessimists' saving, reflecting the asymmetry between optimistic and pessimistic types' contributions to financial distress, formalized in Proposition 1.4.

In the top-right panel, this asymmetry becomes evident in the response of the equilibrium price q_1 to increasing belief heterogeneity. Given that the influence of optimistic behavior is dominant, the equilibrium price declines even though we have not altered the economy's mean belief, but made the two types more heterogeneous in a symmetric manner. The fact that the equilibrium price q_1 is constantly lower under heterogeneity than under homogeneity further implies that financial distress is aggravated by belief disagreements. The social planner improves on the competitive allocation by sustaining a higher price, alleviating the tightness of the financial friction.

The panels in the second row of Figure 1.2 depict the aggregate distributive and collateral externalities associated with each type's investment and the corresponding corrective policies, formalized in equation (1.15). To achieve constrained efficiency, the planner taxes investment by optimists ($\tau_a^o > 0$), and subsidizes asset purchases by pessimists ($\tau_a^p < 0$). The interplay of aggregate distributive and collateral externalities determine the signs of the instruments. The tax on optimists' investment is driven by negative collateral externalities clearly outweighing positive distributive externalities. The latter arise because pessimists, buying claims in $t = 1$, benefit from the price decline induced by optimists' behavior. However, as the collateral price continues falling with increasing heterogeneity, optimists pass over more intense collateral externalities to pessimists. Pessimists, in contrast, are subsidized because their cautious investment decisions tend to mitigate the price decline, benefiting optimists' budget in $t = 1$, and reducing collateral externalities. Since they behave with more precaution the more pessimistic they become, the social planner is less inclined to correct their behavior, and the subsidy reverts to zero.

The lower panels of Figure 1.2 refer to aggregate externalities associated with borrowing and saving and the respective policy instruments, captured by equation (1.16). By the same mechanisms as for the correction of investment, borrowing by optimists is increasingly taxed ($\tau_d^o > 0$), and borrowing by pessimists is subsidized ($\tau_d^p < 0$).[15] If the two types of investors hold the homogeneous belief, their borrowing is slightly taxed.

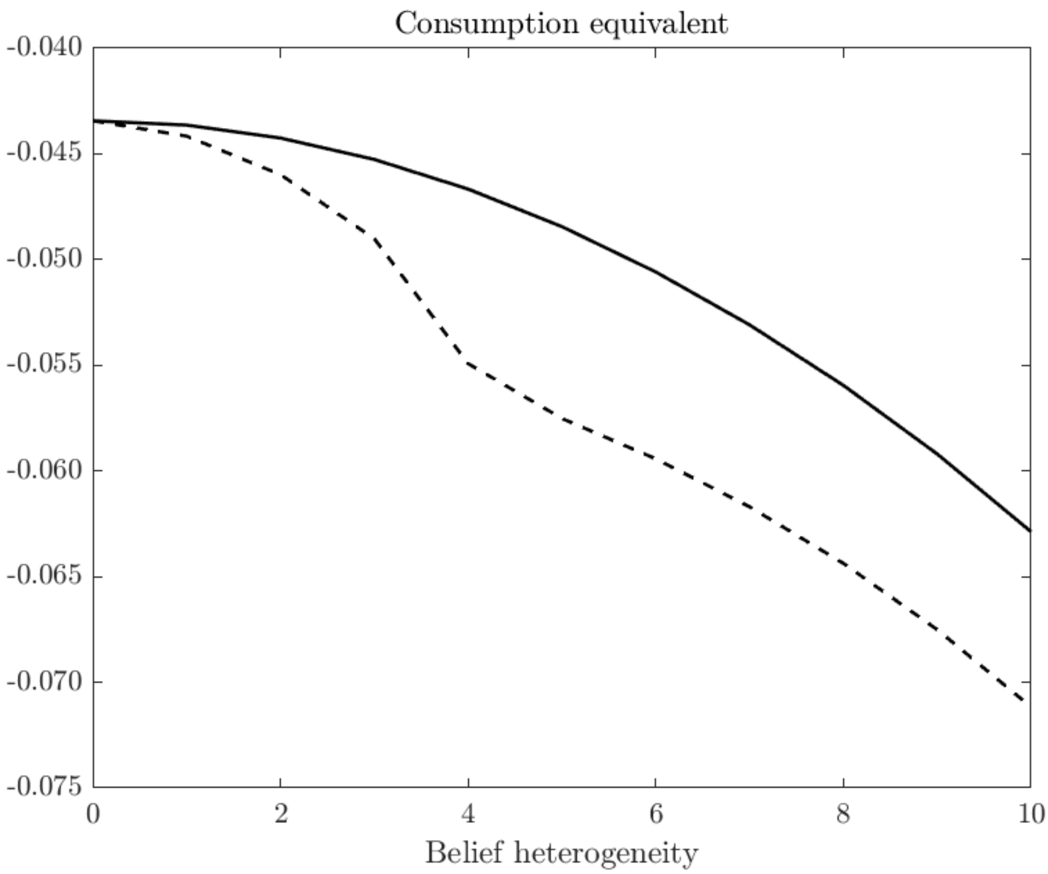
Welfare effects. Thus far, we have qualified both the direction and the extent of corrective taxes. In the following, we turn to the normative question of how the macroprudential

[15] Aggregate distributive and collateral externalities from borrowing turn out to be equal to those from investment in this example due to our assumption $R^b = 0$. In this case, price effects are identical, and so are type-specific externalities (see Definition 1.4).

correction translates into social welfare. We are particularly interested in measuring welfare gains from the non-linear tax policy, addressing individual contributions to financial distress, as opposed to a linear tax system, which is the most frequently proposed instrument in the literature on pecuniary externalities and prudential policy responses, (Bianchi, 2011; Dávila and Korinek, 2018; Jeanne and Korinek, 2019, 2020). This literature typically presumes rational expectations.

In our model, this policy corresponds to the system of linear corrective taxes in the case of homogeneous beliefs (see part (*ii*) of Proposition 1.7). This is when investors feature rational expectations, and the social planner optimally taxes borrowing, while any correction of investment decisions is ineffective. Figure 1.3 displays the welfare effects of this policy in comparison to the non-linear tax system.

Figure 1.3: Welfare effects of linear and non-linear corrective taxes



Notes: This figure shows the consumption equivalents of two types of allocations relative to the unconstrained competitive allocation. The solid line refers to constrained-efficient allocations, which are implemented by means of the system of non-linear taxes proposed in Theorem 1.2. The dotted line refers to allocations implemented by means of the system of linear taxes proposed in part (*ii*) of Proposition 1.7. Each number on the x-axis relates to the N-th heterogeneity step, where $N = 0$ stands for the benchmark case of homogeneous beliefs.

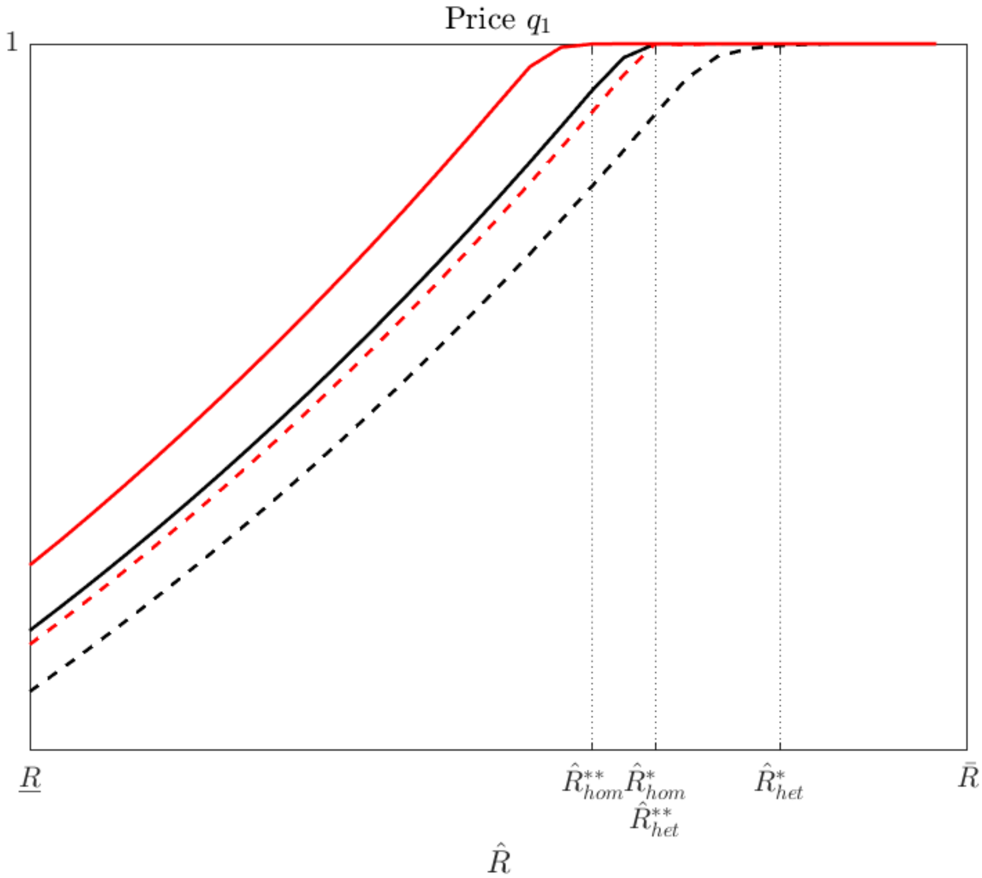
We employ consumption equivalents relative to the unconstrained competitive allocation, which is when no policy intervention is required, as an ex ante social welfare measure. In Figure 1.3, the solid line depicts consumption equivalents of allocations with non-linear corrective taxes, while the dotted line refers to allocations with linear corrective taxes. Each point on the x-axis indicates a specific belief distribution, with beliefs becoming increasingly heterogeneous along the axis.

We find significant efficiency gains of non-linear over linear macroprudential taxes. The planner's intervention contains welfare losses at a level of about four to six percent relative to the unconstrained economy. However, if linear taxes are applied to a heterogeneous population, welfare is well below. As a linear policy cannot address individual contributions to financial distress, the corresponding allocations result in welfare losses which are by up to 14 percent larger than compared to allocations under a non-linear policy.

Probability of financial distress. The last numerical exercise we provide is related to the above evaluation how probable financial distress is in the competitive equilibrium. We have found that belief disagreements across investors do indeed raise the probability that financial distress occurs, relative to the case of rational and homogeneous beliefs. We repeat the simulation from above, but further account for the constrained-efficient allocation. To that end, we first define the lowest possible realization of R such that collateral constraints in the constrained-efficient allocation are slack.

Definition 1.6. *Let* $\hat{R}^{**}_{het} \equiv \min\left\{\hat{R} \mid \tilde{\eta}^j_1 = 0 \textit{ for all } j\right\}$ *and* $\hat{R}^{**}_{hom} \equiv \min\left\{\hat{R} \mid \tilde{\eta}_1 = 0\right\}$ *be the lowest possible realizations of* R *such that the borrowing constraints are slack in the constrained-efficient equilibrium if investors hold heterogeneous beliefs* $\mathcal{F}$ *or the homogeneous belief* F*, respectively.*

Figure 1.4: Mapping from $\hat{\boldsymbol{R}}$ to $\boldsymbol{q_1}$ in the constrained-efficient equilibrium



Notes: This figure shows the mapping from $\hat{R}$ to q_1 for the two cases when investors hold the homogeneous belief $F(R)$ or heterogeneous beliefs $\mathcal{F}$, respectively. Solid lines refer to the homogeneous case, and dashed lines refer to the heterogeneous case. Black lines refer to the competitive equilibrium, and red lines refer to the constrained-efficient equilibrium. $\hat{R}^{*}_{hom}$, $\hat{R}^{*}_{het}$, $\hat{R}^{**}_{hom}$, and $\hat{R}^{**}_{het}$ are thresholds as defined in Definitions 1.3 and 1.6.

Figure 1.4 illustrates the mapping from the realization $\hat{R}$ to q_1 for both the competitive (black lines) and the constrained-efficient equilibrium (red lines). The probability of financial distress is indeed lower under constrained efficiency than in the competitive equilibrium. By manipulating investors' behavior through non-linear taxes, the social planner manages to reduce the thresholds of $\hat{R}$, implying that financial distress in the constrained-efficient equilibrium would only arise in markedly more unfavorable states. Our previous finding that financial distress is generally less likely under the homogeneous belief than under heterogeneity is further robust to the planner intervention.

1.5 Conclusion

This chapter presents a theoretical framework to study the belief channel of financial distress. We build on a model incorporating financial frictions, and enrich it by the heterogeneity of beliefs across economic agents. This framework allows us to characterize individual contributions to financial distress, which is the basis for the subsequent analysis. We employ the model to analyze the competitive equilibrium, its sensitivity to changes in the underlying set of beliefs, as well as its efficiency properties. We derive optimal corrective policies, which are furthermore quantified in a numerical application.

The main result is that belief disagreements increase the likelihood and the extent of financial distress. Specifically, crises occur under less severe macroeconomic shocks, and the associated loss of efficiency is larger than in an economy populated by homogeneous individuals. This finding rests upon the fact that economic agents contribute to financial distress asymmetrically, with optimistic agents making larger contributions than pessimistic agents. That has notable implications for macroprudential policy. We show that, while corrective policy interventions are generally efficiency-enhancing, type-specific policies generate additional efficiency gains. Against this background, we propose a system of non-linear, i. e. type-specific, macroprudential taxes. This policy outperforms linear taxes, which are typically proposed in the literature, in reducing indebtedness and stabilizing collateral prices.

These results add to the literature on financial crises in several ways. We characterize explicitly how financial market participants contribute to distress states. Moreover, in our setting, financial constraints may be binding through ex ante return expectations sufficiently off the ex post realization. This differs from former studies, focusing on financial distress in response to aggregate shocks to investment or net worth. Hence, our framework formalizes a further source of financial distress. Ultimately, our policy proposal improves on linear macroprudential taxes in an economy featuring heterogeneity of beliefs. The latter point is especially relevant when studying optimal financial regulation in booms and busts, which typically go along with high belief divergence and fluctuations.

Our work lays the ground for further research. Whereas we study optimal ex ante policies in a prudential sense, it may be worthwhile examining optimal ex post policies, such as central bank liquidity injections, under belief heterogeneity. In addition, several types of financial frictions are considered in the literature on prudential policies. The collateral constraints used in this chapter link debt issuance to market-valued collateral. However, pecuniary externalities and corrective policies have further been studied in environments with flow constraints, relating to household income or firm cash flows. Their interaction with belief disagreements must still be examined. Ultimately, our three period model may be extended to a dynamic framework, allowing for a more profound quantitative exploration of the effects documented in this chapter.

Appendix 1.A Proofs and derivations

1.A.1 Proof of Proposition 1.1

Models with price-dependent collateral constraints like ours bear the risk that equilibrium prices do not exist. The reason is that these models face downward-sloping supply functions. Constraint agents must sell more if the collateral price is low, but less if it high, and the constraint is less tight.

Existence. We first prove the existence of the equilibrium price. Let

$$S\left(q_1\right) = \sum_{j=1}^{J} \mathbb{1}_{\left\{\tilde{l}_1^j(q_1)>0\right\}} s^j \tilde{l}_1^j\left(q_1\right)$$

denote the supply of claims as a function of q_1. Analogously, define demand as

$$D\left(q_1\right) = -\sum_{j=1}^{J} \mathbb{1}_{\left\{\tilde{l}_1^j(q_1)<0\right\}} s^j \tilde{l}_1^j\left(q_1\right).$$

Let $D\left(q_1\right)$ and $S\left(q_1\right)$ be continuous and differentiable functions on the interval $(0, 1]$. Note that $S\left(q_1\right)$ is bounded from above for any q_1. This follows from the fact that investors cannot sell more claims than they possess, i. e. $\tilde{l}_1^j \leq \tilde{a}_0^j$, and, hence, for any q_1

$$S\left(q_1\right) = \sum_{j=1}^{J} \mathbb{1}_{\left\{\tilde{l}_1^j(q_1)>0\right\}} s^j \tilde{l}_1^j\left(q_1\right) \leq \sum_{j=1}^{J} s^j \tilde{a}_0^j = \bar{a}.$$

Specifically, it follows that $\lim_{q_1\to 0} S\left(q_1\right) \leq \bar{a}$.

We consider two cases when characterizing the demand curve. First, if demand is zero, there is still excess supply. According to the Walrasian equilibrium definition, all prices q_1 are equilibrium prices. Second, if demand is positive, we ensure the existence of an equilibrium price q_1 by showing that demand is infinite as the price approaches zero, i. e. $\lim_{q_1\to 0} D\left(q_1\right) = \infty$. First, note that buyers will exhaust their entire borrowing limit as they trade, i. e. $\tilde{d}_1^j = \phi q_1 \left(\tilde{a}_0^j - \tilde{l}_1^j\right)$, because any price $q_1 < 1$ grants them a pecuniary benefit. From the period-2 budget constraint (1.2), we obtain

$$\tilde{c}_2^j = \left(1 - \phi q_1\right)\left(\tilde{a}_0^j - \tilde{l}_1^j\right). \tag{A1.1}$$

Suppose the price approaches its lower limit of zero, i. e. $q_1 \to 0$. From the price equation (1.6), it follows that either the numerator tends to zero, i. e. $u'\left(\tilde{c}_2^j\right) \to 0$, or the denominator tends to infinity, i. e. $(1-\phi)u'\left(\tilde{c}_1^j\right)+\phi u'\left(\tilde{c}_2^j\right)\to\infty$, or both. If the numerator tends to zero, the concavity of $u\left(\tilde{c}_t^j\right)$ implies that $\tilde{c}_2^j$ becomes infinitely large, i. e. $\tilde{c}_2^j\to\infty$, and, by (A1.1), so does the demand for claims, i. e. $\tilde{l}_1^j\to-\infty$. If, in contrast, the denominator tends to infinity, this can be caused by consumption in $t=1$ and $t=2$ approaching zero, i. e. either $\tilde{c}_1^j\to 0$ or $\tilde{c}_2^j\to 0$. In the first case, all consumption is shifted to the final period, i. e. $\tilde{c}_2^j\to\infty$, from which an infinite demand for claims, i. e. $\tilde{l}_1^j\to-\infty$, follows again. In the second case, both numerator and denominator of the pricing equation (1.6) would tend to infinity, yet the numerator at a faster pace as $\phi<1$, and, consequently, the assumption $q_1\to 0$ would be violated. Thus, at the minimum price of $q_1\to 0$, period-2 consumption $\tilde{c}_2^j$ will tend to infinity and $\tilde{l}_1^j$ will tend to minus infinity for all j with $\tilde{l}_1^j<0$. We conclude that overall demand for claims becomes infinitely large, i. e. $\lim_{q_1\to 0} D\left(q_1\right)=\infty$.

All in all, for $q_1\to 0$, we obtain a bounded supply and an infinitely high demand. It is only required to ensure that this demand exists. We ensure a positive mass of $D(0)$ through assuming that at least one type of investors has had correct expectations ex post, receiving a return that is as high as expected or higher. Formally, $E^j[R]\leq\hat{R}$ for at least one j and all realizations $\hat{R}$ of R ensures that there is at least one group that has sufficient funds available in period $t=1$ to demand claims on the asset.

There are different possibilities how supply and demand can intersect. Either $D\left(q_1\right)$ and $S\left(q_1\right)$ intersect on $(0,1]$ at (possibly multiple) price(s). Then, all prices in this set are equilibrium prices. Or they do not have an intersection on the interval. We have shown that, in this case, demand is permanently larger than supply, i. e. $D\left(q_1\right)>S\left(q_1\right)$ for any $q_1\in(0,1]$ as $D(0)>S(0)$ and there is no intersection on $(0,1]$. Hence, $q_1=1$ is the equilibrium price since, for this price, buying investors are indifferent between all levels of feasible demand, and the bounded supply $S(1)<D(1)$ can be fully met. In conclusion, we have shown that the equilibrium price exists.

Uniqueness. Second, we prove that the equilibrium price is unique and satisfies $q_1\leq 1$ in the case of positive demand. Uniqueness is ensured if, first, $\lim_{q_1\to 0} D\left(q_1\right)=\infty$, second, $D\left(1\right)=S\left(1\right)=0$, and third, if $D\left(q_1\right)$ and $S\left(q_1\right)$ are monotonically decreasing functions on $(0,1]$ with $\frac{\partial D(1)}{\partial q_1}=\frac{\partial S(1)}{\partial q_1}=0$. We continue assuming their continuity and differentiability.

Regarding the first two conditions, we have shown $\lim_{q_1\to 0} D\left(q_1\right)=\infty$ in the previous part, and $D\left(1\right)=S\left(1\right)=0$ follows from our assumption $\tilde{l}_1^j(1)=0$ for all j. Next, we prove that both supply and demand are monotonic functions on $(0,1]$. Specifically, we determine the signs of

$$\frac{\partial S(q_1)}{\partial q_1} = \sum_{j=1}^{J} \mathbb{1}_{\left\{\tilde{l}_1^j(q_1)>0\right\}} s^j \frac{\partial \tilde{l}_1^j}{\partial q_1} \tag{A1.2}$$

$$\frac{\partial D(q_1)}{\partial q_1} = -\sum_{j=1}^{J} \mathbb{1}_{\left\{\tilde{l}_1^j(q_1)<0\right\}} s^j \frac{\partial \tilde{l}_1^j}{\partial q_1}. \tag{A1.3}$$

Using the period-1 equilibrium conditions (1.1), (1.2), (1.3), and (1.9), and applying the implicit function theorem to (1.5), we obtain

$$\frac{\partial \tilde{l}_1^j}{\partial q_1} = \frac{1}{1+(1-2\phi)\,q_1}\left[\frac{1}{(1-\phi q_1)\,Aq_1} - 2\phi\tilde{a}_0^j + (2\phi-1)\tilde{l}_1^j\right] \tag{A1.4}$$

Inserting (A1.4) into (A1.2) and (A1.3) yields

$$\frac{\partial S(q_1)}{\partial q_1} = \frac{1}{1+(1-2\phi)\,q_1}\left[\frac{J^S}{(1-\phi q_1)\,Aq_1} - 2\phi\sum_{j=1}^{J}\mathbb{1}_{\left\{\tilde{l}_1^j(q_1)>0\right\}} s^j\tilde{a}_0^j + (2\phi-1)S(q_1)\right] \tag{A1.5}$$

$$\frac{\partial D(q_1)}{\partial q_1} = -\frac{1}{1+(1-2\phi)\,q_1}\left[\frac{J^D}{(1-\phi q_1)\,Aq_1} - 2\phi\sum_{j=1}^{J}\mathbb{1}_{\left\{\tilde{l}_1^j(q_1)<0\right\}} s^j\tilde{a}_0^j + (2\phi-1)D(q_1)\right], \tag{A1.6}$$

where J^S and J^D are the number of types that are on the supply and the demand side of the market, respectively. We assume that the margin requirement is sufficiently tight, i. e. $\phi < 1/2$.

We first show that the supply curve is a weakly decreasing function of q_1. Recall that $S(q_1)$ is continuous on $(0,1]$, $\lim_{q_1\to 1} S(q_1) = 0$, and an equilibrium with positive demand $D(q_1) > 0$ requires that there is a q_1 such that $S(q_1) > 0$. Hence, there must further be a $q_1^* \equiv \min\left\{q_1 \mid \frac{\partial S(q_1)}{\partial q_1} < 0 \textit{ for all } q_1 > q_1^*\right\}$.

Now we distinguish two cases. If $\frac{\partial S(q_1^*)}{\partial q_1} \neq 0$, there is no $q_1 < q_1^*$ such that $\frac{\partial S(q_1)}{\partial q_1} > 0$, and it follows $\frac{\partial S(q_1)}{\partial q_1} \leq 0$ for all $q_1 \in (0,1]$, making the supply curve monotonically decreasing. If, however, $\frac{\partial S(q_1^*)}{\partial q_1} = 0$, this is equivalent to $S(q_1^*) = \frac{1}{2\phi-1}\left[2\phi\sum_{j=1}^{J}\mathbb{1}_{\left\{\tilde{l}_1^j(q_1^*)\geq 0\right\}} s^j\tilde{a}_0^j - \frac{J^S}{(1-\phi q_1^*)Aq_1^*}\right]$. For $q_1 < q_1^*$, we prove by contradiction that supply is constant.

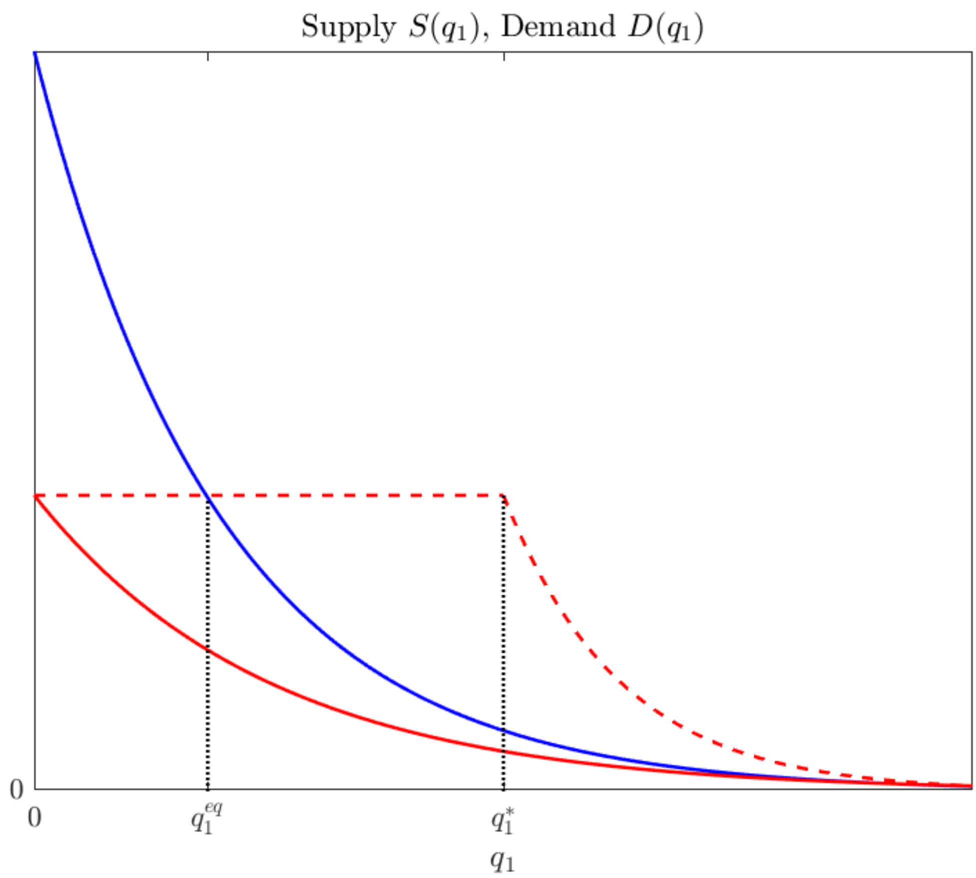
First suppose that $\frac{\partial S(q_1)}{\partial q_1} > 0$. From (A1.5), it follows that $S(q_1) > S(q_1^*)$ in this case, which would imply $\frac{\partial S(q_1)}{\partial q_1} < 0$, violating the assumption. Now suppose that $\frac{\partial S(q_1)}{\partial q_1} < 0$. From (A1.5), it follows that $S(q_1) < S(q_1^*)$ in this case, which would imply $\frac{\partial S(q_1)}{\partial q_1} > 0$, violating the assumption. Therefore, we obtain $\frac{\partial S(q_1)}{\partial q_1} = 0$ for all $q_1 < q_1^*$. The constancy of supply for low collateral prices reflects the fact that supply is bounded from above by the amount invested in $t = 0$. q_1^* is thus the price below which distressed investors are willing to liquidate their entire position.

The slope of the demand curve, i. e. the sign of the left-hand side of equation (A1.6), is determined by the term in brackets. Under the assumption of $\phi < 1/2$, and restricting the initial endowment to $\bar{a} \leq 2$, the term in brackets is positive, yielding $\frac{\partial D(q_1)}{\partial q_1} < 0$ for any $q_1 \in (0, 1]$.

Lastly, equations (A1.5) and (A1.6) reveal that $\frac{\partial D(1)}{\partial q_1} = \frac{\partial S(1)}{\partial q_1} = 0$ because $J^S = J^D = \mathbb{1}_{\{\tilde{l}_1^j(1)>0\}} = \mathbb{1}_{\{\tilde{l}_1^j(1)<0\}} = S(1) = D(1) = 0$ at $q_1 = 1$.

Since all the conditions for uniqueness are satisfied, we deduce that the equilibrium price is unique (see Figure A1.1 for illustration).

Figure A1.1: Supply and demand in $\boldsymbol{t} = \mathbf{1}$



Notes: This figure sketches two possible supply curves and a demand curve in period $t = 1$. Supply curves are depicted in red, while the demand curve is depicted in blue. q_1^{eq} is the equilibrium price, and q_1^* is defined as in the proof of Proposition 1.1.

Equivalences. Third, we show the two equivalences in part (ii). For part (i), suppose $q_1 = 1$. Combining equations (1.4) and (1.5) yields $\tilde{\eta}_1^j = \tilde{\eta}_1^j \phi$. The only solution for the latter condition is $\tilde{\eta}_1^j = 0$. Now, suppose $\tilde{\eta}_1^j = 0$. Equation (1.4) then becomes $u'\left(\tilde{c}_1^j\right) = u'\left(\tilde{c}_2^j\right)$. Substituting out $u'\left(\tilde{c}_2^j\right)$ in equation (1.5) yields $q_1 = 1$.

For part (ii), the equivalence is shown formally:

$$\begin{aligned} q_1 &= \frac{u'\left(\tilde{c}_2^j\right)}{(1-\phi)u'\left(\tilde{c}_1^j\right) + \phi u'\left(\tilde{c}_2^j\right)} < 1 \\ \iff & (1-\phi)u'\left(\tilde{c}_2^j\right) < (1-\phi)u'\left(\tilde{c}_1^j\right) \\ \iff & 0 < u'\left(\tilde{c}_1^j\right) - u'\left(\tilde{c}_2^j\right) = \tilde{\eta}_1^j. \end{aligned}$$

□

1.A.2 Proof of Proposition 1.2

For the proof of part (i), recall that the period-1 equilibrium price satisfies equation (1.6), where $\tilde{c}_1^j$ and $\tilde{c}_2^j$ are given by equations (1.1) and (1.2) for all j. Since the equilibrium price equals one if $\tilde{\eta}_1^j = 0$, we restrict ourselves to price effects in the case of $\tilde{\eta}_1^j > 0$. For the borrowing constraint to be binding, assume that the realization $\hat{R}$ is sufficiently adverse, satisfying $\hat{R} < 1$. Using CARA $A = -\frac{u''\left(\tilde{c}_t^j\right)}{u'\left(\tilde{c}_t^j\right)}$ for all j and t, we obtain the following equilibrium price derivatives:

$$\frac{\partial q_1}{\partial \tilde{a}_0^j} = \frac{(1-\phi)(1-R)\left(q_1\right)^2}{\frac{u'\left(\tilde{c}_2^j\right)}{u''\left(\tilde{c}_1^j\right)} + (1-\phi)\left(q_1\right)^2 \tilde{l}_1^j} \tag{A1.7}$$

$$\frac{\partial q_1}{\partial \tilde{d}_0^j} = \frac{(1-\phi)\left(q_1\right)^2}{\frac{u'\left(\tilde{c}_2^j\right)}{u''\left(\tilde{c}_1^j\right)} + (1-\phi)\left(q_1\right)^2 \tilde{l}_1^j}. \tag{A1.8}$$

The numerators of equations (A1.7) and (A1.8) are positive, and the denominator is negative. To see this, note that $\frac{\partial q_1}{\partial \tilde{c}_1^j} = -(1-\phi)\frac{u''\left(\tilde{c}_1^j\right)}{u'\left(\tilde{c}_2^j\right)}\left(q_1\right)^2 > 0$. For the denominator, it follows

$$\begin{aligned} & \frac{u'\left(\tilde{c}_2^j\right)}{u''\left(\tilde{c}_1^j\right)} + (1-\phi)\left(q_1\right)^2 \tilde{l}_1^j \leq 0 \\ \iff & 1 \geq \frac{\partial q_1}{\partial \tilde{c}_1^j}\tilde{l}_1^j, \end{aligned} \tag{A1.9}$$

which is always satisfied. If $\tilde{l}_1^j \leq 0$, the left-hand side of (A1.9) is negative. But it is exceeded by one even if $\tilde{l}_1^j > 0$. The reason is that $1 \geq \frac{\partial q_1}{\partial \tilde{c}_1^j}\tilde{l}_1^j$ is the condition for finite consumption $\tilde{c}_1^j$. Consider the period-1 budget constraint $\tilde{c}_1^j = R\tilde{a}_0^j + q_1\tilde{l}_1^j + \tilde{d}_1^j - \tilde{d}_0^j$. Increasing the budget by one unit of the consumption good has two effects. First, it directly increases consumption by one unit. Second, it raises q_1, and further increases consumption by $\frac{\partial q_1}{\partial \tilde{c}_1^j}\tilde{l}_1^j$. Suppose $1 < \frac{\partial q_1}{\partial \tilde{c}_1^j}\tilde{l}_1^j$. In this case, the latter effect via q_1 dominates the direct effect, and the initial stimulus initiated an upward loop towards infinite consumption $\tilde{c}_1^j$. Hence, a finite solution requires $1 \geq \frac{\partial q_1}{\partial \tilde{c}_1^j} \tilde{l}_1^j$, concluding the proof of part (i).

Turning to part (ii), for the equilibrium price derivative with respect to borrowing under a homogeneous belief, we obtain

$$\frac{\partial q_1}{\partial \tilde{d}_0} = \frac{(1-\phi)\,(q_1)^2}{\frac{u'(\tilde{c}_2)}{u''(\tilde{c}_1)}},$$

which is negative for a concave utility function. □

1.A.3 Proof of Proposition 1.3

For the proof of part (i), let investors hold heterogeneous beliefs $\mathcal{F}$. The individual type-j decisions for investment and borrowing are governed by equations (1.8) and (1.9), that we rewrite as functions of its belief $F^j(R)$ in the following way:

$$q_0 u'\left(\tilde{c}_0^j\left(\tilde{a}_0^j\left(F^j(R)\right), \tilde{d}_0^j\left(F^j(R)\right)\right)\right) = \int_{\underline{R}}^{\overline{R}} R u'\left(\tilde{c}_1^j\left(\tilde{a}_0^j\left(F^j(R)\right), \tilde{d}_0^j\left(F^j(R)\right)\right)\right) \ldots$$

$$\ldots + u'\left(\tilde{c}_2^j\left(\tilde{a}_0^j\left(F^j(R)\right)\right)\right) + \tilde{\eta}_1^j\left(\tilde{a}_0^j\left(F^j(R)\right), \tilde{d}_0^j\left(F^j(R)\right)\right)\phi q_1 dF^j(R) \qquad \text{(A1.10)}$$

$$u'\left(\tilde{c}_0^j\left(\tilde{a}_0^j\left(F^j(R)\right), \tilde{d}_0^j\left(F^j(R)\right)\right)\right) = \int_{\underline{R}}^{\overline{R}} u'\left(\tilde{c}_1^j\left(\tilde{a}_0^j\left(F^j(R)\right), \tilde{d}_0^j\left(F^j(R)\right)\right)\right) dF^j(R). \qquad \text{(A1.11)}$$

Notably, period-0 choices $\tilde{a}_0^j\left(F^j(R)\right)$ and $\tilde{d}_0^j\left(F^j(R)\right)$ are direct functions of type j's belief, while period-1 and period-2 variables are both indirect functions of $F^j(R)$ via $\tilde{a}_0^j\left(F^j(R)\right)$ and $\tilde{d}_0^j\left(F^j(R)\right)$ direct functions of it through the expectation operator.

In the following, we apply the calculus of variation, as explained in the main text. Consider a perturbation to beliefs of the form $F^j(R)+\epsilon G^j(R)$, where $\epsilon > 0$ is an arbitrary number, and $G^j(R)$ captures the direction of the perturbation. $F^j(R)+\epsilon G^j(R)$ is required to be a valid cdf for small enough ϵ, so we assume it is continuous and differentiable, it satisfies $G(\underline{R}) = G(\overline{R}) = 0$, and $\partial\left(F^j(R) + \epsilon G^j(R)\right)/\partial R \geq 0$ for sufficiently small ϵ. Lastly, let δ denote the operator for functional derivatives.

We characterize the variational derivatives of investment and borrowing choices when beliefs $F^j(R)$ are perturbed with direction $G^j(R)$, i. e. $\frac{\delta \tilde{a}_0^j}{\delta F^j} \cdot G^j$ and $\frac{\delta \tilde{d}_0^j}{\delta F^j} \cdot G^j$. Optimism and pessimism are measured relative to each other in the sense of first order stochastic dominance. A perturbation $G^j(R)$ makes type-j investors more optimistic if and only if it satisfies $F^j(R) + \epsilon G^j(R) \leq F^j(R)$ for all R. It is easy to see that more optimism requires the perturbation to have a negative direction, i. e. $G^j(R) \leq 0$ for all R. Analogously, investors of type j are made more pessimistic through a perturbation with direction $G^j(R) \geq 0$ for all R.

Applying the implicit function theorem to (A1.10) and (A1.11), and combining the resulting expressions yield

$$\frac{\delta \tilde{a}_0^j}{\delta F^j} \cdot G^j = \frac{\int_{\underline{R}}^{\overline{R}} u''\left(\tilde{c}_1^j\right) \tilde{a}_0^j G^j(R) dR \cdot \left(\int_{\underline{R}}^{\overline{R}} (1+\phi) u''\left(\tilde{c}_1^j\right) dF^j(R) + q_0 u''\left(\tilde{c}_0^j\right)\right)}{\left(\int_{\underline{R}}^{\overline{R}} R u''\left(\tilde{c}_1^j\right) dF^j(R) + q_0 u''\left(\tilde{c}_0^j\right)\right) \cdot \left(\int_{\underline{R}}^{\overline{R}} (1+\phi) u''\left(\tilde{c}_1^j\right) dF^j(R) + q_0 u''\left(\tilde{c}_0^j\right)\right)} \cdots$$

$$\cdots \frac{-\int_{\underline{R}}^{\overline{R}} \left(u'\left(\tilde{c}_1^j\right) + \left(R + \phi q_1\right) u''\left(\tilde{c}_1^j\right) \tilde{a}_0^j\right) G^j(R) dR \cdot \left(\int_{\underline{R}}^{\overline{R}} u''\left(\tilde{c}_1^j\right) dF^j(R) + u''\left(\tilde{c}_0^j\right)\right)}{-\left(\int_{\underline{R}}^{\overline{R}} \left(R + \phi q_1\right) R u''\left(\tilde{c}_1^j\right) + \left(1 - \phi q_1\right) u''\left(\tilde{c}_2^j\right) dF^j(R)\right) \cdot \left(\int_{\underline{R}}^{\overline{R}} u''\left(\tilde{c}_1^j\right) dF^j(R) + u''\left(\tilde{c}_0^j\right)\right)} \tag{A1.12}$$

$$\frac{\delta \tilde{d}_0^j}{\delta F^j} \cdot G^j = \frac{-\int_{\underline{R}}^{\overline{R}} u''\left(\tilde{c}_1^j\right) \tilde{a}_0^j G^j(R) dR}{u''\left(\tilde{c}_0^j\right) + \int_{\underline{R}}^{\overline{R}} u''\left(\tilde{c}_1^j\right) dF^j(R)} + \frac{\int_{\underline{R}}^{\overline{R}} R u''\left(\tilde{c}_1^j\right) dF^j(R) + q_0 u''\left(\tilde{c}_0^j\right)}{u''\left(\tilde{c}_0^j\right) + \int_{\underline{R}}^{\overline{R}} u''\left(\tilde{c}_1^j\right) dF^j(R)} \cdot \frac{\delta \tilde{a}_0^j}{\delta F^j} \cdot G^j . \tag{A1.13}$$

First, we further investigate equation (A1.12). Assuming that the choice of parameters ensures a non-zero trading volume, i. e. $A < 1$ and beliefs $\mathcal{F}$ sufficiently divergent such that $\bar{a} - \tilde{a}_0^j \neq 0$ for some j, and that the borrowing constraints bind in response to the adverse shock, i. e. $\hat{R} < 1$ and $\phi < \frac{1}{2}$ such that $\tilde{\eta}_1^j > 0$ for all j, the numerator is negative for $G^j(R) \leq 0$, and positive for $G^j(R) \geq 0$. The denominator is always negative. Hence, the functional derivative $\frac{\delta \tilde{a}_0^j}{\delta F^j} \cdot G^j$ is positive for $G^j(R) \leq 0$ and negative for $G^j(R) \geq 0$.

Given the signs of the components in (A1.13), it follows that $\frac{\delta \tilde{a}_0^j}{\delta F^j} \cdot G^j$ and $\frac{\delta \tilde{d}_0^j}{\delta F^j} \cdot G^j$ have the same sign for each $G^j(R)$. Consequently, the two variational derivatives in (A1.12) and (A1.13) turn out to be positive if investors are more optimistic ($G^j(R) \leq 0$), and negative if they are more pessimistic ($G^j(R) \geq 0$).

Proving part (*ii*), we employ the identical procedure as above. Let investors hold the homogeneous belief $F(R)$. Let further $G(R)$ be the direction of a perturbation of the homogeneous belief. We obtain as the functional derivative of borrowing

$$\frac{\delta \tilde{d}_0}{\delta F} \cdot G = \frac{-\int_{\underline{R}}^{\overline{R}} u''\left(\tilde{c}_1\right) \bar{a} G(R) dR}{u''\left(\tilde{c}_0\right) + \int_{\underline{R}}^{\overline{R}} u''\left(\tilde{c}_1\right) dF(R)},$$

which is as well positive for more optimistic investors ($G^j(R) \leq 0$) and negative for more pessimistic investors ($G^j(R) \geq 0$). □

1.A.4 Proof of Proposition 1.4

With regard to part (*i*), let investors hold heterogeneous beliefs $\mathcal{F}$. Let further $G^j(R)$ be the direction of a perturbation of type-j investors' belief $F^j(R)$, and beliefs $F^i(R)$ be constant for all $i \neq j$.

Recall that the functional derivative $\frac{\delta}{\delta F^j} \cdot G^j$ describes a gradient, so it is identical to a partial derivative if the functional argument is one-dimensional. We write the period-1 equilibrium price as a function of beliefs, i. e. $q_1 = q_1\left(\tilde{a}_0(\mathcal{F}), \tilde{d}_0(\mathcal{F})\right)$. It follows

$$\frac{\delta q_1}{\delta F^j} \cdot G^j = \frac{\delta q_1}{\delta \tilde{a}_0^j} \cdot \frac{\delta \tilde{a}_0^j}{\delta F^j} \cdot G^j + \frac{\delta q_1}{\delta \tilde{d}_0^j} \cdot \frac{\delta \tilde{d}_0^j}{\delta F^j} \cdot G^j = \frac{\partial q_1}{\partial \tilde{a}_0^j} \cdot \frac{\delta \tilde{a}_0^j}{\delta F^j} \cdot G^j + \frac{\partial q_1}{\partial \tilde{d}_0^j} \cdot \frac{\delta \tilde{d}_0^j}{\delta F^j} \cdot G^j. \quad \text{(A1.14)}$$

Using Propositions 1.2 and 1.3, we obtain statement (1) of part (*i*).

For statement (2), let $G^j(R) < 0 < G^i(R)$ with $|G^j(R)| = |G^i(R)|$ for all R be the directions of two perturbations that make investors of type j more optimistic and investors of type i more pessimistic by the same magnitude. We investigate each factor in the two summands on the right-hand side of equation (A1.14) separately. First, note that equations (A1.12) and (A1.13) imply that

$$\left|\frac{\delta \tilde{a}_0^j}{\delta F^j} \cdot G^j\right| = \left|\frac{\delta \tilde{a}_0^i}{\delta F^i} \cdot G^i\right| \text{ and } \left|\frac{\delta \tilde{d}_0^j}{\delta F^j} \cdot G^j\right| = \left|\frac{\delta \tilde{d}_0^i}{\delta F^i} \cdot G^i\right|.$$

Second, taking the derivatives of equations (A1.7) and (A1.8) shows that q_1 is a (decreasing and) concave function of investment and borrowing, i. e. $\frac{\partial^2 q_1}{\partial^2 \tilde{a}_0^j} \leq 0$ and $\frac{\partial^2 q_1}{\partial^2 \tilde{d}_0^j} \leq 0$. As for any concave function, it follows that

$$\left|\frac{\delta q_1}{\delta \tilde{a}_0^j}\right| > \left|\frac{\delta q_1}{\delta \tilde{a}_0^i}\right| \text{ and } \left|\frac{\delta q_1}{\delta \tilde{d}_0^j}\right| > \left|\frac{\delta q_1}{\delta \tilde{d}_0^i}\right|.$$

Inserting the two former results in equation (A1.14) yields statement (2).

To prove part (*ii*), let investors hold the homogeneous belief $F(R)$. Let further $G(R)$ be the direction of a perturbation of the homogeneous belief. Equation (A1.14) simplifies to

$$\frac{\delta q_1}{\delta F} \cdot G = \frac{\partial q_1}{\partial \tilde{d}_0} \cdot \frac{\delta \tilde{d}_0}{\delta F} \cdot G,$$

which is negative for $G(R) \leq 0$ and positive for $G(R) \geq 0$ by the same arguments as in statement (1) of part (*i*). □

1.A.5 Proof of Theorem 1.1

We start out by proving that $\hat{R}^*_{het} > \hat{R}^*_{hom}$, where $\hat{R}^*_{het}$ and $\hat{R}^*_{hom}$ are defined in Definition 1.3. Consider a population with investors holding heterogeneous beliefs $\mathcal{F}$. Let $\hat{R}^{*j}_{het}$ denote the lowest possible realization $\hat{R}$ such that the collateral constraint of type-j investors is slack, i. e. $\tilde{\eta}_1^j = 0$ and $q_1 = 1$, which are equivalent to $\tilde{c}_1^j = \tilde{c}_2^j$. At this point, the borrowing constraint yields $\tilde{d}_1^j = \phi \tilde{a}_0^j$. Using this, and equating the budget constraints (1.1) and (1.2), one obtains $\hat{R}^{*j}_{het} = 1 - 2\phi + \frac{\tilde{d}_0^j}{\tilde{a}_0^j}$.

Given the result from Proposition 1.1, it suffices that one type of investors is constrained to make all investors constrained. We refer to this situation as financial distress, and it follows that $\hat{R}^*_{het} = \max \left\{ \hat{R}^{*j}_{het} \right\}_{j \in \{1,...,J\}}$. Assuming without loss of generality that investors are ordered from more to less optimistic types, i. e. $F^1(R) < ... < F^J(R)$ for all R, we obtain $\hat{R}^*_{het} = \hat{R}^{*1}_{het}$. For the homogeneous case, we derive $\hat{R}^*_{hom} = 1 - 2\phi + \frac{\tilde{d}_0}{\bar{a}}$ equivalently.

To show that $\hat{R}^*_{het} > \hat{R}^*_{hom}$, it is sufficient to prove that $\frac{\tilde{d}^1_0}{\tilde{a}^1_0} > \frac{\tilde{d}_0}{\bar{a}}$. Since type $j = 1$ is the most optimistic type, we know that $\tilde{a}^1_0 > \bar{a}$ and $\tilde{d}^1_0 > \tilde{d}_0$. To prove that $\frac{\tilde{d}^1_0}{\tilde{a}^1_0} > \frac{\tilde{d}_0}{\bar{a}}$, we show that $\tilde{d}^1_0 - \tilde{d}_0 > \tilde{a}^1_0 - \bar{a}$.

The latter statement would follow if a perturbation, making a specific belief more optimistic, i. e. $G^1(R) < 0$ for all R, always increased borrowing by more than investment, i. e. $\frac{\delta \tilde{d}^1_0}{\delta F^1} \cdot G^1 > \frac{\delta \tilde{a}^1_0}{\delta F^1} \cdot G^1$. We deduce from equation (A1.13) that this condition is satisfied provided that

$$\frac{\int_{\underline{R}}^{\overline{R}} Ru'' \left(\tilde{c}^1_1\right) dF^1 + q_0 u'' \left(\tilde{c}^1_0\right)}{u'' \left(\tilde{c}^1_0\right) + \int_{\underline{R}}^{\overline{R}} u'' \left(\tilde{c}^j_1\right) dF^1} > 1. \tag{A1.15}$$

Under the presumption made in Theorem 1.1, requiring the homogeneous belief $F(R)$ to be less optimistic than at least one type's belief in the heterogeneous case, implying $F^1(R) < F(R)$ for all R, inequality (A1.15) is satisfied for any type-1 belief F^1 sufficiently optimistic. Hence, under this assumption, we obtain $\hat{R}^*_{het} > \hat{R}^*_{hom}$.

Ultimately, we derive the corresponding probabilities of financial distress. In our setting, it is for the heterogeneous and the homogeneous case, respectively

$$\Pr\left(\tilde{\eta}^1_1 > 0\right) = \Pr\left(R \le \hat{R}^*_{het}\right) = F\left(\hat{R}^*_{het}\right)$$
$$\Pr\left(\eta_1 > 0\right) = \Pr\left(R \le \hat{R}^*_{hom}\right) = F\left(\hat{R}^*_{hom}\right).$$

Given $\hat{R}^*_{het} > \hat{R}^*_{hom}$ and the strict monotonicity of the cdf F, it follows that $F\left(\hat{R}^*_{het}\right) > F\left(\hat{R}^*_{hom}\right)$. □

1.A.6 Proof of Proposition 1.5

Proposition 1.5 follows from Definition 1.4 and Proposition 1.2. □

1.A.7 Proof of Proposition 1.6

With regard to part (i), let investors hold heterogeneous beliefs $\mathcal{F}$. Let further $G^j(R)$ be the direction of a perturbation of type-j investors' belief $F^j(R)$, and beliefs $F^i(R)$ be constant for all $i \neq j$. We calculate the functional derivatives of distributive and collateral externalities with respect to beliefs in the following way:

$$\frac{\delta D^i_{\tilde{a}^j_0}}{\delta F^j} \cdot G^j = \frac{\delta\left(\frac{q_1}{\partial \tilde{a}^j_0}\right)}{\delta F^j} \cdot G^j \cdot \tilde{l}^j_1 = \left(\frac{\partial^2 q_1}{\partial \tilde{a}^j_0 \partial \tilde{a}^j_0}\frac{\delta \tilde{a}^j_0}{\delta F^j} \cdot G^j + \frac{\partial^2 q_1}{\partial \tilde{a}^j_0 \partial \tilde{d}^j_0}\frac{\delta \tilde{d}^j_0}{\delta F^j} \cdot G^j\right)\tilde{l}^j_1,$$

and analogously for $D^i_{\tilde{d}^j_0}$, $C^i_{\tilde{a}^j_0}$, and $C^i_{\tilde{d}^j_0}$. Since q_1 is strictly decreasing and concave in both $\tilde{a}^j_0$ and $\tilde{d}^j_0$, and using our results from above on the sign of the functional derivatives $\frac{\delta \tilde{a}^j_0}{\delta F^j} \cdot G^j$ and $\frac{\delta \tilde{d}^j_0}{\delta F^j} \cdot G^j$, it follows that the term in brackets is unambiguously negative for $G^j(R) < 0$ and positive for $G^j(R) > 0$. This proves the first statement of part (i).

Statement (2) of part (i), as well as part (ii), follow from the same arguments as those used in the proof of Proposition 1.4. □

1.A.8 Proof of Proposition 1.7

First, we derive the tax formulas in part (i). Consider the period-0 optimization problem of a type-j agent with taxes:

$$\max_{c^j_0, a^j_0 \geq 0, d^j_0} u\left(c^j_0\right) + E^j\left[V^j_1\left(a^j_0, d^j_0 | \tilde{a}_0, \tilde{d}_0\right)\right] \quad \text{s.t.}$$

$$\left(\lambda^j_0\right) \quad c^j_0 = e + \left(1 - \tau^j_d \operatorname{sgn}\left(\tilde{d}^j_0\right)\right) d^j_0 + \left(1 - \tau^j_a \operatorname{sgn}\left(\bar{a} - \tilde{a}^j_0\right)\right) q_0 \left(\bar{a} - a^j_0\right) + T^j.$$

This problem gives rise to the following optimality conditions:

$$\left(1 - \tau^j_a \operatorname{sgn}\left(\bar{a} - \tilde{a}^j_0\right)\right) q_0 u'\left(c^j_0\right) = E^j\left[Ru'\left(c^j_1\right) + u'\left(c^j_2\right) + \eta^j_1 \phi q_1\right] \tag{A1.16}$$

$$\left(1 - \tau^j_d \operatorname{sgn}\left(\tilde{d}^j_0\right)\right) u'\left(c^j_0\right) = E^j\left[u'\left(c^j_1\right)\right]. \tag{A1.17}$$

In a symmetric equilibrium, it will always be the case that $c^j_0 = \tilde{c}^j_0$, $a^j_0 = \tilde{a}^j_0$ and $d^j_0 = \tilde{d}^j_0$ for each j. Combining the latter two conditions with their counterparts from the social planner problem, i. e. equations (1.13) and (1.14), respectively, using the planner's pricing relation $\tilde{\psi} = q_0 \omega^j u'\left(\tilde{c}^j_0\right)$, and solving for the taxes yields the tax formulas (1.15) and (1.16).

Second, it follows that, using these taxes, the competitive allocation is constrained-efficient. Specifically, substituting (1.15) and (1.16) into the optimality conditions of the competitive allocation with taxes, i. e. (A1.16), and (A1.17), replicates the planner's optimality conditions (1.13) and (1.14), as well as $\tilde{\lambda}_0 = \omega^j u'\left(\tilde{c}^j_0\right)$ for each j. Moreover, rebating revenues through T^j for all j ensures that individual period-0 budget constraints are satisfied, and the same holds for the resource constraint in consequence. To summarize, the competitive allocation with taxes satisfies the identical set of conditions, so it turns out to be constrained-efficient.

By the same arguments, we derive the homogeneous tax formula 1.17 in part (*ii*). □

1.A.9 Proof of Theorem 1.2

Theorem 1.2 follows from Propositions 1.3 and 1.7. □

Chapter 2

Urban Spatial Distribution of Housing Liquidity

by Francisco Amaral, Mark Toth and Jonas Zdrzalek

Abstract

We examine the relation between location, liquidity, and prices in urban housing markets. Leveraging a new dataset that encompasses transactions and advertisements of apartments in large German cities over the last decade, we empirically show that housing market liquidity and prices jointly decline with distance to the city center. We build a spatial search model to demonstrate how travel costs to the city center explain this joint spatial distribution. We estimate the model and find that it quantitatively matches the data, and that spatial liquidity differences due to search frictions account for a quarter of the spatial price gradient. Finally, we conduct a counterfactual analysis, showing that prices in the city center are 5.5% higher than in the periphery due to higher liquidity.

Key words: Housing market liquidity, Cities, Spatial equilibrium, House price gradient

JEL codes: G11, G50, R21, R30

2.1 Introduction

Housing markets are local, with little of the variation in house prices being explained by national factors alone. As a result, housing markets are often modeled within spatial equilibrium frameworks in which local market conditions determine prices (Alonso, 1964; Roback, 1982). Housing markets also involve bilateral transactions, typically characterized by significant informational and search frictions (Han and Strange, 2015).

In this paper, we investigate how local market conditions interact with trading frictions. We demonstrate that in urban settings, the cost of traveling to the city center determines both the demand for housing services and local market thickness, thereby simultaneously generating price and liquidity gradients. Similar to liquidity premiums observed in bond and stock markets (Amihud and Mendelson, 1986), we identify and quantify a liquidity premium for properties in city centers compared to those in the outskirts. We conclude that taking into account the interaction between local market conditions and trading frictions significantly enhances our understanding of the factors driving the spatial variation in house prices.

Our knowledge about the spatial variation in housing liquidity is still limited, as detailed data on housing market transactions, especially with measures of liquidity, is scarcely available. Using a novel dataset that combines the universe of real estate transaction contracts and advertisements for apartments in large German cities over the last decade, we provide evidence that housing market liquidity and prices jointly decrease with distance to the city center. Even after controlling for a large set of property characteristics, we find that apartments located farther from the city center take longer to sell and have lower prices.

We then develop a theory to study the effects of location on prices and liquidity. We build a spatial search model of housing within a monocentric city to show how the distance to the city center can affect both the value and the liquidity of housing. The cost of traveling to the city center impact both the demand for housing services and local market thickness. Sellers face a lower expected demand outside the city center, which increases the expected time on the market and decreases house price. We thus show how travel costs can simultaneously explain why prices and liquidity drop with distance to the city center. Using our transaction-level liquidity data, we calibrate the model and are replicate the joint spatial distribution of house prices and liquidity for all cities in our sample with a high degree of certainty.

As hypothesized by Amihud and Mendelson (1986), the discounted value of transaction costs serves as a proxy for the value loss attributed to illiquidity. Consequently, given a certain utility derived from residing in a house, we anticipate that greater illiquidity would correspond to a reduction in its price. However, liquidity is influenced by the house's location, which also affects its fundamental value. Therefore, in order to isolate the effects of liquidity on prices, we require structural estimation. First, we determine the welfare-maximizing allocation by removing search frictions in our model. Comparing the price gradients with and without search frictions allows us estimate a liquidity premium for housing in the city center vis-a-vis the outskirts. Across cities and using different discount factors for robustness, we estimate this liquidity premium to be 5.5% in terms of the house price in the city center.

In our empirical study, we analyze the spatial distribution of housing liquidity. To achieve this, we match housing advertisements with transactions using a nearest-neighbor algorithm. We obtain a dataset at the transaction level with geo-coded information on housing liquidity, covering four large German cities, Hamburg, Munich, Cologne, Frankfurt, and Duesseldorf, from 2012 to 2022. To work with a sample that has consistent coverage across space, we focus our analysis on residential apartments.

We measure the time an apartment spends on the market as the number of weeks the advertisement for this apartment remains online. We start by showing that apartments closer to the city center spend less time on the market. However, housing characteristics in the outskirts differ significantly from those in the inner city. We take these differences into account by controlling for characteristics such as apartment size, building age, or number of bathrooms. The effect magnitude is consistent across cities. In the city center, apartments stay on the market for five weeks less than in the outskirts, while the average time on the market is around 14 weeks.

We demonstrate the robustness of our findings in various alternative specifications. For example, we use an alternative measure of liquidity, the gap between asking and sales prices, and an alternative measure of distance, the travel time to the city center. Furthermore, we show that location is the primary determinant of housing liquidity, even though other aspects of properties also predict housing liquidity.

We rationalize our empirical results within a spatial search model in which a city's frictional housing market clears simultaneously via prices and liquidity. We borrow our notion of space from the monocentric city model in which house prices depend on travel costs to the city center and, consequently, the location of housing units (Alonso, 1964; Mills, 1967; Muth, 1969). We introduce a search mechanism which gives rise to liquidity differentials within the city, building on Krainer (2001). According to our model, liquidity and prices jointly decrease with distance to the city center because buyers are less likely to purchase apartments due to increasing travel costs to the city center. To explain these spatial gradients in detail, we propose the following mechanism.

Buyers search for apartments across the city. The further away an apartment is located from the city center, the higher are the travel costs associated with this property. When deciding whether to buy apartments, buyers want to be compensated for these travel costs. They require higher draws of utility shocks, which represent buyers' idiosyncratic valuation of apartments, the farther an apartment is located from the city center. This results in a lower probability of sale. As a result, the expected time on the market increases with distance to the city center, which results in a negative spatial liquidity gradient.

Sellers, being local monopolists at the distance to the city center at which their apartments are located, then face higher expected demand in the city center. Accordingly, they set higher prices in the city center relative to the outskirts, which gives rise to a negative spatial price gradient.

We calibrate the model using our transaction and advertisement data. For each city, our model only requires information on the average holding period, the average price, the average time on the market, and travel times to the city center. Even though we do not target any equilibrium spatial gradient in the calibration, our model is able to match the spatial price and liquidity gradients from our transaction and advertisement data for all cities with high precision.

Using our calibrated model, we estimate a liquidity premium for apartments in the city center relative to the outskirts. We think of the liquidity premium in the city center relative to some location as the relative price difference in scenarios with search frictions versus without search frictions. We find that the liquidity premium in the in the city center relative to the outskirts amounts to about 5.5% of the house price in the city center. Furthermore, by comparing the model-implied price gradients with and without search frictions with the price gradient from the data, we find that spatial liquidity differences due to search frictions explain about a quarter within-city spatial price gradient.

This paper adds to the extensive body of research on the determinants of housing prices. While previous studies have mainly investigated how location affects the value of land or housing services (for recent studies, see, e.g., Albouy et al. 2018; Gupta et al. 2022; Liotta et al. 2022; Kaas et al. 2024), we show that location also plays a role in determining housing liquidity. In doing so, we document an urban housing liquidity gradient, which adds to the well-documented urban price gradient (for an overview, see Duranton and Puga 2015). From the empirical perspective, a closely related paper is Ruf (2017) which measures liquidity gradients in Swiss rental markets, but the focus is on the implications for investors. By estimating the spatial structure of the housing liquidity premium, we contribute to the literature on liquidity and asset prices, which has mostly been focused on stock and bond markets (Amihud et al., 2012), but also includes studies on housing markets (Lin and Vandell, 2007; He et al., 2015).

By incorporating a spatial equilibrium in a housing search model, we extend current theoretical frameworks of trading frictions in housing markets (see Han and Strange 2015), making them more capable of explaining spatial patterns of prices and liquidity. Duffie et al. (2005) model liquidity in over-the-counter markets via a search framework. Whereas they focus on sales opportunities for sellers and hence the supply side, we focus on the demand side, as we provide arguments for liquidity differentials across space due to travel costs to buyers. Cai et al. (2024) distinguish locations in a spatial search model that is not specific to the housing market via chances for sellers of meeting buyers. They specifically consider the location choice of sellers. In the housing market, locations of sellers are fixed, and we take the spatial distribution of sellers, expressed in distances to the city center, as given.

Our work also contributes to the emerging field of urban finance (e.g., Favilukis et al. 2022) which analyses the role of risk and trading frictions in settings in which location matters for asset prices. Furthermore, we add to the growing body of literature on regional differences in housing markets. While these papers had focused on documenting and explaining differences in housing liquidity across regions (Amaral et al., 2021; Vanhapelto and Magnac, 2023; Jiang et al., 2024), we are the first to document within-city patterns and derive their implications for housing liquidity premiums.

The rest of this paper is organized as follows. Section 2 describes our data and defines our measures of space and liquidity. Section 3 presents our empirical analysis. Section 4 describes our model framework and presents our analytical and quantitative results. Section 5 concludes.

2.2 Data & Measurement

In our empirical analysis, we use two novel datasets on urban housing markets in Germany, one of which is derived from administrative records of housing transactions, and another one which is derived from housing market advertisements. The transaction data gives us information on sales prices. The advertisement data gives us information on advertisement duration from which we obtain our measure of time on the market. From each dataset, we only select apartments for our analysis, which allows us to investigate the role of location consistently within a city, as other types of residential housing are typically scarce in the city center. We match the two datasets for our empirical analysis via a nearest-neighbor algorithm.

2.2.1 Transaction data

Data description. We obtain the transaction data from a novel dataset that covers the universe of residential real estate transactions in large German cities for several decades.

This data source is introduced and described in detail in Amaral et al. (2023c). The authors of this paper compile data from public local real estate committees (*Gutachterausschüsse*). Collecting information on all real estate transactions from notaries, the real estate committees register information on sales prices, contract dates, addresses and precise information on location in the form of coordinates, and various property characteristics.

Data cleaning. We clean the transaction data by filtering out property sales between relatives, leaseholds, package sales involving multiple properties sold together, sales of social housing, foreclosures, and any other sales flagged by the real estate committees as not aligning with genuine market prices. We remove outliers from the sample by excluding transactions of properties with prices or sizes above the 99th percentile or below the 1st percentile within a given year.

2.2.2 Advertisement data

Data description. We obtain data on apartment advertisements via *VALUE Marktdaten*, who collect and processes real estate advertisements from online platforms and real estate agencies.[1] We observe the dates on which the ad was posted and removed, addresses and coarse information on location such as zip code or neighborhood, asking prices, and various property characteristics. The dataset covers the period between January 2012 and December 2022.

Data cleaning. A common issue with online real estate advertisement data is the presence of multiple advertisements for the same property. *VALUE Marktdaten* has developed and implemented an algorithm to identify and exclude duplicates, and as such, this is not an issue in our data cleaning process. We remove outliers by excluding advertisements of properties with asking prices or sizes above the 99th percentile or below the 1st percentile within a given year.

2.2.3 Matching transactions and advertisements

We analyze liquidity and price patterns across space. The transaction data gives us information on sales prices and the location of apartments, while the advertisement data gives us information on liquidity via the advertisement duration. To prepare our analysis, we match the two data sources by applying a nearest-neighbor algorithm, such that we can

[1]We are very grateful to Sebastian Hein from *VALUE Marktdaten* for giving us access to the data and support throughout the process of writing the paper.

associate a sales price and a set of coordinates from the transaction data with a marketing time from the advertisement data. The nearest-neighbor algorithm matches observations from the two datasets by considering locations, contract dates, advertisement dates, and property characteristics. Our goal is to uniquely associate transactions with advertisements. However, we do not observe advertisements for all transacted properties and are therefore only able to match a subset of the universe of transactions with corresponding advertisements. To match the transactions with the online advertisement data, we developed a nearest neighbor algorithm. This algorithm matches observations from both datasets by considering their geographical location, contract and advertisement date, as well as property characteristics. The objective of this matching algorithm is to associate each transaction with one online advertisement. However, it is important to note that not all transacted properties are advertised online. Therefore, we anticipate that only a subset of the transactions will have matches.

Matching algorithm. The algorithm starts by matching observations with complete addresses, that is, addresses which include house numbers and street names. However, for apartments, having information on solely the house number and street name is insufficient for a successful match, as there may be multiple apartment transactions related to the same building. If that is the case, the algorithm excludes ads based on property characteristics if they meet the following criteria, in the given order: 1) The living area differs by more than 10%. 2) The floor number differs by more than 2. We choose these apartment characteristics since they have the lowest number of missing values from the set of variables that are covered by both datasets.[2] We choose the numeric values for the criteria such that we have reasonable buffers for measurement errors due to incorrect user inputs. If, after applying these criteria, there are still multiple potential ads remaining, the algorithm selects the ad that minimizes the distance to the transaction in the aforementioned characteristics.[3] We match the transactions which do not have entries with complete addresses via the same process as for those with complete addresses, but condition sequentially on the following geographical objects: street name, zip code, and neighborhood (*Stadtteil*), until we have a unique match. If there is no unique match, we drop the observation.

[2]When we match based on the building's exact address, we do not exclude matches with different building years. Matching by address is sufficient to identify a building, and typically, the building year is the same for all flats within a building. When this is not the case, we attribute the different building years to measurement error, that is, incorrect user-specified information on the advertisement websites.

[3]We minimize the absolute difference between the value from an advertisement and the value from the transaction. If this difference is the same for the apartment size, we proceed with the difference in the floor number. If the algorithm still does not yield a unique match, we drop the observation.

Data cleaning. We exclude observations that contain implausible information on the sequence of market events. First, we exclude advertisements that were published after the contract date of the transaction. Second, we exclude ads that were removed more than one year before the contract date. A time span of more than one year between the end of the advertisement and the contract date is unlikely. On average, we match and keep about 30% of the transactions across cities.[4] In Table 2.1, we provide further information on the matched observations by city.

Table 2.1: Summary statistics: matched dataset

City	# Transactions	# Ads	# Matched	Avg. sales price (€)	Avg. asking price (€)
Hamburg	74030	69399	22964	401461.5	400359.1
Cologne	35597	41505	14188	236154.3	249582.1
Duesseldorf	34732	30253	10565	304701.5	317618.8
Frankfurt	32828	34171	14696	380930.7	437007.4

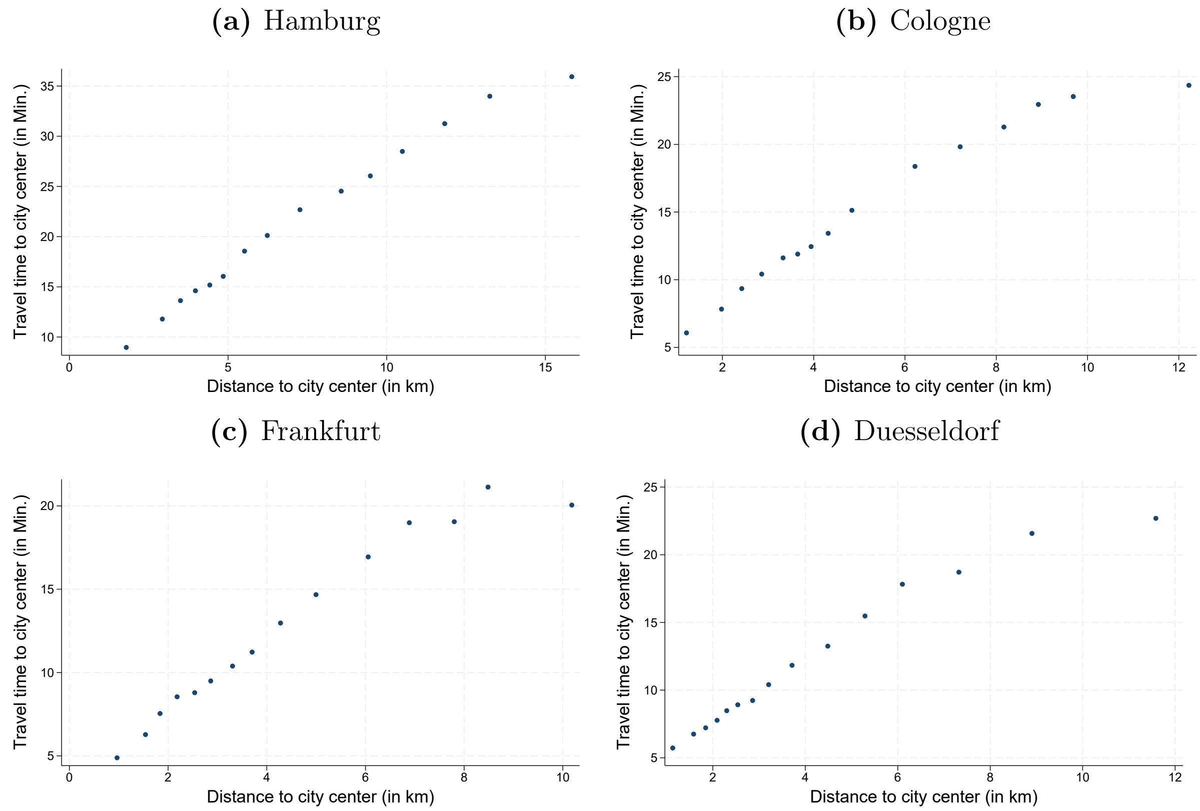
Notes: This table reports summary statistics about the matched transaction and advertisement data for the period January 2012- December 2022.

2.2.4 Measurement of spatial variables

We measure spatial variation in our data using a single variable, the kilometer distance to the city center, an established measure in the urban economics literature (see e.g., Duranton and Puga 2015). We obtain this distance via the coordinates of the city centers and the coordinates of the matched apartments. We select the *Alsterhaus*, a historic shopping quarter, as the city center of Hamburg. For Cologne, we select the Cologne Cathedral (*Kölner Dom*). For Frankfurt, we select the *Willy-Brandt-Platz* and for Duesseldorf, we select the *Marktplatz*. We select these city centers as they are located in the historical centers of these cities. In a robustness check, we show that selecting the centroid of the commercial district with the highest land value (via the *Bodenrichtwerte* land value measurements from the *Gutachterausschüsse* real estate committees) yields almost identical city centers as the ones we choose by hand.

[4]For more details see Appendix 2.A.

Figure 2.1: Travel time to the city center (January 2012- December 2022)



Notes: This figure shows the travel time by car to the city centers of Hamburg (*Alsterhaus*), Cologne (Cologne Cathedral), Frankfurt (*Willy-Brandt-Platz*), and Duesseldorf (*Marktplatz*), in terms of the kilometer distance from the respective city center.

As an alternative spatial measure, we use travel time estimates. The spatial structure of cities can feature rivers or other factors that influence local transportation. Such features can be more accurately represented via the travel time rather than the kilometer distance to the city center. Via an API provided by Openrouteservice, we request the typical travel time to the city center by car for each apartment in our matched dataset. We summarize the resulting travel time estimates in Figure 2.1, using 15 bins with equal numbers of observations.

2.2.5 Measurement of liquidity

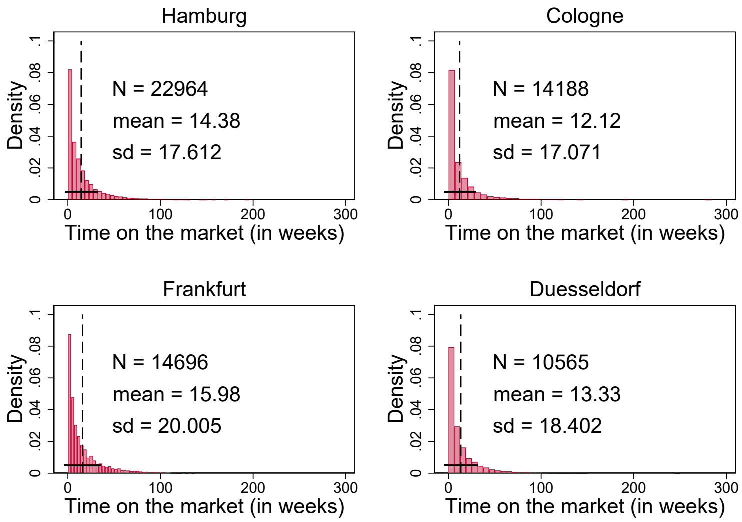
We measure housing market liquidity via the time on the market of apartments, a measure typically used in the literature (Han and Strange, 2015). We define the time that a property stays on the market as the number of weeks between the start and the end of an advertisement.[5]. The time spent on the market of apartment i sold in period t is

[5]We define the start and end dates of the advertisement as the start and end dates of the time an apartment is on the market. By doing so, we make the assumption that the ad is removed when the seller

$$\text{TOM}_{it} = \frac{\text{Number of days advertised}_{it}}{7}, \tag{2.1}$$

where an apartment is defined to have been on the market for $T/7$ weeks if it sells on day number T of being advertised. In Figure 2.2, we plot histograms of our time on the market measurements.

Figure 2.2: Histograms of time on the market (January 2012- December 2022)



Notes: This figure shows histograms for the time on the market, as defined in (2.1), in our matched dataset.

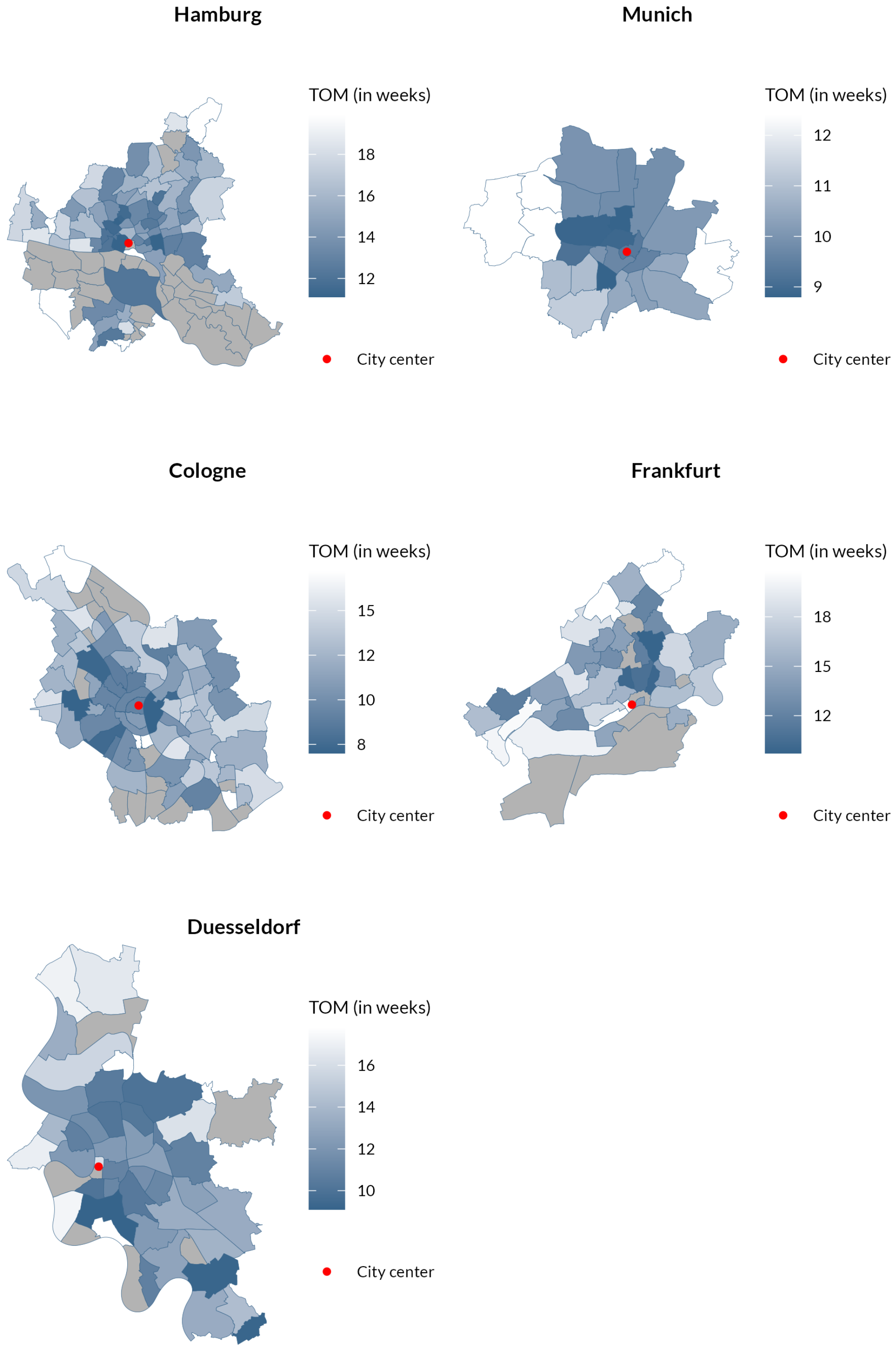
2.3 Empirical results

2.3.1 Time on the market decreases with distance to city center

Visual exploration. Figure 2.3 shows maps with estimates of the average liquidity by district (*Stadtteil*) in Hamburg, Cologne, Frankfurt, and Duesseldorf. In all cities, the time on the market is lowest in the city center and increases with distance to the city center. Close to the city center, apartments stay on the market for about 11 weeks. In the outskirts, apartments stay on the market for about 16 weeks. The difference in marketing time between the city center and the outskirts thus amounts to 5 weeks, consistently across cities. Relative to the average time on the market of 14 weeks, unconditional spatial differences are substantial. Next, we condition our measurements on apartment characteristics to filter out confounding variation.

and the buyer reach an agreement. While we cannot confirm this assumption, there are indications that buyers have strong incentives to ensure the property is taken off the market promptly upon reaching an agreement with the seller (see e.g., Vanhapelto and Magnac 2023)

Figure 2.3: Time on the market across space (January 2012- December 2022)



Notes: The maps show the average time on the market as defined in (2.1) by district (*Stadtteil*) from our matched dataset, controlling for year-quarter fixed effects, on a logarithmic scale. Districts without available data are colored gray.

Regression framework. We want to rule out that these patterns are driven by systematic spatial variation in apartment characteristics. For example, smaller apartments, which are typically traded more easily, are located mostly in the city center.[6] We estimate the association between time on the market and distance to the city center for each city separately via the regression specification:

$$\text{TOM}_{it} = \alpha \cdot \text{distance}_i + \beta \cdot X_i' + f_t + \varepsilon_{it}, \tag{2.2}$$

where i indexes apartments and t indexes the year-quarter of transaction, the distance on the right hand side is the kilometer distance to the city center in our baseline specification and the travel time to the city center (in minutes) in our alternative specification, X_i is a vector of apartment characteristics,[7] f_t is a year-quarter fixed effect to account for common time trends in liquidity within a city, and ε_{it} is the error term.

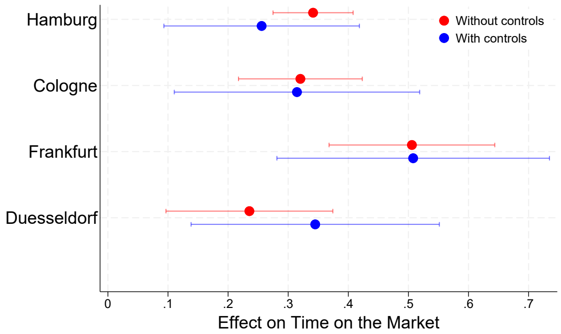
Results. You can find our regression estimates in Figure 2.4. We run 4 specifications in which we use the kilometer distance as the right-hand-side variable without and with controls and the travel time as the right-hand-side variable without and with controls. All our estimates are significant at the 1% level and their economic magnitudes are approximately equal across cities. About 3 additional kilometers of distance to the city center correspond to an additional week on the market. In terms of travel time, about 6 more minutes of car travel time correspond to an additional week on the market. These estimates are sizable, given that the average time on the market is about 14 weeks across cities, and they are almost identical across cities. The repeated pattern calls for a theory which we develop in the second part of the paper. Before doing so, we provide some additional empirical results.

[6] For an overview of the quantitative influence of such characteristics on liquidity, see e.g., Forgey et al. (1996), Anglin et al. (2003), Carrillo (2012), or Hayunga and Pace (2019).

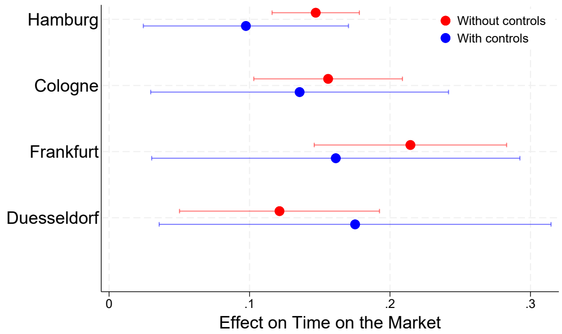
[7] We control for the following variables: living area in m^2, living area squared, number of rooms, year of construction, "Altbau" or not, "Neubau" or not, physical condition of the building, whether the apartment is in the upper floor of the house or not, whether the apartment is rented out or not, type of heating (e.g., central heating), source of heating (e.g., gas), whether the apartment has a fitted kitchen or not, whether the apartment has an open kitchen or not, whether the bathroom has a shower, whether the bathroom has a bathtub, whether the apartment has a terrace or balcony, whether the apartment has a basement, whether the apartment has a garden, and the number of parking spaces. For more details see Appendix 2.A.

Figure 2.4: Effect of distance to city center on time on the market (January 2012-December 2022)

(a) Distance to city center (in km)



(b) Travel time to city center (in min)

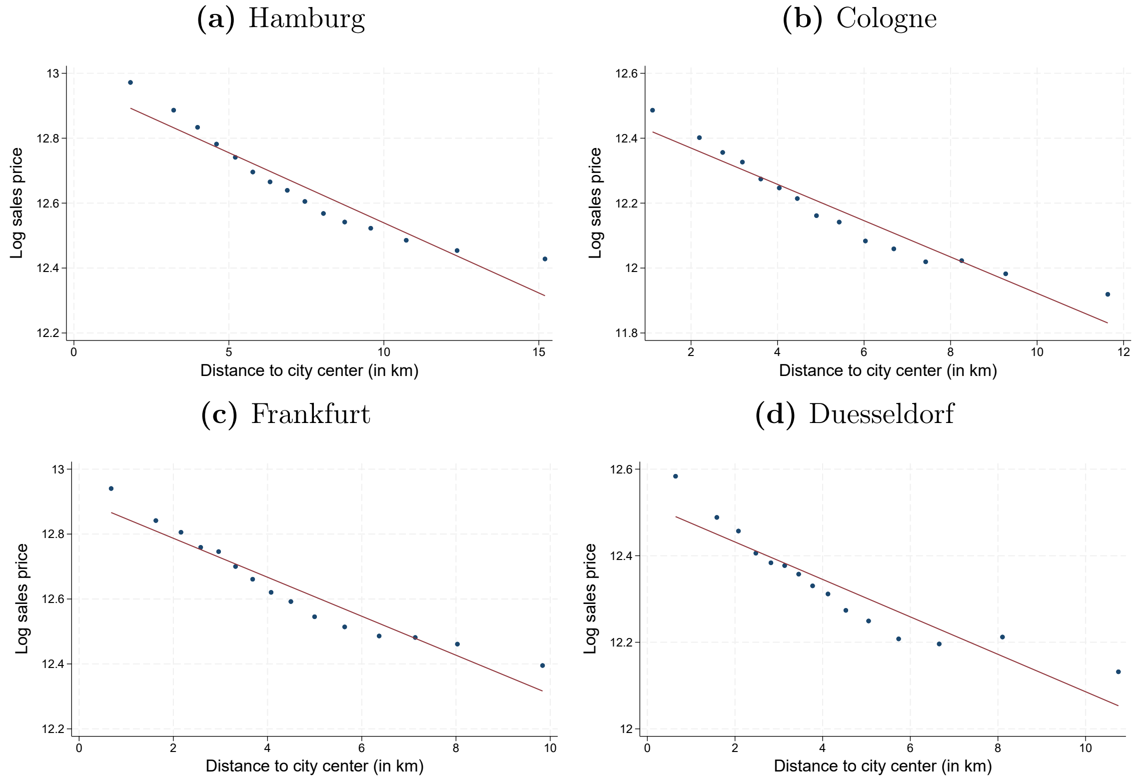


Notes: These figures show the OLS regression coefficients of distance to the city center as specified in (2.2) with 99% confidence intervals. All regressions include year-quarter fixed effects. See Footnote 7 for a full list of apartment characteristics controls.

2.3.2 Sales prices decrease with distance to city center

As an addition to our novel empirical findings, we reproduce a standard finding from the urban economics literature. The monocentric city model predicts a negative spatial price gradient, which is typically tested empirically with prices in logarithms and distances in levels (Duranton and Puga, 2015). We run regression specification (2.2) with log sales prices as the outcome variable and plot the resulting binscatter plots in Figure 2.5. Our results align with the standard findings.

Figure 2.5: Spatial gradients of sales price (January 2012- December 2022)



Notes: These binscatter plots visualize the results of regression (2.2) with log sales price as the outcome variable and 15 equally-sized distance bins. The regressions include year-quarter fixed effects and control for apartment characteristics listed in Footnote 7.

2.3.3 Discussion of external validity

We cannot be sure if we are able to transfer our findings to other cities. The negative spatial price gradient has been observed in monocentric cities of different sizes across the globe (Liotta et al., 2022). If the spatial price gradient is the result of the same economic mechanism as the liquidity gradient, we should be able to generalize our findings about liquidity similarly. In our theoretical framework, we argue that travel costs to the city center give rise to such a mechanism.

Our findings are not transferable to non-monocentric cities. The most established deviation from the monocentric city model is the polycentric city model[8] with Chicago as a classic example (McMillen and McDonald, 1997). We view our findings as applicable to typical monocentric cities and make no statement about spatial liquidity patterns in cities with different structures.

[8]Other non-monocentric models of cities are, for example, the maximum disorder model, the mosaic of live-work communities model, and the constrained dispersal model (Angel and Blei, 2016).

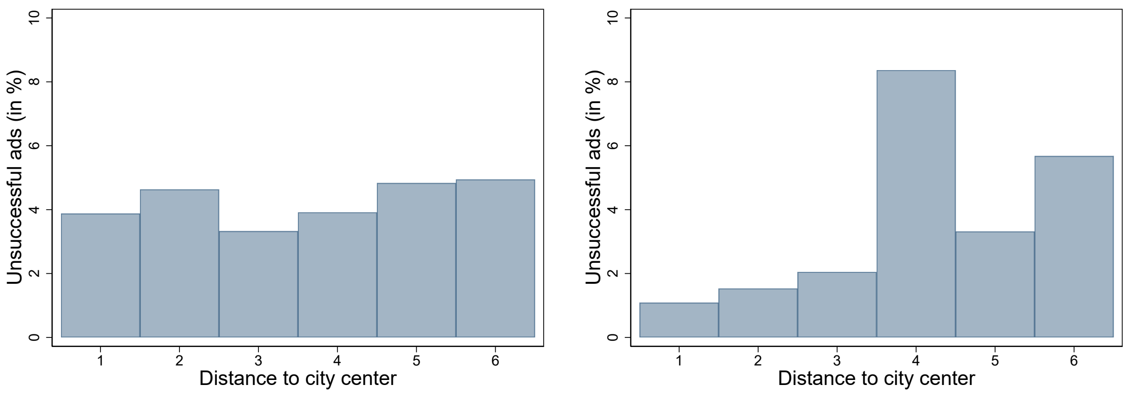
2.3.4 Robustness analysis

Unsuccessful ads. Our baseline analysis is based on the dataset of matched ads. Our results could be biased if the number of ads that did not end up in a sale varies systematically across space. To check this, we run an algorithm to identify the ads that do not end up in a transaction.

We identify ads that did not result in a sale via three steps. First, we match all ads with transactions that occurred within the same neighborhood. Each ad is then associated with a set of potential transactions in the neighborhood. Out of these ads, we identify those as “unsuccessful” that are associated with transactions one year after or before the ad was published. Second, we identify ads as “unsuccessful” that are associated with transactions for which the living area of the matched apartment differs by more than 40%. Finally, we identify ads as “unsuccessful” for which the remaining potential matches have a living area, building year, and floor number that deviate by more than 10%, 10 years, and 2.

Figure 2.6: Unsuccessful ads and distance to the city center

(a) Hamburg **(b)** Cologne



Notes: These figures display the percentage of ads that do not result in a sale by distance to the city center with 6 equally-sized distance bins.

Figure 2.6 plots the percentage of ads that did not result in a sale and shows that this percentage is increasing with distance to the city center. To further quantify this relation, we run a survival analysis using the combined dataset of successful and unsuccessful ads and find that the probability of an ad not resulting in a sale increases with distance to the city center. We display these results in Appendix 2.C. Note that to the best of our knowledge, it is a novel contribution in this paper to identify the ads that result in a sale.

Asking price discount. One potential explanation for the time on the market to be increasing with distance to the city center is that sellers in the city center systematically accept bids below their asking prices, thus accelerating the sale. To check this, we measure the relative spread between the asking price[9] and the transaction price for property i sold in period t, which we define as the asking price discount:

$$\text{Discount}_{it} = 100 \cdot \frac{(\text{Sales price}_{it} - \text{Asking price}_{it})}{\text{Asking price}_{it}}. \tag{2.3}$$

A more negative or less positive discount corresponds to lower liquidity. We find that this spread is generally negative and becomes more negative with distance to the city center. Hence, sellers in the city center can expect a lower time on the market and a final sales price which is closer to the original asking price. We display these results in Appendix 2.C.

Temporal variation. Our analysis covers the period between 2012 and 2022. We check whether our results are driven by a specific time interval by running regression (2.2) on a rolling-window basis to get time-varying coefficients. The coefficients are positive in every period and significant at the 99% confidence level in almost every period (see Figure 2.7). The coefficient sizes are roughly constant over time in every city except for Cologne, where they drop in the second half of the sample.

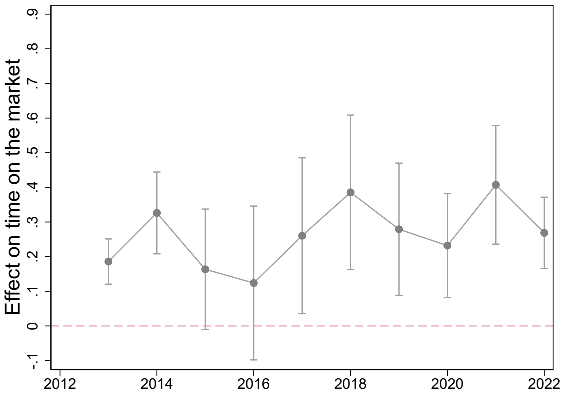
Include unmatched ads. Our matching procedure is conservative, as we try to only keep matches which we consider to be correctly identified with a high degree of certainty. It could, nevertheless, be the case that we are systematically excluding observations that would counteract our findings. To check this, we run regression (2.2) using the complete sample of ads. Figure 2.8 shows that this specification hardly influences our results.

Alternative regression specifications. By definition, the time on the market is positive and as such, a standard OLS regression might not be optimal to use. As we show in Appendix 2.C, our results hold in Poisson regressions and OLS regressions with logarithmic time on the market.

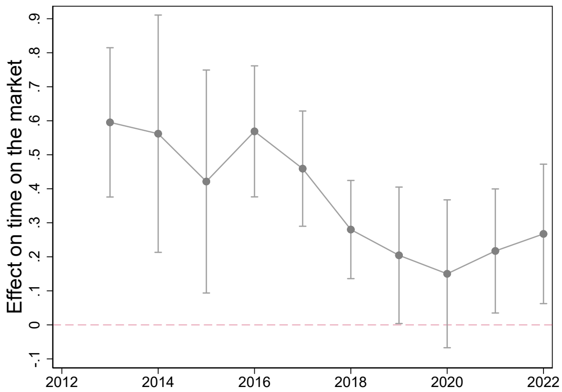
[9] We observe two asking prices in the data: the asking price on the day the ad was initially posted and the asking price on the day the ad was taken down. Our results hold with both of these two asking prices in the calculation of the asking price discount. For simplicity, we only use the asking price on the day the ad was taken down.

Figure 2.7: Effect of distance to the city center on time on the market over time (January 2012–December 2022)

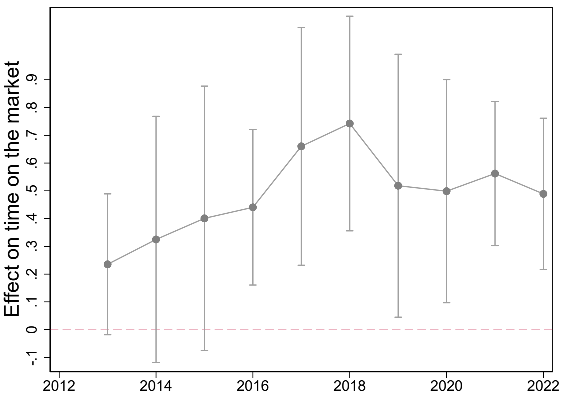
(a) Hamburg



(b) Cologne

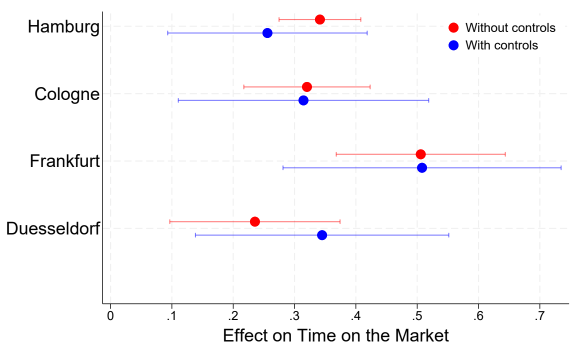


(c) Duesseldorf



Notes: These figures plot the coefficients of regression specification (2.2) for the time on the market as defined in (2.1) over time. The coefficients are based on rolling-window regressions with year-quarter fixed effects and apartment characteristics controls listed in Footnote 7.

Figure 2.8: Effect of distance to city center on time on the market using all ads (January 2012- December 2022)



Notes: These figures show the OLS regression coefficients of distance to the city center as specified in (2.2) with 99% confidence intervals, using a sample with matched an unmatched ads. All regressions include year-quarter fixed effects. See Footnote 7 for a full list of apartment characteristics controls.

2.4 Model

2.4.1 Model setup

2.4.1.1 Overview

We rationalize the empirical facts documented in the previous section by deriving analytical relations between liquidity and distance to the city center in a spatial search model of a city's housing market. Moreover, we show that our model can account for the patterns we observe in the data quantitatively by calibrating the model using our transaction and advertisement data.

We start from a standard housing search model by Krainer (2001) in which housing market clears via prices and liquidity, given by the expected time on the market. To introduce a notion of space, we rely on the monocentric city model (Alonso, 1964; Mills, 1967; Muth, 1969) and measure the distance to the center within a symmetric city. The spatial distribution of housing in the monocentric city model is endogenous. In our model, we take the spatial distribution of housing as exogenously given and do not impose symmetry on the city structure. We abstract from migration between cities.

2.4.1.2 Theoretical framework

Time, agents, and city. Time is discrete and runs forever. A time period equals one day. We focus on a stationary equilibrium and omit time indices. A large number I of infinitely-lived, financially unconstrained agents live in a city. The agents are risk-neutral and discount with a common factor $\beta \in (0, 1)$.

The city has a single city center to which all agents travel every period for work or leisure activities.[10] Agents pay travel costs $\tau(d)$ at distance to the city center d per period, where $\frac{\partial \tau}{\partial d} > 0$ and the distribution of distances is denoted by $\mathcal{D}$. Apart from determining agents' travel costs, space has no economic significance.[11]

Housing. The housing stock is exogenously given by I identical fixed-size apartments. Each apartment is characterized by its distance to the city center d. In the first model period, every agent lives in an apartment. In every period, a match between an agent and an apartment persists with probability π. With probability $1 - \pi$, an agent has to move out, put their apartment up for sale, and search for a new apartment. Then, the agent acts as a buyer and as a seller simultaneously. Risk neutrality of agents implies that we can analyze buyer and seller decisions separately.

Before purchasing an apartment, an agent draws a uniformly distributed[12] idiosyncratic housing dividend $\varepsilon \sim U\left[\tilde{\varepsilon} - 1, \tilde{\varepsilon}\right]$ for this apartment. If they decide to purchase the apartment, they receive this dividend in every period until they are unmatched. The dividend is identically and independently distributed across agents, distances to the city center, and time periods. An agent can only occupy one apartment at a time and can only search for new apartments after they have been unmatched with their old apartment.

While searching for apartments, agents live in the city center in rental units owned by risk-neutral investors from outside the city. Thus, searchers do not incur travel costs and pay rental costs that are equal to the value of their housing service flow, receiving zero net utility. Agents cannot rent out their owned apartments. The rental market is not modeled explicitly.

Search process. A buyer visits a random apartment of those that are currently on the market.[13] When visiting an apartment, the buyer observes their idiosyncratic valuation

[10] In principle, we do not have to assume that agents only travel to the city center. It would be sufficient to assume that jobs and/or leisure activities are concentrated in the city center, such that the city center is the focus of travel within the city. For simplicity, we assume that agents only travel to the city center, which allows us to translate straightforwardly between travel time to the city center in minutes and travel costs $\tau(d)$ in the calibration.

[11] We abstract from other factors that vary with distance to the city center and influence households' location decisions, most importantly, amenities. When calibrating the model, we feed in a precise travel cost function that delivers fundamental economic variation across space. Amenities, on the other hand, are harder to pin down quantitatively. We argue that travel costs to the city center alone are able to explain a large part of the joint spatial distribution of prices and liquidity.

[12] We use a uniform distribution for tractability and thus follow Krainer (2001). We normalize the boundaries of the distribution such that we only need to calibrate a single parameter for every city later in the quantitative exercise.

[13] Search is random in this model. If buyers could choose where to search, they would search with higher intensity closer to the city center, where their ex-ante net utility is higher due to lower commuting costs. All else equal, liquidity would increase in the city center, which would steepen the liquidity gradient.

ε for this apartment and the apartment's distance to the city center d. The seller of the visited apartment posts a price $p(d)$ without observing the buyer's idiosyncratic valuation. The buyer either agrees on the posted price and moves into the apartment in the next period or does not agree on the posted price and continues to search.

Seller's problem. A seller maximizes their profits $\Pi(d)$ over a posting price $p(d)$. In the following, we denote by $\gamma(p(d))$ the probability that the apartment is sold, given that the seller posts a price[14] $p(d)$. The seller's profits for an apartment at distance d are

$$\Pi(d) = \max_{p(d)} \Big\{ \gamma(\cdot)p(d) + (1 - \gamma(\cdot))\beta\Pi(d) \Big\}. \tag{2.4}$$

With probability $\gamma(\cdot)$, the seller receives the amount $p(d)$. With probability $1 - \gamma(\cdot)$, they try to sell the apartment again next period, and their discounted continuation value is $\beta\Pi(d)$. Sellers act as price setters. The probability of sale $\gamma(\cdot)$ reflects the expected demand at distance d, given a price $p(d)$. When optimizing, a seller takes into account the effect of their posted price on the expected demand at the location of their apartment.

An agent can only occupy one apartment at a time, but can have multiple apartments on the market as a seller. Such a scenario occurs if an agent has to move out of an apartment and becomes a searcher, finds a new apartment to live in, but has not been able to sell their old apartment(s).[15]

Buyer's problem. A buyer who purchased an apartment at distance d receives the value

$$V(d, \varepsilon) = \beta \left(\varepsilon - \tau(d) + \pi V(d, \varepsilon) + (1 - \pi) \left(\Pi(d) + W \right) \right), \tag{2.5}$$

where W denotes the value of search. Starting next period, the buyer receives the dividend ε and pays commuting costs $\tau(d)$. With probability π, the buyer keeps on living in the apartment for another period and receives the continuation value. With probability $(1 - \pi)$, the buyer becomes unmatched and receives the sum of the value of the resale value $\Pi(d)$ the value of search W which is given by

$$W = E_{d,\varepsilon} \left[\max \left[V(d, \varepsilon) - p(d), \beta W \right] \right]. \tag{2.6}$$

Either the buyer accepts the posted price and receives $V(d, \varepsilon)$ while paying $p(d)$, or the buyer continues to search and receives βW. We assume that the expectation over distances

[14] Note that with our setup, we give sellers equal price-setting power at every distance to the city center. When calibrating the model, we use distance bins with equal numbers of apartments from the data.

[15] Following Krainer (2001) and Krainer and LeRoy (2002), we ignore the possibility that a single agent accumulates all houses over any finite time interval, in which case this agent would not be able to visit another house if their match fails.

is always formed using the whole distribution of distances $\mathcal{D}$, in other words, buyers do not know the distances of apartments that are on the market.

2.4.2 Equilibrium

Seller's optimization. The first-order condition of the profit function yields

$$p(d) = \beta\Pi(d) - \frac{\gamma(p(d))}{\partial\gamma/\partial p\mid_{p=p(d)}}, \tag{2.7}$$

where the derivative $\partial\gamma/\partial p$ is evaluated at the optimal posting price $p(d)$ at distance d. We prove that the solution of this first order condition provides the required maximum in the Appendix.

Buyer's optimization. We define a reservation dividend $\varepsilon^*(d)$ at which a buyer is indifferent indifferent between buying an apartment and continuing to search:

$$V(d, \varepsilon^*(d)) - p(d) = \beta W. \tag{2.8}$$

The solution of this equation at a given distance to the city center characterizes the reservation dividend at this distance. A buyer purchases an apartment when they draw an idiosyncratic dividend that is larger than or equal to the reservation dividend at the apartment's distance to the city center. If they draw a smaller dividend, they continue to search.

Notion of spatial equilibrium. Equation (2.8) implies that a buyer must be indifferent between buying at different locations, as the left-hand side depends on the distance to the city center, whereas the right-hand side does not. The buyer indifference condition is hence also a *spatial equilibrium condition*. When facing the decision to accept or reject an offer, a buyer has to receive the same net utility (that is, the present value of occupying the apartment minus the price) at all distances to the city center.[16] In line with the interpretation of a spatial equilibrium in housing markets as a spatial no-arbitrage condition (Glaeser and Gyourko, 2008), there is no arbitrage opportunity for buyers across space.

Probability of sale. The equilibrium probability of sale at distance d is equal to the probability that a buyer's idiosyncratic dividend is above the reservation dividend at this distance:

[16]This condition is analogous to typical spatial equilibrium conditions in urban economics. For example, in the Rosen (1979)-Roback (1982) model, agents choose in which city to live, given a rent and a wage in each city. In a spatial equilibrium, the difference between wage and rent must be equal across cities, since otherwise all agents would locate only in the city with the largest difference. Analogously, in our model, searchers are indifferent between accepting and rejecting offers at every location in equilibrium.

$$\gamma(p(d)) = Prob(\varepsilon > \varepsilon^*(d)) = 1 - F(\varepsilon^*(d)) = \tilde{\varepsilon} - \varepsilon^*(d), \tag{2.9}$$

due to the uniform distribution of the idiosyncratic dividend. Thus, for the derivative in the seller optimality condition (2.7) we have that

$$\frac{\partial \gamma}{\partial p}\Big|_{p=p(d)} = -\frac{\partial \varepsilon^*}{\partial p}\Big|_{p=p(d)}. \tag{2.10}$$

Rearranging the buyer's value function (2.5) yields

$$V(d,\varepsilon) = \frac{\beta\big(\varepsilon - \tau(d) + (1-\pi)(\Pi(d) + W)\big)}{1-\pi\beta}. \tag{2.11}$$

With indifference condition (2.8), we get

$$\varepsilon^*(d) = \frac{1-\pi\beta}{\beta}p(d) + \tau(d) - (1-\pi)\Pi(d) + (\pi - \pi\beta)W. \tag{2.12}$$

Hence,

$$\frac{\partial \gamma}{\partial p}\Big|_{p=p(d)} = -\frac{1-\pi\beta}{\beta}, \tag{2.13}$$

where $\frac{\partial \Pi}{\partial p}\big|_{p=p(d)} = 0$ due to the Envelope Theorem.

Equilibrium definition. A *stationary spatial equilibrium* consists of a seller profit function $\Pi(d)$, a price function $p(d)$, a value of search W, a reservation dividend function $\varepsilon^*(d)$, and a conditional sale probability function $\gamma(p(d))$ that satisfy equations (2.4), (2.6), (2.7), (2.8), (2.9) for all distances to the city center $d \in \mathcal{D}$, given a parameter vector $(\beta, \pi, \tilde{\varepsilon})$, a distribution of apartments' distances to the city center $\mathcal{D}$, and a travel cost function $\tau(d)$.

Additional remarks. We do not have a notion of market tightness in the model. Market tightness measures the relation between buyer intensity and seller intensity in the market and can be defined as the ratio of the number of buyers to the number of sellers. Papers that measure market tightness in housing markets typically use buyer online search behavior to approximate the number of buyers (e.g., van Dijk and Francke, 2018). We do not have such data available. Nevertheless, we can interpret our model setup to include relative differences in market tightness within the city, as buyers arrive at random apartments and decide whether to accept or reject offers. Implicitly, this yields a number of potential buyers at a given location, reflected by the probability of sale.

In the Appendix, we extend our model to include a bargaining process, following Carrillo (2012), and provide proofs of the equilibrium's existence, following Krainer (2001), and uniqueness, following Vanhapelto and Magnac (2023). Next, we derive analytical results that rationalize our findings from the empirical part of the paper. The purpose of deriving these results analytically is to show general properties of the model that hold regardless of the calibration.

2.4.3 Analytical results

We start with some auxiliary derivations to get to our main theoretical result, which is that the equilibrium expected time on the market increases with distance to the city center while the equilibrium sales price decreases with distance to the city center.

2.4.3.1 Reservation dividends across space

Lemma 2.1 shows that the buyer reservation dividend is increasing with distance to the city center. In other words, buyers need higher draws of the idiosyncratic dividend to make a purchase the further the apartment they visit is away from the city center. This buyer preference reflects the presence of commuting costs $\tau(d)$, for which buyers want to be compensated with higher idiosyncratic dividend draws.[17]

Lemma 2.1. *The reservation dividend $\varepsilon^*(d)$ increases with distance to the city center d.*

Proof. To show that the reservation dividend increases with distance to the city center, we express it in terms of the travel cost $\tau(d)$, the only variable that exogenously varies across space. We know from (2.12) that

$$\varepsilon^*(d) = \frac{1-\pi\beta}{\beta}p(d) + \tau(d) - (1-\pi)\Pi(d) + (\pi - \pi\beta)W.$$

We reformulate the asking price $p(d)$ and the profits from reselling the apartment $\Pi(d)$ in terms of the reservation dividend $\varepsilon^*(d)$. Combining the seller optimality condition (2.7) and the expression for profits (2.4) evaluated at the equilibrium price, we obtain

$$p(d) = -\frac{(1-\beta)\gamma(\cdot) + \beta\gamma^2(\cdot)}{(1-\beta)\partial\gamma/\partial p\mid_{p=p(d)}}. \tag{2.14}$$

With the equilibrium relations (2.9) and (2.13), we get

$$p(d) = \frac{\beta}{1-\pi\beta}(\tilde{\varepsilon} - \varepsilon^*(d)) + \frac{\beta^2}{(1-\beta)(1-\pi\beta)}(\tilde{\varepsilon} - \varepsilon^*(d))^2. \tag{2.15}$$

Using the seller optimality condition (2.7), profits are then

[17] In the calibrated model, travel costs do not necessarily have to increase with distance to the city center. If that were the case, the following propositions would not apply for the distances to the city center that correspond to the non-increasing part of the travel cost function. In practice, we rarely encounter such cases.

$$\Pi(d) = \frac{\beta}{(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon^*(d))^2. \tag{2.16}$$

Thus, we can express the reservation dividend as

$$\begin{aligned}\varepsilon^*(d) = \frac{1-\pi\beta}{\beta}\left(\frac{\beta}{1-\pi\beta}(\tilde{\varepsilon} - \varepsilon^*(d)) + \frac{\beta^2}{(1-\beta)(1-\pi\beta)}(\tilde{\varepsilon} - \varepsilon^*(d))^2\right) \\ + \tau(d) - (1-\pi)\left(\frac{\beta}{(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon^*(d))^2\right) + (\pi - \pi\beta)W. \end{aligned}\tag{2.17}$$

We simplify and differentiate with respect to the distance to the city center d to obtain that

$$\frac{\partial \varepsilon^*}{\partial d}\underbrace{\left(2 - 2\frac{\pi\beta}{1-\pi\beta}(\tilde{\varepsilon} - \varepsilon^*(d))\right)}_{>0} = \frac{\partial \tau}{\partial d}, \tag{2.18}$$

and therefore $\frac{\partial \varepsilon^*}{\partial d} > 0$, given that $\frac{\partial \tau}{\partial d} > 0$. □

2.4.3.2 Liquidity and prices across space

In line with the measurement of the time on the market in the empirical part of the paper, we define that an apartment has been on the market for T days if it sells on day number T. Via the expected value of the geometric distribution that results from the multiplication of sale probabilities over time, we have that the expected time on the market (in days) at a given distance to the city center d is

$$E[TOM(d)] = \frac{1}{\gamma(p(d))}. \tag{2.19}$$

Proposition 2.1. *The expected time on the market $E[TOM(d)]$ increases with distance to the city center d.*

Proof. Using the equilibrium relation between reservation dividends and probabilities of sale (2.9), we can express the expected time on the market in terms of the reservation dividend:

$$E[TOM(d)] = \frac{1}{\tilde{\varepsilon} - \varepsilon^*(d)}. \tag{2.20}$$

The derivative of the expected time on the market with respect to the distance to the city center amounts to

$$\frac{\partial E[TOM]}{\partial d} = (\tilde{\varepsilon} - \varepsilon^*(d))^{-2}\,\frac{\partial \varepsilon^*}{\partial d} > 0 \tag{2.21}$$

if $\partial\varepsilon^*/\partial d > 0$. We know that this holds from Lemma 2.1. □

Intuition. Reservation dividends increase with distance to the city center, which reflects compensation for travel costs. Hence, probabilities of sale decrease with distance to the city center. A lower probability of sale implies a higher expected time on the market by definition.

Proposition 2.2. *The sales price $p(d)$ decreases with distance to the city center d.*

Proof. Via (2.15) from Lemma 1, we have that

$$p(d) = \frac{\beta}{1-\pi\beta}(\tilde{\varepsilon} - \varepsilon^*(d)) + \frac{\beta^2}{(1-\beta)(1-\pi\beta)}(\tilde{\varepsilon} - \varepsilon^*(d))^2. \tag{2.22}$$

Then, with $\partial\varepsilon^*/\partial d > 0$,

$$\frac{\partial p}{\partial d} = \underbrace{\frac{\beta}{1-\pi\beta}}_{>0}\underbrace{\left(-\frac{\partial\varepsilon^*}{\partial d}\right)}_{<0} + \underbrace{\frac{2\beta^2(\tilde{\varepsilon} - \varepsilon^*(d))}{(1-\beta)(1-\pi\beta)}}_{>0}\underbrace{\left(-\frac{\partial\varepsilon^*}{\partial d}\right)}_{<0} < 0. \tag{2.23}$$

□

Intuition. Sellers expect to sell apartments with a lower probability outside the city center, where they face a lower expected housing demand. Being local monopolists, they find it optimal to post lower prices outside the city center.[18] As in the standard monocentric city model, the underlying factor for prices to decrease with distance to the city center is the cost of travel to the city center.

2.4.4 Solution method

We have established analytically that our model rationalizes the empirical patterns of decreasing liquidity and prices with distance to the city center. These statements are qualitative. We are not able to solve the model in closed form, as we obtain a nonlinear system of equations via the equilibrium conditions. Hence, we solve the model numerically. We show that our model performs well quantitatively by calibrating it with our data on transactions and advertisements.

[18] Moreover, the sensitivity of prices to the probability of sale decreases with distance to the city center. Formally, via (2.14), we get the derivative of prices with respect to the probability of sale at a given location:

$$\frac{\partial p}{\partial \gamma} = \frac{\beta}{1-\pi\beta} + \frac{2\beta^2}{(1-\beta)(1-\pi\beta)}\gamma(\cdot),$$

which decreases with distance to the city center.

Algorithm 1 Solution algorithm: stationary spatial equilibrium

Initialize an iteration tolerance η.
Initialize a value of search W.
Initialize an updated value of search $\tilde{W}$ with $|W - \tilde{W}| > \eta$.
while $|W - \tilde{W}| > \eta$ **do**
    Set $W = \tilde{W}$.
    for $d^\Delta \in \mathcal{D}^\Delta$ **do**
        Solve equations (2.4) to (2.9) for this distance d^Δ, given the value of search W.
    end for
    Update $\tilde{W}$.
end while

Setup. We discretize the distribution of distances to the city center: $\mathcal{D}^\Delta = \{d_1^\Delta, \ldots, d_l^\Delta\}$. The equilibrium condition (2.6), which describes the value of search as an expectation over distances to the city center and idiosyncratic dividends, and the equilibrium conditions (2.4) to (2.9), which have to hold for all distances to the city center $d^\Delta \in \mathcal{D}^\Delta$, constitute a system of non-linear equations. All variables in the system depend on the distance to the city center, except for the value of search W.

Solution algorithm. We solve the model using the solution algorithm described in the environment of Algorithm 1 to obtain the stationary spatial equilibrium. The algorithm iterates over the value of search W. It starts from a guess for the value of search and updates the guess using the expectation of searchers over the apartments they could match with before drawing a random apartment. We update the guess by calculating the searchers' expectation via the expression

$$\tilde{W} = \frac{1}{l} \sum_{d^\Delta \in \mathcal{D}^\Delta} \gamma(p(d^\Delta))\Big(V_m(d^\Delta, E[\varepsilon|_{\varepsilon \geq \varepsilon^*(d^\Delta)}]) - p(d^\Delta)\Big) + (1 - \gamma(p(d^\Delta)))\Big(\beta W\Big).$$

This alternative expression uses the fact that a buyer purchases an apartment with the probability of sale and continues to search with one minus the probability of sale. See Krainer and LeRoy (2002) for details on this approach. The algorithm stops when this expectation is consistent with the guess. We obtain our first guess for the value of search by solving the model without the spatial structure and using the value of search from that solution.

2.4.5 Calibration and estimation of model parameters

We solve the model separately for each city. To get the discretized distance distribution $\mathcal{D}^\Delta$, we group the distances to the city center from our apartment sales data into $l = 15$ bins with equal numbers of observations. We obtain the corresponding travel times from our travel time estimates described in Section 2.4 and convert these into travel costs $\tau(d)$, assuming that $\tau(d) = \mu \cdot \tilde{\tau}(d)$, where $\tilde{\tau}(d)$ is the travel time that we obtain from the data. $\tilde{\tau}(d)$ measures the travel time to the city center in minutes at some distance to the city center. μ measures the cost (in model units, and due to risk-neutrality of agents in terms of utility) of traveling two minutes by car, as agents commute back and forth between their apartment and the city center each day. As such, we relate to similar translations of within-city travel time into travel costs as done by, e.g., Ahlfeldt et al. (2015) and Heblich et al. (2020).

Figure 2.9: Inputs and outputs of the model

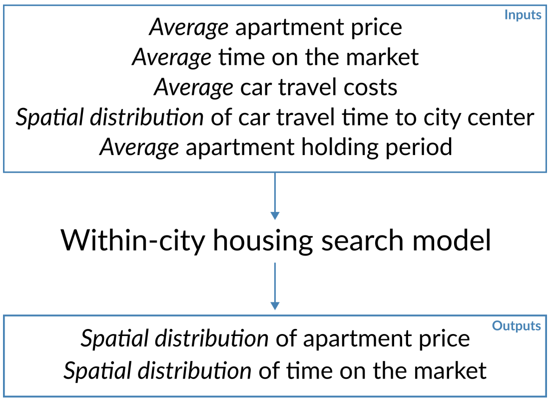


Table 2.2: Calibrated parameters

Parameter	Description	Value	Source
β	Discount factor	0.99986 (yearly: 0.95)	Standard parameter
π^{Hamburg}	Housing match persistence	0.99971 (yearly: 0.90)	Apartment holding periods
π^{Munich}	"	0.99974 (yearly: 0.91)	"
π^{Cologne}	"	0.99970 (yearly: 0.89)	"
$\pi^{\text{Frankfurt}}$	"	0.99971 (yearly: 0.90)	"
$\pi^{\text{Duesseldorf}}$	"	0.99972 (yearly: 0.90)	"

Calibrated parameters. We set $\beta = \sqrt[365]{0.95} \approx 0.99986$, such that the annual discount factor is 0.95. The housing match persistence for Munich is given by $\pi^{\text{Munich}} = 1 - \frac{1}{126 \cdot 30} \approx 0.99974$, as the average holding period for apartments in the data is 126 months. This value refers to observations from January 1990 to December 2022. We increase the time span for the calibration of this parameter to capture the full length of holding periods, which typically span about nine years, as well as possible.[19] The housing match persistence for Cologne is $\pi^{\text{Cologne}} = 1 - \frac{1}{109.5 \cdot 30} \approx 0.99970$, and the housing match persistence for

[19]The housing match persistence exhibits little variation across space when we take into account the spatial variation in holding periods (see Figure A2.10 in the Appendix for the spatial distribution of π^{Cologne} as an example), so we calibrate it using the average holding period.

Duesseldorf is $\pi^{\text{Duesseldorf}} = 1 - \frac{1}{120.7 \cdot 30} \approx 0.99972$. For Hamburg and Frankfurt we do not have data on holding periods available, hence we use the average holding period across all other cities. On a yearly basis, the probability of being unmatched is about 10% in every city. You can find an overview of all calibrated parameters in Table 2.2.

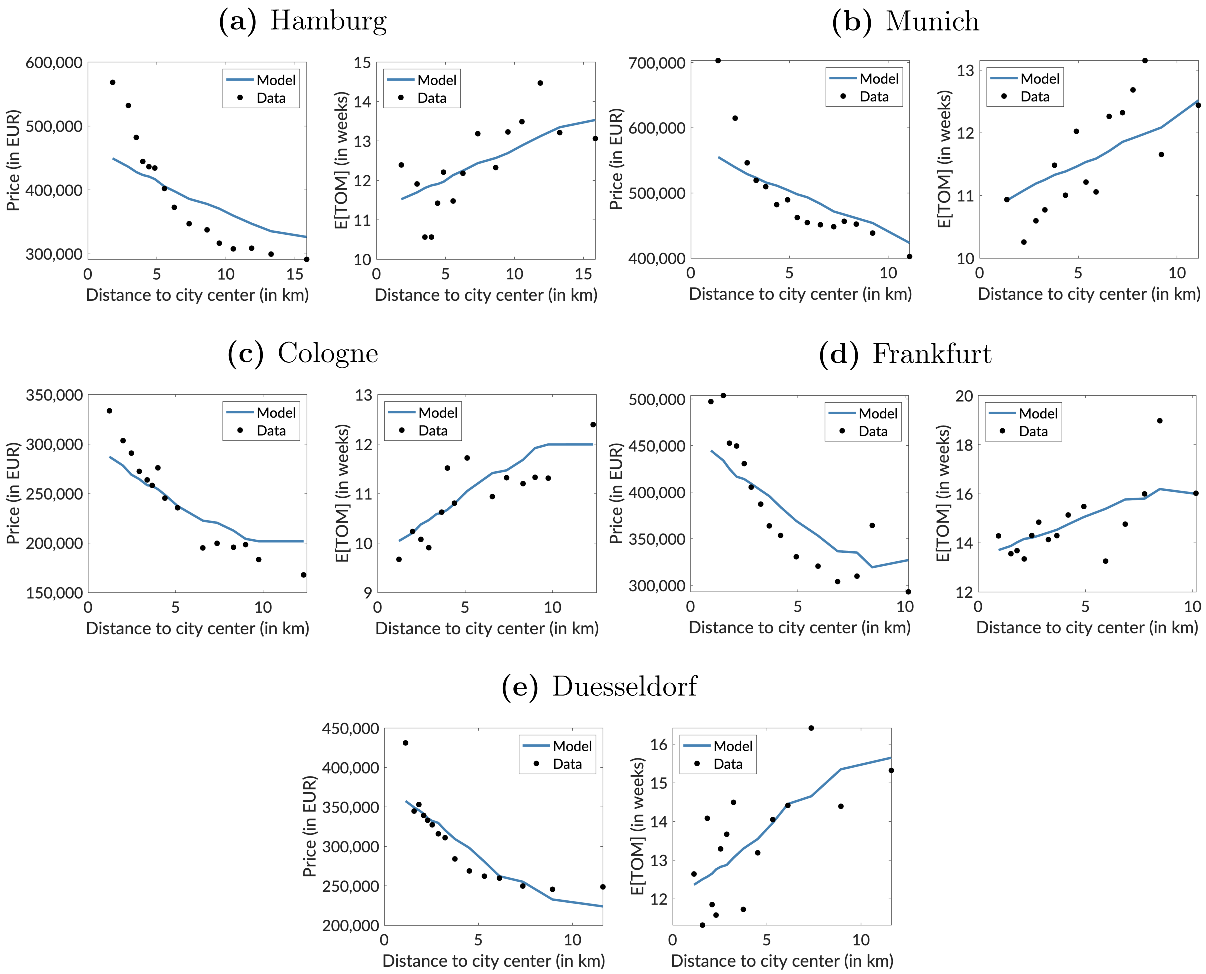
Table 2.3: Estimated parameters

Parameter	Description	Value	Bootstr. 95% CI	Target statistic	Target (model) value
μ^{Hamburg}	Travel time scaling	0.0038	$[0.0037, 0.0041]$	Daily car op. costs	14.17 (14.16) €
μ^{Munich}	—— " ——	0.0047	$[0.0044, 0.0049]$	—— " ——	14.17 (14.16) €
μ^{Cologne}	—— " ——	0.0102	$[0.0096, 0.0109]$	—— " ——	14.17 (14.17) €
$\mu^{\text{Frankfurt}}$	—— " ——	0.0047	$[0.0045, 0.0050]$	—— " ——	14.17 (14.19) €
$\mu^{\text{Duesseldorf}}$	—— " ——	0.0074	$[0.0068, 0.0081]$	—— " ——	14.17 (14.17) €
$\tilde{\varepsilon}^{\text{Hamburg}}$	Dividend dist. bound	0.56	$[0.55, 0.59]$	Avg. time on mkt.	12.38 (12.38) wk
$\tilde{\varepsilon}^{\text{Munich}}$	—— " ——	0.62	$[0.60, 0.64]$	—— " ——	11.59 (11.59) wk
$\tilde{\varepsilon}^{\text{Cologne}}$	—— " ——	0.71	$[0.68, 0.75]$	—— " ——	11.02 (11.02) wk
$\tilde{\varepsilon}^{\text{Frankfurt}}$	—— " ——	0.44	$[0.43, 0.46]$	—— " ——	14.80 (14.81) wk
$\tilde{\varepsilon}^{\text{Duesseldorf}}$	—— " ——	0.53	$[0.50, 0.56]$	—— " ——	13.50 (13.50) wk

Estimated parameters. We estimate the parameters for which we do not have data for calibration by targeting city-specific data moments. We use the method of simulated moments and minimize the euclidean distance of the data moments to the model moments, targeting 2 moments with 2 equally weighted parameters, μ and $\tilde{\varepsilon}$, for every city. You can find an overview of all estimated parameters in Table 2.3. With the travel cost function parameter μ we target average daily car operating costs in Germany of 14.17€ (Andor et al., 2020)[20]. We convert between model units and Euros via the average apartment sales price of each city. With the housing dividend distribution parameter $\tilde{\varepsilon}$ we target the city-specific average time on the market in the data. Intuitively, we allow differences in buyer preferences between cities to affect the average time on the market. We obtain 95% confidence intervals by drawing 1,000 bootstrapped replications of the data with replacement, each sized 1/3 of a city's sample, and using the 3rd and 98th percentile of the resulting parameter distributions as confidence bounds (see, e.g., Gavazza, 2016). Given the non-linearity of the model, these intervals can be asymmetric.

[20]See: supplementary information, Table 3, total monthly car operating costs.

Figure 2.10: Model results: spatial distributions of apartment prices and liquidity



Notes: "TOM" refers to time on the market. The data points are calculated using the regression specification (2.2) with year-quarter fixed effects and apartment characteristics controls.

2.4.6 Model results

Even though we do not target the spatial gradient of any of our two main variables, price and expected time on the market, our results exhibit spatial variation that closely aligns with the data. We plot a comparison of data and model values in Figure 2.10. For each city, we display the average apartment sales price and time on the market by distance to the city center from the data, controlling for apartment characteristics and time fixed effects, together with our model results. Our model matches the spatial price and liquidity gradients with high precision precision in all cities. Figures A2.7 to A2.12 in the Appendix show the model results for other endogenous variables.

2.5 Housing liquidity and asset pricing

In the previous sections, we presented empirical evidence and a theoretical framework showing that apartment prices and liquidity decrease with distance to the city center. Liquidity differences may partly explain the price differences between city center and outskirts, implying that buyers are willing to pay a liquidity premium to own an apartment in the city center. In this section, we measure this liquidity premium and examine how it changes with distance to the city center.

We think of apartment prices being composed of a fundamental value and a liquidity premium. We want to measure the liquidity premium by distance to the city center to quantify how much liquidity matters for the pricing of apartments across space. Both the fundamental value and the liquidity premium depend on the distance to the city center, which makes it challenging to disentangle these two variables empirically.[21]

Hence, we use our model to structurally estimate the liquidity premium. Following Krainer and LeRoy (2002), we remove search frictions from our model and compare the resulting price gradient to the price gradient from the equilibrium with search frictions. Intuitively, an efficient version of the market in our model can be best approximated by a market with a very thick seller side. Buyers in such a market have many outside options, which eliminates the price setting power of sellers. Recall that in our model, search frictions grant sellers local monopoly power, enabling them to set high prices in the city center while selling with a low time on the market.

We compare the allocation with frictional search to the optimal allocation in the spirit of e.g., Gavazza (2016), who quantifies the price impact of trading frictions in a decentralized search market. As an analogy, consider the comparison between optimal unemployment and frictional unemployment. In the efficient allocation, liquidity is not infinite, in other words, the expected time on the market is not equal to zero, similarly to an optimal unemployment rate not equal to 0%. Agents have a benefit from searching until they have found an apartment for which they draw a high enough reservation dividend, and as such, some amount of search and hence illiquidity is optimal.[22]

Efficient allocation. To obtain the efficient allocation, we maximize the discounted average net utility of matched and unmatched agents. Agents have linear utility functions and unmatched agents live in rental housing in the city center, receiving zero net utility.

[21] Measuring the liquidity premium in other asset classes is more straightforward given that the assets' cash flow and maturity are directly observable (e.g. Amihud and Mendelson, 1986). In housing markets, however, the cash flow is typically a latent variable, which needs to be estimated.

[22] Depending on the calibration, there can be too little or too much search implied by the frictional allocation compared to the efficient allocation (see Krainer and LeRoy (2002)). With our calibration, we are clearly in a region of parameter combinations with which agents search too much for every city.

Hence, we maximize the sum of housing dividends, that is, the housing utility of matched agents, net of travel costs. Assuming a steady state, we choose reservation dividends $\varepsilon^{\text{eff}}(d^{\Delta})$ in the discretized model to maximize welfare

$$\mathbb{W} = \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{l} \sum_{d^{\Delta} \in \mathcal{D}^{\Delta}} \left(m(d^{\Delta})(E[\bar{\varepsilon}(d^{\Delta})] - \bar{\tau}(d^{\Delta})) \right) \right), \tag{2.24}$$

where $m(\cdot)$ denotes the unconditional probability of being matched and bars denote average values at a given distance to the city center. Due to the uniform distribution of idiosyncratic dividends, agents at distance d^{Δ} have an average dividend of $\frac{\varepsilon^{\text{eff}}(d^{\Delta})+\tilde{\varepsilon}}{2}$. All agents at distance d^{Δ} pay travel costs $\tau(d^{\Delta})$. As there are no further constraints, we equivalently maximize

$$m(d^{\Delta}) \left(\frac{\varepsilon^{\text{eff}}(d^{\Delta}) + \tilde{\varepsilon}}{2} - \tau(d^{\Delta}) \right) \tag{2.25}$$

at every distance $d^{\Delta} \in \mathcal{D}^{\Delta}$. Next, we calculate the probability of being matched $m(d^{\Delta})$. The transition matrix for the states "matched" (up, left) and "unmatched" (down, right) is

$$\mathbb{T} = \begin{pmatrix} \pi & 1-\pi \\ \pi(\tilde{\varepsilon} - \varepsilon^{\text{eff}}(d^{\Delta})) & 1 - \pi(\tilde{\varepsilon} - \varepsilon^{\text{eff}}(d^{\Delta})) \end{pmatrix}, \tag{2.26}$$

where agents transition from being unmatched to being matched with probability $1 - F(\varepsilon^{\text{eff}}(d^{\Delta})) = \tilde{\varepsilon} - \varepsilon^{\text{eff}}(d^{\Delta})$ and keep the apartment with probability π. The steady-state probability of being matched is $m(d^{\Delta}) = \pi m(d^{\Delta}) + \pi(\tilde{\varepsilon} - \varepsilon^{sp}(d^{\Delta}))(1 - m(d^{\Delta}))$, and thus

$$m(d^{\Delta}) = \frac{\pi(\tilde{\varepsilon} - \varepsilon^{\text{eff}}(d^{\Delta}))}{1 - \pi + \pi(\tilde{\varepsilon} - \varepsilon^{\text{eff}}(d^{\Delta}))}. \tag{2.27}$$

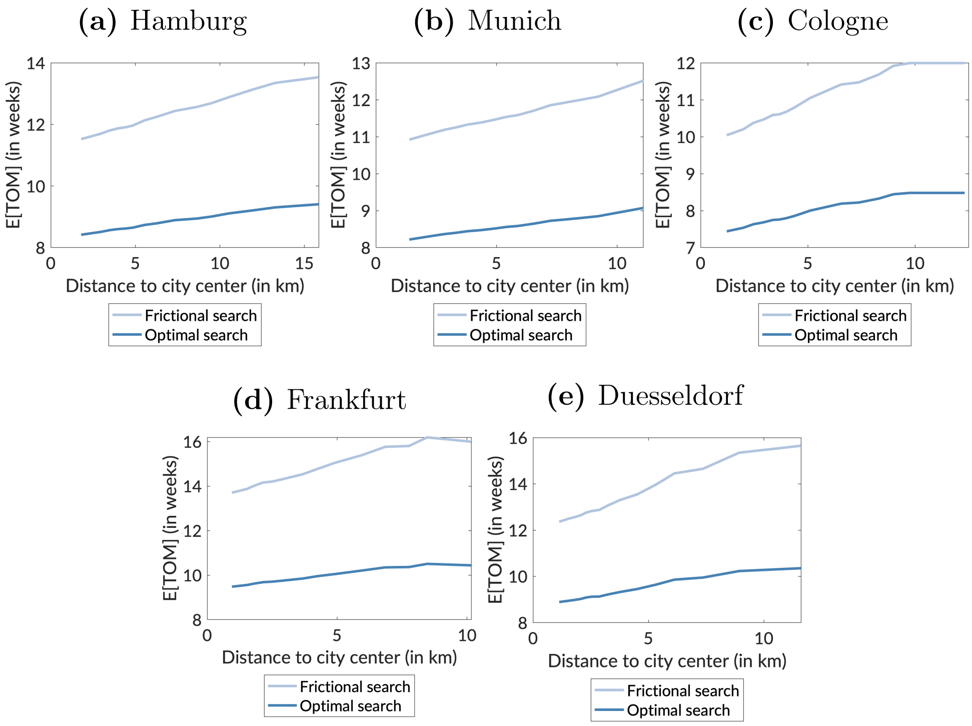
The optimal reservation dividend at distance d^{Δ} is hence given by

$$\underset{\varepsilon^{\text{eff}}(d^{\Delta})}{\operatorname{argmax}} \left(\frac{\pi(\tilde{\varepsilon} - \varepsilon^{\text{eff}}(d^{\Delta}))}{1 - \pi + \pi(\tilde{\varepsilon} - \varepsilon^{\text{eff}}(d^{\Delta}))} \right) \left(\frac{\varepsilon^{\text{eff}}(d^{\Delta}) + \tilde{\varepsilon}}{2} - \tau(d^{\Delta}) \right), \tag{2.28}$$

which we calculate numerically. In Figure 2.11, we plot the associated expected time on the market by distance to the city center. The levels shift down, which implies that agents search too much in the frictional equilibrium. More importantly, the gradients flatten, that is, spatial liquidity differences even out. We want to quantify how this change in the liquidity gradient transmits to a change in the price gradient. To this end, we need the associated prices, which we get via (2.15):

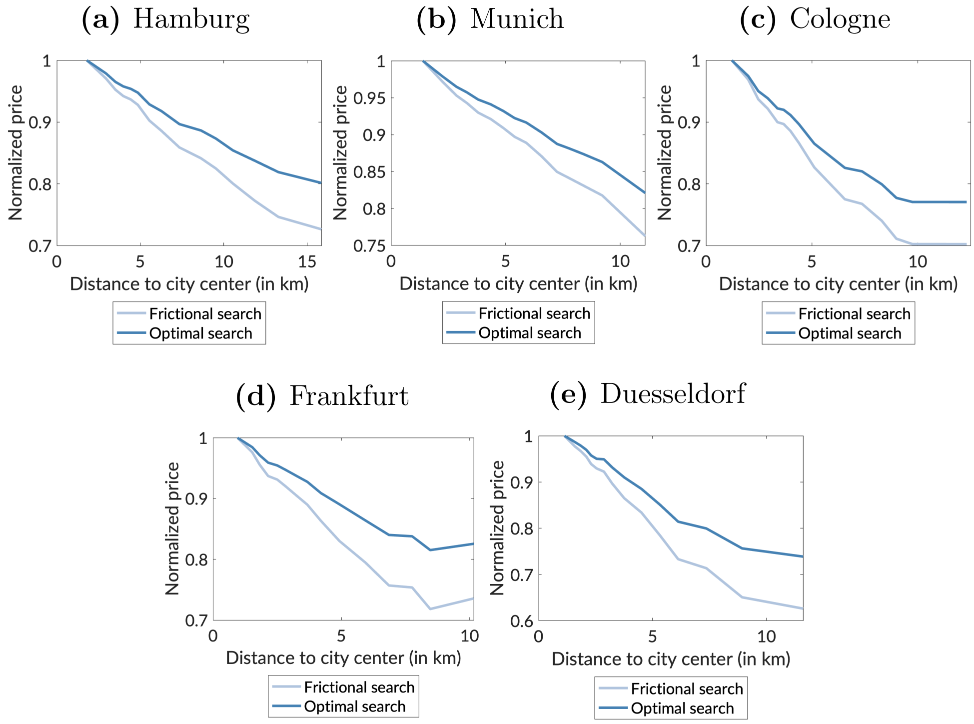
$$p^{\text{eff}}(d^{\Delta}) = \frac{\beta}{1 - \pi\beta}(\tilde{\varepsilon} - \varepsilon^{\text{eff}}(d^{\Delta})) + \frac{\beta^2}{(1-\beta)(1-\pi\beta)}(\tilde{\varepsilon} - \varepsilon^{\text{eff}}(d^{\Delta}))^2. \tag{2.29}$$

Figure 2.11: Spatial liquidity distributions with frictional and optimal search



Notes: The plot shows the equilibrium expected time on the market from our main model ("frictional search") and the expected time on the market from the efficient allocation ("optimal search") as implied by (2.28).

Figure 2.12: Normalized spatial price distributions with frictional and optimal search



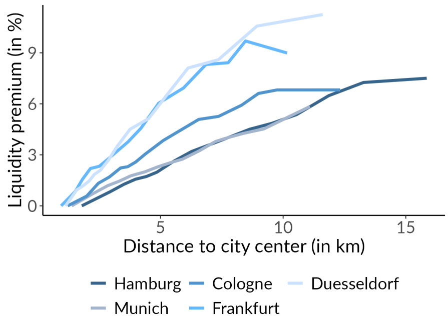
Notes: The plot shows the equilibrium prices from our main model ("frictional search") and the prices from the efficient allocation ("optimal search") as specified in (2.29), normalized by the apartment price at the location closest to the city center.

Liquidity premium. To calculate the liquidity premium in the city center (distance d_1^{Δ}) relative to some distance d^{Δ}, we compare the relative prices by distance to the city center from the efficient allocation $p^{\text{eff}}(d^{\Delta})/p^{\text{eff}}(d_1^{\Delta})$ to the relative prices from the frictional allocation $p(d^{\Delta})/p(d_1^{\Delta})$ and plot the resulting relative price gradients in Figure 2.12. We define the liquidity premium in the city center relative to distance d^{Δ} as

$$l(d^{\Delta}) = p^{\text{eff}}(d^{\Delta})/p^{\text{eff}}(d_1^{\Delta}) - p(d^{\Delta})/p(d_1^{\Delta}). \tag{2.30}$$

We interpret this difference as the cost of illiquidity at distance d^{Δ} relative to the city center. You can find the resulting liquidity premium curves in Figure 2.13. Our model implies liquidity premiums in the city center relative to the outskirts of about 5.5% of the house price in the city center.

Figure 2.13: Liquidity premium across space



Notes: The plot shows the liquidity premium in the city center relative to a given distance to the city center, as defined in (2.30), in terms of the house price in the city center.

Sensitivity analysis. In the calibration of our model, we set the yearly discount factor to 0.95, which is a standard choice. The housing match persistence probabilities are observable in the data, however, we are not able to capture all apartments in our calibration. We cannot calculate holding periods for apartments that were transacted before the beginning of our sample in January 1990. We check to what extent the liquidity premium changes if we use different discount factors or different match persistence probabilities. In Figure 2.14, we plot the liquidity premium at 10km to the city center for all cities, varying the yearly discount factor between 0.935 and 0.965 and the yearly housing match persistence between 0.88 and 0.92 which corresponds to holding periods between 95 and 146 months. On average, across cities, discount factors, and match persistence probabilities, we observe a liquidity premium at 10km to the city center of about 5.5% in terms of the house price in the city center.

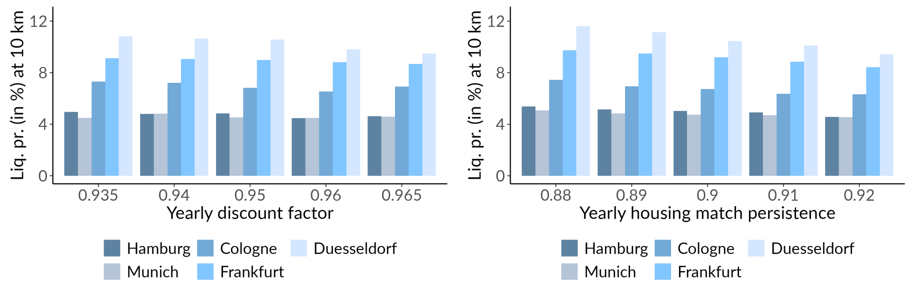
Search frictions and data fit. Using the results on the spatial price gradients with and without search frictions and our spatial price gradients from the data (see Figure 2.5), we can compare if and to what extent search frictions in the model improve the fit to the price data. From Figure 2.12, we observe that the model with search frictions fits the price gradients from the data better than the model without search frictions. The data points in the plot are normalized such that the average price in the data aligns with the average price from the model with search frictions, as in our calibration. To quantify the improvement in the fit to the data, we regress log prices from the two model versions and from the data on the distances to the city center, as done for Figure 2.5. Then, we calculate the difference in the coefficients for the distance to the city center between the version with and without search frictions, relative to the coefficient from the regressions using the data. The resulting measure captures the quantitative role of spatial liquidity differences due to search frictions in explaining the spatial price gradient from the data.

For Hamburg, search frictions explain 21% of the spatial price gradient, and for Munich, Cologne, Frankfurt, and Duesseldorf, they explain 20%, 34%, 33%, and 42% of the spatial price gradient. On average across cities, this amounts to 30%, and we conclude that spatial liquidity differences due to search frictions explain more than a quarter of the within-city spatial price gradient.

Figure 2.14: Liquidity premium at 10km distance to the city center

(a) By discount factor β, with $\pi = \pi^{\text{city}}$

(b) By match persistence π, with $\beta = 0.95$



Notes: The plots show the liquidity premium in the city center compared to a distance of 10km to the city center, as defined in (2.30), in terms of the house price in the city center, for different yearly discount factors and housing match persistence probabilities.

2.6 Conclusion

In this paper, we demonstrate that housing market liquidity decreases with distance to the city center, using a novel dataset with matched apartment transactions and advertisements from large German cities between January 2012 and December 2022. We rationalize our findings in a spatial search model of a housing market in a monocentric city. We show analytically that as an inherent characteristic of the model, the expected time on the market decreases with distance to the city center jointly with the sales price. We calibrate the model with our dataset and obtain a quantitatively precise fit to the data. Using our model, we estimate a liquidity premium in the city center compared to the outskirts ranging from 5.5% in terms of the apartment price in the city center. We conclude that liquidity is priced in a large magnitude across space in urban housing markets.

Appendix 2.A Data cleaning

In this section of the appendix we describe in more detail how we clean the data and prepare the micro data.

We control for the age of the properties by creating a categorical variable that divides the observations into different construction periods. We follow the commonly used categories introduced by the official German appraisers. In particular, we construct the following categories: pre-1950, 1950-1977, 1978–1990, 1990-2005, and post-2005. In addition, we also include a category for properties that are being occupied for the first time and another category that identifies properties where construction is not yet complete. In the regression analysis, we use a categorical variable rather than a continuous variable for the age of the property because the relationship between age and price and liquidity is highly non-linear in the case of the German housing market, as shown in Amaral et al. (2023c).

Following the literature, we divided the heating type of each home into four different categories. We define "brown" dwellings as those that consume energy produced by oil, coal, or use space heating and tile stove heating. We define "standard" dwellings as those that consume energy produced by gas and use central heating. We define "green" properties as those where the energy comes from solar, heat pump or pellets, or use district heating or CHP. We also define an "Other" category, taken directly from the dataset, which includes other energy sources.

We use the categorical variable "aus klassen", provided directly by the platform, to consider the quality of the furnishings and interiors of the property. In addition, we include the categorical variable "zust klassen", also provided by the data platform, to categorize the quality of the construction of the building.

We create a categorical variable to control for the number of rooms in the property. The variable has four categories: 1 room, 2 rooms, 3 rooms, and 4 or more rooms.

We also control for the number of floors on which the apartment is located and the total number of floors in the building where the apartment is located.

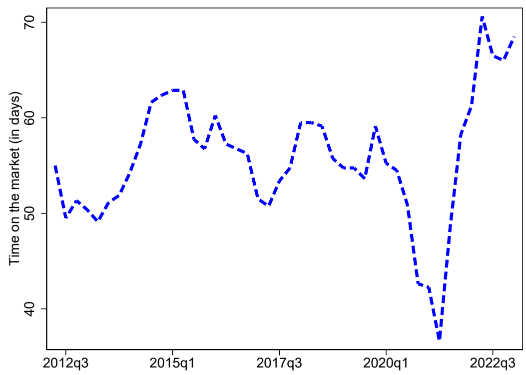
Appendix 2.B Additional empirical results

In this section of the appendix, we provide figures and tables that support the empirical findings in this paper. We begin by presenting the regression output tables, which analyze the relation between time on the market and distance to the city center. We display one table per city. Following this, we present additional empirical results and visualizations regarding the spatial distribution of transaction prices and asking prices.

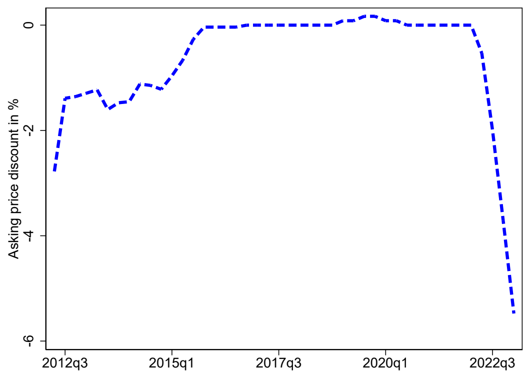
2.B.1 Time series of housing liquidity

Figure A2.1: Time series of liquidity

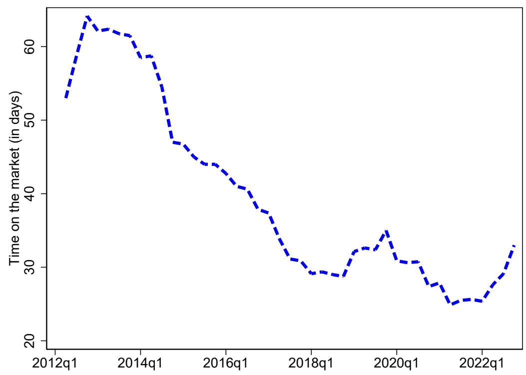
(a) Hamburg – Time on the market



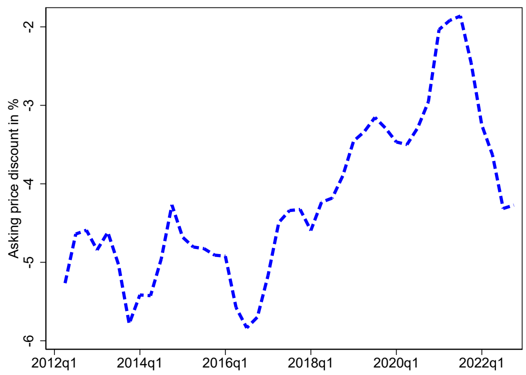
(b) Hamburg – Asking price discount



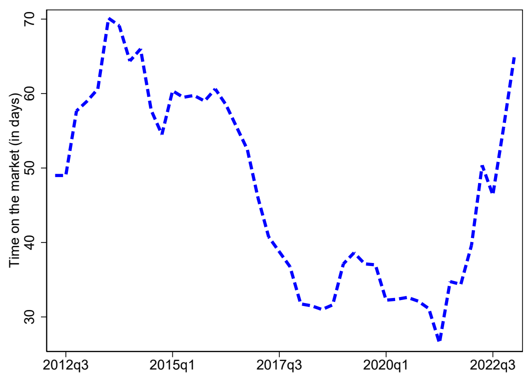
(c) Cologne - Time on the market



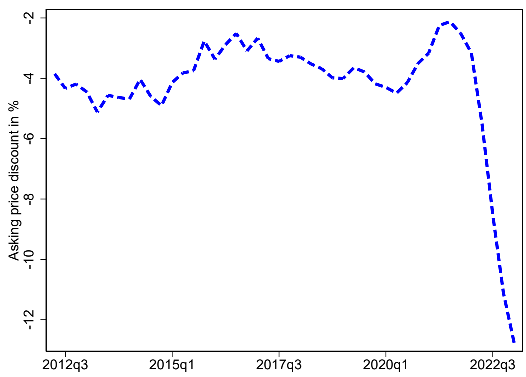
(d) Cologne - Asking price discount



(e) Duesseldorf – Time on the market



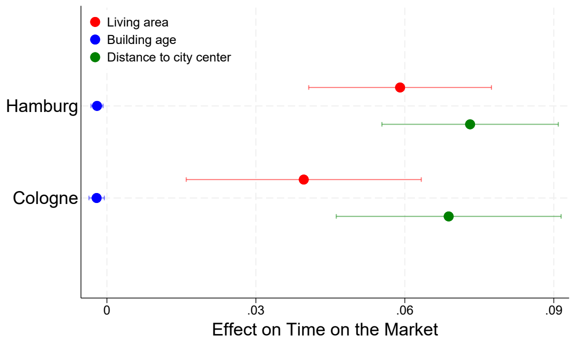
(f) Duesseldorf – Asking price discount



2.B.2 Additional determinants of housing market liquidity

In our main analysis, we exclusively focus on how liquidity affects prices via the location of the house. Nevertheless, houses differ in other dimensions which might also impact their liquidity. In particular, the size and age of the property might be strong determinants of liquidity, as typically the market is also segmented along these dimensions. In other words, the market for larger houses might be thinner than that for smaller houses. Although our focus in this paper is not on these additional dimensions, we also provide evidence that location has a stronger effect on liquidity than these other factors. In Figure A2.2, we plot the standardized coefficients for size as measured by living area square meters, age of the building, and distance to the city center. The coefficients are derived from regression 2.2. As we can see, all coefficients are positive and significant, suggesting that these dimensions have a significant impact on liquidity as measured by time on the market. Nevertheless, as is also evident from the graph, distance to the city center has the largest impact on liquidity.

Figure A2.2: Determinants of time on the market, (January 2012- December 2022)



Notes: These figures show the OLS regression coefficients by city, as well as its respective 99% confidence intervals. See Footnote 7 for a full list of these characteristics. Distance to the city center is measured as the kilometer distance. The coefficients are standardized and thus comparable.

2.B.3 Time on the market – regression output tables

Table A2.1: Time on the market and distance to the city center (All cities, January 2012- December 2022)

	TOM	TOM	TOM	TOM
Distance to center (in km)	0.31***	0.24***		
	(0.05)	(0.04)		
Travel time to center (in min)			0.13***	0.09***
			(0.02)	(0.02)
Year-quarter FEs	Yes	Yes	Yes	Yes
City FEs	Yes	Yes	Yes	Yes
Property characteristics	No	Yes	No	Yes
N	87502	87502	86168	86168
R^2	0.04	0.11	0.04	0.12

Notes: This table shows results for regressions of the time on the market on the distance to the city center as specified in the regression specification (2.2). "TOM" refers to the time on the market in weeks as defined in (2.1). Standard errors (in parentheses) are clustered at the city level. The property characteristics are control variables. See Footnote 7 for a full list of these characteristics. $* : p < 0.1; ** : p < 0.05; *** : p < 0.01$.

Table A2.2: Property prices and distance to the city center (All cities, January 2012-December 2022)

	Price	Price	Price	Price
Distance to center (in km)	-1.9e+04***	-1.9e+04***		
	(2477.84)	(1391.39)		
Travel time to center (in min)			-9112.96***	-8530.82***
			(1655.18)	(659.80)
Year-quarter FEs	Yes	Yes	Yes	Yes
City FEs	Yes	Yes	Yes	Yes
Property characteristics	No	Yes	No	Yes
N	87502	87502	86168	86168
R^2	0.20	0.74	0.20	0.74

Notes: This table shows results for regressions of the sales price on the distance to the city center as specified in the regression specification (2.2). "Price" refers to the sales price in euros. Standard errors (in parentheses) are clustered at the city level. The property characteristics are control variables. See Footnote 7 for a full list of these characteristics. $* : p < 0.1; ** : p < 0.05; *** : p < 0.01$.

Table A2.3: Time on the market and distance to the city center (Cologne, January 2012- December 2022)

	TOM	TOM	TOM	TOM
Distance to center (in km)	0.32***	0.29***		
	(0.09)	(0.07)		
Travel time to center (in min)			0.16***	0.13***
			(0.04)	(0.03)
Quarter FEs	Yes	Yes	Yes	Yes
Property Characteristics	No	Yes	No	Yes
N	14188	14188	14188	14188
R^2	0.08	0.13	0.08	0.13

Notes: This table shows results for regressions of the time on the market on the distance to the city center as specified in the regression specification (2.2). "TOM" refers to the time on the market in weeks as defined in (2.1). Standard errors (in parentheses) are clustered at the borough (*Stadtbezirk*) level. The property characteristics are control variables. See Footnote 7 for a full list of these characteristics. $* : p < 0.1; ** : p < 0.05; *** : p < 0.01$.

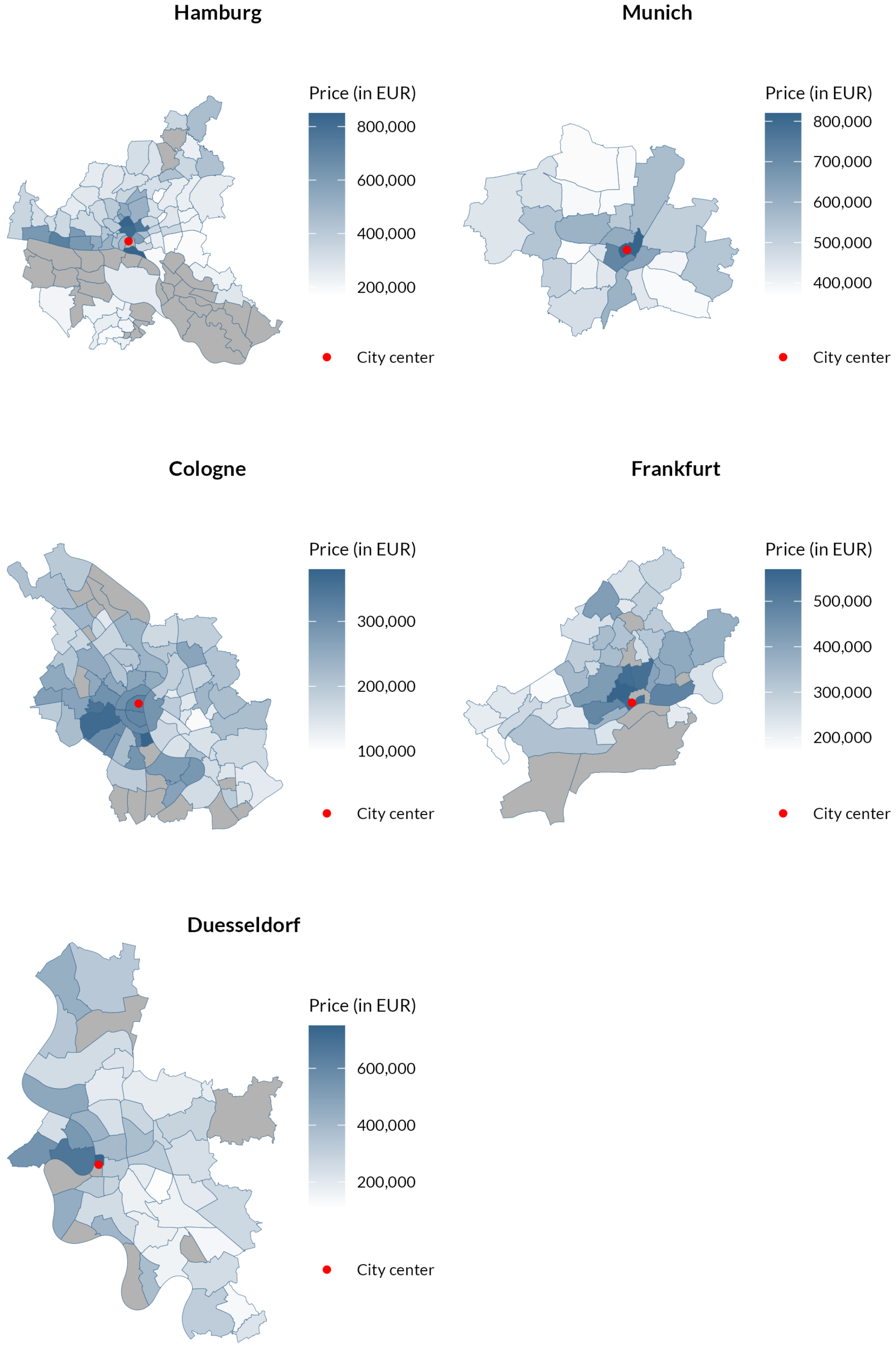
Table A2.4: Time on the market and distance to the city center (Hamburg, January 2012- December 2022)

	TOM	TOM	TOM	TOM
Distance to center (in km)	0.37***	0.26***		
	(0.03)	(0.05)		
Travel time to center (in min)			0.16***	0.11***
			(0.01)	(0.01)
Quarter FEs	Yes	Yes	Yes	Yes
Property Characteristics	No	Yes	No	Yes
N	20672	20672	20672	20672
R^2	0.02	0.11	0.02	0.11

Notes: This table shows results for regressions of the time on the market on the distance to the city center as specified in the regression specification (2.2). "TOM" refers to the time on the market in weeks as defined in (2.1). Standard errors (in parentheses) are clustered at the borough (*Stadtbezirk*) level. The property characteristics are control variables. See Footnote 7 for a full list of these characteristics. $* : p < 0.1; ** : p < 0.05; *** : p < 0.01$.

2.B.4 Spatial distributions of transaction and asking prices

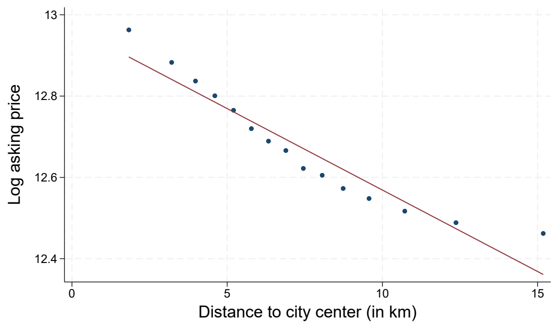
Figure A2.3: Transaction prices across space (January 2012- December 2022)



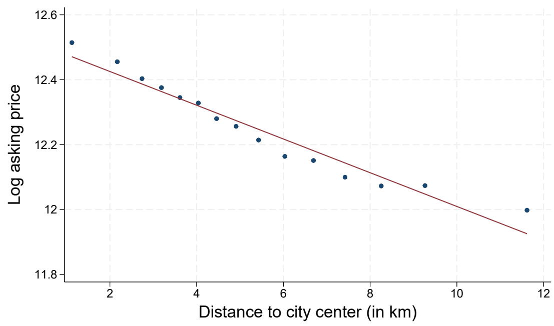
Notes: he maps show the average transaction price by district (*Stadtteil*) from our matched data set, controlling for year-quarter fixed effects. Districts without available data are colored gray.

Figure A2.4: Spatial gradients of asking prices (January 2012- December 2022)

(a) Hamburg



(b) Cologne



Notes: These binscatter plots visualize the results of the regression specification (2.2) with log asking price as the outcome variable and 15 equally-sized distance bins. The city center is the Cologne Cathedral. The regressions control for year-quarter fixed effects and apartment characteristics. See Footnote 7 for a full list of these characteristics.

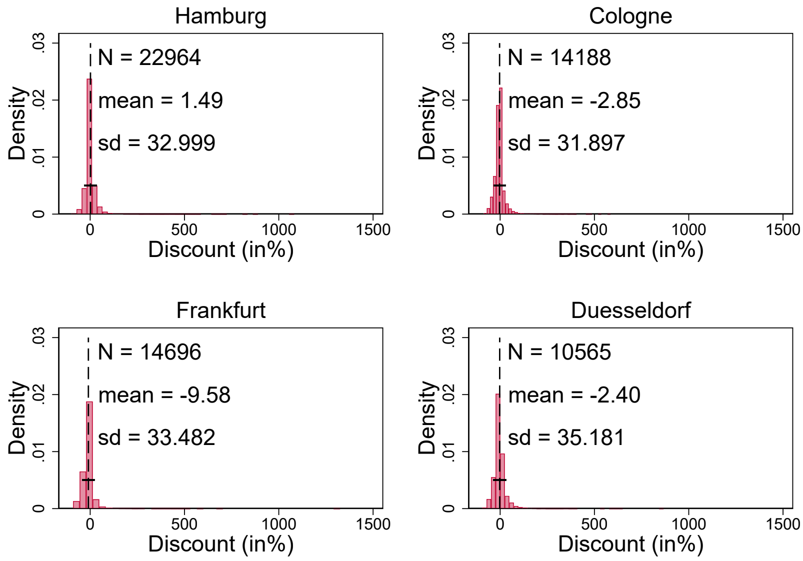
Appendix 2.C Robustness analysis

In this section of the Appendix, we offer additional empirical evidence supporting the robustness analysis section of the paper. Before delving into the empirical analysis, we present histograms with the distribution of asking price discounts by city. We then present evidence showing that the asking price discount becomes increasingly negative with distance from the city center. Here, we provide both binscatters and regression output tables. Next, we demonstrate that our baseline results remain robust across different regression specifications. We present the regression output tables for the alternative specifications outlined in Section 2.3 of the paper, with one table per city.

2.C.1 Asking price discount

In Figure A2.5, we plot a histogram of the asking price discount for our matched sample by city. The majority of transactions exhibit a negative discount, that is, properties typically sell below their asking prices. The distribution resembles a normal distribution but has a more positive skew and thinner tails. On average, a property is transacted at a sales price below its asking price. There is a clear bunching at an asking price discount of 0%. This finding has been documented for other countries as well and reflects that the asking price is a relevant anchor for the bargaining process in housing markets, as it is a partial commitment for the seller (Han and Strange, 2016).

Figure A2.5: Histograms of asking price discount (January 2012- December 2022)



Notes: This figure shows histograms for the asking price discount in our matched data set, where we calculate this measure of liquidity as defined in equation (2.3).

Table A2.5: Relation between asking price discount and distance the to city center (Cologne, January 2012- December 2022)

	Discount	Discount	Discount	Discount
Distance to center (in km)	-0.61**	-0.41		
	(0.19)	(0.23)		
Travel time to center (in min)			-0.29**	-0.18
			(0.10)	(0.13)
Quarter FEs	Yes	Yes	Yes	Yes
Property Characteristics	No	Yes	No	Yes
N	14188	14188	14188	14188
R^2	0.01	0.07	0.01	0.07

Notes: This table shows results for regressions of the asking price discount on the distance to the city center as specified in the regression specification (2.2). "Discount" refers to the asking price discount in percent as defined in equation (2.3). Standard errors (in parentheses) are clustered at the borough (*Stadtbezirk*) level. The property characteristics are control variables. See Footnote 7 for a full list of these characteristics.
$* : p < 0.1; ** : p < 0.05; *** : p < 0.01$.

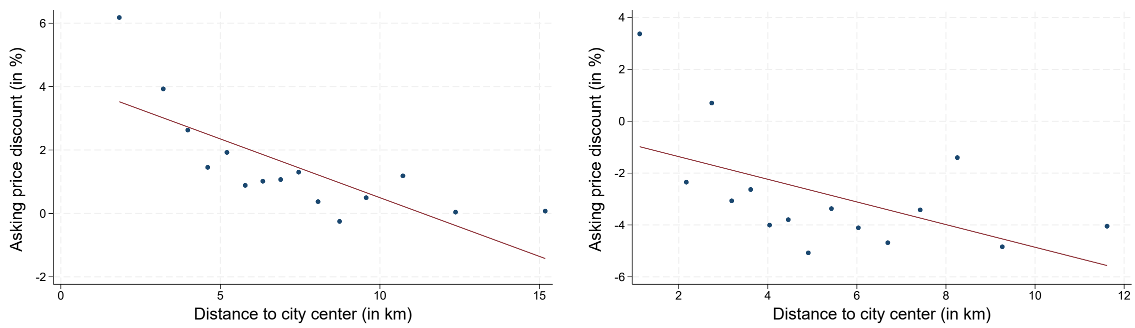
Table A2.6: Relation between asking price discount and distance the to city center (Hamburg, January 2012- December 2022)

	Discount	Discount	Discount	Discount
Distance to center (in km)	-0.40**	-0.39**		
	(0.15)	(0.13)		
Travel time to center (in min)			-0.30***	-0.32***
			(0.06)	(0.05)
Quarter FEs	Yes	Yes	Yes	Yes
Property Characteristics	No	Yes	No	Yes
N	22963	22963	22963	22963
R^2	0.01	0.06	0.02	0.06

Notes: This table shows results for regressions of the asking price discount on the distance to the city center as specified in the regression specification (2.2). "Discount" refers to the asking price discount in percent as defined in equation (2.3). Standard errors (in parentheses) are clustered at the borough (*Stadtbezirk*) level. The property characteristics are control variables. See Footnote 7 for a full list of these characteristics.
$* : p < 0.1; ** : p < 0.05; *** : p < 0.01$.

Figure A2.6: Spatial gradients of asking price discount (January 2012- December 2022)

(a) Asking price discount – Hamburg **(b)** Asking price discount – Cologne



Notes: These binscatter plots visualize the results of regression (2.2) with log asking price as the outcome variable and 15 equally-sized distance bins. The regressions include year-quarter fixed effects and control for apartment characteristics listed in Footnote 7.

2.C.2 Alternative regression specifications

In this section of the Appendix, we present the regression output tables for the alternative specifications as described in Section 2.3 of the paper. Table A2.7 presents the results for Cologne. Table A2.8 presents the results for Hamburg.

Table A2.7: Alternative specifications (Cologne, January 2012- December 2022)

	Poisson	Poisson	Log TOM	Log TOM
Distance to center (in km)	0.03***	0.03***	0.04***	0.04***
	(0.00)	(0.01)	(0.01)	(0.01)
Quarter FEs	Yes	Yes	Yes	Yes
Controls	No	Yes	No	Yes
N	13076	13076	13076	13076

Notes: This table shows results for regressions of the time on the market on the euclidian distance to the city center. The first two columns show the results for the Poisson regressions. The last two columns shows the results for the regression specification where we use log time on the market as dependent variable. Here we follow the specification of regression (2.2). Standard errors (in parentheses) are clustered at the borough (*Stadtbezirk*) level. The property characteristics are control variables. See Footnote 7 for a full list of these characteristics.
$* : p < 0.1; ** : p < 0.05; *** : p < 0.01$.

Table A2.8: Alternative specifications (Hamburg, January 2012- December 2022)

	Poisson	Poisson	Log TOM	Log TOM
Distance to center (in km)	0.03***	0.02***	0.04***	0.02***
	(0.00)	(0.00)	(0.01)	(0.00)
Quarter FEs	Yes	Yes	Yes	Yes
Controls	No	Yes	No	Yes
N	20672	20672	20672	20672

Notes: This table shows results for regressions of the time on the market on the euclidian distance to the city center. The first two columns show the results for the Poisson regressions. The last two columns shows the results for the regression specification where we use log time on the market as dependent variable. Here we follow the specification of regression (2.2). Standard errors (in parentheses) are clustered at the borough (*Stadtbezirk*) level. The property characteristics are control variables. See Footnote 7 for a full list of these characteristics.
$* : p < 0.1; ** : p < 0.05; *** : p < 0.01$.

Appendix 2.D Second-order condition of the seller's problem

The first-order condition of the seller's profit maximization problem is

$$\frac{\partial \Pi}{\partial p} = \gamma(p(d)) + p(d)\frac{\partial \gamma}{\partial p}\Big|_{p=p(d)} - \beta\Pi(d)\frac{\partial \gamma}{\partial p}\Big|_{p=p(d)} = 0. \tag{A2.1}$$

Hence, the second-order condition for a maximum is

$$\frac{\partial^2\Pi}{\partial p^2} = 2\left.\frac{\partial\gamma}{\partial p}\right|_{p=p(d)} + \left.\frac{\partial^2\gamma}{\partial p^2}\right|_{p=p(d)}(p(d) - \beta\Pi(d)) < 0. \tag{A2.2}$$

Using (2.13), we know that

$$\left.\frac{\partial\gamma}{\partial p}\right|_{p=p(d)} = -\frac{1-\pi\beta}{\beta} < 0. \tag{A2.3}$$

Therefore,

$$\left.\frac{\partial^2\gamma}{\partial p^2}\right|_{p=p(d)} = 0, \tag{A2.4}$$

and

$$\frac{\partial^2\Pi}{\partial p^2} = -2\,\frac{1-\pi\beta}{\beta} < 0, \tag{A2.5}$$

which provides the required maximum.

Appendix 2.E Extended model with bargaining

We extend our model by a bargaining process, following Carrillo (2012). With this addition, our model features an asking price and a sales price, which allows us to form a notion of a second measure of liquidity in the model. The asking price discount (APD) measures the relative difference between asking and sales prices, as in the supplementary empirical results. In the model, the asking price discount is always negative, that is, apartments always sell below their asking prices.

The search process in the extended model features the following changes. When a buyer visits an apartment, the buyer and the seller may or may not bargain, which is determined stochastically. With probability θ, the seller does not accept counteroffers, and $p(d)$ is a take-it-or-leave-it offer ("no-counteroffer scenario", subscript n). The buyer accepts or rejects the offer. If the buyer accepts, the seller receives $p(d)$, and the buyer receives their first housing dividend ε in the following period and pays their first commuting costs $\tau(d)$ in the following period. If the buyer rejects, the seller relists the apartment in the following period and the buyer visits a new apartment in the following period. With probability $(1-\theta)$, the buyer can bargain by making a take-it-or-leave-it counteroffer $o(d)$ to the seller ("counteroffer scenario", subscript c). If the buyer makes a counteroffer, the seller accepts or rejects the offer. The outcomes of accepting or rejecting the offer are analogous to those in the no-counteroffer scenario.

Changes in the seller problem. The seller maximizes their profits $\Pi(d)$ over an asking price $p(d)$ and a reservation value $r(d)$. We assume that a buyer has perfect information about a seller's preferences ((see Carrillo, 2012)). Hence, in the counteroffer scenario, the offer $o(d)$ is equal to the seller's reservation value $r(d)$, as this offer corresponds to the lowest price the seller is willing to accept. In the following, we denote by $\gamma_n(p(d))$ the probability that a buyer is willing to buy in the no-counteroffer scenario and the seller posts an asking price $p(d)$. The respective probability in the counteroffer scenario is $\gamma_c(p(d))$. Profits are given by

$$\begin{aligned}\Pi(d) = \max_{p(d),r(d)} \Big\{ &\theta\Big(\gamma_n(\cdot)p(d) + (1-\gamma_n(\cdot))\beta\Pi(d)\Big) \\ &+ (1-\theta)\Big(\gamma_c(\cdot)\max\left[r(d), \beta\Pi(d)\right] + (1-\gamma_c(\cdot))\beta\Pi(d)\Big)\Big\}. \end{aligned} \tag{A2.6}$$

In the no-counteroffer scenario, which happens with probability θ, the seller receives the price $p(d)$ with probability $\gamma_n(\cdot)$ and receives discounted continuation value $\beta\Pi(d)$ of trying to sell the house again next period with probability $(1-\gamma_n(\cdot))$. In the counteroffer scenario, which happens with probability $(1-\theta)$, the seller receives the maximum of the counteroffer $o(d) = r(d)$ and the discounted continuation value $\beta\Pi(d)$, depending on whether they accept or reject the buyer's counteroffer, with probability $\gamma_c(\cdot)$. The seller receives the discounted continuation value of $\beta\Pi(d)$ with probability $(1-\gamma_c(\cdot))$ analogously to the counteroffer scenario.

Changes in the buyer problem. The buyer's search value is given by:

$$W = E_{x,\varepsilon}\left[\theta V_n(d,\varepsilon) + (1-\theta)V_c(d,\varepsilon)\right], \tag{A2.7}$$

With probability θ, the buyer receives the buyer's value in the no-counteroffer scenario $V_n(d,\varepsilon)$. With probability $(1-\theta)$, the buyer receives the buyer's value in the counteroffer scenario $V_c(d,\varepsilon)$. The buyer's value in the no-counteroffer scenario is

$$V_n(d,\varepsilon) = \max[V_m(d,\varepsilon) - p(x), \beta W]. \tag{A2.8}$$

Either the buyer accepts the asking price and receives the continuation value of being matched $V_m(d,\varepsilon)$, which we denote by $V(d,\varepsilon)$ in the main model, while paying $p(d)$, or the buyer continues to search and receives the discounted value of searching next period, βW. The buyer's value in the counteroffer scenario is

$$V_c(d,\varepsilon) = \max[\delta(\cdot)(V_m(d,\varepsilon) - o(d)) + (1-\delta(\cdot))(\beta W), \beta W], \tag{A2.9}$$

where $\delta(o(d))$ denotes the probability that the seller accepts the buyer's counteroffer. If the seller accepts, the buyer receives the continuation value of being matched $V_m(d,\varepsilon)$

while paying the counteroffer price $o(d)$. If the seller rejects the counteroffer, the buyer keeps on searching and receives the discounted value of search. The buyer can also decide not to make a counteroffer and keep on searching by themselves. Note that the seller always accepts the optimal counteroffer $o(d) = r(d)$. Hence, $\delta(\cdot) = 1$ at all distances to the city center.

2.E.1 Equilibrium in the extended model

Seller's optimization. Since the counteroffer $o(d) = r(d)$ is the lowest price that the seller is willing to accept, the seller's reservation value is given by $r(d) = \beta\Pi(d)$. With any reservation value above $\beta\Pi(d)$, it would be better for the seller to reject the buyer's offer and relist the apartment next period. The expression for seller profits (A2.6) then simplifies to

$$\Pi(d) = \max_{p(d),r(d)} \Big\{ \theta\gamma_n(\cdot)p(d) + (1 - \theta\gamma_n(\cdot))r(d) \Big\}. \tag{A2.10}$$

Optimizing with regards to the asking price $p(d)$ yields

$$p(d) = r(d) - \frac{\gamma_n(\cdot)}{\partial\gamma_n/\partial p\mid_{p=p(d)}}, \tag{A2.11}$$

where the derivative $\partial\gamma_n/\partial p$ is evaluated at the optimal asking price $p(d)$. Plugging the condition $r(d) = \beta\Pi(d)$ into (A2.10) evaluated at the seller's optimum, we have that

$$\frac{r(d)}{\beta} = \theta\gamma_n(\cdot)p(d) + (1 - \theta\gamma_n(\cdot))r(d)$$
$$\Rightarrow r(d) = \frac{\beta\theta\gamma_n(\cdot)p(d)}{1 - \beta\left(1 - \theta\gamma_n(\cdot)\right)}. \tag{A2.12}$$

The pair of the optimal asking price and reservation value for a given distance to the city center solves the previous equations (A2.11) and (A2.12) simultaneously.

Buyer's optimization. Via the buyer value function in the no-counteroffer scenario (A2.8), we define a reservation dividend $\varepsilon_n^*(d)$ such that a buyer is indifferent indifferent between buying an apartment and continuing to search:

$$V_m(d, \varepsilon_n^*(d)) - p(d) = \beta W. \tag{A2.13}$$

Analogously, via the buyer value function in the counteroffer scenario (A2.9), we define a reservation dividend $\varepsilon_c^*(d)$ such that

$$V_m(d, \varepsilon_c^*(d)) - r(d) = \beta W. \tag{A2.14}$$

Probability of sale. The probability of sale conditional on a bargaining scenario is equal to the probability that the buyer's idiosyncratic dividend is above their respective reservation dividend. Hence, in the no-counteroffer scenario,

$$\begin{aligned}\gamma_n(p(d)) &= Prob(\varepsilon > \varepsilon_n^*(d)) = 1 - Prob(\varepsilon \leq \varepsilon_n^*(d)) \\ &= \tilde{\varepsilon} - \varepsilon_n^*(d).\end{aligned} \tag{A2.15}$$

Analogously, in the counteroffer scenario,

$$\gamma_c(p(d)) = \tilde{\varepsilon} - \varepsilon_c^*(d). \tag{A2.16}$$

Thus, for the derivative in the seller optimality condition (A2.11) we have that

$$\left.\frac{\partial \gamma_n}{\partial p}\right|_{p=p(d)} = -\left.\frac{\partial \varepsilon_n^*}{\partial p}\right|_{p=p(d)} \tag{A2.17}$$

for all distances to the city center $d \in \mathcal{D}$. By proceeding as in the main derivations, we get

$$\varepsilon_n^*(d) = \frac{1-\pi\beta}{\beta} p(d) + \tau(d) - (1-\pi)\Pi(d) + (\pi - \pi\beta)W \tag{A2.18}$$

and

$$\left.\frac{\partial \gamma_n}{\partial p}\right|_{p=p(d)} = -\frac{1-\pi\beta}{\beta}. \tag{A2.19}$$

Analogous relations hold for the counteroffer scenario.

Equilibrium definition, extended model. A *stationary spatial equilibrium* consists of a seller profit function $\Pi(d)$, an asking price function $p(d)$, a seller reservation value function $r(d)$, a value of search W, buyer reservation dividend functions $\varepsilon_n^*(d)$ and $\varepsilon_c^*(d)$, and conditional sale probability functions $\gamma_n(p(d))$ and $\gamma_c(p(d))$ that satisfy equations (A2.7), (A2.10), (A2.11), (A2.12), (A2.13), (A2.14), (A2.15), and (A2.16) for all distances to the city center $d \in \mathcal{D}$, given a parameter vector $(\beta, \pi, \theta, \tilde{\varepsilon})$, a distribution of apartments' distances to the city center $\mathcal{D}$, and a commuting cost function $\tau(d)$.

2.E.2 Analytical results in the extended model

Again, we start with auxiliary derivations. First, Lemma 2.2 allows to simplify expression with reservation dividends and probabilites of sale.

Lemma 2.2. *The buyer reservation dividends in the counteroffer scenario and the no-counteroffer scenario relate as* $\varepsilon_c^*(d) = 2\varepsilon_n^*(d) - \tilde{\varepsilon}$. *The probabilities of sale in these two scenarios relate as* $\gamma_c(p(d)) = 2\gamma_n(p(d))$.

Proof. Using the buyer indifference condition (A2.14) and the linear expression of the buyer value function (2.11), we have that

$$\varepsilon_c^*(d) = \frac{1-\pi\beta}{\beta} r(d) + \tau(d) - (1-\pi)(\Pi(d) + W) + (1-\pi\beta)W \tag{A2.20}$$

$$= \frac{1-\pi\beta}{\beta}\left(p(d) + \frac{\gamma_n(p(d))}{\frac{\partial \gamma_n}{\partial p}\Big|_{p=p(d)}} \right) + \tau(d) - (1-\pi)(\Pi(d) + W) + (1-\pi\beta)W \tag{A2.21}$$

$$= \varepsilon_n^*(d) - \gamma_n(p(d)), \tag{A2.22}$$

where the last two lines follow due to the seller optimality condition (A2.11), the linear expression of the reservation value (A2.18), and the constant value of the derivative $(\partial\gamma_n/\partial p)|_{p=p(d)}$ with a uniformly distributed idiosyncratic dividend (A2.19). Therefore, we also have that $\varepsilon_c^*(d) = 2\varepsilon_n^*(d) - \tilde{\varepsilon}$, as well as $\gamma_c(p(d)) = 2\gamma_n(p(d))$, via the equilibrium relations between reservation dividends and probabilities of sale (A2.15) and (A2.16). □

Lemma 2.3 shows that both of the buyer reservation dividends are increasing with distance to the city center.

Lemma 2.3. *The reservation dividends in the no-counteroffer scenario $\varepsilon_n^*(d)$ and in the counteroffer scenario $\varepsilon_c^*(d)$ increase with distance to the city center x.*

Proof. We know from (A2.18) that

$$\varepsilon_n^*(d) = \frac{1-\pi\beta}{\beta} p(d) + \tau(d) - (1-\pi)\Pi(d) + (\pi - \pi\beta)W.$$

Analogously to the main derivations, we reformulate the asking price $p(d)$ and the expected profits from reselling the apartment $\Pi(d)$ in terms of the reservation dividend $\varepsilon_n^*(d)$. First, we combine the seller optimality conditions (A2.11) and (A2.12) and get

$$p(d) = -\frac{(1-\beta)\gamma_n(p(d)) + \beta\theta\gamma_n^2(p(d))}{(1-\beta)\partial\gamma_n/\partial p\mid_{p=p(d)}}. \tag{A2.23}$$

Expressing the probability of sale $\gamma_n(p(d))$ and the derivative $(\partial\gamma_n/\partial p)\mid_{p=p(d)}$ in terms of the reservation dividend using the equilibrium relations (A2.15) and (A2.19), we have that

$$p(d) = -\frac{(1-\beta)(\tilde{\varepsilon} - \varepsilon_n^*(d)) + \beta\theta(\tilde{\varepsilon} - \varepsilon_n^*(d))^2}{(1-\beta)\left(-\frac{1-\pi\beta}{\beta}\right)} \tag{A2.24}$$

$$= \frac{\beta}{1-\pi\beta}(\tilde{\varepsilon} - \varepsilon_n^*(d)) + \frac{\beta^2\theta}{(1-\beta)(1-\pi\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d))^2. \tag{A2.25}$$

Next, using the seller's conditions (A2.10) and (A2.11), we get

$$\Pi(d) = p(d) + \frac{\gamma_n(p(d)) - \theta\gamma_n^2(p(d))}{\partial\gamma_n/\partial p\mid_{p=p(d)}}, \tag{A2.26}$$

which, using (A2.25) and again expressing the probability of sale and the derivative in terms of the reservation dividend via (A2.15) and (A2.19), amounts to

$$\begin{aligned}\Pi(d) =& \frac{\beta}{1-\pi\beta}(\tilde{\varepsilon} - \varepsilon_n^*(d)) + \frac{\beta^2\theta}{(1-\beta)(1-\pi\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d))^2 \\ &- \frac{\beta}{1-\pi\beta}\left((\tilde{\varepsilon} - \varepsilon_n^*(d)) - \theta(\tilde{\varepsilon} - \varepsilon_n^*(d))^2\right) &\text{(A2.27)} \\ =& \frac{\beta\theta}{(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d))^2. &\text{(A2.28)}\end{aligned}$$

Therefore, we can express the reservation dividend as

$$\begin{aligned}\varepsilon_n^*(d) = \frac{1-\pi\beta}{\beta}&\left(\frac{\beta}{1-\pi\beta}(\tilde{\varepsilon} - \varepsilon_n^*(d)) + \frac{\beta^2\theta}{(1-\beta)(1-\pi\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d))^2\right) \\ &+ \tau(d) - (1-\pi)\left(\frac{\beta\theta}{(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d))^2\right) + (\pi - \pi\beta)W. \qquad \text{(A2.29)}\end{aligned}$$

After simplifying, we have that

$$2\varepsilon_n^*(d) - 1 + \frac{\pi\beta\theta}{1-\pi\beta}(\tilde{\varepsilon} - \varepsilon_n^*(d))^2 = \tau(d) + (\pi - \pi\beta)W. \tag{A2.30}$$

We take the derivative with respect to the distance to the city center d on both sides and get

$$\frac{\partial\varepsilon_n^*}{\partial d}\underbrace{\left(2 - 2\frac{\pi\beta\theta}{1-\pi\beta}(\tilde{\varepsilon} - \varepsilon_n^*(d))\right)}_{>0} = \frac{\partial\tau}{\partial d} \tag{A2.31}$$

and therefore $\frac{\partial\varepsilon_n^*}{\partial d} > 0$, given that $\frac{\partial\tau}{\partial d} > 0$. Via Lemma 2.2, we have that $\frac{\partial\varepsilon_c^*}{\partial d} = 2\frac{\partial\varepsilon_n^*}{\partial d}$, and hence this relation also applies to the reservation dividend in the counteroffer scenario $\varepsilon_c^*(d)$. □

Via the proof of Lemma 2.3, we directly obtain a description of the spatial variation in other endogenous variables.

Corollary 2.1. *The seller profit $\Pi(d)$, the asking price $p(d)$, the seller reservation value $r(d)$, and the expected sales price $E[Sales\ price(d)] = \theta p(d) + (1-\theta)r(d)$ decrease with distance to the city center d.*

Proof. Using (A2.28), we have that

$$\frac{\partial \Pi}{\partial d} = \frac{2\beta\theta}{(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d))\left(-\frac{\partial \varepsilon_n^*}{\partial d}\right) < 0, \tag{A2.32}$$

where $\frac{\partial \varepsilon_n^*}{\partial d} > 0$ via Lemma 2.3. Next, using (A2.25), we get

$$\frac{\partial p}{\partial d} = -\frac{\beta}{1-\pi\beta}\frac{\partial \varepsilon_n^*}{\partial d} + \frac{2\beta^2\theta}{(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d))\left(-\frac{\partial \varepsilon_n^*}{\partial d}\right) < 0. \tag{A2.33}$$

We express the seller reservation value in terms of the reservation dividend via (A2.25), the seller optimality condition (A2.11), the equilibrium relation between the reservation dividend and probability of sale in the no-counteroffer scenario (A2.15), and the derivative of the probability of sale in the no-counteroffer scenario with respect to the asking price with a uniformly distributed idiosyncratic dividend (A2.19):

$$r(d) = p(d) + \frac{\gamma_n(p(d))}{\partial \gamma_n / \partial p \mid_{p=p(d)}} \tag{A2.34}$$

$$= p(d) - \frac{\beta}{1-\pi\beta}(\tilde{\varepsilon} - \varepsilon_n^*(d)). \tag{A2.35}$$

Then,

$$\frac{\partial r}{\partial d} = \frac{\partial p}{\partial d} + \frac{\beta}{1-\pi\beta}\frac{\partial \varepsilon_n^*}{\partial d} \tag{A2.36}$$

$$= \frac{2\beta^2\theta}{(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d))\left(-\frac{\partial \varepsilon_n^*}{\partial d}\right) < 0, \tag{A2.37}$$

where the second line follows from (A2.33). The expected sales price $E[Sales\ price(d)] = \theta p(d) + (1-\theta)r(d)$ is decreasing with distance to the city center, as both the asking price $p(d)$ and the seller reservation value $r(d)$ are decreasing with distance to the city center. □

Time on the market. The probability $\gamma_{nc}(p(d))$ that an apartment sells in a period is given via the probabilities for the two bargaining scenarios and the corresponding probabilities of sale:

$$\gamma_{nc}(p(d)) = \theta\gamma_n(p(d)) + (1-\theta)\gamma_c(p(d)). \tag{A2.38}$$

The expected time on the market (in days) at a given distance to the city center d is

$$E[TOM(d)] = \frac{1}{\gamma_{nc}(p(d))} = \frac{1}{\theta\gamma_n(p(d)) + (1-\theta)\gamma_c(p(d))}. \tag{A2.39}$$

Proposition 2.3. *The expected time on the market $E[TOM(d)]$ increases with distance to the city center d in the extended model with bargaining.*

Proof. Using Lemma 2.2 and the equilibrium relations between the reservation dividends and the probabilities of sale (A2.15) and (A2.16), we can express the expected time on the market only in terms of the reservation dividend in the no-counteroffer scenario:

$$E[TOM(d)] = \frac{1}{(2-\theta)(\tilde{\varepsilon} - \varepsilon_n^*(d))}. \tag{A2.40}$$

The derivative of the expected time on the market with respect to the distance to the city center amounts to

$$\frac{\partial E[TOM]}{\partial d} = \underbrace{-\left((2-\theta)(\tilde{\varepsilon} - \varepsilon_n^*(d))\right)^{-2}}_{<0} \left(\underbrace{-(2-\theta)}_{<0} \frac{\partial \varepsilon_n^*}{\partial d} \right). \tag{A2.41}$$

□

Intuition. See main text.

Asking price discount. The expected asking price discount at a given distance to the city center is

$$E[Discount(d)] = \theta \cdot Discount_n(d) + (1-\theta) \cdot Discount_c(d) = (1-\theta) \cdot Discount_c(d), \tag{A2.42}$$

where the asking price discount in the no-counteroffer scenario is $Discount_n(d) = 0$ and the asking price discount in the counteroffer scenario is $Discount_c(d)$. We define the asking price discount in the counteroffer scenario analogously to our empirical measure as

$$Discount_c(d) = \frac{r(d) - p(d)}{p(d)}. \tag{A2.43}$$

Proposition 2.4. *Given that the probability of no counteroffer $\theta \in (0,1)$, the expected asking price discount $E[Discount(d)] < 0$ decreases with distance to the city center d.*

Proof. If $\theta = 1$, then the asking price discount is always equal to zero, as the probability of being in the no-counteroffer scenario is equal to one, and hence the asking price is the same as the sales price at all distances to the city center. This corresponds to the setup in the main model. In the following, we consider $\theta < 1$. Plugging in the optimal reservation value $r(d)$ of a seller from (A2.12), we have that

$$Discount_c(d) = \frac{\frac{\beta\theta\gamma_n(d)p(d)}{1-\beta(1-\theta\gamma_n(p(d)))} - p(d)}{p(d)} \tag{A2.44}$$

$$= -\frac{1-\beta}{1-\beta+\beta\theta(\tilde{\varepsilon} - \varepsilon_n^*(d))} < 0, \tag{A2.45}$$

using the equilibrium relation between the reservation dividend and the probability of sale (A2.15) in the second line. Hence, we also have that the expected asking price discount $E[Discount(d)] = (1-\theta)Discount_c(d) < 0$. The derivative of the expected asking price discount with respect to the distance to the city center amounts to

$$\frac{\partial E[Discount]}{\partial d} = \underbrace{-(1-\theta)(\beta-1)\left(1-\beta+\beta\theta(\tilde{\varepsilon}-\varepsilon_n^*(d))\right)^{-2}}_{>0}\left(\underbrace{-\beta\theta}_{<0}\frac{\partial \varepsilon_n^*}{\partial d}\right). \quad \text{(A2.46)}$$

This expression is negative, provided that $\theta > 0$, as the reservation dividend $\varepsilon_n^*(d)$ increases with distance to the city center x via Lemma 2.3. □

Intuition. As in the case of the time on the market, the relevant condition for liquidity to decrease with distance to the city center is that the reservation dividend increases with distance to the city center. Via this condition, we have that the asking price and the seller reservation value both decrease with distance to the city center (see Corollary 2.1). For the expected asking price discount to become more negative with distance to the city center, we need that the seller reservation value decreases more steeply across space than the asking price.[23]

Why is this condition fulfilled? Recall from the seller optimization that the reservation value is equal to discounted profits of the next period in equilibrium, as otherwise, the seller would always reject the buyer's optimal counteroffer. For the asking price discount to become more negative with distance to the city center, we, therefore, need that profits are decrease more steeply across space than asking prices.[24]

A proof of this statement follows at the end of this subsection. Intuitively, we can express profits only in terms of the probability of sale and the asking price. Since both the probability of sale and the asking price decrease with distance to the city center and

[23]Formally,

$$\frac{\partial E[Discount]}{\partial d} = (1-\theta)\frac{\partial(\frac{r-p}{p})}{\partial d} = (1-\theta)\left(\frac{\partial r}{\partial d}\frac{1}{p(d)} - \frac{\partial p}{\partial d}\frac{r}{p^2}\right), \quad \text{(A2.47)}$$

such for the expected discount to be decreasing with distance to the city center, we need

$$\underbrace{\frac{\partial r/\partial d}{r(d)}}_{<0} < \underbrace{\frac{\partial p/\partial d}{p(d)}}_{<0}, \quad \text{(A2.48)}$$

where both sides of the expression are < 0 due to Corollary 2.1.

[24]Formally,

$$\frac{\partial r/\partial d}{r(d)} = \frac{\partial(\beta\Pi)/\partial d}{\beta\Pi(d)} = \frac{\partial \Pi/\partial d}{\Pi(d)} < \frac{\partial p/\partial d}{p(d)}. \quad \text{(A2.49)}$$

profits are composed of the two, profits decrease more steeply with distance to the city center than the asking price alone. All of these variables decrease with distance to the city center because buyers want to be compensated for commuting costs via higher reservation dividends.

Relating this insight back to the asking price discount, we know that the seller reservation value decreases more steeply than the asking price because discounted profits decrease more steeply than the asking price. Hence, the expected asking price discount is decreasing with distance to the city center.

Proof: profits decrease more steeply across space than asking prices. From the seller problem (A2.10), we have that in an equilibrium, we can express profits as

$$\begin{aligned}\Pi(d) &= \theta\gamma_n(\cdot)p(d) + (1-\theta\gamma_n(\cdot))r(d)\\ &= \theta\gamma_n(\cdot)p(d) + (1-\theta\gamma_n(\cdot))\beta\Pi(d),\end{aligned} \tag{A2.50}$$

since the seller's reservation value $r(d) = \beta\Pi(d)$ via the optimal counteroffer of the buyer. Then,

$$\Pi(d) = \frac{\theta\gamma_n(\cdot)p(d)}{1-\beta+\theta\beta\gamma_n(\cdot)} \tag{A2.51}$$

and

$$\frac{\partial\Pi}{\partial d} = \frac{(1-\beta+\theta\beta\gamma_n(\cdot))(\theta\frac{\partial\gamma_n}{\partial d}p(d) + \theta\gamma(\cdot)\frac{\partial p}{\partial d}) - \theta^2\beta\frac{\partial\gamma_n}{\partial d}\gamma_n(\cdot)p(d)}{(1-\beta+\theta\beta\gamma_n(\cdot))^2}. \tag{A2.52}$$

The proportional derivative of profits with respect to the distance to the city center is then

$$\underbrace{\frac{\partial\Pi/\partial d}{\Pi(d)}}_{<0} = \frac{(1-\beta+\theta\beta\gamma_n(\cdot))(\theta\frac{\partial\gamma}{\partial d}p(d) + \theta\gamma(\cdot)\frac{\partial p}{\partial d}) - \theta^2\beta\frac{\partial\gamma_n}{\partial d}\gamma_n(\cdot)p(d)}{\theta\gamma_n(\cdot)p(d)(1-\beta+\theta\beta\gamma_n(\cdot))} \tag{A2.53}$$

$$= \frac{\frac{\partial\gamma_n}{\partial d}p(d) + \frac{\partial p}{\partial d}\gamma_n(\cdot)}{\gamma_n(\cdot)p(d)} - \frac{\theta\beta\frac{\partial\gamma_n}{\partial d}}{1-\beta+\theta\beta\gamma_n(\cdot)} \tag{A2.54}$$

$$= \underbrace{\frac{\partial\gamma_n/\partial d}{\gamma(\cdot)}}_{<0} + \underbrace{\frac{\partial p/\partial d}{p(d)}}_{<0} - \underbrace{\frac{\theta\beta\frac{\partial\gamma_n}{\partial d}}{1-\beta+\theta\beta\gamma_n(\cdot)}}_{<0}. \tag{A2.55}$$

Statement (A2.49) says that

$$\frac{\partial\Pi/\partial d}{\Pi(d)} < \frac{\partial p/\partial d}{p(d)}, \tag{A2.56}$$

for which to hold we therefore need that

$$\frac{\partial \gamma_n / \partial d}{\gamma_n(\cdot)} < \frac{\theta\beta\frac{\partial \gamma_n}{\partial d}}{1-\beta+\theta\beta\gamma_n(\cdot)}. \tag{A2.57}$$

As $\frac{\partial \gamma_n}{\partial d} < 0$, this expression simplifies to

$$\frac{1}{\gamma_n(\cdot)} > \frac{\theta\beta}{1-\beta+\theta\beta\gamma_n(\cdot)}, \tag{A2.58}$$

or

$$1-\beta > 0, \tag{A2.59}$$

which is true, since $\beta \in (0,1)$. Therefore, $\frac{\partial \Pi/\partial d}{\Pi(d)} < \frac{\partial p/\partial x}{p(d)}$, as required. □

Relation between time on the market and asking price discount. Via the proofs of Propositions 2.3 and 2.4, we can directly derive that apartments that spend more time on the market also sell at more negative discounts. Thus, lower liquidity in one measure corresponds to lower liquidity in the other measure, and the two measures of liquidity are interchangeable in the model.

Corollary 2.2. *Given that the probability of no counteroffer $\theta \in (0,1)$, the model correlation between the expected time on the market $E[TOM(d)]$ and the expected asking price discount $E[Discount(d)]$ is negative.*

Proof. We start by expressing the time on the market as a function of the asking price discount. Then, we evaluate the derivative of the time on the market with respect to the discount at a given distance to the city center. First, from the proofs of Propositions 2.3 and 2.4 we have that

$$\tilde{\varepsilon} - \varepsilon_n^*(d) = \frac{1}{(2-\theta)E[TOM(d)]} = \frac{1}{\beta\theta}\left(\frac{(\beta-1)(1-\theta)}{E[Discount(d)]} - 1 + \beta\right). \tag{A2.60}$$

We can hence express the relation between the expected time on the market and the expected asking price discount as

$$E[TOM(d)] = \frac{\beta\theta}{2-\theta}\left(\frac{(\beta-1)(1-\theta)}{E[Discount(d)]} - 1 + \beta\right)^{-1}. \tag{A2.61}$$

The derivative of expected time on the market with respect to the expected asking price discount, evaluated at a given distance to the city center $\bar{d}$ is then

$$\left.\frac{\partial E[TOM]}{\partial E[Discount]}\right|_{d=\bar{d}} = \underbrace{-\frac{\beta\theta}{2-\theta}}_{<0}\underbrace{\left(\frac{(\beta-1)(1-\theta)}{E[Discount(\bar{d})]} - 1 + \beta\right)^{-2}}_{>0}\underbrace{\left(-\frac{(\beta-1)(1-\theta)}{\left(E[Discount(\bar{d})]\right)^2}\right)}_{>0} < 0, \tag{A2.62}$$

provided that $\theta \in (0, 1)$. A less negative asking price discount corresponds to a lower time on the market. $\square$

Appendix 2.F Equilibrium existence and uniqueness

We show existence and uniqueness of an equilibrium in the extended model. The main model can be obtained by setting the probability of the no-counteroffer scenario $\theta = 1$.

2.F.1 Equilibrium existence

First, we argue for the existence of a solution. Evidently, we find a solution numerically, nevertheless, we prove the existence formally, following Krainer (2001). Via (2.11), we can express the buyer's value function as

$$V_m(d, \varepsilon) = \frac{\beta}{1 - \pi\beta}\big(\varepsilon - \tau(d) + (1 - \pi)(\Pi(d) + W)\big). \tag{A2.63}$$

Hence, $V_m(d, \varepsilon)$ is linear in ε and there exist reservation dividends given the linear buyer indifference conditions (A2.13) and (A2.14). In what follows, we express the other endogenous equilibrium objects in terms of the buyer's reservation dividends and the set of model parameters to show the uniqueness of the solution. We then have expressions that only depend on the reservation dividends, given a set of parameters. The fact that reservation dividends exist then implies that a model solution also exists.

2.F.2 Equilibrium uniqueness

To show the uniqueness of the model's solution, we follow Vanhapelto and Magnac (2023). The strategy for the proof of uniqueness is as follows. We show that two possible ways of expressing the value of search allow for only one value of the idiosyncratic reservation dividend (at each distance to the city center) such that both of these expressions hold. The first expression decreases in the idiosyncratic reservation dividends, whereas the second expression increases in the idiosyncratic reservation dividends. Hence, given a set of parameters, the model's solution is unique, as first, only one idiosyncratic reservation dividend can fulfill both of these conditions and second, we express all endogenous model variables in terms of parameters and the idiosyncratic reservation dividend.

The value of search decreases in the reservation dividend. We set up the first expression for the value of search in terms of the buyer's reservation dividends via the definitions (A2.7), (A2.8), and (A2.9):

$$W = E_{d,\varepsilon}\left[\theta V_n(d,\varepsilon) + (1-\theta)V_c(d,\varepsilon)\right] \tag{A2.64}$$
$$= E_{d,\varepsilon}\left[\theta \max\left[V_m(d,\varepsilon) - p(d), \beta W\right] + (1-\theta)\max\left[V_m(d,\varepsilon) - r(d), \beta W\right]\right], \tag{A2.65}$$

and hence

$$W - \beta W = E_{d,\varepsilon}\left[\theta \max\left[V_m(d,\varepsilon) - p(d) - \beta W, 0\right] + (1-\theta)\max\left[V_m(d,\varepsilon) - r(d) - \beta W, 0\right]\right], \tag{A2.66}$$

which simplifies to

$$W = \frac{1}{1-\beta} E_{d,\varepsilon}\left[\theta \max\left[V_m(d,\varepsilon) - p(d) - \beta W, 0\right] + (1-\theta)\max\left[V_m(d,\varepsilon) - r(d) - \beta W, 0\right]\right]. \tag{A2.67}$$

We now express the relations within the max operators in terms of the buyer's reservation dividends. Note that when the buyer indifference conditions (A2.13) and (A2.14) hold, we have that

$$\beta W = V_m(d, \varepsilon_n^*(d)) - p(d) = V_m(d, \varepsilon_c^*(d)) - r(d). \tag{A2.68}$$

Inserting the linear buyer value function from (A2.63), we get

$$\beta W = \frac{\beta}{1-\pi\beta}\big(\varepsilon_n^*(d) - \tau(d) + (1-\pi)(\Pi(d) + W)\big) - p(d) \tag{A2.69}$$
$$= \frac{\beta}{1-\pi\beta}\big(\varepsilon_c^*(d) - \tau(d) + (1-\pi)(\Pi(d) + W)\big) - r(d). \tag{A2.70}$$

Hence,

$$\frac{\beta}{1-\pi\beta}\varepsilon_n^*(d) = \frac{\beta}{1-\pi\beta}\tau(d) - \frac{\beta(1-\pi)}{1-\pi\beta}\Pi(d) + \frac{\pi\beta(1-\beta)}{1-\pi\beta}W + p(d) \tag{A2.71}$$

and

$$\frac{\beta}{1-\pi\beta}\varepsilon_c^*(d) = \frac{\beta}{1-\pi\beta}\tau(d) - \frac{\beta(1-\pi)}{1-\pi\beta}\Pi(d) + \frac{\pi\beta(1-\beta)}{1-\pi\beta}W + r(d). \tag{A2.72}$$

Using the linear buyer value function from (A2.63), we can express the sum within the first max operator from (A2.67) as

$$V_m(d,\varepsilon) - p(d) - \beta W = \frac{\beta}{1-\pi\beta}\big(\varepsilon - \tau(d) + (1-\pi)(\Pi(d) + W)\big) - p(d) - \beta W \tag{A2.73}$$
$$= \frac{\beta}{1-\pi\beta}\varepsilon + -\frac{\beta}{1-\pi\beta}\tau(d) + \frac{\beta(1-\pi)}{1-\pi\beta}\Pi(d) - p(d) - \frac{\pi\beta(1-\beta)}{1-\pi\beta}W.$$

Then, via (A2.71), we get

$$V_m(d,\varepsilon) - p(d) - \beta W = \frac{\beta}{1-\pi\beta}\varepsilon - \frac{\beta}{1-\pi\beta}\varepsilon_n^*(d) = \frac{\beta}{1-\pi\beta}(\varepsilon - \varepsilon_n^*(d)). \tag{A2.74}$$

Analogously, using (A2.72), we have that

$$V_m(d,\varepsilon) - r(d) - \beta W = \frac{\beta}{1-\pi\beta}(\varepsilon - \varepsilon_c^*(d)). \tag{A2.75}$$

We can then express the value of search from (A2.67) as

$$W = \frac{1}{1-\beta} E_{d,\varepsilon}\left[\theta \max\left[\frac{\beta}{1-\pi\beta}(\varepsilon - \varepsilon_n^*(d)), 0\right] + (1-\theta)\max\left[\frac{\beta}{1-\pi\beta}(\varepsilon - \varepsilon_c^*(d)), 0\right]\right] \tag{A2.76}$$

$$= \frac{1}{1-\beta}\frac{\beta}{1-\pi\beta}\left(\theta E_{d,\varepsilon}\Big[\max\left[(\varepsilon - \varepsilon_n^*(d)), 0\right]\Big] + (1-\theta)E_{d,\varepsilon}\Big[\max\left[(\varepsilon - \varepsilon_c^*(d)), 0\right]\Big]\right) \tag{A2.77}$$

$$= \frac{1}{1-\beta}\frac{\beta}{1-\pi\beta}\left(\theta E_{d,\varepsilon}\Big[(\varepsilon - \varepsilon_n^*(d))\mathbb{1}_{\varepsilon \geq \varepsilon_n^*(d)}\Big] + (1-\theta)E_{d,\varepsilon}\Big[(\varepsilon - \varepsilon_c^*(d))\mathbb{1}_{\varepsilon \geq \varepsilon_c^*(d)}\Big]\right), \tag{A2.78}$$

which decreases in $\varepsilon_n^*(d)$ and $\varepsilon_c^*(d)$.

The value of search increases in the reservation dividend. We set up the second expression for the value of search via the buyer indifference conditions (A2.13) and (A2.14). Via (A2.13), we have that

$$V_m(d, \varepsilon_n^*(d)) - p(d) = \beta W \tag{A2.79}$$

for the no-counteroffer scenario. Hence, using the linear buyer value function from (A2.63), we can express this condition as

$$\frac{\beta}{1-\pi\beta}\left(\varepsilon_n^*(d) - \tau(d) + (1-\pi)(\Pi(d) + W)\right) - p(d) = \beta W, \tag{A2.80}$$

and obtain

$$W = \frac{1}{\pi - \pi\beta}\left(\varepsilon_n^*(d) - \tau(d) + (1-\pi)\Pi(d) - \frac{1-\pi\beta}{\beta}p(d)\right). \tag{A2.81}$$

Via Lemma 2.3, we have that

$$p(d) = \frac{\beta}{1-\pi\beta}(\tilde{\varepsilon} - \varepsilon_n^*(d)) + \frac{\beta^2\theta}{(1-\beta)(1-\pi\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d))^2$$

and

$$\Pi(d) = \frac{\beta\theta}{(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d))^2.$$

Then, we are able to calculate the derivative of the value of search with respect to the reservation dividend and show that it is positive. Via (A2.81), we have that

$$\frac{\partial W}{\partial \varepsilon_n^*} = \frac{1}{\pi - \pi\beta} + \frac{1-\pi}{\pi-\pi\beta}\frac{\partial \Pi}{\partial \varepsilon_n^*} - \frac{1-\pi\beta}{\beta(\pi-\pi\beta)}\frac{\partial p}{\partial \varepsilon_n^*} \tag{A2.82}$$

$$= \frac{1}{\pi-\pi\beta} - \frac{2(1-\pi)\beta\theta}{(\pi-\pi\beta)(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d))$$

$$+ \frac{1}{\pi-\pi\beta} + \frac{2(1-\pi\beta)\beta\theta}{(\pi-\pi\beta)(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d)) \tag{A2.83}$$

$$= \frac{2}{\pi-\pi\beta} + \frac{2\pi\beta\theta}{(\pi-\pi\beta)(1-\pi\beta)}(\tilde{\varepsilon} - \varepsilon_n^*(d)) > 0. \tag{A2.84}$$

Via the buyer indifference condition from the counteroffer scenario (A2.14), we have that

$$V_m(d, \varepsilon_c^*(d)) - r(d) = \beta W. \tag{A2.85}$$

Going through the same steps as before yields

$$W = \frac{1}{\pi-\pi\beta}\left(\varepsilon_c^*(d) - \tau(d) + (1-\pi)\Pi(d) - \frac{1-\pi\beta}{\beta}r(d)\right) \tag{A2.86}$$

for the value of search. Via (A2.11), (A2.19), and (A2.23), the seller's reservation value amounts to

$$r(d) = -\frac{(1-\beta)\gamma_n(p(d)) + \beta\theta\gamma_n^2(p(d))}{(1-\beta)\partial\gamma_n/\partial p\mid_{p=p(d)}} + \frac{\gamma_n(p(d))}{\partial\gamma_n/\partial p\mid_{p=p(d)}} \tag{A2.87}$$

$$= \frac{\beta^2\theta}{(1-\pi\beta)(1-\beta)}\gamma_n^2(p(d)) \tag{A2.88}$$

and via Lemma 2.2, we get

$$r(d) = \frac{\beta^2\theta}{(1-\pi\beta)(1-\beta)}\left(\frac{1}{2}\gamma_c(p(d))\right)^2 \tag{A2.89}$$

$$= \frac{\beta^2\theta}{4(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_c^*(d))^2 \tag{A2.90}$$

and we can reformulate profits as

$$\Pi(d) = \frac{\beta\theta}{4(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_c^*(d))^2. \tag{A2.91}$$

Then, via (A2.86), the derivative of the search value with respect to the reservation dividend in the buyer's price scenario amounts to

$$\frac{\partial W}{\partial \varepsilon_c^*} = \frac{1}{\pi - \pi\beta} + \frac{1-\pi}{\pi - \pi\beta}\frac{\partial \Pi}{\partial \varepsilon_c^*} - \frac{1-\pi\beta}{\beta(\pi - \pi\beta)}\frac{\partial r}{\partial \varepsilon_c^*} \tag{A2.92}$$

$$= \frac{1}{\pi - \pi\beta} - \frac{(1-\pi)\beta\theta}{2(\pi - \pi\beta)(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_c^*(d)) + \frac{(1-\pi\beta)\beta\theta}{2(\pi - \pi\beta)(1-\pi\beta)(1-\beta)}(\tilde{\varepsilon} - \varepsilon_c^*(d)) \tag{A2.93}$$

$$= \frac{1}{\pi - \pi\beta} + \frac{\pi\beta\theta}{2(\pi - \pi\beta)(1-\pi\beta)}(\tilde{\varepsilon} - \varepsilon_c^*(d)) > 0. \tag{A2.94}$$

Since the first expression for the value of search decreases in the reservation dividends and the second one is increases in the reservation dividends, there can only be a single pair of reservation dividends to solve the model such that these conditions are fulfilled. Additionally, since the value of search does not depend on the distance to the city center, whereas the reservation dividends do, this relation holds for all distances to the city center. There is hence also a unique value of search that allows to obtain a spatial equilibrium.

Appendix 2.G Additional model results

Figure A2.7: Hamburg: spatial distributions of other endogenous variables

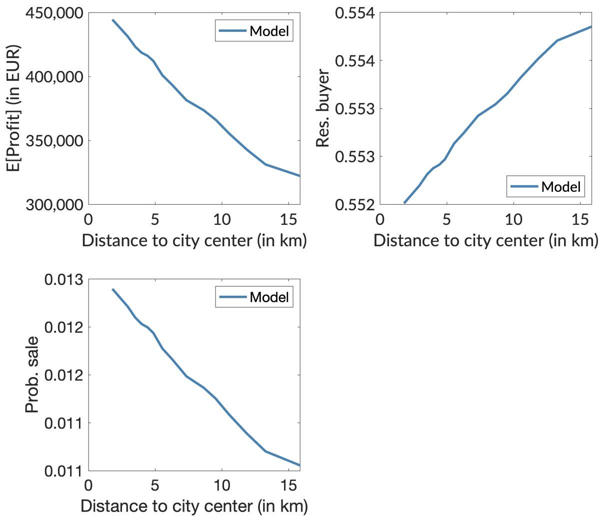


Figure A2.8: Munich: spatial distributions of other endogenous variables

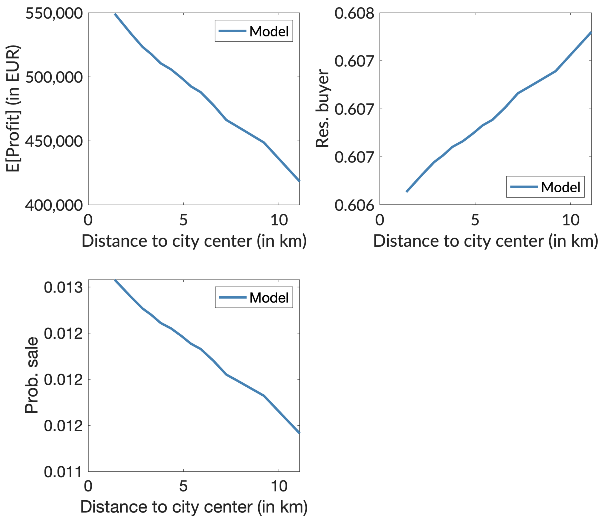


Figure A2.9: Cologne: spatial distributions of other endogenous variables

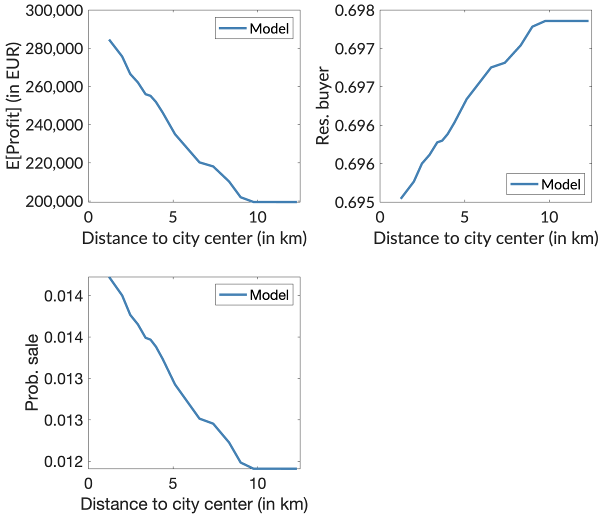


Figure A2.10: Cologne: housing match persistence π across space

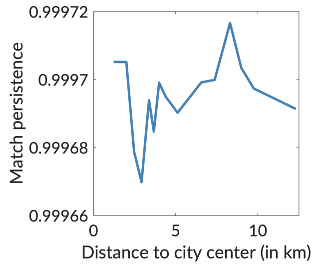


Figure A2.11: Frankfurt: spatial distributions of other endogenous variables

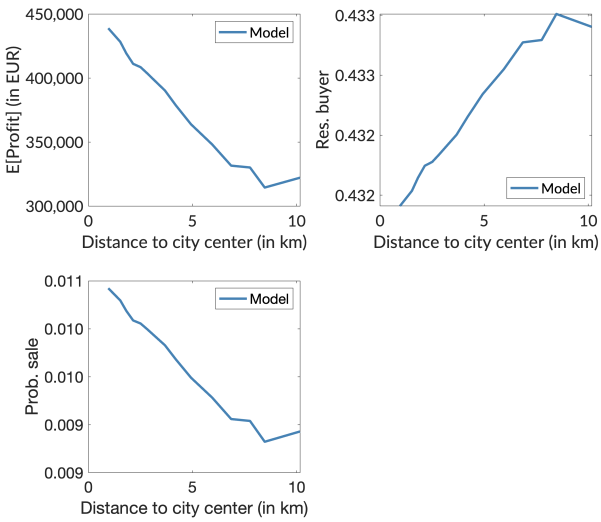
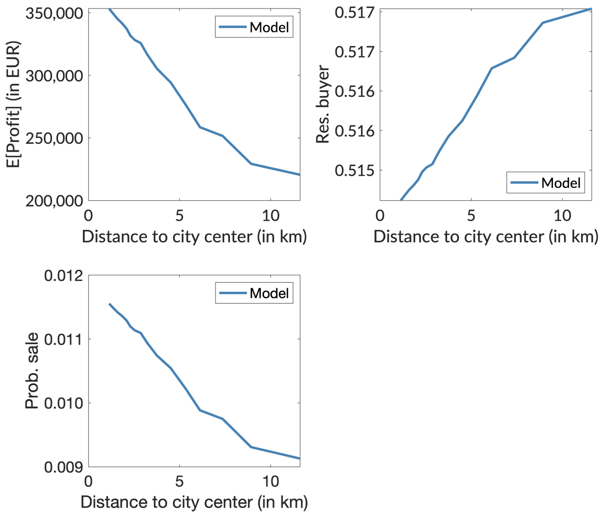


Figure A2.12: Duesseldorf: spatial distributions of other endogenous variables



Chapter 3

Monetary Policy and the Spatially Heterogeneous Response of House Prices

by Francisco Amaral, Martin Kornejew, Jonas Zdrzalek and Steffen Zetzmann

Abstract

This paper investigates the heterogeneous impact of monetary policy on house prices across geographical regions. We document substantial spatial heterogeneity in the sensitivity of house prices to monetary policy shocks. In large superstar agglomerations with low rental yields, prices decline (rise) more strongly in response to contractionary (expansionary) monetary policy compared to peripheral regions. We propose a mechanism which centers pre-exisitng differences in regional discount rates for real estate assets.

Key words: Monetary policy, Regional housing markets, Local housing risk

JEL codes: E43, E52, G12, R21, R30

3.1 Introduction

Real estate collateralizes debt contracts and is a major component of household wealth. Hence, real estate *valuation* matters for economic activity via borrowing capacity and consumption-savings decisions (Piazzesi and Schneider, 2016). Based on these observations, several studies highlighted the role of real estate prices for the transmission of monetary policy (Jordà et al., 2015; Garriga et al., 2017; Agarwal et al., 2022).

However, real estate markets vary along many characteristics across regions (Glaeser et al., 2014; Piazzesi et al., 2020), which can cause local housing markets to respond differently to similar economic shocks, contributing to regional disparities. In this paper, we study spatial heterogeneity in how monetary policy impacts real estate markets.

We document a striking spatial pattern in the response of house prices to monetary policy. We show that housing prices in areas with low rental yields respond more strongly to monetary policy shocks. By embedding high-frequency monetary policy shocks in a state-dependent local projection framework, we show that the effects are quantitatively substantial. Specifically, our results indicate that five years after a contractionary monetary policy shock equivalent to one standard deviation, house prices in the bottom 5% of the rental yield distribution decline by 120% more than in all other MSAs. We further provide evidence consistent with a novel channel centering the role of region-specific discount rates.

We propose that a change in national interest rates leads to larger (smaller) *relative* changes in local discount rates where they have been small (large) to begin with. The change in the *relative* discount rate in turn dictates *relative* price adjustments according to the standing pricing equation for (real estate) assets. Prior literature has documented discount rate differences and connected them to differential local housing market risk. Building on the empirical result that cities with lower housing risk have lower rental yields (Demers and Eisfeldt, 2022; Amaral et al., 2023b), we think in a framework in which the marginal buyer in low rental yield regions discounts future cash-flows at a lower rate than the marginal buyer in other regions, making the valuation of housing more responsive to shocks to interest rates in the less risky housing markets (Amaral et al., 2023b).

We organize our analysis and interpretation of empirical results using the present value equation for housing dating back to Poterba (1984). We show that the regional differences in price changes following a monetary policy shock can be attributed to three channels that may vary regionally: (i) differences or varying responses of economic fundamentals to the monetary policy shock, which influence the value of future housing services, (ii) heterogeneous expectation formation, and (iii) different discount rates.

We find that the responses of economic fundamentals, such as income and rents, to monetary policy shocks differ only slightly across regions. Furthermore, we demonstrate that the differences in rent responses are far too small to account for the variations in price responses. Additionally, our results remain robust after controlling for differences in housing supply elasticities. Similarly, our analysis of expectation formation—based on agents extrapolating from past house price growth—shows that these factors contribute minimally to the spatial heterogeneity in house price responses to monetary policy shocks. On top of that, our baseline framework includes region-specific fixed effects, and the results remain robust to incorporate time fixed effects and even state-time fixed effects.

Instead, our findings strongly support the discount rate channel as the dominant mechanism driving these regional differences. This conclusion is further substantiated by analyzing the role of idiosyncratic risk, where we find that MSAs with lower levels of idiosyncratic risk respond more strongly to monetary policy shocks. This suggests that differences in the discount rate, influenced by local risk factors, are a crucial determinant of the spatially heterogeneous response of house prices to monetary policy.

Our analysis leverages a comprehensive dataset covering 346 Metropolitan Statistical Areas (MSAs) in the United States over a 32-year period (1988Q1 to 2019Q4). The dataset includes variables such as house prices, rents, employment, income, supply elasticities, and idiosyncratic risk, enabling a detailed examination of how different regional housing markets respond to monetary policy shocks. To estimate the spatial heterogeneity in house price responses, we employ a state-dependent local projection framework (Jordà, 2005; Gonçalves et al., 2022), incorporating high-frequency monetary policy shocks from Bauer and Swanson (2023). This approach allows us to capture heterogeneity in impulse responses across the rental yield distribution.[1]

Related Literature. This paper builds on a long line of literature analyzing the impact of monetary policy on asset markets (e.g Thorbecke, 1997; Cochrane and Piazzesi, 2002; Rigobon and Sack, 2004; Bernanke and Kuttner, 2005; Gurkaynak et al., 2005). While this literature has focused primarily on stock and bond markets, several papers have emphasized the effects on housing markets (Del Negro and Otrok, 2007; Goodhart and Hofmann, 2008; Jarocinski and Smets, 2008; Sá et al., 2011).[2] We contribute to this literature by documenting substantial regional heterogeneity in house price responses to monetary policy shocks and by providing a mechanism to explain this heterogeneity based on regional differences in discount rates.

The papers most closely related to ours are Füss and Zietz (2016), Fischer et al. (2021),

[1]Given the skewed distribution of rental yields, with a concentration of very large cities, a linear treatment of rental yields is inappropriate. Instead, we focus our analysis on the bottom 5% of cities by rental yields.

[2]Kuttner (2014) provides a comprehensive review of the literature on interest rates and house prices.

and Aastveit and Anundsen (2022), who find that regional differences in responses are driven by variations in supply elasticities and local regulations. La Cava and He (2021) support this view, but argue that supply elasticities alone do not explain the full cross-sectional variation, pointing to factors such as the share of home investors, income levels, and mortgage debt as additional explanations. In contrast to these papers, our focus is on the discount rate channel, which we show to be robust to these previously established channels.

Our analysis also contributes to the literature on the regional effects of monetary policy (Carlino and DeFina, 1998; Fratantoni and Schuh, 2003; Beraja et al., 2018; Pizzuto, 2020), by emphasizing the role of local housing risk premiums in the pass-through of monetary policy shocks to housing prices. By showing that contractionary monetary policy can increase regional house price dispersion, we also contribute to the long-standing literature on the regional dynamics of real estate markets (Saiz, 2010; Glaeser et al., 2014; Piazzesi et al., 2020) and, in particular, to the work that examines the growing disparity in regional house prices (Van Nieuwerburgh and Weill, 2010; Gyourko et al., 2013; Amaral et al., 2023a).

3.2 Data and Summary Statistics

Our sample contains 316 Metropolitan Statistical Areas (MSAs) and 30 Metropolitan Divisions (MDs) in 49 US states (excluding Alaska and Hawaii, including District of Columbia).[3] The unbalanced panel dataset covers the period from 1988Q1 to 2019Q4.

Monetary Policy Shock. To estimate the effect of monetary policy on the spatial heterogeneity of house prices, we rely on the broad literature of high-frequency monetary policy shocks (Kuttner, 2001; Gertler and Karadi, 2015; Nakamura and Steinsson, 2018; Bauer and Swanson, 2023). In our main specification, we use the orthogonalized monetary policy shock by Bauer and Swanson (2023). Bauer and Swanson (2023) argue that monetary policy surprises are predictable to by publicly available macroeconomic or financial market data prior to the FOMC announcement. More specifically, they find that nonfarm payrolls surprises, employment growth, changes in S&P 500 stock market index, changes in the yield curve slope, changes in commodity prices, and the implied skewness of the 10-year Treasury yield that predate the FOMC announcement explain up to 19% of the variation of the monetary policy shock. Bauer and Swanson (2023) argue that the predictability of monetary policy surprises is due the "Fed's response to

[3]In the following, we use MSAs to refer to the entire sample of 346 MSAs and MDs. See `https://www.census.gov/programs-surveys/metro-micro/about/glossary.html` for detailed definition of the respective geographic entity.

news" channel and uncertainty about Fed's monetary policy rule rather than the "Fed's information channel" as in Nakamura and Steinsson (2018). Independent of the reason of the predictability, the orthogonalized monetary policy shock isolates the pure monetary policy shock. Therefore, it does not suffer from an econometric endogeneity problem. We sum the monetary policy shock to quarterly level similar to Ottonello and Winberry (2020). Since we are interested in the effect of a change in the interest rate on house prices, we are primarily interested in a surprise of the monetary policy shock rather than the "Fed's information channel" or "Fed's response to news channel". As such, we use the orthogonalized monetary policy shocks in our baseline analysis. For robustness checks, we also use the non-orthogonalized monetary policy shock from Bauer and Swanson (2023), as well as another type of high-frequency monetary policy surprise (Nakamura and Steinsson, 2018), and monetary policy shock developed by Romer and Romer (2004) which is based on a narrative approach.

House Prices and Macroeconomic Variables. We use the Quarterly All-Transactions FHFA House Price Index which is repeated-sales house price index based on single-family house sales and appraisal prices from GSE-backed mortgages.[4] To homogenize our sample, we drop all MSAs without house price index before 1990Q1. We deflate the house price index by US CPI from the U.S. Bureau of Labor Statistics.[5]

Unfortunately, it is very difficult to obtain historical and quarterly data on rents at MSA level. The best data we could find are annual time series of median rent series form 2001-2019 at the county-level from Office of Policy Development and Research (HUD User).[6] We aggregate rents to the MSA- and MD-level using the Census Delineation file 2020 such that the geographic areas are the same for all variables. Furthermore, we use linear interpolation to approximate the series at the quarterly level.

We use rental yields to divide the MSAs in different groups. We follow the procedure from Gyourko et al. (2013) to construct rental yields for MSA using housing value and rents at the county-level from the United States Census 1990.[7] We aggregate rents to the MSA- and MD-level using the Census Delineation file 2020.

We use employment, income, and population data at the MSA level from the Bureau of Economic Analysis (BEA). However, since the BEA does not provide quarterly data for these variables at the MSA level, we apply linear interpolation to generate quarterly

[4] Data are publicly available and can be downloaded from the website: `https://www.fhfa.gov/DataTools/Downloads/Pages/House-Price-Index.aspx`.

[5] We downloaded the series form the FRED St. Louis `https://fred.stlouisfed.org/series/CPIAUCSL`.

[6] Data is publicly available at `https://www.huduser.gov/portal/datasets/50per.html`.

[7] The United States Census is not conducted on an annual basis. Hence, we cannot use this source for a time series on rents.

series.

To control for supply side elasticity of MSAs, we use the supply side elasticity measure by Guren et al. (2021).

Idiosyncratic Risk. We obtain the data on idiosyncratic risk for 303 MSAs covering the period from 1990 to 2020 from Amaral et al. (2023b). They use a combination of transaction-level price data for single family homes from Corelogic and price indices on the county level from FHFA and Zillow.com.

Descriptive Statistics. Table 3.1 shows summary statistics for variables related to the housing market (price index, rents), the two monetary policy shocks from Bauer and Swanson (2023), and information about the characteristics of the MSAs. The information about the MSAs includes population size, rental yields, idiosyncratic risk, supply elasticities, employment rate, and per capita income. Particularly relevant to our research questions is the fact that rental yields in 1990 exhibit a certain degree of heterogeneity, ranging from 2.55 to 9.61.[8]

[8]Figure A3.1 in Appendix 3.A shows a map of rental yields in 1990 to illustrate the spatial heterogeneity.

	Mean	SD	Min	p10	p25	Median	p75	p90	Max	N
Housing market: Quarterly growth rate (in %)										
House price index	0.83	1.97	-21.78	-1.30	-0.08	0.89	1.81	2.92	23.52	43832
Median rent 2 bedroom	0.72	1.23	-9.23	-0.61	0.15	0.72	1.23	2.08	9.96	23125
High-frequency Monetary Policy Shock										
Orthogonalized MPS: Bauer Swanson (2023)	-0.00	0.07	-0.24	-0.09	-0.04	0.00	0.04	0.08	0.20	128
MPS: Bauer Swanson (2023)	0.00	0.10	-0.34	-0.11	-0.03	0.01	0.05	0.09	0.28	128
MPS: Nakamura Steinsson (2018)	-0.08	1.42	-5.96	-1.83	-0.61	0.09	0.62	1.47	4.07	100
MPS: Romer Romer (2004)	0.00	0.20	-0.46	-0.23	-0.14	-0.00	0.14	0.27	0.49	80
MSA Characteristics										
Population 1987	553849.00	970874.29	37403.00	88069.00	125180.00	236959.00	553916.00	1405633.00	10417555.00	44178
Rental Yields 1990	6.13	1.24	2.55	4.39	5.53	6.25	6.88	7.52	9.61	44178
St.dev. idio risk annual	0.17	0.04	0.10	0.13	0.14	0.16	0.19	0.22	0.34	38746
Supply Side Elasticity by Guren et al. (2021)	1.66	1.09	0.43	0.73	0.95	1.39	1.94	2.81	7.63	39727
Total employment/Population	0.57	0.08	0.30	0.47	0.52	0.57	0.62	0.68	0.94	44178
Personal Income per capita (thousands of $)	32.75	12.90	8.67	18.30	22.90	31.20	40.21	48.10	140.55	11054

Table 3.1: Summary statistics

The table shows some summary statistics of 316 MSAs and 30 MDs. The period ranges from 1988Q1 to 2019Q4. The MSA characteristics Personal Income and employment ratio are on yearly basis.

3.3 Empirical Results

In this section, we show that housing prices in cities with lower rental yields are more sensitive to monetary policy shocks. First, we introduce our empirical framework. Second, we discuss why differences in rental yields can explain the heterogeneous responses to monetary policy shocks through a valuation effect. Third, we provide evidence that the effect is not driven by potential alternative channels, such as (i) economic fundamentals and (ii) heterogeneous expectations.

3.3.1 Empirical Framework

To analyze the spatial heterogeneity of house prices responses to monetary policy shocks, we use local projections (Jordà, 2005) including a state-dependent term in style of Gonçalves et al. (2022).[9]

We estimate the cumulative percentage change to the dependent variable for h quarters after a monetary policy shock with $h \in \{0, 1, 2, \ldots, 20\}$. Our baseline empirical model takes the following form:

$$\log(\mathrm{y})_{i,t+h} - \log(\mathrm{y})_{i,t-1} = \alpha_i^h + \theta_h \mathrm{MPS}_t \times X_i + \beta_h \mathrm{MPS}_t + Z_{i,t} + \varepsilon_{i,t+h}. \tag{3.1}$$

The logarithm of the dependent variable for MSA i at quarter t with local projection horizon h is represented by $\log(\mathrm{y})_{i,t+h}$. Thus, the cumulative percentage change of the dependent variable is expressed as $\log(\mathrm{y})_{i,t+h} - \log(\mathrm{y})_{i,t-1}$. MPS_t represents the high-frequency monetary policy shocks aggregated to quarter t. X_i is our state-dependence measure in which $X_i \in \{0, 1\}$ if either MSA i belongs to a specific group of the rental yield distribution or not i.e. in our baseline specification $X_i = 1$ if MSA i is in the bottom 5% of the rental yield distribution.[10] The variable $Z_{i,t}$ denotes the lag-augmentation variables, specifically including the dependent and all independent variables lagged by eight quarters, following the lag augmentation approach of Olea and Plagborg-Møller (2021). MSA-fixed effects are expressed as α_i^h in order to control for all time-invariant MSA-specific characteristics.[11] The residual term is denoted by $\varepsilon_{i,t+h}$. We cluster the standard errors at the MSA-level.

[9] Jordà (2005) shows that local projections can be used to estimate impulse response functions with fewer assumptions on the data-generating process compared to a standard vector autoregression (VAR) framework. Additionally, since local projections can be estimated using simple univariate equations, they can be easily calculated using standard regression techniques.

[10] In Appendix 3.A we provide a list of treated MSAs. In 1990 the MSAs in the bottom 5% of the rental yield distribution accounted for approximately 19% of the population.

[11] In practice, we employ a double interaction of the monetary policy shock MPS_t with the state-dependence measure X_i. However, the variable X_i drops out because it is time-invariant and, therefore, perfectly collinear with the MSA-fixed effects.

3.3.2 Theoretical Foundation

Various studies have shown that returns on housing investments differ significantly across regions within the same country (Demers and Eisfeldt, 2022; Amaral et al., 2023b). Specifically, these studies highlight substantial variation in rental yields, which can be attributed to the fact that housing prices exhibit far greater variation than rents across regions (Demers and Eisfeldt, 2022). More recently, Liu et al. (2022) have shown that assets with longer durations—where there is a larger difference between the asset's payoff and price—may be more sensitive to changes in interest rates. In this paper, we examine the extent to which this mechanism affects the impact of monetary policy on housing prices. Specifically, we test whether housing prices in cities with lower rental yields are more sensitive to monetary policy shocks. To fix ideas, consider the present value equation for a house in city i:

$$P_t^i = \sum_{j=1}^{\infty} \mathbb{E}^{\boldsymbol{i}} \left(Rent_{t+j}^i \times \left(\frac{1}{1+r_t^i} \right)^j \right), \tag{3.2}$$

where P_t^i is the real house price for city i at time t and $\sum_{j=1}^{\infty} Rent_{t+j}^i$ is the future rental flow, net of costs in city i and r_t^i is the real discount rate and can be written as $r_t^i = \text{risk-free-rate}_t + \text{risk-premium}^i$. This means that r_t^i will only vary across cities if the risk premium varies across cities.

From this, it becomes clear that if monetary policy influences the risk-free rate, then it directly affects housing prices. The question is then whether the impact of monetary policy on housing prices can differ across cities. Following Amaral et al. (2023a), the impact of monetary policy should vary across cities if the risk premium component differs by city. In particular, we expect cities with lower risk premiums to exhibit stronger responses to monetary policy shocks. This is because the same risk-free rate change will have a larger impact on a lower discount rate [12]. As shown by Amaral et al. (2023b), cities where housing investments are less risky tend to have lower rental yields. Building on this result, we test in the following sections whether lower rental yields are associated with a stronger response of housing prices to monetary policy shocks.

A crucial aspect to consider for this mechanism to work is the persistence of the monetary policy shock on the risk-free rate. If we consider the most extreme case, where the shock is only a one-period transitory shock with no impact in the following period, the resulting price changes would be minimal. Moreover, investors would likely not re-evaluate if shocks are merely transitory. Thus, for substantial effects on house prices, the monetary policy shock needs to be persistent. The literature shows that the surprise component

[12] In Appendix 3.B we provide further intuition on this mechanism by (i) an illustration in Figure A3.2 taken from Amaral et al. (2023a), and (ii) a straightforward numerical example.

of monetary policy shocks is highly persistent (Gertler and Karadi, 2015; Nakamura and Steinsson, 2018).

Furthermore, from equation 3.2, it becomes clear that monetary policy can also influence housing prices if it affects future rents or expectations. Rents are determined by economic fundamentals such as income, employment, and supply and demand for housing. If monetary policy were to impact, for instance, income differently across regions, it could affect housing demand, and consequently explain differences in house price responses. Thus, the discount rate is theoretically not the only channel through which monetary policy can shape housing prices.[13] In the following sections, we demonstrate that monetary policy has a stronger impact on house prices in cities with lower rental yields and provide evidence that this mechanism is due to lower discount rates in these cities.

3.3.3 Results

Visual exploration. Figure 3.1 shows a map of the US displaying the cumulative house price changes in MSAs two years after a one standard deviation contractionary monetary policy shock. The map clearly illustrates that regional house prices react heterogeneously to a monetary policy shock, as the average response to a one standard deviation monetary policy shock is -0.69%. The change in house prices ranges from -2% in coastal areas of California, Florida, and the Northeast to little or even slightly positive growth in MSAs located in central states of the US and/or further inland from the coast.

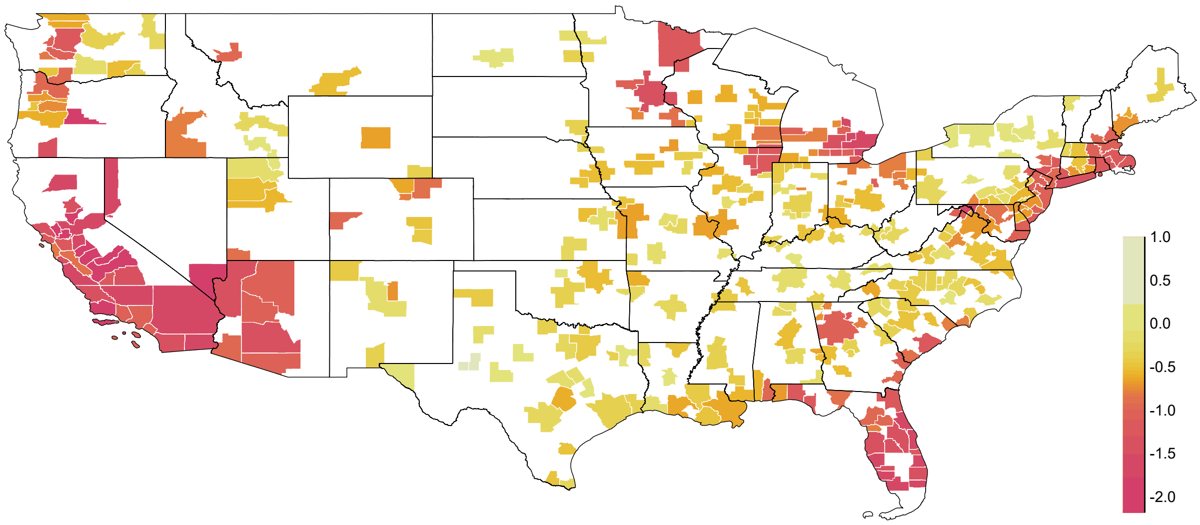
Figure 3.2 combines the results illustrated in Figure 3.1 with the spatial differences in rental yields. This map provides the first indication that the magnitude of the reaction to monetary policy shocks is correlated with rental yields. MSAs with stronger house price reactions, shown in red, are more often marked with green squares or triangles, indicating that these MSAs fall within the lower quartiles of the rental yield distribution. Examples include California and coastal MSAs in the Northeast. On the other hand, MSAs where house prices experienced smaller declines, illustrated by lighter colors, are more likely to be marked with blue circles or diamonds, indicating that these MSAs belong to the higher end of the rental yield distribution.

Thus, we can conclude that spatial differences in reactions to monetary policy are substantial and are negatively correlated with rental yields. In the next section, we aim to achieve two objectives: (i) to move beyond simple correlations by employing the more formal analysis outlined in the next paragraph to precisely identify the differential

[13] While monetary policy should have no long-run effect on the expected growth rate of rents, our empirical results support this, as the initial difference in the response of rents to a monetary policy shock diminishes after 8 years. However, it may have a contemporaneous effect on rents.

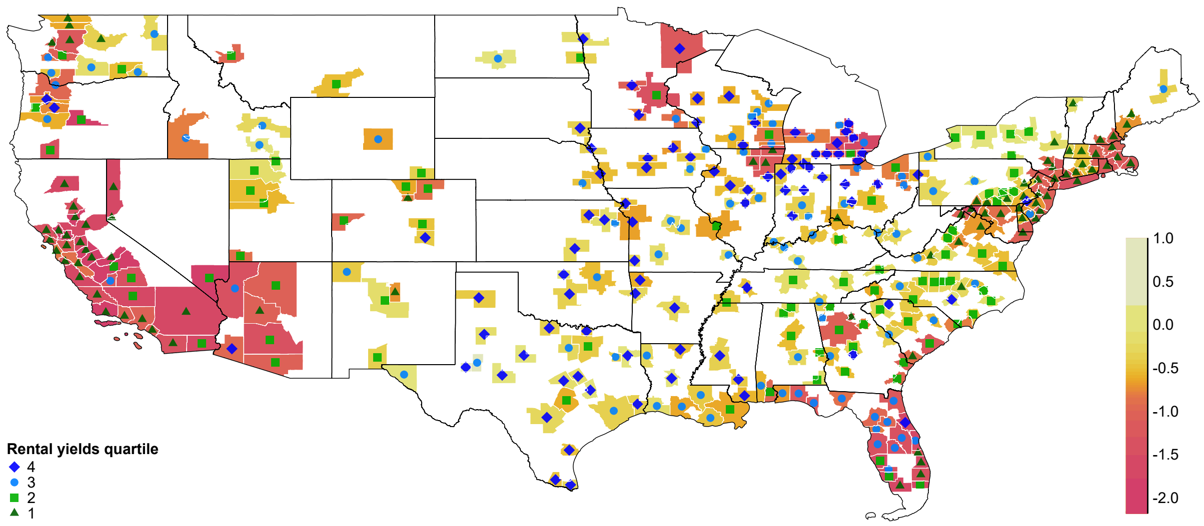
response of house prices with respect to rental yields, and (ii) to demonstrate how these results evolve over time.

Figure 3.1: Spatial house price response to monetary policy shock



Notes: This figure shows the cumulative house price response two years after a one standard deviation monetary policy shock. Standard errors are clustered at the MSA level. These results are displayed for 346 MSAs in the period between 1988 and 2019.

Figure 3.2: Spatial house price response to monetary policy shock and corresponding rental yield quartile



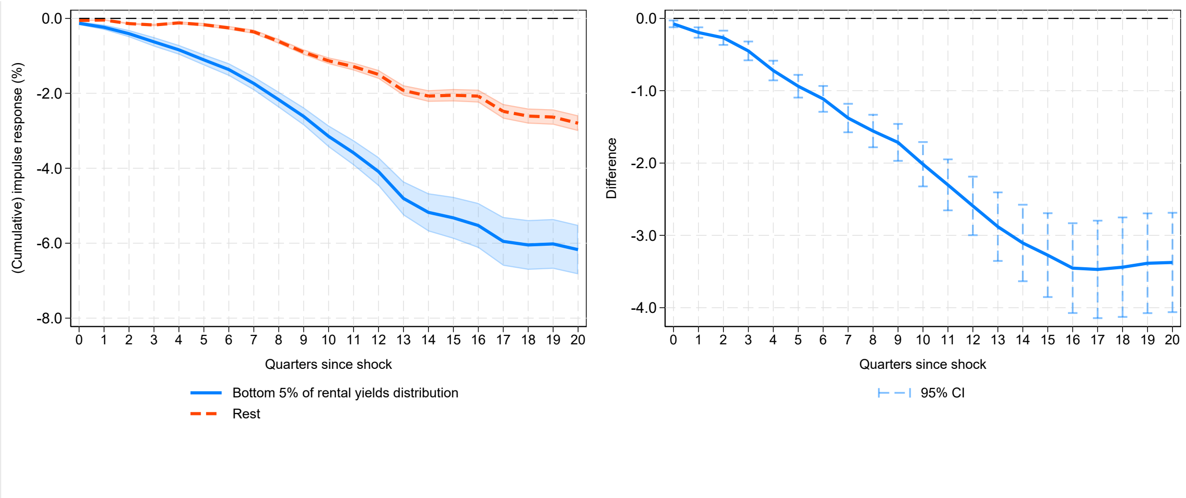
Notes: This figure shows the cumulative house price response two years after a one standard deviation monetary policy shock and the corresponding rental yield quartile. Standard errors are clustered at the MSA level. These results are displayed for 346 MSAs in the period between 1988 and 2019.

Empirical Results. We aim to conduct a more formal analysis to investigate, beyond correlations, how different rental yields shape the response of house prices to monetary policy shocks. This exercise is carried out by employing equation 3.1. Additionally, this equation allows us to track how the gap with respect to rental yields evolves over time.

The main result is presented in Figure 3.3, which displays the outcome of equation 3.1 for a one standard deviation contractionary monetary policy shock. The state-dependence measure consists of two groups: the MSAs in the bottom 5% of the rental yield distribution and the remaining MSAs. The left panel of the figure illustrates the cumulative house price response for both groups, while the right panel shows the difference between these groups, including the 95% confidence interval.

Consistent with the hypothesis derived from Figure 3.2, Figure 3.3 demonstrates that house prices in MSAs in the bottom 5% of the rental yield distribution exhibit a substantially more pronounced decline after a one standard deviation contractionary monetary policy shock than all other MSAs. The change in house prices in the lowest rental yield MSAs accumulates to around -6% after 5 years compared to the period before the shock. In contrast, the house price change in all other MSAs amounts to around -3%. The right panel of Figure 3.3 emphasizes this point by showing the gap between both groups. It illustrates that the gap consistently grows over time and is significantly different from zero at a 95% confidence level in all quarters. The gap stabilizes after 4 years at a difference of approximately -3.5%.

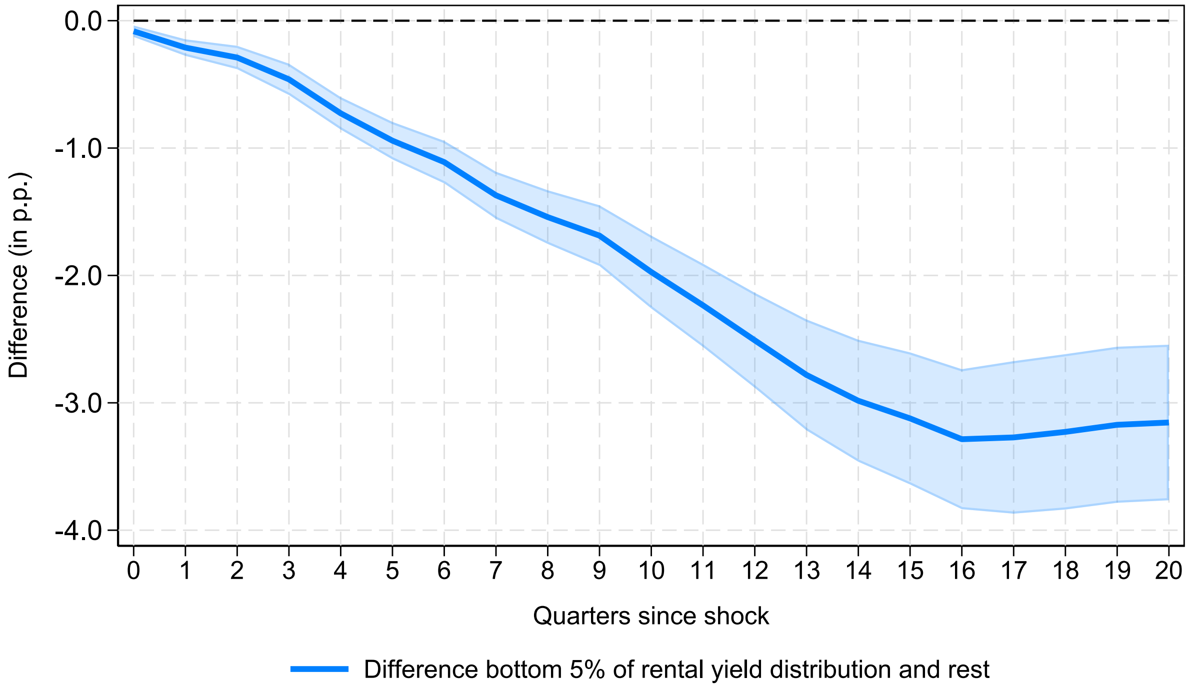
Figure 3.3: House prices and monetary policy shock



Notes: This figure shows the cumulative impulse response function (IRF) of house prices to a one standard deviation monetary policy shock. The left panel compares the IRFs for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs. The solid blue line shows the IRF of house prices in the bottom 5% of the rental yield distribution with the respective 95% confidence interval. The dashed red line shows the IRF for MSAs in the remaining 95% of the rental yield distribution with the respective 95% confidence interval. The right panel shows the difference between the two IRFs as well as the 95% confidence interval of the difference. Standard errors are clustered at the MSA level. These results are based on 346 MSAs in the period between 1988 and 2019.

Additionally, we can exploit the panel structure of our dataset by including time-fixed effects. This specification controls for common shocks across all MSAs. However, due to the perfect collinearity between the time-fixed effects and the monetary policy shock, we can no longer interpret the results in terms of level responses but only in terms of differences between the groups. Figure 3.4 shows the IRFs with time-fixed effects included. Controlling for common shocks across regions does not change our results, as the heterogeneous house price response remains statistically and economically significant, with a gap of about -3% after 5 years since the shock between the MSAs in the bottom 5% of the distribution compared to the remaining MSAs.

Figure 3.4: House prices and monetary policy shock controlling for time-fixed effects



Notes: This figure shows the cumulative difference of house prices to a one standard deviation monetary policy shock for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs with the respective 95% confidence interval. The regression includes time-fixed effects and standard errors are clustered at the MSA level. These results are based on 346 MSAs in the period between 1988 and 2019.

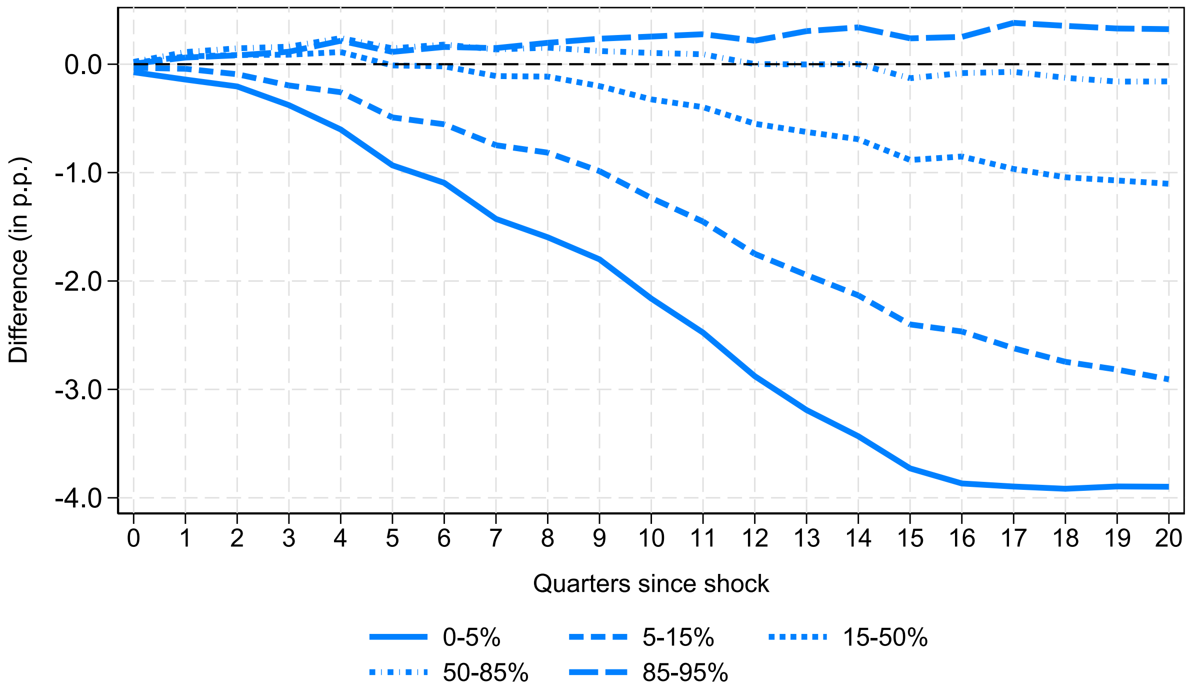
To ensure that the result is not driven by an arbitrary choice of different groups, we estimate the effect of a monetary policy shock on the cumulative house price response for five groups of the rental yield distribution of 1990.

Figure 3.5 shows that the effects (i) are driven by the MSAs in the bottom 5% of the rental yield distribution and (ii) are not linear. The figure is again based on equation 3.1, but the state-dependence measure consists of different groups. We compare the top 5% of MSAs in the rental yield distribution with five other groups, plotting only the discrepancies in house price reactions to monetary policy shocks due to controlling for time-fixed effects.

Starting with the assessment relative to the MSAs in the bottom 5% of the rental yield distribution, it is evident that the difference is even more pronounced compared to our baseline result in Figure 3.4, accumulating to about -4% after 20 quarters. Hence, comparing the MSAs in the tails of the distribution supports the previous results by showing that larger differences in rental yields lead to even larger differences in house price changes in response to monetary policy shocks.

However, Figure 3.5 goes further by revealing that the heterogeneity in house price changes after a monetary policy shock is specific to the MSAs in the bottom of the rental yield distribution, and the effect is non-linear. Comparing the base group to MSAs in the rental yield distribution between the bottom 5% and 15% illustrates that the heterogeneity persists, as the gap in house prices accumulates to approximately 3%. In contrast, assessing the gap between the base group and all other groups, starting with the MSAs between the bottom 15% and 50%, and continuing with two groups from the top half of the rental yield distribution, shows that the house price changes are not significantly different.

Figure 3.5: House prices and monetary policy shock for multiple groups



Notes: This figure shows the cumulative difference in house prices in response to a one standard deviation monetary policy shock for five groups of the rental yield distribution in 1990, always comparing them with the top 5% of the rental yield distribution. The regression includes time-fixed effects, and standard errors are clustered at the MSA level. These results are based on 346 MSAs in the period between 1988 and 2019.

Consistent with our theoretical framework, a uniform change in the risk-free rate leads to a larger response of house prices in areas with low rental yields. However, to provide evidence for the discount rate channel, we have to ensure that the result is not due to other channels. In the following subsection, we provide evidence that economic fundamentals as well as non-rational expectations cannot fully explain the heterogeneity. As such, the results seem to be driven by the discount rate channel.

3.3.4 Economic Fundamentals

In this subsection, we will show that economic fundamentals as well as non-rational expectation formation cannot fully explain our findings. We will focus on four previously mentioned elements: (i) supply elasticities, (ii) income, (iii) rents, and (iv) expectations.[14] We will evaluate the results for a one standard deviation monetary policy shock using the MSAs in the bottom 5% of the rental yield distribution and the remaining MSAs as our groups. This ensures comparability to our main findings.

Supply Elasticities. One explanation for the locally different reactions of house prices to monetary policy shocks is provided by Aastveit and Anundsen (2022). They show in their paper that spatially heterogeneous price developments can be explained by regional differences in supply elasticities. The authors argue that house prices in areas with low supply elasticities react significantly more to expansionary monetary policy shocks than in areas with higher supply elasticities. However, they find no heterogeneous response to contractionary monetary policy surprises.

Since regional differences in rental yields are positively correlated with supply elasticities, the results we find might not be driven by differences in rental yields but rather by supply elasticities.[15] To rule out this possibility, we add supply elasticities as additional control variables to our baseline equation (3.1). Specifically, to control for the entire distribution of supply elasticities, we include each decile of the distribution of supply elasticities interacted with the monetary policy shocks in the local projection.

Figure 3.6 shows the results of this exercise. Due to data limitations of the supply elasticities by Guren et al. (2021), the sample in both local projections contains only 312 MSAs. Thus, the left panel shows our baseline result for this sample without controlling for supply elasticities. The right panel displays the result, including the supply elasticity control variable described above. Both graphs exclusively show the difference between the two rental yield groups. The right graph indicates that while the differences in house price developments between the groups are smaller, the difference compared to the left graph is only marginal, and the heterogeneity across rental yields is still significant for all quarters. The difference between the groups, including the additional control variable, is only about 1 percentage point smaller compared to the baseline result.[16]

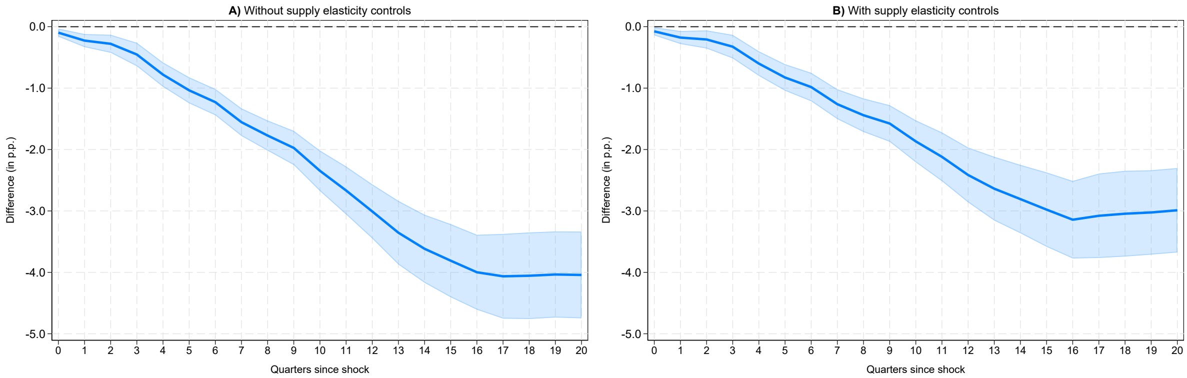
Therefore, we can conclude that regional differences in supply elasticities are not sufficient to explain the entire heterogeneity of house price responses to monetary policy.

[14] We will also assess a combination of multiple components.

[15] The correlation coefficient between rental yields in 1990 and supply elasticities provided by Guren et al. (2021) is 0.1395. The correlation is significant at a 1% confidence level.

[16] The IRF of the baseline results without supply elasticity controls is slightly different compared to the previous Figure 3.4 due to the slightly different sample.

Figure 3.6: House prices and monetary policy shock controlling for supply elasticity



Notes: This figure shows the cumulative difference in house prices in response to a one standard deviation monetary policy shock for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs, along with the respective 95% confidence interval. The left panel shows our baseline result, while the right panel displays the result controlling for supply elasticities. The regressions include time-fixed effects, and standard errors are clustered at the MSA level. These results are based on 312 MSAs in the period between 1988 and 2019.

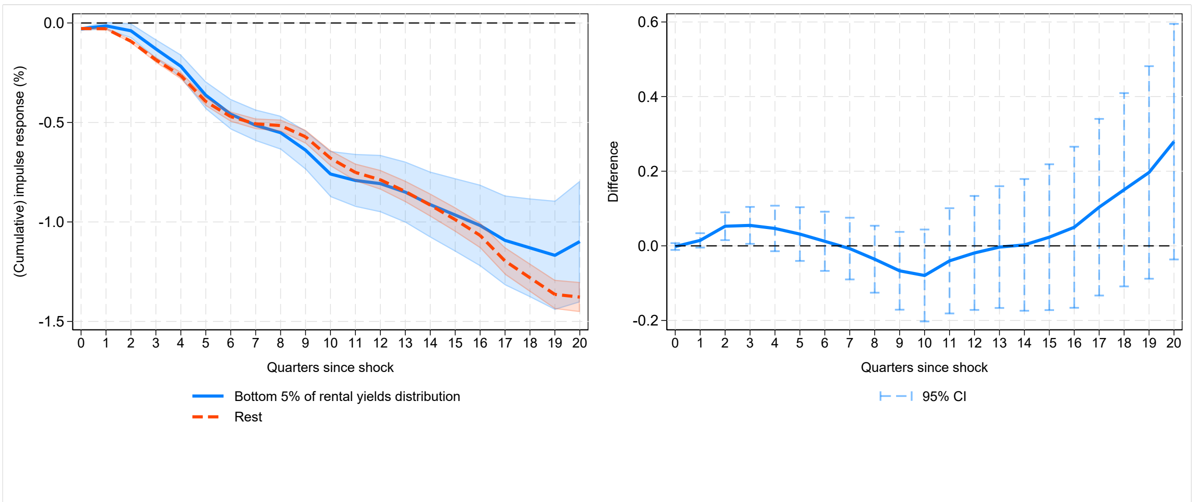
Income. In the previous paragraph, we showed that differences in supply elasticity can only explain a small fraction of the heterogeneous effect on house prices between the different groups. Nevertheless, it could be that the effects we observe are not explained by the varying discount rates but rather by other economic fundamentals, such as income, that confound our analysis.

To analyze whether monetary policy has a heterogeneous effect on income across the two groups, we use the same local projection framework described by equation (3.1). However, we are now interested in the cumulative impulse response function of income to a monetary policy shock. More specifically, we are interested in the difference between the response of MSAs with low rental yields and the rest.

The right panel of Figure 3.7 shows the cumulative effect of a monetary policy shock on income for MSAs in the bottom 5% of the rental yield distribution in 1990 and the response of income in all the other MSAs, with their respective 95% confidence intervals. The figure shows that incomes decline equally in both groups as a result of a contractionary monetary policy shock. It is only toward the end of the observation period that the decline stops in the MSAs with low rental yields. Overall, the differences are never statistically significant, as measured by the 95% confidence interval (see left panel of Figure 3.7).[17]

[17] Even after including time-fixed effects in the local projection, the difference is not significant.

Figure 3.7: Income and monetary policy shock



Notes: This figure shows the cumulative impulse response function (IRF) of income to a one standard deviation monetary policy shock. The left panel compares the IRFs for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs. The solid blue line shows the IRF of income in the bottom 5% of the rental yield distribution with the respective 95% confidence interval. The dashed red line shows the IRF for MSAs in the remaining 95% of the rental yield distribution with the respective 95% confidence interval. The right panel shows the difference between the two IRFs as well as the 95% confidence interval of the difference. Standard errors are clustered at the MSA level. These results are based on 346 MSAs in the period between 1988 and 2019.

In summary, the heterogeneous development of house prices in relation to rental yields cannot be explained by income developments, as these show no relevant differences across the two groups.[18]

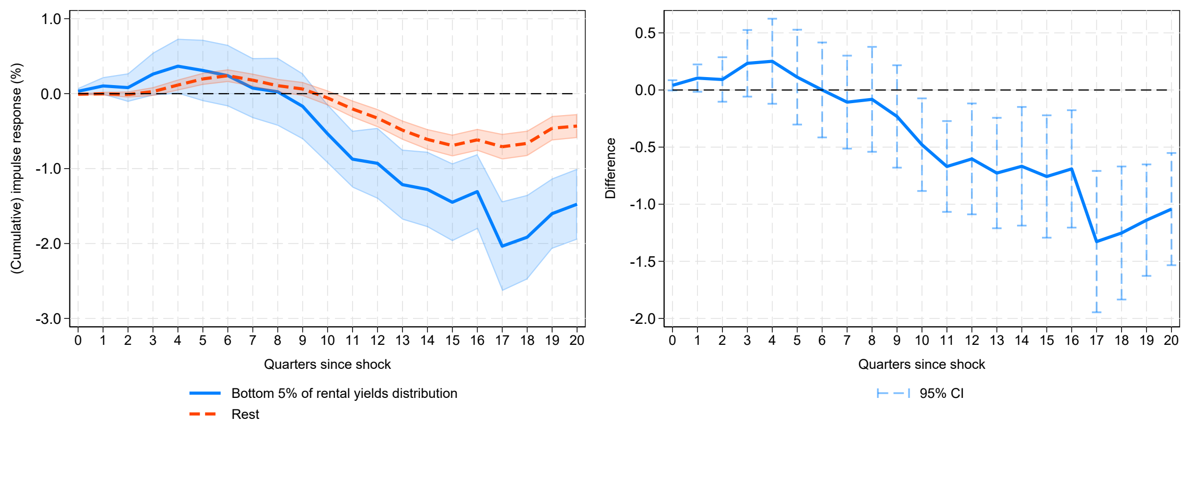
Rents. Ultimately, we aim to provide evidence that there is no heterogeneous response across regions in rents. We leverage the same empirical approach used for our main outcomes, as given by equation (3.1). In this case, the dependent variable is the cumulative change in log median rents compared with the period prior to the shock.

The results are displayed in Figure 3.8. The left panel illustrates the cumulative response of both groups, while the right panel displays the differences, including the confidence interval. The left panel highlights that rents, after a contractionary monetary policy shock, take some time to react. In the first few quarters, rents in both groups are rather stable or even slightly increasing. Only after about 8 quarters do rents in the MSAs with the lowest rental yields drop, while rents decline only after around 10 quarters in the other group. The rents in low rental yield MSAs also fall faster than in the comparison group. This aspect is highlighted by the right panel, which shows that the difference in

[18]In Appendix 3.B, we show that other economic fundamentals, such as employment and population, also do not provide a basis for explaining the heterogeneous development of house prices following a monetary policy shock.

rents is statistically significant at a 95% confidence interval after 10 quarters and remains so until the end of the observation period. The cumulative response of rents in MSAs at the bottom of the rental yield distribution after 20 quarters is about −1.5%, whereas the drop in the remaining MSAs is only −0.5%. Thus, the difference at the end of the observation period is about −1 percentage point.

Figure 3.8: Rents and monetary policy shock



Notes: This figure shows the cumulative impulse response function (IRF) of median rents for 2 bedroom apartment to a one standard deviation monetary policy shock. The left panel compares the IRFs for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs. The solid blue line shows the IRF of rents in the bottom 5% of the rental yield distribution with the respective 95% confidence interval. The dashed red line shows the IRF for MSAs in the remaining 95% of the rental yield distribution with the respective 95% confidence interval. The right panel shows the difference between the two IRFs as well as the 95% confidence interval of the difference. Standard errors are clustered at the MSA level. These results are based on 320 MSAs in the period between 2001 and 2019.

The graphic thus emphasizes that the rents of the respective groups develop in the same direction as house prices. Therefore, it can be summarized that the development of rents can explain part of the heterogeneous development of house prices. However, the question arises as to how large a share the development of rents can explain.

To quantify the extent to which rents explain the heterogeneous house price developments, we perform a back-of-the-envelope calculation. We reformulate equation (3.2) following Amaral et al. (2023a) to obtain the rental yields.[19] Afterwards we take (i) the logarithm on both sides of the equation, (ii) the first differences, and finally (iii) the difference between the two groups, i.e., the MSAs in the bottom 5% of the rental yield distribution and the remaining MSAs. Under the assumption of constant expected growth

[19] The simplifications to obtain the rental yields are presented in Appendix 3.B. Equation (A3.1) presents the result from which we start the next steps.

rates g_t^i at time t, the equation becomes:[20]

$$\Delta \log(P_t^5) - \Delta \log(P_t^{95}) = \Delta \log(\text{Rent}^5) - \Delta \log(\text{Rent}^{95}) - \frac{\Delta \rho_t^5}{\rho_t^5 - g^5} + \frac{\Delta \rho_t^{95}}{\rho_t^{95} - g^{95}} \qquad (3.3)$$

While the left-hand side of the equation shows the overall heterogeneous price reactions across the different groups illustrated in Figures 3.3 and 3.4, the right-hand side splits the effect into (i) the first part, which captures the effect generated by the contemporaneous response of rents to monetary policy, and (ii) the discount rate channel. Comparing the difference between the two groups in the development of house prices, which is -3.5 percentage points at the end of the observation period, with the difference in rent developments presented here, which is -1 percentage point, shows that the rent channel can only explain part of the overall difference. Specifically, it accounts for less than one-third of the total difference.

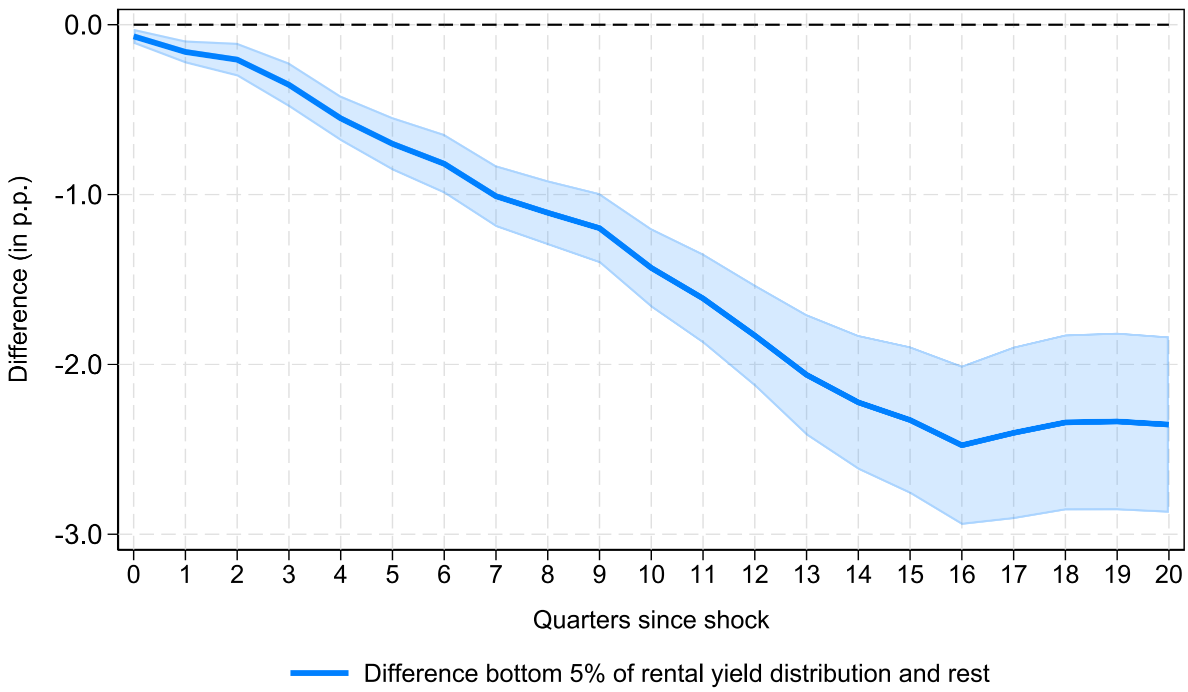
Expectations. The last aspect that we neglected in the previous analyses is the possibility that expectation formation may differ between the MSAs. Bordalo et al. (2018) argue that economic agents form expectations by extrapolating from past experiences, which, in turn, influences asset values. Heterogeneous expectations might explain the heterogeneous response of house prices to monetary policy.

If expectation formation differs across regions in a time-invariant manner, we already control for this effect by including MSA-fixed effects in the local projections. However, if market participants adjust their expectations following a monetary policy shock, this could explain the spatially heterogeneous response of house prices. To test this hypothesis, we exploit the finding by Bordalo et al. (2018) that agents form their expectations by extrapolating. Therefore, we control for time-varying changes in expectations at the MSA level by including the previous house price growth over the past 5 years, interacted with the monetary policy shock, as a control variable in the local projections.

The results of this exercise are shown in Figure 3.9. As usual, the figure shows the difference between house price developments in the low rental yield group compared to the rest. Compared to our baseline result, we observe that the effect is less pronounced (about -2.5 percentage points compared to -3 percentage points at the end of the observation period). However, the heterogeneous price development between the two groups, depending on their rental yield, remains statistically significant at the 95% confidence interval throughout the entire observation period.

[20] After the three steps, we apply the logarithmic properties to simplify the equation.

Figure 3.9: House prices and monetary policy shock controlling for expectation formation



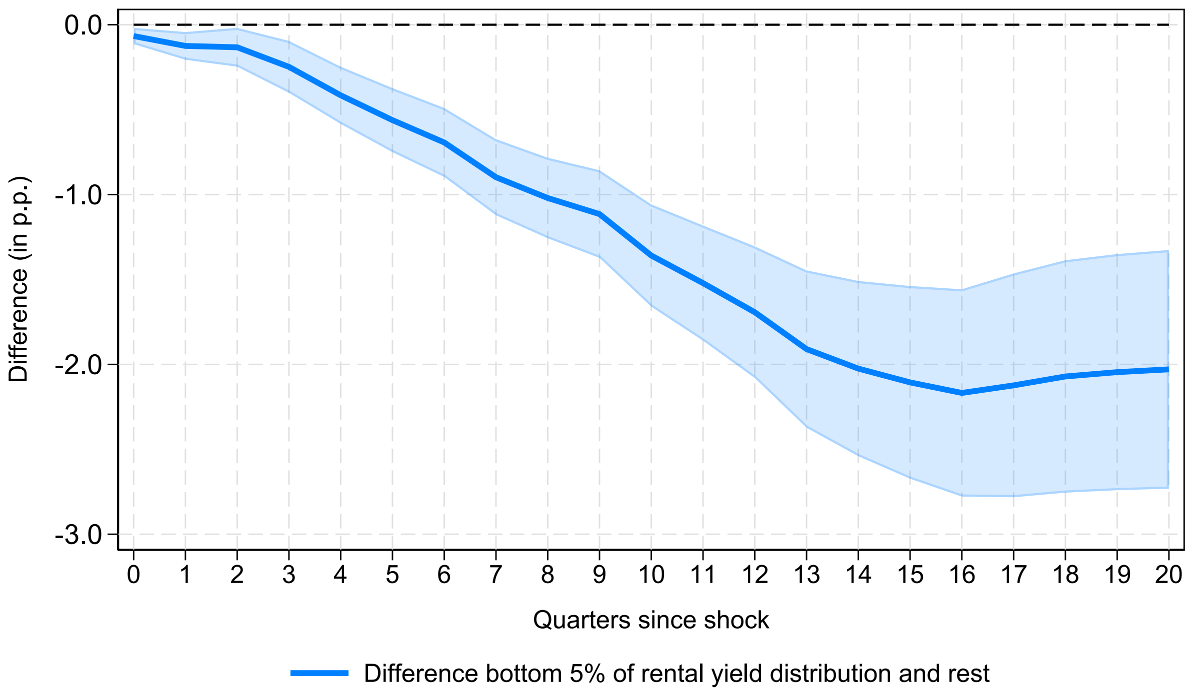
Notes: This figure shows the cumulative difference in house prices in response to a one standard deviation monetary policy shock for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs, along with the respective 95% confidence interval controlling for previous house price growth. The regressions include time-fixed effects, and standard errors are clustered at the MSA level. These results are based on 346 MSAs in the period between 1988 and 2019.

Therefore, we can conclude that differences in expectation formation are not sufficient to explain the observed patterns of regional differences in house price developments following a monetary policy shock, based on their respective rental yields.

Combination of Multiple Components. Finally, we control for the combined effect of income, supply elasticities, and non-rational expectations. For this, we include the logarithm of income, the deciles of the supply elasticities, and previous house price growth, all interacted with the monetary policy shock, as controls in the local projection.

As Figure 3.10 shows, even when controlling for the combined effect of income, supply elasticities, and 5-year previous house price growth, the entire effect cannot be explained. The difference between the response of house prices in the bottom 5% of the rental yield distribution and the remaining MSAs decreases from -3 percentage points to -2 percentage points.

Figure 3.10: House prices and monetary policy shock controlling for income, supply elasticity, and expectation formation



Notes: This figure shows the cumulative difference in house prices in response to a one standard deviation monetary policy shock for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs, along with the respective 95% confidence interval controlling for income, supply elasticities and previous house price growth. The regressions include time-fixed effects, and standard errors are clustered at the MSA level. These results are based on 346 MSAs in the period between 1988 and 2019.

3.4 The Discount Rate Channel

In the previous section, we showed that economic fundamentals can only to small extent explain the heterogeneous response of house prices to monetary policy. As such, given the present value equation, the remaining variation can be rationalized by the discount rate channel. In this section, we provide further evidence that monetary policy has an uneven effect on house prices through a pure valuation effect due to heterogeneous risk premia across regions.

For this purpose, it is important to recall that, as previously described, Amaral et al. (2023b) show that the lower rental yields in superstar cities can be rationalized by the lower risk. They identify two sources of housing risk that can exhibit local variation and thus influence the discount rate through the risk premium: (i) the covariance between local housing returns and consumption growth, and (ii) idiosyncratic risk.

In this section, we will focus on idiosyncratic risk. The literature provides good reasons to assume that idiosyncratic risk is priced into real estate. Real estate is a large, indivisible, and illiquid asset, typically owned by individual users who generally own only one property in a specific area, and therefore do not hold a diversified portfolio (Piazzesi and Schneider, 2016). Thus, in real estate markets, the standard assumptions of diversified portfolios do not apply, and homeowners are exposed to this idiosyncratic risk. The literature shows that at least half of the price volatility is specific to the property and therefore idiosyncratic (Piazzesi and Schneider, 2016; Giacoletti, 2021).

The task we want to undertake is to explicitly test whether house prices react heterogeneously to a monetary policy shock in MSAs with different levels of idiosyncratic risk. To emphasize once again, the idea is to directly show that a component of the pricing equation (3.2), which is not attributed to economic fundamentals but rather to the discount rate, can explain our results. In this case, the component is the idiosyncratic risk as part of the risk premium. However, before we can carry out this exercise, we need to determine the idiosyncratic risk in the MSAs. We follow the approach of Amaral et al. (2023b), which builds on the work of Giacoletti (2021).

In this approach, idiosyncratic housing risk is defined as the price change between two sales of the same property that cannot be explained by (i) the general price developments at the MSA level, and (ii) the general characteristics of the transaction and the property. The following equation describes this relationship:

$$\Delta p_{j,i,t} = \Delta v_{i,t} + BX_i + \sigma_{i,\text{idio}}\varepsilon_{j,t}, \tag{3.4}$$

where $\Delta v_{i,t}$ is the change in local MSA i house prices, BX_j is a vector of transaction and property characteristics of property j, including time and MSA fixed effects, and $\sigma_{i,\text{idio}}\varepsilon_{j,t}$ is the sales-specific shock we are interested in. Idiosyncratic risk is measured as the standard deviation of sales-specific shocks for properties within a specific MSA. This is obtained by running a regression at the property level j, including the first two terms of equation (3.4), recovering the residuals, and calculating the standard deviation of the residuals for all properties per MSA.[21]

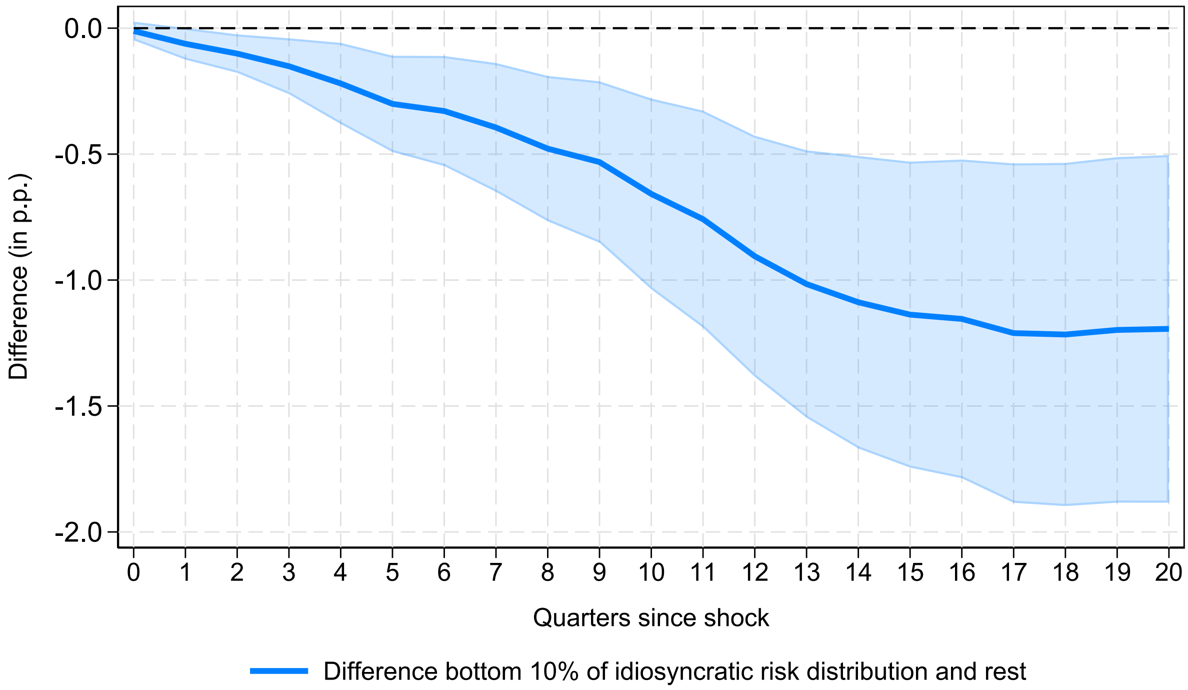
Figure 3.11 presents our results, repeating our baseline approach using the obtained idiosyncratic risk as the state-dependence measure, with clustering into two groups, namely the bottom 10% of the idiosyncratic risk distribution and the rest.[22] The figure displays only the difference, as we control for time-fixed effects. It becomes clear that the pattern from our baseline result is repeated. House prices in the bottom 10% of the idiosyncratic risk distribution decline more sharply than in the remaining MSAs. The difference is statistically significant for almost all quarters of the observation period. After 20 quarters,

[21] Detailed explanation are given by Amaral et al. (2023b).

[22] Since this is a smaller sample compared to our baseline result, we decided to use the bottom 10% of the distribution instead of the bottom 5%. The analysis is robust to using only the bottom 5% of the idiosyncratic risk distribution.

the difference between the two groups is around −1 percentage point.

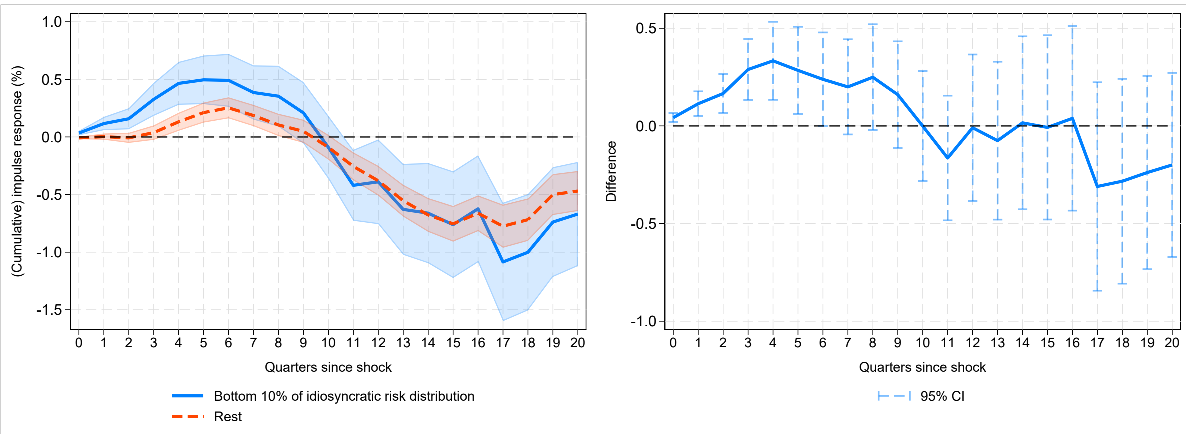
Figure 3.11: House Prices and monetary policy shock grouped by the idiosyncratic risk distribution



Notes: This figure shows the cumulative difference in house prices in response to a one standard deviation monetary policy shock for the bottom 10% of the idiosyncratic risk distribution and the remaining MSAs, along with the respective 95% confidence interval. The distribution is constructed as an average over the entire time span. The regressions include time-fixed effects, and standard errors are clustered at the MSA level. These results are based on 303 MSAs in the period between 1988 and 2019.

Moreover, the results hold even when controlling for the distribution of supply elasticity (see Figure A3.5 in the Appendix 3.B). Figure 3.12 shows that rents increase more in MSAs within the bottom 10% of the idiosyncratic risk distribution compared to other MSAs during the first six quarters following the shock. However, after six quarters, this difference is no longer significant. The response in rents in the initial period after the shock goes against our findings, as house prices in MSAs with lower idiosyncratic risk should decline less, given the rent response. As such, we provide a lower bound on the difference in the house price response to a monetary policy shock attributable to the discount rate channel.

Figure 3.12: Rents and monetary policy shock grouped by the idiosyncratic risk distribution



Notes: This figure shows the cumulative impulse response function (IRF) of median rents for 2 bedroom apartment to a one standard deviation monetary policy shock for the bottom 10% of the idiosyncratic risk distribution and the remaining MSAs, along with the respective 95% confidence interval. The distribution is constructed as an average over the entire time span. The regressions include time-fixed effects, and standard errors are clustered at the MSA level. These results are based on 303 MSAs in the period between 2001 and 2019.

Therefore, we can conclude that we provide direct evidence that the discount rate channel is the main mechanism through which the effect operates. We show that a component of the discount rate, in this case, the idiosyncratic risk as part of the risk premium, responds in alignment with the response of the rental yield groups to a monetary policy shock.

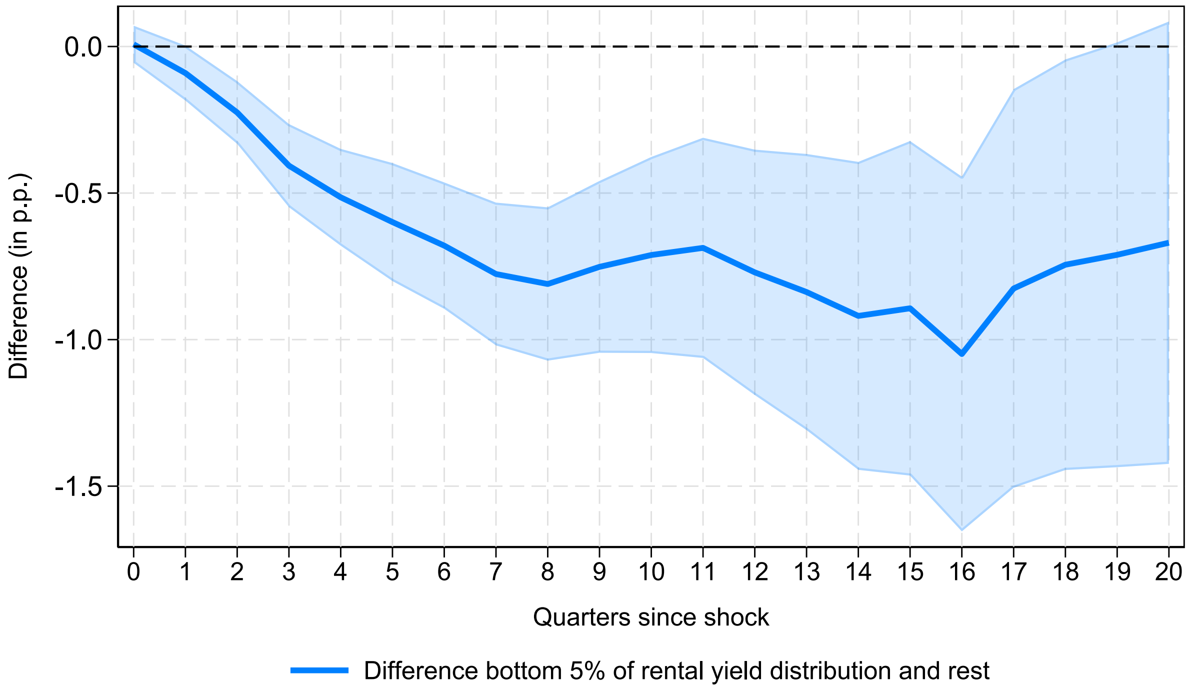
3.5 Robustness

In this section, we aim to perform additional robustness checks to eliminate aspects that might potentially confound our results. Furthermore, we want to test the sensitivity of our analyses by examining further variations of the data.

State-Time Fixed Effects. Multiple papers (Mian and Sufi, 2009; Favara and Imbs, 2015) highlight the role of different lending standards and their impact on regional house price developments. The variation in these lending standards arises at the state level. One option would be to directly control for a deregulation index created by Rice and Strahan (2010). However, this approach would ignore other shocks that equally affect geographically close MSAs. Therefore, we include state-quarter fixed effects in equation

3.1 to provide a more general control.[23] This robustness check is much more spatially granular compared to the existing literature (see Aastveit and Anundsen 2022). To emphasize this point, when we control for state-time fixed effects, it means that we are only measuring the variation between the MSAs within one state. This implies that the effects we observe are likely to be significantly smaller, as there is often clustering of MSAs with similar rental yields within states (see Figure 3.2).

Figure 3.13: IRFs of House Prices and Monetary Policy Shock Including State-Time Fixed Effects



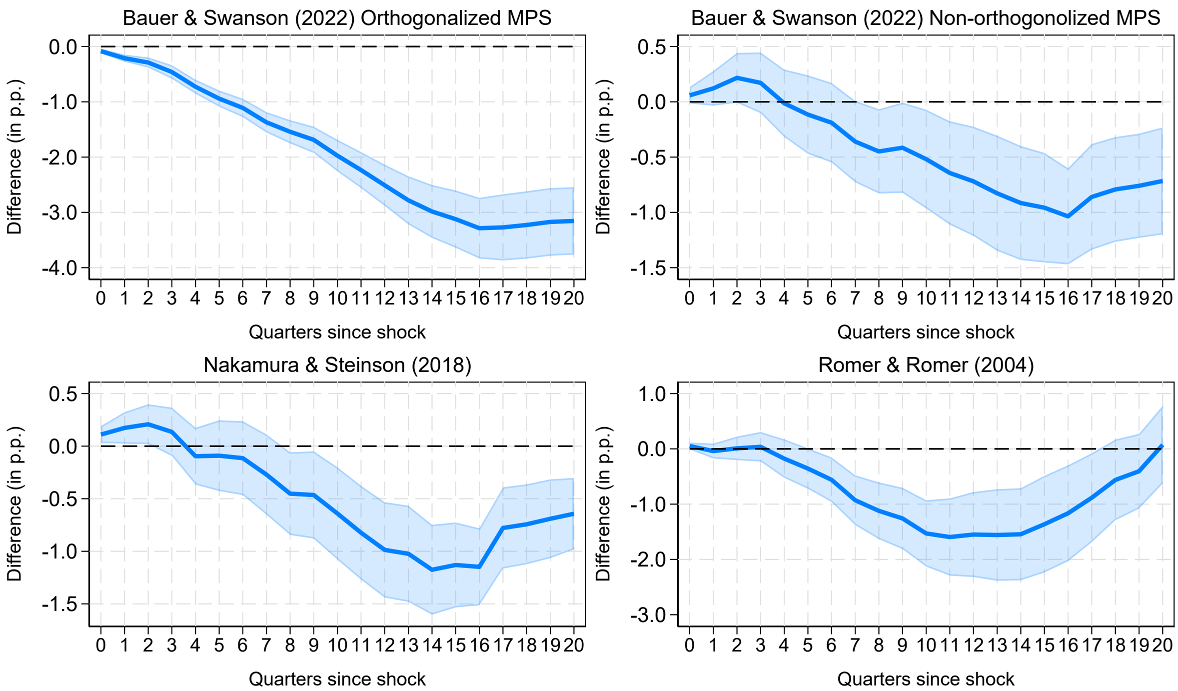
Notes: This figure shows the cumulative difference in house prices in response to a one standard deviation monetary policy shock for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs, along with the respective 95% confidence interval. The regressions include state-time-fixed effects, and standard errors are clustered at the MSA level. These results are based on 342 MSAs in the period between 1988 and 2019.

Nonetheless, even with these strict control variables, Figure 3.13 shows that our main results remain robust. House prices in the group of MSAs with low rental yields decline more than in the rest. As expected, the difference is smaller, but with the exception of the first two and last two quarters, the difference remains statistically significant at the 95% confidence level.

[23]Since some MSAs are in multiple states, we use the state where most people of the MSA live as the main state. Thus, more precisely, we use main-state-time-fixed effects.

Alternative Monetary Policy Shocks. As we already explained in Section 3.2, we follow the reasoning of Bauer and Swanson (2023), who suggest that orthogonalized monetary policy shocks are more appropriate. However, it has been documented in the literature that responses after a monetary policy shock can be sensitive to the selection of the specific shock (Ramey, 2016). Therefore, we want to test whether our results are also robust to the choice of the monetary policy shock.

Figure 3.14: House prices and different monetary policy shocks



Notes: This figure shows the cumulative difference in house prices in response to a one standard deviation of the respective monetary policy shock for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs, along with the respective 95% confidence interval for four different monetary policy shocks. The regressions are based on 346 MSAs. The regression includes time-fixed effects, and standard errors are clustered at the MSA level. The top-left panel shows our baseline result with the orthogonalized monetary policy shock. The top-right panel presents the results for the non-orthogonalized monetary policy shock by Bauer and Swanson (2023). The results in both panels are based on the period between 1988 and 2019. The bottom-left panel displays the results based on the high-frequency monetary policy shock by Nakamura and Steinsson (2018), which is based on the period between 1995 and 2019. The bottom-right panel shows the results for the monetary policy shock of Romer and Romer (2004), based on the period between 1988 and 2007.

We compare our baseline result with the non-orthogonalized monetary policy shock from Bauer and Swanson (2023), as well as another high-frequency monetary policy shock from Nakamura and Steinsson (2018), and the monetary policy shock from Romer and Romer (2004), which is based on a narrative approach.

The results are presented in Figure 3.14. All regressions include time-fixed effects. It becomes evident that, even for the other monetary policy shocks, house prices in regions with lower rental yields decline more than in the rest. However, the effects of a 1 standard deviation contractionary shock are often considerably smaller for the other monetary policy shocks.

Furthermore, it becomes apparent that the difference between the two groups at the beginning of the observation period is not statistically significant. We attribute this to the fact that, as Bauer and Swanson (2023) argue, estimates with conventional, non-orthogonalized monetary policy shocks in local projections tend to be significantly less precisely estimated. In general, the authors show that this can also be observed with other macroeconomic variables.

Overall, the results are supportive, as we can demonstrate that, despite the existing issues with conventional shocks, the clear trends of our baseline analysis remain present, and the effects in all cases are significant from the middle to the end of the observation period.

Moreover, we provide evidence that our results are not driven by the period after the Great Financial Crisis. As Figure A3.6 in the appendix shows, restricting the sample from 1988Q1 to 2006Q4 does not change the results substantially.

3.6 Conclusion

In this paper we measure the response the response of regional house prices to monetary policy shocks. Our findings indicate substantial heterogeneity in house price responses to monetary policy shocks across cities. We demonstrate that the variation in house price developments depends on differences in rental yields. Regions with lower rental yields react more strongly to monetary policy shocks than regions with higher rental yields. Our results are particularly driven by the superstar cities, which exhibit the lowest rental yields. We introduce a new mechanism that operates through a discount rate channel and functions via heterogeneous risk premia across space. Additionally, we can rule out that the observed effects are driven by economic fundamentals or the previously known supply elasticity channel (Aastveit and Anundsen, 2022).

The findings clearly indicate that monetary policy transmits heterogeneously to different regions via the housing market. We emphasize that the uneven transmission of monetary policy operates through a pure, mechanical valuation effect. Since housing valuation matters as it collateralizes debt, this varying development can subsequently trigger second-round effects. In addition, cities with low rental yields tend to be the most expensive cities. This means that a contractionary monetary policy has the potential to widen the gap in housing values, benefiting the richer regions. This finding highlights another channel through which monetary policy can have distributional effects.

Appendix 3.A Data

In this section of the appendix we illustrate in more detail parts of the data.

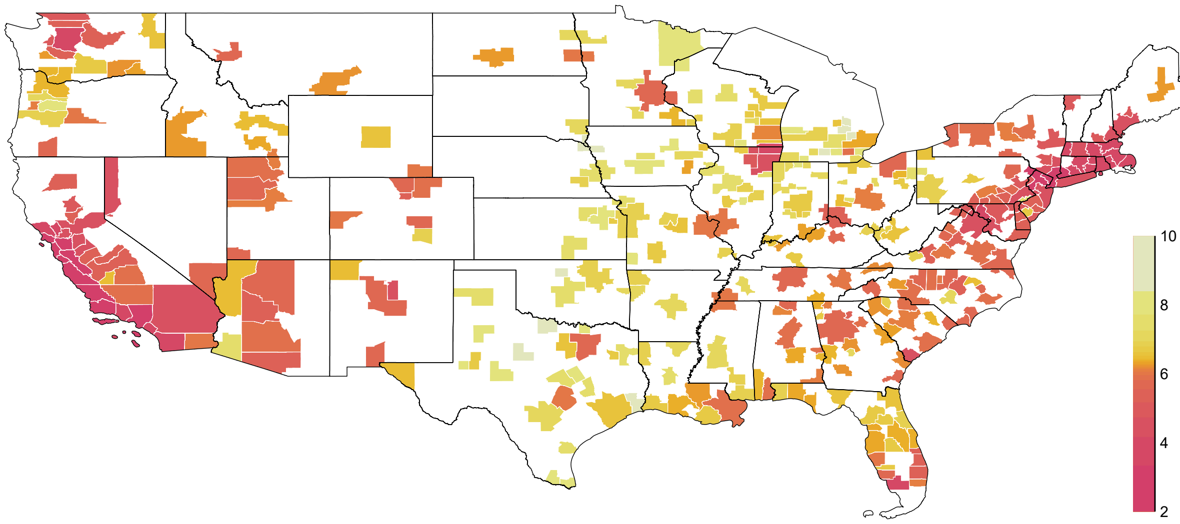
Table A3.1 gives an overview of the treated MSAs regarding the rental yield and the idiosyncratic risk distribution. It is apparent that there is a group of MSAs that belong to the treated units in both distributions, but there is also a substantial part that is only treated in one of the exercises.

Table A3.1: Treated MSAs

	Bottom 5% rental yield 1990	Bottom 5% idiosyncratic risk	Top 5% population density 1987
Anaheim-Santa Ana-Irvine, CA (MSAD)	1	1	1
Oxnard-Thousand Oaks-Ventura, CA	1	1	0
San Luis Obispo-Paso Robles, CA	1	1	0
San Diego-Chula Vista-Carlsbad, CA	1	1	0
Cambridge-Newton-Framingham, MA (MSAD)	1	0	1
Los Angeles-Long Beach-Glendale, CA (MSAD)	1	0	1
Oakland-Berkeley-Livermore, CA (MSAD)	1	0	1
Bridgeport-Stamford-Norwalk, CT	1	0	1
New York-Jersey City-White Plains, NY-NJ (MSAD)	1	0	1
San Francisco-San Mateo-Redwood City, CA (MSAD)	1	0	1
Santa Maria-Santa Barbara, CA	1	0	0
Newark, NJ-PA (MSAD)	1	0	0
Napa, CA	1	0	0
Salinas, CA	1	0	0
San Jose-Sunnyvale-Santa Clara, CA	1	0	0
Santa Rosa-Petaluma, CA	1	0	0
San Rafael, CA (MSAD)	1	0	0
Santa Cruz-Watsonville, CA	1	0	0
Frederick-Gaithersburg-Rockville, MD (MSAD)	0	1	0
Spokane-Spokane Valley, WA	0	1	0
Seattle-Bellevue-Kent, WA (MSAD)	0	1	0
Corvallis, OR	0	1	0
Portland-Vancouver-Hillsboro, OR-WA	0	1	0
Bremerton-Silverdale-Port Orchard, WA	0	1	0
Colorado Springs, CO	0	1	0
Olympia-Lacey-Tumwater, WA	0	1	0
Montgomery County-Bucks County-Chester County, PA (MSAD)	0	1	0
Raleigh-Cary, NC	0	1	0
Lancaster, PA	0	1	0
Tacoma-Lakewood, WA (MSAD)	0	1	0
Detroit-Dearborn-Livonia, MI (MSAD)	0	0	1
Nassau County-Suffolk County, NY (MSAD)	0	0	1
Philadelphia, PA (MSAD)	0	0	1
New Brunswick-Lakewood, NJ (MSAD)	0	0	1
Fort Lauderdale-Pompano Beach-Sunrise, FL (MSAD)	0	0	1
Milwaukee-Waukesha, WI	0	0	1
Trenton-Princeton, NJ	0	0	1
Cleveland-Elyria, OH	0	0	1
New Haven-Milford, CT	0	0	1
Chicago-Naperville-Evanston, IL (MSAD)	0	0	1
Boston, MA (MSAD)	0	0	1

Figure A3.1 illustrates the spatial heterogeneity of rental yields in 1990. The rental yields are in particular low in the MSAs in California and the Northeast with higher rental yields in MSAs located in central states of the US and/or further inland from the coast.

Figure A3.1: Regional variation in rental yields of MSAs in 1990



Notes: The figure shows the spatial variation in rental yields of 346 MSAs in 1990. The geographic boundaries of the MSAs, MDs, and states are based on U.S. Census Bureau TIGER/Line Shapefiles 2021. Rental yields are constructed based on Census data 1990 on county level according to Gyourko et al. (2013). Data provided by Gyourko et al. (2013). County level aggregated to MSA level based on 2020 Census Delineation file.

Appendix 3.B Additional Results

In this section of the appendix, we provide additional information that support the mechanism and the empirical findings in this paper.

Theoretical Foundation. We want to provide additional intuition for the ideas of Amaral et al. (2023b).

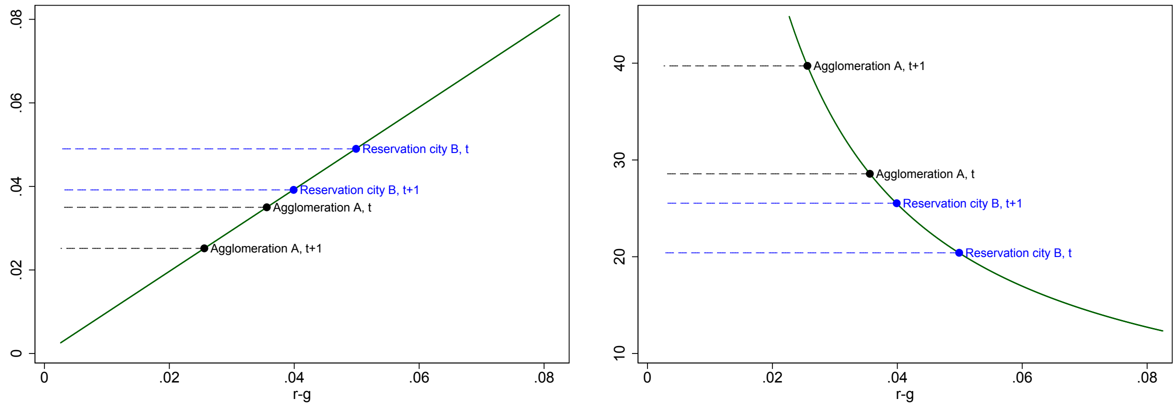
The rental yields from equation (3.2) are obtained in the following way. They make further assumptions to simplify the equation: (i) expected rents at time t in MSA i are growing at a constant rate g_t^i, and (ii) $r_t^i > g_t^i$ to ensure house prices are finite. Using these assumptions, equation 3.2 can be simplified to express the rental yield:

$$\text{Rental yield}_t^i = \frac{\text{Rent}_t^i}{P_t^i} = \frac{r_t^i - g^i}{1 + g^i}. \tag{A3.1}$$

The graphical intuition for the mechanism is depicted in Figure A3.2. It becomes evident that the rent-price ratios change in a linear fashion, while the price-rent ratios, as the inverse function, change non-linearly. Thus, a uniform fall in r results in a larger increase of the price-rent ratio in agglomeration A vis-à-vis reservation city B due to the initially higher price-rent ratios. Ultimately, the price dispersion between agglomeration A and reservation city B increases.

Figure A3.2: Effect of a Uniform Fall in Discount Rate

(a) Rent-Price Ratios **(b)** Price-Rent Ratios



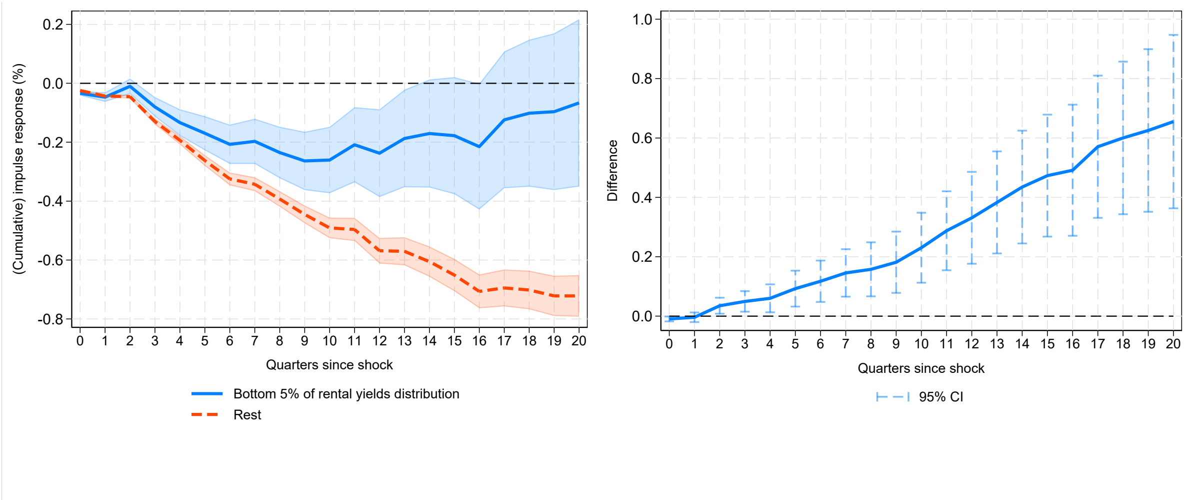
Notes: This figure taken from Amaral et al. (2023a) plots in panel (a) rent-price ratio as in their model as a function of $r - g$. They assume that $g = 0.0175$. Panel (b) shows the corresponding price-rent ratio. For both ratios the response to a uniform change in the discount rate for agglomeration A and reservation city B are displayed.

On top of the we want to provide a straightforward numerical example of the mechanism via the present value equation. In our example, we look at a low rental yield city, e.g. New York City, in contrast to a high rental yield city, e.g. Enid, Oklahoma. Consider the following values: $rent^{\text{New York City}} = 8,400\$$, $rent^{\text{Enid}} = 4,200\$$, risk-free rate$_t = 0.02$, risk-premium$_t^{\text{New York City}} = 0.02$, and risk-premium$_t^{\text{Enid}} = 0.08$. Hence, the price in New York City is equal to $P_t^{\text{New York City}} = 210,000\$$, and in Enid, it equals $P_t^{\text{Enid}} = 42,000\$$. Now assume that a monetary policy shock increases the risk-free rate by 100 basis points: risk-free rate$_t = 0.03$. As a result, the price in New York City drops to $P_t^{\text{New York City}} = 168,000\$$, while the price in Enid only drops to $P_t^{\text{Enid}} \approx 38,182\$$. The relative price change in New York City is: $\Delta^{\text{New York City}} = -20\%$, as opposed to a much smaller price change in Enid: $\Delta^{\text{Enid}} \approx -9\%$.

Economic Fundamentals. In the main text, we have only presented the development of income as the sole macroeconomic variable. However, it is possible that other macroeconomic variables could evolve in a way that might explain why house prices decline more sharply in regions with lower rental yields compared to other regions.

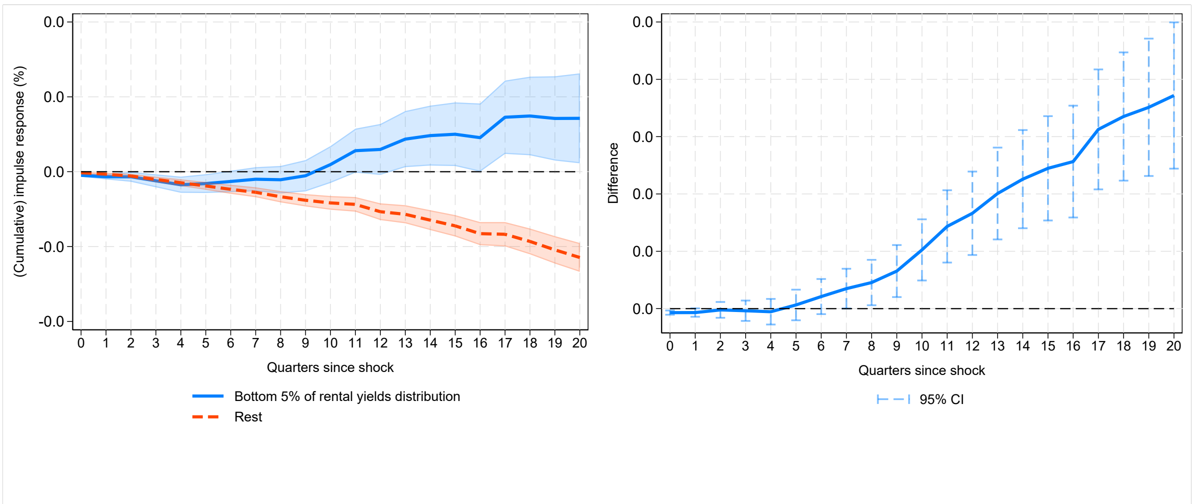
Therefore, we will focus on two additional macroeconomic variables that could potentially support this hypothesis: (i) employment, and (ii) population.

Figure A3.3: Employment and monetary policy shock



Notes: This figure shows the cumulative impulse response function (IRF) of employment to a one standard deviation monetary policy shock. The left panel compares the IRFs for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs. The solid blue line shows the IRF of income in the bottom 5% of the rental yield distribution with the respective 95% confidence interval. The dashed red line shows the IRF for MSAs in the remaining 95% of the rental yield distribution with the respective 95% confidence interval. The right panel shows the difference between the two IRFs as well as the 95% confidence interval of the difference. Standard errors are clustered at the MSA level. These results are based on 346 MSAs in the period between 1988 and 2019.

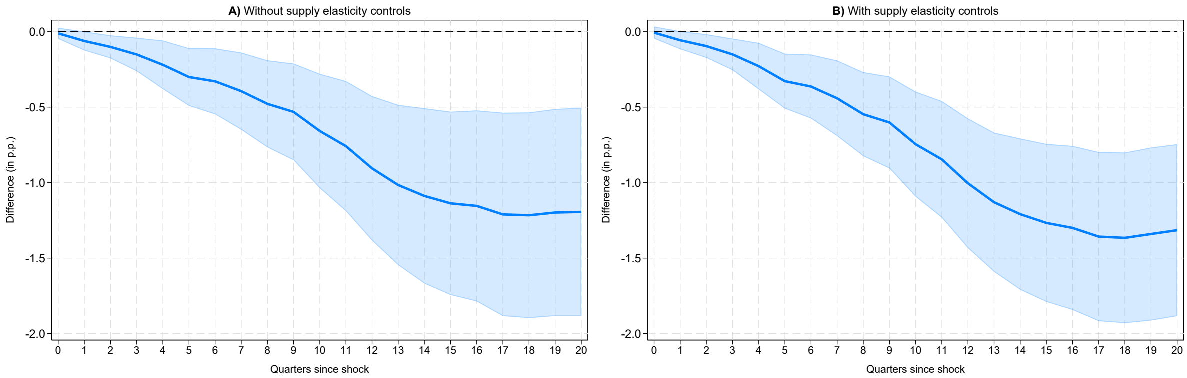
Figure A3.4: Population and monetary policy shock



Notes: This figure shows the cumulative impulse response function (IRF) of population to a one standard deviation monetary policy shock. The left panel compares the IRFs for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs. The solid blue line shows the IRF of income in the bottom 5% of the rental yield distribution with the respective 95% confidence interval. The dashed red line shows the IRF for MSAs in the remaining 95% of the rental yield distribution with the respective 95% confidence interval. The right panel shows the difference between the two IRFs as well as the 95% confidence interval of the difference. Standard errors are clustered at the MSA level. These results are based on 346 MSAs in the period between 1988 and 2019.

Discount rate channel. We provide further evidence for the discount rate channel by dividing the MSAs according to the idiosyncratic risk distribution. Figure A3.5 shows that the results are unaffected by controlling for the deciles of the supply elasticity distribution interacted with the monetary policy shock.

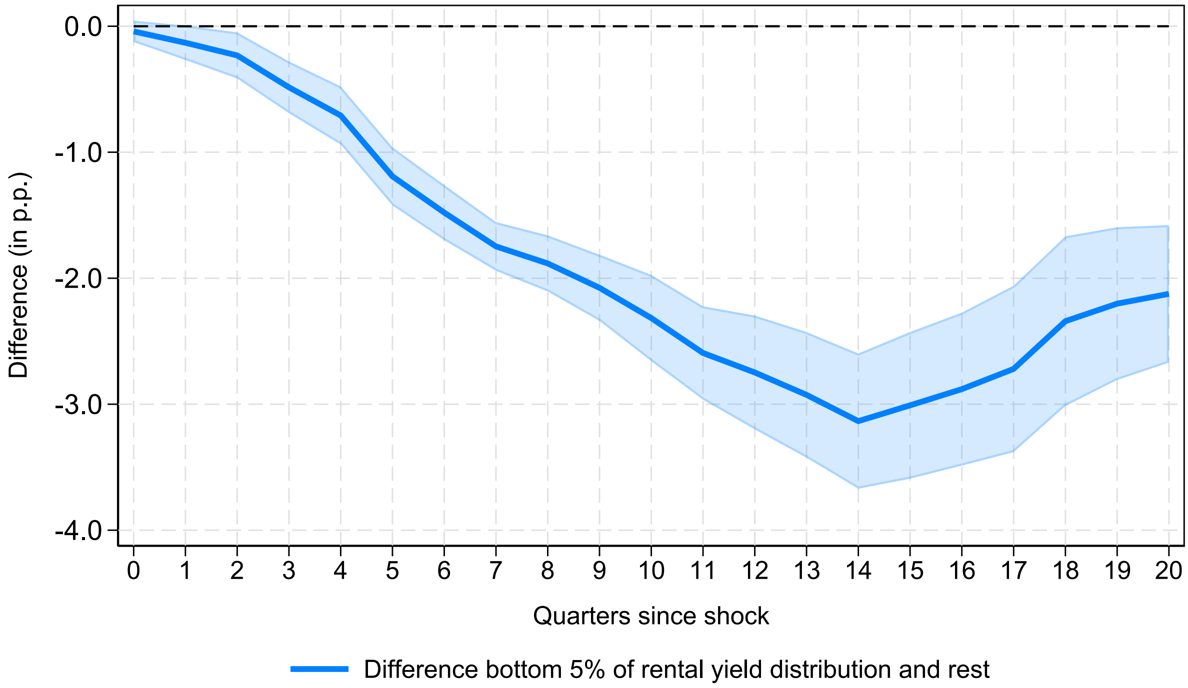
Figure A3.5: House prices after monetary policy shock by the idiosyncratic risk distribution controlling for supply elasticity



Notes: This figure shows the cumulative difference in house prices in response to a one standard deviation monetary policy shock for the bottom 10% of the idiosyncratic risk distribution and the remaining MSAs, along with the respective 95% confidence interval. The distribution is constructed as an average over the entire time span. The regressions include the decile of the supply elasticity distribution interacted with the monetary poliy shock as controls, time-fixed effects, and standard errors are clustered at the MSA level. These results are based on 303 MSAs in the period between 1988 and 2019.

Pre-Finacial Crisis. We show that our results remain robust when excluding the period after the Great Financial Crisis by limiting the time span to 1988Q1 to 2006Q4. Figure A3.6 presents the results for this exercise.

Figure A3.6: House prices and monetary policy shock (1988-2006)



Notes: This figure shows the cumulative difference of house prices to a one standard deviation monetary policy shock for the bottom 5% of the rental yield distribution in 1990 and the remaining MSAs with the respective 95% confidence interval. The regression includes time-fixed effects and standard errors are clustered at the MSA level. These results are based on 346 MSAs in the period between 1988 and 2006.

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List of Applied Software

Matlab R2022A was applied for the numerical application of the model in chapter 1.

Python was used for geolocating the apartments for the empirical analysis in chapter 2.

Openroute was used to obtain the car travel time to the city center for the empirical analysis in chapter 2.

Excel 2016 was applied for data organization and cleaning in chapter 2 and 3.

StataMP 18 was applied for the empirical analysis in chapter 2 and 3.

Texmaker 5.1.4 was applied for the compilation of this dissertation.

Jonas Zdrzalek

Curriculum Vitae

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Current Positions

Kiel Institute for the World Economy
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PhD Candidate
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Education

University of Cologne, PhD Studies in Economics, since Oct 2019
Supervisors: Andreas Schabert & Tom Zimmermann

University of Oslo, Norway, Master of Philosophy in Economics, Oct 2017 – Dez 2019
Grade: A

Humboldt-University of Berlin, Erasmus+, Apr 2019 – Aug 2019

University of Pennsylvania, USA, Exchange Semester, Jan 2019 – Apr 2019

University of Cologne, Bachelor of Science in Economics, Oct 2013 – March 2017
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University of Otago, New Zealand, Exchange Semester, Jul 2015 – Nov 2015

Franz-Meyers-Gymnasium, Abitur (High School Certificate), Jun 2013

Experience

Kiel Institute for the World Economy, Doctoral Researcher,
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Center for Macroeconomic Research, University of Cologne, Research and Teaching Assistant,
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Institute for Macroeconomics and Econometrics, University of Bonn, Research and Teaching Assistant,
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ECONtribute: Markets and Public Policy – Cluster of Excellence, Young ECONtribute Program (YEP)
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German Economic Institute, Economist for Monetary Policy, Financial and Real Estate Markets,
Mar 2021 - Nov 2022

University of Oslo, Teaching Assistant "Welfare and Trade" (Econ 2610),
Aug 2018 - Dec 2018

HSBC Trinkaus and Burkhardt, Internship in Private Banking,
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HSBC Trinkaus and Burkhardt, Student Employee in Credit Services,
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EGC Eurogroup Consulting, Internship in a Consultancy Agency,
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Awards and Scholarships

Erasmus+ Grant, University of Oslo, 2019

Scholarship in Advanced Methods and Research, University of Oslo, 2018

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Affidavit

according to Article 9 (5) of the Doctoral Regulation of 1 August 2022

I hereby affirm that I have written this dissertation independently and without the use of other aids and literature than those indicated. No other person, apart from any co-authors listed in the thesis, were involved in the substantive preparation of this thesis. All passages that have been taken verbatim or in spirit from published and unpublished works of others are marked as such. I affirm that this dissertation has not yet been submitted to any other faculty or university for examination; that it has not yet been published — apart from the partial publications and incorporated articles and manuscripts indicated — and that I will not publish the dissertation before completion of the doctorate without the approval of the doctoral examination board. I am aware of the provisions of these regulations. Furthermore, I hereby declare that I have read the guidelines of the University of Cologne for ensuring good scientific practice and that I have observed them in carrying out the work underlying the dissertation and in writing the dissertation, and I hereby undertake to observe and implement the guidelines stated therein in all scientific activities. I affirm that the submitted electronic version corresponds completely to the submitted printed version. I affirm that to the best of my knowledge I have been truthful and have not concealed anything.

Cologne, October 2024