ESSAYS ON THE INTEGRATION OF RENEWABLES IN ELECTRICITY MARKETS

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1 Introduction

Energy consumption is an essential part of every day life and a foundation for the functioning of economies worldwide. The conversion of energy makes it possible to satisfy the demand for heating, cooling, transportation, lighting and information, which puts energy consumption at the center of societies in the 21st century. In the past, this demand has primarily been satisfied by the combustion of fossil fuels, such as oil, gas or coal. This has lead to large carbon dioxide emissions, which are regarded as one of the main drivers that are causing global climate change (IPCC, 2013). In order to circumvent these negative externalities of fossil fuel combustion new ways of energy provision are being developed and deployed. In the Paris agreement countries agreed to limit the worldwide temperature increase to well below 2°C (UNFCC, 2015). This essentially means that the energy supply needs to be restructured in order to limit the implications of global climate change for future generations.

One possibility to reduce carbon dioxide emissions is to improve energy efficiency in the conversion process. For fossil fuel technologies, however, reduction potentials are limited and new ways need to be found to satisfy the demand for energy (IEA, 2016a). A possible pathway, that is currently seen as very promising, is the substitution of fossil fuel based technologies with technologies that are able to satisfy the demand based on electricity generated from renewable energy technologies, such as wind or solar power. This means for example that transportation in the future will be provided by electric vehicles which are charged with electricity generated from renewable energies. The heat demand furthermore may be satisfied by heat pumps or electric stoves which are also powered by electricity. This would mean that a large part of the energy demand in the future will be provided based on electricity. The efficient organization of electricity markets is therefore of high importance for future societies.

The electricity sector, as such, has already experienced tremendous changes in the last three decades. The liberalization has lead to a restructuring of the whole sector. Formerly vertically-integrated monopolies were transformed into unbundled firms focusing each on generation, transmission, distribution and retailing. This restructuring was undertaken with the aim to increase competition and emphasized the
importance of markets for the efficient allocation of resources to meet demand in electricity systems. Different markets, that will be described in more detail within this thesis, have historically evolved in Europe and contribute to the operation of the whole electricity system. Wholesale markets enable the efficient allocation of resources for electricity generation in order to satisfy demand just before real-time. Retail markets enable consumers to choose among different suppliers to take charge of their electricity supply. Balancing power markets enable transmission system operators to procure balancing power for the secure operation of the electricity grid in real-time.

After the liberalization, the electricity sector has seen a second transformation which is aiming at the decarbonization of electricity systems. Besides the introduction of new additional markets such as the market for CO$_2$ emission allowances, governments are setting ambitious targets to increase the share of renewable generation in the electricity system. Because of these efforts and due to technological developments, renewable capacities surpassed the cumulative installed capacities of coal in 2015. This was mainly driven by newly build onshore wind (63 GW) and solar photovoltaic (49 GW) capacities being deployed worldwide in 2015 (IEA, 2016b). Obama (2017) expects that this trend is going to continue because renewable capacities are already cost-competitive compared to fossil fuel based power generation in many parts of the world. Furthermore, Obama expects that renewable energies and efficiency technologies will make it possible to decouple emissions from economic growth.

The increasing share of renewables in electricity systems is transforming electricity markets worldwide. Wind and solar generation technologies differ in essential economic terms from conventional power generation technologies. Besides high initial fixed costs, the electricity is generated at short-term marginal costs of almost zero and generation highly depends on weather conditions at the location (wind speed and solar radiation). Especially, the dependence on weather conditions introduces two important characteristics into electricity markets. First, the generation of renewables is highly fluctuating in time. In order to reliably balance supply and demand in electricity markets with renewables, demand needs to be able to quickly respond to these changes or conventional capacities need to be able to adjust their production in shorter time intervals. This increases the need for markets with higher product granularity in terms of time resolution, where fluctuating supply and demand can be matched efficiently. Second, the generation of renewables can only be predicted to a certain degree ahead of time. The uncertainty about the final production level
resolves over time until electricity is finally generated. This creates a need for markets in which the arrival of new information can be traded in order to allow for the efficient allocation of renewable generation and conventional generation to meet demand.

The Winter Package (COM(2016) 860) by the European Commission acknowledges these profound impacts of renewable generation on electricity markets and calls for an improved design of wholesale and retail electricity markets (EU Commission, 2016). For wholesale markets, the Commission suggests that short-term markets should be made overall more flexible and responsive for being able to integrate the increasing share of renewable generation. Furthermore, consumers should be given the possibility to actively participate in electricity markets by equipping them with smart-meters and offering dynamic retail tariffs that reflect the changing wholesale prices.

The dissertation at hand sheds light onto some of these important aspects for the integration of renewable generation in electricity markets. Chapters 2 and 3 investigate the impact of renewables on short-term wholesale markets, whereas Chapters 4 and 5 focus on the role of the demand side in wholesale and retail markets. Chapter 6 analyses the role of balancing power markets in electricity systems, which may become more important with an increasing share of renewables. Each chapter is based on a single article to which the authors contributed equally:

- Chapter 2: How to Sell Renewable Electricity - Strategic Interaction in Sequential Markets (based on Knaut and Obermüller (2016))
- Chapter 3: Price Volatility in Commodity Markets with Restricted Participation (based on Knaut and Paschmann (2017))
- Chapter 4: When Are Consumers Responding to Electricity Prices? An Hourly Pattern of Demand Elasticity (based on Knaut and Paulus (2016))
- Chapter 5: Retail Tariff Design in Electricity Markets with Variable Renewable Production (based on Knaut (2017))
- Chapter 6: Tender Frequency and Market Concentration in Balancing Power Markets (based on Knaut et al. (2017)).

The content of each chapter will be outlined in the following before the methodological approaches are discussed.
1 Introduction

1.1 Outline

The uncertainty of renewable production and the optimal trading strategies of renewable generators in sequential markets are investigated in Chapter 2. We formulate a model in which renewable generators trade their production in two sequential markets, which can be regarded as the day-ahead and intraday markets. Trading in the first market takes place under uncertainty about the final production level of renewable generation. Renewable producers choose quantities to sell into the day-ahead market under uncertainty and can adjust their position after learning about the final production level in the intraday market. Based on the model, we find that it is optimal for renewable producers to sell less than the expected quantity in the day-ahead market. A renewable monopolist, for example, would maximize her profit if she sells half of the expected quantity in the day-ahead market. However, if additional renewable producers are competing in the market, the optimal quantity tends towards the overall expected quantity. In addition, we investigate the impact of short-term flexibility that can be provided by conventional power plants on the market outcome. If the conventional power plant fleet is less flexible, which means costs for short-term adjustments increase, renewable producers will have an incentive to increase the quantity traded in the first stage. Regarding the uncertainty of renewable production, we show that improved forecast quality of renewable production increases social welfare.

Chapter 3 focuses on the high variability in production from renewable electricity and its effect on prices. It is motivated by the highly fluctuating prices in the German market for quarter-hourly products. We first develop a model for the allocation of hourly and quarter-hourly electricity generation, assuming that the participation in the market for quarter-hourly products is restricted. The assumption of restricted participation is primarily motivated by the missing possibilities for cross-border trade in the market for quarter-hourly products in Germany. Based on the model we can explain the highly volatile quarter-hourly prices, which are caused by restricted participation in combination with sub-hourly variations in demand and renewable supply. The model is verified based on empirical observations for the German day-ahead and intraday auction in 2015. By estimating the supply curves for the hourly and quarter-hourly market, we are able to quantify the efficiency loss that is caused by restricted participation. We find that restricted participation in the market for quarter-hourly products caused welfare losses of EUR 96 million in 2015.

While the demand side in electricity markets is commonly assumed as being com-
pletely price inelastic, this assumption is questioned in **Chapter 4**. We empirically estimate the hourly price elasticity of demand for electricity in the German day-ahead market. In a first step, we focus on the main drivers of both the demand and supply sides in order to obtain a general understanding of the causal relationships that alter demand or supply in the day-ahead market. In a second step, we construct an econometric model that accounts for endogeneity in the form of simultaneity of price and quantity. Here we make use of the stochastic character of renewable generation that primarily affects the supply side but not the demand side. Based on a two-stage least squares regression with hourly data on the feed-in from renewable energies as the instrumental variable, we are able to estimate the level of demand elasticity as well as the hourly variation. The empirical results indicate a high level of variation of price elasticity of demand throughout the day ranging from -0.02 to -0.13 depending on the time of the day in the German day-ahead market in 2015.

Whereas the previous chapters focus on price formation in wholesale markets, **Chapter 5** investigates the effects of different tariff designs in retail markets. The predominant tariff scheme that is currently offered to end consumers in Germany is time-invariant pricing. Within this tariff scheme consumers pay the same price for electricity regardless of when it is consumed. This leads to inefficiencies compared to the efficient case of real-time pricing. In order to analyze the inefficiency, we quantify the resulting deadweight losses in a theoretical model. The model accounts for the variability in generation from renewables and is thus able to draw a connection between the deadweight losses from time-invariant pricing and characteristics of renewable generation. We find that deadweight losses from time-invariant pricing increase with an increase of the variability in demand and renewable generation. A positive correlation between demand and renewable generation leads to a reduction of deadweight losses. Motivated by the recent announcement of the German government to increase the demand responsiveness of the demand side, we conduct an illustrative case study on the implications of real-time, time-of-use and time-invariant pricing at the example of Germany. We find that the deadweight loss amounts to about EUR 97.1 million, depending on the price responsiveness of consumers. Furthermore, we find that time-of-use pricing can just achieve a fraction of the efficiency gains that could be achieved under real-time pricing.

**Chapter 6** takes a look at an additional market that is crucial for the efficient functioning of the electricity system, namely the balancing power market. Balancing power is procured by Transmission System Operators (TSOs) in order for being able to balance short-term deviations of demand and supply that occur after the
gate-closure of the wholesale market. Due to unbundling restrictions TSOs are not allowed to own generation assets and need to procure balancing power from generators that are also active in the wholesale market. Because balancing power currently can only be provided by few large operators, an efficient market design that limits the possibility of market power abuse is essential. We develop a numerical model of the German wholesale and balancing power market which is able to represent the operator structure of power plants as well as different tender frequencies for balancing power. With the model, we are able to analyze the implications of different market designs for balancing power markets on efficiency and market concentration. Based on the model results, we find that shorter tender frequencies could lower the costs of balancing power procurement by up to 15%. While market concentration decreases in many markets with shorter provision duration, we – surprisingly – identify cases in our model where shorter time spans lead to higher concentration.

1.2 Discussion of Methodological Approaches

The chapters within this thesis address different aspects of electricity markets. Depending on the research question that is posed different methods and assumptions are applied. The choice of each methodology and set of assumptions has been made in order to keep the analysis tractable without losing the essential aspects that are relevant for answering the research question at hand. Nevertheless each choice of methodology or assumption implicates a loss in generality. In the following, the assumptions that were chosen as well as possible implications are being discussed.

Assumptions about the model setup and the level of competition are an essential part of theoretical analyses in economics. A common assumption which reduces the effort of the analysis, is the assumption of perfectly competitive markets. We rely on this assumption especially in Chapters 3 and 6. In both chapters, we analyze the German electricity market for which the German regulator states that there is currently no issue for potential market power abuse. In Chapter 6 we, however, find based on concentration measures that there could be periods with high market concentration in the balancing power market, which may facilitate the possibility of market power abuse. Within this work we cannot draw a final conclusion on the severity of market power abuse and leave it open for further research. In Chapter 2 we deviate from the assumption of perfect competition and investigate the possibility of strategic behavior in a Cournot game, where we observe a renewable monopolists to withhold quantities in order to increase profits.
While competitive markets in absence of market failures will lead in many cases to the welfare maximizing outcome, this is not clear in every industry setting. Competitive markets may not always lead to the same outcome that would be chosen by a social planner. In Chapter 5, we analyze the optimal tariff design for electricity consumers from a welfare perspective. Nevertheless, we can only conjecture that this welfare optimal tariff would be offered in a competitive market depending on the vertical industry structure. We therefore discuss the implications of different vertical structures ranging from regulated retailers to integrated firms.

Within Chapter 2, 3 and 5, the research question is primarily addressed within a theoretical model framework. In order to keep the model tractable, linear supply functions are assumed in all three chapters, representing the conventional electricity supply curve. In reality the supply curve in electricity markets may be much more diverse depending on the marginal costs of technologies that are being used for electricity generation (e.g. nuclear, coal, gas, oil, pumped storage etc.). This may lead to a shape that differs from the linear assumption and may alter the results in some aspects. Within the empirical parts of Chapter 3 and 5, however, the linear assumption seems to be valid for the supply curve in Germany for the year 2015. In future scenarios or in analyses conducted for different countries the result may be different. The linear setup nevertheless helps in gaining valuable insights in all three chapters.

Besides the supply side, also assumptions about the demand side are a crucial part in all models. A common assumption in electricity markets is that the demand side can be regarded as perfectly price inelastic. This assumption is also part of the analysis in Chapters 2, 3 and 5. We also question the applicability of this assumption by empirically estimating the level of price elasticity in Chapter 4 and find that it depends on the respective time of the day. In general, however, we observe that the price elasticity is low with values ranging from -0.02 to -0.13. Chapter 5 additionally investigates the implications of different tariff designs and demand elasticities on price formation and the resulting welfare implications.

All these assumptions are an important part of the analyses and need to be kept in mind for the proper interpretation of the results.
2 How to Sell Renewable Electricity - Strategic Interaction in Sequential Markets

Uncertainty about renewable production increases the importance of sequential short-term trading in electricity markets. We consider a two-stage market where conventional and renewable producers compete in order to satisfy the demand of consumers. The trading in the first stage takes place under uncertainty about production levels of renewable producers, which can be associated with trading in the day-ahead market. In the second stage, which we consider as the intraday market, uncertainty about the production levels is resolved. Our model is able to capture different levels of flexibility for conventional producers as well as different levels of competition for renewable producers. We find that it is optimal for renewable producers to sell less than the expected production in the day-ahead market. In situations with high renewable production it is even profitable for renewable producers to withhold quantities in the intraday market. However, for an increasing number of renewable producers, the optimal quantity tends towards the expected production level. More competition as well as a more flexible power plant fleet lead to an increase in overall welfare, which can even be further increased by delaying the gate-closure of the day-ahead market or by improving the quality of renewable production forecasts.

2.1 Introduction

The electricity sector is currently experiencing rapid changes, especially due to the deployment of large capacities for electricity generation from renewables with the aim of decarbonizing economies. This leads to a transformation of the producer side, away from conventional generation technologies (such as coal, gas, and nuclear) towards an increasing share of variable renewable electricity generation (especially wind and solar). Whereas these technologies were highly subsidized in the past and therefore not well integrated into the market, it is now high on the European Union’s policy agenda to integrate renewable generation into the market (EU Commission (2009), EU Commission (2013)). This means in the future, renewable producers are expected to sell their entire production at the existing sequential wholesale elec-
In electricity markets, demand and supply need to be balanced at all times. Therefore it is essential for all market participants to announce their foreseeable production and consumption in advance. The largest share of electricity is currently traded in the day-ahead market, which can be considered as a kind of forward market. Trading commonly takes place at noon one day before physical delivery. This is necessary to signal the regional supply and demand situations to the transmission system operators in advance, such that they can guarantee grid stability. In contrast, the intraday market provides the opportunity to trade electricity down to 30 minutes before physical delivery. Hence, adjustments to the day-ahead market clearing result can be traded which may occur due to (uncertain) short term deviations in electricity systems (e.g. demand forecast errors, renewable forecast errors, and unforeseen power plant shortages).

The characteristics of renewable electricity generation have increased the importance of sequential short-term trading and are affecting the competition in electricity markets. Renewable energy technologies differ in two important aspects from classic conventional technologies. First, renewables produce electricity at short run marginal costs of zero whereas conventional technologies have short run marginal costs greater than zero. Second, renewable electricity production depends on weather conditions that can only be predicted to a certain level. The uncertainty diminishes with a shorter time duration to the physical delivery. Thus, volatile renewable producers have a higher uncertainty if they trade in the day-ahead market. Therefore, the optimal bidding strategy for renewable energy producers in the intraday and day-ahead market under uncertainty is not clear and focus of the following investigations.

Electricity markets are known to be especially vulnerable to the potential abuse of market power (Borenstein et al., 2002, Green and Newbery, 1992). The demand can be regarded as very price inelastic in the short-run and therefore participants may be able to increase prices above the competitive level. While this has been an issue of large conventional generators in the past, we also can expect large renewable producers as being able to act strategically in sequential electricity markets. The size of renewable aggregators who aggregate renewable generation plants and sell the production in the market is steadily increasing especially because they are able to lift significant scale effects by increasing their renewable portfolio (e.g. reduction in the long run).
2.1 Introduction

In this paper, we analyze the competition between conventional and renewable producers that interact in two sequential stages by using an analytic model. The first stage is considered as the day-ahead and the second as the intraday market. The electricity production of the renewable producer is uncertain in the first stage and is realized in the second stage. In particular, this affects renewable producers in choosing the optimal quantity to trade in both stages. Furthermore, we account for flexibility constraints of conventional power producing technologies, because not all conventional technologies are flexible enough to change their production schedules in short time intervals (e.g. 30 minutes before physical delivery). These flexibility constraints are included in our model to measure effects on profit maximizing quantities and prices. We analyze the results based on different levels of competition for the renewable producers, ranging from a monopoly to oligopolies under a flexible and less flexible power plant fleet.

Our investigation is strongly related to the branch of two-stage Cournot games as well as the literature of optimal bidding strategies for renewable producers. Concerning two-stage Cournot games, a fundamental work is given by Allaz (1992) and Allaz and Vila (1993) who investigate Cournot competition of a duopoly in sequential markets. Their subject of investigation is the forward market which, however, can be transferred to our idea of a day-ahead auction before the market is finally cleared in an intraday auction. The setting differs to our model with respect to the type of players. In Allaz and Vila (1993) both players have increasing marginal costs of production and no uncertainty associated with their level of production. In Allaz (1992), uncertainty is incorporated in the two-stage model such that risk hedging influences the optimal production. However, Allaz (1992) and Allaz and Vila (1993) assume implicitly infinite production capacity, which is not true for our renewable producer. Similar to Allaz and Vila (1993), Saloner (1987) developed an extension of the classical Cournot one-shot duopoly to a model with two production stages in which the market clears only once after the second stage. In this framework Saloner showed the existence of a unique Nash-Cournot equilibrium under the possibility of a second stage response action. Nevertheless, the model does not account for different player types or uncertainty of production. Twomey and Neuhoff (2010) consider the case of renewable and conventional producers that are competing in electricity markets. They analyze the case when conventional players use market power to increase prices. With their model they are able to show that renewable producers are worse off in settings with market power. However, they do not consider the strategic
behavior of renewable producers and abstract from uncertainty.

The other branch of relevant literature covers optimal bidding strategies under uncertain production of one single player. Many papers in this field analyze numerical models from a price taker perspective and focus on wind power producers. For instance, Botterud et al. (2010) numerically analyze the optimal bidding for a wind power producer in a two-stage market (day-ahead and real-time market) under certain risk assumptions. They find that the optimal bid on the day-ahead market depends on risk behavior and the respective market prices. Furthermore, it tends towards the expected production as a deviation penalty between the day-ahead and the real-time market is introduced. Botterud et al. (2010) focus on one specific wind power producer without considering the implications of adjusted bidding strategies on the market equilibrium. Those effects can influence the optimal bidding strategy as we will show in the investigated oligopoly cases. Further literature similar to Botterud et al. (2010) can be found in Bathurst et al. (2002), Usaola and Angarita (2007), Pinson et al. (2007), and Morales et al. (2010).

In parallel but independent work, which was just published while our paper was about to be finalized, Ito and Reguant (2016) deal with a similar problem and come to very similar conclusions. Our basic model setup is essentially identical to Ito and Reguant (2016) and therefore also many of the theoretical insights coincide. Their case of "no arbitrage" is similar to our monopolist case and the case of strategic arbitrage is similar to the introduction of additional renewable players. Our work nevertheless, adds some important insights to the topic that can not be found in Ito and Reguant (2016). We explicitly consider the role of uncertainty in our model. While this has no effect (at least for linear marginal costs) on the optimal strategies, we are able to quantify the effect of uncertainty on overall welfare. We find that welfare is decreased if uncertainty about final production levels is large. This signifies the importance of forecast uncertainty and market design for the efficient functioning of electricity markets. In addition, we also consider the effect of a convex marginal cost function and show that this increases the incentive for strategic withholding of quantities. Furthermore, our analysis sheds light on the role of strategic behavior in oligopolistic markets instead of focusing solely on the monopolist case (as in Ito and Reguant (2016)). We are therefore able to illustrate distributional and welfare effects for different numbers of strategic players, which can not be found in Ito and Reguant (2016). Besides providing additional intuitions for the results of Ito and Reguant (2016), the paper is therefore also able to shed light on some important

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2Here, real-time market means the ancillary grid services for balancing supply and demand.
additional aspects.

The latter of the paper is organized as followed: In Section 2.2 we develop the basic model framework. Section 2.3 analyzes the Cournot competition and the basic model is applied to the monopolistic as well as the oligopolistic case. Section 2.4 focuses on the impact of flexibility constraints for conventional power technologies. Section 2.5 sheds light onto the incentives of renewable producers to withhold capacity in the intraday market. In Section 2.6 we show the effects on welfare, producer and consumer surplus by numerical examples. In Section 2.7 we conclude our results and discuss possible policy implications.

2.2 The Model

We consider two players that interact at two stages in the wholesale market for electricity, namely, conventional producers ($c$) and renewable producers ($r$). The consumers are assumed to behave completely price-inelastic in the short-run and demand a quantity $D$. The demand of consumers is satisfied already in the first stage, since we assume consumers as being myopic and risk-averse. On the supply side, we distinguish between conventional producers and renewable producers.

Conventional producers in the model are represented as competitive fringe. They are able to produce electricity at total costs of $C(q_c)$ where $q_c$ is the quantity produced. These quantities are sold into the market at a uniform price of the marginal production costs. The conventional producers also act as market makers which means they always satisfy the residual demand in both stages$^3$.

Renewable producers produce electricity at zero marginal costs. Their final production level $Q$ is uncertain in the first stage with the probability density function $f(Q)$. The uncertainty about the production level resolves over time (from stage 1 to 2; cf. Figure 2.1).

Throughout our analysis we assume the probability function $f(Q)$ as symmetric. In our view this assumption is reasonable, since well behaved forecasting models should be able to produce a symmetric distribution.$^4$

$^3$Conventional producers have a strong incentive to sell their production in a market as long as the price is above their marginal production costs. This makes it seem to be a reasonable assumption that conventional producers always satisfy the residual demand when prices are above or equal to their marginal generation costs.

$^4$Of course the distribution would not be symmetric in cases where production is expected to be extreme in the sense of a very low (close to zero) or very high (close to the capacity limit) production. Further information on wind forecasts and uncertainty can be found in Zhang et al. (2014).
Conventional and renewable producers can trade electricity in the two stages ($t = 1$ and $t = 2$). For the conventional producers quantities are denoted by $q_{ct}$ and for the renewable producer by $q_{rt}$. Here, we allow for $q_{ct}$ and $q_{rt}$ to be positive or negative. This allows producers, e.g. to sell too much production in the first stage and buy back quantities in the second stage. As already mentioned, we assume the demand of consumers ($D$) to get satisfied in the first stage. In the second stage, conventional and renewable producers can adjust their positions, e.g. conventional producers buy quantities from the renewable producer in order to replace their more expensive conventional production with renewable electricity. In this setting it is unclear what quantity ($q^*_1$ and $q^*_2$) is optimal to trade in the first and second stage for the renewable producer.

The market clearing conditions at both stages can be written as

\begin{align*}
\text{Stage 1:} & \quad D = q_{c1} + q_{r1} \quad (2.1) \\
\text{Stage 2:} & \quad D = q_{c1} + q_{c2} + q_{r1} + q_{r2}. \quad (2.2)
\end{align*}

The conventional producers produce electricity based on linear increasing marginal cost functions in both stages. A linear marginal cost function abstracts from real cost functions in electricity markets in two important assumptions. The first model assumption is the linearity. In reality, the cost function is usually a monotonic increasing function (with a mainly stepwise convex-similar shape). Therefore, in theory, a usual simplifying assumption is a convex cost function. In contrast to this, we assume linearity since it simplifies the theoretical analysis. Similar results can be obtained with a convex cost function (e.g. arbitrary second order quadratic functions monotonic increasing in $R^+$). We will exemplarily discuss possible implications for the case of renewable monopolist facing a convex marginal cost function.
Second, in reality, marginal costs of production may change with time, which can have multiple reasons. In electricity markets this may be due to technical constraints of power plants (start-up costs, minimum load restrictions or partload-efficiency losses) or due to transaction costs of participants that do not engage in short-term trading in short intervals before production. In the end, this may lead to a reduction of electricity supply that is available on short notice.

We account for a change of the supply side by considering two different marginal cost functions $MC_1(q)$ and $MC_2(q)$ with different inclinations $a_1$ and $a_2$. Since the number of flexible power plants is lowered the closer we get to physical delivery (or less power plant operators participate in the second market), $a_2$ has to be greater than $a_1$. As explained before, the supply curve may change due to two reasons. First, technical constraints of power plants which are not able to adjust their power output in short intervals before production can lead to reduced supply. Second, there may be transaction costs for power plant operators to participate in the intraday market which is why supply is also reduced. This approach is similar to Henriot (2014) and has been empirically verified for the German intraday market by Knaut and Paschmann (2017).

For the analysis we have to define the properties of the marginal cost function in the second stage. Besides the increase of the slope to $a_2$, the whole curve needs to cross the market clearing point from the first stage. Because if there are no adjustments in quantities the price of the first and second stage are identical. Thus the marginal cost function for the second stage can be obtained by a rotation around the market clearing point from stage 1 (cf. Figure 2.2). This means an increase in production comes at additional costs and a decrease in production at less savings of production costs. In combination with the market clearing conditions, this leads to
the following two equations for price formation in the two stages:

\[ p_1(q_{r1}) = a_1(D - q_{r1}) + b_1 \]  
\[ E[p_2(q_{r1}, q_{r2})] = a_2(D - q_{r1} - q_{r2}) + b_1 + (D - q_{r1})(a_1 - a_2), \]  

where \( b_1 \) is the offset, \( a_1 \) the gradient in the first stage and \( a_2 \) the gradient in the second stage of the marginal cost function.

In a next step, we will derive the respective profit functions for the conventional and renewable producer. The conventional producer’s profit function is defined as

\[ \Pi_c(q_{c1}, q_{c2}) = p_1(q_{r1})q_{c1} + p_2(q_{r1}, q_{r2})q_{c2} - C_1(q_{c1}) - C_2(q_{c1} + q_{c2}) + C_2(q_{c1}). \]  

Revenues in both stages are the products of the respective prices and quantities. Production costs depend on the power plants utilized for production. Since the marginal costs of production may change with time, the costs consist of the sum of quantities planned for production in each stage.

The profit function of the renewable producer

\[ \Pi_r(q_{r1}, q_{r2}) = p_1(q_{r1})q_{r1} + p_2(q_{r1}, q_{r2})q_{r2} \]  

consists of the quantities traded at the respective prices in the first and second stage without associated production costs.

We are able to show how competition between renewable producers and conventional producers can be modeled by applying this framework to different settings. In this paper, we will consider three cases:

- Competition in the first stage with identical cost functions: \( q_r = Q, a_1 = a_2 = a \)
- Competition in the first stage with changing cost functions: \( q_r = Q, a_2 > a_1 \)
- Competition in the first and second stage with changing cost functions: \( q_r \leq Q, a_2 > a_1 \).
2.3 Cournot Competition of Renewable Producers

Throughout this paper we focus on a linear marginal cost function which can be regarded as the simplest case. In this section, we will first give an intuition for the results of the model based on the simple case of identical cost functions and a renewable monopolist who acts strategically in the first stage. For this part of the analysis, we assume that the renewable producer sells the complete remaining production in the second stage, meaning \( q_r = Q \). In a next step, we will extend the analysis from the renewable monopoly to an oligopoly.

We can parametrize the linear marginal cost function \( MC(q_c) = aq_c + b \) by the gradient \( a \in \mathbb{R}_{>0} \) and an offset \( b \in \mathbb{R}_{\geq 0} \) with variable \( q_c \in \mathbb{R}_{\geq 0} \) as the produced quantity from conventional producers. Because demand is assumed to be price inelastic, we can write the prices in both stages as a function of renewable quantities:

\[
p_1(q_{r1}) = a(D - q_{r1}) + b \quad (2.7)
\]

and

\[
p_2(Q) = a(D - Q) + b. \quad (2.8)
\]

2.3.1 Renewable Producer Monopoly

First, we focus on the simple case in which all renewable production is traded by one firm. From economic literature it is well known that under the assumptions of Cournot competition, the monopolist has incentives to deviate from welfare optimal behavior in order to maximize its own profits. In our sequential market setting, this can be observed as well. By Proposition 2.1 we show that the optimal bidding strategy for a renewable producer under a monopoly is to bid half the expected production in the first stage.

**Proposition 2.1.** The profit maximizing quantity for a renewable monopolist is

\[
q_{r1}^* = \frac{\mu_q}{2} \quad \text{with} \quad \mu_q \text{ the expected renewable production.}
\]

\[^5\text{The main results also hold for convex second-order cost functions. However, the exact results may slightly deviate (i.e. it has a slightly shifting influence to the profit maximizing bidding strategy, but comparable small impact on the main results).}\]

\[^6\text{Note that we assume additionally } Q \leq D. \text{ If } Q > D \text{ and renewable producers have to sell their whole production in stage 2, we would force producers to bid negative prices. In such cases, we would expect that renewable producers reduce their production to avoid too low prices, e.g. below 0. This will be discussed in Section 2.5 in which we extend the model and allow for } q_r \leq Q.\]
Proof. The basic profit function of a renewable producer in our theoretical model framework is described in (2.6). For identical marginal cost functions, we derive the following expected profit function

\[ E[\Pi_r(q_{r1})] = q_{r1} (a(D - q_{r1}) + b) + \int (Q - q_{r1})f(Q)(a(D - Q) + b)\,dQ. \quad (2.9) \]

Where the first derivative results in

\[ \frac{d}{dq_{r1}} E[\Pi_r(q_{r1})] = a(D - q_{r1}) + b - aq_{r1} - Da \int f(Q)\,dQ + a \int Qf(Q)\,dQ - b \int f(Q)\,dQ. \quad (2.10) \]

Since \( f(Q) \) is symmetric and the marginal cost function is linear, we can further simplify the expected profit function by the following substitutes:

Expected value for \( Q \): \[ \int Qf(Q)\,dQ = \mu_q \quad (2.11) \]

Distribution function has a total probability of 1: \[ \int f(Q)\,dQ = 1. \quad (2.12) \]

This leads to the simplified necessary condition for the profit maximizing quantity \( q_{r1}^* \) as

\[ \frac{d}{dq_{r1}} E[\Pi_r(q_{r1})] = -Da + a\mu_q - aq_{r1} + a(D - q_{r1}) = 0. \quad (2.13) \]

Now we can solve this equation for \( q_{r1} \) which results in the profit maximizing quantity

\[ q_{r1}^* = \frac{\mu_q}{2}. \quad (2.14) \]

In order for this being a maximum the second derivative has to be negative. This can easily be checked by calculating

\[ \frac{d^2}{dq_{r1}^2} E[\Pi_r(q_{r1})] = -2a. \quad (2.15) \]

Since \( a \) is defined as the slope of the marginal cost function and is positive by definition, \( q_{r1}^* = \frac{\mu_q}{2} \) indeed describes the profit maximizing quantity for the renewable producer.

The motivation of the renewable producer to bid half her expected quantity in the first stage becomes clear by analyzing Figure 2.3. Since we consider a linear marginal cost function, we can abstract from the uncertainty in renewable production \( f(Q) \)
and only consider the expected production $\mu_q$. The profit of the renewable producer can be split into two parts. One part stems from selling the expected production into the market, as can be seen in Figure 2.3i (single hatched area). This part can be considered as a lower bound to the profit of the renewable producer and does not depend on the strategy of the renewable producer because she has to sell all production to the market in the final stage. The resulting price in the second stage is thus given by $\mathbb{E}[p_2]$. The second part of the renewable producer profit can be obtained by selling a quantity forward in the first stage at a price $p_1$. In order to increase her profit, the quantity in the first stage needs to be between $D - \mu_q$ and $D$ to obtain a higher price compared to $\mathbb{E}[p_2]$. Since the marginal cost function is linear and we have a monopolist selling forward, it is optimal to sell half her expected production because it maximizes the additional profit in Figure 2.3ii (cross hatched area).

**Proposition 2.2.** The optimal strategy of a renewable monopolist selling its renewable production in sequential markets with multiple stages is to sell it in small quantities at decreasing prices.

**Proof.** The triangle in Figure 2.3i can be considered as the maximum profit which can be gained by selling the expected production of the renewable producer. When the renewable producer is able to sell this production in multiple stages, it is optimal to sell it little by little in order to maximize her profit. This means prices in multiple sequential market stages would be declining until the price of $\mathbb{E}[p_2]$ is reached in the final stage. In this case, the renewable producer would be able to increase its profit by the triangle in Figure 2.3i compared to selling the expected quantity already in the first stage. \[\square\]
For the case of multiple market stages also conventional producers would be able to increase their profit. In this case, they would be able to obtain a higher profit in the first stage, where they can sell a larger quantity at a higher price. On the other hand, consumer surplus would be lowered due to higher prices.

This leads us to the conclusion that with a renewable monopolist, different market designs can have a large impact on distributional effects between producers and consumers. Consumers loose if producers trade electricity in multiple stages. Thus, continuous trading in short-term markets lowers consumer surplus. From the view of consumers, a few separate auctions should be preferred to a continuous auction since this limits strategic behavior of a renewable monopolist.

Strategic production withholding is commonly observed by market participants at the margin (see, for instance, Fabra et al. (2006), Ausubel et al. (2014)). The reason is that it is most profitable to reduce the production at the margin if the corresponding price increase overcompensates the production withholding. The production close to the margin has generally the lowest profits and thus the profit for the whole production fleet can be increased. In contrast to this, our results show that strategic production withholding may also occur for infra-marginal production with our underlying model assumptions (two-stage trading possibility, zero marginal costs for the renewable producers, positive marginal costs for the perfect competitive conventional producers). Unlike usual, it is not dependent on a higher steepness of the cost function for extra-marginal production but also holds for the basic case of a linear cost function. This spans a new dimension of strategic behavior and could also be investigated in further research.

### 2.3.2 Renewable Producer Monopoly in the Context of a Strict Convex Marginal Cost Function

The results, so far, stem from an analysis with a linear marginal cost function for conventional producers. This has been mainly due to practical reasons, in order to show first effects. In reality, however, the assumption of a linear marginal cost curve may not be valid in every situation. The marginal cost curve in electricity markets is generally assumed to be strict convex and monotonic increasing. Whereas the

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7 Additional to pure production withholding, strategic behavior at the margin can also be exerted with bids above marginal costs to increase the market clearing price.

8 This is due to the different cost structures of power plants. For example in high demand situations gas turbines are needed to satisfy the demand with high variable costs. This leads to a steep increase of the marginal cost function.
parametrization of a linear function is straightforward, a strict convex and monotonic increasing function can be parametrized in various ways. One way, for example, can be using a quadratic second order function.

In this section we will analyze the general effects of a strict convex marginal cost function on our results for the case of a renewable monopolist. There are basically two important differences which stem from the different shape of the marginal cost function. One difference is that the expected price in the second stage is greater than the price for the realization of the expected production \( \mathbb{E}[p_2(Q)] > p_2(\mu_q) \). Whereas in the case of a linear marginal cost function both were equal and we could abstract from the uncertainty in renewable production, this is not the case for a different marginal cost shape. Realizations below the expected production \( \mu_q \) lead to a higher increase in the second stage price \( p_2 \), compared to higher realization than the expected production. Therefore, the expected price \( \mathbb{E}[p_2] \) in the strict convex case will be greater than the price for a realization of the expected production. The second difference is that the shape has also an impact on the optimal quantity \( q^{* r} \).

We will try to give the intuition for the second difference based on Figure 2.4. In Figure 2.4i we plot the profit when the renewable producer bids the optimal quantity from the linear case \( \mu_q \). This is compared to the case of optimal trading in Figure 2.4ii in the first stage under a strict convex marginal cost function. The single hatched area represents the lower bound for the expected profit, as explained in Section 2.3.1. This area is equal in both settings, regardless of the traded quantity in the first stage.

The double hatched areas represent the additional profit that can be obtained from
trading a quantity in the first stage. In Figure 2.4i, $\mu/2$ is traded in the first stage which is the result of the former optimal amount under a linear merit order.

Now, in the case of a convex merit order, the profit can further be increased by trading even less than half the expected production $\mu q/2$ (as it can be seen in Figure 2.4ii). The double hatched area is greater than in Figure 2.4i. The magnitude of the impact depends on the shape of the merit order, the demand, and the expected renewable production as well as the uncertainty (standard deviation) of the renewable production. This reasoning can also be proofed for a strict convex polynomial of second order and results in Proposition 2.3.

**Proposition 2.3.** For a quadratic merit order, the optimal first stage offer $q_{r1}$ of a renewable monopolist is strictly below $\mu q/2$.

**Proof.** See Appendix 2.8.1.

We show that a strict convex merit order leads to a stronger withholding of quantities in the first stage compared to the linear case. Therefore, the optimal quantities of the renewable producers, which we derive for the linear marginal cost curve can be considered as an upper bound. For the sake of simplicity we will stick to the analysis of a linear marginal cost curve in the following sections. But based on the results from Figure 2.4 we have to keep in mind, that the results from this special case should be considered as an upper bound to the optimal quantities of renewable producers.

### 2.3.3 Renewable Producer Oligopoly

In this section, we extend the monopoly case to the case of multiple symmetric renewable producers that form an oligopoly. The symmetry implies that the renewable producers have perfectly correlated generation as well as forecast errors. The remaining approach and notation are similar to previous sections. As we learned from before, the conventional producer reacts to the decision of the renewable producers and can be considered as a price taker. So we can focus on the optimal quantities of the renewable producers. We still consider a linear marginal cost function $MC(q) = aq + b$ and define the players $i = 1, ..., N$ with their corresponding quantities in stage 1 as $q_{ir1}$. Furthermore, we define the sum of the quantities of all players but $i$ as $q_{-ir1} = \sum_{j \neq i} q_{jr1}$. We find that the optimal bid of a renewable producer in the first stage is still driven by strategic behavior but tends towards the expected production level as the number of producers increases.
2.4 Flexibility and its Role in Short-term Markets

Proposition 2.4. The optimal quantity traded in the first stage for each player is
\[ q_{ir1}^* = \frac{1}{N+1} \mu_q \] with \( \mu_q \) the total expected renewable production of all players.

Proof. See Appendix 2.8.2. \( \square \)

As a direct implication from the optimal first stage bid we see that for the linear marginal cost function, the optimal strategy is still independent of the gradient or the uncertainty of production.

Corollary 2.1. The profit maximizing traded quantity in stage 1 of the above setting is identical for all players. Furthermore, \( q_{ir1}^* \) is independent of the steepness \( a \in [0, \infty) \) of the marginal cost function, the offset \( b \in [0, \infty) \) of the marginal cost function, and the probability distribution function \( f(Q_i) \).

According to Proposition 2.4 it is optimal for renewable producers to always trade less than the expected production in the first stage since this maximizes their profits. The overall quantity tends towards the overall expected quantity as the number of players increases.\(^9\)

In stage 1, this leads to an overall traded quantity of renewable production of
\[ q_{r1} = \sum_{j=1}^{N} q_{jr1} = \frac{N}{N+1} \mu_q \] (2.16)

with \( \mu_q := \sum_{j=1}^{N} \mu_{jq} \). In two sequential markets, renewable producers have an incentive to trade less than the total expected renewables production in the first stage. The more players enter the market the stronger the competition and thus the traded amount in the first stage tends towards the expected production. Our results of the first stage show that, under the described setting, a renewable producer acts exactly as predicted in a standard one-shot oligopolistic Cournot game.

2.4 Flexibility and its Role in Short-term Markets

In this section we shed light on the implications of changing cost functions in short-term markets. As mentioned before, this can happen for essentially two reasons. One reason is that not all conventional power plants are flexible enough to adjust

\(^9\)Note that, for the moment, we assumed a linear marginal cost function which does not change between the first and the second stage.
their production capacity in stage 2 in the short run. The second reason is that there can be transaction costs for power plant operators associated with the trading in the intraday market.

The difference between the cost function of the first and second stage has implications for the optimal quantity of the renewable producers in the first stage, which we analyze here in more detail. The nomenclature corresponds to the previous sections.

**Proposition 2.5.** The optimal quantity traded in the first stage for each renewable player is 

$$q^{*}_{ir1} = \frac{1}{N+1} \mu_{iq} (N + 1 - \frac{a_1}{a_2}) $$

with the ratio \( \frac{a_1}{a_2} \) representing the degree of flexibility of the supply side in both stages.\(^{10}\)

**Proof.** In a first step we will derive the optimal quantity of a player \( i \) who competes against \( N-1 \) identical players\(^{11}\). According to the setup, the prices in the first and second stage can be defined as:

$$p_1(q_{ir1}, q_{-ir1}) = a_1(D - q_{ir1}) - q_{-ir1} + b_1 \tag{2.17}$$

$$p_2(q_{ir1}, q_{-ir1}) = a_2(D - Q_{N}) + b_1 + (a_1 - a_2)(D - q_{ir1} - q_{-ir1}). \tag{2.18}$$

Again, we can define the expected profit function for player \( i \), take the first derivative and integrate over \( f_i \) (which is assumed as being identical for all players). Setting the first derivative equal to zero leads us to the necessary condition for an optimal quantity:

$$- a_1 \mu_{iq} + a_2 \mu_{iq} N + a_2 q_{-ir1} - a_2 q_{ir1} - 2a_2 q_{ir1} = 0. \tag{2.19}$$

Under the assumption that all players are identical we can set \( q_{-ir1} = (N - 1)q_{ir1} \) and solve for \( q_{ir1} \) which leads to:

$$q^{*}_{ir1} = \left( 1 - \frac{1}{N + 1} \frac{a_1}{a_2} \right) \mu_{iq}. \tag{2.20}$$

The second derivative of the expected profit function is negative, which proves \( q^{*}_{ir1} \) being a maximum for the expected profit function. \( \square \)

This means that all renewable producers together submit a quantity of

$$q^{*}_{r1} = \mu_q - \frac{1}{N + 1} \frac{a_1}{a_2} \mu_q \tag{2.21}$$

\(^{10}\)Small values of \( \frac{a_1}{a_2} \) represent a very inflexible supply side in the second stage.

\(^{11}\)The sum over all other players is still denoted by the quantity \( q_{-ir1} = \sum_{j \neq i} q_{jr1} \)
in the first stage (with $a_2 > a_1$).

From Equation (2.21) we can conclude the following: (1) $q^*_1$ increases if conventional producers are less flexible ($a_2 \gg a_1$); (2) $q^*_1$ increases with an increasing number of renewable producers $N$. For a perfectly competitive market (with $N \rightarrow \infty$) it is optimal for each player to trade its share of the total expected quantity in the first stage.

By looking at the example of a renewable monopolist in Figure 2.5, we can get a deeper understanding of the motives for a renewable producer who faces a market with inflexible conventional producers. As explained before, the marginal cost curve for the second stage rotates around the market clearing point of the first stage. The total production of the renewable producer that needs to be sold after both stages however does not change. Thus, the renewable producer has to decide what quantity to sell at a respective price in the first stage and sell the remaining quantity at a lower price in the second stage. The price is lower in the second stage due to the additional renewable quantities that are sold by the renewable producer. Basically, in Figure 2.5, the sum of the cross hatched area and the single hatched area needs to be maximized. The renewable producer is able to maximize both areas by a parallel shift of the marginal cost function for stage 2 (green dotted line). This means, the renewable producer has to optimize the quantity in the first stage in such a way that the profit from both stages is maximized. Summarizing, a less flexible power plant fleet shifts the total optimal first stage bidding quantity of a renewable producer towards the expected production.

The described effects on the optimal quantity hold true for different numbers of renewable producers and different degrees of flexibility. This is shown exemplarily

![Figure 2.5: Profit of a renewable monopolist facing a inflexible conventional producers](image-url)
in Figure 2.6. Here, the optimal quantity converges more slowly to the expected production in the perfectly flexible case \((\frac{a_2}{a_1} = 1)\) compared to a highly inflexible conventional power plant mix \((\frac{a_2}{a_1} = 4)\). An increase in the number of renewable producers leads to a similar effect of a higher overall renewable quantity in the first stage.

![Graph](image)

Figure 2.6: Optimal renewable quantity \(q_{r1}\) dependent on the number of players \(N\) and the ratio \(\frac{a_2}{a_1}\)

### 2.5 Incentives of Renewable Producers to Withhold Production

In this section we extend the analysis of strategic competition in the first stage by investigating the case in which renewable producers are allowed to withhold production in the second stage. Therefore, we relax the assumption that the renewable producer needs to sell all her realized production in the second stage. This means \(q_{r1} + q_{r2} \leq Q\) instead of \(q_{r1} + q_{r2} = Q\). We still assume that renewable producers strictly avoid being short after stage 2, i.e. selling more production than they produce. The rational is that high financial penalties need to be payed in case of an imbalance. All other model assumptions stay the same.

The motivation for the relaxation of the second stage restriction to sell the whole production is threefold. First, we note that, in general, it is technically possible to reduce production for renewable producers. This happens for photovoltaic in critical grid situation if the voltage level extends a critical value (automatic shut down around 50.2 Hertz) or for wind turbines during storms. Second, a reduced production could be economically profitable in specific situation. Especially if prices
are negative or, like in the investigated case, if market prices could be increased profitably by withholding production. Third, market manipulation by a withhold of renewable production is not easy to prove by the regulator. It is hard to detect whether a wind turbine does not produce due to maintenance, local wind conditions or strategic production withholding.

We extend the model with cost functions by replacing the constraint \( q_{r1} + q_{r2} = Q \) with \( q_{r1} + q_{r2} \leq Q \). Based on this model we obtain the following results.

**Proposition 2.6.** If renewable producers are allowed to withhold production, they only withhold production after the second stage if the expected production of all producers is high compared to the demand \( D \), i.e. if \( \mu_q > \frac{a_2 N (N+1)}{a_2 (N+1)^2 - a_1 N} D + \frac{a_2 N (N+1)}{a_1 (a_2 (N+1)^2 - a_1 N)} b \). This means the expected renewable production needs to be at least \( \frac{D}{2} \). Otherwise, renewable producers sell the total realized production into the market (same result as of Proposition 2.5).

**Proof.** We use the same model as in Section 2.4 (and corresponding Proposition 2.5). The only difference is the relaxed constraint \( q_{r1} + q_{r2} = Q \) by \( q_{r1} + q_{r2} \leq Q \). This allows the renewable producer to withhold production and to increase prices in the second stage. Since we adjusted an equality constraint by an inequality constraint, we face now a convex optimization problem with inequalities and can use the Karush-Kuhn-Tucker (KKT) conditions to solve it. The full proof can be found in the Appendix 2.8.3.

The main finding is that renewable producers have an incentive to withhold production after the second stage only if the (expected) production exceeds a threshold value which is at least \( \frac{D}{2} \) (but dependent on \( a_1, a_2, b \) and \( N \)).\(^{12}\) The exact threshold value is

\[
Q_{\text{threshold}} := \frac{a_2 N (N+1)}{a_2 (N+1)^2 - a_1 N} D + \frac{a_2 N (N+1)}{a_1 (a_2 (N+1)^2 - a_1 N)} b. \tag{2.22}
\]

As long as the (expected) production is below this threshold, the renewable producers will sell their total realized production in the second stage. Nevertheless, the production is split between first and second stage to increase profits. By analyzing this threshold we find the following

\(^{12}\)In stage 1, the expected production is the relevant quantity while in stage 2 the realized production is the relevant quantity. If both, expected and realized production, deviate from each other, it is possible that the renewable producers pursue a different strategy in each stage.
Q_{threshold} \text{ is increasing in } N: \text{ The more producers exist, the higher the threshold. Therefore, more competition between renewable producers limits the incentive for renewable producers to withhold quantities in the second stage.}

\* Q_{threshold} \text{ is decreasing in } a_2 (\text{with } a_1 \text{ fixed}): \text{ The more inflexible the power plant fleet, the lower is the threshold. Therefore, renewable producers start to withhold production at a lower level of expected renewable production.}

\* Q_{threshold} \text{ converges to } \frac{N}{N+1} \left( D + \frac{b}{a_1} \right) \text{ for } a_2 \to \infty \text{ but is strictly above } \frac{D}{2}.

To sum up, renewable producers only have an incentive to withhold quantities in situations with very high renewable generation compared to the demand. Additional renewable producers as well as more flexible conventional producers increase the threshold \( Q_{threshold} \) to withhold production quantities.

### 2.6 Prices, Welfare, Producer Surplus and Consumer Surplus

Trading in the day-ahead and intraday market has implications for overall welfare, producer surplus and consumer surplus. So far, we focused on the quantities of the renewable producers that maximize their respective profits. They determine the quantities that are traded by the conventional producers and thereby the prices in both stages. In order to disentangle the effects on overall welfare, producer and consumer surplus, we will first analyze the effects on prices in the two stages.

Since we found in Section 2.5 that renewable players only withhold production at very high production levels compared to demand \( D \), we focus on the case in which renewable producers sell all their production after stage 2 (the case \( q_{r1} + q_{r2} = Q \)).

#### 2.6.1 Prices and the Role of Arbitrageurs

By plugging in the optimal quantity from Equation (2.20) into the price equations for the case with flexibility constraints (Equation (2.3) and (2.4)) we obtain the

\[ Q_{threshold} \text{ is at least } 0.85D. \]

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13 For a realistic number of renewable players \( N > 5 \) and an arbitrary ratio of \( a_2 \) to \( a_1 \), the threshold \( Q_{threshold} \) is at least 0.85\( D \).
following prices:

\[ p_1 = Da_1 + b_1 - \frac{a_1}{a_2 N + 1} \left( a_2(N + 1) - a_1 \right) \]  \hspace{1cm} (2.23)

\[ \mathbb{E}[p_2] = Da_1 + b_1 - \frac{a_1}{a_2 N + 1} (a_2(N + 2) - a_1). \]  \hspace{1cm} (2.24)

From these two equations we can already see that the price in the first stage is higher than in the second stage. This becomes obvious by taking the difference between the two prices:

\[ p_1 - \mathbb{E}[p_2] = \frac{a_1 \mu_q}{N + 1}. \]  \hspace{1cm} (2.25)

We can observe the following implications: First, the price difference between stage 1 and 2 is independent of the change in the slope of the marginal cost function \( a_2 \). The renewable producers choose their quantity dependent on the slope \( a_2 \). This has an effect on the absolute prices in the two stages but the price delta stays constant. Second, with a higher overall expected production from renewables \( \mu_q \) also the price difference increases. The quantity that is withheld from trading in the first stage increases with the expected production and, thereby, the price difference increases. Third, the price difference decreases with an increasing number of renewable producers \( N \). In a perfectly competitive market (with \( N \rightarrow \infty \)), prices in both stages are equal. As we can observe in Figure 2.6, the quantity in the first stage tends towards the overall expected quantity and hereby prices in both stages converge.

Based on the price difference in both stages one could suspect arbitrageurs to be entering the market. By obtaining a short position in the day-ahead market and adjusting their position in the intraday market, they would be able to make a profit. The optimal strategy of an arbitrageur is therefore identical with the strategy of the renewable players. The only difference is that arbitrageurs do not necessarily own production assets. Each additional arbitrageur that would enter the market can nevertheless be regarded as an additional renewable player. This would in turn decrease the price difference between the day-ahead and intraday market (cf. Figure 2.7).

Still, electricity markets have some unique features that may prevent arbitrageurs from engaging in short-term electricity markets. First, the assets that are traded are not only financial but physical obligations to produce and deliver electricity. Therefore, some short-term market platforms restrict the participation to firms with physical production assets. This prevents for example banks from entering these mar-
Figure 2.7: Prices in the two stages for an example with \( D = 70, \mu_q = 20, \sigma_q = 5, b_1 = 20 \) and \( a_1 = 0.5 \)

With two markets. Second, there may be information asymmetries between renewable producers and arbitrageurs that may be hard to overcome. For example renewable producers can be assumed as having better knowledge about the expected production level of their assets. For the following discussions we will thus not focus on the case of additional arbitrageurs entering the market. Nevertheless, the implications of arbitrageurs entering the market can be observed implicitly by considering an increase in the number of renewable players (\( N \)).

In order to gain a deeper understanding of the effects from changing cost functions and increased competition on prices, we plot this relationship in Figure 2.7 for an exemplary case. The direction of the effects will stay the same for arbitrary \( a_1, a_2 \) with \( a_2 \geq a_1 \) and arbitrary \( D, Q \) and \( b \) with \((D - Q)a_1 + b \geq 0 \) (\( Q \sim \mathcal{N}(\mu_q, \sigma_q) \)).

In Figure 2.7, we chose the values such that one can easily find similarities to the German electricity market. A demand \( D \) of 70 GW can be observed during peak times, where also an expected renewable production \( \mu_q \) of 20 GW is quite common. Furthermore the parameters of the marginal cost function were chosen such that they represent common price levels.\(^{14}\)

We can see that the prices in stage 1 and 2 converge to the same value with an increasing number of players. This benchmark is set by the perfectly flexible case (\( \frac{a_2}{a_1} = 1 \)), where the price in the second stage stays constant. In the next sections we will analyze the effects on producer surplus, consumer surplus and overall welfare.

\(^{14}\)Of course a linear marginal cost function is a crude assumption in this case, but it allows us to show the overall effects.
2.6 Prices, Welfare, Producer Surplus and Consumer Surplus

2.6.2 Producer Surplus

The producer surplus is defined as the sum of the renewable producer surplus and the conventional producer surplus. For the case with a changing marginal cost function, the conventional producer surplus can be defined as

\[ \mathbb{E}[\Pi_c(q_{r1})] = p_1(D - q_{r1}) + p_2 \int (q_{r1} - Q)f(Q)dQ - C_1(q_{r1}) - \int C_2(q_{r1})f(Q)dQ. \]  

(2.26)

It is the difference between the income from sold quantities in stage 1 and 2 and the associated costs with the production of electricity.

The first stage costs \( C_1 \) in our model depend on the quantities offered by the renewable producers \( q_{r1} \). We can thus obtain the costs in the first stage by integrating over the marginal cost function \( MC_1 \)

\[ C_1(q_{r1}) = \frac{1}{2}a_1(D - q_{r1})^2 + b_1(D - q_{r1}). \]  

(2.27)

The formulation is more complex for the costs that are associated with the second stage of production. First, it depends on the quantity that is traded in the first stage by the renewable producer \( q_{r1} \). Second, it depends on the realization of the final renewable production \( Q \). In the first stage, the conventional producers plan to produce a certain quantity \( D - q_{r1} \). In the second stage, this quantity has to be adjusted to meet the total residual demand of \( D - Q \). This means if the renewable production turns out to be higher than the traded quantity in the first stage, the conventional producers need to reduce their planned production and can buy back quantities at a lower price. Meanwhile the slope of the cost function has changed from \( a_1 \) to \( a_2 \). This leads us to the following expected cost function for the second stage:

\[ \mathbb{E}[C_2(q_{r1})] = \int \int_{Q_{r1}} (a_2q_{c2} + (a_1 - a_2)(D - q_{r1}) + b_1)dq_{c2}f(Q)dQ \]  

(2.28)

\[ = (Da_1 - a_1q_{r1} + b_1)(\mu_q - q_{r1}) + a_2q_{r1}(\mu_q - \frac{q_{r1}}{2}) - \frac{a_2n^2}{2} \left( \mu_{iq}^2 + \sigma_{iq}^2 \right). \]  

(2.29)

What is especially noticeable in this equation, is that for the first time in our analysis also the standard deviation (\( \sigma_{iq} \)) of the expected renewable production plays a role. The reason for this lies in the non-linear cost function of the conventional producers. Here, deviations from the expected value are not multiplied by a linear curve and
weighted equally but weighted by the non-linear function. This is why the standard deviation plays an important role. By inserting Equation (2.27) and (2.29) in (2.26), we obtain the total conventional producer surplus.

In the same way, we can also derive the producer surplus for the renewable producers.

\[
E[\Pi_r(q_{r1})] = p_1 q_{r1} + \int p_2(Q - q_{r1})f(Q)dQ \tag{2.30}
\]

By plugging in the results from Equation (2.20) it is possible to quantify the renewable and conventional producer surplus. We plot this for an exemplary cases in Figure 2.8.

Figure 2.8: Expected producer surplus for an example with \( D = 70, \mu_q = 20, \sigma_q = 5, b_1 = 20 \) and \( a_1 = 0.5 \).

As we could already see from Figure 2.7, prices in the first stage decrease with an increase in competition or a less flexible supply curve. At the same time prices in the second stage increase. This results in both, a dampening and an increasing effect on producer surplus. From Figure 2.8 we can observe that the decreasing effect of the first stage outweighs the increasing effect in the second stage. Overall, we see that the producer surplus decreases with the number of renewable producers \( N \) and with a less flexible power plant mix. Especially the decrease in conventional producer surplus is noticeable. For renewable producers the decrease in surplus is not as prominent, since they are able to reduce the effects by adjusting their optimal quantity \( q_{r1} \). For example the overall quantity traded by renewable producers \( q^{*}_{r1} \) is increased when more renewable producers compete in the first stage. Also a less flexible power plant mix leads to a higher optimal quantity for renewable producers in the first stage (cf. Figure 2.6).
2.6.3 Consumer Surplus

In our model, consumers are represented as being completely inelastic in their demand behavior. In electricity markets it is common practice to assume consumers as completely price inelastic and consuming electricity up to the point when the price exceeds the value of lost load (VOLL). We therefore slightly adjust our assumptions by introducing the price $p^{\text{VOLL}}$ which can be regarded as the upper limit for the willingness-to-pay for electricity consumption.

As consumers are assumed to be risk-averse, demand is already satisfied in the first stage at price $p_1$, as long as $p_1 < p^{\text{VOLL}}$. The consumer surplus can therefore be expressed as $(p^{\text{VOLL}} - p_1)D$. By plugging in the price formulation for the first stage from Equation (2.3), we get

$$CS = D \left( p^{\text{VOLL}} - Da_1 - b_1 + \frac{a_1}{a_2} \frac{\mu_q}{N + 1} (a_2(N + 1) - a_1) \right). \quad (2.31)$$

We can now compare the consumer surplus for the different combinations of $N$ and $a_2/a_1$. In order to circumvent an assumption for the upper price limit $p^{\text{VOLL}}$, we focus our analysis on changes in consumer surplus compared to a reference point. We therefore choose the reference point where consumer surplus is the lowest. This is the case for a renewable monopolist and perfectly flexible conventional producers ($a_1 = a_2$).

![Graphs showing consumer surplus and overall welfare](image)

(i) Delta in consumer surplus  
(ii) Delta in expected overall welfare

Figure 2.9: Delta in consumer surplus and expected overall welfare for an example with $D = 70$, $\mu_q = 20$, $\sigma_q = 5$, $b_1 = 20$ and $a_1 = 0.5$

As one could already expect from the decrease in prices with an increasing number of players in Figure 2.7, the consumer surplus increases with the number of players. What may be counterintuitive is that consumers can profit from a less flexible power plant mix. The lower flexibility of conventional producers leads renewable
producers to adjust their quantity, which has a price dampening effect for the first stage. Consumers can therefore profit from the lower prices in the first stage as it is shown exemplarily in Figure 2.9i.

2.6.4 Welfare

Combining the effects on producer and consumer surplus leads to changes in overall welfare. As we can only analyze differences in consumer surplus this also holds for the case of overall welfare. Again, we define the perfectly flexible case with a monopolistic renewable producer as a reference point for the analysis (cf. Section 2.6.3). The difference in overall expected welfare to the monopolistic case can be defined as

$$\Delta E[W(q_{r1})] = -\Delta E[CE(q_{r1})] + \Delta E[\Pi_p(q_{r1})].$$  \hspace{1cm} (2.32)

In Figure 2.9ii we can observe these effects on overall welfare. The overall welfare stays constant for the case of a perfectly flexible power plant mix. In this case, the demand is always satisfied at the same costs which does not lead to a change in overall welfare. Negative effects on overall welfare occur only if the total production costs for electricity increase, i.e. if conventional power producers are less flexible. Especially if the power plant mix is highly inflexible, as in the case with \(a_2/a_1 = 4\), it will lead to a substantial decrease in overall welfare. Generally we can observe two effects. First, the effect on welfare has a smaller magnitude than the isolated effects on producer surplus or consumer surplus. The increase in consumer surplus and decrease in producer surplus counteract each other and lead only to a slightly reduced effect on overall welfare. Second, the welfare is generally decreased in a setting with less flexible power plants.

In a last step, we analyze the effects of uncertainty on overall welfare. So far, we assumed the production of the renewable producer in the final stage to be forecasted with a standard deviation of \(\sigma_q = 5\) in the numerical examples. Now, we assume that if forecasts are improved or trading time is delayed, the standard deviation decreases, as to Foley et al. (2012). A decrease in standard deviation could also be accomplished by delaying trading of the first stage (e.g. by trading in the evening of the day before physical delivery instead of at noon). We quantify the welfare effects by comparing them to the case with no uncertainty (\(\sigma_q = 0\)) and a perfectly competitive market (\(q_{r1} = \mu_q\)). From Figure 2.10 we can observe that a larger standard deviation results in welfare losses. From this we can conclude that it is
2.7 Concluding Remarks

desirable to increase the quality of forecasts or to change the timing of trading in order to increase overall welfare.

Figure 2.10: Delta in expected overall welfare for varying standard deviation of the forecast \( \sigma_q \) \((D = 70, \mu_q = 20, b_1 = 20, a_1 = 0.5 \text{ and } a_2 = 1)\)

2.7 Concluding Remarks

We derive the optimal quantities for renewable producers that are strategically selling their production in a two-stage game with uncertainty about production in stage 1 and knowledge about the realization of their production in stage 2. It is profit maximizing for renewable producers to bid less than their expected total quantity in the first stage, which we consider as the day-ahead market. Renewable producers are able to increase their profits by selling only part of their expected production in the first stage and thus raising the price in the first stage. The optimal quantity in the first stage tends towards the overall expected quantity with an increasing number of renewable producers. Conventional producers are considered as a competitive fringe that satisfies the residual demand in both markets. If conventional power producers are less flexible in their operation, renewable producers have a larger incentive to increase the traded quantity in the first stage. In general, prices in the first stage (day-ahead) are higher compared to the second stage (intraday), but with an increasing number of renewable producers or with arbitrageurs entering the market this difference decreases. In situations with very high production levels, that are at least able to serve half of the demand, renewable producers have an incentive to withhold production in the second stage. This effect is decreased by an increasing number of players but increases in a setting with low flexibility of conventional producers.
A reduced forecast uncertainty leads to an increase in overall welfare. This leads us to two conclusions. First, overall welfare can be increased by delaying the trade in the day-ahead market closer to the time of physical delivery. For example by shifting the auction from noon to the evening. Second, an increase in forecast quality has a positive effect on overall welfare.

Based on the results it becomes obvious that in a future electricity system with high shares of renewables, regulators need to pay attention to the possible abuse of market power by large renewable producers. In situations with low liquidity and the absence of arbitrageurs this could lead to significant distributional effects and even welfare losses.

In our whole analysis, we assumed the generation of all renewable producers to be perfectly correlated, as well as their forecast errors. This is not the case in reality and could be further investigated. Additionally, it would be possible to quantify welfare implications of improved forecast quality and alternative market designs at concrete examples.

The role of uncertainty only plays a minor role in our analysis since we mainly focus on the case of linear marginal cost functions and risk-neutrality. In reality, however, participants may be acting more risk-averse which would increase the importance of accounting for uncertainty. This could be especially interesting when the analysis is extended to players with mixed portfolios of renewable and conventional power production. The optimization within a generation portfolio (maybe in combination with risk-averse behavior) could lead to interesting insights on the potential use of market power in electricity markets in a more general setting.

2.8 Appendix

2.8.1 Proof of Proposition 2.3

Proof. Let $MC(q) = aq^2 + bq + c$ with $a > 0$ and $b, c \geq 0$ be a strictly monotonic increasing convex (quadratic) marginal cost function. The optimal first stage trading amount for a monopolistic renewable producer is

$$q_{r1}^* = \frac{2}{3} D + \frac{1}{3} \frac{b}{a} - \frac{2}{3} \sqrt{\left[D - \frac{3}{4} \mu_q + \frac{1}{2} \frac{b}{a}\right]^2 + \frac{3}{16} \mu_q^2 + \frac{3}{4} \sigma_q}$$

(this can be derived analogously to the optimal amount of the linear case in Proposition 2.1). Then the following
holds:
\[ q^*_r = \frac{2}{3}D + \frac{b}{3a} - \frac{2}{3} \sqrt{\left[ D - \frac{3}{4} \mu_q \pm \frac{1}{2} \frac{b}{a} \right]^2 + \frac{3}{16} \mu_q^2 + \frac{3}{4} \sigma_q^2} \]

\[ > \frac{2}{3}D + \frac{b}{3a} - \frac{2}{3} \sqrt{\left[ D - \frac{3}{4} \mu_q \pm \frac{1}{2} \frac{b}{a} \right]^2} \]

\[ = \frac{1}{2} \mu_q. \]  \hspace{1cm} (2.33)

The inequality is strict since the square root is a strict monotonic function on positive numbers. Therefore, under a convex merit order, it holds that \( q^*_r < \frac{1}{2} \mu_q \). Note that we assumed \( \mu_q < D \) in the model setup.

\[ \square \]

### 2.8.2 Proof of Proposition 2.4

**Proof.** Because all players are symmetric we can denote the total traded renewable production of all players in stage 1 by \( q_{r1} = q_{r1} + q_{-r1} \) (where \( q_{-r1} \) aggregates all players but not player \( i \), the realized production in stage 2 by \( Q = NQ_i \), and the expected quantity by \( \mu_q = N \mu_{iq} \). With these definitions, Equation (2.7) and (2.8) still hold for the oligopoly case.

The profit function of renewable producer \( i \) can be derived by plugging in those values into
\[ \Pi_{ir}(q_{r1}) = p_1(q_{r1})q_{ir1} + p_2(D - Q)q_{ir2} \]  \hspace{1cm} (2.34)

so that the profit function results in

\[ \Pi_{ir}(q_{r1}) = (a(D - q_{ir1} - q_{-ir1}) + b)q_{ir1} + (a(D - NQ_i) + b)(Q_i - q_{ir1}). \]  \hspace{1cm} (2.35)

Remember that \( q_{ir2} = Q_i - q_{ir1} \) and that we assume \( Q_i \) to be uncertain. In order to derive the expected profit function we have to integrate for \( Q_i \) over the distribution \( f(Q_i) \), where \( f(Q_i) \) is the probability density function for \( Q_i \). After taking the first derivative, setting it equal to zero and replacing the expected values (analogous to Equations (2.11) and (2.12)), we get the necessary conditions

\[ \frac{d}{dq_{ir1}} \mathbb{E}[\Pi_{ir}(q_{ir1})] = a(\mu_q - q_{ir1} - q_{-ir1}) \parallel 0 \]  \hspace{1cm} (2.36)
and the corresponding solution is

\[ q_{ir_1}^* = \frac{1}{2} N \mu_{jq} - \frac{1}{2} q_{-ir_1} \]  \quad (2.37)

for \( i = 1, ..., N \).

In an equilibrium of identical players we have identical solutions which results in \( q_{-ir_1} = (N - 1)q_{ir_1} \). With this, we derive

\[ q_{ir_1}^* = \frac{1}{2} N \mu_{iq} - \frac{1}{2} (N - 1)q_{ir_1}^* \]  \quad (2.38)

\[ \iff q_{ir_1}^* = \frac{1}{N + 1} \mu_{iq} \]  \quad (2.39)

Because the second derivative of Equation (2.35) is negative, we found the profit maximizing quantity \( q_{ir_1}^* \).

### 2.8.3 Proof of Proposition 2.6

**Proof.** As before, we assume \( N \) identical (symmetric) renewable producers. Let us define our inequality constraint for producer \( i \) by

\[ g(q_{ir_1}, q_{ir_2}) := q_{ir_1} + q_{ir_2} - Q_i \leq 0 \]  \quad (2.40)

Then the Lagrange function is

\[
L(q_{ir_1}, q_{ir_2}, \lambda) := q_{ir_1} \left( a_1 \left( D - q_{ir_1} - q_{jr_1} (N - 1) \right) + b \right) + \\
q_{ir_2} \left( a_2 \left( D - q_{ir_1} - q_{ir_2} - q_{jr_1} (N - 1) - q_{jr_2} (N - 1) \right) \right) + \\
q_{ir_2} \left( b + (a_1 - a_2) \left( D - q_{ir_1} - q_{jr_1} (N - 1) \right) \right) \int f_i(Q_i) dQ_i + \\
\lambda (Q_i - q_{ir_1} - q_{ir_2}),
\]

which is the corresponding profit function of the first and second stage minus the function \( g \).
The conditions of the KKT which need to be fulfilled are

Stationarity: \[ \frac{\partial L}{\partial q_{irk}} = 0 \quad k = \{1, 2\} \tag{2.42} \]

Primal feasibility: \[ q_{ir1} + q_{ir2} \leq Q_i \tag{2.43} \]

Dual feasibility: \[ \lambda \geq 0 \tag{2.44} \]

Complementary slackness: \[ \lambda (q_{ir1} + q_{ir2} - Q_i) = 0. \tag{2.45} \]

We need to consider two cases: \( \lambda = 0 \) or \( q_{ir1} + q_{ir2} = Q_i \) (binding capacity constraint).

To case 1 (\( \lambda = 0 \)):

From (2.42) we derive two equations which we can solve for \( q_{ir1} \) and \( q_{ir2} \). Since we focus on symmetric probability distribution functions \( f_i \) for the renewable production, we can substitute \( \int f_i(Q_i) dQ_i = 1 \). Furthermore, due to symmetric renewable producers, we can plug in \( q_{ir1} = q_{jr1} \) and \( q_{ir2} = q_{jr2} \) for all renewable producers \( i \) and \( j \). Therefore, the equilibrium solution aggregated for all identical renewable producers are

\[ q_{r1}^* = \frac{a_2 N (N + 1) - a_1 N}{a_2 (N + 1)^2 - a_1 N} D + \frac{1}{a_1} \frac{a_2 N (N + 1) - a_1 N}{a_2 (N + 1)^2 - a_1 N} b. \tag{2.46} \]

\[ q_{r2}^* = \frac{a_1 N}{a_2 (N + 1)^2 - a_1 N} D + \frac{N}{a_2 (N + 1)^2 - a_1 N} b. \tag{2.47} \]

Note that the individual quantities are \( q_{irk} = q_{rk}/N \) for \( k = \{1, 2\} \).

Now, we can plug the optimal quantities into the equation of the investigated case, i.e. into \( q_{r1} + q_{r1} < Q \). This gives us the threshold value above which the renewable producers start to withhold production to increase prices. The threshold is

\[ Q_{\text{threshold}} := \frac{a_2 N (N + 1)}{a_2 (N + 1)^2 - a_1 N} D + \frac{a_2 N (N + 1)}{a_1 (a_2 (N + 1)^2 - a_1 N)} b. \tag{2.48} \]

If the overall expected renewable production \( \mu_q \) exceeds this threshold, the renewable producers withhold production. Otherwise, the sold quantities are constraint and we are in case 2.

Note that the expected production has to reach a high level relative to the demand such that renewable producers withhold production. \( \mu_q \) has to be at least \( \frac{D}{2} \) (for the monopoly situation with an infinite inflexible power plant fleet) but increases with increasing number of players or more flexible power plant fleet (for a duopoly it is
at least \( \frac{2D}{3} \).

To case 2 \((q_{ir1} + q_{ir2} = Q_i)\): This is the same case as shown in Proposition 2.5. Therefore the optimal quantities for each individual renewable producer is

\[
q_{ir1}^\ast = \frac{1}{N + 1} \left( N + 1 - \frac{a_1}{a_2} \right) \mu q
\]

\[
q_{ir2}^\ast = \frac{1}{N + 1} \frac{a_1}{a_2} \mu q.
\]

and for all renewable producers together are

\[
q_{r1}^\ast = \mu q - \frac{a_1}{a_2(N + 1)} \mu q
\]

\[
q_{r2}^\ast = \frac{a_1}{a_2(N + 1)} \mu q
\]

if \( \mu q \leq \frac{a_2(N+1)}{a_2(N+1)^2-a_1N} D + \frac{a_2(N+1)}{a_1(a_2(N+1)^2-a_1N)} b \). Remember that \( N\mu q_i = \mu q \). This closes the proof. \( \square \)
3 Price Volatility in Commodity Markets with Restricted Participation

In commodity markets price volatility tends to increase with shorter contract duration. We derive a theoretical model that reproduces the price formation in two markets with altering product granularity and restricted participation. The model is empirically validated based on evidence from German electricity markets for hourly and quarter-hourly products. We find that the high price volatility is triggered by restricted participation in the market for quarter-hourly products as well as by sub-hourly variations of renewable supply and demand. Welfare implications reveal efficiency losses of EUR 96 million in 2015 that may be reduced if markets are coupled.

3.1 Introduction

Prices in commodity markets mostly reveal high price volatility, especially when contracts are settled close to physical delivery. This is particularly applicable to energy commodities such as oil, gas or electricity (Regnier, 2006). Moreover, electricity markets have additional characteristics that favor high price volatility. First, demand and supply have to be balanced at each point in time. Second, there is only limited potential to store large quantities of energy, especially in the short run. The increasing intermittent electricity generation from renewable energies, which are prone to forecast uncertainty and highly fluctuating feed-in profiles, has increased the need of short-term trading opportunities. This has lead to an establishment of new trading opportunities on the exchange where market participants are granted the option to trade products with shorter contract duration close to the point of physical delivery. In these markets, electricity is traded first with hourly and afterwards with quarter-hourly contract duration. Price variations between the respective products can be huge. Figure 3.1 illustrates the price volatility observed in first, the German day-ahead auction for hourly products and second, the intraday auction for quarter-hourly contracts on an exemplary day\(^1\). As both auctions are cleared in rapid succession, 12:00 day-ahead and 15:00 intraday, one day before physical delivery, the

\(^1\)This is the 13th of March 2015.
information of participants is almost identical. Nevertheless, Figure 3.1 seems puzzling as we observe an apparently systematic price pattern. Prices for quarter-hourly products fluctuate around the previously settled prices in the day-ahead auction and are much more volatile. In this article, we derive a fundamental explanatory approach in order to model the price relations observed.

Because price signals in short-term electricity markets may reflect an additional need for electricity market flexibility or indicate an inefficient market design, it is important to gain a deeper understanding of the underlying drivers. We therefore develop a theoretical framework to model the price formation in the day-ahead and intraday auction and empirically validate it for the German market.

![Figure 3.1: Exemplary price time series of German short-term electricity markets (2015-03-15)](image)

Research into sequential market design and price volatility has a long history. In general, the article at hand builds on the literature in the field of sequence economies. More precisely, the literature has emphasized the importance of sequential market organization in order to allocate commodities efficiently. A large and growing body of literature has investigated the interaction of sequential markets such as Green (1973) and Veit et al. (2006). Pindyck (2001) analyzes the short-term dynamics of commodity markets as well as prices and Pindyck (2004) depicts the impact of volatility on commodity prices. Closely related, Kawai (1983) derives a model in order to explain the impact of future trading on spot market dynamics. Electricity markets represent a special subset of commodity markets and previous research into sequential electricity markets has focused on short-term trading opportunities on the exchange. Against this backdrop, von Roon and Wagner (2009) as well as Borggreve and Neuhoff (2011) outline the importance of functioning short-term markets in or-
der to deal with the increasing share of renewable energies in the German power supply system and the corresponding forecast uncertainty. Ito and Reguant (2016) and Knaut and Obermüller (2016) focus on strategic behavior in sequential short-term electricity markets. Their main findings are that, under restricted market entry and imperfect competition, a systematic price premium analogous to Bernhardt and Scoones (1994) may occur in the first market stage. Additionally, there is a vast body of literature investigating the price formation in short-term electricity markets based on forecasting techniques such as time series analysis or artificial neural networks (Karakatsani and Bunn (2008), Hagemann (2013), Weron (2014), Kiesel and Paraschiv (2015)).

We analyze the price formation in sequential short-term electricity markets based on a fundamental approach. To the best of our knowledge there is no prior literature with focus on the fundamental interaction of sequential markets with differing product granularities. It has to be stressed that we neglect the influence of uncertainty due to rapid succession of both investigated markets. Rather to the contrary, we derive a theoretical model illustrating that the high volatility of quarter-hourly intraday prices is mainly driven by two aspects. First, the main purpose of trading in the intraday auction is to balance sub-hourly variations of demand and renewable generation. Second, we find an average increase of the quarter-hourly gradient of the supply curve compared to the day-ahead auction due to restricted participation in the intraday auction. We apply an empirical analysis of historical price data and validate our theoretical considerations. Furthermore, we quantify the increase of the supply curve gradients. Based on the respective estimates, we relate restricted participation in the intraday auction to welfare losses of about EUR 96 million in 2015. We expect these inefficiencies to increase with an augmented share of renewable energies, as they increase the need for trading of sub-hourly contracts. These losses could be reduced by the introduction of market coupling for sub-hourly products or by increased short-term power market flexibility, such as storage technologies.

The article is structured as follows. First, we briefly depict the price formation in the markets of interest (Section 3.2). We then address our main research questions by conducting empirical analyses that are outlined in Section 3.3. We use historical price data for the intraday auction in Germany. Finally, conclusions are drawn in Section 3.4.
3.2 Price Formation in the Day-Ahead and Intraday Auction

Electricity is traded sequentially at various points in time. Trading opportunities increase closer to the time of physical delivery and the contract duration for different products decreases. Figure 3.2 depicts the time line of trading for the German wholesale electricity market. Trading on the exchange starts with futures that are traded for yearly, quarterly, monthly or weekly time intervals. These markets are mainly used for risk hedging purposes and financial trading. In contrast, in the day-ahead auction physical electricity is traded at hourly time intervals. The respective auction is held at noon (12:00), one day before physical delivery. Historically, the day-ahead price has been the most important reference price for all electricity market participants. At the end of 2014, the intraday auction has been implemented which is settled at 3pm and first allows for trading 15-minute contracts. As a consequence, market participants are now able to balance sub-hourly variations of supply and demand. Subsequently, trading is organized in a continuous intraday market, where trade takes place on a first-come-first-serve basis via an open order book. Gate closure is 30 minutes before physical delivery and the respective products include hourly as well as 15-minute contracts. The continuous intraday market is mainly used to balance forecast errors based on updated information until delivery. The end of the intraday trading period marks the end of electricity trading in the wholesale market.

In this article, we focus on the interaction of the day-ahead and intraday auction. Both markets are settled in rapid succession and differ in terms of product granularity (hourly/quarter-hourly). As the intraday auction is settled three hours after the day-ahead auction, we consider new information to be negligible between both market stages. Based on additional empirical evidence, we abstract from the impact of forecast errors that are expected to rather influence continuous intraday trade. In contrast, we find that the relation of prices in both markets under consideration is
mainly driven by restricted participation in the intraday auction. This may especially be the case because participation in the intraday auction is restricted to a national level and cross-border trade is not possible. Markets with quarter-hourly contracts are not coupled within the internal European electricity market in contrast to the hourly day-ahead auction. Additional reasons for restricted participation in the intraday auction may also be a lack of short-term flexibility regarding different types of conventional power plants, additional costs of market entry, and a slow adjustment of market participants to newly emerging trading opportunities. Our explanatory approach that aims at modeling the price formation in the intraday auction is consequently based on restricted participation as the main driver of the price relations under consideration.

### 3.2.1 Theoretical Model

We use a stylized theoretical model in order to depict the market interaction as well as the price formation in the day-ahead and intraday auction. In general, we consider two types of suppliers (restricted and unrestricted) which interact in two markets (day-ahead and intraday auction) that differ in terms of product granularity and participation. Both types of suppliers participate in the market for hourly products, which can be regarded as the day-ahead auction. In the second market (intraday auction) products are traded with shorter contract duration and only unrestricted suppliers are able to participate. More precisely, the common product that can be supplied by both types of suppliers is further split into \( n \) different sub-products in the intraday auction which are identified by \( \tau \in 1, 2, ..., n \).

Consumers may demand a different quantity \( D_\tau \) in each time interval \( \tau \). The demand is satisfied under perfect competition by both restricted and unrestricted suppliers. Both suppliers operate generation plants with increasing marginal costs of generation. The unrestricted suppliers offer the quantity \( q^u_\tau \) reflecting the production level in \( \tau \) that results from supply in both markets. The respective total costs are \( C_u(q^u_\tau) \). In contrast, the restricted players are not able to participate in the sub-hourly market and do not vary their production level along the time intervals \( \tau \). Thus, the respective supply is kept constant at a level of \( q^r \) over \( n \) time intervals. The total costs for the restricted players in time interval \( \tau \) amount to \( C_r(q^r) \). Due to rapid succession of both market settlements, we assume information in both markets to be identical. As the quantities of both types of suppliers are chosen under perfect competition, we formulate the following optimization problem minimizing the total
costs of electricity generation such that supply meets demand:

\[
\min z = \sum_{\tau} [C_u(q^u_\tau) + C_r(q^r_\tau)] \tag{3.1}
\]

s.t. \( D_\tau = q^u_\tau + q^r \quad \forall t. \tag{3.2} \)

In order to derive an optimal solution, we transform the problem into its Lagrangian representation by introducing the shadow prices \( p_\tau \):

\[
\mathcal{L} = \sum_{\tau} [C_u(q^u_\tau) + C_r(q^r_\tau) + p_\tau (D_\tau - q^u_\tau - q^r)] \tag{3.3}
\]

Applying the Karush-Kuhn-Tucker conditions, we derive the necessary conditions that characterize the cost minimal solution. We get the optimal quantities \( q^u_\tau \) and \( q^r \) as well as the respective shadow prices \( p_\tau \).

\[
\frac{\partial \mathcal{L}}{\partial q^r} = \sum_{\tau} [C_r'(q^r) - p^*_\tau] = 0 \quad \rightarrow C_r'(q^r) = \frac{\sum_{\tau} p^*_\tau}{n} \tag{3.4}
\]

\[
\frac{\partial \mathcal{L}}{\partial q^u_\tau} = C_u'(q^u_\tau) - p^*_\tau = 0 \quad \rightarrow p^*_\tau = C_u'(q^u_\tau) \tag{3.5}
\]

Due to illustration purposes, we apply the general model to a framework assuming linear marginal cost functions of both restricted and unrestricted suppliers. However, the following considerations could analogically be applied to different types of supply functions. Exemplary linear marginal cost functions are displayed in Figure 3.3. We formulate the respective marginal cost functions for both suppliers as

**Restricted suppliers:** \( C_r'(q^r) = a_0 + a_r^t q^r \tag{3.6} \)

**Unrestricted suppliers:** \( C_u'(q^u_\tau) = a_0 + a_u^t q^u_\tau \), \( \tag{3.7} \)

where \( a_0 \) is the offset, \( a_r^t \) is the gradient of the restricted supply curve and \( a_u^t \) is the gradient of the unrestricted supply curve.\(^2\) Adding both functions horizontally we attain the aggregate supply function as

\[
C'(q) = a_0 + \frac{a_r^t a_u^t}{a_r^t + a_u^t} q = a_0 + a_1 q, \tag{3.8}
\]

with \( a_1 = \frac{a_r^t a_u^t}{a_r^t + a_u^t} \) being the gradient of the aggregate supply function. We now

\(^2\)We assume the offset \( (a_0) \) of both marginal cost functions to be identical.
3.2 Price Formation in the Day-Ahead and Intraday Auction

solve the linear model with respect to optimal quantities and prices.

![Diagram](image)

Figure 3.3: Marginal cost functions of restricted and unrestricted suppliers and the resulting aggregate marginal cost function

**Proposition 3.1.** The average price over all periods ($\bar{p}$) is determined by the intersection of the aggregate supply function (including restricted as well as unrestricted suppliers) and the average demand ($\bar{D}$)

$$\bar{p} = a_0 + a_1 \bar{D}. \quad (3.9)$$

**Proof.** Making use of the linear marginal cost functions, we can plug in (3.5) and (3.2) into (3.4). As a result, we get

$$a_0 + a_1 q_{r*} = \frac{1}{n} \sum_{\tau} a_0 + a_1^u (D_{\tau} - q_{r*}). \quad (3.10)$$

Defining the average demand over $n$ periods as $\bar{D} = \frac{1}{n} \sum_{\tau} D_{\tau}$ and solving for $q_{r*}$, we obtain the quantity that is produced by the restricted suppliers as

$$q_{r*} = \frac{\bar{D}a_1^u}{a_1^r + a_1^u}. \quad (3.11)$$

Furthermore, based on (3.4), the average price $\bar{p} = \frac{1}{n} \sum_{\tau} p_{\tau}$ is determined by the marginal generation costs of the restricted suppliers. By plugging in the result for $q_{r*}$, we obtain the average price as the previously calculated aggregate marginal cost function for the average demand ($\bar{D}$) in (3.9).

The average price $\bar{p}$ may be regarded as the settlement price in the first market where both types of suppliers are able to participate. In a next step, we derive the prices for each time period $\tau$ that are settled in the second market where only unrestricted suppliers are participating.
3. Price Volatility in Commodity Markets with Restricted Participation

**Proposition 3.2.** The price in each time period \( \tau \) depends on the difference between the average demand and the demand in each time period \( \tau \) \( (D_\tau) \) as well as the gradient of the unrestricted supply curve.

\[
p^*_\tau = a_0 + a_1 \bar{D} + (D_\tau - \bar{D})a_1^u = p + (D_\tau - \bar{D})a_1^u \tag{3.12}
\]

**Proof.** Based on the previously derived quantity \( q_r \) from (3.11) and (3.2) in (3.5), we obtain

\[
p^*_\tau = a_0 - \frac{(a_1^u)^2}{a_1^r + a_1^u} \bar{D} + a_1^u D_\tau. \tag{3.13}
\]

Here the first term is the offset of the aggregate supply function \( (a_0) \). Furthermore, we make use of the following equation

\[
\frac{(a_1^u)^2}{a_1^r + a_1^u} = a_1^u - \frac{a_1^u a_1^r}{a_1^r + a_1^u} = a_1^u - a_1 \tag{3.14}
\]

and introduce the gradient of the aggregate supply function \( (a_1) \). By inserting the term into (3.13) and reformulating, we obtain (3.12).

The optimal prices and quantities relate to the second-best outcome, given that restricted suppliers are not able to change their production level at a temporal resolution \( \tau \). If the restricted suppliers were able to adjust their production level, efficiency would be increased.

**Proposition 3.3.** The welfare loss due to restricted participation is given by

\[
\Delta W_\tau = W^{eff}_\tau - W^{ineff}_\tau = \frac{1}{2}(a_1^u - a_1)(\bar{D} - D_\tau)^2 \geq 0. \tag{3.15}
\]

**Proof.** Because we assume a perfectly inelastic demand, we derive welfare implications based on cost considerations. Assuming restricted participation of some suppliers, the total costs to satisfy demand in period \( \tau \) amount to

\[
C^{ineff}(D_\tau) = C_u(q^u_\tau) + C_r(q^r)
\]

\[
= a_0(D_\tau - q^*_\tau) + \frac{(a_1^u)^2}{2}(D_\tau - q^*_\tau)^2 + a_0 q^*_\tau + \frac{(a_1^r)^2}{2}(q^*_r)^2. \tag{3.16}
\]

The efficient outcome could be achieved if both suppliers were able to adjust their production level in each time period \( \tau \) without restrictions. As a result, this would
lead to costs that are determined by plugging in \( D_\tau \) into the aggregate supply function (3.8).

\[
C^{\text{eff}}(D_\tau) = a_0 + \frac{a_1^r a_1^u}{a_1^r + a_1^u} D_\tau. \tag{3.17}
\]

Analyzing the difference between costs in the efficient and inefficient cases and inserting the result from (3.11), we get the total deadweight loss defined as

\[
\Delta W_\tau = C^{\text{indef}}(D_\tau) - C^{\text{eff}}(D_\tau)
= \frac{1}{2a_1^r + 2a_1^u} \left( \overline{D}^2 (a_1^u)^2 - 2\overline{D}(a_1^u)^2D_\tau + (a_1^u)^2D_\tau^2 \right). \tag{3.18}
\]

By rewriting and simplifying we finally obtain (3.15).

Welfare losses from restricted participation essentially depend on (1) the difference between the gradient of the supply curve of unrestricted suppliers and the aggregate supply function \((a_1^u - a_1^r)\), and (2) the volatility of demand \((D_\tau - D)\). We thus identify two major drivers of welfare losses and derive the following relations. First, if fewer suppliers are participating in both markets, this will increase the gradient \(a_1^u\) and lead to an increase in welfare losses. Second, the higher the volatility of demand in time periods \(\tau\), the higher the overall welfare losses.

The consumers that determine the inelastic demand and suppliers are affected in different ways.

**Proposition 3.4.** Compared to the case of unrestricted participation, restricted participation leads to losses in consumer surplus and producer surplus of restricted suppliers. Producer surplus of unrestricted suppliers increases.

**Proof.** See Appendix 3.5.1.

In Appendix 3.5.1 we derive that consumer surplus is significantly reduced compared to the efficient outcome. The respective consumer losses are twice as high as the total welfare losses \((2\Delta W = 2\sum_{\tau=1}^n \Delta W_\tau)\). On the opposite side, suppliers altogether profit from the inefficiency. Taking a closer look at the distributional effects between restricted and unrestricted suppliers, we find that only unrestricted suppliers face a higher surplus if market participation is restricted. The surplus of restricted suppliers is lower compared to the efficient case.
3.2.2 Application to Intraday Auction Prices

Applying the previous model to real-world electricity market dynamics, we are able to depict the fundamental causal relations that drive the price relations between the German day-ahead and intraday auction. Therefore, it is first necessary to comment on some basic assumptions made in the stylized theoretical framework.

In the context of electricity market analyses, the demand side is most commonly modeled using the term residual demand. We follow this approach and define the residual demand as total demand minus the electricity generation from wind and solar power:

\[ D_{res}^t = D_t - Wind_t - Solar_t. \]

The generation from renewable energies is subtracted because they are characterized by short-term marginal costs close to zero and the respective electricity generation corresponds to the availability of wind and solar power at each point in time. Trade in electricity markets is performed by balancing responsible parties which are responsible to balance supply and demand within their balancing group. Because balancing group operators have the incentive to be balanced in each time interval to avoid penalties, the residual demand is expected to drive the level of trade volumes in electricity spot markets. Furthermore, in the existing literature evidence is given that demand in electricity markets can be assumed to be rather price inelastic, especially in the short-run (Knaut and Paulus, 2016, Lijesen, 2007). Within our model, we thus do not consider any price elasticity of demand.

The residual demand is supplied by conventional generation units with increasing marginal costs depending on the underlying energy carrier. In our model we assume the marginal cost functions to be linear. As far as the day-ahead auction is concerned, we clearly observe a rather linear relation of residual demand and the respective prices in historical data (for more details see Section 3.5.2). In contrast, the structure of the intraday auction supply curve may vary in individual hours as the underlying market dynamics are crucially depending on the day-ahead market clearing point. However, within the scope of this article, we use an aggregate explanatory approach that focuses on general price relations. We find empirical evidence that these relations can be adequately mapped based on the assumption of linear relations. Further details are given in the empirical part of this article.

In general, the assumption of perfect competition seems approximately appropriate for the German day-ahead and intraday markets.\(^3\)

We additionally assume a simultaneous decision of restricted and unrestricted sup-

\(^3\) See the findings of the Monitoring Report by the German regulator (Bundesnetzagentur, 2016).
3.2 Price Formation in the Day-Ahead and Intraday Auction

pliers regarding their production quantities. In reality, however, the settlement of the day-ahead and intraday auction is determined in sequential order and the unrestricted suppliers reflect the subset of all suppliers that are able to participate in both markets. Based on a continuous interaction of all market participants, we assume an absence of unexploited arbitrage opportunities between both markets following general economic theory (see, e.g., Harrison and Kreps (1979) and Delbaen and Schachermayer (1994)). More precisely, sequential markets should exhibit identical average price levels under the following conditions (Mercadal, 2015):

- First, prices should be transparent, unambiguous and accessible to each market participant.
- Second, prices should refer to identical products and the respective products should be perfect substitutes. More precisely, they should be valid for electricity supply at the same point in time.
- Third, prices should be based on the same and latest available information.

The three conditions are crucial in order to expect mean price equivalence between the day-ahead and intraday auction. Going into detail, trade in both auctions is processed on the exchange and information transparency is given at each point in time. Sequential settlement goes hand in hand with day-ahead prices being reference prices for bids in the subsequent intraday auction. Furthermore, there is no discrimination of individual players. As a consequence, we claim that the first condition is met. Second, intraday auction products combined represent perfect substitutes for day-ahead contracts. Additionally, contracts in both auctions refer to the physical delivery of electricity. As a consequence, the second condition is valid as well. Finally, the day-ahead and intraday auction are settled in rapid succession. Forecast errors that appear until delivery are rather balanced within continuous intraday trade that starts after the intraday auction gate closure. We have tested and validated these assumptions empirically. To sum up, we suggest that the three conditions as listed above are valid regarding day-ahead and intraday auction market dynamics. In fact, a descriptive analysis of historical price data reveals that the average day-ahead and intraday auction prices equal within our period of observations (see Section 3.3.1). Based on the previous considerations, we equate the hourly average price in Equation (3.12) and the hourly day-ahead auction price.
3.2.3 Illustrative Insights Derived From the Theoretical Model

Based on our theoretical model, we gain insights into the price relations in markets with low and high product granularity under restricted participation. For the case of the day-ahead and intraday auction this means that hourly products are further divided into quarter-hourly products ($\tau \in 1, 2, 3, 4$). We illustrate the respective implications for an exemplary hour in Figure 3.4 and describe the price formation in more detail. The day-ahead supply curve reflects the aggregate marginal cost function ($C'(q)$) because market participation is considered to be unrestricted. In contrast, the gradient of the intraday auction supply curve equals the gradient of the supply curve of unrestricted producers ($a_{u1}$). As we model intraday auction prices as deviations from the respective day-ahead prices, we can project this gradient into the day-ahead market clearing point according to Equation (3.12). Differences between the quarter-hourly and hourly mean of the residual demand ($D_{\tau} - \bar{D}$) are now transferred into movements along the 15-minute supply curve and result in quarter-hourly intraday auction prices.

![Figure 3.4: Supply and demand in the hourly and quarter-hourly market](image)

When we transfer these relations to subsequent hours as depicted in Figure 3.5, one can observe a distinct pattern of prices. Prices for quarter-hourly products fluctuate around the respective prices for hourly contracts as illustrated by the green price time series. If the participation in the intraday auction would not be restricted, the gradients of the supply curves would be equal in both markets and prices would follow the curve of the fictitious quarter-hourly residual demand level as marked in blue.
3.3 Empirical Analysis

Based on Figure 3.5, we observe three typical price movements. First, for an increasing residual demand, prices in the first quarter-hour are significantly lower compared to the respective ones in the last quarter-hourly time interval of the hour. Second, with a decreasing demand, the opposite is the case. Third, a flat demand profile leads to a low price variation.

So far, the model suggests that the high price volatility in sequential electricity markets is mainly driven by two aspects. First, quarter-hourly prices are driven by quarter-hourly deviations of the residual demand from the respective hourly means. Second, the high volatility of prices stems from restricted participation of some suppliers which results in an inclination of the supply curve from the first (hourly) to the second market stage (quarter-hourly) ($a^u_1 > a_1$).

3.3 Empirical Analysis

By analyzing the time period from January 2015 to the end of February 2016, we are able to test the applicability of our theoretical model to historical data. Furthermore, we intend to quantify the inclination of the supply curve between the day-ahead and intraday auction and gain insights on welfare implications. We first give a short
overview on the historical data used. We then describe our estimation approach and, finally, we depict and evaluate the empirical results.

3.3.1 Data

This section gives an overview on relevant data included in the empirical estimation and the respective references. Due to the recent implementation of the intraday auction on 9 December 2014, the analysis includes data from January 2015 until the end of February 2016.

A detailed list of all variables that are used in the empirical analyses in the following sections is presented in Table 3.1. The table includes a brief explanation for each variable and the symbols that we use in order to depict our empirical models and the respective estimation equations. Additionally, Table 3.2 provides information on the most relevant descriptive statistics.

Price data for German electricity markets can be obtained from the European Power Exchange (EPEX, 2016). Trades in the day-ahead and intraday auction take place one day before physical delivery and are based on expectations for the level of demand and generation from wind and solar power. Forecasts for renewable generation are provided by the four German transmission system operators (TSOs) who are in charge of the reliable operation of the power system. We make use of the day-ahead forecasts for wind and solar power based electricity generation published on the transparency platform of the European Energy Exchange (EEX, 2016).

In addition to forecasts on the renewable energy feed-in, the four TSOs generate and publish load forecasts. Load is commonly considered as the best proxy for electricity demand and is therefore used within the framework of our empirical analysis. We use load data that is published on the transparency platform of the European Network of Transmission System Operators for Electricity (ENTSO-E, 2016).

3.3.2 Empirical Estimations

We apply a multistage approach in order to analyze the validity of our underlying theoretical model empirically. In more detail, we are first interested in the applicability of the model with respect to historical prices observed in the intraday auction. Several robustness checks are made. Second, we set up an empirical approach that

5 More information on load can be found in Schumacher and Hirth (2015)
3.3 Empirical Analysis

Table 3.1: List of variables and references

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Label</th>
<th>Variable</th>
<th>Measure</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{ida}^{t}$</td>
<td>id auction price</td>
<td>Uniform settlement price for a 15-minute product in the German intraday auction</td>
<td>EUR/MWh</td>
<td>EPEX (2016)</td>
</tr>
<tr>
<td>$p_{da}^{t}$</td>
<td>day-ahead price</td>
<td>Hourly German day-ahead auction price</td>
<td>EUR/MWh</td>
<td>EPEX (2016)</td>
</tr>
<tr>
<td>$D_{i}^{res}; D_{i}^{res}$</td>
<td>residual demand</td>
<td>Day-ahead forecast for the residual demand in a 15-minute period and the respective hourly mean (ex-ante value)</td>
<td>GW</td>
<td>EEX (2016), ENTSO-E (2016)</td>
</tr>
<tr>
<td>$\Delta D_{i}^{res}$</td>
<td>residual demand deviation</td>
<td>Difference of the 15-minute residual demand and the respective hourly mean</td>
<td>GW</td>
<td>EEX (2016), ENTSO-E (2016)</td>
</tr>
<tr>
<td>$Solar_{i}; Solar_{i}$</td>
<td>solar power 15; solar power 60</td>
<td>Day-ahead forecast for the 15-minute solar power and the respective hourly mean (ex-ante value)</td>
<td>GW</td>
<td>EEX (2016)</td>
</tr>
<tr>
<td>$\Delta Solar_{i}$</td>
<td>solar power deviation</td>
<td>Difference of the 15-minute solar power and the respective hourly mean</td>
<td>GW</td>
<td>EEX (2016)</td>
</tr>
<tr>
<td>$Wind_{i}; Wind_{i}$</td>
<td>wind power 15; wind power 60</td>
<td>Day-ahead forecast for the 15-minute wind power and the respective hourly mean (ex-ante value)</td>
<td>GW</td>
<td>EEX (2016)</td>
</tr>
<tr>
<td>$\Delta Wind_{i}$</td>
<td>wind power deviation</td>
<td>Difference of the 15-minute wind power and the respective hourly mean</td>
<td>GW</td>
<td>EEX (2016)</td>
</tr>
<tr>
<td>$D_{i}; D_{i}$</td>
<td>load 15; load 60</td>
<td>Day-ahead forecast for the 15-minute load and the respective hourly mean (ex-ante value)</td>
<td>GW</td>
<td>ENTSO-E (2016)</td>
</tr>
<tr>
<td>$\Delta D_{i}$</td>
<td>load deviation</td>
<td>Difference of the 15-minute load and the respective hourly mean</td>
<td>GW</td>
<td>ENTSO-E (2016)</td>
</tr>
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</table>

Table 3.2: Descriptive statistics

<table>
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<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Std.Dev.</th>
<th>Min</th>
<th>25%</th>
<th>Median</th>
<th>75%</th>
<th>Max</th>
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<tr>
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<td>30.9</td>
<td>14.8</td>
<td>-164.5</td>
<td>21.7</td>
<td>30.5</td>
<td>40.1</td>
<td>464.4</td>
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<td>23.9</td>
<td>29.9</td>
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<td>99.8</td>
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<td>41.7</td>
<td>11.0</td>
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<td>34.2</td>
<td>41.6</td>
<td>49.6</td>
<td>70.6</td>
</tr>
<tr>
<td>residual demand 60</td>
<td>38,640</td>
<td>41.7</td>
<td>11.0</td>
<td>6.3</td>
<td>34.2</td>
<td>41.6</td>
<td>49.6</td>
<td>70.6</td>
</tr>
<tr>
<td>residual demand deviation</td>
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<td>0.0</td>
<td>0.9</td>
<td>-9.1</td>
<td>-0.4</td>
<td>0.0</td>
<td>0.4</td>
<td>9.5</td>
</tr>
<tr>
<td>solar power 15</td>
<td>38,640</td>
<td>3.6</td>
<td>5.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>5.4</td>
<td>25.8</td>
</tr>
<tr>
<td>solar power deviation</td>
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<td>0.5</td>
<td>-6.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>4.5</td>
</tr>
<tr>
<td>wind power 15</td>
<td>38,640</td>
<td>9.6</td>
<td>7.6</td>
<td>0.2</td>
<td>3.7</td>
<td>7.4</td>
<td>13.9</td>
<td>33.6</td>
</tr>
<tr>
<td>wind power deviation</td>
<td>38,640</td>
<td>0.0</td>
<td>0.2</td>
<td>-1.6</td>
<td>-0.1</td>
<td>0.0</td>
<td>0.0</td>
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</tr>
<tr>
<td>load 15</td>
<td>38,640</td>
<td>55.0</td>
<td>9.9</td>
<td>31.7</td>
<td>46.7</td>
<td>54.5</td>
<td>64.1</td>
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<td>load deviation</td>
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<td>0.0</td>
<td>0.8</td>
<td>-8.3</td>
<td>-0.4</td>
<td>0.0</td>
<td>0.4</td>
<td>9.5</td>
</tr>
</tbody>
</table>
allows us to derive conclusions on the fundamental relation of the gradient of the aggregate supply curve in the day-ahead market and the supply curve of unrestricted suppliers in the subsequent intraday auction with quarter-hourly contract duration.

Empirical Framework

As outlined above, the econometric approach adopted within this article aims at depicting the price formation for quarter-hourly products in the intraday auction. The general estimation procedure is formulated in Equation (3.19):

$$p_t = X'_{i,t} \beta_i + \nu + \epsilon_t$$

with $\epsilon_t \sim N(0, \sigma^2)$,

(3.19)

where $p_t$ denotes the quarter-hourly price in period $t = 1, 2, ..., T$. $X'_{i,t}$ includes the exogenous variables of the model, namely the hourly day-ahead price as well as the quarter-hourly deviation of the residual demand from its respective hourly mean value. We consider the intercept $\nu$ being the estimated constant assuming that the underlying supply function is time-invariant. $\epsilon_t$ denotes the error term. In order to choose a suitable estimation methodology, we first test for basic assumptions that would be required if applying Ordinary Least Squares Regression techniques. These are standard assumptions such as predetermination or exogeneity of regressors and $p_t, X_{i,t}$ being ergodic and jointly stationary.

Beginning with stationarity, we apply two different statistical tests for unit roots. The respective results of an Augmented Dickey Fuller test and a Phillips-Perron test are depicted in detail in Appendix 3.5.2. The statistics clearly reject the assumption of non-stationary processes. This is especially plausible because we only include data for a limited period of observations. During these 14 months the underlying drivers of demand and supply as well as prices in the markets of interest only changed slightly. These are, e.g., fuel prices and the share of renewable power plants. A significant time trend is not identified.

By using forecasted data, we guarantee exogeneity of the residual demand by construction. We furthermore conduct a Durbin-Wu-Hausman test in order to control for the exogeneity of the day-ahead auction price. The test results reject the assumption of exogeneity $^6$ and we thus use a Two-Stage Least Squares (2SLS) Regression Analysis including the hourly average of the residual demand as an instrument for

$^6$In more detail, the test suggests that $\text{Cov}(X'_{i,t}, \epsilon_t) \neq 0$
3.3 Empirical Analysis

the day-ahead price. The hourly residual demand is the main driver of demand in the day-ahead auction and thus is highly correlated with the respective prices \((\text{Cov}(X_{i,t}, Z_{i,t}) \neq 0, \text{where } Z_{i,t} \text{ is the instrument})\). This assumption is supported by the first stage regression results giving clear empirical evidence for a strong instrument. Additionally, we argue that our underlying estimation approach directly accounts for the exclusion restriction \((\text{Cov}(Z_{i,t}, \epsilon_t) = 0)\). All information from the first market that can be expected to influence quarter-hourly product prices in the second market is incorporated by the inclusion of the day-ahead price. Finally, we use robust standard errors in order to account for heteroscedasticity.

Empirical Validation

In a first step, the aim of our empirical analysis is to validate the theoretical model as depicted in Section 3.2.1. Based on the model Equation (3.12) and according to Section 3.3.2, we apply Equation (3.20) using a Two-Stage Least Squares Regression:

\[
p^\text{ida}_t = \beta_1 \cdot p^\text{da}_t + \beta_2 \cdot (D^\text{res}_t - \overline{D^\text{res}_t}) + \nu + \epsilon_t
\]

\[
= \beta_1 \cdot p^\text{da}_t + a^u_1 \cdot \Delta D^\text{res}_t + \nu + \epsilon_t. \tag{3.20}
\]

The difference between the residual demand on a quarter-hourly and hourly level (residual demand deviation \((\Delta D^\text{res}_t))\) is included as the main explanatory variable. Besides, the day-ahead auction price for hourly products (day-ahead price \((p^\text{da}_t))\) is used. We use forecast values for the residual demand as trading decisions in the day-ahead and intraday auction are made under uncertainty. The coefficient \(\beta_2\) can be interpreted as the gradient of the unrestricted supply curve \((a^u_t)\).

The resulting estimates are depicted in column (1) of Table 3.3. Additional robustness checks have been conducted and we show the respective results in columns (2) - (3). The latter tests will be explained in more detail below.

The estimates in column (1) of Table 3.3 indicate that our theoretical model is applicable to actual price relations observed in the intraday and day-ahead auction. We observe an adjusted \(R^2\) that is close to 85% and thus a large part of the variance of intraday auction prices can be explained by the model. Additionally, the t-values of the coefficients validate that the difference in prices is influenced significantly by the deviation of the residual demand on a quarter-hourly level from its hourly mean. Furthermore, the estimated coefficient with respect to the day-ahead auction price is close to one, as suggested by the model. Thus, the regression results confirm the
### Table 3.3: Regression estimates for intraday auction price data

<table>
<thead>
<tr>
<th>Explanatory variable</th>
<th>IV (1)</th>
<th>IV (2)</th>
<th>IV (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>day-ahead price ($p_{t}^{da}$)</td>
<td>0.94***</td>
<td>0.94***</td>
<td>0.94***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>residual demand deviation ($\Delta P_{t}^{res}$)</td>
<td>7.80***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>positive residual demand deviation</td>
<td>7.96***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>negative residual demand deviation</td>
<td>7.65***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>wind power deviation ($\Delta Wind_{t}$)</td>
<td></td>
<td>-9.38***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.21)</td>
<td></td>
</tr>
<tr>
<td>solar power deviation ($\Delta Solar_{t}$)</td>
<td></td>
<td>-10.08***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.08)</td>
<td></td>
</tr>
<tr>
<td>load deviation ($\Delta D_{t}$)</td>
<td></td>
<td>6.0***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.12)</td>
<td></td>
</tr>
<tr>
<td>intercept ($\nu$)</td>
<td>1.99***</td>
<td>1.91***</td>
<td>2.10***</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.14)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>observations</td>
<td>38,640</td>
<td>38,640</td>
<td>38,640</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.84</td>
<td>0.84</td>
<td>0.85</td>
</tr>
<tr>
<td>F</td>
<td>46,650</td>
<td>33,840</td>
<td>30,390</td>
</tr>
</tbody>
</table>

Notes to Table 3.3: Robust standard errors in parentheses. ∗ / ∗∗ / ∗∗∗ : significant at the 0.05 / 0.02 / 0.01 error level respectively. The term positive residual demand deviation in column (2) is constructed using a dummy variable that equals one if the residual demand deviation is positive. The term negative residual demand deviation is constructed using a dummy variable that equals one if the residual demand deviation is negative. Due to indication of endogeneity of day-ahead price we use residual demand deviation 60 as instrumental variable and apply a 2SLS Regression. In general, we use data from January 2015 until the end of February 2016.

Validity of day-ahead auction prices as reference prices for intraday auction prices. If we estimate the model without an intercept, the coefficient of the day-ahead price even equals one.\(^7\)

The estimated coefficient for residual demand deviation reveals a positive sign and can be interpreted as the gradient of the supply curve in the intraday auction. The positive coefficient means that a positive deviation of the residual demand leads to

\(^7\)In more detail, the respective estimation results show a coefficient for day-ahead price that is 0.998 and a robust standard error of 0.0001
an increase of quarter-hourly prices compared to the respective hourly day-ahead price. This causal relation is in line with our theoretical model assumptions. The average absolute value for *residual demand deviation* amounts to 0.590 GW and can be transferred into an absolute price difference of 4.6 EUR/MWh. To sum up, we find evidence that restricted participation in combination with highly variable demand indeed triggers the high volatility of intraday auction prices observed.

In a next step, we are interested in the robustness of the results. One underlying assumption of our model considers identical gradients of the supply curve for a positive as well as a negative deviation of the residual demand. More precisely, we assume a linear relation of prices and quantities in the intraday auction reflecting the underlying supply curves. We test this assumptions by distinguishing between positive and negative differences of the residual demand in the regression (*positive residual demand deviation* and *negative residual demand deviation*). The respective results are shown in column (2) of Table 3.3. The coefficients for the positive and negative residual demand deviation only exhibit slight differences. However, the overall picture strongly supports the hypothesis of a continuous linear relation between supply and prices in the intraday auction.\(^8\)

In order to gain additional insights with respect to the different drivers of the residual demand, we conduct an additional regression. The results are displayed in column (3). Here, we decompose the residual demand deviation into its three elements *wind power deviation*, *solar power deviation* and *load deviation*. The respective estimates reveal variations as has to be expected because high variations of wind and solar power as well as load do not fully coincide. For illustration purposes, electricity generation from solar power is only present in distinct hours when the sun is shining. As a consequence, when disentangling the individual drivers, we measure the average coefficient of the quarter-hourly supply curve only in a subset of hours. Based on these considerations, different coefficients for solar power, wind power and load are not surprising. On the contrary, it is rather important to evaluate whether the signs of the coefficients match the underlying causal relations. A positive deviation of the renewable energy generation implies oversupply which in turn causes lower prices in the intraday auction. In line, the respective coefficients are negative whereas the coefficient for load is positive. Looking at the value distribution of solar and wind power as well as load, it is revealed that the volatility of intraday auction prices is mainly driven by the quarter-hourly variation of load. However, very high differences in prices can also result from a high gradient of so-

\(^8\)We also tested for deviant types of relations such as quadratic ones but found no empirical evidence for applicability.
lar power generation. Besides these explicitly outlined robustness tests, additional insights into seasonality, alternative hypotheses, and the methodological approach are given in Appendix 3.5.2. To sum up, we find model validity and robustness of our findings.

### Econometric Analysis of the Supply Curve Gradients

As a further part of the empirical analysis, we conduct a comparative analysis for the gradients of the supply curve in the day-ahead and intraday auction. The theoretical model as formulated in Section 3.2.1 suggests that the high price volatility is triggered by restricted participation which leads to differing supply curve gradients. This section aims at giving empirical evidence supporting this hypothesis. In order to do so, the day-ahead spot market price in Equation (3.20) is substituted by the hourly residual demand according to Equation (3.9). The purpose is to estimate $a_1$ as a proxy for the gradient of the aggregate supply curve. We thus obtain Equation (3.21):

$$p_{id}^{da} = a_1 \cdot \overline{D^{res}}_t + a_1^u \cdot \Delta D^{res}_t + \xi + \epsilon_t,$$

where the constant intercept of the hourly supply curve is shifted into the constant $\xi$ and the error-term of the estimation equation. Again, we use forecast values for the construction of the hourly and quarter-hourly residual demand in order to circumvent endogeneity. Based on these considerations, we apply an Ordinary Least Squares Regression using robust standard errors. The empirical results indicate explanatory power and a significant impact of the respective explanatory variables. We observe a slight decrease of the adjusted $R^2$ due to a loss of information by using a less informative variable ($\overline{D^{res}}_t$ instead of $p_{id}^{da}$). Furthermore, we are now able to comment on the average difference of the aggregate and unrestricted supply curve by comparing the coefficients $a_1$ and $a_1^u$. The estimation results are depicted in Table 3.4. The estimated coefficient for the impact of the quarter-hourly residual demand deviation (residual demand deviation) on intraday auction prices is more than eight times higher than the influence of the hourly residual demand (residual demand 60) on the proxy for day-ahead spot prices.

Based on the estimates for the gradients of the aggregate and unrestricted supply curve ($a_1$ and $a_1^u$), we are now able to estimate the welfare losses as derived in Equation (3.15). In 2015 the total welfare losses from restricted participation amounted
Table 3.4: Regression estimates for intraday auction price data (2)

<table>
<thead>
<tr>
<th>Explanatory variable</th>
<th>OLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>hourly residual demand ( (D_{h,t}^{res}) )</td>
<td>0.94***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
</tr>
<tr>
<td>residual demand deviation ( (\Delta D_{q-h,t}^{res}) )</td>
<td>7.8***</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
</tr>
<tr>
<td>intercept ( (\xi) )</td>
<td>-8.2***</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>observations</th>
<th>38,640</th>
</tr>
</thead>
<tbody>
<tr>
<td>adj. ( R^2 )</td>
<td>0.70</td>
</tr>
<tr>
<td>( F )</td>
<td>24,440</td>
</tr>
</tbody>
</table>

Notes to Table 3.3: Robust standard errors in parentheses. \( * / ** / *** \) : significant at the 0.05 / 0.02 / 0.01 error level respectively. We use data from July 2013 until the end of July 2015.

to EUR 96 million. When taking a closer look at the distributional effects, as derived in Appendix 3.5.1, consumer surplus is reduced by EUR 192 million. On the supply side, the surplus of unrestricted producers is increased by EUR 107 million and surplus of restricted suppliers is reduced by EUR 11 million compared to the efficient case of unrestricted participation.

We note that these calculations do not include actual costs of market entry and thus have to be regarded as an upper bound for the welfare and distributional effects from restricted participation. As a lack of market coupling is one driver of restricted participation, we may regard German power plant operators as the unrestricted suppliers. In this case, the German suppliers profit from non-coupled markets. In contrast, power plant operators in neighboring countries and German consumers suffer from the lack of market coupling. As the implementation of cross-border trade of 15-minute and even shorter contracts is planned for 2017 (Cross-Border Intraday Market Project XBID), welfare losses may decrease in the future. However, ensuring sufficient cross-border intraday capacities as well as an efficient coupling mechanism are crucial pillars that should be urged by policy makers.
3.4 Conclusion

After identifying a concurrence of strongly increasing price volatility and shortened contract duration in German short-term electricity markets, we derive a theoretical model to illustrate the respective price formation based on a fundamental approach. We consider two markets that are characterized by altering product granularity and a change in the set of suppliers along the sequential market settlement. We apply our model to the German day-ahead and intraday auction that allow for trading hourly and quarter-hourly products respectively. Our empirical results clearly indicate validity of our theoretical considerations. More precisely, we find that the high price volatility that is observed in historical price data basically is triggered by two factors. First, the variability of demand and renewable electricity generation causes a need to trade sub-hourly contracts. Second, we find that the supply curve in the intraday auction inclines compared to the day-ahead auction due to restricted market participation. Based on our estimates, we relate restricted intraday auction participation to welfare losses that amounted to EUR 96 million in 2015.

The main findings presented within the scope of this article provide a better understanding of markets with restricted participation and differing product granularity. The identification and classifications of reasons why the current spot market design reveals inefficiencies are indispensable to derive appropriate strategies of how to reduce such welfare losses. This is extremely important because the increasing share of renewable energies will lead to additional needs for sub-hourly short-term trade and thus may increase efficiency losses if the short-term flexibility potential provided in the electricity markets of interest is not increasing accordingly. Policy makers should tackle issues related to intraday market participation. Above all, a market opening may be a first step towards a more efficient market outcome. Against this backdrop, the Cross-Border Intraday Market Project planning to implement cross-border intraday trade with 15-minute and potentially even lower contract duration by 2017, is expected to make trading needs stemming from renewable electricity generation and flexibility offered in electricity markets more compatible. Our results show that this is expected to clearly reduce welfare losses. However, the provision of sufficient cross-border intraday capacity as well as the implementation of an efficient coupling mechanism should be urged.

On a more micro-economic level, a fundamental understanding of the price relations in the markets of interest can be transferred into price forecasts and may be used in order to evaluate the future market developments. Market participants need
to understand long-term drivers of price spreads in short-term electricity markets in order to assess investment decisions with respect to more flexible generation units. As of today, an exemplary profitability calculation for a battery storage unit revealed that the price volatility observed does not allow for profitable operation.

Finally, as we observe the respective price patterns not only in electricity markets, it would be worthwhile to analyze the applicability of the model to further market settings e.g. for commodities such as gas, coal or oil. However, due to the market structures being fundamentally different we leave this open for future research.
3.5 Appendix

3.5.1 Proof of Proposition 3.4 on Distributional Effects

Proof. Before taking a closer look into the distributional effects that result from restricted participation, we first derive the respective surplus of consumers and producers. We therefore consider consumers being price inelastic up to a certain threshold where the electricity price exceeds the value of lost load (VOLL). We link the VOLL to the price $p^{VOLL}$ which marks the upper limit of the willingness-to-pay regarding electricity consumption (this definition is analogous to Knaut and Obermüller (2016)). On the supply side, the producer surplus is determined by the difference of the unique market price and the marginal costs of electricity generation of each producer.

In the case of restricted participation ($ineff$) the consumer ($CS$) and producer surplus ($PS$) in each period $\tau$ are calculated as

$$CS_{\tau}^{ineff} = p^{VOLL} D_{\tau} - p_{\tau}(D_{\tau} - \overline{D})$$  \hspace{1cm} (3.22)
$$PS_{\tau}^{ineff} = D_{\tau} p_{\tau} + (D_{\tau} - \overline{D})p_{\tau} - C^{ineff}(D_{\tau}).$$  \hspace{1cm} (3.23)

If all suppliers were able to supply at resolution $\tau$, the efficient outcome ($eff$) would lead to the following consumer and producer surplus

$$CS_{\tau}^{eff} = (p^{VOLL} - p_{\tau}^{eff}) D_{\tau}$$  \hspace{1cm} (3.24)
$$PS_{\tau}^{eff} = p_{\tau}^{eff} D_{\tau} - C(D_{\tau}).$$  \hspace{1cm} (3.25)

The price in the efficient case ($p_{\tau}^{eff} = a_0 + a_1 D_{\tau}$) directly depends on the aggregate marginal cost function. The difference in consumer and producer surplus can therefore be derived as

$$\Delta CS_{\tau} = CS_{\tau}^{eff} - CS_{\tau}^{ineff}$$
$$= (a_{\tau}^u - a_1)(\overline{D} - D_{\tau})^2 + a_1 \overline{D}(\overline{D} - D_{\tau})$$
$$= 2\Delta W_{\tau} + a_1 \overline{D}(\overline{D} - D_{\tau})$$  \hspace{1cm} (3.26)
\[ \Delta PS_{\tau} = P_{\tau}^{eff} - P_{\tau}^{ineff} \]
\[
= -\frac{1}{2}(a_{\tau}^{u} - a_{\tau}^{d})(\overline{D} - D_{\tau})^{2} - a_{\tau}^{d}\overline{D}(\overline{D} - D_{\tau}) 
\]
\[
= -\Delta W_{\tau} - a_{\tau}^{d}\overline{D}(\overline{D} - D_{\tau}). \tag{3.27}
\]

We insert the previously derived welfare losses \(\Delta W_{\tau}\) for both the change in consumer and producer surplus. Summing up over the \(n\) time periods, this results in

\[ \Delta CS = \sum_{\tau=1}^{n} \Delta CS_{\tau} = 2 \sum_{\tau=1}^{n} \Delta W_{\tau} + a_{\tau}^{d}\overline{D} \sum_{\tau=1}^{n}(\overline{D} - D_{\tau}) = 2 \sum_{\tau=1}^{n} \Delta W_{\tau} \geq 0 \tag{3.28} \]

\[ \Delta PS = \sum_{\tau=1}^{n} \Delta PS_{\tau} = -\sum_{\tau=1}^{n} \Delta W_{\tau} - a_{\tau}^{d}\overline{D} \sum_{\tau=1}^{n}(\overline{D} - D_{\tau}) = -\sum_{\tau=1}^{n} \Delta W_{\tau} \leq 0. \tag{3.29} \]

The consumer surplus decreases due to restricted participation in the second market. It is twice as high as the overall welfare losses. In contrast, the producers face an increasing surplus. The respective increase amounts to the total sum of welfare losses along all time periods. As these considerations differ across restricted and unrestricted suppliers, we now analyze the respective surplus in more detail. In the inefficient case we get the following relations

\[ PS_{\tau}^{r,ineff} = \overline{p} q_{\tau}^{r} - C^{r}(q_{\tau}^{r}) \tag{3.30} \]
\[ PS_{\tau}^{u,ineff} = \overline{p}(q_{\tau}^{u}) + p_{\tau}(D_{\tau} - \overline{D}) - C^{u}(q_{\tau}^{u}). \tag{3.31} \]

In the efficient case the respective surplus would be as follows

\[ PS_{\tau}^{r,eff} = p_{\tau}^{eff} q_{\tau}^{r,eff} - C^{r}(q_{\tau}^{r,eff}) \tag{3.32} \]
\[ PS_{\tau}^{u,eff} = p_{\tau}^{eff} q_{\tau}^{u,eff} - C^{u}(q_{\tau}^{u,eff}). \tag{3.33} \]

We derive the optimal quantities supplied by restricted and unrestricted suppliers
3 Price Volatility in Commodity Markets with Restricted Participation

in the efficient case based on the aggregate supply function

\[ q_{\tau}^{r,\text{eff}} = \frac{a_1^u}{a_r + a_1^u} D_{\tau} \]  
(3.34)

\[ q_{\tau}^{u,\text{eff}} = \frac{a_1^u}{a_r + a_1^u} D_{\tau}. \]  
(3.35)

For restricted suppliers we now derive the difference in surplus

\[ \Delta PS_{\tau}^r = PS_{\tau}^{r,\text{eff}} - PS_{\tau}^{r,\text{ineff}} \]  
(3.36)

\[ \Delta PS_{\tau}^r = \frac{1}{2} a_1 (1 - \frac{a_1}{a_r^2})(D_{\tau}^2 - \bar{D}^2). \]  
(3.37)

Summing up over all time periods, we can further simplify the expression

\[ \Delta PS^r = \sum_{\tau=1}^{n} \Delta PS_{\tau}^r = \frac{1}{2} a_1 (1 - \frac{a_1}{a_r^2}) \sum_{\tau=1}^{n} (D_{\tau}^2 - \bar{D}^2) \geq 0. \]  
(3.38)

Because the variance of demand is always positive, we conclude that restricted suppliers have a lower surplus in the inefficient case. In a next step, we derive the difference in surplus for unrestricted suppliers. As we have already derived the difference in surplus for all suppliers and the respective one for restricted suppliers, we just derive the following expression.

\[ \Delta PS^u = \Delta PS - \Delta PS^r \leq 0. \]  
(3.39)

To sum up, we prove that restricted participation leads to a reduction in consumer surplus. On the other hand, unrestricted suppliers face a higher surplus in the inefficient case whereas restricted suppliers suffer from restricted participation. \( \Box \)
3.5 Appendix

3.5.2 Supplementary Information on the Econometric Approach

The Relation of Quantities and Prices in the Day-Ahead Auction

![Graph showing the relation of day-ahead quantities and prices in 2015](image)

Figure 3.6: Relation of day-ahead quantities and prices in 2015

**Unit Root Tests**

We apply both, an Augmented Dickey Fuller test and a Phillips-Perron test for unit roots which are displayed in Table 3.5 (Dickey and Fuller, 1979, Phillips and Perron, 1979). The latter test uses Newey-West standard errors in order to account for serial correlation. The null hypothesis of both is that there is a unit root in the respective period of observation. We use the Akaike Information Criterion (AIC) in order to determine the optimal lag lengths. However, the AIC results are ambiguous for some variables and tend to indicate using as many lags as tested for. In these cases we use the Schwert rule of thumb and consider a leg length of 55 (Schwert, 1989). We prefer making a slight error due to including too many lags since Monte Carlo experiments suggest that this procedure is preferable to including too few lags. In order to give evidence for the robustness of our results, we repeat the tests for different lag lengths. Within the scope of the Augmented Dickey Fuller test, we extend the basic test of a random walk against a stationary autoregressive process by including a drift and trend term. As far as the listed results are concerned, we decide whether to include a trend or constant by checking the significance of the trend/constant...
parameters at a 5% significance threshold. The parameter residuals refers to the estimation results for Equation (3.20) using a 2SLS regression.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Augmented Dickey Fuller (Levels)</th>
<th>Philipps-Perron Test (Levels)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>statistic p-value lags</td>
<td>statistic p-value lags</td>
</tr>
<tr>
<td>id auction price</td>
<td>-17.50 0.00 55</td>
<td>-152.25 0.00 55</td>
</tr>
<tr>
<td>day ahead price</td>
<td>-17.30 0.00 55</td>
<td>-18.65 0.00 55</td>
</tr>
<tr>
<td>residual demand 60</td>
<td>-13.15 0.00 53</td>
<td>-15.18 0.00 53</td>
</tr>
<tr>
<td>residual demand deviation 15</td>
<td>-40.19 0.00 55</td>
<td>-492.70 0.00 55</td>
</tr>
<tr>
<td>residuals (ε)</td>
<td>-17.42 0.00 54</td>
<td>-204.50 0.00 54</td>
</tr>
</tbody>
</table>

### Additional Information with Respect to Robustness, Alternative Hypotheses, and Methodological Variation

Besides explicitly outlined robustness checks, further variations of the basic estimation procedure were evaluated in order to get further insights into the underlying causal relations of short-term price formation in electricity markets. First, the four specific 15-minute intervals of each hour are addressed via a dummy variable in order to analyze whether the estimated coefficients for the quarter-hourly deviation of the residual demand from its hourly mean differ significantly across the 15-minute time intervals of each hour. The estimation results depict that the respective coefficients only differ slightly at a level of approximately ten percent. However, due to a small absolute difference we value the sub-hourly variation of coefficients as negligible. Second, the intra-day variation of the coefficients for the 15-minute residual demand deviation is analyzed by referring to each specific hour of a day via a dummy variable. The results only give slight evidence for significant intra-day deviation in hour two. Thus, we conclude that the causal relations are robust against intra-day variation. Third, the causal relations in winter and summer basically are the same.

In a next step, we want to comment on additional impact factors that may influence quarter-hourly intraday auction prices and thus should be listed in order to complete our explanatory approach. First, we analyze the impact of forecast errors. In more detail, forecast errors reveal after day-ahead gate closure and are balanced within subsequent intraday trade. However, continuous intraday trade is assumed to be more favourable to balance these forecast errors. This is due to both, market design and gate closure closer to physical delivery. In line with these considerations, we find empirical evidence that the impact of forecast errors on intraday auction
prices is insignificant. Additionally, strategic behavior could have impact on the price formation of interest but based on Bundesnetzagentur (2016) we reject this hypothesis.

Finally, our empirical approach is based on crucial assumptions with regard to exogeneity and stationarity of data. However, a simple Ordinary Least Squares Regression and the 2SLS Regression basically provide identical estimates. Furthermore, an application of a Vector Error Correction Model after an initial test for cointegration of the respective variables gives additional evidence for significance of the included parameters.
4 When Are Consumers Responding to Electricity Prices? An Hourly Pattern of Demand Elasticity

System security in electricity markets relies crucially on the interaction between demand and supply over time. However, research on electricity markets has been mainly focusing on the supply side arguing that demand is rather inelastic. Assuming perfectly inelastic demand might lead to delusive statements regarding the price formation in electricity markets. In this article, we quantify the short-run price elasticity of electricity demand in the German day-ahead market and show that demand is adjusting to price movements in the short-run. We are able to solve the simultaneity problem of demand and supply for the German market by incorporating variable renewable electricity generation for the estimation of electricity prices in our econometric approach. We find a daily pattern for demand elasticity on the German day-ahead market where price-induced demand response occurs in early morning and late afternoon hours. Consequently, price elasticity is lowest at night times and during the day. Our measured price elasticity peaks at a value of approximately -0.13 implying that a one percent increase in price reduces demand by 0.13 percent.

4.1 Introduction

Understanding the price elasticity of demand is important since demand adjustments based on price movements contribute to the functioning of electricity markets. In electricity markets it is worth stressing that balancing demand and supply occurs on a high temporal frequency which, not only in Germany, results in debates on whether or not it is possible to match demand and supply at all times. An inelastic price elasticity of demand assumption, as often argued for the short-run, would imply that the burden of balancing electricity consumption and generation at all times rests with the supply side.

The empirical literature estimating long-run and short-run price elasticity of demand in electricity markets is extensive. For the short-run, peer-reviewed studies have estimated the elasticity for different sectors and time intervals. Table 4.1 shows...
that estimates of price elasticity vary from -0.02 to -0.3 depending on the chosen approach, the country-specific data and the sector. Taylor et al. (2005), for instance, find that short-run elasticity ranges from -0.05 to -0.26 for the industrial sector in North Carolina by using annual data. He et al. (2011) confirm this finding whereas Bardazzi et al. (2014) measure a slightly higher elasticity in terms of magnitude for the Italian industry sector. For the residential sector, numerous studies have been performed as well (i.e. Ziramba (2008), Dergiades and Tsoulfidis (2008) and Hosoe and Akiyama (2009)). However, little attention has been devoted to the price response of the whole market with respect to wholesale prices. So far, this market has only been investigated by Genc (2014) and Lijesen (2007). Whereas Genc (2014) applies a bottom-up Cournot modeling framework, Lijesen (2007) uses a regression approach in order to quantify the price elasticity during peak hours. Genc and Lijesen conclude from their chosen approaches that the hourly price elasticity is rather small. They furthermore argue that in peak hours demand switching behavior of consumers barely occurs in practice.

In this article we extend the existing literature on short-run elasticity with respect to the wholesale price in two ways. First, we use wind generation as an instrument variable to solve the simultaneity problem of demand and supply. Second, we account for the variation in utility from electricity consumption during the day. Using hourly data on load, temperature, prices and wind generation for the German day-ahead market in 2015, we quantify the level of price elasticity and its variation throughout the day.

Our results show that the short-run price elasticity of demand in the German electricity market is not perfectly inelastic. Even though our obtained short-run price elasticity of demand is generally low, consumers still react to price movements. Measuring the price elasticity of demand can give a more meaningful understanding of the contribution of demand reactions to system security. However, we stress that a price elasticity of demand with respect to the day-ahead price is not explicitly showing the contribution of each consumer group. The daily pattern of our estimate of price elasticity reveals some prominent peaks in the morning and evening, where the price elasticity of demand is highest. As expected, these hours show overall high price levels providing incentives to consumers for a reduction of their consumption. In the morning and evening hours, price elasticity varies between -0.08 and -0.13. Thus, we infer that demand adjustments in these hours are to some extent beneficial for consumers. On the contrary, we measure a lower price elasticity of demand at

1 The approach is similar to Bönte et al. (2015).
4.1 Introduction

Table 4.1: Literature review of estimated short-run elasticity

<table>
<thead>
<tr>
<th>Source</th>
<th>Type of model</th>
<th>Type of data</th>
<th>Elasticity</th>
<th>Sector</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Garcia-Cerrutti (2000)</td>
<td>Dynamic random variables model</td>
<td>Annual</td>
<td>-0.79 to 0.01, mean -0.17</td>
<td>Residential</td>
<td>California</td>
</tr>
<tr>
<td>Al-Faris (2002)</td>
<td>Dynamic cointegration and Error Correction Model</td>
<td>Annual, 1970-1997</td>
<td>-0.04 / -0.18</td>
<td>Residential</td>
<td>Oman</td>
</tr>
<tr>
<td>Boisvert et al. (2004)</td>
<td>Generalized Leontief Cointegration and Error Correction Model</td>
<td>Annual, 1955-1996</td>
<td>Peak: -0.05</td>
<td>TOU</td>
<td>Taiwan</td>
</tr>
<tr>
<td>Bernstein et al. (2006)</td>
<td>dynamic demand model with lagged variables and fixed effects</td>
<td>Panel, 1977-1999</td>
<td>-0.24 to -0.21</td>
<td>Residential, Commercial</td>
<td>US</td>
</tr>
<tr>
<td>Rapanos and Polemis (2006)</td>
<td>Bounds testing approach to cointegration within ARDL model</td>
<td>1965-1999</td>
<td>-0.31</td>
<td></td>
<td>Greece</td>
</tr>
<tr>
<td>Halicioglu (2007)</td>
<td>Bounds testing approach to cointegration within ARDL model</td>
<td>1968-2005</td>
<td>-0.33</td>
<td></td>
<td>Turkey</td>
</tr>
<tr>
<td>Lijsesen (2007)</td>
<td>Reduced form regression linear, loglinear</td>
<td></td>
<td>-0.0014 / -0.0043</td>
<td>Wholesale</td>
<td>Netherlands</td>
</tr>
<tr>
<td>Dergiades and Tsouflias (2008)</td>
<td>Bounds testing approach to cointegration within ARDL model</td>
<td>1965-2006</td>
<td>-1.06</td>
<td>Residential</td>
<td>US</td>
</tr>
<tr>
<td>Ziramba (2008)</td>
<td>Bounds testing approach to cointegration within ARDL model</td>
<td>1978-2005</td>
<td>-0.02</td>
<td>Residential</td>
<td>South Africa</td>
</tr>
<tr>
<td>Hosoe and Akiyama (2009)</td>
<td>OLS/Translog cost function</td>
<td>1976-2006</td>
<td>0.09 to 0.3</td>
<td>Residential</td>
<td>Japan</td>
</tr>
<tr>
<td>He et al. (2011)</td>
<td>General equilibrium analysis</td>
<td>2007</td>
<td>-0.017 to -0.019, -0.293 to -0.311, -0.0624 to -0.0634</td>
<td>Industry, residential, agriculture</td>
<td>China</td>
</tr>
<tr>
<td>Bardazzi et al. (2014)</td>
<td>Two-stage translog model</td>
<td>Panel, 2000-2005</td>
<td>-0.561 to -0.299</td>
<td>Industry</td>
<td>Italy</td>
</tr>
<tr>
<td>Genc (2014)</td>
<td>Cournot competition model</td>
<td>Hourly 2007, 2008</td>
<td>-0.144 to -0.013, -0.019 to -0.083</td>
<td>Wholesale</td>
<td>Ontario</td>
</tr>
</tbody>
</table>

Night times and during the day. A lower elasticity indicates less willingness of consumers to adjust the consumption due to high or low electricity prices. This can be due to the fact that economic activity in general is higher during daytime.

The remainder of the paper is organized as follows. Section 4.2 deepens the understanding of supply and demand in electricity markets. Section 4.3 describes the data and presents the applied econometric approach. Section 4.4 discusses the estimation results. Section 4.5 concludes.
4.2 Measuring Market Demand Reactions Based on Wholesale Prices

In order to specify our econometric model capturing demand reactions due to electricity wholesale price movements, knowledge about the supply and demand functions in electricity markets is pivotal. In this section, we therefore describe the functioning of the retail and wholesale electricity market before arguing that demand elasticity can be estimated based on market demand being defined as aggregated demand of all end consumer groups and wholesale electricity prices. We further specify the drivers of demand and supply by setting up the respective functions.

4.2.1 The Retail Market for Electricity

Consumers commonly sign contracts with retailers to take charge of their electricity demand. These contracts are subject to different possible tariff schemes ranging from time-invariant pricing to real-time pricing. Tariff structures vary depending on the consumer group and metering facilities. Small end consumers (e.g. households, businesses, or small industries) in Germany are mostly on time-invariant tariffs. This means that the price of electricity for these consumer groups is at the same level for every hour over the entire year. These consumers therefore have little incentive to adjust their demand in the short-run. For larger consumers, such as big industrial companies, contracts are differently designed allowing them to benefit from adjusting consumption in the short run.

In Germany, the retail price that consumers pay for electricity consists of several components. The most important component is the price for electricity generation, which is the price that generators charge for the generation of electricity. Besides paying for the generation of electricity, end consumers also pay for the transmission and distribution of electricity, as well as for additional taxes and levies. In Germany, for instance the retail price consists of network charges, the renewable support levy, and taxes which are added to the wholesale price. Some of these additional price

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2 The electricity consumption of many end consumers is not observable over time because the metering facilities only display the amount of electricity consumed but not during which period measurement is performed.

3 According to Bundesnetzagentur (2016), consumers can be grouped by their metering profile into customers with and without interval metering. Only consumers with interval metering have the technical capability to be billed depending on the time of usage. For Germany in 2014, 268 TWh were supplied to interval metered customers and 160 TWh to customers without interval metering.
4.2 Measuring Market Demand Reactions Based on Wholesale Prices

Components vary substantially depending on the consumer group. The differing retail prices for each consumer group lead to a total electricity demand of all consumers that varies over the year. This aggregated demand of all end consumers is equal to the observed load in the total electricity system.

4.2.2 The Wholesale Market for Electricity

The price for electricity generation is determined in the wholesale market. In principal, the wholesale market allows different players to place bids that eventually either result in produced quantities or demanded quantities for a specific point in time. Participants in these markets are for example utilities, retailers, power plant operators and large industrial consumers.

Figure 4.1i gives an exemplary overview of the five different players and their corresponding electricity demand and supply on the wholesale market. The first two players are two different utilities, A and B. As such, utility A and B illustrate cases for players with different generation assets while at the same time each of them possesses different customer bases. However, for both utilities, we would expect that generation for their own customer base depends on the marginal cost of generation. In other words, if the wholesale price is above the marginal cost of the utility’s marginal cost of generation, the utility chooses to supply their customer base instead of demanding quantities from the wholesale market.

The next player in the market we refer to is the retailer. As a retailer, supplying electricity is by default not an option and therefore we expect them to demand electricity quantities only. The opposite is true for renewable and conventional generation players. With marginal costs of zero, renewable generation players offer their production at very low cost compared to conventional generation players where marginal costs are greater than zero and vary depending on the generation technology.

Figure 4.1ii horizontally aggregates all demand and supply curves from each player we identified. It thus shows the aggregated demand and supply, as well as the realized equilibrium electricity price of 20 EUR/MWh.

Figure 4.1iii shows the resulting supply and demand bids by the individual players in the wholesale market. First, players that can only supply electricity, such as renewable or conventional generators, appear in ascending order on the supply side.

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4In Germany, for example, electricity intensive industries are exempted from paying the renewable support levy.
when are consumers responding to electricity prices? an hourly pattern of demand elasticity

figure 4.1: electricity price formation on the wholesale market

(i) Wholesale market players

(ii) Supply and demand aggregation

(iii) Supply and demand in the wholesale market

only. Second, retailers demand quantities and generally more, if prices are low. Third, players that own generation assets and also have customers, net their supply and demand positions internally before submitting bids. This is the case for utility A and B. The bids for the demand and supply side depend on the internal netting of supply and demand. In total this results in four possible outcomes for placing bids which can be describes as follows

- sell bid on the supply side for generation units that have not been internally matched and could satisfy the demand of other participants
- purchase bid on the demand side for demand that has not been internally matched
- sell bid on the supply side, resulting from demand that has been matched internally but would be able to reduce consumption if the price rises above a given threshold (see e.g. demand of utility B with 90 EUR/MWh)
- purchase bid on the demand side for generation units that have internally be matched but that would substitute their production if the price falls below their marginal costs of generation.

Whereas the first two outcomes are intuitively straightforward, outcomes three and four may seem counter intuitive at first. Due to the internal matching of supply and demand, parts of the demand and supply curve that have been internally
matched result in bids on the opposite side. By placing these bids, utilities can optimize their position and choose to substitute formally demanded quantities to supplied quantities or vice versa, above or below a certain wholesale price.

The supply and demand curves in Figure 4.1ii and 4.1iii look very different from a first glance, but both result in the same price for electricity and lead to the same allocation of resources. Nevertheless, both provide a very different impression of the price responsiveness of the demand side. Based on Figure 4.1ii the demand side can be characterized as rather price inelastic. In the example, the level of demand would not change if prices stay within a range of 5 to 80 EUR/MWh. Figure 4.1iii may however lead to the misleading conclusion that the demand side in electricity markets is rather price elastic. Within the submitted supply and demand bids at the wholesale market it is not possible to identify separate bids that actually stem from generators or actual consumers of electricity. It is therefore not possible to estimate the demand elasticity of actual electricity consumers based on the curves observed in the wholesale market. In order to estimate the demand elasticity of the actual electricity consumers it is, however, possible to combine the wholesale equilibrium price with the total load observed.

4.2.3 The Interaction of Wholesale and Retail Markets

Within this article we are interested in the reaction of electricity demand to electricity prices. Because disaggregated load data for each consumer group with the respective retail prices are not available, we focus our attention on the interaction of total hourly demand and hourly wholesale electricity prices. Figure 4.2 shows the relation we are interested in for an exemplary hour. The blue line depicts the supply curve for electricity generation. The red line is the aggregated demand curve of all consumers for electricity consumption. Consumers pay an average retail price of \( p_r \), which is made up of the wholesale price for electricity \( p_w \) and additional price components \( c \). When we account for the effect of the additional price components, we obtain the demand function that is observable in the wholesale market (wholesale demand, red dashed line). The intersection of wholesale demand and wholesale supply leads to point A and determines the wholesale price \( p_w \), as well as the quantity consumed and produced \( q^{el} \). By inferring the relationship illustrated in Figure 4.2 and using the wholesale price and total electricity demand, we are able to estimate the point elasticity of the red dashed demand curve.

\(^5\)In Germany, most additional price components are added to the wholesale price independent on the price level or quantity consumed.
The relations of the demand and supply curve in electricity markets are only vaguely sketched in Figure 4.2. In reality, demand is fluctuating over time due to varying utility levels throughout the day. The demand for electricity can be regarded as a function of various inputs and the relation can be written as

\[ q_{el} = f(p^w, \text{HDD, time-of-the-day}), \]  

(4.1)

where \( q_{el} \) is the quantity consumed, \( p^w \) is the wholesale price for electricity, HDD are heating degree days capturing the seasonality within the data. HDD measure the temperature difference to a reference temperature. The variable therefore captures the seasonal variation of electricity demand. For example, if outside temperature is low, heating processes consume more electricity compared to warmer weather conditions. In addition, electricity consumption depends on the time of usage. This is mainly driven by the variation of the consumer’s utility function over the day. Additional variables determining the level of demand, such as economic activity, may also alter demand but are assumed to be time-invariant on an hourly basis and within the considered time span. Therefore, we abstract from including additional variables for the demand side in the short run.

Like the demand function, the supply of electricity can also be regarded as a function of multiple inputs with the wholesale price \( p^w \) being one of them. We define the supply function as:

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6The data in Section 4.4 reveals that this relation is true for Germany, however it may not be applicable to other countries. In warmer climates also cooling degree days (CDD) determine the demand for electricity.
\[ q^{el} = f(p^w, p^{fuel}, r), \]

where \( q^{el} \) is the quantity produced, \( p^{fuel} \) is a vector of fuel prices and \( r \) is the production of variable renewable energy.

In electricity markets, the structure of the supply side is commonly represented by the merit order curve. It represents the marginal generation costs of all conventional (fossil) power plants. The shape of the curve mainly depends on the technologies being used for power generation and their respective fuel prices \( p^{fuel} \). However, variable renewable electricity generation is becoming increasingly important within the generation portfolio. This is particularly true for the German market region. Since renewable technologies do not rely on fossil fuel inputs to generate electricity, their fuel costs are close to zero. Additionally, its stochastic nature that is driven by wind speeds and solar radiation makes generation vary throughout time. We will later make use of the stochastic nature and by using wind generation as an instrument variable within our econometric model.

### 4.3 Empirical Framework

#### 4.3.1 Data

Our data set consists of hourly data for 2015. We include hourly data for load, day-ahead-prices and the forecast of production from variable renewables for Germany. In addition, HDD are calculated based on hourly temperatures that we obtain from the NASA Goddard Institute for Space Studies (GISS). Summary statistics for all variables are provided in Table 4.2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load [GWh]</td>
<td>61.688</td>
<td>9.428</td>
<td>38.926</td>
<td>77.496</td>
<td>ENTSO-E</td>
</tr>
<tr>
<td>Wind Generation [GWh]</td>
<td>8.574</td>
<td>6.864</td>
<td>0.153</td>
<td>32.529</td>
<td>EEX Transparency</td>
</tr>
<tr>
<td>Day-ahead price [EUR/MWh]</td>
<td>35.6</td>
<td>11.5</td>
<td>-41.74</td>
<td>99.77</td>
<td>EPEX Spot</td>
</tr>
<tr>
<td>Temperature [°C]</td>
<td>10.4</td>
<td>7.9</td>
<td>-6.3</td>
<td>34.6</td>
<td>NASA MERRA</td>
</tr>
<tr>
<td>Heating degree days [K]</td>
<td>10.1</td>
<td>6.9</td>
<td>0</td>
<td>26.3</td>
<td>NASA MERRA</td>
</tr>
</tbody>
</table>

The hourly load profile for Germany was taken from ENTSO-E. According to ENTSO-E, load is the power consumed by the network including network losses but exclude-

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7 Common power plant types and fuels are hydro power, nuclear, lignite, coal, gas and oil.
ing consumption of pumped storage and generating auxiliaries.\(^8\) The load data includes all energy that is sold by German power plants to consumers.\(^9\) Load therefore is the best indicator on the level of demand in the German market area since almost all energy sold has to be transferred through the grid to consumers. Figure 4.3i shows average hourly values for weekdays in the German market area in a box plot. The plot shows significant differences in the level for night hours (00:00-6:00, 19:00-00:00) compared to daytime. Also load peaks in the morning (9:00-12:00) and evening hours (16:00-18:00). Especially in the evening, variation in load levels is higher than at other times. The average load level is 62 GW and the maximum peak load is 77 GW in the early evening hours.

Figure 4.3: Hourly data for load, electricity price, wind and solar generation for 2015

We obtain the hourly day-ahead price for electricity from the European Power Exchange (EPEX) which is the major trading platform for Germany. Historically the day-ahead price has evolved as the most important reference price on an hourly level

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\(^8\) ENTSO-E collects the information from the four German transmission system operators (TSO) and claims that the data covers at least 91% of the total supply. These quantities may also be reflected in the day-ahead price which we can not account for.

\(^9\) To a small amount load may also include energy that is sold from neighboring countries to the German market. These trade flows impact the domestic electricity price and load. However, we expect this impact to be rather small.
in the wholesale electricity market. The day-ahead market run by EPEX Spot is by far the most liquid trading possibility close to the point of physical delivery. The price is determined in a uniform price auction at noon one day before electricity is physically delivered. We follow this perspective and use the day-ahead price as our reference price for electricity generation. Although not all electricity is sold through the day-ahead-auction, the price reflects the value of electricity in the respective hours and contains all available information on demand and supply at that specific point in time. Figure 4.3ii shows a box plot for the hourly day-ahead electricity price for each hour of the day. The average hourly day-ahead electricity price is at 36 EUR/MWh over the 24 hours time interval and for weekdays (without public holidays and Christmas time). The electricity price pattern is similar to the load pattern emphasizing the fact that higher demand levels tend to increase prices in the day-ahead market. Especially during peak times in the morning and evening one can observe higher standard deviations and peaking prices. Standard deviation over all hours is around 12 EUR/MWh.

Electricity generation from wind and solar power is taken from forecasts published on the transparency platform by the European Energy Exchange (EEX). These forecasts result from multiple TSO data submissions to the EEX. Since they are submitted one day before physical delivery, they contain all information that is relevant for participants in the day-ahead market. Figure 4.3iii and 4.3iv show box plots for electricity generation from wind and solar power. Due to weather dependent generation volatility, we observe a larger amount of volatility in the hourly data. Wind generation varies steadily throughout the day with a small increase during the day. Solar generation shows its typical daily pattern with no generation at night and peak generation values for midday.

The level of demand does not only depend on the electricity price which in return is partially influenced by generation from wind. We add temperature as an additional parameter to our investigation of electricity demand since the level of temperature is a main driver for the seasonal variation of demand. We compute a Germany wide average temperature based on the reanalysis MERRA data set provided by NASA (NASA, 2016). The hourly values are based on different grid points within Germany that are spatially averaged in order to obtain a consistent hourly value for Germany. Based on the hourly temperature we derive HDD that are relevant for the seasonal

---

10 In 2015 264 TWh have been traded in the day-ahead market, compared to 37 TWh traded in the continuous intraday market (EPEX Spot, 2016).
11 We also considered taking the actual generation from renewables but reckon that the ex-ante forecasts are reflecting the causal relationship in a better way since decisions made on the day-ahead market are based on forecast values.
variation of demand in electricity markets.\textsuperscript{12}

\textbf{4.3.2 Econometric Approach}

Due to the fact that the electricity price is endogenously determined by the interaction of demand and supply, we choose a two-stage approach to solve the simultaneity problem.\textsuperscript{13} As we are interested in estimating the demand function (4.1), possible instruments affecting the price but not the level of demand have to be determined. Possible instruments can be found on the supply side in (4.2), where fuel prices ($p_{\text{fuel}}$) and the production of variable renewable energy ($r$) are considered. Although fuel prices are one of the major drivers for generation decisions, a closer look reveals that they show little variation over the year 2015 (cf. Figure 4.6 in the Appendix). Therefore, we exclude them from a further analysis within our framework.

The production of variable renewable energy ($r$) can further be split into wind ($w$) and solar ($s$) generation. Figure 4.4 depicts the respective correlations of renewable generation with prices and load for each hour interval of the day. In Figure 4.4i, we observe that the correlation between solar generation and load is higher in absolute values than the correlation between wind generation and load. However, wind and solar generation are correlated opposite in sign with load being positively correlated with wind generation and solar generation negatively correlated with load.

Figure 4.4ii shows the correlation between renewable generation and electricity price. Both, wind and solar generation are negatively correlated with the electricity price, however their patterns are different throughout the day. The correlation between wind generation and electricity price weakens over the day until 17:00 where the correlation is lowest with an absolute value of -0.45. From 17:00 on the correlation between wind generation and price increases again. The pattern for the correlation between solar generation and electricity price is reversed whereas the increasing correlation until 17:00 is mainly driven by an increasing solar radiation. Based on the generally high correlation of wind and prices and at the same time low correlation of wind and load, we choose wind generation as an instrument for the price.\textsuperscript{14}

\textsuperscript{12}We calculate HDDs based on a reference temperature of 20 °C.

\textsuperscript{13}Durbin and Wu–Hausman test statistics show highly significant p-values. The null hypothesis tests for all variables in scope being exogenous. With p-values for both test of both equal to 0.000 we reject the null of exogeneity implying that prices and demand are endogenous.

\textsuperscript{14}Statistically speaking, weak instruments may cause estimation bias if the correlation with the endogenous explanatory variable (in our case $p_{w,t}$) is very low.
4.3 Empirical Framework

More formally, wind generation as a variable fulfills the two conditions (1) $\text{cov}[w, p^w] \neq 0$ and (2) $\text{cov}[w, \mu] = 0$, where $w$ is wind generation, $p^w$ the wholesale electricity price and $\mu$ the error term. The first condition is needed in order to provide unbiased electricity price estimates. In our context the chosen instrument $w$ correlates with the electricity price (c.f. Figure 4.4ii). From the second condition it follows that $w$ and $\mu$ are not correlated.\(^{15}\) Because wind can be regarded as a stochastic variable especially throughout the day and load inhibits strong daily patterns, both can be regarded as independent (c.f. Figure 4.4i). With these two conditions fulfilled we are now able to postulate the first and second stage equations. The first stage can be written as

$$p^w_{h,t} = \gamma_{0,h} + \gamma_{1,h} \cdot w_{h,t} + \epsilon_{h,t}$$  \hspace{1cm} (4.3)

and the second stage as

$$q^{el}_{h,t} = \beta_{0,h} + \beta_{1,h} \cdot p^w_{h,t} + \beta_{2} \cdot HDD_{t} + \beta_{3} \cdot MON_{t} + \beta_{4} \cdot FRI_{t} + \mu_{h,t}.$$  \hspace{1cm} (4.4)

We estimate price coefficients $\beta_{1,h}$ and dummy coefficients $\beta_{0,h}$ on an hourly basis $h$. We do this, because we expect the utility of electricity consumption to be different in each hour of the day. Here, $\beta_{0,h}$ captures the price independent change of utility from electricity consumption throughout the day. Since we observe a different demand pattern for working days and week-ends, we eliminate week-ends and hol-

\(^{15}\)Testing for validity expressed by $\text{cov}[w, \mu] = 0$ within our framework is not feasible since our model is exactly identified.
idays from the data. Furthermore, we add dummies for Monday (MON) and Friday (FRI)\(^\text{16}\) to capture differing demand levels at the beginning and end of the working week. Based on our estimates, we can calculate the average hourly price elasticity of electricity demand according to

\[
\epsilon_h = \frac{\bar{p}_h^w}{\bar{q}_h} \frac{\partial q_h}{\partial p_h} = \frac{\bar{p}_h^w}{\bar{q}_h} \beta_{1,h},
\]  
(4.5)

where \(\epsilon_h\) is the hourly elasticity using the average price \(\bar{p}_h^w\) and average demand \(\bar{q}_h\) in the respective hour of the day \((h)\).

### 4.4 Empirical Application

By applying the econometric framework, we are able to estimate the level of price elasticity of demand for the German day-ahead market on an hourly basis. The regression is based on levels and elasticity is calculated with respect to the average prices and quantities in each hour.\(^\text{17}\)

The results of the estimation can be found in Table 4.3. When taking a look at the price coefficients in Table 4.4a, we can see that all price coefficients are negative in sign and are significant at least at the 1% level. We note that coefficients during morning hours (9:00-12:00) are lower in absolute values. The highest value can be found at 17:00. In this particular hour, a wholesale price increase of 1 EUR/MWh leads to a demand reduction of 201.8 MWh. The hourly dummy coefficients in Table 4.4a capture the varying level of utility throughout the day. During the day, hourly coefficients are higher than at other times. In the evening, we can observe a peak in the level of utility, especially between 16:00 - 20:00 (c.f. Figure 4.5i). Beside the hourly coefficients, we also account for the influence of temperature and weekdays on electricity demand. All coefficients are significant at the 0.1% level and can be explained in their sign. HDD have a positive sign and thus increase electricity demand. Mondays and Fridays are negative in sign, indicating that demand is generally lower at the start of the week and at the end compared to other working days.

Since the focus of our work is on the hourly price elasticity of demand, we estimate

\(^{16}\)For Mondays the dummy is positive for the time between 0:00 and 9:00. For Fridays the time frame is from 17:00 to 23:00.

\(^{17}\)In a previous version of the paper, we normalized our data to the median, which is why previous estimates differ from this version. Furthermore, elasticity was calculated with respect to the average price and quantity level including values of zero. As we are running a pooled regression many observations of zero were included which resulted in low estimates of the elasticity.
### 4.4 Empirical Application

Table 4.3: Regression results

<table>
<thead>
<tr>
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<table>
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<tr>
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</tr>
<tr>
<td>Friday dummy</td>
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</tr>
<tr>
<td>Constant</td>
<td>46.57*** (84.62)</td>
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Observations: 5760  
$R^2$: 0.940  
Adjusted $R^2$: 0.939

---

(a) Dummy and price coefficients

$^t$ statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

(b) Regression coefficients

(c) Elasticity

---

85
the elasticity based on the results from the basic regression. The results are displayed in Figure 4.5ii and the numerical values can be found in Table 4.4c.\textsuperscript{18}

As observed before, all coefficients are negative in sign and significant at a strict 1\% level. With the elasticity estimates at hand, we are able to plot a distinctive pattern for the hourly price elasticity of demand for the German day-ahead market. The unique shape of the hourly price elasticity of demand pattern is depicted above in Figure 4.5ii. Our results show that demand reactions are rather small. However, a perfect inelastic demand assumption can also be neglected. More precisely, the elasticity is the lowest during night times (22:00 - 6:00). During these hours electricity demand and utility from electricity consumption is generally lower (as we can also observe from Table 4.4a). The graph shows two prominent peaks of price elasticity of demand in the morning and in the evening. At these times working hours start and end. Possible reasons for a high elasticity of demand at those times is the shifting or delaying of consumption. When prices are low in the morning, some processes may be able to start the operation earlier and thereby circumventing a time with a higher electricity price level. The same might be true for the evening, when the workday ends. Here working hours may be extended to lower price levels at other times. Throughout the day, the price elasticity of demand remains relatively low and is less significant. At those hours, economic activity is high and the option to shift or delay electricity consumption might not be feasible for consumers. In other

\textsuperscript{18}It is important to note that elasticity is calculated with respect to the wholesale price level and not the retail price level, as represented by the dashed red demand curve in Figure 4.2. The elasticity with respect to retail prices would be higher. For example if we consider the sum of additional price components ($c$) to be 150 EUR/MWh, which is an average value based on Eurostat (2016) for Germany, the highest elasticity measured would be -0.58 at hour 17:00-18:00. Without the sum of additional price components, we obtain an elasticity of -0.13 as indicated in Table 4.4c.
words, consumers are bound to consume electricity which results in high electricity consumption regardless of the price level.

4.5 Conclusion

We estimate the hourly pattern of price elasticity of demand for the German day-ahead market, using hourly data on load, price, generation of wind and temperature. By doing this, we are able to determine the degree of short-run demand response within this market. To the best of our knowledge, a market-wide hourly analysis of the price elasticity of demand has not been conducted so far.

Based on our two-stage regression approach which uses wind generation as an instrument to proxy the electricity price, we find that hourly price elasticity of demand is not completely price inelastic. Especially during the morning and evening demand is responding to price signals. Values for price elasticity range from approximately -0.02 to -0.13 depending on the investigated hour. The hourly price elasticity pattern reveals that elasticity is lowest in the night hours and around mid day. Low values for price elasticity during night time (22:00 - 06:00) indicate that consumers are less likely to react. Around middle day economic activity is high which may explain the low elasticity values. Price elasticity of demand is the highest in the early morning (04:00 - 07:00) and late afternoon (16:00 - 20:00) hours, with levels between -0.08 and -0.13.

The empirical results indicate a high level of variation in the price elasticity of demand throughout the day in the German day-ahead market. Although the hourly elasticity is low from a first glance, load shifting accumulates over the year. The found elasticity pattern helps to understand when demand shifting occurs and when demand may be able to contribute to system security in situations of low supply. We find that especially during critical situations, such as peak times in the morning and evening, price elasticity of demand is high and may contribute to a secure electricity system.

Our research sheds some light on how flexible the German electricity market has already been in 2015, given the underlying renewable generation of the German day-ahead market. It may also give policy makers a starting point for evaluating the interaction of supply and demand in electricity markets. In addition to the analysis of the day-ahead market, we reckon that further research on demand response could focus on short-term markets, such as the intraday market. These markets are
essential to the integration of large amounts of renewable electricity because they are able to balance forecast errors of wind and solar electricity. Whereas this additional research would gain further insights onto the short-term demand response, we argue that currently the day-ahead market remains the most important market where demand and supply are balanced.

### 4.6 Appendix

![Figure 4.6: Prices for coal, gas and co2 certificates from January to December 2015](image-url)
5 Retail Tariff Design in Electricity Markets with Variable Renewable Production

Time-invariant pricing (TIP) is still widely applied for the billing of consumers in electricity markets even though real-time pricing (RTP) is known to be superior in terms of efficiency. In this paper, we investigate the inefficiency stemming from the second-best TIP in electricity markets accounting for the impact of renewable electricity generation and its correlation with demand. Compared to existing literature we do not restrict our analysis to a vertical structure with unbundled retailers and allow vertical integration. We find that deadweight losses from TIP result primarily from fluctuations of demand and renewable generation. Losses increase if (i) the standard deviation of demand and renewable generation is high, and (ii) the correlation of both variables is low. In an illustrative case study for Germany, we are able to confirm these theoretical findings: low levels of solar capacities are able to reduce the deadweight loss from TIP because of a positive correlation with demand; higher levels of wind capacity lead to an increase of the deadweight loss. Furthermore, we find that time-of-use pricing is able to reduce deadweight losses only by up to 58% compared to RTP.

5.1 Introduction

Electricity markets are characterized by highly fluctuating demand and non-storability. In spite of high demand volatility, in most markets time-invariant pricing (TIP) is still very common, while it is straightforward to show that real-time pricing (RTP) is needed to implement the first-best. Additionally, in many markets the share of weather dependent and therefore stochastic production (wind, solar) is increasing. At the same time, the market is subject to many regulations. One important field of regulation are restrictions for the vertical industry structure. Borenstein and Holland (2005) have demonstrated that, even with competitive retail markets, the second-best TIP can not be obtained if there is retail unbundling, i.e., if producers and retailers are required to be separate entities; a regulation which is in place in some US markets (Meyer, 2012). Their methodology has since then also been applied
to other markets (see e.g. Allcott (2012), Joskow and Tirole (2006), Pahle et al. (2016)).

In this paper, we investigate the second-best TIP without retail unbundling and focus on the effect of increasing shares of stochastic renewable electricity production. The case appears to be particularly relevant for European electricity markets: Like in the US, TIP is still the predominant tariff scheme of end consumers; In Europe, retail unbundling does usually not apply; and Europe experiences a large increase in renewable energies, especially from wind and solar. In this context, looking at the second-best TIP is interesting at least for two reasons. First, we are able to derive results on the upper bound of the welfare losses due to a lack of RTP. Second, we conjecture that the second-best TIP can be a (competitive) market outcome when retailers are not obliged to unbundle. Even though technical complexities in the representation of strategic interactions prohibit a formal proof that guarantees our second-best TIP to be the (competitive) market outcome, we will argue that compared to Borenstein and Holland (2005), prices are closer to the second-best TIP in the absence of retail unbundling. Moreover, our model extends the one by Borenstein and Holland by representing simultaneous fluctuations of demand and variable renewable energies, characterized by their joint probabilities.

Based on our theoretical model, we show that outcomes highly depend upon the simultaneous interaction of demand and renewable generation when analyzing the implications of tariff designs. Both variables are (at least partly) weather driven and inhibit strong diurnal cycles (e.g. demand is high during the day, when also solar generation is high). Regarding the deadweight loss stemming from the second-best TIP we observe two effects: First, the deadweight loss increases for higher standard deviations of demand and renewable generation. Second, if both variables are positively correlated this can lead to a decrease in deadweight losses.

After gaining first insights from our theoretical model, we calibrate it with data from Germany, a country which sees a strong increase in stochastic renewable production. Our results indicate that – at least in this case – the second-best TIP may indeed be the competitive market outcome. We show that deadweight losses from TIP generally increase for large shares of renewable capacities. However, the effects of wind and solar on overall welfare are very different due to their correlation with demand. Low levels of installed solar capacities might even lead to a reduction of deadweight losses because solar generation is positively correlated (0.322) with demand. Wind on the opposite strictly increases deadweight losses as generation is only marginally correlated with demand (0.035). When thinking about realistic ap-
approximation for RTP for final customers, we find that even with quite sophisticated tariffs (with up to 576 different prices over the whole year), the welfare loss reduces only by 58% compared to perfect RTP.

The paper is structured as follows: Section 5.2 presents the model. In Section 5.3, we derive the welfare maximizing TIP and the resulting deadweight losses in electricity systems with variable renewable generation and compare the results to Borenstein and Holland (2005). The results are applied to the German electricity market in Section 5.4 before we conclude in Section 5.5.

5.2 The Model

Our model builds on the formulation by Borenstein and Holland (2005). However, instead of modeling unbundled retailers and generators that compete in the wholesale and retail market, we do not impose any specific vertical industry structure and focus on the welfare-optimal short-term equilibrium. Possible implications of different vertical industry structures will nevertheless be discussed. Furthermore, we extend the model from Borenstein and Holland by incorporating the properties of variable renewable electricity generation, i.e., we explicitly account for the stochastic nature and zero short-term marginal costs of wind and solar power.

Consumers demand electricity according to the time-varying demand \( D_t(p_t) \), with \( t \in [0, T] \) being the number of time periods \( T \) per year and \( D_t' < 0 \). Two different tariffs, RTP and TIP, are offered to end consumers. The consumers on RTP make up a fraction of \( \alpha \) of all consumers and demand electricity according to \( D_t(p_t) \). Within this share \( \alpha \) could for example be large industrial costumers that are already on RTP. A fraction of consumers \( (1-\alpha) \) is on TIP at a price of \( p \) and accordingly demands \( D_t(p) \). The total demand thus sums up to \( \hat{D}_t(p, \bar{p}) = \alpha D_t(p_t) + (1-\alpha)D_t(p) \). The formulation is identical to Borenstein and Holland (2005). We are aware that in reality there are more than two consumer groups on different tariff schemes. However, two tariffs are analytically tractable and still allow to derive the most important implications for consumers on different tariff structures. Note that the formulation of demand furthermore assumes that consumption in time period \( t \) only depends on price \( p_t \) but not on prices in other time periods. Hence, we do not account for the possibility of load shifting by consumers from one period to another.\(^1\)

\(^1\)Load shifting is an additional driver for demand response in reality. The model could potentially be extended by introducing cross-price elasticities that would account for load shifting behavior of consumers.
Electricity is generated in conventional and renewable power plants by generators. The generators are assumed to be identical in technology and size with each one operating and owning a small fraction of the generation facilities. The costs of electricity generation depend on $q$ the total quantity produced, and $r_t$ the quantity generated from variable renewable resources, such as wind or solar. The total costs sum up to $C(q, r_t)$ with the following properties of the partial derivatives:

(i) $C_q' > 0$, with $r_t$ being constant, marginal generation costs increase with an increase of production;

(ii) $C_{qq''} > 0$, the short-run marginal costs are increasing in quantity for a given renewable generation level $r_t$;

(iii) $C_r' < 0$, the costs of generation are decreasing with additional renewable generation;

(iv) $C_{qr''} < 0$, the short-run marginal costs are decreasing in renewable generation.

Whereas (i) and (ii) are almost identical to the assumptions of Borenstein and Holland (2005), (iii) and (iv) are added to the model and introduce the first property of renewable electricity generation, namely zero short-term marginal costs of generation. A second property of renewable generation is the weather dependency of generation. We therefore assume, that $r_t$ is stochastic according to the distribution $f(r_t)$. The properties of this distribution will be introduced in Section 5.3.2, where the focus is put on the role of renewables.

Let $P_t(q)$ be the inverse demand function, $D_t^{-1}(p)$. Then the utility of consumers on RTP and TIP can be defined as

$$\tilde{U}_t(p_t, p) = \alpha U_t(D_t(p_t)) + (1 - \alpha) U_t(D_t(p))$$

$$= \alpha \int_0^{D_t(p_t)} P_t(q) dq + (1 - \alpha) \int_0^{D_t(p)} P_t(q) dq. \quad (5.1)$$

The overall welfare from electricity consumption with consumers on TIP and RTP can therefore be written as

$$W = \sum_t \left[ \tilde{U}_t(p_t, p) - C_D(D_t(p_t), r_t) \right]. \quad (5.2)$$

Electricity generation costs from gas power plants are for example higher compared to coal or nuclear power plants.
5.3 Theoretical Analysis

In our theoretical analysis, we will first focus on the welfare optimal TIP and discuss the implications of different industry structures on a competitive market outcome (especially in comparison to the results of Borenstein and Holland). After this, we will focus on the implications of renewable generation on the second-best TIP. Furthermore we analyze the impact of a tax on the optimal TIP and the resulting deadweight losses.

Before we begin with the analysis of the second-best TIP it is important to stress that the first-best market outcome will only be achieved if all consumers are on RTP for $\alpha = 1$. In this case consumers demand electricity according to their marginal utility of consumption and generators supply electricity at marginal costs of generation at every point in time $p_t^e = C'_q(D(p_t^e), r_t)$. The total welfare in this case amounts to:

$$W^e = \sum_t [U_t(D_t(p_t^e)) - C(D_t(p_t^e), r_t)].$$

(5.3)

The first-best electricity allocation will in the following enact as a reference point for the comparison of alternative tariff designs, when not all consumers are on RTP.

5.3.1 The Welfare Optimal Time-Invariant Price

If not all consumers are on RTP ($\alpha < 1$) deadweight losses occur, because not all consumers respond to the marginal costs of generation. For simplicity we will first focus on the case when all consumers are on TIP ($\alpha = 0$) and no renewable generation ($r_t = 0$), which results in welfare as

$$W = \sum_t [U_t(D_t(\overline{p})) - C(D_t(\overline{p}))].$$

(5.4)

Maximizing over $\overline{p}$ and assuming an interior solution, the optimal time invariant tariff $\overline{p}$ satisfies$^3$:

$$\frac{\partial W}{\partial \overline{p}} = \sum_t D'_t(\overline{p}) - C'_q(D_t(\overline{p})) \cdot D'_t(\overline{p}) = 0.$$

(5.5)

$^3$Note that we can rewrite $U'(D_t(\overline{p})) = \overline{p}$.
This implies that the optimal time invariant tariff is implicitly defined (note that \( p \) appears on both sides of the following equation):

\[
\bar{p} = \frac{\sum_t C_q' (D_t(\bar{p})) \cdot D'_t(\bar{p})}{\sum_t D'_t(\bar{p})}.
\] (5.6)

Thus, it is characterized as a weighted average of marginal costs, where the weights stem from the slope of the demand function in the different time periods. Why the slope of the demand function plays a role becomes clear when looking at a simple two period (peak/off-peak) example with constantly increasing marginal costs. In this case, an interior optimal \( p \) satisfies:

\[
\left(\bar{p} - C_q' (D_{op}(\bar{p}))\right) D'_{op}(\bar{p}) = \left(C_q' (D_p(\bar{p})) - \bar{p}\right) D'_p(\bar{p}).
\] (5.7)

In any case, \( \bar{p} \) will be between the two time dependent optimal tariffs. Now start by the special case where the slope of the demand functions is the same in both periods. Then, clearly, \( \bar{p} \) will be exactly in the middle between the (higher) marginal cost of the peak quantity and the (lower) marginal cost of the off-peak quantity, as displayed in Figure 5.1i. Now let the peak demand become less price sensitive, i.e., the inverse demand function of the peak period is steeper, as in Figure 5.1ii. Then welfare can be increased by lowering \( \bar{p} \). Lowering \( \bar{p} \) leads only to a small increase of excess consumption in the peak period, while the welfare gain from reducing underconsumption off-peak is much larger, due to the higher price sensitivity off-peak. The deadweight losses stemming from TIP are depicted in the grey shaded areas.

![Figure 5.1: Welfare optimal time-invariant price for two exemplary demand situations (peak/off-peak) and the resulting deadweight losses shaded in grey.](image)

When the analysis is not restricted to two periods, different shares of consumers on RTP and TIP (\( \alpha > 0 \)) are considered and a possible feed-in of renewables (\( r_t \geq 0 \),
the optimal TIP generalizes to

\[ \bar{p} = \frac{\sum t p^*_t D'_t(\bar{p}^* \mid \bar{p})}{\sum t D'_t(\bar{p}^* \mid \bar{p})}, \text{ with } p^*_t = C'_q(D_t(p^*_t \mid \bar{p}), r_t). \]  

(5.8)

Let us reflect upon the meaning and implications of the price derived in Equation (5.8) in the context of different industry structures. To ensure maximized overall welfare, \( \bar{p}^* \) could be offered by a social planner. In reality, of course, most electric industries are organized by means of competitive generation and retail sectors. Borenstein and Holland present a model assuming retailers which offer tariffs to end consumers to be unbundled from generation. This reflects current regulatory practice, e.g., in some states in the US (see, e.g., Meyer (2012)). In order to model the competitive outcome for the case of retail unbundling, they consider two markets, namely the wholesale and retail market. In the wholesale market, generators engage in Cournot competition by choosing quantities. This results in wholesale prices \( (w_t) \) equal to short-term marginal costs of generation \( w_t = C'_q(D_t(p_t \mid \bar{p}), r_t) \). In the retail market, retailers buy electricity from the wholesale market and sell it to end consumers engaging in Bertrand competition. Based on a zero profit condition for retailers Borenstein and Holland (2005) derive the following TIP for the case of retail unbundling

\[ \bar{p}^{RU} = \frac{\sum t p_t D_t(\bar{p}^{RU} \mid \bar{p}^{RU})}{\sum t D_t(\bar{p}^{RU} \mid \bar{p}^{RU})}, \text{ with } p^{RU}_t = C'_q(D_t(p^{RU}_t \mid \bar{p}^{RU}), r_t). \]  

(5.9)

In this case the TIP is also a weighted average of marginal costs but in this case weighting is based on absolute demand levels instead of the slope of the demand function. The TIP offered by retailers is therefore different from the welfare optimal price and Borenstein and Holland (2005) conclude that the second-best can not be achieved in a competitive market setting. The result has nevertheless to be put into perspective as it only holds under the assumption that retailers and generators need to be unbundled. In many markets, such as in countries in the EU, this is not the case and may lead to different vertical industry structures as displayed in Figure 5.2.

In the case of vertically integrated suppliers that own generation assets and offer retail tariffs, we argue that the competitive outcome may be different from Borenstein and Holland (2005). Compared to the profit function of unbundled retailers, the profit function of vertically integrated suppliers does not depend on the wholesale price. Integrated suppliers are able to generate electricity with their own assets
and do not need to buy it from the wholesale market at the short-term marginal cost of generation. Suppliers could possibly be able to offer lower tariffs and deadweight losses may be reduced. Figure 5.3 shows the TIP in the two period case for the second-best (as derived in Equation (5.8)), retail unbundling (based on Borenstein and Holland (2005), see Equation (5.9)) and a possible TIP offered by integrated suppliers. In addition shaded areas for the profit conditions of retailers and vertically integrated suppliers are illustrated in green (profit) and red (loss).

With retail unbundling, retailers make zero profits. In Figure 5.3ii this implies that the losses in the peak period are equal to the profits in the off-peak period. This defines $p_{RU}^{P}$. At the same time, generators upstream make (short-term) profits. For the case of integrated suppliers, $p_{RU}^{P}$ can no longer be the equilibrium price in a competitive setting, since integrated suppliers make strictly positive (short-run) profits while engaging in price competition. These profits are depicted in triangles (a) and (b) in Figure 5.3ii. In price competition between integrated suppliers prices will decrease, and therefore be lower (and closer to $p_{RU}^{P}$), as shown in Figure 5.3iii.
5.3 Theoretical Analysis

When taking a closer look at Figure 5.3iii, it is important to note that a part of area (c) is the result of profits in the peak and off-peak period. The profits that occur in both periods are shaded in darker green. It may still be the case that under $\bar{p}^*$ the integrated suppliers make (even short-term) losses. This occurs if area (d) is larger than (c). However, the calibration for Germany in Section 5.4 shows that, at least for this application, this is not the case for any realistic parameter assumption.

In this paper, we are interested in the deadweight losses that stem from the additional feed-in of stochastic renewables and will focus in the following on the welfare optimal price, as derived in Equation (5.8) and graphically displayed in Figure 5.3i. Regardless of the industry structure and competitive model setup, the inefficiencies of TIP derived in this paper are a lower bound for the deadweight losses from TIP for any vertical industry structure, as they are based on the welfare optimal price. Any TIP that differs from the welfare optimal outcome will lead to an increase of deadweight losses.

5.3.2 The Impact of Variable Renewable Electricity Generation

In order to handle the additional complexities introduced by the renewable production, we will from now on restrict our analysis to the case of linear demand and supply functions.

The short-run costs of electricity generation essentially depend on the quantity generated by conventional technologies. Conventional technologies are defined as dispatchable power plants whose generation does not primarily depend on weather conditions. The short-run costs of electricity generation from weather dependent technologies, such as wind and solar, can assumed to be zero. A commonly used term in electricity markets is therefore the residual demand $q_{\text{res}}$, defined as the total demand subtracted by the quantity generated from renewables ($q_{\text{res}} = q - r_t$). This is basically the quantity that needs to be generated by conventional technologies. In the linear case, the marginal cost function therefore can be written as $C'_{q_{\text{res}}}(q_{\text{res}}) = a_0 + a_1 q_{\text{res}}$ for $q_{\text{res}} > 0$. The total costs of electricity generation for the short-run can thus be defined as

$$C(q_{\text{res}}) = a_0 q_{\text{res}} + \frac{1}{2} a_1 q_{\text{res}}^2, \quad \forall q_{\text{res}} > 0.$$ (5.10)

The demand of consumers is assumed to be linear according to
where $d_t$ represents the hourly reference demand for electricity and $\epsilon$ is the gradient of the demand curve. The hourly reference demand is assumed to be varying throughout time but we assume the gradient to be constant. When $\epsilon$ is not varying in time, there is a linear relation between the hourly reference demand and the willingness-to-pay for electricity consumption ($wtp = \frac{d_t}{\epsilon}$). Because the relation is linear and the term willingness-to-pay may be more intuitive for the reader, we will use it as a synonym at some points. The willingness-to-pay can assumed to be higher during the day, when economic activity is high, and comparably low during the night. This is also described in Knaut and Paulus (2016). Additionally, there may also be seasonal components captured in $d_t$ depending on the climate conditions. In Germany, for example, there is a higher willingness-to-pay for electricity in the winter, as some of it is used for heating, compared to the summer. For warmer climates this may be different, because air conditioning is driving a lot of the electricity consumption in the summer. Therefore $d_t$ in warmer climates would be higher in the summer compared to the winter.

Since renewable production ($r_t$) and the willingness-to-pay for electricity consumption ($d_t$) both depend on weather conditions and the time of the day, we assume the probability space of the model to be characterized by the joint distribution $f(d_t, r_t)$, the marginal distributions $f_d(d_t), f_r(r_t)$ and the correlation of both stochastic variables $\rho(d_t, r_t)$. This accounts for the fact that the utility of electricity consumption and renewable generation are correlated. For example, solar generation is highest during the day when also the willingness-to-pay of consumers is the highest. Thus, we can infer that the variables $d_t$ and $r_t$ are not completely independent of each other. We therefore assume the joint distribution to be characterized by the expected values ($\mu_d, \mu_r$), the standard deviations ($\sigma_d, \sigma_r$) and the correlation of both variables ($\rho_{d,r}$) (see also Appendix 5.6.1).

**The Second-best Tariff and the Resulting Deadweight Losses**

For the previously discussed properties of the joint probability distribution we can solve the linear model and derive the second-best TIP offered to end consumers.

**Proposition 5.1.** The optimal tariff offered to end consumers under TIP depends on the expected values of demand ($\mu_d$) and renewable generation ($\mu_r$) without being affected by the share of consumers on TIP, the standard deviations or the correlation. The welfare
5.3 Theoretical Analysis

optimal time-invariant tariff for any given distribution \( f(d_t, r_t) \) is

\[
\overline{p}^* = \frac{a_0 + a_1(\mu_d - \mu_r)}{1 - a_1 \varepsilon}, \quad \text{as long as } \tilde{D}_t(p_t, \overline{p}) > r_t. \tag{5.12}
\]

Proof. The second-best tariff was previously derived as a weighted average of the short-term marginal costs of electricity generation, with the weighting being based on the gradient of the demand curve. For the linear case, as previously defined, the gradients are identical \((D'(p) = \varepsilon)\). The optimal price is therefore identical to the expected marginal costs of generation and we can write the optimal price as

\[
\overline{p} = \int C'_q(\tilde{D}_t(p_t, \overline{p}), r_t) f(d_t, r_t) d(d_t, r_t)
= \int \left[ a_0 + a_1(\tilde{D}_t(p_t, \overline{p}) - r_t) \right] f(d_t, r_t) d(d_t, r_t)
\tag{5.13}
\]

Besides depending on \( \overline{p} \), the overall quantity demanded \( \tilde{D}_t(p_t, \overline{p}) \) depends on the short-term equilibrium with consumers also being on RTP. The overall quantity consumed can be derived by solving

\[
C'_q(\tilde{D}_t(p_t, \overline{p}), r_t) = \tilde{D}_t^{-1}(p_t, \overline{p}). \tag{5.14}
\]

which results in

\[
\tilde{D}_t(p_t, \overline{p}) = \frac{\alpha \varepsilon (\overline{p} + a_1 r_t - a_0) + d_t - \varepsilon \overline{p}}{a_1 \alpha \varepsilon + 1}. \tag{5.15}
\]

We can now use the result from (5.15) in (5.13) and solve this for the optimal price by making use of the properties for the joint distribution from Appendix 5.6.1. By plugging in these properties and solving for \( \overline{p} \), we obtain the optimal price as written in Equation (5.12) (the extensive derivation can be found in the Appendix 5.6.2).

Interestingly, the optimal tariff only depends on the expected values of demand \((\mu_d)\) and renewable generation \((\mu_r)\) without being affected by the share of consumers on TIP, the standard deviations or the correlation. Furthermore, we can observe the following intuitive results: (1) an increase of the supply curve gradient \((a_1)\) leads to a higher tariff; (2) a higher willingness-to-pay \((\mu_d)\) increases the tariff; (3) an increase of renewable generation \((\mu_r)\) lowers the tariff; and (4) a lower gradient of the demand curve \((\varepsilon)\) leads to a lower tariff.
The derived tariff represents the second-best outcome, when not all consumers are on RTP \((\alpha < 1)\). Because the first-best outcome can only be achieved with all consumers being on RTP, it is of interest to quantify the deadweight losses resulting from TIP.

**Proposition 5.2.** The deadweight loss induced by a time-invariant tariff is mainly driven by the standard deviations of demand \((\sigma_d)\), renewable generation \((\sigma_r)\) and their respective correlation \((\rho_{dr})\). The welfare loss from TIP amounts to

\[
\Delta W = T \frac{a_1^2 \epsilon (1 - \alpha)}{2(a \alpha_1 \epsilon + 1)(a_1 \epsilon + 1)} \left( \sigma_d^2 + \sigma_r^2 - 2 \rho_{dr} \sigma_d \sigma_r \right), \quad \text{as long as } \hat{D}_t(p_t, \bar{p}) > r_t.
\]

\(\text{(5.16)}\)

**Proof.** The welfare at time \(t\) can be written as

\[
W_t = \hat{U}_t(p_t, \bar{p}) - C(\hat{D}_t(p_t, \bar{p}), r_t).
\]

The price \(p_t\) under RTP is equal to the intersection of the short-run marginal costs of generation and the respective demand of consumers. For the linear case it depends on the total quantity from (5.15) and can be derived as

\[
p_t^* = C'\left(\hat{D}_t(p_t, \bar{p}), r_t\right) = a_0 + a_1 d_t - a_0 \alpha \epsilon - r_t - (1 - \alpha) \epsilon p_t a_1 \alpha \epsilon + 1 .
\]

\(\text{(5.18)}\)

The first-best outcome would be obtained with all consumers being on RTP for \(\alpha = 1\), which results in a real-time price of

\[
p_t^e = a_0 + \frac{a_1 (d_t - a_0 \epsilon - r_t)}{a_1 \epsilon + 1} .
\]

\(\text{(5.19)}\)

The welfare for all consumers being on RTP can be written as

\[
W_t^e = \int_0^{\hat{D}(p_t^e)} P_t(q) dq - C(D_t(p_t^e), r_t).
\]

\(\text{(5.20)}\)

Welfare under the second-best for only a share of consumers being on RTP sums up to
The deadweight loss from consumers being on TIP results in

$$W_t^* = \alpha \int_0^{D(p_t^*)} P_t(q) dq + (1 - \alpha) \int_0^{D(\overline{p}^t)} P_t(q) dq - C(D_t(p_t^*, \overline{p}^t), r_t). \quad (5.21)$$

The deadweight loss from consumers being on TIP results in

$$\Delta W_t = W_t^e - W_t^* \quad (5.22)$$

In order to assess the overall deadweight loss over all time periods $t$, we need to integrate over the respective distributions for $d_t$ and $r_t$. Because we assume $f(d_t, r_t)$ being normalized, we can multiply it with the number of all considered time periods $T$ to obtain the deadweight loss over the whole timespan

$$W = T \int \Delta W_t f(d_t, r_t) d(d_t, r_t). \quad (5.23)$$

The linear formulations for the demand, supply and prices can be used to calculate the deadweight loss. By making use of the additional properties of the distributional function $f(d_t, r_t)$ (see Appendix 5.6.1) we are able to obtain the deadweight loss from Equation (5.16). The detailed calculations can be found in Appendix 5.6.3.

The deadweight loss induced by TIP in the linear case does not depend on the expected level of demand ($\mu_d$) or renewable generation ($\mu_r$), but depends on the variability of demand and renewable generation and their respective correlation over time. Based on Proposition 5.2, we can say that the following drivers lead to an increase of deadweight losses from TIP: (1) a lower share of consumers on RTP ($\alpha$); (2) a steeper supply curve ($a_1$); (3) a higher demand responsiveness of consumers ($\epsilon$); (4) an increasing demand variation ($\sigma_d$); (5) an increasing variation of renewable generation ($\sigma_r$); (5) a lower correlation of demand and renewable generation ($\rho_{d,r}$).

It is especially interesting how the variability of demand and renewable generation are interacting. When we consider the effect of each parameter individually, we see that an increase in variance leads to a higher deadweight loss. Nevertheless, the deadweight loss could also be reduced if the correlation between the two parameters is increased. In the extreme case where the standard deviation is equal
Retail Tariff Design in Electricity Markets with Variable Renewable Production

\( (\sigma_d = \sigma_r) \) and the correlation is one \( (\rho_{d,r} = 1) \), the deadweight loss from TIP would be zero. In this case, marginal generation costs would not fluctuate in time and stay constant. The variation in demand would be perfectly compensated by the variation of renewable generation on the supply side.

Besides grasping the drivers for a welfare optimal time-invariant tariff that is limiting deadweight losses, we can also draw conclusions on welfare optimal TOU tariffs based on Proposition 5.2.

**Corollary 5.1.** TOU tariffs should aim at the clustering of time periods with low variability in demand \( (\sigma_d) \) and renewable generation \( (\sigma_r) \), as well as a high correlation between both variables \( (\rho_{d,r}) \) to reduce deadweight losses.

This conclusion is quite intuitive and TOU rates have historically followed this attempt. A common design for TOU pricing are for example peak and off-peak tariffs, as well as a distinction between working and non-working days. The variation in demand is lower within these peak and off-peak periods and therefore leads to a reduction of deadweight losses. Whereas TOU pricing was in the past primarily focused on the variation of the demand side, renewable electricity generation is going to also play a major role in future electricity systems. With an increasing share of solar generation, this does for example not affect the peak/off-peak pricing, since the variation of solar driven renewable generation also follows this pattern. But it may for example increase the importance of seasonal tariffs, as solar and wind generation inhibit strong seasonal variations. We will take a closer look at TOU rates in a case study in Section 5.4.3.

**The Taxing of Electricity**

Consumers of electricity are often not only charged for the electricity they consume but also additional taxes and levies are added to the price for electricity generation. These may for example be charges for the transmission of electricity through the network or taxes and levies raised by the state. Commonly these additional price components are added to the price that is charged by the utility company. We will therefore analyze the implications of an additional price component \( c_{\text{tax}} \) that is added to the price for end consumers.

**Proposition 5.3.** The introduction of an additional tax on electricity leads to an adjustment of the welfare optimal TIP that depends on the demand and supply gradients. When the price is adjusted in this way, deadweight losses that stem from TIP with taxing
are the same as in the case with no taxing. The welfare optimal time-invariant price charged on end consumers for a tax of \((c_{tax})\) is

\[
\bar{p}_{tax}^* = \frac{1}{a_1 \epsilon + 1} \left( a_0 + a_1 \mu_d - a_1 \epsilon c_{tax} \right) + c_{tax}, \quad \text{as long as } \tilde{D}_t(p_t, \bar{p}_{tax}^*) > r_t.
\] (5.24)

**Proof.** See Appendix 5.6.4.

As we can see from Equation (5.24), the additional price component \(c_{tax}\) does not only increase the TIP previously derived in Equation (5.12). Besides increasing the price charged by \(c_{tax}\), it also leads to a reduction of the welfare optimal tariff by \(\frac{a_1 \epsilon}{a_1 \epsilon + 1} c_{tax}\). This result is quite intuitive and by adjusting the TIP in this way, deadweight losses resulting from the tax can be reduced.

In addition to the effect on the optimal price level, we show in the Appendix 5.6.4 the effects of a tax on deadweight losses. Whereas the introduction of a tax generally leads to a higher deadweight loss, the deadweight loss that stems solely from TIP is not affected by an additional price component \((c_{tax})\). Due to the linear model setup, quantity adjustment by the demand are independent of the absolute price level. Basically this means that the deadweight losses from TIP with a tax occur at a higher (lower) price (demand) level, but the magnitude of the deadweight loss is not changed.

### 5.4 Empirical Analysis

The implications of TIP compared to RTP were so far analyzed from a theoretical perspective. Thereby we were already able to identify the main drivers for the optimal level of TIP and the resulting inefficiencies. In a next step we can apply the new findings to the German electricity system and gain insights on the magnitude of the respective inefficiency. Based on the model we can furthermore get insights on the short-term profits of integrated suppliers as discussed in Section 5.3.1.

Germany can hold as a good practical example for three reasons. First, only transmission and distribution in Germany is being regulated. This means that retailers and generators do not need to be modeled as separate entities and the second-best may be offered by integrated suppliers. Second, the electricity system is currently under transition with a large increase of renewable capacities that have an effect on the efficiency of TIP. Third, a major share of household consumers in Germany
is currently still on TIP. An aim of the German government and also of many other countries in Europe is to increase the demand side participation in the electricity system (BMWi, 2016, EU Commission, 2014). This could for example be achieved by installing smart meters in many households and changing the tariff scheme from TIP towards RTP. In a first step this could also mean offering consumers TOU pricing in order to lift a fraction of the potential efficiency gains from RTP. We will therefore apply the previous analysis to the German system in order to quantify the potential benefits of RTP and the way towards RTP by implementing different TOU pricing schemes.

It is important to stress that the analysis builds on a very stylized framework and will not be able to address all peculiarities of the German electricity sector. Nevertheless, it helps to put the previously derived theoretical results into perspective.

### 5.4.1 The German Electricity System

The German electricity system has seen a tremendous increase of renewable capacities in recent years. The share of renewable electricity generation from solar and wind increased from 9.2% in 2010 to 18.3% in 2015 (A. G. Energiebilanzen eV, 2016). Besides generation from variable renewable sources, a large share of electricity is generated from lignite (24.0%), hard coal (18.3%), nuclear (14.2%) and gas (9.4%) (A. G. Energiebilanzen eV, 2016). In order to grasp the properties of the demand and supply side, we will make use of hourly data on the demand, prices, as well as generation from wind and solar. Table 5.1 summarizes the data used for 2015 in the German electricity system.\(^4\)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>31.6</td>
<td>9.005</td>
<td>3.985</td>
<td>12.990</td>
</tr>
<tr>
<td>Std</td>
<td>12.7</td>
<td>7.227</td>
<td>6.044</td>
<td>8.609</td>
</tr>
</tbody>
</table>

Based on the provided data we are in a first step able to estimate the conventional supply curve for the German electricity system. We do so by explaining the day-ahead prices in Germany based on the residual demand of consumers that is satisfied by mainly conventional electricity generation. The residual demand is defined

\(^4\)Price data is taken from EPEX (2016), renewable data from EEX (2016) and load data from ENTSO-E (2016).
as \( \text{ResDemand}_t = \text{Load}_t - \text{Wind}_t - \text{Solar}_t \). The scatter plot in Figure 5.4 displays the relation between prices and residual demand for Germany. In addition we added the result of a linear estimation. The previously mentioned conventional power plants can all be assumed at supplying electricity at increasing marginal costs of generation. For example nuclear power plants produce electricity at lower marginal costs compared to hard coal power plants. The whole conventional supply (electricity generation without solar and wind) adds up to the marginal cost function. The detailed results of the OLS regression can be found in Appendix 5.6.5. It is important to note that we are assuming a perfectly price inelastic demand for this estimation of the marginal cost function. With all consumers being on TIP this assumption would be true. In Germany this is not necessarily true as Knaut and Paulus (2016) show that there is some price responsiveness of end consumers present with respect to the day-ahead price. Nevertheless the price elasticity is comparably small. For this illustrative case study we will therefore rely on the estimation assuming an inelastic demand. The coefficients of the regression can be applied in our model as the gradient of the marginal cost function \( a_1 \) and the offset \( a_0 \).

![Figure 5.4: Marginal generation costs of conventional power technologies in Germany.](image)

The offset \( a_0 \) is estimated as being negative. In Figure 5.4, we can see that negative prices can also be observed in the German day-ahead market. This is due to the fact that some power plants may even accept generating electricity at negative prices. These are for example combined heat and power plants that are not only generating electricity but also supply heat to end consumers. Additional information on generation capacities with a minimum level of output in Germany can be found in Hirth (2015).
With this approach we are already able to model the relationship between wholesale (day-head) prices and demand. For end consumers in Germany the wholesale price is only one of many additional components on the electricity bill. In addition consumers are charged taxes and levies, as well as costs for the network. In Germany, network charges depend on the peak capacity and on the energy supplied. Here, we will assume an average level of 68.6 EUR/MWh. Taxes and levies are also added to the wholesale price and sum up to an average level of 151.9 EUR/MWh.\(^5\) A large part of this sum is made up by the so called EEG levy that is raised for the support of renewable electricity generation (61.7 EUR/MWh) (Netztransparenz, 2015).

The load in the electricity system can be regarded as the best indicator of the overall consumption of end consumers. Based on the load we can calculate the reference demand \(d_t\) of end consumers \(d_t = \text{Load} + \epsilon (\bar{p}_{\text{da}} + c_{\text{tax}})\), where \(\bar{p}_{\text{da}}\) is the average day-ahead price and \(c_{\text{tax}}\) are the additional charges (network, taxes and levies). The only parameter that we can not observe in the data is the hypothetical demand responsiveness of end consumers (\(\epsilon\)). Because most end consumers are on TIP and we are not able to distinguish between consumers being on TIP, TOU or RTP in our data, we can draw no conclusion on the level of demand responsiveness. We will therefore rely on values that are in line with short-run demand elasticities from the literature and vary the level in a sensitivity analysis.\(^6\) Besides the gradient of the demand curve being constant, the elasticity is changing from time to time since it also depends on the absolute level of demand. Throughout our analysis we will make use of constant gradients but also give an indication for the average elasticity throughout the year.

The joint probability distribution of demand and renewables plays a major role in the analysis, as we have learned in Section 5.3.2. In Table 5.1 we can already observe the expected values (\(\mu_d, \mu_r\)) and the respective standard deviations (\(\sigma_d, \sigma_r\)). Because both variables are not independent of each other, we also need to consider the correlation (\(\rho_{d,r}\)), that is 0.256 in 2015 for renewable generation from wind and solar (just for wind the correlation are 0.035 and for solar 0.322).

With this model configuration, we can now take a closer look at the implications of RTP and the inefficiency of TIP.

\(^5\)Network costs and levies are taken from Eurostat (2016) based on consumers with a consumption between 2500 kWh and 5000 kWh (Band DC) in 2015.

\(^6\)For a review of demand elasticities see for example Knaut and Paulus (2016).
5.4 Empirical Analysis

5.4.2 Consumption and Welfare with TIP

RTP and TIP can be regarded as two extreme tariff schemes for end consumers. Under TIP consumers pay an average price throughout the year. The variation in electricity consumption is therefore mainly driven by their time-varying utility with respect to one price. In contrast on RTP consumers also observe different prices in each hour and may adjust their electricity consumption. The respective consumption patterns for all consumers either being on RTP or TIP are displayed for an average week in Figure 5.5.

We can see the distinctive pattern of electricity consumption in Germany with a peak in the early morning and another peak in the early evening. The pattern evolves from the two extreme cases with all consumers either being on RTP or TIP. When consumers are on TIP, electricity consumption at peak times is increased compared to RTP because the price on TIP is much lower. Interestingly the peak at the early evening almost disappears when all consumers are on RTP. At off-peak times, the electricity consumption on TIP on the other hand is lower because the price of consumers on TIP is relatively higher compared to RTP.

The changes in electricity consumption especially affect overall welfare. For a demand gradient of 0.2 (average elasticity of -0.088), the deadweight loss over the whole year in 2015 would have amounted to EUR 97.1 million. The total welfare with all consumers on TIP is about EUR 198000 million. If all consumers would
switch from TIP to RTP, this would result in an efficiency increase of 0.05%.\footnote{It is important to note that in our model demand is characterized by the same (linear) function. This makes overall welfare extremely large and hence all changes to it relatively small. In reality there may be price levels at which consumers become more elastic and the resulting overall welfare would therefore probably be lower. This would result in a higher efficiency increase (in relative terms) from RTP.} In order to gain additional insights, we conduct sensitivity analysis with respect to the demand gradient and the level of renewable installed capacities. The results are displayed in Figure 5.6.

As mentioned there is no clear empirical evidence on the level of demand responsiveness of end consumers to real-time prices. We therefore vary the demand gradient ($\varepsilon$) in Figure 5.6.\footnote{In terms of demand elasticity the value of $\varepsilon = 3$ corresponds to an average elasticity of -0.195 and the previously used gradient of $\varepsilon = 0.2$ to an average elasticity of -0.088.} The difference in welfare between RTP and TIP increases with a higher demand gradient. This means that the deadweight loss of TIP increases when consumers are more price responsive. The overall welfare loss increases for a higher gradient and approaches a value of about EUR 400 million for a demand gradient of 3.

![Graph](image)

(i) Demand gradient sensitivity  
(ii) Renewable sensitivity for $\varepsilon = 0.2$ (Reference capacities (100%) are 44.67 GW for wind and 39.79 GW for solar.)

Figure 5.6: Sensitivities for different demand gradients and renewable generation

As the share of electricity generation from renewables is expected to increase in the upcoming years, we also conduct a sensitivity analysis regarding the installed capacities of wind and solar power. The previous calculations were based on installed capacities of wind and solar in 2015 which amounted to 44.67 GW and 39.79 GW respectively. In order to shed light on the deadweight losses in combination with TIP, we vary the installed capacity starting from zero to a doubling. This is done for each technology separately and for the portfolio of both technologies. All variations pri...
5.4 Empirical Analysis

5.4.1 Impact of Renewables on Portfolio Variation

The results are displayed in Figure 5.6ii. In the portfolio variation, the correlation of demand and renewables stays constant because the structure of renewable generation over time is not changing. Therefore the shape of the curve is primarily driven by the increase in standard deviation changing from zero to 17.2 GWh for a 200% renewable portfolio. Interestingly the increase in renewables first leads to a reduction in deadweight losses up to a level of about 30%. When we compare this with the sensitivities for only solar and only wind, we can conclude that the reduction in deadweight losses is mainly stemming from solar generation. The higher correlation of demand and solar (0.322) compared to wind (0.035) results in a reduction of deadweight losses. This effect, however, diminishes for increasing shares of renewable generation when shares of wind and solar are simultaneously increased. Based on the renewable sensitivity we can learn that the value of both technologies, wind and solar, may be very different when combined with TIP as they lead to very different effects on overall welfare.

The profits of integrated suppliers in all sensitivities are strictly positive which underlines our previous conjecture that the second-best TIP may be offered by integrated suppliers in a competitive market. However, when we consider the profit function of unbundled retailers, we find that they would incur losses when offering the second-best TIP. This underlines the statement by Borenstein and Holland that the second-best TIP will not be offered under retail unbundling. Nevertheless, this result only holds for countries where regulation prohibits the vertical integration of generators and retailers, which is not the case in Germany.

The results illustrate the effect when all consumers are either on RTP or TIP. The difference in welfare thus reflects the most extreme case. Whenever already a share of consumers is on RTP, the implications would be lower. This can also be observed based on a sensitivity analysis for \( \alpha \) (cf. Figure 5.8 in the Appendix). If for example a share of 50% of consumers is already on RTP, the welfare loss from TIP reduces to EUR 41.8 million.

5.4.3 On the Way to the First-best: Introducing Time-of-use Tariffs

Previously we found that the deadweight loss primarily depends on the standard deviations \( (\sigma_d, \sigma_r) \) and the correlation \( (\rho_{d,r}) \). This finding can also be used to analyze different TOU pricing schemes, by grouping time spans with low variability and high correlation, as mentioned in Corollary 5.1.
Table 5.2: Optimal prices and welfare implications for different time-of-use pricing schemes

<table>
<thead>
<tr>
<th>Unit</th>
<th>TIP</th>
<th>Peak/Off-Peak</th>
<th>Peak/Off-peak/Week</th>
<th>Peak/Off-peak/Weekend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{p}$ EUR/MWh</td>
<td>216.4</td>
<td>218.8</td>
<td>212.5</td>
<td>222.2</td>
</tr>
<tr>
<td>$\mu_d$ GWh</td>
<td>64.0</td>
<td>69.2</td>
<td>55.4</td>
<td>73.2</td>
</tr>
<tr>
<td>$\mu_r$ GWh</td>
<td>13.0</td>
<td>15.2</td>
<td>9.3</td>
<td>15.1</td>
</tr>
<tr>
<td>$\sigma_d$ GWh</td>
<td>10.2</td>
<td>8.5</td>
<td>6.4</td>
<td>5.9</td>
</tr>
<tr>
<td>$\sigma_r$ GWh</td>
<td>8.6</td>
<td>8.7</td>
<td>7.0</td>
<td>8.8</td>
</tr>
<tr>
<td>$\rho_{d,r}$</td>
<td>0.256</td>
<td>0.05</td>
<td>0.059</td>
<td>0.112</td>
</tr>
<tr>
<td>$N$ h</td>
<td>8760</td>
<td>5475</td>
<td>3285</td>
<td>3915</td>
</tr>
</tbody>
</table>

$\Delta W_t$ mEUR | 91.7          | 61.2          | 14.5              | 11.0                 |

Table 5.2 shows the optimal price level and the respective properties for demand and renewable generation under different TOU schemes. Peak is here chosen to be from 7:00 - 22:00 and Off-Peak from 22:00 - 7:00. The optimal prices ($\bar{p}$) for both time periods can be calculated based on the expected values $(\mu_d, \mu_r)$ (see Equation (5.12)). Furthermore we can directly calculate the deadweight loss induced by TOU pricing compared to RTP using Equation (5.16). Here we can see that a pricing scheme of peak and off-peak leads to a lower deadweight loss of about EUR 10 million by defining a higher price in the peak and a lower price for the off-peak period.

The deadweight loss can further be reduced by increasing the granularity of the tariff design and distinguishing between week and weekend. In this case the deadweight loss can further be reduced by EUR 20 million to EUR 61.2 million.

Based on this simple approach different pricing schemes can be evaluated. The
5.5 Conclusion

results for pricing schemes ranging from two prices (Peak/Off-peak) to 576 different prices (Hourly;Week/Weekend;Monthly) are calculated and displayed in Figure 5.7. We can see that TOU pricing is able to decrease the deadweight loss significantly from EUR 91.7 million to EUR 38.5 million. Nevertheless the tariff with the highest granularity still makes up for a deadweight loss of 42% compared to TIP. Only a tariff with an even higher number of different prices for periods would be able to further decrease deadweight losses.

5.5 Conclusion

This paper discusses different tariff designs for a good with highly fluctuating demand. The analysis focuses on the electricity sector that is undergoing a major transition due to the deployment of variable renewable generation with the aim of decarbonization. In this context it becomes more important that price signals are transmitted to end consumers by appropriate pricing schemes. We, therefore, analyze the inefficiency introduced by time-invariant pricing (TIP) compared to real-time pricing (RTP) and also extend the analysis to time-of-use (TOU) pricing.

TIP leads to deadweight losses since consumers do not respond to the marginal costs of generation. The increased generation from renewable energies may lead to an increase of the deadweight loss of TIP compared to RTP. Depending on the variation in demand and renewable generation we can observe two effects. First, higher variations of demand and renewable generation lead to an increase of deadweight losses. Second, a high correlation between both variables dampens their effect on deadweight losses.

In an illustrative case study for Germany, we find that the deadweight loss increases for high shares of renewable generation. Especially wind generation leads to an increase of losses because the correlation with demand is low. Increasing shares of solar capacity on the other hand may even be able to decrease deadweight losses because correlation with demand is relatively high (0.322). Furthermore, we find that TOU pricing is only able to lift a fraction of the efficiency gains that can be achieved under RTP.

Our results have important implications for policy makers. Smart metering and the introduction of real-time or TOU pricing is so far seen as an important instrument for the decarbonization of the electricity sector (EU Commission, 2014, FERC, 2011). In this paper, however, we show that the expected efficiency increases may
be relatively small. If all consumers would switch from TIP to RTP, welfare could be increased by EUR 97.1 million which essentially translates to EUR 1.2 per German inhabitant per year. The results, however, depend on the characteristics of the demand function of consumers.

We assumed consumers as risk-neutral and consumption being based on a linear demand function. In reality, consumers of electricity are probably rather risk-averse and also have preferences regarding the granularity of their pricing scheme. Thus, it may be the case that consumers prefer being on TIP instead of RTP, as they do not need to worry about possible price spikes. Maybe there are also some similarities to preferences in other sectors with fluctuating demand over time. In the telecommunications sector, for example, flat rate tariffs are predominantly chosen by end consumers. In flat rate tariffs consumers pay a price for an unlimited quantity. Recently, also utility companies in the electricity sector start offering similar tariffs, whereas the conditions are much more complex compared to the telecommunications sector (Reid, 2016). Accounting for risk-averse behavior of consumers or considering different shapes of demand functions could therefore be an interesting extensions of our model, which we leave open for further research.

---

9Flat rate tariffs for electricity are currently only being offered by utility companies when consumers commit to an investment in generation equipment such as solar panels and electricity storage (providers in Germany are for example Beegy, innogy and sonnen.)
5.6 Appendix

5.6.1 Properties of the distributional function $f(d_t, r_t)$

The probability distribution $f(d_t, r_t)$ is defined with the following properties:

- Zeroth moment: $\int f(d_t, r_t) d(d_t, r_t) = 1$ (5.25a)

- First moment of $d_t$: $\int d_t f(d_t, r_t) d(d_t, r_t) = \mu_d$ (5.25b)

- First moment of $r_t$: $\int r_t f(d_t, r_t) d(d_t, r_t) = \mu_r$ (5.25c)

- First joint moment: $\int r_t d_t f(d_t, r_t) d(d_t, d_r) = \mu_r \mu_d + \sigma_d \sigma_r \rho_{d,r}$ (5.25d)

- Second central moment of $d_t$: $\int d_t^2 f(d_t, r_t) d(d_t, d_r) = \mu_d^2 + \sigma_d^2$ (5.25e)

- Second central moment of $r_t$: $\int r_t^2 f(d_t, r_t) d(d_t, d_r) = \mu_r^2 + \sigma_r^2$. (5.25f)
5.6.2 Extensive Proof of Proposition 5.1

The inverse of the demand function is $\hat{P}_t(q) = \frac{1}{\alpha e}(\hat{p} \alpha e - \hat{p}e + d_t - q)$. Setting this equal to the short-term marginal costs of generation we obtain the total quantity demanded in (5.15).

Inserting the total demand into (5.13) and simplifying, we obtain

$\bar{p} = \int \left[ a_0 + a_1 \left( \hat{D}_t(p_t, \hat{p}) - r_t \right) \right] f(d_t, r_t) d(d_t, r_t)$

$= \int \left[ a_0 + a_1 \left( \frac{ae(\hat{p} - a_0) + d_t - e\hat{p}}{a_1 \alpha e + 1} \right) \right] f(d_t, r_t) d(d_t, r_t)$

$= a_0 + a_1 \frac{ae(\hat{p} - a_0) - e\hat{p}}{a_1 \alpha e + 1} + \int \left[ a_1 \frac{ae a_1 r_t + d_t - a_1 r_t}{a_1 \alpha e + 1} \right] f(d_t, r_t) d(d_t, r_t)$

$= \frac{1}{a_1 \alpha e + 1} \left[ a_0 + a_1 a_1 e \hat{p} - a_1 e \hat{p} + a_1 \int [d_t - r_t] f(d_t, r_t) d(d_t, r_t) \right]$ (5.26)

$= \frac{a_0 + a_1 \mu_d - a_1 \mu_r}{a_1 \alpha e + 1} + \hat{p} \frac{a_1 a_1 e - a_1 e}{a_1 \alpha e + 1}$.

We can now rearrange this and solve for $\bar{p}$

$\bar{p} \left( 1 - \frac{a_1 a_1 e - a_1 e}{a_1 \alpha e + 1} \right) = \frac{a_0 + a_1 \mu_d - a_1 \mu_r}{a_1 \alpha e + 1}$

$\bar{p} \frac{a_1 e + 1}{a_1 \alpha e + 1} = \frac{a_0 + a_1 \mu_d - a_1 \mu_r}{a_1 \alpha e + 1}$ (5.27)

$\bar{p} = \frac{a_0 + a_1 \mu_d - a_1 \mu_r}{a_1 e + 1}$.

5.6.3 Extensive Proof of Proposition 5.2

The welfare in time period $t$ can be derived as

$W_t = \hat{U}_t(p_t, \hat{p}) - C(\hat{D}_t(p_t, \hat{p}), r_t)$

$= a \hat{D}_t(p_t) d_t - \frac{D_t(p_t)^2}{2} + \frac{1 - a}{\epsilon} \left( \hat{D}_t(\hat{p}) d_t - \frac{D_t(\hat{p})^2}{2} \right)$

$- a_0 (\hat{D}_t(p_t, \hat{p}) - r_t) - \frac{a_1}{2} (\hat{D}_t(p_t, \hat{p}) - r_t)^2$. (5.28)
The quantity demanded by consumers on TIP and RTP can be written as

\[ D(p_t) = d_t - \frac{\epsilon}{a_1 \alpha e + 1} (\bar{p} a_1 \alpha e - \bar{p} a_1 e + a_0 + a_1 d_t - a_1 r_t) \]  

(5.29)

\[ D(\bar{p}) = d_t - \epsilon \bar{p}. \]  

(5.30)

By inserting the demand of consumers and simplifying, we obtain the welfare in time period \( t \) dependent on \( \bar{p} \) and \( \alpha \)

\[
W_t = \frac{1}{\epsilon (a_1 \alpha e + 1)} \left[ \frac{a_0^2 \alpha}{2} e^2 - a_0 \epsilon (d_t - r_t) + a_1 d_t \epsilon r_t - \frac{a_1 \epsilon}{2} r_t^2 + \frac{d_t^2}{2} \right] 
+ (\alpha - 1) \left( \frac{\bar{p}^2 a_1}{2} e^3 + \frac{\bar{p}^2 e^2}{2} - \bar{p} a_0 e^2 - \bar{p} a_1 e^2 (d_t - r_t) + \frac{a_1 \epsilon}{2} d_t^2 \right). 
\]  

(5.31)

For the first-best outcome with all consumers on RTP (\( \alpha = 1 \)), the welfare in time period \( t \) is

\[
W_t^e = \frac{1}{\epsilon (a_1 \alpha e + 1)} \left[ \frac{a_0^2 \alpha}{2} e^2 - a_0 \epsilon (d_t - r_t) + a_1 d_t \epsilon r_t - \frac{a_1 \epsilon}{2} r_t^2 + \frac{d_t^2}{2} \right]. 
\]  

(5.32)

When consumers on TIP are offered the second-best TIP \( \bar{p}^{*} \), this results in

\[
W_t^{*} = \frac{1}{2 \epsilon (a_1 \alpha e + 1)} \left[ e^2 (a_0 + a_1 \mu_d - a_1 \mu_r)^2 (\alpha - 1) 
+ 2 e^2 (a_0 + a_1 \mu_d - a_1 \mu_r) (-a_0 \alpha + a_0 - a_1 \alpha d_t + a_1 \alpha r_t + a_1 d_t - a_1 r_t) 
+ (a_1 \epsilon + 1) (a_0^2 \alpha e^2 - 2 a_0 d_t \epsilon + 2 a_0 \epsilon r_t + a_1 \alpha d_t^2 \epsilon - a_1 d_t^2 \epsilon + 2 a_1 d_t \epsilon r_t - a_1 \epsilon r_t^2 + d_t^2) \right] 
\]  

(5.33)

The deadweight loss in time period \( t \) therefore can be written as

\[
\Delta W_t = W_t^e - W_t^{*} 
= \frac{a_0^2 \epsilon (1 - \alpha)}{2 (a_1 \epsilon + 1) (a_1 \alpha e + 1)} \left[ d_t^2 - 2 d_t \mu_d + 2 d_t \mu_r - 2 d_t r_t + \mu_d^2 - 2 \mu_d \mu_r 
+ 2 \mu_d r_t + \mu_r^2 - 2 \mu_r r_t + r_t^2 \right] 
\]  

(5.34)
When we now integrate over the probability space for \(d_t\) and \(r_t\), this results in

\[
\Delta W = T \int \Delta W_t f(d_t, r_t) d(d_t, r_t)
\]

\[
= T \frac{a_1^2 \varepsilon_1 (1 - \alpha)}{2(a_1^2 \varepsilon_1 + 1)(a_1^2 \alpha \varepsilon_1 + 1)} \left( \mu_d^2 - 2 \mu_d \mu_r + \mu_r^2 \right)
\]

\[
+ \int \left( d_t^2 - 2d_t \mu_d + 2d_t \mu_r - 2d_t r_t + 2 \mu_d r_t - 2 \mu_r r_t + r_t^2 \right) f(d_t, r_t) d(d_t, r_t)
\]

\[
= T \frac{a_1^2 \varepsilon_1 (1 - \alpha)}{2(a_1^2 \varepsilon_1 + 1)(a_1^2 \alpha \varepsilon_1 + 1)} \left( \sigma_d^2 + \sigma_r^2 - 2 \rho \sigma_d \sigma_r \right).
\]

(5.35)

### 5.6.4 Proof of Proposition 5.3

**Proof.** The introduction of a tax leads to a shift of the supply function that consumers react to. The short-term marginal costs of generation including the tax therefore increase to

\[
\tilde{C}'_{q,tax}(\tilde{D}_t(p_t, \overline{p}), r_t) = a_0 + a_1 \left( \tilde{D}_t(p_t, \overline{p}) - r_t \right) + c_{tax}. 
\]

This changes the total quantity that is consumed to

\[
\tilde{D}_{t,tax}(p_t, \overline{p}) = \frac{1}{a_1^2 \varepsilon_1 + 1} \left( \overline{p} a_1 \alpha \varepsilon_1 - \overline{p} a_1 \alpha \varepsilon_1 + a_0 a_1 \alpha \varepsilon_1 - a_0 a_1 \alpha \varepsilon_1 + d_t \right). 
\]

(5.36)

Based on this, we can solve for welfare maximizing TIP by solving

\[
\overline{p} = \int \left[ a_0 + c_{tax} + a_1 \left( \tilde{D}_{t,tax}(p_t, \overline{p}) - r_t \right) \right] f(d_t, r_t) d(d_t, r_t)
\]

\[
= \frac{1}{a_1^2 \alpha \varepsilon_1 + 1} \left( \overline{p} a_1 \alpha \varepsilon_1 - \overline{p} a_1 \alpha \varepsilon_1 + a_0 + a_1 \mu_d - a_1 \mu_r + c_{tax} \right). 
\]

(5.37)

By rearranging, we obtain the optimal price

\[
\overline{p} \left( 1 - \frac{a_1 \alpha \varepsilon_1 + a_1 \varepsilon_1}{a_1 \alpha \varepsilon_1 + 1} \right) = \frac{1}{a_1^2 \alpha \varepsilon_1 + 1} \left( a_0 + a_1 \mu_d - a_1 \mu_r + c_{tax} \right)
\]

\[
\overline{p} = \frac{1}{a_1^2 \varepsilon_1 + 1} \left( a_0 + a_1 \mu_d - a_1 \mu_r + c_{tax} \right),
\]

(5.38)

\[
= \frac{1}{a_1^2 \varepsilon_1 + 1} \left( a_0 + a_1 \mu_d - a_1 \mu_r - a_1 \varepsilon c_{tax} \right) + c_{tax}.
\]
For the deadweight loss from TIP, we first need to derive the prices and quantities analogous to Proposition 5.2. The price for consumers on RTP and the quantity demanded by consumers on TIP and RTP can be written as

\[ p^{*}_{t,\text{tax}} = \frac{1}{a_1 \alpha e + 1} \left( \bar{p} a_1 e (\alpha - 1) + a_0 + a_1 d_t - a_1 r_t + c_{\text{tax}} \right) \]  
(5.39)

\[ D(p^{*}_{t,\text{tax}}) = \frac{1}{a_1 \alpha e + 1} \left( d_t (a_1 \alpha e + 1) - e \left( \bar{p} a_1 e (\alpha - 1) + a_0 + a_1 d_t - a_1 r_t + c_{\text{tax}} \right) \right) \]  
(5.40)

\[ D(\bar{p}_{\text{tax}}) = d_t - e \bar{p}_{\text{tax}}. \]  
(5.41)

The welfare in time period \( t \) dependent on \( p \) and alpha can be derived as

\[ W_{t,\text{tax}} = \frac{1}{2e (a_1 \alpha e + 1)} \left( (\alpha - 1) \left[ \bar{p}^2 a_1 e^3 + \bar{p} \alpha e^2 - 2\bar{p} a_0 e^2 - 2\bar{p} a_1 d_t e^2 + 2\bar{p} a_1 e^2 r_t \right. \right. \]
\[ + a_1 d_t^2 e \left. \right] + a_0^2 \alpha e^2 - 2a_0 d_t e + 2a_0 e r_t + 2a_1 d_t e r_t - a_1 e r_t^2 - c^2_{\text{tax}} e^2 + d_t^2 \right) \]  
(5.42)

In the efficient case with all consumers on RTP, welfare with a tax is

\[ W^e_{t,\text{tax}} = \frac{1}{2e (a_1 \alpha e + 1)} \left( a_0^2 e^2 - 2a_0 d_t e + 2a_0 e r_t + 2a_1 d_t e r_t - a_1 e r_t^2 - c^2_{\text{tax}} e^2 + d_t^2 \right) \]  
(5.43)

When we now make use of the previously derived TIP \( \bar{p}^{*}_{\text{tax}} \), we can calculate the deadweight loss to

\[ \Delta W_{\text{tax}} = T \int W^e_{t,\text{tax}} - W_{t,\text{tax}} f(d_t, r_t) d(d_t, r_t) \]  
(5.44)

\[ = T \frac{a_1^2 e (1 - \alpha)}{2(\alpha a_1 e + 1)(a_1 \alpha e + 1)} \left( \sigma_d^2 + \sigma_r^2 - 2\rho_{d,r} \sigma_d \sigma_r \right), \]

which is identical to the deadweight loss from TIP in the case of no taxes in Proposition 5.2. \( \Box \)
5.6.5 Supply Curve Regression

Table 5.3: Ordinary least squares regression

<table>
<thead>
<tr>
<th>Dependent variable: da_price</th>
</tr>
</thead>
<tbody>
<tr>
<td>ResLoad</td>
</tr>
<tr>
<td>Constant</td>
</tr>
</tbody>
</table>

Observations 8,759  
R² 0.779  
Adjusted R² 0.779  
Residual Std. Error 5.956 (df = 8757)  
F Statistic 30,844.950*** (df = 1; 8757)

Note: *p<0.1; **p<0.05; ***p<0.01

5.6.6 Sensitivity on the Share of Consumers on RTP

Figure 5.8: RTP share sensitivity for $e = 0.2$
6 Tender Frequency and Market Concentration in Balancing Power Markets

Balancing power markets ensure the short-term balance of supply and demand in electricity markets and their importance may increase with a higher share of fluctuating renewable electricity production. While it is clear that shorter tender frequencies, e.g. daily or hourly, are able to increase the efficiency compared to a weekly procurement, it remains unclear in which respect market concentration will be affected. Against this background, we develop a numerical electricity market model to quantify the possible effects of shorter tender frequencies on costs and market concentration. We find that shorter time spans of procurement are able to lower the costs by up to 15%. While market concentration decreases in many markets, we – surprisingly – identify cases in which shorter time spans lead to higher concentration.

6.1 Introduction

In electricity markets supply and demand need to be equal at all times and commonly transmission system operators (TSOs) are in charge of balancing supply and demand. Due to unbundling policies TSOs are not allowed to own generation assets and need to procure short-term flexibility from operators of power plants. These power plants need to be able to adjust their production on short notice to balance supply and demand.

In Germany, balancing power is currently procured on a weekly basis for primary and secondary balancing power. Operators that offer for example positive balancing power therefore need to withhold production capacities over the time span of a whole week and can not sell their full capacity into the spot market. The costs that arise from balancing power provision are thus based on the opportunity costs with respect to selling the capacity in the spot market, namely the foregone profits from spot market operation.

In this paper, we take a closer look at the German balancing power markets with a special focus on two problems that may arise from the current (weekly) market
design. First, the weekly procurement leads to inefficiencies as operators need to withhold capacities for a whole week and can not fully participate in the hourly spot market. There is a missing market for hourly balancing power products that could be solved by an hourly procurement of balancing power. Secondly, we observe that large players with a broad portfolio of power plants are able to provide balancing power at lower costs, especially in a weekly auction. These economies of scale for large players may lead to highly concentrated markets and the possible abuse of market power.

Whereas in theory it is well understood that shorter time spans lower costs and may increase market concentration, the magnitude of a change in market design towards shorter time spans remains unclear. In order to assess the possible impact, we develop a numerical model that accounts for the operator structure in the balancing power market and considers different time spans for balancing power procurement. Based on the model we are able to quantify the effects of different market designs (weekly, daily, hourly) on system costs and market concentration.

The modeling of balancing power markets is complex, as it is driven by the opportunity costs of operators. Just and Weber (2008) started to write down this problem analytically and solved the simplified model numerically. Later the methodology was again applied by Just (2011) to analyze the implications of different tender frequency on the procurement costs but without considering the operator structure. Richter (2012) bases his analysis on the model developed by Just and Weber (2008) and is able to show the existence of a competitive simultaneous equilibrium in spot and balancing power markets that is unique and efficient. He finds out that the bids of the capacity providers form a u-shaped bidding function around the spot demand. This work shows that the integrated modeling of spot and balancing power markets in a fundamental model as it is done in the analysis at hand yields meaningful results. In addition, the equilibrium of the spot and balancing power market was further analyzed by Mürgens et al. (2014) in the context of the German market design.

The procurement of balancing power is commonly organized via auctions. A special characteristic of the balancing power procurement process is that the cost structure of participants can be divided into two parts. One part is fixed for a period and stems from withholding capacity for balancing purposes. The second part are variable costs for the supply of energy in the case of being called during operation. Bushnell and Oren (1994) were the first to analyze the auction design of balancing power markets. Their work was later extended by Chao and Wilson (2002) in order
to design incentive compatible scoring and settlement rules. They found that incentive compatible auctions can be designed by considering only the capacity bid for scoring in a uniform price auction. Nevertheless many of the implemented auction designs in Europe differ from their proposals.

The auction design of balancing markets was also studied by Müsgens et al., who analyzed the importance of timing and feedback (Müsgens and Ockenfels, 2011, Müsgens et al., 2012). The development in the tertiary reserve market and the change in rules was analyzed by Haucap et al. (2012). They find that the cooperation of the four TSOs in Germany decreased costs for the procurement of tertiary reserve.

Whereas previous literature focuses on the efficient design, high market concentrations are an additional issue in balancing power markets with few big operators. In 2010, Growitsch et al. (2010) analyzed the operator structure in the tertiary balancing power market. They find high market concentration in certain situations of the tertiary balancing power market. Heim and Götz (2013) looked at the market outcomes in the German secondary reserve market based on exclusive data provided by the German regulator and find that the price increase in 2010 can be traced back to the bidding behavior of the two largest firms.

While the general effects of a design change towards shorter spans is well understood, the empirical importance is less clear. To contribute to filling this gap, we simulate a design change for the German balancing market. We compare simulation results for the current market design to simulation results for shorter time spans. From the comparison of the results, we derive a difference of 15% balancing cost in favor of shorter time spans. With respect to concentration, our model results indicate that an hourly market design for balancing power leads to periods with higher market concentration. This means that in some hours market concentration could increase by a change of market design from weekly to hourly and policy makers should be aware of this.

The paper is organized as follows: In Section 6.2 we focus on the background informations which include, among others, the general electricity market structure, bidding behavior for balancing power and the concepts of market concentration indices. Section 6.3 introduces the methodology, namely a unit-commitment model for electricity markets and the model specifications to account for the balancing power markets. Section 6.4 presents the modeling results as to the system costs and the market concentration indices. Section 6.5 concludes.
6.2 Background

6.2.1 On the Functioning of the Balancing Power Market

The balancing power market is an additional market for electricity generators, besides the classic spot markets like the day-ahead and intraday market and is divided into products depending on the urgency and the direction of power provision. In Germany, the markets are divided into primary, secondary and tertiary balancing power provision which differ mainly in reaction time. In the primary balancing power market, power plants need to be able to adjust their output in both directions (upward and downward). Secondary and tertiary balancing power markets are divided into products for positive and negative balancing power. The secondary balancing power market is further divided into a peak and off-peak product. Additional information on the current market design can also be found in Hirth and Ziegenhagen (2015).

Because the balancing of imbalances has to occur in very short time periods before physical delivery, providers of balancing power have to reserve capacity for balancing purposes. This means for example that an operator for positive balancing power cannot sell all her production capacity into the spot market and needs to operate power plants below the maximum capacity level. When being called for the supply of balancing power, the power plant needs to increase its output. For the case of negative balancing power provision, operators need to run their plants above their minimum production capacity and when negative balancing power is called, these plants have to be able to decrease their electricity production.

The cost structure of participants in the balancing power market is thus different compared to the spot market. On the one side, participants must account for opportunity costs that arise from the opportunity of marketing the spare capacity in other power markets (such as day-ahead and intraday) and on the other side participants need to pay fuel costs in case that their plants are being called for balancing purposes. The opportunity costs for capacity provision mainly depend on the type of power plant and the prices that are being observed in the markets where the power could also be sold. For example, a power plant that has marginal generation costs a bit lower than the spot market price, has very low opportunity costs for positive balancing power provision. If this power plant decreases its spot market production in order to offer positive balancing power, the income from the spot market is only slightly lowered. The opportunity costs for the provision of positive balancing power...
6.2 Background

are thus close to zero.¹ In contrast to this, if the power plant has very low marginal costs of production compared to the spot price, the opportunity costs for positive balancing power provision are very high, as the forgone spot market profits are very high. Figure 6.1 shows the opportunity costs for different ranges of the merit order.

![Figure 6.1: Capacity bidding behavior for balancing power markets is theoretically based on an opportunity cost strategy to the spot market (here: positive balancing power)](image)

The demand of electricity depends mainly on the time of consumption and fluctuates throughout the day. Therefore prices fluctuate as well. This means opportunity costs of single power plants are constantly changing and providers of balancing power need to take this into account. For the case of operators owning multiple power plants with a well diversified portfolio this effect is not as severe because in the best case they are always operating a power plant with marginal costs close to the spot price that has very low opportunity costs. This makes it obvious that bigger power plant portfolios may have significant cost advantages compared to small players.

In order to illustrate the effect of the portfolio on the opportunity costs, we consider the following example which is visualized schematically in Figure 6.2: Let us assume that there are three power plants A, B, and C with the same capacity but different marginal costs of 10, 20 and 30 EUR/MWh. With an ordering according to the marginal costs, we derive the simplified spot market merit order. The spot market clearing price is thus the intersection of the demand function with the merit order. The opportunity costs for positive balancing power arise by the difference of the power plants’ marginal costs to the spot market clearing price. Thus, the opportu-

¹This holds only true if efficiency losses due to partial load operation are neglected.
nity costs are dependent on the spot market demand situation. Now, let us consider two demand situations: A low and a high spot market demand situation. In the low demand situation, the demand is lower than the total capacity of plant A. Hence, the cheapest power plant A can satisfy the total spot market demand resulting in a spot market clearing price of 10 EUR/MWh. This leads to opportunity costs of 0, 10 and 20 EUR/MWh for A, B and C respectively (shown in Figure 6.2 on the lower y-axis part). In the high demand situation, the demand exceeds the joint capacity of plant A and B. Therefore, plant C determines the spot price of 30 EUR/MWh, which results in opportunity costs of 20, 10 and 0 EUR/MWh for A, B and C respectively.

![Figure 6.2: Schematic situation of the portfolio effect](image)

If we assume that power plants need to provide the positive balancing power for both situations, the opportunity costs in each situation sum up for each power plant:

$$TotalOpportunityCosts(p) = \sum_{i=\text{low,high}} OpportunityCosts_i(p), \forall p \in \{A, B, C\}$$ (6.1)

This results in total opportunity costs of 20 EUR/MWh for each power plant. A coalition of two power plants could reduce the joint opportunity costs. Power plants A and B could cooperate, e.g. belong to the same operator. Then, in each situation the operator can provide balancing power by her power plant with the lowest opportunity costs. She would use plant A in the low demand situation, and plant B in the high demand situation. The joint opportunity costs for power plant A and B for both situations is 10 EUR/MWh, which is lower than the individuals’ 20 EUR/MWh.

For the negative balancing power, this portfolio effect does not hold in general. The 2

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2We assume that power plants need to run in order to provide positive balancing power (e.g. due to minimum load or ramping constraints). If plants B and C would not need to run, their opportunity costs would be 0 EUR/MWh.
6.2 Background

Opportunity costs are 0 for inframarginal power plants and usually monotonically increasing for extramarginal power plants. This leads to monotonically increasing opportunity costs in each demand situation. The sum of monotonically increasing functions is still monotonically increasing. Thus, the cheapest power plants to provide negative balancing power are always in the left segment of the merit order and there is no possibility to get better off with a coalition with another power plant. Note that we assumed no part load efficiency and attrition costs in this example. Furthermore, we assumed the balancing power demand to be small such that the marginal power plant can fully provide the balancing power demand.

The portfolio effect only occurs if balancing power is procured over a long time horizon that differs from the hourly spot market tender frequency. Here, large players may have significant cost advantages because they can provide balancing power at lower costs from their portfolio. For shorter time periods of balancing power procurement, the portfolio effect is reduced.

In Figure 6.3, an exemplary merit-order for Germany divided into the main operators is shown. Power plants that do not belong to the largest five companies are considered as power plants of a fringe.3

![Figure 6.3: Merit Order in Germany colored as to the operators](image)

As previously explained, opportunity costs in the balancing power market do strongly depend on the intersection of supply and demand in the spot market. There-

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3Throughout the paper we use the following abbreviation for the operators: RWE (RWE), E.ON (E.ON), Vattenfall (VAT), STEAG (STE), EnBW (ENB), fringe (FRI).
fore, to investigate market concentration, we need to consider the power plant portfolio of all operators in the merit order (cf. Figure 6.3). We can see that several ranges are covered by only a few operators. Especially, in the left part of the merit order, there are only two to three operators covering a range of up to several Gigawatts. These are operators owning nuclear and lignite power plants with high investment costs and low marginal costs. Those ranges with few operators tend to favor market concentration. By incorporating the operators and their power plant portfolio into our modeling, we are able to show the effect of different provision duration on market concentration.

6.2.2 Market Concentration

In order to compare different levels of market concentration, we apply typical market concentration indices from the economic literature. Those indices are the Herfindahl-Hirschmann-Index (HHI, Hirschman (1964)) and the residual supplier index (RSI).

The HHI uses the market shares of operators as an indicator for market concentration. It is defined as

$$HHI := \sum_{i=1}^{n} MS_i^2,$$

where $MS_i$ is the market share of operator $i$ in % and $n$ the total number of operators.

Note, that we use the decimal representation of the market shares (50% = 0.5). Therefore, our HHI index is in the range between 0 and 1. Comparable high market shares have an higher impact to the HHI due to the squared functional representation. If we would have only five operators in the electricity market, the HHI could not be lower than 0.2 which would be the case of equally shared capacity. Since we also consider a fringe in our numerical analysis, these lower bounds are not necessarily holding. Based on the described indices we are able to compare the effects of different market designs on market concentration.

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4Note that the fringe at the right of the merit order does not cause higher market concentration, because those plants do not belong to a single firm.

5We do not focus on the pivotal supplier index (PSI), since the non-binary RSI is a refinement of the binary PSI. Furthermore, we do not investigate market concentration indices which involve prices, e.g. Lerner-Index (Elzinga and Mills, 2011) Because we are applying a mixed-integer model, prices cannot be easily derived from the results due to the convexity problem (cf. (Bjørndal and Jörnsten, 2008, Ruiz et al., 2012)). Technical restrictions like minimum load or start-up costs in mixed-integer problems lead to non-convexities. Therefore, the marginal of the supply-demand-equilibrium cannot directly be interpreted as an estimator for electricity prices. Power plant specific variable costs can be above the system marginal costs of mixed-integer problems.

6The HHI is broadly applied in energy economics, see for instance Hogan (1997) and Twomey et al. (2006). A general discussion on concentration indices can be found in Green et al. (2006).
6.3 Methodology

The RSI for operator $x$ measures the remaining capacity without supplier $x$’s capacity to fulfill the demand. It is defined as

$$\text{RSI}(x) := \frac{\text{TotalCapacity} - \text{Capacity}_x}{\text{Demand}}, \quad (6.3)$$

where $\text{Capacity}_x$ is the capacity of operator $x$ (cf. Twomey et al. (2006)). In our analysis, we account only for active capacity which means capacity that is already operating. Non-operating capacities cannot provide balancing power in time or have additional start-up costs which make the opportunity costs non-competitive. For comparison reasons, we focus on the inverse value, i.e. $\text{RSI}^{-1}$. Thus, similar to HHI, a higher value indicates higher market concentration.

The HHI represents a market concentration index based on the market share while the RSI represents a market concentration index based on the residual supply (remaining capacity). Both measures therefore give different insights on the level of market concentration.

6.3 Methodology

In this section, details of the basic modeling approach as well as data and assumptions are presented.

6.3.1 Modeling Approach

The analysis is performed with a unit-commitment model for the German power market.\textsuperscript{7} The basic model formulation is based on the work by Ostrowski et al. (2012) and Morales-Espeña et al. (2013) and is extended for the modeling of balancing power provision.

In this article, we explain the general modeling approach for unit-commitment models but abstract from the detailed formulation that can be found in the literature on unit-commitment models (e.g. Ostrowski et al. (2012) and Morales-Espeña et al. (2013)). The focus is set on the introduction of additional equations that account for the characteristics of balancing power markets.

\textsuperscript{7}The model builds on the modelling framework MORE (Market Optimization for Electricity with Redispatch in Europe) that was developed at ewi Energy Research and Scenarios gGmbH and is written in GAMS (further information can be found at http://www.ewi.research-scenarios.de/en/models/more/).
The overall goal of the unit-commitment model is to derive the cost minimal production schedule of power plants to satisfy the demand for electricity. Power plants are modeled blockwise on an hourly time resolution. Power plant blocks are denoted by index $p$ and hourly timesteps by index $t$. The objective function of the unit-commitment model is to minimize the total costs of electricity production and can be expressed as

$$\min \text{TotalCosts} = \sum_{t,p} (\text{VarCosts}(t,p) + \text{StartUpCosts}(t,p)). \quad (6.4)$$

StartUpCosts arise if a power plant is not producing in time step $t$ but produces electricity in time step $t + 1$. The actual StartUpCosts are dependent on the power plant $p$ as well as on the non-production duration (time steps since last time operating). Power plants produce electricity to satisfy the demand. This essential constraint is represented as

$$\forall sm : \sum_{p_{sm}} \text{production}(p_{sm}) + \text{import}(sm) - \text{export}(sm) = \text{demand}(sm) \quad (6.5)$$

and holds for every time step $t$ and every spot market $sm$. Here, $p_{sm}$ are the power plants in spot market $sm$, import considers the electricity flow from other countries (spot markets) to the respective one and vice versa for exports. The exogenous demand is assumed to be perfectly inelastic.

Technical characteristics of power plants are modeled via different constraints. An important modeling aspect of unit-commitment models is that it accounts for different states of power plants that can be incorporated by using binary variables. This makes the model a mixed-integer model. For example, each power plant has a range of feasible production which can be described by

$$\text{production}(p) = 0 \quad \text{or} \quad \text{minload}(p) \leq \text{production}(p) \leq \text{capacity}(p). \quad (6.6)$$

Additional technical constraints of power plant blocks can also be incorporated, such as part load efficiency losses, load change rates, combined heat and power operation and start up times.

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8In the analysis at hand, only the German spot market is considered. Imports and exports are given exogenously as explained later.

9If this assumption would be relaxed, we expect a similar outcome with respect to balancing power provision, since the intersection point of demand and supply curve at the spot market, and hence the relevant opportunity costs would not change.
6.3 Methodology

The basic model is extended to account for the unique characteristics of balancing power markets. These characteristics are essentially given by (i) different provision intervals and (ii) operator structures. We therefore need to map the hourly timesteps to the balancing provision intervals as well as the different power plant blocks to operators.

Table 6.1 gives an overview of the sets, parameters and variables used for the modeling of balancing power. In the following, the equations of the model will be discussed.

<table>
<thead>
<tr>
<th>Set</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPi</td>
<td>interval for balancing power provision, e.g. week, day or hour</td>
</tr>
<tr>
<td>op</td>
<td>operator</td>
</tr>
<tr>
<td>t</td>
<td>hour</td>
</tr>
<tr>
<td>p</td>
<td>powerplant</td>
</tr>
<tr>
<td>t_BPi</td>
<td>set of hours that are in the respective interval for balancing power provision</td>
</tr>
<tr>
<td>p_OP</td>
<td>set of plants that belong to respective operator</td>
</tr>
<tr>
<td>FRI</td>
<td>Fringe operators</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(BPi)</td>
<td>balancing power demand in interval</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP_O(BPi, op)</td>
<td>balancing power provision by operator in interval</td>
</tr>
<tr>
<td>BP(t, p)</td>
<td>balancing power provision by plant and hour</td>
</tr>
<tr>
<td>BP_F(BPi, p)</td>
<td>balancing power provided by fringe operators in the interval</td>
</tr>
</tbody>
</table>

The total demand for balancing power during a provision interval must be satisfied by the sum of the provision of all operators:

$$\forall \ BPi : \sum_{op} BP_O(BPi, op) = D(BPi). \quad (6.8)$$

The balancing power provision of all operators during a provision interval is constituted by the provision of the plants of the operators in each hour:

$$\forall \ BPi, \ t \in t_BPi, \ op : \sum_{p\in p_OP} BP(t, p) = BP_O(BPi, op). \quad (6.9)$$

The balancing power provision of the fringe during the provision interval is con-
stituted by the fringe power plants without the option to pool:

$$\forall \ BP_{i} : \sum_{p \in p_{OP}("FRI")} BP_{F}(BP_{i}, p) = BP_{O}(BP_{i}, "FRI"). \tag{6.10}$$

The power plant specific balancing power provision of fringe power plants is fixed in each hour of the provision interval:

$$\forall \ BP_{i}, \ t \in t_{BP_{i}}, \ p \in p_{OP}("FRI") : \ BP_{F}(BP_{i}, p) = BP(t, p). \tag{6.11}$$

Thus, the model allows the fundamental modeling of power plants that provide balancing power accounting for the operator structure. However, calls of balancing power are not modeled. Model outputs are the hourly production per power plant, as well as, balancing power provision by operator and power plant. In combination with the operator structure, we can evaluate market concentration indices in an ex-post analysis.

### 6.3.2 Input Data and Assumptions

We model two representative weeks in 2014, i.e. a winter week and a summer week. Figure 6.4i shows the demand, residual demand, solar feed-in and wind feed-in during the winter week. This winter week represents a typical situation of high demand in the early evening hours combined with no or very few solar radiation during the day. Especially at the beginning of the week, the wind production is low as well. As a result, there are situations with a residual demand of up to 71.2 GW in which the conventional power plant fleet (nuclear and fossil power plants, pumped storage plants) is utilized up to 69.3%. In the last three days of the week, the residual demand is low due to low demand during the weekend and high wind feed-in. In such a situation of low residual demand, the base load power plants supply a large share of the spot market demand. Since the base load plants are owned by the large operators, situations with low demand may show a high market concentration in the spot market. This has implications for the market concentration on the balancing power markets as well.

Figure 6.4ii shows the demand, residual demand and renewable feed-in in the summer week. It can be seen that there is a contrast to the conditions of the winter week. The demand in summer is typically low and there is high solar radiation during the day. This combination leads to a reduced utilization of the power plant
fleets and therefore to lower prices. Here, even base load and mid load German power plants (lignite and hard coal power plants) reduce their production. Wind feed-in is on a relatively low level (below 10 GW in every hour), but increases during the weekend when the demand is already especially low. This leads to a low residual demand of only 24.3 GW on the Sunday.

Typical weeks during spring and autumn can be interpreted as a combination of the situations in those weeks. The varying demand and renewable feed-in in every single hour of those weeks cover a broad range of situations and therefore reflect also average situation with medium demand and/or renewable feed-in.

The assumptions on power plant capacities are based on Bundesnetzagentur (2014). Only German power plants are modeled. Imports and exports are exogenously given based on ENTSO-E data. Fuel costs and CO₂ prices are based on historical data. Installed capacities, fuel costs and techno-economic parameters of power plants can be found in the Appendix 6.6.1.

Power plants are also constrained in their balancing power provision. We consider primary and secondary balancing power in our model, but abstract from tertiary balancing power provision.

We assume that all running plants can provide a certain share of their capacity as balancing power. For the fossil and nuclear power plants, this share is derived by information about the ramping speeds multiplied by the time duration until the power adjustment needs to be realized. The ramping speed deviates by the year of

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(i) Winter Week (Monday-Sunday)  
(ii) Summer Week (Monday-Sunday)  

Figure 6.4: Demand, Residual Demand, Solar Feed-In and Wind Feed-In

We do not consider tertiary balancing power since (i) technical restrictions are lower for the tertiary market and it tends to be compensated by the intraday-market (30 min before physical delivery), (ii) the current market design of tertiary balancing power has already a high tender frequency (provision duration of four hours), and (iii) there are many competitors in the tertiary market which reduces the risk of market power. Therefore, primary and secondary balancing power are in the focus of our analysis.
construction of the technology. Furthermore, we assume that the capacity (share) for positive balancing power is the same as for negative balancing power. Table 6.2 shows the maximum allowed share of the capacity to provide balancing power for different power plant technologies.\textsuperscript{11}

We assume that power plants that are not running have high starting costs, e.g. due to attrition and fuel consumption, and thus are not competitive in offering balancing power.\textsuperscript{12} We do not consider balancing power provision by renewables and demand side management, because those technologies were not important for the balancing power market in 2014 (Dena, 2014).

| Table 6.2: Share of total capacity that can be used for balancing power provision |
|---------------------------------|---------------------------------|
|                                 | primary balancing power         | secondary balancing power       |
| CCGT                            | 2.50 - 4.00%                    | 25.00 - 40.00%                  |
| Coal                            | 1.00 - 2.50%                    | 5.00 - 12.5%                    |
| Lignite                          | 1.00 - 2.50%                    | 5.00 - 12.5%                    |
| Nuclear                          | 2.00 - 2.50%                    | 10.00%                          |
| OCGT                            | 5.00 - 12.50%                   | 50.00 - 60.00%                  |
| Oil                             | 2.00%                           | 20.00%                          |
| Pumped Storage                  | 10.00%                          | 15.00%                          |

There is only one product that is procured for primary balancing power. However, in the case of secondary balancing power, we consider a positive and negative product for peak and off-peak times, respectively. Additionally we investigate the cases of shorter tendering times, namely daily and hourly. In the case of a weekly provision, the peak time are working days between 8 am and 8 pm. All other hours (night and weekends) are off-peak time. In the case of a daily provision, the peak time is the time between 8 am and 8 pm on every day (including weekends). In an hourly auction, the distinction between of peak and off-peak products disappears.

We map the information about the ownership to each power plant. We consider the German power plant operators E.ON, RWE, EnBW, Vattenfall and STEAG in our model. All other power plants are mapped to the fringe. We obtain information\textsuperscript{11}\textsuperscript{13}

\textsuperscript{11}Pumped storage plants have a high ramping speed. Therefore, they have a high technical potential to provide balancing power (up to 30 % of the capacity for the primary balancing power, and up to 45% for the secondary balancing power for a single plant). However, due to multiple bidding strategies and prequalification requirements, we assume that not all pumped storage plants are bidding their total technical potential into the balancing power markets.

\textsuperscript{12}Start-up costs for a cold start can be up to 60,000 Euro for e.g. a 500 MW CCGT or OCGT power plant with 2010 cost data (Schill, 2016). These costs would have to be reimbursed by the revenue in the balancing power markets. Additionally, a faster start-up than usually increases the attrition and has a higher consumption of equivalent operating hours (EOH).
about ownership of plants from a list of the German regulator Bundesnetzagentur.\textsuperscript{13}

E.ON, RWE, EnBW, Vattenfall and STEAG can use pooling to provide balancing power over a time period, e.g. they can offer a certain volume of balancing power during the provision period and use different power plants within their pool to fulfill their commitment. The fringe is not allowed to pool meaning that each power plant of the fringe has to provide the balancing power of the whole provision period. This is the most restrictive assumption for the pooling of the fringe. Indeed, there are several pooling companies which aggregate smaller producers to a virtual power plant and therefore allow for pooling for subsets of the fringe. However, if we allow that the whole fringe may use pooling effects, the fringe would operate as an additional big producer. Therefore, we expect that the general results for market concentration hold and only the absolute level of market concentration deviates.\textsuperscript{14}

\section*{6.4 Results}

In this section, we present the model results for a weekly, daily and hourly provision duration. The weekly provision duration represents the status quo which is currently in operation in Germany. Daily and hourly provision duration are currently discussed as alternative market designs for the German balancing power market. We analyze the balancing power provision in three dimensions. First, we focus on the efficiency gains by a shortened provision duration which are captured in the total system costs. Second, we analyze the balancing power provision by technology and operator which enables us to shed light onto the level of market concentration for the different provision duration using the indices HHI and RSI\textsuperscript{−1}.\textsuperscript{15}

\subsection*{6.4.1 System Costs}

Power system costs of different model configurations are a benchmark for the efficiency of the market design. In order to assess the costs of balancing power provision, we additionally model the electricity system without balancing power provi-

\textsuperscript{13}Each power plant is mapped to only one owner. This corresponds to the assumption that even if several owners have shares in one plant, only one owner is responsible for marketing balancing power.

\textsuperscript{14}Furthermore, fringe power plants are typically gas fired power plants. Therefore, the effect on market concentration affects only situation with high residual demand as to the opportunity cost bidding strategy and the merit order.

\textsuperscript{15}Note that we use RSI\textsuperscript{−1} instead of RSI. Thus, a higher value of RSI\textsuperscript{−1} indicates higher market concentration, similar to the interpretation of HHI.
sion. The difference between this baseline run and the model runs with balancing power provision can thus be considered as the extra costs of balancing power provision.\footnote{When referred to balancing power in this section, primary and secondary balancing power is meant.}

Table 6.3 gives an overview of the total system costs in the simulated summer and winter week with different designs of the balancing power markets. Irrespective of if and how balancing power is provided, it can be seen that the system costs in the winter are more than EUR 50 million higher than in the summer.

\begin{table}[h]
\centering
\caption{Total System Cost in Reference Scenario in Million Euros}
\begin{tabular}{lcccccc}
\hline
 & in mio. Euro & no provision & hourly & daily & weekly & weekly (no pooling) \\
\hline
Winter & 175.6 & 176.7 & 176.8 & 177.0 & 178.0 \\
Summer & 124.6 & 125.1 & 125.2 & 125.2 & 125.6 \\
\hline
\end{tabular}
\end{table}

As outlined above, the major power plant operators are allowed to pool their portfolio in order to provide balancing power. In order to quantify the efficiency gain resulting from pooling, a sensitivity with weekly balancing power provision in which pooling is not allowed is simulated additionally to a weekly configuration with pooling and hence included in Table 6.3.

The difference between the system costs without balancing power provision and the system costs of a configuration with hourly / daily / weekly balancing power provision can be understood as the respective costs of balancing power provision. Figure 6.5 illustrates those costs. It can be seen that not only the total modeled system costs are higher in winter, but also the costs of balancing power provision. This is expected given the higher residual demand levels in the winter.

If pooling would not be allowed, the cost of balancing power provision would be EUR 2.361 million in the winter week and EUR 0.995 million in the summer week. The modeled costs of the current weekly market design (with pooling of major operators) amount to EUR 1.328 million in the winter week, and EUR 0.677 million in the summer week. The cost difference between the weekly configuration with pooling and without pooling, that can be interpreted as the efficiency gain of pooling, is EUR 1.033 million in the winter and EUR 0.319 million in the summer.\footnote{An additional sensitivity analysis not included in figure 6.5 in which pooling of all fringe operators in one common fringe pool would be allowed shows no significant further efficiency gain.}

The difference between the system costs of a configuration with weekly balancing power provision and a configuration with hourly balancing power provision (from
Figure 6.5: Costs of primary and secondary balancing power (compared to no provision)

now on we only consider configurations with pooling) can be interpreted as the maximum efficiency gain from shortening the provision duration. This cost difference is EUR 222 k in the winter week, and EUR 96 k in the summer week.\textsuperscript{18} The system costs of the daily balancing power provision are between the system costs for the hourly and weekly balancing power provision. Compared to the efficiency gain from pooling, this further efficiency gain by a shortened provision duration is small.

The level of renewable feed-in can influence those results. Therefore, we consider a sensitivity in which we double the values of the historically observed renewable feed-in in the simulated weeks. The detailed results are shown in Appendix 6.6.2. A higher renewable feed-in leads to higher costs of balancing power provision especially in the summer week compared to the configuration with less renewables. For instance, in the case of weekly provision in the summer, the balancing power costs increase by EUR 559k if the renewable feed-in doubles. Due to the lower residual demand, more power plants have to be operational only in order to provide balancing power. The order of magnitude of the efficiency gain from pooling, however, remains unchanged by doubling the renewable feed-in.

The German expenses for the provision of primary and secondary balancing power were EUR 331 million in 2014 (Bundesnetzagentur, 2016) corresponding to average expenses of EUR 6.37 million per week.\textsuperscript{19} This means that the average real expenses were higher than the simulated costs for the balancing power market with the weekly market design (EUR 1.328 million in the winter and EUR 0.677m in the summer). Our model calculates total costs for power plants to provide balancing power under

\textsuperscript{18}Due to solver inaccuracies (difference between current best integer solution and optimal value of LP relaxation), we cannot resolve the exact effect. However, we can be sure about the order of magnitude of the effect.

\textsuperscript{19}This figure is calculated based on capacity bids, not energy bids. This is consistent with our modeling approach in which we consider only provision and not calling of balancing power.
perfect competition and foresight. Those can be interpreted as a lower bound for producers’ costs for the balancing power provision. The Bundesnetzagentur publishes the total expenditures for the balancing power provision. These expenditures also include producers’ surplus. If every operator would bid their real costs in the pay-as-bid auction (under perfect foresight and perfect information), both results should be the same. However, since it is profit maximizing for the operators to estimate and bid the system marginal costs instead of own marginal costs (see for instance Müsgens et al. (2014)), the real expenditures are higher than the modeled costs for provision. Furthermore, the exercise of market power (e.g. withholding of volumes) could even lead to higher system marginal costs and hence higher producers’ surplus. Effects like strategic bidding between capacity and electricity bid or sub-optimal behavior due to information asymmetries could further increase the cost difference between real expenditures and the model results. Additionally, uncertainty for e.g. residual demand, prices, and power plant shortages of the next week are included in the bids which increase costs. These aspects are not considered by the cost minimizing model under perfect foresight. Therefore, we would expect our results to be a lower bound for the possible cost reductions.

6.4.2 Provision of Balancing Power

Balancing power is provided by different types of power plants within the portfolio of operators. Depending on the portfolio of operators and the pooling within the portfolio, the balancing power provision by technology changes from hour to hour. This effect can be observed in the graphs in Figure 6.6i for different provision durations at the example of positive secondary balancing power in the winter week.

For the weekly provision, we see a strong hourly fluctuation within the technologies although operators are restricted to a weekly provision duration. This indicates that the operators make significant use of the pooling option. The operators can freely select the power plants that shall provide balancing power in certain hours of the week. Therefore, the operators choose those power plants in their portfolio which have the lowest opportunity costs with respect to the spot market. Here, obviously, operators with a large portfolio have an advantage compared to small operators. For primary balancing power as well as for the case of the summer week, the fluctuation of balancing power providing technologies are similar to the Figure 6.6i.

If we take a look at the provision by technology for daily or hourly provision duration, we find a surprisingly similar structure to the weekly provision duration.
However, small differences in the diagrams can be identified. CCGT (in orange), for instance, have a more important role in peak hours with the hourly provision compared to the outcomes with longer provision duration. In the daily configuration, coal power plants (in grey) provide more often balancing power compared to the other configurations. The hourly provision duration can be expected to be the efficient benchmark where the owner structure of power plants does not matter. This means that the most cost efficient power plants in each hour provide balancing power. Since the capacity provision by technology of the weekly and daily cases are similar to the hourly benchmark, we conclude that the pooling possibilities allow a provision pattern that is close to the most efficient outcome. Even with a weekly provision duration, almost the same cost efficient technologies provide balancing power as in the case with an hourly provision.\(^{20}\) This interpretation is in line with the results presented in Section 6.4.1 where the efficiency gain from pooling was quantified to be EUR 1.382 million in the winter week whereas the respective efficiency gain from shortening the provision duration from a weekly to an hourly market design was found to be EUR 0.222 million.

Figure 6.6ii shows the modeled capacity provision by operator for positive secondary balancing power for a weekly, daily and hourly provision duration. Compared to the modeled provision by technology, the modeled provision by operators differs more significantly for the three market designs. The fluctuation of market shares becomes higher with a shorter provision duration.

The capacity provision by operator can be considered as a first indicator for the market concentration indices. Therefore, we expect stronger fluctuation of the market concentration indices for shorter provision duration. Drivers for this are:

- the absolute residual demand level at a given time point in the time frame,
- the volatility of the residual demand level in the provided time frame,
- the steepness of the marginal cost function of the power plants and therefore the steepness of the opportunity cost function,
- the operator structure of the opportunity cost function, i.e. whether operators capacities are in blocks or spread in the opportunity costs merit order.

Thus, the capacity provision by operator is typically dependent on the specific market circumstances, e.g. the product definition, the annual season, and the provision duration. Hence, we investigate the different market designs based on market con-

\(^{20}\)This result does not only hold for the case of positive secondary balancing power, but also for the other investigated products.
6.4.3 Market Concentration

Based on the balancing power provision by operator observed in Figure 6.6(ii) we compute market indices for the three balancing power products, primary, secondary positive and secondary negative balancing power. The indices vary depending on the market design and provision duration. In order to assess the different ranges of market concentration indices, we analyze the model results in histograms for the HHI (cf. Figures 6.7, 6.9 and 6.10). Those diagrams show the HHI values in the weekly market design as a red line. In the case of secondary balancing power, two red lines are present due to the two contract durations (HT and NT, as described in Section 6.2). For the hourly provision duration, 168 different products are defined and hence 168 HHI values. The histograms show the distribution of those hourly HHI values. Similar histograms for the RSI\(^{-1}\) are evaluated (cf. Figures 6.8, 6.12).
6.4 Results

For the interpretation of the results, we also add dotted lines into the histograms which indicate threshold values for high market concentration. For the HHI, a strong market concentration exists at a value of 25% according to US Department of Justice, Federal Trade Commission (2010, §5.3) and at 20 % (with further restrictions) as to EUR-lex (2004, 19. and 20.). In the case of the $\text{RSI}^{-1}$ we consider a threshold value of 1.11 (which corresponds to a threshold value of 0.9 for the original RSI definition).

The indices are no absolute measures in which one index would be sufficient to indicate market concentration. Nevertheless, high market concentration is more likely if both discussed indices point to a critical level.

**Market Concentration for Primary Balancing Power Provision**

For the modeled provision of primary balancing power, the HHI values are displayed in Figure 6.7. We observe that the summer seems to be slightly more concentrated in balancing power provision than the winter. The reason for this lies in the different demand profiles and the increasing production of solar generation (cf. Figure 6.4i). In the summer, a lower electricity demand and higher solar generation lead to less demand of generation from conventional power plants and therefore there are less power plants available (i.e. running) that are able to provide primary balancing power. This is also indicated by high values of the $\text{RSI}^{-1}$ that can be seen in Figure 6.8.

Based on the model results we can infer that the primary balancing power market is prone to high market concentration. When the market design is changed from weekly provision to hourly provision we observe that the indices take on a broader range of values. This means there are hours in which market concentration is increased and hours when market concentration is lowered. An increase in market concentration may occur if the level of demand is at a level where only few operators are close to the marginal production level. As previously explained in Section 6.2 and shown in Figure 6.3, there are intervals in the merit order where only some operators own power plants. This is for example the case for lignite power plants that are owned by Vattenfall and RWE. When demand is low and lignite power plants are

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21Additionally, an analysis for the concentration indices CR1 and CR3 was conducted. The CR for $m$ firms is defined as $\text{CR}(m) := \sum_{i=1}^{m} MS_i$ where $MS_i$ is the market share of operator $i$ in % for the $m$ largest firms. The analysis for CR1 and CR2 did not lead to different conclusions compared to the analysis based on HHI and $\text{RSI}^{-1}$.
marginal in their production, they can provide balancing power at lowest cost. Since this effect only depends on one single demand period in the hourly provision case instead of multiple demand periods in the weekly design, the modeled market concentration increases in some hours. In addition, market concentration is higher in the summer because of lower demand levels and therefore less conventional power plants that are operating. These baseload power plants which are still operating are owned by fewer operators, which increases market concentration.

There is no clear trend observable to conclude whether shorter provision duration structurally mitigates or favors market concentration. The \( RSI^{-1} \), however, that can be seen in Figure 6.8, decreases in average with shorter provision duration especially in the winter week. This means that the average market concentration is reduced because there is more active capacity that could provide balancing power. Nevertheless, there are some hours when the \( RSI^{-1} \) indicates a slightly higher concentration compared to the weekly provision. The number of hours with critically high values can be significantly reduced if the market design is changed to an hourly balancing power provision. In the winter this leads to \( RSI^{-1} \) values below the threshold. In the summer, however, the \( RSI^{-1} \) can only be decreased below the threshold in some hours. Based on the model results, the primary balancing power market seems to be highly concentrated such that even in the case with an hourly balancing power provision the average market concentration in the summer is still modeled as critically high.
Market Concentration for Positive Secondary Balancing Power Provision

Whereas primary balancing power is mostly provided by baseload power plants that are able to increase and decrease their generation, secondary balancing power is divided into positive and negative balancing power. In the case of positive balancing power, power plants provide the ability to increase their generation when being called. For the winter we see the respective technology and operator mix in Figure 6.6. The result for the summer week is similar which is the reason why it is not shown additionally. The main difference is that more lignite power plants are providing balancing power instead of CCGTs than in the winter week. Especially the high provision of balancing power from lignite power plants leads to a high market share by RWE and Vattenfall.

The market concentration indices in Figure 6.9 show a high market concentration based on the HHI. Here, again, concentration seems to be higher in the summer compared to the winter. Nevertheless, the story is a bit different compared to the provision of primary balancing power because in the case of positive secondary balancing power there is a larger proportion of active power plants that could potentially provide balancing power. The respective $RSl^{-1}$ indicates that the market is not too concentrated because the providing power plants could be replaced by the provision from power plants that are currently not delivering balancing power (the histogram for the $RSl^{-1}$ can be found in the Appendix). Therefore the market can be considered as not as concentrated compared to the primary balancing power market. When the provision duration is lowered to an hourly level, the average modeled
market concentration based on the RSI\textsuperscript{-1} is further reduced. In the case of the HHI, there is, however, no clear evidence for a reduction in average market concentration by reducing provision durations. There are single hours with very high modeled market concentrations in the hourly case.

![Histogram of the hourly HHI values for positive secondary balancing power in winter week (left) and summer week (right)](image)

**Figure 6.9:** Histogram of the hourly HHI values for positive secondary balancing power in winter week (left) and summer week (right)

### Market Concentration for Secondary Negative Balancing Power Provision

The HHI values for secondary negative balancing power that can be seen in Figure 6.10 have similar characteristics as the values for the positive secondary balancing power. Nevertheless, in the negative secondary balancing power market, we would expect no abuse of market power even with a high market concentration. The rational for this is as follows: As to Section 6.2, the costs for capacity bids for balancing power are driven by opportunity cost compared to the spot market. Thus, for one hour, all operating power plants have zero costs for offering negative balancing power. For a longer provision duration, the costs would increase if the power plant would not be inframarginal all the time. However, due to pooling effects, operators can choose power plants which are operating in a specific situation. Therefore, the opportunity costs for each provider can be assumed to be (almost) zero. Many fringe operators can potentially participate in the auction, since e.g. wind producers could also provide negative balancing power. This means that the resulting supply curve for negative balancing power is very flat. If operators would try to withhold quantities in an attempt to increase prices, fringe operators with similar small costs would provide the balancing power. Hence, prices of (almost) zero for negative bal-
6.4 Results

Balancing power should be the consequence. Note that in reality, there is uncertainty (e.g. power plant outages) which leads to slightly positive capacity bids. With our model, we can find the cost minimal provision of balancing power but we would expect fierce competition. Therefore, even high shares of market concentration that can be observed in the model results should not lead to the abuse of market power because all providers face the same low level of opportunity costs.

Figure 6.10: Histogram of the hourly HHI values for negative secondary balancing power in winter week (left) and summer week (right)
6.5 Conclusion

Currently, the German primary and secondary balancing power markets have a weekly tender frequency. In a weekly market design, large power plant operators make use of pooling within their portfolio in order to provide balancing power. Fringe operators, however, do not have pooling options and need to withhold the capacity of their plants from the spot market for a whole week to provide balancing power which can lead to inefficiencies. Hence, fringe operators could potentially benefit from a shortened provision duration. The analysis at hand focuses on (1) efficiency gains from a shorter provision duration in primary and secondary balancing power markets, and (2) market concentration in market designs with different provision duration. Since it is known from the literature that simultaneous equilibria in spot and balancing power markets are efficient and unique (Richter, 2012), our methodology is based on a cost minimizing unit-commitment model for the electricity market in which we account for the ownership of power plants.

We quantify the efficiency gain from allowing pooling in a weekly market design to be EUR 1.033 million in a winter week and EUR 0.139 million in a summer week. Compared to this, the further efficiency gains that can be realized by shortening the provision duration from a week to an hour are small. An hourly market design would lower the costs of balancing power provision by EUR 222 k in a winter week and EUR 96 k in a summer week. Relative to the total simulated cost of balancing power provision in the weekly market design with pooling, the efficiency gain is 17% in the winter week, and 14% in the summer week.

Besides the efficiency gains, we identify effects on the market concentration. Here, we investigate the HHI and RSI\(^{-1}\) indices which are based on the market share and the residual supply, respectively. According to the model results, we see the potential for high market concentration in the primary balancing power market due to the technical requirements power plants need to fulfill in order to participate in this market. In the market for positive secondary balancing power, the model results indicate less concentration because there is more available capacity that could potentially replace the providing power plants. For the negative secondary balancing power, our results are quantitatively similar to the other products. However, we consider concentration in the market for negative balancing power not to be an issue due to the low opportunity costs for providing negative balancing power. Based on the model results, we find a higher market concentration in the summer than in the winter in all considered markets. The higher market concentration in the summer is
driven by a lower level of demand, which reduces the number of active power plants and also the number of operators that are providing balancing power.

Our results reveal a tendency towards decreasing average market concentration by shortening the provision duration. However, the market concentration indices take on a broader range of values in the case of a shorter provision duration depending on the residual demand level and its volatility. There are single provision periods with a very high market concentration in the hourly and daily market design that could favor the potential for market power abuse.

Although market concentration can be an indicator for market power, it does not necessarily identify market power. The characteristics of the supply curve for balancing power determine the potential for market power abuse. If high market concentration is found in a flat segment of the supply curve, prices cannot be raised significantly. The goal of further research should be to comprehensively understand market imperfections in balancing power markets. Besides market concentration, aspects like e.g. strategic bidding between capacity and energy bid and uncertainty about the renewable feed-in or demand should be considered.

As a policy implication, we recommend to monitor market concentration and price levels carefully after a change of the market design in the balancing power market. In specific situations, single operators may have a cost advantage compared to their competitors.
6.6 Appendix

6.6.1 Input Data for Modeling

Since we model the year 2014, we are able to use realistic data according to public available sources. Assumptions that are made are inline with typical assumptions for modeling the electricity generation sector. The installed power plant capacities of different fuel types are shown in Table 6.4 and are based on Bundesnetzagentur (2014).\(^{22}\)

<table>
<thead>
<tr>
<th>Fuel Type</th>
<th>Capacity [GW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear</td>
<td>12.1</td>
</tr>
<tr>
<td>Lignite</td>
<td>21.3</td>
</tr>
<tr>
<td>Coal</td>
<td>25.5</td>
</tr>
<tr>
<td>Gas</td>
<td>26.9</td>
</tr>
<tr>
<td>Oil</td>
<td>2.4</td>
</tr>
<tr>
<td>Pumped Storage</td>
<td>6.4</td>
</tr>
<tr>
<td>PV</td>
<td>32.7</td>
</tr>
<tr>
<td>Wind onshore</td>
<td>31.4</td>
</tr>
<tr>
<td>Wind offshore</td>
<td>0.4</td>
</tr>
<tr>
<td>Biomass</td>
<td>7.5</td>
</tr>
<tr>
<td>Hydro</td>
<td>4.4</td>
</tr>
<tr>
<td>Others</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The actual input of installed capacities is further separated as to the year of construction: This gives further technical characteristics and parameters like full load and part load efficiency. The newer a power plant, the better are its technical parameters.

<table>
<thead>
<tr>
<th>Fuel Type</th>
<th>EUR/MWh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear</td>
<td>3.6</td>
</tr>
<tr>
<td>Lignite</td>
<td>1.5</td>
</tr>
<tr>
<td>Coal</td>
<td>13.2</td>
</tr>
<tr>
<td>Gas</td>
<td>22.8</td>
</tr>
<tr>
<td>Oil</td>
<td>49.4</td>
</tr>
<tr>
<td>Biomass</td>
<td>31.8</td>
</tr>
<tr>
<td>Others</td>
<td>22.8</td>
</tr>
</tbody>
</table>

\(^{22}\)The actual input of installed capacities is further separated as to the year of construction: This gives further technical characteristics and parameters like full load and part load efficiency. The newer a power plant, the better are its technical parameters.
The CO₂ emission certificate costs are assumed to be 6.20 EUR/t CO₂. We assume those costs to be static over the whole year. Table 6.6 shows the assumed technical power plant parameters (particularly dependent on the year of construction).

<table>
<thead>
<tr>
<th>Table 6.6: Techno-economic parameters for conventional power plants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net efficiency</td>
</tr>
<tr>
<td>[%]</td>
</tr>
<tr>
<td>Coal</td>
</tr>
<tr>
<td>Lignite</td>
</tr>
<tr>
<td>CCGT</td>
</tr>
<tr>
<td>OCGT</td>
</tr>
<tr>
<td>Nuclear</td>
</tr>
<tr>
<td>Biomass</td>
</tr>
</tbody>
</table>

### 6.6.2 Robustness Checks

As a robustness check, a model run is considered in which the values of renewable feed-in is doubled. Table 6.7 gives an overview of the total system costs, and Figure 6.11 illustrated the costs for providing primary and secondary balancing power compared to a model run without balancing power provision.

<table>
<thead>
<tr>
<th>Table 6.7: Total system cost in scenario with doubled renewable feed-in in million Euros</th>
</tr>
</thead>
<tbody>
<tr>
<td>in mio. Euro</td>
</tr>
<tr>
<td>Winter</td>
</tr>
<tr>
<td>Summer</td>
</tr>
</tbody>
</table>

![Figure 6.11: Costs of primary and secondary balancing power (compared to no provision) in scenario with doubled renewable feed-in](image-url)
6.6.3 RSI Concentration Index for Secondary Balancing Power

Figure 6.12 and 6.13 show the RSI\(^{-1}\) market concentration indices for secondary balancing power (positive and negative, respectively).

Figure 6.12: Histogram of the hourly concentration index RSI\(^{-1}\) for positive secondary balancing power in winter week (left) and summer week (right)

Figure 6.13: Histogram of the hourly concentration index RSI\(^{-1}\) for negative secondary balancing power in winter week (left) and summer week (right)
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