The Roles of Demographic Changes on Labor Market Dynamics and Consumption Inequality

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To my parents,

my lovely wife,

and my son

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Chapter 1

Introduction

This thesis consists of three essays on the impacts of demographic changes on labor markets and consumption goods markets. More specifically, the first two essays examine the role of demographic changes on labor market dynamics with empirical evidence and a theoretical framework, respectively. The third essay investigates the differential impacts of demographic changes on the consumption of retirees and workers.

The impacts on labor market dynamics come from the substantial changes in the demographic composition of labor markets, which happen among the vast majority of developed economies. In the U.S., for instance, the share of young workers (aged 15–24 years) in the labor force, let us name it the youth share afterward, has increased from 19% since 1960s to 25% by the late 1970s due to the postwar baby boom. Recently, this value decreased to less than 15% as the inflow of young workers dwindled. In Japan, this value declined from 23% in the late 1960s to less than 10% currently. If workers at different age levels are disproportionately affected by the business cycle, then changes in the youth share will have a direct composition effect on aggregate labor market dynamics, even if age-specific labor market dynamics remain unchanged during demographic changes.

It is often thought that young workers would be more easily affected by the business cycle, as they lose jobs more often and their (un)employment rate is generally more cyclically sensitive. In the first essay, I first show that this guess is supported by data. In particular, the labor market volatility of young workers, in terms of unemployment volatility, is significantly higher than those of other age groups, and the corresponding values of prime-age and old workers are relatively close. This age-specific difference points out the importance of demographic changes in accounting for aggregate business cycle volatility. Besides, I observe that there is a hump-shaped pattern in the youth share in the U.S. from 1960 to 2007, at the same time, aggregate unemployment volatility comoves with this youth share over time. More importantly, I find an even closer comovement between the unemployment volatility of young workers and this youth share. This suggests a new channel: Unemployment volatility within the group of young workers increases with their share in the labor force.

I then test empirically this new channel. Using an unbalanced panel of 20 OECD countries from 1960 to 2007, I find that the variation in the youth share has a quantitatively large and statistically significant, positive effect on the unemployment volatility of young workers. The causality of this relationship is identified by the exogeneity of the youth share, as it is predetermined at least 15 years prior. I refer to this novel fact as a spillover effect.

To quantify the relative importance of this spillover effect with respect to the composition effect as mentioned before, which is proposed as the only demographic explanation for business cycle volatility by Jaimovich and Siu (2009), I decompose the overall effect of demographic changes on aggregate unemployment volatility. In contrast to Jaimovich and Siu (2009), I find that the spillover effect accounts for most of the effect of demographic changes. In accounting for "The Great Moderation"—the substantial decline in cyclical volatility experienced in the U.S. since the mid-1980s, I find that demographic changes can explain one quarter of the decline in unemployment volatility. Of this, the spillover effect accounts for two-thirds of the overall effect, while the composition effect accounts for only a third.

In the second essay, I explore a potential explanation of this novel fact. I argue that an increase in the youth share depresses the price of goods produced by young workers. This generates a congestion in their labor market dynamics and raise their cyclical unemployment volatility. To be specific, I build on Jaimovich, Pruitt, and Siu (2013) by incorporating a job matching model with endogenous job separation into the real business cycle model, to connect labor demand with the dynamics of labor market transition rates.

I start by distincting goods produced by young and old workers. Then, these two distinct goods are combined with capital for the production of final goods. Therefore, this distinction generates a labor demand structure that differentiates between young and old workers. With this labor demand structure, a greater share of young workers is associated with a lower relative price of goods produced by them, lowering firms' profits. This induces spillover effects in labor market dynamics, both in the job separation and finding rates. For the former, the volatility of the job separation rate increases with the youth share, because young workers become more vulnerable to productivity shocks. For the latter, the volatility of job finding rate also increases with the youth share, since firms' vacancy posting also becomes more sensitive to productivity shocks, which works the same way as Hagedorn and Manovskii (2008). Therefore, both aspects imply that the unemployment volatility of young workers increases with the youth share.

As corroborating evidence for this mechanism, I find that the decline in the relative price of goods produced by young workers due to a greater share of young workers is supported by empirical data. Further evidence comes from the empirical dynamics of transition rates: The second moments of the transition rates of young workers are higher if the share of young workers is higher, and vice versa. This empirical pattern is exactly the same as my model predicts.

The dependence of the labor market dynamics of young workers on demographic structure have important policy implications. First, it points out the necessity for policy-makers to pay attention to the current state of demographic structure and its projected path, if they need information about the labor market response to potential shocks, especially for young workers. Second, the positive correlation between business cycle volatility and demographic changes also suggests that the Great Moderation was more likely driven by structural factors other than good policies.

The third essay focuses on consumption goods markets, by assessing the differential impacts of demographic changes on the consumption of retirees and workers. The research topic is of particular interest, not only because of the increasing income and wealth inequality in industrialized countries in the past few decades (Piketty, 2015), but also because these countries will experience considerable growths in their older population while significant declines in their working population. In the U.S., the dependency ratio—the number of population over 65 years to working age population, is expected to grow from 0.22 in 2015 to 0.40 in 2060. Some other countries have already stepped into the society of aging. In Japan, the dependency ratio is already up to 0.47 in 2017, almost three times as much as that in 1990. The current ratios for Italy, Germany, and France are all over 0.3.

Using a New Keynesian model featuring the life-cycle behavior of Gertler (1999), I predict an increase in consumption inequality between retirees and workers in the U.S., measured by the ratio of consumption per retiree to consumption per worker, using its projected demographic evolution. In my calibrated model, this ratio is predicted to decline by 40% for the U.S., from 0.68 to 0.41, between 1990 and 2060, due to population aging. As the zero lower bound will bind more frequently with an older population, in addition, I also investigate the distributional effects of the zero lower bound during cyclical downturns. I find that the presence of the zero lower bound can mitigate the asymmetric effects of shocks on workers and retirees in dynamics, because there is a lower decline in the real return on assets, which particularly benefits retirees.

Chapter 2

Demographic Changes and Unemployment Volatility

2.1 Introduction

The demographic composition of the labor market has changed substantially among developed economies. In the U.S., the persistent inflow of young workers (aged 15–24 years) into the labor market since 1960s has pushed the youth share, the share of young workers in the labor force, to be more than one quarter by the late 1970s, and then as the inflow dwindled, this value decreased to less than 15% recently. In Japan, the share of old workers (aged 55–64 years) in the labor force increased from 10% in the 1960s to more than 20% currently[1]. The implications of demographic changes in the labor market are far reaching. In this paper, I focus on the effect of demographic changes on unemployment volatility.

It is often thought that young workers would be more easily affected by the business cycle, while the opposite is considered true for prime-age workers. My first contribution consists in showing that this guess is supported by data. There are sharp age-specific differences. In particular, the unemployment volatility of young workers is significantly higher than those of other age groups, and the corresponding values of prime-age and old workers are relatively close.

Given the size of the unemployment volatility of young workers, changes in the youth share have a direct composition effect on aggregate unemployment

[1] If the labor force is evenly distributed across age groups, the youth share will be 20% and so will the share of old workers.

volatility. Ceteris paribus, more young workers in the labor force naturally mean higher aggregate unemployment volatility even if age-specific unemployment volatility remains unchanged. The prediction of the impact of demographic changes would be a simple task if it was only due to the composition effect. But, if age-specific unemployment volatility is also affected by demographic changes, estimating the effect on each age group is of vital importance for precise prediction. Besides, the estimation results could have important implications for policy makers when it comes to coping with age-specific labor market policies.

My second contribution lies in showing that there is also a spillover effect among young workers. The variation in the youth share has a quantitatively large and statistically significantly positive effect on the unemployment volatility of young workers. This further contributes to the variation in aggregate unemployment volatility. After decomposing the overall effect of demographic changes on aggregate unemployment volatility into composition and spillover effects, I find that the spillover effect accounts for most of the effect of demographic changes. In accounting for "The Great Moderation"—the substantial decline in cyclical volatility experienced in the U.S. since the mid-1980s, I show that demographic changes can explain 24.2% of the decline in unemployment volatility. Of this, the spillover effect accounts for 16.6%, while the composition effect only 7.6%.

To measure unemployment volatility, I use time-varying volatility based on a stochastic volatility process with an autoregression (SV with AR) used in recent studies (see e.g. Fernández-Villaverde et al., 2011; Born and Pfeifer, 2014). Compared with the rolling-window standard deviation, a conventional measure used in the literature (see e.g. Blanchard and Simon, 2001; Jaimovich and Siu, 2009; and Carvalho and Gabaix, 2013), it has two advantages: First, as a dependent variable, it does not suffer from serial correlation in the residuals, which is inevitable for the conventional measure that uses overlapping data for the construction of consecutive values. Second, it does not have any loss in the amount of observations, while for the other, a 10-year window period means a loss of nine observations for each country. Furthermore, in comparison with the instantaneous standard deviation based on Stock and Watson (2003) (Stock/Watson) used in Jaimovich and Siu (2009), it is still preferred. For the measure based on Stock/Watson, the unit root assumption used for the variance equation is not empirically supported[2]. Besides, it uses a mixture of different normal distributions for the shock in the variance equation to ensure a better fit with the data, but this practice makes it suffer from subjectivity in the selection of weights for this shock.

In addition to the demographic variable, which is used as the sole explanatory variable in Jaimovich and Siu (2009), Lugauer and Redmond (2012), and Lugauer (2012b), I include labor market institutions to explain unemployment volatility. Union density and the centralization of wage bargaining reflect real wage rigidities, which play an important role in the amplification of unemployment fluctuation (Hall, 2005). In addition, according to Hagedorn and Manovskii (2008), high unemployment benefit suggests a low value for firms' profit, and therefore a high cyclical unemployment fluctuation. Neglecting the effects of labor market institutions might lead to biased estimation or even spurious regression. Although changes in labor market institutions could also partially reflect demographic pressures, this does not affect the consistency of the estimation.

As pointed out by Everaert and Vierke (2016), the negligence of the time series property of panel data might lead to spurious inferences. In view of this, I also carry out extensive stationarity analyses on relevant series. Another reason is the relatively monotoneous variation in the demographics and unemployment volatility over the sample period. Hypothetically, the two could merely share a similar trend in coincidence. The results show that this possibility is excluded as stationarity tests reject the null hypothesis of a unit root in any of them. But I detect cross-sectional dependence in both series. This could lead to biased upward stationarity statistics and falsely reject the null hypothesis. Thus, I also run panel cointegration tests to ensure the validity of regression results in case of non-stationarity. Besides, I use the common correlated effects (CCE) estimators proposed by Pesaran (2006) to take care of cross-sectional dependence.

[2] The estimates of the coefficient of the AR(1) process of the variance equation are smaller than unity for all countries in my sample. Therefore, the unit root assumption is not supported. Results are available upon request.

2.1.1 Related literature

This paper contributes to four strands of literature. First, this paper is closely related to the studies initiated by Jaimovich and Siu (2009) on the role of demographic changes in aggregate business cycle volatility. By linking demographic changes to the aggregate cyclical volatility of real GDP after World War II in G7 countries, they claim that the change in the age composition of the labor force over time is the main demographic reason for the variation in aggregate cyclical volatility. As follow-ups, Lugauer and Redmond (2012) and Lugauer (2012b) also found similar results with, respectively, a balanced panel dataset for 51 countries and state-level data of the U.S. But the replication by Everaert and Vierke (2016) indicates that these regression results may be spurious, as the series used in these papers for demographic change and cyclical volatility are found to be non-stationary. Besides, no co-integrating relation is detected between these two series.

Building on these studies, this paper confirms the positive impact of demographic changes on aggregate unemployment volatility. To obtain this result, I deviate from the literature in three aspects. First, I use the youth share as a measure for demographics instead of the compact index, which is the share of young and old workers in the labor force, used in Jaimovich and Siu (2009). Although their analysis shows that the volatilities of hours worked and employment for young and old workers are obviously higher than that of prime-age workers, it is too restrictive to assume the regression coefficients for the young and the old as being the same. This is because the underlying reasons are different: The fact that the cyclical volatility of young workers is high is related to their low work experience, which is a persistent feature for young workers in most countries; for old workers, on the other hand, it depends on labor market reforms such as changes in the generosity of unemployment benefits and firing cost, which are heterogeneous in terms of timing and scale among different countries. Second, I use the time-varying volatility based on SV with AR to measure unemployment volatility instead of a rolling-window standard deviation[3] or the

[3] Besides, the rolling-window standard deviation in Jaimovich and Siu (2009) is filtered by Hodrick-Prescott (HP) filter. Since HP filter itself already has several flaws (see Hamilton, 2017), a volatility measure which further uses a rolling-window standard deviation is at a disadvantage.

measure based on Stock/Watson used in Jaimovich and Siu (2009) with its advantages mentioned before. In fact, with this measure, I find that the difference in unemployment volatility between old and prime-age workers is not a general worldwide phenomenon; this corroborates my first deviation. Finally, I also control the effect of labor market institutions, which is overlooked in the literature and is likely to be the reason for the non-stationarity detected by Everaert and Vierke (2016) in the series used by Jaimovich and Siu (2009) and their follow-ups.

Despite there being rich literature on the impact of demographic changes on aggregate business cycle volatility, there are few studies that examine the impact on the age-specific unemployment volatility. Han (2018b) happens to be the only one to offer a theoretical explanation of this spillover effect. In that paper, I show that the explanation lies in the price changes of goods typically produced by young workers. An increase in the youth share depresses this price. Since the decline in price can shrink profits, firms raise their selection criteria of young workers, thus pushing the cutoff of the productivity of young workers close to the center of its distribution. This implies that more young workers will be affected when aggregate productivity changes. Furthermore, firms' profit and thereby their vacancy postings also become more sensitive to aggregate productivity since their profit shrinks. Therefore, the unemployment volatility of young workers increases with the youth share. This paper ties in with this study by providing empirical evidence to Han (2018b).

Third, this paper is related to the literature which studies the demographic differences in labor market fluctuations. Clark and Summers (1981) document the demographic differences in unemployment variation in the U.S. from 1950 to 1976. They conclude that the cyclical variation of the unemployment rate of young workers is higher and thereby accounts more of aggregate unemployment variation[4]. Jaimovich, Pruitt, and Siu (2013) extend this results to hours worked and wage in the U.S. They find the volatilities of both hours and wages for young workers to be higher than those of the prime-age. This paper enriches the literature by generalizing the pattern of the demographic differences in unemployment volatility to 20 OECD countries. In addition, this paper shows that

[4]In Clark and Summers (1981), the cyclical variation of the young is measured by the sensitivity of the change rate of the unemployment rate of young workers with respect to aggregate demand.

this pattern exists not only in summary statistics, which is the measure used in the literature, but also in the time-varying measure.

As already mentioned, this paper is also related to the literature that investigates the impact of labor market institutions on unemployment dynamics. The conventional explanation for cross-country differences in the unemployment rate and unemployment volatility considers the variation in labor market institutions. Studies by Blanchard and Wolfers (2000) and Krause and Uhlig (2012) focus on the long-run effect of labor market institutions on the unemployment rate, while studies by Bertola, Blau, and Kahn (2007) and Abbritti and Weber (2010) explore the impact of labor market institutions on cyclical unemployment dynamics. Their findings suggest that it is hard to detect the true relationship between demographic changes and business cycle volatility without controlling the effects of labor market institutions on the latter. Therefore, in this paper, I include the effects of labor market institutions while focusing on the role of demographic changes.

The paper proceeds as follows. The next section is about the measure of unemployment volatility and provides a description of the data. Section 2.3 documents the differences in age-specific unemployment volatility. Section 2.4 lays out the empirical evidence of the existence of the spillover effect. Section 2.5 presents a decomposition of the overall effect of demographic changes in the Great Moderation. Section 2.6 concludes.

2.2 Measure of unemployment volatility

A time-varying volatility measure, with which one can easily disentangle the influence of temporary shocks, is crucial to identify persistent demographic features in unemployment volatility. I use a measure based on SV with AR for unemployment volatility.

The unemployment rate is assumed to follow an AR(1) process[5]:

$$u_t = \rho u_{t-1} + e^{\sigma_t} v_t, \tag{2.1}$$

[5] I omit the country specific subscript for simplicity.

and the volatility equation follows an AR(1) stochastic volatility process (see e.g. Fernández-Villaverde et al. 2011; Born and Pfeifer, 2014):

$$\sigma_t = (1-\rho^\sigma)\bar{\sigma} + \rho^\sigma \sigma_{t-1} + \eta\varepsilon_t, \tag{2.2}$$

where v_t and $\varepsilon_t \overset{iid}{\sim} N(0,1)$, ρ and ρ^σ are respectively the coefficients of the level and volatility equations, Equations (2.1) and (2.2), $\bar{\sigma}$ is an unconditional mean, and η is a scale factor.[6]

The error term of the level equation is a product of the instantaneous innovation variance and an i.i.d. random process with unity variance. I use the unemployment rate itself instead of the log unemployment rate in the level equation. This is because the unemployment rate is already in percentage and usually considered as stationary, more importantly, according to Andreasen (2010), log-transformation of the level equation leads to an infinite expectation of the unemployment rate[7].

The time-varying volatility of the unemployment rate is e^{σ_t} and what one needs is the estimation of the stochastic volatility σ_t. With the non-linear setup of the shocks, the Sequential Importance Resampling particle filter (with 10,000 particles) is used to evaluate the likelihood. Thereafter, a standard Metropolis-Hastings algorithm is used to maximize the posterior, followed by a backward-smoothing routine to get the fitted value for σ_t. The detailed procedure can be found in Born and Pfeifer (2014).[8]

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[6] As priors for the stochastic volatility processes in the sample, I assume the coefficient of the level equation follows uniform distribution: $\rho \sim \mathrm{U}(-0.9999, 0.9999)$, the unconditional mean in the volatility equation follows uniform distribution: $\bar{\sigma} \sim \mathrm{U}(-11,-3)$, the coefficient of the volatility equation follows beta distribution: $\rho^\sigma \sim \mathrm{Beta}(0.9, 0.01)$, and the scale factor of the shock in the volatility equation follows gamma distribution: $\eta \sim \mathrm{Gamma}(0.5, 0.01)$.

[7] See Appendix A.1 for proof.

[8] The code for generating the time-varying volatility based on SV with AR is publicly available on Pfeifer's personal website.

2.2.1 Data

I use an unbalanced panel of data consisting of annual[9] observations of 20 OECD countries[10] from 1960 to 2007. The cross-sectional coverage is limited to 20 countries because the data for labor market institutions are derived from the ICTWSS database (Visser, 2016) and the database of CEP-OECD institutions (Nickell, 2006), which only cover these countries[11]. The data are up to 2007, because unemployment behavior afterward contains noisy signals of the recent financial crisis. Besides, it is technically difficult to disentangle the effect of the financial crisis from the effect of demographic changes.

The dependent variables in regressions are the age-specific unemployment volatility and aggregate unemployment volatility, respectively. Aggregate measure covers people aged 15-64 years. The main explanatory variable is the share of the corresponding age group in the labor force, while the youth share for aggregate unemployment volatility. In addition, I also use the population share as well as the native population share to deal with potential endogeneity problems.[12]

To characterize labor market institutions, I use five indicators. These are: (i) union density, which is the percentage of wage and salary earners who are union members; (ii) the union centralization of wage bargaining, which is related to real wage rigidities; (iii) the strictness of employment protection legislation, which is associated with firing cost; (iv) tax wedge, which is a measure of the deviation of the actual wage from the labor cost to employer due to taxation; (v) gross replacement rate, which is the ratio of unemployment benefits received when not working over wages earned when employed, as a measure of the generosity of unemployment benefit.

Additionally, I include a measure for the world demand shock, which is proxied by the log-difference of the sum of real GDP of other 19 countries.

[9] The quarterly data for the age-specific unemployment rate are only available for limited countries, besides, the data for the labor force share of each age group and labor market institutions are only available annually.

[10] Table 2.1 includes a list of these 20 countries.

[11] See Appendix A.2 for detailed information on data sources.

[12] See Appendix A.3 for the summary statistics of key variables.

2.3 Unemployment volatility by age

I start with the analysis of differences in unemployment volatility across age groups. Young workers form the age group of 15-24 years, while the old comprise the age group of 55-64 years. I find that the unemployment volatility of young workers is far higher than those of the other age groups, and the corresponding values of old workers are quite close to those of prime-age workers in general.

To be consistent with the literature started by Jaimovich and Siu (2009), in addition to the main measure for unemployment volatility as mentioned, I also use the standard deviation of the cyclical component of the unemployment rate as an extra measure. For this I apply the one-sided HP filter with a smoothing parameter of 100 on the unemployment rate [13]. If one filters the log unemployment rate and uses the percentage standard deviation to measure unemployment volatility, the volatility will be over-sensitive to real shocks when the unemployment rate is at a low level[14].

Table 2.1 reports the cyclical volatility of the unemployment rate by age group for 20 OECD countries[15]. For easy comparison, I normalize the value for the age group of 25-54 years to unity. From Table 2.1 we can see an almost strictly declining[16] trend in the age-specific unemployment volatility before age 55. For the age group of 15-19 years, the unemployment rate is about three times volatile than that of prime-age workers, and more than four times in Austria, Belgium, France, and Italy; for the age group of 20-24 years, the unemployment rate is still about twice as volatile as that of prime-age workers. However, this

[13] A parameter of 100 is commonly used in the literature for annual data, see e.g. Cooley and Ohanian (1991) and Rogerson and Shimer (2011). It is also the value used throughout this paper. Besides, I have also repeated the analyses with a parameter of 6.25, as suggested by Ravn and Uhlig (2002), and got similar results.

[14] An extreme case is Germany. From the early 1960s to the mid-1970s, the unemployment rate remains less than 1 percent in Germany. This makes it highly sensitive to any real shock. The percentage standard deviation of the unemployment rate stays at a quite high level before the mid-1970s.

[15] In fact, the unemployment rate data of the age groups of 15-19 years and 20-24 years are only available for Netherlands from 1987 and the data of the age group of 55-64 years are only available for Portugal from 1975. For Ireland, the age-specific unemployment rate has several missing values before 1983. I use the average of the neighboring values to replace the missing if both are available; otherwise, it is left as missing. In this way, missing values are filled back to 1975.

[16] One exception is Switzerland, for whom the unemployment volatility of the age group of 15-19 years is slightly lower than that of the age group of 20-24 years, but still the value is much higher for the young than that of the prime-age.

Table 2.1: Cyclical volatility of unemployment by age group, 20 OECD countries

Country	Period	15-19	20-24	25-54	55-64	15-64
Australia	1966-2007	2.88	1.97	1.00	1.18	1.29
Austria	1994-2007	4.86	2.08	1.00	0.78	1.15
Belgium	1983-2007	4.27	2.76	1.00	1.10	1.13
Canada	1976-2007	1.95	1.83	1.00	0.97	1.15
Denmark	1983-2007	2.17	2.27	1.00	0.96	1.03
Finland	1963-2007	2.15	1.95	1.00	1.40	1.11
France	1968-2007	4.72	3.51	1.00	1.11	1.21
Germany	1970-2007	1.82	1.77	1.00	1.34	1.10
Ireland	1975-2007	3.02	1.99	1.00	0.77	1.18
Italy	1970-2007	5.24	3.53	1.00	1.26	1.47
Japan	1968-2007	3.16	1.75	1.00	1.67	1.15
Netherlands	1971-2007	3.26	1.75	1.00	0.62	1.11
New Zealand	1986-2007	2.12	2.10	1.00	0.86	1.19
Norway	1972-2007	3.42	2.30	1.00	0.68	1.11
Portugal	1974-2007	3.46	2.85	1.00	0.70	1.22
Spain	1972-2007	3.00	2.21	1.00	0.70	1.22
Sweden	1963-2007	3.32	2.85	1.00	0.92	1.16
Switzerland	1991-2007	2.28	2.55	1.00	1.02	0.99
U.K.	1984-2007	2.32	1.97	1.00	0.92	1.14
U.S.	1960-2007	2.02	1.64	1.00	0.83	1.12

Notes: Cyclical volatility of the unemployment rate is filtered by a one-sided HP filter with a smoothing parameter of 100. It is expressed relative to the prime-age (25-54).

trend reverses in several countries[17] when it comes to old workers. Among these, the unemployment volatility of old workers is relatively high for Japan, Germany, and Finland, which are currently facing serious aging of the labor force[18]. I also report aggregate unemployment volatility (15-64) in the last column, which is higher than that of prime-age workers for almost all 20 countries[19] because of the

[17] Which include Australia, Belgium, Finland, France, Germany, Italy, Japan, and Switzerland.

[18] The labor force share of the old (55-64) for these three countries in 2015 are, respectively, 19.91%, 18.71%, and 18.37%, ranking top 3 among the 20 OECD countries.

[19] Again, the exception is Switzerland. The aggregate unemployment volatility is lower than all age-specific volatilities. The reason for this puzzling observation could be due to the negative relation between age-specific unemployment rates. As the aggregate unemployment rate can be expressed as the average of age-specific unemployment rates weighted by the corresponding labor force share. Therefore, the aggregate unemployment volatility is composed of the volatility of $u_t^i s_t^i$ and the covariance of $u_t^i s_t^i$ and $u_t^j s_t^j$, where i and j

volatile behavior of young workers. In summary, the unemployment volatility of young workers is significantly higher than that of other age groups.

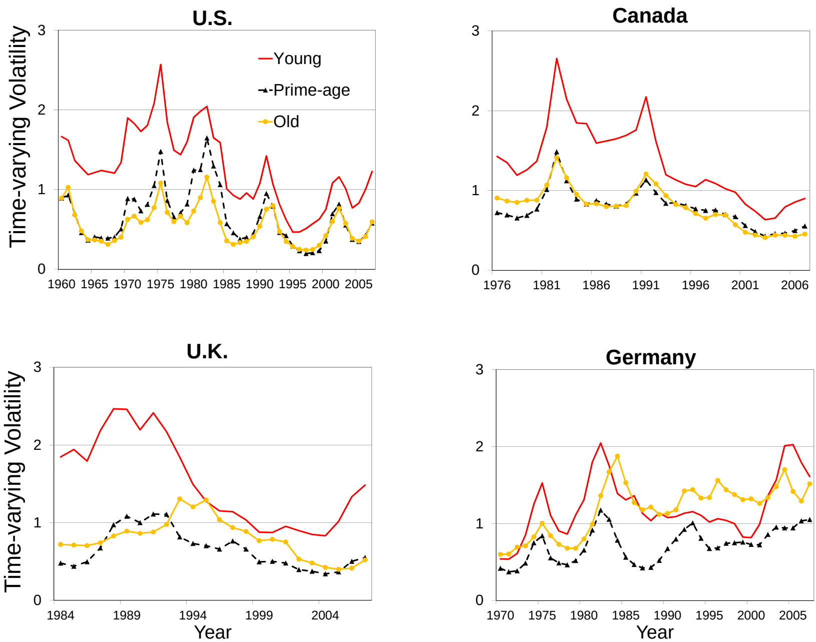

Figure 2.1: Time-varying unemployment volatility by age group

Notes: Solid line is for young workers (15-24); dotted, triangle-hatched line is for prime-age workers (25-54); and solid, square-hatched line is for old workers (55-64).

The aforementioned measure is straightforward and easy to compare across age groups, but as summary statistics, it could lead to misleading interpretation. One might wonder whether the observed pattern is a persistent demographic feature or is simply due to temporary shocks. For example, right before the

are the indexes for different age groups. If the unemployment rates of young workers and the prime-age move in opposite direction, suppose that it is due to certain labor market institutions which only favor the job finding probability of young workers, we will see a negative covariance of $u_t^Y s_t^Y$ and $u_t^P s_t^P$. And summation of these terms gives a lower value for aggregate unemployment volatility. This is also one of the reasons that I add labor market institutions into the analysis.

beginning of 2005, there was a big spike in the unemployment rate of the age group of 60-64 years in Germany. The reason is that January 1, 2005 marked the effective date of the Hartz IV reform, which shortened the duration and lowered the level of unemployment benefits. This creates an incentive for old workers to enter the unemployment pool and grab the last chance for high unemployment benefit without having much to lose. Potentially, this could be the reason for the higher unemployment volatility of old workers compared to that of prime-age workers for Germany as shown in Table 2.1, rather than any essential difference among workers of different ages. A similar logic also applies to the high values for young workers, which might be the result of changes in certain labor market institutions that only impact young workers. Therefore, it is also necessary to ascertain the time-varying measure of unemployment volatility.

Figure 2.1 shows the volatility measure based on SV with AR by age group for four countries. It displays distinctive features. For the U.S. and Canada, the unemployment volatility of young workers (solid line) is obviously above the values for prime-age (dotted and triangle-hatched line) and old workers (solid and square-hatched line). The value for prime-age workers is slightly higher than that for old workers over most of the time for the U.S., while the two almost coincide for Canada. For the U.K. and Germany, the value for old workers even overtakes that for young workers. For the U.K., this happens only once; while for Germany, though, this lasts for about two decades. Besides, we see there is indeed a sudden spike around 2005 in the unemployment volatility of old workers in Germany, but its long-lasting high value does not seem to be driven by the labor market reform in 2005.

Based on the examination of the time-varying unemployment volatilities by age group also for the other 16 countries[20], I conclude that the unemployment volatility of young workers is the highest in general, and this is more likely to be a persistent demographic feature over time. The value for old workers frequently overtakes that for prime-age workers for a few countries (Japan, Germany, and Finland), but overall these two are quite close.

[20]See Appendix A.4 for the measures of other sixteen countries.

2.4 Empirical identification of the role of demographic changes

In this section, I employ the panel data model to study the relationship between unemployment volatility and demographic changes in the 20 OECD countries. For age-specific unemployment volatility, I find the following: (1) the variation in the youth share has a quantitatively large and statistically significantly positive effect on the unemployment volatility of *young* workers, and (2) no statistically significant relationship is detected among prime-age and old workers. After the spillover effect is identified, I go on to check for aggregate unemployment volatility and find that the variation in the youth share also has a statistically significant effect on *aggregate* unemployment volatility.

2.4.1 Unemployment volatility of young workers

In this subsection, I first present a brief look at the comovement between the unemployment volatility of young workers and the youth share. I then check the time series properties of these two series, and finally report the regression results.

Figure 2.2 presents the time series of the time-varying unemployment volatility of young workers and the youth share for the U.S., U.K., Canada, and Germany. In the U.S., the cyclical behavior can be divided into two distinct periods, one with a persistent rise from 1960s to the mid-1970s, and the other with a decline from the mid-1970s. Coincidentally, the evolution of the youth share can also be characterized by a similar pattern—a rise with the entry of baby boomers into the workforce in the 1960s and the 1970s, and then a decline from the late 1970s. In Canada and the U.K., both series share a monotonically decreasing trend over time. For Germany, both series share a common decline from the mid-1980s to the early 2000s.

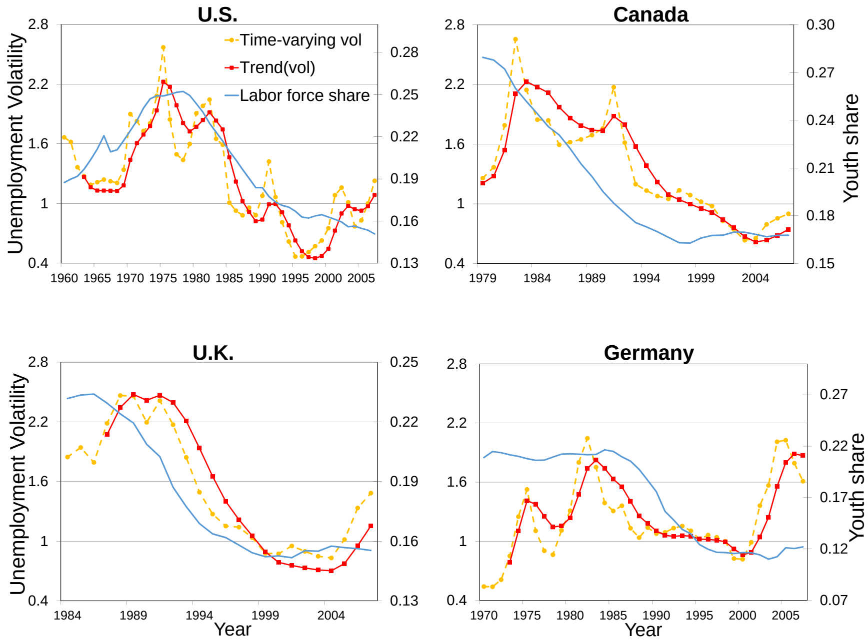

Figure 2.2: Youth share and unemployment volatility of young workers

Notes: Solid line is the youth share; dotted, circle-hatched line is the time-varying unemployment volatility of young workers; solid, square-hatched line is the trend of unemployment volatility which is filtered using a one-sided HP filter with a smoothing parameter of 100.

2.4.1.1 Time series properties

Before moving to regression analysis, it is necessary to check whether unemployment volatility is intrinsically driven by demographic changes or only coincidentally shares a similar trend. This concern comes from the insufficient variation in both series[21]. The rather monotonous variation could arouse the suspicion of spurious regression.

[21] Due to the long-term decline in birth rate and increase in life expectancy, the youth share decreases persistently from the 1980s, at the same time, there is also a decline in the unemployment volatility of the young in most OECD countries.

To take care of this, I first check the stationarity of the relevant series. The upper part of Table 2.2 reports the results of panel unit root tests of the youth share and the unemployment volatility of young workers. For the youth share, after projecting out fixed effects (FE), I show that the null hypothesis of a unit root is rejected as a panel by the Maddala and Wu (1999) test (MW)[22], and it is rejected at 5 percent level of significance in six countries (out of 20) by country-specific augmented Dickey and Fuller (1979) tests (ADF). For unemployment volatility of young workers, after projecting out fixed effects and time effects (FETE), along with the effects of labor market institutions (LMIs) and external shock[23], the null hypothesis is rejected at 1 percent level of significance by the MW test as a panel. Besides, it is rejected in six countries by country-specific tests. These suggest the two series are more likely to be stationary.

However, I also detect cross-sectional dependence in the data. Columns 1 and 2 in the middle part show that, although the values for the average cross-sectional correlation $\bar{\rho}$ are low, the Pesaran (2004) cross-sectional dependence (CD) tests are significant. This strongly suggests the existence of cross-sectional dependence in both series. As the panel unit root tests are biased toward stationarity when cross-sectional dependence is neglected according to O'Connell (1998), we cannot fully rule out non-stationarity. In Column 3, I also carry out the same test for unemployment volatility by projecting out only fixed effects and time effects. The results are similar. The reason for this exercise is to show that, with a model setup similar to Jaimovich and Siu (2009), cross-sectional dependence is likely to be a potential reason for biased estimates.

Therefore, I also carry out cointegration tests as shown in the lower part of Table 2.2. Even if these two series were non-stationary, panel regression is still capable of detecting the true relationship if they are cointegrated[24]. Technically, if panel cointegration tests reject the null hypothesis of a unit root in the error terms of the regressions, the panel regressions still offer meaningful results even with the presence of cross-sectional dependence. The results show that there

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[22] It combines the p-values of the country-specific ADF tests.

[23] I use a fixed effect model and regress the unemployment volatility of each country on time dummies, the corresponding labor market institution indicators, and an external demand shock. Then, I use the residuals to calculate the country-specific unit root tests.

[24] Phillips and Moon (1999) show that a consistent estimation can also be obtained with neither stationarity nor cointegration, but their result relies on cross-sectional independence.

is indeed a cointegrating relationship between the youth share and the unemployment volatility of young workers, both shown by the cointegration tests as a panel and the ADF tests of individual countries.[25]

Table 2.2: Specification tests

Model	FE (1)	FETE+LMIs (2)	FETE (3)
Variable	youth share	unem. vol. (young)	unem. vol. (young)
ADF unit root tests of the series			
Panel unit root test	73.74	83.81	91.74
P-value	[0.00]	[0.00]	[0.00]
# $p_i \leqslant 5\%$	6/20	6/20	7/20
Cross-sectional dependence			
Average correlation	0.04	-0.03	-0.03
CD test	3.24	-2.37	-2.09
P-value	[0.00]	[0.02]	[0.04]
Panel cointegration test			
Panel unit root test	-	87.47	93.02
P-value	-	[0.00]	[0.00]
# $p_i \leqslant 5\%$	-	7/20	7/20

Notes: Sample period: 1960-2007, unbalanced panel of 20 OECD countries.
FE refers to a model which projects out fixed effects; FETE refers to a model which projects out both fixed effects and time effects; FETE + LMIs refers to a model which not only projects out fixed effects and time effects but also excludes the effects from labor market institutions and external demand shock.
CD test refers to cross-sectional dependence test. Details for the calculation of test statistics can be found in Appendix A.5.

[25] I regress $\sigma_{it}^Y - \bar{\sigma}_{it}^Y$ on the regressors, which includes $s_{it}^Y - \bar{s}_{it}^Y$, time dummies, labor market indicators, an external demand shock, and a constant term for each country. Then, I carry out the country-specific ADF tests on the error terms and count the number of countries with a p-value lower than 5%. The panel cointegration tests are constructed similarly as the panel unit root tests in the top left of Table 2.2, by combining the p-values from the country-specific ADF tests.

2.4.1.2 Regression results

The regression setup is a panel data model with fixed effects and time effects (FETE):

$$\sigma_{it} = \alpha_i + \beta_t + \gamma s_{it} + \lambda X_{it} + \varepsilon_{it}, \tag{2.3}$$

where σ_{it} is the volatility measure based on SV with AR for country i at time t; s_{it} is the share of the corresponding age group in the labor force; X_{it} includes the five indicators of the labor market institutions and the external shock as mentioned; α_i is a country fixed effect; and β_t denotes a full set of time dummies to control for time effects.

Table 2.3: Unemployment volatility and the share of labor force: young workers

Model	FETE (1)	MG (2)	CCEP (3)	CCEMG (4)
Coef. of the share	2.24	3.28	8.97***	8.80*
	(2.64)	(5.81)	(3.43)	(4.82)
LMIs & shock	Yes	Yes	Yes	Yes
FETE	Yes		Yes	
Nobs	644	644	644	644
R^2	0.31	0.96	0.77	0.62
CD test	-2.45	2.31	0.60	-0.76
	[0.02]	[0.02]	[0.55]	[0.45]

Notes: Sample period: 1960-2007, unbalanced panel of 20 OECD countries. The dependent variable is the unemployment volatility of young workers.
All models exclude the effects of labor market institutions and external demand shock. FETE refers to a model which also projects out fixed effect and time effects; MG refers to mean group estimator with trends; CCEP refers to a model that projects out fixed effects and excludes the effects of the unobserved common factors; and CCEMG refers to the mean group model, which also excludes the effects of the unobserved common factors.
Driscoll-Kraay standard errors for Columns 1 and 2 and Newey-West standard errors for Columns 3 and 4 are in parentheses. p values are in square brackets.
**** and * stand for significance levels of 1% and 10%, respectively.*

Table 2.3 presents the regression results for young workers. Columns 1 and 2 give the results of fixed effects and time effects (FETE) and the mean group (MG) estimations. With both estimators reported, we can reduce the risk of model misspecification[26]. I also include a trend component in the MG estimator, as there appears to be a trend in the unemployment volatility of young workers for some countries. Besides, I report the Driscoll and Kraay (1998)[27] standard error for both, which is robust to the missing of unobserved common factors unrelated to regressors. The results in the first row show that the coefficients are both positive but insignificant. It seems that the youth share does not have any statistically significant impact on the unemployment volatility of young workers. At the same time, though, I detect cross-sectional dependence, which suggests that the unobserved common factors in the error term is correlated with regressors. In this case, both FETE and MG estimators are biased. Still the former is used as the main estimator in Jaimovich and Siu (2009).

To deal with this problem, I follow the approach proposed by Pesaran (2006) with common correlated effects (CCE) estimators[28]. Here, the unobserved common factors are proxied by the cross-sectional averages of all observed variables, including both the dependent variable and the regressors. As a result, the unobserved common factors are eliminated and the consistency of panel estimator is ensured. Column 3 reports the pooled common correlated effect (CCEP) estimator, the fixed-effect version of the CCE estimator. Now, the coefficient becomes significant at the 1 percent level, meaning the youth share has a positive impact on the unemployment volatility of young workers[29]. The magnitude of the

[26]The FETE estimator pools the data together and assumes that the slope coefficients, the intercepts, and error variances of different age groups are all identical. Thus it may produce inconsistent estimates in case of heterogeneity among age groups. While the MG estimator by Pesaran and Smith (1995) fits the model separately for each group, therefore the estimated parameters are allowed to be heterogeneous across groups.

[27]Driscoll and Kraay (1998) propose a non-parametric covariance matrix estimator for the consistent standard errors. They use the cross-sectional average of the orthogonality moments of the product of the regressors and the residual to act as a Newey-West type weight in the estimation.

[28]Other approaches include the seemingly unrelated regressions (SUR) approach and the principal components approach. Among all, the CCE estimator is the most attractive because of its simplicity. One only needs to add the cross-sectional averages of observed variables in the fixed effects panel estimation and corrects the standard errors accordingly.

[29]Meanwhile, I also report the CCEP estimators of the coefficients of labor market institutions in Appendix A.6. Of which, the coefficients of union density and employment protection legislation are sizable and statistically significant. The effect of union density on unemployment volatility is positive, because union density is positively related to real wage rigidities and firms have to cut employees in case of recession; while the effect of employment protection legislation is negative, which is consistent with the finding of Blanchard and Portugal (2001).

coefficient suggests that a 10% increase in the youth share would increase the unemployment volatility of young workers by almost 0.90. Since the time-varying measure of the unemployment volatility of young workers for the U.S. is 2.57 in 1975 and 0.46 in 1995, the regression result suggests that: If there were a decline of 23.5% less young workers in the labor force over these 12 years, and all other conditions unchanged, then we would have the same decline in unemployment volatility of young workers. In other words, the actual decline in the youth share is 8.2%, meaning demographic changes account for approximately a third of the variation of the unemployment volatility of young workers.

To further check the robustness of the results and allow for more flexibility in the model setup, I also report the results of the common correlated effects mean group (CCEMG) estimator in Column 4, the mean group version of the CCE estimator. It allows for heterogeneous slope coefficients and also controls for cross-sectional dependence. The results confirm the positive impact of the youth share on unemployment volatility as implied by the CCEP estimator. The magnitude of the coefficient is slightly lower and less significant. Considering that the country-specific coefficients are relaxed to be heterogeneous, one should expect a lower significance level. The null assumption of cross-sectional independence cannot be rejected either, which ensures the validity of the estimation result.

As the youth share also depends on the participation rate of young workers, its validity as a measure of demographics lies in the exogeneity condition that the participation rate of young workers plays a minor role in the unemployment dynamics of young workers. To verify this, I use the share of young population as a proxy for the youth share. As the share of young population is independent of their labor force participation decision, the endogeneity bias is likely to be excluded. Column 1 of Table 2.4 reports the results of CCEP estimation result. The coefficient is positive and significant at the 10% level. Moreover, the level of the coefficient is similar to that in Table 2.3. The CD test also suggests that cross-sectional dependence is well taken care of by the CCEP estimator.

Table 2.4: Labor force participation and international migration: further robust check

Proxy for youth share	population share (young)	native population share (young)
Coef. of the proxy	6.53*	9.41**
	(3.47)	(4.45)
LMIs & shock	Yes	Yes
FETE	Yes	Yes
Nobs	659	627
R^2	0.77	0.78
CD test	-0.05	-0.67
	[0.96]	[0.50]

Notes: Unbalanced panel of 20 OECD countries, sample period: 1960-2007 for Column 1 and 1970-2007 for Column 2. The dependent variable is the unemployment volatility of young workers.
*Model used in this table is the pooled common correlated effect (CCEP) estimator. Newey-West standard errors are in parentheses. p values are in square brackets. ** and * stand for significance levels of 1% and 5%, respectively.*

However, if the response of the labor market activity of a foreign young worker to shocks differs from the response of that of a native young worker, the young population share is still likely to be endogenous as international migration decision is unaccounted for. To take care of this, Jaimovich and Siu (2009) suggest using the share of the native population[30]. As the historical data of this distribution are directly unavailable, they proxy it by the projection of the labor force share on the lagged birth rates. Instead, I use directly the age distribution of native population calculated from data on lagged birth rates and population size. With historical data on birth rates and population from Mitchell (2008) and Maddison (2003), I first get the historical series of the size of newborns. Since the population size of the young native population today is just the sum of newborns 15 to 24 years ago, I calculate the share of native young population as

$$b_{it}^{Y} = \frac{\sum_{j=15}^{j=24} b_{i,t-j} p_{i,t-j}}{\sum_{j=15}^{j=64} b_{i,t-j} p_{i,t-j}}, \tag{2.4}$$

[30]Since the past fertility decision is made long before the realization of shocks affecting the current unemployment volatility, the distribution of native young population is not likely to be influenced by international migration.

where $b_{i,t}$, $p_{i,t}$ are, respectively, the birth rate and population size at time t for country i.

Still, this measure does not account for the dynamics of the mortality rate for people below 64, and assumes that all newborns can live up to the age of 64. Therefore, I start this series from 1970 to mitigate the effects of the variation in the mortality rate and longevity. Column 2 in Table 2.4 reports the corresponding results of the CCEP estimation when the youth share is replaced by the share of young native population. The coefficient is significant at the 1% level and the magnitude of the coefficients is also similar to the other two cases. Taken together, I interpret the results of this subsection as convincing evidence of the existence of the spillover effect.

2.4.2 Unemployment volatilities of prime-age and old workers

In this section, I verify whether a similar relationship is also present among workers in other age levels.

Table 2.5 reports the results of CCEP estimations. Column 1 presents the estimation results when I regress the unemployment volatility of prime-age workers on their labor force share and other regressors. The share of the prime-age group in the labor force has a negative but insignificant impact on the unemployment volatility of prime-age workers. When I use the share of prime-age population as a proxy in Column 2, the impact becomes weaker and the coefficient is even less significant. Furthermore, when I use the share of native prime-age population as a proxy in Column 3, the results are similar: The coefficient is insignificant and the impact becomes even weaker. These, in turn, confirm the non-existence of any statistically significant relationship among prime-age workers. The validity of these results is ensured as cross-sectional dependence is rejected in all three cases.

In the last three columns, I redo the same exercises for old workers. For the share of old workers and the share of old population, the coefficients are positive but insignificant. For the share of the native old population, the coefficient becomes significant at the 5% level, but CD test is rejected at the 10% level. Thus, in Appendix A.7, I also report the corresponding CCEMG estimator, which becomes insignificant and whose coefficient also changes direction. This

Table 2.5: Unemployment volatility and the share of labor force: prime-age and old workers

Age-specific share	s_{it}^P (1)	p_{it}^P (2)	b_{it}^P (3)	s_{it}^O (4)	p_{it}^O (5)	b_{it}^O (6)
Coef. of the share	-3.54	-1.30	-0.64	3.26	3.57	4.50**
	(2.25)	(2.17)	(2.37)	(3.73)	(3.04)	(2.14)
Nobs	658	658	627	658	658	627
CD test	-0.07	0.23	-1.00	-1.17	-1.20	-1.89
	[0.95]	[0.81]	[0.32]	[0.24]	[0.23]	[0.06]

Notes: Unbalanced panel of 20 OECD countries, sample period: 1970-2007 for Columns 3 and 6 and 1960-2007 for other columns. The dependent variable for Columns 1-3 is the unemployment volatility of prime-age workers, and for Columns 4-6, it is the unemployment volatility of old workers.
The model used in this table is the pooled common correlated effect (CCEP) estimator. s_{it}^P, p_{it}^P, and b_{it}^P refer, respectively, to the share of prime-age workers, the share of prime-age population, and the share of native prime-age population. s_{it}^O, p_{it}^O, and b_{it}^O are the corresponding measures for old workers.
See Tables 3 and 4 for further notes.

indicates that the variation in the unemployment volatility of old workers is unlikely to be due to any persistent demographic phenomenon. Another reason could be the lack of enough variation in the unemployment volatility of prime-age and old workers, which makes it hard to detect any statistical relationship.

2.4.3 Aggregate unemployment volatility

In this section, I regress aggregate unemployment volatility on the youth share[31]. Concerning the potential endogeneity problems as mentioned already, I also use the share of young population and that of young native population as proxies for the youth share.

Table 2.6 reports the regression results by using the CCEP estimator. The coefficient of the youth share is significant at the 1% level. When I use the share of young population, the magnitude of the coefficient decreases slightly but is still significant at the 5% level. Even with the share of young native population,

[31] I do not consider the shares of other age groups as part of the regressors. This is because the effect of demographic changes on aggregate unemployment volatility is mainly due to the spillover effect, rather than the composition effect, as shown in the decomposition analysis in Section 2.5.

the coefficient is still significant and has similar magnitude. The CD test results also suggest the absence of cross-sectional dependence.

Table 2.6: Aggregate unemployment volatility and the share of labor force

Age-specific share	youth share	population share (young)	native population share (young)
Coef. of the share	6.52***	4.88**	5.08*
	(1.86)	(1.79)	(2.63)
LMIs & shock	Yes	Yes	Yes
FETE	Yes	Yes	Yes
Nobs	658	658	627
R^2	0.77	0.77	0.78
CD test	0.48	-0.09	-0.18
	[0.63]	[0.93]	[0.86]

Notes: Sample period: 1960-2007, unbalanced panel of 20 OECD countries. The dependent variable is aggregate unemployment volatility.
The model used in this table is the pooled common correlated effect (CCEP) estimator.
See Tables 3 and 4 for further notes.

2.4.3.1 Changes in composition of the sample and redefinition of the age group

As one of the main counterarguments of Everaert and Vierke (2016) is that the results of Jaimovich and Siu (2009), Lugauer and Redmond (2012) and Lugauer (2012b) are sensitive to the changes in the composition of the sample, I carry out 20 exercises which drop one country from the sample each time when I regress aggregate unemployment volatility on the youth share.

We see that the estimation results remain unchanged, as shown in Figure 2.3, which gives the distributions of four key parameters of the CCEP estimations. The coefficients are concentrated around 6, which is very close to 6.52, the corresponding coefficient of the youth share in Table 2.6. Although its value deviates when I exclude Ireland, Japan, or Sweden from the sample, the deviation is still acceptable, considering that the consistency of the CCEP estimator potentially relies on the assumption of infinite cross-sectional observations. Moreover, the distribution of the standard errors is quite narrow. The distribution of the P-values of the coefficients shows that the coefficients are basically significant at the 1% level, only with slightly lower significance level for two cases, when Germany or Ireland is excluded. In addition, the lowest p-value of the CD test is 0.15 when New Zealand is excluded from the sample, suggesting that cross-sectional dependence is well taken care of in all 20 exercises.

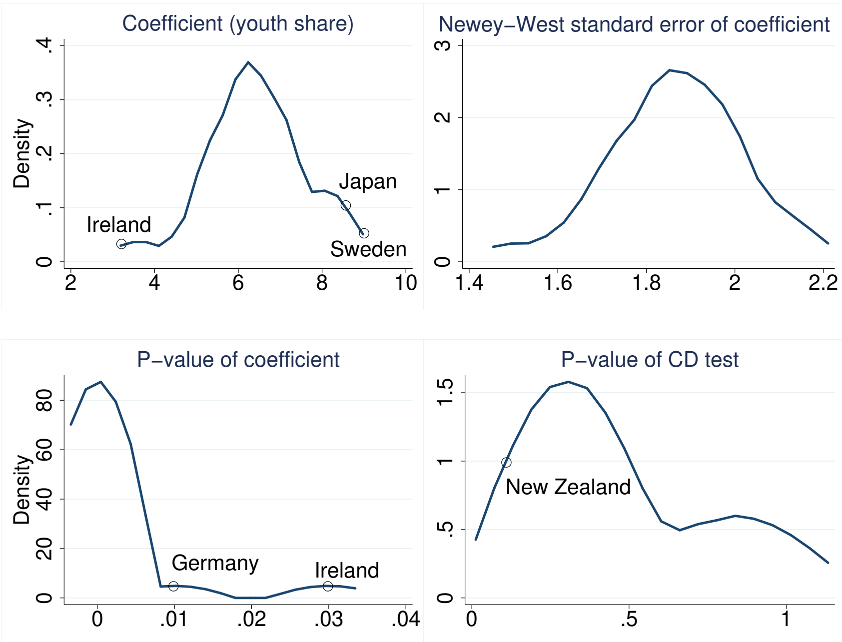


Figure 2.3: Robustness: distribution of key parameters after dropping one country at a time

Notes: The dependent variable is aggregate unemployment volatility. Graphs are drawn using Stata v11.2. Distribution is approximated using the nonparametric method with a Epanechnikov kernel function.

Except for the definition of the age groups used earlier, which categorizes the age group of 15-24 years as the young, 25-54 as the prime-age, and 55-64 as the old, I also take the specification used in Jaimovich and Siu (2009) and define the age group of 15-29 years as the young and the age group of 60-64 years as the old. The estimation results suggest that the changes in the youth share still have a statistically significantly positive effect on the unemployment volatility of young workers and aggregate unemployment volatility, only with a slightly smaller magnitude[32].

To summarize, I find convincing evidence that the age composition of the labor force not only has an effect on aggregate unemployment volatility but also makes a spillover effect on that of the young.

2.5 Decomposition of the effect of demographic changes in the Great Moderation

Since the mid-1980s, there has been a substantial decline in unemployment volatility in the U.S. It can be partially accounted for by demographic changes as shown in the earlier analysis. In this section, I decompose the overall effect of demographic changes in the Great Moderation into composition and spillover effects.

To this end, I first calculate the counterfactual aggregate unemployment volatility for the U.S. during the Great Moderation (1985-2007) if the labor force composition were kept at an average preceding the Great Moderation (1970-1984)[33]. As the aggregate unemployment rate can be expressed as a weighted average of the age-specific unemployment rates[34]:

$$u_t = \sum_{i=Y,P,O} s_t^i u_t^i, \tag{2.5}$$

where u_t^i is the unemployment rate of group i; and the weight, s_t^i, is the labor force share of i. This counterfactual aggregate unemployment volatility can be constructed as

$$\hat{\sigma}_t^2 = \sum_{i=Y,P,O} (s_{pre}^i \sigma_t^i)^2 + \rho_t^{i,-i} (s_{pre}^i \sigma_t^i)(s_{pre}^{-i} \sigma_t^{-i}), \tag{2.6}$$

[32]See Appendix A.8 for details.

[33]The youth share remains at a high level from 1970 to 1984 and only slightly varies.

[34]The aggregate unemployment rate can be written as: $u_t = (U_t^Y + U_t^P + U_t^O)/L_t = u_t^Y s_t^Y + u_t^P s_t^P + u_t^O s_t^O$.

where s^i_{pre} is the pre-great-moderation average of the labor force share of group i and ρ_t is the correlation coefficient between different age-specific unemployment rates measured by a five-year moving correlation coefficient.

Thereafter, the composition effect can be defined as

$$\text{Composition effect} \equiv \frac{\bar{\hat{\sigma}}_t - \bar{\sigma}_{post}}{\bar{\sigma}_{pre} - \bar{\sigma}_{post}}, \tag{2.7}$$

where $\bar{\sigma}_{pre}$ and $\bar{\sigma}_{post}$ are, respectively, the average of aggregate unemployment volatility before and after 1985; and $\bar{\hat{\sigma}}_t$ stands for the average of counterfactual aggregate unemployment volatility $\hat{\sigma}_t$. Hence the denominator, $\bar{\sigma}_{pre} - \bar{\sigma}_{post}$, is the actual decline during the Great Moderation, and the numerator, $\bar{\hat{\sigma}}_t - \bar{\sigma}_{post}$, is the reduction if the labor force composition were kept at an average preceding the Great Moderation and the age-specific unemployment volatility were independent of demographic changes.

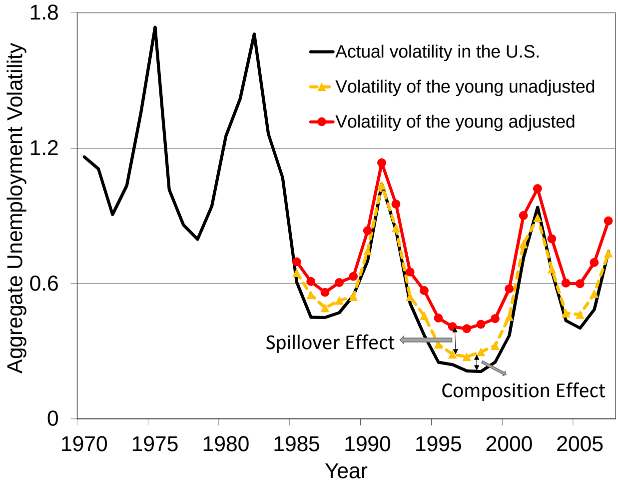
To get the spillover effect, I first adjust the age-specific unemployment volatility to match the demographic structure at the pre-great-moderation average. As the regression analyses suggest the existence of the spillover effect among young workers, the adjusted value for the unemployment volatility of young workers is

$$\sigma_t^{YAdj} = \sigma_t^Y + 8.97(s^Y_{pre} - s^Y_t). \tag{2.8}$$

Here, 8.97 is the CCEP estimator for the coefficient of the youth share in Table 2.3. If we use this adjusted value for the calculation of the counterfactual aggregate unemployment volatility, the spillover effect can be written as

$$\text{Spillover effect} \equiv \frac{\bar{\hat{\sigma}}^{adj}_t - \bar{\hat{\sigma}}_t}{\bar{\sigma}_{pre} - \bar{\sigma}_{post}}, \tag{2.9}$$

where $\bar{\hat{\sigma}}^{adj}_t$ is the average of the adjusted counterfactual aggregate unemployment volatility and $\bar{\hat{\sigma}}^{adj}_t - \bar{\hat{\sigma}}_t$ stands for the reduction in the average of aggregate unemployment volatility due to the effect of actual changes in age structure on the unemployment volatility of young workers.



Notes: Solid line is the actual aggregate unemployment volatility in the U.S.; dotted, triangle-hatched line is the first counterfactual series for the composition effect, which assumes that age composition were unchanged and the age-specific unemployment volatility were independent of demographic changes; solid, circle-hatched line is the second counterfactual series for the spillover effect, which adjusts the unemployment volatility of young workers if the age composition were unchanged.

Figure 2.4: Decomposition of the effect of demographic changes in the Great Moderation

Figure 2.4 shows the decomposition of the overall effect of demographic changes in the Great Moderation. Comparing the value for the actual aggregate unemployment volatility (solid line) across 1970-84 with the one across 1985-2007, we see that it has dropped significantly since 1985, with a decrease of 0.66. The counterfactual series without adjusting the unemployment volatility of young workers (dotted, triangle-hatched line) lies in the middle of the actual series and the counterfactual series with the unemployment volatility of young workers adjusted (solid, circle-hatched line). It states that, if the age composition remained constant after 1985 and the age-specific unemployment volatility were unaffected, aggregate unemployment volatility would have fallen comparatively less and the average would have fallen by 0.61, suggesting that the composition effect can explain 7.6%[35] of the moderation. For the second counterfactual series, had only the age composition remained constant, aggregate unemployment volatility would have fallen by 0.50. Thus, the overall effect of demographic changes explains 24.2%[36] of the moderation, leaving 16.6% to be attributed to the spillover effect.[37]

My result for the overall effect of demographic changes is close to that of Jaimovich and Siu (2009), who claim that 24% of the moderation is due to demographic changes, but the underlying mechanism is different. They attribute it exclusively to the composition effect, while I conclude that the spillover effect is the main demographic contributor.

2.6 Conclusion

In this paper, I address the question of how demographic changes affect unemployment volatility. First, I document the sharp differences in age-specific unemployment volatility: The unemployment volatility of young workers is significantly higher than those of other age groups, while the corresponding values of prime-age and old workers are quite close[38]. Based on this persistent age-specific difference, aggregate unemployment volatility can increase with the share of young workers in the workforce through a direct composition effect.

Second, I document a new stylized fact: Unemployment volatility with the group of young workers increases with their share in the labor force. This spillover effect further contributes to the variation in aggregate unemployment volatility due to demographic changes. My estimation controls for the effects of labor market institutions and is robust to changes in the composition of the

[35] Which is calculated by (0.66-0.61)/0.66.

[36] It is calculated in the same way as the former case: (0.66-0.50)/0.66.

[37] The above analysis is based on the assumption that changes in age structure do not have any effect on the correlation coefficients ρ_t between different age-specific unemployment rates. To check if this is true, I regress the three correlation coefficients, ρ_t^{YP}, ρ_t^{YO}, and ρ_t^{PO}, respectively, on the corresponding share of the labor force and other regressors. I do not find any statistically significant relationship between the correlation coefficients and the youth share, neither with fixed effects and time effects (FETE) estimator nor with common correlated effects (CCE) estimators.

[38] Exceptions include countries which are currently facing serious aging of the labor force (Germany, Japan, and Finland). For these three countries, the values for old workers are clearly higher than that of prime-age workers.

sample. Furthermore, it takes care of the cross-sectional dependence detected in the data by using common correlated effects (CCE) estimators.

Finally, taking the Great Moderation in the U.S. as an example, I decompose the overall effect of demographic changes on aggregate unemployment volatility into composition and spillover effects. I find that the spillover effect accounts for about two-thirds of the effect of demographic changes.

Chapter 3

Demographic Structure and the (Un)employment Volatility of Young Workers

3.1 Introduction

Unemployment volatility increases with a greater share of young workers in the labor force, as young workers lose jobs more often and their unemployment (rate) is generally more cyclically sensitive. This age-specific difference in the cyclical labor market volatility has been well documented and explanations have been offered by several authors[1]. With the existence of this difference, change in the relative share of young workers naturally has a direct composition effect, even if age-specific labor market volatility remains unchanged.

In this paper, I focus on a new fact: Unemployment volatility within a group of young workers also increases with their share in the labor force, which I refer to as a spillover effect in Chapter 2. Empirically, this phenomenon does not exist in other age groups as I show in Han (2018a). The reason, as argued in this paper, lies in a capital-experience complementarity, which is observed in the data I use.

Figure 3.1 shows the time series for unemployment volatility (along with its trend, filtered using a one-sided HP filter with a smoothing parameter of 100)[2]

[1] The empirical evidence is available both in intensive (see e.g. Gomme et al., 2004; Jaimovich and Siu, 2009) and extensive margin (see Han, 2018). Theoretical explanations include the capital-experience complementarity by Jaimovich, Pruitt, and Siu (2013), the higher initial training sunk cost for more educated workers by Cajner and Cairo (2013), and the difference in time spent for a good match by Lugauer (2012a) and Engbom (2018).

[2] Unemployment volatility is calculated by the author and measured by a stochastic volatility process which assumes that the unemployment rate follows an AR(1) process and

and the share of young workers[3] in the labor force (hereafter referred to as the youth share) for the U.S. from 1960 to 2007. We see that aggregate unemployment volatility comoves with the youth share over time (left panel). Moreover, an even closer comovement exists between this share and the unemployment volatility of the young (right panel). By contrast, the common underlying assumption used in the literature is that changes in the labor force composition do not have any impact on the labor market outcomes of a specific group[4]. Ignoring this spillover effect automatically leads to an overestimation of the composition effect. In addition, it could lead one to overlook the necessity of age-specific labor market policies in maintaining macroeconomic stability. Therefore, an investigation into this spillover effect and a possible explanation of this effect have significance for both empirical estimation and policy evaluation.

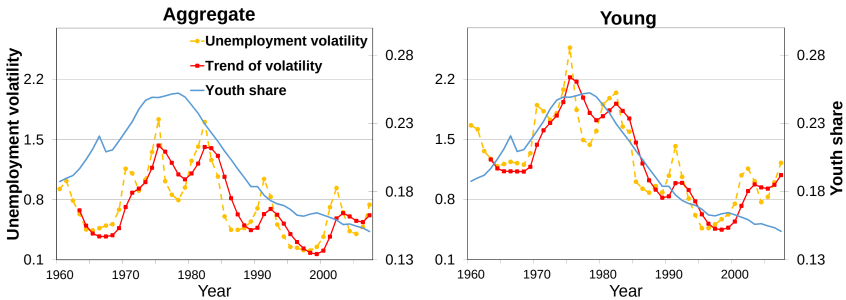


Figure 3.1: Unemployment volatility and the youth share (U.S.)

Notes: Dotted, circle-hatched line is the time-varying unemployment volatility (left axis), solid, square-hatched line is the trend of this measure (left axis), and the solid line is the youth share (right axis).

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the volatility equation follows an AR(1) stochastic volatility process. The unemployment rate and volatility are at an annual frequency. Throughout this paper, I use a one-sided HP filter with a smoothing parameter of 100 for annual data, which is commonly used in the literature, see e.g. Cooley and Ohanian (1991) and Rogerson and Shimer (2011). Besides, I have also repeated the analysis with a smoothing parameter of 6.25, as suggested by Ravn and Uhlig (2002), and got similar results.

[3]Young workers refer to those aged 15-24 years and aggregate measure covers people aged 15-64 years.

[4]See e.g. Jaimovich and Siu (2009) and Cajner and Cairo (2013).

To explain this new finding, I incorporate the Mortensen–Pissarides (MP) job matching model with endogenous job separation into the real business cycle (RBC) model. Workers search for jobs in labor markets, which are segmented by experience levels, and produce distinct intermediate goods with labor as the only input. This distinction is motivated by two empirical facts: First, there is an experience premium, which exists among workers of different age levels if we equate age with experience. Second, the spillover effect only exists among young workers[5]. Different intermediate goods are then combined with capital for the production of final goods, thereby giving rise to age-specific difference in labor demand, when age is equated with experience.

A greater share of young workers is associated with a bigger supply and thus a lower relative price of goods produced by them. This price change triggered by demographic change is the key to understanding the spillover effect. First, since the decline in price can lower profits, firms raise their selection criteria of young workers, thus pushing the productivity cutoff of young workers to the mode of its distribution. This implies a greater number of young workers will be affected when aggregate productivity changes. Furthermore, as Hagedorn and Manovskii (2008) have shown, firms' profits and thereby their vacancy posting also become more sensitive to aggregate productivity since profitability shrinks. Therefore, both aspects imply that the unemployment volatility of young workers increases with the youth share.

One contribution of this paper lies in bridging the theoretical gap between demographic changes and the response of age-specific labor market to productivity shocks. Filling this gap is not only important for our understanding of the uniqueness of the labor market dynamics of young workers but also helpful in explaining the falling macroeconomic volatility in the U.S. existing since the mid-1980s, referred to as "The Great Moderation." I demonstrate that when the youth share increases, the unemployment rate of young workers responds more strongly to productivity shocks. This is important as it establishes a new channel for the explanation of the Great Moderation. In contrast to Jaimovich and Siu (2009), who proposed a composition effect as its main demographic explanation, I show theoretically that the changes in the unemployment volatility of the young due to demographic changes can also be an important contributor.

[5] Intuitively, inexperienced workers are more suitable for less sophisticated products and vice versa.

In my simulations, I find that demographic changes can account for at least 23% of this moderation. Of this, the spillover effect accounts for three-quarters of the overall effect[6], while the composition effect accounts for only a quarter.

As already mentioned, Jaimovich and Siu (2009) started the recent literature on the role of demographic changes in labor market dynamics. They provide empirical evidence for the age-specific difference in labor market volatility and claim that a direct composition effect among different age groups can account for a sizable fraction of the Great Moderation. Lugauer and Redmond (2012) and Lugauer (2012b) confirm that changes in the age composition of the labor force have a large and significant effect on cyclical volatility[7]. Jaimovich, Pruitt, and Siu (2013) offer an explanation for this age-specific difference. They detect a capital-experience complementarity in the data. As capital is in inelastic supply in the short-run, the employment of old workers, who are rich in experience, features rigidity in the short-run. But their model fails in generating any spillover effect. Because productivity shock is fully transmitted to labor market, thus age-specific labor market dynamics are independent of demographic composition. This paper solves this by introducing labor market frictions into a RBC model with age-specific difference in labor demand. It shows that both the unemployment volatility of young workers and the dynamics of their labor market transition rates increase as the youth share increases.

Meanwhile, this paper is related to the growing number of studies on the state dependence of labor market fluctuations over the business cycle. Michaillat (2014) and Cacciatore et al. (2016) show that labor market reforms have different effects on labor market fluctuations across phases of business cycles. Pizzinelli, Theodoridis, and Zanetti (2018) develop a search model with endogenous job separation and on-the-job search, replicating the asymmetries in the fluctuations of labor market variables across different states of aggregate productivity. As an

[6]Compared with the empirical value for the contribution from the spillover effect, which is two-thirds as shown in Chapter 2, my simulation result is in general consistent with this empirical finding, only slightly overestimates the relative importance of the spillover effect.

[7]Although the replication of these results by Everaert and Vierke (2016) suggests that the relationship identified in these papers between variation in the age composition of the labor force and cyclical volatility may well be spurious, Han (2018a) reconfirms the validity of this relationship with a broader dataset by measuring unemployment volatility with a stochastic volatility process proposed by Fernández-Villaverde et al. (2011) and accounting for the effects of labor market institutions.

extension, I show that the fluctuations of labor market variables also depend on the state of demographics.

Finally, this paper is also related to the study conducted by Hagedorn and Manovskii (2008), who provide a different calibration method[8] to remedy the failure of basic search and matching model in generating sufficient volatility to match empirical data, a puzzle raised by Shimer (2005). As an alternative solution, Hall (2005) proposes introducing wage stickiness[9]. This paper starts from a different angle and explores whether the Mortensen and Pissarides (1994) job matching model can generate unemployment volatility after accounting for the variation in the youth share.

The paper proceeds as follows. The next section gives further evidence for the existence of the spillover effect. Section 3.3 lays out the model. Section 3.4 shows the calibration strategy. Section 3.5 discusses the model's performance. Section 3.6 shows the robustness of the results when jobs are exogenously separated. Section 3.7 provides supporting evidence for the mechanism. The final section summarizes.

3.2 Empirical evidence

The right panel of Figure 3.1 shows a hump-shaped demographic trend for the youth share. It climbs up to its maximum in the mid-1970s as baby boomers start entering the labor force from the 1960s. The trend falls persistently from 1980 with a decreasing inflow of young workers. Meanwhile, this hump-shaped trend is closely accompanied by the unemployment volatility of the young over the whole period. This comovement serves as a preliminary evidence for the relationship between the youth share and the unemployment volatility of the young.

To identify the causality, further evidence is provided in this part. First, I show that the cyclicality of labor force participation is not of primary concern

[8]Their calibration method relies on the strict specification of unemployment benefit and worker's bargaining power. The result is an unrealistically high opportunity cost of unemployment, suggesting that workers are motivated only by a small share of profit, as pointed out by Mortensen and Nagypal (2007).

[9]However, Haefke, Sonntag, and Van Rens (2013) find little evidence of the existence of wage rigidity in empirical data.

for the cyclical unemployment volatility of the young. With this potential endogeneity concern taken care of, I go on to show the empirical evidence that I find in Chapter 2, that is, there is a statistically significant relationship existing between the unemployment volatility of the young and the youth share, using data from 20 OECD countries. As the timing and extent of demographic changes are different among these countries, together with the exogeneity of the youth share, which is predetermined at least 15 years prior, the validity of the empirical identification is ensured.

3.2.1 The role of labor force participation

It is generally acknowledged that labor market dynamics are affected disproportionately across sex and age groups by the labor force participation decision (see e.g. Clark and Summers, 1982). An increase in the participation rate, mostly during booms, will push up the unemployment rate, while a fall in the participation rate during recession will drag down the unemployment rate. As young workers have more options in case of job separation, their unemployment rate might display greater cyclicality than those of other age groups. For instance, with worsening labor market conditions, unemployed young workers are more likely to reconsider a trade-off between job searching and further education. This cyclicality of participation might be more pronounced for females, given that the participation rate of females is more sensitive to labor market conditions. Therefore, in the following, I examine the role of the cyclicality of participation in the unemployment dynamics of the young.

Note that the unemployment rate of a specific group can be expressed as a ratio between the unemployment-to-population ratio and the labor force participation rate of this group,

$$u_i \equiv \frac{U_i}{L_i} = \frac{U_i}{P_i} \Big/ \frac{L_i}{P_i} = \frac{up_i}{lp_i}, \tag{3.1}$$

where U_i is the size of unemployment of group i, L_i is the size of its labor force, P_i is the size of its population, up_i is the unemployment-to-population ratio and lp_i stands for the labor force participation rate. Thus, unemployment volatility can be decomposed as

$$\mathrm{Var}\,(\ln u_i) = \mathrm{Var}\,(\ln up_i) + \mathrm{Var}\,(\ln lp_i) - 2\mathrm{Cov}\,(\ln up_i, \ln lp_i). \tag{3.2}$$

If the unemployment volatility from the variation of labor force participation rate accounts for only a minor share, the cyclicality of participation rate is less of a concern.

Table 3.1: Decomposition of unemployment volatility: share of participation

	Female		Male		Both	
	15-24	15-64	15-24	15-64	**15-24**	15-64
Cov. excl.	2.67	0.39	1.11	0.05	**1.53**	0.10
Cov. incl.	7.60	1.68	4.01	1.30	**4.93**	1.19

Notes: "Cov. excl." means that the covariance terms, $Cov\,(1-e_i, lfpr_i)$, are excluded for the calculation of the contribution of participation to unemployment volatility. "Cov. incl." means that total variance also includes covariance terms. Annual U.S. data from the OECD Labour Force Statistics database, 1960–2017. Cyclical volatility of each series is the percentage standard deviation of each series filtered by one-sided HP-filter with a smoothing parameter 100.

Table 3.1 reports the share of unemployment volatility, which can be attributed to the participation rate of young workers and the aggregate of both sex groups in the U.S. I use a one-sided HP filter to keep the time ordering of the data undisturbed. The decomposition results in the first row do not take into account the effect of the covariance terms. As expected, the contribution of the participation decision to unemployment volatility is higher for young and female workers. But even for young females, who have the highest value, the contribution is only 2.67%. For young workers of both sex groups, the contribution of participation only accounts for 1.53%, leaving the rest unexplained to the unemployment-to-population ratio. When the effect of the covariance between the unemployment-to-population ratio and the labor force participation rate is also considered, still about 95% of the unemployment volatility of the young can be attributed to the unemployment-to-population ratio of young workers.

Thus, even for young workers, the cyclicality of the participation rate is not of primary importance for cyclical unemployment volatility[10].

3.2.2 Identifying the role of demographic changes

To identify the relationship between the unemployment volatility of the young and the youth share, I consider the following regression:

$$\sigma_{it} = \alpha_i + \beta_t + \gamma s_{it} + \lambda X_{it} + \varepsilon_{it}, \tag{3.3}$$

where σ_{it} stands for the unemployment volatility of the young for country i at time t measured by a stochastic volatility process[11]; s_{it} is the youth share; and X_{it} includes indicators for labor market institutions and an external shock. Of which, the indicators of labor market institutions include union density, union centralization of wage bargaining, the strictness of employment protection legislation, tax wedge, and gross replacement rate; and the external shock is a world demand shock, which is proxied by the log-difference of the sum of real GDP of other 19 countries. α_i is a country fixed effect and β_t is a full set of time dummies to control for time effects.

As mentioned in Chapter 2, the data covers up to 2007 because unemployment dynamics afterward contain too much noisy signal from the very recent financial crisis. It is technically difficult to disentangle its effect from that of demographic changes.

[10]This result is robust to the redefinition of young workers as those aged 15-29 years. See Appendix B.1 for details.

[11]The unemployment rate is assumed to follow an AR(1) process and the level equation for the unemployment rate is

$$u_t = \rho u_{t-1} + e^{\sigma_t} v_t,$$

and the volatility equation follows an AR(1) stochastic volatility process (see Fernández-Villaverde et al., 2011; Born and Pfeifer, 2014; and Han, 2018):

$$\sigma_t = (1-\rho^\sigma)\bar{\sigma} + \rho^\sigma \sigma_{t-1} + \eta \varepsilon_t,$$

where v_t and $\varepsilon_t \overset{iid}{\sim} N(0,1)$, ρ and ρ^σ are respectively the coefficients of the level and volatility equations, $\bar{\sigma}$ is an unconditional mean and η is a scale factor. The time-varying volatility of unemployment rate is e^{σ_t} and what one needs is the estimation of the time-varying parameter σ_t. I omit the country specific subscript in the level and volatility equations for simplicity. See Chapter 2 for details.

The first column of Table 3.2 reports the pooled common correlated effects (CCEP) estimator[12], which projects out fixed effects and eliminates the effects of the unobserved common factors. The coefficient of the youth share is positive and significant at the 1% level, suggesting that changes in the youth share have a statistically significantly positive impact on the unemployment volatility of the young. In the second column, I take care of the potential participation endogeneity by replacing the youth share by the share of young population. The coefficient is still significant at the 10% level and at a similar level. Besides, in the last column, I use the share of the native young population[13] as a proxy to deal with the potential endogeneity from migration and get similar result.

Table 3.2: Unemployment volatility of the young and demographic changes

	Youth share	Young population share	Native young population share
Coefficient	8.97***	6.53*	9.41**
	(3.43)	(3.47)	(4.45)
Nobs	644	659	627
R^2	0.77	0.77	0.78

*Notes: Unbalanced panel of 20 OECD countries, sample period: 1970-2007. Dependent variable is the unemployment volatility of the young. Newey-West standard errors are in the parentheses. ***, **, and * stand for significance levels of 1%, 5%, and 10%, respectively. Model used in this table is the CCEP estimator, which projects out fixed effects and eliminates the effects of the unobserved common factors.*

However, there is not any statistically significant relationship detected in other age groups[14], in support of the age-specific difference in labor demand structure.

[12]For both the fixed effects and time effects estimator and the mean group estimator, I detect cross-sectional dependence in the data. Because its existence can induce biased estimation, I use the CCEP estimator to take care of this.

[13]The share of young population and the share of native young population are calculated using data from Maddison (2003) and Mitchell (2008).

[14]See Appendix B.2 for the regression results of other age groups.

3.3 The model

In this section, I lay out the details of the model. There are three sectors in the economy: households, firms producing intermediate goods, and firms producing final goods. Households provide labor as the only input for the production of intermediate goods and lease capital to firms producing final goods. Household members search for jobs in segmented labor markets. The setup of job searching behavior shares similarities with Krause and Lubik (2007) and Trigari (2009). Distinct intermediate goods and capital are then used as inputs for the production of final goods.

3.3.1 Households

Time is discrete and households have an infinite time horizon. Household members are characterized by age and experience levels. There are two types of age levels, young and old, and two types of experience levels, inexperienced and experienced. As the accumulation of experience takes time, for the sake of simplicity, I assume that young members are all inexperienced and old members are all experienced, deriving only two types of workers. Following Merz (1995), I use the pooling assumption for the consumption of each type of members. Suppose the utility function is $u^i(c_t^i) = \frac{(c_t^i)^{1-\tau}-1}{1-\tau}$, where c_t^i stands for the consumption of i at time t, $i \in \{Y, O\}$ denotes either young or old, and τ is the inverse of the intertemporal elasticity of substitution. Let us denote S_t^Y as the youth share[15] and normalize the labor force to one.

Taking labor income and capital holdings at time t as given, a representative household decides the amount of consumption for each member type and the amount of capital to hold at time $t+1$. Thus, the problem of a representative

[15] A special case is that the youth share remains constant, which can be realized if the birth rate of the young, ρ^B, the exit rate of the old, ρ^D, and the transition rate from young to old, ρ^{YO}, satisfy

$$\rho^{YO} = \frac{\rho^B}{S_t^Y} - S_t^Y \rho^D.$$

It is just the solution of

$$\frac{L_{t+1}^Y}{L_{t+1}} = \frac{\rho^B L_t + (1-\rho^{YO})L_t^Y}{(1+\rho^B)L_t - \rho^D L_t^O} = \frac{L_t^Y}{L_t}.$$

In my simulations, I abstract from these transition probabilities by focusing on this special case and compare the results under different demographic structures.

household can be written as

$$\max_{\{c_t^Y, c_t^O, K_{t+1}\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t [S_t^Y u^Y(c_t^Y) + (1 - S_t^Y) u^O(c_t^O)],$$

subject to[16]

$$S_t^Y c_t^Y + (1 - S_t^Y) c_t^O + K_{t+1} = r_t K_t + (1 - \delta) K_t + S_t^Y n_t^Y \omega_t^Y + (1 - S_t^Y) n_t^O \omega_t^O,$$

where $\beta \in (0,1)$ is the household's discount factor; K_t denotes capital holdings at time t; r_t is the rental rate; $\delta \in (0,1)$ is the depreciation rate of capital holdings; n_t^Y and n_t^O are the employment rates of young and old workers, respectively; and ω_t^Y and ω_t^O are wage rates. With these setups, optimality conditions imply that consumption is the same for both types of family members,

$$c_t^Y = c_t^O = c_t, \tag{3.4}$$

and the intertemporal first-order condition is

$$c_t^{-\tau} = \beta E_t [c_{t+1}^{-\tau} (r_{t+1} + 1 - \delta)], \tag{3.5}$$

from which we can construct the implied stochastic discount factor, $E_t \beta_{t,t+1} = \beta E_t c_{t+1}^{-\tau} / c_t^{-\tau}$, to evaluate the activities of firms and workers.

3.3.2 Labor markets

Labor markets are segmented by experience levels. A worker–firm match can produce an output level of $A_t a_t$, where A_t and a_t are aggregate and match-specific productivities, respectively. The aggregate productivity follows an AR(1) process: $\ln A_t = \psi \ln A_{t-1} + e_t$. Match-specific productivities are drawn independently from a time-invariant lognormal distribution with cumulative distribution function $F(a_t)$ and positive support. At the beginning of each period, both productivity shocks are realized, then workers face job separation shocks, an exogenous separation shock at rate ρ^x in both markets and an endogenous separation shock in each labor market. Endogenous separation happens if the

[16] Here I assume that unemployment benefit is financed by the tax income on wage. These two cancel out each other and thus do not appear in the budget constraint.

realization of the match-specific productivity is lower than a certain threshold for workers at age i, $\widetilde{a}_t^i$, leading to an overall separation rate $\rho_t^i = \rho^x + (1-\rho^x)F(\widetilde{a}_t^i)$. If separation occurs, production does not take place; if not, production starts. Labor markets open afterward. Figure 3.2 describes the timing of events.

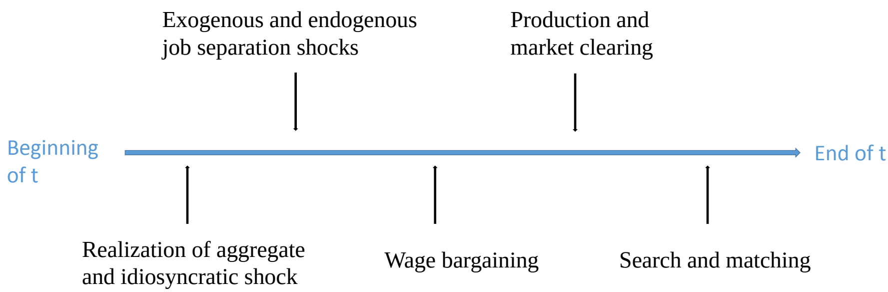

Figure 3.2: Timing of events

The unmatched firm posts exactly one vacancy at the cost of either ς^Y or ς^O, depending on where the vacancy is posted. New firms can enter both labor markets without restrictions until the value of posting an additional vacancy equals to zero. The matching process is depicted by a constant-returns-to-scale matching function:

$$m_t^i = \phi^i (v_t^i)^{1-\mu} (u_t^i)^{\mu}, \tag{3.6}$$

where ϕ^i stands for matching efficiency of labor market i and μ is the elasticity of the matching function with respect to unemployment. The probability of a vacancy being filled equals to $q_t^i = m_t^i / v_t^i = \phi^i {\theta_t^i}^{-\mu}$ and the probability for an unemployed worker to be matched equals $f_t^i = m_t^i / u_t^i = \phi^i {\theta_t^i}^{1-\mu}$, where $\theta_t^i \equiv v_t^i / u_t^i$ is the market tightness of labor market i at time t. As the unemployment benefit is usually proportional to the experience level, I assume that the value for old workers, z^O, is higher than that of young workers, z^Y.

With the sequence of events, employment rate at each segmented labor market evolves according to the following

$$n_t^i = (1-\rho_t^i)(n_{t-1}^i + f_{t-1}^i u_{t-1}^i). \tag{3.7}$$

It shows that employment at t is given by the sum of the existing and re-employed workers at $t-1$ contingent on unseparation at t. As the number of unemployed workers when production starts is the same as that when job searching starts, this setup ensures that the ratio of workers searching for jobs at t, u_t^i, is equal to the unemployment rate at t, $1-n_t^i$.

3.3.3 Firms

I assume that the goods produced by young and old workers are distinct intermediate goods. Both goods use labor as the only input and are subject to common aggregate productivity shock. As the labor force is normalized to one, the size of young labor force equals to S_t^Y and the size of young matched workers is $n_t^Y S_t^Y$. The total output of intermediate goods produced by young workers is

$$Y_t^Y = A_t n_t^Y S_t^Y \int_{\widetilde{a}_t^Y}^{\infty} a \frac{dF(a)}{1-F(\widetilde{a}_t^Y)}. \tag{3.8}$$

Here, the term with integral is simply the conditional expectation of match-specific productivity, $E(a|a \geqslant \widetilde{a}_t^Y)$, as only matches above the threshold, $\widetilde{a}_t^Y$, can survive. Similarly, the output of intermediate goods produced by old workers is

$$Y_t^O = A_t n_t^O (1-S_t^Y) \int_{\widetilde{a}_t^O}^{\infty} a \frac{dF(a)}{1-F(\widetilde{a}_t^O)}. \tag{3.9}$$

Both intermediate goods, together with capital, are used as inputs for the production of final goods in perfectly competitive firms. To comply with the empirical observation that the unemployment volatility of young workers is higher than that of old workers, I follow Jaimovich, Pruitt, and Siu (2013)'s strategy by introducing a nested constant-elasticity-of-substitution (CES) functional form for final output:

$$Y_t = \left[\lambda_1 (Y_t^Y)^\sigma + (1-\lambda_1)\Big(\lambda_2 (K_t)^\rho + (1-\lambda_2)(Y_t^O)^\rho\Big)^{\sigma/\rho}\right]^{1/\sigma}, \tag{3.10}$$

where $\rho, \sigma \in (0,1)$; $(1-\rho)^{-1}$ is the elasticity of substitution between intermediate goods produced by old workers and capital; $(1-\sigma)^{-1}$ is the elasticity of substitution between intermediate goods produced by young workers and the composite of other two inputs; and $\lambda_1, \lambda_2 \in (0,1)$ are the factor share parameters.[17] If ρ is far smaller than σ, the final production features capital–experience complementarity. As capital is in inelastic supply in the short-run, complementarity suggests that the intermediate goods produced by the old do not respond simultaneously to productivity shock, and so does the unemployment rate of the old.

Firms in the final goods sector maximize profit by choosing the optimal levels of intermediate goods and capital:

$$\max_{Y_t^Y, Y_t^O, K_t} Y_t - P_t^Y Y_t^Y - P_t^O Y_t^O - r_t K_t,$$

and the FOCs are as follows:

$$P_t^Y = (Y_t)^{1-\sigma} \lambda_1 (Y_t^Y)^{\sigma-1} \tag{3.11}$$

$$P_t^O = (Y_t)^{1-\sigma} (1-\lambda_1) \Omega_t (1-\lambda_2) (Y_t^O)^{\rho-1} \tag{3.12}$$

$$r_t = (Y_t)^{1-\sigma} (1-\lambda_1) \Omega_t \lambda_2 (K_t)^{\rho-1}, \tag{3.13}$$

where $\Omega_t \equiv [\lambda_2 (K_t)^\rho + (1-\lambda_2)(Y_t^O)^\rho]^{(\sigma-\rho)/\rho}$.

To close the model, it is easy to see that the aggregate resource constraint is

$$C_t + K_{t+1} = Y_t + (1-\delta) K_t - C_t^h \tag{3.14}$$

where $C_t = S_t^Y c_t^Y + (1 - S_t^Y) c_t^O$ is the aggregate consumption; and $C_t^h \equiv v_t^Y S_t^Y \varsigma^Y + v_t^O (1 - S_t^Y) \varsigma^O$ is the aggregate hiring costs incurred by firms.

3.3.4 Characterization of agents' asset values

Bellman equations, which characterize the problems of firms and workers, are similar to those in the standard search and matching model with endogenous

[17] Notice that final output does not feature a constant marginal product of labor. If labor enters directly into the final production, the marginal product of labor exhibits diminishing returns, implying the match surplus depends on the size of employees within a firm. As a result, firms cannot bargain with each worker independently and the bargaining solution is more complicated than the Nash bargaining solution. See Elsby and Michaels (2013) for details. Here, I make a distinction between intermediate and final goods, and assume that labor is the only input for intermediate goods. In this way, the simplicity of the bargaining solution is kept.

separation. The value of a vacancy in terms of final goods is

$$V_t^i = -\varsigma^i + E_t\beta_{t,t+1}\Big[q_t^i(1-\rho_{t+1}^i)\int_{\widetilde{a}_{t+1}^i}^{\infty} J_{t+1}^i(A_{t+1},a)\frac{dF(a)}{1-F(\widetilde{a}_{t+1}^i)} + (1-q_t^i)V_{t+1}^i\Big], \tag{3.15}$$

where ς^i is the hiring cost in the current period. In the next period, a vacancy is filled and production happens with probability $q_t^i(1-\rho_{t+1}^i)$. Free entry ensures that $V_t^i = 0$ for both types of vacancies at any time t, which yields

$$\frac{\varsigma^i}{q_t^i} = E_t\beta_{t,t+1}\Big[(1-\rho_{t+1}^i)\int_{\widetilde{a}_{t+1}^i}^{\infty} J_{t+1}^i(A_{t+1},a)\frac{dF(a)}{1-F(\widetilde{a}_{t+1}^i)}\Big], \tag{3.16}$$

suggesting that the expected cost of a vacancy equals to the expected benefit.

The asset value of a job with productivity (A_t, a_t) is

$$\begin{aligned} J_t^i(A_t,a_t) \quad = \quad & P_t^i A_t a_t - \omega_t^i(A_t,a_t) + E_t\beta_{t,t+1} \\ & \Big[(1-\rho_{t+1}^i)\int_{\widetilde{a}_{t+1}^i}^{\infty} J_{t+1}^i(A_{t+1},a)\frac{dF(a)}{1-F(\widetilde{a}_{t+1}^i)}\Big], \end{aligned} \tag{3.17}$$

where P_t^i and ω_t^i are the relative price of the intermediate good and the wage rate at time t, respectively. The value of a job is composed of the current profits $P_t^i A_t a_t - \omega_t^i$ and the continuation value. In the next period, jobs survive with probability $1-\rho_{t+1}^i$, and with probability ρ_{t+1}^i they are destroyed, whereupon firms get a future value of zero.

For workers, the asset value of being matched is

$$\begin{aligned} W_t^i(A_t,a_t) \quad = \quad & \omega_t^i(A_t,a_t) + E_t\beta_{t,t+1}\Big[(1-\rho_{t+1}^i)\int_{\widetilde{a}_{t+1}^i}^{\infty} W_{t+1}^i(A_{t+1},a) \\ & \frac{dF(a)}{1-F(\widetilde{a}_{t+1}^i)} + \rho_{t+1}^i U_{t+1}^i\Big]. \end{aligned} \tag{3.18}$$

It includes the current inflow ω_t^i and the continuation value. When not separated from the labor market, employed workers get an expected value W_{t+1}^i in the next period, otherwise they get U_{t+1}^i.

The unemployed worker gets a utility of z^i in the current period, forms a match with probability $f_t^i(1-\rho_{t+1}^i)$, and stays unemployed with probability

$1-f_t^i(1-\rho_{t+1}^i)$:

$$\begin{aligned} U_t^i &= z^i + E_t\beta_{t,t+1}\Big[f_t^i(1-\rho_{t+1}^i)\int_{\widetilde{a}_{t+1}^i}^{\infty} W_{t+1}^i(A_{t+1},a)\frac{dF(a)}{1-F(\widetilde{a}_{t+1}^i)} \\ &\quad +\big(1-f_t^i(1-\rho_{t+1}^i)\big)U_{t+1}^i\Big]. \end{aligned} \tag{3.19}$$

With Nash bargaining, wage is chosen to satisfy the optimality condition:

$$\eta J_t^i(A_t,a_t) = (1-\eta)\big(W_t^i(A_t,a_t) - U_t^i\big), \tag{3.20}$$

where $\eta \in (0,1)$ reflects the relative bargaining power of workers. Together with the free-entry conditions, it is straightforward to get the wage equation, after substituting the equations for the asset values of workers and firms into the above optimality condition:

$$\omega_t^i(A_t,a_t) = \eta(P_t^i A_t a_t + \varsigma^i\theta_t^i) + (1-\eta)z^i. \tag{3.21}$$

Summing up Bellman equations, match surplus, $S_t^{Mi}(A_t,a_t) \equiv J_t^i(A_t,a_t) + W_t^i(A_t,a_t) - V_t^i - U_t^i$, can be written as

$$\begin{aligned} S_t^{Mi}(A_t,a_t) &= P_t^i A_t a_t - z^i + E_t\beta_{t,t+1} \\ &\quad (1-\rho^x)(1-f^i\eta)\int_{\widetilde{a}^i}^{\infty} S_{t+1}^{Mi}(A_{t+1},a)dF(a). \end{aligned} \tag{3.22}$$

It is easy to see that $\frac{\partial S_t^{Mi}(A_t,a_t)}{\partial a_t} = P_t^i A_t > 0$, which ensures the existence and uniqueness of reservation productivity $\widetilde{a}^i$.

3.3.5 Comparative statics and the youth share

In this section, I first show how price and reservation productivity interact with the youth share. Based on these results, I present how the responses of market tightness and youth unemployment with respect to aggregate productivity change with the youth share.

Proposition 1. ***If the employment rate of the old is irresponsive to the changes in the youth share, then the price of intermediate goods***

produced by young workers decreases with the youth share, that is, $\frac{\partial P_t^Y}{\partial S_t^Y} < 0$**.**

Proof: See the Appendix B.3. ■

The intuition of Proposition 1 is straightforward: When the youth share increases, the output by young workers increases accordingly; with the increase of its supply, the price drops. The assumption is satisfied if the final production features capital–experience complementarity. As capital is in inelastic supply in the short-run, the employment rate of the old does not respond to the youth share.

Notice that the youth share is associated not only with the availability of young workers but also the reservation productivity of the young, which could then induce changes in the average productivity and the employment rate of the young. Suppose the reservation productivity of the young increases with the youth share, then the average productivity of remaining young workers increases while the employment rate decreases. Although the decline in employment rate can partially cancel the effect of the youth share on the output by young workers, the direct increase in the availability of young workers plays a dominant role. This results in a higher output by the young and a lower output by the old if the youth share increases. Since final output has first-degree homogeneity and capital is in inelastic supply in the short-run, there is an oversupply of goods produced by the young and its price decreases accordingly. It is important because the response of price to demographic changes also affects firms' profits, which then influences their vacancy-posting behavior.

Corollary 1. ***If we further assume that the distribution of the match-specific productivity of young workers is concentrated such that the expected value,*** $E(a|a \geqslant \widetilde{a}_t^Y)$***, barely changes with the youth share, then the per-match output of young workers in terms of final goods decreases with the youth share, that is,*** $\frac{\partial P_t^Y E(a|a \geqslant \widetilde{a}_t^Y)}{\partial S_t^Y} < 0$.

In fact, this assumption can be satisfied by most distributions for match-specific productivity used in the literature. As endogenous separation rate is about 0.03 per quarter (Krause and Lubik, 2007), the expected value is barely affected by the young share.

Proposition 2. ***If the employment rate of the old is irresponsive to the changes in the youth share, then the reservation productivity of the young increases with the youth share, that is,*** $\frac{\partial \widetilde{a}_t^Y}{\partial S_t^Y} > 0$**.**

Proof: See the Appendix B.4. ■

Intuitively, as a greater number of young workers means a decrease in the price of goods by the young, marginal young workers need to be more productive, which automatically pushes up the reservation productivity of the young.

If there is no aggregate uncertainty, it can be shown that the elasticity of market tightness of the young with respect to aggregate productivity is[18]

$$\frac{\partial \ln \theta^Y}{\partial \ln A} = \frac{1-\beta(1-\widetilde{\rho}^Y)+\eta\beta(1-\rho^x)\widetilde{f}^Y}{\mu[1-\beta(1-\widetilde{\rho}^Y)]+\eta\beta(1-\rho^x)\widetilde{f}^Y} \frac{\frac{\partial \ln P^Y}{\partial \ln A}+1}{1-z^Y/[P^Y AE(a|a \geqslant \widetilde{a}^Y)]}, \quad (3.23)$$

where $\widetilde{f}^Y \equiv f^Y[1-F(\widetilde{a}^Y)]$, $\widetilde{\rho}^Y \equiv \rho^Y[1-F(\widetilde{a}^Y)]$. The first term of this elasticity is a function of the overall separation rate, the elasticity of the matching function with respect to unemployment, workers' bargaining power, and the overall job-finding rate. Among which, only the job-finding rate responds to the youth share. The increase in the number of young workers could lead to overcompetition for the young and a decrease in the job-finding rate. This implies a higher value of the first term when the youth share increases. But the limited variation of this term is well discussed in Shimer (2005).

The second term bears some resemblance to Hagedorn and Manovskii (2008) in the sense that its value depends on the gap between unity and a function containing the unemployment benefit of the young[19]. The difference lies in that this function is negatively related to the price of goods produced by the young, as shown in Corollary 1. The increase in the youth share can lower the profits generated by the match between firm and young worker, since the price of goods produced by the young decreases. In a boom, there is a large percentage increase in firms' profits due to the hiring of young workers, and firms react by posting more vacancies. In a recession, firms' profits decline further and they rarely post any vacancy. Therefore, a greater share of young workers generates a congestion in their job-finding, and firms' vacancy posting to young workers becomes more

[18] Details of the derivations of $\frac{\partial \ln \theta^Y}{\partial \ln A}$ and $\frac{\partial \ln u^Y}{\partial \ln A}$ can be found in the Appendix B.5.

[19] Notice that one necessary condition for the existence of match between firm and young worker is $P^Y AE(a|a \geqslant \widetilde{a}_t^Y) > z^Y$, which ensures the non-negativeness of the denominator of the second term.

sensitive to aggregate productivity, so does the labor market tightness of the young[20].

Besides, it can be shown that the elasticity of the youth unemployment rate with respect to aggregate productivity is

$$\begin{aligned}\frac{\partial \ln u^Y}{\partial \ln A} &= \frac{\tilde{f}^Y}{\tilde{\rho}^Y + (1-\tilde{\rho}^Y)\tilde{f}^Y}\Big[-(1-\tilde{\rho}^Y)(1-\mu)\frac{\partial \ln \theta^Y}{\partial \ln A} \\ &\quad + F'(\widetilde{a}^Y)\widetilde{a}^Y(1-\rho^x)(1+\frac{1}{\tilde{\rho}^Y})\frac{\partial \ln \widetilde{a}^Y}{\partial \ln A}\Big].\end{aligned} \tag{3.24}$$

It can be considered as a sum of two terms. The first term is a function of the elasticity of market tightness with respect to aggregate productivity. It represents the response of the job-finding rate to aggregate productivity. As mentioned before, the elasticity of market tightness increases as the youth share increases.

The second term contains the probability density function at reservation productivity and other functions of reservation productivity. It represents the response of the job separation rate to aggregate productivity. The density function at reservation productivity is highly responsive to reservation productivity, as long as the distribution of idiosyncratic productivity is concentrated and does not have any fat tail. According to Proposition 2, a higher youth share means a higher reservation productivity of the young. The level increase in this threshold could be mild, but it can induce a large increase in the density function, suggesting a dramatic increase in the mass of marginal workers. As more marginal young workers are prone to aggregate productivity shock, the unemployment rate of the young becomes more volatile. Therefore, the volatilities of both the job-finding and separation rates of the young increase with the youth share, and jointly contribute to the spillover effect.

[20] Although the second term in this elasticity also depends on the elasticity of the price of goods produced by young workers to aggregate productivity, this second elasticity is close to one and has limited variation when the youth share changes if the goods produced by young workers are substitutable for the composite of other inputs.

3.4 Calibration

This section describes the parameter specification of the model for the U.S. I begin by estimating the two parameters governing the elasticities of substitution in the production of final goods. Thereafter, other parameter values are set to be consistent with the literature or empirical observations.

3.4.1 The two parameters governing the elasticities of substitution

As there is no first moment in the data for parameters $\rho$and σ, which describe the elasticity of substitution among inputs, we need to estimate them. For this, I take advantage of the FOCs in final goods production in the same way as Jaimovich, Pruitt, and Siu (2013).

Notice that the FOC with respect to Y_t^Y can be written in the following form

$$\triangle \ln (P_t^Y Y_t^Y / Y_t) = \sigma \triangle \ln (N_t^Y / Y_t) + \sigma \triangle \ln (A_t), \tag{3.25}$$

where N_t^Y is the employment size of the young at time t. $P_t^Y Y_t^Y$ stands for the total value created by young workers in term of final goods, which can be measured by the labor income of the young. Note that the ratio of $P_t^Y Y_t^Y$ to Y_t is just the share of final goods created by the young. Therefore, σ can be obtained by estimating the response of the share of final goods produced by the young with respect to the number of young workers needed per final good, conditional on that the latter is uncorrelated to $\triangle \ln (A_t)$, a function of current and past aggregate productivity shocks[21].

Similarly, from the FOCs with respect to Y_t^O and K_t, we get

$$\triangle \ln \left(P_t^O Y_t^O / (r_t K_t)\right) = \rho \triangle \ln (N_t^O / K_t) + \rho \triangle \ln (A_t). \tag{3.26}$$

It is easy to see that $P_t^O Y_t^O$ is the total value produced by old workers in terms of final goods and $r_t K_t$ is the total amount paid for capital. The ratio of these

[21]From the AR(1) process for aggregate productivity: $\ln A_t = \psi \ln A_{t-1} + e_t$, we have

$$\triangle \ln (A_t) = e_t - (1-\psi)(e_{t-1} + \psi e_{t-2} + ...).$$

two is just the ratio of the share of final goods produced by old workers to the share by capital. Thereafter, ρ can be estimated similarly as σ.

Table 3.3: Estimation results of the elasticities of substitution in final goods production

	σ	ρ
Point Estimate	0.701***	0.195***
	(0.056)	(0.025)
Hansen's J-test	1.474 [0.479]	
$\sigma = \rho$	47.19 [0.000]	

*Notes: Data from March CPS, 1964–2007. Model used for estimation is GMM with IVs. HAC standard errors are in parentheses. p-values are in square brackets. *** stands for a significance level of 1%.*

As employment size, final output, and aggregate capital are all correlated with aggregate productivity shocks, I use lagged birth rates as instruments for the regressors in both equations. Considering that aggregate productivity shocks enter the error terms of both, I use GMM for simultaneous estimation using data from CPS up to 2007.

Table 3.3 reports the estimation results. The point estimate of ρ is 0.195, indicating that the elasticity of substitution between intermediate goods by old workers and capital $(1-\rho)^{-1}$ is slightly more than one. The point estimate of σ is 0.701, suggesting that the elasticity of substitution between intermediate goods produced by young and the composite of other two inputs $(1-\sigma)^{-1}$ is much larger than one. The p-value of Hansen's J-test is 0.479, suggesting no over-identification. Besides, the null hypothesis $\sigma = \rho$ is statistically rejected. Notice that $\sigma > \rho$ is not imposed as an assumption in the former analysis; it is a fact supported by data, which supports capital–experience complementarity.

3.4.2 Other parameters

The model is simulated at a quarterly frequency. Table 3.4 summarizes parameter values.

Table 3.4: Parameter values in simulation (quarterly)

Parameter	Interpretation	Value	Rationale
σ	$(1-\sigma)^{-1}$	0.701	Estimated
ρ	$(1-\rho)^{-1}$	0.195	Estimated
β	Discount factor	0.99	
δ	Depreciation rate	0.025	Jaimovich, Pruitt, and Siu (2013)
τ	Inverse of EIS	0.5	
ρ^x	Exog. separation rate	0.055	Unemployment rate 7%
ϕ	Matching efficiency	0.86	Job-finding rate 79%
μ	Matching elasticity	0.5	Petrongolo and Pissarides (2001)
η	Worker's barg. power	0.5	Hosios condition
ς	Vacancy posting cost	0.11	Cajner and Cairo (2013)
z^O	Unem. benefit- old	0.76	Literature midpoint
z^Y	Unem. benefit- young	0.55	Ratio of the young to the old (0.7)
λ_1	Factor share by young	0.335	National income share by young
λ_2	Factor share of capital	0.275	National income share of capital
ψ	Coef for productivity	0.81	Labour productivity (BLS)
σ^A	SD for productivity	0.011	Labour productivity (BLS)
μ_{LN}	Mean of log normal	0	Standardization
σ_{LN}	SD of log normal	0.12	Krause and Lubik (2007)

In the benchmark calibration, the youth share S^Y is set at 0.20, which is the average from 1960 to 2007 for the U.S. In a standard manner, discount rate β is set at 0.99. The depreciation rate δ takes the value of 0.025 and is the same as that in Jaimovich, Pruitt, and Siu (2013). The intertemporal elasticity of substitution is chosen to be 2, which is commonly used in the literature. The exogenous job separation rate ρ^x is set at 0.055 to target an average unemployment rate of 7%. The parameter of match efficiency ϕ is set to be the same for both labor markets and equals to 0.86 to match the empirical average job-finding rate of 0.79 in Elsby, Hobijn, and Sahin (2010). Following Petrongolo and Pissarides (2001), the elasticity of market tightness with respect to vacancies μ is set at 0.5. To satisfy the Hosios (1990) condition, I assume that workers' bargaining power η is equal to the elasticity of matching function.[22] The vacancy posting

[22]The Hosios condition makes sure the externalities inflicted by searchers and firms offset each other optimally. Although there is *ex ante* heterogeneity in the model, workers of different age and experience search in segmented labor market, thus no further externality is inflicted and the Hosios condition is guaranteed by equalizing the worker's bargaining power to the elasticity of the matching function with respect to unemployment.

cost ς is set to be the same for both labor markets and equals to 0.11 based on the evidence in the Employment Opportunity Pilot Project survey[23] of 1982.

It is demonstrated by Hagedorn and Manovskii (2008) that a high value for non-market activity can generate higher unemployment dynamics, with $z = 0.995$ for their case. But a high value for non-market activity also implies an unrealistically high opportunity cost of employment and an unrealistic sensitivity of unemployment to changes in unemployment benefits. Here, I set the unemployment benefit of old workers z^O at 0.76, which is close to the value used by Hall and Milgrom (2008), 0.71. As unemployment benefit is proportional to experience level, I assume that the unemployment benefit of the young is 70% of that of the old, which gives the value for the young z^Y at 0.55. For the specification of the factor share parameters λ_1 and λ_2, I follow Krusell et al. (2000) and calibrate these parameters to match the national income share from the CPS, which shows that the share created by old workers amounts to 50%, and the share by capital amounts to 36%. Parameters for the aggregate productivity shock are calibrated to match the seasonally adjusted real output in the non-farm business sector. I take log transformation and then use a one-sided HP filter to detrend the series with smoothing parameter[24] of 1600. The standard deviation of productivity is 0.013 and the quarterly autocorrelation is 0.81, implying that the standard deviation of error term σ^A equals to 0.11[25]. Concerning the parameters of the lognormal distribution of idiosyncratic productivity a_t, I follow Krause and Lubik (2007) by normalizing its mean μ_{LN} at 1 and setting the standard deviation σ_{LN} at 0.12.

Table 3.5 shows that the simulated first moments are well targeted, close to their empirical counterparts.

3.5 Simulation results

This section presents the quantitative results of the study. First, I report the simulation results for the aggregate economy. Second, I check the existence of

[23] See Cajner and Cairo (2013) for details.

[24] I follow the standard smoothing parameter used for quarterly data, see e.g. Fujita and Ramey (2012).

[25] As aggregate productivity shock follows an AR(1) process in the model: $ln\,A_t = \psi ln\,A_{t-1} + e_t$, it is easy to see that $\mathrm{var}\,(ln\,A_t) = \frac{(\sigma^A)^2}{1-\psi^2}$.

Table 3.5: Summary of the first moments in benchmark calibration (quarterly)

	Data	Model
Unemployment rate	0.06	0.07
Unemployment rate of the young	0.12	0.09
Unemployment rate of the old	0.05	0.06
Job-finding rate	0.81	0.79
Separation rate	0.06	0.06

Notes: Aggregate and age-specific unemployment rates are from St. Louis Fed, while transition rates are from CPS. All series are from 1960Q1 to 2007Q4.

the spillover effect in the simulation of each age group, by changing the value for the youth share and comparing the age-specific unemployment volatility across demographic states. Finally, I present impulse response functions with respect to aggregate productivity shock to disentangle the contribution of job-finding and separation rates in the spillover effect. To solve the model numerically, I log-linearize the model in Section 3.3 around its steady state. Details can be found in the Appendix B.6.

3.5.1 Aggregate volatility across demographic states

Table 3.6 reports the standard deviations (SDs) of aggregate labor market variables in the data[26] from 1960 to 2007 in Column 1, as well as the simulation results when the youth share varies around its empirical average, 20%, in Columns 2-4. As productivity shock is the main but not the only reason for labor market fluctuations, to make sure the comparability of the empirical and simulated moments, I identify the empirical business cycle component as the projection on the current and lagged detrended output.

The results show that all the simulated moments increase with the youth share, except that the SD of productivity is targeted at its empirical level. When the youth share increases from 18 to 20%, the simulated SD of the unemployment rate increases by 9%, from 0.15 to 0.17; when this share further increases from 20 to 22%, the simulated value for unemployment increases by 24%, from 0.17 to 0.21. Similarly, the simulated SDs of the job-finding rate, the separation rate,

[26] I use the seasonally adjusted unemployment rate, the quarterly average of monthly data. Productivity is the real output per person of nonfarm business sector. Both data series are from St. Louis Fed. Vacancy rate is seasonally adjusted help-wanted advertising index from the Conference Board.

and market tightness also increase with the youth share. The model predicts a positive relationship between aggregate labor market volatilities and the youth share.

Table 3.6: Aggregate second moments in the data and the model

	Data	Model				
Youth share	20%	18%	20%		22%	
	(1)	(2)	(3)	↗	(4)	↗
Unemployment rate	0.64	0.15	0.17	9%	0.21	24%
Job-finding rate	5.16	2.05	2.12	3%	2.28	7%
Separation rate	0.10	0.02	0.02	16%	0.04	83%
Market tightness	21.67	4.91	5.09	4%	5.50	8%
Productivity	1.30	1.30	1.30	-	1.30	-

Notes: Sample period: 1960Q1–2007Q4. Rates are in percentage points, market tightness and productivity in percent. All quarterly series are filtered by a one-sided HP filter with a smoothing parameter of 1600. ↗ stands for the percentage change after a 2% increase in the youth share.

If we calculate the empirical SD of the unemployment rate across 1970-84 and the one across 1985-2007, we see that it drops significantly from 0.89 to 0.43, with a decrease of 0.46. The simulation results suggest a decrease of 0.11 if the youth share decreases from 24 to 17% as in the data over these two periods[27]. Therefore, the model predicts that demographic changes account for 23% of the Great Moderation. Considering that the model underpredicts[28] labor market volatilities by comparing Columns 1 and 3, which have the same youth share, the contribution of demographic changes should be even larger.

3.5.2 Age-specific volatility across demographic states

This section assesses if the model is capable of generating the spillover effect as observed in the data.

[27] Which is calculated from (0.21-0.15)*(0.24-0.17)/(0.22-0.18). I also recalculate this difference by setting the youth share as 0.17 and 0.24, respectively, in the simulation and get similar result.

[28] This underprediction is well known in the search and matching literature. For model with exogenous separation, see Shimer (2005); for model with endogenous separation, see Mortensen and Nagypal (2007) and Fujita and Ramey (2012).

Table 3.7: Age-specific second moments in the data and the model

	Data	Model				
Youth share	20%	18%	20%		22%	
	(1)	(2)	(3)	↗	(4)	↗
Unemployment rate						
Young	1.03	0.22	0.31	41%	0.49	61%
Old	0.54	0.14	0.14	-4%	0.13	-3%
Job-finding rate						
Young	5.26	2.16	2.24	4%	2.26	1%
Old	4.06	2.02	2.02	0%	2.03	0%
Separation rate						
Young	0.15	0.01	0.03	391%	0.13	273%
Old	0.08	0.02	0.02	-14%	0.01	-13%
Market tightness						
Young	-	6.63	7.83	18%	8.96	14%
Old	-	4.62	4.55	-2%	4.51	-1%

Notes: The age-specific unemployment rate is from St. Louis Fed, 1960Q1–2007Q4, and age-specific transition rates are from the CPS, 1976Q3–2007Q4. The young include workers aged 15–24 years, and the old cover workers aged 25–64 years. See Table 3.6 for further notes.

Table 3.7 presents the SDs of age-specific labor market variables in the data in Column 1 and the simulation results when I vary the youth share from 18 to 22% in Columns 2-4. We can see a clear pattern for the spillover effect: The simulated SD of the unemployment rate of the young increases from 0.22 to 0.31 as the youth share increases from 18 to 20%; it further increases to 0.49 as the youth share reaches 22%. As unemployment volatility can be attributed to variations in job-finding and separation rates, I also report the results for these two. The simulated SDs increase for both rates of the young as the youth share increases[29]. This is especially true for the separation rate, because its volatility becomes about four times larger when the share increases from 18 to 20% and

[29]Notice that, the SDs of age-specific job-finding rates, in Column 4 of Table 3.7, are both lower than the SD of aggregate job-finding rate, in Column 4 of Table 3.6, even for young workers. This seemingly counterintuitive result is due to the fact that the latter is not a simple weighted average of the former two. In addition to the former two, the dynamics of aggregate job-finding rate also depend on the dynamics of age-specific

about three times larger when it increases from 20 to 22%. A similar pattern is also found for the market tightness of the young, with a relatively less increment.

While for old workers, the volatilities of most labor market variables (the unemployment rate, the separation rate, and market tightness) decrease as the youth share increases, since the increase in the youth share naturally means a decrease in the share of the old. But the volatility of the job-finding rate remains unchanged, besides, the declines in other variables are mild. The reason lies in the complementarity between capital and experience, as the data show. It is also worth noting that the simulated SD of the unemployment rate of the young is higher than that of the old across all demographic states.

Clearly, the model can generate the spillover phenomenon as observed in the data. It predicts that there is approximately a 20% increase in the unemployment volatility of the young when the youth share increases by 1%. With the hump-shaped youth share across 1960-2007, the model can also generate the hump-shaped unemployment volatility of the young over time as shown in Figure 3.1.

To see the relative importance of the spillover effect, I further decompose the overall effect of demographic changes. I shut down the spillover channel by assuming that the SDs of age-specific unemployment rates remain the same as those when the youth share lies in 20%. This gives us the counterfactual values for the SD of the unemployment rate when the youth share changes. This counterfactual value is 0.165 if the youth share declines from 20 to 18%, and 0.18 if the youth share increases from 20 to 22%. From Table 3.6, we already know that the SD of the unemployment rate changes from 0.17 to 0.15 if the youth share declines from 20 to 18%, and changes from 0.17 to 0.21 if the youth share increases from 20 to 22%. Therefore, based on the simulation results, we see that the composition effect accounts for only a quarter of the overall effect of demographic changes, while spillover effect accounts for three-quarters.

3.5.3 Dynamic response of labor market variables of the young

To highlight the mechanism for the spillover effect, Figure 3.3 plots the impulse response functions (IRFs) of key labor market variables of the young with respect

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unemployment rates, whose weight increases as the youth share increases, due to the spillover effect.

to a positive aggregate productivity shock when the youth share varies, from 18 (dotted line) to 20% (solid line), and further to 22% (dotted, circle-hatched line).

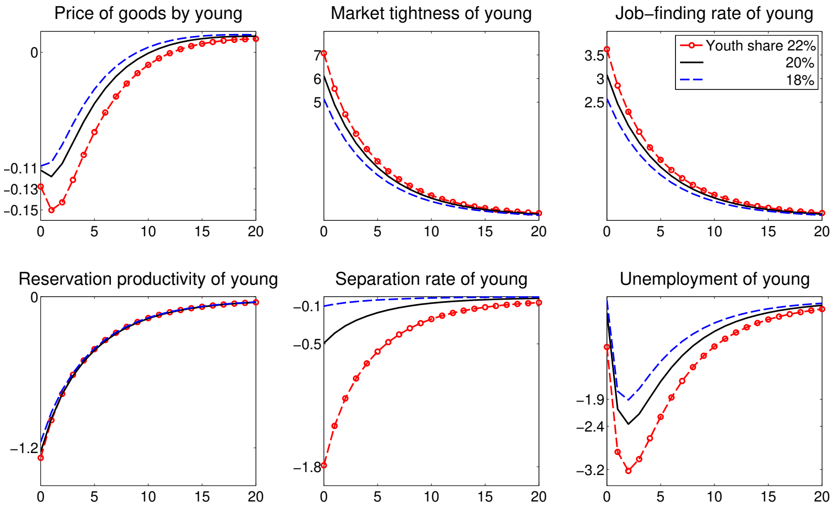

Figure 3.3: IRFs of variables of the young with respect to aggregate productivity shock across demographic states

The response of the job-finding rate to productivity shock depends mainly on the response of market tightness. A higher youth share is associated with a lower price of goods produced by the young, moreover, this price declines more steeply when there are more young workers (top-left panel). As the decrease in price lowers profits generated by the match between firm and young worker, the percentage increase in profits by hiring young workers is larger in case of a positive productivity shock. Therefore, firms' vacancy posting becomes more sensitive to aggregate productivity when the youth share increases, and so do the labor market tightness and the job-finding rate of the young (top-right panel).

To understand how the response of the separation rate to productivity shock is affected by demographic changes, we need to look at the corresponding changes in the size of marginal workers. A higher youth share induces a higher reservation productivity of the young. Since a higher reservation productivity is associated with a larger mass of marginal workers as pointed out by Pizzinelli, Theodoridis, and Zanetti (2018), there are more marginal young workers affected by aggregate shock when the youth share increases. Furthermore, the response of the reservation productivity of the young to productivity shock remains almost unchanged when the youth share changes[30] (bottom-left panel). This means the impact of productivity shock on each marginal young worker remains equally strong at different values for the youth share. Thus, the volatility of the separation rate of the young increases with the youth share (bottom-middle panel).

Hence, the volatilities of both the job-finding and separation rates of the young increase with the youth share, jointly raising the unemployment volatility of the young.

3.6 Introducing exogenous separation

In this section, I discuss the robustness of the former results in the case of exogenous job separation. This exercise is necessary because the dynamics of the unemployment rate in the U.S. comes mainly from the dynamics of the job-finding rate, especially for the young. The purpose of this exercise is about the generality of the spillover effect in different theoretical frameworks rather than about the selection of a suitable job-matching model.

Table 3.8: Further decomposition of unemployment volatility

	15-24	25-54	55-64	15-64
Job-finding rate	**80.91**	68.80	84.97	73.36
Separation rate	**12.42**	26.21	15.86	19.38

Notes: Quarterly data from CPS, 1976Q3–2007Q4. All series are detrended by one-sided HP-filter with a smoothing parameter of 1600.

In Table 3.8, I decompose unemployment volatility for each age group to quantify the contribution of the job-finding and separation rates in accounting for age-specific unemployment volatility[31]. The results show that the fluctuation

[30] The reservation productivity of the young depends mainly on its price and aggregate productivity. Since the response of this price to productivity shock is low in level and only slightly varies when the youth share changes, the response of the reservation productivity barely changes with the youth share.

[31] Following Shimer (2012) in the construction of worker flow rates with CPS data, I decompose the cyclical unemployment volatility of different age groups into contributions from the job-finding and separation rates. Due to the effect of labor force participation, the numbers do not necessarily add up to 100.

in the job-finding rate accounts for almost all the unemployment volatility for young workers. Even for other age groups, the role of the job-finding rate is dominant.

The Bellman equations of the system with exogenous job separation are provided in the Appendix B.7. If there is no aggregate uncertainty, it can be shown that the elasticity of market tightness of the young with respect to aggregate productivity is[32]

$$e^Y_{\theta,A} = \frac{\partial \ln \theta^Y}{\partial \ln A} = \frac{1-\beta(1-\rho^x)+\eta\beta(1-\rho^x)f^Y}{\mu[1-\beta(1-\rho^x)]+\eta\beta(1-\rho^x)f^Y} \frac{\frac{\partial \ln P^Y}{\partial \ln A}+1}{1-z^Y/(P^Y A)}. \tag{3.27}$$

The first term has limited variation as discussed before, besides, the numerator in the second term also has limited variation when the youth share changes, as the estimation results show that the goods produced by young workers are substitutable for the composite of other inputs. Therefore, changes in this elasticity come mainly from changes in the price of goods produced by young workers. Since this price decreases when there are more young workers, as shown in Proposition 1, firms hiring young workers get lower profits and there are more fluctuations in the labor market tightness of the young with respect to the aggregate productivity shock[33].

Table 3.9 reports the second moments when job is exogenously separated. Similar to the former case, for the young, the simulated SD of the unemployment rate is higher when there are more young workers in the labor force; for the old, it is relatively stable, with a slight decline; at the aggregate level, it also increases with the youth share. The dynamics of the job-finding rate and market tightness also exhibit a similar pattern. Notice that the values for unemployment volatilities (both aggregate and age-specific) are lower than those in the model with endogenous job separation[34], which is consistent with Fujita and Ramey (2012).

[32]The proof can be found in Appendix B.8.

[33]Notice that one necessary condition for the existence of match between firm and young worker is $P^Y A > z^Y$, which ensures the non-negativeness of $1-z^Y/(P^Y A)$.

[34]The reason lies in the irresponsiveness of job separation rate to productivity shock in the model with exogenous job separation. With a positive shock, the value for a new match increases and firms post more vacancies. This increases worker's job-finding rate and reduces unemployment. If separation rate is endogenous determined, firms lower the threshold of individual productivity and therefore separation rate, which further decreases unemployment.

Table 3.9: Second moments in the data and the model (with exogenous separation)

	Data	Model				
Youth share	20%	18%	20%		22%	
	(1)	(2)	(3)	↗	(4)	↗
Unemployment rate						
Young	1.03	0.22	0.29	30%	0.38	33%
Old	0.54	0.13	0.13	-3%	0.12	-3%
Aggregate	0.64	0.15	0.16	8%	0.18	13%
Job-finding rate						
Young	5.26	2.30	2.40	4%	2.50	4%
Old	4.06	2.10	2.09	0%	2.09	0%
Market tightness						
Young	-	6.87	8.30	21%	10.22	23%
Old	-	4.70	4.61	-2%	4.52	-2%

Notes: Sources of data are the same as that in Table 3.7. See Table 3.6 for further notes.

Besides, I plot the IRFs with respect to the aggregate productivity shock when job is exogenously separated, which can be found in the Appendix B.9. The results are similar to the case with endogenous job separation. The price of goods produced by the young decreases more when the youth share increases, shrinking the level of firms' profits. Therefore, firms' vacancy posting becomes more sensitive to aggregate productivity. The responses of market tightness and the job-finding rate of the young also become stronger when the youth share increases. As the dynamics of unemployment are now determined by the dynamics of the job-finding rate, unemployment volatility of the young also increases.

To summarize, even without the dynamics of the separation rate, there is still a positive relationship between the volatility of the job-finding rate of the young and the youth share, and it leads to the spillover effect.

3.7 Empirical assessment of the mechanism

This section provides supporting evidence for the aforementioned mechanism. First, I investigate the causal relationship between the price of goods produced by the young and the youth share; then, I check the relationship between the dynamics of transition rates and the youth share.

The first evidence is important since changes in price connect demographic changes and labor market dynamics. In particular, as I claim in Proposition 1, there is a decrease in the price of goods produced by the young when the youth share increases. Since this price level is not directly available in the data, I use the wage of the young as a proxy, which is

$$\omega_t^Y(A_t, a_t) = \eta(P_t^Y A_t a_t + \varsigma^Y \theta_t^Y) + (1-\eta) z^Y. \tag{3.28}$$

This shows that the wage of the young is positively related to the price of goods produced by them. If this price decreases when there is a greater share of young workers, we should also see a decrease in the wage of the young.

As this wage also depends on aggregate and individual productivities, as well as the market tightness of the young, I regress the log wage of the young on the youth share, the lag of the log real GDP per capita, and the job-finding rate of the young[35] using annual data from the U.S. Besides, I also include a trend in the regression, as wage seems to have a long-term trend in the data.

Table 3.10: The price of goods produced by the young and the share of the young

	Youth share	Young population share	Native young population share
Coefficient	-1.71*	-1.28*	-1.35*
	(0.86)	(0.71)	(0.77)
Nobs	31	31	31
R^2	0.98	0.98	0.98

*Notes: Newey–West standard errors are in parentheses. * stands for a significance level of 10%.*

[35] The wage of the young is per capita wage, which is not affected by the size of the young. I put the lag of the log real GDP per capita as an independent variable, as Real GDP per capita is likely to be correlated with the error term. I use the job-finding rate of the young as a proxy, because data of the age-specific market tightness is not available.

The results are reported in Table 3.10. The first column shows that the youth share has a statistically significant, negative effect on the wage of the young. Besides, to take care of the potential endogeneity problems due to labor force participation and migration, I use the share of young population and the share of native young population, respectively, as proxy for the youth share. The results are similar, a statistically significant, negative impact from demographic composition on the wage of the young, supporting my claim in Proposition 1. Given the exogeneity of the youth share and its proxies, the causality of this relationship is ensured.

Table 3.11: Second moments of the transition rates of the young over time

Period Youth share	1976Q3-1986Q2 23.3%	1986Q3-1996Q2 18.0%	1996Q3-2006Q2 15.9%
Job-finding rate (young)	7.59	3.98	3.86
Separation rate (young)	0.22	0.11	0.10

Table 3.11 shows the SDs of the transition rates of the young between 1976Q3 and 2006Q2. This period is characterized by a persistent and sharp decline in the youth share. It can be divided into three subperiods with a 10-year span for each. From the first subperiod to the second, the youth share decreases from 23.3 to 18.0%, with a decline of 5.3% in level; from the second subperiod to the third, it further declines but only by 2.1% in level. Similarly, the SDs of the job-finding and separation rates of the young also have a decreasing trend over the whole period, with more decrease from the first subperiod to the second than from the second subperiod to the third[36]. These results are consistent with the prediction of the model, which shows that the variations in the dynamics of the

[36]Notice that, the variation in the simulated job-finding rate of the young is less obvious than its empirical counterpart: The empirical results suggest a decrease of 13% in the SD of job-finding rate with one percent less young workers, while the model only predicts a corresponding value of 1%. The opposite is true for separation rate: The empirical results suggest a decrease of 16%, while the model predicts a value of 50%. This inconsistency in level is left for future research. Still it does not invalidate the mechanism generating the spillover effect, since both the volatilities of job-finding rate and separation rate in the model share the same pattern as their empirical counterparts, that is, they increase with the youth share.

job-finding and separation rates of the young, when triggered by demographic changes, jointly contribute to the spillover effect.

3.8 Summary

In this paper, I highlight the role of the demographic structure of the workforce in shaping aggregate and age-specific cyclical business cycles. I document a new stylized fact: Unemployment volatility within the group of young workers increases with their share in the labor force, generating a spillover effect.

More importantly, I provide a theoretical explanation of this new fact. By incorporating a job-matching model into the RBC model featuring age-specific differences in labor demand, I show that the dynamics of the transition rates of the young depend on demographic state. These two are connected through the general equilibrium price of the intermediate goods produced by the young. Quantitative results show that the responses of the job-finding and separation rates of the young to productivity shock become stronger as there are more young workers in the labor force, jointly contributing to the spillover effect. This result has important implications for policy-makers. First, it points out the necessity to pay attention to the current state of demographic structure and its projected path, either when policy-makers want information about the labor market response to potential shocks, especially for young workers, or when they need to properly evaluate the impacts of age-specific labor market policies. Second, the positive correlation between business cycle volatility and demographic changes also suggests that the Great Moderation was more likely driven by structural factors other than good policies.

Chapter 4

Population Aging, Monetary Policy, and Consumption Inequality

4.1 Introduction

Population aging is one of the biggest challenges most industrialized countries are currently facing. In the U.S., the average 65-year-old can expect to live another 24 years in 2060, with an increase of seven years from that in 1990, while population growth is expected to decline persistently even allowing for immigration (left panel of Figure 4.1). These two trends will inevitably push up the dependency ratio—the number of population over 65 years to working age population[1] (right panel of Figure 4.1), and this will be 0.4 in 2060 in the U.S. Some other countries have already stepped into the society of aging. In Japan, the dependency ratio is already up to 0.47 in 2017, almost three times as much as that in 1990. The current ratios for Italy, Germany, and France are all over 0.3.

The demographic trends have far-reaching implications for both monetary policy and consumption inequality. For the former, there is a long-term downward pressure on real and nominal interest rates as agents facing a longer retirement period tend to save more (see e.g. Krueger and Ludwig, 2007; Carvalho, Ferrero, and Nechio, 2016), besides, the zero lower bound (ZLB) is more likely to bind following a shock with the declining trend of the nominal interest rate (see

[1]The working age population refers to people aged 15-64 years. Although the full benefit age is 66 years and 2 months in 2017 in the U.S., 65 is still a common full retirement age in many other countries.

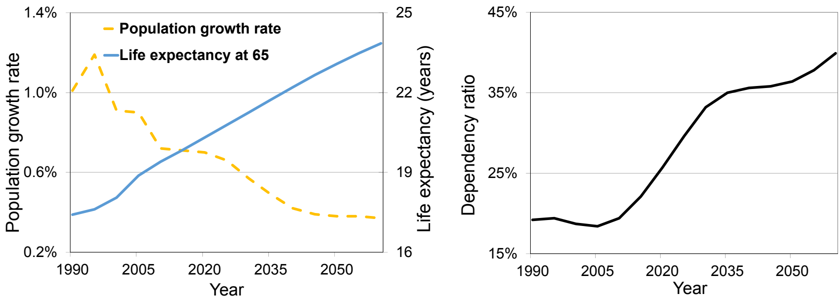

Figure 4.1: Demographic transition in the U.S. projected until 2060

Note: In the left panel, the dotted line represents the population growth rate and the solid line shows the life expectancy at 65. In the right panel, the solid line gives the dependency ratio. All series are from 1990 and projection starts from 2020.
Source: United Nations World Population Prospects (2017 Revision).

e.g. Jones, 2018). For the latter, there is a trend toward increasing consumption inequality between retirees and workers as the pension system comes under strain with a shrinking percentage of workers. This paper connects these two and examines the impact of population aging on consumption inequality between retirees and workers over time through its impact on monetary policy[2]. While the previous research (see e.g. Deaton and Paxson, 1994) has focused on the impact of aging on consumption inequality within a fixed cohort, I restrict my analysis to the narrower question of between-cohort consumption inequality[3].

The connection follows from that population aging can induce higher consumption inequality through the downward adjustment of the real interest rate. With the increasing life expectancy of retirees, workers increase their labor supply, reduce consumption, and save more to insure against a longer retirement

[2]It needs to be pointed out that my analysis does not consider the role of taxation, particularly progressive taxes, which contributes to a large part of the resurgence of inequality after 1980 (see Piketty, 2015). Potentially, this fiscal shift could also be related to demographic changes as there is an increasing need to finance pension systems with more retirees. But here I focus on the role of monetary policy in connecting population aging and consumption inequality.

[3]In the paper, if unspecified, the term consumption inequality refers exclusively to consumption inequality between retirees and workers.

period. At the same time, retirees postpone current consumption and increase savings to support their future consumption. The accumulation of assets leads to a decrease in the real interest rate[4]. Workers' willingness to carry assets into retirement is partially discouraged, as assets become less attractive with lower real returns. With fewer assets carried by workers, retirees further decrease their consumption. Therefore, consumption inequality has an increasing[5] trend as population ages. On the contrary, the binding of the ZLB can mitigate this inequality in dynamics. Since retirees rely typically on the returns of assets, they benefit more when the fall in the nominal interest rate is bounded.

To quantitatively assess the effect of population aging on consumption inequality, I consider a dynamic stochastic general equilibrium (DSGE) model with sticky prices, investment adjustment costs, and the life-cycle feature of Gertler (1999); however I deviate in terms of the modeling of labor supply. I endogenize the labor supply of workers while letting retirees enjoy full leisure. The main purpose of this deviation is to capture the insufficient flexibility of the labor supply of retirees, which is either a necessary result of declining health or a reflection of a strong willingness to be outside of the labor market[6]. Another concern is that rehiring is not easily available for retirees, since post-pension employees usually undertake highly complex tasks requiring proficient knowledge and skills, and constitute only a small part of the retired population. Moreover, as shown by Fujiwara and Teranishi (2008), a variable labor supply of retirees implies that the real interest rate can increase with life expectancy, which contradicts the empirical finding of Carvalho, Ferrero, and Nechio (2016), that is the real interest rate declines monotonically when life expectancy increases.

[4]This is consistent with the empirical finding in Carvalho, Ferrero, and Nechio (2016) and Kara and von Thadden (2016).

[5]The main context here is that workers are wealthier than retirees. This is not an assumption, but rather a result from simulations under reasonable parameter specification. Instead of focusing on the very few asset rich, this paper gives its attention to how population aging adds to the strain on pension systems and the demand for self-insurance.

[6]Although there is an upward trend in the labor force participation rate of older people (65+) in some countries recently, this is more likely to be the result of the extension of retirement age, e.g., from 65 to 67 between 2012 to 2029 in Germany, and from 66 to 67 in the U.S. The labor force participation rate of retirees is still quite low and rigid. The supporting evidence can be found in Italy and Spain, who currently have no plan to extend retirement age. The labor force participation rate of older people (65+) is lower than 4% for Italy and 2% for Spain, and remains almost unchanged for both over time.

Consumption inequality is measured by the ratio of consumption per retiree to consumption per worker (hereafter referred to as the relative consumption ratio). Compared with an inequality measure of the whole population, it is independent of the distribution of the population. In my calibrated model, this ratio for the U.S. is predicted to decline by 40%, from 0.68 to 0.41, between 1990 and 2060, due to population aging. This suggests an increasing consumption inequality over time. Consumption per retiree declines over time, while consumption per worker increases in the long term. This result is mainly due to the decline in the real interest rate as population ages, which is predicted to decrease from 3.7% to -0.3% over this period for the U.S. Retirees, relying only on the returns on assets, lose more because of the decline in the real interest rate. Decomposing the effect of population aging into the effect of the prolonging life expectancy and that of the decreasing population growth, I find that both the increase in consumption inequality and the decline in the real interest rate are mainly due to the prolonging life expectancy.

To investigate the distributional effect of the ZLB, I consider two types of shocks which can push the nominal interest rate against the lower bound: a positive preference shock, which can be considered as a contractionary demand shock, and an expansionary monetary policy shock. For the former, the downward adjustment of the nominal interest rate is partially hindered with the presence of the ZLB. This decreases labor demand and the real wage, and induces a corresponding fall in consumption per worker relative to consumption per retiree. For the latter, the expansionary shock induces an instantaneous decrease in the nominal interest rate. Retirees suffer more due to their full reliance on assets. With the presence of the ZLB, the change in the nominal interest rate is limited and so is the loss of retirees. Therefore, consumption inequality between retirees and workers is mitigated in dynamics after both shocks.

The relationship between population aging and the long-term trend of overall consumption inequality has been examined by several studies. Deaton and Paxson (1994) show that consumption inequality within a fixed cohort increases with age, using data from the U.S., the U.K., and Taiwan, since idiosyncratic shocks accumulate throughout the life cycle in an incomplete market. Abe and Yamada (2009) and Badel and Huggett (2014) provide further evidence of this positive relationship. One of its implications is that population aging can lead to greater overall consumption inequality as the share of older cohorts rises due to a direct composition effect, as shown by Ohtake and Saito (1998) using Japanese household data. This paper enriches this literature by investigating the implication of aging for between-cohort consumption inequality.

The theoretical framework of this paper shares similarity with Fujiwara and Teranishi (2008), Carvalho, Ferrero, and Nechio (2016), and Kara and von Thadden (2016), who follow Gertler's suggestion for demographic developments as time-dependent exogenous processes in a New Keynesian model. The first one studies the asymmetry of the effects of structural shocks, technology and monetary policy shocks, over the life cycle, through the changes in the labor supply of retirees as population ages. However, it does not systematically address the impact of population aging on this asymmetry. The other two focus on the impact of demographic changes on the long-term transition of the real interest rate. The present paper connects the impact of population aging on consumption inequality and its implication for the real interest rate. Besides, I deviate from them[7] in labor supply specifications to capture the low and rigid labor supply of retirees.

This paper is also related to a growing literature studying the distributional effect of monetary policy shocks[8]. By tracing overall consumption and income inequality in the U.S. since 1980, Coibion et al. (2017) find that monetary policy shocks, particular contractionary monetary policy shocks, account for a nontrivial part of the cyclical variation in both inequality. In addition, by measuring the effect of unanticipated changes in the nominal interest rate on income inequality for a large sample of advanced and emerging economies, Furceri, Loungani, and Zdzienicka (2018) find that contractionary monetary policy shocks increase inequality, in both cyclical and long-term variations. The main channel works through heterogeneity across households in terms of their primary sources of income. The present paper adds to this literature by investigating the distributional effect of shocks, a preference shock and a monetary policy shock, with

[7]Labor supply is endogenously determined for both workers and retirees in Fujiwara and Teranishi (2008) and Kara and von Thadden (2016) while exogenously given for both in Carvalho, Ferrero, and Nechio (2016). In my modeling setup, it is endogenously determined for workers while exogenously given for retirees.

[8]On the theoretical side, based on DSGE models, Auclert (2017) considers the asymmetric effect of short-term changes in the real interest rate on aggregate consumption and Doepke, Schneider, and Selezneva (2015) study the heterogeneous effect of changes in the nominal interest rate generated by a persistent change in the inflation target.

the binding of the ZLB. I find that redistribution due to the ZLB works through similar channels as that due to monetary policy shocks.

The paper proceeds as follows. The next section lays out the model. Section 4.3 gives the calibration strategy. Section 4.4 reports the main results and explains the mechanism. The final section concludes the paper.

4.2 The model

The model is a canonical New Keynesian DSGE model with Rotemberg (1982) type price adjustment cost and the life-cycle feature of Gertler (1999). Time is discrete and households have an infinite time horizon. There are six sectors in the economy: households, final goods producers, intermediate goods producers, capital producers, a fiscal authority, and a monetary authority. I restrict the analysis to a perfect-foresight environment.

4.2.1 Households

Households invest their assets in capital producers to produce capital, which is then sold to firms producing intermediate goods. Besides, households own all producers and receive their profits[9]. Individuals are born as workers. They choose the optimal levels of consumption, labor supply, and asset holdings to maximize their utility. With an exogenous probability $1-\omega_{t+1}$, workers retire in the next period and enter into retirement[10]. Retirees choose the optimal levels of consumption and asset holdings. They survive with an exogenous probability γ_{t+1} in the next period and live on their asset holdings.

4.2.1.1 Retirees

The value function of retirees, r, who were born at j and become retired at k, is written as

$$V_t^{rjk} = \max_{C_t^{rjk}, A_t^{rjk}} \left[(C_t^{rjk})^\rho + \beta_t \gamma_{t+1} (V_{t+1}^{rjk})^\rho \right]^{1/\rho},$$

[9]This ownership structure is also used in Jaimovich and Floetotto (2008).

[10]This setup is similar as the perpetual youth model by Yaari (1965) and Blanchard (1985).

subject to their intertemporal budget constraint:

$$C_t^{rjk} + \frac{A_t^{rjk}}{P_t} + T_t^{rjk} = \frac{1}{\gamma_t}\frac{R_{t-1}A_{t-1}^{rjk}}{P_t} + \Pi_t^{rjk},$$

where ρ determines the inter-temporal elasticity of substitution[11], β_t is a time-varying subjective discount factor and defined by $\beta_t \equiv \beta \exp(\eta_t)$, in which β is the scale factor and η_t stands for an inter-temporal preference shock[12] with $\eta_t = \rho^\eta \eta_{t-1} + e_t^\eta$.

Retirees carry A_{t-1}^{rjk} units of nominal assets into period t, which is the amount invested in capital producers and which is predetermined in period $t-1$. They receive a nominal gross return of $R_{t-1}A_{t-1}^{rjk}$ from their investment and real profits of Π_t^{rjk} from their ownership of firms[13] and capital producers in period t. Given investment returns and profits, retirees decide the current level of consumption and the amount to lend to capital producers in the next period. T_t is a lump-sum tax which is the same for all households. A perfectly competitive life-insurance company makes sure the bequests of assets are evenly distributed among all survived retirees and pays a premium $1/\gamma_t$ over the market return. Besides, the consistency of asset holdings requires that the retirees' initial asset holdings upon retirement correspond to the assets held in the last period as a worker, namely, $A_t^{rj(t+1)} = A_t^{wj}$. From the first-order conditions, we can get the Euler equation for the consumption of retirees:

$$\frac{R_t}{\pi_{t+1}}\beta_t\Big(\frac{C_{t+1}^{rjk}}{C_t^{rjk}}\Big)^{\rho-1} = 1,$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ denotes the gross inflation rate.

Let ξ_t^r denote the retirees' marginal propensity to consume (MPC) out of wealth, then the retirees' consumption at time t can be written as a linear function of total wealth at t:

$$C_t^{rjk} = \xi_t^r\Big(\frac{1}{\gamma_t}\frac{R_{t-1}A_{t-1}^{rjk}}{P_t} + NP_t^{rjk}\Big),$$

[11] $\sigma = (1-\rho)^{-1}$ is the elasticity of inter-temporal substitution.

[12] The preference shock is intended to generate output contraction and push the nominal interest rate against the ZLB. It is widely used in the ZLB literature, see e.g. Christiano, Eichenbaum, and Rebelo (2011) and Krause and Moyen (2016).

[13] Firms include final and intermediate goods producers.

where NP_t^{rjk} is the present discounted value of net profits of retirees. These net profits are the difference between profits from firms and capital producers, Π_t^{rjk}, and tax payment of retirees, T_t^{rjk}. The dynamic equation for NP_t^{rjk} is

$$\begin{aligned} NP_t^{rjk} &\equiv \Pi_t^{rjk} - T_t^{rjk} + \sum_{\nu=1}^{\infty} \frac{\Pi_{t+\nu}^{rjk} - T_{t+\nu}^{rjk}}{\prod_{s=0}^{\infty} \left(\frac{R_{t+s}}{\pi_{t+s}\gamma_{t+s}}\right)} \\ &= \Pi_t^{rjk} - T_t^{rjk} + \frac{\pi_{t+1}\gamma_{t+1}}{R_t} NP_{t+1}^{rjk}, \end{aligned}$$

where γ_{t+1} is the survival rate of a retiree and $\frac{\pi_{t+1}\gamma_{t+1}}{R_t}$ represents the real discount rate for a retiree. It follows that the dynamic equation of ξ_t^r is

$$\xi_t^r = 1 - \gamma_{t+1}\beta_t^{\frac{1}{1-\rho}} \left(\frac{R_t}{\pi_{t+1}}\right)^{\frac{\rho}{1-\rho}} \frac{\xi_t^r}{\xi_{t+1}^r}. \tag{4.1}$$

Retirees' MPC depends on the real interest rate, the discount factor and its survival probability. If $\rho = 0$, we can have $\xi_t^r = 1 - \gamma_{t+1}\beta_t$. If the life expectancy of retirees increases, retirees tend to consume less of their total wealth and decrease their MPC, as in Yaari (1965) and Blanchard (1985). If the discount factor increases, which represents a positive preference shock, retirees tend to postpone their consumption, and their MPC declines accordingly.

4.2.1.2 Workers

The value function of workers born at j is written as

$$V_t^{wj} = \max_{C_t^{wj}, L_t^{wj}, A_t^{wj}} \left\{ \left[(C_t^{wj})^v (1 - L_t^{wj})^{1-v}\right]^\rho + \beta_t \left[\omega_{t+1} V_{t+1}^{wj} + (1-\omega_{t+1}) V_{t+1}^{rj}\right]^\rho \right\}^{1/\rho},$$

subject to their intertemporal budget constraint:

$$C_t^{wj} + \frac{A_t^{wj}}{P_t} + T_t^{wj} = \frac{R_{t-1}A_{t-1}^{wj}}{P_t} + \frac{W_t}{P_t} L_t^{wj} + \Pi_t^{wj},$$

where v is a parameter related to the marginal rate of substitution between consumption and leisure.

The form of the instantaneous utility of workers is consistent with that of retirees. To see this, note that the instantaneous utility of retirees can be written as

$$[(C_t^{rjk})^v(1-L_t^{rjk})^{1-v}]^\rho.$$

Because retirees enjoy full-time leisure, we have $L_t^{rjk}=0,\forall j,k$. Besides, the marginal rate of substitution between consumption and leisure goes to infinity, since labor supply is constant for retirees, thus we have $\nu=1$ for retirees and their instantaneous utility becomes $(C_t^{rjk})^\rho$.

In addition to the investment returns from capital producers and the profits from the ownership of firms and capital producers, workers get one unit of labor in each period, from which they choose to supply L_t^{wj}, and get wage payment $\frac{W_t}{P_t}L_t^{wj}$. In the next period, workers retire with probability $1-\omega_{t+1}$. If it happens, they get a value of $V_{t+1}^{rj(t+1)}$, which is the value function of retirees born at j and retire at $t+1$.

From the first-order conditions, we can get the equation for labor supply:

$$L_t^{wj}=1-\frac{1-v}{v}\frac{P_t}{W_t}C_t^{wj},$$

from which, we see that labor supply increases with wage and declines with consumption. As consumption is proportionate to total wealth at t, there is a negative wealth effect on labor supply.

As proved in the Appendix C.1, the Euler equation for workers' consumption is

$$\begin{aligned}\omega_{t+1}\left(\tfrac{1-v}{v}\tfrac{P_{t+1}}{W_{t+1}}\right)^{1-v}C_{t+1}^{wj}+(1-\omega_{t+1})\left(\tfrac{\xi_{t+1}^r}{\xi_{t+1}^w}\right)^{-1/\rho}C_{t+1}^{rj(t+1)}\\=\left[\beta_t\tfrac{R_t}{\pi_{t+1}}\Omega_{t+1}\left(\tfrac{1-v}{v}\tfrac{P_t}{W_t}\right)^{\rho(v-1)}\right]^{\frac{1}{1-\rho}}C_t^{wj},\end{aligned}$$

where ξ_t^w is the MPC of workers, $\Omega_{t+1}\left(\frac{1-v}{v}\frac{P_t}{W_t}\right)^{\rho(v-1)}$ is a weight for the real interest rate, and Ω_{t+1} is defined by $\Omega_{t+1}\equiv\omega_{t+1}\left(\frac{1-v}{v}\frac{P_{t+1}}{W_{t+1}}\right)^{1-v}+(1-\omega_{t+1})\nu^{-1}\left(\frac{\xi_{t+1}^r}{\xi_{t+1}^w}\right)^{-\frac{1-\rho}{\rho}}$. Workers take into account the probability of retirement in the next period when they decide the optimal consumption for current period.

Accordingly, the decision rule for workers' consumption is

$$C_t^{wj} = \xi_t^w \Big(\frac{R_{t-1} A_{t-1}^{wj}}{P_t} + H_t^{wj} + NP_t^{wj} \Big),$$

where ξ_t^w is the MPC of workers, H_t^{wj} is human wealth, NP_t^{wj} is the present discounted value of net profits of workers. Combining this decision rule with the Euler equation for workers' consumption, we get the dynamic equation for the MPC of workers ξ_t^w :

$$\xi_t^w \quad = \quad 1 - \Big[\Omega_{t+1} \Big(\frac{1-v}{v} \frac{P_t}{W_t} \Big)^{\rho(v-1)} \beta_t \Big(\frac{R_t}{\pi_{t+1}} \Big)^{\rho} \Big]^{\frac{1}{1-\rho}} \Big(\Omega_{t+1}^{'} \Big)^{-1} \frac{\xi_t^w}{\xi_{t+1}^w}, \quad (4.2)$$

and the dynamic equations for human wealth H_t^{wj} and the present discounted value of net profits of workers NP_t^{wj} :

$$\begin{aligned} H_t^{wj} &= \frac{W_t}{P_t} L_t^{wj} + \frac{\pi_{t+1}}{R_t} \frac{\omega_{t+1}}{\Omega_{t+1}^{'}} \Big(\frac{1-v}{v} \frac{P_{t+1}}{W_{t+1}} \Big)^{1-v} H_{t+1}^{wj} \\ NP_t^{wj} &= \Pi_t^{wj} - T_t^{wj} + \frac{\pi_{t+1}}{R_t} \frac{\omega_{t+1}}{\Omega_{t+1}^{'}} \Big(\frac{1-v}{v} \frac{P_{t+1}}{W_{t+1}} \Big)^{1-v} NP_{t+1}^{wj} \\ &\quad + \frac{\pi_{t+1}}{R_t} \frac{1-\omega_{t+1}}{\Omega_{t+1}^{'}} \Big(\frac{\xi_{t+1}^r}{\xi_{t+1}^w} \Big)^{1-1/\rho} NP_{t+1}^{rj(t+1)}, \end{aligned}$$

where $\Omega_{t+1}^{'} \equiv \omega_{t+1} \Big(\frac{1-v}{v} \frac{P_{t+1}}{W_{t+1}} \Big)^{1-v} + (1-\omega_{t+1}) \Big(\frac{\xi_{t+1}^r}{\xi_{t+1}^w} \Big)^{-\frac{1-\rho}{\rho}}$. The term $\frac{\pi_{t+1}}{R_t} \frac{\omega_{t+1}}{\Omega_{t+1}^{'}} \Big(\frac{1-v}{v} \frac{P_{t+1}}{W_{t+1}} \Big)^{1-v}$ constitutes the real discount rate for a worker, which captures the positive probability of retiring and losing labor income.

4.2.1.3 Aggregation

Suppose that there are N_t^w workers and N_t^r retirees at time t, the size of new born is $(1-\omega_{t+1}+n_{t+1})N_t^w$ and the size of remaining workers is $\omega_{t+1}N_t^w$, then the law of motion for aggregate labor force is

$$N_{t+1}^w = (1-\omega_{t+1}+n_{t+1})N_t^w + \omega_{t+1}N_t^w = (1+n_{t+1})N_t^w.$$

For retirees, the inflow is $(1-\omega_{t+1})N_t^w$ and the size of remaining retirees is $\gamma_{t+1}N_t^r$, thus the law of motion for aggregate retirees is

$$N_{t+1}^r = (1-\omega_{t+1})N_t^w + \gamma_{t+1}N_t^r.$$

As dependency ratio is the ratio of the size of retirees to the size of workers, N_t^r/N_t^w, we get the dynamic equation for dependency ratio as

$$(1+n_{t+1})d_{t+1} = 1-\omega_{t+1}+\gamma_{t+1}d_t. \tag{4.3}$$

Since the MPC of retirees, ξ_t^r, is the same for all retirees at each time t, the aggregate consumption of retirees follows a similar form as C_t^{rjk}. The only difference is that the aggregate asset holdings of retirees do not need to be divided by γ_t, because redistribution of requests does not play a role in aggregate. Thus, the equation for the aggregate consumption of retirees is

$$C_t^r = \xi_t^r\Big(\frac{R_{t-1}A_{t-1}^r}{P_t} + PP_t^r - PT_t^r\Big). \tag{4.4}$$

Similarly, the aggregate consumption of workers and aggregate labor supply are

$$\begin{aligned} C_t^w &= \xi_t^w\Big(\frac{R_{t-1}A_{t-1}^w}{P_t} + H_t^w + PP_t^w - PT_t^w\Big) && (4.5)\\ L_t &= N_t^w - \frac{1-v}{v}\frac{P_t}{W_t}C_t^w. && (4.6)\end{aligned}$$

The aggregate asset holdings of retirees in period t are composed of the assets of retirees who survive at t and the assets of workers who retire at t. Thus the law of motion of the aggregate asset holdings of retirees is

$$\begin{aligned}\frac{A_t^r}{P_t} &= \frac{R_{t-1}A_{t-1}^r}{P_t} + \mathit{\Pi}_t^r - \frac{d_t}{1+d_t}T_t - C_t^r\\ &\quad +(1-\omega_{t+1})\Big(\frac{R_{t-1}A_{t-1}^w}{P_t} + \frac{W_t}{P_t}L_t + \mathit{\Pi}_t^w - \frac{T_t}{1+d_t} - C_t^w\Big),\end{aligned}$$

and the aggregate asset holdings of workers depend only on the asset holdings of the remaining works in the labor force, which gives

$$\frac{A_t^w}{P_t} = \omega_{t+1}\Big(\frac{R_{t-1}A_{t-1}^w}{P_t} + \frac{W_t}{P_t}L_t + \mathit{\Pi}_t^w - \frac{T_t}{1+d_t} - C_t^w\Big).$$

Combining the last two equations, we get

$$\begin{aligned}\frac{A_t^r}{P_t} &= \frac{R_{t-1}A_{t-1}^r}{P_t}(1-\xi_t^r)+\Pi_t^r-\frac{d_t}{1+d_t}T_t-\xi_t^r(PP_t^r-PT_t^r)\\&+\frac{1-\omega_t}{\omega_t}\frac{A_t^w}{P_t}. \qquad (4.7)\end{aligned}$$

With population growth, the discounting rate for aggregate human wealth and the present values of net profits needs to be adjusted by $1/(1+n_{t+1})$. Therefore, the dynamic equation of aggregate human wealth is

$$H_t^w=\frac{W_t}{P_t}L_t^w+\frac{\pi_{t+1}}{(1+n_{t+1})R_t}\frac{\omega_{t+1}}{\Omega'_{t+1}}\left(\frac{1-v}{v}\frac{P_{t+1}}{W_{t+1}}\right)^{1-v}H_{t+1}^w, \qquad (4.8)$$

and the dynamic equations of the present discounted value of net profits are

$$NP_t^r = \Pi_t^r-\frac{d_t}{1+d_t}T_t+\frac{\pi_{t+1}\gamma_{t+1}}{(1+n_{t+1})R_t}NP_{t+1}^r \qquad (4.9)$$

$$\begin{aligned}NP_t^w &= \Pi_t^w-\frac{1}{1+d_t}T_t+\frac{\pi_{t+1}}{(1+n_{t+1})R_t}\frac{\omega_{t+1}}{\Omega'_{t+1}}\left(\frac{1-v}{v}\frac{P_{t+1}}{W_{t+1}}\right)^{1-v}NP_{t+1}^w\\&+\frac{\pi_{t+1}}{(1+n_{t+1})R_t}\frac{1-\omega_{t+1}}{\Omega'_{t+1}}\left(\frac{\xi_{t+1}^r}{\xi_{t+1}^w}\right)^{1-1/\rho}NP_{t+1}^r. \qquad (4.10)\end{aligned}$$

4.2.2 Firms

Final goods are produced by perfectly competitive firms with a continuum of intermediate goods, indexed by $i\in[0,1]$, purchased from intermediate goods producers. The production of intermediate goods follows a Cobb-Douglas function. It is under monopolistic competition and subject to Rotemberg-type price adjustment cost.

4.2.2.1 Final goods producers

Taking the price of each input $P_t(i)$ and the price of final goods P_t as given, the final goods producers choose optimal input $Y_t(i)$ for all $i\in[0,1]$ to maximize profits. The problem of the final goods producers is

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di,$$

subject to a Dixit-Stiglitz aggregation technology:

$$Y_t = \Big(\int_0^1 Y_t(i)^{(\kappa-1)/\kappa} di \Big)^{\kappa/(\kappa-1)},$$

where κ is the constant elasticity of substitution among goods and $\kappa/(\kappa-1)$ is the gross price markup. From the first-order condition, we get the demand function for intermediate goods in the form of a downward sloping curve:

$$Y_t(i) = \Big(\frac{P_t(i)}{P_t}\Big)^{-\kappa} Y_t.$$

4.2.2.2 Intermediate goods producers

Monopolistically competitive intermediate goods producers purchase labor $L_t(i)$ at price W_t/P_t from households and capital $K_{t-1}(i)$ at price r_t^K from capital producers to produce intermediate good i, which is then sold to final goods producers at price $P_t(i)$. In order to eliminate the distortion from monopolistic competition, government provides a subsidy τ to each intermediate goods producer with the support of a lump-sum tax.

The problem of a producer producing good i can be written as[14]

$$\min_{L_t(i), K_{t-1}(i)} \frac{W_t}{P_t} L_t(i) + r_t^K K_{t-1}(i),$$

subject to

$$Y_t(i) = L_t(i)^{1-\alpha} K_{t-1}(i)^{\alpha}. \tag{4.11}$$

[14]In aggregate, we have $\int_i K_{t-1}(i) = K_{t-1}$. Although the aggregate capital is determined at $t-1$, its distribution is realized at t.

The first-order conditions with respect to capital and labor supply are

$$\frac{W_t}{P_t} = (1-\alpha)\varphi_t L_t(i)^{-\alpha} K_{t-1}(i)^{\alpha} \tag{4.12}$$

$$r_t^K = \alpha\varphi_t L_t(i)^{1-\alpha} K_{t-1}(i)^{\alpha-1} \tag{4.13}$$

where $\varphi_t = [\frac{W_t}{(1-\alpha)P_t}]^{1-\alpha}(\frac{r_t^K}{\alpha})^{\alpha}$ is the Lagrangian multiplier of the optimization problem and denotes real marginal cost. Notice that the real marginal cost is the same for each intermediate goods producer.

Besides, producers also need to choose the optimal price $P_t(i)$ given the Rotemberg price adjustment cost and a target of unity for gross inflation rate. The optimal price is set by the maximization of the sum of current and future profits,

$$\underset{P_t(i)}{max} \sum_{s=0}^{\infty} \Phi_{t,t+s} \Pi_t^M(i),$$

subject to the demand function for intermediate goods. Of which, $\Phi_{t,t+1}$ is a discount factor given by $1 = \Phi_{t,t+1}\frac{R_t}{\pi_{t+1}}$ and $\Pi_t^M(i)$ is the monopoly profit of producer i at t with

$$\Pi_t^M(i) = \Big[(1+\tau)\frac{P_t(i)}{P_t} - \varphi_t\Big] Y_t(i) - \frac{\phi}{2}\Big(\frac{P_t(i)}{P_{t-1}(i)} - 1\Big)^2 Y_t. \tag{4.14}$$

Because of symmetry across intermediate goods producers, each one charges the same price in equilibrium, $P_t(i) = P_t$, given that the real marginal cost is the same for each producer. The first-order condition is

$$\kappa(\varphi_t - 1) - \phi(\pi_t - 1)\pi_t + \frac{\pi_{t+1}}{R_t}\phi(\pi_{t+1} - 1)\pi_{t+1} = 0. \tag{4.15}$$

4.2.3 Capital producers

The behavior of capital producers shares similarity with Fujiwara and Teranishi (2008)[15]. But instead of introducing financial intermediaries who borrow from households and invest in capital producers, I assume that the households invest

[15]It is also similar as the behavior of type II firms in Ljungqvist and Sargent (2012).

directly in capital producers. This simplification of the funding structure offers very similar result as Fujiwara and Teranishi (2008)[16].

Perfectly competitive capital producers receive a nominal payment of $P_t r_t^K K_t$ from intermediate goods producers and pay a nominal amount of $R_t A_{t-1}$ to households at the beginning of period t, then they choose the amount of assets to borrow from households for the next period, A_t, and the amount of new investment I_t for the production of new capital. The capital producers' real dividend is given by

$$\Pi_t^K = \frac{A_t}{P_t} + r_t^K K_{t-1} - I_t - R_{t-1}\frac{A_{t-1}}{P_t}, \tag{4.16}$$

subject to the law of motion for capital,

$$K_t = (1-\delta)K_{t-1} + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{4.17}$$

where S is the investment adjustment cost and of similar form as Fujiwara and Teranishi (2008):

$$S\left(\frac{I_t}{I_{t-1}}\right) = (1+n_t)^2 S"\left[\frac{\left(\frac{I_t}{I_{t-1}}\right)^2}{2(1+n_t)^2} - \frac{I_t}{I_{t-1}(1+n_t)} + \frac{1}{2}\right],$$

in which $S"$ is the parameter of investment adjustment cost[17]. Accordingly, the aggregate dividend of households, which is the sum of the dividends from intermediate goods producers and capital producers, is

$$\Pi_t = \Pi_t^M + \Pi_t^K.$$

[16] I fix some obvious typos in parameter specifications in their Table 4.1: θ in the first row should be κ, α should be $1-0.667$ instead of 0.667, and $\delta = 1.1^{.25} - 1$ instead of $\delta = 1.1^{2.5} - 1$.

[17] The detrended version of this equation (aggregate investment I divided by $(1+n_t)$) is:

$$S(\frac{i_t}{i_{t-1}}) = [(1+n_t)]^2 S"[(\frac{i_t}{i_{t-1}})^2/2 - \frac{i_t}{i_{t-1}} + \frac{1}{2}].$$

It satisfies the properties specified in Christiano, Eichenbaum, and Evans (2005): $S(1) = S'(1) = 0$, and $S"(1) > 0$.

The first-order conditions with respect to capital and investment are

$$Q_t = \frac{\pi_{t+1}}{R_t}[Q_{t+1}(1-\delta)+r^K_{t+1}] \tag{4.18}$$

$$1 = Q_t[1-S(\frac{I_t}{I_{t-1}})-S'(\frac{I_t}{I_{t-1}})\frac{I_t}{I_{t-1}}]+\frac{\pi_{t+1}}{R_t}Q_{t+1}S'(\frac{I_{t+1}}{I_t})(\frac{I_{t+1}}{I_t})^2, \tag{4.19}$$

where Q_t is the Lagrange multiplier on the capital formation, Tobin's Q. Equation (4.19) gives the arbitrage condition. In steady state, we get $R = 1-\delta+r^K$. Equation (4.20) gives the dynamic equation for the Lagrange multiplier Q_t.

4.2.4 Government's policy

4.2.4.1 Fiscal policy

The government collects a lump-sum tax and subsidizes intermediate goods producers in order to eliminate monopolistic distortion. Therefore, its budget constraint is

$$T_t = \tau Y_t. \tag{4.20}$$

4.2.4.2 Monetary policy

I assume that the gross nominal interest rate follows a truncated Taylor rule:

$$R_t = max\left[1,\, R_{ss}+1.5(\pi_t-1)+\frac{0.5}{4}\left(\frac{1}{1+n_t}\frac{Y_t}{Y_{t-1}}-1\right)+e^M_t\right], \tag{4.21}$$

where R_{ss} is the short-term nominal interest rate.

4.2.5 Market clearing

The market clearing condition for final goods is given by

$$Y_t = C^r_t + C^w_t + I_t, \tag{4.22}$$

and the market clearing condition for asset holdings is

$$\frac{A_t}{P_t} = Q_t K_t, \tag{4.23}$$

where $A_t = A_t^r + A_t^w$.

4.2.6 Equilibrium

An equilibrium consists of sequences of endogenous variables for all periods, a sequence of quantities $\{C_t^w, PP_t^w, PT_t^w, H_t^w, A_t^w, C_t^r, PP_t^r, PT_t^r, A_t^r, \Pi_t^M, \Pi_t^K, T_t,$ and $K_t, L_t, Y_t, I_t, \varphi_t\}$, marginal propensities to consume $\{\xi_t^w, \xi_t^r\}$, and prices $\{R_t, \pi_t, r_t^K, Q_t, W_t\}$, that satisfy the system of Equations (4.1)-(4.23), taking the dynamics of the demographic processes as given. As there is population growth, which could be the source of non-stationarity, I detrend the equation system to get individual measures for both workers and retirees. To be precise, C_t^w, PP_t^w, PT_t^w, H_t^w, K_t, L_t, Y_t , and I_t are detrended by N_t^w; C_t^r, PP_t^r, and PT_t^r are detrended by N_t^r; Π_t^M, Π_t^K, and T_t are detrended by N_t; A_t^w is detrended by $N_t^w P_t$; A_t^r is detrended by $N_t^r P_t$; and W_t is detrended by P_t.[18]

4.3 Calibration

This section describes the parameter specification of the model for the U.S. from 1990 to 2060 at a quarterly frequency. Individuals are born at age 16 as workers and enter into retirement at age 65, implying an average quarterly rate of remaining as workers, ω, as[19] $1.021^{-.25}$. The projected data for population growth, n_t, and dependency ratio, d_t, are only available at 5-year intervals. To get their quarterly measures, I use a similar method[20] as Carvalho, Ferrero, and Nechio (2016) for the generation of these two demographic processes and focus on the impact of their long-term demographic trends. Their initial values in 1990 are from the United Nations World Population Prospects (the 2017 revision),

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[18] Appendix C.2 lists the detrended counterparts of Equations (4.1)-(4.23).

[19] The annual rate is $1 - 1/(65-16) = 1.021^{-1}$, with its quarterly counterpart as $1.021^{-.25}$.

[20] I assume that n_t and d_t follow

$$\begin{aligned} n_t &= n_{1990} \exp(n_t^1 - n_t^2) \\ d_t &= d_{1990} \exp(d_t^1 - d_t^2), \end{aligned}$$

where n_t^i and d_t^i (for $i = \{1,2\}$) are stationary AR(1) processes. n_t^1 and n_t^2 share common innovation ε_t^n, similarly a common innovation ε_t^d for d_t^1 and d_t^2. The persistence parameters and innovations of these four processes are chosen such that the generated processes for n_t and d_t match their empirical counterparts.

with the growth rate of population equal to the growth rate of the labor force. As shown in Figure 4.2, the generated processes for both fit their data well. The survival probability of retirees, γ_t , is backed out from Equation (4.3), with its initial value calibrated with the steady-state counterpart of this equation:

$$\gamma = 1 + n - \frac{1-\omega}{d}.$$

The demographic structure changes gradually from a relatively young population, represented by the population of the U.S. in 1990, to a relatively old population, represented by that in 2060.

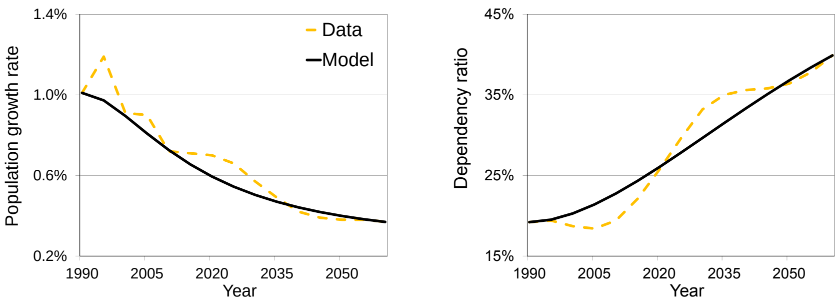

Figure 4.2: Demographic processes in the model and in the data

Note: In the left panel, the solid line and the dotted line represent the population growth rate in the model and in the data, respectively. In the right panel, the solid line and the dotted line are the dependency ratio in the model and in the data, respectively.

I set the scale factor of the subjective discount factor $\beta = 0.96^{.25}$. The capital share, α, is set to 0.45 given the declining trend of the U.S. labor share[21] as shown by Elsby, Hobijn, and Sahin (2013) and Karabarbounis and Neiman (2013). The elasticity of inter-temporal substitution, σ, is set to 0.25, the same

[21]The labor share in the U.S. decreases from 0.64 in 1990 to 0.57 in 2012 according to Karabarbounis and Neiman (2013) due to advances in information technology. As population ages, the accumulation of assets will push down the real interest rate and the rental price of capital. More firms will shift away from labor to capital. Therefore, for the period from 1990 to 2060, I use 0.45 for the capital share, which is higher than 0.33, the value used in Gertler (1999).

as Gertler (1999). The weight of consumption compared to leisure, υ, is set to 0.64 according to Kara and von Thadden (2016).

Table 4.1: Parameter Values in Simulation

	Interpretation	Value	Rationale
ω	Prob. of remaining as worker	$1.021^{-.25}$	
β	Scale of subj. discount factor	$0.96^{.25}$	Gertler (1999)
α	Capital share	0.45	Karabarbounis and Neiman (2013)
σ	Ela. of intertemporal subs.	0.25	Gertler (1999)
ν	For MRS (cons. & leisure)	0.64	Kara and von Thadden (2016)
δ	Capital depreciation rate	$1.1^{.25}-1$	Gertler (1999)
κ	Ela. of subs. among goods	10	Chari, Kehoe, and McGrattan (2000)
ϕ	Price adjustment cost	24	Keen and Wang (2007)
S"	Second derivative of inv. adj. cost	2.48	Christiano, Eichenbaum, and Evans (2005)
ρ^{η}	Persistence of pref. shock	0.9	Krause and Moyen (2016)

On the firm side, I set the capital depreciation rate equal to $1.1^{.25}-1$, with an annual depreciation rate of 10%, the same as Gertler (1999). I follow Chari, Kehoe, and McGrattan (2000) and set the elasticity of substitution among goods $\kappa = 10$, which suggests a 11% markup. I set the price adjustment cost $\phi = 24$, implying a duration of nine months for firms to re-optimize price in a Calvo-type model, as shown by Keen and Wang (2007). The parameter governing the investment adjustment cost is taken from Christiano, Eichenbaum, and Evans (2005) with $S" = 2.48$. The persistence of preference shock is taken from Krause and Moyen (2016), with the value of 0.9. Table 4.1 summarizes the parameter values.

4.4 Simulation results

This section analyzes the effect of population aging on consumption inequality measured by the relative consumption ratio, which is the ratio of consumption per retiree to consumption per worker. I first study the transition path of this ratio under the projected demographic transition path from 1990 to 2060 for the U.S. Meanwhile, I carry out counterfactual experiments to disentangle the effects of the increasing life expectancy and the decreasing population growth. I go on to examine the effects of the ZLB on the asymmetry of consumption with respect to shocks. Finally, I discuss the robustness of the results when the retirement age is extended.

4.4.1 Transition path

Figure 4.3 shows the transition paths of simulated series from the initial steady state in 1990 to the terminal steady state in 2060 given the assumed demographic shocks. To see the overall effects of population aging, I first simulate the model under scenario (I): Both the survival probability of retirees and population growth rate evolve as the generated processes described in Section 4.3 (solid line in Figure 4.3). In addition, in order to disentangle the effects of the increasing life expectancy and the decreasing population growth, I further simulate the model under the following scenarios: (II) Only survival probability changes while population growth rate remains at its initial level in 1990 (dotted, circle-hatched line), and (III) only population growth rate changes while survival probability remains at its initial level in 1990 (dotted line).

The top two panels present one of the main results of this paper: Consumption inequality between retirees and workers increases in response to the projected population aging. With full demographic changes, the relative consumption ratio[22] is expected to fall from 0.68 in 1990 to 0.41 in 2060. This means that a retiree consumes 68% of the consumption level of a worker in 1990, but only 41% of that in 2060, with a 40 percentage decline. At the same time, the annual real interest rate is predicted to drop from 3.7% to -0.3% over this period. Besides, we see that the change of the survival probability of retirees plays a dominant role in the decline in the relative consumption ratio and the real interest rate.

[22] This ratio is smaller than unity over time. This is consistent with Fernández-Villaverde and Krueger (2007): Consumption per worker is higher than consumption per retiree.

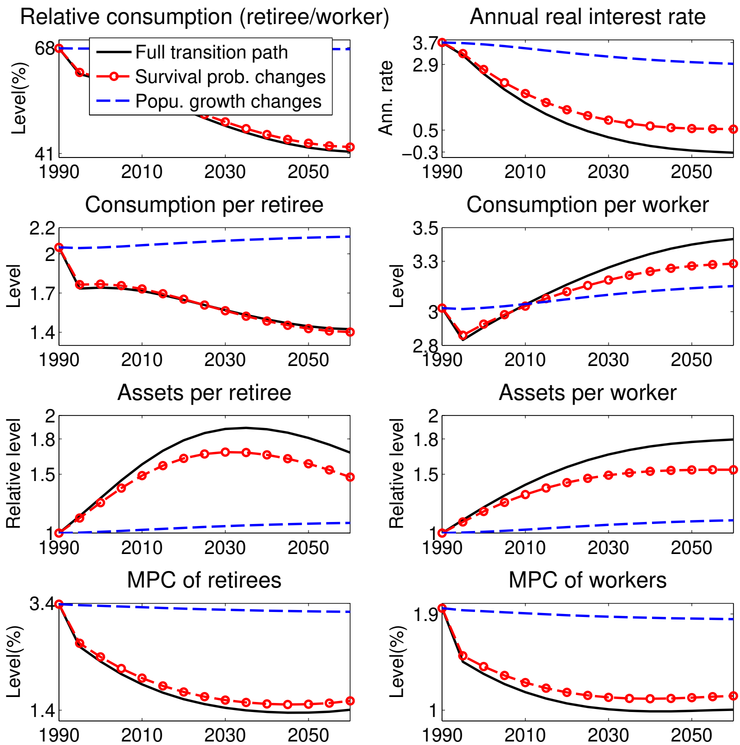

Figure 4.3: Simulations following different paths of demographic transition

Note: Solid line represents the full demographic transition; dotted, circle-hatched line represents the case with only the increase in survival probability of retirees; and dotted line represents the case with only the decrease in the labor force. Assets per retiree and assets per worker are expressed relative to their corresponding values in 1990.

The real interest rate declines because of the mismatch between the demand and supply of assets. Population aging suggests insufficient availability of labor and a sluggish demand of capital and assets. On the other side, both workers and retirees tend to increase their asset holdings, as shown in the second bottom panels. This naturally leads to an oversupply of assets. Assets per worker increase for two reasons: First, the real wage increases with the deepening of aging due to the relative scarcity of labor, so do labor supply and human wealth per worker, which induces workers to increase savings to smooth consumption over the life cycle. Second, workers save more in order to insure against a longer retirement period. For retirees, they also increase savings in order to support future consumption, as their effective discounting factor increases with the survival probability.

Notice that the increase in assets per retiree is higher than the increase in assets per worker almost for the whole period. Due to population aging, it should be just the opposite, since there are more retirees splitting the assets carried by workers when the dependency ratio increases. This seemingly counterintuitive result is due to the decline in the real interest rate.

When the real interest rate falls, assets become less attractive as a value-preserving technology. This partially discourages workers' incentive to carry assets into retirement, which are used instead for workers' consumption. In fact, in the long run, the effect of the declining real interest rate dominates the incentive to carry assets into retirement, and results in an increase in consumption per worker, as shown in the second top panels. Without enough assets carried by workers, retirees decreases their consumption to self-insure. Thus, from the two bottom panels, we see that the MPC of retirees falls more than that of workers, with a 59 percentage decrease for retirees, from 3.4% to 1.4%, and a 47 percentage decrease for the latter, from 1.9% to 1.0%. Therefore, consumption inequality between retirees and workers increases. With the deepening of population aging, the real interest rate is even likely to be pushed into negative territory, which will induce even higher consumption inequality between retirees and workers.

Counterfactual experiments (scenarios II and III) show that the increase in the survival probability is almost the only demographic reason for the increase in consumption inequality. It is responsible for almost all changes in the relative consumption ratio. The reason is that the variation of population growth is relatively small in size over the whole period, thus quantitatively unimportant for the increasing dependency ratio[23] and the declining real interest rate.

[23] In Appendix C.3, I carry out another decomposition exercise which illustrates the relative importance of population growth rate and the survival probability on dependency ratio. The result shows that the change in the survival probability plays a dominant role.

4.4.2 The asymmetric dynamic effects

To investigate the distributional effect of the ZLB, I consider two types of unexpected shocks: a positive preference shock and an expansionary monetary policy shock, which can force the nominal interest against the ZLB. I find that in both cases the asymmetric effects of shocks on workers and retirees are mitigated in dynamics, as retirees fully rely on the return of assets for consumption, therefore can benefit from the binding of the ZLB.

4.4.2.1 Preference shock

Figure 4.4 presents the impulse response functions (IRFs) to an unexpected positive preference shock[24], which brings the subjective discount factor β from an annual rate of 0.96 up to 1.00 in period 6, with an increase of about 4%. The shock size generates enough contraction of output and inflation, resulting in three quarters during which the ZLB binds.

The demographic structure is kept the same as that of the U.S. in 1990. Most of the variables are reported in percentage deviation from steady state, except for interest rates and inflation, which are in absolute deviation. In the IRFs, I use the solid lines to represent the case when the central bank follows a Taylor rule and the dotted lines when it follows a truncated Taylor rule.

For the time being, I ignore the constraint imposed by the ZLB. An increase in the discount factor makes households become more patient and postpone their consumption. In the short run, consumption declines immediately, both for workers and retirees. Output drops instantaneously due to either the sluggish response of investment or sticky prices. The asset holdings of both agents increase to support more future consumption. Firms adjust downward their labor demand because capital is inelastic in the short run. Decline in the prices of inputs follows, so does inflation. The increase in the discount factor can be interpreted as a contractionary demand shock since it induces decline in both

[24] I use the toolkit proposed by Guerrieri and Iacoviello (2015), "occbin" , to solve the DSGE model with a binding constraint. The IRFs, which compare the effects of shocks (a positive preference shock and an expansionary monetary policy shock) under different demographic structures, are available in Appendix C.4.1. In addition, I also present this comparison under the assumption of a variable labor supply of retirees, which is used in Fujiwara and Teranishi (2008), in Appendix C.4.2.

output and inflation. The central bank follows a Taylor rule and decreases the nominal interest rate to mitigate the effects of this contractionary shock.

Figure 4.4: Impulse responses to a positive preference shock

Note: Solid lines represents the case without a binding ZLB constraint and dotted lines represents the case with a binding ZLB constraint. The shock is an unexpected increase in β *and occurs in period 6.*

When the central bank follows a truncated Taylor rule, the downward adjustment of the nominal interest rate is partially hindered as it cannot be pushed into negative territory. With the presence of sticky prices, the equilibrium allocations of real variables depend on monetary policy and the presence of the ZLB influences real allocations. The real interest rate increases, so does the rental price of capital via the no arbitrage condition. The demand for capital and labor decreases in comparison to the case when the ZLB is not binding, therefore output and consumption fall more in the short run. From the impulse responses, we see that output recovers with a lower speed, so do labor supply and the real wage. For workers, human wealth recovers more slowly, which results in an even lower adjustment of consumption. While for retirees, consumption actually jumps back faster due to the rigidity of the nominal interest rate.

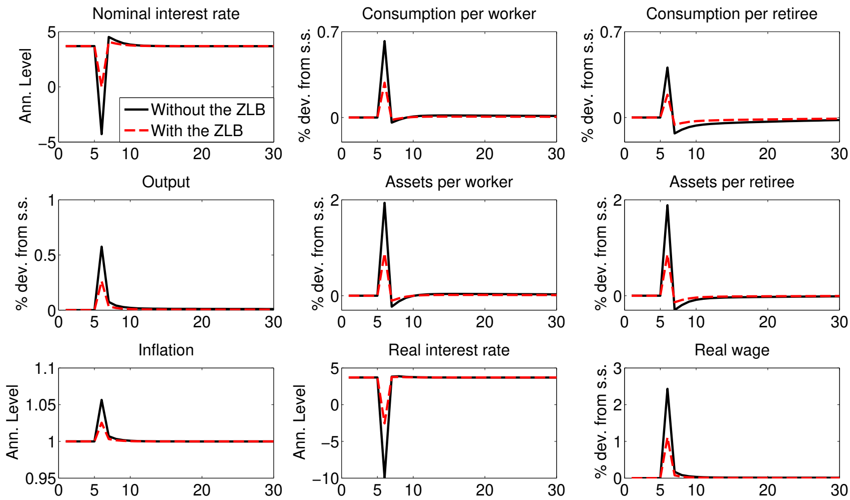

Figure 4.5: Impulse responses to an expansionary monetary policy shock

Note: It is an unexpected negative shock in the Taylor rule and occurs in period 6. See Figure 4.4 for further notes.

Therefore, with respect to a positive preference shock, workers experience a greater loss in consumption than retirees when the ZLB is binding. The reason lies in the negative impact of the ZLB on labor earnings. Its binding partially hinders the downward adjustment of the nominal interest rate, therefore gives rise to further reduction in labor demand and the real wage. Given that workers consume more than retirees in equilibrium, the presence of the ZLB mitigates consumption inequality with respect to a positive preference shock.

4.4.2.2 Monetary shock

Figure 4.5 presents the IRFs to an unexpected expansionary monetary policy shock, which brings the nominal interest rate from an annual rate of 3.7% down to -4.3% in period 6, with a decline of 8% in level. The shock size ensures that the IRFs with the ZLB binding are apparently different from those without the binding constraint. The demographic structure is kept the same as that of the U.S. in 1990.

Without the ZLB on the nominal interest rate, an expansionary monetary policy shock induces an instantaneous, sharp decrease in the nominal interest rate. Due to the inertia of inflation in the presence of nominal rigidity, inflation rate changes mildly. Investment, consumption, and output increase instantaneously via a lower real interest rate. Real wage and labor supply also increase in support of the increase in output, so does capital. Accordingly, asset holdings increase to clear the asset market.

After this monetary shock, the decline in investment is sluggish due to its adjustment costs, but output falls with a faster speed. This necessitates the reduction in consumption. From the IRF of consumption per retiree, we see that the consumption of retirees is lower than its steady state for several quarters after the monetary shock. While for workers, their consumption moves back to its steady state much faster, because real wage and labor supply do not fall back instantaneously after the shock with the sluggish adjustment of capital.

With the presence of the ZLB, the minimum of the nominal interest rate is zero, thus the effect of an expansionary monetary policy is limited. The real interest rate declines only mildly, which induces only mild changes in consumption and asset holdings. Therefore, the asymmetric effects of monetary shock are relatively small in size when the ZLB is binding, so does consumption inequality.

4.4.2.3 The effects of the ZLB under an older demographic structure

To verify the above results under an older demographic structure, I redo the analyses for an older population with a 15% higher dependency ratio[25]. The dependency ratio of the U.S. is 0.19 in 1990. With an increase of 15%, it adds up to 0.22, corresponding to the dependency ratio in 2015. The increase in dependency ratio is implemented by the corresponding increase in the survival probability of retirees. The sizes of preference shock and monetary shock remain the same.

Figure 4.6 reports the IRFs of the relative consumption ratio with and without the ZLB under two these different demographic structures, with the top panels for a positive preference shock and the bottom for an expansionary monetary policy shock. The left panels, corresponding to the younger population of

[25]Instead of considering the whole transition path from 1990 to 2060, for simplicity, I only compare the results under these two different demographic structures.

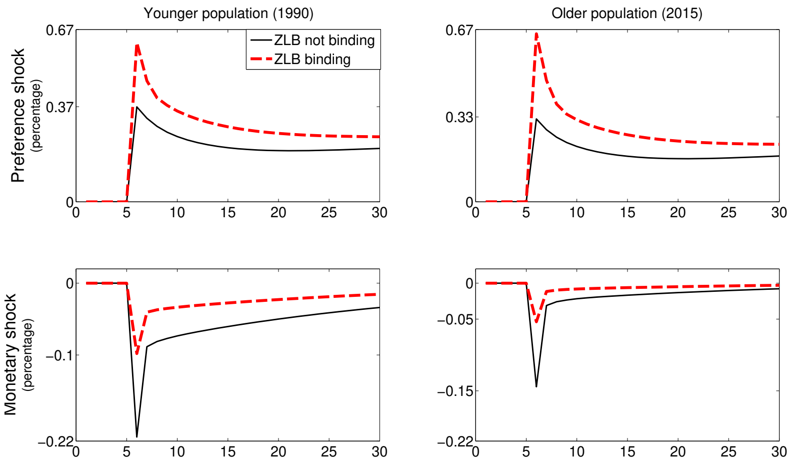

Figure 4.6: The IRFs of the relative consumption ratio to shocks under different demographic structures

Note: The top panels give the IRFs with respect to a preference shock, and the bottom panels with respect to a monetary shock. From the left panels to the right, population becomes older.

the U.S. in 1990, are just restating the main results in Figures 4.4 and 4.5 in terms of the relative consumption ratio. Comparing with their counterparts in the right panels, which correspond to that in 2015, we see that the mitigation effect of the ZLB is unchanged as population ages, that is, the presence of the ZLB can always mitigate consumption inequality between retirees and workers in dynamics.

In summary, either with respect to a preference shock or a monetary shock, the binding of ZLB can always decrease the asymmetric effects of shocks on retirees and workers in dynamics, and reduce the consumption inequality between them. Moreover, this result is robust to population aging.

4.4.3 Extension of retirement age

In this subsection, I discuss the robustness of the former results when the retirement age is extended, a direct way to mitigate the effects of aging. The mitigation might be weakened, especially in European countries, by a trend toward early retirement[26]. But research by Hairault, Sopraseuth, and Langot (2010) and Chéron, Hairault, and Langot (2013) show that the retirement decision of older workers depends on the distance to retirement. Because, with a short distance to retirement, older job searchers have a low continuation value of a job, thereby a low search effort. Then the extension of retirement age is expected to postpone retirement and neutralize the threat of aging.

I consider an extension of the retirement age to 67 in 2020 in the U.S. The probability of remaining as workers, ω, is changed[27] from $1.021^{-.25}$ to $1.02^{-.25}$. Dependency ratio becomes the ratio of the population over 67 to the new working population[28]. The growth rate of labor force remains the same, and the survival probability of retirees is backed out from the steady-state equation of dependency ratio in the same way as the benchmark specification.

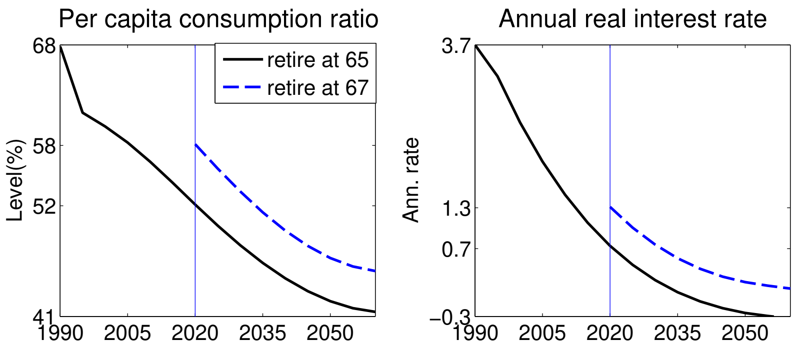

Figure 4.7: Transition paths of the relative consumption ratio and the real interest rate after an extension of retirement age ($65 \rightarrow 67$)

[26]This trend could be due to the insufficient adaptability of older workers required by technical progresses, but this is a debated issue. The empirical findings are rather mixed. See Hairault, Sopraseuth, and Langot (2010) for details.

[27]After the extension, the the annual rate of ω is $1-1/51$, and the quarterly rate is $(1-1/51)^{0.25}$.

[28]The new working age refers to people aged 15-66 years. A direct measure of the new dependency ratio is not available. I use the projection of age-specific population from the United Nations World Population Prospects (2017 Revision) to calculate this ratio. As the projected data are at 5-years intervals, we only have data of the age group of 65-69 years. I treat 40% of the age group of 65-69 years as working age population and the other 60% as retirees.

Figure 4.7 presents the comparison of the transition paths of the relative consumption ratio and the real interest rate under different retirement arrangements. The solid line represents the transition path when workers retire at age 65, and the dotted line at age 67 from 2020. We see that the transition path for the relative consumption ratio shifts upward after the extension, so does the transition path of the real interest rate, but the declining trend in both is still present. Therefore, the extension of retirement age can mitigate the effects of population aging, but does not change the increasing trend in consumption inequality[29].

Besides, I also recheck the distributional effect of the ZLB in dynamics when retirement age is extended to 67. I find that the extension of retirement age does not change the conclusion either: With respect to both types of shocks, the binding of ZLB can always decrease asymmetric effects and consumption inequality in dynamics.

4.5 Conclusion

This paper examines the relationship between population aging and consumption inequality between retirees and workers. The channels connecting these two are the two implications of population aging—a downward pressure on the real interest rate and the more frequent binding of the ZLB. The former makes assets less attractive as a value-preserving technology and partially discourages workers' incentive to carry assets into retirement, leading to an increasing transition path of consumption inequality. In my calibrated model, the relative consumption ratio, which is the ratio of consumption per retiree to consumption per worker, is predicted to fall from 0.68 in 1990 to 0.41 in 2060, with a 40% decline for the U.S. The latter decreases the asymmetric effects of shocks (preference and monetary shocks) on retirees and workers, and reduces the consumption inequality between them. Because the real return on assets decreases less with the presence of the ZLB, which particularly benefits retirees.

My conclusions about the predicted transition path depend heavily on the argument for the adjustment of asset holdings per worker to changes in the real

[29]Theoretically, a series of continuous and further extensions of retirement age is likely to fully cancel out the declining trend of the real interest rate, thereby counteracting the increasing consumption inequality in transition.

interest rate. Therefore, one urgent future endeavor would be to find empirical evidence on this asset adjustment due to population aging or changes in labor market policies. A further task is the empirical identification of the relationship between the relative consumption ratio and population aging[30].

[30]The data of consumption per worker and consumption per retiree over time are not easily available. The difficulty in identifying the empirical relationship makes this theoretical research more valuable.

Appendix A

Appendices to Chapter 2

A.1 Specification of Equation (2.1)

With log-transformation of the level equation, Equation (2.1) becomes

$$\log\, u_t = \rho \log\, u_{t-1} + e^{\sigma_t} v_t, \tag{A.1}$$

together with Equation (2.2), we have

$$u_t = (u_{t-1})^{\rho} exp\{exp[(1-\rho^{\sigma})\bar{\sigma} + \rho^{\sigma}\sigma_{t-1}]exp(\eta\varepsilon_t)v_t\}. \tag{A.2}$$

From Proposition 1 in Andreasen (2010), we have

$$E(exp[exp(\eta\varepsilon_t)v_t]) \to \infty,$$

as its corresponding probability density function

$$g(\varepsilon_t, v_t) \equiv \frac{1}{2\pi} exp[e^{\eta\varepsilon_t} v_t - 0.5(\varepsilon_t)^2 - 0.5(v_t)^2], \tag{A.3}$$

is an increasing function of ε_t for certain range of ε_t , η and v_t[1]. Therefore, the moment $E(exp[exp(\eta\varepsilon_t)v_t])$ does not exist. This means that the expected unemployment rate goes to infinity. Clearly this is impossible to justify.

Similarly if we assume log $(1-u_{it})$ follows an AR(1) in the level equation,

$$\log\,(1-u_t) = \rho \log\,(1-u_{t-1}) + e^{\sigma_t} v_t, \tag{A.4}$$

[1] For example, if all three are bigger than unity.

from which we can get the time-varying percentage standard deviation of employment rate, we will encounter the same problem.

A.2 Data sources

The age-specific unemployment rate data are from OECD Labour Force Statistics database at an annual frequency. These are available from the age of 15 years with a 5-years interval. The main explanatory variable, the share of each age group in the labor force, is also from this database. Besides, I also use the share of population and the share of native population for each age group to deal with potential endogeneity problems. For the former, I use the population size of each age group from the OECD Demography and Population database[2]; for the latter, I use the data of birth rate for each countries from Mitchell (2008).

Data for union density are from the OECD database trade union section since 1960. Data for union centralization of wage bargaining are from the ICTWSS database (Visser (2016), 2016) since 1960 for 20 OECD countries. Data for the strictness of employment protection legislation are available from 1960 to 1995 in Blanchard and Wolfers (2000) and from 1985 to 2013 in OECD Employment Protection Legislation, but with different scales. I take the one from Blanchard and Wolfers (2000) as the benchmark measure and extrapolate them using the second series, such that the annual percentage change from 1996 in the extrapolated data is the same as that in the other. Data for tax wedge are from Nickell (2006) CEP-OECD institutions database since 1960 for 20 OECD countries, but particularly unbalanced. I extrapolate these data until 2007 using the tax wedge data from Bassanini and Duval (2006) and OECD Tax Statistics. Data for gross replacement rate are from the benefits and wages section of the OECD database since 1960.

[2]Under the link: https://stats.oecd.org/Index.aspx?DataSetCode=POP_PROJ#

A.3 Summary statistics of key variables

Table A.1: Unemployment volatility and demographic statistics

	Young 15-24	Prime-age 25-54	Old 55-64
Unemployment volatility	1.71	0.71	0.76
Share of labor force	0.18	0.71	0.12
Share of population	0.23	0.61	0.16
Share of native population	0.17	0.60	0.23

Notes: Sample period: 1960-2007.

A.4 Time-varying unemployment volatility by age group for other sixteen countries

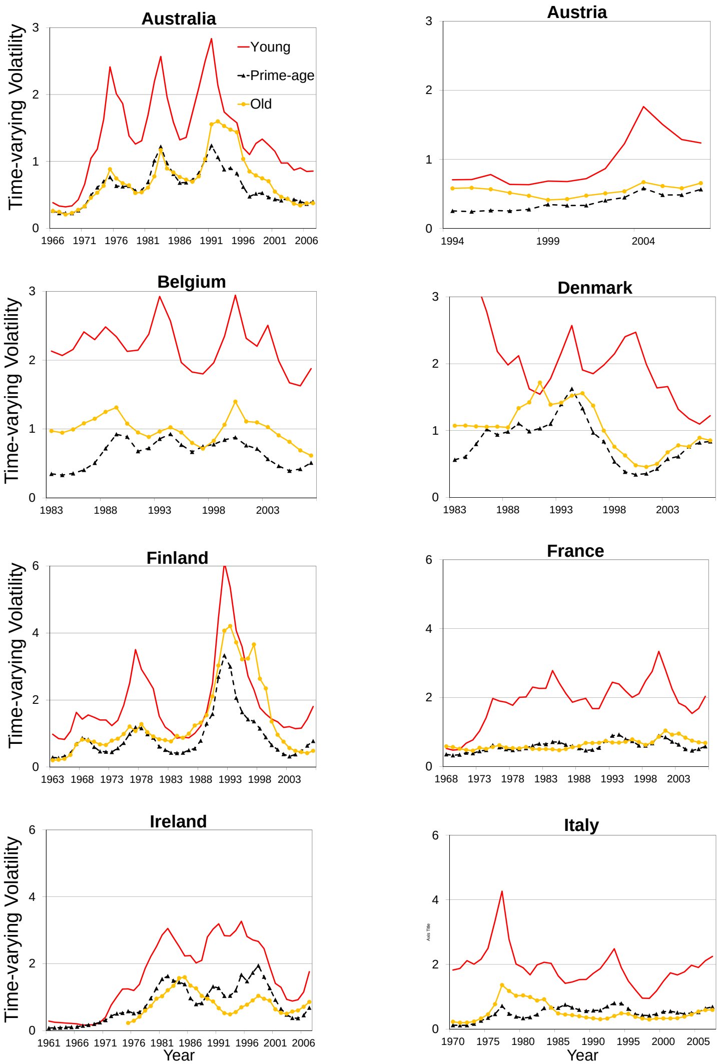

Figure A.1: Time-varying unemployment volatility by age group (1/2)

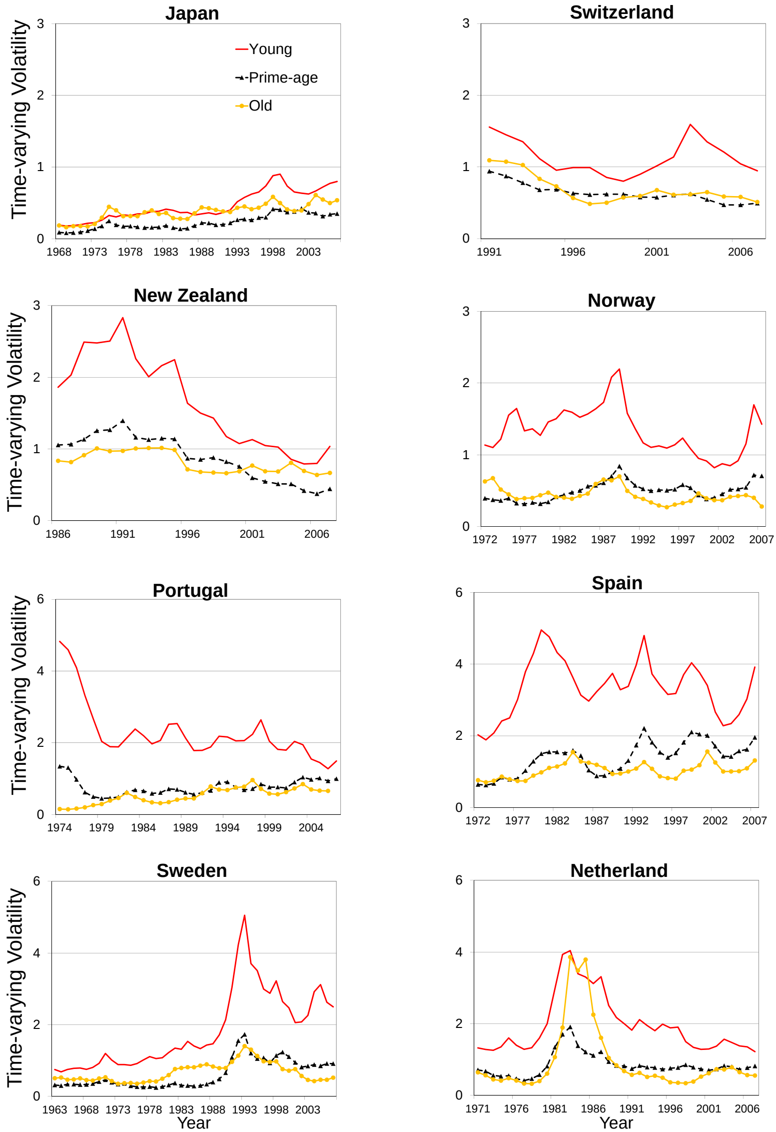

Figure A.2: Time-varying unemployment volatility by age group (2/2)

Notes: Solid line is for young workers (15-24); dotted, triangle-hatched line is for prime-age workers (25-54); and solid, square-hatched line is for old workers (55-64).

A.5 Calculation of test statistics

Panel MW test refers to the panel Maddala and Wu (MW) unit root test. The test statistic is defined as $-2\sum_{i=1}^{N} ln(p_i)$, where p_i is the p-value of ADF test for country i. For the country-specific ADF test, it has a constant term and its lag length is selected by the Akaike information criterion.

Average correlation refers to the average cross-sectional correlation coefficient. It is defined as $2/(N(N-1))\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho_{ij}}$, where $\hat{\rho_{ij}}$ is the cross-sectional correlation coefficient between country i and country j. For unbalanced panel, according to Pesaran (2004), the calculation of $\hat{\rho_{ij}}$ should also take care of the fact that the residuals for subsets of t are not necessarily mean zero.

CD test is the Pesaran (2004) cross-sectional dependence (CD) test, which is defined as $\sqrt{2/(N(N-1))}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\sqrt{T_{ij}}\hat{\rho_{ij}}$, where T_{ij} is the number of common time-series observations between country i and j.

MW test refers to the panel Maddala and Wu (MW) cointegration test. The test statistic is defined in a similar way as that of a panel unit root test. Now the p-values are from the ADF unit root tests on the residuals of the model.

A.6 The role of labor market institutions in the unemployment volatility of young workers

Table A.2: Unemployment volatility and the share of labor force: young workers - with the coefficients of LMIs

Model	CCEP
Youth share	8.97***
	(3.43)
Union density	0.14***
	(0.02)
Union centralization	-0.61
	(0.90)
Employment protection legislation	-1.05***
	(0.24)
Tax wedge	0.04***
	(0.01)
Gross replacement rate	-0.01
	(0.01)
Demand shock	0.42**
	(0.19)
Nobs	644
R^2	0.77
CD test	0.60
	[0.55]

Notes: Sample period: 1960-2007, unbalanced panel of 20 OECD countries. The dependent variable is the unemployment volatility of young workers.
See Tables 3 and 4 for further details.

A.7 CCEMG estimators for prime-age and old workers

Table A.3: Unemployment volatility and the share of labor force: prime-age and old workers

Age-specific share	native population share (prime-age)	native population share (old)
Coef. of share	4.22	-2.04
	(6.46)	(4.51)
Observations	627	627
CD test	-0.49	-1.16
	[0.62]	[0.24]

Notes: Sample period: 1970-2007, unbalanced panel of 20 OECD countries. Model used in this table is the common correlated effects mean group (CCEMG) estimator.

A.8 Redefinition of age group: the young (15-29)

Table A.4: Unemployment volatility and the share of labor force: young workers (15-29)

Model	CCEP		
Age-specific share	labor force share (young)	population share (young)	native population share (young)
Coef. of share	5.45***	4.73***	4.87**
	(1.62)	(1.42)	(1.39)
LMIs & shock	Yes	Yes	Yes
FETE	Yes	Yes	Yes
Nobs	658	658	627
CD test	[0.58]	[0.57]	[0.43]

Notes: Sample period: 1970-2007, unbalanced panel of 20 OECD countries. The dependent variable is aggregate unemployment volatility.

Appendix B

Appendices to Chapter 3

B.1 Participation's share in unemployment volatility (with different age groups categorization)

Table B.1: Decomposition of unemployment volatility: participation's share

	Female		Male		Both	
	15-24	15-64	15-24	15-64	**15-24**	15-64
Cov. excl.	1.87	0.39	0.40	0.05	**0.81**	0.10
Cov. incl.	4.72	1.68	2.38	1.30	**2.82**	1.19

Notes: Robustness check for Table 1. See Table 1 for further notes.

B.2 Unemployment volatilities of other age groups and demographic changes

Table B.2: Unemployment volatility and demographic changes: prime-age and old workers

	Labor force share	Population share	Labor force share	Population share
	Prime-age		Old	
Coefficient	-3.54	-1.30	3.26	3.57
	(2.25)	(2.17)	(3.73)	(3.04)
Nobs	658	658	658	658
R^2	0.80	0.80	0.79	0.79

Notes: Unbalanced panel of 20 OECD countries, sample period: 1960-2007. The dependent variable for Columns 1-2 is the unemployment volatility of prime-age workers (25-59 years old), and for Columns 3-4, it is the unemployment volatility of old workers (60-64).
See Table 2 for further notes.

B.3 Proof of Proposition 1.

Taking partial derivative of $P_t^Y = (Y_t)^{1-\sigma}\lambda_1(Y_t^Y)^{\sigma-1}$ with respect to S_t^Y gives

$$\begin{aligned}\frac{\partial P_t^Y}{\partial S_t^Y} &= (1-\sigma)\big[(Y_t)^{-\sigma}\lambda_1(Y_t^Y)^{\sigma-1}\frac{\partial Y_t}{\partial S_t^Y} - (Y_t)^{1-\sigma}\lambda_1(Y_t^Y)^{\sigma-2}\frac{\partial Y_t^Y}{\partial S_t^Y}\big] \\ &= (1-\sigma)P_t^Y(\frac{1}{Y_t}\frac{\partial Y_t}{\partial S_t^Y} - \frac{1}{Y_t^Y}\frac{\partial Y_t^Y}{\partial S_t^Y}),\end{aligned}$$

from which we see that the effect of demographic changes on the price of output produced by young workers is negatively related to its effect on output produced by young workers and positively related to its effect on final goods.

As the output produced by young workers can be written as

$$Y_t^Y = A_t n_t^Y S_t^Y E(a|a \geqslant \widetilde{a}_t^Y),$$

we have

$$\frac{\partial Y_t^Y}{\partial S_t^Y} = A_t\Big(n_t^Y E(a|a \geqslant \widetilde{a}_t^Y) + S_t^Y E(a|a \geqslant \widetilde{a}_t^Y)\frac{\partial n_t^Y}{\partial S_t^Y} + n_t^Y S_t^Y \frac{\partial E(a|a \geqslant \widetilde{a}_t^Y)}{\partial S_t^Y}\Big).$$

Besides, we have

$$\frac{\partial n_t^Y}{\partial S_t^Y} = -\frac{n_t^Y F'(\widetilde{a}_t^Y)}{1-F(\widetilde{a}_t^Y)}\frac{\partial \widetilde{a}_t^Y}{\partial S_t^Y}$$

and[1]

$$\frac{\partial E(a|a \geqslant \widetilde{a}_t^Y)}{\partial S_t^Y} = \Big[E(a|a \geqslant \widetilde{a}_t^Y) - \widetilde{a}_t^Y\Big]\frac{F'(\widetilde{a}_t^Y)}{1-F(\widetilde{a}_t^Y)}\frac{\partial \widetilde{a}_t^Y}{\partial S_t^Y},$$

thus $\frac{\partial Y_t^Y}{\partial S_t^Y}$ can be written as

$$\frac{\partial Y_t^Y}{\partial S_t^Y} = A_t\Big[n_t^Y E(a|a \geqslant \widetilde{a}_t^Y) + \widetilde{a}_t^Y S_t^Y \frac{\partial n_t^Y}{\partial S_t^Y}\Big].$$

Similarly, we can get the corresponding expression for $\frac{\partial Y_t^O}{\partial S_t^Y}$:

$$\frac{\partial Y_t^O}{\partial S_t^Y} = A_t\Big[-n_t^O E(a|a \geqslant \widetilde{a}_t^O) + \widetilde{a}_t^O(1-S_t^Y)\frac{\partial n_t^O}{\partial S_t^Y}\Big].$$

For $\frac{\partial Y_t}{\partial S_t^Y}$, as capital is also in inelastic supply in the short-run, we have

$$\begin{aligned}\frac{\partial Y_t}{\partial S_t^Y} &= (Y_t)^{1-\sigma}\Big[\lambda_1(Y_t^Y)^{\sigma-1}\frac{\partial Y_t^Y}{\partial S_t^Y} + (1-\lambda_1)\Omega_t(1-\lambda_2)(Y_t^O)^{\rho-1}\frac{\partial Y_t^Y}{\partial S_t^Y}\Big] \\ &= P_t^Y\frac{\partial Y_t^Y}{\partial S_t^Y} + P_t^O\frac{\partial Y_t^Y}{\partial S_t^Y}.\end{aligned}$$

[1]For the derivative of the conditional expectation, I used the fact that

$$\int_{\widetilde{a}^Y}^{\infty} a\frac{dF(a)}{1-F(\widetilde{a}^Y)} = \widetilde{a}^Y + \frac{\int_{\widetilde{a}^Y}^{\infty}[1-F(\widetilde{a}^Y)]da}{1-F(\widetilde{a}^Y)}$$

is true for any function F such that $F(\infty)=1$ and the Leibniz integral rule.

Plugging the equations for $\frac{\partial Y_t^Y}{\partial S_t^Y}$, $\frac{\partial Y_t^O}{\partial S_t^Y}$, and $\frac{\partial Y_t}{\partial S_t^Y}$ back into the equation for $\frac{\partial P_t^Y}{\partial S_t^Y}$, one gets[2]

$$\begin{aligned}\frac{\partial P_t^Y}{\partial S_t^Y} &= (1-\sigma)P_t^Y A_t\Big[\big(E(a|a \geqslant \widetilde{a}_t^Y)n_t^Y + \widetilde{a}_t^Y S_t^Y \frac{\partial n_t^Y}{\partial S_t^Y}\big)\big(\frac{1}{Y_t}P_t^Y - \frac{1}{Y_t^Y}\big) \\ &\quad -\frac{1}{Y_t}P_t^O n_t^O E(a|a \geqslant \widetilde{a}_t^O)\Big].\end{aligned}$$

From the optimization of final firm's problem, it is easy to see that $Y_t > P_t^Y Y_t^Y$, which leads to $\frac{1}{Y_t}P_t^Y A_t n_t^Y - \frac{1}{Y_t^Y}A_t n_t^Y = A_t n_t^Y(\frac{1}{Y_t}P_t^Y - \frac{1}{Y_t^Y}) < 0$. Besides, it is easy to see that $n_t^Y > S_t^Y \frac{\partial n_t^Y}{\partial S_t^Y}$, which means that the change in the employment rate due to demographic changes cannot be greater than itself. Therefore, we have $\frac{\partial P_t^Y}{\partial S_t^Y} < 0$, since $\sigma \in (0,1)$. ■

B.4 Proof of Proposition 2.

In the following, I try to identify the sign of $\frac{\partial \widetilde{a}^Y}{\partial S_t^Y}$. With free entry condition and wage equation, the asset value of a job for the young can be written as

$$J_t^Y(A_t, a_t) = (1-\eta)(P_t^Y A_t a_t - z) - \eta\varsigma^Y\theta_t^Y + \frac{\varsigma^Y}{q_t^Y}.$$

Besides, from $J_t^Y(A_t, \widetilde{a_t}^Y) = 0$, we can get the reservation productivity for the young explicitly as

$$\widetilde{a_t}^Y = \frac{1}{P_t^Y A_t}[z + \frac{1}{1-\eta}(\eta\varsigma^Y\theta_t^Y - \frac{\varsigma^Y}{q_t^Y})].$$

Taking derivative with respect to S_t^Y yields

$$\frac{\partial \widetilde{a}_t^Y}{\partial S_t^Y} = -\frac{\widetilde{a}_t^Y}{P_t^Y}\frac{\partial P_t^Y}{\partial S_t^Y} + \frac{\varsigma^Y}{P_t^Y A_t}\frac{1}{1-\eta}(\eta - \frac{\mu}{f_t^Y})\frac{\partial \theta_t^Y}{\partial S_t^Y}.$$

[2]Here I use the assumption that the employment rate of the old is irresponsive to the changes in the youth share.

which suggests the effect of the youth share on the reservation productivity of the young depends on not only its effect on the price of intermediate goods produced by young workers, but also its effect on the market tightness of young labor market. It is necessary to express the second effect with known expressions. After abstracting from aggregate productivity shock, the asset equation for young can be written as

$$S^{MY}(A,a) = P^Y Aa - z + \beta(1-\rho^x)(1-f^Y\eta)\int_{\widetilde{a}^Y}^{\infty} S^{MY}(A,a)dF(a),$$

and free entry condition as

$$\frac{\varsigma^Y}{q^Y} = \beta(1-\rho^x)(1-\eta)\int_{\widetilde{a}^Y}^{\infty} S^{MY}(A,a)dF(a).$$

As $S^{MY}(A,\widetilde{a}^Y) = 0$, subtracting $S^{MY}(A,\widetilde{a}^Y)$ from $S^{MY}(A,a)$, we get

$$S^{MY}(A,a) = P^Y A(a-\widetilde{a}^Y),$$

then free entry condition becomes

$$\frac{\varsigma^Y}{q^Y} = \beta(1-\rho^x)(1-\eta)AP^Y\int_{\widetilde{a}^Y}^{\infty}(a-\widetilde{a}^Y)dF(a).$$

Taking derivative with respect to S_t^Y yields

$$\frac{\partial\theta^Y}{\partial S_t^Y} = \frac{f^Y\beta(1-\rho^x)(1-\eta)A}{\varsigma^Y\mu}[(E(a|a>\widetilde{a}^Y)-(1-F(\widetilde{a}^Y))\widetilde{a}^Y)\frac{\partial P^Y}{\partial S_t^Y} - (1-F(\widetilde{a}^Y))P^Y\frac{\partial\widetilde{a}^Y}{\partial S_t^Y}].$$

Combining the above two equations with partial derivatives gives

$$\frac{\partial\widetilde{a}^Y}{\partial S_t^Y} = \frac{\beta(1-\rho^x)\Big[E(a|a>\widetilde{a}^Y) - \big(1-F(\widetilde{a}^Y)\big)\widetilde{a}^Y\Big] + \widetilde{a}^Y/(1-f^Y)}{P^Y M}\frac{\partial P^Y}{\partial S_Y},$$

where $M = -1/(1-f^Y) + \beta(1-\rho^x)\big(1-F(\widetilde{a}^Y)\big)$.

As the probability of a young worker being matched is lower than 1, we can get that $1/(1-f^Y) > 1$ and the second part of M is smaller than 1, thus it is easy to see that $M < 0$. Besides, the conditional expectation of idiosyncratic productivity is higher than the reservation productivity, thus, $E(a|a > \widetilde{a}^Y) -$

$(1-F(\widetilde{a}^Y))\widetilde{a}^Y > 0$. Together with the result from Proposition 1, which suggests that higher supply of young labor will drive down the price of intermediate goods produced by young, $\frac{\partial P^Y}{\partial S_Y} < 0$, we can conclude that the youth share has a positive effect on the reservation productivity of the young.■

B.5 Derivation of the equations for $\frac{\partial \ln \theta^Y}{\partial \ln A}$ and $\frac{\partial \ln u^Y}{\partial \ln A}$ in case of endogenous job separation

Part 1: $\frac{\partial \ln \theta^Y}{\partial \ln A}$

In steady state, the free entry condition becomes[3]

$$\frac{\varsigma^i}{q^i} = \beta(1-\rho^x)(1-\eta)\int_{\widetilde{a}^i}^{\infty} S^{Mi}(A,a)dF(a) = \beta(1-\rho^x)(1-\eta)P^iA\int_{\widetilde{a}^i}^{\infty}[1-F(a)]da. \tag{B.1}$$

Evaluating the equation for match surplus at $\widetilde{a}^i$ gives the condition for reservation productivity

$$P^iA(\widetilde{a}^i + \beta(1-\rho^x)\int_{\widetilde{a}^i}^{\infty}[1-F(a)]da) = z^i + \frac{\eta\theta^i\varsigma^i}{1-\eta}. \tag{B.2}$$

Equations (B.1) and (B.2) uniquely determine the steady values of market tightness, θ^i, and reservation productivity, $\widetilde{a}^i$. With these two equations, I can derive the comparative statics for the responsiveness of market tightness with respect to aggregate productivity.

From Equation (B.1), we have

$$\frac{\varsigma^Y}{q^Y} = \beta(1-\rho^x)(1-\eta)P^YA\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da. \tag{B.3}$$

Taking log form and totally differentiating with respect to $\ln A$ gives

$$\frac{\partial \ln \theta^Y}{\partial \ln A}\mu = 1 + \frac{\partial \ln P^Y}{\partial \ln A} - \frac{[1-F(\widetilde{a}^Y)]\widetilde{a}^Y}{\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da}\frac{\partial \ln \widetilde{a}^Y}{\partial \ln A}. \tag{B.4}$$

[3]The second equality follows from integration by parts for any function such that $F(\infty) = 0$ and the condition $\frac{\partial S_t^i(A_t,a_t)}{\partial a_t} = P_t^iA_t$.

From Equation (B.2), we have

$$P^Y A\{\widetilde{a}^Y + \beta(1-\rho^x)\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da\} = z + \frac{\eta\theta^Y\varsigma^Y}{1-\eta}. \tag{B.5}$$

Taking log form and totally differentiating with respect to $\ln A$ gives

$$\begin{aligned}(1+\tfrac{\partial \ln P^Y}{\partial \ln A})\{\widetilde{a}^Y + \beta(1-\rho^x)\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da\}+\\ \tfrac{\partial \ln \widetilde{a}^Y}{\partial \ln A}\widetilde{a}^Y\{1-\beta(1-\rho^x)[1-F(\widetilde{a}^Y)]\} = \tfrac{\partial \ln\theta^Y}{\partial \ln A}\tfrac{\eta\theta^Y\varsigma^Y}{P^Y A(1-\eta)}.\end{aligned} \tag{B.6}$$

Combining Equations (B.3) and (B.4) to substitute out $\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da$, and the same for Equations (B.5) and (B.6), then solving for $\frac{\partial \ln\theta^Y}{\partial \ln A}$ and get

$$\frac{\partial \ln\theta^Y}{\partial \ln A} = (\frac{\partial \ln P^Y}{\partial \ln A}+1)\frac{\frac{z(1-\eta)\beta(1-\rho^x)\widetilde{f}^Y}{\varsigma^Y\theta^Y} + 1 - \beta(1-\widetilde{\rho}^Y) + \eta\beta(1-\rho^x)\widetilde{f}^Y}{\mu[1-\beta(1-\widetilde{\rho}^Y)] + \eta\beta(1-\rho^x)\widetilde{f}^Y}.$$

Denote $\tilde{z}^Y \equiv z/(P^Y A\int_{\widetilde{a}^Y}^{\infty} a\frac{dF(a)}{1-F(\widetilde{a}^Y)})$ and $\widetilde{f}^Y \equiv f^Y[1-F(\widetilde{a}^Y)]$, and use the fact that[4] $\int_{\widetilde{a}^Y}^{\infty} a\frac{dF(a)}{1-F(\widetilde{a}^Y)} = \widetilde{a}^Y + \frac{\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da}{1-F(\widetilde{a}^Y)}$, then $\tilde{z}^Y$ becomes

$$\tilde{z}^Y = \frac{z}{\widetilde{a}^Y + \frac{F(\widetilde{a}^Y)\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da}{1-F(\widetilde{a}^Y)} + \int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da}.$$

Using Equation (B.3) to substitute out the first integral in the denominator and Equation (B.5) to substitute out the second integral, then get

$$\tilde{z}^Y = \frac{z\beta(1-\rho^x)\widetilde{f}^Y(1-\eta)}{P^Y A\widetilde{a}^Y\widetilde{f}^Y(1-\eta)[\beta(1-\rho^x)-1] + \theta^Y\varsigma^Y - (\widetilde{f}^Y\eta - [1-F(\widetilde{a}^Y)])\theta^Y\varsigma^Y + z\widetilde{f}^Y(1-\eta)}.$$

[4] Which can be obtained from the integral of $\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da$ by parts. In fact, there is a limit for the upper bound, as a follows a lognormal distribution.

Combining Equations (B.3) and (B.5) to get

$$(\widetilde{f}^Y \eta - [1 - F(\widetilde{a}^Y)])\theta^Y \varsigma^Y = (P^Y A \widetilde{a}^Y - z)(1 - \eta)\widetilde{f}^Y.$$

Substituting this term into the expression of $\tilde{z}^Y$ to get

$$\frac{z(1-\eta)\beta(1-\rho^x)\widetilde{f}^Y}{\varsigma^Y \theta^Y} = \frac{\tilde{z}^Y \beta(1-\rho^x)(\widetilde{f}^Y \eta - [1 - F(\widetilde{a}^Y)]) + \tilde{z}^Y}{1 - \tilde{z}^Y}.$$

After the substitution of this term into the expression of $\frac{\partial \ln \theta^Y}{\partial \ln A}$, it can be simplified as

$$\frac{\partial \ln \theta^Y}{\partial \ln A} = \frac{1 - \beta(1 - \widetilde{\rho}^Y) + \eta\beta(1-\rho^x)\widetilde{f}^Y}{\mu[1 - \beta(1 - \widetilde{\rho}^Y)] + \eta\beta(1-\rho^x)\widetilde{f}^Y} \frac{\frac{\partial \ln P^Y}{\partial \ln A} + 1}{1 - \tilde{z}^Y}.$$

Part 2: $\frac{\partial \ln u^Y}{\partial \ln A}$

Notice that the elasticity of the overall job-finding rate with respect to aggregate productivity can be written as

$$\begin{aligned} \frac{\partial \ln \tilde{f}^Y}{\partial \ln A} &= \frac{\partial \ln([1 - F(\widetilde{a}^Y)] f^Y)}{\partial \ln A} \\ &= (1-\mu)\frac{\partial \ln \theta^Y}{\partial \ln A} - \frac{f(\widetilde{a}^Y)\widetilde{a}^Y}{1 - F(\widetilde{a}^Y)} \frac{\partial \ln \widetilde{a}^Y}{\partial \ln A}, \end{aligned}$$

and the elasticity of the overall separation rate with respect to aggregate productivity can be written as

$$\begin{aligned} \frac{\partial \ln \tilde{\rho}^Y}{\partial \ln A} &= \frac{\partial \ln [\rho^x + (1-\rho^x) F(\widetilde{a}^Y)]}{\partial \ln A} \\ &= \frac{(1-\rho^x) f(\widetilde{a}^Y)\widetilde{a}^Y}{\rho^x + (1-\rho^x)F(\widetilde{a}^Y)} \frac{\partial \ln \widetilde{a}^Y}{\partial \ln A}. \end{aligned}$$

With endogenous separation, employment rate at each segmented labor market evolves according to

$$n_t^Y = (1 - \tilde{\rho}^Y)(n_{t-1}^Y + \tilde{f}_{t-1}^Y u_{t-1}^Y).$$

Therefore the steady state value for unemployment rate is

$$u^Y = \frac{\tilde{\rho}^Y}{\tilde{\rho}^Y + (1-\tilde{\rho}^Y)\tilde{f}^Y}.$$

Thus the elasticity of unemployment with respect to aggregate productivity is

$$\begin{aligned}
\frac{\partial \ln u^Y}{\partial \ln A} &= \frac{\tilde{f}^Y}{\tilde{\rho}^Y + (1-\tilde{\rho}^Y)\tilde{f}^Y}\frac{\partial \ln \tilde{\rho}^Y}{\partial \ln A} - \frac{(1-\tilde{\rho}^Y)\tilde{f}^Y}{\tilde{\rho}^Y + (1-\tilde{\rho}^Y)\tilde{f}^Y}\frac{\partial \ln \tilde{f}^Y}{\partial \ln A} \\
&= \frac{\tilde{f}^Y}{\tilde{\rho}^Y + (1-\tilde{\rho}^Y)\tilde{f}^Y}\{f(\widetilde{a}^Y)\widetilde{a}^Y[(1-\rho^x)(1+\frac{1}{\tilde{\rho}^Y})]\frac{\partial \ln \widetilde{a}^Y}{\partial \ln A} \\
&\quad -(1-\tilde{\rho}^Y)(1-\mu)\frac{\partial \ln \theta^Y}{\partial \ln A}\}. \blacksquare
\end{aligned}$$

B.6 (Linearized) equation system for model with endogenous separation

1. Euler equation:

$$c_t^{-\tau} = \beta E_t[c_{t+1}^{-\tau}(r_{t+1}+1-\delta)]$$

$$0 = E_t[\tau(\hat{c}_t - \hat{c}_{t+1}) + \frac{r}{r+1-\delta}\hat{r}_{t+1}]$$

2. Output of intermediate goods:

$$Y_t^Y = A_t n_t^Y S^Y \int_{\widetilde{a}_t^Y}^{\infty} a \frac{dF(a)}{1-F(\widetilde{a}_t^Y)}$$

$$Y_t^O = A_t n_t^O (1-S^Y) \int_{\widetilde{a}_t^O}^{\infty} a \frac{dF(a)}{1-F(\widetilde{a}_t^O)}$$

$$\hat{Y}_t^i = \hat{A}_t + \hat{n}_t^i + \frac{H'(\widetilde{a}^i)\widetilde{a}^i}{H(\widetilde{a}^i)}\hat{\widetilde{a}}_t^i$$

where $H(\widetilde{a}^i) \equiv \int_{\widetilde{a}^i}^{\infty} a \frac{dF(a)}{1-F(\widetilde{a}^i)}$ and $H'(\widetilde{a}^i) = \frac{dH(\widetilde{a}^i)}{d\widetilde{a}^i} = \frac{f(\widetilde{a}^i)[H(\widetilde{a}^i)-\widetilde{a}^i]}{1-F(\widetilde{a}^i)}$[5]

3. Output of final good:

$$Y_t = [\lambda_1 (Y_t^Y)^\sigma + (1-\lambda_1)(\lambda_2 (K_t)^\rho + (1-\lambda_2)(Y_t^O)^\rho)^{\sigma/\rho}]^{1/\sigma}$$

$$\hat{Y}_t = \lambda_1 (\frac{Y^Y}{Y})^\sigma \hat{Y}_t^Y + \frac{1-\lambda_1 (Y^Y/Y)^\sigma}{\lambda_2 (K)^\rho + (1-\lambda_2)(Y^O)^\rho} (\lambda_2 K^\rho \hat{K}_t + (1-\lambda_2)(Y^O)^\rho \hat{Y}_t^O)$$

4. Price of intermediate goods produced by young:

$$P_t^Y = (Y_t)^{1-\sigma} \lambda_1 (Y_t^Y)^{\sigma-1}$$

$$\hat{P}_t^Y = (1-\sigma)\hat{Y}_t + (\sigma-1)\hat{Y}_t^Y$$

5. Price of intermediate goods produced by old:

$$P_t^O = (Y_t)^{1-\sigma}(1-\lambda_1)\Omega_t (1-\lambda_2)(Y_t^O)^{\rho-1}$$

$$\hat{P}_t^O = (1-\sigma)\hat{Y}_t + \hat{\Omega}_t + (\rho-1)\hat{Y}_t^O$$

where $\hat{\Omega}_t = \frac{\sigma-\rho}{\lambda_2 (K)^\rho + (1-\lambda_2)(Y^O)^\rho}(\lambda_2 K^\rho \hat{K}_t + (1-\lambda_2)(Y^O)^\rho \hat{Y}_t^O)$.

6. Price of capital:

$$r_t = (Y_t)^{1-\sigma}(1-\lambda_1)\Omega_t \lambda_2 (K_t)^{\rho-1}$$

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[5] As $H(\widetilde{a}^i) \equiv \int_{\widetilde{a}^i}^{\infty} a \frac{dF(a)}{1-F(\widetilde{a}^i)}$, $H'(\widetilde{a}^i) = \frac{d\int_{\widetilde{a}^i}^{\infty} a\, dF(a)/d\widetilde{a}^i}{1-F(\widetilde{a}^i)} - \frac{\int_{\widetilde{a}^i}^{\infty} a\, dF(a)}{(1-F(\widetilde{a}^i))^2}(-f(\widetilde{a}^i))$. With Leibniz integral rule, it is easy to see that:

$$d\int_{\widetilde{a}^i}^{\infty} a\, dF(a)/d\widetilde{a}^i = -f(\widetilde{a}^i)\widetilde{a}^i$$

therefore,

$$\begin{aligned} H'(\widetilde{a}^i) &= \frac{-f(\widetilde{a}^i)\widetilde{a}^i}{1-F(\widetilde{a}^i)} + \frac{f(\widetilde{a}^i)H(\widetilde{a}^i)}{1-F(\widetilde{a}^i)} \\ &= \frac{f(\widetilde{a}^i)[H(\widetilde{a}^i)-\widetilde{a}^i]}{1-F(\widetilde{a}^i)} \end{aligned}$$

.

$$r_t = (1-\sigma)\hat{Y}_t + \hat{\Omega}_t + (\rho - 1)\hat{K}_t$$

7. Aggregate resource constraint:

$$c_t + K_{t+1} = Y_t + (1-\delta)K_t - v_t^Y S^Y \varsigma^Y - v_t^O (1-S^Y)\varsigma^O$$

$$c\hat{c}_t + K\hat{K}_{t+1} = Y\hat{Y}_t + (1-\delta)K\hat{K}_t - S^Y \varsigma^Y v^Y \hat{v}_t^Y - (1-S^Y)\varsigma^O v^O \hat{v}_t^O$$

8. Evolution of employment rate:

$$n_t^i = (1-\rho_t^i)(n_{t-1}^i + \phi^i (v_{t-1}^i)^{1-\mu}(u_{t-1}^i)^{\mu})$$

$$\hat{n}_t^i + \frac{\rho^i}{1-\rho^i}\hat{\rho}_t^i = (1-\rho^i)\hat{n}_{t-1}^i + \rho^i(1-\mu)\hat{v}_{t-1}^i + \rho^i \mu \hat{u}_{t-1}^i)$$

9. Evolution of unemployment rate:

$$u_t^i = 1 - n_t^i$$

$$u^i \hat{u}_t^i = -n^i \hat{n}_t^i$$

10. Labor market tightness:

$$\theta_t^i = v_t^i / u_t^i$$

$$\hat{\theta}_t^i = \hat{v}_t^i - \hat{u}_t^i$$

11. Overall separation rate:

$$\rho_t^i = \rho^x + (1-\rho^x)F(\widetilde{a}_t^i)$$

$$\hat{\rho}_t^i = \frac{(1-\rho^x)\widetilde{a}^i f(\widetilde{a}^i)}{\rho^i}\hat{\widetilde{a}}_t^i$$

12. Overall job-finding rate:

$$fr_t^i = (1-\rho^x)[1 - F(\widetilde{a}_{t+1}^i)]\phi^i \theta_t^{i\,1-\mu}$$

$$\hat{fr}_t^i = -\frac{\widetilde{a}^i f(\widetilde{a}^i)}{1-F(\widetilde{a}^i)}\hat{\widetilde{a}}_{t+1}^i + (1-\mu)\hat{\theta}_t^i$$

13. Reservation productivity:

$$\widetilde{a_t}^i = \frac{1}{P_t^i A_t}[z + \frac{\varsigma^i}{1-\eta}(\eta\theta_t^i - (\theta_t^i)^\mu/\phi^i)]$$

$$\hat{\widetilde{a}}_t^i = -\hat{P}_t^i - \hat{A}_t + \frac{\frac{\varsigma^i}{1-\eta}(\eta\theta^i - \mu(\theta_t^i)^\mu/\phi^i)}{z + \frac{\varsigma^i}{1-\eta}(\eta\theta^i - (\theta^i)^\mu/\phi^i)}\hat{\theta}_t^i$$

14. Free entry condition in differential equation:

$$\begin{aligned}
\frac{\varsigma^i(\theta_t^i)^\mu}{\phi^i} \quad &= \quad \beta E_t\{\frac{c_{t+1}^{-\tau}}{c_t^{-\tau}}(1-\rho_{t+1}^i)[(1-\eta)P_{t+1}^i A_{t+1} H(\widetilde{a}_{t+1}^i) \\
&\quad -\varsigma^i\eta\theta_{t+1}^i - (1-\eta)z + \frac{\varsigma^i(\theta_{t+1}^i)^\mu}{\phi^i}]\}
\end{aligned}$$

$$\begin{aligned}
\mu\hat{\theta}_t^i - \tau\hat{c}_t \quad &= \quad -\tau E_t\hat{c}_{t+1} - \frac{\rho^i}{1-\rho^i}E_t\hat{\rho}_{t+1}^i + \frac{\varsigma^i(\mu(\theta^i)^\mu/\phi^i - \eta\theta^i)E_t\hat{\theta}_{t+1}^i}{B} \\
&\quad + \frac{(1-\eta)P^i A[H(\widetilde{a}^i)(E_t\hat{P}_{t+1}^i + E_t\hat{A}_{t+1}) + H'(\widetilde{a}^i)\widetilde{a}^i E_t\hat{\widetilde{a}}_{t+1}^i]}{B}
\end{aligned}$$

where $B = (1-\eta)P^i AH(\widetilde{a}^i) - \varsigma^i\eta\theta^i - (1-\eta)z + \varsigma^i(\theta^i)^\mu/\phi^i$.

15. Aggregate productivity shock:

$$ln\,A_t = \psi^A ln\,A_{t-1} + e_t^A$$

$$\hat{A}_t = \psi^A\hat{A}_{t-1} + e_t^A$$

16. Aggregate unemployment rate:

$$u_t = u_t^O(1-S^Y) + u_t^Y S^Y$$

$$u\hat{u}_t = u^O(1-S^Y)\hat{u}_t^O + u_t^Y S^Y\hat{u}_t^Y$$

17. Aggregate vacancies:

$$v_t = v_t^O(1-S^Y) + v_t^Y S^Y$$

$$v\hat{v}_t = v^O(1-S^Y)\hat{v}_t^O + v_t^Y S^Y \hat{v}_t^Y$$

18. Aggregate market tightness:

$$\theta_t = v_t/u_t$$

$$\hat{\theta}_t = \hat{v}_t - \hat{u}_t$$

19. Aggregate job-finding rate:

$$fr_t = [u_t^O(1-S^Y)fr_t^O + u_t^Y S^Y fr_t^Y]/u_t$$

$$\hat{fr}_t = \frac{u_t^O(1-S^Y)fr_t^O(\hat{fr}_t^O + \hat{u}_t^O) + u_t^Y S^Y fr_t^Y(\hat{fr}_t^Y + \hat{u}_t^Y)}{u_t^O(1-S^Y)fr_t^O + u_t^Y S^Y fr_t^Y} - \hat{u}_t$$

20. Aggregate separation rate[6]:

$$\rho_t = 1-(1-u_t)/[(1-S^Y)(n_{t-1}^O + f_{t-1}^O u_{t-1}^O) + S^Y(n_{t-1}^Y + f_{t-1}^Y u_{t-1}^Y)]$$

$$\begin{aligned} -\frac{\tilde{\rho}}{1-\tilde{\rho}}\hat{\rho}_t^i \quad &= \quad -\frac{u}{1-u}\hat{u}_t - \frac{(1-S^Y)[n^O\hat{n}_{t-1}^O + \tilde{f}^O u^O((1-\mu)\hat{\theta}_{t-1}^O + \hat{u}_{t-1}^O)]}{(1-S^Y)(n^O + f^O u^O) + S^Y(n^Y + f^Y u^Y)} \\ &\quad -\frac{S^Y[n^Y\hat{n}_{t-1}^Y + f^Y u^Y((1-\mu)\hat{\theta}_{t-1}^Y + \hat{u}_{t-1}^Y)]}{(1-S^Y)(n^O + f^O u^O) + S^Y(n^Y + f^Y u^Y)} \end{aligned}$$

B.7 Bellman equations for model with exogenous job separation

The Bellman equation for the asset value of a job in term of final good is

$$J_t^i = P_t^i A_t - \omega_t^i + (1-\rho^x)E_t\beta_{t,t+1}J_{t+1}^i.$$

The value of a vacancy is

$$V_t^i = -\varsigma^i + E_t\beta_{t,t+1}[q_t^i(1-\rho^x)J_{t+1}^i + (1-q_t^i)V_{t+1}^i].$$

[6]It comes from the evolution of age-specific employment rate, which gives $1-\tilde{\rho}_t = \frac{n_t}{n_{t-1}+f_{t-1}u_{t-1}}$.

For workers, the asset value of being matched is

$$W_t^i = \omega_t^i + E_t\beta_{t,t+1}[(1-\rho^x)W_{t+1}^i + \rho U_{t+1}^i].$$

The expected value of being unemployed is

$$U_t^i = z + E_t\beta_{t,t+1}[f_t^i(1-\rho^x)W_{t+1}^i + (1-f_t^i(1-\rho^x))U_{t+1}^i].$$

B.8 Derivation of the equation for $\frac{\partial \ln \theta^Y}{\partial \ln A}$ in case of exogenous job separation

In case of no aggregate uncertainty ($A_t = A$), we can solve for the asset value of a job,

$$J^i = \frac{(1-\eta)(P^iA - z^i) - \eta\varsigma^i\theta^i}{1-\beta(1-\rho^x)}.$$

Plugging this into the free entry condition, we can get a implicit function for market tightness θ^i

$$\frac{\varsigma^i}{\beta(1-\rho^x)q^i} = \frac{(1-\eta)(P^iA - z^i) - \eta\varsigma^i\theta^i}{1-\beta(1-\rho^x)}.$$

Denote aggregate productivity $\alpha \equiv [1-\beta(1-\rho^x)]/[\beta(1-\rho^x)]$ and rearrange the equation,

$$\alpha\frac{\varsigma^i}{q^i} + \eta\varsigma^i\theta^i = (1-\eta)(P^iA - z^i).$$

Totally differentiate the above equation,

$$-\alpha\varsigma^i\frac{[q^i(\theta^i)]'}{[q^i(\theta^i)]^2}d\theta^i + \eta\varsigma^i d\theta^i = (1-\eta)d(P^iA),$$

then, we can get

$$\begin{aligned}
e^Y_{\theta,A} &= \frac{\partial\theta^Y}{\partial A}\frac{A}{\theta^Y} \\
&= \frac{\partial\theta^Y}{\partial(P^YA-z^Y)}\frac{P^YA-z^Y}{\theta^Y}\frac{\partial(P^YA-z^Y)}{\partial A}\frac{A}{P^YA-z^Y} \\
&= \frac{1-\eta}{\eta\varsigma^Y-\alpha\varsigma^Y\frac{[q^Y(\theta^Y)]'}{[q^Y(\theta^Y)]^2}}\frac{\alpha\frac{\varsigma^Y}{q^Y\theta^Y}+\eta\varsigma^Y}{1-\eta}\frac{\partial\ln(P^YA-z^Y)}{\partial\ln A} \\
&= \frac{\alpha+\eta f^Y(\theta^Y)}{\alpha\mu+\eta f^Y(\theta^Y)}\frac{\partial\ln(P^YA-z^Y)}{\partial\ln A}.
\end{aligned}$$

For the second term, we can get that $\frac{\partial\ln(P^YA-z^Y)}{\partial\ln A}=\frac{\partial(P^YA-z^Y)}{\partial A}\frac{A}{P^YA-z^Y}=(A\frac{\partial P^Y}{\partial A}+P^Y)\frac{A}{P^YA-z^Y}$. Then, we have

$$e^Y_{\theta,A}=\frac{\partial\ln\theta^Y}{\partial\ln A}=\frac{1-\beta(1-\rho^x)+\eta\beta(1-\rho^x)f^Y}{\mu[1-\beta(1-\rho^x)]+\eta\beta(1-\rho^x)f^Y}\frac{\frac{\partial\ln P^Y}{\partial\ln A}+1}{1-z^Y/(P^YA)}.\blacksquare$$

B.9 IRFs with respect to an aggregate productivity shock (with exogenous job separation)

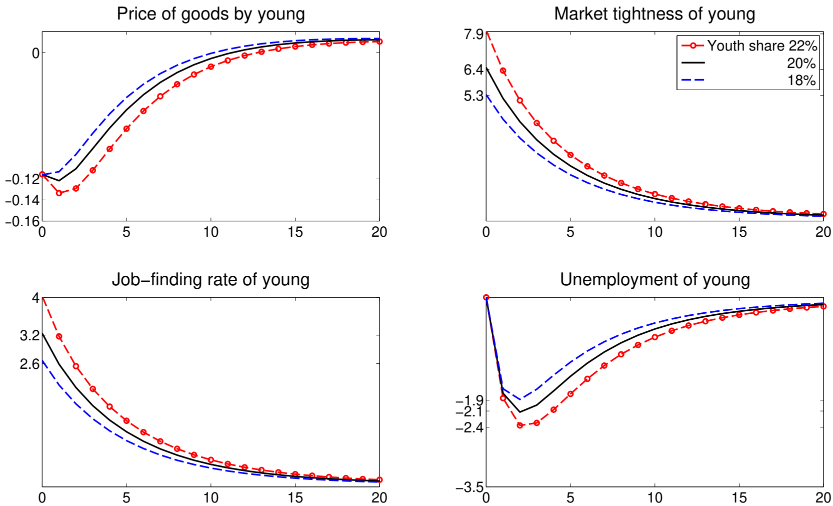

Figure B.1: IRFs of variables of the young with respect to an aggregate productivity shock across demographic states (with exogenous separation)

Appendix C

Appendices to Chapter 4

C.1 Derivations of Euler equations for households and FOCs for final goods producers

C.1.1 Retirees' problem

C.1.1.1 Derivation of the dynamic equations

Substituting in the objective function with C_t^{rjk} using in the constraint, we get the first-order condition with respect to A_t^{rjk} as

$$\beta_t \gamma_{t+1} (V_{t+1}^{rjk})^{\rho-1} \frac{\partial V_{t+1}^{rjk}}{\partial A_t^{rjk}} = \frac{(C_t^{rjk})^{\rho-1}}{P_t}.$$

The Envelope condition is

$$\frac{\partial V_{t+1}^{rjk}}{\partial A_t^{rjk}} = \frac{(V_{t+1}^{rjk})^{1-\rho} (C_{t+1}^{rjk})^{\rho-1}}{\gamma_{t+1}} \frac{R_t}{P_{t+1}}. \tag{C.1}$$

Combining the FOC with Equation (C.1) yields the Euler equation:

$$\frac{R_t}{\pi_{t+1}} \beta_t \left(\frac{C_{t+1}^{rjk}}{C_t^{rjk}} \right)^{\rho-1} = 1.$$

Substitution of the guess function for C_t^{rjk} into the budget constraint yields the dynamic equation for asset holdings:

$$\frac{A_t^{rjk}}{P_t} = \frac{1-\xi_t^r}{\gamma_t}\frac{R_{t-1}A_{t-1}^{rjk}}{P_t} + \Pi_t^{rjk} - T_t - \xi_t^r(PP_t^{rjk} - PT_t^{rjk}).$$

Combining the guess function for C_t^{rjk} with the Euler equation yields the dynamic equation for the marginal propensity to consume of retirees, ξ_t^r:

$$\begin{aligned} & \xi_{t+1}^r \quad \left[\frac{1}{\gamma_{t+1}}\left(\frac{R_t A_t^{rjk}}{P_{t+1}}\right) + PP_{t+1}^{rjk} - PT_{t+1}^{rjk}\right] \\ = \quad & \xi_t^r\left(\beta_t \frac{R_t}{\pi_{t+1}}\right)^{\frac{1}{1-\rho}}\left(\frac{1}{\gamma_t}\frac{R_{t-1}A_{t-1}^{rjk}}{P_t} + PP_t^{rjk} - PT_t^{rjk}\right). \end{aligned}$$

With the dynamic equations for the present discounted value of profits and tax payment,

$$\begin{aligned} PP_t^{rjk} &= \Pi_t^{rjk} + \frac{\pi_{t+1}\gamma_{t+1}}{R_t}PP_{t+1}^{rjk} \\ PT_t^{rjk} &= T_t^{rjk} + \frac{\pi_{t+1}\gamma_{t+1}}{R_t}PT_{t+1}^{rjk}, \end{aligned}$$

and the dynamic equation for asset holdings, the dynamic equation for the marginal propensity to consume can be simplified to

$$\xi_t^r = 1 - \gamma_{t+1}\beta_t^{\frac{1}{1-\rho}}\left(\frac{R_t}{\pi_{t+1}}\right)^{\frac{\rho}{1-\rho}}\frac{\xi_t^r}{\xi_{t+1}^r}.$$

C.1.1.2 Verification of the conjecture of the value function of retirees

Suppose that $V_{t+1}^{rjk} = (\xi_{t+1}^r)^{-\frac{1}{\rho}}C_{t+1}^{rjk}$ is true. We need to check whether $V_t^{rjk} = (\xi_t^r)^{-\frac{1}{\rho}}C_t^{rjk}$ is also true. From the Bellman equation of the value function of retirees, we can have

$$\begin{aligned} (V_t^{rjk})^\rho &= (C_t^{rjk})^\rho + \beta_t\gamma_{t+1}(V_{t+1}^{rjk})^\rho \\ &= (C_t^{rjk})^\rho + \beta_t\gamma_{t+1}(\xi_{t+1}^r)^{-1}(C_{t+1}^{rjk})^\rho \\ &= (C_t^{rjk})^\rho + \beta_t^{\frac{1}{1-\rho}}\gamma_{t+1}(\xi_{t+1}^r)^{-1}\left(\frac{R_t}{\pi_{t+1}}\right)^{\frac{\rho}{1-\rho}}(C_t^{rjk})^\rho. \end{aligned}$$

In the last step, I use the Euler equation for retirees. Then, it is easy to see that the dynamic equation for ξ_t^r in Equation (4.1) is sufficient to justify that $V_t^{rjk} = (\xi_t^r)^{-\frac{1}{\rho}} C_t^{rjk}$ is true.

C.1.2 Workers' problem

C.1.2.1 Derivation of the dynamic equations

The first-order conditions with respect to labor supply and assets, respectively, are

$$\begin{aligned}
L_t^{wj} &= 1 - \frac{1-v}{v}\frac{P_t}{W_t}C_t^{wj} \\
\frac{v(C_t^{wj})^{\rho v-1}(1-L_t^{wj})^{\rho(1-v)}}{P_t} &= \beta_t[\omega_{t+1}V_{t+1}^{wj} + (1-\omega_{t+1})V_{t+1}^{rj(t+1)}]^{\rho-1} \\
&\quad \left[\omega_{t+1}\frac{\partial V_{t+1}^{wj}}{\partial A_t^{wj}} + (1-\omega_{t+1})\frac{\partial V_{t+1}^{rj(t+1)}}{\partial A_t^{wj}}\right].
\end{aligned}$$

The Envelope condition is

$$\frac{\partial V_{t+1}^{wj}}{\partial A_t^{wj}} = v(C_{t+1}^{wj})^{\rho v-1}(1-L_t^{wj})^{\rho(1-v)}(V_{t+1}^{wj})^{1-\rho}\frac{R_t}{P_{t+1}}. \tag{C.2}$$

As the initial asset holdings of a retiree equal to the asset holdings of a worker before she becomes a retiree, which is $A_t^{wj} = A_t^{rj(t+1)}$. It is easy to see that

$$\frac{\partial V_{t+1}^{rj(t+1)}}{\partial A_t^{wj}} = \frac{\partial V_{t+1}^{rj(t+1)}}{\partial A_t^{rj(t+1)}} = (V_{t+1}^{rj(t+1)})^{1-\rho}(C_{t+1}^{rj(t+1)})^{\rho-1}\frac{R_t}{P_{t+1}}. \tag{C.3}$$

Combining the second FOC with Equations (C.2) and (C.3) yields the Euler equation for consumption[1]:

$$
\begin{aligned}
(C_t^{wj})^{\rho v-1}(1-L_t^{wj})^{\rho(1-v)} \quad = \quad & \beta_t\Big(\omega_{t+1}V_{t+1}^{wj}+(1-\omega_{t+1})V_{t+1}^{rj(t+1)}\Big)^{\rho-1} \\
& \frac{R_t}{\pi_{t+1}}\Big[\omega_{t+1}(V_{t+1}^{wj})^{1-\rho}(C_{t+1}^{wj})^{\rho v-1}(1-L_{t+1}^{wj})^{\rho(1-v)} \\
& +(1-\omega_{t+1})\nu^{-1}(V_{t+1}^{rj(t+1)})^{1-\rho}(C_{t+1}^{rj(t+1)})^{\rho-1}\Big].
\end{aligned}
$$

Conjecture that the value function of workers takes the form

$$
V_t^{wj}=(\xi_t^w)^{-1/\rho}C_t^{wj}\Big(\frac{1-v}{v}\frac{P_t}{W_t}\Big)^{1-v},
$$

where ξ_t^w is the marginal propensity to consume of workers. Then, the Euler equation for consumption can be written as

$$
\begin{aligned}
& \omega_{t+1}\Big(\tfrac{1-v}{v}\tfrac{P_{t+1}}{W_{t+1}}\Big)^{1-v}C_{t+1}^{wj}+(1-\omega_{t+1})\Big(\tfrac{\xi_{t+1}^r}{\xi_{t+1}^w}\Big)^{-1/\rho}C_{t+1}^{rj(t+1)} \\
& =\Big(\beta_t\tfrac{R_t}{\pi_{t+1}}\Omega_{t+1}\Big)^{\frac{1}{1-\rho}}\Big(\tfrac{1-v}{v}\tfrac{P_t}{W_t}\Big)^{\frac{\rho(v-1)}{1-\rho}}C_t^{wj},
\end{aligned}
$$

where $\Omega_{t+1}\equiv\omega_{t+1}\Big(\tfrac{1-v}{v}\tfrac{P_{t+1}}{W_{t+1}}\Big)^{1-v}+(1-\omega_{t+1})\nu^{-1}\Big(\tfrac{\xi_{t+1}^r}{\xi_{t+1}^w}\Big)^{-\frac{1-\rho}{\rho}}$.

Guess that consumption of workers is a fraction ξ_t^w of the total wealth:

$$
C_t^{wj}=\xi_t^w\Big(\frac{R_{t-1}A_{t-1}^{wj}}{P_t}+H_t^{wj}+PP_t^{wj}-PT_t^{wj}\Big),
$$

where H_t^{wj} is the present discounted value of human wealth for a worker, and PP_t^{wj}, PT_t^{wj} are, respectively, the present discounted values of profits from firms and tax payment for a worker.

[1] If a worker becomes a retiree at time $t+1$, the survival probability is 1 for current period, namely:

$$
\frac{\partial V_{t+1}^{rj(t+1)}}{\partial A_t^{rj(t)}}=\lambda_{t+1}\frac{R_t}{P_{t+1}}
$$

Substitution of guess functions for $C_{t+1}^{rj(t+1)}$ and C_t^{wj} into the Euler equation for the consumption of workers yields[2]

$$\begin{aligned}
&\left[1-\xi_t^w-\beta_t^{\frac{1}{1-\rho}}\left(\tfrac{R_t}{\pi_{t+1}}\right)^{\frac{\rho}{1-\rho}}\left(\Omega_{t+1}\right)^{\frac{1}{1-\rho}}\left(\Omega_{t+1}^{'}\right)^{-1}\left(\tfrac{1-v}{v}\tfrac{P_t}{W_t}\right)^{\frac{\rho(v-1)}{1-\rho}}\tfrac{\xi_t^w}{\xi_{t+1}^w}\right] \\
&\qquad \frac{R_{t-1}A_{t-1}^{wj}}{P_t} \\
= \quad &\left[-1+\xi_t^w+\beta_t^{\frac{1}{1-\rho}}\left(\tfrac{R_t}{\pi_{t+1}}\right)^{\frac{\rho}{1-\rho}}\left(\Omega_{t+1}\right)^{\frac{1}{1-\rho}}\left(\Omega_{t+1}^{'}\right)^{-1}\left(\tfrac{1-v}{v}\tfrac{P_t}{W_t}\right)^{\frac{\rho(v-1)}{1-\rho}}\tfrac{\xi_t^w}{\xi_{t+1}^w}\right] \\
&\qquad (H_t^{wj}+PP_t^{wj}-PT_t^{wj}) \\
&+H_t^{wj}+PP_t^{wj}-PT_t^{wj}-\left(\tfrac{W_t}{P_t}L_t^{wj}+\mathit{\Pi}_t^{wj}-T_t\right)-\tfrac{\pi_{t+1}}{R_t}\tfrac{\omega_{t+1}}{\Omega_{t+1}^{'}}\left(\tfrac{1-v}{v}\tfrac{P_{t+1}}{W_{t+1}}\right)^{1-v} \\
&(H_{t+1}^{wj}+PP_{t+1}^{wj}-PT_{t+1}^{wj})-\tfrac{\pi_{t+1}}{R_t}\tfrac{1-\omega_{t+1}}{\Omega_{t+1}^{'}}\left(\tfrac{\xi_{t+1}^r}{\xi_{t+1}^w}\right)^{1-1/\rho}(PP_{t+1}^{rj(t+1)}-PT_{t+1}^{rj(t+1)}),
\end{aligned}$$

where $\Omega_{t+1}^{'} \equiv \omega_{t+1}\left(\frac{1-v}{v}\frac{P_{t+1}}{W_{t+1}}\right)^{1-v}+(1-\omega_{t+1})\left(\frac{\xi_{t+1}^r}{\xi_{t+1}^w}\right)^{-\frac{1-\rho}{\rho}}$.

This equation holds if

$$\begin{aligned}
\xi_t^w &= 1-\left[\Omega_{t+1}\left(\frac{1-v}{v}\frac{P_t}{W_t}\right)^{\rho(v-1)}\beta_t\left(\frac{R_t}{\pi_{t+1}}\right)^{\rho}\right]^{\frac{1}{1-\rho}}\left(\Omega_{t+1}^{'}\right)^{-1}\frac{\xi_t^w}{\xi_{t+1}^w} \\
H_t^{wj} &= \frac{W_t}{P_t}L_t^{wj}+\frac{\pi_{t+1}}{R_t}\frac{\omega_{t+1}}{\Omega_{t+1}^{'}}\left(\frac{1-v}{v}\frac{P_{t+1}}{W_{t+1}}\right)^{1-v}H_{t+1}^{wj} \\
PP_t^{wj} &= \mathit{\Pi}_t^{wj}+\frac{\pi_{t+1}}{R_t}\frac{\omega_{t+1}}{\Omega_{t+1}^{'}}\left(\frac{1-v}{v}\frac{P_{t+1}}{W_{t+1}}\right)^{1-v}PP_{t+1}^{wj} \\
&\quad +\frac{\pi_{t+1}}{R_t}\frac{1-\omega_{t+1}}{\Omega_{t+1}^{'}}\left(\frac{\xi_{t+1}^r}{\xi_{t+1}^w}\right)^{1-1/\rho}PP_{t+1}^{rj(t+1)} \\
PT_t^{wj} &= T_t+\frac{\pi_{t+1}}{R_t}\frac{\omega_{t+1}}{\Omega_{t+1}^{'}}\left(\frac{1-v}{v}\frac{P_{t+1}}{W_{t+1}}\right)^{1-v}PT_{t+1}^{wj} \\
&\quad +\frac{\pi_{t+1}}{R_t}\frac{1-\omega_{t+1}}{\Omega_{t+1}^{'}}\left(\frac{\xi_{t+1}^r}{\xi_{t+1}^w}\right)^{1-1/\rho}PT_{t+1}^{rj(t+1)}.
\end{aligned}$$

[2]Note that: $C_t^{rj(t)} = \xi_t^r(\frac{R_t A_t^{wj}}{P_t}+PP_t^{rj(t)}-PT_t^{rj(t)})$, as the initial asset holdings for a retiree is the same as that of a worker.

C.1.2.2 Verification of the conjecture of the value function of workers

Suppose that $V_{t+1}^{rjk} = (\xi_{t+1}^r)^{-\frac{1}{\rho}} C_{t+1}^{rjk}$ and $V_{t+1}^{wj} = (\xi_{t+1}^w)^{-1/\rho} C_{t+1}^{wj}\Big(\frac{1-v}{v}\frac{P_{t+1}}{W_{t+1}}\Big)^{1-v}$ are true. We need to check whether $V_t^{wj} = (\xi_t^w)^{-1/\rho} C_t^{wj}\Big(\frac{1-v}{v}\frac{P_t}{W_t}\Big)^{1-v}$ is also true. From the Bellman equation of the value function of workers, we can have:

$$
\begin{aligned}
(V_t^{wj})^\rho &= \Big[(C_t^{wj})^v(1-L_t^{wj})^{1-v}\Big]^\rho + \beta_t\Big[\omega_{t+1}V_{t+1}^{wj} + (1-\omega_{t+1})V_{t+1}^{rj(t+1)}\Big]^\rho \\
&= \Big[(C_t^{wj})^v\Big(\frac{1-v}{v}\frac{P_t}{W_t}C_t^{wj}\Big)^{1-v}\Big]^\rho + \beta_t\Big[\omega_{t+1}(\xi_{t+1}^w)^{-1/\rho}C_{t+1}^{wj}\Big(\frac{1-v}{v}\frac{P_{t+1}}{W_{t+1}}\Big)^{1-v} \\
&\quad +(1-\omega_{t+1})(\xi_{t+1}^r)^{-1/\rho}C_{t+1}^{rj(t+1)}\Big]^\rho \\
&= \Big(\frac{1-v}{v}\frac{P_t}{W_t}\Big)^{(1-v)\rho}(C_t^{wj})^\rho \\
&\quad +\beta_t\Big[(\xi_{t+1}^w)^{-1/\rho}\Big(\beta\frac{R_t}{\pi_{t+1}}\Omega_{t+1}\Big)^{\frac{1}{1-\rho}}\Big(\frac{1-v}{v}\frac{P_t}{W_t}\Big)^{\frac{\rho(v-1)}{1-\rho}}C_t^{wj}\Big]^\rho.
\end{aligned}
$$

In the last step, I use the Euler equation for workers. Then, it is easy to see that, the dynamic equation for ξ_t^w in Equation (4.2) is sufficient to justify $V_t^{wj} = (\xi_t^w)^{-1/\rho}C_t^{wj}\Big(\frac{1-v}{v}\frac{P_t}{W_t}\Big)^{1-v}$ is true.

C.1.3 Final goods producer's Problem

$$
\max_{Y_t(i)} P_t\Big(\int_0^1 Y_t(i)^{(\kappa-1)/\kappa}di\Big)^{\kappa/(\kappa-1)} - \int_0^1 P_t(i)Y_t(i)di
$$

The first-order conditions with respect to input $Y_t(i)$:

$$
\frac{\kappa}{\kappa-1}P_t\Big(\int_0^1 Y_t(i)^{(\kappa-1)/\kappa}di\Big)^{\kappa/(\kappa-1)-1}\frac{\kappa-1}{\kappa}Y_t(i)^{-1/\kappa} = P_t(i),
$$

which can be written as:

$$
Y_t(i) = \Big(\frac{P_t(i)}{P_t}\Big)^{-\kappa}Y_t.
$$

C.2 Detrended system of equations

1. Marginal propensity to consume of retirees:

$$\xi_t^r = 1-\gamma_{t+1}(\beta \exp(\eta_t))^{\frac{1}{1-\rho}}(\frac{R_t}{\pi_{t+1}})^{\frac{\rho}{1-\rho}}\frac{\xi_t^r}{\xi_{t+1}^r}$$

2. Marginal propensity to consume of workers:[3]

$$\xi_t^w = 1-\left[\tilde{\Omega}_{t+1}\left(\frac{1-v}{vw_t}\right)^{\rho(v-1)}\beta\exp(\eta_t)\left(\frac{R_t}{\pi_{t+1}}\right)^{\rho}\right]^{\frac{1}{1-\rho}}\left(\tilde{\Omega}_{t+1}'\right)^{-1}\frac{\xi_t^w}{\xi_{t+1}^w}$$

3. Dependency ratio:

$$(1+n_t)d_t = 1-\omega_t+\gamma_t d_{t-1}$$

4. Consumption of retirees:[4]

$$c_t^r = \xi_t^r(\frac{R_{t-1}a_{t-1}^r}{(1+n_t)\pi_t}+pp_t^r-pt_t^r)$$

5. Consumption of workers:

$$c_t^w = \xi_t^w[\frac{R_{t-1}a_{t-1}^w}{(1+n_t)\pi_t}+h_t^w+pp_t^w-pt_t^w]$$

6. Labor supply:

$$l_t = 1-\frac{1-v}{v}\frac{c_t^w}{w_t}$$

7. Law of motion for asset holdings of retirees:

$$\begin{aligned} a_t^r &= \frac{R_{t-1}a_{t-1}^r(1-\xi_t^r)}{(1+n_t)\pi_t}+\tilde{\mathit{\Pi}}_t^M+\tilde{\mathit{\Pi}}_t^K-t_t-\xi_t^r(pp_t^r-pt_t^r) \\ &\quad +\frac{1-\omega_{t+1}}{\omega_{t+1}d_t}a_t^w \end{aligned}$$

[3]Where the detrended Ω_{t+1} is $\tilde{\Omega}_{t+1} = \omega_{t+1}(\frac{1-v}{vw_{t+1}})^{1-v}+(1-\omega_{t+1})\nu^{-1}(\frac{\xi_{t+1}^r}{\xi_{t+1}^w})^{-\frac{1-\rho}{\rho}}$, and the detrended Ω_{t+1}' is $\tilde{\Omega}_{t+1}' = \omega_{t+1}(\frac{1-v}{vw_{t+1}})^{1-v}+(1-\omega_{t+1})(\frac{\xi_{t+1}^r}{\xi_{t+1}^w})^{-\frac{1-\rho}{\rho}}$.

[4]Detrended variables are denoted by lower-case letters. For the profits from firms, I use $\tilde{\mathit{\Pi}}_t$ to distinguish from inflation rate π_t. Similarly, the detrended Ω_t is denoted as $\tilde{\Omega}_t$, to distinguish from the probability of remaining as worker ω_t.

8. Human wealth:

$$h_t^w = w_t l_t + \frac{\pi_{t+1}}{R_t}\frac{\omega_{t+1}}{\tilde{\Omega}'_{t+1}}(\frac{1-v}{vw_t})^{1-v}h_{t+1}^w$$

9. Present discounted value of profits of retirees:

$$pp_t^r = \tilde{\Pi}_t^M + \tilde{\Pi}_t^K + \frac{\pi_{t+1}\gamma_{t+1}}{R_t}pp_{t+1}^r$$

Present discounted value of lump-sum tax of retirees:

$$pt_t^r = t_t + \frac{\pi_{t+1}\gamma_{t+1}}{R_t}pt_{t+1}^r$$

10. Present discounted value of profits of workers:

$$\begin{aligned} pp_t^w &= \tilde{\Pi}_t^M + \tilde{\Pi}_t^K + \frac{\pi_{t+1}}{R_t}\frac{\omega_{t+1}}{\tilde{\Omega}'_{t+1}}(\frac{1-v}{vw_t})^{1-v}pp_{t+1}^w \\ &\quad + \frac{d_{t+1}\pi_{t+1}}{R_t}\frac{1-\omega_{t+1}}{\tilde{\Omega}'_{t+1}}(\frac{\xi_{t+1}^r}{\xi_{t+1}^w})^{\frac{\rho-1}{\rho}}pp_{t+1}^r \end{aligned}$$

Present discounted value of lump-sum tax of workers:

$$\begin{aligned} pt_t^w &= t_t + \frac{\pi_{t+1}}{R_t}\frac{\omega_{t+1}}{\tilde{\Omega}'_{t+1}}(\frac{1-v}{vw_t})^{1-v}pt_{t+1}^w \\ &\quad + \frac{d_{t+1}\pi_{t+1}}{R_t}\frac{1-\omega_{t+1}}{\tilde{\Omega}'_{t+1}}(\frac{\xi_{t+1}^r}{\xi_{t+1}^w})^{\frac{\rho-1}{\rho}}pt_{t+1}^r \end{aligned}$$

11. Output:

$$y_t = {l_t}^{1-\alpha}(\frac{k_{t-1}}{1+n_t})^{\alpha}$$

12. Real wage:

$$w_t = (1-\alpha)\varphi_t y_t l_t^{-1}$$

13. Cost of capital:

$$r_t^K = \alpha\varphi_t y_t(\frac{k_{t-1}}{1+n_t})^{-1}$$

14. Profits from intermediate goods producers:

$$(1+d_t)\tilde{\Pi}_t^M = [1+\tau-\varphi_t-\frac{\phi}{2}(\pi_t-1)^2]y_t$$

15. New Keynesian Phillips curve:

$$\kappa(\varphi_t-1)-\phi(\pi_t-1)\pi_t+\frac{\pi_{t+1}}{R_t}\phi(\pi_{t+1}-1)\pi_{t+1}=0$$

16. Profits of capital producers:

$$(1+d_t)\tilde{\Pi}_t^K = a_t^w+a_t^r d_t+\frac{r_t^K k_{t-1}}{(1+n_t)}-i_t-\frac{R_{t-1}(a_{t-1}^w+a_{t-1}^{\mathrm{r}}d_{t-1})}{\pi_t(1+n_t)}$$

17. Law of motion for capital:

$$k_t=(1-\delta)\frac{k_{t-1}}{1+n_t}+\{1-(1+n_t)^2S"[(\frac{i_t}{i_{t-1}})^2/2-\frac{i_t}{i_{t-1}}+\frac{1}{2}]\}i_t$$

18. Rental cost of capital:

$$Q_t=\frac{\pi_{t+1}}{R_t}[Q_{t+1}(1-\delta)+r_{t+1}^K]$$

19. Lagrange multiplier on the capital formation:

$$\begin{aligned}1 \quad = \quad & Q_t\{1-(1+n_t)^2S"[(\frac{i_t}{i_{t-1}})^2/2-\frac{i_t}{i_{t-1}}+\frac{1}{2}]-(1+n_t)^2S"(\frac{i_t}{i_{t-1}}-1)\frac{i_t}{i_{t-1}}\}\\ & +\frac{\pi_{t+1}Q_{t+1}}{R_t}(1+n_{t+1})^3S"(\frac{i_{t+1}}{i_t})^2(\frac{i_{t+1}}{i_t}-1)\end{aligned}$$

20. Government budget constraint:

$$(1+d_t)t_t=\tau y_t$$

21. Truncated monetary policy:

$$R_t=max\,[1,\,R_{ss}+1.5(\pi_t-1)+\frac{0.5}{4}(\frac{y_t}{y_{t-1}}-1)+e_t^M]$$

22. Market clearing for final goods:

$$y_t = c_t^r d_t + c_t^w + i_t$$

23. Market clearing for assets:

$$a_t^r d_t + a_t^w = Q_t k_t$$

C.3 Decomposition of dependency ratio

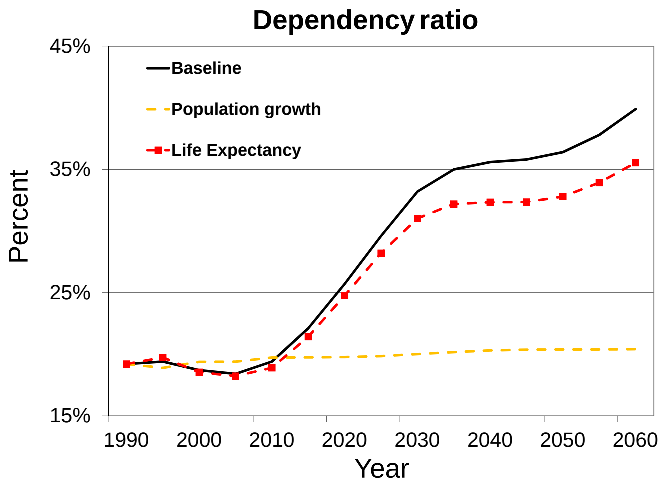

Figure C.1: Decomposition of dependency ratio

Note: The solid line is the actual transition path of dependency ratio, with the joint effects of population growth and life expectancy. The dotted, square-hatched line is a counterfactual transition path of dependency ratio, which assumes that only the survival probability of retirees changes while the population growth rate remains at its initial level in 1990. The dotted line represents another counterfactual transition path, which assumes that only the population growth rate changes while the other remains at its initial level in 1990.

C.4 Effects of population aging

C.4.1 When retirees enjoy full leisure

Response to a positive preference shock as population gets older[5]:

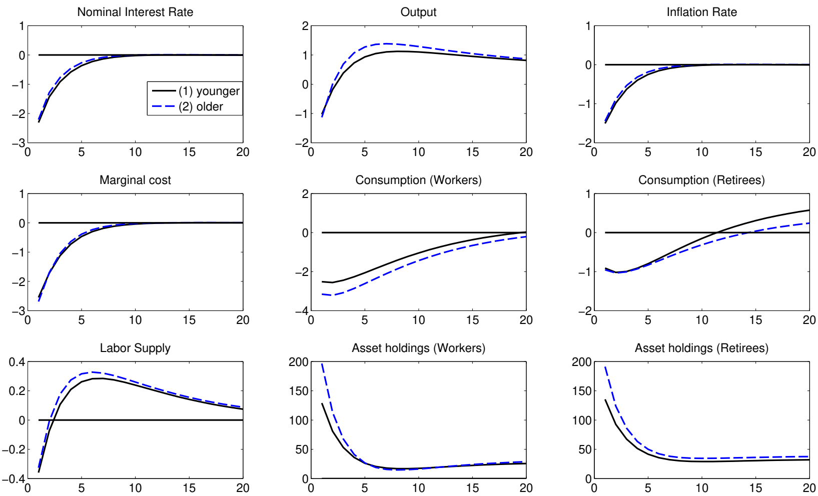

Figure C.2: Response to a preference shock when retirees enjoy full leisure

Response to an expansionary monetary policy shock as population gets older:

[5]For easy comparison, different demographic structures are defined in the same way as Fujiwara and Teranishi (2008): Younger population refers to their baseline scenario, with the average life expectancy equal to 75, and older population refers to the scenario with this value equal to 85.

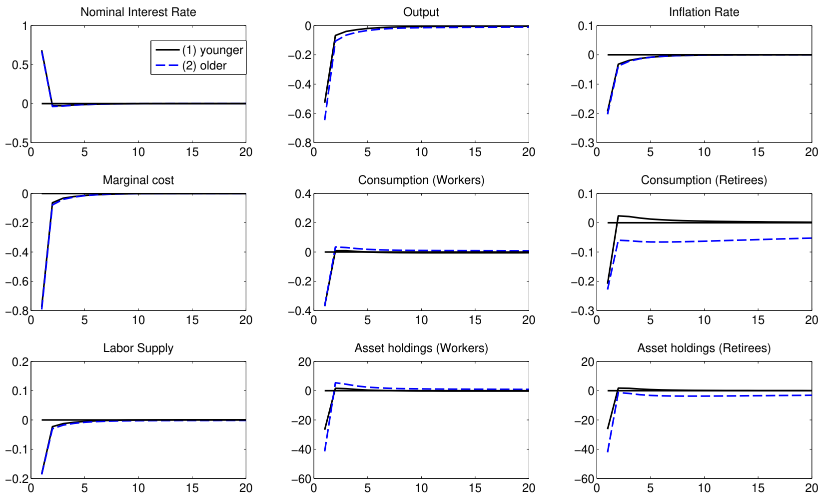

Figure C.3: Response to a monetary shock when retirees enjoy full leisure

C.4.2 When retirees endogenously decide their labor supply

Stationary results corresponding to Table 2 in Fujiwara and Teranishi (2008):

Table C.1: Values under stationary population

	c/y	k/y	R	a^r/k	ξ^w	ξ^r	l^w	l^r
(i)	0.70	10.3	0.01	0.16	0.016	0.03	0.41	0.05
(ii)	0.70	10.3	0.01	0.22	0.013	0.02	0.41	0.12

Notes: All variables are detrended in the same way as Fujiwara and Teranishi (2008), divided by the size of labor force, thus low-case letters do not necessarily mean the per capita counterparts. The values of marginal propensity to consume of workers, ξ^w, are slightly lower than those in Fujiwara and Teranishi (2008). In their case, these two are 0.017 and 0.014, respectively.

IRFs corresponding to Figure 4 in Fujiwara and Teranishi (2008):

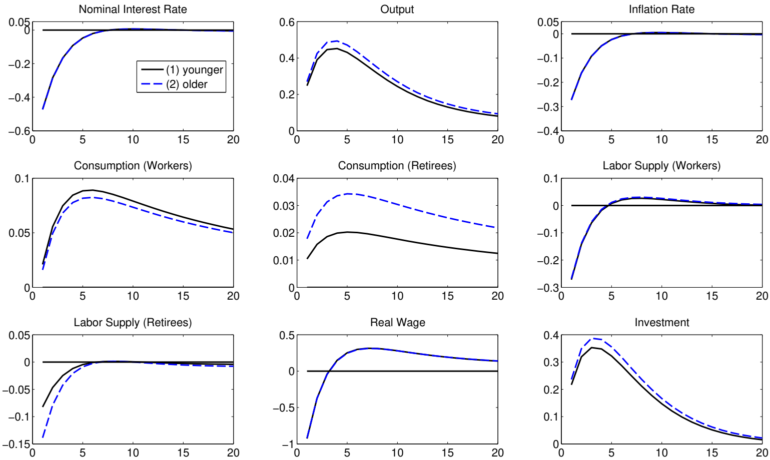

Figure C.4: Response to a technology shock with flexible labor supply from retirees

IRFs corresponding to Figure 8 in Fujiwara and Teranishi (2008):

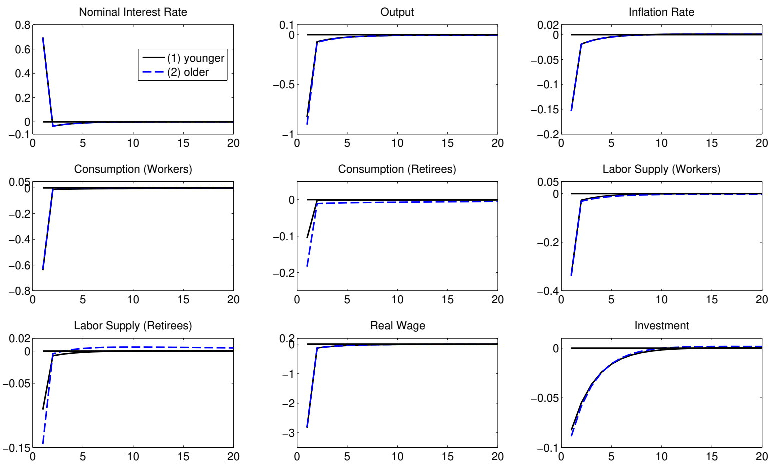

Figure C.5: Response to a monetary shock with flexible labor supply from retirees

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