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To my parents

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Chapter 1

Introduction

This thesis consists of three self-contained chapters that contribute to the research fields of monetary and international macroeconomics. The chapters cover questions on business cycle stabilization policy in monetary unions, on the effects of unconventional monetary policy instruments, and on the macroeconomic impact of demographic change. Common to all three of these topics is their high relevance for economic policy in the recent past and ongoing present, on which I elaborate in the following.

Chapters 2 and 3 share the joint theme of studying business cycle policy in situations where the conduct of monetary policy is constrained, either due to membership of a monetary union or when the main monetary policy rate cannot be reduced any further in proximity of the zero lower bound on interest rates.

Chapter 2 addresses questions related to the conduct of stabilization policy in monetary unions. The recent experiences of the euro area after 2010 revived both the scientific and the public debate about which additional policy instruments are required in a monetary union, where national central banks are no longer capable to pursue individual monetary policies to accommodate country-specific shocks. As a result, cross-country imbalances between member countries can no longer be reduced by exchange rate adjustments. The last years witnessed the emergence of a consensual view that, amongst other reforms, more coordination of national fiscal policies is needed. Farhi and Werning (2017) provide a theoretical justification for the implementation of fiscal transfers between the member countries of a currency union in a very general framework, based on earlier work by, amongst others, Galí and Monacelli (2008) or Ferrero (2009). Due to the severe moral hazard problems and the numerous political constraints that are associated with cross-country transfers, their implementation is still protracted.

Complementing a fiscal transfer system, so-called fiscal devaluation policies can be part of the response to reduce imbalances in a monetary union. The term of fiscal devaluation refers to policies that aim at affecting international relative prices, i.e. the terms of trade and the real exchange rate, in a similar way as the nominal exchange rate does. The idea goes back to Keynes (1931) and has been brought back to the scientific and public debate by Farhi, Gopinath and Itskhoki (2014), who show that the effects of the nominal exchange rate on the allocation of an economy can be replicated entirely using a sufficient number of tax instruments.

The research in Chapter 2 contributes to this debate. Specifically, I study optimal monetary and fiscal policy in a New Keynesian 2-country open economy framework, which I use to assess to what degree tax policies can substitute for the role of a flexible exchange rate in a monetary union. To this end, I describe the conduct of optimal policy in response to country-specific shocks and provide welfare comparisons between various policy scenarios. I show that even under a minimum set of fiscal instruments being a member of a monetary union does not have to be unduly painful. The optimal use of a single tax instrument per country already reduces the welfare costs of giving up exchange rate flexibility significantly. Fiscal devaluation policies are not only viable, but can be observed as the optimal policy response to country-specific shocks in such a setting – even in presence of several competing objectives for the tax instrument. These objectives are, besides replicating the effects of a flexible exchange rate system, in particular the financing of government expenditures in the least distortionary way and the stabilization of firms' marginal costs in order to avoid welfare-reducing price dispersion.

As the main goal of fiscal devaluation policies is to affect relative goods prices, they are usually centred on adjustments of value added taxes (VAT), which are therefore also in the focus of my analysis in Chapter 2. The intuition for the effect of the simplest form of such a VAT-based fiscal devaluation policy is that, say, an increase of the domestic relative to the foreign VAT rate induces firms to charge higher prices for goods sold at home, resulting in higher prices of domestic imports relative to exports, for the latter are subject to the relatively reduced foreign VAT. Comparable to a nominal devaluation, this fiscal devaluation policy leads to a deterioration of the terms of trade.

The model framework features staggered price setting and it differentiates between the case where prices are sticky only in the country of the producer, such that the law of one price holds internationally, and the case where firms are capable of pricing-to-market, implying an additional sticky price friction for imported goods. This distinction is important, as the welfare costs of fixed exchange rates as well as the capabilities of fiscal policy to reduce these costs depend decisively on the pricing scheme. The model allows for home bias and asymmetries between the countries along several dimensions, such as country size, the degree of competition, and the size of the public sector. Optimal policy is characterized using a Ramsey approach. This procedure involves finding sequences for the policy instruments that support the welfare-maximizing competitive equilibrium.

I calibrate the model to characteristics of the euro area and then address the research question along the two dimensions of welfare and conduct of policy. In terms of welfare, I find that optimal fiscal policy reduces the welfare costs of pegged exchange rates by 86% in case the law of one price holds and by 69% in case of pricing-to-market, by only adjusting the VAT rates of both countries. The order of magnitude of these results is highly robust to changes in the parametrization and also if payroll taxes are used instead of the VAT. In terms of the conduct of optimal policy, I show that for a broad set of different shock processes, fiscal devaluations are an optimal policy response to country-specific shocks in a monetary union. The results imply the policy recommendation that whenever it were optimal to devalue the currency of a country in a monetary union, optimal fiscal policy is to increase the VAT of that country relative to the VAT rates of the other countries in the union.

Chapter 3 adds to the understanding of the effects of unconventional monetary policy instruments that become relevant in situations where the adjustment of conventional tools, i.e. the main policy rate, is not feasible or not sufficient to fulfil the objectives of the central bank. The most important unconventional monetary policy measures are large scale asset purchase programmes and forward guidance. The latter type of policy refers to the way how central banks manage market expectations about the future stance of their policy. The analysis of forward guidance stands in the focus of Chapter 3.

Unconventional policies became highly relevant in the aftermath of the financial crisis of 2007 and 2008, when the main monetary policy rates in both the United States and Europe fell to levels close to their effective lower bound. Despite highly accommodative conventional policy, the economies of both currency areas ran into danger of deflationary spirals. To avoid such an adverse situation, unconventional policies were applied on a massive scale. In the recent past, the Federal Reserve System already started tapering its asset purchases and began to increase the federal funds rate in line with the ongoing recovery of the US economy. The European Central Bank however continues to maintain their highly accommodative policy stance for the time being.

An important role of scientific research is to assess the success and effectiveness of these types of policies. Eggertsson and Woodford (2003) provide an early theoretical analysis of the effects of forward guidance in a basic New Keynesian framework. Assuming full commitment of the central bank, they show that this type of policy is highly effective in the conventional model. Even small announcements about future reductions of the policy rate lead to massive responses of output and inflation today. In this canonical model, whose basic structure is also widely applied by central banks in models for policy analysis, such an announcement can suffice to terminate a deflationary contractionary episode at the zero lower bound immediately. Recent empirical studies emphasize however that New Keynesian models massively overstate the effects of forward guidance announcements on the economy. Del Negro, Giannoni and Patterson (2015) coined this mismatch between theoretical predictions and empirical evidence the forward guidance puzzle.

The research in Chapter 3, which is joint work with Christian Bredemeier and Andreas Schabert, provides a solution to this puzzle that is motivated by novel empirical findings. We offer empirical evidence that liquidity premia, measured by spreads between assets that are similar in terms of safety, but different in the degree of liquidity services they offer, rise systematically in response to accommodative forward guidance statements of the US Federal Reserve. The measure for highly liquid near-money assets are US Treasuries. Nagel (2016) shows that these feature a liquidity premium compared to other assets due to their unique feature of being eligible to serve as collateral for obtaining liquidity from the central bank. The fact that forward guidance leads to changes of liquidity premia implies that announcements of future policy rate changes are passed-through only imperfectly to other market interest rates that are more relevant for savings and investment decisions of the private sector than the federal funds rate. After establishing this empirical finding, we show that a New Keynesian model that features endogenous liquidity premia delivers responses to forward guidance events that are substantially smaller than in the conventional New Keynesian model.

To analyse the response of liquidity premia, we apply the method by Gürkaynak, Sack and Swanson (2005) to extract and quantify the surprise component of policy announcements in press releases after meetings of the Federal Open Market Committee (FOMC) using daily data between 1990 and 2016. We examine the responses of various spreads, which have been suggested in the literature to measure liquidity premia, to the surprise components and we also construct a common liquidity factor that is based on these spreads, as in Del Negro, Eggertsson, Ferrero and Kiyotaki (2017). All of these liquidity measures increase systematically after FOMC announcements that financial markets consider to be accommodative.

To incorporate endogenous responses of liquidity spreads into the model, we add a stylized banking sector to a New Keynesian model with an explicit specification of central bank operations. Banks are required to hold reserves from the central bank to meet the liquidity demands of their depositors. Reserves can only be obtained in open market operations against assets that are eligible to serve as collateral, namely Treasuries, where the central bank controls the price of money by setting the policy rate. Returns on eligible assets then follow the policy rate closely, whereas interest rates on non-eligible assets, such as corporate bonds, tend to be higher due to a liquidity premium.

The intuition for adjustments of liquidity premia in this model framework can be given as follows. By announcing a lower future monetary policy rate, the central bank announces to provide more means of payment for a given amount of collateral. Thereby, the central bank increases the liquidity value of assets, which are eligible for open-market operations. In an arbitrage-free equilibrium, non-eligible assets then have to provide a relatively higher return, such that the interest rate spread between these asset classes, i.e. the liquidity premium, increases. These less liquid assets serve as the actual store of wealth for the agents in the economy. The return on these assets directly affects consumption-savings decisions via a classical Euler equation. As the announced policy accommodation leads to an increase of the liquidity premium, the response of aggregate demand is dampened compared to the conventional New Keynesian model, where liquidity premia are absent.

We are able to show analytically in our model that forward guidance leads to a rise in the liquidity premium and to moderate increases of output and inflation. In a subsequent quantitative analysis, we calibrate the model to US data, in order to match the response of liquidity premia to forward guidance that we observe in our empirical analysis. Compared to the predictions of the standard New Keynesian model without a liquidity premium, we find the immediate output effects in response to forward guidance in our model to be seven times smaller. The order of magnitude of our results is well in line with empirical evidence from VAR-models. Moreover, also in opposition to the conventional New Keynesian model, the impact responses of output and inflation do not increase with the time horizon of the forward guidance.

Chapter 4 addresses another topic of highest relevance for policy making. While Chapters 2 and 3 study economic phenomena at business cycle frequency, this chapter adopts a longer perspective by contributing to the literature on the macroeconomic effects of demographic change.

Demographic change, understood as the combination of increased life expectancies and low fertility rates that lead to population ageing, particularly in developed countries, is considered to be a key economic driving force in the course of this century. The population age structure of a society has considerable effects on aggregate savings and investment, the composition of consumption demand, or factor prices (see, for instance, Krueger and Ludwig, 2007). The ongoing debate about secular stagnation (see Summers, 2013 and Eggertsson and Mehrotra, 2014) discusses demographic change and the resulting excess savings in the developed world as one possible explanation for the present low levels of interest rates. As a consequence, central banks may be forced to set their policy rates at levels close the zero lower bound in order to ensure stable prices and full employment. The long-term development of the population age structure can therefore also have immediate impact on the relevance of the policies discussed in Chapter 3. It is hence of decisive importance to consider the consequences of demographic change thoroughly.

The research in Chapter 4, which is joint work with Max Groneck, contributes to the analysis of demographic change by focussing on its effect on the relative prices of different types of goods and services. The results imply a direct relation between demographic change and real exchange rates. This chapter is linked to Chapter 2 by emphasizing the role of international relative prices in a different context.

The starting point of our analysis is that preferences for different categories of goods change over the life cycle, which we document using micro data for the United States. We then show that demographic change raises demand for non-tradable old-age related services relative to tradable consumption goods. This demand shift increases the relative price of non-tradables and thereby causes real exchange rates to appreciate. The analysis therefore relates to the literature on structural determinants of real exchange rates, originally initiated by Balassa (1964) and Samuelson (1964). We argue that age-related changes in demand affect prices because of imperfect intersectoral factor mobility. In the main part of the chapter, we analyse the relation between demographic change and relative prices empirically and we also test the relevance of imperfect factor mobility for the transmission of the effects.

The intuition for the mechanism that we consider is the following. The higher demand of elderly people for non-traded services relative to people in working age implies an increase in overall demand for those goods due to population ageing. At the same time, the old-age population has lower saving rates than younger cohorts, such that aggregate savings of an ageing society decline, while aggregate consumption increases. Likewise this rise in spending is also biased towards non-tradable goods. If the additional demand for non-traded services of an ageing society is not associated with an equally strong increase of supply, the relative price of non-tradables increases. We claim that persistent imperfect intersectoral mobility of production factors hampers a reallocation of factor inputs to the non-tradable sector. Since we are concentrating on OECD countries with highly developed capital markets and since the production of non-traded services tends to be labour-intensive, labour market rigidities are – as we show – most important.

To demonstrate the relationship between sectoral prices and population ageing formally, we construct a stylized small open economy model with overlapping generations (OLG) and two production sectors. The model allows us to discuss conditions under which demand effects in general translate into changes of relative prices. We use the model to illustrate the intuition of the effects and to provide guidance for the empirical test of the mechanism.

The basic econometric specification arises from the theoretical model and shows that the relative price of non-tradables depends on the old-age dependency ratio. To analyse whether imperfect labour mobility is relevant for the transmission of the effect, we introduce interactions of indices of labour market rigidity with the OADR. We construct a panel of 15 OECD countries that are followed from 1970 to 2009. Our estimation strategy explicitly takes into account the non-stationarity and cross-sectional dependence of the data. To this end, we use methods by Pesaran (2006) and Kapetanios, Pesaran and Yamagata (2011). Our results indicate a significant link between population ageing and relative sectoral prices. According to our main estimate, up to one fifth of the average increase in relative prices between 1970 and 2009 can be attributed to population ageing. We are able to identify labour market rigidity as the driving force for the transmission of this demand effect by showing that countries with more rigid labour markets experience stronger price effects. Various robustness checks underpin the validity of our findings and demonstrate the importance of labour market frictions relative to other possible channels. Further results widen the analysis to the whole population age structure.

Chapter 2

Optimal Fiscal Substitutes for the Exchange Rate in a Monetary Union

This chapter is based on Kaufmann (2016).[1]

2.1 Introduction

Freely floating exchange rates are generally regarded as an important shock absorber for countries facing macroeconomic turmoil. Giving up this instrument by joining a monetary union (MU) or committing to a peg clearly reduces the abilities of business cycle stabilization policy in reacting to country-specific shocks, as an independent monetary policy is no longer feasible anymore. The fixed exchange rate regime of the European Monetary Union is also blamed for the slack or even missing recovery of some southern European countries in the aftermath of the global financial crisis.

Within a monetary union, fiscal policies can take up the role of the exchange rate, since taxes can in principle affect international relative prices, namely the terms of trade and the real exchange rate, in a similar fashion as the exchange rate does. Policies of this type are referred to as fiscal devaluations. Setting a theoretical benchmark, Farhi et al. (2014) show that the effects of the nominal exchange rate on the allocation of an economy can be replicated entirely using a sufficient number of tax instruments. Following a related approach, Adao, Correia and Teles (2009) conclude that the exchange rate regime can be

[1] I wish to thank Andreas Schabert, Klaus Adam, Christian Bredemeier, Andrea Ferrero, Mathias Hoffmann, Mathias Klein, Michael Krause, Morten Ravn, Dominik Sachs, Thomas Schelkle, Mirko Wiederholt as well as conference and seminar participants at Deutsche Bundesbank, the European Winter Meeting of the Econometric Society (Edinburgh), the European Economic Association (Geneva), the Spring Meeting of Young Economists (Lisbon), the German Economic Association (Augsburg), and University of Cologne for helpful comments and suggestions.

completely irrelevant for stabilization policy.

In this chapter, I show that even under a minimum set of fiscal instruments being in a monetary union does not have to be unduly painful. In a common New Keynesian 2-country open economy framework, the optimal use of only one tax instrument per country reduces the welfare costs of giving up exchange rate flexibility in a MU already significantly. Fiscal devaluation policies are not only feasible, but can be identified as the optimal policy response to country-specific shocks.

As their aim is to affect relative goods prices, fiscal devaluation policies are usually centred around adjustments of the value added tax (VAT). The intuition for the simplest form of a fiscal devaluation policy is that, say, an increase of the domestic relative to the foreign VAT rate induces firms to charge higher prices for goods sold at home, resulting in higher prices of domestic imports relative to exports, for the latter are subject to the relatively reduced foreign VAT.[2] Comparable to a nominal devaluation, this fiscal devaluation policy accordingly leads to a deterioration of the terms of trade. As shown by Farhi et al. (2014), reproducing the depreciation of the real exchange rate that would emerge simultaneously under a nominal devaluation, but not under this VAT-based fiscal devaluation, and stabilizing internal prices of domestically produced goods that are distorted by the change in the VAT requires additional instruments, though.

The model that is used in this chapter features 2 countries with complete international capital markets and staggered price setting a là Calvo (1983). I differentiate between the case where prices are sticky in the country of the producer only such that the law of one price (LOOP) holds internationally, and the case where firms are capable of pricing-to-market (PTM), implying an additional sticky price friction for imported goods.[3] This is important as the welfare costs of fixed exchange rates as well as the capabilities of fiscal policy to reduce these costs depend decisively on the pricing scheme. The model allows for home bias and asymmetries between the countries along several dimensions, such as country size, the degree of competition, and the size of the public sector. Each country has a fiscal authority, whose objective is to finance a given amount of public spending by collecting distortionary taxes and issuance of debt. Each authority controls its own VAT rate that is payable by firms and that is levied on all goods sold within a country. Optimal policy is characterized using a Ramsey approach. This procedure involves to find sequences for the policy instruments that support the welfare-maximizing

[2] In most legislations, including those of both the EU and the US, export revenues are exempted from the VAT, but are subject to the taxation rules of the buyer's country.

[3] In case of a flexible exchange rate regime, these two pricing schemes are also referred to as producer currency pricing and local currency pricing. Regarding the high empirical relevance of both schemes and a recent overview of the literature on international price setting, see Burstein and Gopinath (2014).

competitive equilibrium.

Calibrating the model to characteristics of the euro area, I find that optimal fiscal policy reduces the welfare costs of pegged exchange rates by 86% in case the law of one price holds and by 69% in case of pricing-to-market. The order of magnitude of these results is highly robust to changes in the parametrization and also if payroll taxes are used instead of the VAT.

Besides analysing the welfare effects, I describe the conduct of optimal stabilization policy depending on the exchange rate regime and the way prices are set. In general, under flexible exchange rates, taxes aim to finance public expenditures in the least distortionary way. At the same time, they can be used to stabilize the marginal costs of firms and, hence, inflation via the New Keynesian Phillips curve. This trade-off involves a further dimension in case of a monetary union, where taxes can additionally substitute for the role of the nominal exchange rate, e.g., by inducing expenditure switching effects. In this way, optimal fiscal policy can compensate at least partially for the loss of country-specific monetary policy as a stabilization instrument, thereby bringing the economy closer to the efficient allocation.

In a monetary union, I find that optimal fiscal policy is indeed engaged actively in replicating the flexible exchange rate allocation. Optimal policy favours replicating the behaviour of the terms of trade under a free float over reproducing the response of the real exchange rate, in line with the intuition given above. In situations where a nominal devaluation of a region were optimal, optimal fiscal policy in a MU is a relative increase of the VAT of that region, i.e., to conduct a fiscal devaluation. Although the transmission of fiscal policy is different under LOOP and PTM due to the limited pass-through of tax changes on prices in the latter case, this finding is independent of the pricing scheme. Simulating the economy under both exchange rate regimes yields correlations between the hypothetical optimal exchange rate response and the ratio of VAT rates in the MU of 81% when the LOOP holds and of 59% under PTM.

The reaction of the level of tax rates depends on the specific types of shocks, though. In case of shocks for which an efficient response could be attainable under flexible exchange rates (I consider productivity, government spending, and demand preference shocks), replicating the effects of the exchange rate does not conflict with marginal cost stabilization—an instance of "divine coincidence" for fiscal policy under fixed exchange rates. This manifests in correlations between tax rate increases in the MU and the counterfactual nominal devaluations of about 90%. Translated into a general policy recommendation, a MU member should increase its VAT whenever its currency should be devaluated and vice-versa. In case of mark-up shocks, optimal policy needs to trade-off the objective of stabilizing firms' marginal costs with the incentive to replicate the effect of the exchange rate. Correlations between the hypothetical exchange rate and taxes also depend on the origin of the shock in this instance.

This chapter contributes to the literature on optimal stabilization policy for monetary unions in a New Keynesian framework.[4] Benigno (2004) offers a description of optimal monetary policy in a 2-country setting. He finds that inflation should be stabilized at the level of the union, with a higher weight attached to the country with more rigid prices. The efficient response is generally not achievable, though. Lombardo (2006) builds on the model of Benigno, focusing in particular on the role of different degrees of imperfect competition for monetary policy. Beetsma and Jensen (2005) add fiscal policy to the model in the form of lump-sum financed government spending that enters households' utility. In this setting, optimal monetary policy is still used to stabilize aggregate inflation, while fiscal policy aims at affecting cross-country inflation differentials. Using a similar fiscal setting, Galí and Monacelli (2008) study optimal policy in a monetary union consisting of a continuum of small open economies.

The closest antecedent to this chapter is by Ferrero (2009). In his model, fiscal policy also needs to finance an exogenous stream of government spending by distortionary taxes and debt. Optimal policy is described by targeting rules using a linear-quadratic approach. The focus of the paper lies on the question how far simple policy rules can approximate optimal policy in a monetary union. My analysis assesses how far optimal fiscal policy can reduce the welfare costs of a fixed exchange rate regime. I further show that fiscal devaluation policies can be an optimal policy response to country-idiosyncratic shocks. To this end, I generalize the modelling framework of Ferrero (2009) by adding PTM, by allowing for asymmetries between countries, and by comparing policy scenarios that differ in terms of the exchange rate regime and the availability of fiscal policy as a stabilization device.

This chapter further contributes to the literature on fiscal devaluations. Besides the work of Farhi et al. (2014), this entails, amongst others, Lipinska and von Thadden (2012), and Engler, Ganelli, Tervala and Voigts (2017), who study the quantitative effects of tax swaps from direct (payroll taxes) to indirect taxation (VATs). In general, this literature studies the economic effects of given fiscal policies, but it does not provide a normative analysis. I show that in fixed exchange rate regimes it is not only feasible, but indeed optimal to use fiscal devaluations as a substitute for the exchange rate.

[4]Noteworthy, a monetary union always makes the economy worse off in this literature, as its sole focus lies on the cost-side of giving up flexible exchange rates. For an overview of other (beneficial) aspects of monetary unions, see Beetsma and Giuliodori (2010), and Santos Silva and Tenreyro (2010).

The rest of the chapter is structured as follows. Section 2.2 presents the open economy model. The setup of the Ramsey policy problem is described in Section 2.3, while Section 2.4 features a description of the calibration of the model to the euro area. All results are provided in Section 2.5, with a description of the steady state in Section 2.5.1, the analysis of welfare costs of giving up exchange rate flexibility in 2.5.2, and results on optimal policy conduct in 2.5.3. A conclusion including a discussion of the results is given in Section 2.6.

2.2 The Model

The model economy consists of two countries or regions i, labelled as the core ($i = H$) and the periphery ($i = F$), that can form a monetary union. The world population of households (indexed by h) and firms (indexed by k) each sums up to one, of which a fraction $n \in (0, 1)$ of households and firms lives in the core and a fraction $1 - n$ in the periphery. In each region, households choose consumption of domestic and foreign goods, supply labour, which is mobile only within the region, and trade assets internationally. Firms demand labour to produce tradable goods under monopolistic competition. Price setting is subject to a Calvo-type friction. International prices are either set according to the law of one price or taking into account local market conditions. Fiscal authorities levy distortionary taxes and issue debt to finance an exogenously given amount of public spending. Depending on whether the countries form a monetary union, there are two separate or one single central bank, whose policy instrument is the nominal interest rate. The economy operates at the cashless limit. Periphery variables are denoted by an asterisk (*). The following exposition focuses on the core region; the periphery economy is modelled symmetrically.

2.2.1 Households

A representative household h living in region H derives utility from consumption and disutility from work effort. The consumption bundle $C_t(h)$ consists of tradable goods only and is defined as a composite index over domestic- and foreign-produced consumption goods,

$$C_t(h) = \left[\gamma_H^{\frac{1}{\xi}} C_{Ht}(h)^{\frac{\xi-1}{\xi}} + \gamma_F^{\frac{1}{\xi}} C_{Ft}(h)^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}, \tag{2.1}$$

with $\xi > 0$ being the Armington elasticity of substitution between core and periphery goods, and $\gamma_H = 1 - \gamma_F \in (0,1)$ the share of domestic goods in the consumption bundle. If $\gamma_H > n$, a home bias in preferences exists. Consumption of domestic and imported goods by household h itself is given via Dixit-Stiglitz aggregators over imperfectly substitutable individual varieties k,

$$C_{Ht}(h) = \left[\left(\frac{1}{n}\right)^{\frac{1}{\rho}} \int_0^n C_{Ht}(k,h)^{\frac{\rho-1}{\rho}}\,\mathrm{d}k\right]^{\frac{\rho}{\rho-1}}, \tag{2.2}$$

$$C_{Ft}(h) = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\rho^*}} \int_n^1 C_{Ft}(k,h)^{\frac{\rho^*-1}{\rho^*}}\,\mathrm{d}k\right]^{\frac{\rho^*}{\rho^*-1}}, \tag{2.3}$$

where $\rho, \rho^* > 1$ are the elasticities of substitution between the varieties in each country. To express specialization of countries in production, the elasticity of substitution between varieties within a country is assumed to be greater than between goods of different origin, i.e. $\rho > \xi$.

The corresponding price indices can be shown to equal:

$$P_t = \left[\gamma_H P_{Ht}^{1-\xi} + \gamma_F P_{Ft}^{1-\xi}\right]^{\frac{1}{1-\xi}}, \tag{2.4}$$

$$P_{Ht} = \left[\left(\frac{1}{n}\right) \int_0^n P_{Ht}(k)^{1-\rho}\,\mathrm{d}k\right]^{\frac{1}{1-\rho}}, \tag{2.5}$$

$$P_{Ft} = \left[\left(\frac{1}{1-n}\right) \int_n^1 P_{Ft}(k)^{1-\rho^*}\,\mathrm{d}k\right]^{\frac{1}{1-\rho^*}}. \tag{2.6}$$

P_t denotes the core's consumer price index (CPI), P_{Ht} the producer price index (PPI) of core goods, and P_{Ft} the price index of imported goods. Given the definitions of the price indices, it is easy to show that consumer expenditures are given by $P_t C_t(h) = P_{Ht} C_{Ht}(h) + P_{Ft} C_{Ft}(h)$ with $P_{Ht} C_{Ht}(h) = \int_0^n P_{Ht}(k) C_{Ht}(k,h)\mathrm{d}k$ and $P_{Ft} C_{Ft}(h) = \int_n^1 P_{Ft}(k) C_{Ft}(k,h)\mathrm{d}k$. Consumption demand functions are characterized by:

$$C_{Ht}(h) = \gamma_H \left(\frac{P_{Ht}}{P_t}\right)^{-\xi} C_t(h), \quad C_{Ft}(h) = \gamma_F \left(\frac{P_{Ft}}{P_t}\right)^{-\xi} C_t(h), \tag{2.7}$$

$$C_{Ht}(k,h) = \frac{1}{n} \left(\frac{P_{Ht}(k)}{P_{Ht}}\right)^{-\rho} C_{Ht}(h), \quad C_{Ft}(k,h) = \frac{1}{1-n} \left(\frac{P_{Ft}(k)}{P_{Ft}}\right)^{-\rho^*} C_{Ft}(h). \tag{2.8}$$

Each household h maximizes the utility function

$$\mathbb{U}_0(h) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \frac{C_t(h)^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\eta}}{1+\eta} \right], \tag{2.9}$$

subject to the flow budget constraint

$$P_t C_t(h) + \mathbb{E}_t \left\{ Q_{t,t+1} \left[D_{t+1}(h) + B_{t+1}(h) \right] \right\} \leq W_t N_t(h) + D_t(h) + B_t(h) + \Pi_t(h), \tag{2.10}$$

where ζ_t^c denotes a demand preference shock, $N_t(h)$ labour supply, W_t the wage rate, and $\Pi_t(h)$ the profit share of a well-diversified portfolio of firms in possession of household h. Asset markets are complete within and across countries. $Q_{t,t+1}$ is the period t price of one unit of domestic currency in a particular state of period $t+1$, normalized by the probability of occurrence of that state, i.e., the stochastic discount factor. Accordingly, $\mathbb{E}_t Q_{t,t+1}$ is the price of an asset portfolio that pays off one unit of domestic currency in every state of period $t+1$ and, therefore, equals the inverse of the risk-free gross nominal interest rate, $R_t = 1/\mathbb{E}_t Q_{t,t+1}$. $D_{t+1}(h)$ is the quantity of an internationally-traded state-contingent private asset portfolio denominated in domestic currency, while $B_{t+1}(h)$ denotes holdings of government debt. It is assumed without loss of generality that sovereign debt of country i can be held only by agents of that country.

The first-order conditions of the household's problem imply the Euler equation,

$$Q_{t,t+1} = \beta \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}(h)}{C_t(h)} \right)^{-\sigma} \frac{P_t}{P_{t+1}}, \tag{2.11}$$

as well as an intratemporal consumption-leisure trade-off, given by

$$\frac{N_t(h)^{\eta}}{\zeta_t^c C_t(h)^{-\sigma}} = \frac{W_t}{P_t}. \tag{2.12}$$

Besides, the following transversality conditions hold:

$$\lim_{s \to \infty} \mathbb{E}_t \left[Q_{t,s} D_{t+s}(h) \right] = 0 \quad \text{and} \quad \lim_{s \to \infty} \mathbb{E}_t \left[Q_{t,s} B_{t+s}(h) \right] = 0, \tag{2.13}$$

where $Q_{t,s} = \prod_{z=t}^{s} Q_{t,z}$ denotes the stochastic discount factor from period s to period t.

Foreign households behave analogously and in particular hold a quantity $D_{t+1}^*(h)$ of the internationally-traded asset portfolio. From the periphery's perspective, the stochastic discount factor is priced as

$$Q_{t,t+1} = \beta \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*(h)}{C_t^*(h)} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*} \frac{E_t}{E_{t+1}}, \tag{2.14}$$

where P_t^* is the CPI of the periphery, and E_t is the nominal exchange rate, which is defined as the price of one unit of periphery currency in terms of core currency ($E_t = [H]/[F]$). An increase in E_t accordingly implies a nominal devaluation of the core region. In case the countries form a monetary union, the exchange rate is fixed at unity ($\overline{E} = 1$). Combining (2.11) and (2.14) yields the well-known condition of international risk sharing that links consumption of the two countries and determines their (real) exchange rate:

$$q_t = \frac{\zeta_t^{c*}}{\zeta_t^c} \left(\frac{C_t^*(h)}{C_t(h)} \right)^{-\sigma} \kappa. \tag{2.15}$$

The real exchange rate is defined as the nominal exchange rate weighted ratio of the CPIs, $q_t = (E_t P_t^*)/P_t$, while $\kappa = q_0 (C_0/C_0^*)^{-\sigma}$ is a positive constant that depends on preferences and the initial asset distribution. As pointed out, among others, by Faia and Monacelli (2004), $\kappa = 1$ if markets are complete, the initial net foreign indebtedness is zero ($D_{t+1}(h) = D_{t+1}^*(h) = 0 \; \forall h$), and preferences are symmetric across countries.

2.2.2 Firms and Price Setting Assumptions

In the core a continuum of firms $k \in [0, n]$ operates under monopolistic competition. Each firm produces a variety k according to the production plan

$$Y_t(k) = A_t N_t^{\alpha}(k), \tag{2.16}$$

where $Y_t(k)$ is total supply of variety k, A_t a country-specific stochastic productivity level, and $N_t(k)$ the firm's labour demand. Labour is the sole input of production, and α is the input elasticity of production. Labour supply by households is perfectly mobile across firms within the country, but immobile between countries. Total demand for the good produced by firm k is given by the demand of domestic ($C_{Ht}(k)$) and foreign ($C_{Ht}^*(k)$) households as well as public demand by the domestic government ($G_t(k)$):

$$Y_t(k) = \int_0^n C_{Ht}(k,h) \, \mathrm{d}h + \int_n^1 C_{Ht}^*(k,h) \, \mathrm{d}h + G_t(k). \tag{2.17}$$

The period t profit function of firm k reads

$$\begin{aligned}\Pi_t(k) &= (1-\tau_t^v)\, P_{Ht}(k) \left[\int_0^n C_{Ht}(k,h)\,\mathrm{d}h + G_t(k)\right] \\ &\quad + (1-\tau_t^{v*})\, E_t P_{Ht}^*(k) \int_n^1 C_{Ht}^*(k,h)\,\mathrm{d}h - W_t N_t(k), \qquad (2.18)\end{aligned}$$

where $P_{Ht}^*(k)$ is the price of core good k abroad. τ_t^v and τ_t^{v*} are country-specific value-added taxes (VAT) in region H and F respectively. As common in existing tax systems, τ_t^v is levied on all goods sold within the Home country, but not on exports. The latter are taxed at the border with the foreign VAT rate τ_t^{v*}.

Price setting of firms is impaired by Calvo-type price stickiness. Each period t, a firm can adjust prices with probability $1-\theta$, independent of the date of previous price changes. With probability θ the firm has to maintain last period's prices. Optimal prices are set as to maximize the net present value of future profits

$$\sum_{s=t}^{\infty} \theta^{s-t} \mathbb{E}_t \left[Q_{t,s} \Pi_s(k)\right] \qquad (2.19)$$

subject to the production technology and demand. Prices always include taxes. The price of domestic goods sold within the core, $P_{Ht}(h)$, is always set in domestic currency. The setting of export prices for the periphery, $P_{Ht}^*(k)$, is conducted according to the assumption of either the law of one price or pricing-to-market.

Law of One Price (LOOP)

Under this pricing scheme, firms set a price for their good in domestic currency, while the price in the other region satisfies the law of one price, adjusted for tax rates:

$$\begin{aligned}(1-\tau_t^{v*})\, E_t P_{Ht}^*(k) &\overset{!}{=} (1-\tau_t^v)\, P_{Ht}(k) \\ \Leftrightarrow P_{Ht}^*(k) &= \frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{1}{E_t} P_{Ht}(k). \qquad (2.20)\end{aligned}$$

Following Farhi et al. (2014), this expression is derived from the assumption that one unit of sales should yield the same revenue to the firm, independent of the origin of the buyer. (2.20) implies complete and immediate pass-through of both exchange rates and taxes on international prices. A relative increase of the core's VAT rate has the same effect on prices abroad as a nominal devaluation.

The optimality condition for the price set in period t, $\overline{P}_{Ht}(k)$, is derived in Appendix 2.A.1 and reads

$$\mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\overline{P}_{Ht}(k)}{P_{Hs}} \right)^{-1-\rho} \frac{Y_s}{P_{Hs}} \left[\frac{\rho}{\rho-1} \mu_s MC_{Hs}(k) - (1-\tau_s^v)\, \overline{P}_{Ht}(k) \right] = 0, \quad (2.21)$$

where $MC_{Ht}(k) = W_t / \left[\alpha A_t N_t^{\alpha-1}(k) \right]$ denotes marginal costs, and μ_t a stochastic mark-up shock. The equation shows the standard result that the optimal price is set equal to a mark-up over a weighted average of current and future marginal costs.

Pricing-to-Market (PTM)

Under the alternative assumption of PTM, firms set separate prices at home, $\overline{P}_{Ht}(k)$, and abroad, $\overline{P}^*_{Ht}(k)$, each of them subject to a Calvo friction. As a result, there is only limited direct pass-through of exchange rates and taxes on international prices, and the law of one price can be violated. Optimal price setting is now described by two conditions, also derived in Appendix 2.A.1:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\overline{P}_{Ht}(k)}{P_{Hs}} \right)^{-1-\rho} \frac{(nC_{Hs} + G_s)}{P_{Hs}} \left[\frac{\rho}{\rho-1} \mu_s MC_{Hs}(k) - (1-\tau_s^v)\, \overline{P}_{Ht}(k) \right] = 0, \quad (2.22)$$

$$\mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\overline{P}^*_{Ht}(k)}{P^*_{Hs}} \right)^{-1-\rho} \frac{C^*_{Hs}}{P^*_{Hs}} \left[\frac{\rho}{\rho-1} \mu_s MC_{Hs}(k) - (1-\tau_s^{v*})\, E_s \overline{P}^*_{Ht}(k) \right] = 0. \quad (2.23)$$

A devaluation of the domestic currency has the same effect on the firm's pricing decision for exports as a reduction in marginal costs, since every unit sold abroad leads to higher revenues than selling on the domestic market. Note that reducing the periphery's VAT rate τ^{v*} induces, ceteris paribus, the same effect on import prices in the periphery as a rise in E_t.

Foreign Firms

Foreign firms are modelled symmetrically. Under the LOOP, they set a price $\overline{P}^*_{Ft}(k)$ at which periphery goods are sold in F. The price at which goods are sold internationally is again determined by the law of one price, adjusted for taxes:

$$P_{Ft}(k) = \frac{(1-\tau^{v*})}{(1-\tau^v)} E_t P^*_{Ft}(k). \quad (2.24)$$

In case of PTM, firms in the periphery can also set separate prices for their domestic and the international market. Optimal prices are implicitly given by:

$$
\begin{aligned}
\mathbb{E}_t \sum_{s=t}^{\infty} \theta^{*s-t} Q^*_{t,s} \left(\frac{\overline{P}^*_{Ft}(k)}{P^*_{Fs}} \right)^{-1-\rho^*} & \frac{\left((1-n)\, C^*_{Fs} + G^*_s\right)}{P^*_{Fs}} \\
\cdot \left[\frac{\rho^*}{\rho^*-1} \mu^*_s MC^*_{Fs}(k) - \left(1-\tau^{v*}_s\right) \overline{P}^*_{Ft}(k) \right] &= 0, \qquad (2.25) \\
\mathbb{E}_t \sum_{s=t}^{\infty} \theta^{*s-t} Q^*_{t,s} \left(\frac{\overline{P}_{Ft}(k)}{P_{Fs}} \right)^{-1-\rho^*} & \frac{C_{Fs}}{P^*_{Fs}} \\
\cdot \left[\frac{\rho^*}{\rho^*-1} \mu^*_s MC^*_{Fs}(k) - \frac{(1-\tau^v_s)}{E_s} \overline{P}_{Ft}(k) \right] &= 0, \qquad (2.26)
\end{aligned}
$$

where $MC^*_{Ft}(k) = W^*_t / \left[\alpha A^*_t N^{*\alpha-1}_t(k)\right]$.

2.2.3 Monetary and Fiscal Authorities

The public sector consists of separate fiscal authorities and central banks at the country level. The policy instruments of the central banks are their nominal interest rates, R_t and R^*_t. If the regions share the same currency, only one central bank for the union as a whole exists, whose policy instrument is denoted by R^{MU}_t.

The task of the fiscal authorities is to finance an exogenously given stochastic amount of public spending G_t. In each country, government spending consists of an index of locally produced goods only,

$$
G_t = \left[\left(\frac{1}{n}\right) \int_0^n G_t(k)^{\frac{\rho-1}{\rho}} \, \mathrm{d}k \right]^{\frac{\rho}{\rho-1}}, \qquad (2.27)
$$

with corresponding demand functions for each variety k, given by

$$
G_t(k) = \frac{1}{n} \left(\frac{P_{Ht}(k)}{P_{Ht}} \right)^{-\rho} G_t. \qquad (2.28)
$$

These expenditures are financed by distortionary value-added taxes and state-contingent public debt. The budget constraint of the domestic government reads

$$
P_{Ht} G_t + B_t \;\; \leq \;\; \mathbb{E}_t Q_{t,t+1} B_{t+1} + \tau^v_t \int_0^n P_{Ht}(k) \left(\int_0^n C_{Ht}(k,h)\, \mathrm{d}h + G_t(k) \right) \mathrm{d}k
$$

$$+\tau_t^v \int_n^1 \int_0^n P_{Ft}(k) C_{Ft}(k,h)\,\mathrm{d}h\,\mathrm{d}k. \tag{2.29}$$

Note that the VAT is not only levied on domestically produced goods, but also on imports C_{Ft}.

2.2.4 Aggregation and Equilibrium

Due to symmetry among agents within a country, households and firms, respectively, will in each situation come to the same decisions. In the process of aggregation, one can, therefore, drop indices h and k.

By the law of large numbers, today's PPIs consist of the prices set today and last period's price index, weighted with the probabilities of adjustment and non-adjustment, respectively. As shown in Appendix 2.A.2, the law of motion for P_{Ht} can be expressed as

$$\widetilde{p}_{Ht} = \frac{\overline{P}_{Ht}}{P_{Ht}} = \left(\frac{1-\theta\pi_{Ht}^{\rho-1}}{1-\theta}\right)^{\frac{1}{1-\rho}}, \tag{2.30}$$

where $\pi_{Ht} = P_{Ht}/P_{Ht-1}$ denotes the PPI inflation rate of domestically produced goods in H.

(2.21) gives an expression for the Phillips curve of core goods inflation under the LOOP. In order to solve the model, it is required to rewrite the Phillips curve in a recursive way, which avoids the use of infinite sums. To do so, I follow Schmitt-Grohé and Uribe (2006) and restate (2.21)by defining two auxiliary variables, $X1_{Ht}$ and $X2_{Ht}$ (for a derivation, see Appendix 2.A.3), such that

$$\frac{\rho}{\rho-1}\mu_t X1_{Ht} = X2_{Ht}, \tag{2.31}$$

where

$$X1_{Ht} = \widetilde{p}_{Ht}^{-1-\rho} Y_t mc_{Ht} + \theta\beta\mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{\pi_{Ht+1}^{1+\rho}}{\pi_{t+1}} \left(\frac{\widetilde{p}_{Ht}}{\widetilde{p}_{Ht+1}}\right)^{-1-\rho} X1_{Ht+1}, \tag{2.32}$$

$$X2_{Ht} = \widetilde{p}_{Ht}^{-\rho} Y_t (1-\tau_t^v) + \theta\beta\mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{\pi_{Ht+1}^{\rho}}{\pi_{t+1}} \left(\frac{\widetilde{p}_{Ht}}{\widetilde{p}_{Ht+1}}\right)^{-\rho} X2_{Ht+1}, \tag{2.33}$$

with $\pi_t = P_t/P_{t-1}$ being the CPI inflation rate of the core, and $mc_{Ht} = MC_{Ht}/P_{Ht}$ being real marginal costs.

The resource constraint of the economy can be obtained by integrating the production function (2.16) over firms. The result differs depending on whether pricing follows the law of one price or firms can engage in pricing-to-market:[5]

$$
\begin{aligned}
A_t n N_t^{\alpha} &= \Delta_{Ht}\left(nC_{Ht} + (1-n)C_{Ht}^{*} + G_t\right) && (2.34)\\
A_t n N_t^{\alpha} &= \Delta_{Ht}\left(nC_{Ht} + G_t\right) + \Delta_{Ht}^{*}(1-n)C_{Ht}^{*} && (2.35)
\end{aligned}
$$

The first equation holds under the LOOP, the latter one under PTM. Δ_{Ht} and Δ_{Ht}^{*} are indices of price dispersion that render inflation costly in efficiency terms. They are defined as

$$
\begin{aligned}
\Delta_{Ht} &= \frac{1}{n}\int_0^n \left(\frac{P_{Ht}(k)}{P_{Ht}}\right)^{-\rho} dk, && (2.36)\\
\Delta_{Ht}^{*} &= \frac{1}{n}\int_0^n \left(\frac{P_{Ht}^{*}(k)}{P_{Ht}^{*}}\right)^{-\rho} dk. && (2.37)
\end{aligned}
$$

Their laws of motion are given by

$$
\begin{aligned}
\Delta_{Ht} &= (1-\theta)\widetilde{p}_{Ht}^{-\rho} + \theta\pi_{Ht}^{\rho}\Delta_{Ht-1}, && (2.38)\\
\Delta_{Ht}^{*} &= (1-\theta)\widetilde{p}_{Ht}^{*-\rho} + \theta\pi_{Ht}^{*\rho}\Delta_{Ht-1}^{*}. && (2.39)
\end{aligned}
$$

As clarified by Schmitt-Grohé and Uribe (2006), price dispersion is irrelevant for the allocation if the non-stochastic (steady state) level of inflation is zero and only a first-order approximation to the equilibrium conditions is used.

An equilibrium in this economy is characterized by prices and quantities that fulfil the optimality conditions of households and firms in both countries such that all markets clear, given stochastic processes for all shocks, and sequences for the policy instruments. Goods markets under LOOP and markets for private assets clear at the international level; goods markets under PTM, government bond, and labour markets clear at national levels. A complete list of all equilibrium conditions under both LOOP and PTM is given in Appendix 2.B.

The following definition of the terms of trade will be useful for the rest of the analysis. The terms of trade indicate how much of exports the economy has to give for one unit of imports,

[5]For derivations, see Appendix 2.A.4.

$$z_t = \frac{P_{Ft}}{P_{Ht}} \overset{(LOOP)}{=} \underbrace{\frac{(1-\tau_t^{v*})}{(1-\tau_t^{v})}}_{FD_t} E_t \frac{P_{Ft}^*}{P_{Ht}}, \tag{2.40}$$

where the second equality sign holds under the law of one price. In this case of complete pass-through only, exchange rate and tax adjustments translate directly into changes of the terms of trade. The formula shows that under LOOP an increase of H's VAT relative to F's has the same effect on z_t as a nominal devaluation. The term $FD_t = (1-\tau_t^{v*})/(1-\tau_t^{v})$ will, therefore, also be referred to as the fiscal devaluation factor.

In case of pricing-to-market, the pass-through of the exchange rate and taxes on the terms of trade is limited by their effect on P_{Ft} and P_{Ht}. The price setting conditions (2.23) and (2.26) make clear that the tax rates can have the same effect on import prices as the nominal exchange rate. The speed of pass-through depends on the degree of price stickiness, with the law of one price and, so, the second part of (2.40) only holding in the long-run. The short-run efficacy of fiscal devaluation policies to affect the terms of trade will, therefore, be higher under LOOP than under PTM.

2.3 The Ramsey Problem

Optimal monetary and fiscal policy is determined using a Ramsey approach. This procedure involves finding the sequences of the available policy instruments that support the welfare-maximizing competitive equilibrium. All policy authorities can credibly commit to their announced policies, and I assume full cooperation between all entities. The objective of the Ramsey planner is a utilitarian world welfare function that weights utility of core and periphery households according to their population size:

$$\mathbb{W}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ n \left(\zeta_t^c \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right) + (1-n) \left(\zeta_t^{c*} \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{N_t^{*1+\eta}}{1+\eta} \right) \right\}. \tag{2.41}$$

If prices were flexible, the optimal policy problem could be described by maximizing (2.41) subject to one implementability and one resource constraint for each country only. Using this so-called primal approach to the Ramsey problem, proposed by Lucas and Stokey (1983) also in the context of optimal stabilization policy, the planner directly chooses an equilibrium allocation, from which prices and instruments can be backed out afterwards. In presence of a sticky price friction, this reduction of the problem to just two constraints per country is generally not possible anymore, as the Phillips curves now effectively constrain the evolution of prices.[6] The dual approach to the Ramsey problem, which involves choosing prices and instruments directly, has to be used instead.

In the following analysis, I compare optimal policy under various scenarios to assess the consequences of being in a monetary union. The scenarios differ by the type of price setting (LOOP vs. PTM) and by the availability of different policy instruments: flexible exchange rates vs. monetary union, and monetary and fiscal policy vs. monetary policy only. In all of these scenarios, the dual solution to the policy problem is found by maximizing (2.41) subject to the relevant equilibrium conditions, described in Appendix 2.B. If fiscal policy is an instrument to the Ramsey planner, the time path of the VAT rates, $\{\tau_t^v, \tau_t^{v*}\}_{t=0}^{\infty}$, has to ensure solvency of the fiscal authorities in both countries. To this end, the problem is augmented with the intertemporal fiscal budget constraints of both countries. As an example, I describe the solution to the Ramsey problem by means of its first-order conditions for the case of a monetary union, where the law of one price holds, with fiscal policy in detail in Appendix 2.C.[7]

2.4 Calibration

I calibrate the model to characteristics of the euro area using quarterly data between 2001:1 and 2014:4 from Eurostat. In the calibration, the core (region H) comprises Austria, Belgium, Finland, France, Germany, the Netherlands, and Slovakia. The periphery (region F) consists of Greece, Ireland, Italy, Portugal, and Spain. This leads to a population share of the core of 60%; hence, $n = 0.6$. In total, these 12 countries cover 98% of euro area GDP in 2014.

The discount factor β is set to 0.99, which is the standard value in the business-cycle literature for quarterly data, implying an annual real interest rate of about 4% in steady state. Risk aversion and the inverse Frisch elasticity are both set equal to 2, also following conventions of the literature. A mild home bias in demand preferences of 20% exists in both countries, yielding $\gamma_H = 1.2n = 0.72$ and $\gamma_F^* = 1.2(1-n) = 0.48$. Following estimates by Feenstra, Luck, Obstfeld and Russ (2014), I set the Armington elasticity between goods of different origin to $\xi = 1.2$. Initial international private debt in steady state is set to match the average trade balance surplus relative to GDP of the core of 2% between 2001 and 2014.

The elasticities of substitution between individual goods varieties, ρ and ρ^*, are set

[6] The work of, for instance, Schmitt-Grohé and Uribe (2004), and Faia and Monacelli (2004) is also subject to this issue.

[7] Solutions to all other scenarios are available on request.

Table 2.1: Parameter Values

Parameter	Core	Periphery
Size of region	$n = 0.6$	$(1 - n) = 0.4$
Discount factor	$\beta = 0.99$	
Risk aversion	$\sigma = 2$	
Inverse Frisch elasticity	$\eta = 2$	
Home bias	$\gamma_H = 0.72$	$\gamma_F^* = 0.48$
Armington elasticity (Home-Foreign goods)	$\xi = 1.2$	
Elasticity of substitution between varieties	$\rho = 6$	$\rho^* = 4$
Labor input elasticity of production	$\alpha = 1$	
1 – Probability of price adjustment	$\theta = 0.75$	$\theta^* = 0.75$
Gov. spending ratio to GDP in steady state	$G/Y = 0.21$	$G^*/Y^* = 0.19$
Annual gov. debt to GDP ratio in steady state	$B/Y = 0.78$	$B^*/Y^* = 1.08$

to match aggregate mark-ups. Høj, Jimenez, Maher, Nicoletti and Wise (2007) provide estimates for several OECD countries that suggest a mark-up of 1.2 in the core and 1.3 in the periphery, which implies $\rho = 6$ and $\rho^* = 4$. The labour input elasticity of production is set to one, which implies that the production technology is linear in labour. Following empirical evidence by ECB (2005), the probability of price stickiness is set to $\theta, \theta^* = 0.75$ so that price contracts last on average 4 quarters.[8] Cross-country evidence by Druant, Fabiani, Kezdi, Lamo, Martins and Sabbatini (2012) confirms that the frequency of price adjustments is similar across core and periphery countries.

The ratio of government spending to GDP in steady state (G/Y) is set to the average values between 2001 and 2014, which are 21% for the core and 19% for the periphery. The government debt to GDP ratio in annualized steady state (B/Y) matches the 2010-2014 average debt-to-GDP-ratios of the core (78%) and the periphery (108%). This calibration requires a steady state primary surplus relative to quarterly GDP of 3.1% in the core and of 4.3% in the periphery. Balanced public budgets imply steady state VAT rates of 24.6% and 22.6%, respectively. Table 2.1 summarizes all parameter values.

The evolution of the economy outside steady state is driven by region-specific stochastic processes for productivity A_t and government spending G_t, the demand preference shocks ζ_t^c, and the mark-up shocks μ_t in both countries. All but the mark-up shocks are modelled as AR(1)-processes, while the latter are assumed to be white noise.[9] Persis-

[8]The average time until a firm gets a chance to adjust its price is given by $1/(1-\theta)$, as Calvo-type price stickiness implies a Poisson process, where time until next adjustment is an exponentially-distributed random variable.

[9]Allowing the mark-up shock to follow an AR(1)-process as well yields persistence parameters of (μ_t, μ_t^*) close to zero and does not affect the moments of the other processes significantly, which is in line with results of Smets and Wouters (2003).

Table 2.2: Shock Processes

Parameter	Core	Periphery
Persistence of productivity shocks (φ_A, φ_{A*})	0.9301	0.9434
Persistence of demand preference shocks (φ_C, φ_{C*})	0.8135	0.8990
Persistence of government spending shocks (φ_G, φ_{G*})	0.7731	0.6439
Std. dev. of productivity shocks (σ_A, σ_{A*})	0.0034	0.0032
Std. dev. of demand preference shocks (σ_C, σ_{C*})	0.0139	0.0209
Std. dev. of government spending shocks (σ_G, σ_{G*})	0.0071	0.0194
Std. dev. of mark-up shocks ($\sigma_\mu, \sigma_{\mu*}$)	0.0057	0.0140

Notes: Parameters calibrated to match autocorrelations and standard deviations of GDP, government spending, private consumption, and wage data between 2001:1 and 2014:4.

tence and Variance of the shocks are calibrated to match autocorrelations and standard deviations of seasonally adjusted and quadratically detrended data on GDP, government spending, private consumption, and average wage rates of the core and periphery between 2001:1 and 2014:4. The resulting parameters are given in Table 2.2. Details on the used data, including the target moments, are shown in Appendix 2.D.

2.5 Results

The solution to the Ramsey problem, calibrated to the euro area, is assessed quantitatively next. Section 2.5.1 provides a brief description of the steady state. Section 2.5.2 analyses to what extent optimal fiscal policy reduces the welfare costs of giving up flexible exchange rates within the European Monetary Union. The conduct and mechanisms of optimal policy are subsequently described in Section 2.5.3.

2.5.1 The Allocation in Steady State

Gross inflation rates in all sectors, domestic goods and imports, are equal to one in the Ramsey-optimal steady state, since price dispersion that would arise otherwise impairs an efficient bundling of individual goods. Given this result, optimal price setting of domestic firms in steady state when the LOOP holds is described by

$$\frac{\rho}{\rho-1}\frac{1}{(1-\tau^v)}MC_H = P_H, \tag{2.42}$$

while under PTM the following condition for export prices additionally holds:

$$\frac{\rho}{\rho-1}\frac{1}{(1-\tau^{v*})}\frac{1}{E}MC_H = P_H^*. \tag{2.43}$$

Combining (2.42) and (2.43) immediately yields the law of one price (2.20). Hence, there are no long-run deviations from the law of one price, which would distort the composition of consumption between domestic and imported goods.

Also visible from (2.42) and (2.43), the distortions that render the long-run allocation different from its first-best level are the reduction in activity due to monopolistic competition and the necessity to use distortionary taxation to finance public expenditures. As taxes have to be positive in steady state, they cannot be used for mark-up elimination. Instead, taxes exacerbate the wedge driven by the mark-up between prices and marginal costs. The steady state is, therefore, in general not efficient.

2.5.2 The Welfare Costs of Giving up Exchange Rate Flexibility

The welfare comparison of the various policy scenarios is discussed next. The welfare measure used to assess the scenarios is units of steady state consumption that households are willing to give up in order to live in the deterministic steady state economy instead of a stochastic economy—that is a percentage amount of steady state consumption $\omega_I^{\mathcal{E}}$ satisfying

$$\frac{1}{1-\beta}\left\{n\left(\frac{\left[C\left(1+\omega_I^{\mathcal{E}}\right)\right]^{1-\sigma}}{1-\sigma}-\frac{N^{1+\eta}}{1+\eta}\right)+(1-n)\left(\frac{\left[C^*\left(1+\omega_I^{\mathcal{E}}\right)\right]^{1-\sigma}}{1-\sigma}-\frac{N^{*1+\eta}}{1+\eta}\right)\right\}\stackrel{!}{=}\mathbb{W}_0^{\mathcal{E},I},$$

where $\mathbb{W}_0^{\mathcal{E},I}$ is the expected net present value of aggregate welfare as defined by (2.41) for a given exchange rate regime $\mathcal{E}\in\{MU, FLEX\}$ and a given set of policy instruments $I\in\{MP, MFP\}$. To evaluate welfare, the model is solved by a second-order approximation to the policy functions and simulated for $T = 1000$ periods.[10] I average the welfare measure over $J = 100$ simulations with different stochastic seeds to obtain ergodic means.

Results are given in Table 2.3. The evaluated scenarios differ along 3 dimensions. A first distinction is made in terms of the pricing scheme, law of one price or pricing-to-market. Second, the two columns, headed MU and FLEX, indicate whether the exchange rate regime is a monetary union or flexible. Third, rows mark if only monetary policy is available for stabilization purposes (abbreviated by MP) or if both monetary and fiscal policy can be used (MFP). To allow for comparisons between these scenarios, the underlying steady state is calibrated to be identical across all 8 scenarios. This implies for

[10] I use the Dynare toolbox to solve the model. The second-order simulations are obtained using the pruning algorithm proposed by Kim, Kim, Schaumburg and Sims (2008).

the MP scenarios that VAT rates in steady state have to be set on the optimal values obtained under MFP.

Table 2.3: Welfare Costs of Fixed Exchange Rates

(A) Benchmark				
LOOP		MU	FLEX	Difference
Monetary Policy (MP)	$10^{-2}*$	-5.1612	-4.5269	0.6343
Monetary+Fiscal Policy (MFP)	$10^{-2}*$	-4.4485	-4.3582	0.0903
		Reduction of Welfare Costs:		**85.76**%
PTM				
Monetary Policy	$10^{-2}*$	-5.1593	-5.0605	0.0988
Monetary+Fiscal Policy	$10^{-2}*$	-4.9095	-4.8785	0.0310
		Reduction of Welfare Costs:		**68.66**%
(B) Productivity, Preference, Gov. Spending Shocks				
LOOP				
Monetary Policy	$10^{-2}*$	-4.1227	-3.6759	0.4468
Monetary+Fiscal Policy	$10^{-2}*$	-3.7696	-3.6983	0.0713
		Reduction of Welfare Costs:		**84.03**%
PTM				
Monetary Policy	$10^{-2}*$	-4.1212	-4.0689	0.0523
Monetary+Fiscal Policy	$10^{-2}*$	-4.0826	-4.0593	0.0233
		Reduction of Welfare Costs:		**55.42**%
(C) Mark-up Shocks				
LOOP				
Monetary Policy	$10^{-2}*$	-0.9617	-0.7742	0.1875
Monetary+Fiscal Policy	$10^{-2}*$	-0.6064	-0.5868	0.0194
		Reduction of Welfare Costs:		**89.58**%
PTM				
Monetary Policy	$10^{-2}*$	-0.9613	-0.9148	0.0465
Monetary+Fiscal Policy	$10^{-2}*$	-0.7505	-0.7428	0.0077
		Reduction of Welfare Costs:		**83.43**%

Notes: Welfare measure: consumption equivalents between deterministic and stochastic world economy. Exchange rate regime either monetary union (MU) or flexible (FLEX). Panel (A): productivity, demand preference, government spending, & mark-up shocks in both countries. Panel (B): all but mark-up shocks. Panel (C): mark-up shocks only. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

As is well-known, absolute numbers calculated for the welfare costs of business cycles are in general small in representative agent models.[11] The focus of this analysis, yet, lies on the comparison across different scenarios, which yields significant outcomes. Results for the benchmark calibration are given in Panel (A) of the table. Under LOOP and exclusive

[11] An exemption is the analysis of Schmitt-Grohé and Uribe (2016), which relies on downward nominal wage rigidity.

availability of monetary policy, households are willing to give up $\omega_{MP}^{MU} = 5.16 * 10^{-2}\%$ of steady state consumption (hereafter c%) to avoid living in the stochastic economy of a monetary union and $\omega_{MP}^{FLEX} = 4.53 * 10^{-2}$c% with flexible exchange rates. The difference between these two numbers, $\Delta\omega_{MP} = \omega_{MP}^{MU} - \omega_{MP}^{FLEX} = 0.63 * 10^{-2}$c%, given in the last column, shows the welfare costs of giving up exchange rate flexibility in a monetary union. Allowing for the VAT rates of both countries as a stabilization tool reveals that fiscal policy is almost irrelevant under flexible exchange rates—welfare costs are reduced from $\omega_{MFP}^{MU} = 4.45 * 10^{-2}$c% to $\omega_{MFP}^{FLEX} = 4.36 * 10^{-2}$c%. By contrast, fiscal policy is an effective instrument in a monetary union: welfare costs of entering a MU are reduced by 85.76% from $\Delta\omega_{MP} = 0.63 * 10^{-2}$c% to $\Delta\omega_{MFP} = 0.09 * 10^{-2}$c%.

Engel (2011) shows that optimal exchange rate volatility is lower in presence of pricing-to-market since in this case exchange rate movements do not directly translate into changes of international relative prices as they would under the LOOP. Instead, exchange rate changes merely distort price mark-ups of firms, thereby leading to inefficient deviations from the law of one price.[12] Welfare costs of fixed exchange rate regimes are, therefore, strictly lower with PTM than under LOOP, a point also raised by Corsetti (2008). Additionally, as shown in Section 2.2, fiscal policy can potentially be much more effective in manipulating the terms of trade when the LOOP holds due to the assumption of full pass-through than under PTM. To take into account the effect of the pricing scheme on the welfare costs of exchange rate pegs on the one hand, and to avoid an overestimation of the beneficial effect of fiscal policy because of full pass-through on the other hand, the reduction of welfare costs is studied next for the case of PTM. The welfare costs of entering a MU are now about $\Delta\omega_{MP} = 0.099 * 10^{-2}$c% under monetary policy only, which is about 6.4 times smaller than when the law of one price holds. Adding fiscal policy to the set of instruments also helps to reduce welfare costs considerably by 68.66%. Hence, even under PTM, fiscal policy is capable of reducing the welfare costs of fixed exchange rates substantially.

The two bottom panels, (B) and (C), of Table 2.3 decompose the shocks into those, for which the efficient response is attainable by the use of monetary policy only when the law of one price holds and exchange rates are fully flexible (productivity, demand preferences, and government spending shocks), and the mark-up shocks, which cannot be fully stabilized. While the size of the welfare costs in the various cases naturally depends on the type and number of shocks considered, the percentage reduction of the welfare

[12] Under very specific conditions, it can even be optimal to completely stabilize the nominal exchange rate in presence of PTM, as shown by Devereux and Engel (2003). Duarte and Obstfeld (2008) emphasize that this extreme result holds only under a restrictive set of assumptions. Among these are one period in advance price stickiness and the absence of home bias.

costs of the fixed exchange rate regime is of comparable magnitude as in the benchmark (Panel A). Under the law of one price, using the tax instruments for stabilization policy purposes reduces the welfare costs of the monetary union by 84% in Panel (B) and by almost 90% in presence of the mark-up shocks. Under pricing-to-market, the reduction in welfare costs depends to a larger extent on the type of shock. Allowing for active fiscal policy reduces welfare costs by 55.42% in Panel (B) and by 83.43% in Panel (C). The cause for the effective stabilization of mark-up shocks can directly be understood from the firms' first-order conditions (2.21) to (2.23). The VAT rates can directly offset the effect of the mark-up shocks μ_t on the firms' price setting.

Various sensitivity checks confirm that the results of Table 2.3 are very robust to changes in the parametrization of the model. Table 2.7 in Appendix 2.E provides results, where standard deviations of all shocks are doubled compared to the benchmark calibration. Increasing the shock size naturally raises the shares of steady state consumption that households are willing to give up to avoid living in the stochastic economy. The percentage reduction of the welfare costs of fixed exchange rates by using fiscal policy, however, remains virtually the same. Optimal exchange rate volatility and the costs of pegs also depend on the structural parameters of the model. For instance, Lombardo and Ravenna (2014) and Faia and Monacelli (2008) emphasize the role of trade openness for the exchange rate, while De Paoli (2009) analyses the impact of the Armington elasticity. Results in Table 2.8 show that the findings of this section regarding the reduction of welfare costs are qualitatively fully maintained for changes in all structural parameters as well as the amount of government spending and debt.

Instead of VAT rates, policymakers could in principle also use payroll taxes as the fiscal instrument to substitute for the effect of the exchange rate.[13] A change in the labour tax implies that the prices of all goods produced within a country are affected equally, while changes of the domestic VAT alter only prices of goods sold at home, but not of exports. To analyse whether these differences influence the capability of fiscal policy to reduce the welfare costs of a fixed exchange rate regime, I repeat the welfare analysis of Table 2.3 with a payroll tax in each country levied on firms instead of a VAT.

Table 2.9 in Appendix 2.E presents the results of that exercise. To ensure comparability, I use the same calibration as before. Most importantly, the reduction of welfare costs by the additional use of fiscal policy remains to be high with payroll taxes. Under

[13] If a payroll tax τ_t^n is levied on the employers, profits of firm k become

$$\Pi_t(k) = (1 - \tau_t^v)\, P_{Ht}(k)\, [nC_{Ht}(k,h) + G_t(k)] + (1 - \tau_t^{v*})\, E_t P_{Ht}^*(k) C_{Ht}^*(k,h) - (1 + \tau_t^n) W_t N_t(k).$$

the benchmark calibration, welfare costs of a peg can be reduced by about 60% under the LOOP and by 80% under PTM. The relatively smaller reduction of welfare costs under the LOOP is driven by the low reduction for productivity, demand preference, and government spending shocks in Panel (B) of 41% only compared to 84% with VATs. The main reason for these different results is that, in opposition to VATs, payroll tax changes do not directly pass through on the terms of trade via the law of one price (recap equations 2.20 and 2.40). In case of mark-up shocks, on the other hand, the welfare costs of fixed exchange rates can be avoided almost completely—by 99% under LOOP and 97% under PTM. A change in the domestic payroll tax suffices to neutralize the effect of a domestic mark-up shock, while with VATs the rates of both countries would have to adjust for stabilization of the shock along all relevant margins.

In sum, these results suggest that optimal use of only one fiscal instrument per country could substantially reduce welfare costs in the euro area that arise from the fixed exchange rate regime.

2.5.3 Optimal Fiscal Substitutes for the Exchange Rate

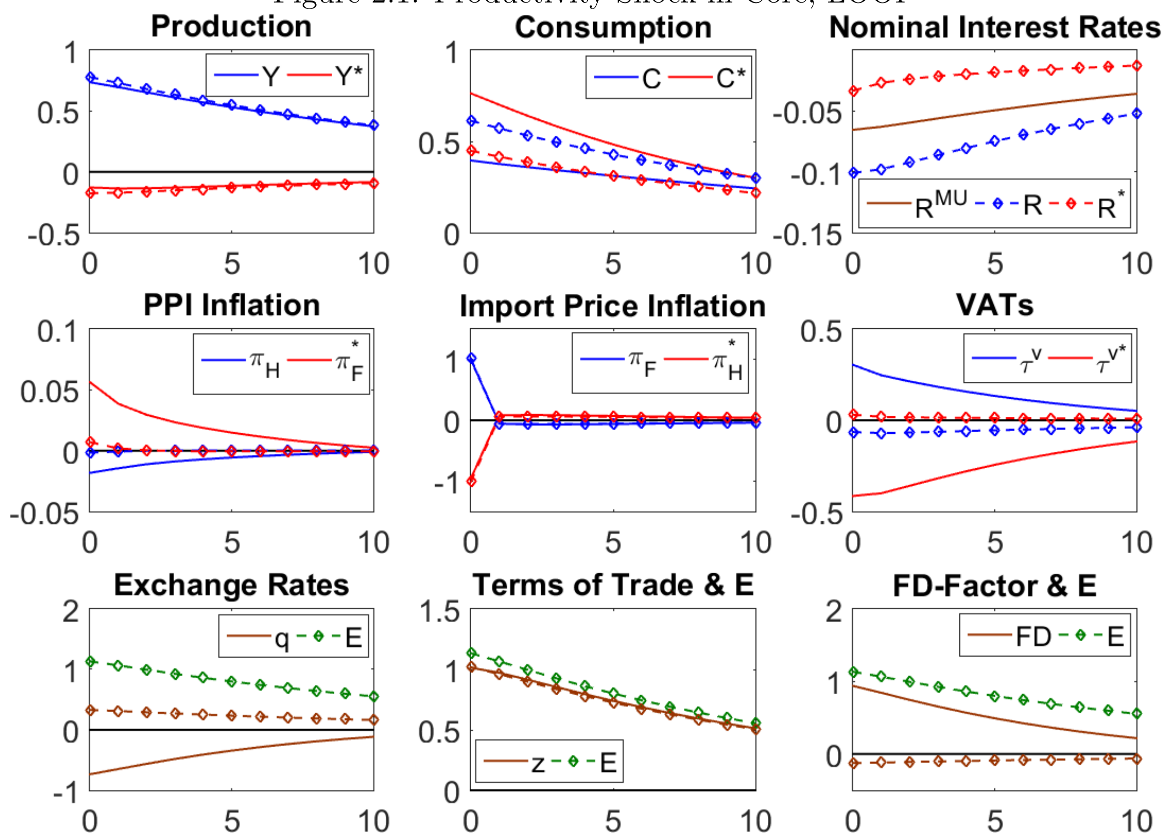
This section describes the conduct of optimal policy and shows how taxes should be used to substitute for the nominal exchange rate inside the euro area. Optimal fiscal policy in the monetary union depicts a fiscal devaluation policy: in case it would be optimal to devalue the exchange rate of a region, it is optimal to increase its VAT relative to the other region of the monetary union.

Dynamic Response to a Productivity Shock

To gain intuition for the findings of the welfare analysis, Figure 2.1 compares the impulse response to a 1% productivity shock in the core under LOOP in the monetary union (solid lines) with the counterfactual response under flexible exchange rates (dashed diamond lines). As shown by Corsetti, Dedola and Leduc (2010), the latter case constitutes the benchmark of "divine coincidence" in open economies, where stabilizing PPI inflation by monetary policy in both regions is sufficient to obtain the efficient allocation in presence of the shock.

The increase of productivity implies that it is efficient to produce a larger share of world output in the core. Y_t increases strongly, while Y_t^* declines on impact (Panel 1). To induce the required expenditure-switching towards core goods, the terms of trade of the core have to deteriorate (i.e. z_t has to increase). According to (2.40), this can be achieved by changes of the PPIs (P_{Ht},P_{Ft}^*), by nominal or by fiscal devaluation. As long

Figure 2.1: Productivity Shock in Core, LOOP



Notes: Comparison of impulse responses to 1% productivity shock in the core under the law of one price. Solid lines: monetary union. Dashed diamond lines: flexible exchange rate. Blue lines: core. Red lines: periphery. Unit of y-axis is % deviation from steady state (p.p. deviation in Panels 3 & 6). X-axis indicates quarters after impulse.

as exchange rates are flexible (dashed diamond lines), this shift in the terms of trade is generated by the nominal exchange rate due to its feature of immediate pass-through under LOOP (see Panel 8), while PPI inflation rates are kept constant to avoid welfare-reducing price dispersion among goods (Panel 4). The adjustment of the exchange rate leads to strong effects on the prices of imports (Panel 5). As imports behave as under flexible prices, inflation in that sector does not have to be minimized to avoid welfare losses. The VAT rates are basically unused under flexible E_t (Panels 6 and 9) since the efficient response to the shock can in this case be brought about by monetary policy alone.

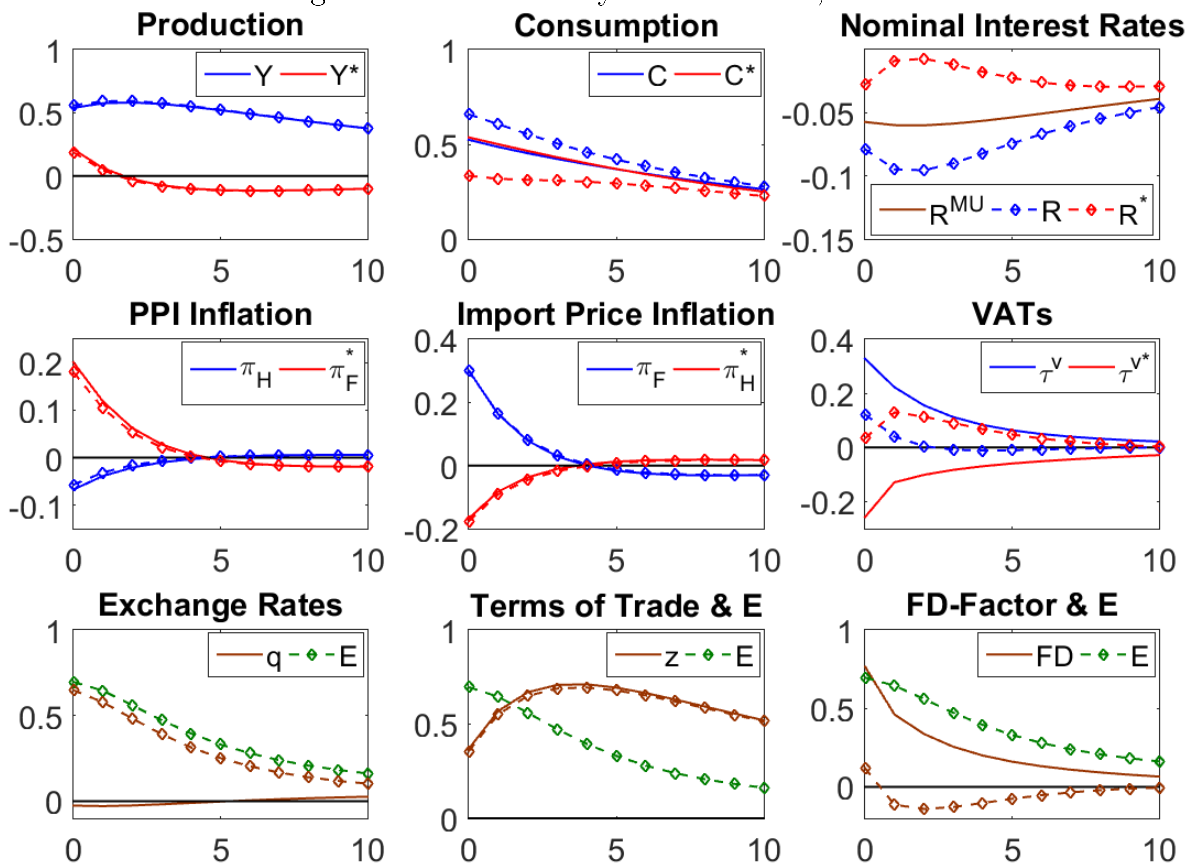
These dynamics change altogether in the monetary union (solid lines). Monetary policy on its own is not able to reach the efficient response any more. The reaction of the nominal interest rate is now in between the responses of core and periphery under flexible exchange rates, which implies a rate too low for the periphery and rate too high for the core (Panel 3). As a consequence, deviations of PPI inflation from steady state are now slightly larger than under exchange rate flexibility. The reaction is somewhat stronger in the periphery due to its relatively lower weight in the welfare function of the Ramsey planner. Even though E_t is now fixed, the efficient response of the terms of trade can again be reached (the brown solid and dashed lines in Panel 8 cover up each other perfectly). The way the reaction of the terms of trade is induced is completely different, though. The VAT rates are now used actively to substitute for the effect of E_t on the terms of trade. Panel 9 shows that the response of the fiscal devaluation factor, FD_t, in the monetary union is very close to the counterfactual flexible exchange rate response. On impact, 93% of the response of z_t in the monetary union are due to a fiscal devaluation policy. Only the remaining 7% are due to changes in PPIs. To implement the fiscal devaluation, the VAT of the core increases, while the VAT of the periphery decreases. Besides its effect on the terms of trade, these tax responses at the same time help to stabilize firm mark-ups. The increase of τ_t^v supports monetary policy in fighting back deflationary pressures in the core that arise from the increased productivity, while the decrease of τ_t^{v*} reduces inflationary pressures in the periphery, which are the result of the relatively loose monetary policy for that region.

Under the free floating regime, the real exchange rate q_t depreciates because the devaluation of the core's currency dominates the relative increase of the core's CPI (Panel 7). In the monetary union, the real exchange rate appreciates instead. The fiscal devaluation also increases the CPI of the core relative to the periphery by making core imports more expensive, but the relative currency value between the regions now remains fixed. As a result, in case of the monetary union consumption in the periphery increases by more than in the core due to international risk sharing (Panel 2).

Taken together, the optimal fiscal devaluation policy focusses on replicating the behaviour of the terms of trade under flexible exchange rates to induce exenditure switching effects, but it does not reproduce the response of the real exchange rate that affects levels of consumption via the international risk sharing condition (2.15). The policymaker thereby favours production efficiency over an efficient allocation of aggregate consumption in the monetary union. Adressing the latter would require an additional instrument to affect the real exchange rate. Farhi et al. (2014) show that a consumption subsidy payable to households could succeed to that task. They also prove that a complete replication of the allocation under flexible exchange rates lacks even further instruments. A payroll subsidy to firms would be needed to stabilize internal prices of domestically produced goods, which are distorted by the VAT, while a labour income tax levied household would be required to neutralize distortions by the consumption subsidy on wages.[14]

[14] Depending on the specific model setting, still more instruments may be required. See also Adao et

Figure 2.2: Productivity Shock in Core, PTM



Notes: Comparison of impulse responses to 1% productivity shock in the core under pricing-to-market. Solid lines: monetary union. Dashed diamond lines: flexible exchange rate. Blue lines: core. Red lines: periphery. Unit of y-axis is % deviation from steady state (p.p. deviation in Panels 3 & 6). X-axis indicates quarters after impulse.

Figure 2.2 compares impulse responses of the monetary union with the flexible exchange rate scenario for the case of pricing-to-market. Engel (2011) shows for the case of flexible exchange rates and PTM that CPI inflation (as the weighted average of PPI and import price inflation) instead of PPI inflation only ought to be stabilized, since the import sector is now also subject to a sticky price friction. However, avoiding inflation and closing output gaps is not sufficient to obtain the efficient allocation, because deviations of the terms of trade from their efficient level and deviations from the law of one price can still occur. These wrong price signals translate into inefficient shifts in the level and composition of consumption between the regions. Accordingly, in opposition to the LOOP case, the response to the productivity shock under flexible exchange rates and PTM does not reach the efficient allocation.

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al. (2009) on that point.

Beginning the description with the case of flexible exchange rates again (dashed diamond lines), PPI inflation is more pronounced under PTM, but import inflation is decisively weaker, leading to a terms of trade deterioration which falls short of its efficient response. Under PTM, z_t rises by 0.36% in the first quarter, while the efficient response under the LOOP renders 1%. Expenditure switching from periphery to core is, therefore, not sufficient. The reason for the dampened reaction of the terms of trade is that exchange rate pass-through on international prices is now limited by the sticky price friction in the import sector, visible in the low co-movement between z_t and E_t (Panel 8). Policymakers generate a weaker devaluation of the nominal exchange rate under PTM (0.7% on impact) than under LOOP (1.1% on impact), for they now have to trade off the costs of additional import price dispersion against the benefits of deteriorated terms of trade due to higher import prices P_{Ft}. Taxes have a comparable effect on prices as monetary policy (confer equations 2.22 and 2.26 for the perspective of the core). Increasing the domestic VAT, τ_t^v, dampens the deflationary pressure on the core's PPI, but incentivizes higher import prices also. As long as exchange rate flexibility is given, taxes are used only to a limited extent for stabilization purposes.

In the monetary union (solid lines), the response of the terms of trade under flexible exchange rates can again be replicated entirely by fiscal policy. The VAT rates are used to induce the same price setting behaviour as the flexible exchange rate would. Relative VAT rates, i.e. the fiscal devaluation factor, are highly correlated with the counterfactual exchange rate (Panel 9), in order to shift relative prices and to reduce deviations from the law of one price. On impact, FD_t even overshoots the response of E_t by 7%. The pass-through of these tax changes on prices and the terms of trade remains, however, limited again.

Fiscal policy in the monetary union is successful in replicating the path of the terms of trade under flexible exchange rates, but again the fiscal devaluation policy does not keep track of the respective real exchange rate path. Since CPIs are implicitly stabilized under PTM, q_t barely moves in the monetary union (Panel 7), leading to almost perfectly correlated reactions of consumption in the core and periphery because of international risk sharing. Under flexible exchange rates instead, the real exchange rate follows the depreciation of E_t closely. Consumption in the core, hence, increases by more than in the periphery in this case.

Business-Cycle Properties of the Ramsey Allocations

In this section, I show to what extent the findings and intuitions obtained under the productivity shock generalize to the other shocks as well. To do so, I analyse second moments of key variables, generated from simulated business cycle data.

Table 2.4 presents correlations between the counterfactual flexible exchange rate and various tax measures in the monetary union for both types of price setting and different types of shocks. Correlations are calculated with the fiscal devaluation factor, FD_t, and with the tax rates in levels, τ_t^v and τ_t^{v*}.

Table 2.4: Correlations between Exchange Rates and Taxes

LOOP	Benchmark	No Mark-up Shocks	Mark-up, Core	Mark-up, Periphery
$Corr(E_t, FD_t)$	0.81	0.89	0.52	0.52
$Corr(E_t, \tau_t^v)$	0.03	0.81	0.43	-0.49
$Corr(E_t, \tau_t^{v*})$	-0.41	-0.91	0.40	-0.50
PTM				
$Corr(E_t, FD_t)$	0.59	0.89	0.86	0.65
$Corr(E_t, \tau_t^v)$	0.11	0.88	0.90	-0.73
$Corr(E_t, \tau_t^{v*})$	-0.31	-0.86	0.93	-0.69

Notes: Correlations between tax measures obtained in monetary union scenario and counterfactual flexible exchange rate. Columns indicate shock processes used for simulation: 'Benchmark' includes productivity, demand preference, government spending, & mark-up shocks in both countries. 'No Mark-up' includes all but mark-up shocks. Last two columns include mark-up shocks in the respective region only. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

The correlation between FD_t and E_t is generally found to be high. In the benchmark scenario with all shocks, it reads 81% when the LOOP holds and 59% under PTM. It is even higher at 89% for both pricing schemes when looking at the productivity, demand preference, and government spending shocks, and it ranges between 52% and 86% for the mark-up shocks. These results indicate that the policymaker actively uses fiscal policy to replicate the path of the terms of trade in absence of a flexible exchange rate.

The results regarding tax rates in levels do not allow for general conclusions in the benchmark scenario, both under the LOOP and PTM.[15] A more detailed inspection reveals that the correlations—and, hence, the exact conduct of tax policy—depend decisively on the type of shocks. With the productivity, demand preference, and government spending shocks, the correlations with taxes under the LOOP (PTM) read 81% (88%) and -91% (-86%), respectively. Whenever it were optimal to devalue the exchange rate of a region in the monetary union, its VAT ought to be increased, while the tax of the

[15]The asymmetry between the correlations of τ_t^v and τ_t^{v*} is mainly driven by the different shock sizes in core and periphery.

other (re-valuing) region should decrease. Fiscal devaluation policies as outlined in the introduction can accordingly be observed, independent of the type of price setting.

Under mark-up shocks, the tax responses additionally depend on the origin of the shock. The VAT rates of both regions are now positively correlated with the exchange rate of that region which experiences a mark-up shock.[16] In response to a positive mark-up shock, e.g., in the core, it is efficient to shift production to the periphery, which requires an appreciated exchange rate (i.e. a decline of E_t) for the core. The optimal response in a monetary union is to decrease taxes in both regions. Under the LOOP, this policy attenuates the higher mark-up in the core and fosters the expenditure-switch by reducing prices for periphery goods, while still taking heed of solvency of the fiscal authority. In order to achieve a decline of FD_t nevertheless, the VAT of the periphery should decline by less than its core counterpart. Under PTM, it is clear from (2.23) that a rise in τ_t^{v*} aimed at replicating the decline in E_t, would even exacerbate the mark-up distortion for the periphery's import goods. τ_t^{v*}, therefore, also declines instead, which explains the positive correlation of τ_t^{v*} with E_t.

Altogether, policymakers in the monetary union always adjust the ratio of tax rates between the regions to induce relative price shifts in a similar fashion as the exchange rate would. The behaviour of tax rates in levels and their correlation with the exchange rate crucially depends on the type of shocks.

Table 2.5 compares standard deviations of international relative prices and of taxes in the monetary union and the flexible exchange rate regime, for both types of price setting and different shock compositions. The following observations stand out.

In all scenarios, standard deviations of the terms of trade, z_t, in the monetary union are found to be close to their counterpart under flexible exchange rates, e.g. 1.69% versus 1.68% in the benchmark with LOOP. The volatility of real exchange rates, instead, differs markedly between MU and FLEX. This indicates a generalization of the finding, obtained from the productivity shock, that optimal fiscal devaluation policies focus on replicating the time path of the terms of trade, but not of the real exchange rate.

Confirming Engel's (2011) result, nominal exchange rate volatility is in all panels found to be lower under PTM than under the LOOP, at least by a factor of two.

Also in line with the results obtained from the analysis of the productivity shock, volatilities of the fiscal devaluation factor are smaller under flexible exchange rates than in the monetary union in all scenarios. In the benchmark (Panel A), the volatility increases from 0.26% to 1.19% when the LOOP holds and from 1.33% to 1.67% under PTM. The

[16]Note that E_t denotes the exchange rate from the perspective of the core. Correlations are, therefore, negative for a mark-up shock in the periphery.

Table 2.5: Standard Deviations over the Business-Cycle

(A) Benchmark						
LOOP	q_t	z_t	E_t	FD_t	τ_t^v	τ_t^{v*}
Monetary Union (MU)	0.89	1.69		1.19	1.46	1.73
Flexible Exchange Rate (FLEX)	0.49	1.68	1.84	0.26	1.52	1.53
PTM						
Monetary Union	0.11	1.29		1.67	1.43	1.95
Flexible Exchange Rate	0.69	1.27	0.81	1.33	1.43	1.87
(B) Productivity, Preference, Gov. Spending Shocks						
LOOP						
Monetary Union	0.75	1.44		1.00	0.32	0.46
Flexible Exchange Rate	0.46	1.42	1.58	0.17	0.11	0.09
PTM						
Monetary Union	0.10	1.21		0.56	0.26	0.17
Flexible Exchange Rate	0.63	1.19	0.77	0.15	0.09	0.13
(C) Mark-up Shocks						
LOOP						
Monetary Union	0.47	0.87		0.64	1.43	1.67
Flexible Exchange Rate	0.14	0.88	0.89	0.19	1.52	1.53
PTM						
Monetary Union	0.03	0.39		1.57	1.41	1.94
Flexible Exchange Rate	0.28	0.39	0.39	1.33	1.42	1.86

Notes: Standard deviations are measured in percentage points. Exchange rate regime either monetary union (MU) or flexible (FLEX). Panel (A): productivity, demand preference, government spending, & mark-up shocks in both countries. Panel (B): all but mark-up shocks. Panel (C): mark-up shocks only. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

volatility of the tax rates itself is found to be of similar size in the MU as well as the FLEX scenario in Panel (A). The decomposition into the different shock types in Panel (B) and (C) reveals that this is primarily driven by the mark-up shocks, for the latter require an active fiscal policy response even under flexible exchange rates. In case of the productivity, demand preference, and government spending shocks, the intuition, obtained from the impulse responses, is restored that taxes are used only mildly under flexible exchange rates, but intensely in the monetary union.

The volatility of tax rates is of the same order of magnitude as the volatility of E_t. In the benchmark of panel (A), this implies that taxes on average do not have to fluctuate more than about 2 percentage points for an optimal policy response to the business cycle—

thereby rendering fiscal devaluations as a practically implementable policy option.[17]

2.6 Conclusion

This chapter analyses to what extent fiscal policy can compensate for the absence of nominal exchange rate adjustment in a monetary union in terms of business cycle stabilization. Various Ramsey-optimal policy scenarios are studied in a New Keynesian 2-region model, calibrated to the euro area, that differ regarding the exchange rate regime and the availability of fiscal policy for stabilization purposes. Optimal use of only one tax instrument per country enables policymakers to reduce the welfare costs of giving up flexible exchange rates in a monetary union by up to 86% when the law of one price holds for traded goods, and up to 69% when different prices can be set for the regions. Fiscal devaluations arise as an outcome of optimal fiscal policy. Whenever a nominal exchange rate devaluation were optimal for a region, a relative increase of the region's VAT is the optimal fiscal policy in the monetary union. In particular in case of mark-up shocks, policymakers face a trade-off between replicating the effects of the nominal exchange rate and stabilizing firms' costs, however. Optimal fiscal policy in the monetary union focusses on the reproduction of the flexible exchange rate path of the terms of trade, but not of the real exchange rate. The policymaker thereby favours production efficiency over an efficient allocation of aggregate consumption in the monetary union.

A practical challenge is that fiscal policy, as considered in the model, requires tax changes at a business cycle frequency, whose implementation surely poses political economy issues. However, first steps in direction of a unified VAT framework for all member states of the European Union are already taken that will facilitate a higher degree of coordination in fiscal policy in future.[18]

The analysis of optimal policy studies the relevant benchmark of full cooperation between the central bank(s) and the fiscal authorities at the region level. It remains open to future research to analyse the strategic interactions between these different authorities,

[17] Naturally, the standard deviations of both exchange rates and taxes increase with the size of the underlying shocks. The seemingly small volatility of E_t found in the simulations, nevertheless, does not need to be entirely unrealistic. The model provides an optimal policy response that reacts to changes in fundamentals only, compared to actual exchange rate data, which notoriously entails a sizeable amount of unexplainable volatility. Regarding this point, see also the vast literature on the "exchange rate disconnect" puzzle following Obstfeld and Rogoff (2001).

[18] See in particular European Council Directive 2006/112/EC, which lays down a common system of value added tax regulation for the EU. It covers aspects such as the tax base, the allowed number of reduced tax rates besides the standard rate, and also defines which types of goods are eligible for exemptions. It further regulates which country's rate applies to imported goods, and even directs upper and lower bounds for tax rates.

e.g., in form of a dynamic Nash game.

The chapter focuses on VAT-based fiscal devaluation policies. Further research could also study the optimality of more general fiscal devaluation policies in the sense of tax swaps from direct to indirect taxation (e.g. an increase in the VAT, paired with a reduction of payroll taxes of employers), which can be revenue neutral to public budgets. An analysis of such policies is, however, impeded in the present class of models due to an indeterminacy between consumption and income taxes. The inclusion of another, untaxed, production factor could possibly remedy this issue.

Appendix 2.A Derivations for Section 2.2

2.A.1 Optimal Firm Price Setting

In order to derive the conditions for optimal price setting, one first needs to derive an aggregate demand equation $Y_t(k)$ for firm k. Consider (2.17), which can be rewritten as

$$\begin{aligned} Y_t(k) &= nC_{Ht}(k) + (1-n)C^*_{Ht}(k) + G_t(k) \\ &= \frac{1}{n}\left(\frac{P_{Ht}(k)}{P_{Ht}}\right)^{-\rho}(nC_{Ht}+G_t) + \frac{(1-n)}{n}\left(\frac{P^*_{Ht}(k)}{P^*_{Ht}}\right)^{-\rho} C^*_{Ht}. \end{aligned} \tag{2.44}$$

In the first line, I integrated over households using the fact that agents within a country behave identically. In the second line, I applied consumption demand functions (2.8) of both households and the domestic government.

Law of One Price: Using the law of one price (2.20) and the fact that the law also holds for price indices under the given structure, (2.44) reduces to

$$Y_t(k) = \frac{1}{n}\left(\frac{P_{Ht}(k)}{P_{Ht}}\right)^{-\rho} Y_t, \tag{2.45}$$

where $Y_t = nC_{Ht} + (1-n)C^*_{Ht} + G_t$. By means of the law of one price again and (2.45), firm profits (2.18) change to

$$\Pi_t(k) = (1-\tau^v_t)\, P_{Ht}(k)\frac{1}{n}\left(\frac{P_{Ht}(k)}{P_{Ht}}\right)^{-\rho} Y_t - W_t N_t(k). \tag{2.46}$$

The optimal price $\overline{P}_{Ht}(k)$ is then determined by maximizing the expected present discounted value of profits subject to the production technology (2.16) and demand (2.45):

$$\begin{aligned} \max_{\overline{P}_{Ht}(k),N_s(k)} \mathbb{L}^{LOOP} = \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{Hs} \Bigg\{ &\left[(1-\tau^v_s)\frac{1}{n}\left(\frac{\overline{P}_{Ht}(k)}{P_{Hs}}\right)^{1-\rho} Y_s - w_s N_s(k)\right] \\ &+ mc_{Hs}(k)\left[A_s N_s(k)^{\alpha} - \frac{1}{n}\left(\frac{\overline{P}_{Ht}(k)}{P_{Hs}}\right)^{-\rho} Y_s\right]\Bigg\}, \end{aligned}$$

where $w_t = W_t/P_{Ht}$ is the producer real wage. The associated first-order conditions are:

$$\begin{aligned}\frac{\partial \mathbb{L}^{LOOP}}{\partial \overline{P}_{Ht}(k)} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\overline{P}_{Ht}(k)}{P_{Hs}} \right)^{-1-\rho} Y_s \left\{ (1-\tau_s^v)(1-\rho) \frac{\overline{P}_{Ht}(k)}{P_{Hs}} + mc_{Hs}(k)\rho \right\} = 0, \\ \frac{\partial \mathbb{L}^{LOOP}}{\partial N_s(k)} &= \mathbb{E}_t \theta^{s-t} Q_{t,s} P_{Hs} \left[-w_s + mc_{Hs}(k) \alpha A_s N_s(k)^{\alpha-1} \right] = 0.\end{aligned}$$

Combining the two conditions and rearranging the result yields the optimal pricing condition under LOOP (2.21).

Pricing-to-Market: The price setting problem of the firm under PTM implies maximizing profits (2.18) subject to (2.16) and (2.44):

$$\begin{aligned}&\max_{\overline{P}_{Ht}(k), \overline{P}^*_{Ht}(k), N_s(k)} \mathbb{L}^{PTM} = \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left\{ (1-\tau_s^v) \overline{P}_{Ht}(k) \frac{1}{n} \left(\frac{\overline{P}_{Ht}(k)}{P_{Hs}} \right)^{-\rho} [nC_{Hs} + G_s] \right. \\ &+ (1-\tau_s^{v*}) E_s \overline{P}^*_{Ht}(k) \frac{1}{n} \left(\frac{\overline{P}^*_{Ht}(k)}{P^*_{Hs}} \right)^{-\rho} (1-n) C^*_{Hs} - W_s N_s(k) \\ &\left. + MC_{Hs}(k) \left[A_s N_s(k)^{\alpha} - \frac{1}{n} \left(\frac{\overline{P}_{Ht}(k)(k)}{P_{Hs}} \right)^{-\rho} [nC_{Hs} + G_s] - \frac{(1-n)}{n} \left(\frac{\overline{P}^*_{Ht}(k)}{P^*_{Hs}} \right)^{-\rho} C^*_{Hs} \right] \right\}.\end{aligned}$$

The associated first-order conditions are:

$$\begin{aligned}\frac{\partial \mathbb{L}^{PTM}}{\partial \overline{P}_{Ht}(k)} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\overline{P}_{Ht}(k)}{P_{Hs}} \right)^{-1-\rho} [nC_{Hs} + G_s] \\ &\quad \cdot \left\{ (1-\tau_s^v) \frac{\overline{P}_{Ht}(k)}{P_{Hs}} - \frac{\rho}{\rho-1} \frac{MC_{Hs}(k)}{P_{Hs}} \right\} = 0, \\ \frac{\partial \mathbb{L}^{PTM}}{\partial \overline{P}^*_{Ht}(k)} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\overline{P}^*_{Ht}(k)}{P^*_{Hs}} \right)^{-1-\rho} C^*_{Hs} \\ &\quad \cdot \left\{ (1-\tau_s^{v*}) E_s \left(\frac{\overline{P}^*_{Ht}(k)}{P^*_{Hs}} \right) - \frac{\rho}{\rho-1} \frac{MC_{Hs}(k)}{P^*_{Hs}} \right\} = 0, \\ \frac{\partial \mathbb{L}^{PTM}}{\partial N_s(k)} &= \mathbb{E}_t \theta^{s-t} Q_{t,s} \left[-W_s + MC_{Hs}(k) A_s \alpha N_s(k)^{\alpha-1} \right] = 0.\end{aligned}$$

Combining the conditions and rearranging the results yields the optimal pricing conditions under PTM, (2.22) and (2.23).

2.A.2 Evolution of Price Indices

Price index (2.5) can be written as

$$
\begin{aligned}
& nP_{Ht}^{1-\rho} = \int_0^{n\theta} P_{Ht-1}^{1-\rho}(k)\,dk + \int_{n\theta}^{n} \overline{P}_{Ht}^{1-\rho}(k)\,dk \\
\Leftrightarrow\ & nP_{Ht}^{1-\rho} = n\theta P_{Ht-1}^{1-\rho} + n(1-\theta)\,\overline{P}_{Ht}^{1-\rho} \\
\Leftrightarrow\ & 1 = \theta\pi_{Ht}^{\rho-1} + (1-\theta)\left(\frac{\overline{P}_{Ht}}{P_{Ht}}\right)^{1-\rho} \\
\Leftrightarrow\ & \widetilde{p}_{Ht} = \frac{\overline{P}_{Ht}}{P_{Ht}} = \left(\frac{1-\theta\pi_{Ht}^{\rho-1}}{1-\theta}\right)^{\frac{1}{1-\rho}},
\end{aligned}
$$

which is (2.30) in the main text. Similar expressions hold for P_{Ft}^{*} and, under PTM, also for P_{Ft} and P_{Ht}^{*}.

2.A.3 Recursive Phillips Curves

The recursive form of a Phillips curve is derived here by way of example for the PPI of home goods under the LOOP. The optimal pricing condition (2.21) can be written as

$$
\begin{aligned}
\frac{\rho}{\rho-1}\mu_s\mathbb{E}_t\sum_{s=t}^{\infty}\theta^{s-t}Q_{t,s}\left(\frac{\overline{P}_{Ht}}{P_{Ht}}\right)^{-1-\rho}Y_s mc_{Hs} &= \mathbb{E}_t\sum_{s=t}^{\infty}\theta^{s-t}Q_{t,s}\left(\frac{\overline{P}_{Ht}}{P_{Ht}}\right)^{-\rho}Y_s\left(1-\tau_s^{v}\right) \\
\frac{\rho}{\rho-1}\mu_t X1_{Ht} &= X2_{Ht}
\end{aligned}
$$

with

$$
\begin{aligned}
X1_{Ht} &= \mathbb{E}_t\sum_{s=t}^{\infty}\theta^{s-t}Q_{t,s}Y_s\left(\frac{\overline{P}_{Ht}}{P_{Hs}}\right)^{-1-\rho}mc_{Hs} \\
&= \left(\frac{\overline{P}_{Ht}}{P_{Ht}}\right)^{-1-\rho}Y_t mc_{Ht} + \mathbb{E}_t\sum_{s=t+1}^{\infty}\theta^{s-t}Q_{t,s}\left(\frac{\overline{P}_{Ht}}{P_{Hs}}\right)^{-1-\rho}Y_s mc_{Hs} \\
&= \left(\frac{\overline{P}_{Ht}}{P_{Ht}}\right)^{-1-\rho}Y_t mc_{Ht} \\
&\quad +\theta\mathbb{E}_t Q_{t,t+1}\left(\frac{\overline{P}_{Ht}}{\overline{P}_{Ht+1}}\right)^{-1-\rho}\mathbb{E}_{t+1}\sum_{s=t+1}^{\infty}\theta^{s-t-1}Q_{t+1,s}\left(\frac{\overline{P}_{Ht+1}}{P_{Hs}}\right)^{-1-\rho}Y_s mc_{Hs}
\end{aligned}
$$

$$= \left(\frac{\overline{P}_{Ht}}{P_{Ht}}\right)^{-1-\rho} Y_t mc_{Ht} + \theta \mathbb{E}_t Q_{t,t+1} \left(\frac{\overline{P}_{Ht}}{\overline{P}_{Ht+1}}\right)^{-1-\rho} X1_{Ht+1}$$

and

$$\begin{aligned} X2_{Ht} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} Y_s \left(\frac{\overline{P}_{Ht}}{P_{Hs}}\right)^{-\rho} (1-\tau_s^v) \\ &= \left(\frac{\overline{P}_{Ht}}{P_{Ht}}\right)^{-\rho} Y_t (1-\tau_t^v) + \theta \mathbb{E}_t Q_{t,t+1} \left(\frac{\overline{P}_{Ht}}{\overline{P}_{Ht+1}}\right)^{-\rho} X2_{Ht+1}. \end{aligned}$$

Inserting the definition of the stochastic discount factor (2.11) and the law of motion of the PPI (2.30) yields equation (2.32) and (2.33) in the text.

Corresponding expressions for PPIs and import price indices under PTM can be derived accordingly from (2.22), (2.23), (2.25), and (2.26).

2.A.4 Aggregate Resource Constraint

To derive the aggregate resource constraints, combine production (2.16) with demand (2.44), and integrate over firms:

$$\begin{aligned} A_t N_t^{\alpha}(k) &= \frac{1}{n}\left(\frac{P_{Ht}(k)}{P_{Ht}}\right)^{-\rho} (nC_{Ht} + G_t) + \frac{1}{n}\left(\frac{P_{Ht}^*(k)}{P_{Ht}^*}\right)^{-\rho} (1-n) C_{Ht}^* \\ A_t \int_0^n N_t^{\alpha}(k)\, dk &= \frac{1}{n}\int_0^n \left(\frac{P_{Ht}(k)}{P_{Ht}}\right)^{-\rho} dk\, (nC_{Ht} + G_t) \\ &\quad + \frac{1}{n}\int_0^n \left(\frac{P_{Ht}^*(k)}{P_{Ht}^*}\right)^{-\rho} dk\, (1-n) C_{Ht}^*. \end{aligned}$$

As $(P_{Ht}(k)/P_{Ht}) = (P_{Ht}^*(k)/P_{Ht}^*)$ if the law of one price holds, this reduces to (2.34) under LOOP, but to (2.35) under PTM.

The law of motion for price dispersion emerges from (2.36) as follows:

$$\begin{aligned} \Delta_{Ht} &= \frac{1}{n}\int_0^n \left(\frac{P_{Ht}(k)}{P_{Ht}}\right)^{-\rho} dk \\ &= \frac{1}{n}\left[n(1-\theta)\left(\frac{\overline{P}_{Ht}}{P_{Ht}}\right)^{-\rho} + n(1-\theta)\theta \left(\frac{\overline{P}_{Ht-1}}{P_{Ht}}\right)^{-\rho} + \ldots \right] \end{aligned}$$

$$
\begin{aligned}
&= (1-\theta)\sum_{j=0}^{\infty}\theta^{j}\left(\frac{\overline{P}_{Ht-j}}{P_{Ht}}\right)^{-\rho} \\
&= (1-\theta)\left(\frac{\overline{P}_{Ht}}{P_{Ht}}\right)^{-\rho} + (1-\theta)\sum_{j=1}^{\infty}\theta^{j}\left(\frac{\overline{P}_{Ht-j}}{P_{Ht}}\right)^{-\rho} \\
&= (1-\theta)\left(\frac{\overline{P}_{Ht}}{P_{Ht}}\right)^{-\rho} + \theta\left(\frac{P_{Ht-1}}{P_{Ht}}\right)^{-\rho}(1-\theta)\sum_{j=1}^{\infty}\theta^{j-1}\left(\frac{\overline{P}_{Ht-j}}{P_{Ht-1}}\right)^{-\rho} \\
&= (1-\theta)\,\widetilde{p}_{Ht}^{-\rho} + \theta\pi_{Ht}^{\rho}\Delta_{Ht-1}.
\end{aligned}
$$

Appendix 2.B Competitive Equilibrium

This appendix lists equilibrium conditions for the cases of LOOP and PTM. All prices are expressed in relative terms.

2.B.1 Law of One Price

Let $p_{Ht} = P_{Ht}/P_t$ and $p^*_{Ft} = P^*_{Ft}/P^*_t$ be the PPI-CPI ratios, and $w_t = W_t/P_{Ht}$ and $w^*_t = W^*_t/P^*_{Ft}$ the producer real wages. A competitive equilibrium under the LOOP and autonomous monetary policy in both countries is a set of sequences $\{C_t,\, C_{Ht},\, C_{Ft},\, C^*_t,\, C^*_{Ht},\, C^*_{Ft},\, Y_t,\, Y^*_t,\, N_t,\, N^*_t,\, q_t,\, p_{Ht},\, p^*_{Ft},\, w_t,\, w^*_t,\, \pi_t,\, \pi^*_t,\, \pi_{Ht},\, \pi^*_{Ft},\, \Delta_{Ht},\, \Delta^*_{Ft},\, \widetilde{p}_{Ht},\, \widetilde{p}^*_{Ft},\, X1_{Ht},\, X2_{Ht},\, X1^*_{Ft},\, X2^*_{Ft}\}_{t=0}^{\infty}$, satisfying

- Demand for Home and Foreign goods:

$$
\begin{aligned}
C_{Ht} &= \gamma_H p_{Ht}^{-\xi} C_t\ , \qquad C_{Ft} = \gamma_F\left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^{v})}p^*_{Ft}q_t\right)^{-\xi} C_t \\
C^*_{Ht} &= \gamma^*_H\left(\frac{(1-\tau_t^{v})}{(1-\tau_t^{v*})}\frac{p_{Ht}}{q_t}\right)^{-\xi} C^*_t\ , \qquad C^*_{Ft} = \gamma^*_F p^{*-\xi}_{Ft} C^*_t
\end{aligned}
$$

- Euler equations and international risk sharing:

$$
\begin{aligned}
\frac{1}{R_t} &= \beta\mathbb{E}_t\left[\frac{\zeta^c_{t+1}}{\zeta^c_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{\pi_{t+1}}\right] \\
\frac{1}{R^*_t} &= \beta\mathbb{E}_t\left[\frac{\zeta^{c*}_{t+1}}{\zeta^{c*}_t}\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\frac{1}{\pi^*_{t+1}}\right] \\
q_t &= \kappa\frac{\zeta^{c*}_t}{\zeta^c_t}\left(\frac{C^*_t}{C_t}\right)^{-\sigma}
\end{aligned}
$$

- Labour supply:

$$
\begin{aligned}
N_t^{\eta} C_t^{\sigma} &= \zeta_t^c w_t p_{Ht} \\
N_t^{*\eta} C_t^{*\sigma} &= \zeta_t^{c*} w_t^* p_{Ft}^*
\end{aligned}
$$

- Aggregate demand:

$$
\begin{aligned}
Y_t &= nC_{Ht} + (1-n)C_{Ht}^* + G_t \\
Y_t^* &= nC_{Ft} + (1-n)C_{Ft}^* + G_t^*
\end{aligned}
$$

- Resource constraints:

$$
\begin{aligned}
A_t n N_t^{\alpha} &= \Delta_{Ht} Y_t \\
A_t^* (1-n) N_t^{*\alpha} &= \Delta_{Ft}^* Y_t^*
\end{aligned}
$$

- Phillips curves:

$$
\frac{\rho}{\rho-1} \mu_t X1_{Ht} = X2_{Ht},
$$

$$
\begin{aligned}
X1_{Ht} &= \widetilde{p}_{Ht}^{-1-\rho} Y_t \frac{w_t}{A_t \alpha N_t^{\alpha-1}} + \theta\beta\mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{\pi_{Ht+1}^{1+\rho}}{\pi_{t+1}} \left(\frac{\widetilde{p}_{Ht}}{\widetilde{p}_{Ht+1}}\right)^{-1-\rho} X1_{Ht+1} \\
X2_{Ht} &= \widetilde{p}_{Ht}^{-\rho} Y_t (1-\tau_t^v) + \theta\beta\mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{\pi_{Ht+1}^{\rho}}{\pi_{t+1}} \left(\frac{\widetilde{p}_{Ht}}{\widetilde{p}_{Ht+1}}\right)^{-\rho} X2_{Ht+1}
\end{aligned}
$$

$$
\frac{\rho^*}{\rho^*-1} \mu_t^* X1_{Ft}^* = X2_{Ft}^*
$$

$$
\begin{aligned}
X1_{Ft}^* &= \widetilde{p}_{Ft}^{*-1-\rho^*} Y_t^* \frac{w_t^*}{A_t^* \alpha N_t^{*\alpha-1}} + \theta^*\beta\mathbb{E}_t \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{\pi_{Ft+1}^{*1+\rho^*}}{\pi_{t+1}^*} \left(\frac{\widetilde{p}_{Ft}^*}{\widetilde{p}_{Ft+1}^*}\right)^{-1-\rho^*} X1_{Ft+1}^* \\
X2_{Ft}^* &= \widetilde{p}_{Ft}^{*-\rho^*} Y_t^* (1-\tau_t^{v*}) + \theta^*\beta\mathbb{E}_t \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{\pi_{Ft+1}^{*\rho^*}}{\pi_{t+1}^*} \left(\frac{\widetilde{p}_{Ft}^*}{\widetilde{p}_{Ft+1}^*}\right)^{-\rho^*} X2_{Ft+1}^*
\end{aligned}
$$

- Consumer price indices:

$$
1 = \gamma_H p_{Ht}^{1-\xi} + \gamma_F \left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} p_{Ft}^* q_t\right)^{1-\xi}
$$

$$1 = \gamma_H^* \left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{1-\xi} + \gamma_F^* p_{Ft}^{*1-\xi}$$

- Evolution of PPIs:

$$\begin{aligned}
\widetilde{p}_{Ht} &= \left(\frac{1-\theta\pi_{Ht}^{\rho-1}}{1-\theta} \right)^{\frac{1}{1-\rho}} \\
\widetilde{p}_{Ft}^* &= \left(\frac{1-\theta^* \left(\pi_{Ft}^*\right)^{\rho^*-1}}{1-\theta^*} \right)^{\frac{1}{1-\rho^*}}
\end{aligned}$$

- Evolution of price dispersion:

$$\begin{aligned}
\Delta_{Ht} &= (1-\theta)\, \widetilde{p}_{Ht}^{-\rho} + \theta\pi_{Ht}^{\rho}\Delta_{Ht-1} \\
\Delta_{Ft}^* &= (1-\theta^*)\, \widetilde{p}_{Ft}^{*-\rho^*} + \theta^* \left(\pi_{Ft}^*\right)^{\rho^*} \Delta_{Ft-1}^*
\end{aligned}$$

- Evolution of relative prices:

$$\frac{p_{Ht}}{p_{Ht-1}} = \frac{\pi_{Ht}}{\pi_t}, \qquad \frac{p_{Ft}^*}{p_{Ft-1}^*} = \frac{\pi_{Ft}^*}{\pi_t^*},$$

given the transversality conditions, sequences of the policy instruments $\{R_t, R_t^*, \tau_t^v, \tau_t^{v*}\}_{t=0}^{\infty}$ and of the shocks $\{A_t, A_t^*, \mu_t, \mu_t^*, \zeta_t^c, \zeta_t^{c*}, G_t, G_t^*\}_{t=0}^{\infty}$.

If the two countries form a monetary union, the equation defining R_t^* drops out. Instead, an expression that restricts the evolution of the real exchange rate needs to be added:

$$\frac{q_t}{q_{t-1}} = \frac{\pi_t^*}{\pi_t}.$$

2.B.2 Pricing-to-Market

Let $p_{Ft} = P_{Ft}/P_t$ and $p_{Ht}^* = P_{Ht}^*/P_t^*$ be the import-price-to-CPI ratios. A competitive equilibrium under PTM and autonomous monetary policy in both countries is a set of sequences $\{C_t$, C_{Ht}, C_{Ft}, C_t^*, C_{Ht}^*, C_{Ft}^*, N_t, N_t^*, q_t, E_t, p_{Ht}, p_{Ft}, p_{Ht}^*, p_{Ft}^*, w_t, w_t^*, π_t, π_t^*, π_{Ht}, π_{Ft}, π_{Ht}^*, π_{Ft}^*, Δ_{Ht}, Δ_{HF}, Δ_{Ht}^*, Δ_{Ft}^*, $X1_{Ht}$, $X2_{Ht}$, $X1_{Ft}$, $X2_{Ft}$, $X1_{Ht}^*$, $X2_{Ht}^*$, $X1_{Ft}^*$, $X2_{Ft}^*\}_{t=0}^{\infty}$, satisfying

- Demand for Home and Foreign goods:

$$\begin{aligned} C_{Ht} &= \gamma_H p_{Ht}^{-\xi} C_t \ , \qquad C_{Ft} = \gamma_F p_{Ft}^{-\xi} C_t \\ C_{Ht}^* &= \gamma_H^* p_{Ht}^{*-\xi} C_t^* \ , \qquad C_{Ft}^* = \gamma_F^* p_{Ft}^{*-\xi} C_t^* \end{aligned}$$

- Euler equations and international risk sharing:

$$\begin{aligned} \frac{1}{R_t} &= \beta \mathbb{E}_t \left[\frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \right] \\ \frac{1}{R_t^*} &= \beta \mathbb{E}_t \left[\frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{1}{\pi_{t+1}^*} \right] \\ q_t &= \kappa \frac{\zeta_t^{c*}}{\zeta_t^c} \left(\frac{C_t^*}{C_t} \right)^{-\sigma} \end{aligned}$$

- Labour supply:

$$\begin{aligned} N_t^{\eta} C_t^{\sigma} &= \zeta_t^c w_t p_{Ht} \\ N_t^{*\eta} C_t^{*\sigma} &= \zeta_t^{c*} w_t^* p_{Ft}^* \end{aligned}$$

- Resource constraints:

$$\begin{aligned} A_t n N_t^{\alpha} &= \Delta_{Ht} \left(n C_{Ht} + G_t \right) + \Delta_{Ht}^* \left(1 - n \right) C_{Ht}^* \\ A_t^* \left(1 - n \right) N_t^{*\alpha} &= \Delta_{Ft}^* \left(\left(1 - n \right) C_{Ft}^* + G_t^* \right) + \Delta_{Ft} n C_{Ft} \end{aligned}$$

- Phillips curves:

$$\frac{\rho}{\rho - 1} X1_{Ht} = X2_{Ht}$$

$$\begin{aligned} X1_{Ht} &= \left(\frac{1 - \theta \pi_{Ht}^{\rho-1}}{1 - \theta} \right)^{\frac{\rho+1}{\rho-1}} \left(n C_{Ht} + G_t \right) \frac{w_t}{A_t \alpha N_t^{\alpha-1}} \\ &\quad + \theta \beta \mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^{1+\rho}}{\pi_{t+1}} \left(\frac{1 - \theta \pi_{Ht}^{\rho-1}}{1 - \theta \pi_{Ht+1}^{\rho-1}} \right)^{\frac{\rho+1}{\rho-1}} X1_{Ht+1} \\ X2_{Ht} &= \left(\frac{1 - \theta \pi_{Ht}^{\rho-1}}{1 - \theta} \right)^{\frac{\rho}{\rho-1}} \left(n C_{Ht} + G_t \right) \left(1 - \tau_t^v \right) \end{aligned}$$

$$+\theta\beta\mathbb{E}_t\frac{\zeta_{t+1}^c}{\zeta_t^c}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\pi_{Ht+1}^{\rho}}{\pi_{t+1}}\left(\frac{1-\theta\pi_{Ht}^{\rho-1}}{1-\theta\pi_{Ht+1}^{\rho-1}}\right)^{\frac{\rho}{\rho-1}}X2_{Ht+1}$$

$$\frac{\rho}{\rho-1}X1_{Ht}^{*}=X2_{Ht}^{*}$$

$$\begin{aligned}
X1_{Ht}^{*} &= \left(\frac{1-\theta\pi_{Ht}^{*\rho-1}}{1-\theta}\right)^{\frac{\rho+1}{\rho-1}}C_{Ht}^{*}E_t\frac{p_{Ht}}{q_tp_{Ht}^{*}}\frac{w_t}{A_t\alpha N_t^{\alpha-1}} \\
&\quad +\theta\beta\mathbb{E}_t\frac{\zeta_{t+1}^c}{\zeta_t^c}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\pi_{Ht+1}^{*1+\rho}}{\pi_{t+1}}\left(\frac{1-\theta\pi_{Ht}^{*\rho-1}}{1-\theta\pi_{Ht+1}^{*\rho-1}}\right)^{\frac{\rho+1}{\rho-1}}X1_{Ht+1}^{*} \\
X2_{Ht}^{*} &= \left(\frac{1-\theta\pi_{Ht}^{*\rho-1}}{1-\theta}\right)^{\frac{\rho}{\rho-1}}C_{Ht}^{*}E_t\left(1-\tau_t^{v*}\right) \\
&\quad +\theta\beta\mathbb{E}_t\frac{\zeta_{t+1}^c}{\zeta_t^c}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\pi_{Ht+1}^{*\rho}}{\pi_{t+1}}\left(\frac{1-\theta\pi_{Ht}^{*\rho-1}}{1-\theta\pi_{Ht+1}^{*\rho-1}}\right)^{\frac{\rho}{\rho-1}}X2_{Ht+1}^{*}
\end{aligned}$$

$$\frac{\rho^*}{\rho^*-1}X1_{Ft}^{*}=X2_{Ft}^{*}$$

$$\begin{aligned}
X1_{Ft}^{*} &= \left(\frac{1-\theta^*\pi_{Ft}^{*\rho^*-1}}{1-\theta^*}\right)^{\frac{\rho^*+1}{\rho^*-1}}\left(\left(1-n\right)C_{Ft}^{*}+G_t^{*}\right)\frac{w_t^{*}}{A_t^{*}\alpha N_t^{*\alpha-1}} \\
&\quad +\theta^*\beta\mathbb{E}_t\frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}}\left(\frac{C_{t+1}^{*}}{C_t^{*}}\right)^{-\sigma}\frac{\pi_{Ft+1}^{*1+\rho^*}}{\pi_{t+1}^{*}}\left(\frac{1-\theta^*\pi_{Ft}^{*\rho^*-1}}{1-\theta^*\pi_{Ft+1}^{*\rho^*-1}}\right)^{\frac{\rho^*+1}{\rho^*-1}}X1_{Ft+1}^{*} \\
X2_{Ft}^{*} &= \left(\frac{1-\theta^*\pi_{Ft}^{*\rho^*-1}}{1-\theta^*}\right)^{\frac{\rho^*}{\rho^*-1}}\left(\left(1-n\right)C_{Ft}^{*}+G_t^{*}\right)\left(1-\tau_t^{v*}\right) \\
&\quad +\theta^*\beta\mathbb{E}_t\frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}}\left(\frac{C_{t+1}^{*}}{C_t^{*}}\right)^{-\sigma}\frac{\pi_{Ft+1}^{*\rho^*}}{\pi_{t+1}^{*}}\left(\frac{1-\theta^*\pi_{Ft}^{*\rho^*-1}}{1-\theta^*\pi_{Ft+1}^{*\rho^*-1}}\right)^{\frac{\rho^*}{\rho^*-1}}X2_{Ft+1}^{*}
\end{aligned}$$

$$\frac{\rho^*}{\rho^*-1}X1_{Ft}=X2_{Ft}$$

$$X1_{Ft} = \left(\frac{1-\theta^*\pi_{Ft}^{\rho^*-1}}{1-\theta^*}\right)^{\frac{\rho^*+1}{\rho^*-1}}\frac{C_{Ft}}{E_t}\frac{q_tp_{Ft}^{*}}{p_{Ft}}\frac{w_t^{*}}{A_t^{*}\alpha N_t^{*\alpha-1}}$$

$$
\begin{aligned}
&\quad +\theta^* \beta \mathbb{E}_t \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{1+\rho^*}}{\pi_{t+1}^*} \left(\frac{1-\theta^* \pi_{Ft}^{\rho^*-1}}{1-\theta^* \pi_{Ft+1}^{\rho^*-1}} \right)^{\frac{\rho^*+1}{\rho^*-1}} X1_{Ft+1} \\
X2_{Ft} &= \left(\frac{1-\theta^* \pi_{Ft}^{\rho^*-1}}{1-\theta^*} \right)^{\frac{\rho^*}{\rho^*-1}} \frac{C_{Ft}}{E_t} (1-\tau_t^v) \\
&\quad +\theta^* \beta \mathbb{E}_t \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{\rho^*}}{\pi_{t+1}^*} \left(\frac{1-\theta^* \pi_{Ft}^{\rho^*-1}}{1-\theta^* \pi_{Ft+1}^{\rho^*-1}} \right)^{\frac{\rho^*}{\rho^*-1}} X2_{Ft+1}
\end{aligned}
$$

- Consumer price indices:

$$
\begin{aligned}
1 &= \gamma_H p_{Ht}^{1-\xi} + \gamma_F p_{Ft}^{1-\xi} \\
1 &= \gamma_H^* p_{Ht}^{*1-\xi} + \gamma_F^* p_{Ft}^{*1-\xi}
\end{aligned}
$$

- Evolution of price dispersion:

$$
\begin{aligned}
\Delta_{Ht} &= (1-\theta) \left(\frac{1-\theta \pi_{Ht}^{\rho-1}}{1-\theta} \right)^{\frac{\rho}{\rho-1}} + \theta \pi_{Ht}^{\rho} \Delta_{Ht-1} \\
\Delta_{Ht}^* &= (1-\theta) \left(\frac{1-\theta \pi_{Ht}^{*\rho-1}}{1-\theta} \right)^{\frac{\rho}{\rho-1}} + \theta \pi_{Ht}^{*\rho} \Delta_{Ht-1}^*
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{Ft}^* &= (1-\theta^*) \left(\frac{1-\theta^* \left(\pi_{Ft}^*\right)^{\rho^*-1}}{1-\theta^*} \right)^{\frac{\rho^*}{\rho^*-1}} + \theta^* \left(\pi_{Ft}^*\right)^{\rho^*} \Delta_{Ft-1}^* \\
\Delta_{Ft} &= (1-\theta^*) \left(\frac{1-\theta^* \left(\pi_{Ft}\right)^{\rho^*-1}}{1-\theta^*} \right)^{\frac{\rho^*}{\rho^*-1}} + \theta^* \left(\pi_{Ft}\right)^{\rho^*} \Delta_{Ft-1}
\end{aligned}
$$

- Evolution of relative prices:

$$
\frac{p_{Ht}}{p_{Ht-1}} = \frac{\pi_{Ht}}{\pi_t}, \quad \frac{p_{Ft}}{p_{Ft-1}} = \frac{\pi_{Ft}}{\pi_t}, \quad \frac{p_{Ht}^*}{p_{Ht-1}^*} = \frac{\pi_{Ht}^*}{\pi_t^*}, \quad \frac{p_{Ft}^*}{p_{Ft-1}^*} = \frac{\pi_{Ft}^*}{\pi_t^*}
$$

- Evolution of the real exchange rate

$$
\frac{q_t}{q_{t-1}} = \frac{E_t}{E_{t-1}} \frac{\pi_t^*}{\pi_t},
$$

given the transversality conditions, sequences of the policy instruments $\{R_t, R_t^*, \tau_t^v, \tau_t^{v*}\}_{t=0}^{\infty}$

and of the shocks $\{A_t, A_t^*, \mu_t, \mu_t^*, \zeta_t^c, \zeta_t^{c*}, G_t, G_t^*\}_{t=0}^{\infty}$, and an initial $E_{-1} = 1$. Unlike with the LOOP, the nominal exchange rate is itself a relevant argument to the equilibrium.

If the two countries form a monetary union, the equation defining R_t^* drops out, and the nominal exchange rate is fixed, i.e. $E_t = 1 \; \forall t$.

Appendix 2.C The Ramsey Problem

2.C.1 Derivation of the Intertemporal Fiscal Budget Constraint

Integrating (2.29) over h and k, and dividing by P_t yields

$$b_t = \mathbb{E}_t Q_{t,t+1} \pi_{t+1} b_{t+1} + s_t, \tag{2.47}$$

where $b_t = B_t / P_t$ is real debt and the primary surplus reads

$$s_t = \frac{1}{P_t} \left[\tau_t^v n \left(P_{Ht} C_{Ht} + P_{Ft} C_{Ft} \right) - \left(1 - \tau_t^v\right) P_{Ht} G_t \right].$$

Repeatedly iterating on (2.47) using successive future terms of it, beginning in period $t = 0$, yields the present-value fiscal budget constraint

$$b_0 = \mathbb{E}_0 \sum_{t=0}^{T} Q_{0,t} \pi_{0,t} s_t + \mathbb{E}_0 Q_{0,T+1} \pi_{0,T+1} b_{T+1},$$

where $\pi_{0,T+1} = P_{T+1}/P_0$ is the product of inflation rates between $t = 0$ and $t = T + 1$. Imposing the transversality condition

$$\lim_{T \to \infty} \mathbb{E}_0 Q_{0,T+1} \pi_{0,T+1} b_{T+1} = 0$$

and using the definition of $Q_{0,t}$, one ends up with

$$\zeta_0^c C_0^{-\sigma} b_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^c C_t^{-\sigma} s_t.$$

2.C.2 The Lagrangian of the Ramsey Problem

The scenario under study assumes the law of one price, the availability of fiscal policy as an instrument, and that the countries form a monetary union. The objective of the policy planner is, hence, to find sequences $\left\{R_t^{MU}, \tau_t^v, \tau_t^{v*}\right\}_{t=0}^{\infty}$.

$$
\begin{aligned}
\mathbb{V} \;=\; & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ n \left(\zeta_t^c \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right) + (1-n) \left(\zeta_t^{c*} \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{N_t^{*1+\eta}}{1+\eta} \right) \right. \\
& + \Lambda^H \zeta_t^c C_t^{-\sigma} \left[\tau_t^v n \left(p_{Ht} C_{Ht} + \frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} p_{Ft}^* q_t C_{Ft} \right) - (1-\tau_t^v)\, p_{Ht} G_t \right] \\
& + \Lambda^F \zeta_t^{c*} C_t^{*-\sigma} \left[\tau_t^{v*} (1-n) \left[p_{Ft}^* C_{Ft}^* + \frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} C_{Ht}^* \right] - (1-\tau_t^{v*})\, p_{Ft}^* G_t^* \right] \\
& + \lambda_t^1 \left[\gamma_H p_{Ht}^{-\xi} C_t - C_{Ht} \right] + \lambda_t^2 \left[\gamma_F \left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} p_{Ft}^* q_t \right)^{-\xi} C_t - C_{Ft} \right] \\
& + \lambda_t^3 \left[\gamma_H^* \left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{-\xi} C_t^* - C_{Ht}^* \right] + \lambda_t^4 \left[\gamma_F^* p_{Ft}^{*-\xi} C_t^* - C_{Ft}^* \right] \\
& + \lambda_t^5 \left[N_t^\eta C_t^\sigma - \zeta_t^c w_t p_{Ht} \right] + \lambda_t^6 \left[N_t^{*\eta} C_t^{*\sigma} - \zeta_t^{c*} w_t^* p_{Ft}^* \right] \\
& + \lambda_t^7 \left[A_t n N_t^\alpha - \Delta_{Ht} \left(n C_{Ht} + (1-n) C_{Ht}^* + G_t \right) \right] \\
& + \lambda_t^8 \left[A_t^* (1-n) N_t^{*\alpha} - \Delta_{Ft}^* \left(n C_{Ft} + (1-n) C_{Ft}^* + G_t^* \right) \right] \\
& + \lambda_t^9 \left[\frac{\rho}{\rho-1} \mu_t X1_{Ht} - X2_{Ht} \right] \\
& + \lambda_t^{10} \left[\widetilde{p}_{Ht}^{-1-\rho} \left(n C_{Ht} + (1-n) C_{Ht}^* + G_t \right) \frac{w_t}{\alpha A_t N_t^{\alpha-1}} \right. \\
& \left. + \theta\beta \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^{1+\rho}}{\pi_{t+1}} \left(\frac{\widetilde{p}_{Ht}}{\widetilde{p}_{Ht+1}} \right)^{-1-\rho} X1_{Ht+1} - X1_{Ht} \right] \\
& + \lambda_t^{11} \left[\widetilde{p}_{Ht}^{-\rho} \left(n C_{Ht} + (1-n) C_{Ht}^* + G_t \right) (1-\tau_t^v) \right. \\
& \left. + \theta\beta \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^{\rho}}{\pi_{t+1}} \left(\frac{\widetilde{p}_{Ht}}{\widetilde{p}_{Ht+1}} \right)^{-\rho} X2_{Ht+1} - X2_{Ht} \right] \\
& + \lambda_t^{12} \left[\frac{\rho^*}{\rho^*-1} \mu_t^* X1_{Ft}^* - X2_{Ft}^* \right] \\
& + \lambda_t^{13} \left[\widetilde{p}_{Ft}^{*-1-\rho^*} \left(n C_{Ft} + (1-n) C_{Ft}^* + G_t^* \right) \frac{w_t^*}{\alpha A_t^* N_t^{*\alpha-1}} \right. \\
& \left. + \theta^*\beta \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*1+\rho^*}}{\pi_{t+1}^*} \left(\frac{\widetilde{p}_{Ft}^*}{\widetilde{p}_{Ft+1}^*} \right)^{-1-\rho^*} X1_{Ft+1}^* - X1_{Ft}^* \right] \\
& + \lambda_t^{14} \left[\widetilde{p}_{Ft}^{*-\rho^*} \left(n C_{Ft} + (1-n) C_{Ft}^* + G_t^* \right) (1-\tau_t^{v*}) \right. \\
& \left. + \theta^*\beta \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*\rho^*}}{\pi_{t+1}^*} \left(\frac{\widetilde{p}_{Ft}^*}{\widetilde{p}_{Ft+1}^*} \right)^{-\rho^*} X2_{Ft+1}^* - X2_{Ft}^* \right] \\
& + \lambda_t^{15} \left[\gamma_H p_{Ht}^{1-\xi} + \gamma_F \left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} p_{Ft}^* q_t \right)^{1-\xi} - 1 \right] + \lambda_t^{16} \left[\gamma_H^* \left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{1-\xi} + \gamma_F^* p_{Ft}^{*1-\xi} - 1 \right]
\end{aligned}
$$

$$
\begin{aligned}
&+\lambda_t^{17}\left[\left(\frac{1-\theta\pi_{Ht}^{\rho-1}}{1-\theta}\right)^{\frac{1}{1-\rho}}-\widetilde{p}_{Ht}\right]+\lambda_t^{18}\left[\left(\frac{1-\theta^*\pi_{Ft}^{*\rho^*-1}}{1-\theta^*}\right)^{\frac{1}{1-\rho^*}}-\widetilde{p}_{Ft}^*\right] \\
&+\lambda_t^{19}\left[\kappa\frac{\zeta_t^{c*}}{\zeta_t^c}\left(\frac{C_t^*}{C_t}\right)^{-\sigma}-q_t\right] \\
&+\lambda_t^{20}\left[(1-\theta)\,\widetilde{p}_{Ht}^{-\rho}+\theta\pi_{Ht}^{\rho}\Delta_{Ht-1}-\Delta_{Ht}\right]+\lambda_t^{21}\left[(1-\theta^*)\,\widetilde{p}_{Ft}^{*-\rho^*}+\theta^*\pi_{Ft}^{*\rho^*}\Delta_{Ft-1}^*-\Delta_{Ft}^*\right] \\
&+\lambda_t^{22}\left[\frac{p_{Ht}}{p_{Ht-1}}-\frac{\pi_{Ht}}{\pi_t}\right]+\lambda_t^{23}\left[\frac{p_{Ft}^*}{p_{Ft-1}^*}-\frac{\pi_{Ft}^*}{\pi_t^*}\right]+\lambda_t^{24}\left[\frac{q_t}{q_{t-1}}-\frac{\pi_t^*}{\pi_t}\right]\Bigg\} \\
&-\Lambda^H\zeta_0^c C_0^{-\sigma}b_0-\Lambda^F\zeta_0^{c*}C_0^{*-\sigma}b_0^*.
\end{aligned}
$$

2.C.3 First-order Conditions for $t \geq 1$

The solution to the optimal policy problem can be described by the first-order conditions with respect to all Lagrange multipliers and with respect to all endogenous variables of the model:

- W.r.t. C_t :

$$
\begin{aligned}
0 \;=\;& n_H\zeta_t^c C_t^{-\sigma}-\Lambda^H\zeta_t^c\sigma C_t^{-\sigma-1}s_t+\lambda_t^1\gamma_H p_{Ht}^{-\xi}+\lambda_t^2\gamma_F\left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)}p_{Ft}^*q_t\right)^{-\xi}+\lambda_t^5 N_t^{\eta}\sigma C_t^{\sigma-1} \\
&+\frac{\sigma}{C_t}\left[\lambda_t^{10}\theta\beta\frac{\zeta_{t+1}^c}{\zeta_t^c}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\pi_{Ht+1}^{1+\rho}}{\pi_{t+1}}\left(\frac{\widetilde{p}_{Ht}}{\widetilde{p}_{Ht+1}}\right)^{-1-\rho}X1_{Ht+1}\right. \\
&\left.-\lambda_{t-1}^{10}\theta\frac{\zeta_t^c}{\zeta_{t-1}^c}\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\frac{\pi_{Ht}^{1+\rho}}{\pi_t}\left(\frac{\widetilde{p}_{Ht-1}}{\widetilde{p}_{Ht}}\right)^{-1-\rho}X1_{Ht}\right] \\
&+\frac{\sigma}{C_t}\left[\lambda_t^{11}\theta\beta\frac{\zeta_{t+1}^c}{\zeta_t^c}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\pi_{Ht+1}^{\rho}}{\pi_{t+1}}\left(\frac{\widetilde{p}_{Ht}}{\widetilde{p}_{Ht+1}}\right)^{-\rho}X2_{Ht+1}\right. \\
&\left.-\lambda_{t-1}^{11}\theta\frac{\zeta_t^c}{\zeta_{t-1}^c}\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\frac{\pi_{Ht}^{\rho}}{\pi_t}\left(\frac{\widetilde{p}_{Ht-1}}{\widetilde{p}_{Ht}}\right)^{-\rho}X2_{Ht}\right]+\lambda_t^{19}\kappa\frac{\sigma}{C_t}\frac{\zeta_t^{c*}}{\zeta_t^c}\left(\frac{C_t^*}{C_t}\right)^{-\sigma}
\end{aligned}
$$

- W.r.t. C_{Ht} :

$$
0=\Lambda^H\zeta_t^c C_t^{-\sigma}\tau_t^v p_{Ht}n_H-\lambda_t^1-\lambda_t^7\Delta_{Ht}n_H+\lambda_t^{10}\widetilde{p}_{Ht}^{-1-\rho}\frac{n_H w_t}{\alpha A_t N_t^{\alpha-1}}+\lambda_t^{11}\widetilde{p}_{Ht}^{-\rho}n_H\left(1-\tau_t^v\right)
$$

- W.r.t. C_{Ft} :

$$
0=\Lambda^H\zeta_t^c C_t^{-\sigma}\tau_t^v\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)}p_{Ft}^*q_t n_H-\lambda_t^2-\lambda_t^8\Delta_{Ft}^*n_H+\lambda_t^{13}\widetilde{p}_{Ft}^{*-1-\rho^*}\frac{n_H w_t^*}{\alpha A_t^* N_t^{*\alpha-1}}+\lambda_t^{14}\widetilde{p}_{Ft}^{*-\rho^*}n_H\left(1-\tau_t^{v*}\right)
$$

- W.r.t. C_t^* :

$$\begin{aligned}
0 &= n_F \zeta_t^{c*} C_t^{*-\sigma} - \Lambda^F \zeta_t^{c*} \sigma C_t^{*-\sigma-1} s_t^* + \lambda_t^3 \gamma_H^* \left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{-\xi} + \lambda_t^4 \gamma_F^* p_{Ft}^{*-\xi} + \lambda_t^6 N_t^{*\eta} \sigma C_t^{*\sigma-1} \\
&\quad + \frac{\sigma}{C_t^*} \left[\lambda_t^{13} \theta^* \beta \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*1+\rho^*}}{\pi_{t+1}^*} \left(\frac{\widetilde{p}_{Ft}^*}{\widetilde{p}_{Ft+1}^*} \right)^{-1-\rho^*} X1_{Ft+1}^* \right. \\
&\quad \left. - \lambda_{t-1}^{13} \theta^* \frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{\pi_{Ft}^{*1+\rho^*}}{\pi_t^*} \left(\frac{\widetilde{p}_{Ft-1}^*}{\widetilde{p}_{Ft}^*} \right)^{-1-\rho^*} X1_{Ft}^* \right] \\
&\quad + \frac{\sigma}{C_t^*} \left[\lambda_t^{14} \theta^* \beta \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*\rho^*}}{\pi_{t+1}^*} \left(\frac{\widetilde{p}_{Ft}^*}{\widetilde{p}_{Ft+1}^*} \right)^{-\rho^*} X2_{Ft+1}^* \right. \\
&\quad \left. - \lambda_{t-1}^{14} \theta^* \frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{\pi_{Ft}^{*\rho^*}}{\pi_t^*} \left(\frac{\widetilde{p}_{Ft-1}^*}{\widetilde{p}_{Ft}^*} \right)^{-\rho^*} X2_{Ft}^* \right] - \lambda_t^{19} \kappa \frac{\sigma}{C_t^*} \frac{\zeta_t^{c*}}{\zeta_t^c} \left(\frac{C_t^*}{C_t} \right)^{-\sigma}
\end{aligned}$$

- W.r.t. C_{Ht}^* :

$$0 = \Lambda^F \zeta_t^{c*} C_t^{*-\sigma} \tau_t^{v*} \frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} n_F - \lambda_t^3 - \lambda_t^7 \Delta_{Ht} n_F + \lambda_t^{10} \widetilde{p}_{Ht}^{-1-\rho} \frac{n_F w_t}{\alpha A_t N_t^{\alpha-1}} + \lambda_t^{11} \widetilde{p}_{Ht}^{-\rho} n_F (1-\tau_t^v)$$

- W.r.t. C_{Ft}^* :

$$0 = \Lambda^F \zeta_t^{c*} C_t^{*-\sigma} \tau_t^{v*} p_{Ft}^* n_F - \lambda_t^4 - \lambda_t^8 \Delta_{Ft}^* n_F + \lambda_t^{13} \widetilde{p}_{Ft}^{*-1-\rho^*} \frac{n_F w_t^*}{\alpha A_t^* N_t^{*\alpha-1}} + \lambda_t^{14} \widetilde{p}_{Ft}^{*-\rho^*} n_F (1-\tau_t^{v*})$$

- W.r.t. p_{Ht} :

$$\begin{aligned}
0 &= \Lambda^H \zeta_t^c C_t^{-\sigma} \left(\tau_t^v n_H C_{Ht} - (1-\tau_t^v) G_t \right) + \Lambda^F \zeta_t^{c*} C_t^{*-\sigma} \tau_t^{v*} \frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{n_F C_{Ht}^*}{q_t} \\
&\quad - \lambda_t^1 \gamma_H \xi p_{Ht}^{-\xi-1} C_t - \lambda_t^3 \gamma_H^* \left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{-\xi} \frac{\xi C_t^*}{p_{Ht}} - \lambda_t^5 \zeta_t^c w_t \\
&\quad + \lambda_t^{15} \gamma_H (1-\xi) p_{Ht}^{-\xi} + \lambda_t^{16} \gamma_H^* \left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{1-\xi} \frac{(1-\xi)}{p_{Ht}} + \frac{\lambda_t^{22}}{p_{Ht-1}} - \lambda_{t+1}^{22} \beta \frac{p_{Ht+1}}{p_{Ht}^2}
\end{aligned}$$

- W.r.t. p_{Ft}^* :

$$\begin{aligned}
0 &= \Lambda^H \zeta_t^c C_t^{-\sigma} \tau_t^v \frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} q_t n_H C_{Ft} + \Lambda^F \zeta_t^{c*} C_t^{*-\sigma} \left(\tau_t^{v*} n_F C_{Ft}^* - (1-\tau_t^{v*}) G_t^* \right) \\
&\quad - \lambda_t^2 \gamma_F \left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} p_{Ft}^* q_t \right)^{-\xi} \frac{\xi C_t}{p_{Ft}^*} - \lambda_t^4 \gamma_F^* \xi p_{Ft}^{*-\xi-1} C_t^* - \lambda_t^6 \zeta_t^{c*} w_t^*
\end{aligned}$$

$$+\lambda_t^{15}\gamma_F\left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^{v})}p_{Ft}^*q_t\right)^{1-\xi}\frac{(1-\xi)}{p_{Ft}^*}+\lambda_t^{16}\gamma_F^*(1-\xi)\,p_{Ft}^{*-\xi}+\frac{\lambda_t^{23}}{p_{Ft-1}^*}-\lambda_{t+1}^{23}\beta\frac{p_{Ft+1}^*}{p_{Ft}^{*2}}$$

- W.r.t. q_t :

$$\begin{aligned}0 &= \Lambda^H\zeta_t^c C_t^{-\sigma}\tau_t^v\frac{(1-\tau_t^{v*})}{(1-\tau_t^{v})}p_{Ft}^*n_H C_{Ft}-\Lambda^F\zeta_t^{c*}C_t^{*-\sigma}\tau_t^{v*}\frac{(1-\tau_t^{v})}{(1-\tau_t^{v*})}\frac{p_{Ht}}{q_t^2}n_F C_{Ht}^*\\ &\quad-\lambda_t^2\gamma_F\left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^{v})}p_{Ft}^*q_t\right)^{-\xi}\frac{\xi C_t}{q_t}+\lambda_t^3\gamma_H^*\left(\frac{(1-\tau_t^{v})}{(1-\tau_t^{v*})}\frac{p_{Ht}}{q_t}\right)^{-\xi}\frac{\xi C_t^*}{q_t}\\ &\quad+\lambda_t^{15}\gamma_F\left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^{v})}p_{Ft}^*q_t\right)^{1-\xi}\frac{(1-\xi)}{q_t}-\lambda_t^{16}\gamma_H^*\left(\frac{(1-\tau_t^{v})}{(1-\tau_t^{v*})}\frac{p_{Ht}}{q_t}\right)^{1-\xi}\frac{(1-\xi)}{q_t}\\ &\quad-\lambda_t^{19}+\frac{\lambda_t^{24}}{q_{t-1}}-\lambda_{t+1}^{24}\beta\frac{q_{t+1}}{q_t^2}\end{aligned}$$

- W.r.t. π_t :

$$\begin{aligned}0 &= -\lambda_{t-1}^{10}\theta\frac{\zeta_t^c}{\zeta_{t-1}^c}\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\frac{\pi_{Ht}^{1+\rho}}{\pi_t^2}\left(\frac{\widetilde{p}_{Ht-1}}{\widetilde{p}_{Ht}}\right)^{-1-\rho}X1_{Ht}\\ &\quad-\lambda_{t-1}^{11}\theta\frac{\zeta_t^c}{\zeta_{t-1}^c}\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\frac{\pi_{Ht}^{\rho}}{\pi_t^2}\left(\frac{\widetilde{p}_{Ht-1}}{\widetilde{p}_{Ht}}\right)^{-\rho}X2_{Ht}+\lambda_t^{22}\frac{\pi_{Ht}}{\pi_t^2}+\lambda_t^{24}\frac{\pi_t^*}{\pi_t^2}\end{aligned}$$

- W.r.t. π_t^* :

$$\begin{aligned}0 &= -\lambda_{t-1}^{13}\theta^*\frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}}\left(\frac{C_t^*}{C_{t-1}^*}\right)^{-\sigma}\frac{\pi_{Ft}^{*1+\rho^*}}{\pi_t^{*2}}\left(\frac{\widetilde{p}_{Ft-1}^*}{\widetilde{p}_{Ft}^*}\right)^{-1-\rho^*}X1_{Ft}^*\\ &\quad-\lambda_{t-1}^{14}\theta^*\frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}}\left(\frac{C_t^*}{C_{t-1}^*}\right)^{-\sigma}\frac{\pi_{Ft}^{*\rho^*}}{\pi_t^{*2}}\left(\frac{\widetilde{p}_{Ft-1}^*}{\widetilde{p}_{Ft}^*}\right)^{-\rho^*}X2_{Ft}^*+\lambda_t^{23}\frac{\pi_{Ft}^*}{\pi_t^{*2}}-\frac{\lambda_t^{24}}{\pi_t}\end{aligned}$$

- W.r.t. π_{Ht} :

$$\begin{aligned}0 &= \lambda_{t-1}^{10}\theta\frac{\zeta_t^c}{\zeta_{t-1}^c}\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\frac{(1+\rho)\,\pi_{Ht}^{\rho}}{\pi_t}\left(\frac{\widetilde{p}_{Ht-1}}{\widetilde{p}_{Ht}}\right)^{-1-\rho}X1_{Ht}\\ &\quad+\lambda_{t-1}^{11}\theta\frac{\zeta_t^c}{\zeta_{t-1}^c}\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\frac{\rho\pi_{Ht}^{\rho-1}}{\pi_t}\left(\frac{\widetilde{p}_{Ht-1}}{\widetilde{p}_{Ht}}\right)^{-\rho}X2_{Ht}\\ &\quad+\lambda_t^{17}\left(\frac{1-\theta\pi_{Ht}^{\rho-1}}{1-\theta}\right)^{\frac{\rho}{1-\rho}}\frac{\theta}{1-\theta}\pi_{Ht}^{\rho-2}+\lambda_t^{20}\theta\rho\pi_{Ht}^{\rho-1}\Delta_{Ht-1}-\frac{\lambda_t^{22}}{\pi_t}\end{aligned}$$

- W.r.t. π^*_{Ft} :

$$\begin{aligned} 0 &= \lambda^{13}_{t-1}\theta^*\frac{\zeta^{c*}_t}{\zeta^{c*}_{t-1}}\left(\frac{C^*_t}{C^*_{t-1}}\right)^{-\sigma}\frac{(1+\rho^*)\,\pi^{*\rho^*}_{Ft}}{\pi^*_t}\left(\frac{\widetilde{p}^*_{Ft-1}}{\widetilde{p}^*_{Ft}}\right)^{-1-\rho^*}X1^*_{Ft} \\ &+\lambda^{14}_{t-1}\theta^*\frac{\zeta^{c*}_t}{\zeta^{c*}_{t-1}}\left(\frac{C^*_t}{C^*_{t-1}}\right)^{-\sigma}\frac{\rho^*\pi^{*\rho^*-1}_{Ft}}{\pi^*_t}\left(\frac{\widetilde{p}^*_{Ft-1}}{\widetilde{p}^*_{Ft}}\right)^{-\rho^*}X2^*_{Ft} \\ &+\lambda^{18}_t\left(\frac{1-\theta^*\pi^{*\rho^*-1}_{Ft}}{1-\theta^*}\right)^{\frac{\rho^*}{1-\rho^*}}\frac{\theta^*}{1-\theta^*}\pi^{*\rho^*-2}_{Ft}+\lambda^{21}_t\theta^*\rho^*\pi^{*\rho^*-1}_{Ft}\Delta^*_{Ft-1}-\frac{\lambda^{23}_t}{\pi^*_t} \end{aligned}$$

- W.r.t. Δ_{Ht} :

$$0=-\lambda^7_t\left(n_H C_{Ht}+n_F C^*_{Ht}+G_t\right)-\lambda^{20}_t+\lambda^{20}_{t+1}\beta\theta\pi^{\rho}_{Ht+1}$$

- W.r.t. Δ^*_{Ft} :

$$0=-\lambda^8_t\left(n_H C_{Ft}+n_F C^*_{Ft}+G^*_t\right)-\lambda^{21}_t+\lambda^{21}_{t+1}\beta\theta^*\pi^{*\rho^*}_{Ft+1}$$

- W.r.t. $\widetilde{p}_{Ht}$:

$$\begin{aligned} 0 &= -\lambda^{10}_t\frac{1+\rho}{\widetilde{p}_{Ht}}\left[\widetilde{p}^{-1-\rho}_{Ht}\left(n_H C_{Ht}+n_F C^*_{Ht}+G_t\right)\frac{w_t}{\alpha A_t N^{\alpha-1}_t}\right. \\ &\left.+\theta\beta\frac{\zeta^c_{t+1}}{\zeta^c_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\pi^{1+\rho}_{Ht+1}}{\pi_{t+1}}\left(\frac{\widetilde{p}_{Ht}}{\widetilde{p}_{Ht+1}}\right)^{-1-\rho}X1_{Ht+1}\right] \\ &+\lambda^{10}_{t-1}\frac{1+\rho}{\widetilde{p}_{Ht}}\theta\frac{\zeta^c_t}{\zeta^c_{t-1}}\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\frac{\pi^{1+\rho}_{Ht}}{\pi_t}\left(\frac{\widetilde{p}_{Ht-1}}{\widetilde{p}_{Ht}}\right)^{-1-\rho}X1_{Ht} \\ &-\lambda^{11}_t\frac{\rho}{\widetilde{p}_{Ht}}\left[\widetilde{p}^{-\rho}_{Ht}\left(n_H C_{Ht}+n_F C^*_{Ht}+G_t\right)\left(1-\tau^v_t\right)\right. \\ &\left.+\theta\beta\frac{\zeta^c_{t+1}}{\zeta^c_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\pi^{\rho}_{Ht+1}}{\pi_{t+1}}\left(\frac{\widetilde{p}_{Ht}}{\widetilde{p}_{Ht+1}}\right)^{-\rho}X2_{Ht+1}\right] \\ &+\lambda^{11}_{t-1}\frac{\rho}{\widetilde{p}_{Ht}}\theta\frac{\zeta^c_t}{\zeta^c_{t-1}}\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\frac{\pi^{\rho}_{Ht}}{\pi_t}\left(\frac{\widetilde{p}_{Ht-1}}{\widetilde{p}_{Ht}}\right)^{-\rho}X2_{Ht}-\lambda^{17}_t-\lambda^{20}_t\left(1-\theta\right)\rho\widetilde{p}^{-\rho-1}_{Ht} \end{aligned}$$

- W.r.t. $\widetilde{p}^*_{Ft}$:

$$0 = -\lambda^{13}_t\frac{1+\rho^*}{\widetilde{p}^*_{Ft}}\left[\widetilde{p}^{*-1-\rho^*}_{Ft}\left(n_H C_{Ft}+n_F C^*_{Ft}+G^*_t\right)\frac{w^*_t}{\alpha A^*_t N^{*\alpha-1}_t}\right.$$

$$+\theta^*\beta\frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{\pi_{Ft+1}^{*1+\rho^*}}{\pi_{t+1}^*}\left(\frac{\widetilde{p}_{Ft}^*}{\widetilde{p}_{Ft+1}^*}\right)^{-1-\rho^*}X1_{Ft+1}^*\Bigg]$$
$$+\lambda_{t-1}^{13}\frac{1+\rho^*}{\widetilde{p}_{Ft}^*}\theta^*\frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}}\left(\frac{C_t^*}{C_{t-1}^*}\right)^{-\sigma}\frac{\pi_{Ft}^{*1+\rho^*}}{\pi_t^*}\left(\frac{\widetilde{p}_{Ft-1}^*}{\widetilde{p}_{Ft}^*}\right)^{-1-\rho^*}X1_{Ft}^*$$
$$-\lambda_t^{14}\frac{\rho^*}{\widetilde{p}_{Ft}^*}\Big[\widetilde{p}_{Ft}^{*-\rho^*}\left(n_HC_{Ft}+n_FC_{Ft}^*+G_t^*\right)\left(1-\tau_t^{v*}\right)$$
$$+\theta^*\beta\frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{\pi_{Ft+1}^{*\rho^*}}{\pi_{t+1}^*}\left(\frac{\widetilde{p}_{Ft}^*}{\widetilde{p}_{Ft+1}^*}\right)^{-\rho^*}X2_{Ft+1}^*\Bigg]$$
$$+\lambda_{t-1}^{14}\frac{\rho^*}{\widetilde{p}_{Ft}^*}\theta^*\frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}}\left(\frac{C_t^*}{C_{t-1}^*}\right)^{-\sigma}\frac{\pi_{Ft}^{*\rho^*}}{\pi_t^*}\left(\frac{\widetilde{p}_{Ft-1}^*}{\widetilde{p}_{Ft}^*}\right)^{-\rho^*}X2_{Ft}^*-\lambda_t^{18}-\lambda_t^{21}\left(1-\theta^*\right)\rho^*\widetilde{p}_{Ft}^{*-\rho^*-1}$$

- W.r.t. $X1_{Ht}$:

$$0=\lambda_t^9\frac{\rho}{\rho-1}\mu_t-\lambda_t^{10}+\lambda_{t-1}^{10}\theta\frac{\zeta_t^c}{\zeta_{t-1}^c}\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\frac{\pi_{Ht}^{1+\rho}}{\pi_t}\left(\frac{\widetilde{p}_{Ht-1}}{\widetilde{p}_{Ht}}\right)^{-1-\rho}$$

- W.r.t. $X2_{Ht}$:

$$0=-\lambda_t^9-\lambda_t^{11}+\lambda_{t-1}^{11}\theta\frac{\zeta_t^c}{\zeta_{t-1}^c}\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\frac{\pi_{Ht}^{\rho}}{\pi_t}\left(\frac{\widetilde{p}_{Ht-1}}{\widetilde{p}_{Ht}}\right)^{-\rho}$$

- W.r.t. $X1_{Ft}^*$:

$$0=\lambda_t^{12}\frac{\rho^*}{\rho^*-1}\mu_t^*-\lambda_t^{13}+\lambda_{t-1}^{13}\theta^*\frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}}\left(\frac{C_t^*}{C_{t-1}^*}\right)^{-\sigma}\frac{\pi_{Ft}^{*1+\rho^*}}{\pi_t^*}\left(\frac{\widetilde{p}_{Ft-1}^*}{\widetilde{p}_{Ft}^*}\right)^{-1-\rho^*}$$

- W.r.t. $X2_{Ft}^*$:

$$0=-\lambda_t^{12}-\lambda_t^{14}+\lambda_{t-1}^{14}\theta^*\frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}}\left(\frac{C_t^*}{C_{t-1}^*}\right)^{-\sigma}\frac{\pi_{Ft}^{*\rho^*}}{\pi_t^*}\left(\frac{\widetilde{p}_{Ft-1}^*}{\widetilde{p}_{Ft}^*}\right)^{-\rho^*}$$

- W.r.t. N_t :

$$0=-n_HN_t^{\eta}+\lambda_t^5\eta N_t^{\eta-1}C_t^{\sigma}+\lambda_t^7A_tn_H\alpha N_t^{\alpha-1}+\lambda_t^{10}\widetilde{p}_{Ht}^{-1-\rho}\left(n_HC_{Ht}+n_FC_{Ht}^*+G_t\right)\frac{(1-\alpha)}{\alpha}\frac{w_t}{A_tN_t^{\alpha}}$$

- W.r.t. N_t^* :

$$0 \;=\; -n_FN_t^{*\eta}+\lambda_t^6\eta N_t^{*\eta-1}C_t^{*\sigma}+\lambda_t^8A_t^*n_F\alpha N_t^{*\alpha-1}$$

$$+\lambda_t^{13}\widetilde{p}_{Ft}^{*-1-\rho^*}\left(n_H C_{Ft}+n_F C_{Ft}^*+G_t^*\right)\frac{(1-\alpha)}{\alpha}\frac{w_t^*}{A_t^* N_t^{*\alpha}}$$

- W.r.t. w_t :

$$0=-\lambda_t^5\zeta_t^c p_{Ht}+\lambda_t^{10}\widetilde{p}_{Ht}^{-1-\rho}\left(n_H C_{Ht}+n_F C_{Ht}^*+G_t\right)\frac{N_t^{1-\alpha}}{\alpha A_t}$$

- W.r.t. w_t^* :

$$0=-\lambda_t^6\zeta_t^{c*}p_{Ft}^*+\lambda_t^{13}\widetilde{p}_{Ft}^{*-1-\rho^*}\left(n_H C_{Ft}+n_F C_{Ft}^*+G_t^*\right)\frac{N_t^{*1-\alpha}}{\alpha A_t^*}$$

- W.r.t. τ_t^v :

$$\begin{aligned}
0 \;=\;& \Lambda^H\zeta_t^c C_t^{-\sigma}\left(p_{Ht}\left(n_H C_{Ht}+G_t\right)+\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)^2}p_{Ft}^* q_t n_H C_{Ft}\right)-\frac{\Lambda^F\zeta_t^{c*}C_t^{*-\sigma}\tau_t^{v*}}{(1-\tau_t^{v*})}\frac{p_{Ht}}{q_t}n_F C_{Ht}^*\\
&-\lambda_t^2\gamma_F\left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)}p_{Ft}^* q_t\right)^{-\xi}\frac{\xi C_t}{(1-\tau_t^v)}+\lambda_t^3\gamma_H^*\left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})}\frac{p_{Ht}}{q_t}\right)^{-\xi}\frac{\xi C_t^*}{(1-\tau_t^v)}\\
&-\lambda_t^{11}\widetilde{p}_{Ht}^{-\rho}\left(n_H C_{Ht}+n_F C_{Ht}^*+G_t\right)\\
&+\frac{(1-\xi)}{(1-\tau_t^v)}\left[\lambda_t^{15}\gamma_F\left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)}p_{Ft}^* q_t\right)^{1-\xi}-\lambda_t^{16}\gamma_H^*\left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})}\frac{p_{Ht}}{q_t}\right)^{1-\xi}\right]
\end{aligned}$$

- W.r.t. τ_t^{v*} :

$$\begin{aligned}
0 \;=\;& -\Lambda^H\zeta_t^c C_t^{-\sigma}\tau_t^v\frac{p_{Ft}^* q_t n_H C_{Ft}}{(1-\tau_t^v)}+\Lambda^F\zeta_t^{c*}C_t^{*-\sigma}\left(p_{Ft}^*\left(n_F C_{Ft}^*+G_t^*\right)+\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})^2}\frac{p_{Ht}}{q_t}n_F C_{Ht}^*\right)\\
&+\lambda_t^2\gamma_F\left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)}p_{Ft}^* q_t\right)^{-\xi}\frac{\xi C_t}{(1-\tau_t^{v*})}-\lambda_t^3\gamma_H^*\left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})}\frac{p_{Ht}}{q_t}\right)^{-\xi}\frac{\xi C_t^*}{(1-\tau_t^{v*})}\\
&-\lambda_t^{14}\widetilde{p}_{Ft}^{*-\rho^*}\left(n_H C_{Ft}+n_F C_{Ft}^*+G_t^*\right)\\
&-\frac{(1-\xi)}{(1-\tau_t^{v*})}\left[\lambda_t^{15}\gamma_F\left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)}p_{Ft}^* q_t\right)^{1-\xi}-\lambda_t^{16}\gamma_H^*\left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})}\frac{p_{Ht}}{q_t}\right)^{1-\xi}\right]
\end{aligned}$$

Appendix 2.D Data Sources and Calibration Targets

All data is taken from Eurostat (`http://ec.europa.eu/eurostat/data/database`). Population shares of the core and periphery are calculated using time averages of total population between 2001-2014 (variable name in source: `[demo_pjan]`). Government debt over GDP in steady state (G/Y) is calculated as time average of general government consolidated gross debt as percentage of GDP using annual data between 2010-2014 (variable name in source: `[gov_10dd_edpt1]`). Government spending and trade balance relative to GDP in steady state are constructed analogously as time averages on quarterly data between 2001:1 and 2014:4 (variables in source from category: `[namq_10_gdp]`).

Table 2.6: Empirical and Theoretical Second Moments

	GDP	Cons.	Gov.	Wage
Target Moments				
Core:				
Autocorrelation:	0.87	0.81	0.77	
Std. Dev. (in p.p.):	1.67	0.96	1.07	0.99
Periphery:				
Autocorrelation:	0.82	0.88	0.64	
Std. Dev. (in p.p.):	2.06	1.78	2.42	1.78
Model-Generated Moments				
Core:				
Autocorrelation:	0.87	0.81	0.77	
Std. Dev. (in p.p.):	0.88	1.17	1.12	1.39
Periphery:				
Autocorrelation:	0.82	0.88	0.64	
Std. Dev. (in p.p.):	1.11	2.11	2.54	1.96

Notes: Empirical target moments (upper panel) calculated using quarterly data from Eurostat for the period 2001:1 to 2014:4. All series in logs, seasonally adjusted and quadratically detrended. Theoretical moments (lower panel) from calibrated model (see Tables 2.1 and 2.2). Available policy instruments: monetary policy at union level only.

The data series used to construct the calibration targets for GDP, consumption, and government spending also stem from `[namq_10_gdp]`. The variable names are "Gross domestic product at market prices", "Final consumption expenditure of households", and "Final consumption expenditure of general government". The raw series are not seasonally adjusted and measured in current prices. Data on aggregate wages are proxied by the labour cost index (LCI) for the business economy sector (variable name in source: `[lc_lci_r2_q]`), which provides observations for all required countries but France. The index is given at a quarterly frequency, not seasonally adjusted, and it takes on a value of 100 in 2012. Before calculating the target moments (autocorrelations, standard deviations) for the calibration, the log of all series is quadratically detrended and seasonally adjusted. An overview of the second moments generated from the data and from the model is given in Table 2.6.

Appendix 2.E Sensitivity Analysis

Table 2.7: Welfare Costs of Fixed Exchange Rates – Increased Shock Size

(A) Benchmark				
LOOP		MU	FLEX	Difference
Monetary Policy (MP)	$10^{-2}*$	-21.059	-18.530	2.5295
Monetary+Fiscal Policy (MFP)	$10^{-2}*$	-18.219	-17.858	0.3604
		Reduction of Welfare Costs:		**85.75**%
PTM				
Monetary Policy (MP)	$10^{-2}*$	-21.052	-20.658	0.3938
Monetary+Fiscal Policy (MFP)	$10^{-2}*$	-20.058	-19.934	0.1235
		Reduction of Welfare Costs:		**68.64**%
(B) Productivity, Preference, Gov. Spending Shocks				
LOOP				
Monetary Policy	$10^{-2}*$	-17.069	-15.286	1.7827
Monetary+Fiscal Policy	$10^{-2}*$	-15.654	-15.371	0.2836
		Reduction of Welfare Costs:		**84.09**%
PTM				
Monetary Policy	$10^{-2}*$	-17.063	-16.854	0.2086
Monetary+Fiscal Policy	$10^{-2}*$	-16.911	-16.818	0.0930
		Reduction of Welfare Costs:		**55.43**%
(C) Mark-up Shocks				
LOOP				
Monetary Policy	$10^{-2}*$	-3.8470	-3.0972	0.7498
Monetary+Fiscal Policy	$10^{-2}*$	-2.4248	-2.3467	0.0781
		Reduction of Welfare Costs:		**89.58**%
PTM				
Monetary Policy	$10^{-2}*$	-3.8454	-3.6595	0.1859
Monetary+Fiscal Policy	$10^{-2}*$	-3.0018	-2.971	0.0308
		Reduction of Welfare Costs:		**83.43**%

Notes: Welfare measure: consumption equivalents between deterministic and stochastic world economy. Exchange rate regime either monetary union (MU) or flexible (FLEX). Panel (A): productivity, demand preference, government spending, & mark-up shocks in both countries. Panel (B): all but mark-up shocks. Panel (C): mark-up shocks only. Shock standard deviations in all scenarios doubled compared to benchmark calibration. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

Table 2.8: Percentage Reduction of Welfare Costs of Fixed Exchange Rates – Parameter Changes

	Benchmark	$\sigma = 4$	$\eta = 5$	Home Bias 30%	$\xi = 3$	$\rho, \rho^* = 10$	$\theta, \theta^* = 0.85$	$G/Y, G^*/Y^* = 0.30$	$B/Y, B^*/Y^* = 1.80$
(A) All Shocks									
LOOP	85.76	93.01	90.10	85.23	89.78	93.17	87.12	87.20	85.66
PTM	68.66	84.12	49.99	65.21	83.26	67.32	63.56	78.93	68.61
(B) Productivity, Preference, Gov. Spending Shocks									
LOOP	84.03	90.62	88.66	81.99	91.15	92.00	86.45	86.64	84.29
PTM	55.42	76.84	42.25	54.22	67.57	53.27	54.68	71.74	55.23
(C) Mark-up Shocks									
LOOP	89.58	97.74	96.04	92.95	79.89	94.34	89.37	88.34	88.63
PTM	83.43	89.90	68.18	84.13	96.05	77.21	79.85	91.53	83.71

Notes: Table shows percentage reduction of welfare costs of fixed exchange rates by using optimal fiscal policy. Underlying numbers of the welfare measure for all policy scenarios are available on request. Welfare measure: consumption equivalents between deterministic and stochastic world economy. Column 'Benchmark' repeats results of Table 2.3. Panel (A): productivity, demand preference, government spending, & mark-up shocks in both countries. Panel (B): all but mark-up shocks. Panel (C): mark-up shocks only. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

Table 2.9: Welfare Costs of Fixed Exchange Rates – Payroll Taxes

(A) Benchmark				
LOOP		MU	FLEX	Difference
Monetary Policy (MP)	$10^{-2}*$	-5.0352	-4.4208	0.6144
Monetary+Fiscal Policy (MFP)	$10^{-2}*$	-4.0039	-3.747	0.2569
		Reduction of Welfare Costs:		**58.20**%
PTM				
Monetary Policy (MP)	$10^{-2}*$	-5.0348	-4.9412	0.0936
Monetary+Fiscal Policy (MFP)	$10^{-2}*$	-4.006	-3.988	0.0180
		Reduction of Welfare Costs:		**80.77**%
(B) Productivity, Preference, Gov. Spending Shocks				
LOOP				
Monetary Policy	$10^{-2}*$	-4.0539	-3.6155	0.4385
Monetary+Fiscal Policy	$10^{-2}*$	-3.8918	-3.6351	0.2567
		Reduction of Welfare Costs:		**41.45**%
PTM				
Monetary Policy	$10^{-2}*$	-4.0531	-4.0023	0.0507
Monetary+Fiscal Policy	$10^{-2}*$	-3.8749	-3.8582	0.0168
		Reduction of Welfare Costs:		**66.96**%
(C) Mark-up Shocks				
LOOP				
Monetary Policy	$10^{-2}*$	-0.9066	-0.7306	0.176
Monetary+Fiscal Policy	$10^{-2}*$	-0.0453	-0.0442	0.0012
		Reduction of Welfare Costs:		**99.34**%
PTM				
Monetary Policy	$10^{-2}*$	-0.9071	-0.8642	0.0429
Monetary+Fiscal Policy	$10^{-2}*$	-0.0565	-0.0553	0.0012
		Reduction of Welfare Costs:		**97.11**%

Notes: Payroll taxes as fiscal instrument instead of VATs. Welfare measure: consumption equivalents between deterministic and stochastic world economy. Exchange rate regime either monetary union (MU) or flexible (FLEX). Panel (A): productivity, demand preference, government spending, & mark-up shocks in both countries. Panel (B): all but mark-up shocks. Panel (C): mark-up shocks only. Shock standard deviations in all scenarios doubled compared to benchmark calibration. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

Chapter 3

Interest Rate Spreads and Forward Guidance

This chapter is based on joint work with Christian Bredemeier and Andreas Schabert.

3.1 Introduction

Ever since the onset of the financial crisis in 2007 and with monetary policy rates close to zero, forward guidance – the communication of central banks about the likely future course of their policy stance – has gained considerable importance for the conduct of monetary policy by major central banks, including the US Federal Reserve and the European Central Bank. This way of expectations management aims at steering longer-term interest rates, e.g., a flattening of the yield curve, by providing guidance about future real short-term interest rates. Based on the New Keynesian paradigm, this should stimulate aggregate demand today and may even break deflationary spirals at zero interest rates (see, e.g., Eggertsson and Woodford, 2003). Recent empirical studies, however, emphasize that New Keynesian models massively overstate the effects of forward guidance announcements,[1] which has led Del Negro et al. (2015) to coin this the "forward guidance puzzle".[2]

In this chapter, we show that the puzzle is resolved when the effects of forward guidance on those interest rates that are actually relevant for private sector consumption and investment decisions, are taken into account. Our analysis is motivated by the empirical observation that interest rates on various assets respond differently to forward guidance announcements. For example, Del Negro et al. (2015) examine asset price effects around

[1]See, for example, Del Negro et al. (2015), Carlstrom, Fuerst and Paustian (2015), or Kiley (2016).

[2]Several contributions, for instance McKay, Nakamura and Steinsson (2016) or Del Negro et al. (2015), have already addressed this puzzle.

forward guidance announcements by the Federal Open Market Committee (FOMC) at three dates in 2011 and 2012 with an explicit calendar-based forward guidance. At these dates, the press releases state that the federal funds rate stays at low levels at least through a period of 2 or 3 years ahead (see Appendix 3.A.1 for details). Extending the list of asset prices examined by Del Negro et al. (2015), Table 3.1 presents changes in asset prices in a one-day window around the three FOMC dates (as in Krishnamurthy and Vissing-Jorgensen, 2011). While the magnitude of the announcement effects varies between dates due to expectations and economic conditions, the columns 2-4 show that Treasury yields fall at all dates. As also found by Del Negro et al. (2015), corporate bond yields (see columns 5-8) tend to fall by less compared to yields of treasuries with the same maturity (and might even rise), implying that the corporate-treasury spreads unambiguously increase.[3] Since the underlying assets mainly differ by liquidity, but are similar in terms of safety, as argued by Krishnamurthy and Vissing-Jorgensen (2012), forward guidance seems to alter liquidity premia.[4] Given that borrowing and saving decisions are typically related to interest rates on less liquid assets, the observation that these interest rates respond to a smaller extent than treasury rates is indicative for forward guidance to be less effective than suggested by New Keynesian models without a liquidity premium.

Table 3.1: One-Day Changes of Asset Returns

	Treasuries			Corporate Bonds			
Dates	3Y	5Y	10Y	3Y	5Y	10Y(A)	10Y(B)
2011-08-09	-12	-20	-20	-2	-11	-9	-7
2012-01-25	-5	-11	-7	-3	-5	1	-1
2012-09-13	-1	-5	-2	-2	-1	3	11

Notes: Table shows absolute changes of asset returns in a one-day window around selected FOMC announcement (end-of-day minus day before). All numbers are given in basis points rounded to integers. Maturity is measured in years (Y). Corporate Bond 10Y(A) and (B) refer to long-term bonds with AAA and BAA rating, respectively.

In the first part of the chapter, we present an econometric analysis revealing that liquidity premia rise systematically in response to stimulative forward guidance announcements.

[3]The forward guidance announcement on 2012-09-13 seems to have been anticipated by market participants. In our econometric analysis in Section 2 that focusses on the identification of unanticipated effects of forward guidance, we do not observe a clear reduction in futures rates, which we do for the first two dates. This strongly points towards anticipation. It is therefore not surprising that rate changes are less pronounced on 2012-09-13 compared to the other two dates.

[4]Similar findings are reported by Campbell, Evans, Fisher and Justiniano (2012) on the effect of forward guidance in the period from 2007 to 2011.

We apply the method of Gürkaynak et al. (2005) to extract a surprise component in the announcements of FOMC meetings between 1990 and 2016,[5] and examine how measures of liquidity premia, which have been suggested in the literature, respond to the changes in the anticipated future paths of the monetary policy instrument. Decomposing policy announcements into a target factor and a path factor, we find that changes in these two factors affect interest rates in different ways and, in particular, that a stimulative forward guidance shock increases interest rate spreads, which are suggested as measures for liquidity premia by Krishnamurthy and Vissing-Jorgensen (2012) and Nagel (2016). Furthermore, we construct a common liquidity factor, as Del Negro et al. (2017), and find that it increases as well.

Based on this empirical evidence, the second part of the chapter presents a macroeconomic model that rationalizes the increase of liquidity premia after stimulative central bank announcements. We show that this separation of the policy rate from interest rates on stores of wealth substantially weakens the GDP response to these announcements, providing a solution to the forward guidance puzzle. To this end, we introduce an endogenous liquidity premium to a New Keynesian model by accounting for differential pledgeability of assets, as in Schabert (2015) and Williamson (2016). To understand why forward guidance is so powerful in standard New Keynesian models, recall that current consumption depends on all expected future real monetary policy rates in these models. For given inflation expectations, an anticipated reduction of the short-term nominal policy rate for one single period in, say, k periods ahead will, therefore, increase consumption in all k periods before the interest rate change actually takes place, while the impact effect is – counterfactually – predicted to grow with the horizon (larger k). A central assumption for this effectiveness of monetary policy announcements is that interest rates that are relevant for private agents' intertemporal choices move – up to first order – one-for-one with the monetary policy rate. This however neglects the empirical observation that other interest rates, which are more relevant for private-sector transactions than the federal funds rate, are separated by spreads that might change endogenously with the state of the economy and with monetary policy (see Krishnamurthy and Vissing-Jorgensen, 2012 and Nagel, 2016).

By contrast, our model that features a liquidity premium allows the policy rate set by the central bank to differ from other interest rates. We incorporate a stylized banking sector in a New Keynesian model with an explicit specification of central bank operations.

[5]This method has widely been used to analyse the effects of monetary policy and forward guidance on financial markets and has for instance, also been applied by Campbell et al. (2012), Swanson (2017), or Gertler and Karadi (2015). Our results are qualitatively unchanged when we consider a sample ending in 2008, i.e., a sample excluding the recent zero lower bound episode.

Banks are required to hold reserves from the central bank to meet the liquidity demands of their depositors. Reserves can only be obtained in open market operations against assets that are eligible to serve as collateral (i.e., Treasury bills), where the central bank controls the price of money by setting the policy rate.[6] Thus, returns on eligible assets closely follow the policy rate, whereas interest rates on non-eligible assets (such as corporate debt) tend to be higher due to an (il-)liquidity premium. Given this separation, an announcement of future policy rate reductions is passed-through to Treasury rates in a more pronounced way than to interest rates on non-eligible assets, which serve as agents' preferred store of wealth due to their higher returns.

We show analytically that a reduction of the policy rate accompanied with an announcement to keep future policy rates low leads to a rise in the liquidity premium and moderate increases in output and inflation.[7] We decompose these effects into those of the reduction in the current policy rate and of the announcement of low future interest rates. The reduction of the current policy rate in isolation has conventional effects also in our model, i.e., it leads to surges in output, inflation, and consumption, while it also raises the liquidity premium.[8] The announcement of low future policy rates stimulates future output and also induces the liquidity premium to rise already today, whereas it dampens the immediate response of consumption and output.

The intuition for the rise of the liquidity premium in response to forward guidance announcements is fairly simple. The monetary policy rate determines the price of money in terms of eligible assets in our model. By announcing a lower future monetary policy rate, the central bank thus announces to provide more means of payment against a given amount of collateral. Thereby, the central bank increases the liquidity value (and hence the non-pecuniary return) of assets that are eligible as collateral for open-market operations. In an arbitrage-free equilibrium, non-eligible assets therefore have to provide a relatively higher pecuniary return, such that the interest rate spread between these asset classes, i.e., the liquidity premium, increases. Given that these less liquid assets actually serve as agents' stores of wealth, their consumption-saving decisions are not directly related to the policy rate, such that the rise in the liquidity premium tends to dampen aggregate demand and output compared to a case where liquidity premia are absent.

[6]While we abstract from an interbank market for federal funds, the model features a federal reserve treasury repo rate. We consider the latter as the policy rate, which is supported by the observation that it hardly differs from the federal funds rate, see Bredemeier et al. (2017).

[7]Notably, the increase in the liquidity premium relates to the increase in risk premia due to forward guidance in Caballero and Farhi (2017).

[8]The rise in the liquidity premium in response to a conventional, contemporaneous reduction in the policy rate is in line with the results of empirical analysis where we identify reactions to forward guidance controlling for changes in current policy rates which we find to increase liquidity premia.

In a quantitative analysis, we calibrate our model for US data with the objective to quantitatively match the response of the liquidity premium to an isolated forward guidance announcement that we identified in our econometric analysis. We then study the macroeconomic effects of announcements to keep the monetary policy rate 25 basis points below steady state for one or two years, respectively. We find that such an announcement triggers output to increase by about 0.1 percent relative to its steady state value from the time of the announcement until the policy rate is raised again. Compared to the prediction of a model version without a liquidity premium, which corresponds to a standard New Keynesian model, we find the immediate output effects of the four-quarter forward guidance in our model that features the liquidity premium to be seven times smaller. Moreover, the length of the guidance period hardly affects the impact output response, which again clearly differs from the prediction of a model version without liquidity premia. Overall, this confirms that a standard New Keynesian model vastly overestimates the output effects of forward guidance. By contrast, in our model with an endogenous liquidity premium, the quantitative predictions align well with empirical evidence. Specifically, Gertler and Karadi (2015) find that forward guidance has moderate and only delayed expansionary output effects, while the length of the guidance period hardly has any effect. Quantitatively, the output effects are slightly larger than in our model but substantially weaker than predicted by New Keynesian models.

This chapter relates to several empirical studies. The econometric analysis applied in this chapter is based on the approach of Gürkaynak et al. (2005), who analyze the effects of US monetary policy on asset prices using high-frequency data and show that forward guidance is capable of affecting bond yields and stock prices. Campbell et al. (2012) and Campbell, Fisher, Justiniano and Melosi (2016) extend this analysis to further assets and also to private sector forecasts of inflation and unemployment. While Campbell et al. (2012) find counterintuitive reactions of private sector expectations (for instance, unemployment expectations rise after an announced interest rate reduction), the findings of Campbell et al. (2016) are qualitatively consistent with predictions of the basic New Keynesian model. Quantitatively, though, the effects are considerably weaker than predicted by the New Keynesian model. Del Negro et al. (2015) also analyze the effects of forward guidance on forecasts and find that an announced 15 basis point decrease in short-term rates in 4 quarters leads to increases in GDP growth forecasts by about 0.3 percentage points. Gertler and Karadi (2015) analyze the effects of monetary policy shocks using a high-frequency identification procedure in a VAR that includes quarterly US data on real activity and various financial variables. Some of their results are suggestive for effects of forward guidance, which seem, however, to be quantitatively limited. Bundick and Smith

(2016) examine zero lower bound episodes and apply a high-frequency identification of forward guidance changes in futures contracts, which they use as shocks in a VAR with monthly data. D'Amico and King (2015) identify forward guidance shocks in their VAR based on sign restrictions. All of these papers find that forward guidance about future interest rate reductions tend to lead to moderate and gradual, rather than strong and sudden, increases in output and inflation that peak after a few quarters. These effects are consistent with the results of our quantitative analysis, but not with a standard New Keynesian model that predicts much stronger effects that peak immediately after the announcement.

The chapter further relates to theoretical studies that address the effects of forward guidance on macroeconomic outcomes. Del Negro et al. (2015) handle the excess response to policy announcements in the New Keynesian model by introducing a perpetual youth structure, which leads to a higher discounting of future events and thereby reduces current responses. McKay et al. (2016, 2017) show that the effects of forward guidance are much more limited in a model with heterogeneous agents that face the risk of hitting a borrowing constraint. A further set of papers by Carlstrom et al. (2015), Kiley (2016), and Chung, Herbst and Kiley (2015) demonstrate that the effects are dampened when firms are subject to sticky information instead of a direct sticky price friction, as this confines the forward-lookingness of the Phillips curve. Relatedly, Wiederholt (2015) shows that forward guidance has limited effects in a model where households have dispersed inflation expectations. Campbell et al. (2016) differentiate between Delphic and Odyssean forward guidance and find that the predictions of their medium scale model, in which government bond holdings provide direct utility, do not reflect the forward guidance puzzle. Caballero and Farhi (2017) construct a model where the economy is pushed to the zero lower bound because of a shortage of safe assets. In this model, forward guidance does not foster recovery, but only leads to increases in risk premia in their setting, which relates to the rise in liquidity premia implied by our model.

The remainder of this chapter is structured as follows. Section 3.2 provides empirical evidence on the response of liquidity premia on monetary policy announcements. Section 3.3 presents the model. We derive analytical results on forward guidance effects for a simplified version and present impulse responses for a calibrated version of the model in Section 3.4. A conclusion is given in Section 3.5.

3.2 Empirical Effects of Forward Guidance on Liquidity Premia

In this section, we document empirically that liquidity premia on near-money assets tend to rise in response to forward guidance announcements that financial markets consider to be accommodative. We explain how we measure the value of liquidity services of near-money assets in the data in Section 3.2.1. In Section 3.2.2, we provide an analysis of asset returns and interest rate spreads at all FOMC meeting dates between 1990 and 2016 using the approach of Gürkaynak et al. (2005). This method allows to separate the effects of unanticipated forward guidance announcements from those of simultaneously announced changes in other monetary policy instruments, such as the current federal funds rate or asset purchase programmes. We apply this approach to identify the response of liquidity premia to forward guidance.

3.2.1 Measurement of Liquidity Premia

We use various market-based measures of the value of liquidity services of near-money assets by calculating interest rate spreads between assets that differ in the degree of liquidity in financial markets, but feature similar characteristics in terms of safety and maturity. In this way, we rule out that movements in the spreads are mainly determined by differences in credit risk or term premia. As the measure for highly liquid near-money assets, we use US-Treasuries at various maturities. Those can be seen as close substitutes for money, as, typically, Treasuries are allowed to serve as collateral for obtaining liquidity from the Federal Reserve system. The less liquid assets that we consider were suggested and applied for this purpose in the related literature.

Specifically, we use the following spreads as measures of liquidity premia. Krishnamurthy and Vissing-Jorgensen (2012) state that the spread between high rated corporate bonds and Treasuries is primarily driven by liquidity. We therefore use the spreads between high rated commercial papers and corporate bonds with maturities of 3 months and 3, 5, and 10 years on the one hand and Treasuries of the same maturities on the other hand. As some credit risk may remain even in very high rated corporate bonds, we also follow Krishnamurthy and Vissing-Jorgensen (2012) in using spreads between relatively illiquid certificates of deposit (CD), which are very safe due to coverage by the Federal Deposit Insurance Corporation (FDIC), and Treasury bills at maturities of 3 and 6 months. Finally, we use the spread between the rate on 3-month general collateral repurchase agreements (GC repos, hereafter) and the 3-month T-bill. Nagel (2016) considers this spread to be a particularly clean measure of the value of liquidity, as the repos are secured by collateralization. We end up with 8 different spreads, for which we collect daily data with observations ranging from January 1990 to September 2016. A detailed description of the data set and the construction of the spreads is given Section 3.A.2 of the appendix.

We acknowledge that these spreads also contain a small noise component, for instance due to small remaining differences in credit risk or additional safety attributes of Treasuries as discussed by Krishnamurthy and Vissing-Jorgensen (2012). We therefore follow Del Negro et al. (2017) and construct a factor model with all spreads to extract their common component over time, which can be interpreted as a purified liquidity premium. This further yields the advantage of having one single summary measure for the value of liquidity. We calculate the liquidity factor for a sample from 1990-01-02 to 2016-09-16 using principle component analysis. To account for missing values in our data, we employ a method by Stock and Watson (2002) that relies on an expectation maximization algorithm.[9] To give the resulting factor f_t a quantitative interpretation as a measure of the liquidity premium in basis points, we assume that f_t is related to the liquidity premium LP_t by

$$LP_t = a + bf_t, \tag{3.1}$$

where a and b are unknown parameters. We apply the same assumptions as Del Negro et al. (2017) to obtain values for a and b. First, we assume that the average value of the liquidity premium before the outbreak of the financial crisis in July, 2007 equals 46 basis points. This number is a long-run estimate for the liquidity value of Treasuries by Krishnamurthy and Vissing-Jorgensen (2012) for a sample from 1926 to 2008. Second, Del Negro et al. (2017) argue that the asset in their sample with the highest spread to Treasuries at the peak of the crisis (a BBB rated bond, whose credit risk is hedged by a credit default swap) was essentially illiquid. The average size of this spread of 342 basis points in the last quarter of 2008 therefore gives a value for the liquidity premium at this time. Using these two assumptions, we can construct a daily time series for the liquidity premium in equation (3.1) that we plot in Figure 3.5 in the Appendix. The mean value of the liquidity premium in the figure reads 54 basis points with a standard deviation of 49 basis points (see also Table 3.4 in the Appendix). For a very short period at the height of the financial crisis, the premium rises up to values of about 450 basis points.[10] There

[9] As a robustness check for our treatment of missing values, we also calculated the common factor for the maximum balanced sample of our data, which ranges from 1997-01-02 to 2013-06-28. We find that the common factor is very similar to the one estimated on the whole data set.

[10] The focus of the analysis by Del Negro et al. (2017) lies on this episode in the end of 2008 as well as

are only a few days with a negative value for the liquidity premium in the whole sample of over 16 years, all of which occur in the first years of the 1990s. Figures 3.6 and 3.7 in the Appendix provide time series plots of all individual liquidity spreads along with a linear projection of the common factor and a constant on each spread. They show that the common liquidity factor captures a large part of the variation for the majority of the series.

3.2.2 Regression Analysis

We now analyze the effect of forward guidance on the valuation of liquidity in financial markets using the approach of Gürkaynak et al. (2005). This approach takes into account the following points. First, forward guidance announcements are usually given simultaneously with announcements about the federal funds rate or – at least in the years following the financial crisis – asset purchases, which requires to separate the individual effects. Second, since financial markets are forward looking, only unanticipated components of the policy changes should matter for market interest rates and spreads and hence those components need to be identified. Anticipated policy actions should already be priced into the markets ex ante, therefore leading to only limited reactions after publication. Ignoring this may mislead to concluding that a policy had no effect. Related to this issue, a by words accommodative policy announcement can actually have negative effects on markets when the press release was interpreted as bad news for the economy. Finally, the Federal Reserve can affect markets by refraining from taking action in a situation, where a policy adjustment was expected – i.e., also reactions on the non-appearance of a forward guidance announcement can be informative for the effects of forward guidance if such announcement had been expected by market participants. The method by Gürkaynak et al. (2005) addresses all of these identification issues and it allows to quantify the content of forward guidance announcements. We extend their analysis to the time period from January 1990 to September 2016 and to different types of assets and liquidity spreads.

The method extracts the surprise component of forward guidance announcements by looking at the changes in futures rates around FOMC meetings. Gürkaynak et al. (2005) show, based on work by Kuttner (2001), how federal funds and eurodollar futures data can be used for this purpose. After constructing such monetary surprise measures for futures with maturities between 1 and 12 months, we extract their first two principle components. A transformation of these two factors allows us to give them a structural interpretation. Following the terminology of Gürkaynak et al. (2005), we denote the first

its aftermath.

one as the "target factor", which measures the unanticipated change in the current federal funds rate, and the second one as the "path factor", which measures the unanticipated change of expectations about the path of the federal funds rate.[11] The path factor can be interpreted as a quantitative measure of forward guidance. In a last step, we regress the change of asset returns and our liquidity measures on the target and the path factor to study the effects of forward guidance.

In detail, we collect daily data on federal funds futures that expire in the current and the next 3 months as well as eurodollar futures with maturities of 6, 9 and 12 months around all FOMC meetings between January 1990 and September 2016.[12] Federal Funds futures settle at a rate that is calculated as the average daily effective federal funds funds rate for the delivery month. Changes of the current month futures rate will then reflect adjustments in the expectations of market participants about the federal funds rate in the rest of the month, while changes in futures rates with longer maturities reflect expectation adjustments about the federal funds rate in the month when the contract expires. In Appendix 3.A.3, we provide details on the futures data and we show how rate changes at FOMC dates need to be scaled with respect to the day of the month at which the meeting takes place, in order to extract the surprise component of the FOMC press release for current and future monetary policy. We follow the related literature to use eurodollar instead of federal funds futures for maturities of more than 6 months, as Gürkaynak, Sack and Swanson (2007) show that eurodollar futures provide a better measure of market expectations about future federal funds rates at those longer horizons.

We compile the surprise changes of the various futures in a matrix X of size $[T \times v]$, where T denotes the number of FOMC dates and v the number of different futures. Our sample covers $T = 237$ FOMC dates in total and we use $v = 5$ futures with maturities of 1, 3, 6, 9, and 12 months. Each row of X measures the expectation changes about monetary policy between the end-of-day value at the FOMC meeting date and the end-of-day value at the day before for the v futures. We then assume that X can be described by a factor model of the form

$$X = F\Lambda + \epsilon, \tag{3.2}$$

where F is a $[T \times f]$ matrix of $f < v$ unobserved factors, Λ is a $[f \times v]$ matrix of factor

[11] Swanson (2017) also uses the approach by Gürkaynak et al. (2005), but estimates three factors, giving the third one the interpretation to capture changes in asset purchase programmes. We also address the separate effect of quantatitive easing policies in our analysis, though in a different way.

[12] Gürkaynak et al. (2005) use intraday data with windows of 30 minutes around the FOMC meetings, which is not available to us. Using data at this high frequencies reduces the risk of endogeneity problems that can occur when other news of importance to financial markets are released at the day of the meeting. They show, however, that all of their results are highly robust to the usage of daily data.

loadings, and ϵ a $[T \times v]$ matrix of white noise. Using the same selection of futures, Gürkaynak et al. (2005) show that X is appropriately described by 2 factors. We therefore set $f = 2$ and, after demeaning and standardizing X, estimate two factors in F, named F_1 and F_2, by principle component analysis. Without further transformation, the factors F are a statistical decomposition that explains a maximal fraction of the variance of X, but they lack an economic interpretation. In order to give F a meaningful interpretation, we follow Gürkaynak et al. (2005) and rotate it according to

$$\widetilde{F} = FU, \tag{3.3}$$

where U is a $[2 \times 2]$ matrix, to obtain two new factors $\widetilde{F}_1$ and $\widetilde{F}_2$. In line with Gürkaynak et al. (2005), the elements of U are chosen such that the columns of $\widetilde{F}$ remain orthogonal to each other and that the second factor, $\widetilde{F}_2$, has no effect on the current federal funds rate.[13] This rotation implies that the unexpected change of the current target of the federal funds rate is tightly linked to $\widetilde{F}_1$, while $\widetilde{F}_2$ covers all other aspects of FOMC announcements that change the expectations about the path of the federal funds rate in the next 12 months. Following Gürkaynak et al. (2005), we name $\widetilde{F}_1$ the target factor and $\widetilde{F}_2$ the path factor, where the latter constitutes our quantitative measure of forward guidance shocks after FOMC meetings. We find the correlation between $\widetilde{F}_1$ and the first column of X, which measures the surprises in the current federal funds rate target, to be 0.93.[14] To allow for an interpretation in basis points, we normalize the elements of $\widetilde{F}$ as in Campbell et al. (2012), such that an increase of 0.01 in $\widetilde{F}_1$ corresponds to a surprise change of 1 basis point in the federal funds target and that an increase of 0.01 in $\widetilde{F}_2$ corresponds to a surprise change of 1 basis point in the 12-months ahead eurodollar futures rate.[15]

We now estimate the effect of the target and the path factor on the change of the asset returns and liquidity spreads with the regression model

$$\Delta y_t = \beta_0 + \beta_1 \widetilde{F}_{1,t} + \beta_2 \widetilde{F}_{2,t} + \beta_3 qe_t + e_t, \tag{3.4}$$

where Δy_t is the one-day change of an asset return or spread around the FOMC meeting

[13]Details on this transformation are given in Appendix 3.A.3.

[14]Notably, Gürkaynak et al. (2005), who apply a different sample period. report the almost identical value of 0.95.

[15]Given our estimate of the path factor, we can now rationalize our findings for the three dates discussed in the introduction. On 2011-08-09, the path factor assumes a value equivalent to a -2.3 standard deviation innovation, while the values on 2012-01-25 and 2012-09-13 read -1 and -0.4 standard deviations. This indicates that the forward guidance given on the first date was the least expected announcement of the three and thereby explains the relatively large response of asset returns and spreads on that date.

at time $t \in T$, β_0 is a constant, β_1 and β_2 are the coefficients on target and path factor, respectively, and e_t is an error term. β_3 is the coefficient on the dummy variable qe_t, which takes a value of 1 at FOMC meetings with important decisions regarding quantitative easing.[16] This variable ensures that our results are not driven by these events, which were shown, e.g. by Krishnamurthy and Vissing-Jorgensen (2011), to have affected financial markets considerably.

Results for the asset returns are given in Table 3.2. The first row shows the effect of a change in the current federal funds rate, as measured by the target factor, while the second row shows the effect of a change in forward guidance, as measured by the path factor. The coefficients can be interpreted in the following way. As an example, the return on the 1 year Treasury increases by 0.62% to a 1% increase of the target factor (which measures a 1% surprise increase of the current federal funds rate) and by 0.28% to a 1% increase of the path factor (which implies a 1% surprise interest rate increase in one year). For the Treasuries and the corporate bonds, the effect of changes in the current federal funds rate is very strong and highly significant for short maturities, but becomes smaller as the term to maturity increases. The opposite holds true for the effect of changes in forward guidance. Coefficients are relatively small for maturities below one year and then evolve hump-shaped over longer horizons with a peak at 5 years of remaining maturity. These results are in line with previous findings by Gürkaynak et al. (2005), Campbell et al. (2012), and Swanson (2017).[17] The explanatory power of the regressions, as measured by the R^2-statistic, also evolves hump-shaped with especially high values of about 0.80 in case of the Treasuries with longer maturities. The certificates of deposit and the GC repo react to the target factor in a similar fashion as the short-run commercial paper rate, while the response to the path factor is relatively small and mostly insignificant due to the relatively short maturities of these assets. The LIBOR does not react significantly on changes either in the current federal funds rate or in forward guidance. The limited relevance of monetary policy changes on bank rates is also reflected in relatively small values of R^2. Taken together, rates on Treasuries tend to react stronger to both $\widetilde{F}_1$ and $\widetilde{F}_2$ than the rates on the various less liquid assets at the same maturity.

This finding is confirmed in Table 3.3, which shows the response of our liquidity measures to the surprise changes in monetary policy. First and foremost, we present

[16]The variable qe_t takes a value of 1 at the following 6 dates. 2009-03-18: Announcement of QE1. 2010-11-03: Announcement of QE2. 2011-09-21: Announcement of "Operation Twist" 2012-09-13: Announcement of QE3. 2012-12-12: Announcement of additional long-term Treasury purchases. 2013-12-18: Begin to taper asset purchases.

[17]The absolute size of the coefficients can, however, not be compared one-to-one with all papers of the related literature due to differing unit normalizations of $\widetilde{F}_1$ and $\widetilde{F}_2$.

results on the liquidity premium from our factor model (3.1). We find that the premium reacts strongly on both, changes in the current and the expected path of the federal funds rate. A 1% reduction of the current federal funds rate target increases the valuation of liquidity by 0.41%, while the liquidity premium rises by 0.28% today to a 1%-reduction of the expected federal funds rate over the next year. Accordingly, markets value the liquidity property of near-money assets higher in response to all types of expansionary monetary policy. This finding constitutes the main result of our empirical analysis. Regressions of the individual spreads provide additional supportive evidence. In line with the relatively stronger response of Treasuries, observed in Table 3.2, coefficients on the target and path factor have a negative sign in the majority of cases. Also following the pattern of the asset returns, the coefficients as well as the significance of forward guidance changes become stronger for longer maturities, whereas the effect of the current federal funds rate on liquidity spreads is particularly strong for shorter maturities.

Note that Tables 3.5 and 3.6 in the Appendix repeat the above analysis for a sample excluding the recent zero lower bound episode (sample end in December 2008). Overall, the results are similar.

Table 3.2: Response of Asset Returns to Changes in Monetary Policy

	Treasuries					GC
	3M	1Y	3Y	5Y	10Y	3M
Change in Federal Funds Rate $\left(\widetilde{F}_1\right)$	0.65***	0.62***	0.33***	0.19***	0.028	0.31***
	(0.079)	(0.065)	(0.043)	(0.037)	(0.037)	(0.077)
Change in Forward Guidance $\left(\widetilde{F}_2\right)$	0.16***	0.38***	0.69***	0.79***	0.70***	0.0087
	(0.041)	(0.041)	(0.058)	(0.058)	(0.054)	(0.038)
R^2	0.54	0.76	0.81	0.84	0.77	0.23
Number of Observations (T)	237	237	237	237	237	213
	Commercial Paper / Corporate Bonds					CD
	3M	3Y	5Y	10Y(A)	10Y(B)	3M
Change in Federal Funds Rate $\left(\widetilde{F}_1\right)$	0.27**	0.38***	0.15**	-0.025	-0.0037	0.38***
	(0.11)	(0.11)	(0.060)	(0.040)	(0.031)	(0.14)
Change in Forward Guidance $\left(\widetilde{F}_2\right)$	0.034	0.52***	0.58***	0.39***	0.39***	0.15*
	(0.069)	(0.083)	(0.070)	(0.038)	(0.037)	(0.077)
R^2	0.10	0.43	0.65	0.50	0.50	0.22
Number of Observations (T)	122	165	165	237	237	212

Notes: Table shows responses of asset returns to changes in the federal funds rate, measured by the target factor, and to changes in forward guidance, measured by the path factor, at FOMC meetings between January 1990 and September 2016. Constant and QE-Dummy included in all regressions. Heteroskedasticity-robust (White) standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Maturity is measured either in months (M) or in years (Y). Corporate Bond 10Y(A) and (B) refer to long-term bonds with AAA and BAA rating, respectively. CD: Certificate of Deposit; GC: General Collateral Repo.

Table 3.3: Response of Liquidity Spreads to Changes in Monetary Policy

	Liquidity Premium	Commercial Paper / Corporate Bond - Spread				
	Common Factor	3M	3Y	5Y	10Y(A)	10Y(B)
Change in Federal Funds Rate $\left(\widetilde{F}_1\right)$	-0.41***	-0.30***	0.15*	0.043	-0.053	-0.032
	(0.13)	(0.11)	(0.088)	(0.055)	(0.056)	(0.042)
Change in Forward Guidance $\left(\widetilde{F}_2\right)$	-0.28***	-0.11*	-0.13***	-0.17***	-0.30***	-0.31***
	(0.059)	(0.068)	(0.048)	(0.035)	(0.037)	(0.035)
R^2	0.29	0.10	0.09	0.19	0.46	0.51
Number of Observations (T)	237	122	165	165	237	237

	GC-Spread	CD-Spread	
	3M	3M	6M
Change in Federal Funds Rate $\left(\widetilde{F}_1\right)$	-0.37**	-0.26*	-0.35*
	(0.15)	(0.15)	(0.18)
Change in Forward Guidance $\left(\widetilde{F}_2\right)$	-0.16***	-0.011	-0.11
	(0.060)	(0.084)	(0.12)
R^2	0.55	0.21	0.41
Number of Observations (T)	213	212	212

Notes: Table shows responses of liquidity spreads to changes in the federal funds rate, measured by the target factor, and to changes in forward guidance, measured by the path factor, at FOMC meetings between January 1990 and September 2016. Constant and QE-Dummy included in all regressions. Heteroskedasticity-robust (White) standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Maturity is measured either in months (M) or in years (Y). Corporate Bond 10Y(A) and (B) refer to long-term bonds with AAA and BAA rating, respectively. CD: Certificate of Deposit; GC: General Collateral Repo. All spreads are calculated relative to Treasuries of the same maturity.

3.3 The Model

In this section, we present a macroeconomic New Keynesian model with an endogenous liquidity premium for the analysis of forward guidance, which is based on Schabert (2015) and Bredemeier, Juessen and Schabert (2017), from which we adopt most of the notation. To endogenize the liquidity premium, we consider commercial banks that demand high powered money, i.e., reserves, that are supplied by the central bank via open market operations against eligible securities to serve withdrawals of demand deposits, which relate to households' goods market transactions. Our model distinguishes between several assets in order to account for rates of return, which respond differently to forward guidance shocks in the data. Decisively, assets differ with respect to liquidity, i.e. to their ability to serve as substitutes for central bank money. The price of reserves equals the monetary policy rate and is set by the central bank. The interest rate on eligible assets (i.e. Treasury bills) is closely related to the policy rate, as they are close substitutes to central bank money, whereas interest rates on non-eligible assets differ by a liquidity premium. Given that the latter assets (rather than money or Treasury bills) actually serve as agents' store of value, their real interest rates reflect private agents' intertemporal consumption and investment choices. To isolate the main mechanism, we neither model frictions that justify the existence of banks nor other financial market frictions. In fact, the model is constructed to feature only a single non-standard element in form of the liquidity premium.

In each period, the timing of events in the economy, which consists of households, banks, intermediate goods producing firms, retailers, and the public sector unfolds as follows: At the beginning of each period, aggregate shocks materialize. Then, banks can acquire reserves from the central bank via open market operations. Subsequently, the labor market opens, goods are produced, and the goods market opens, where money is used as a means of payment. At the end of each period, the asset market opens. Throughout the paper, upper case letters denote nominal variables and lower case letters real variables.

3.3.1 Households

There is a continuum of infinitely lived and identical households of mass one. It maximizes the expected sum of a discounted stream of instantaneous utilities $u_t = u(c_t, n_t)$,

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \tag{3.5}$$

where $u(c_t, n_t) = [c_t{}^{1-\sigma}/(1-\sigma)] - \theta n_t^{1+\sigma_n}/(1+\sigma_n)$ with $\sigma \geq 1$, σ_n, $\theta \geq 0$. c_t denotes consumption, n_t working time, E_0 the expectation operator conditional on the time 0 information set, and $\beta \in (0,1)$ is the subjective discount factor. Households can store their wealth in shares of firms $z_t \in [0,1]$ valued at the price V_t with the initial stock of shares $z_{-1} > 0$. The budget constraint of the household reads

$$\left(D_t/R_t^D\right) + V_t z_t + P_t c_t + P_t \tau_t \leq D_{t-1} + (V_t + P_t \varrho_t) z_{t-1} + P_t w_t n_t + P_t \varphi_t, \tag{3.6}$$

where P_t denotes the price level, w_t the real wage rate, τ_t a lump-sum tax, ϱ_t dividends from intermediate goods producing firms, φ_t profits from banks and retailers, and D_t demand deposits that are offered by a banking sector at the price $1/R_t^D$. We assume that households rely on money for purchases of consumption goods, while we abstract from purchases of goods via credit for simplicity. To purchase goods, households could in principle hold cash, which is dominated by the rate of return of other assets. Instead, we consider the demand deposits to serve the same purpose. Households typically hold more deposits than necessary for consumption expenditures such that the goods market constraint, which resembles a standard cash in advance constraint, can be summarized as

$$P_t c_t \leq \mu D_{t-1}, \tag{3.7}$$

where $D_{t-1} \geq 0$ denotes holdings of bank deposits at the beginning of period t and $\mu \in [0,1]$ denotes an exogenously determined fraction of deposits withdrawn by the representative household. Given that households can withdraw deposits at any point in time, they have no incentive to hold non-interest-bearing money. Maximizing the objective (3.5) subject to the budget constraint (3.6), the goods market constraint (3.7), and $z_t \geq 0$ for given initial values leads to the following first-order conditions for working time, consumption, shares, and deposits:

$$\begin{aligned} -u_{n,t} &= w_t \lambda_t, & (3.8)\\ u_{c,t} &= \lambda_t + \psi_t, & (3.9)\\ \beta E_t\left[\lambda_{t+1} R_{t+1}^q \pi_{t+1}^{-1}\right] &= \lambda_t, & (3.10)\\ \beta E_t\left[\left(\lambda_{t+1} + \mu \psi_{t+1}\right) \pi_{t+1}^{-1}\right] &= \lambda_t / R_t^D, & (3.11) \end{aligned}$$

where $u_{n,t} = \partial u_t / \partial n_t$ and $u_{c,t} = \partial u_t / \partial c_t$ denote marginal (dis-)utility from labor and consumption, $R_t^q = (V_t + P_t \varrho_t)/V_{t-1}$ is the nominal rate of return on equity, and λ_t and ψ_t denote the multipliers on the budget constraint (3.6) and the goods market constraint

(3.7). Finally, the complementary slackness conditions that hold in the household's optimum are $0 \leq \mu d_{t-1}\pi_t^{-1} - c_t$, $\psi_t \geq 0$, $\psi_t \left(\mu d_{t-1}\pi_t^{-1} - c_t\right) = 0$, where $d_t = D_t/P_t$, as well as (3.6) with equality and associated transversality conditions. Under a binding goods market constraint (3.7) that implies $\psi_t > 0$, the deposit rate tends to be lower than the expected return on equity (see 3.10 and 3.11), as demand deposits provide transaction services.

3.3.2 Commercial Banks

There is a continuum of perfectly competitive banks $i \in [0,1]$. A bank i receives demand deposits $D_{i,t}$ from households and supplies risk-free loans to firms $L_{i,t}$ at the price $1/R_t^L$. Bank i further holds short-term government debt (i.e., treasury bills) $B_{i,t-1}$ and reserves $M_{i,t-1}$ for withdrawals of deposits by households. The central bank supplies reserves via open market operations either outright or temporarily under repurchase agreements. The latter correspond to a collateralized loan offered by the central bank. In both cases, treasury bills serve as collateral for central bank money, while the price of reserves in open market operations in terms of treasuries (the repo rate) equals R_t^m. Specifically, reserves are supplied by the central bank only in exchange for treasuries $\Delta B_{i,t}^C$, while the price of money is the repo rate R_t^m:

$$I_{i,t} = \Delta B_{i,t}^C / R_t^m \quad \text{and} \quad \Delta B_{i,t}^C \leq B_{i,t-1}, \tag{3.12}$$

where $I_{i,t}$ denotes additional money received from the central bank. Hence, (3.12) describes a central bank money supply constraint, which shows that a bank i can acquire reserves $I_{i,t}$ in exchange for the discounted value of Treasury bills carried over from the previous period $B_{i,t-1}/R_t^m$. We abstract from modelling an interbank market for overnight loans in terms of reserves and the associated (federal funds) rate and assume – consistent with US data (see Bredemeier et al., 2017) – that the Treasury repo rate and the federal funds rate are identical, implying that the central bank sets the repo rate R_t^m. Reserves are demanded by bank i to meet liquidity demands from withdrawals of deposits

$$\mu D_{i,t-1} \leq I_{i,t} + M_{i,t-1}. \tag{3.13}$$

By imposing the constraint (3.13), we implicitly assume that a reserve requirement is either identical to the expected withdrawals or slack. Banks supply one-period risk-free loans $L_{i,t}$ to firms at a period t price $1/R_t^L$ and a payoff $L_{i,t}$ in period $t+1$. Thus, R_t^L denotes the rate at which firms can borrow. Banks can further invest in short-term government bonds that are issued at the price $1/R_t$, which are eligible for open market operations. Given that bank i transferred T-bills to the central bank under outright sales and that it repurchases a fraction of T-bills, $B_{i,t}^R = R_t^m M_{i,t}^R$, from the central bank, its holdings of T-bills before it enters the asset market equal $B_{i,t-1} + B_{i,t}^R - \Delta B_{i,t}^C$ and its money holdings equal $M_{i,t-1} - R_t^m M_{i,t}^R + I_{i,t}$. Hence, bank i's profits $P_t \varphi_{i,t}^B$ are given by

$$
\begin{aligned}
P_t \varphi_{i,t}^B \;=\;& \left(D_{i,t}/R_t^D\right) - D_{i,t-1} - M_{i,t} + M_{i,t-1} - I_{i,t}\left(R_t^m - 1\right) \\
& - \left(B_{i,t}/R_t\right) + B_{i,t-1} - \left(L_{i,t}/R_t^L\right) + L_{i,t-1}.
\end{aligned} \tag{3.14}
$$

Notably, the aggregate stock of reserves only changes with the central bank money supply, $\int_0^1 M_{i,t} di = \int_0^1 (M_{i,t-1} + I_{i,t} - M_{i,t}^R) di$, and is fully backed by Treasury bills, whereas demand deposits can be created by the banking sector subject to (3.13). Banks maximize the sum of discounted profits, $E_t \sum_{k=0}^{\infty} p_{t,t+k} \varphi_{i,t+k}^B$, where $p_{t,t+k}$ denotes the stochastic discount factor $p_{t,t+k} = \beta^k \lambda_{t+k}/\lambda_t$, subject to the money supply constraint (3.12), the liquidity constraint (3.13), the budget constraint (3.14), and the borrowing constraints $\lim_{s\to\infty} E_t[p_{t,t+k} D_{i,t+s}/P_{t+s}] \geq 0$, $B_{i,t} \geq 0$, and $M_{i,t} \geq 0$. The first-order conditions with respect to deposits, T-bills, corporate and interbank loans, money holdings, and reserves can be written as

$$
\frac{1}{R_t^D} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1 + \mu \varkappa_{i,t+1}}{\pi_{t+1}}, \tag{3.15}
$$

$$
\frac{1}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1 + \eta_{i,t+1}}{\pi_{t+1}}, \tag{3.16}
$$

$$
\frac{1}{R_t^L} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}^{-1}, \tag{3.17}
$$

$$
1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1 + \varkappa_{i,t+1}}{\pi_{t+1}}, \tag{3.18}
$$

$$
\varkappa_{i,t} + 1 = R_t^m \left(\eta_{i,t} + 1\right), \tag{3.19}
$$

where $\eta_{i,t}$ and $\varkappa_{i,t}$ denote the multipliers on the money supply constraint (3.12) and the liquidity constraint (3.13), respectively. Further, the following complementary slackness conditions hold in the bank's optimum *i.*) $0 \leq b_{i,t-1}\pi_t^{-1} - R_t^m i_{i,t}$, $\eta_{i,t} \geq 0$, $\eta_{i,t}\left(b_{i,t-1}\pi_t^{-1} - R_t^m i_{i,t}\right) = 0$, and *ii.*) $0 \leq i_{i,t} + m_{i,t-1}\pi_t^{-1} - \mu d_{i,t-1}\pi_t^{-1}$, $\varkappa_{i,t} \geq 0$, $\varkappa_{i,t}\left(i_{i,t} + m_{i,t-1}\pi_t^{-1} - \mu d_{i,t-1}\pi_t^{-1}\right) = 0$, where $d_{i,t} = d_{i,t}/P_t$, $m_{i,t} = M_{i,t}/P_t$, $b_{i,t} = B_{i,t}/P_t$, and $i_{i,t} = I_{i,t}/P_t$, and the associated transversality conditions.

3.3.3 Production Sector

The production sector of the economy consists of intermediate goods producing firms, which sell their goods to monopolistically competitive retailers that are subject to a Calvo-type sticky price friction. The retailers sell a differentiated good to bundlers, who assemble final goods using a Dixit-Stiglitz technology.

The intermediate goods producing firms are identical, perfectly competitive, owned by the households, and produce an intermediate good y_t^m with labor n_t according to the production function

$$y_t^m = n_t^\alpha,$$

with the labor elasticity of production α. They sell the intermediate good to retailers at the price P_t^m. We neglect retained earnings and assume that firms rely on bank loans to finance wage outlays before goods are sold. The firms' loan demand satisfies

$$L_t/R_t^L \geq P_t w_t n_t. \tag{3.20}$$

Firms are committed to fully repay their liabilities, such that bank loans are default risk-free. The problem of a representative firm can then be summarized as $\max E_t \sum_{k=0}^{\infty} p_{t,t+k}\varrho_{t+k}$, where ϱ_t denotes real dividends $\varrho_t = (P_t^m/P_t)n_t^\alpha - w_t n_t - l_{t-1}\pi_t^{-1} + l_t/R_t^L$, subject to (3.20). The first-order conditions for loan and labor demand are then given by

$$\begin{aligned} 1+\gamma_t &= R_t^L E_t[p_{t,t+1}\pi_{t+1}^{-1}], & (3.21)\\ P_t^m/P_t \alpha n_t^{\alpha-1} &= (1+\gamma_t)\, w_t, & (3.22) \end{aligned}$$

where γ_t denotes the multiplier on the loan demand constraint (3.20). Given that we abstract from financial market frictions, the Modigliani-Miller theorem applies here, such that the multiplier γ_t equals zero. This can immediately be seen from combining the banks' loan supply condition (3.17) with the firm's loan demand condition (3.21), which implies $\gamma_t = 0$. Hence, the loan demand constraint (3.20) is slack, such that the firm's labor demand (3.22) will be undistorted and read $P_t^m/P_t = w_t/\left(\alpha n_t^{\alpha-1}\right)$.

Monopolistically competitive retailers, indexed with $k \in [0,1]$ buy intermediate goods y_t^m at the price P_t^m to relabel them to a good $y_{k,t}$. The latter are sold at a price $P_{k,t}$ to perfectly competitive bundlers. Only a random fraction $1-\phi$ of the retailers is able to reset their price $P_{k,t}$ in an optimizing way each period., while the remaining retailers of mass ϕ have to keep the price of the previous period, $P_{k,t} = P_{k,t-1}$. The problem of a price adjusting retailer reads $\max_{\widetilde{P}_{k,t}} E_t \sum_{s=0}^{\infty} \phi^s \beta^s \phi_{t,t+s} \left(\left(\Pi_{k=1}^s \widetilde{P}_{k,t}/P_{t+s}\right) - mc_{t+s}\right) y_{k,t+s}$, where marginal costs are $mc_t = P_t^m/P_t$. The first-order condition can be written as $\tilde{Z}_t = \frac{\varepsilon}{\varepsilon-1}Z_t^1/Z_t^2$, where $\tilde{Z}_t = \widetilde{P}_t/P_t$, $Z_t^1 = \xi_t c_t^{-\sigma} y_t mc_t + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1$ and $Z_t^2 = \xi_t c_t^{-\sigma} y_t + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2$.

The perfectly competitive bundlers combine the various $y_{k,t}$ to the final consumption good y_t using the technology $y_t^{\frac{\varepsilon-1}{\varepsilon}} = \int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}} dk$, where $\varepsilon > 1$ is the elasticity of substitution between the different varieties. The cost minimizing demand for each good is given by $y_{k,t} = (P_{k,t}/P_t)^{-\varepsilon} y_t$. The bundlers sell the final good y_t to the households at the price P_t, which can be written as the consumer price index (CPI) $P_t^{1-\varepsilon} = \int_0^1 P_{k,t}^{1-\varepsilon} dk$.

The evolution of this price index equals $1 = (1-\phi)\tilde{Z}_t^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}$. In a symmetric equilibrium, $y_t^m = \int_0^1 y_{k,t} dk$ and $y_t = a_t n_t^{\alpha}/s_t$ will hold, where $s_t = \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} dk$ is an index of price dispersion that evolves according to $s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1} (\pi_t)^{\varepsilon}$ for a given s_{-1}.

3.3.4 Public Sector

The public sector consists of a government and a central bank. The government issues one-period bonds B_t^T and obtains potential profits of the central bank $P_t\tau_t^m$. Revenues beyond those used to repay debt from last period are transferred to the households in a lump-sum fashion, $P_t\tau_t$, to balance the budget. The government budget constraint is then given by

$$\left(B_t^T/R_t\right) + P_t\tau_t^m = B_{t-1}^T + P_t\tau_t.$$

Given that one period equals one quarter in our setting, this debt corresponds to 3-month Treasury bills. Government debt is held by banks in the amount of B_t and by the central bank in the amount of B_t^C, such that $B_t^T = B_t + B_t^C$. Following Schabert (2015), we assume that the supply of Treasury bills is exogenously determined by a constant growth rate Γ

$$B_t^T = \Gamma B_{t-1}^T, \tag{3.23}$$

where $\Gamma > \beta$. (3.23) describes the supply of money market instruments that the central bank declares eligible. There is only short-term government debt in the model for simplicity. To appropriately account for the role of long-term Treasury debt, which in particular have been purchases by the US Federal reserve in their large scale asset purchase programmes, we would specify them as partially eligible for central bank operations. It can be shown in a straightforward way that the associated yields would then behave like a combination of the T-bill rate and corporate debt rate.

The central bank supplies money in exchange for Treasury bills either outright, M_t, or under repos M_t^R. At the beginning of each period, the central bank's stock of Treasuries equals B_{t-1}^C and the stock of outstanding money equals M_{t-1}. It then receives an amount ΔB_t^C of Treasuries in exchange for newly supplied money $I_t = M_t - M_{t-1} + M_t^R$. After repurchase agreements are settled, its holdings of Treasuries and the amount of outstanding money are reduced by B_t^R and by M_t^R, respectively. Before the asset market opens, where the central bank can reinvest its payoffs from maturing securities in T-bills B_t^C, it holds an amount equal to $B_{t-1}^C + \Delta B_t^C - B_t^R$. Its budget constraint is thus given by $\left(B_t^C/R_t\right) + P_t\tau_t^m = \Delta B_t^C + B_{t-1}^C - B_t^R + M_t - M_{t-1} - \left(I_t - M_t^R\right)$, which after substituting out I_t, B_t^R, and ΔB_t^C using $\Delta B_t^C = R_t^m I_t$, can be simplified to $\left(B_t^C/R_t\right) - B_{t-1}^C = R_t^m \left(M_t - M_{t-1}\right) + \left(R_t^m - 1\right) M_t^R - P_t\tau_t^m$. Following central bank practice, we assume that interest earnings are transferred to the government, $P_t\tau_t^m = B_t^C \left(1 - 1/R_t\right) + \left(R_t^m - 1\right)\left(M_t - M_{t-1} + M_t^R\right)$, such that holdings of Treasuries evolve according to $B_t^C - B_{t-1}^C = M_t - M_{t-1}$. Restricting the initial values to $B_{-1}^C = M_{-1}$ leads to the central bank balance sheet

$$B_t^C = M_t. \tag{3.24}$$

Regarding the implementation of monetary policy, we assume that the central bank sets the policy rate R_t^m following a Taylor-type feedback rule, while respecting the zero lower bound:

$$R_t^m = \max\left\{1; \left[R_{t-1}^m\right]^{\rho_R} \left[R_m \left(\frac{\pi_t}{\pi}\right)^{\rho_\pi} \left(\frac{y_t}{\widetilde{y}_t}\right)^{\rho_y}\right]^{1-\rho_R} \exp\left(\varepsilon_t^m \cdot \prod_{k=1}^{K} \varepsilon_{t,t-k}^m\right)\right\}, \tag{3.25}$$

where $\widetilde{y}_t$ is the efficient level of output, $\rho_\pi \geq 0, \rho_y \geq 0$, $0 \leq \rho_R < 1$, $R^m \geq 1$, and ε_t^m denotes a contemporaneous monetary policy shock. Following Laséen and Svensson (2011), $\prod \varepsilon_{t,t-k}^m$ describes a series of anticipated policy shocks, which materialize in period t, but were announced in period $t-k$, that are used to model forward guidance.

The target inflation rate π is controlled by the central bank and is assumed to equal the growth rate of Treasuries Γ. This assumption is made for convenience only and is not associated with a loss of generality, as the central bank can implement its inflation targets even if $\pi \neq \Gamma$, as shown in Schabert (2015). Finally, the central bank fixes the fraction of money supplied under repurchase agreements relative to money supplied outright at $\Omega \geq 0 : M_t^R = \Omega M_t$. For the subsequent analysis, Ω will be set at a sufficiently large value to ensure that central bank money injections I_t are non-negative.

3.3.5 Equilibrium Properties

Given that households, firms, retailers, and banks behave in an identical way, we can omit indices. A definition of the rational expectations equilibrium can be found in Appendix 3.B. It should be noted that the Modigliani-Miller theorem applies here as financial markets are frictionless. The main difference to a standard New Keynesian model is the money supply constraint (3.12), which ensures that reserves are fully backed by Treasuries. The model in fact reduces to a New Keynesian model with a conventional cash-in-advance constraint if the money supply constraint (3.12) is slack, which is summarized in Definition 3 in Appendix 3.B.[18]

In our model, rates of return on non-eligible assets (i.e., corporate debt and equity) exceed the policy rate and the Treasury rate by a liquidity premium if (3.12) is binding. This is the case when the central bank supplies money at a lower price than households are willing to pay, $R_t^m < R_t^{IS}$, where R_t^{IS} denotes the nominal marginal rate of intertemporal substitution of consumption

$$R_t^{IS} = u_{c,t}/\beta E_t \left(u_{c,t+1}/\pi_{t+1} \right), \tag{3.26}$$

which measures the marginal valuation of money by the private sector.[19] For $R_t^m < R_t^{IS}$, households thus earn a positive rent and are willing to increase their money holdings. Given that access to money is restricted by holdings of Treasury bills, the money supply constraint (3.12) is then binding. To see this, compare (3.15) with (3.11) to get $E_t[\frac{\lambda_{t+1}+\mu\psi_{t+1}}{\lambda_t}\pi_{t+1}^{-1}] = E_t[\frac{\lambda_{t+1}}{\lambda_t}\left(1+\varkappa_{t+1}\mu\right)\pi_{t+1}^{-1}]$, which is satisfied if $\varkappa_t = \psi_t/\lambda_t$. Hence, the equilibrium versions of the conditions (3.18) and (3.19) imply $\left(\psi_t + \lambda_t\right)/\lambda_t = R_t^m \left(\eta_t + 1\right)$ and $\beta\pi_{t+1}^{-1}\left(\lambda_{t+1} + \psi_{t+1}\right) = \lambda_t$, which can – by using the equilibrium version of condition (3.9) – be combined to

$$\eta_t = \left(R_t^{IS}/R_t^m \right) - 1. \tag{3.27}$$

Condition (3.27) implies that the money supply constraint (3.12) is binding, i.e. $\eta_t > 0$, if the central bank sets the policy rate R_t^m below R_t^{IS}. Given that short-term Treasuries and money are close substitutes, the T-bill rate R_t relates to the expected future policy rate, which can be seen from combining (3.16) with (3.18) and (3.19), $R_t \cdot E_t\varsigma_{1,t+1} = E_t[R_{t+1}^m \cdot \varsigma_{1,t+1}]$, where $\varsigma_{1,t+1} = \lambda_{t+1}\left(1+\eta_{t+1}\right)/\pi_{t+1}$. Thus, the Treasury bill rate equals

[18] It should be noted that a binding money supply constraint does not imply that monetary policy is inferior compared to a regime, where money is supplied in an unbounded way, as shown by Schabert (2015).

[19] Agents are willing to spend $R_t^{IS} - 1$ to transform one unit of an illiquid asset, i.e. an asset that is not accepted as a means of payment today and delivers one unit of money tomorrow, into one unit of money today.

the expected policy rate up to first order,

$$R_t = E_t R^m_{t+1} + \text{h.o.t.}, \tag{3.28}$$

where h.o.t. represents higher order terms. Notably, the relation (3.28) accords to the empirical evidence provided by Simon (1990). The bank's first order conditions (3.15), (3.17), and (3.18) further imply that the deposit rate R^D_t exceeds one and is smaller than the interest rates on loans to firms R^L_t when liquidity is positively valued, i.e., if $\psi_t > 0$. Combining (3.17), with $\beta E_t \pi^{-1}_{t+1} (\lambda_{t+1} + \psi_{t+1}) = \lambda_t$ (see 3.17) shows that the loan rate R^L_t relates to the expected marginal rate of intertemporal substitution $(1/R^L_t) \cdot E_t \varsigma_{2,t+1} = E_t[(1/R^{IS}_{t+1}) \cdot \varsigma_{2,t+1}]$, where $\varsigma_{2,t+1} = (\lambda_{t+1} + \psi_{t+1})/\pi_{t+1}$. Likewise, (3.11) implies that the expected rates of return on equity is related to the expected marginal rate of intertemporal substitution: $E_t \varsigma_{2,t+1} = E_t \left[\left(R^q_{t+1}/R^{IS}_{t+1} \right) \cdot \varsigma_{2,t+1} \right]$. Hence, the loan rate equals to the expected marginal rate of intertemporal substitution up to first order,

$$R^L_t = E_t R^{IS}_{t+1} + \text{h.o.t.}, \tag{3.29}$$

as well as to the expected rate of return on equity, $E_t R^q_{t+1} = E_t R^{IS}_{t+1} +$ h.o.t. Accordingly, the spread between the marginal rate of intertemporal substitution and the monetary policy rate, $R^{IS}_t - R^m_t$, captures how rates of return of non-eligible assets deviate from the monetary policy rate and summarizes how interest rates in the current model differ from those of a standard model. Accordingly, $R^{IS}_t - R^m_t$ constitutes an endogenous liquidity premium. When we derive analytical results in the subsequent section, we therefore focus on the difference between R^{IS}_t and R^m_t to unveil the main mechanism at work.

It should further be noted that as long as the nominal marginal rate of intertemporal substitution R^{IS}_t (rather than the policy rate R^m_t) does not hit the zero lower bound, i.e., $R^{IS}_t > 1$, the demand for money is well defined, as the liquidity constraints of households (3.7) and banks (3.13) are binding. This can be seen by substituting out $\varkappa_t$ in the equilibrium version of (3.18) with $\varkappa_t = \psi_t/\lambda_t$ and combining with the equilibrium version of (3.9), which leads to

$$\psi_t = u_{c,t} \left(1 - 1/R^{IS}_t \right). \tag{3.30}$$

Thus, (3.30) implies that the household's liquidity constraint (3.7) as well as the bank's liquidity constraint (3.13) are binding if R^{IS}_t is strictly larger than one. Notably, liquidity might still be positively valued by households and banks, i.e., $R^{IS}_t > 1$, even when the policy rate is at the zero lower bound, $R^m_t = 1$.

3.4 The Effect of Forward Guidance in the Model

In this section, we examine the models' predictions regarding the macroeconomic effects of forward guidance. We begin with deriving some analytical results in Section 3.4.1. Subsequently, we calibrate the model and study its quantitative predictions in Section 3.4.2. Throughout these sections, we separately analyze two versions of the model, which differ with regard to the relation between the monetary policy rate and the marginal rate of intertemporal substitution.

3.4.1 Analytical Results

We separately analyze the cases where the money supply constraint (3.12) is either binding, which leads to an endogenous liquidity premium, or where money supply is de-facto unconstrained, implying that the policy rate R_t^m equals the marginal rate of intertemporal substitution R_t^{IS}. Technically, this means that we assume that the central bank sets the policy rate in the long-run either below or equal to $R^{IS} = \pi/\beta$ (where time indices are omitted to indicate steady state values) and examine the local dynamics in the neighborhood of the particular steady state.[20] In a neighborhood of a steady state, the equilibrium sequences are approximated by the solutions to the linearized equilibrium conditions, where $\widehat{a}_t$ denotes relative deviations of a generic variable a_t from its steady state value $a : \widehat{a}_t = \log(a_t/a)$. To facilitate the derivation of analytical results, we assume that outright money supply is negligible, $\Omega \to \infty$, which reduces the set of endogenous state variables. We further assume for convenience that the central bank targets long-run price stability $\pi = 1$, which is further supported by the supply of eligible government debt $\Gamma = 1$[21].

Definition 1. *A rational expectations equilibrium for $\Omega \to \infty$, $\Gamma = \pi = \alpha = 1$, and $\rho_{R,y} = 0$ is a set of convergent sequences $\{\widehat{c}_t,\, \pi_t,\, \widehat{b}_t,\, \widehat{R}_t^{IS},\, \widehat{R}_t^m\}_{t=0}^{\infty}$ satisfying*

$$\widehat{c}_t = \widehat{b}_{t-1} - \widehat{\pi}_t - \widehat{R}_t^m \text{ if } R_t^m < R_t^{IS}\ , \tag{3.31}$$

$$\text{or } \widehat{c}_t \leq \widehat{b}_{t-1} - \widehat{\pi}_t - \widehat{R}_t^m \text{ if } R_t^m = R_t^{IS},$$

$$\sigma\widehat{c}_t = \sigma E_t\widehat{c}_{t+1} - \widehat{R}_t^{IS} + E_t\widehat{\pi}_{t+1}, \tag{3.32}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \chi\left[(\sigma_n + \sigma)\,\widehat{c}_t + \widehat{R}_t^{IS}\right], \tag{3.33}$$

[20] We further assume that shocks are sufficiently small such that the ZLB is never binding.

[21] Notably, the latter assumption is not necessary for the implementation of long-run price stability, since the central bank can in principle adjust the share of short-term treasuries that are eligible for money supply operations to implement the desired inflation target, as shown by Schabert (2015).

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{3.34}$$

where $\chi = (1-\phi)(1-\beta\phi)/\phi$ *for a monetary policy rate satisfying*

$$\widehat{R}_t^m = \rho_\pi \widehat{\pi}_t + \widehat{\varepsilon}_t^m + \sum_{k=1}^{K} \widehat{\varepsilon}_{t,t-k}^m, \tag{3.35}$$

where $\rho_\pi \geq 0$*, for a given* $b_{-1} > 0$.

Consider first the case, where the money supply constraint (3.12) is not binding, such that the policy rate equals the marginal rate of intertemporal substitution, $R_t^m = R_t^{IS}$, and there is no liquidity premium. This will be the case if eligible assets are supplied abundantly or if there are no collateral requirements in open market operations. Given that condition (3.31) is then slack, the model reduces to a standard New Keynesian model with a cash-in-advance constraint. This constraint implies that the policy rate affects the marginal rate of substitution between consumption and working time and therefore enters the aggregate supply constraint (3.33). In this setting, forward guidance exerts the stark effects that were criticized in the literature (see Del Negro et al., 2015), such as large initial output and inflation effects as well as cumulative output responses that are growing in the horizon of forward guidance.

In case the policy rate is set below the marginal rate of intertemporal substitution, i.e. $R_t^m < R_t^{IS}$, the money supply constraint and, hence, (3.31) is binding, which implies a positive liquidity premium. As shown by Bredemeier et al. (2017), there exist unique locally convergent equilibrium sequences, if but not only if

$$\rho_\pi < [(1+\beta)\chi^{-1} + 1 - \sigma]/\sigma \tag{3.36}$$

is satisfied. Condition (3.36) implies that an active monetary policy ($\rho_\pi > 1$) is not relevant for equilibrium determinacy and that the central bank can even peg the policy rate ($\rho_\pi = 0$) without inducing indeterminacy. It should further be noted that the sufficient condition (3.36) is far from being restrictive for a broad range of reasonable parameter values.

Forward guidance announcements of the FOMC in the last years stated to keep policy rates at low levels for a specific period of time. To assess the effect of this kind of forward guidance in our model, we consider the following simple experiment: The central bank announces in period t to reduce the policy rate for the periods t and $t+1$. Formally, this forward guidance consists of to components: a shock to the policy rate in t, i.e. $\widehat{\varepsilon}_t^m < 0$, and a shock in $t+1$ that is announced in t, i.e. $\widehat{\varepsilon}_{t+1,t}^m < 0$ and $K = 1$ in (3.35). For the linearized model given in Definition 1, we are able to present some analytical results for this experiment that we summarize in the following proposition.[22]

Proposition 1. *Suppose that* $R_t^m < R_t^{IS}$, $\sigma = \sigma_n = 1$, *and* $\rho_\pi < \beta\chi^{-1}$ *which guarantees that (3.36) is satisfied. The effect of a forward guidance announcement in period* t *that reduces the monetary policy rate in* t *and* $t+1$ *can be separated into the partial effects of a conventional monetary policy shock in* t, $\widehat{\varepsilon}_t^m < 0$, *and an in period* t *announced shock for* $t+1$, $\widehat{\varepsilon}_{t+1,t}^m < 0$.

1. *The reduction of the policy rate* $\widehat{R}_t^m$ *leads in period* t *to rise of consumption* $\widehat{c}_t$, *inflation* $\widehat{\pi}_t$, *and in the liquidity premium* $\widehat{R}_t^{IS} - \widehat{R}_t^m$.

2. *The reduction of the policy rate* $\widehat{R}_{t+1}^m$ *in* $t+1$, *announced in period* t, *leads*

   (a) *in period* t *to a fall of consumption* $\widehat{c}_t$, *a rise of inflation* $\widehat{\pi}_t$, *and a rise of the liquidity premium* $\widehat{R}_t^{IS} - \widehat{R}_t^m$, *and*

   (b) *in period* $t+1$ *to a rise of consumption* $\widehat{c}_{t+1}$, *inflation* $\widehat{\pi}_{t+1}$ *and in the liquidity premium* $\widehat{R}_{t+1}^{IS} - \widehat{R}_{t+1}^m$.

3. *In total, forward guidance leads to an increase of consumption* $\widehat{c}_t$, *inflation* $\widehat{\pi}_t$ *and the liquidity premium* $\widehat{R}_t^{IS} - \widehat{R}_t^m$ *in both periods,* t *and* $t+1$.

Proof. See Appendix 3.C.1. □

In line with the evidence presented in Section 3.2, both reductions in the current monetary policy rate as well as announced reductions in future policy rates lead to rising liquidity premia in our model. The intuition for the spread responses in a period $t+k$ to a reduction in the monetary policy rate R_{t+k}^m (see Case 1. and 2.(b) in Proposition 1) is as follows. A temporary reduction in the policy rate increases the amount of money available per unit of eligible asset held by private agents, such that contemporaneous consumption increases (compared to previous and future consumption). To clear the market for commodities, the real interest rate on (non-eligible) assets that serve as a store of wealth declines. For an elasticity of intertemporal substitution $1/\sigma$ that is not too low (which is the case for $\sigma = 1$), the decline in the marginal rate of intertemporal substitution is less pronounced than the fall in the policy rate, such that the liquidity premium increases. For an announced reduction in the future policy rate the response

[22] Note that the parameter restriction $\rho_\pi < \beta\chi^{-1}$ is hardly restrictive, given that in our calibration used in Section, $\beta\chi^{-1} = 19.72$ which is by far larger than values typically applied for ρ_π of about 1.5.

of the current liquidity premium (see Case 2.(a) in Proposition 1) can also easily be understood. As eligible assets can be exchanged against a larger amount of reserves in the subsequent period, the liquidity value of newly issued treasuries rises. Given that the valuation of non-eligible assets is, in contrast, not directly affected by the policy measure, agents' demand for these asset fall, which tends to reduce their price. Hence, their current interest rates increase (while the current policy rate is unchanged), such that the liquidity premium rises.

This interest-rate increasing property of forward guidance has important implications for its aggregate effects. The additional announcement of a reduction in tomorrow's monetary policy rate does not per-se re-enforce the expansionary effects of a reduction in today's policy rate. In fact, the rise in liquidity premia exerts a dampening effect on today's consumption, since upward pressure on the returns on non-eligible assets induces households to postpone consumption. This prediction is in stark contrast to that of a standard New Keynesian model where increased inflation today due to the announcement of low future interest rates unambiguously reduces the relevant real interest rate since the nominal rate is directly controlled by the central bank. This additional reduction in the real interest rate reinforces increases in consumption and can make output responses to forward guidance very strong (see Carlstrom et al. (2015) and our quantitative evaluations below). While the standard New Keynesian model has been is criticized for predicting effects of forward guidance which are too strong compared to empirical evidence (e.g., Gertler and Karadi, 2015), the dampening effect stemming from the responses of liquidity premia helps matching empirical findings. We will evaluate this point more deeply in the context of our quantitative results in Section 3.4.2.

3.4.2 Numerical Results

In this section, we describe the calibration of the model and present quantitative effects of forward guidance. The model is calibrated to match the empirical response of the liquidity premium to an announcement shock as analyzed in Section 3.2. Motivated by forward guidance announcements of the FOMC in the last years that stated to keep policy rates at low levels over a period of a 1 to 3 years, we study the effects of policy rate reductions that last several quarters. We show that our model with the liquidity premium generates moderate output and inflation effects that are substantially smaller than in a model version without the liquidity premium, which corresponds to a conventional New Keynesian model.

Calibration

We calibrate the model to selected characteristics of the US economy and a period is assumed to be one quarter. For a first set of parameters, we apply values that are standard in the literature on business cycle analysis. The elasticity of substitution between individual varieties of the intermediate goods producing firms ϵ is set to 6, which implies a steady state mark-up of 20%, the inverse Frisch elasticity σ_n is set to 2, and the labor income share α is set to 2/3. Consistent with broad empirical evidence, the probability that firms are not able to reset prices in the Calvo model is set to $\phi = 0.8$, and the reaction coefficients of the interest rate rule (3.25) are set to $\rho_\pi = 1.5$, $\rho_y = 0.05$, and $\rho_R = 0.8$.

A second set of parameters is set to match mean observations in our data set from Section 3.2 (January 1990 to September 2016). The rate of inflation and the policy rate in steady state are set to the average values of the CPI inflation and the federal funds rate.[23] The corresponding values are $\pi = 1.02426^{1/4}$ and $R^m = 1.0304^{1/4}$. We calibrate the long-run liquidity premium between Treasuries that are eligible for open market operations and the less liquid assets that are non-eligible, $\eta = R^L/R - 1$, to 53 basis points, which is the mean value of the common liquidity factor from Section 3.2.1 between January 1990 and September 2016. This implies $\eta = 0.001322$, which requires a steady-state value of $R^{IS} = 1.03586^{1/4}$. Since $R^{IS} = \pi/\beta$ in steady state, we set $\beta = 0.9972$ to achieve this target. The growth rate Γ of the T-bills in (3.23) is set to the long-run inflation rate, which roughly accords to the average T-bill growth rate in the pre crisis sample. As in Bredemeier et al. (2017), we assume the ratio of money supplied under repos Ω to equal 1.5 which is based on data about the mean fraction of repos to total reserves of depository institutions in the US between 2003 and 2007. This value further ensures that money injections by the central bank I_t are, in line with the data, always positive.

Finally, the elasticity of intertemporal substitution $1/\sigma$ is set to match the response of the empirical liquidity premium factor, LP_t, to an innovation in the path factor $\widetilde{F}_2$ as presented in Section 3.2 with the response of a model-implied long-term liquidity premium, $\widehat{\eta}_t^{LT} = \prod_s^q (\widehat{R}_{t+s}^L - \widehat{R}_{t+s})^{1/q}$. For $\sigma = 1.5$ and $q = 4$, the model generates an increase of $\widehat{\eta}_t^{LT}$ by 25 basis points to an (isolated) announced reduction of the policy rate R_t^m by 100 basis points in four quarters. This is close to the corresponding empirical response of the common liquidity factor LP_t by 28 basis points to a 1%-reduction of $\widetilde{F}_2$, where the latter is normalized to the effect of a 100 basis point reduction of the expected policy rate in one year.

[23] We use monthly data from FRED between January 1990 and December 2016 that we aggregate to quarterly values as the basis for the long-run means. For the CPI we take the series [CPIAUCSL] and for the federal funds rate we take the series [FEDFUNDS].

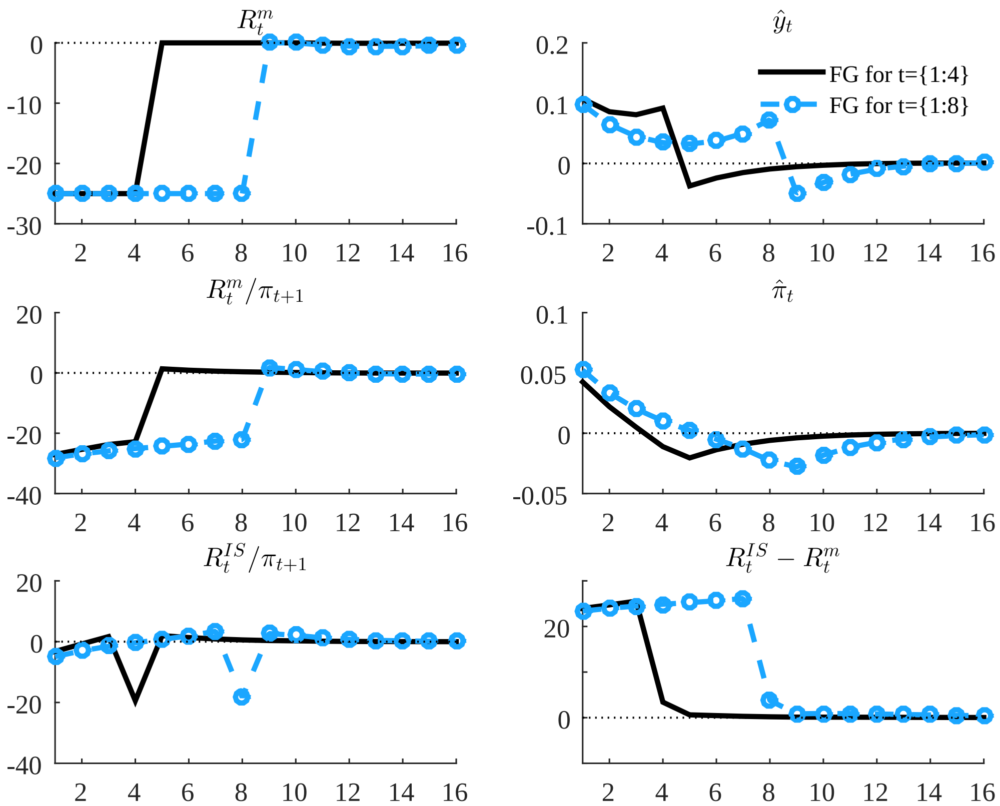
For the policy experiments, we consider paths of the monetary policy rate announced in advance. For this, it is convenient to assume that the contemporaneous shock, ε_t^m, and all anticipated monetary policy shocks, $\prod \varepsilon_{t,t-k}^m$, in (3.25) are completely transitory white-noise innovations that are identically and independently distributed as $N(0, \sigma_{m,k}^2)$. The assumption that all anticipated shocks are uncorrelated is innocuous in our analysis and could be relaxed without consequences for the results. We model forward guidance a path for the monetary policy rate $\left\{R_{T+h}^m\right\}_{h=1}^H$ in the upcoming H periods that the central bank announces at the beginning of period $T+1$, before which the economy is assumed to rest in steady state. We then back out a sequence of present and anticipated future monetary policy innovations $\varepsilon_{T+1}^m = \left\{\varepsilon_{T+1}^m, \varepsilon_{T+1+k,T+1}^m\right\}_{k=1}^K$ that yields this desired interest rate path. The calculation of the shocks is based on a procedure by Laséen and Svensson (2011) and Del Negro et al. (2015) that we adjust to our application. We provide details in Section 3.C.2 of the appendix.

Impulse Responses to Forward Guidance

Figure 3.1 shows impulse responses to different forward guidance scenarios in our model with the endogenous liquidity premium. The two scenarios shown in the figure are credible announcements of the central bank to reduce the policy rate R_t^m by 25 annualized basis points for the next 4 and 8 quarters, respectively. This resembles recent forward guidance experiences, where central banks stated to keep policy rates at low levels over a horizon of about two years, and also relates to the VAR analyses of forward guidance by Gertler and Karadi (2015). The central bank resets the policy rate to its steady state value after the guidance period until quarter 10. After that, monetary policy is governed by the Taylor rule (3.25), which then implies values in close proximity of the steady state. The assumed path of the nominal interest rate can be seen in the upper left panel of Figure 3.1. The interest rate reduction leads to a moderate increase of output by about 0.1%, see upper right panel of the figure. Output remains close to this level until the end of the guidance period. Once the policy rate increases, output experiences a brief dip before returning to its steady state value.

The real policy rate (middle left panel) behaves similar to the nominal rate where differences reflect the endogenous response of inflation. Inflation (middle right panel) rises on impact by about 0.05 percentage points but it starts decreasing already before the end of the guidance period. Households' real marginal rate of intertemporal substitution, R_t^{IS}/π_{t+1} (lower left panel), which is related to private-sector interest rates via (3.29), barely moves on impact and only experiences a negative spike at the end of the guidance

Figure 3.1: Effects of Forward Guidance

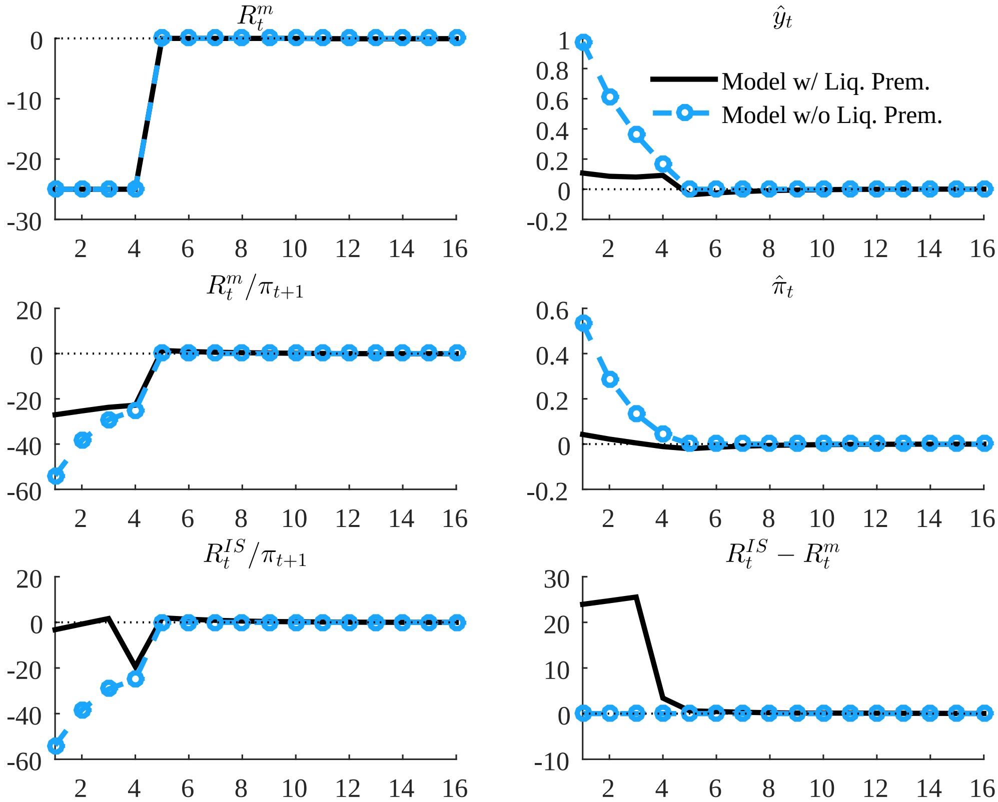


Notes: Impulse responses to forward guidance about policy rate R_t^m announced before quarter 1 in model with endogenous liquidity premium: production $\hat{y}_t$, inflation $\hat{\pi}_t$, real policy rate R_t^m/π_{t+1}, private-sector real rate R_t^{IS}/π_{t+1}, liquidity premium $R_t^{IS} - R_t^m$. Y-axis: Deviations from steady state in percent ($\hat{y}_t$, $\hat{\pi}_t$) or in basis points (else). X-axis: quarters. Black line: Announced policy rate reduction of 25 basis points in quarters 1 to 4. Blue circled line: Announced policy rate reduction of 25 basis points in quarters 1 to 8.

period, reflecting the change in consumption. The liquidity premium $R_t^{IS} - R_t^m$ (lower right panel) instead increases sharply on impact and remains on that level until output drops. Comparing the scenario of forward guidance about 4 quarters with that about 8 quarters reveals that differences in terms of the impact responses of output and the liquidity premium are small while inflation is slightly higher on impact in case of the longer horizon. Notably, this observation differs from the critized prediction of the conventional New Keynesian model without an endogenous liquidity premium that the impact responses of output and inflation increase with the horizon of the forward guidance (see McKay et al., 2016). Intuitively, cumulated output effects are more pronounced for the longer forward-guidance experiment.

Figure 3.2 compares the effects of forward guidance in the model featuring the en-

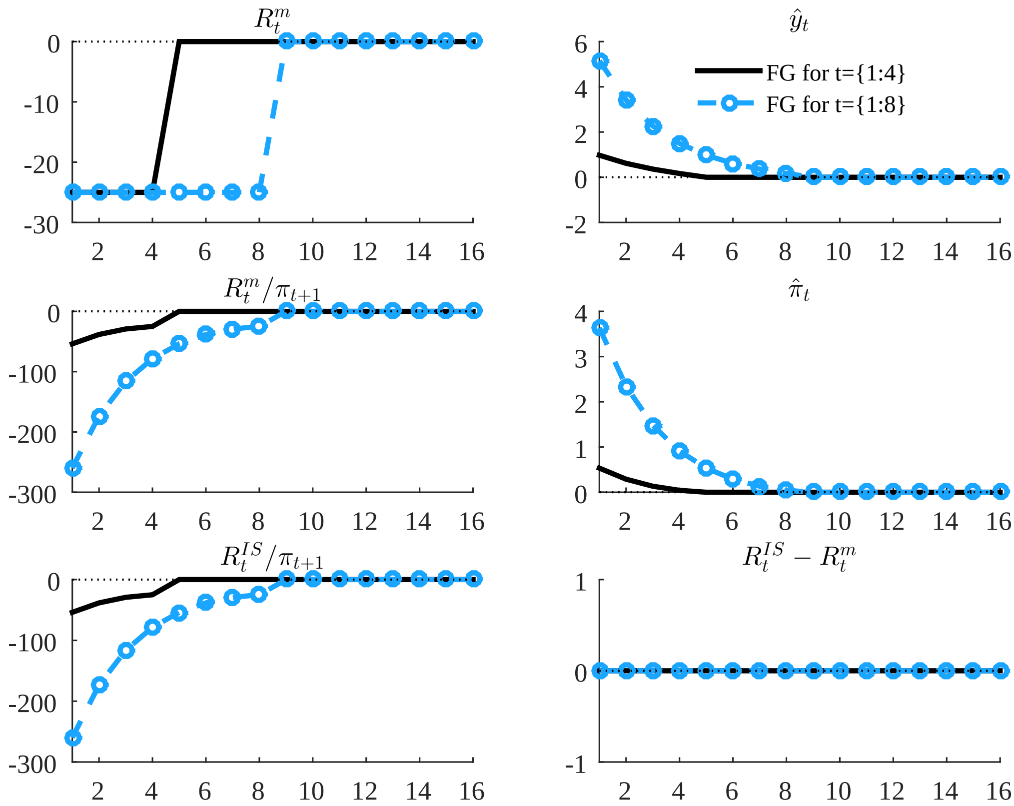
Figure 3.2: Comparison with Standard New Keynesian Model



Notes: Impulse responses to policy rate (R_t^m) reduction of 25 basis points in quarters 1 to 4, announced before quarter 1: production $\hat{y}_t$, inflation $\hat{\pi}_t$, real policy rate R_t^m/π_{t+1}, private-sector real rate R_t^{IS}/π_{t+1}, liquidity premium $R_t^{IS} - R_t^m$. Y-axis: Deviations from steady state in percent ($\hat{y}_t$, $\hat{\pi}_t$) or in basis points (else). X-axis: quarters. Black line: Baseline model with endogenous liquidity premium. Blue circled line: Conventional New Keynesian model.

dogenous liquidity premium with a version of the model without the liquidity premium ($\eta_t = 0$, see 3.27), which corresponds to a conventional New Keynesian model. In both cases, the central bank announces to reduce the policy rate by 25 basis points for the next 4 quarters and to return to steady state afterwards. The results for the model with interest rate spreads are those from the first scenario of Figure 3.1. Output and inflation in the model version without the liquidity premium increase sharply on impact, in line with the findings by Carlstrom et al. (2015) and others but are too large compared to the empirical effects of forward guidance (see for example Gertler and Karadi, 2015). Compared to the model version with the liquidity premium, the responses on impact are about 10 times higher. In the model version without the liquidity premium, the central bank can steer the growth rate of consumption directly by adjusting the policy rate. The real interest rate falls by more on impact than the nominal rate due to the increase in inflation and, hence, add to the increase of consumption and output.

Figure 3.3: Forward Guidance in the Standard New Keynesian Model

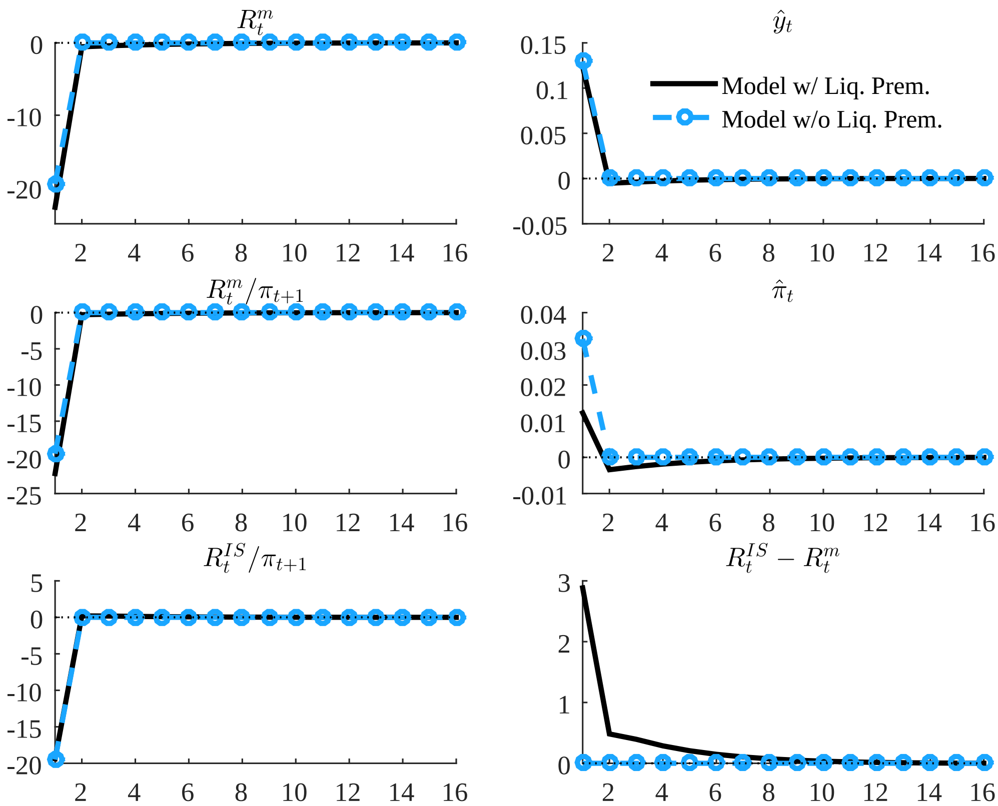


Notes: Impulse responses to forward guidance about policy rate R_t^m announced before quarter 1 in standard New Keynesian model without liquidity premia: production $\hat{y}_t$, inflation $\hat{\pi}_t$, real policy rate R_t^m/π_{t+1}, private-sector real rate R_t^{IS}/π_{t+1}, liquidity premium $R_t^{IS} - R_t^m$. Y-axis: Deviations from steady state in percent ($\hat{y}_t$, $\hat{\pi}_t$) or in basis points (else). X-axis: quarters. Black line: Announced policy rate reduction of 25 basis points in quarters 1 to 4. Blue circled line: Announced policy rate reduction of 25 basis points in quarters 1 to 8.

Figure 3.8 in the Appendix presents responses to a similar policy, where the central bank provides forward guidance for four quarters about the real instead of the nominal policy rate, which we perform for comparability to McKay et al. (2016). The results for both model versions are similar to the ones presented in Figure 3.1. For this reason, we continue to consider policies, where guidance is provided in terms of nominal policy rates.

Figure 3.3 shows the two forward-guidance experiments in the model version without the liquidity premium, which corresponds to a standard New Keynesian model. We see that the length of the guidance period has a huge impact on the output effect of monetary policy. Specifically, announcing low interest rates also for quarters 5 through 8 increases the impact response of output by factor of 5, while there is almost no change in the impact

Figure 3.4: Effects of Conventional Monetary Policy



Notes: Impulse responses to conventional monetary policy of 25 basis points on monetary policy rate (R_t^m) in quarter 1 only: production $\hat{y}_t$, inflation $\hat{\pi}_t$, real policy rate R_t^m/π_{t+1}, private-sector real rate R_t^{IS}/π_{t+1}, liquidity premium $R_t^{IS} - R_t^m$. Y-axis: Deviations from steady state in percent ($\hat{y}_t$, $\hat{\pi}_t$) or in basis points (else). X-axis: quarters. Black line: Baseline model with endogenous liquidity premium. Blue circled line: Conventional New Keynesian model.

response in the model version with the liquidity premium.

Finally, Figure 3.4 shows the responses to a standard unannounced one-time monetary policy shock in the model versions with and without the liquidity premium. The responses of the two models are very similar, except for the model with the liquidity premium generating a rise in the premium in line with the evidence presented in Section 3.2. Comparing this standard shock without forward guidance, we see that in the model without the liquidity premium, the impact output responses to both shocks are similar while the forward-guidance policy intuitively triggers a longer expansion in output. In the standard model without the liquidity premium, the shock without forward guidance triggers an output expansion which is almost ten times smaller than the one induced by the one-year forward guidance policy.

To sum up, in our model with the liquidity premium, forward guidance prolongs the output effects of monetary policy but does not substantially foster the immediate output effects. By contrast, in a standard New Keynesian model, forward guidance affects the immediate output responses of monetary policy very strongly.

Our quantitative model evaluations help understanding the VAR results of Gertler and Karadi (2015). They present three sets of VAR responses to gauge the importance of forward guidance for the output effects of monetary policy. In particular, they estimate the responses to unanticipated 25 bp changes in the Federal Funds rate, the return on one-year government bonds, and the returns on two-year government bonds. These changes include different degrees of forward guidance with the change in the two-year rate containing the highest degree of forward guidance and the change in Federal Funds rate the lowest degree of forward guidance. The results of Gertler and Karadi (2015) show that output responses to monetary policy shocks are affected by forward guidance only in the medium run and even slightly weakened on impact. The responses to the change in the two-year rate are very similar to the ones to the one-year rate suggesting that the length of forward guidance is not of primary importance for the output effects of monetary policy shocks.

These findings are in line with the predictions of our liquidity-premium model where forward guidance has strong output effects only with a delay but leaves contemporaneous output almost unchanged. By contrast, the standard New Keynesian model predicts very strong impact effects of forward guidance which are rejected by the empirical evidence of Gertler and Karadi (2015).

3.5 Conclusion

We show empirically that liquidity premia tend to rise after forward guidance announcements. We augment the conventional New Keynesian model by an endogenous liquidity premium that separates the monetary policy rate from other interest rates that are more relevant for private-sector transactions. We show both analytically and numerically that forward guidance is a much less powerful policy tool in this setting. The forward guidance puzzle can be solved in our framework and we provide a theoretical rationale for the increases in liquidity premia that are present in the data.

Appendix 3.A Empirical Analysis

3.A.1 The Case Study

The most relevant content of the FOMC press releases on the three events of the case study in the introduction is the following:

2011-08-09: Economic growth has been "considerably lower" than expected. The FFR is unchanged at 0-0.25 percent. "[...] economic conditions [...] are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013."

2012-01-25: The economy has been "expanding moderately". The FFR is unchanged at 0-0.25 percent. "[...] economic conditions [...] are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014."

2012-09-13: Economic activity has "continued to expand at a moderate pace". The FFR is unchanged at 0-0.25 percent. "[...] exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015." Additional purchases of mortgage-backed securities at a pace of $40 billion per month ("QE3") are announced.

For the full text of the press releases, see `www.federalreserve.gov`. See also Table 1 in Del Negro et al. (2015) for further details.

3.A.2 Measurement of Liquidity Premia

In this appendix, we describe the data sources and the construction of all liquidity spreads. We also provide summary statistics and figures of all our liquidity measures.

We collect daily return data on various assets to construct the spreads that aim at measuring liquidity premia. All spreads are calculated as the difference in annualized daily returns between Treasuries as the liquid near-money asset and an illiquid asset of similar safety and maturity. We use data from FRED (`https://fred.stlouisfed.org`) and from Bloomberg. Original mnemonics in the data source are given in square brackets.

- The data for the Treasury rates stems from FRED. We use the 'Treasury Constant Maturity Rates' with the mnemonic [DGS'xx'], where 'xx'$= \{3MO, 6MO, 1, 3, 5, 10\}$ refers to the maturity in months (MO) or years (else). We collect daily data from 1990-01-02 to 2016-09-16.

- Following Krishnamurthy and Vissing-Jorgensen (2012) as well as Del Negro et al. (2017), we construct several spreads between the rates on investment grade rated commercial papers or corporate bonds and Treasuries for different maturities. All series are taken from FRED.

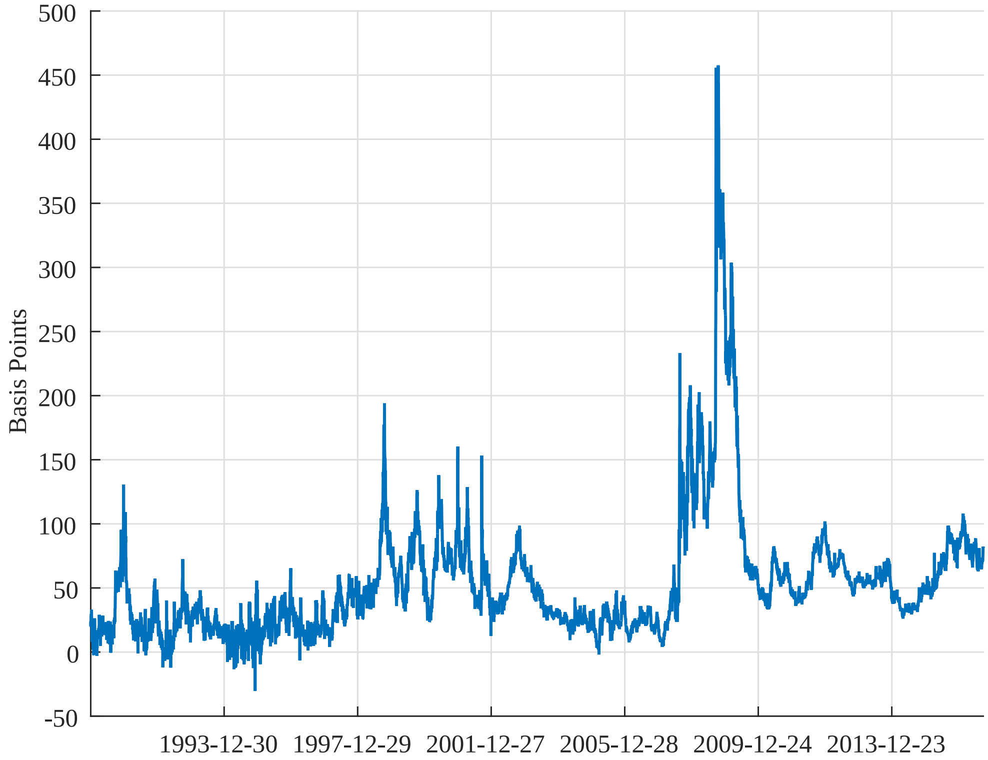
As a short-run measure, we use the '3-Month AA/P1 Nonfinancial Commercial Paper Rate' with mnemonic [DCPN3M] and we calculate the spread relative to the series [DGS3MO].
For longer maturities, we employ the following four corporate bond indexes: (1) The 'Bank of America (BofA) Merrill Lynch US Corporate 1-3 Year Effective Yield', mnemonic [BAMLC1A0C13YEY], which is a subset of the 'BofA Merrill Lynch US Corporate Master Index' that includes investment grade rated corporate bonds that were publically issued in the United States. The series that we use includes all securities with a remaining term to maturity between 1 and 3 years. We calculate the spread as [BAMLC1A0C13YEY] – [DGS3]. (2) The 'BofA Merrill Lynch US Corporate AAA Effective Yield', mnemonic [BAMLC0A1CAAAEY], which is a subset of the 'BofA Merrill Lynch US Corporate Master Index' that covers securities with an AAA rating. We calculate the spread as [BAMLC0A1CAAAEY] – [DGS5]. (3) 'Moody's Seasoned Aaa Corporate Bond Yield', mnemonic [DAAA], which consists of bonds with an AAA rating and long remaining terms to maturity. We construct the spread relative to the series [DGS10]. (4) 'Moody's Seasoned Baa Corporate Bond Yield', mnemonic [DBAA], which consists of US bonds with an BAA rating and long remaining terms to maturity. We construct the spread relative to the series [DGS10].
The series on commercial papers and the indexes from BofA Merrill Lynch are available to us from 1997-01-02 onwards. We collect data on the indexes by Moody's beginning on 1990-01-02.

- Krishnamurthy and Vissing-Jorgensen (2012) explain that the spread between the rates on certificates of deposit (CD) and Treasury bills can only reflect a liquidity attribute, since the certificates are basically risk-free due to its coverage by the FDIC. CDs are relatively illiquid, as withdrawals before maturity usually imply large contractual penalties. We collect the series 'Certificate of Deposit: Secondary Market Rate' with maturities of 3 and 6 months from FRED with the mnemonics [DCD90] and [DCD6M]. We calculate the spreads relative to the Treasury-series [DGS3MO] and [DGS6MO], respectively. Daily data is available to us from 1990-01-02 to 2013-06-28.

- Nagel (2016) suggests the spread between the rates on general collateral repurchase agreements (GC repos, hereafter) and the 3-month T-bill as an excellent measure of the "premium for the liquidity services by near-money assets". He notes that these repos are very illiquid, as the term loan is locked in until maturity, which is also reflected by relatively wide bid-ask spreads. Since GC repos are collateralized with a portfolio of Treasuries, they are essentially risk-free. We collect data from Bloomberg with the mnemonic [USRGCGC ICUS Curncy] from 1991-05-21 to 2016-09-16. We follow Nagel (2016) in calculating averages between bid and ask prices. We construct the spread relative to the series [DGS3MO].

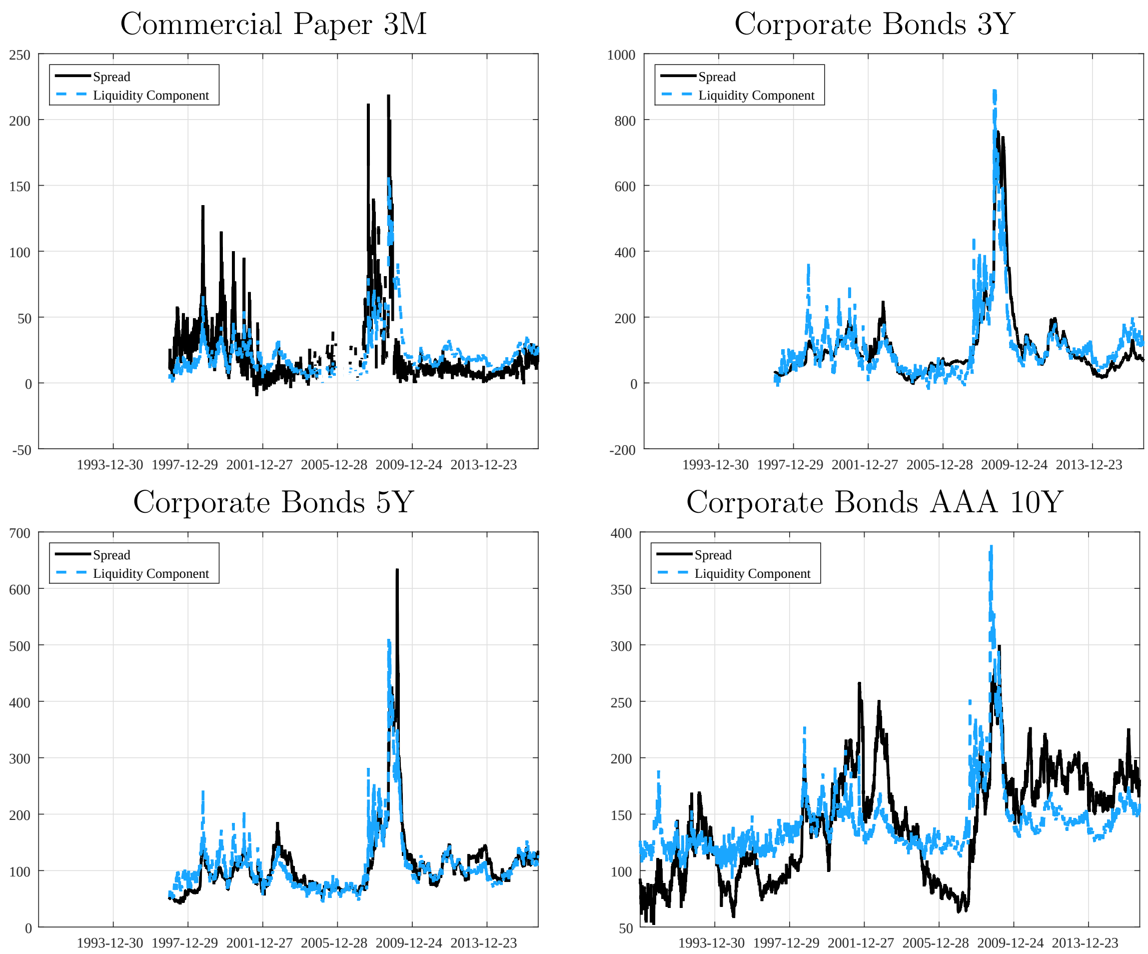
Figure 3.5 shows the times series of the liquidity premium in equation (3.1). Figures 3.6 and 3.7 provide time series plots of all spreads along with a linear projection of the common factor and a constant. Summary statistics on all spreads and the liquidity premium derived from the factor model are given in Table 3.4.

Figure 3.5: Time Series of the Liquidity Premium



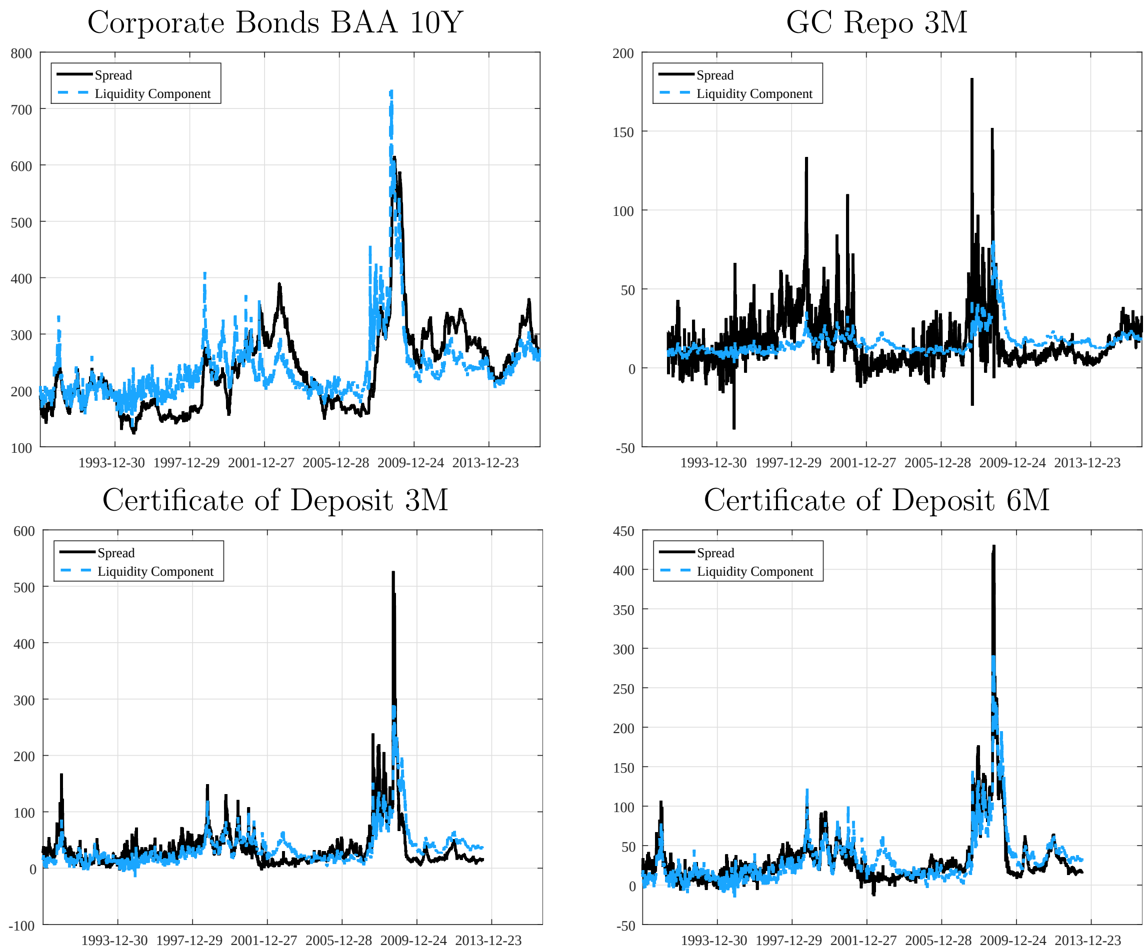
Notes: Plot of a time series of the liquidity premium in equation (3.1) in basis points using daily data from 1990-01-2 to 2016-09-16, constructed from a panel of 8 liquidity spreads using principal component analysis.

Figure 3.6: Time Series of Liquidity Spreads and Common Factor



Notes: Figure shows daily time series of liquidity spreads (black lines) along with their linear projections on the common factor and a constant (blue lines).

Figure 3.7: Time Series of Liquidity Spreads and Common Factor – Continued



Notes: Figure shows daily time series of liquidity spreads (black lines) along with their linear projections on the common factor and a constant (blue lines).

Table 3.4: Summary Statistics of Liquidity Spreads

Spread	Time Range	Mean	Std. Dev.
Commercial Paper 3M	1997-01-02 to 2016-09-16	21.82	24.79
Corporate Bonds 3Y	1997-01-02 to 2016-09-16	110.99	120.10
Corporate Bonds 5Y	1997-01-02 to 2016-09-16	108.89	60.61
Corporate Bonds AAA 10Y	1990-01-02 to 2016-09-16	141.55	47.74
Corporate Bonds BAA 10Y	1990-01-02 to 2016-09-16	238.00	77.47
Certificate of Deposit 3M	1990-01-02 to 2013-06-28	35.69	40.97
Certificate of Deposit 6M	1990-01-02 to 2013-06-28	31.83	37.49
GC Repo 3M	1991-05-21 to 2016-09-16	16.04	16.24
Liquidity Premium (Factor)	1990-01-02 to 2016-09-16	53.47	49.45

Notes: Mean and Standard Deviation (Std. Dev.) given in basis points.

3.A.3 Estimation of the Target and the Path Factor

In this appendix, we describe the data sources of the federal funds and eurodollar futures that we use. We explain how futures can be used to extract the surprise component of monetary policy at FOMC meeting dates and how we derive the target and the path factor.

Data Sources

All futures data is taken from Quandl (`https://www.quandl.com`).

- For the federal funds rate, we use the '30 Day Federal Funds Futures, Continuous Contract' series for the front month and the next 3 months thereafter. The mnemonics read [CHRIS/CME_FF'X'], where 'X'= $\{1, 2, 3, 4\}$ is the number of months until delivery of the contract. The raw data for the continuous contract calculation is from the Chicago Mercantile Exchange, where the futures are traded. We extract the daily settlement price (series 'settle'), which is given as 100 minus the average daily federal funds overnight rate for the delivery month, between 1990-01-02 to 2016-09-16.
- For the eurodollars, we use the 'Eurodollar Futures, Continuous Contract' series with the mnemonic [CHRIS/CME_ED'X'], where 'X'= $\{6, 9, 12\}$ gives the number of months until delivery of the contract. The raw data for the continuous contract calculation is from the Chicago Mercantile Exchange, where the futures are traded. We extract the daily settlement price (series 'settle'), which is given as 100 minus the 3-month London interbank offered rate for spot settlement on the 3rd Wednesday of the contract month, between 1990-01-02 to 2016-09-16.

Construction of the Monetary Surprise Components

We now explain how the elements of the data matrix X in equation (3.2) are constructed. The rows correspond to the 237 FOMC meeting dates between January 1990 and September 2016. The five columns of X refer to the different futures contracts. The third to fith column gives the one-day change of the eurodollar futures contracts with 6, 9, and 12 months until delivery around the FOMC meetings. Due to the spot settlement of these contracts, this difference directly gives a measure for the change in expectations about interest rates in 6, 9, and 12 months, respectively. The first two columns entail the surprise changes of expectations using mainly the 1- and the 3-month federal funds futures, whose calculation is more involved, since these contracts settle on the average federal funds rate in the delivery month. The following exposition is based on Gürkaynak et al. (2005) and Gürkaynak (2005).

Given the specification of the federal funds future contracts, the current month future settlement rate at the day before the FOMC meeting in t, $ff^1_{t-\Delta 1}$, can be written as

$$ff^1_{t-\Delta 1} = \frac{d_1}{m_1} r_{t-\Delta 1} + \frac{m_1 - d_1}{m_1} E_{t-\Delta 1}\left(r_t\right) + \varpi^1_{t-\Delta 1}, \tag{3.37}$$

where $r_{t-\Delta 1}$ is the average federal funds rate that has prevailed in this month until the day before the meeting (i.e. day $t-\Delta 1$), $E_{t-\Delta 1}\left(r_t\right)$ is the expectation at $t-\Delta 1$ about the federal funds rate for the rest of the month, d_1 the day of the FOMC meeting t in the current month with length m_1, and $\varpi^1_{t-\Delta 1}$ any potentially present term or risk premia. Analogously, the settlement rate at the day of the meeting itself reads

$$ff^1_t = \frac{d_1}{m_1} r_{t-\Delta 1} + \frac{m_1 - d_1}{m_1} r_t + \varpi^1_t. \tag{3.38}$$

Defining the surprise change in the target of the federal funds rate after the current meeting as $mp^1_t \equiv r_t - E_{t-\Delta 1}\left(r_t\right)$, allows its calculation according to

$$mp^1_t = \left(ff^1_t - ff^1_{t-\Delta 1}\right) \frac{m_1}{m_1 - d_1}, \tag{3.39}$$

which assumes that term and risk premia ϖ^1 do not change significantly between t and $t-\Delta 1$, which Gürkaynak et al. (2005) argue to be in line with empirical evidence. The change in the futures rates is scaled with the factor $m_1/\left(m_1 - d_1\right)$, since the surprise change of the federal funds rate only applies to the remaing $m_1 - d_1$ days of the month. For meeting dates very close to the end of the month, the scaling factor becomes relatively big, which can be problematic when there is too much noise in the data. We therefore follow Gürkaynak (2005) and use the unscaled change in the futures that are due in the next month, $mp^1_t = \left(ff^2_t - ff^2_{t-\Delta 1}\right)$, when the meeting is within the last 7 days of the month. Another special case are FOMC meetings at the first day of the month. In this case, the monetary surprise has to be calculated as $mp^1_t = \left(ff^1_t - ff^2_{t-\Delta 1}\right)$.

In a next step, we determine the change of expectations about the federal funds rate that will prevail after the second FOMC meeting $(t+1)$ from the perspective of $t-\Delta 1$, r_{t+1}. These values form the entries in the second column of X. Since there are 8 regularly scheduled FOMC meetings per year, the next meeting $(t+1)$ will be in $j = \{1, 2\}$ months.[24]

[24]In case of additional unscheduled meetings, the next meeting can also be in the same month. 23 of the 237 FOMC meetings in our sample are unscheduled intermeeting moves. Most of these observations occured in the early 1990s and some happened after surprising financial turmoil, e.g. 2001 and 2007/8.

At date $t-\Delta 1$, the futures rate that covers the second meeting from now is then given by

$$ff_{t-\Delta 1}^{1+j} = \frac{d_{1+j}}{m_{1+j}} E_{t-\Delta 1}(r_t) + \frac{m_{1+j} - d_{1+j}}{m_{1+j}} E_{t-\Delta 1}(r_{t+1}) + \varpi_{t-\Delta 1}^{1+j}, \tag{3.40}$$

where ff^{1+j} refers to the futures contract that expires in $1+j$ months, while d_{1+j} and m_{1+j} refer to the day and the length of the month of the second FOMC meeting from now, respectively. Analogously to the procedure above, we calculate the change in the expected target of the federal funds rate after the next meeting as

$$mp_t^{1+j} \equiv E_t(r_{t+1}) - E_{t-\Delta 1}(r_{t+1}) = \left[\left(ff_t^{1+j} - ff_{t-\Delta 1}^{1+j}\right) - \frac{d_{1+j}}{m_{1+j}} mp_t^1\right] \frac{m_{1+j}}{m_{1+j} - d_{1+j}}. \tag{3.41}$$

We apply the same corrections as above in case the meeting $t+1$ is on the first day or within the last week of the month.

Factor Estimation and Transformation

We normalize each column of X to have a zero mean and a unit variance before extracting the first two principal components. As there is a very small number of missing values for the 12-month eurodollar future, we apply the method of Stock and Watson (2002). This gives us the two factors F_1 and F_2, which we again normalize to have a unit variance. Next, we determine the elements of the $[2 \times 2]$ transformation matrix U to obtain $\widetilde{F}_1$ and $\widetilde{F}_2$ in (3.3). The matrix U is given by the four elements

$$U = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix},$$

whose identification requires four restrictions that we adopt from Gürkaynak et al. (2005).

We normalize the columns of U to unit length, which leads to the conditions

$$a_1^2 + a_2^2 = 1, \tag{3.42}$$

$$b_1^2 + b_2^2 = 1. \tag{3.43}$$

This assumption implies that the variance of $\widetilde{F}_1$ and $\widetilde{F}_2$ is unity. The next restriction demands that $\widetilde{F}_1$ and $\widetilde{F}_2$ remain orthogonal to each other, i.e. $E\left(\widetilde{F}_1, \widetilde{F}_2\right) = 0$. This can

Following Gürkaynak (2005), we assume that on every FOMC meeting, future intermeeting moves are assumed to occur with zero probability.

be shown to imply that the scalar product of the columns of U equals zero,

$$\langle U \rangle = a_1 b_1 + a_2 b_2 = 0. \tag{3.44}$$

The final restriction is that the second factor $\widetilde{F}_2$ does not affect the current monetary policy surprise, mp_t^1, that forms the first column of X. This is implemented as follows. Starting from $F = \widetilde{F} U^{-1}$, we write F_1 and F_2 as functions of $\widetilde{F}_1$ and $\widetilde{F}_2$, which yields

$$F_1 = \frac{1}{\det(U)} \left(b_2 \widetilde{F}_1 - a_2 \widetilde{F}_2 \right), \tag{3.45}$$

$$F_2 = \frac{1}{\det(U)} \left(a_1 \widetilde{F}_2 - b_1 \widetilde{F}_1 \right). \tag{3.46}$$

The current monetary surprise can be written as

$$mp_t^1 = \lambda_1 F_1 + \lambda_2 F_2,$$

where λ_1 and λ_2 are elements of the estimated loading matrix Λ in (3.2). Using (3.45) and (3.46), mp_t^1 can be rearranged to

$$mp_t^1 = \frac{1}{\det(U)} \left[(\lambda_1 b_2 - \lambda_2 b_1) \widetilde{F}_1 + (\lambda_2 a_1 - \lambda_1 a_2) \widetilde{F}_2 \right]. \tag{3.47}$$

Setting the coefficient of $\widetilde{F}_2$ in (3.47) to zero, then implements the restriction as

$$\lambda_2 a_1 - \lambda_1 a_2 = 0. \tag{3.48}$$

Using (3.42)-(3.44) and (3.48), we can solve for the elements of U to obtain the series for the target and the path factor, $\widetilde{F}_1$ and $\widetilde{F}_2$.

3.A.4 Additional Regression Results

Tables 3.5 and 3.6 are the counterparts to Tables 3.2 and 3.3 for the sample 1990-2008. Results for this sample excluding the recent zero lower bound episode are similar to those for the total sample.

Table 3.5: Response of Asset Returns to Changes in Monetary Policy in a Sample Ending 2008-12-16

	Treasuries						GC
	3M	6M	1Y	3Y	5Y	10Y	3M
Change in Federal Funds Rate $\left(\widetilde{F}_1\right)$	0.65***	0.67***	0.60***	0.32***	0.20***	0.042	0.31***
	(0.080)	(0.065)	(0.059)	(0.040)	(0.040)	(0.041)	(0.078)
Change in Forward Guidance $\left(\widetilde{F}_2\right)$	0.18***	0.35***	0.48***	0.74***	0.77***	0.66***	-0.001
	(0.054)	(0.046)	(0.050)	(0.069)	(0.067)	(0.058)	(0.057)
R^2	0.55	0.77	0.82	0.82	0.84	0.79	0.24
Number of Observations (T)	175	175	175	175	175	175	152

	Commercial Paper / Corporate Bonds					CD	
	3M	3Y	5Y	10Y(A)	10Y(B)	3M	6M
Change in Federal Funds Rate $\left(\widetilde{F}_1\right)$	0.27***	0.38***	0.15**	-0.033	-0.0047	0.37***	0.32*
	(0.097)	(0.11)	(0.064)	(0.040)	(0.031)	(0.14)	(0.17)
Change in Forward Guidance $\left(\widetilde{F}_2\right)$	-0.003	0.62***	0.58***	0.37***	0.36***	0.18*	0.22
	(0.12)	(0.13)	(0.083)	(0.046)	(0.045)	(0.10)	(0.16)
R^2	0.11	0.44	0.61	0.53	0.54	0.22	0.120
Number of Observations (T)	73	103	103	175	175	175	175

Notes: Table shows responses of asset returns to changes in the federal funds rate, measured by the target factor, and to changes in forward guidance, measured by the path factor, at FOMC meetings between January 1990 and December 2008. Constant included in all regressions. Heteroskedasticity-robust (White) standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Maturity is measured either in months (M) or in years (Y). Corporate Bond 10Y(A) and (B) refer to long-term bonds with AAA and BAA rating, respectively. CD: Certificate of Deposit; GC: General Collateral Repo.

Table 3.6: Response of Liquidity Spreads to Changes in Monetary Policy in a Sample Ending 2008-12-16

	Liquidity Premium (Factor)	Commercial Paper / Corporate Bond - Spread 3M	3Y	5Y	10Y(A)	10Y(B)
Change in Federal Funds Rate $\left(\widetilde{F}_1\right)$	-0.41***	-0.28***	0.154*	0.031	-0.075	-0.047
	(0.13)	(0.10)	(0.090)	(0.060)	(0.061)	(0.045)
Change in Forward Guidance $\left(\widetilde{F}_2\right)$	-0.32***	-0.15	-0.063	-0.14***	-0.29***	-0.30***
	(0.081)	(0.12)	(0.076)	(0.047)	(0.042)	(0.034)
R^2	0.30	0.09	0.05	0.10	0.43	0.53
Number of Observations (T)	175	73	103	103	175	175
		GC-Spread 3M		CD-Spread 3M	6M	
Change in Federal Funds Rate $\left(\widetilde{F}_1\right)$		-0.36**		-0.27*	-0.35*	
		(0.15)		(0.15)	(0.19)	
Change in Forward Guidance $\left(\widetilde{F}_2\right)$		-0.20**		-0.0014	-0.13	
		(0.088)		(0.11)	(0.16)	
R^2		0.21		0.08	0.10	
Number of Observations (T)		152		175	175	

Notes: Table shows responses of liquidity spreads to changes in the federal funds rate, measured by the target factor, and to changes in forward guidance, measured by the path factor, at FOMC meetings between January 1990 and December 2008. Constant included in all regressions. Heteroskedasticity-robust (White) standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Maturity is measured either in months (M) or in years (Y). Corporate Bond 10Y(A) and (B) refer to long-term bonds with AAA and BAA rating, respectively. CD: Certificate of Deposit; GC: General Collateral Repo. All spreads are calculated relative to Treasuries of the same maturity.

Appendix 3.B Definition of Equilibrium

Definition 2. *A rational expectations equilibrium is a set of sequences* $\{c_t, y_t, n_t, w_t, \lambda_t, m_t^R, m_t, b_t, b_t^T, mc_t, Z_{1,t}, Z_{2,t}, Z_t, s_t, \pi_t, R_t^{IS}\}_{t=0}^{\infty}$ *satisfying*

$$c_t = m_t + m_t^R, \text{ if } R_t^{IS} > 1, \quad \text{or } c_t \leq m_t + m_t^R, \text{ if } R_t^{IS} = 1, \tag{3.49}$$

$$b_{t-1}/\left(R_t^m \pi_t\right) = m_t - m_{t-1}\pi_t^{-1} + m_t^R, \text{ if } R_t^{IS} > R_t^m, \tag{3.50}$$

$$\text{or } b_{t-1}/\left(R_t^m \pi_t\right) \geq m_t - m_{t-1}\pi_t^{-1} + m_t^R, \text{ if } R_t^{IS} = R_t^m,$$

$$m_t^R = \Omega m_t, \tag{3.51}$$

$$b_t = b_t^T - m_t, \tag{3.52}$$

$$b_t^T = \Gamma b_{t-1}^T/\pi_t, \tag{3.53}$$

$$\theta n_t^{\sigma_n} = u_{c,t} w_t / R_t^{IS}, \tag{3.54}$$

$$1/R_t^{IS} = \beta E_t\left[u_{c,t+1}/\left(u_{c,t}\pi_{t+1}\right)\right], \tag{3.55}$$

$$w_t/\left(\alpha n_t^{\alpha-1}\right) = mc_t, \tag{3.56}$$

$$\lambda_t = \beta E_t\left[u_{c,t+1}/\pi_{t+1}\right], \tag{3.57}$$

$$Z_{1,t} = \lambda_t y_t mc_t + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{1,t+1}, \tag{3.58}$$

$$Z_{2,t} = \lambda_t y_t + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{2,t+1}, \tag{3.59}$$

$$Z_t = \left[\varepsilon/\left(\varepsilon-1\right)\right] Z_{1,t}/Z_{2,t}, \tag{3.60}$$

$$1 = (1-\phi) Z_t^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}, \tag{3.61}$$

$$s_t = (1-\phi) Z_t^{-\varepsilon} + \phi s_{t-1}\pi_t^{\varepsilon}, \tag{3.62}$$

$$y_t = n_t^{\alpha}/s_t, \tag{3.63}$$

$$y_t = c_t, \tag{3.64}$$

(where $u_{c,t} = c_t{}^{-\sigma}$*), the transversality conditions, a monetary policy* $\{R_t^m \geq 1\}_{t=0}^{\infty}$, $\Omega > 0$, $\pi \geq \beta$, *and a fiscal policy* $\Gamma \geq 1$, *for given initial values* $M_{-1} > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, *and* $s_{-1} \geq 1$.

Given a rational expectations equilibrium as summarized in Definition 2, the equilibrium sequences $\{R_t, R_t^D, R_{t+1}^q, R_t^L\}_{t=0}^{\infty}$ can be determined by

$$R_t = E_t[u_{c,t+1}\pi_{t+1}^{-1}]/[E_t\left(R_{t+1}^m\right)^{-1} u_{c,t+1}\pi_{t+1}^{-1}], \tag{3.65}$$

$$\lambda_t/R_t^D = \beta E_t[(u_{c,t+1} + (1-\mu)\lambda_{t+1})/\pi_{t+1}], \tag{3.66}$$

$$1 = \beta E_t\left[\left(R_{t+1}^q/\pi_{t+1}\right)\left(\lambda_{t+1}/\lambda_t\right)\right], \tag{3.67}$$

$$1/R_t^L = E_t\left[1/R_{t+1}^{IS}\right], \tag{3.68}$$

If the money supply constraint (3.12) is not binding, which is the case if $R_t^m = R_t^{IS}$ (see 3.27), the model given in Definition 2 reduces to a standard New Keynesian model with a cash-in-advance constraint, where government liabilities can be determined residually.

Definition 3. *A rational expectations equilibrium under a non-binding money supply constraint (3.12) is a set of sequences* $\{c_t, y_t, n_t, w_t, \lambda_t, mc_t, Z_{1,t}, Z_{2,t}, Z_t, s_t, \pi_t, R_t^{IS}\}_{t=0}^{\infty}$ *satisfying* $R_t^{IS} = R_t^m$, *(3.54)-(3.64), the transversality conditions, and a monetary policy* $\{R_t^m \geq 1\}_{t=0}^{\infty}$, $\pi \geq \beta$, *for a given initial value* $s_{-1} \geq 1$.

Appendix 3.C Details to Section 3.4

3.C.1 Analytical Results

Proof of Proposition 1. To establish the claims made in the proposition, the model given in Definition 1 for the version with $R_t^m < R_t^{IS}$, i.e., (3.31)-(3.35), is further simplified by substituting out $\widehat{R}_t^{IS}$ and $\widehat{R}_t^m$. Without loss of generality, we restrict ourselves to $K = 1$ in (3.35):

$$\widehat{\pi}_t = \delta E_t \widehat{\pi}_{t+1} + \left(\widehat{c}_t + \widehat{b}_t - \widehat{\varepsilon}_{t+1,t}^m\right), \tag{3.69}$$

$$\widehat{c}_t = \widehat{b}_{t-1} - (1 + \rho_\pi)\widehat{\pi}_t - \widehat{\varepsilon}_{t,t-1}^m, \tag{3.70}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{3.71}$$

where $\delta = \beta - \chi\rho_\pi > 0$. We now prove the claims of the proposition one after another. For this, we assume that (3.36) is satisfied, which ensures existence and uniqueness of a locally stable solution.

1. We begin with analyzing the reduction of the policy rate in period t, i.e. $\widehat{\varepsilon}_t^m < 0$, by applying the following solution form for the system (3.69)-(3.71):

$$\widehat{\pi}_t = \gamma_{\pi b}\widehat{b}_{t-1} + \gamma_{\pi\varepsilon}^m \widehat{\varepsilon}_t^m, \tag{3.72}$$

$$\widehat{b}_t = \gamma_b \widehat{b}_{t-1} + \gamma_{b\varepsilon}^m \widehat{\varepsilon}_t^m, \tag{3.73}$$

$$\widehat{c}_t = \gamma_{cb}\widehat{b}_{t-1} + \gamma_{c\varepsilon}^m \widehat{\varepsilon}_t^m. \tag{3.74}$$

Substituting out the endogenous variables in (3.69)-(3.71) with generic solutions in (3.72)-(3.74), leads to the following conditions for $\gamma_{\pi b}$, γ_{cb}, γ_b, $\gamma_{\pi\varepsilon}^m$, $\gamma_{c\varepsilon}^m$, and $\gamma_{b\varepsilon}^m$:

$$\gamma_{\pi b} = \delta\gamma_{\pi b}\gamma_b + \chi\gamma_b + \chi\gamma_{cb},\ 1 = (1 + \rho_\pi)\,\gamma_{\pi b} + \gamma_{cb},\ 1 = \gamma_b + \gamma_{\pi b}, \tag{3.75}$$

$$\gamma_{\pi\varepsilon}^{m} = (\delta\gamma_{\pi b} + \chi)\,\gamma_{b\varepsilon}^{m} + \chi\gamma_{c\varepsilon}^{m},\ \gamma_{c\varepsilon}^{m} = -\,(1+\rho_{\pi})\,\gamma_{\pi\varepsilon}^{m} - 1,\ \gamma_{b\varepsilon}^{m} = -\gamma_{\pi\varepsilon}^{m}. \tag{3.76}$$

Using the three conditions in (3.75) and substituting out $\gamma_{\pi b}$ with $\gamma_{\pi b} = 1 - \gamma_b$, gives $0 = (\delta\gamma_b - 1)(1-\gamma_b) + \chi\gamma_b + \chi\gamma_{cb}$, $1 = (1+\rho_\pi)(1-\gamma_b) + \gamma_{cb}$, and further eliminating γ_{cb}, leads to $0 = (\delta\gamma_b - 1)(1-\gamma_b) + \chi\gamma_b + \chi(1-(1+\rho_\pi)(1-\gamma_b))$, which is a quadratic equation in γ_b that reads $\gamma_b^2 - (\delta + \chi + \chi(\rho_\pi + 1) + 1)\,\delta\gamma_b + (\rho_\pi\chi + 1)\,\delta = 0$. Condition (3.36) ensures that there exists exactly one stable and positive solution (see proof of Lemma 1 in Bredemeier et al., 2017). Assigning the stable root to $\gamma_b \in (0,1)$, such that $\gamma_{\pi b} = 1 - \gamma_b \in (0,1)$, we can easily identify the effects of the monetary policy shock $\widehat{\varepsilon}_t^m$ on inflation and consumption in t, i.e. $\gamma_{\pi\varepsilon}^m$ and $\gamma_{c\varepsilon}^m$. Combining the three conditions in (3.76) yields

$$\gamma_{\pi\varepsilon}^{m} = -\frac{\chi}{1+\delta\gamma_{\pi b} + \chi(2+\rho_\pi)}, \tag{3.77}$$

which is negative since whose sign depends on the sign of denominator since $\chi, \delta, \gamma_{\pi b}, \rho_\pi > 0$ and, hence, $\gamma_{\pi\varepsilon}^m < 0$. Inflation in t, thus, increases in response to an expansionary conventional monetary policy shock.

The effect on consumption can be obtained by using (3.77) in $\gamma_{c\varepsilon}^m = -(1+\rho_\pi)\,\gamma_{\pi\varepsilon}^m - 1$, which gives

$$\gamma_{c\varepsilon}^{m} = \frac{\chi(1+\rho_\pi)}{1+\delta\gamma_{\pi b} + \chi(2+\rho_\pi)} - 1, \tag{3.78}$$

where the fraction in (3.78) is less than one such that the total term is negative. Hence, consumption in t also increases in response to the conventional monetary policy shock, $\gamma_{c\varepsilon}^m < 0$.

Next, we show the effects of the monetary policy shock on the liquidity premium $\widehat{R}_t^{IS} - \widehat{R}_t^m$. Using the generic solutions (3.72)-(3.74) in the equilibrium conditions (3.32) and (3.35), we can write $\widehat{R}_t^{IS}$ and $\widehat{R}_t^m$ as

$$\widehat{R}_t^{IS} = [\gamma_{cb}(\gamma_b - 1) + \gamma_{\pi b}\gamma_b]\,\widehat{b}_{t-1} + [\gamma_{cb}\gamma_{b\varepsilon}^m - \gamma_{c\varepsilon}^m + \gamma_{\pi b}\gamma_{b\varepsilon}^m]\,\widehat{\varepsilon}_t^m, \tag{3.79}$$

$$\widehat{R}_t^{m} = \rho_\pi\gamma_{\pi b}\widehat{b}_{t-1} + (1+\rho_\pi\gamma_{\pi\varepsilon}^m)\,\widehat{\varepsilon}_t^m. \tag{3.80}$$

Given these solutions, the response of the liquidity premium to the monetary policy shock reads

$$\frac{\partial\left(\widehat{R}_t^{IS} - \widehat{R}_t^m\right)}{\partial\widehat{\varepsilon}_t^m} = (\gamma_{cb}\gamma_{b\varepsilon}^m - \gamma_{c\varepsilon}^m) + \gamma_{\pi b}\gamma_{b\varepsilon}^m - \rho_\pi\gamma_{\pi\varepsilon}^m - 1. \tag{3.81}$$

Using $\gamma_{c\varepsilon}^m = -(1+\rho_\pi)\,\gamma_{\pi\varepsilon}^m - 1$, $\gamma_{b\varepsilon}^m = -\gamma_{\pi\varepsilon}^m$, and $1 = (1+\rho_\pi)\,\gamma_{\pi b} + \gamma_{cb}$, we can show that (3.81) is negative since $\rho_\pi\gamma_{\pi b} \geq 0$ which follows from $\rho_\pi \geq 0$ and $\gamma_{\pi b} > 0$ (see above). Hence, the liquidity premium rises in response to the expansionary monetary policy shock.

2. We now aim at identifying the responses in periods t and $t+1$ to a negative monetary policy shock in $t+1$ that is announced in t, $\widehat{\varepsilon}^m_{t+1,t} < 0$. This time, we apply the solution form

$$\begin{aligned}
\widehat{\pi}_t &= \gamma_{\pi b}\widehat{b}_{t-1} + \gamma_{\pi\varepsilon}\widehat{\varepsilon}^m_{t+1,t} + \gamma'_{\pi\varepsilon}\widehat{\varepsilon}^m_{t,t-1}, && (3.82)\\
\widehat{b}_t &= \gamma_b\widehat{b}_{t-1} + \gamma_{b\varepsilon}\widehat{\varepsilon}^m_{t+1,t} + \gamma'_{b\varepsilon}\widehat{\varepsilon}^m_{t,t-1}, && (3.83)\\
\widehat{c}_t &= \gamma_{cb}\widehat{b}_{t-1} + \gamma_{c\varepsilon}\widehat{\varepsilon}^m_{t+1,t} + \gamma'_{c\varepsilon}\widehat{\varepsilon}^m_{t,t-1} && (3.84)
\end{aligned}$$

for the system (3.69)-(3.71), where $\gamma_{\pi\varepsilon}$, $\gamma_{b\varepsilon}$, and $\gamma_{c\varepsilon}$ yield the effects of the announcement on the endogenous variables in t, while $\gamma'_{\pi\varepsilon}$, $\gamma'_{b\varepsilon}$, and $\gamma'_{c\varepsilon}$ show the effect in $t+1$. Following the procedure from above and substitute for the variables in (3.69)-(3.71) with the generic solutions in (3.82)-(3.84), leads to the conditions

$$\begin{aligned}
\gamma_{\pi b} &= \delta\gamma_{\pi b}\gamma_b + \chi\gamma_b + \chi\gamma_{cb},\ 1 = (1+\rho_\pi)\,\gamma_{\pi b} + \gamma_{cb},\ 1 = \gamma_b + \gamma_{\pi b}, && (3.85)\\
\gamma'_{\pi\varepsilon} &= (\delta\gamma_{\pi b} + \chi)\,\gamma'_{b\varepsilon} + \chi\gamma'_{c\varepsilon},\ \gamma'_{c\varepsilon} = -(1+\rho_\pi)\,\gamma'_{\pi\varepsilon} - 1,\ \gamma'_{b\varepsilon} = -\gamma'_{\pi\varepsilon}, && (3.86)\\
\gamma_{\pi\varepsilon} &= (\delta\gamma'_{\pi\varepsilon} - \chi) + (\delta\gamma_{\pi b} + \chi)\,\gamma_{b\varepsilon} + \chi\gamma_{c\varepsilon},\ \gamma_{c\varepsilon} = -(1+\rho_\pi)\,\gamma_{\pi\varepsilon},\ \gamma_{b\varepsilon} = -\gamma_{\pi\varepsilon}. && (3.87)
\end{aligned}$$

First note that lines (3.85) and (3.86) correspond to (3.75) and (3.76) with $\gamma'_{x\varepsilon}$ replacing $\gamma^m_{x\varepsilon}$ for a generic variable $x = \pi, b, c$. The system (3.85), (3.86) contains six conditions in the six unknown coefficients $\gamma'_{\pi\varepsilon}$, $\gamma'_{b\varepsilon}$, $\gamma'_{c\varepsilon}$, $\gamma_{\pi b}$, γ_b, and γ_{cb} while (3.75), (3.76) is the same system of conditions in $\gamma^m_{\pi\varepsilon}$, $\gamma^m_{b\varepsilon}$, $\gamma^m_{c\varepsilon}$, $\gamma_{\pi b}$, γ_b, γ_{cb}. If follows that $\gamma'_{\pi\varepsilon}$, $\gamma'_{b\varepsilon}$, and $\gamma'_{c\varepsilon}$, respectively, equal $\gamma^m_{\pi\varepsilon}$, $\gamma^m_{b\varepsilon}$, and $\gamma^m_{c\varepsilon}$, respectively, from step 1 and that $\gamma_{\pi b}$,γ_b,γ_{cb} are the same as in step 1.

We start by identifying the effects of the forward guidance shock on inflation and consumption in period t, i.e. $\gamma_{\pi\varepsilon}$ and $\gamma_{c\varepsilon}$. Combining the three conditions in (3.87) yields

$$\gamma_{\pi\varepsilon} = \frac{\delta\gamma'_{\pi\varepsilon} - \chi}{1 + \delta\gamma_{\pi b} + \chi\,(2+\rho_\pi)}. \tag{3.88}$$

The denominator is the same as in (3.77) and therefore positive. The sign of the numerator, $\delta\gamma'_{\pi\varepsilon} - \chi$, which can by using the definitions of δ be rewritten as

$$-\frac{\chi\,(\beta+\chi) + \chi\,(1 + \delta\gamma_{\pi b} + \chi)}{1 + \delta\gamma_{\pi b} + \chi\,(2+\rho_\pi)} < 0$$

is unambiguously negative, such that $\gamma_{\pi\varepsilon} < 0$.

The consumption response in period t is easy to determine. With $\gamma_{c\varepsilon} = -(1+\rho_\pi)\,\gamma_{\pi\varepsilon}$ from (3.87), it follows directly that $\gamma_{c\varepsilon} > 0$. In response to the forward guidance shock, consumption falls and inflation rises in period t.

Now, we turn to period $t+1$. The total effect on inflation in this period is given by

$$\frac{\partial \widehat{\pi}_{t+1}}{\partial \widehat{\varepsilon}^m_{t,t-1}} = \gamma^{'}_{\pi\varepsilon} + \gamma_{\pi b}\gamma_{b\varepsilon}$$

which we claim to be negative. To prove this claim, we use $\gamma^{'}_{\pi\varepsilon} = \gamma^{m}_{\pi\varepsilon}$, $0 < \gamma_{\pi b} < 1$ and $\gamma_{b\varepsilon} = -\gamma_{\pi\varepsilon}$ from above such that it is sufficient to show that

$$\gamma^m_{\pi\varepsilon} < \gamma_{\pi\varepsilon}.$$

Using the results on these two coefficients from above, this is equivalent to

$$-\frac{\chi}{1+\delta\gamma_{\pi b}+\chi\left(2+\rho_\pi\right)} < \frac{\delta\gamma'_{\pi\varepsilon}-\chi}{1+\delta\gamma_{\pi b}+\chi\left(2+\rho_\pi\right)}.$$

The common denominator is positive (see above) such that this condition is equivalent to

$$-\chi < \delta\gamma'_{\pi\varepsilon} - \chi.$$

which is equivalent to

$$0 < \delta\gamma'_{\pi\varepsilon}$$

which is true as $\delta > 0$ and $\gamma'_{\pi\varepsilon} < 0$.

Now, we consider the total effect on consumption in period $t+1$ which is given by

$$\frac{\partial c_{t+1}}{\partial \widehat{\varepsilon}^m_{t,t-1}} = \gamma'_{c\varepsilon} + \gamma_{cb}\gamma_{b\varepsilon} = \gamma'_{c\varepsilon} - \gamma_{cb}\gamma_{\pi\varepsilon}$$

which we claim is negative. Using results on the $\gamma'_{c\varepsilon}$ and $\gamma_{\pi\varepsilon}$ from above and $\gamma_{cb} = 1-\left(1+\rho_\pi\right)\gamma_{\pi b}$, the claim is equivalent to

$$\frac{\chi\left(1+\rho_\pi\right)}{1+\delta\gamma_{\pi b}+\chi\left(2+\rho_\pi\right)} - 1 < \left(1-\left(1+\rho_\pi\right)\gamma_{\pi b}\right)\cdot\frac{\delta\gamma'_{\pi\varepsilon}-\chi}{1+\delta\gamma_{\pi b}+\chi\left(2+\rho_\pi\right)}$$

Again, the common denominatior is positive, hence the condition is equivalent to

$$\chi\left(1+\rho_\pi\right) - \left(1+\delta\gamma_{\pi b}+\chi\left(2+\rho_\pi\right)\right) < \left(1-\left(1+\rho_\pi\right)\gamma_{\pi b}\right)\cdot\left(\delta\gamma'_{\pi\varepsilon}-\chi\right)$$

which can be simplified to

$$1 + \delta\gamma_{\pi b} > \rho_\pi \gamma_{\pi b} \delta \gamma'_{\pi\varepsilon} - \rho_\pi \gamma_{\pi b} \chi - \chi.$$

The left hand side of this condition is positive. All terms on the right hand side are negative which is obvious for the final two and also holds for the first one as $\delta > 1$ as $\gamma'_{\pi\varepsilon} < 0$. This completes the proof of the claim that $\frac{\partial c_{t+1}}{\partial \widehat{\varepsilon}^m_{t,t-1}} < 0$.

Finally, we consider the responses of the liquidity premium $\widehat{R}^{IS} - \widehat{R}^m$. Using the generic solutions (3.82)-(3.84) in the equilibrium conditions as above, we can now write $\widehat{R}^{IS}_t$ and $\widehat{R}^m_t$ as:

$$\begin{aligned} \widehat{R}^{IS}_t &= [\gamma_{cb}(\gamma_b - 1) + \gamma_{\pi b}\gamma_b]\,\widehat{b}_{t-1} + [\gamma_{cb}\gamma_{b\varepsilon} + \gamma'_{c\varepsilon} - \gamma_{c\varepsilon} + \gamma_{\pi b}\gamma_{b\varepsilon} + \gamma'_{\pi\varepsilon}]\,\widehat{\varepsilon}^m_{t+1,t} \\ &\quad + [\gamma_{cb}\gamma'_{b\varepsilon} - \gamma'_{c\varepsilon} + \gamma_{\pi b}\gamma'_{b\varepsilon}]\,\widehat{\varepsilon}^m_{t,t-1} \qquad (3.89) \\ \widehat{R}^m_t &= \rho_\pi\gamma_{\pi b}\widehat{b}_{t-1} + \rho_\pi\gamma_{\pi\varepsilon}\widehat{\varepsilon}^m_{t+1,t} + (1 + \rho_\pi\gamma'_{\pi\varepsilon})\,\widehat{\varepsilon}^m_{t,t-1} \qquad (3.90) \end{aligned}$$

In the announcement period, the response of the liquidity premium in t to the forward guidance shock reads

$$\begin{aligned} \frac{\partial\left(\widehat{R}^{IS}_t - \widehat{R}^m_t\right)}{\partial\widehat{\varepsilon}^m_{t+1,t}} &= \gamma_{cb}\gamma_{b\varepsilon} + \gamma'_{c\varepsilon} - \gamma_{c\varepsilon} + \gamma_{\pi b}\gamma_{b\varepsilon} + \gamma'_{\pi\varepsilon} - \rho_\pi\gamma_{\pi\varepsilon} \\ &= [(1+\rho_\pi) - \gamma_{\pi b} - (1 - (1+\rho_\pi)\gamma_{\pi b}) - \rho_\pi]\,\gamma_{\pi\varepsilon} + \gamma'_{c\varepsilon} + \gamma'_{\pi\varepsilon} \qquad (3.91) \\ &= \rho_\pi\gamma_{\pi b}\gamma_{\pi\varepsilon} + \gamma'_{c\varepsilon} + \gamma'_{\pi\varepsilon}, \end{aligned}$$

where we use $\gamma_{cb} = 1 - (1+\rho_\pi)\gamma_{\pi b}$ from (3.85) and $\gamma_{c\varepsilon} = -(1+\rho_\pi)\gamma_{\pi\varepsilon}$ and $\gamma_{b\varepsilon} = -\gamma_{\pi\varepsilon}$ from (3.87) to obtain the second line. Since $\gamma_{\pi\varepsilon}$, $\gamma'_{\pi\varepsilon}$, and $\gamma'_{c\varepsilon}$ are all negative and ρ_π and $\gamma_{\pi b}$ are positive. Hence, the sign of (3.91) is negative and the liquidity premium rises on impact in response to the negative forward guidance shock.

In period $t+1$, the response of the premium is governed by

$$\begin{aligned} \frac{\partial\left(\widehat{R}^{IS}_{t+1} - \widehat{R}^m_{t+1}\right)}{\partial\widehat{\varepsilon}^m_{t+1,t}} &= [\gamma_{cb}\gamma_{b\varepsilon} + \gamma'_{c\varepsilon} - \gamma_{c\varepsilon} + \gamma_{\pi b}\gamma_{b\varepsilon} + \gamma'_{\pi\varepsilon} - (1 + \rho_\pi\gamma'_{\pi\varepsilon})] \\ &\quad + [\gamma_{cb}(\gamma_b - 1) + \gamma_{\pi b}\gamma_b - \rho_\pi\gamma_{\pi b}] \cdot \frac{\partial\widehat{b}_t}{\partial\widehat{\varepsilon}^m_{t+1,t}} \\ &= [(\gamma_{cb}\gamma_{b\varepsilon} + \gamma'_{c\varepsilon} - \gamma_{c\varepsilon}) + \gamma_{\pi b}\gamma_{b\varepsilon} + \gamma'_{\pi\varepsilon} - (1 + \rho_\pi\gamma'_{\pi\varepsilon})] \\ &\quad + [\gamma_{cb}(\gamma_b - 1) + \gamma_{\pi b}\gamma_b - \rho_\pi\gamma_{\pi b}] \cdot \gamma_{b\varepsilon}. \end{aligned}$$

Substituting out $\gamma_{b\varepsilon}$, $\gamma_{c\varepsilon}$, $\gamma'_{c\varepsilon}$, $\gamma_{\pi b}$, and γ_{cb} through the conditions in (3.85), (3.86), and (3.87) and rearranging, we can simplify this derivative to

$$\frac{\partial\left(\widehat{R}^{IS}_{t+1}-\widehat{R}^{m}_{t+1}\right)}{\partial\widehat{\varepsilon}^{m}_{t+1,t}}=-2\left(\gamma'_{\pi\varepsilon}+1\right)+\left(2-\gamma_b^2\right)\rho_\pi\gamma_{\pi\varepsilon}$$

which is unambiguously negative as $\gamma'_{\pi\varepsilon}=-\frac{\chi}{1+\delta\gamma_{\pi b}+\chi(2+\rho_\pi)}>-1$. □

3.C.2 Calculation of Anticipated Monetary Policy Shocks

In this appendix, we describe how to calculate the sequence of current and anticipated policy shocks $\varepsilon^m_{T+1}=\left\{\varepsilon^m_{T+1},\varepsilon^m_{T+1+k,T+1}\right\}_{k=1}^{K}$ of length $H=K+1$, which are announced in period $T+1$ for all periods until $T+H$ that yields a desired interest rate path $\left\{R^m_{T+h}\right\}_{h=1}^{H}$ that we want to study in a policy experiment about forward guidance.

We solve our model using standard perturbation techniques, yielding policy functions of the type

$$Y_{T+1}=g_c+g_p\widehat{\mathbf{s}}_T+g_\varepsilon\varepsilon^m_{T+1} \tag{3.92}$$

for a generic endogenous variable Y_t, such as the policy rate, that depends on a constant (steady state) value g_c, a vector of state variables $\widehat{\mathbf{s}}_T=\mathbf{s}_T-\mathbf{s}$ that is formulated in deviations from steady state $\mathbf{s}$ with dimension $[S\times 1]$, and the corresponding coefficient vector $g_p\,[1\times S]$. $\varepsilon^m_t\,[H\times 1]$ is a vector of one current and K anticipated policy shocks for period from $T+1$ to $T+H$ with the corresponding coefficients vector $g_\varepsilon\,[1\times H]$. Analogously, the vector of policy functions for the state variables reads

$$\mathbf{s}_{T+1}\,[S\times 1]=g_{sc}\,[S\times 1]+g_{sp}\,[S\times S]\,\widehat{\mathbf{s}}_T\,[S\times 1]+g_{s\varepsilon}\,[S\times H]\,\varepsilon^m_{T+1}\,[H\times 1] \tag{3.93}$$

with the coefficient matrices g_{sc}, g_{sp}, and $g_{s\varepsilon}$. The coefficient matrices and the steady state are all known objects. Using (3.92) and (3.93) and assuming that ε^m_{T+1} has non-zero entries only in the initial period $T+1$ (i.e. forward guidance is only provided in $T+1$ until $T+H$) allows us to write down solutions for the policy rate for H periods ahead that depend on the steady state of the state variables (in period T) and the policy shocks that are announced in $T+1$ only:[25]

$$R_{T+1}\left(\widehat{\mathbf{s}}_T,\varepsilon^m_{T+1}\right)\;=\;g_c+g_p\widehat{\mathbf{s}}_T+g_\varepsilon\varepsilon^m_{T+1}$$

[25] The procedure can obviously be applied to any other endogenous variable also, e.g. the policy rate in real terms.

$$
\begin{aligned}
R_{T+2}\left(\widehat{\mathbf{s}}_T, \varepsilon^m_{T+1}\right) &= g_c + g_p g_{sp} \widehat{\mathbf{s}}_T + g_p g_{s\varepsilon} \varepsilon^m_{T+1} \\
&\dots \\
R_{T+H}\left(\widehat{\mathbf{s}}_T, \varepsilon^m_{T+1}\right) &= g_c + g_p g_{sp}^{H-1} \widehat{\mathbf{s}}_T + g_p g_{sp}^{H-2} g_{s\varepsilon} \varepsilon^m_{T+1}.
\end{aligned}
$$

This constitutes a system of H linear equations in H unknown elements of ε^m_{T+1} for a given sequence $\left\{R^m_{T+h}\right\}_{h=1}^{H}$, which can be rewritten as

$$
b = M\varepsilon^m_{T+1}, \tag{3.94}
$$

where

$$
b\,[H \times 1] = \begin{bmatrix} R_{T+1} \\ R_{T+2} \\ \dots \\ R_{T+H} \end{bmatrix} - g_c \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix} - \begin{bmatrix} g_p \\ g_p g_{sp} \\ \dots \\ g_p g_{sp}^{H-1} \end{bmatrix} \widehat{\mathbf{s}}_T,
$$

and

$$
M\,[H \times H] = \begin{bmatrix} g_\varepsilon \\ g_p g_{s\varepsilon} \\ \dots \\ g_p g_{sp}^{H-2} g_{s\varepsilon} \end{bmatrix}.
$$

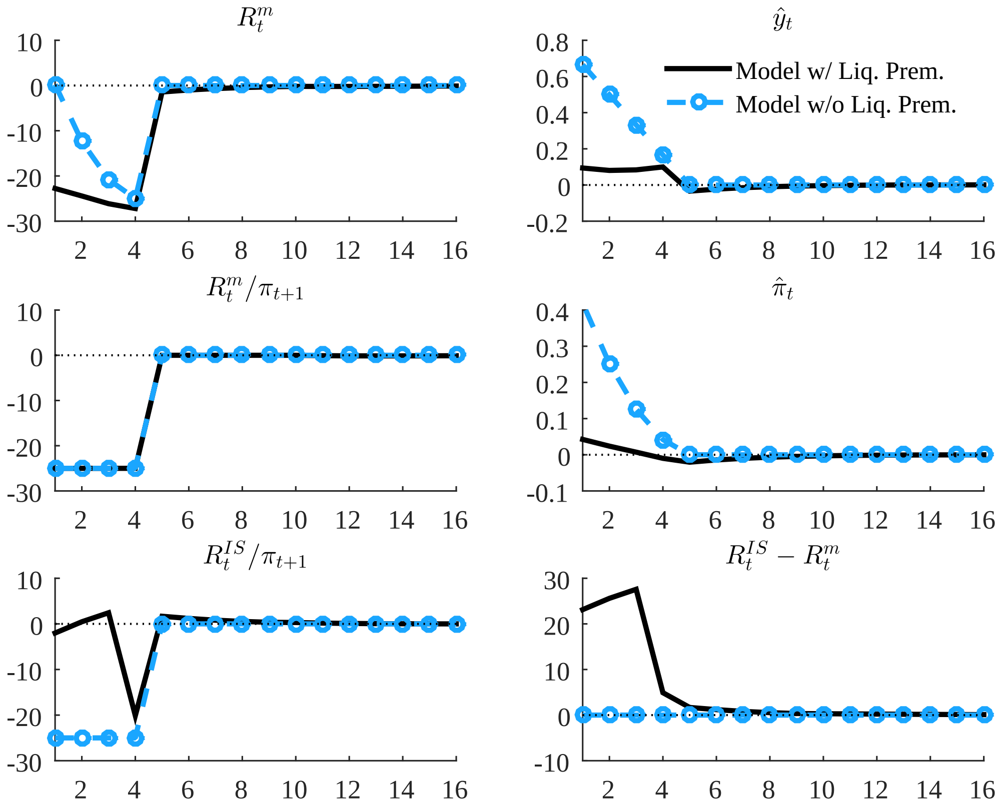
Rearranging (3.94) then allows to back out the H shocks as

$$
\varepsilon^m_{T+1} = M^{-1} b.
$$

3.C.3 Additional Numerical Results

Figure 3.8 complements our analysis by repeating the comparison of Figure 3.2, but now the central bank provides forward guidance about the real instead of the nominal policy rate. In this case, the real rate is announced to be reduced by 25 basis points for 4 quarters. We conduct this analysis because central banks in the end aim at steering real rates that are directly relevant for intertemporal consumption decisions. Moreover, we make our analysis thereby comparable to the analysis of McKay et al. (2016), who focus solely on real policy rates. It turns out that whether guidance is in terms of the real instead of the nominal rate does not make much of a difference for the model with the endogenous liquidity premium. The corresponding responses for output, inflation, and the liquidity premium in Figures 3.2 and 3.8 line up almost completely. The difference is larger for the conventional New Keynesian model, as the exacerbating effect via higher inflation that endogenously lowers real rates is now absent. The responses of real activity and inflation are nevertheless still much stronger than in the model with the liquidity premium.

Figure 3.8: Comparison with Standard New Keynesian Model – Real Policy Rate



Notes: Impulse responses to *real* policy rate (R_t^m/π_{t+1}) reduction of 25 basis points in quarters 1 to 4, announced before quarter 1: production $\hat{y}_t$, inflation $\hat{\pi}_t$, policy rate R_t^m, private-sector real rate R_t^{IS}/π_{t+1}, liquidity premium $R_t^{IS} - R_t^m$. Y-axis: Deviations from steady state in percent ($\hat{y}_t$, $\hat{\pi}_t$) or in basis points (else). X-axis: quarters. Black line: Baseline model with endogenous liquidity premium. Blue circled line: Conventional New Keynesian model.

Chapter 4

Determinants of Relative Sectoral Prices: The Role of Demographic Change

This chapter is based on Groneck and Kaufmann (2017).[1]

4.1 Introduction

The relative price of non-tradable services to tradable commodities is well-known to be an important determinant of real exchange rates. According to the famous Balassa-Samuelson hypothesis, which dates back to 1964, movements in these relative sectoral prices can be attributed to sectoral differences in productivity growth. Empirical studies tend to find support in favour of the hypothesis.[2] Further determinants of the relative price beyond the Balassa-Samuelson effect operate over the demand-side of the economy. The literature discusses non-homothetic preferences (Bergstrand, 1991), government demand (De Gregorio et al., 1994; Galstyan and Lane, 2009) and net foreign assets (Lane and Milesi-Ferretti, 2002, 2004; and Christopoulos et al., 2012).[3]

In this chapter, we propose a country's demographic structure as an additional economic fundamental for the relative price of non-traded goods and we study this relation-

[1]We thank Tino Berger, Alexander Ludwig, Øivind Nilsen, Matthias Schön, three anonymous referees, as well as seminar and conference participants at Cologne, Dortmund, and Paris for helpful comments. We also thank Julie Graf for excellent research assistance.

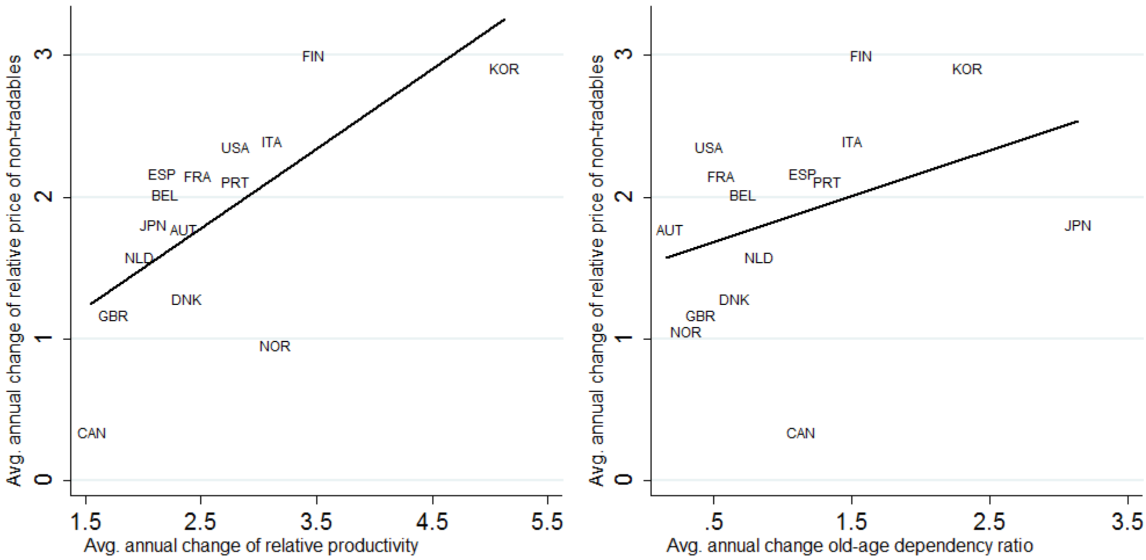
[2]See, amongst others, Canzoneri, Cumby and Diba (1999), Kakkar (2003), Berka, Devereux and Engel (2014), and Coto-Martinez and Reboredo (2014), where the latter also consider the role of imperfect competition.

[3]A synopsizing study is conducted by Ricci, Milesi-Ferretti and Lee (2013).

ship empirically. Figure 4.1 highlights the importance of this determinant. As a point of reference, the left panel depicts the cross-sectional relation between changes in relative prices of non-tradables and productivity growth differentials between tradables and non-tradables for a set of industrialized countries. The strong positive correlation illustrates the Balassa-Samuelson effect. The right panel plots relative-price changes against the average growth rates of old-age dependency ratios (hereafter named OADR), which are defined as the fraction of population aged 65+ to the population of age 15-64. This highlights our proposed channel: changes in the age structure of the population are positively correlated with the growth rate of the relative price of non-tradables. In particular, countries with stronger growth of the OADR experience higher growth in the relative price.

Figure 4.1: Cross-Sectional Correlations of Relative Price Changes

(a) Relative Productivity (b) Old-age Dependency Ratio



Notes: Average annual changes for 15 OECD countries between 1970 and 2009. Abscissa left panel: productivity in tradable relative to non-tradable sector. Abscissa right panel: old-age dependency ratio (population aged 65+ divided by population aged 15-64). Details on the construction of all variables are given in Appendix 4.B. Country codes are explained in Table 4.1.

There are several mechanisms how ageing can lead to higher relative prices. In this chapter, we focus on the following demand effects. We present evidence that elderly people consume more non-traded services relative to people in working age. This implies an increase in overall demand for those goods due to population ageing. At the same time, the old-age population has lower saving rates than younger cohorts, such that aggregate savings of an ageing society decline (see for instance Higgins 1998 and Yoon et al. 2014), while aggregate consumption increases. Likewise this rise in spending is also biased towards non-tradable goods. If the additional demand for non-traded services of an ageing society is not fully met by higher supply, the relative price of non-tradables increases. We claim that persistent imperfect intersectoral mobility of production factors hampers a reallocation of factor inputs to the non-tradable sector. Since we are concentrating on OECD countries with highly developed capital markets and since the production of non-traded services tends to be labour-intensive, labour market rigidities are – as we show – most important. The empirical literature supports this reasoning. For instance Wacziarg and Wallack (2004), Lee and Wolpin (2006), and Artuc, Chaudhuri and McLaren (2010) all provide evidence of substantial and long-lasting intersectoral worker immobility in response to labour demand shocks.[4]

Demographic change may also influence relative prices by other channels, which our analysis accounts for but which we argue to be of less importance. Besides its aforementioned impact on savings, ageing can also influence national investment and, hence, net foreign asset positions. Changes in the latter may imply relative price shifts due to the classical transfer effect. For instance, Lane and Milesi-Ferretti (2002, 2004) argue that higher net foreign assets can generate wealth effects, which lower labour supply. The resulting upward pressure on wages can lead to relative price increases if one sector is relatively labour-intensive, which is usually assumed to be the case for non-tradable services. Simulation results by Krueger and Ludwig (2007) show that changes in net foreign asset positions due to demographic change remain small for the United States and Europe, though. A further consequence of ageing could be an increased scarcity of labour relative to capital. This may also result in an upward pressure on wages with corresponding effects on the relative price, given the non-tradable sector is relatively labour-intensive. According to Krueger and Ludwig (2007), Ludwig, Schelkle and Vogel (2012) and Heijdra and Reijnders (2012) the impact of demographic change on factor prices is expected to be limited.[5] Nevertheless, our empirical analysis considers these channels as well.

To illustrate the relation between sectoral prices and population ageing and in order to provide guidance for the subsequent empirical analysis, we construct a stylized small open economy model with overlapping generations (OLG). We assume two production

[4] Cardi and Restout (2015) demonstrate the importance of labour market rigidities for the transmission of the Balassa-Samuelson effect.

[5] The literature shows that endogenous human capital adjustments, increased labour market participation of women, migration, international capital flow adjustments, as well as pay-as-you-go pension systems financed by payroll taxes all dampen the effect of demographic change on factor prices. In fact, the number of employees has risen in most countries over the past decades.

sectors. As is well known at least since Rogoff (1992), a precondition for any demand effects to matter for relative prices in such a setting is a deviation from full intersectoral factor mobility.[6] Otherwise, supply factors would just shift to the sector that experiences the increase in demand, leaving relative goods prices unchanged. The majority of the related literature merely assumes fixed amounts of sectoral production, thereby ruling out any kind of factor mobility. For instance, Rose, Supaat and Braude (2009) rely in a related model on the strong assumption of completely inelastic labour supply, both in the aggregate and between sectors.[7] In contrast, our model features an endogenous labour supply decision of households, and we explicitly allow for different degrees of imperfect intersectoral mobility of labour. We obtain two testable key results from our model. First, an increase in the old-age dependency ratio leads to an increase in the relative price of non-tradables. The reason is that workers do not reallocate their labour as much as needed to let supply keep up with changing demand. Second, we show that price effects are more pronounced for higher degrees of labour market rigidity.

The basic econometric specification arises from the theoretical model and shows that relative prices depend on the old-age dependency ratio, which is the regressor of main interest. To analyse whether imperfect labour mobility is relevant for the transmission of the effect, we introduce interactions of indices of labour market rigidity with the OADR. We construct a panel of 15 OECD countries that are followed from 1970 to 2009. Detailed sector-specific data is classified into tradables and non-tradables to construct relative prices and productivities. To quantify labour market immobility we use the index by Botero, Djankov, Porta, Lopez-De-Silanes and Shleifer (2004) that includes measures of institutional flexibility of the labour market. Our estimation strategy explicitly takes into account the non-stationarity and cross-sectional dependence of the data. To this end, we use the method by Pesaran (2006) and Kapetanios et al. (2011), which finds increasing use in macro panel studies.

Our results indicate a significant link between population ageing and relative sectoral prices. A one percent increase of the old-age dependency ratio inflates the relative price of non-tradables by 0.34 percent. This implies that about one fifth of the average increase in relative prices between 1970 and 2009 can be attributed to the increase of the OADR.

[6] Instead of assuming imperfect factor mobility, introducing deviations from the assumption of a small open economy or diminishing returns to scale in at least one sector (see Galstyan and Lane (2009) for an example) is also sufficient for both supply and demand factors to matter in determining relative prices and real exchange rates.

[7] Cantor and Driskill (1999) analyse the effect of a change in death rates on the real exchange rate in a stylized OLG model and find that the direction of the effect depends on the long-run net foreign asset position of a country. However, they do not consider non-tradable goods and their model also does not feature endogenous production.

The results indicate that the aforementioned demand effects of ageing are predominantly responsible for this finding. Moreover, we identify labour market rigidity as the driving force for the transmission of this demand effect. While price effects are close to zero for countries with very flexible labour markets, they increase monotonically with the degree of rigidity. Various robustness checks underpin the validity of our findings, and demonstrate the importance of this transmission mechanism relative to other possible channels. Further results widen the analysis to the whole population age structure.

We add to the literature that studies the determination of relative sectoral prices and real exchange rates by proposing a demand effect induced by population ageing. Few studies have analysed demographic change in this context. Complementary to our study, Rose et al. (2009) examine the effect of fertility on the real exchange rate arguing that a higher share of the dependent young population leads to lower savings and a higher demand for non-tradables. They confirm their theoretical prediction by finding a depreciating effect of declines in fertility on real exchange rates. Bettendorf and Dewachter (2007) analyse the impact of changes in the whole population age structure on the relative price of non-tradables, but their empirical findings remain in parts insignificant and inconclusive. Using a structural model, Aloy and Gente (2009) focus on Japan and find that declines in population growth are able to explain a large share of the real appreciation since 1970. Andersson and Österholm (2005, 2006) perform reduced-form regressions of real exchange rates on the population age structure. They show that using demographic data can improve forecasts of real exchange rates. For a detailed overview on this literature, see Hassan, Salim and Bloch (2011). Recently, Yoon, Kim and Lee (2014) and Juselius and Takáts (2015) study the effect of demographic change on inflation using post-war data with mixed results.

Overall, the above mentioned literature studying demand effects on relative prices and real exchange rates is silent about the precise mechanism through which changes in demand translate into price effects.[8]

Our contribution to the literature is, hence, twofold. First, by making use of recent advances in statistical methods, we are able to establish population ageing as a demand-driven determinant of relative sectoral prices. Second, in our empirical specification we pay particular attention to the theoretical transmission mechanism of the old-age related

[8] Although the importance of certain transmission channels like non-homothetic preferences, decreasing returns to scale or production factor immobility is generally acknowledged theoretically, it is usually not further investigated in the empirical specifications, cf. De Gregorio, Giovannini and Wolf (1994), Galstyan and Lane (2009), and Lane and Milesi-Ferretti (2002, 2004) and Rose et al. (2009), for example. One exception are Christopoulos, Gente and León-Ledesma (2012), who explicitly evaluate the importance of frictions on capital markets in developing countries for real exchange rate determination both theoretically and empirically.

demand effects by introducing labour market rigidity. We show that labour market rigidities are indeed a driving factor for the transmission.

The rest of the chapter proceeds as follows. In Section 4.2, we present stylized facts about age-specific consumption patterns of tradable and non-tradable goods. Section 4.3 lays out the theoretical model and derives two testable implications. Section 4.4 translates the theory into an econometric model, describes the data and explores data properties. Section 4.5 presents the main results of the paper, while various sensitivity checks are shown in Section 4.6. We conclude the paper in Section 4.7.

4.2 Ageing and Consumption

The demand effects of demographic change on sectoral prices rely on the premise that the elderly consume a higher fraction of non-tradable services than the population in working age does. Micro studies on the United States and some European countries all detect this age pattern in consumption data. Hobijn and Lagakos (2003), Börsch-Supan (2003) as well as van Ewijk and Volkerink (2012) present cross-sectional overviews of consumption-age profiles of several different expenditure groups for the U.S., Germany and the Netherlands, respectively. Lührmann (2005, 2008) investigates consumption-age profiles by means of panel data from Germany and the U.K. that enable her to control for all kinds of cohort-, time-, income-, and household-effects. The essence of these studies is that when people become older, they tend to reduce their expenses on tradable goods categories like 'transportation', 'furniture and home electronics' and 'clothing', while demand for non-tradables, such as 'housing' and 'health care goods and services' increases. Based on their findings, Hobijn and Lagakos yet discuss the introduction of an additional CPI for the elderly in the U.S. that takes into account their differing consumption spending patterns.

However, household data does not cover the full scope of changes in consumption patterns at the aggregate level. In particular, it does not take into account the substantial public spending on health and long-term care. According to OECD data, average health care spending of member states amounts to about ten percent of GDP in recent years, of which on average only 30 percent are financed by the private sector. Hagist and Kotlikoff (2005) estimate age profiles of health care spending for a sample of ten OECD countries and show that expenditures at old age are a multiple of those in working age. For instance, average health care expenses already double between the age groups 50-64 and 65-69.

As an illustration we quantify the difference between aggregate tradable and non-tradable consumption shares at working age and during retirement exemplarily for a specific country. We choose the United States in the year 2011. To this end, we combine micro data of the U.S. Consumer Expenditure Survey (CE) with aggregate data on Medicare and Medicaid health care spending. First, we map the numerous expenditure categories in the CE data on the tradable and non-tradable sector, based on sector classifications by De Gregorio et al. (1994), to obtain expenditures per capita on tradable and non-tradable goods for young (aged 15-64) and old people (aged 65+).[9] Next, we add Medicare and Medicaid spending per capita, which are classified to be non-tradable, to consumption expenditures of the two age groups. The resulting share of aggregate non-tradable consumption of the older people (83 percentage points) is on average about eight percentage points higher than in case of the younger people (75 percentage points). The magnitude of the difference depends on the chosen country and time period, though. Braude (2000) and van Ewijk and Volkerink (2012) conduct similar exercises for the United States in 1990 and the Netherlands in 2010 respectively and quantify the difference in shares to be, even larger, about 20 (70 versus 50) and 13 (70 versus 57) percentage points. In sum, the available evidence suggests substantial differences in aggregate consumption shares of tradables and non-tradables over the life cycle. Changes in the age distribution of the population are therefore expected to induce non-negligible changes in aggregate demand.

4.3 Model

We employ a model with overlapping generations and two production sectors $j = \{T, N\}$ to study the effect of population ageing on the relative price of non-tradable goods. Following the literature on structural real exchange rate determination, we assume a small open economy, where interest rates are taken as given by world markets.

A continuum of households lives for at most two periods, in each period t a young and an old generation is alive. Every young individual faces a probability π_t of growing old. The population size of the young generation is normalized to unity. Therefore, π_t is the ratio of old relative to young households. It can be interpreted both in an aggregate perspective as the OADR and in an individual perspective as the life expectancy of the household in period t for $t+1$. In order to capture the observations made in the last section in an abstract fashion, young households receive utility from the consumption of tradable commodities C_t^T and disutility of labour effort L_t, whereas the elderly enjoy the consumption of non-tradable services C_t^N.[10] Households maximize lifetime utility given

[9] Regarding details on data sources, the reader may be referred to Appendix 4.B.

[10] A generalized setting in which both generations consume both types of goods does not change the results qualitatively as long as preferences of the elderly are biased in favour of non-tradables.

by

$$U\left(C_t^T, L_t\right) + \beta\pi_t U\left(C_{t+1}^N\right), \tag{4.1}$$

where $\beta \in (0,1)$ is the subjective discount factor. Utility in working age is given by $U\left(C_t^T, L_t\right) = \ln C_t^T - \ln L_t$, while utility of the elderly is given by $U\left(C_{t+1}^N\right) = \ln C_{t+1}^N$. The assumed preference structure is chosen to obtain analytical solutions and implies both an intertemporal elasticity of substitution and a Frisch-elasticity of one. Labour supply can be allotted to both sectors of production. Following Horvath (2000) and Cardi and Restout (2015), households have a preference to work in both sectors, which drives a wedge between sectoral wages. Total labour in the utility function is defined by the CES-aggregate

$$L_t = \left[\left(L_t^T\right)^{\frac{\rho+1}{\rho}} + \left(L_t^N\right)^{\frac{\rho+1}{\rho}}\right]^{\frac{\rho}{\rho+1}}, \tag{4.2}$$

where L_t^j denotes hours worked in the tradable ($j = T$) and non-tradable ($j = N$) sector respectively. $\rho > 0$ measures the elasticity of substitution between labour supplies in both sectors. For $\rho \to \infty$, hours worked are perfect substitutes and workers would devote all working time to the sector that pays the highest wage, while for $\rho < \infty$, workers have a preference to diversify their labour supply and are willing to work in both sectors even in the presence of wage differentials. Hence, ρ measures the degree of imperfect intersectoral labour mobility, where small values of ρ imply less mobility. In line with empirical evidence cited above, this modelling choice generates persistent labour market frictions and shall be regarded as a short-cut for more comprehensive models of labour market rigidities, in order to allow for explicit analytical solutions and comparative statics.

Temporal budget constraints are given by

$$C_t^T = L_t^T W_t^T + L_t^N W_t^N - S_t \tag{4.3}$$

and

$$P_{t+1} C_{t+1}^N = \frac{(1+R^*)}{E_t \pi_t} S_t. \tag{4.4}$$

W_t^T and W_t^N label wages in the two sectors, S_t denotes household savings that are invested on international capital markets and that yield an exogenously given return of R^*, which is the world interest rate. In addition, we assume a perfect annuity market, where assets of those who deceased are passed to the survivors, so that the return on savings is $(1+R^*)/\pi_t$.[11] Finally, we let P_t denote the relative price of non-tradable ser-

[11] Alternatively, one could enrich the model by a warm-glow bequest motive. This would imply less dissaving in old age, but at the same time already more savings in young age due to anticipation effects. In sum, effects on the consumption path of households are limited or even absent.

vices to tradable commodities and assume the price of tradable goods to be given by world markets and normalized to unity.[12] The first-order conditions of the household's maximization problem yields the standard Euler-equation, $C^N_{t+1}/C^T_t = \beta\left(1+R^*\right)/P_{t+1}$, and an equation on how to optimally supply labour in the two sectors, given by

$$\frac{L^T_t}{L^N_t} = \left(\frac{W^T_t}{W^N_t}\right)^{\rho}. \tag{4.5}$$

Condition (4.5) states that relative hours worked depend on the wage ratio $\left(W^T_t/W^N_t\right)$ and the elasticity of substitution ρ.

Both in the tradable and the non-tradable sector a homogeneous consumption good is produced by perfectly-competitive firms using labour L^j_t and physical capital K^j_t as inputs within the Cobb-Douglas technology

$$Y^j_t = F\left(A^j_t, K^j_t, L^j_t\right) = A^j_t \left(K^j_t\right)^{\alpha_j} \left(L^j_t\right)^{1-\alpha_j}, \tag{4.6}$$

where Y^j_t and A^j_t are output and productivity in sector $j = \{T, N\}$, respectively. Firms borrow capital on international markets, which is assumed to fully depreciate within one period. In this small open economy setting, profit maximization and perfect competition among firms yield that optimal sectoral capital intensities are tied down by productivity and the world interest rate, while real sectoral wages depend on exogenous parameters and exogenous stochastic processes for A^j_t and π_t only.

A competitive equilibrium in this economy is defined as a sequence of prices and quantities such that optimality conditions of all agents hold and markets clear in each period for a given interest rate R^*, a given price of tradable goods and sectoral productivities A^j_t. All agents operate as price takers. Households choose consumption C^T_t and C^N_{t+1}, savings S_t and sectoral labour supplies L^j_t, while firms decide on their labour and capital demand $\left(L^j_t, K^j_t\right)$ in both sectors. Labour markets clear every period. Savings of households are fully invested at international capital markets and firms borrow all capital from abroad. Supply of tradable commodities Y^T_t has to equal domestic demand C^T_t plus net exports. The market clearing condition of the non-tradable sector is given by

$$Y^N_t = \pi_t C^N_t. \tag{4.7}$$

[12] As discussed repeatedly in the literature (for instance in Froot and Rogoff 1995), the relative price of non-tradables is directly related to the real exchange rate in such a setting.

In steady state, the relative price of non-tradables can be shown to evolve according to

$$P = k\left(\frac{\left(A^T\right)^{\frac{1-\alpha_N}{1-\alpha_T}}}{A^N}\right)\left(\frac{\pi\varkappa}{1-\pi\varkappa}\right)^{\frac{1-\alpha_N}{1+\rho}}, \tag{4.8}$$

where k and $\varkappa$ are positive constants and $P > 0$ as long as $1-\pi\varkappa > 0$.[13] Accordingly, P is driven by two main components in this framework. The first term of (4.8) in parentheses illustrates the well-known Balassa-Samuelson effect. It states that an increase in productivity in the tradable sector generates an increase in the relative price of non-tradables and appreciation of the real exchange rate, while productivity growth in the non-traded sector yields a decline in the relative price and real depreciation. The second component of the equation, on which this chapter lays its focus, highlights the effect of population ageing as well as the role of labour market rigidities in its transmission on the relative price. In particular, we are able to show the following:

Proposition 2. *In steady state, the effect of ageing on the relative price of non-tradables is positive:*

$$\frac{\partial P}{\partial \pi} > 0 \tag{4.9}$$

Proposition 3. *In steady state, the effect of ageing on relative prices is the higher, the lower intersectoral labour mobility,*

$$\frac{\partial\left(\partial P/\partial\pi\right)}{\partial\rho} < 0, \tag{4.10}$$

Formal proofs are given in Appendix 4.A.2.

Increasing the old-age dependency ratio leads to higher demand for non-tradable services: Currently young households will consume less tradables and save more for old-age. Once old, they consume the proceedings of their higher savings in form of more non-tradable goods. In case of perfect factor mobility ($\rho \to \infty$) these demand shifts are fully met by higher supply as labour moves immediately to the service sector due to positive wage pressure. Production of services increase until wages are equal in both sectors again, leaving relative goods prices unchanged. The last term of (4.8) converges to one. In contrast, in case of imperfect intersectoral labour mobility ($\rho < \infty$), higher demand in the non-tradable service sector entails a positive effect on the relative price of non-tradables, since reallocation of labour is not exhaustive: increased demand is only partly met by changes in supply and partly by an increase in the relative price.

[13] A derivation of this equation is given in Appendix 4.A.1.

4.4 Estimation Procedure and Data

4.4.1 Econometric Model

Propositions 2 and 3 constitute the two main hypotheses we intend to test. To this end, we set up a reduced form econometric specification based on (4.8), which is given by

$$\ln(p_{it}) = c_i + \gamma_1 \ln(oadr_{it}) + \gamma_2 \ln(oadr_{it}) \cdot lri_i + \gamma' X_{it} + u_{it}. \tag{4.11}$$

Sub-indices denote country i and time period t respectively, c_i labels country-specific intercepts and u_{it} is an error term, whose structure will be discussed below. To allow for a convenient interpretation as (semi-)elasticities, variables are, when sensible, used as natural logarithms. The dependent variable $\ln(p_{it})$ is the natural log of a measure of the relative price of non-tradables. The covariate of main interest, $\ln(oadr_{it})$, is the empirical counterpart of π_t and denotes the log of the old-age dependency ratio. According to our first hypothesis, based on Proposition 2, its coefficient γ_1 should possess a positive sign. Our second hypothesis, deduced from Proposition 3, claims that imperfect labour mobility leads to higher price effects of ageing. This is tested for by including an interaction term of $\ln(oadr_{it})$ and a measure of labour market rigidities, lri_i, which is considered to be the empirical counterpart of ρ. For expositional reasons, we begin with using a binary variable, $\widetilde{lri}_i = \{0, 1\}$, for lri_i in the interaction term. This binary index takes on a value of one for countries with an LRI-value above the sample mean ($\widetilde{lri}_i = \mathbf{1}_{lri_i \geq \overline{lri}_i}$) and a value of zero else ($\widetilde{lri}_i = \mathbf{0}_{lri_i < \overline{lri}_i}$). This way the coefficient γ_1 can be interpreted as the effect of ageing for countries with an LRI-value below mean, while $\gamma_1 + \gamma_2$ indicates the effect for countries with an LRI-value above mean. In a next step, we estimate the interaction effect by using the actual country-specific values of lri_i. In this case, $\partial \ln(p_{it}) / \partial \ln(oadr_{it}) = \gamma_1 + \gamma_2 lri_i$ gives the partial effect of ageing for the respective value of lri_i. According to Proposition 3, the partial effect of the old-age dependency ratio on the relative price is thus expected to be positive and to increase for higher degrees of labour market rigidity.

Further explanatory variables are summarized in the vector X_{it}. Its elements are motivated by (4.8) and by the related empirical literature. We consider variables, for which there exists broad consensus on their importance in determining real exchange rates and relative prices of non-tradables.[14] First, we include productivity in the tradables relative to the non-tradables sector (relative sectoral productivity) to account for the classic Balassa-Samuelson effect. Next, we add GDP per capita to control for demand-side effects,

[14]Ricci et al. (2013) provide an overview about which variables may belong to this canonical set of determinants.

for instance due to non-homothetic preferences that regard non-tradable services as luxuries and tradable commodities as necessities – an approach proposed first by Bergstrand (1991). Moreover, GDP per capita is capable of capturing effects of changes in factor endowments in the spirit of Bhagwati (1984) as GDP is strongly correlated with the capital-labour ratio of the economy. Higher capital-labour ratios lead to higher wages, in particular in the labour-intensive non-tradable sector and, thereby, to a higher relative price. To this extent, GDP per capita also controls for supply-side effects of demographic change, such as changes in the relative scarcity of labour. In the presence of these effects, the coefficient of GDP is expected to be positive. The third factor we consider is government consumption relative to GDP to control for further demand effects, since public expenditures are known to be biased towards non-tradables. Given this reading, its coefficient should also be positive. Evidence on this effect is for instance provided by Ricci et al. (2013) and Galstyan and Lane (2009). Lane and Milesi-Ferretti (2002, 2004) deal with wealth effects of net foreign asset positions on real exchange rates. According to their argument, which is related to Keynes' classical transfer problem, an increase in net foreign assets induces wealth effects that reduce labour supply. This hits labour-intensive non-tradable sectors relatively stronger, thereby leading to an increase of the relative price of non-tradables. To control for this kind of effect, we augment our specifications with a variable on net foreign assets relative to GDP. As demographic change can influence net foreign asset positions via its effect on savings, this variable can also capture these indirect price effects of population ageing. Since net foreign assets can also attain negative values, it is the only element of X_{it}, which is not used in logs.

By the inclusion of net foreign assets and GDP per capita, we control for the transfer and supply-side effects of demographic change that are discussed in the introduction. As a consequence, the coefficient of the old-age dependency ratio will predominantly capture the demand effects of ageing on relative prices.

4.4.2 Data Description

The empirical analysis is based upon a new-constructed panel data set of 15 OECD countries with annual observations beginning earliest in 1970 and ending at the latest in 2009. No country is followed for less than 30 years. On average we have 36 annual observations per country. Overall, we command 546 usable observations in the benchmark model. The choice of countries is restricted by the availability of sufficiently detailed data on sectoral prices and productivity over sufficiently long time horizons. An overview of the sample dimensions is given in Table 4.1. All data stems from publicly available sources,

Table 4.1: Sample Overview

Country	Abbrev.	Coverage	Country	Abbrev.	Coverage
Austria	AUT	1976-2009	Korea	KOR	1971-2009
Belgium	BEL	1975-2009	Netherlands	NLD	1977-2009
Canada	CAN	1970-2006	Norway	NOR	1970-2009
Denmark	DNK	1970-2009	Portugal	PRT	1977-2006
Finland	FIN	1970-2009	Spain	ESP	1980-2009
France	FRA	1970-2008	United Kingdom	GBR	1971-2007
Italy	ITA	1970-2009	United States	USA	1977-2009
Japan	JPN	1970-2008	*Full Sample (avg.)*	–	1973-2008

Notes: 546 usable observations in the benchmark model.

such as the OECD STAN data base or the Penn World Tables. Details on the sources and regarding the construction of all variables are shifted to Appendix 4.B. A list of all variables used throughout the text and summary statistics can be found in Table 4.2.

The relative price of non-tradable goods is constructed as the ratio of price indices of the non-tradables and the tradables sector. Vice versa, the relative sectoral productivity refers to the productivity ratio of the tradables relative to the non-tradables sector. As in Canzoneri et al. (1999) and Ricci et al. (2013), sectoral productivities are measured as value added per worker. The old-age dependency ratio, which is one of the standard measures in population economics, is defined as population aged older than 65 divided by population in working age (15-64). The the old-age population share measures the amount of the population aged 65+ relative to total population, while the working-age population share accounts for the ages 15-64 to total population. Similar to the OADR, the young-age dependency ratio (hereafter named YADR) measures the amount of dependent children aged 0-14 relative to the population in working age (15-64). The total fertility rate is defined as the number of children that would be born to a woman during her childbearing years if she bears children in accordance with current age-specific fertility rates.

Table 4.2 illustrates the magnitude and evolution of the data over time. As most variables in our sample feature clear upward trends, we present means and standard deviations at the beginning and end of the observation period instead of the less meaningful overall sample statistics.

The labour market rigidity index, LRI, in the main results is taken from Botero et al. (2004). This measure is widely used both in academia and by institutions such as the World Bank. It is defined as the average of four other indices, namely alternative employment contracts, cost of increasing hours worked, cost of firing workers, and dismissal procedures. This composite index can attain values between zero and one, where higher

Table 4.2: Summary Statistics

	Unit	1970 Mean	1970 Std. Dev.	2009 Mean	2009 Std. Dev.	Average Annual Change[a]
Relative Price on Non-Tradables	Index	63.80	17.28	121.86	18.82	1.86
Old-Age Dependency Ratio	%	15.04	5.25	23.51	4.30	1.06
Old-Age Population Share	% of Pop.	9.57	3.37	15.82	2.63	1.24
Working-Age Population Share	% of Pop.	63.14	4.16	67.29	2.01	0.17
Young-Age Dependency Ratio	%	43.95	1.41	25.12	0.30	-1.37
Total Fertility Rate	%	2.54	0.83	1.65	0.27	-0.84
Labour Market Rigidity Index, LRI	[0,1]	0.53	0.22	0.58	0.17	–
Labour Market Rigidity Index, LRI^{EPI}	[0,6]	2.33[b]	1.19[b]	2.14	0.72	-0.36
Capital Openness, KAOPEN	[0,1]	0.39	0.28	0.96	0.13	3.70
Economic Freedom, EconFree	[0,100]	69.18[c]	6.21[c]	73.26	5.36	0.53
Labour Market Freedom, LabFree	[0,100]	67.83[d]	17.97[d]	65.64	20.06	-0.13
Relative Sectoral Productivity	Index	52.86	21.73	148.75	42.01	2.65
GDP per capita	2005 Int$	14,242	4943	34,650	7425	2.31
Government Consumption	% of GDP	7.36	1.35	7.20	1.18	-0.01
Net Foreign Assets	% of GDP	-1.94	34.85	-1.39	48.94	0.12

Notes: [a] Cross-sectional mean of average annual growth rates in percent. [b]Mean and Std. Dev. in 1985 instead of 1970 due to data limitations. [c] Mean and Std.Dev. in 1995. [d]Mean and Std.Dev. in 2005. The relative price of non-tradables is defined relative to the price of tradables, whereas relative sectoral productivity is defined as productivity in the tradables relative to the non-tradables sector.

values represent larger rigidities. Table 4.3 reveals a wide variation of the index in our sample. As one would expect, the index takes on substantially lower values for Anglo-American than for continental European countries (e.g. United States 0.22 versus France 0.74). Yet, a drawback of this measure is that it does not reflect changes of these rigidities over time, since it is merely a fixed number per country. This issue is addressed by the OECD Indicators of Employment Protection. These contain a time-*variant* measure of the strictness of employment protection for years following 1985. We denote this index as LRI^{EPI} and use it for a robustness check. This alternative index is on a scale from 0 (least rigid) to 6 (most rigid). As Table 4.3 reveals, means per country of the LRI^{EPI} yield a similar ranking as the index by Botero et al. (2004). Further indices that measure other economic and legal characteristics (see KAOPEN, EconFree, LabFree in Table 4.2) will be described within the sensitivity analysis.

Table 4.3: Labour Market Rigidity per Country

Country	LRI	Mean(LRI^{EPI})	Country	LRI	Mean(LRI^{EPI})
Austria	0.50	2.64	Korea	0.45	2.64
Belgium	0.51	1.78	Netherlands	0.73	2.93
Canada	0.26	0.92	Norway	0.69	2.33
Denmark	0.57	2.15	Portugal	0.81	4.68
Finland	0.74	2.43	Spain	0.74	2.83
France	0.74	2.39	United Kingdom	0.28	1.01
Italy	0.65	2.76	United States	0.22	0.26
Japan	0.16	1.67	*Full Sample (avg.)*	0.54	2.23

Notes: LRI denotes Labour Market Rigidity Index and has a range of [0,1], see Botero et al. (2004). LRI^{EPI} has a range of [0,6], see OECD Indicators of Employment Protection. For both indices lower values mean lower degrees of rigidity.

4.4.3 Non-Stationarity and Cross-Sectional Dependence

In order to determine the appropriate estimation techniques, we test the data for cross-sectional correlation and its trend behaviour. Macroeconomic variables are notoriously affected by these two issues, which can seriously distort inference and consistency of estimations.[15]

To check for cross-sectional dependencies, Table 4.4 presents average (absolute) cross-sectional correlation coefficients and results of Pesaran's (2004) CD_P test statistic, which is $N(0,1)$-distributed under the null hypothesis of cross-sectional independence. The CD_P statistics rejects cross-sectional independence for all variables and the computed average correlation coefficients reveal strong cross-sectional correlations for all variables, except for government consumption and net foreign assets. Altogether, the results leave no doubt that cross-sectional correlation is indeed a problem in this data set.

The trend behaviour of the data is explored by means of three different panel unit root tests. We apply the tests by Im, Pesaran and Shin (2003) (IPS), Maddala and Wu (1999) (MW), and by Pesaran (2007) (CIPS). The latter is a panel unit root test of the second generation that can account for cross-sectional correlations, which is important as tests that neglect this issue can have non-negligible size distortions. Results are shown in Table 4.5. Under the null hypothesis of all three tests, the variable in question entails a unit root, while under the alternative hypothesis at least one series of the panel is stationary. All three tests are based on standard augmented Dickey-Fuller regressions on

[15] As shown, for instance, by O'Connell (1998) in the context of tests for purchasing power parity, disregarding cross-sectional dependence can come at high costs and may even revert outcomes of empirical studies.

Table 4.4: Cross-Sectional Dependence Tests

	CD_P	$avg.\,(r_{ij})$	$avg.\,(\lvert r_{ij}\rvert)$
Relative Price on Non-Tradables	51.03***	0.86	0.86
Old-Age Dependency Ratio	37.90***	0.64	0.70
Old-Age Population Share	45.73***	0.77	0.78
Working-Age Population Share	27.34***	0.46	0.56
Young-Age Dependency Ratio	51.87***	0.87	0.87
Total Fertility Rate	14.05***	0.23	0.60
Relative Sectoral Productivity	57.92***	0.97	0.97
GDP per capita	58.17***	0.98	0.98
Government Consumption (% of GDP)	10.34***	0.17	0.52
Net Foreign Assets (% of GDP)	-2.48**	-0.041	0.58

Notes: All variables except net foreign assets are measured in logs. CD_P denotes Pesaran (2004) cross-sectional dependence test statistic. Asterisks indicate rejection of the null hypothesis of cross-sectional independence at 10%(*), 5% (**) and 1% (***). $avg.\,(r_{ij})$ and $avg.\,(|r_{ij}|)$ denote average and average absolute cross-sectional correlation coefficients.

the individual time series, but in case of the CIPS these are extended by cross-sectional averages in lagged levels and first-differences of the variable in question to address issues of cross-sectional correlation. In order to control for serial correlation, all tests can include various autoregressive lags. The optimal lag length for each variable is determined by the Akaike and Bayesian information criteria searching between 0 and 4 lags. As Table 4.5 reveals all variables but the working-age population share, the YADR, (and fertility) are found to be non-stationary. It is, therefore, important to examine the time series properties of the regression residuals as well in order to rule out spurious regression results.

4.4.4 Econometric Methodology

Given the presence of cross-sectional correlation in the data, we follow Pesaran (2006) in assuming an error term of multi-factorial structure for our panel regression model (4.11). We describe the error term, u_{it}, by

$$u_{it} = \delta_i' \mathbf{f}_t + \varepsilon_{it}, \quad \varepsilon_{it} \sim \text{i.i.d. } N\left(0, \sigma^2\right), \tag{4.12}$$

where $\mathbf{f}_t$ is a vector of unobserved, potentially non-stationary common factors, which represent events that appear to influence all countries at the same time. By the vector of individual-specific factor loadings δ_i', different countries are still allowed to react differently to these common effects. The covariates $x_{it} \in (oadr_{it}, X_{it})$ in (4.11) can be correlated with

Table 4.5: Panel Unit Root Tests

	CIPS	IPS	MW
Relative Price on Non-Tradables	1.27	1.46	24.35
Old-Age Dependency Ratio	5.17	1.94	35.67
Old-Age Population Share	1.11	1.19	34.88
Working-Age Population Share	-4.32***	-2.35***	61.96***
Young-Age Dependency Ratio	-6.68***	-7.70***	49.05**
Total Fertility Rate	1.66	-2.34***	81.66***
Relative Sectoral Productivity	0.46	2.85	25.20
GDP per capita	-0.37	-0.28	22.81
Government Consumption (% of GDP)	1.19	0.032	19.07
Net Foreign Assets (% of GDP)	6.24	5.19	11.30

Notes: All variables except net foreign assets are measured in logs. Results of CIPS (Pesaran 2007), IPS (Im et al. 2003), and MW (Maddala and Wu 1999) panel unit root test statistics. Asterisks indicate rejection of the null hypothesis of a unit root at 10%(*), 5% (**) and 1% (***). Optimal lag length determined by Akaike and Bayesian information criteria searching between 0 and 4 lags.

the same unobserved factors $\mathbf{f}_t$ as u_{it}, and may be described as a process of the type

$$x_{it} = a_i + \eta_i' \mathbf{f}_t + v_{it}, \tag{4.13}$$

which is assumed to depend on a fixed effect a_i, the factors $\mathbf{f}_t$ with country-specific factor loadings η_i' and a random component v_{it}. In case of economic macro variables, examples for the factors $\mathbf{f}_t$ are common business cycles, the world financial crisis, or the effects of globalization. In case of demographic variables, one may think of changes in working environments, habits, or medical innovations that increase longevity or reduce birth rates such as the contraceptive pill. If common factors are present in u_{it}, but u_{it} and x_{it} are uncorrelated ($\delta_i \neq \mathbf{0}, \eta_i = \mathbf{0}$), error terms in (4.11) will be cross-sectionally correlated and the use of conventional estimators will yield inefficient standard errors. If u_{it} and x_{it} are correlated additionally ($\delta_i \neq \mathbf{0}, \eta_i \neq \mathbf{0}$), coefficient estimates itself are biased due to a type of omitted variable bias.

A promising approach to remedy the problem is to apply the Common Correlated Effects Pooled (referred to as CCEP) estimator developed by Pesaran (2006). This estimator is practically computed as an ordinary least squares regression, augmented with a set of additional regressors that consists of the cross-sectional averages of the dependent and independent variables, which are interacted with country dummies. As the data are found to be non-stationary, it is likely that at least some components of $\mathbf{f}_t$ are integrated of order one. Kapetanios et al. (2011) show that the CCEP estimator is consistent in presence of unit roots in the unobservable factors. Using Monte Carlo studies, they demonstrate the superiority of the CCEP estimator over other commonly used ones, even in small samples as ours. A further appealing feature of the estimator is that by controlling for the potentially non-stationary common factors the approach helps to deal with the problems of non-stationary data, such as biased inference that can lead to spurious regressions. Given these issues, estimations of (4.11) are conducted by means of Pesaran's CCEP approach and, for comparison reasons, using the DOLS procedure (see Stock and Watson 1993, Kao and Chiang 2000), which is a widely used methodology for non-stationary panels. It is calculated as a two-way fixed effects model that employs additional leads and lags of first differences of the independent variables.

In order to rule out spurious regressions, we control for stationarity of the regression residuals, again taking into account the possibility of cross-sectional dependence. To this end, we again apply the CIPS test.[16] As is well known, we cannot directly use the critical values from Pesaran (2007) that were constructed for the case of raw data, since regression residuals are calculated as to minimize the sum of their squares. Instead, we generate critical values directly from our sample by applying the Continuous-Path Block Bootstrap method developed by Paparoditis and Politis (2000, 2003), assuming a fixed block length of 10 percent of the overall observation period.[17] This method is explicitly designed to preserve non-stationarity and cross-sectional dependence of the data. We generate 500 bootstrap redraws of the estimated regression residuals. For each redraw we compute the CIPS statistic and thereby generate a distribution of the test statistic.

4.5 Results

In this section, we provide evidence for the two hypotheses deduced from Propositions 2 and 3. Table 4.6 presents results of model (4.11) without (columns I, II) and with (columns III, IV) the interaction term using the CCEP and the DOLS approach.

4.5.1 Ageing and the Relative Price of Non-Tradables

The coefficient of the old-age dependency ratio using the CCEP estimator, cf. column (I), implies that an increase in the OADR by one percent leads to an increase in the relative price of non-tradables of 0.34 percent. The estimate is statistically significant at the 1 percent level and constitutes good evidence for our first hypothesis – population ageing

[16] For instance, Holly, Pesaran and Yamagata (2010) use it for the same purpose.

[17] We adopt and modify code by Fachin (2007), such that it can be used for an unbalanced panel as ours.

Table 4.6: Main Regressions

Dependent Variable: Relative Price of Non-Tradables	(I) CCEP	(II) DOLS	(III) CCEP	(IV) DOLS
Old-Age Dependency Ratio (OADR)	0.34***	0.20***	0.15	0.20***
	(0.093)	(0.040)	(0.111)	(0.040)
OADR × Labour Market Rigidity (LRI)			0.54***	0.16**
			(0.175)	(0.065)
Relative Sectoral Productivity	0.59***	0.58***	0.58***	0.55***
	(0.046)	(0.034)	(0.045)	(0.038)
GDP per capita	0.41***	-0.054	0.32***	-0.018
	(0.072)	(0.040)	(0.076)	(0.041)
Government Consumption (% of GDP)	0.11**	-0.055	0.073	-0.10**
	(0.051)	(0.039)	(0.051)	(0.043)
Net Foreign Assets (% of GDP)	-0.00083***	-0.0022***	-0.00081***	-0.0020***
	(0.00017)	(0.00017)	(0.00017)	(0.00020)
Residual diagnostics				
CD_P	-2.08**	-3.26***	-2.51**	-3.33***
CIPS	-7.095***	-0.20	-7.21***	0.014
F(OADR,Interaction= 0)			11.91***	16.19***
Observations	546	501	546	501

Notes: All variables except net foreign assets are measured in logs. Regressions based on (4.11). In regressions (III) & (IV) , OADR is interacted with a binary variable with value one in case of labour market rigidity above average ($\widetilde{lri}_i = \mathbf{1}_{lri_i \geq \overline{lri}_i}$) and zero else ($\widetilde{lri}_i = \mathbf{0}_{lri_i < \overline{lri}_i}$). Country dummies included in all regressions, time dummies in (II) and (IV). Standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Residual diagnostics: CD_P cross-sectional dependence test statistic by Pesaran (2004), Residual stationarity tested by Pesaran's (2007) CIPS test using bootstrapped critical values. F(OADR,Interaction= 0) denotes F-test about joint significance of OADR and the interaction term.

leads to a higher relative price of non-tradables. Since our regression model controls for the transfer and supply-side effects of demographic change, we infer that this finding is mainly driven by changes in demand. Our results complement the empirical finding by Rose et al. (2009), who find that increases in fertility lead to appreciated real exchange rates.

All of the control variables enter the regression significantly, most of them with both qualitatively and quantitatively reasonable coefficients. An increase in relative sectoral productivity by one percent leads to an increase in the relative price by 0.59 percent, thereby offering evidence in favour of the Balassa-Samuelson effect. The effect of GDP per capita is positive also, which indicates the presence of price effects due to non-homothetic preferences (Bergstrand 1991) or relative factor endowments (Bhagwati 1984), as discussed in Section 4.4.1. Because of the latter, the variable also captures supply-side effects of demographic change, such as changes in the size of the labour force relative to the capital stock. In line with the related literature, rises in government spending inflate the relative price, though its coefficient is relatively small. As opposed to the intuition given in Section 4.4.1, the effect of an increase in net foreign assets tends to reduce relative prices, though at a very low rate. An increase in net foreign assets over GDP by one percentage point lowers the relative price by 0.083 percent only. However, according to the findings of Christopoulos et al. (2012), transfer effects are generally negligible for developed countries and only gain importance in case of capital-constraint developing countries, which are not included in our sample. This result also implies that the effects of demographic change on relative prices that operate over the transfer channel are very small. This finding is also in line with simulation results by Krueger and Ludwig (2007).

As diagnostic tests of the regressions, we provide cross-sectional dependence and stationarity tests of the residuals. Cross-sectional independence is rejected at the 5 percent level in column (I), which implies that it is important to consider this type of correlations when testing for residual stationarity. According to the bootstrapped critical values of the CIPS test, a unit root in the residuals can be rejected at the 1 percent level, so that we can rely on the inference of the estimated effects.

In column (II), the CCEP results are contrasted with the same regression using a DOLS estimator.[18] Our finding regarding the effect of ageing on the relative price remains qualitatively unchanged. An increase of the OADR by one percent raises the relative price of non-tradables by 0.20 percent, which is slightly smaller than the CCEP estimate. Turning to the control variables, the effects of relative productivity and net foreign assets can be found close to the CCEP estimates, while the effects of GDP per capita and government consumption turn insignificant. However, in reference to the residual diagnostics, regression (II) appears to be spurious as we are not able to reject the null hypothesis of non-stationarity in the residuals. Though this does not need to imply inconsistent estimates of the coefficients (see Phillips and Moon 1999), inference can be highly misleading and should not be relied on. This difference between the two estimation approaches underlines vividly the capabilities of the CCEP estimator to filter out unobservable non-stationary components of the data.

[18]Due to the limited sample length, we use one lead and lag.

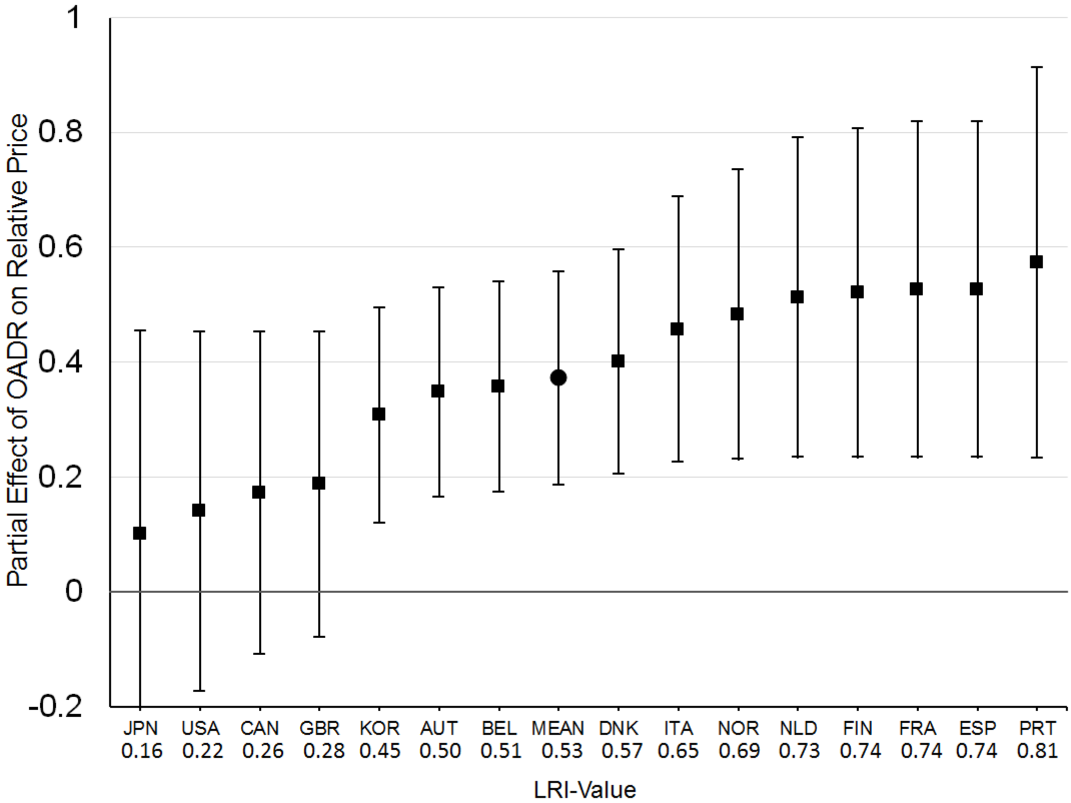
4.5.2 The Importance of Labour Market Rigidities

Columns (III) and (IV) in Table 4.6 show results for the regressions augmented with an interaction term of the OADR and the binary variable $\widetilde{lri}_i$ that indicates rigid labour markets. It is constructed with the index by Botero et al. (2004), which is a standard measure in the labour economics literature. In line with our prediction in Proposition 3, the effect of ageing indeed depends on labour market rigidities. In column (III), the coefficient of the OADR now gives the effect of ageing on relative prices for the less rigid countries, where $lri_i < \overline{lri}_i$. For these countries a one percent increase of the OADR implies an increase of the relative price by 0.15 percent, which is less than half the size of the effect in column (I). Statistically this effect is not significantly different from zero. Such findings regularly occur in models with interaction effects due to the inherent multicollinearity between the interaction and its basis variable. The coefficient of the interaction itself is positive and highly significant as theory predicts. The effect of a one percent increase of the OADR for the rigid countries reads 0.15+0.54=0.69 percent. An F-test implies joint significance of the OADR-coefficient and the interaction term at the 1 percent level. In sum, the estimates imply that ageing has a small insignificant effect in less rigid countries, while the effect is strong and positive for rigid countries. In the DOLS model in column (IV) the results are qualitatively similar, though the coefficients are again slightly different from the CCEP estimation. For the countries with less rigid labour markets, the effect of ageing reads 0.20 percent and for those with rigid labour markets it rises to 0.36 percent. In terms of control variables and residual diagnostics both regressions behave similarly to their counterparts in (I) and (II). Again, inference in the DOLS regression has to be questioned due to a rejection of residual stationarity.

To further investigate the effect of the interaction, we reestimate model (4.11) using the untransformed country-specific values of the labour market rigidity index and the CCEP estimator. By means of this model, we calculate the partial effect of ageing $\partial \ln(p_{it}) / \partial \ln(oadr_{it}) = \gamma_1 + \gamma_2 lri_i$ for the LRI-values of each country in the sample. The resulting effects of the old-age dependency ratio for each country along with their 95 percent confidence band are plotted in Figure 4.2.[19] Confirming the theoretical predictions of Proposition 3, the countries with the most flexible labour markets, appearing on the

[19] In order to compute significance levels of the partial effect $\partial \ln(p_{it}) / \partial \ln(oadr_{it})$, the standard approach is to reparameterize model (4.11) by subtracting the respective value of the rigidity index for country j before using it in the interaction term. Model (4.11) is then given by $\ln(p_{it}) = c_i + \widetilde{\gamma}_1 \ln(oadr_{it}) + \gamma_2 \ln(oadr_{it}) \cdot (lri_i - lri_j) + \gamma' X_{it} + u_{it}$. The coefficient of OADR can then be shown to be the partial effect of OADR for country j, i.e. $\widetilde{\gamma}_1 = \gamma_1 + \gamma_2 lri_j$. This way standard errors can be directly read out to evaluate statistical significance of the partial effect at the respective rigidity value of country j.

Figure 4.2: Visualisation of Interaction Effects



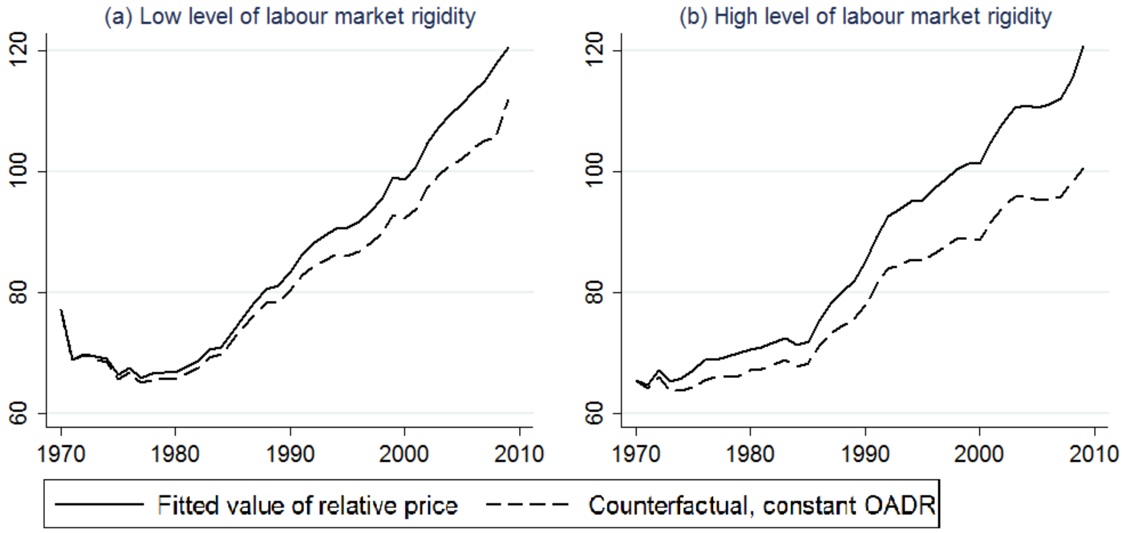
Notes: Partial effects of OADR, $\partial \ln (p_{it}) / \partial \ln (oadr_{it}) = \gamma_1 + \gamma_2 lri_i$, and corresponding 95% confidence band evaluated at the country-specific LRI-value using model (4.11). For the underlying regression, see Table 4.12 in Appendix 4.C. Country codes are explained in Table 4.1 in the text, LRI values for each country depicted on the abscissa.

left side of the figure, undergo small price effects of ageing, while countries with higher degrees of rigidity experience larger effects. In particular, for LRI-values up to 0.3 that are related with Anglo-American countries, price effects are estimated to be not statistically different from zero. In case of LRI-values about 0.7 - 0.8, which correspond to (Southern) European countries as France, Spain and Portugal, relative price effects rise up to nearly 0.60 percent. These findings underpin the empirical relevance of Proposition 3 and support the validity of the proposed transmission via imperfect labour market mobility.

4.5.3 The Economic Significance of Ageing

In terms of economic significance, the estimate of our main regression in column (I) of Table 4.6 implies the following. As the average old-age dependency ratio in our sample increased from a value of 15 to 23.5 (see Table 4.2) by about 56 percent between 1970

Figure 4.3: Fitted Values and Counterfactual Experiments



Notes: Fitted and counterfactual values of the relative price of non-tradables, based on model (4.11), for two groups of countries. Left panel (a): Austria, Belgium, Canada, Japan, Korea, United Kingdom, United States. Right panel (b): Denmark, Finland, France, Italy, Netherlands, Norway, Portugal, Spain.

and 2009, the coefficient of the OADR of 0.34 percent implies a surge of relative prices due to demographic change of about 19 percent. As in the same time period the relative price increased on average by 91 percent, about one fifth of the price movements can be attributed to population ageing. Hence, the effect of ageing appears to be of reasonable and non-negligible magnitude.

Economic significance is further studied by means of a set of counterfactual experiments, presented in Figure 4.3. The figure provides fitted values (solid lines) of the relative price of non-tradables, as implied by model (4.11), and compares these to a counterfactual scenario, where the OADR is kept constant at its 1970 value (dashed lines). The difference between the solid and the dashed line can be interpreted as the ceteris paribus effect of ageing on the relative price over the sample period. This is done for two groups of countries. In Panel (a) of Figure 4.3 the average LRI-value of countries whose LRI is below sample mean are used for calculating fitted values and the counterfactual simulation, while Panel (b) employs the average of countries with LRI values above mean. The figure confirms the relative importance of population ageing and the transmission via labour market frictions. While in both groups relative price effects would be dampened in case of a constant age structure, the difference between actual and counterfactual lines is by far more pronounced for countries with a high degree of labour market rigidity. In numbers, in Panel (a) the difference in the increase of the solid (57 percent) and the dashed line (45 percent) sums up to about 12 percentage points between 1970 and 2009, while in Panel (b) the difference is about 31 percentage points. Simulations for the whole sample and all individual countries can be found in Section 4.C.1 of the appendix. They also support the relevance of ageing for relative prices.

4.6 Sensitivity Analysis

To provide further evidence of the importance of demographic change for sectoral prices and the proposed channel via labour market rigidities, we analyze various alternative specifications. First, we evaluate the effect of changes in the whole demographic structure on relative prices instead of using the aggregated old-age dependency ratio. Second, we provide robustness checks for our proposed transmission mechanism. To this end, we use an alternative labour market rigidity index, as well as indices that account for further economic and legal factors. The result of the latter exercise is that other potential transmission mechanisms do not have the same empirical support as labour market rigidities do.

4.6.1 Disentangled Age Effects

To complement the finding of an appreciating effect of young cohorts on the real exchange rate by Rose et al. (2009), we test for the effect of the young-age dependency ratio (the fraction of population aged 0-14 to the population of age 15-64) on the relative price of non-tradables. The intuition for a higher share of the population at young-age is analogue to the impact of the OADR: the young dependent population consumes a higher share of non-tradable goods – such as education – with an upward effect on relative prices. Column (I) of Table 4.7 shows the results using the CCEP estimator. Consistent with the intuition provided by Rose et al. (2009) for real exchange rates, the young-age dependency ratio is also found to increase the relative price of non-tradables. A 1 percent increase of YADR inflates the relative price by 0.19 percent.[20]

The old-age dependency ratio as the explanatory variable of main interest in Section 4.5 subsumes two opposing effects. Being defined as the ratio of the retired to the

[20]We also successively added two different demographic measures of young cohorts, the YADR and the fertility rate, to our main model in column (I) of Table 4.6. The effect of OADR on relative prices remains qualitatively the same in all cases. The coefficient of the YADR also stays close to the one estimated in Table 4.7. The fertility rate, however, is not found to have a significant effect on relative prices. Full results on these checks can be found in Section 4.C.2 of the appendix.

working-age population, this variable simultaneously captures the effect of changes in two population shares. According to our hypothesis both age groups should yield opposite effects on relative sectoral prices: A higher share of the working-age population is expected to generate a negative impact due to a relatively lower demand for non-tradables.[21] On the contrary, a higher share of the elderly relative to total population should have a positive impact. To study the impact of a countries' demographic structure on the relative sectoral price in more detail, we decompose the old-age dependency ratio in its components, and use the share of population in working age (15-64) and the old-age population share (65+) relative to total population instead of the OADR.

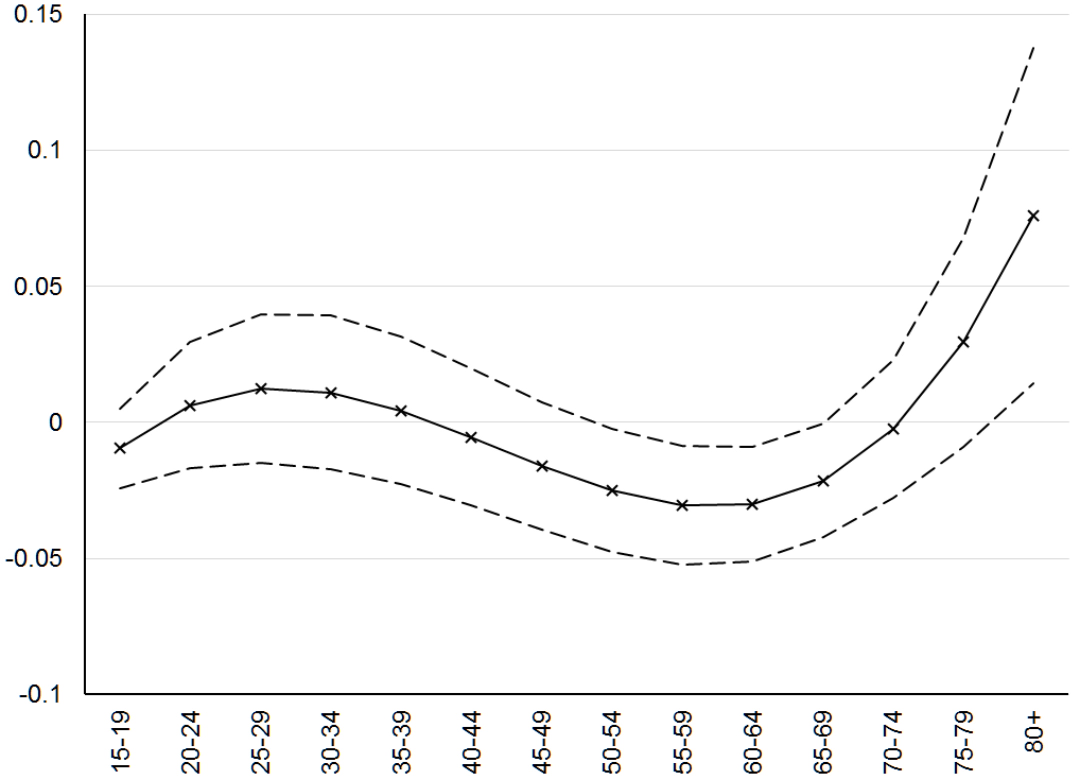
Columns (II) and (III) of Table 4.7 present results of using the two alternative demographic variables. In line with theory, the working-age population share has a negative and significant coefficient in column (II). A 1 percent increase leads to a decline of relative prices by 1.62 percent. Column (II) shows – similar to the results in Table 4.6 – that population ageing significantly inflates relative prices. An increase of the old-age population share by 1 percent increases relative prices by 0.26 percent. The quantitative importance of the relatively high coefficient of the working-age population share is put into perspective by the fact that changes in the working-age share are far less pronounced than in the old-age population share (see Table 4.2).

The coefficients of the control variables in all three columns are close to those estimated in our main regression in Section 4.5. Inference of the regressions is valid, since non-stationarity of the residuals can be rejected. The CD_P test indicates presence of cross-sectional correlations in (I) and (II), but not in (III). Regression results using the DOLS estimator are again close to CCEP estimates, but residual stationarity has to be rejected following bootstrapped critical values to the CIPS test. Therefore, all results using the DOLS methodology are relegated to Section 4.C.2 of the appendix.

Having established the opposing effects of the different age groups, we now investigate the effect of the demographic structure in more detail. To this end, we group the population in smaller bins. In particular, we construct $\mathcal{L} = 14$ population groups of 5-year intervals starting at age 15 until age 80+, where the last age bin covers all households at ages 80 and older. For each country i and time period t we compute the fraction of the age interval relative to the total population at ages 15 onwards. The age variables enter the estimation equation as $\sum_{l=1}^{\mathcal{L}} \nu_l \cdot age_{lit}$, where age_{lit} is the population share of age bin l in country i at period t and ν_l is the corresponding coefficient. Because of our relatively small sample we approximate this detailed demographic information by an age polynomial as it is done by Fair and Dominguez (1991) and Higgins (1998). In particular,

[21] In the model of Section 4.3 this would imply a decline in π_t.

Figure 4.4: Approximated Coefficients for 5-year Age-Bins



Notes: Coefficients ν_l for 5-year age bins approximated with the cubic age polynomial according to equations (4.14) and (4.15). Dashed lines represent the 95% confidence band. The inverted S-shape implies negative coefficients for the working-age population and positive coefficients for older ages.

we assume that the coefficients ν_l lie along a cubic polynomial[22]

$$\nu_l = \omega_0 + \omega_1 l + \omega_2 l^2 + \omega_3 l^3. \tag{4.14}$$

We can estimate ω_1, ω_2 and ω_3 by introducing auxiliary age variables Z_{it} in an estimation model related to (4.11) as follows:[23]

$$\ln(p_{it}) = c_i + \sum_{k=1}^{3} \omega_k Z_{kit} + \gamma' X_{it} + u_{it}, \tag{4.15}$$

[22]We have to approximate the demographic structure by a polynomial in order to mimimize the losses of degrees of fredom. For each additional explanatory variable the CCEP estimator involves interaction terms of cross-sectional averages with all country dummies. Hence, three addional variables that are required for a cubic polynomial already imply 48 additional variables in the estimation.

[23]Imposing a zero-sum restriction for ν_l, $\sum_{l=1}^{\mathcal{L}} \nu_l = 0$, implies $\omega_0 = -\frac{1}{\mathcal{L}} \left[\omega_1 \sum_{l=1}^{\mathcal{L}} l + \omega_2 \sum_{l=1}^{\mathcal{L}} l^2 + \omega_3 \sum_{l=1}^{\mathcal{L}} l^3\right]$. This removes the perfect colinearity between the constant and the age shares in the regression.

where the Z_{kit} are defined as

$$Z_{kit} = \sum_{l=1}^{\mathcal{L}} l^k \cdot age_{lit} - \frac{1}{\mathcal{L}} \sum_{l=1}^{\mathcal{L}} l^k \sum_{l=1}^{\mathcal{L}} age_{lit} \text{ with } k = 1, 2, 3;$$

and X_{it} is again a vector of additional control variables. Once the coefficients ω_1, ω_2 and ω_3 are estimated, we can approximate the coefficients ν_l for each age bin l by equation (4.14). Since the age bin coefficients ν_l are linear transformations of our estimated coefficients ω_k, we can employ the delta method to compute standard errors and confidence bands for ν_l. The results of our estimation are depicted in Figure 4.4. A full set of results from regression (4.15) including coefficients for the auxiliary demographic variables Z_{kit} and residual diagnostics are relegated to Section 4.C.2 of the appendix.

The approximated coefficients ν_l of the age bins are in line with our theory. Coefficients for age bins during working age are negative from ages 40 onwards. At older ages past 70, the coefficients turn positive. Overall, 5 out of 14 age bins are significant at the 5 percent level.[24] A test for joint significance of all Z_{kit} reveals that demography is jointly significant at the 1 percent level. To evaluate the economic significance, we employ the same counterfactual experiment as in Section 4.5 and thus keep the whole age structure constant at their values in 1970. We find that demographic changes can account for 17 percentage points of the increase in the relative price between 1970 and 2009, which is of the same order of magnitude as in our main specification.

[24] We also tried different degrees for the polynomials. A quadratic polynomial yields similar results with respect to the shape (u-shaped) and significance of the coefficients. 4th and 5th order polynomials yield a similar shape but mostly insignificant coefficients for the age bins. As explained before, this might be due to the loss of too many degrees of freedom associated with the inclusion of additional variables.

Table 4.7: Alternative Demographic Variables

Dependent Variable: Relative Price of Non-Tradables	(I)	(II)	(III)
Young-Age Dependency Ratio	0.19**		
	(0.10)		
Working-Age Population Share		-1.62***	
		(0.34)	
Old-Age Population Share			0.26***
			(0.096)
Relative Sectoral Productivity	0.56***	0.50***	0.63***
	(0.046)	(0.045)	(0.046)
GDP per capita	0.40***	0.51***	0.30***
	(0.067)	(0.071)	(0.067)
Government Consumption (% of GDP)	0.13***	0.16***	0.099*
	(0.049)	(0.048)	(0.052)
Net Foreign Assets (% of GDP)	-0.0005***	-0.0005***	-0.0008***
	(0.00017)	(0.00017)	(0.00018)
Residual diagnostics			
CD_P	-2.39**	-3.00***	-1.19
CIPS	-8.958***	-8.48***	-7.43***
Observations	546	546	546

Notes: All variables except net foreign assets are measured in logs. Method of estimation: CCEP. Regressions based on (4.11) without interaction term using alternative demographic variables. Country dummies included in all regressions. Standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Residual diagnostics: CD_P cross-sectional dependence test statistic by Pesaran (2004), Residual stationarity tested by Pesaran's (2007) CIPS test using bootstrapped critical values.

Table 4.8: Alternative LRI and Sample Split

Dependent Variable: Relative Price of Non-Tradables	(I) Alternative Variable	(II) Sample Split	(III)
		LRI $<$ $\overline{\text{LRI}}$	LRI $\geq$ $\overline{\text{LRI}}$
Old-Age Dependency Ratio	0.42**	0.17*	0.53***
	(0.17)	(0.090)	(0.10)
OADR $\times$ Labour Market Rigidity (LRI^{EPI})	0.016**		
	(0.0079)		
Relative Sectoral Productivity	0.27***	0.36***	0.75***
	(0.081)	(0.077)	(0.058)
GDP per capita	0.45***	0.039	0.19**
	(0.11)	(0.083)	(0.088)
Government Consumption (% of GDP)	0.22***	0.086	-0.0067
	(0.065)	(0.061)	(0.075)
Net Foreign Assets (% of GDP)	-0.000086	-0.0012***	-0.00093***
	(0.00020)	(0.00039)	(0.00019)
Residual diagnostics			
CD_P	-1.52	-2.95***	1.11
CIPS	-6.26**	-4.49***	-5.46***
F(OADR,Interaction= 0)	4.45**		
Observations (countries)	360	254 (7)	292 (8)

Notes: All variables except net foreign assets are measured in logs. Method of estimation: CCEP. Regressions based on (4.11). Country dummies included in all regressions. Interpretation of interaction in column (I): Partial effect of ageing is given by $0.42{+}0.016{\cdot}lri_{it}^{EPI}$. The low rigidity set (column II) contains Austria, Belgium, Canada, Japan, Korea, United Kingdom, and United States, while the high rigidity set (column III) covers Denmark, Finland, France, Italy, Netherlands, Norway, Portugal, and Spain. Standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Residual diagnostics: CD_P cross-sectional dependence test statistic by Pesaran (2004), Residual stationarity tested by Pesaran's (2007) CIPS test using bootstrapped critical values. F(OADR,Interaction= 0) denotes F-test about joint significance of OADR and the interaction term.

4.6.2 Robustness of the Transmission Mechanism

To further validate our proposed transmission channel we present results of a further set of robustness analyses. First, in our main regression, we constructed the interaction term using the old-age dependency ratio and the labour market rigidity index by Botero et al. (2004) to study the transmission mechanism of demand effects. Although widely used in the literature, a shortcoming of this index is that it is constant over time and therefore cannot reflect changes in these rigidities due to, e.g., labour market reforms. In this section, we employ the time-varying labour market rigidity index by the OECD, denoted by LRI^{EPI}, instead. In a second exercise, we perform a different estimation strategy to test for the influence of labour market rigidities. We split the sample into countries with high and low degrees of rigidity, using the measure of Botero et al. (2004) again, and run separate regressions on the subsamples. We assign countries to the low rigidity sample, whose index value is below the sample mean, $\overline{\text{LRI}}$, and to the high rigidity sample else. A third group of results aims at testing whether other factors besides labour market rigidities, such as capital market frictions or imperfect competition, could also drive or be confounded with the transmission of the age effects.

Results on the first two points are shown in Table 4.8 where the CCEP estimator is used for all regressions, while results from the DOLS method are relegated to Section 4.C.2 of the appendix. Column (I) shows results of using the time-varying labour market rigidity index. Due to limited data availability for the LRI^{EPI}, the sample size is reduced by about one third to 360 observations. Both the coefficient of the OADR and of the interaction term enter the regression statistically significant. The coefficients can now be interpreted as follows. The partial effect of the OADR on the relative price is given by $\partial \ln(p_{it}) / \partial \ln(oadr_{it}) = 0.42 + 0.016 \cdot lri_{it}^{EPI}$. For instance at the sample mean of the rigidity index ($\overline{lri}^{EPI} = 2.23$), the partial effect is 0.46 percent, which is of the same order of magnitude as the result shown in Figure 4.2 (0.37 percent). Evaluating the effect of the OADR at different points of LRI^{EPI} also yields qualitatively analogue outcomes to those shown in Figure 4.2, albeit the differences between countries are smaller.

Columns (II) and (III) show results for the two subsamples with high and low rigidities, respectively. As hypothesized, the effect of ageing is very marked for the rigid countries, $\text{LRI} \geq \overline{\text{LRI}}$, and about 50 percent larger than the effect found for the full sample in Table 4.6. For the subsample with lower degrees of rigidity, $\text{LRI} < \overline{\text{LRI}}$, the coefficient of the old-age dependency ratio is about half the size of the coefficient in Table 4.6. The effects of the controls are in all cases broadly comparable to those found earlier, though government consumption enters the regressions insignificantly when splitting the sample. In terms of residual diagnostics, non-stationarity is again rejected at high rates for all regressions.

Table 4.9: Tests for other Transmission Mechanisms

Dependent Variable:	(I)	(II)	(III)	(IV)
Relative Price of Non-Tradables	Capital Market Openness	Economic Freedom	Labor-Market Freedom	All Interactions
Old-Age Dependency Ratio	0.31***	0.37***	0.51***	0.54***
	(0.09)	(0.13)	(0.12)	(0.16)
OADR × KAOPEN	-0.0036			-0.0041
	(0.0032)			(0.0032)
OADR × EconFree w/o Labour		-0.044		-0.097
		(0.18)		(0.18)
OADR × LabFree			-0.36**	-0.38**
			(0.17)	(0.18)
Relative Sectoral Productivity	0.61***	0.59***	0.55***	0.57***
	(0.047)	(0.045)	(0.05)	(0.051)
GDP per capita	0.36***	0.40***	0.41***	0.35***
	(0.074)	(0.08)	(0.071)	(0.081)
Government Consumption (% of GDP)	0.091*	0.10**	0.09*	0.074
	(0.051)	(0.052)	(0.051)	(0.052)
Net Foreign Assets (% of GDP)	-0.00078***	-0.00082***	-0.00079***	-0.00075***
	(0.00017)	(0.00017)	(0.00017)	(0.00017)
Residual diagnostics				
CD_P	-2.33**	-2.12**	-2.36**	-2.68***
CIPS	-7.33**	-7.10***	-6.82***	-7.07***
F(OADR,Interactions= 0)	5.79***	6.88***	9.10***	4.10***
Observations	542	546	546	542

Notes: Method of estimation: CCEP. All variables except net foreign assets are measured in logs. Regressions based on (4.11). In each column OADR is interacted with binary variables with value one in case the respective index value is above average and zero else. Country dummies included in all regressions. Standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Residual diagnostics: CD_P cross-sectional dependence test statistic by Pesaran (2004), Residual stationarity tested by Pesaran's (2007) CIPS test using bootstrapped critical values. F(OADR,Interaction= 0) denotes F-test about joint significance of OADR and the interaction terms.

In Table 4.9 we evaluate if other factors besides labour market rigidities are also relevant for the size of the price effects of population ageing.

Similar to our proposed mechanism via labour market rigidities, capital market frictions could impair intersectoral and international adjustments of inputs after demand shifts due to population ageing. To test for the relevance of this channel, we re-estimate our main result from Table 4.6 using the index of capital market openness (KAOPEN) by Chinn and Ito (2006) instead of the LRI. The KAOPEN index measures restrictions on cross-border financial transactions on a scale from 0 to 1, where higher values imply more open capital markets. Results using the index can be found in column (I) of Table 4.9. As with the LRI, the interaction is constructed with a binary variable that takes on a value of one in case of capital market openness above average and zero else. The coefficient of OADR is highly significant and of the same size as in our main regressions. The effect of ageing does not change for different degrees of capital market rigidities though: the interaction term is insignificant and close to zero.

In presence of imperfect competition, demand shifts could lead to relative price changes, even with completely elastic supply. To test for such a mechanism and to cover a broad set of potential other factors, we employ the Index of Economic Freedom (EconFree) by Miller and Kim (2015) next. This index is based on 4 categories, each consisting of several sub-dimensions. The categories are: rule of law (including items like property rights and corruption), limited government (taxation, government spending), regulatory efficiency (business, market entry, and labour market regulation; monetary policy), and open markets (trade policy, banking and investment regulation). Each sub-dimension is graded on a scale of 0 to 100, where higher values mean more economic freedom. The index itself is an unweighted average of all sub-dimensions. To avoid confounding effects with labour market rigidities, we construct a sub-index with all components of the EconFree excluding labour market regulation. Regression results using a binary interaction term are shown in column (II) of Table 4.9. The OADR is found to be highly significant and of similar size as in the benchmark results, while the interaction with the Economic Freedom index remains small and insignificant. Countries with less general regulation do not experience significantly different price effects of ageing. In opposition, using an interaction between the labour market regulation component (LabFree) of the EconFree in column (III) confirms our previous finding of a significant effect of the OADR that decreases in case of less labour market regulation (note that the interaction effect is now negative since higher values mean less regulation for the LabFree). Adding all 3 interaction terms – LabFree, EconFree, and KAOPEN – jointly in column (IV) does not alter these findings. In Appendix 4.C.2 we show that our baseline result from column (III) in Table 4.6, i.e. using the labour market rigidity index from Botero et al. (2004) instead of LabFree, is also not affected by the inclusion of interaction terms with additional rigidity indices.

We value this as further evidence for our proposed transmission mechanism: higher labour market rigidity implies stronger effects of population ageing on the relative price of non-tradables, even after controlling for frictions and regulations on other markets, which on itself do not play any apparent role.

4.7 Conclusion

This chapter analyses the impact of demographic change on the relative price of non-tradable services to tradable commodities. We illustrate by means of a simple model how changes in demand induced by population ageing can affect relative prices. Imperfect labour market mobility is key for the transmission of changes in demand in this setup. We test these hypotheses empirically for a panel of 15 OECD countries. By making use of the CCEP estimator, we account for non-stationary and cross-sectional dependence, which is present in our data. Our results indicate a statistically and economically significant relation of reasonable size between the old-age dependency ratio and the relative price of non-tradables, which implies an appreciation of the real exchange rate in case of population ageing. Our model predicts that about one fifth of relative price movements between 1970 and 2009 can be attributed to demographic change. We further find support for our proposed transmission mechanism through labour market rigidities. In particular, Southern European countries like Portugal, Spain and France with more rigid labour markets experience stronger price effects due to population ageing than Anglo-American countries that feature lower degrees of rigidity.

This chapter extends the literature on structural real exchange rate determination by offering the demographic structure of the population as a further complementary explanation for international inflation differentials apart from existing ones like relative productivities or government spending. Furthermore, we stress the role of labour market imperfections for the transmission of effects on relative prices and real exchange rates. As trends in population ageing for the countries of our sample are forecasted to exacerbate in the upcoming decades, one can expect considerable price changes due to demographic change.

Appendix 4.A Details to the Model

4.A.1 Derivation of Equation 4.8

From the market clearing condition for non-tradables in period t, $Y_t^N = \pi_t C_t^N$, and the first-order conditions of households and firms, we obtain a dynamic implicit function of the relative price of non-tradables given by

$$\Phi\left(P_t, P_{t-1}\right) = L_t^N \frac{W_t^N}{P_t\left(1-\alpha_N\right)} - \pi_t \frac{\beta\left(1+R^*\right)}{P_t} W_{t-1}^N L_{t-1}^N \left[1 + \left(\frac{W_{t-1}^T}{W_{t-1}^N}\right)^{1+\rho}\right] = 0, \quad (4.16)$$

where wages in both sectors are given by

$$\begin{aligned} W_t^T &= \left(1-\alpha_T\right)\left(\frac{\alpha_T}{R^*}\right)^{\frac{\alpha_T}{1-\alpha_T}} \left(A_t^T\right)^{\frac{1}{1-\alpha_T}} && (4.17) \\ W_t^N &= \left(1-\alpha_N\right)\left(\frac{\alpha_N}{R^*}\right)^{\frac{\alpha_N}{1-\alpha_N}} \left(P_t A_t^N\right)^{\frac{1}{1-\alpha_N}}. && (4.18) \end{aligned}$$

We evaluate Φ at steady state. Time indices may be omitted now and (4.16) simplifies to

$$\Phi\left(P\right) = \frac{1}{\left(1-\alpha_N\right)\pi\beta\left(1+R^*\right)} - 1 - \left(\frac{W^T}{W^N}\right)^{1+\rho} = 0 \quad (4.19)$$

Inserting wages, (4.17) and (4.18), and solving for P immediately yields (8)

$$P = k\left(\frac{\left(A^T\right)^{\frac{1-\alpha_N}{1-\alpha_T}}}{A^N}\right)\left(\frac{\pi\varkappa}{1-\pi\varkappa}\right)^{\frac{1-\alpha_N}{1+\rho}}, \quad (4.20)$$

where

$$\begin{aligned} k &= \left(\frac{\left(1-\alpha_T\right)\alpha_T^{\frac{\alpha_T}{1-\alpha_T}}}{\left(1-\alpha_N\right)\alpha_N^{\frac{\alpha_N}{1-\alpha_N}}}\right)^{1-\alpha_N} \left(R^*\right)^{\frac{\alpha_N-\alpha_T}{1-\alpha_T}}, \\ \varkappa &= \left(1-\alpha_N\right)\beta\left(1+R^*\right). \end{aligned}$$

To rule out negative prices, it must hold that

$$1 - \pi\varkappa > 0, \quad (4.21)$$

which is the case even for extreme parameter constellations and which we hence assume to be fulfilled continuously.[25]

4.A.2 Proofs of Proposition 2 and 3

To assess the effect of an increase in the OADR π on the relative price P in steady state and to prove Proposition 2, we differentiate (4.8) with respect to π :

$$\begin{aligned}\frac{\partial P}{\partial \pi} &= k\left(\frac{\left(A^{T}\right)^{\frac{1-\alpha_N}{1-\alpha_T}}}{A^{N}}\right)\frac{1-\alpha_N}{1+\rho}\left(\frac{\pi\varkappa}{1-\pi\varkappa}\right)^{\frac{1-\alpha_N}{1+\rho}-1}\frac{\varkappa\left(1-\pi\varkappa\right)+\pi\varkappa\varkappa}{\left(1-\pi\varkappa\right)^{2}} \\ &= P\frac{1-\alpha_N}{1+\rho}\frac{1}{\pi\left(1-\pi\varkappa\right)}, \qquad (4.22)\end{aligned}$$

which is greater than zero as long as (4.21) holds. ■

In order to prove Proposition 3, we take the derivative of (4.22) with respect to ρ :

$$\frac{\partial\left(\frac{\partial P}{\partial \pi}\right)}{\partial \rho} = -K\left(\frac{\pi\varkappa}{1-\pi\varkappa}\right)^{\frac{1-\alpha_N}{1+\rho}}\frac{1-\alpha_N}{(1+\rho)^{2}}\left[\ln\left(\frac{\pi\varkappa}{1-\pi\varkappa}\right)\frac{1-\alpha_N}{1+\rho}+1\right], \qquad (4.23)$$

where

$$K = k\left(\frac{\left(A^{T}\right)^{\frac{1-\alpha_N}{1-\alpha_T}}}{A^{N}}\right)\frac{1}{\pi\left(1-\pi\varkappa\right)}.$$

This expression is less than zero (i.e. Proposition 3 is fulfilled) if

$$\begin{aligned}\ln\left(\frac{\pi\varkappa}{1-\pi\varkappa}\right) &> -\frac{1+\rho}{1-\alpha_N} \\ \ln\left(\frac{\pi\varkappa}{1-\pi\varkappa}\right) &> 0\end{aligned}$$

where the last step followed since $\rho > 0$ and $0 < \alpha_N < 1$. The only restriction for this to be case is again that (4.21) has to hold. ■

[25] As an example consider the following values in the upper range of the parameters involved: A capital share in the non-tradables sector of 20% ($\alpha_N = 0.2$), an annual real interest rate of 4% and an annual discount factor of 0.99 with a length of a generation of 30 years ($1 + R^{*} = 1.04^{30}$, $\beta = 0.99^{30}$) yield $\varkappa = 1.92$. The highest value of the OADR in our sample is about 0.35, such that $\pi\varkappa = 0.672 < 1$.

Appendix 4.B Data Sources and Construction

An overview of all data sources is given in Table 4.11. In case of all variables but relative prices and productivities no further data transformations (except for taking logarithms) are needed. In case of these two exemptions, the variables have to be constructed by hand from raw data. Below, we describe the procedure to construct the relative price and productivity measures, which is based on De Gregorio et al. (1994). The Structural Analysis (STAN) database by the OECD publishes detailed production data of some of its member states, where total value added is decomposed into nine standardized sectors. Series are provided both in current and constant prices using the base year 2000, allowing the calculation of sectoral deflators. In order to classify sectors to be tradable or non-tradable, De Gregorio et al. compute average ratios of exports to production for each sector. If this measure exceeds a given threshold, they use 10 percent, a sector is marked as tradable. These classifications are still used frequently, for instance by Ricci et al. (2013), and we also stick to it. An overview of all sectors with their original notation by the OECD and their classification of tradability are given in Table 4.10. Accordingly, five sectors, accountable for 65 percent of total value added in the year 2000, are classified as non-tradable, the four remaining sectors as tradable. As one can see, all service sectors except for '*Transport, Storage and Communications*' that is accountable for only 6.7 percent of total value added, are marked as non-tradable.

Table 4.10: Sector Classifcations

Sector	Share of Value Added	Classification
Agriculture, hunting, forestry and fishing	3.2	T
Mining and quarrying	0.3	T
Manufacturing	24.8	T
Electricity, gas and water supply	3.0	N
Construction	7.0	N
Wholesale and retail trade - restaurants and hotels	15.0	N
Transport, storage and communications	6.7	T
Finance, insurance, real estate and business services	22.9	N
Community, social and personal services	17.1	N

Notes: Share of Value Added in % based on own calculations, defined as unweighted cross-sectional average over whole sample in 2000 using data in constant prices. N and T denote non-tradability and tradability, respectively. Classifications are taken from De Gregorio et al. (1994).

To obtain the relative price of non-tradables, we first compute separate price indices for non-tradable services and tradable commodities using the following formula:

$$p^j = \frac{\sum_{s\in j} VALU^s}{\sum_{s\in j} VALK^s} \text{ for } j = \{T, N\}, \tag{4.24}$$

where s is an index running over all sub-sectors in sector j, and $VALU$ and $VALK$ denote value added in current and constant prices, respectively. Subsequently, the deflator of non-tradables is divided by its counterpart of tradable goods to obtain the relative price $p_{it} = p_{it}^N / p_{it}^T$, which is – after taking logs – employed in the regressions. Data on relative productivity, which we compute as value added per worker, also stems from the STAN database. First, productivity measures for both the non-tradable and the tradable sector are calculated by dividing sectoral value added at constant prices ($VALK$) by sectoral total employment ($EMPN$):

$$spr^j = \frac{VALK^j}{EMPN^j} \text{ for } j = \{T, N\} \tag{4.25}$$

Relative sectoral productivity, rpr_{it}, as used in the regression analysis is then constructed by $rpr_{it} = spr_{it}^T / spr_{it}^N$ and taking logs of the result.

Table 4.11: Data Sources Overview

Type of Data / Variable	Source	Name in Source
OECD Macro Health Care Data	OECD Health Data: http://www.oecd.org/health/health-systems/ oecdhealthdata.htm	HCTOT-PARPIB-HFTOT, HCTOT-PARTOT-HF2
Consumer Expenditure Survey 2011	http://www.bls.gov/cex/	Various
Medicare spending per capita	Health and Health Care of the Medicare Population 2009: http://www.cms.gov/Research-Statistics-Data-and-Systems/ Research/MCBS/Data-Tables.html	Table 4.1
Medicaid spending per capita	http://www.census.gov/compendia/statab/cats/health_nutrition/ medicare_medicaid.html	Table 151
Relative prices and productivities	OECD - Structural Analysis (STAN): http://stats.oecd.org/	VALU, VALK, EMPN
Demographic data	United Nations - World Population Prospects (WPP): http://esa.un.org/wpp/	Various
Labour market rigidity index (LRI)	Botero et al. (2004): http://faculty.tuck.dartmouth.edu/rafael-laporta/research-publications	index_labor7a
Labour market rigidity index (LRI^{EPI})	OECD Indicators of Employment Protection: www.oecd.org/employment/protection	EPRC_V1
Capital market openness index	Chinn and Ito (2006): http://web.pdx.edu/~ito/Chinn-Ito_website.htm	KAOPEN
Index of Economic Freedom	Miller and Kim (2015): http://www.heritage.org/index/	Various
GDP per capita, government consumption (% of GDP)	Heston et al. (2012) - Penn World Table (PWT) 7.1: http://pwt.econ.upenn.edu/	rgdpl, kg
Net foreign assets (% of GDP)	Lane and Milesi-Feretti (2007): http://www.philiplane.org/EWN.html	NFA/GDP

Appendix 4.C Additional Results

Table 4.12: Continuous Interaction Effects

Dependent Variable: Relative Price of Non-Tradables	(I)
Old-Age Dependency Ratio	-0.020
	(0.25)
OADR × Labour Market Rigidity(continuous)	0.73
	(0.47)
Relative Sectoral Productivity	0.58***
	(0.046)
GDP per capita	0.37***
	(0.076)
Government Consumption (% of GDP)	0.091*
	(0.052)
Net Foreign Assets (% of GDP)	-0.00082***
	(0.00017)
Residual diagnostics	
CD_P	-2.51**
CIPS	-7.028***
F(OADR,Interaction= 0)	8.13***
Observations (countries)	546

Notes: Table provides underlying regression to Figure 4.2. All variables except net foreign assets are measured in logs. Method of estimation: CCEP. Regressions based on (4.11). Country dummies included. Standard errors in parentheses. Interpretation of interaction: Partial effect of ageing is given by $-0.02+0.73 \cdot lri_i$. Standard error of the partial effect is obtained as described in footnote 18. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Residual diagnostics: CD_P cross-sectional dependence test statistic by Pesaran (2004), Residual stationarity tested by Pesaran's (2007) CIPS test using bootstrapped critical values. F(OADR,Interaction= 0) denotes F-test about joint significance of OADR and the interaction term.

4.C.1 Fitted Values and Counterfactual Simulation

Figure 4.5: Full Sample and Individual Countries

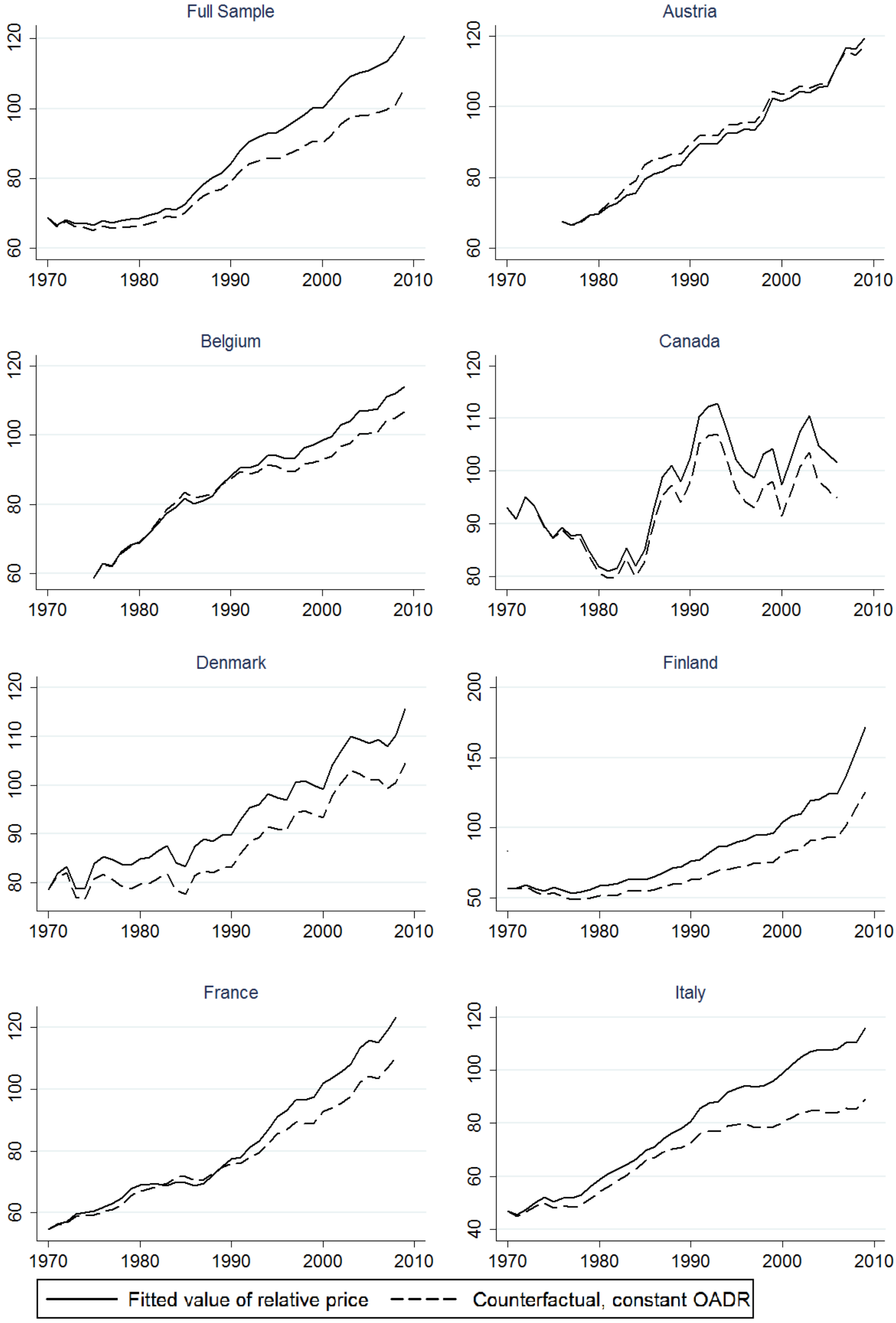
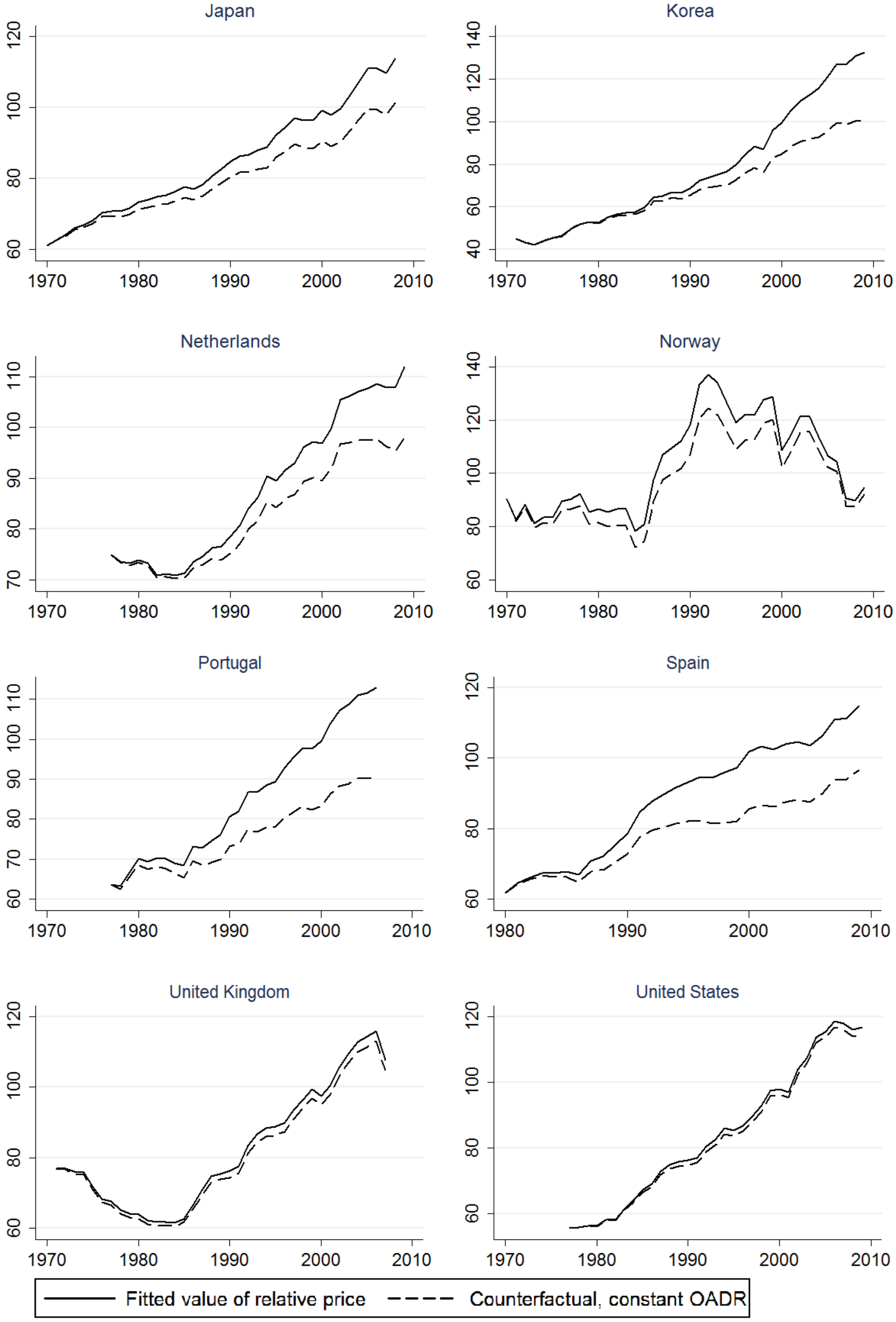


Figure 4.5 (continued): Full Sample and Individual Countries



Notes: Fitted values and counterfactual values of the relative price of non-tradables for whole sample and all individual countries. All graphs are derived from regression (4.11) using the continuous LRI and the CCEP estimator.

4.C.2 Sensitivity Analysis

Additional Results to Section 4.6.1

Table 4.13: Alternative Demographic Variables - DOLS

Dependent Variable: Relative Price of Non-Tradables	(I)	(II)	(III)
Young-Age Dependency Ratio	-0.087		
	(0.054)		
Working-Age Population Share		-0.81***	
		(0.25)	
Old-Age Population Share			0.19***
			(0.041)
Relative Sectoral Productivity	0.57***	0.55***	0.58***
	(0.036)	(0.035)	(0.035)
GDP per capita	-0.038	0.16***	-0.076*
	(0.050)	(0.052)	(0.043)
Government Consumption (% of GDP)	0.015	0.054	-0.059
	(0.041)	(0.035)	(0.040)
Net Foreign Assets (% of GDP)	-0.0021***	-0.0023***	-0.0021***
	(0.00018)	(0.00018)	(0.00017)
Residual diagnostics			
CD_P	-3.19***	-3.04***	-3.24***
CIPS	-0.25	-1.34	0.13
Observations	501	501	501

Notes: All variables except net foreign assets are measured in logs. Method of estimation: DOLS. Regressions based on (4.11) without interaction term using alternative demographic variables. Country and time dummies included in both regressions. Standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Residual diagnostics: CD_P cross-sectional dependence test statistic by Pesaran (2004), Residual stationarity tested by Pesaran's (2007) CIPS test using bootstrapped critical values.

Table 4.14: Alternative Demographic Variables - Joint Effects

Dependent Variable: Relative Price of Non-Tradables	(I) CCEP	(II) DOLS	(III) CCEP	(IV) DOLS
Young-Age Dependency Ratio	0.24***	0.023		
	(0.092)	(0.061)		
Fertility Rate			0.023	-0.049
			(0.057)	(0.055)
Old-Age Dependency Ratio	0.45***	0.19***	0.23**	0.17***
	(0.12)	(0.044)	(0.10)	(0.042)
Relative Sectoral Productivity	0.47***	0.57***	0.34***	0.57***
	(0.056)	(0.035)	(0.049)	(0.035)
GDP per capita	0.52***	-0.046	0.24***	-0.056
	(0.087)	(0.049)	(0.070)	(0.050)
Government Consumption (% of GDP)	0.17***	-0.048	0.083*	-0.042
	(0.056)	(0.043)	(0.049)	(0.043)
Net Foreign Assets (% of GDP)	-0.00057***	-0.0022***	-0.00029*	-0.0022***
	(0.00018)	(0.00018)	(0.00016)	(0.00017)
Residual diagnostics				
CD_P	-3.10***	-3.14***	-2.97***	-3.33***
CIPS	-10.01***	-0.48	-9.92***	-0.40
Observations	546	501	546	501

Notes: All variables except net foreign assets are measured in logs. Method of estimation: DOLS. Regressions based on (4.11) without interaction term using alternative demographic variables. Country dummies included in all regressions, time dummies in (II) and (IV). Standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Residual diagnostics: CD_P cross-sectional dependence test statistic by Pesaran (2004), Residual stationarity tested by Pesaran's (2007) CIPS test using bootstrapped critical values.

Table 4.15: Age Polynomial

Dependent Variable: Relative Price of Non-Tradables	CCEP
Auxiliary Demographic Variable Z_{1it}	0.035***
	(0.0091)
Auxiliary Demographic Variable Z_{2it}	-0.0071***
	(0.0019)
Auxiliary Demographic Variable Z_{3it}	0.00037***
	(0.000098)
Relative Sectoral Productivity	0.28***
	(0.056)
GDP per capita	0.38***
	(0.073)
Government Consumption (% of GDP)	0.10**
	(0.052)
Net Foreign Assets (% of GDP)	-0.00052***
	(0.00020)
Residual diagnostics	
CD_P	-2.68***
CIPS	-12.391***
$F(Z_{1it}, Z_{2it}, Z_{3it} = 0)$	5.04***
N	546

Notes: Relative Sectoral Productivity, GDP per capita and Government Consumption are measured in logs. Method of estimation: CCEP. Regression based on (4.15). Country dummies included. Standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Residual diagnostics: CD_P cross-sectional dependence test statistic by Pesaran (2004), Residual stationarity tested by Pesaran's (2007) CIPS test using bootstrapped critical values. $F(Z_{1it}, Z_{2it}, Z_{3it} = 0)$ denotes F-test about joint significance of all Z_{kit} variables.
The coefficients ω_k of the auxiliary demographic variables are highly significant. The control variables behave similar to those in the main regressions. Non-stationarity of residuals can be rejected.

Table 4.16: Approximated Coefficients of the Age Bins

Age Bin	Coefficient ν_l	Standard Errors
$15-19$	-0.009	0.0074
$20-24$	0.006	0.012
$25-29$	0.012	0.014
$30-34$	0.011	0.014
$35-39$	0.004	0.014
$40-44$	-0.005	0.013
$45-49$	-0.016	0.012
$50-54$	-0.025**	0.011
$55-59$	-0.030***	0.011
$60-64$	-0.030***	0.011
$65-69$	-0.021**	0.011
$70-74$	-0.002	0.0013
$74-79$	0.029	0.020
80+	0.076**	0.031

Notes: The table shows the approximated coefficients ν_l and standard errors of the age bins age_{lit}, on which Figure 4.4 is based. Coefficients on age bins age_{lit} extracted from the regression in Table 4.15 according to the procedure in (4.14). Asterisks mark significance at 10% (*), 5% (**), 1% (***).

Additional Results to Section 4.6.2

Table 4.17: Alternative LRI and Sample Split - DOLS

Dependent Variable: Relative Price of Non-Tradables	(I) Alternative Variable	(II) Sample split	(III)
		LRI $<$ $\overline{\text{LRI}}$	LRI $\geq$ $\overline{\text{LRI}}$
Old-Age Dependency Ratio	0.12	-0.060	0.49***
	(0.079)	(0.057)	(0.096)
OADR $\times$ Labour Market Rigidity (LRI^{EPI})	0.00032		
	(0.0083)		
Relative Sectoral Productivity	0.66***	0.49***	0.72***
	(0.064)	(0.061)	(0.060)
GDP per capita	-0.14	0.20***	-0.22*
	(0.10)	(0.063)	(0.11)
Government Consumption (% of GDP)	-0.17**	0.23***	0.088
	(0.070)	(0.085)	(0.070)
Net Foreign Assets (% of GDP)	-0.0018***	-0.0027***	-0.0014***
	(0.00021)	(0.00038)	(0.00028)
Residual diagnostics			
CD_P	-2.28**	1.76*	-0.08
CIPS	-0.033	-0.11	-0.69
F(OADR,Interaction= 0)	1.29		
Observations (countries)	315	233 (7)	268 (8)

Notes: All variables except net foreign assets are measured in logs. Method of estimation: DOLS. Regressions based on (4.11). Country and time dummies included in all regressions. The low rigidity set (column II) contains Austria, Belgium, Canada, Japan, Korea, United Kingdom, and United States, while the high rigidity set (column III) covers Denmark, Finland, France, Italy, Netherlands, Norway, Portugal, and Spain. Standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Residual diagnostics: CD_P cross-sectional dependence test statistic by Pesaran (2004), Residual stationarity tested by Pesaran's (2007) CIPS test using bootstrapped critical values. F(OADR,Interaction= 0) denotes F-test about joint significance of OADR and the interaction term.

Table 4.18: Tests for other Transmission Mechanisms

Dependent Variable:	(I)	(II)
Relative Price of Non-Tradables	Benchmark	All Interactions
Old-Age Dependency Ratio	0.15	-0.38*
	(0.111)	(0.21)
OADR × LRI (Botero et al. 2004)	0.54***	1.01***
	(0.175)	(0.22)
OADR × KAOPEN		-0.0039
		(0.0031)
OADR × EconFree w/o Labour		0.57**
		(0.225)
Relative Sectoral Productivity	0.58***	0.59***
	(0.045)	(0.046)
GDP per capita	0.32***	0.33***
	(0.076)	(0.08)
Government Consumption (% of GDP)	0.073	0.062
	(0.051)	(0.051)
Net Foreign Assets (% of GDP)	-0.00081***	-0.0008***
	(0.00017)	(0.00017)
Residual diagnostics		
CD_P	-2.51**	-2.52**
CIPS	-7.21***	-7.34***
F(OADR,Interactions= 0)	11.91***	8.15***
Observations	546	542

Notes: Method of estimation: CCEP. All variables except net foreign assets are measured in logs. Regressions based on (4.11). In each column OADR is interacted with binary variable(s) with value one in case the respective index value is above average and zero else. Column (I) repeats benchmark result from Table 6 in the paper. Country dummies included in all regressions. Standard errors in parentheses. Asterisks mark significance at 10% (*), 5% (**), 1% (***). Residual diagnostics: CD_P cross-sectional dependence test statistic by Pesaran (2004), Residual stationarity tested by Pesaran's (2007) CIPS test using bootstrapped critical values. F(OADR,Interaction= 0) denotes F-test about joint significance of OADR and the interaction terms.

Table 4.18 shows that the benchmark result of the chapter (repeated in column I) continues to hold when further interactions of the OADR with indices of capital market frictions and general economic freedom are added (column II). The negative coefficient of the OADR results from a change in interpretation because of the inclusion of the additional indices. The coefficient now gives the effect of OADR for observations where all 3 binary dummies adopt a value of 0 simultaneously, which applies to 28 data points in our sample only. Confirming the main result, countries with labour markets that are more rigid than the sample mean experience signifcantly stronger price effects of ageing. The coefficient for those countries reads -0.38+1.01=0.63, which is close to the benchmark result of 0.15+0.54=0.69 in column (I). Capital market frictions do not affect the size of the price effect of ageing. As opposed to Table 4.9, the interaction between OADR and the EconFree index is significant. The size of the coefficient is, however, still rather low. Countries with an EconFree index value above mean experience a relative price effect of -0.38+0.57=0.19.

Bibliography

Adao, Bernardino, Isabel Correia, and Pedro Teles, "On the Relevance of Exchange Rate Regimes for Stabilization Policy," *Journal of Economic Theory*, 2009, *144* (4), 1468 – 1488.

Aloy, Marcel and Karine Gente, "The Role of Demography in the Long-Run Yen/USD Real Exchange Rate Appreciation," *Journal of Macroeconomics*, 2009, *31* (4), 654 – 667.

Andersson, Andreas and Pär Österholm, "Forecasting Real Exchange Rate Trends Using Age Structure Data - the Case of Sweden," *Applied Economics Letters*, 2005, *12* (5), 267 – 272.

_ **and** _ , "Population Age Structure and Real Exchange Rates in the OECD," *International Economics Journal*, 2006, *20* (1), 1 – 18.

Artuc, Erhan, Shubham Chaudhuri, and John McLaren, "Trade Shocks and Labor Adjustment: A Structural Empirical Approach," *American Economic Review*, 2010, *100* (3), 1008 – 1045.

Balassa, Bela, "The Purchasing-Power Parity Doctrine: A Reappraisal," *Journal of Political Economy*, 1964, *72* (6), 584 – 596.

Beetsma, Roel and Massimo Giuliodori, "The Macroeconomic Costs and Benefits of the EMU and Other Monetary Unions: An Overview of Recent Research," *Journal of Economic Literature*, 2010, *48* (3), 603 – 41.

Beetsma, Roel M.W.J. and Henrik Jensen, "Monetary and Fiscal Policy Interactions in a Micro-founded Model of a Monetary Union," *Journal of International Economics*, 2005, *67* (2), 320 – 352.

Benigno, Pierpaolo, "Optimal Monetary Policy in a Currency Area," *Journal of International Economics*, 2004, *63* (2), 293 – 320.

Bergstrand, Jeffrey H., "Structural Determinants of Real Exchange Rates and National Price Levels: Some Empirical Evidence," *American Economic Review*, 1991, *81* (1), 325 – 334.

Berka, Martin, Michael B. Devereux, and Charles Engel, "Real Exchange Rates and Sectoral Productivity in the Eurozone," Working Paper 20510, National Bureau of Economic Research September 2014.

Bettendorf, Leon and Hans Dewachter, "Ageing and the Relative Price of Nontradables," Discussion Paper 2007-064/2, Tinbergen Institute July 2007.

Bhagwati, Jagdish N., "Why are Services Cheaper in the Poor Countries?," *Economic Journal*, 1984, *94* (374), 279 – 286.

Börsch-Supan, Axel, "Labor Market Effects of Population Aging," *Labour*, 2003, *17* (Special Issue), 5 – 44.

Botero, Juan, Simeon Djankov, Rafael Porta, Florencio C. Lopez-De-Silanes, and Andrei Shleifer, "The Regulation of Labor," *Quarterly Journal of Economics*, November 2004, *119* (4), 1339 – 1382.

Braude, Jacob, "Age Structure and the Real Exchange Rate," Discussion Paper 2000.10, Bank of Israel October 2000.

Bredemeier, Christian, Falko Juessen, and Andreas Schabert, "Fiscal Multipliers and Monetary Policy: Reconciling Theory and Evidence," 2017. mimeo.

Bundick, Brent and A. Lee Smith, "The Dynamic Effects of Forward Guidance Shocks," Research Working Paper 16-02, Federal Reserve Bank of Kansas City 2016.

Burstein, Ariel and Gita Gopinath, "International Prices and Exchange Rates," in Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, eds., *Handbook of International Economics*, Vol. 4, Elsevier, 2014, chapter 7, pp. 391 – 451.

Caballero, Ricardo J. and Emmanuel Farhi, "The Safety Trap," *Review of Economic Studies*, 2017, *forthcoming.*

Calvo, Guillermo A., "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 1983, *12* (3), 383 – 398.

Campbell, Jeffrey R., Chalres L. Evans, Jonas D.M. Fisher, and Alejandro Justiniano, "Macroeconomic Effects of FOMC Forward Guidance," *Brookings Papers on Economic Activity*, 2012, *2012* (Spring), 1 – 80.

_ , Jonas D. M. Fisher, Alejandro Justiniano, and Leonardo Melosi, "Forward Guidance and Macroeconomic Outcomes Since the Financial Crisis," *NBER Macroeconomics Annual*, 2016, *31*, forthcoming.

Cantor, Richard and Robert Driskill, "Effects on the Real Exchange Rate of Demographic and Fiscal Policy-induced Changes in National Savings: Is the Conventional Academic Wisdom Always Right?," 1999. mimeo.

Canzoneri, Matthew B., Robert E. Cumby, and Behzad Diba, "Relative Labor Productivity and the Real Exchange Rate in the Long Run: Evidence for a Panel of OECD Countries," *Journal of International Economics*, 1999, *47* (2), 245 – 266.

Cardi, Olivier and Romain Restout, "Imperfect Mobility of Labor Across Sectors: A Reappraisal of the Balassa-Samuelson Effect," *Journal of International Economics*, 2015, *97* (2), 249 – 265.

Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian, "Inflation and Output in New Keynesian Models with a Transient Interest Rate Peg," *Journal of Monetary Economics*, 2015, *76*, 230 – 243.

Chinn, Menzie D and Hiro Ito, "What Matters for Financial Development? Capital Controls, Institutions, and Interactions," *Journal of Development Economics*, 2006, *81* (1), 163 – 192.

Christopoulos, Dimitris K., Karine Gente, and Miguel A. León-Ledesma, "Net Foreign Assets, Productivity and Real Exchange Rates in Constrained Economies," *European Economic Review*, 2012, *56* (3), 295 – 316.

Chung, Hess, Edward Herbst, and Michael T. Kiley, "Effective Monetary Policy Strategies in New Keynesian Models: A Reexamination," *NBER Macroeconomics Annual*, 2015, *29* (1), 289 – 344.

Corsetti, Giancarlo, "A Modern Reconsideration of the Theory of Optimal Currency Areas," Economic Papers 308, European Commission March 2008.

_ , **Luca Dedola, and Sylvain Leduc**, "Optimal Monetary Policy in Open Economies," in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics*, Vol. 3, Elsevier, 2010, chapter 16, pp. 861 – 933.

Coto-Martinez, Javier and Juan C. Reboredo, "The Relative Price of Non-Traded Goods under Imperfect Competition," *Oxford Bulletin of Economics and Statistics*, 2014, *76* (1), 24 – 40.

D'Amico, Stefania and Thomas King, "What Does Anticipated Monetary Policy Do?," Working Paper 2015-10, Federal Reserve Bank of Chicago 2015.

De Gregorio, José, Alberto Giovannini, and Holger C. Wolf, "International Evidence on Tradables and Nontradables Inflation," *European Economic Review*, 1994, *38* (6), 1225 – 1244.

De Paoli, Bianca, "Monetary Policy and Welfare in a Small Open Economy," *Journal of International Economics*, 2009, *77* (1), 11 – 22.

Del Negro, Marco, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki, "The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities," *American Economic Review*, March 2017, *107* (3), 824 – 857.

_ , **Marc Giannoni, and Christina Patterson**, "The Forward Guidance Puzzle," Staff Report 574, Federal Reserve Bank of New York 2015.

Devereux, Michael B. and Charles Engel, "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility," *The Review of Economic Studies*, 2003, *70* (4), 765 – 783.

Druant, Martine, Silvia Fabiani, Gabor Kezdi, Ana Lamo, Fernando Martins, and Roberto Sabbatini, "Firms' Price and Wage Adjustment in Europe: Survey Evidence on Nominal Stickiness," *Labour Economics*, 2012, *19* (5), 772 – 782.

Duarte, Margarida and Maurice Obstfeld, "Monetary Policy in the Open Economy Revisited: The Case for Exchange-Rate Flexibility Restored," *Journal of International Money and Finance*, 2008, *27* (6), 949 – 957.

ECB, "Sticky Prices in the Euro Area: A Summary of New Micro Evidence," Working Paper Series no. 563, European Central Bank 2005.

Eggertsson, Gauti B. and Michael Woodford, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003, *2003* (Spring), 139 – 233.

_ **and Neil R. Mehrotra**, "A Model of Secular Stagnation," Working Paper 20574, National Bureau of Economic Research October 2014.

Engel, Charles, "Currency Misalignments and Optimal Monetary Policy: A Reexamination," *American Economic Review*, 2011, *101* (6), 2796 – 2822.

Engler, Philipp, Giovanni Ganelli, Juha Tervala, and Simon Voigts, "Fiscal Devaluation in a Monetary Union," *IMF Economic Review*, 2017, *65* (2), 241 – 272.

Fachin, Stefano, "Long-Run Trends in Internal Migrations in Italy: A Study in Panel Cointegration with Dependent Units," *Journal of Applied Econometrics*, 2007, *22* (2), 401 – 428.

Faia, Ester and Tommaso Monacelli, "Ramsey Monetary Policy and International Relative Prices," 2004. mimeo.

_ **and** _ , "Optimal Monetary Policy in a Small Open Economy with Home Bias," *Journal of Money, Credit and Banking*, 2008, *40* (4), 721 – 750.

Fair, Ray C and Kathryn M Dominguez, "Effects of the Changing U.S. Age Distribution on Macroeconomic Equations," *American Economic Review*, 1991, *81* (5), 1276 – 1294.

Farhi, Emmanuel and Ivan Werning, "Ficsal Unions," *American Economic Review*, 2017, *forthcoming.*

_ **, Gita Gopinath, and Oleg Itskhoki**, "Fiscal Devaluations," *The Review of Economic Studies*, 2014, *81* (2), 725 – 760.

Feenstra, Robert C., Philip A. Luck, Maurice Obstfeld, and Katheryn N. Russ, "In Search of the Armington Elasticity," Working Paper 20063, National Bureau of Economic Research April 2014.

Ferrero, Andrea, "Fiscal and Monetary Rules for a Currency Union," *Journal of International Economics*, 2009, *77* (1), 1 – 10.

Froot, Kenneth A. and Kenneth Rogoff, "Perspectives on PPP and Long-Run Real Exchange Rates," in Gene M. Grossman and Kenneth Rogoff, eds., *Handbook of International Economics*, Vol. 3, Elsevier, 1995, chapter 32, pp. 1647 – 1688.

Galí, Jordi and Tommaso Monacelli, "Optimal Monetary and Fiscal Policy in a Currency Union," *Journal of International Economics*, 2008, *76* (1), 116 – 132.

Galstyan, Vahagn and Philip R. Lane, "The Composition of Government Spending and the Real Exchange Rate," *Journal of Money, Credit and Banking*, 2009, *41* (6), 1233–1249.

Gertler, Mark and Peter Karadi, "Monetary Policy Surprises, Credit Costs, and Economic Activity," *American Economic Journal: Macroeconomics*, January 2015, *7* (1), 44 – 76.

Groneck, Max and Christoph Kaufmann, "Determinants of Relative Sectoral Prices: The Role of Demographic Change," *Oxford Bulletin of Economics and Statistics*, 2017, *79* (3), 319 – 347.

Gürkaynak, Refet S., "Using Federal Funds Futures Contracts for Monetary Policy Analysis," Working Paper 2015-29, Federal Reserve Board 2005.

_ , Brian P. Sack, and Eric T. Swanson, "Market-Based Measures of Monetary Policy Expectations," *Journal of Business & Economic Statistics*, 2007, *25* (2), 201 – 212.

_ , Brian Sack, and Eric T. Swanson, "Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements," *International Journal of Central Banking*, 2005, *1* (1), 55 – 93.

Hagist, Christian and Laurence Kotlikoff, "Who's Going Broke? Comparing Growth in Healthcare Costs in Ten OECD Countries," Working Paper 11833, National Bureau of Economic Research December 2005.

Hassan, AFM, Ruhul Salim, and Harry Bloch, "Population Age Structure, Saving, Capital Flows and the Real Exchange Rate: a Survey of the Literature," *Journal of Economic Surveys*, 2011, *25* (4), 708 – 736.

Heijdra, Ben J. and Laurie S. M. Reijnders, "Human Capital Accumulation and the Macroeconomy in an Ageing Society," Working Paper 4046, CESifo 2012.

Heston, Alan, Robert Summers, and Bettina Aten, "Penn World Table Version 7.1, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania," http://pwt.econ.upenn.edu/ 2012.

Higgins, Matthew, "Demography, National Savings, and International Capital Flows," *International Economic Review*, May 1998, *39* (2), 343 – 369.

Hobijn, Bart and David Lagakos, "Social Security and the Consumer Price Index for the Elderly," *FRB of New York: Current Issues in Economics and Finance*, 2003, *9* (5), 1 – 6.

Høj, Jens, Miguel Jimenez, Maria Maher, Giuseppe Nicoletti, and Michael Wise, "Product Market Competition in the OECD Countries: Taking Stock and Moving Forward," Economics Department Working Paper 575, OECD 2007.

Holly, Sean, M. Hashem Pesaran, and Takashi Yamagata, "A Spatio-Temporal Model of House Prices in the USA," *Journal of Econometrics*, 2010, *158* (1), 160 – 173.

Horvath, Michael, "Sectoral Shocks and Aggregate Fluctuation," *Journal of Monetary Economics*, 2000, *45* (1), 69 – 106.

Im, Kyung So, M. Hashem Pesaran, and Yongcheol Shin, "Testing for Unit Roots in Heterogeneous Panels," *Journal of Econometrics*, July 2003, *115* (1), 53 – 74.

Juselius, Mikael and Elod Takáts, "Can Demography Affect Inflation and Monetary Policy?," Working Paper 485, Bank for International Settlements February 2015.

Kakkar, Vikas, "The Relative Price of Nontraded Goods and Sectoral Total Factor Productivity: An Empirical Investigation," *Review of Economics and Statistics*, 2003, *85* (2), 444 – 452.

Kao, Chihwa and Min-Hsien Chiang, "On the Estimation and Inference of a Cointegrated Regression in Panel Data," *Advances in Econometrics*, 2000, *15*, 179 – 222.

Kapetanios, G., M. Hashem Pesaran, and T. Yamagata, "Panels with Non-Stationary Multifactor Error Structures," *Journal of Econometrics*, February 2011, *160* (2), 326 – 348.

Kaufmann, Christoph, "Optimal Fiscal Substitutes for the Exchange Rate in a Monetary Union," Discussion Paper 44/2016, Deutsche Bundesbank 2016.

Keynes, John Maynard, "Addendum to: Great Britain. Committee on Finance and Industry Report," [Macmillan Report], His Majestys Stationery Office 1931.

Kiley, Michael T., "Policy Paradoxes in the New Keynesian Model," *Review of Economic Dynamics*, 2016, *21*, 1 – 15.

Kim, Jinill, Sunghyun Kim, Ernst Schaumburg, and Christopher A. Sims, "Calculating and Using Second-Order Accurate Solutions of Discrete Time Dynamic Equilibrium Models ," *Journal of Economic Dynamics and Control*, 2008, *32* (11), 3397 – 3414.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," *Brookings Papers on Economic Activity*, 2011, *2011* (Fall), 215 – 287.

_ **and** _ , "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 2012, *120* (2), 233–267.

Krueger, Dirk and Alexander Ludwig, "On the Consequences of Demographic Change for Rates of Returns to Capital, and the Distribution of Wealth and Welfare," *Journal of Monetary Economics*, January 2007, *54* (1), 49 – 87.

Kuttner, Kenneth N., "Monetary Policy Surprises and Interest Rates: Evidence from the Fed Funds Futures Market," *Journal of Monetary Economics*, 2001, *47* (3), 523 – 544.

Lane, Philip R. and Gian M. Milesi-Ferretti, "The Transfer Problem Revisited: Net Foreign Assets and Real Exchange Rates," *Review of Economics and Statistics*, 2004, *86* (4), 841 – 857.

_ **and** _ , "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004," *Journal of International Economics*, 2007, *73* (2), 223 – 250.

_ **and Gian Maria Milesi-Ferretti**, "External Wealth, the Trade Balance, and the Real Exchange Rate," *European Economic Review*, 2002, *46* (6), 1049 – 1071.

Laséen, Stefan and Lars E.O. Svensson, "Anticipated Alternative Policy Rate Paths in Policy Simulations," *International Journal of Central Banking*, 2011, *7* (3), 1 – 35.

Lee, Donghoon and Kenneth I. Wolpin, "Intersectoral Labor Mobility and the Growth of the Service Sector," *Econometrica*, 2006, *74* (1), 1 – 46.

Lipinska, Anna and Leopold von Thadden, "On the (In)effectiveness of Fiscal Devaluations in a Monetary Union," 2012. mimeo.

Lombardo, Giovanni, "Inflation Targeting Rules and Welfare in an Asymmetric Currency Area," *Journal of International Economics*, 2006, *68* (2), 424 – 442.

_ **and Federico Ravenna**, "Openness and Optimal Monetary Policy," *Journal of International Economics*, 2014, *93* (1), 153 – 172.

Lucas, Robert E. and Nancy L. Stokey, "Optimal Fiscal and Monetary Policy in an Economy without Capital," *Journal of Monetary Economics*, 1983, *12* (1), 55 – 93.

Ludwig, Alexander, Thomas Schelkle, and Edgar Vogel, "Demographic Change, Human Capital and Welfare," *Review of Economic Dynamics*, January 2012, *15* (1), 94 – 107.

Lührmann, Melanie, "Population Aging and the Demand for Goods & Services," Discussion Paper 95-05, Munich Center for the Economics of Ageing (MEA) November 2005.

_ , "Effects of Population Ageing on Aggregated UK Consumer Demand," 2008. Mimeo.

Maddala, Gangadharrao S. and Shaowen Wu, "A Comparative Study of Unit Root Tests with Panel Data and a New Simple Test," *Oxford Bulletin of Economics and Statistics*, 1999, *61* (Special Issue), 631 – 652.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson, "The Power of Forward Guidance Revisited," *American Economic Review*, October 2016, *106* (10), 3133 – 3158.

_ **,** _ **, and** _ , "The Discounted Euler Equation: A Note," *Economica*, 2017, *forthcoming.*

Miller, Terry and Anthony B. Kim, "Index of Economic Freedom, The Heritage Foundation & The Wall Street Journal," 2015.

Nagel, Stefan, "The Liquidity Premium of Near-Money Assets," *The Quarterly Journal of Economics*, 2016, *131* (4), 1927 – 1971.

Obstfeld, Maurice and Kenneth Rogoff, "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," in Ben S. Bernanke and Kenneth Rogoff, eds., *NBER Macroeconomics Annual 2001*, Vol. 15, MIT Press, 2001, chapter 6, pp. 339 – 412.

O'Connell, Paul G.J., "The Overvaluation of Purchasing Power Parity," *Journal of International Economics*, 1998, *44* (1), 1 – 19.

Paparoditis, Efstathios and Dimitris N. Politis, "The Continuous-Path Block-Bootstrap," in Madan L. Puri, ed., *Asymptotics in Statistics and Probability*, VSP International Science Publishers, 2001, pp. 305 – 320.

_ **and** _ , "Residual-Based Block Bootstrap for Unit Root Testing," *Econometrica*, 2003, *71* (3), 813 – 855.

Pesaran, M. Hashem, "General Diagnostic Tests for Cross Section Dependence in Panels," Discussion Paper 1240, IZA 2004.

_ , "Estimation and Inference in Large Heterogeneous Panels with a Multifactor Error Structure," *Econometrica*, 2006, *74* (4), 967 – 1012.

_ , "A Simple Panel Unit Root Test in the Presence of Cross-Section Dependence," *Journal of Applied Econometrics*, 2007, *22* (2), 265 – 312.

Phillips, Peter C. B. and Hyungsik R. Moon, "Linear Regression Limit Theory for Nonstationary Panel Data," *Econometrica*, 1999, *67* (5), 1057 – 1111.

Ricci, Luca Antonio, Gian Maria Milesi-Ferretti, and Jaewoo Lee, "Real Exchange Rates and Fundamentals: A Cross-Country Perspective," *Journal of Money, Credit and Banking*, 2013, *45* (5), 845 – 865.

Rogoff, Kenneth, "Traded Goods Consumption Smoothing and the Random Walk Behavior of the Real Exchange Rate," Working Paper 4119, National Bureau of Economic Research July 1992.

Rose, Andrew K., Saktiandi Supaat, and Jacob Braude, "Fertility and the Real Exchange Rate," *Canadian Journal of Economics*, 2009, *42* (2), 496 – 518.

Samuelson, Paul A., "Theoretical Notes on Trade Problems," *Review of Economics and Statistics*, 1964, *46* (2), 145 – 154.

Santos Silva, J.M.C. and Silvana Tenreyro, "Currency Unions in Prospect and Retrospect," *Annual Review of Economics*, 2010, *2*, 51 – 74.

Schabert, Andreas, "Optimal Central Bank Lending," *Journal of Economic Theory*, 2015, *157*, 485 – 516.

Schmitt-Grohé, Stephanie and Martín Uribe, "Optimal Fiscal and Monetary Policy under Sticky Prices," *Journal of Economic Theory*, 2004, *114* (2), 198 – 230.

_ **and** _ , "Optimal Simple and Implementable Monetary and Fiscal Rules: Expanded Version," Working Paper 12402, National Bureau of Economic Research August 2006.

_ **and** _ , "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 2016, *124* (5), 1466 – 1514.

Simon, D., "Expectations and the Treasury Bill-Federal Funds Rate Spread over Recent Monetary Policy Regimes," *Journal of Finance*, 1990, *45*, 567 – 577.

Smets, Frank and Raf Wouters, "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 2003, *1* (5), 1123 – 1175.

Stock, James H. and Mark W. Watson, "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems," *Econometrica*, 1993, *61* (4), 783 – 820.

Stock, James H and Mark W Watson, "Macroeconomic Forecasting Using Diffusion Indexes," *Journal of Business & Economic Statistics*, 2002, *20* (2), 147 – 162.

Summers, Lawrence, "Why Stagnation Might Prove to be the New Normal," *The Financial Times* 2013. Published online: December 15, 2013.

Swanson, Eric T., "Measuring the Effects of Federal Reserve Forward Guidance and Asset Purchases on Financial Markets," Working Paper 23311, National Bureau of Economic Research April 2017.

van Ewijk, Casper and Maikel Volkerink, "Will Ageing Lead to a Higher Real Exchange Rate for the Netherlands?," *De Economist*, 2012, *160* (1), 59 – 80.

Wacziarg, Romain and Jessica Seddon Wallack, "Trade Liberalization and Intersectoral Labor Movements," *Journal of International Economics*, 2004, *64* (2), 411 – 439.

Wiederholt, Mirko, "Empirical Properties of Inflation Expectations and the Zero Lower Bound," 2015. mimeo.

Williamson, Stephen D., "Scarce Collateral, the Term Premium, and Quantitative Easing," *Journal of Economic Theory*, 2016, *164*, 136 – 165.

Yoon, Jong-Won, Jinill Kim, and Jungjin Lee, "Impact of Demographic Changes on Inflation and the Macroeconomy," Working Paper 14/210, International Monetary Fund November 2014.